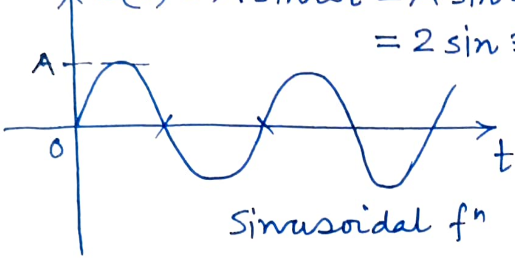
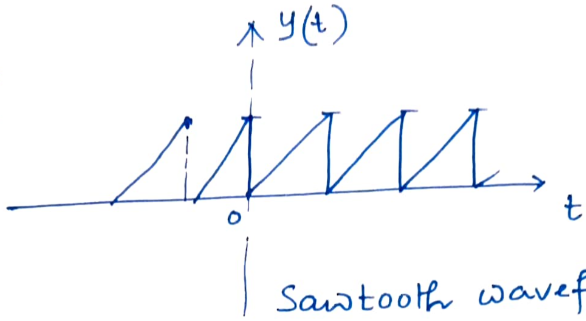
AC FUNDAMENTALS

Periodic waveforms.

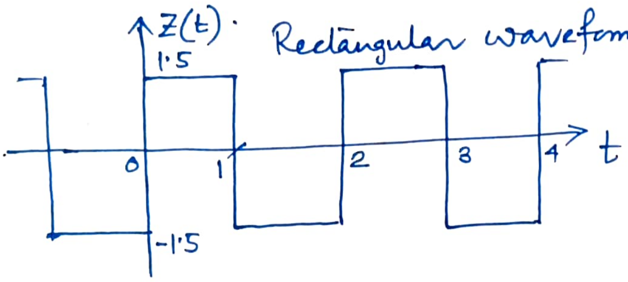
$x(t) = A\sin\omega t = A\sin 2\pi f t$
$= 2\sin 314t$



Sinusoidal f^n



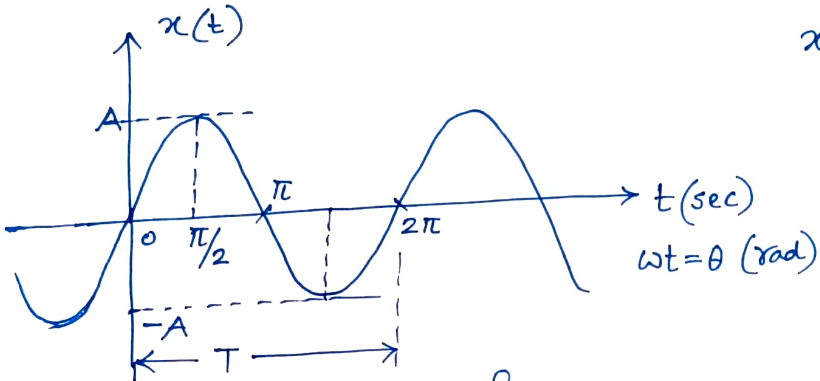
Sawtooth waveform



Rectangular waveform.

$z(t) = 1{\cdot}5, \quad 0 \leqslant t \leqslant 1$
$= -1{\cdot}5 \quad 1 \leqslant t \leqslant 2$

Sinusoidal function



$x(t) = A\sin\omega t$
$= A\sin 2\pi f t$
$= A\sin\omega t$

A = amplitude/peak
f = freq in cycles/sec or Hz
t = independent variable (time)
$\omega = 2\pi f$ = angular freq in rad/s
$= \frac{2\pi}{T}$

$f = \frac{1}{T}$, T = time period in sec

In the above waveform.

$A = 2$ (say) $\quad f = 50$ Hz

$T = \frac{1}{f} = \frac{1}{50} = 0{\cdot}02$ sec
$= 20$ ms.

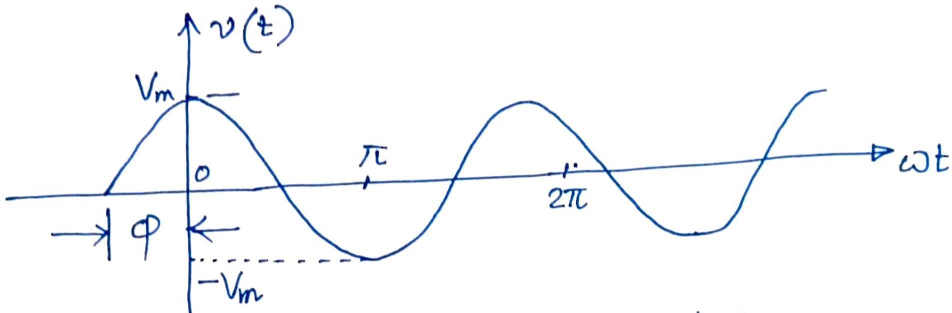
$\omega = 2\pi f = 2\pi \times 50 = 314$ rad/s.

$\therefore x(t) = 2\sin 314t$

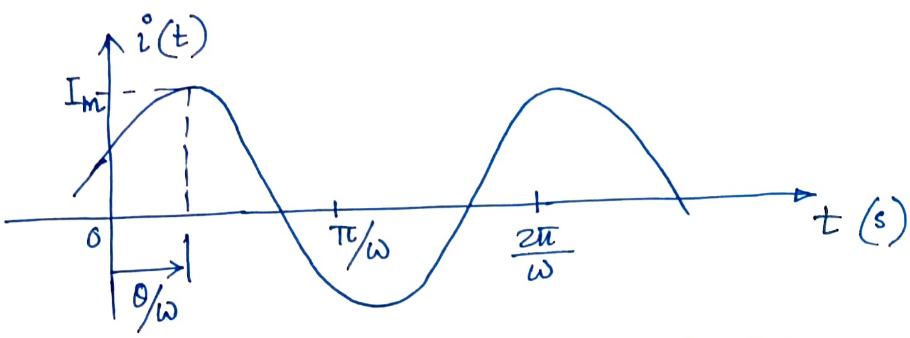
- Sinusoidal waveform has half wave and quarter wave symmetries
- Sinusoidal waveform has an associated phase which depends upon the reference time selected.

Consider a voltage sine wave, as shown in the following figure, where the maximum value is V_m and where φ, the phase angle, is the phase of the wave at $t=0$. The function may be written

$$v(t) = V_m \sin(\omega t + \varphi) \quad \text{or} \quad v(t) = V_m \cos(\omega t + \varphi - 90^\circ)$$



A current cosine wave is shown below



$$i(t) = I_m \cos(\omega t - \theta) \quad \text{or} \quad i(t) = I_m \sin(\omega t - \theta + 90^\circ)$$

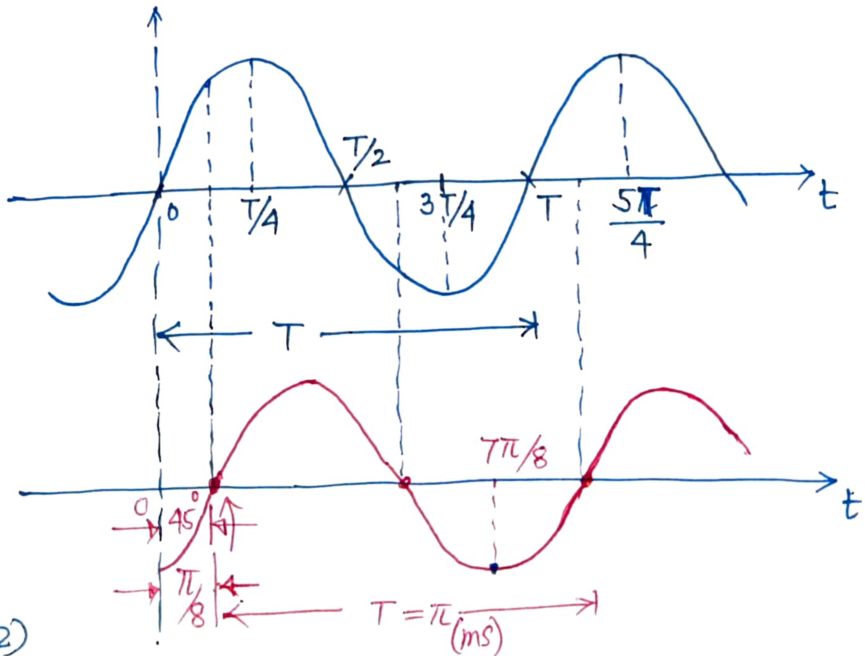
Ex. A sinusoidal current i reaches its first negative maximum, -50 mA, at $t = \frac{7\pi}{8}$ (ms) and has period $T = \pi$ (ms). Express i as a sine and as a cosine.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi \times 10^{-3}}$$

$$= 2000 \text{ rad/s}.$$

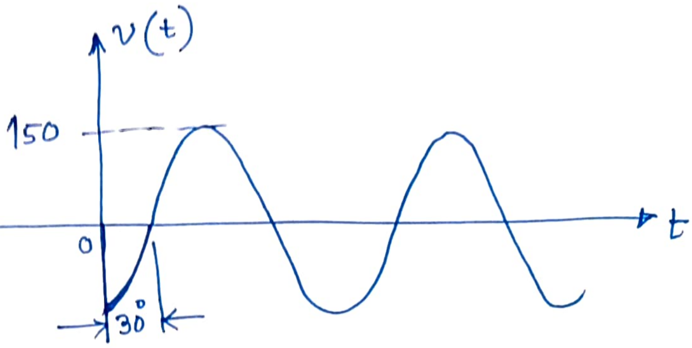
$$i(t) = 50(\text{mA}) \sin(2000t - 45^\circ)$$

$$= 50 \cos(2000t - 135^\circ)$$



Ex. For the voltage function, $v(t) = 150 \sin(500t - 30^\circ)$ (V) find (a) the voltage at $t = 2.09$ ms, (b) the times at which the positive maxima occur.

(a)



$v(t = 2.09 \text{ ms})$

$= 150 \sin\left[500 \times 2.09 \times 10^{-3} \times \frac{180^\circ}{\pi} - 30^\circ\right]$

$= 150 \sin 29.9^\circ = 74.7$ V

(b) With the phase expressed in radians, positive maxima occur when

$$500t - \frac{\pi}{6} = \frac{\pi}{2} + n\,2\pi \quad , \quad n = 0, \pm 1, \pm 2, \ldots$$

$$\Rightarrow t = 4.19(1 + 3n) \text{ (ms)}$$

Ex A voltage sine wave passes through zero at $t = 0$ and each 3.93 ms thereafter. At $t = 3.12$ ms, the voltage is 30.0 V. obtain ω, f, T and V_{max}.

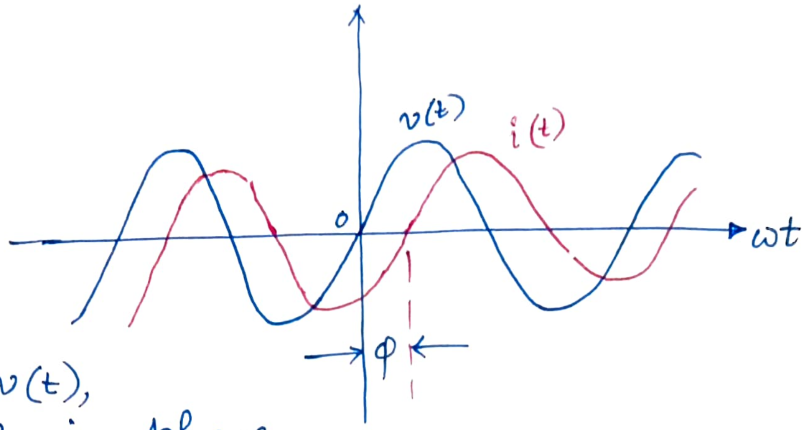
[Ans. 799 rad/s, 127.2 Hz, 7.86 ms, 49.7 V]

Ex A cosine current function has phase angle -26°, period 4.19 ms, and magnitude 1.41 mA at $t = 0.826$ ms. Obtain the cosine function. [Ans. $2 \cos(1500t - 26^\circ)$]

Phase difference

$v(t) = V_m \sin \omega t$

$i(t) = I_m \sin(\omega t - \varphi)$



Waveform of $i(t)$ is displaced in time or angle from that of $v(t)$, i.e. $v(t)$ and $i(t)$ differ in phase.

Here the phase difference is φ

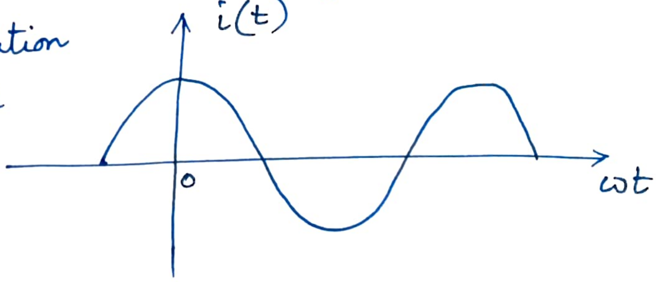
As the $v(t)$ reaches maximum earlier than $i(t)$ so the current $i(t)$ lags behind the voltage $v(t)$.

In other words, $v(t)$ is leading the current $i(t)$. The leading or lagging angle is φ and it is phase difference.

RMS (effective) value

In specifying a varying voltage or current, the root mean square (rms) value of the alternating voltage or current is used in practice to specify the quantity.

General expression for calculation of rms value of a periodic wave is



$$I_{eff} = I_{rms} = \sqrt{\text{average } i^2(t)}$$

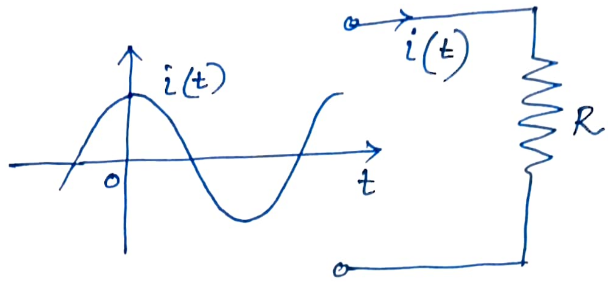
$$= \sqrt{\left(\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}\right)} \quad \text{(considering one cycle)}$$

General expression for the average value of current is

$$I_{av} = \frac{i_1 + i_2 + \cdots + i_n}{n}, \quad \text{over positive half cycle}$$

RMS, Average value of sinusoidal waveform.

Let $i(t) = I_m \cos \omega t$ is a current passing through a resistance R



Instantaneous power dissipation in R

$$p(t) = i^2(t) \cdot R$$

Average power dissipation over one cycle (time period T) is

$$P = \frac{\int_0^T i^2(t) R \, dt}{T} = I^2 R$$

$$\therefore \; I^2 = \frac{\int_0^T i^2(t) \, dt}{T} \Rightarrow I = \sqrt{\frac{1}{T} \int_0^T i^2(t) \, dt}$$

This value of current is called as the RMS value of the current.

$$\therefore \; I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

$$= \sqrt{\frac{1}{T} \int_0^T I_m^2 \cos^2 \omega t \, dt}$$

$$= \sqrt{\frac{1}{T} \int_0^T \frac{I_m^2}{2} \left(2\cos^2 \omega t\right) dt}$$

$$= \sqrt{\frac{1}{T} \int_0^T \frac{I_m^2}{2} \left(1 + \cos 2\omega t\right) dt}$$

$$= \sqrt{\frac{1}{T} \int_0^T \frac{I_m^2}{2} dt + \frac{1}{T} \int_0^T \cos 2\omega t \, dt}$$

$$= \frac{I_m}{\sqrt{2}}$$

The average value of sinusoidal waveform is taken over positive half cycle

$$I_{avg} = \frac{2 I_m}{\pi}$$

Form Factor and Peak Factor

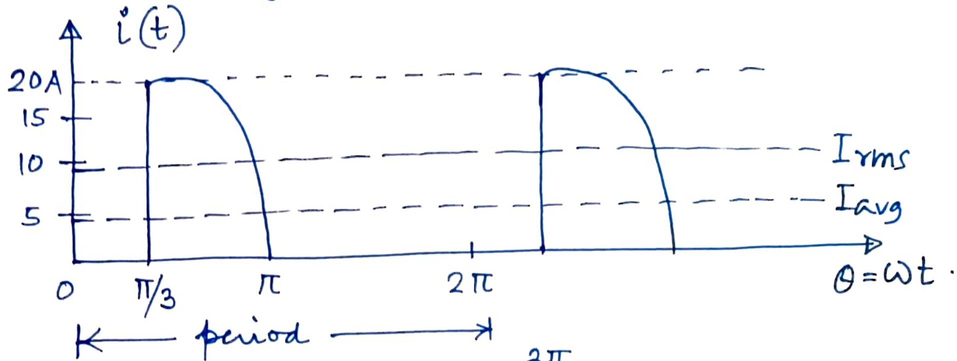
$$\text{Form Factor} = \frac{I_{rms}}{I_{avg}} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = 1.11 \text{ for sinusoidal wave.}$$

$$\text{Peak Factor} = \frac{I_{max}}{I_{rms}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414 \quad ''$$

Observation:

An alternating current will deliver to a resistance the same power as a direct current of value equal to the rms (or effective) value of the alternating current.

Ex. Calculate average and rms value of current for the following waveform.



$$I_{avg} = \frac{1}{T}\int_0^T i(t)\,dt = \frac{1}{2\pi}\int_0^{2\pi} 20\sin\omega t\, d(\omega t)$$

$$= \frac{1}{2\pi}\int_{\pi/3}^{\pi} 20\sin\omega t\, d(\omega t) = 4.775\text{ A}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} 20^2\sin^2\omega t\, d(\omega t)}$$

$$= 8.97\text{ A}$$

Phasor representation :-

Euler's identity : $e^{j\theta} = \cos\theta + j\sin\theta$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Using the above

$$Ae^{j(\theta+\omega t)} = A\cos(\omega t+\theta) + jA\sin(\omega t+\theta)$$

So, $\mathrm{Re}\left[Ae^{j(\theta+\omega t)}\right] = A\cos(\omega t+\theta)$

= Representing a sinusoidal qty.

$\mathrm{Im}\left[Ae^{j(\theta+\omega t)}\right] = A\sin(\omega t+\theta)$

= Representing a sinusoidal qty

Assuming "A" as rms quantity

$$A\cos(\omega t+\theta) = \mathrm{Re}\left[Ae^{j\theta}\cdot e^{j\omega t}\right]$$

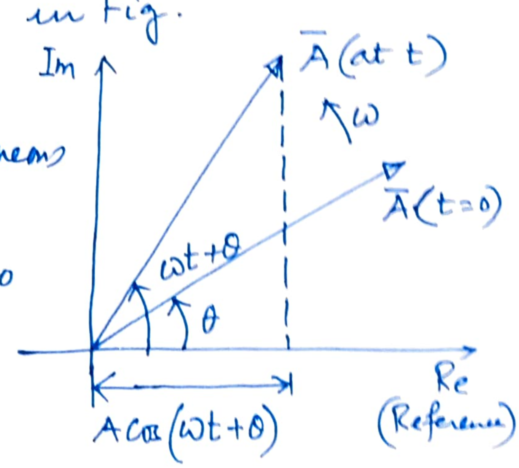
$$= \mathrm{Re}\left[\bar{A}\, e^{j\omega t}\right]$$

where $\bar{A} = Ae^{j\theta} = A\angle\theta$ = phasor

The expression $e^{j\omega t}$ imparts rotation to the phasor in the complex plane, as shown in Fig.

The projection of the phasor tip on the real axis yields the instantaneous value of the original sine wave.

Similarly, this illustration can also be done using the imaginary quantity.



Ex. Express the following wave form in phasor form.

(i) $i_1(t) = 2\sqrt{2}\cos(\omega t + 30^\circ)$

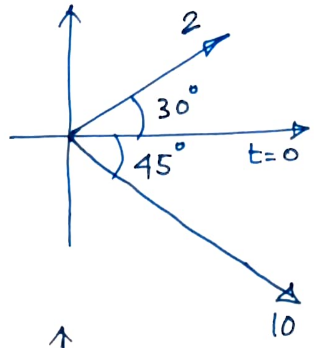
(ii) $i_2(t) = 10\sqrt{2}\cos(\omega t - 45^\circ)$

(iii) $v_1(t) = 20\sqrt{2}\sin\omega t$

(iv) $v_2(t) = 1.414\sin(\omega t + 60^\circ)$

Sol^n. (i) $I_m = 2\sqrt{2}$ $\quad I_{rms} = \dfrac{I_m}{\sqrt{2}} = 2$

In phasor notation, $\bar{I}_1 = 2\angle 30^\circ$

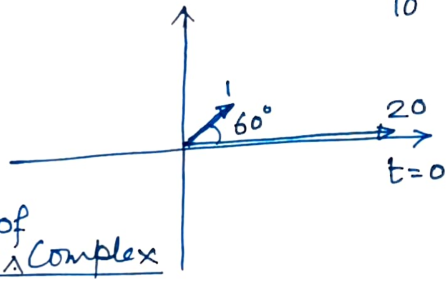


(ii) $I_m = 10\sqrt{2}$ $\quad I_{rms} = \dfrac{I_m}{\sqrt{2}} = 10$

In phasor notation, $\bar{I}_2 = 10\angle -45^\circ$

(iii) $\bar{V}_1 = 20\angle 0^\circ$

(iv) $\bar{V}_2 = 1\angle 60^\circ$



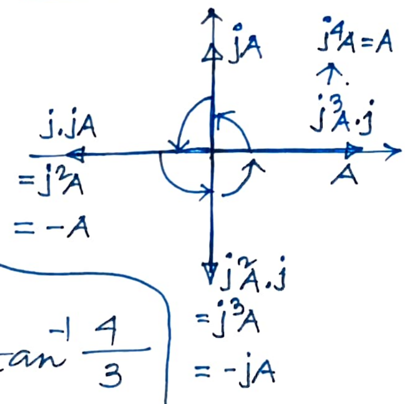
Rectangular, Polar and exponential form of complex quantity

$\bar{Z}_1 = a \pm jb$ (Rectangular form)

$\bar{Z}_2 = r\angle \pm\theta$ (Polar form)

$\bar{Z}_3 = re^{\pm j\theta}$ (Exponential form)

j-operator



Conversion :

$\bar{Z}_1 = 3 + j4 \Rightarrow |z| = \sqrt{3^2 + 4^2}, \ \theta = \tan^{-1}\dfrac{4}{3}$

$\bar{Z}_1 = 5\angle \tan^{-1}\left(\dfrac{4}{3}\right)$

So, $\bar{Z}_1 = 3 + j4$ is equivalent to $5\angle \tan^{-1}\dfrac{4}{3}$.

* For addition and subtraction keep compts in rectangular
* For multiplication and division keep compts in polar.

Ex: $\bar{Z}_1 = a + jb$, $\bar{Z}_2 = c - jd$ $\quad \therefore \bar{Z}_1 + \bar{Z}_2 = (a+c) + j(b-d)$

$\bar{Z}_1 - \bar{Z}_2 = (a-c) + j(b+d)$

Ex. $\bar{Z}_1 = r_1\angle\theta_1$ $\quad \bar{Z}_2 = r_2\angle\theta$ $\quad \bar{Z}_1\bar{Z}_2 = (r_1\angle\theta_1)(r_2\angle\theta_2)$

$= r_1 r_2\angle(\theta_1 + \theta_2)$

$\bar{Z}_1/\bar{Z}_2 = \dfrac{r_1\angle\theta_1}{r_2\angle\theta_2} = \dfrac{r_1}{r_2}\angle\theta_1 - \theta_2$

Ex Add the following two sine waves.

$v_1(t) = 100\sqrt{2}\cos(314t - 30^\circ)$
$v_2(t) = 200\sqrt{2}\cos(314t - 60^\circ)$

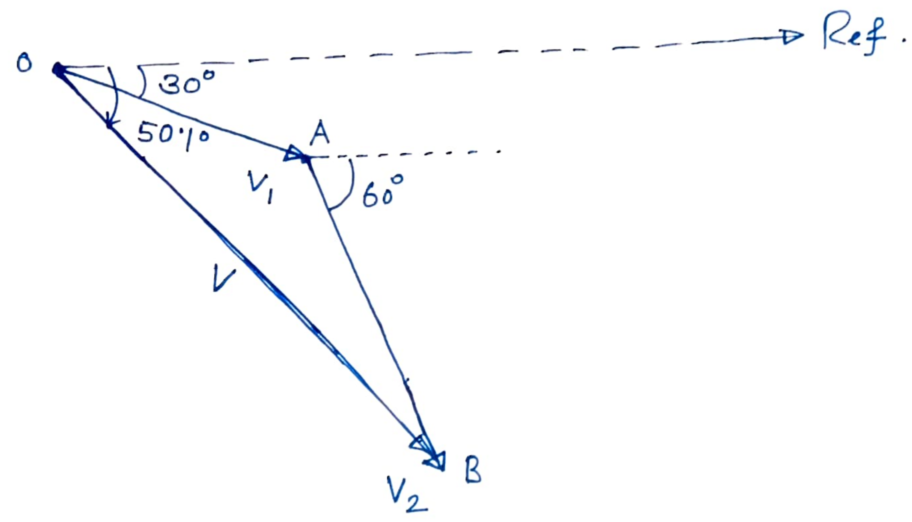
In polar form $\bar{V_1} = 100\angle{-30^\circ}$, $\bar{V_2} = 200\angle{-60^\circ}$

$= 86.6 - j50$ $\qquad = 100 - j173.2$

$\therefore \bar{V} = \bar{V_1} + \bar{V_2} = (86.6 - j50) + (100 - j173.2)$
$= 186.6 - j223.2$
$= 290.9\angle{-50.1^\circ}$

$\therefore v(t) = 290.9 \times \sqrt{2}\cos(314t - 50.1^\circ)$ //

The above problem can also be solved graphically



→ Draw V_1 at O (OA)
→ Draw V_2 at the tip of V_1 (AB)
→ Join OB
→ OB is the resultant V