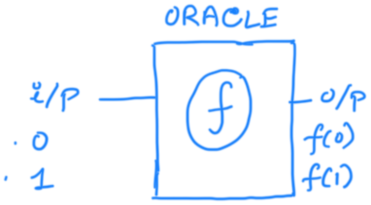
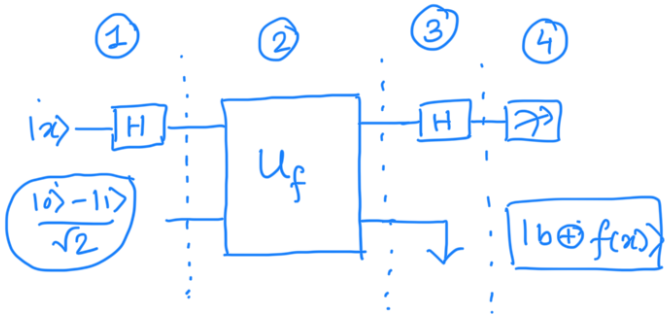
# Introduction to Quantum Algorithms

## Deutsch :




$f(0) = 0$, $f(1) = 0$; $f(0) = 1$, $f(1) = 1$ } constant

$f(0) = 0$, $f(1) = 1$; $f(0) = 1$, $f(1) = 0$ } balanced




$$H|0\rangle = \frac{1}{\sqrt2}(|0\rangle + |1\rangle)$$
$$H|1\rangle = \frac{1}{\sqrt2}(|0\rangle - |1\rangle)$$

$f(0) \oplus f(1) = 0$

$f(0) \oplus f(1) = 1$

$$\frac{1}{\sqrt2}|x\rangle|0\rangle - \frac{1}{\sqrt2}|x\rangle|1\rangle \xrightarrow{U_f} \frac{1}{\sqrt2}U_f(|x\rangle|0\rangle - |x\rangle|1\rangle)$$

$$= \frac{1}{\sqrt2}(|x\rangle|0\oplus f(x)\rangle - |x\rangle|1\oplus f(x)\rangle)$$

$x=0$, $f(x)=0 \xrightarrow{+1} \frac{1}{\sqrt2}|0\rangle(|0\rangle - |1\rangle) \rightarrow (-1)^0 = 1$

$(-1)^1 = -1$

$x=0$, $f(x)=1 \rightarrow \frac{1}{\sqrt2}|0\rangle(|1\rangle - |0\rangle) = -\frac{1}{\sqrt2}|0\rangle(|0\rangle - |1\rangle)$

$x=1$, $f(x)=0$

$x=1$, $f(x)=1$

$$U_f\left(|x\rangle\frac{1}{\sqrt2}(|0\rangle - |1\rangle)\right) = (-1)^{f(x)}|x\rangle\frac{1}{\sqrt2}(|0\rangle - |1\rangle)$$ Phase Kickback

$$|x\rangle = |0\rangle \xrightarrow{H} \frac{1}{\sqrt2}(|0\rangle + |1\rangle)\frac{1}{\sqrt2}(|0\rangle - |1\rangle) \cdots (1)$$

$$= \frac{1}{\sqrt2}\cdot|0\rangle\cdot\frac{|0\rangle - |1\rangle}{\sqrt2} + \frac{1}{\sqrt2}\cdot|1\rangle\frac{|0\rangle - |1\rangle}{\sqrt2}$$

$|\psi\rangle$, $e^{i\theta}|\psi\rangle$

(2) $$U_f \rightarrow \frac{1}{\sqrt2}\cdot(-1)^{f(0)}|0\rangle\cdot\frac{|0\rangle - |1\rangle}{\sqrt2} + \frac{1}{\sqrt2}\cdot(-1)^{f(1)}\frac{|0\rangle - |1\rangle}{\sqrt2}$$

$$= \frac{1}{\sqrt2}\left[(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right]\cdot\cancel{\frac{|0\rangle - |1\rangle}{\sqrt2}}$$

$f(0)=0$

$f(1)=0/1$

Global phase $\leftarrow = (-1)^{f(0)} \frac{1}{\sqrt{2}} \cdot \left[ |0\rangle + (-1)^{f(0) \oplus f(1)} |1\rangle \right]$ | $f(0) = 1.$ $f(1) = 0/1$

Constant: $(-1)^{f(0)} \cdot \frac{1}{\sqrt{2}} \cdot \left[ |0\rangle + (-1)^{0} \cdot |1\rangle \right] = \frac{1}{\sqrt{2}} \cdot \left[ |0\rangle + |1\rangle \right] \xrightarrow{H} |0\rangle$ ;

Balanced: $(-1)^{f(0)} \cdot \frac{1}{\sqrt{2}} \cdot \left[ |0\rangle + (-1)^{1} |1\rangle \right] = \frac{1}{\sqrt{2}} \cdot \left[ |0\rangle - |1\rangle \right] \xrightarrow{H} |1\rangle$

Phase kickback: $U_f |x\rangle \frac{|0\rangle - |1\rangle}{\sqrt{2}} = (-1)^{f(x)} \cdot |x\rangle \cdot \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

$$\left. \begin{aligned} H|0\rangle &= \frac{|0\rangle + |1\rangle}{\sqrt{2}} \\ H|1\rangle &= \frac{|0\rangle - |1\rangle}{\sqrt{2}} \end{aligned} \right\} \quad H|x\rangle = \frac{1}{\sqrt{2^n}} \sum_z (-1)^{x \cdot z} |z\rangle \rightarrow \text{Any no. of qubit}$$

$H|0\rangle = (-1)^{0.0} \cdot |0\rangle + (-1)^{0.1} \cdot |1\rangle = |0\rangle + |1\rangle$

$H|1\rangle = (-1)^{1.0} |0\rangle + (-1)^{1.1} |1\rangle = |0\rangle - |1\rangle$

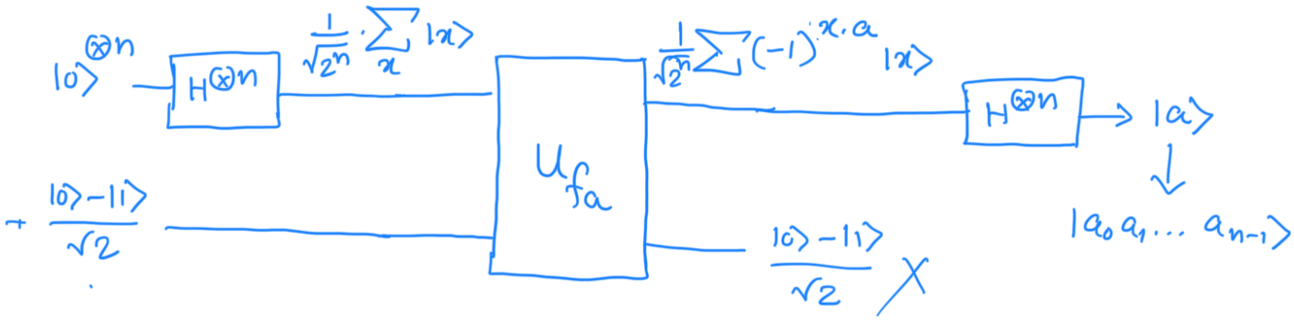
\# $f_a(x) = x \cdot a$ $\quad x = 1011$ $\quad x \cdot a = 1001$

$a = 1101 \rightarrow$ only the oracle knows.



$a_0 0 0 0$
$0 a_1 0 0$

n-queries




Bernstein-Vazirani Algorithm