Operator Backpropagation

Problem
- Classically simulate
  - $\xrightarrow{B}$ No noise.
  - $\xrightarrow{D}$ small circuit
- Execute on Quantum
  - $\xrightarrow{B}$ large circuit
  - $\xrightarrow{D}$ noise

Issue: higher depth $\Rightarrow$ higher noise.

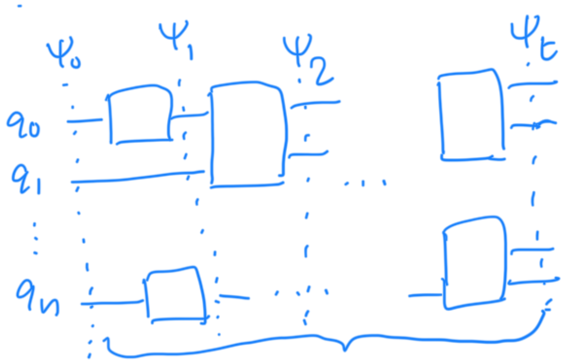
$\langle O \rangle$

- Schrodinger:
  i) Observable is fixed.
  ii) wavefunction evolves with time

$$|\psi_0\rangle \to |\psi_1\rangle \to \cdots \to \boxed{|\psi_t\rangle \Rightarrow \langle O_0\rangle}$$

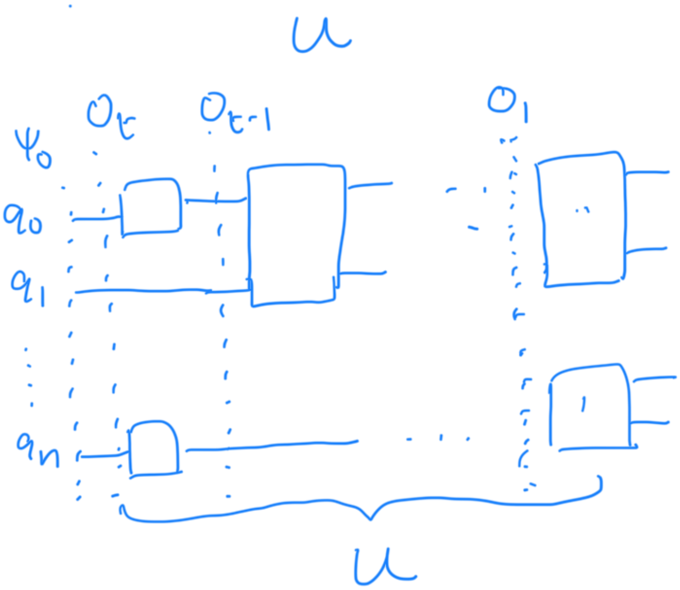
- Heisenberg:
  i) wavefunction is fixed.
  ii) Observable evolves with time

$$\langle O_0\rangle \to \langle O_1\rangle \to \cdots \to \boxed{\langle O_t\rangle \Rightarrow |\psi_0\rangle}$$



$\langle O \rangle$

$\Rightarrow$ Schrodinger

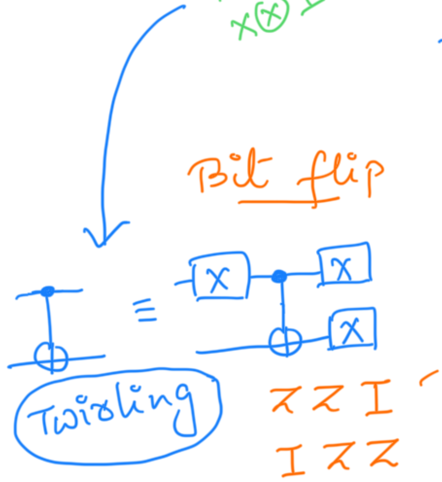


$\langle O \rangle$ $\Rightarrow$ Heisenberg

Pauli : $\{ I, X, Y, Z \}^{\otimes n}$ $\pm i, \pm 1$.

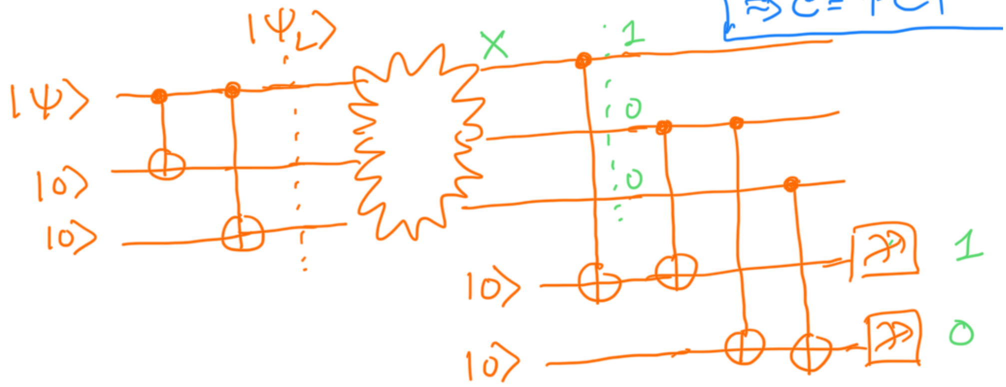
Clifford : C s.t. $\boxed{PC = CP'}$, P, P' are Pauli

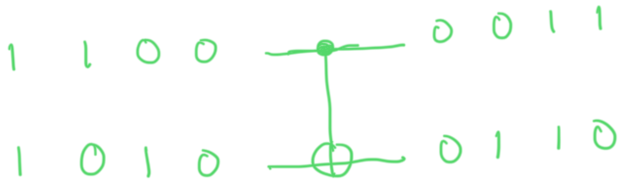
$PC = CP'$
$\Rightarrow P^\dagger P C = P^\dagger C P'$
$\Rightarrow C = PCP'$

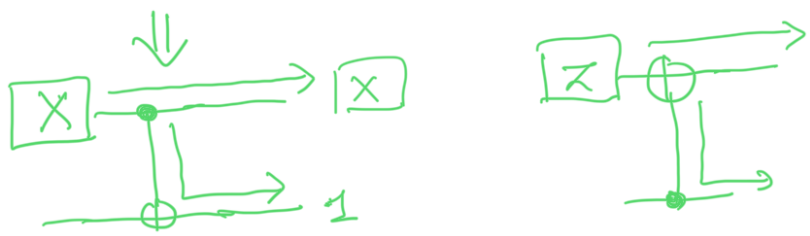


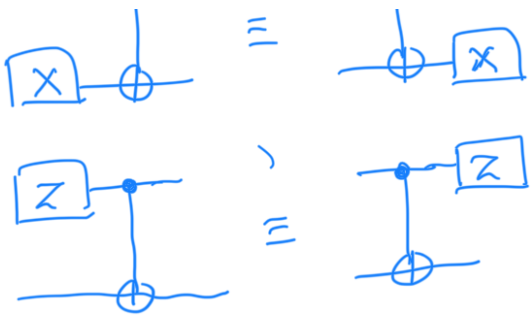
Bit flip

ZZI
IZZ









$\boxed{C_n}$ $\Rightarrow$ $\underbrace{2^n \times 2^n}$. (exact)

n-qubit gate

clifford $\Rightarrow$ $PC_n = C_n P'$

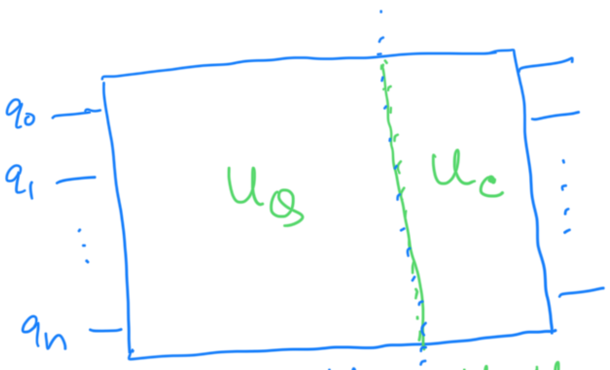
P can be represented in $\underline{O(n)}$ size

if you know every pair $\underline{(P, P')}$ you know everything about C_n.

$\Rightarrow$ Only Pauli & clifford gates cannot do universal quantum computing.

$\{\underbrace{X, SX, CX, ID}_{\text{clifford}}, RZ\}$

$RZ \hookrightarrow$ non-clifford.



$U = U_c U_Q$

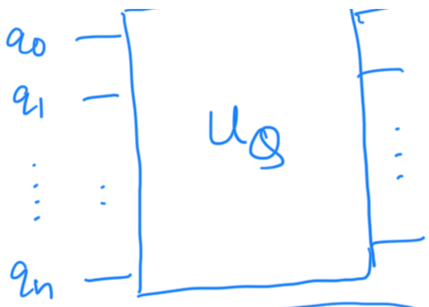
$\Downarrow$

$\langle O \rangle$

$\hookrightarrow \langle O_{new} \rangle = \underline{U_c O U_c^{\dagger}}$

$\downarrow$

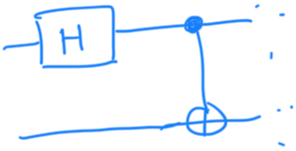
classical (no noise)



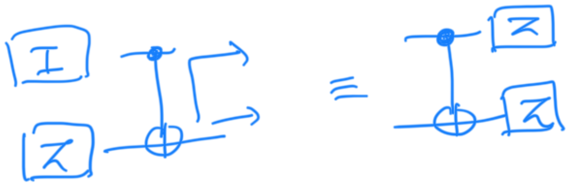
$\langle O_{new} \rangle = U_c O U_c^{\dagger}$ (OBP)

$\boxed{\begin{array}{l} \text{depth of } U_Q \\ \# \text{ gates in } U_Q \end{array}}$ $\begin{array}{l} < \text{depth of } U \\ < \# \text{ gates in } U \end{array}$

noise is less



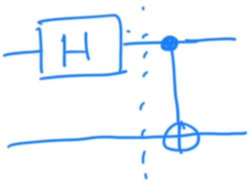
$\langle ZZ \rangle$



$\Downarrow$

$\langle IZ \rangle$

Q)



[II, IX, IY, IZ,
XI, XX, XY, XZ,
YI, YX, YY, YZ,
ZI, ZX, ZY, ZZ]

$\Downarrow$

?

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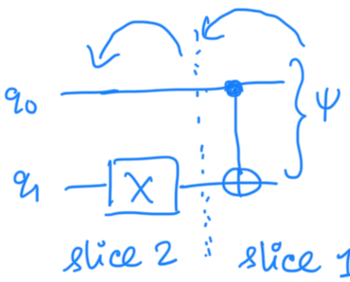


$\langle ZZ \rangle$

$\langle \psi | ZZ | \psi \rangle$

$$\langle \psi | ZZ | \psi \rangle = \underbrace{\langle 00 | (I \otimes X)\, CNOT}_{\langle \psi |}\, (ZZ)\, \underbrace{CNOT\, (I \otimes X) | 00 \rangle}_{| \psi \rangle}$$

$\langle \ldots$

$$= \langle 00| (I\otimes X)(I\otimes Z)\underbrace{CNOT \cdot CNOT}(I\otimes X)|00\rangle$$

$$= \langle 00|(I\otimes X)(I\otimes Z)(I\otimes X)|00\rangle$$

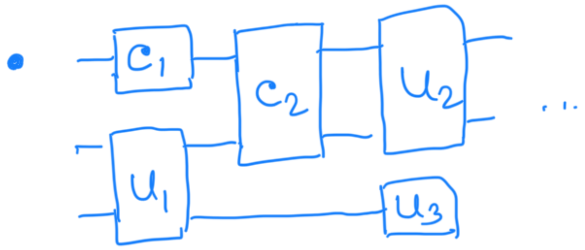
$$= \langle 00|(-I\otimes Z)\underbrace{(I\otimes X)(I\otimes X)}|00\rangle$$

$$= \langle 00|(-I\otimes Z)|00\rangle$$

q_0 –

q_1 –

$\langle -IZ\rangle$

- 

A general circuit:

$$U_1 C_1 U_2 C_2 \ldots U_N C_N |0\rangle^n$$

- $C \Rightarrow$ clifford : may be identity
- $U \Rightarrow$ non-clifford

Non-clifford gate:

$$R_X(\theta) = \exp\left(-i\frac{\theta}{2}X\right)$$

$$R_Z(\theta) = \exp\left(-i\frac{\theta}{2}Z\right)$$

$$U_i = \exp\left(-i\frac{\theta_i}{2}\cdot P_i\right)$$ ↳ Pauli

$$R_{ZZ}(\theta) = \exp\left(-i\frac{\theta}{2}\cdot ZZ\right)$$

$$|\theta_i| \le \pi/4$$

$$\theta_j > \pi/4 \Rightarrow \theta_j = \pi/4 + \tilde{\theta}_j$$

clifford

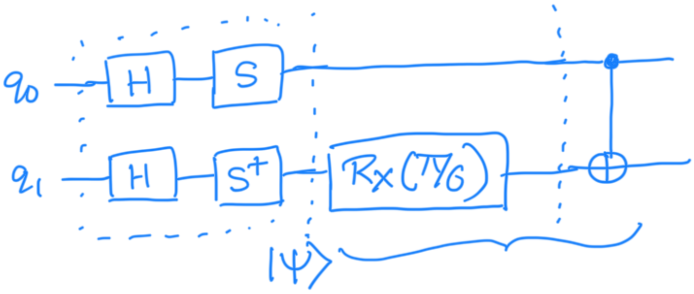
non-clifford

$$O \rightarrow U O U^\dagger$$

$$e^{i\theta P/2}\,\Theta\,e^{-i\theta P/2} = \begin{cases} \Theta & [P,\Theta]=0 \end{cases}$$

$\cos\theta \cdot O + i \sin\theta \cdot P \cdot O \qquad \{P, O\} = 0$

$$O \to \cos\theta \cdot \hat{O} + i \cdot \sin\theta \; P \cdot O$$



$\langle ZZ \rangle$

$$\langle\psi| \left(I \otimes R_x(-\pi/6)\right) CNOT\, (ZZ)\, CNOT\, \left(I \otimes R_x(\pi/6)\right) |\psi\rangle$$

$$= \langle\psi| \left(I \otimes R_x(-\pi/6)\right) (IZ)\, \underline{CNOT \cdot CNOT} \cdot \left(I \otimes R_x(\pi/6)\right) |\psi\rangle$$

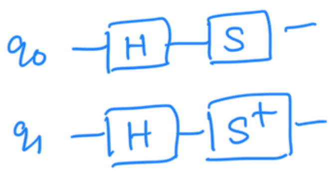
$$= \langle\psi| \left(I \otimes R_x(-\pi/6)\right) (IZ) \left(I \otimes R_x(\pi/6)\right) |\psi\rangle$$

$$= \langle\psi| \left(I \otimes \exp\left(i \tfrac{1}{2} \cdot \tfrac{\pi}{6} \cdot X\right)\right) (IZ) \left(I \otimes \exp\left(-i \tfrac{1}{2} \cdot \tfrac{\pi}{6} \cdot X\right)\right) |\psi\rangle$$

$$= \langle\psi| \cos\tfrac{\pi}{6}\, IZ + i \cdot \sin\tfrac{\pi}{6}\, IY |\psi\rangle$$

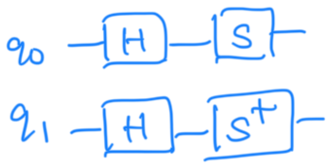
$$= \cos\tfrac{\pi}{6} \cdot \underbrace{\langle\psi| IZ |\psi\rangle} + i \cdot \sin\tfrac{\pi}{6} \cdot \underbrace{\langle\psi| IY |\psi\rangle}$$

Circuit 1 :



$\langle IZ \rangle$

Circuit 2 :



$\langle IY \rangle$