Query: $P(G = Pass \mid A = True, F = True, S = True)$

Query variable: $G = Pass$ (Grade is Pass)

Evidence variables: $A = True, F = True, S = True$ (Attendance, Final Exam score, study Hours)

Hidden variables: M, E, D, I (Midterm Score, Effort, Difficulty, Intelligence)

$$P(G = Pass \mid Evidence) = \frac{P(G = pass \cap Evidence)}{P(Evidence)}$$

$$P(G, A, F, S, M, E, D, I) = P(G \mid M, F, A) \cdot P(F \mid M, S) \cdot P(S \mid E, D) \cdot P(M \mid E, I) \cdot P(A) \cdot P(E) \cdot P(D) \cdot P(I)$$

Marginalize: $P(G = Pass \cap Evidence) = \sum_M \sum_E \sum_D \sum_I P(G = Pass, A = True, F = True, S = True, M, E, D, I)$

Normalize: $P(G = Pass \mid Evidence) = \frac{P(G = pass \cap Evidence)}{P(Evidence)}$

$$P(Evidence) = P(G = Pass \cap Evidence) + P(G = Fail \cap Evidence)$$

$P(A = True) = 0.60$
$P(A = False) = 0.40$
$P(E = True) = 0.53$
$P(E = False) = 0.47$
$P(D = True) = 0.57$
$P(D = False) = 0.43$
$P(I = True) = 0.34$
$P(I = False) = 0.66$

$P(M \mid E, I)$:
$P(M = True \mid E = True, I = True) = 0.85$
$P(M = True \mid E = True, I = False) = 0.45$
$P(M = True \mid E = False, I = True) = 0.65$
$P(M = True \mid E = False, I = False) = 0.25$

$P(S \mid E, D)$:
$P(S = True \mid E = True, D = True) = 0.44$
$P(S = True \mid E = True, D = False) = 0.74$
$P(S = True \mid E = True, D = True) = 0.19$
$P(S = True \mid E = True, D = False) = 0.36$

$P(F \mid M, S)$:
$P(F = True \mid M = True, S = True) = 0.79$
$P(F = True \mid M = True, S = False) = 0.69$
$P(F = True \mid M = True, S = True) = 0.39$
$P(F = True \mid M = True, S = False) = 0.17$

$P(G \mid M, F, A)$:
$P(G = Pass \mid M = True, F = True, A = True) = 0.92$
$P(G = Pass \mid M = True, F = True, A = False) = 0.78$
$P(G = Pass \mid M = True, F = False, A = True) = 0.58$
$P(G = Pass \mid M = True, F = False, A = False) = 0.38$
$P(G = Pass \mid M = False, F = True, A = True) = 0.46$
$P(G = Pass \mid M = False, F = True, A = False) = 0.28$
$P(G = Pass \mid M = False, F = False, A = True) = 0.16$
$P(G = Pass \mid M = False, F = False, A = False) = 0.05$

Joint Probabilities:

1. M=True, E=True, D=True, I=True:
P(G=0.92) * P(F=0.79) * P(S=0.44) * P(M=0.85) * P(A=0.6) * P(E=0.53) * P(D=0.57) * P(I=0.34)
Joint Probability: 0.016752028898880004
2. M=True, E=True, D=True, I=False:
P(G=0.92) * P(F=0.79) * P(S=0.44) * P(M=0.45) * P(A=0.6) * P(E=0.53) * P(D=0.57) * P(I=0.66)
Joint Probability: 0.017215752882240004
3. M=True, E=True, D=False, I=True:
P(G=0.92) * P(F=0.79) * P(S=0.74) * P(M=0.85) * P(A=0.6) * P(E=0.53) * P(D=0.43) * P(I=0.34)
Joint Probability: 0.021253969679520004
4. M=True, E=True, D=False, I=False:
P(G=0.92) * P(F=0.79) * P(S=0.74) * P(M=0.45) * P(A=0.6) * P(E=0.53) * P(D=0.43) * P(I=0.66)
Joint Probability: 0.021842314860960006
5. M=True, E=False, D=True, I=True:
P(G=0.92) * P(F=0.79) * P(S=0.19) * P(M=0.65) * P(A=0.6) * P(E=0.47) * P(D=0.57) * P(I=0.34)
Joint Probability: 0.00490551668568
6. M=True, E=False, D=True, I=False:
P(G=0.92) * P(F=0.79) * P(S=0.19) * P(M=0.25) * P(A=0.6) * P(E=0.47) * P(D=0.57) * P(I=0.66)
Joint Probability: 0.0036624898331999996
7. M=True, E=False, D=False, I=True:
P(G=0.92) * P(F=0.79) * P(S=0.36) * P(M=0.65) * P(A=0.6) * P(E=0.47) * P(D=0.43) * P(I=0.34)
Joint Probability: 0.007011763462080002
8. M=True, E=False, D=False, I=False:
P(G=0.92) * P(F=0.79) * P(S=0.36) * P(M=0.25) * P(A=0.6) * P(E=0.47) * P(D=0.43) * P(I=0.66)
Joint Probability: 0.005235027019200001
9. M=False, E=True, D=True, I=True:
P(G=0.46) * P(F=0.39) * P(S=0.44) * P(M=0.35) * P(A=0.6) * P(E=0.53) * P(D=0.57) * P(I=0.34)
Joint Probability: 0.0017026447838399998
10. M=False, E=True, D=True, I=False:
P(G=0.46) * P(F=0.39) * P(S=0.44) * P(M=0.75) * P(A=0.6) * P(E=0.53) * P(D=0.57) * P(I=0.66)
Joint Probability: 0.007082429983200001
11. M=False, E=True, D=False, I=True:
P(G=0.46) * P(F=0.39) * P(S=0.74) * P(M=0.35) * P(A=0.6) * P(E=0.53) * P(D=0.43) * P(I=0.34)
Joint Probability: 0.0021602135973600002
12. M=False, E=True, D=False, I=False:
P(G=0.46) * P(F=0.39) * P(S=0.74) * P(M=0.75) * P(A=0.6) * P(E=0.53) * P(D=0.43) * P(I=0.66)
Joint Probability: 0.008985762442800003
13. M=False, E=False, D=True, I=True:
P(G=0.46) * P(F=0.39) * P(S=0.19) * P(M=0.55) * P(A=0.6) * P(E=0.47) * P(D=0.57) * P(I=0.34)
Joint Probability: 0.00102456994068
14. M=False, E=False, D=True, I=False:
P(G=0.46) * P(F=0.39) * P(S=0.19) * P(M=0.75) * P(A=0.6) * P(E=0.47) * P(D=0.57) * P(I=0.66)
Joint Probability: 0.0027120969017999995
15. M=False, E=False, D=False, I=True:
P(G=0.46) * P(F=0.39) * P(S=0.36) * P(M=0.55) * P(A=0.6) * P(E=0.47) * P(D=0.43) * P(I=0.34)
Joint Probability: 0.00146448224208
16. M=False, E=False, D=False, I=False:
P(G=0.46) * P(F=0.39) * P(S=0.36) * P(M=0.75) * P(A=0.6) * P(E=0.47) * P(D=0.43) * P(I=0.66)
Joint Probability: 0.0038765706407999996

Marginalization Formula: $P(Y) = \sum_X P(X, Y)$

Marginalization over M: $P(E = True, D = True, I = True) = P(M = True, E = True, D = True, I = True) + P(M = False, E = True, D = True, I = True)$

Marginalization over I: $P(E = True, D = True) = \sum_I (P(I = True, E = True, D = True) + P(I = False, E = True, D = True))$

Marginalized over M:
E=True, D=True, I=True: 0.018454673682720003
0.016752028898880004 + 0.0017026447838399998 = 0.018454673682720003
E=True, D=True, I=False: 0.024298182865440006
0.017215752882240004 + 0.007082429983200001 = 0.024298182865440006
E=True, D=False, I=True: 0.023414183276880003
0.021253969679520004 + 0.0021602135973600002 = 0.023414183276880003
E=True, D=False, I=False: 0.03082807730376001
0.021842314860960006 + 0.008985762442800003 = 0.03082807730376001
E=False, D=True, I=True: 0.00593008662636
0.00490551668568 + 0.00102456994068 = 0.00593008662636
E=False, D=True, I=False: 0.006374586734999999
0.0036624898331999996 + 0.0027120969017999995 = 0.006374586734999999
E=False, D=False, I=True: 0.008476245704160001
0.007011763462080002 + 0.00146448224208 = 0.008476245704160001
E=False, D=False, I=False: 0.00911159766
0.005235027019200001 + 0.0038765706407999996 = 0.00911159766

Marginalized over I:
M=True, E=True, D=True: 0.033967781781120004
0.016752028898880004 + 0.017215752882240004 = 0.033967781781120004
M=True, E=True, D=False: 0.04309628454048001
0.021253969679520004 + 0.021842314860960006 = 0.04309628454048001
M=True, E=False, D=True: 0.00856800651888
0.00490551668568 + 0.0036624898331999996 = 0.00856800651888
M=True, E=False, D=False: 0.012246790481280002
0.007011763462080002 + 0.005235027019200001 = 0.012246790481280002
M=False, E=True, D=True: 0.008785074767040001
0.0017026447838399998 + 0.007082429983200001 = 0.008785074767040001
M=False, E=True, D=False: 0.011145976040160002
0.0021602135973600002 + 0.008985762442800003 = 0.011145976040160002
M=False, E=False, D=True: 0.0037366668424799994
0.00102456994068 + 0.0027120969017999995 = 0.0037366668424799994
M=False, E=False, D=False: 0.00534105288288
0.00146448224208 + 0.0038765706407999996 = 0.00534105288288

Normalization Formula: $P(Y|Evidence) = P(Y) / \sum P(Y)$

$$P(E=True, D=True, I=True | Evidence) = \frac{P(E=True, D=True, I=True)}{\sum P(E=True, D=True, I=True)}$$

Normalization probabilities marginalized over M:
Total sum before normalization: 0.12688763385432003
E=True, D=True, I=True: 0.018454673682720003 / 0.12688763385432003 = 0.14544107350845437
E=True, D=True, I=False: 0.024298182865440006 / 0.12688763385432003 = 0.19149370295088647
E=True, D=False, I=True: 0.023414183276880003 / 0.12688763385432003 = 0.18452691224238507
E=True, D=False, I=False: 0.03082807730376001 / 0.12688763385432003 = 0.2429557267901598
E=False, D=True, I=True: 0.00593008662636 / 0.12688763385432003 = 0.046734945291582515
E=False, D=True, I=False: 0.006374586734999999 / 0.12688763385432003 = 0.05023804559488181
E=False, D=False, I=True: 0.008476245704160001 / 0.12688763385432003 = 0.06680119604004593
E=False, D=False, I=False: 0.00911159766 / 0.12688763385432003 = 0.07180839758160393

Normalization probabilities marginalized over I:
Total sum before normalization: 0.12688763385432
M=True, E=True, D=True: 0.033967781781120004 / 0.12688763385432 = 0.26769970208537813
M=True, E=True, D=False: 0.04309628454048001 / 0.12688763385432 = 0.3396413285628681
M=True, E=False, D=True: 0.00856800651888 / 0.12688763385432 = 0.06752436197775544
M=True, E=False, D=False: 0.012246790481280002 / 0.12688763385432 = 0.09651681656654243
M=False, E=True, D=True: 0.008785074767040001 / 0.12688763385432 = 0.06923507437396277
M=False, E=True, D=False: 0.011145976040160002 / 0.12688763385432 = 0.08784131046967684
M=False, E=False, D=True: 0.0037366668424799994 / 0.12688763385432 = 0.02944862890870891
M=False, E=False, D=False: 0.00534105288288 / 0.12688763385432 = 0.04209277705510748