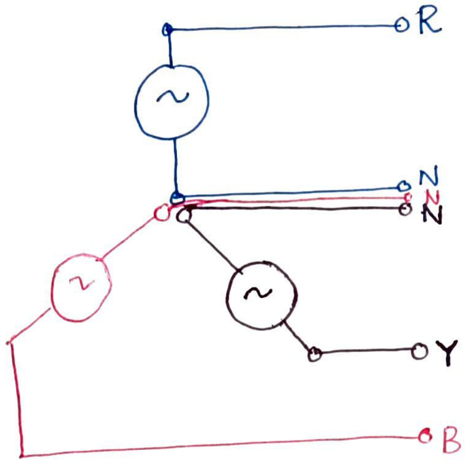
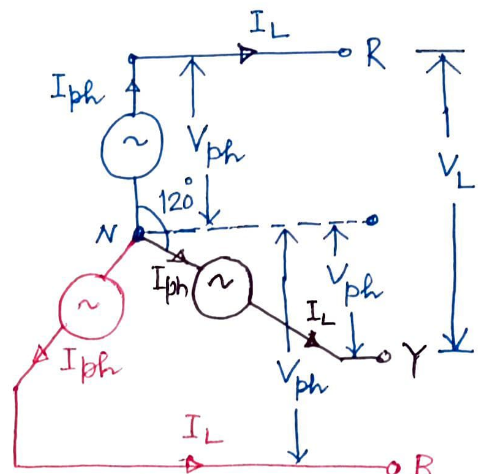
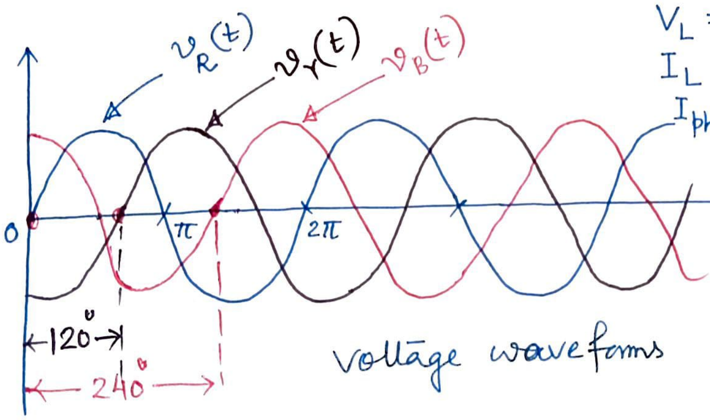
THREE PHASE CIRCUIT

Almost all the electric power used in this country is generated and transmitted in the form of balanced three-phase (3-φ) voltage systems. The single-phase (1-φ) voltage sources originate as a part of the 3-φ system. A balanced 3-φ voltage system is composed of three 1-φ voltages having the same amplitude and frequency of variation but time displaced from one another by $120°$.

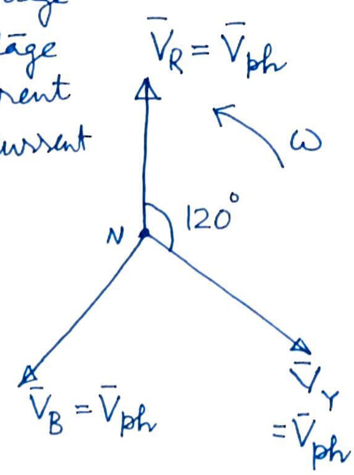




V_{ph} = phase voltage
V_L = line voltage
I_L = line current
I_{ph} = phase current



voltage waveforms



Phasor diagram

$$v_R(t) = V_m \sin \omega t$$

$$v_Y(t) = V_m \sin(\omega t - 120°)$$

$$v_B(t) = V_m \sin(\omega t - 240°)$$

The three 1-φ voltages appear in a Y configuration but a Δ configuration is also possible.

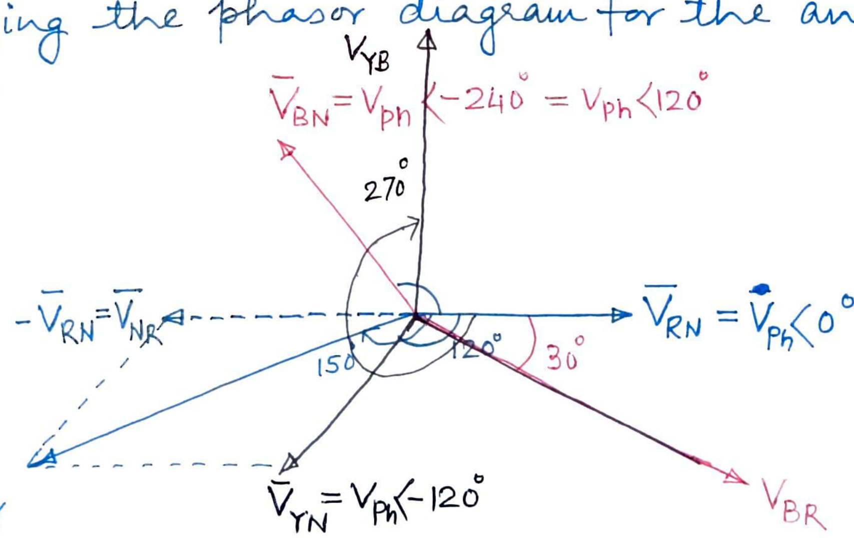
These single-phase voltages are generated by a common rotating flux field in three identical windings which are separated from each other by 120° inside the housing of the electric generator. When we join one end of each winding together to form terminal N, the Y (star) connection results.

As shown in the diagrams, when the phasors revolve at the angular frequency ω with respect to the ref. line in the counterclockwise (CCW) direction, the complete time diagrams are generated. The +ve maximum value occurs for phase R and then in succession for Y and B. For this reason the 3-ϕ voltages that obtained is said to have the phase order RYB. This is called phase sequence. If the phasors revolve in clockwise direction then its phase order or sequence will change and it will be BYR.

Voltage across a single phase is called as phase voltage (V_{ph}) and voltage across two phases (say R and Y) is called as line voltages (V_L).

Relationship between line and phase quantities in Y-connected 3-ϕ source

Rearranging the phasor diagram for the analysis we have



According to K's law (voltage)

$$\bar{V}_{RY} = -\bar{V}_{RN} + \bar{V}_{YN}$$

$$(= \bar{V}_{NR} + \bar{V}_{YN})$$

$$= -V_{ph}\angle 0^\circ + V_{ph}\angle -120^\circ$$

$$= V_{ph}\left[-1\angle 0^\circ + 1\angle -120^\circ\right]$$

$$= V_{ph}\left(-\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3}\, V_{ph}\angle -150^\circ$$

lly, $\bar{V}_{YB} = \sqrt{3}\, V_{ph}\angle -270^\circ$ and $\bar{V}_{BR} = \sqrt{3}\, V_{ph}\angle -30^\circ$

Here $|\bar{V}_{RY}| = |\bar{V}_{YB}| = |\bar{V}_{BR}| = V_L$ (Line voltage)

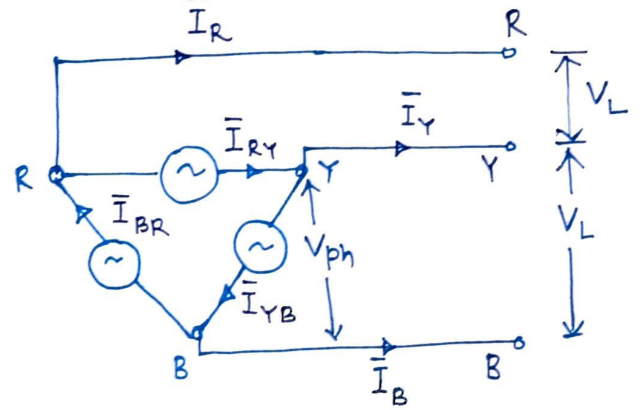
$$= \sqrt{3}\, V_{ph}$$

$\therefore$ In a Y- connected 3-ϕ ckt $\boxed{V_L = \sqrt{3}\, V_{ph}}$

From the diagram it is clear that

$$\boxed{I_L = I_{ph}}$$

Δ- Connected 3-ϕ voltage source



In phasor notation

$$\bar{I}_R = \bar{I}_{BR} - \bar{I}_{RY}$$

$$\bar{I}_Y = \bar{I}_{RY} - \bar{I}_{YB}$$

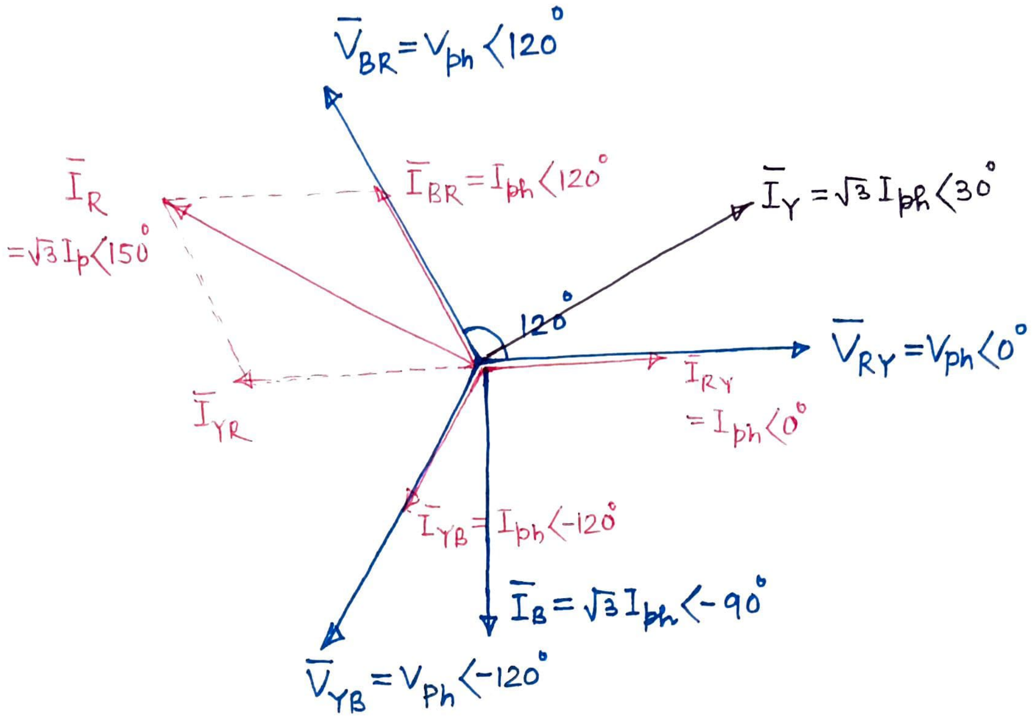
$$I_B = \bar{I}_{YB} - \bar{I}_{BR}$$

(applying K's current law)

To illustrate the relationship existing between line and phase currents, let us consider that the nature of the load ckt is such that it causes the 3-ϕ currents to be expressed as

$$\bar{I}_{RY} = I_{ph}\angle 0^\circ$$
$$\bar{I}_{YB} = I_{ph}\angle -120^\circ$$
$$\bar{I}_{BR} = I_{ph}\angle -240^\circ = I_{ph}\angle 120^\circ$$

Now $\bar{I}_R = \bar{I}_{BR} - \bar{I}_{RY}$



$$\bar{I}_R = \bar{I}_{BR} - \bar{I}_{RY}$$
$$= I_{ph}\angle 120^\circ - I_{ph}\angle 0^\circ$$
$$= I_{ph}\left(-\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}\, I_{ph}\angle 150^\circ$$

lly, $\bar{I}_Y = \sqrt{3}\, I_{ph}\angle 30^\circ$ and $\bar{I}_B = \sqrt{3} I_{ph}\angle -90^\circ$

Here $|\bar{I}_R| = |\bar{I}_Y| = |\bar{I}_B| = I_L$ = Line current
$= \sqrt{3}\, I_{ph}$

$\therefore$ In a Δ-connected 3-ϕ ckt $\boxed{I_L = \sqrt{3}\, I_{ph}}$

From the diagram it is clear that
$\boxed{V_L = V_{ph}}$

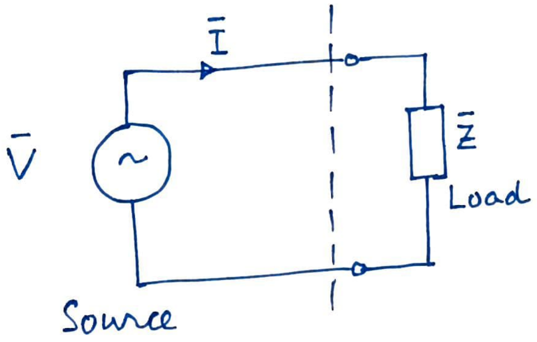
In summary, for 3-ϕ voltage source

Y-connected : $V_L = \sqrt{3}\, V_{ph}$ & $I_L = I_{ph}$

Δ-connected : $V_L = V_{ph}$ & $I_L = \sqrt{3}\, I_{ph}$

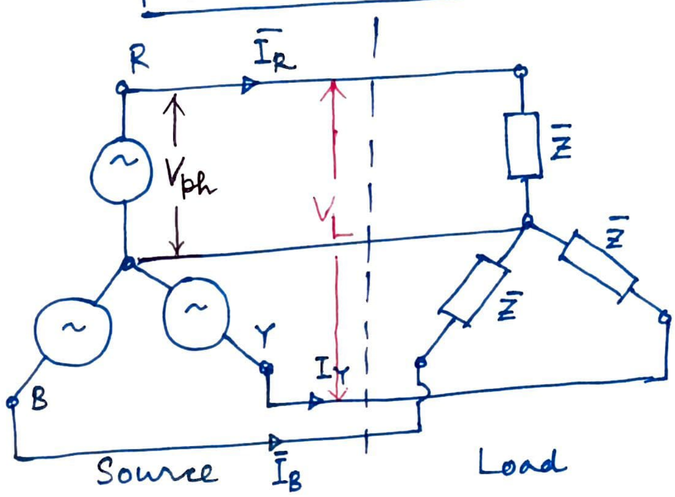
Load connection

In 1-ϕ ckt



In 3-ϕ ckt

Y-connected load



Ex. In the Y-connected load given in the above figure, if the source phase voltage is 200V and the load impedance is $100\angle 60^\circ$, calculate all the phase voltages, line voltages and line currents.

Sol^n. $\bar{Z} = 100\angle 60^\circ$, $V_{ph} = 200\,V$ $\qquad V_L = \sqrt{3}\, V_{ph} = 346.4\,V$

$$I_L = I_{ph} = \frac{200}{100} = 2\,A$$

Let $\bar{V}_{RN} = V_{ph}\angle 0^\circ$ $\quad \bar{V}_{YN} = V_{ph}\angle -120^\circ$ and $\bar{V}_{BN} = V_{ph}\angle -240^\circ$

$$\bar{I}_R = \cancel{I_{ph}}\, \frac{\bar{V}_{RN}}{\bar{Z}} = 2\angle -60^\circ, \quad \bar{I}_Y = 2\angle -180^\circ, \quad \bar{I}_B = 2\angle -300^\circ$$

$$\bar{V}_{RY} = 346.4\angle -150^\circ, \quad \bar{V}_{YB} = 346.4\angle -270^\circ = 346.4\angle 90^\circ \quad \bar{V}_{BR} = 346.4\angle -30^\circ$$

Three phase power

The 3-ϕ voltages and currents in a balanced ckt can be written in the instantaneous form as

$$v_R(t) = \sqrt{2}\, V_{ph} \sin \omega t$$

$$v_Y(t) = \sqrt{2}\, V_{ph} \sin(\omega t - 120^\circ)$$

$$v_B(t) = \sqrt{2}\, V_{ph} \sin(\omega t - 240^\circ)$$

and

$$i_R(t) = \sqrt{2}\, I_{ph} \sin(\omega t - \theta)$$

$$i_Y(t) = \sqrt{2}\, I_{ph} \sin(\omega t - \theta - 120^\circ)$$

$$i_B(t) = \sqrt{2}\, I_{ph} \sin(\omega t - \theta - 240^\circ)$$

The instantaneous power in each phase is

$$p_R(t) = v_R(t)\, i_R(t) = V_{ph} I_{ph} [\cos\theta - \cos(2\omega t - \theta)]$$

$$p_Y(t) = v_Y(t)\, i_Y(t) = V_{ph} I_{ph} [\cos\theta - \cos(2\omega t - \theta - 240^\circ)]$$

$$p_B(t) = v_B(t)\, i_B(t) = V_{ph} I_{ph} [\cos\theta - \cos(2\omega t - \theta - 480^\circ)]$$

In the above expressions the sum of three 2nd harmonic oscillating terms which have a progressive phase difference of 120° is zero.

$\therefore$ Total instantaneous 3-ϕ power.

$$p(t) = p_R(t) + p_Y(t) + p_B(t)$$

$$= 3\, V_{ph}\, I_{ph} \cos\theta$$

= constant and equals to three times of avg. power of each pase = P

For Y-connected system.

$$P = 3 \cdot \frac{V_L}{\sqrt{3}} \cdot I_L \cos\theta = \sqrt{3}\, V_L I_L \cos\theta$$

For Δ-connected system

$$P = 3 \cdot V_L \frac{I_L}{\sqrt{3}} \cos\theta = \sqrt{3}\, V_L I_L \cos\theta$$

where θ is the phase different bet^n V_{ph} & I_{ph}