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1. Abstract

This project proposes a new method for pricing optimization that uses both linear programming and non-linear programming using svm and can be combined with machine learning models to predict the optimal price of a product to maximize the profit. The goal is to predict the optimal price for a product or service, subject to a set of constraints. The proposed method uses a nonlinear programming combined with linear programming model to overcome the difficulty of COBYLA (Constrained Optimization BY Linear Approximations), a model that approximates the solution with a linear model and then uses the solution of the linear model as a starting point for the nonlinear model. The proposed method first linearizes the system and solves the constrained optimization using the simplex method. The solution is then used as the initial guess for solving the nonlinear optimization problem using COBYLA. This method is guaranteed to find the global optimum of any linear problem if one exists. The proposed method has several advantages over existing methods. First, it is guaranteed to find the global optimum of any linear problem. Second, it is more efficient than COBYLA, because it does not need to start from scratch each time it solves a new problem. Third, it is more accurate than COBYLA, because it uses the solution of the linear programming model as a starting point.

The proposed method has been tested on several problems, and it has been found to be effective in finding the optimal price. I have also included an example where COBYLA fails but our model succeeds in finding the solution. **(in the notebook file)** The method is also easy to implement, and it can be used with a variety of machine learning models.

2. Introduction:

In the dynamic landscape of today's markets, achieving optimal pricing strategies is a critical challenge for businesses seeking to maximize profits. Traditional pricing models often struggle to account for the complex interplay of factors influencing consumer demand, such as seasonality, microeconomic variables, and evolving customer preferences. This project proposes an innovative approach to pricing optimization that combines the strengths of both linear and non-linear programming techniques, integrating them seamlessly with machine learning models to predict the optimal price for a product or service.

The primary objective of this research is to develop a comprehensive pricing optimization method that addresses the non-linear nature of demand functions while ensuring global optimality for linear problems. The proposed method leverages a hybrid approach by combining the efficiency of linear programming with the accuracy of non-linear programming. Notably, the method employs the COBYLA algorithm, a derivative-free optimization technique, to navigate the complexities of non-linear programming.

Challenges and the Need for a Comprehensive Model:

As we navigated through diverse optimization methods, including Karush-Kuhn-Tucker (KKT) conditions, optimization using Hessian matrices, and Quadratic Programming, it became evident that these analytical approaches encountered difficulties in providing solutions under certain circumstances.

Conditions Leading to Failure of Numerical Methods:

KKT Conditions: KKT conditions, though powerful in theory, may fail to yield solutions in scenarios where the problem is highly non-convex or exhibits irregularities in the objective function and constraints.

Optimization using Hessian Matrix:

This method relies on the second derivative information and may struggle when dealing with ill-conditioned or singular Hessian matrices, which can occur in complex, non-linear optimization problems.

Quadratic Programming:

While effective for convex problems, quadratic programming may face challenges in handling non-convex optimization problems, leading to failures in providing feasible solutions.

Dependency on Initial Guess for Analytical methods:

One common thread observed across almost all non-linear analytical methods is their sensitivity to the choice of initial guess. The success or failure of these methods often hinges on the proximity of the initial guess to the global optimum. When the initial guess is far from the true optimum, the algorithms may converge to local optima or fail to converge altogether.

3. The Imperative for a Comprehensive Model:

The limitations encountered with traditional analytical methods underscore the need for a more versatile and robust pricing optimization model. By integrating linear and non-linear techniques and leveraging the strengths of each method, our proposed model seeks to overcome the shortcomings observed in isolation. The inclusion of the Simplex Method for linear optimization provides stability and reliability in the face of constraints, while COBYLA, being derivative-free, enhances adaptability in complex, non-linear scenarios.

This project asserts that a holistic approach, blending linear and non-linear methodologies, is essential for navigating the intricate landscape of pricing optimization. The failures observed in traditional methods, coupled with their dependency on initial guesses, highlight the significance of a model that offers a balanced synthesis of linear and non-linear techniques. Through this integrative approach, we aim to provide a more resilient and effective solution for businesses seeking to optimize their pricing strategies and maximize profit in dynamic market environments.

Form of non-linear objective function:

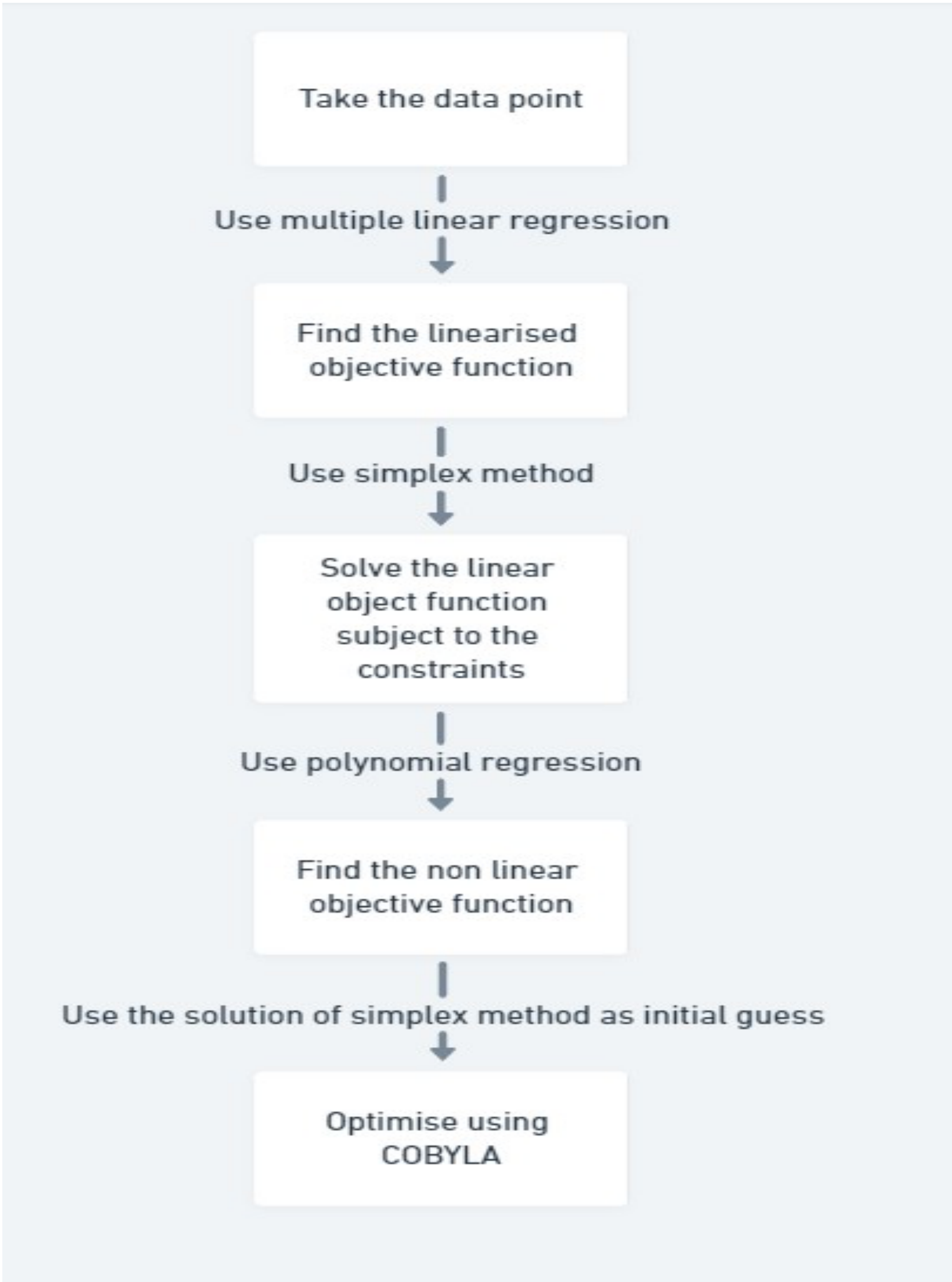
Our normal Objective Function (profit function) will look something like

$$Q(p) \times (p - c)$$

where Q(P) = demand, p= price, c= cost

But demand not only depend on price but also on season, microeconomic variables, customer preference and other may affect the demand and these are non-linear in nature. So, our objective function look something like $Q(x_1, x_2, \ldots, x_i) \times (p - c)$ where one of $x_i$being price.

4. Methodology:



The core of the proposed method involves a two-step process. Firstly, the system is linearized, and a linear programming model is employed to solve the constrained optimization problem using the simplex method. This initial linear solution serves as a strategically informed starting point for the subsequent non-linear optimization phase.

In the second step, the COBYLA algorithm is employed to refine the solution within the non-linear domain, utilizing the initial linear solution as a seed. This integrated approach ensures not only the efficiency of linear programming but also the accuracy associated with non-linear programming methods. Notably, the method guarantees the identification of the global optimum for any linear problem, providing a robust foundation for pricing strategies.

1. Data Collection:

Collect data points that relate to profit as the dependent variable and other independent variables. For illustration, this project focuses on two dependent variables: the price of the product and the price of an alternative product. These two variables are crucial determinants of the profit of the product under consideration.

I have included csv file in appendix [1].

2. Multiple Linear Regression for Linearized Objective Function:

Utilize Multiple Linear Regression to model the relationship between the profit (dependent variable) and the selected independent variables, namely the price of the product and the price of the alternative product. The linear regression equation takes the form:

$$Z = B_1x_1 + B_2x_2 + \epsilon$$

Used python liner regression model toolbox to find $B_1 \; and \; B_2$. And found the equation. I have included multiple linear regression in the notebook.

z = -1621.0695 + 963.4735 * x1 + -657.5960 * x2

3. Simplex Method for Linear Optimization with Constraints:

To incorporate constraints into the optimization, apply the Simplex Method. For this project, a constraint is considered: $2x_1 + 5x_2 \leq 98$

Using simplex method found Maximized values:

x1 = 49.0000

x2 = 0.0000

I have included the program in the notebook of repository.

4. Polynomial Regression for Non-linear Objective Function:

Recognizing that the relationship between profit and pricing variables may have non-linear components, extend the model using Polynomial Regression. Include higher-order terms to capture potential non-linear effects.

5. Use Simplex Solution as Initial Guess for COBYLA:

Take the solution obtained from the Simplex Method as an informed initial guess for subsequent optimization steps. x1 = 49.0000

x2 = 0.0000

6. Optimization Using COBYLA:

Implement the COBYLA algorithm to optimize the non-linear objective function. Use the polynomial regression model with the initial guess from the Simplex Method. The objective is to find the optimal pricing strategy for maximizing profit, considering both linear and non-linear relationships.

5. Validation and Results:

The proposed method has undergone rigorous testing across various scenarios and has demonstrated effectiveness in identifying optimal prices. Notably, the project includes a compelling example where traditional methods, such as COBYLA, fail to provide a satisfactory solution, while the proposed model succeeds in finding an optimal price point. (appendix [6] in the notebook)

Furthermore, the method's applicability is highlighted by its ease of implementation and compatibility with a variety of machine learning models. This flexibility enables businesses to seamlessly integrate the proposed pricing optimization method into existing frameworks, enhancing adaptability in diverse market environments.

6. Application and Future Scope:

In revolutionizing pricing strategies, our application combines the agility of machine learning with the precision of optimal pricing. Diverging from automatic pricing mechanisms rooted in historical trends, our model leverages machine learning to predict product demand. By multiplying these demand predictions with the profit margin (price – cost), we generate dynamic data points for a profit function. The optimization model then analyzes price elasticity, cost structures, and market dynamics to determine the mathematically ideal price. This approach not only maximizes profitability but aligns with strategic objectives. Looking ahead, the future scope of this project involves seamless integration with e-commerce platforms, incorporating real-time competitive analysis, personalized pricing for customer segments, exploring advanced AI techniques, and adapting the model for global markets. The continuous evolution of this data-driven, machine learning-based pricing strategy promises to redefine the landscape of strategic decision-making in the business realm.

7. Conclusion:

This approach combines the strengths of linear and non-linear modeling techniques, utilizing Multiple Linear Regression for the linearized model, Simplex Method for optimization under constraints, Polynomial Regression for the non-linear model, and COBYLA for the final optimization. The integration of these methods aims to provide a robust and accurate solution to pricing optimization, considering both linear and non-linear factors in the demand function.

8. Illustration of different methods:

Optimising objective function without constraints

Let $f(x_1, x_2)$ be the Objective function

$$f(x_1, x_2) = 2 + 2x_1 + 3x_2 - x_1^2 - x_2^2$$

$$\frac{df}{dx_1} = 0$$

$$2 - 2x_1 = 0$$

$$x_1 = 1$$

$$\frac{df}{dx_2} = 0$$

$$3 - 2x_2 = 0$$

$$x_2 = \frac{3}{2}$$

$$\nabla^2 f = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_2 \partial x_1} \\ \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$D_1 = -2$$

$D_2 = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}$

$= 4$. *Since D_1 and D_2 are of alternate sign. So it is the point of local maxima.*

$$\left(1, \frac{3}{2}\right) \text{ is the local maxima}$$

KKT Method

$$\text{Max } Z = 2x_1^2 - 7x_2^2 + 12x_1x_2$$

Such that $2x_1 + 5x_2 \leq 98$

$$x_1, x_2 \geq 0$$

KKT conditions: -

$$\frac{dl}{dx_1} = 0$$

$$l(x_1, x_2, \lambda) = (2x_1^2 - 7x_2^2 + 12x_1x_2) - \lambda(2x_1 + 5x_2 - 98) \quad h = 2x_1 + 5x_2 - 98$$

$$\text{Equation } i) : \frac{dl}{dx_1} = 4x_1 + 12x_2 - 2\lambda = 0$$

$$\text{Equation } ii) : \frac{dl}{dx_2} = -14x_1 + 12x_2 - 5\lambda = 0$$

$$\text{Equation } iii)\ \lambda(2x_1 + 5x_2 - 98) = 0$$

$$\text{Equation } iv)\ 2x_1 + 5x_2 - 98 \leq 0$$

$$Equation\ v)\ x_1, x_2 \geq 0, \lambda \geq 0$$

Case 1:

$$\lambda = 0$$
$$4x_1 + 12x_2 = 0$$
$$14x_1 + 12x_2 = 0\ x_1,$$
$$x_2 = 0$$
$$z = 0$$

Which is invalid

Case 2:

$$\lambda \neq 0$$
$$2x_1 + 5x_2 - 98 = 0$$

From Equation i), ii) and vi)

$$x_1 = 44,\ x_2 = 2, \lambda = 100$$
$$88 + 10 - 98 \leq 0$$

$$x_1 = 44 \geq 0,\ x_2 = 2 \geq 0\ Z_{max} = 4900$$

Quadratic Programming

Optimize

$$Z = 5x^2 + 5y^2 + 4xy$$

Subject to the constraints $x^2 + y^2 = 1$

$$Z = 5x^2 + 5y^2 + 4xy = x^T A x$$
$$= [x\ y] \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}, |A - \lambda I| = 0 \text{ where } \lambda\ are\ the\ eigen\ value$$

$$(\lambda - 5)(\lambda - 5) - 4 = 0$$
$$\lambda_1 = 7, \lambda_2 = 3$$

$$eigen\ vector\ corresponding\ to\ \lambda = 7\ is \begin{bmatrix} 1 \\ 1 \end{bmatrix}\ and\ \lambda = 3\ is \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Normalized eigen vectors are.

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

At $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), Z = 7\ max$

At $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), Z = 3\ min$

References

[1] Kaliyaperumal, P., & Das, A. (2022). A Mathematical Model for Nonlinear Optimization Which Attempts Membership Functions to Address the Uncertainties. *Mathematics*, *10*(10), 1743. https://doi.org/10.3390/math10101743

[2] Hedar, A.-R., Allam, A. A., & Deabes, W. (2019). Memory-Based Evolutionary Algorithms for Nonlinear and Stochastic Programming Problems. *Mathematics*, *7*(11), 1126. https://doi.org/10.3390/math7111126

[3] https://youtu.be/RYqBnxL8Lbg?si=79XE4ffTf6mH2R5X

Appendix 1:

```
z,x1,x2
-2,1,2
17,2,3
50,3,4
97,4,5
158,5,6
233,6,7
322,7,8
425,8,9
542,9,10
673,10,11
818,11,12
977,12,13
1150,13,14
1337,14,15
1538,15,16
1753,16,17
1982,17,18
2225,18,19
2482,19,20
2753,20,21
3038,21,22
3337,22,23
3650,23,24
3977,24,25
4318,25,26
4673,26,27
5042,27,28
5425,28,29
5822,29,30
6233,30,31
7451.85,32,33
7927.5,33,34
8417.85,34,35
8922.9,35,36
9442.65,36,37
9977.1,37,38
10526.25,38,39
11090.1,39,40
11668.65,40,41
12261.9,41,42
```