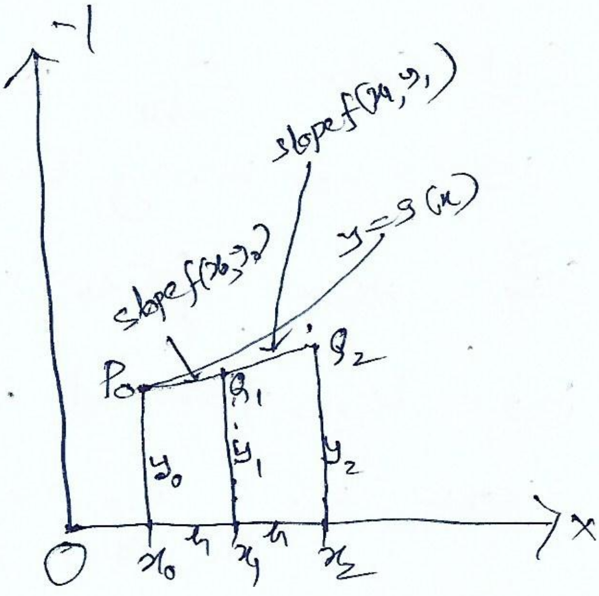
§ Euler's eq^{n}s

Consider the eq^n $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0 \rightarrow (1)$

Let $y = g(x)$ be the sol^n of (1). Let $x_0, x_1, x_2 \ldots$ where $x_1 = x_0 + h$, $x_2 = x_1 + h = x_0 + 2h \ldots x_n = x_0 + nh$ be equidistant values of x.

On a small interval, a curve is nearly a st. line. This is the property used in Euler's method.



The eq^n of the tangent at of $P_0(x_0, y_0)$ is

$$\frac{y - y_0}{x - x_0} = \left.\frac{dy}{dx}\right)_{P_0}$$

or, $y = y_0 + f(x,y)(x - x_0)$ $\left[\because \frac{dy}{dx} = f(x,y)\right] \rightarrow (2)$

This gives the y-coordinate of any pt. on the tangent. Since the curve is approximated by the tangent in the interval (x_0, x_1), the value of y on the curve corresponding to $x = x_1$ is given by the above value of y in (2) approximately.

$\therefore$ putting $x = x_1 = x_0 + h$ in (2) we have $y_1 = y_0 + h f(x_0, y_0)$

Thus Q_1 is (x_1, y_1). Similarly approximating the curve in the next interval (x_1, y_1) by a line through $Q_1(x_1, y_1)$ whose slope is $\left(\frac{dy}{dx}\right)_{Q_1} = f(x_1, y_1)$ we get $y_2 = y_1 + h f(x_1, y_1)$.

Repeating this proces we have $y_3 = y_2 + h f(x_2, y_2)$ and so on. In general $y_{n+1} = y_n + h f(x_n, y_n)$.

This is called Euler's algorithm.

This process is very slow and to obtain reasonable accuracy with Euler's method, we need to take a smaller value of h.

Ex1 Solve $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$ in the range $0 \le x \le 0.2$ using Euler's method. Take $h = 0.1$.

Sol: Here $f(x, y) = y - \frac{2x}{y}$, $x_0 = 0$, $y_0 = 1$ & $h = 0.1$

By Euler's formula. $y_1 = y_0 + h f(x_0, y_0) \longrightarrow (1)$

Here x: 0, 0.1, 0.2
x_0, x_1, x_2

So we are to find y corresponding to $x = x_2$.

Now $f(x_0, y_0) = 1 - 0 = 1$

$\therefore y_1 = 1 + 0.1 \times 1 = 1.1$

$y_2 = y_1 + h f(x_1, y_1)$ where $f(x_1, y_1) = 1.1 - \frac{2 \times 0.1}{1.1} = .9181$
$\cancel{= 1.1918}$

$\therefore y_2 = 1.1 + 0.1 \times \cancel{1.1918}\ .9181 = 1.1918$

& $y_3 = y_2 + h f(x_2, y_2) = 1.1918 + 0.1 f(0.2, 1.1918)$
$= 1.1918 + .1 (1.1918 - .3356)$
$= 1.2274$

Ex2 Using Euler's method solve for y at $x = 0.1$ from $\frac{dy}{dx} = x + y + xy$, $y(0) = 1$ taking $h = 0.025$