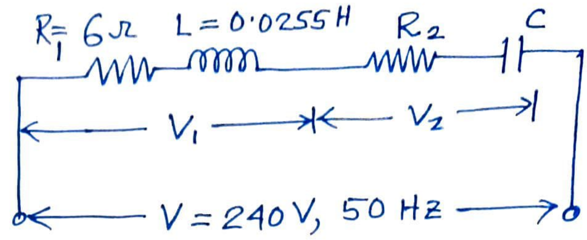
Ex

In the ckt,

$V_1 = 3V_2$

and V_1 and V_2 are in quadrature.



Find R_2 and C. Also draw the phasor diagram.

Sol^n. $X_L = 2\pi f L = 2\pi \times 50 \times 0.0255 = 8\ \Omega$ (inductive reactance)

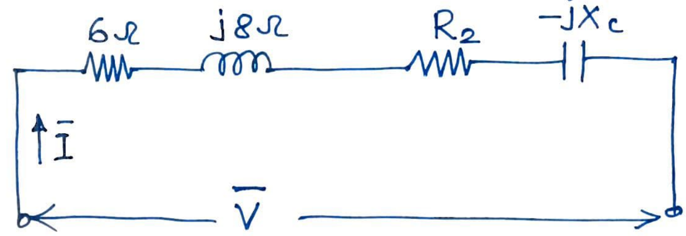
$\bar{V}_1$ and $\bar{V}_2$ are in quadrature and $\bar{V}_1 + \bar{V}_2 = \bar{V}$

$\therefore \quad V_1^2 + V_2^2 = V^2$

$\Rightarrow (3V_2)^2 + V_2^2 = (240)^2$

$\Rightarrow \quad V_2 = 75.9\ V \qquad$ and $V_1 = 3V_2 = 227.7\ V$

The ckt can also be drawn as shown



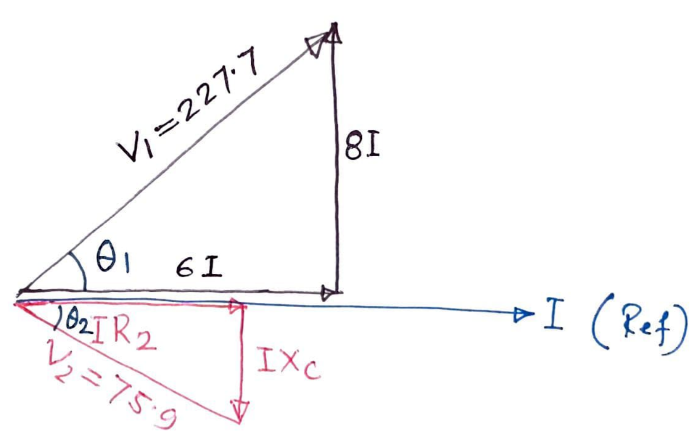
Taking $\bar{I}$ as reference phasor, the phasor diagram is drawn.

$\tan\theta_1 = \dfrac{8I}{6I} \Rightarrow \theta_1 = 53.1^\circ$

$\tan\theta_2 = \dfrac{IX_C}{IR_2} = \dfrac{X_C}{R_2}$

But $\theta_1 + \theta_2 = 90^\circ$

[$\because$ V_1 and V_2 in quadrature]



$\theta_2 = 90^\circ - 53.1^\circ = 36.9^\circ$

Again $(6I)^2 + (8I)^2 = (227.7)^2 \Rightarrow I = 22.8\ A$ (upper Δ)

$\dfrac{X_C}{R_2} = \tan 36.9^\circ = 0.75 \Rightarrow X_c = 0.75 R_2$

$(IR_2)^2 + (IX_c)^2 = (75.9)^2$ (lower Δ)

$\Rightarrow R_2 = 2.664\ \Omega$ and $X_c = 0.75 \times 2.664 = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$\Rightarrow C = 1.594$ mF

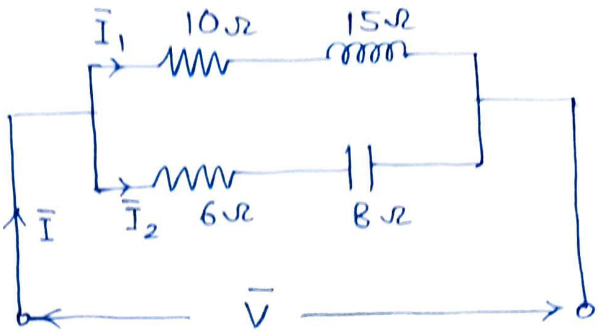
Ex

$|\bar{I}| = 15A$

Calculate (i) $\bar{I}_1$ (ii) $\bar{I}_2$

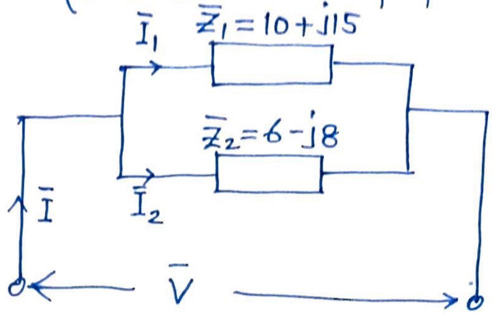
(iii) $\bar{V}$

Draw the phasor diagram.



Sol^n.
$\bar{Z}_1 = 10 + j15$, $\bar{Z}_2 = 6 - j8$

Assume that $\bar{I} = I\angle 0^\circ = 15\angle 0^\circ$ (current is ref. phasor)



$\bar{I} = \bar{I}_1 + \bar{I}_2$

According to current div^n method

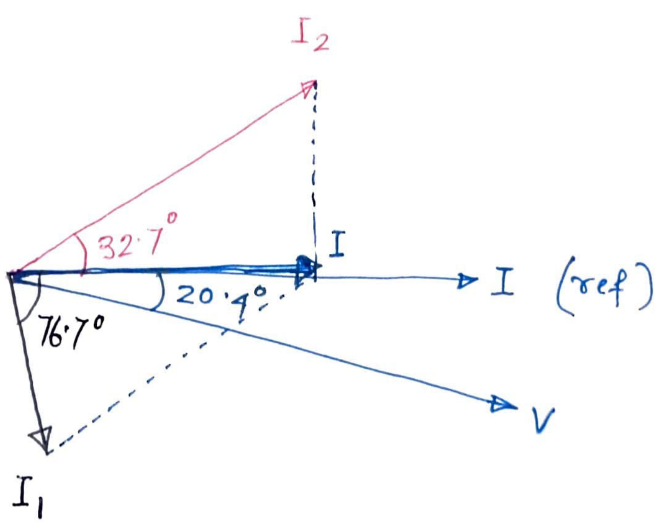
$$\bar{I}_1 = \frac{\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} \times \bar{I}$$

$$\bar{I}_2 = \frac{\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2} \times \bar{I}$$

$$\therefore \bar{I}_1 = \frac{6 - j8}{10 + j15 + 6 - j8} \times 15\angle 0^\circ = 8.59\angle -76.7^\circ \text{ A}$$

lly, $\bar{I}_2 = 15.49\angle 32.7^\circ$

And
$$\bar{V} = \bar{I}_1 \bar{Z}_1 = \bar{I}_2 \bar{Z}_2$$
$$= (8.59\angle -76.7^\circ)(10 + j15)$$
$$= (8.59\angle -76.7^\circ)(18.03\angle 56.3^\circ)$$
$$= 154.9\angle -20.4^\circ \text{ V}$$



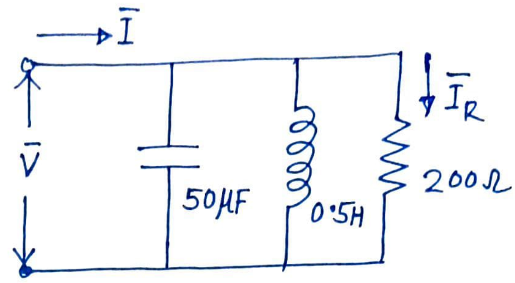
Ex. In the ckt find

(i) $\bar{I}$ if $\bar{I}_R = 0.02\angle 30^\circ$ A

(ii) $\bar{I}_R$ if $\bar{I} = 2\angle -40^\circ$ A

(iii) Applied voltage

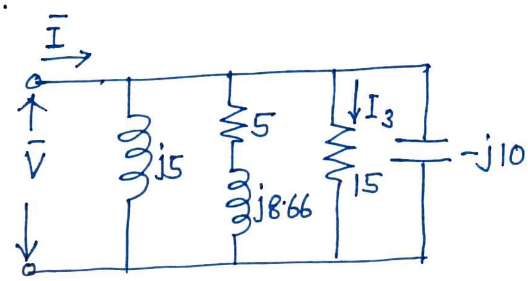
Also draw phasor diagram.



[Ans. $\bar{I} = 0.02\angle 30^\circ$ A, $\bar{I}_R = 2\angle -40^\circ$ A, $\bar{V} = 400\angle -40^\circ$ V]

Ex. In the above ckt find admittance, $\bar{Y}_T$ (equivalent), $\bar{Y}_1$, $\bar{Y}_2$, $\bar{Y}_3$ and solve the above problem.

Ex. Compute the equivalent impedance $\bar{Z}_{eq}$ and admittance $\bar{Y}_{eq}$ for the four branch ckt.



[Ans. $\bar{Z}_{eq} = 4.53\angle 58.0^\circ$

$\bar{Y}_{eq} = 0.221\angle -58.0^\circ$]

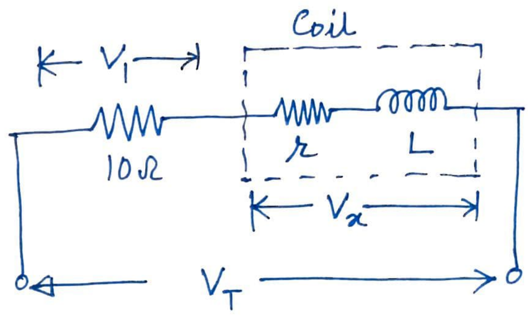
If the total current I entering the ckt is $33\angle -13^\circ$ find branch current $\bar{I}_3$ and voltage $\bar{V}$.

[Ans. $\bar{V} = 149.5\angle 45^\circ$ V, $\bar{I}_3 = 9.97\angle 45^\circ$ A]

Ex. In the given ckt we have measured V_1, V_T and V_x using a voltmeter

$V_1 = 20$V, $V_T = 36$V, $V_x = 22.4$V

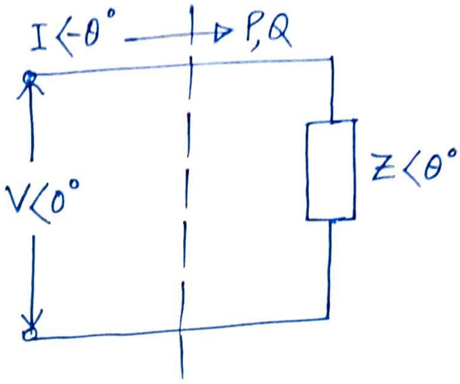
Find r and L.



[Ans. $r = 4.92\,\Omega$, $L = 26.7$ mH]

Power in sinusoidal steady state

A single-phase AC source supplying a load of impedance $Z\angle\theta$



Let $v(t) = \sqrt{2}V\sin\omega t \Rightarrow \bar{V} = V\angle 0^\circ$

$i(t) = \sqrt{2}\,I\sin(\omega t-\theta) \Rightarrow \bar{I} = I\angle -\theta$

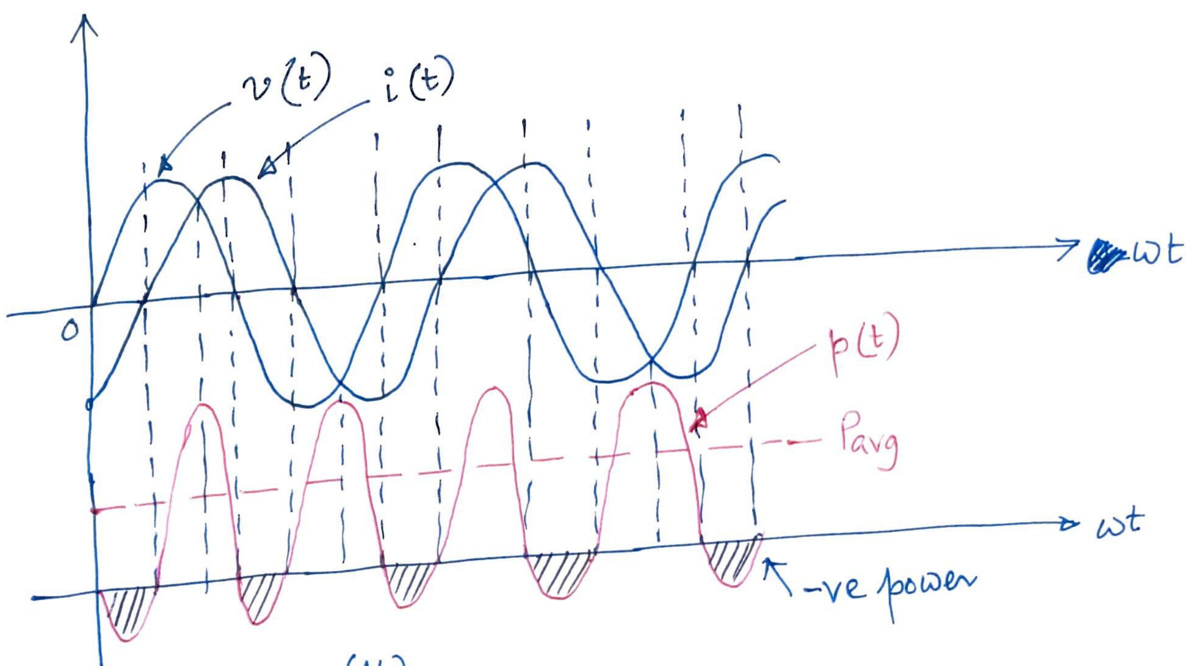
$\bar{Z} = Z\angle\theta \qquad \bar{I} = \dfrac{\bar{V}}{\bar{Z}}$

The instaneons power delivered to the load is given by

$$p(t) = v(t)\,i(t) = 2VI\sin\omega t\cdot\sin(\omega t-\theta)$$
$$= VI\left(2\sin\omega t\cdot\sin(\omega t-\theta)\right)$$
$$= VI\left(\cos\theta - \cos(2\omega t-\theta)\right)$$
$$= VI\cos\theta - VI\cos(2\omega t-\theta)$$

$\therefore$ Average power,

$$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T\left[VI\cos\theta - VI\cos(2\omega t-\theta)\right]dt$$
$$= \frac{1}{T}\int_0^T VI\cos\theta\,dt - \frac{1}{T}\int_0^T VI\cos(2\omega t-\theta)\,dt$$
$$= VI\cos\theta - 0$$
$$= VI\cos\theta$$



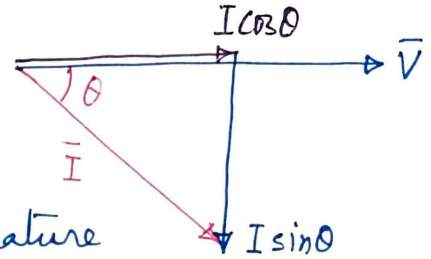
From equation

$$p(t) = VI\cos\theta - VI\cos(2\omega t - \theta)$$

we can write

$$p(t) = V\left[I\cos\theta(1-\cos 2\omega t) + I\sin\theta . \sin 2\omega t\right]$$

In the above equation we found two compts of current which are also marked in the phasor diagram.



$I\cos\theta$: current compt in phase with voltage

$I\sin\theta$: current compt in quadrature (at 90°) to voltage

The in phase compt $I\cos\theta$ is actually feeding real power or active power (average) to the load given by

$$P_{avg} = P = VI\cos\theta$$ where V = rms value of voltage, I = " " " current

$\cos\theta$ is defined as power factor.

Again the product of V & I is called apparent power (S).

$$\therefore \quad S = VI$$

Thus the power factor is defined as

$$pf = \cos\theta = \frac{\text{Active power}}{\text{Apparent power}} = \frac{VI\cos\theta}{VI}$$

The unit of active power is watt/W but the unit of apparent power is Volt-Ampere (VA).

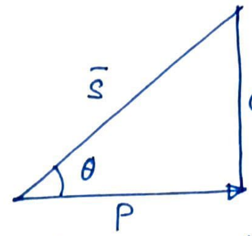
The quadrature compt $I\sin\theta$ is responsible for another power which is known as reactive power, Q.

$$Q = VI\sin\theta$$

The unit of reactive power is volt-amp-reactive (VAR).

Reactive power will be taken as positive for lagging and negative for leading power factor load. The complex power phasor diagram is shown where

$\bar{S} = P + jQ$



Complex power

It is convenient to express power in complex form.

$\bar{S} = \bar{V}\bar{I}^*$ where $\bar{I}^*$ is the conjugate of $\bar{I}$

Let $\bar{V} = V\angle 0 = Ve^{j0}$

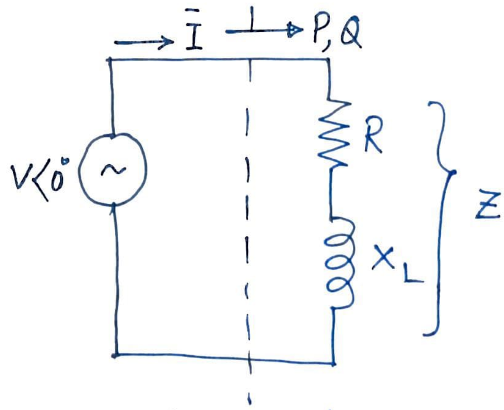
$\bar{I} = I\angle -\theta = Ie^{-j\theta}$ $\quad (\because \bar{I}$ lags $\bar{V}$ by $\theta)$

$$
\begin{aligned}
\therefore \bar{S} &= \bar{V}\bar{I}^* \\
&= Ve^{j0} . Ie^{j\theta} \\
&= VIe^{j\theta} \\
&= VI(\cos\theta + j\sin\theta) \\
&= VI\cos\theta + jVI\sin\theta \\
&= P + jQ \qquad [Q \text{ is +ve for lagging pf}]
\end{aligned}
$$

$S = \sqrt{P^2 + Q^2}$

and $pf = \cos\theta = \cos\left[\tan^{-1}\frac{Q}{P}\right]$; lagging for +ve Q
leading for −ve Q.

Consider an RL ckt where



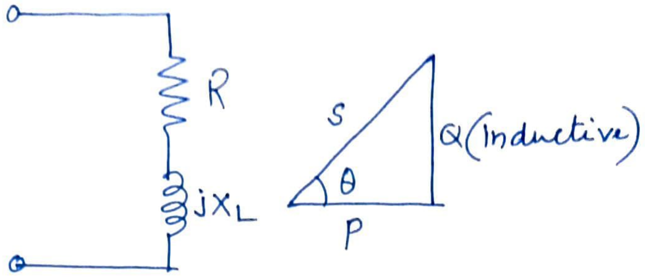
$$
\begin{aligned}
\bar{S} &= \bar{V}\bar{I}^* \\
&= \bar{I}\bar{Z}\bar{I}^* \\
&= \bar{Z}\bar{I}^2 \\
&= (R + jX_L)I^2 \\
&= RI^2 + jX_LI^2
\end{aligned}
$$

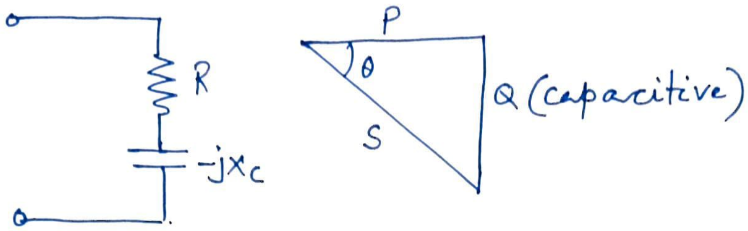
$\therefore P = RI^2$ = real power consumed in resistive element

$Q = X_LI^2$ = reactive power consumed in reactive element (inductive)

[Note: It is customary always to take Q as nonnegative. The −ve VAR is represented by VAR (capacitive)]

Power triangle





Ex. A certain passive network has equivalent impedance $\bar{Z} = 3 + j4\ (\Omega)$ and an applied voltage $v(t) = 42.5\cos(1000t + 30^\circ)$

Give complete power information.

$$\bar{V} = V\angle\theta = \frac{42.5}{\sqrt{2}}\angle 30^\circ\ (V)$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{(42.5/\sqrt{2})\angle 30^\circ}{3+j4} = \frac{(42.5/\sqrt{2})\angle 30^\circ}{5\angle 53.13^\circ} = \frac{8.5}{\sqrt{2}}\angle -23.13^\circ\ (A)$$

$$\bar{S} = \bar{V}\bar{I}^* = \left(\frac{42.5}{\sqrt{2}}\angle 30^\circ\right)\left(\frac{8.5}{\sqrt{2}}\angle 23.13^\circ\right)$$

$$= 180.6\angle 53.13^\circ = 108.4 + j144.5$$

$$= P + jQ$$

$\therefore$ Active power, $P = 108.4$ W,

Reactive power, $Q = 144.5$ VAR, (inductive)

Apparent " $S = 180.6$ VA

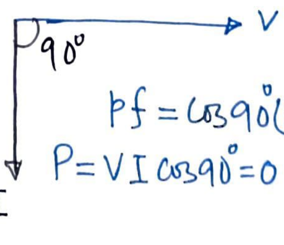
and power factor, $pf = \cos 53.13^\circ = 0.6$ (lagging)

Ex.

Resistive ckt	Pure Inductive ckt	Pure Capacitive ckt
R	L	C

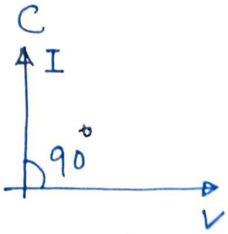
$pf = \cos 0^\circ = 1$ (unity power factor)

$P = VI\cos 0^\circ = VI$



$pf = \cos 90^\circ$ (Lag)

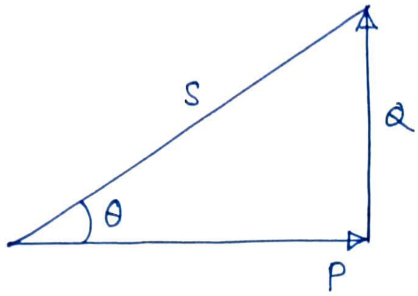
$P = VI\cos 90^\circ = 0$

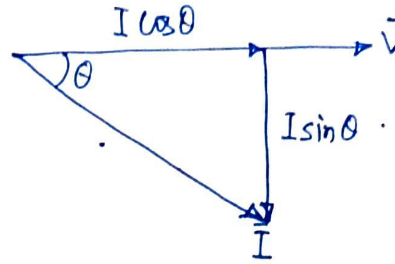


$pf = 90^\circ$ (lead)

$P = VI\cos 90^\circ = 0$

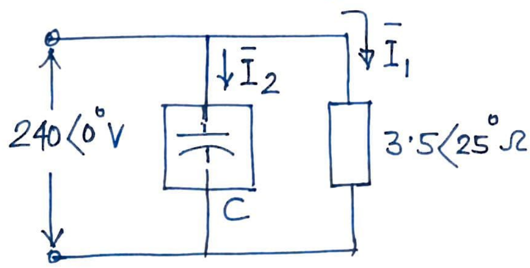
Power factor improvement

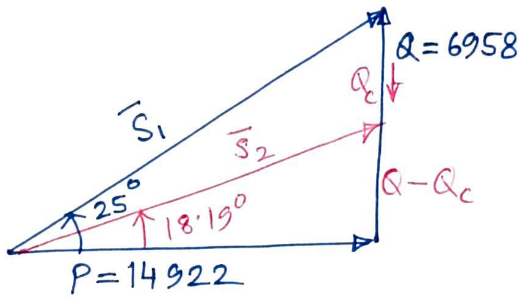




We can improve the pf by reducing Q.

<u>Ex</u>. How much capacitive Q must be provided by the capacitor bank in the ckt to improve the pf to 0.95 lagging?





Before capacitor bank is added, $pf = \cos 25^\circ = 0.906$ (lag)

$$\bar{I}_1 = \frac{240\angle 0^\circ}{3.5\angle 25^\circ} = 68.6\angle -25^\circ \text{ (A)}$$

$$\bar{S}_1 = \bar{V}\,\bar{I}_1^* = (240\angle 0^\circ)(68.6\angle 25^\circ) = 16464\angle 25^\circ$$
$$= 14922 + j\,6958$$

After the improvement, the triangle has the same P, but its angle is $\cos^{-1}(0.95) = 18.19$

Let Q is reduced by Q_c

$$\therefore \tan 18.19 = \frac{Q - Q_c}{P} = \frac{6958 - Q_c}{14922}$$

$$\Rightarrow Q_c = 2054.8 \text{ VAR (capacitive)}$$

The new value of apparent power is

$$S_2 = \sqrt{P^2 + Q^2} = \sqrt{(14922)^2 + (4904)^2} = 15707.2 \text{ VA}$$
$$\approx 15707 \text{ VA}$$

Total decrease in apparent power after improvement of power factor to 0.95 (lag) is 757 VA (around 4.6%)