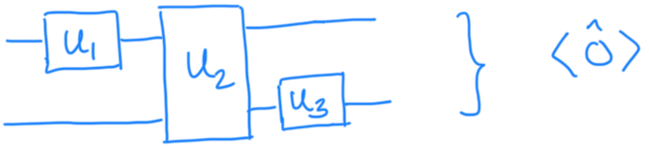
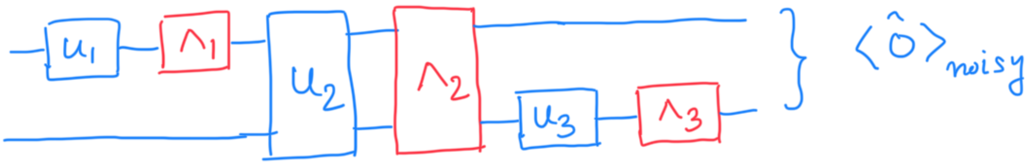
ZERO NOISE EXTRAPOLATION (ZNE)



But each gate is noisy

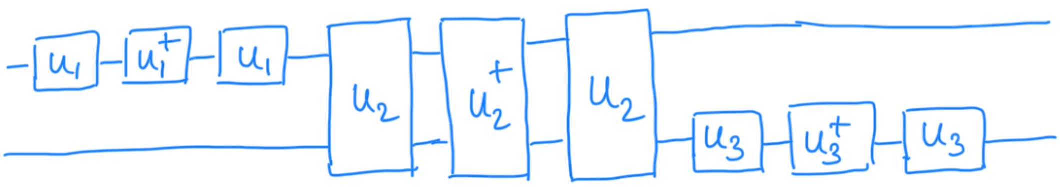


Q) Can we get an estimate of $\langle \hat{O} \rangle$ from $\langle \hat{O} \rangle_{noisy}$?

$U \rightarrow U(U^\dagger U)$ These two are functionally equivalent.

$\rightarrow U(U^\dagger U)^{\lambda-1}$ for any $\lambda \geq 1$ | $\lambda = 1$ is the original circuit

In ideal circuit:



But in noisy circuit? The noise gets amplified!

Assume depolarizing noise

N qubit circuit, density matrix ρ

Let [illegible] be a 2 qubit operation U between qubits k & l.

Let there

$$\rho \rightarrow (1-\epsilon_{kl})\, u_{kl}\, \rho\, u_{kl}^{+} + \epsilon_{kl}\left(\frac{I_{kl}}{4} \otimes \rho_{\cancel{kl}}\right)$$

when there is no noise, $\epsilon_{kl} = 0 \Rightarrow \rho \rightarrow u_{kl}\, \rho\, u_{kl}^{+}$

If $u_{kl} \rightarrow u_{kl}\,(u_{kl}^{+} u_{kl})$ i.e. $\lambda = 3$

$$\rho \rightarrow (1-\epsilon_{kl})^{3}\, u_{kl}\, \rho\, u_{kl}^{+} + \left[1-(1-\epsilon_{kl})^{3}\right]\left(\frac{I_{kl}}{4} \otimes \rho_{\cancel{kl}}\right)$$

$$= \left[1 - 3\epsilon_{kl} + O(\epsilon_{kl}^{2})\right] u_{kl}\, \rho\, u_{kl}^{+} + \left[1 - 1 + 3\epsilon_{kl} - O(\epsilon_{kl}^{2})\right]\left(\frac{I_{kl}}{4} \otimes \rho_{\cancel{kl}}\right)$$

$$= (1-3\epsilon_{kl})\,\underbrace{u_{kl}\, \rho\, u_{kl}^{+}}_{\text{gives ideal expectation value.}} + 3\epsilon_{kl}\,\underbrace{\left(\frac{I_{kl}}{4} \otimes \rho_{\cancel{kl}}\right)}_{\text{gives noisy expectation value}} + O(\epsilon_{kl}^{2})$$

$$\Rightarrow (1-3\epsilon_{kl})\,\langle \hat{o} \rangle + 3\epsilon_{kl}\,\langle \hat{o} \rangle_{noisy} + O(\epsilon_{kl}^{2})$$

Then for any $\lambda \rightarrow (1-\lambda\epsilon_{kl})\,\langle \hat{o} \rangle + \lambda . \epsilon_{kl}\,\langle \hat{o} \rangle_{noisy} + O(\epsilon_{kl}^{2})$

If there are M gates :

$$\langle o \rangle_{\lambda} = \left(1 - \lambda \sum_{i=1}^{M} \epsilon_i\right)\langle \hat{o} \rangle + \left(\lambda . \sum_{i=1}^{M} \epsilon_i\right)\langle \hat{o} \rangle_{noisy} + O\left(\sum_{i=1}^{M} \epsilon_i^{2}\right)$$

dominated by the

maximum error probability

$$= \left(1-\lambda\sum_{i=1}^{M}\epsilon_i\right)\langle\hat{O}\rangle + \left(\lambda.\sum_{i=1}^{M}\epsilon_i\right)\langle\hat{O}\rangle_{noisy} + O(\epsilon_{max}^2) \quad \cdots (i)$$

If we can set $\lambda=0 \Rightarrow \left(1-\lambda\sum_{i=1}^{M}\epsilon_i\right)\langle\hat{O}\rangle + O(\epsilon_{max}^2)$

1) we get an improved estimate of $\langle\hat{O}\rangle$

2) ZNE is a biased estimator

But how to <u>physically</u> set $\lambda=0$?

for simplicity, take $\epsilon_i=\epsilon \quad \forall i$

$$\langle\hat{O}\rangle_\lambda = (1-\lambda.M.\epsilon)\langle\hat{O}\rangle + \lambda.M.\epsilon.\langle\hat{O}\rangle_{noisy} + O(\epsilon^2)$$

$$\lambda=1 \Rightarrow \langle\hat{O}\rangle_1 = (1-M.\epsilon)\langle\hat{O}\rangle + M.\epsilon.\langle\hat{O}\rangle_{noisy} + O(\epsilon^2)$$

$$= \langle\hat{O}\rangle + M.\epsilon.\left[\langle\hat{O}\rangle_{noisy} - \langle\hat{O}\rangle\right] + O(\epsilon^2) \quad \cdots (ii)$$

$$\lambda=3 \Rightarrow \langle\hat{O}\rangle_3 = (1-3.M.\epsilon).\langle\hat{O}\rangle + 3.M.\epsilon\,\langle\hat{O}\rangle_{noisy} + O(\epsilon^2)$$

$$= \langle\hat{O}\rangle + 3M.\epsilon\left[\langle\hat{O}\rangle_{noisy} - \langle\hat{O}\rangle\right] + O(\epsilon^2) \quad \cdots (iii)$$

Take

$$\frac{3}{2}\langle\hat{O}\rangle_1 - \frac{1}{2}\langle\hat{O}\rangle_3 = \frac{3}{2}\langle\hat{O}\rangle + \frac{3}{2}M.\epsilon.\langle\hat{O}\rangle_{noisy} - \frac{3}{2}M.\epsilon.\langle\hat{O}\rangle + O(\epsilon^2)$$

$$- \frac{1}{2}\langle\hat{O}\rangle - \frac{3}{2}M.\epsilon.\langle\hat{O}\rangle_{noisy} + \frac{3}{2}M.\epsilon.\langle\hat{O}\rangle + O(\epsilon^2)$$

$$= \langle\hat{O}\rangle + O(\epsilon^2)$$ <u>Richardson Extrapolation</u>

$$\boxed{\frac{3}{2}\langle\hat{O}\rangle_1 - \frac{1}{2}\langle\hat{O}\rangle_3 = \langle\hat{O}\rangle + O(\epsilon^2)}$$

$$\boxed{\frac{3}{2}\langle \hat{o}\rangle_1 - \frac{1}{2}\langle \hat{o}\rangle_3 = \langle \hat{o}\rangle + \ldots}$$

Summarize :

1. Prepare the circuit at different values of λ and calculate the expectation values $\langle \hat{o}\rangle_\lambda$ for each λ.

2. Extrapolate to $\lambda = 0$

3. Can use higher order extrapolators