Part 3.
Question1.

a)

I^+ = DFGI so I is not a super key and I → DFG violates BCNF.
H^+ = ACEH so H is not a super key and H → CEA violates BCNF.
BI^+ = ABCDEFGHIJK so BI is a super key and BI → J does not violate BCNF.
B^+ = ABCEH so B is not a super key and B → H violates BCNF.
CI^+ = CDFGIK so CI is not a super key and CI → K violates BCNF.

b)

-Decompose R using FD B → H. B^+ = ABCEH, so this yields two relations: R_1 = ABCEH and R_2 = BFGIJK.
-Project the FDs onto R_1 = ABCEH.

A	B	C	E	H	closure	FDs
✓					A^+ = A	nothing
	✓				B^+ = BHCEA	B → ACEH
		✓			C^+ = C	nothing
			✓		E^+ = E	nothing
				✓	H^+ = HCEA	H → ACE: violates BCNF;abort the projection

-Decompose R_1 using FD H → ACE. H^+ = HCEA, so this yields two relations: R_3 = ACEH and R_4 = BH.
-Project the FDs onto R_3 = ACEH.

A	C	E	H	closure	FDs
✓				A^+ = A	nothing
	✓			C^+ = C	nothing
		✓		E^+ = E	nothing
			✓	H^+ = HCEA	H → ACE;H is a super key of R_3.

-Project the FDs onto R_4 = BH.

B	H	closure	FDs
✓		B^+ = ABCEH	B → H;B is a super key of R_4.
	✓	H^+ = ACEH	nothing
superset of B		irrelevant	can only generate weaker FDs than what we already have

-Return to R_2.
-Project the FDs onto R_2 = BFGIJK.

B	F	G	I	J	K	closure	FDs
✓						B^+ = ABCEH	nothing
	✓					F^+ = F	nothing
		✓				G^+ = G	nothing
			✓			I^+ = DFGI	I → GF: violates BCNF;abort the projection

-Decompose R_2 using FD I → GF.I^+ = IGF, so this yields two relations: R_5 = IGF and R_6 = BIJK.
-Project FDs onto R_5 = IGF.

I	G	F	closure	FDs
✓			I^+ = DFGI	I → GF;I is a super key of R_5
	✓		G^+ = G	nothing
		✓	F^+ = F	nothing
	✓	✓	GF^+ = GF	nothing
supersets of I			irrelevant	can only generate weaker FDs than what we already have

-Project FDs onto R_6 = BIJK.

B	I	J	K	closure	FDs
✓				B^+ = ABCEH	nothing
	✓			I^+ = DFGI	nothing
		✓		J^+ = J	nothing
			✓	K^+ = K	nothing
✓	✓			BI^+ = ABCDEFGHIJK	BI → JK;BI is a super key of R_6.
subsets of IJK				we can't possibly get B since it's not on RHS	nothing
subsets of BJK				we can't possibly get I since it's not on RHS	nothing
supersets of BI				irrevant	can only generate weaker FDs than what we already have

Final decomposition:
-R_3 = ACEH with FD H → ACE
-R_4 = BH with FD B → H
-R_6 = BIJK with FD BI → JK
-R_5 = FGI with FD I → FG

Question2.
a)
T = {ACDE → B, BF → AD, B → CF, CD → AF, ABF → CDH}.

Step 1: Split the RHSs to get our initial set of FDs, S1:
a. ACDE → B
b. BF → A
c. BF → D
d. B → C
e. B → F
f. CD → A
g. CD → F
h. ABF → C
i. ABF → D
j. ABF → H

Step 2: For each FD, try to reduce the LHS:
a. No singleton LHS yields anything. AC^+ = AC, AD^+ = AD, AE^+ = AE, CD^+ = CDAF, so none of them yields B either. We cannot reduce the LHS of this FD.
b. B^+ = BCFA yields the new FD B → A.
c. B^+ = BCFAD yields the new FD B → D.
d. Singleton LHS cannot be reduced.
e. Singleton LHS cannot be reduced.
f. No singleton LHS yields anything.
g. No singleton LHS yields anything.
h. B^+ = BC yields the new FD B → C.
i. B^+ = BFD yields the new FD B → D.
j. BF^+ = BFAH yields the new FD BF → H.

Our new sets of FDs, lets call it S2, is:
a. ACDE → B
b. B → A
c. B → D
d. B → C
e. B → F
f. CD → A
g. CD → F
h. B → C
i. B → D
j. BF → H

Step 3: Try to eliminate each FD.
a. $ACDE^+_{S2\text{-}a}$ = ACDE. We need this FD.
b. $B^+_{S2\text{-}b}$ = BDCF<u>A</u>. We can remove this FD.
c. $B^+_{S2\text{-}\{b,c\}}$ = B<u>D</u>. We can remove this FD.
d. $B^+_{S2\text{-}\{b,c,d\}}$ = B<u>C</u>. W can remove this FD.
e. $B^+_{S2\text{-}\{b,c,d,e\}}$ = BCD<u>F</u>. We can remove this FD.
f. $CD^+_{S2\text{-}\{b,c,d,e,f\}}$ = CDF. We need this FD.
g. $CD^+_{S2\text{-}\{b,c,d,e,g\}}$ = CDA. We need this FD.
h. $B^+_{S2\text{-}\{b,c,d,e,h\}}$ = BD. We need this FD.
i. $B^+_{S2\text{-}\{b,c,d,e,i\}}$ = BC. We need this FD.
j. $BF^+_{S2\text{-}\{b,c,d,e,j\}}$ = BFCD. We need this FD.

Our final sets of FDs is:
ACDE → B
B → C
B → D
BF → H
CD → A
CD → F

b)

Attribute	Appears on		Conclusion
	LHS	RHS	
G	-	-	must be in every key
E	✓	-	must be in every key
H	-	✓	is not in any key
A,B,C,D,F	✓	✓	must check

-GEA^+ = GEA. This is not a key.
-GEB^+ = GEBCDAFH. So GEB is a key.
-GEC^+ = GEC. This is not a key.
-GED^+ = GED. This is not a key.
-GEF^+ = GEF. This is not a key.
-All other possibilities include GEB.

c)
-First, merge the RHS sides to get one relation for each FD.
-Lets call the revised FDs S3:
ACDE → B
B → CD
BF → H
CD → AF

-The set of relations that would result would have these attributes:
R1(A,B,C,D,E)
R2(B,C,D)
R3(B,F,H)
R4(A,C,D,F)

-Since the attributes BCD occur within R1, we don't need to keep the relation R2.
-Since the attributes in none of the relations in S3 form a super key for T, we need to add another relation to T whose schema is a key for R.
-So the final set of relations is:
R1(A,B,C,D,E)
R3(B,F,H)
R4(A,C,D,F)
R5(B,E,G)

d)
-Because we formed each relation from an FD, the LHS of those FDs are obviously super keys for their relations. However,there may be other FDs that violate BCNF and allow redundancy. The only way to find out is to project the FDs onto each relation.
-Obviously, if we project $B \rightarrow CD$ onto R1, $B^+ = BCDA$, so B is not a super key for this relation.
-So yes, these schema allows redundancy.