UCLA
Dept. of Electrical Engineering
EE 114, Fall 2017
Due: October 11, 2017

Problem Set 1

1. Consider the discrete time sequence:

$$x(n) = [\underline{4}, -1, 2]$$

   (a) Compute the Z-transform $X(z)$.

   (b) Let $y(n) = x(n) * x(n)$. Determine $y(n)$ by (i) convolution in the time domain, and (ii) by transformation into the Z-domain.

2. Consider the discrete-time signal:

$$x(n) = [\underline{1}, 0, -2, 0]$$

   (a) Compute the 4-point DFT, $X(k)$ of the above signal.

   (b) Using $X(k)$, evaluate the inverse transform $\hat{x}(n)$. Is $\hat{x}(n)$ equal to $x(n)$ for $n < 0$ and $n > 3$? Write down $\hat{x}(n)$ for $n = -6$ through 6.

3. Consider a continuous-time signal consisting of a cosine at 1 Hz, $x_{\mathrm{a}}(t) = \cos(2\pi t)$, sampled with a sampling rate of $F_{\mathrm{s}} = 2/3$ Hz:

$$x(n) = x_{\mathrm{a}}(t_0 + nT),\ n = \ldots, -1, 0, 1, \ldots$$

   (a) What is the output of the sampler if $t_0 = 0$ s?

   (b) The sampled signal (with $t_0 = 0$ s) is then input to an ideal D/A converter. What is the frequency of the signal that will emerge from the D/A converter?

   (c) What is the output of the sampler if $t_0 = 0.25$ s?

4. Let $x_{\mathrm{a}}(t)$ be a continuous-time speech signal 50 ms in duration, that is sampled at $F_s = 16$ kHz. We wish to perform spectral analysis using a radix-2 FFT, with spacing between adjacent frequency bins no greater than 20 Hz. What is the minimum length, N, of the FFT used? If the original signal was 75 ms in duration, what value of N would you choose?

5. Consider the discrete-time signal:

$$x(n) = [1, 2, \underline{3}, 2, 1],$$

   where the underscore denotes the origin. Compute the following quantities without explicitly computing the DTFT $X(\omega)$. Include all steps that led to your answer.

   (a) Find the phase of $X(\omega)$.

(b) $\int_{-\pi}^{\pi} X(\omega)\,\mathrm{d}\omega$

(c) $\int_{-\pi}^{\pi} |X(\omega)|^2\,\mathrm{d}\omega$

(d) Let $y(n) = x(n - n_0)$. Compute $Y(\omega)$.

(e) Let $\hat{x}(n) = x(n)\,\mathrm{e}^{-\mathrm{j}\omega_0 n}$. Compute $\hat{X}(\omega)$.