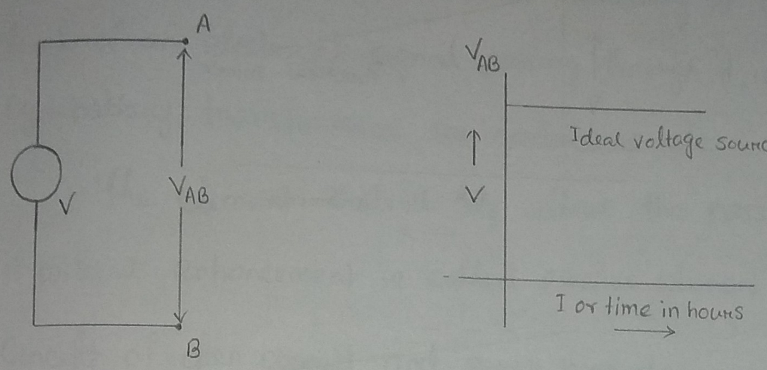
Module 1

DC circuits

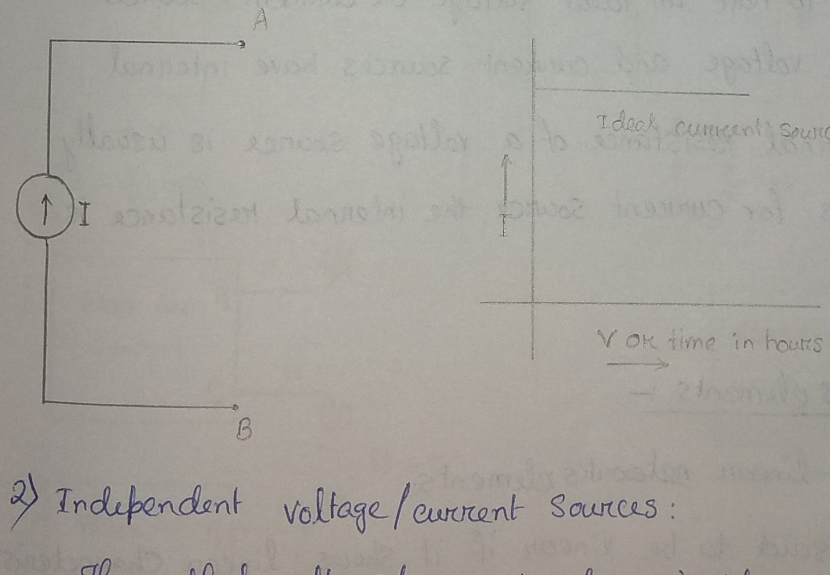
I. Voltage and current sources –

1) Characteristics of ideal voltage and current sources:

i) In an ideal voltage source, the terminal voltage of the source is constant, whereas current drawn from the source can vary depending upon the connected load. An ideal voltage source has zero internal resistance and can never be short circuited.



ii) An ideal current source delivers constant current into the circuit, whereas the voltage across the ideal current source varies depending on the circuit parameters. An ideal current source has infinite internal resistance and can never be open circuited.

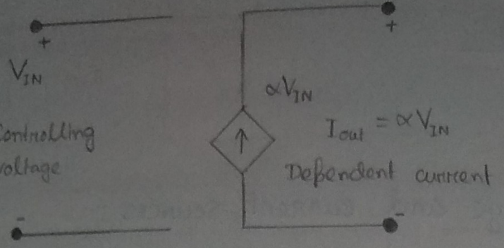


2) Independent voltage/current sources:

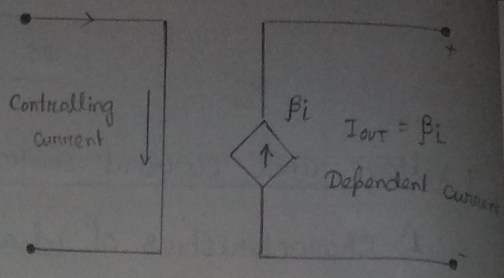
The output voltage/current of an independent voltage/current source does not depend on any circuit element.

3) Dependent or controlled sources:

The output depends on some other variables (voltage or current) in a circuit.

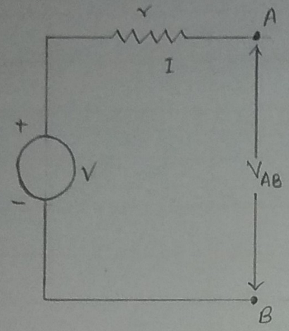


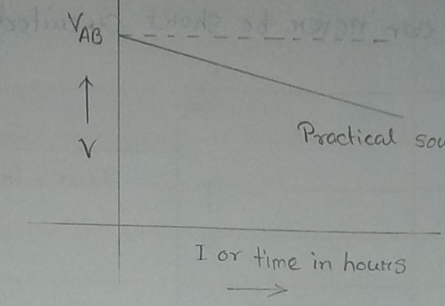
Voltage controlled current source

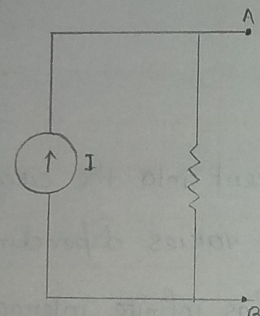


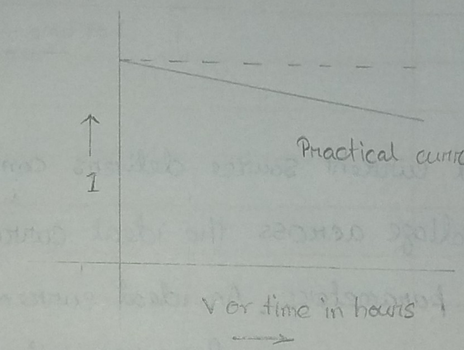
Current controlled Current Source

4) Practical voltage and current Sources









It is impossible to have an ideal voltage or current sources because all the real voltage and current sources have internal impedance. The internal resistance of a voltage source is usually very small, whereas for current source, the internal resistance is very high.

II. Types of networks elements :-

1) Linear and non-linear ~~networks~~ elements:

A ~~network~~ element is said to be linear if it shows linear characteristics of voltage vs current. Eg - Resistors, inductors, capacitors etc.

A ~~network~~ element is said to be non-linear if its ~~elements~~ parameters chang with applied voltag and current change. Eg - Semiconductor ~~diods~~ devices like diodes, transistors, thyristors etc.

A non-linear network does not follow ohm's law.

2) Unilateral and bilateral ~~elements~~ networks elements:

If the magnitude of the current passing through an element is affected due to change in polarity of the applied voltage, the ~~elme~~ network is called unilateral ~~network~~ element.

If the current magnitude remains the same even if the polarity of the applied voltage is reversed, it is called bilateral element element.

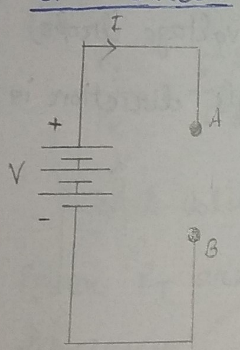
3) Active and passive networks element (Lumped and distributed networks)

If a circuit network has the capability of enhancing the energy level of an electrical signal passing through it, is called an active element. Eg - battery, transformers, semiconductors etc.

The element that simply allows the passage of the signal through it without enhancement is called passive element. Eg - resistors, inductors etc.

III. Concept of open circuit and short circuit:-

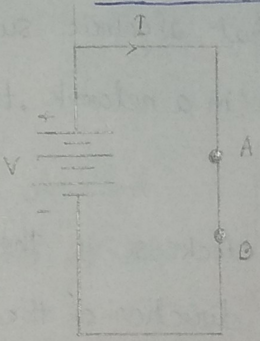
Open circuit



$R \to \infty$

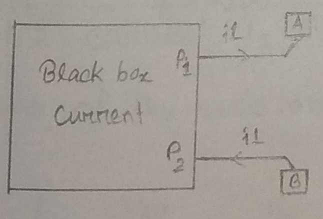
$I = \frac{V}{R} = 0$

Short circuit

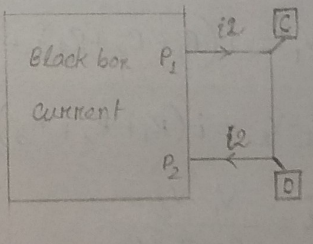


$R \to 0$

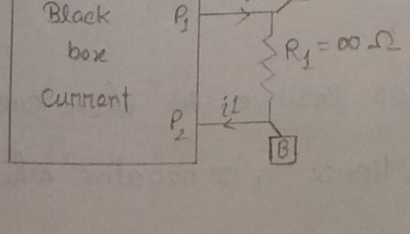
$I = \frac{V}{R} = \infty$

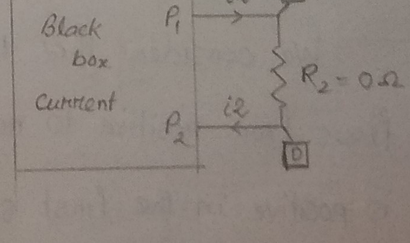


$i1 = 0$



$V_{CD} = 0$



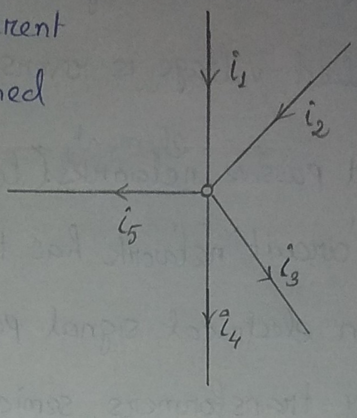


IV. Kirchhoff's law :-

1) Kirchhoff's current law (KCL)

It states that in any electrical network the algebric sum of currents meeting at any node of a circuit is zero.

Eg - Here i_1 and i_2 are the inward current towards the junction o and are assumed as negative currents. Currents i_3, i_4 and i_5 are outward current and are taken as positive. As per KCL



$$-i_1 - i_2 + i_3 + i_4 + i_5 = 0$$

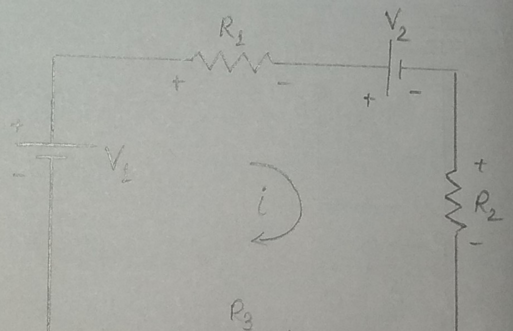
$$\Rightarrow i_1 + i_2 = i_3 + i_4 + i_5$$

∴ The algebraic sum of currents entering a node must be equal to the algebraic sum of currents leaving that node.

2) Kirchhoff's voltage law (KVL)

It states that algebric sum of voltages (or voltage drops) in any closed path, in a network, traversed in a single direction is zero.

If we travel clockwise in the network along the direction of the current, application of KVL yields.



$$-V_1 + iR_1 + V_2 + iR_2 + iR_3 = 0$$

$$\Rightarrow V_1 = i(R_1 + R_2 + R_3) + V_2$$

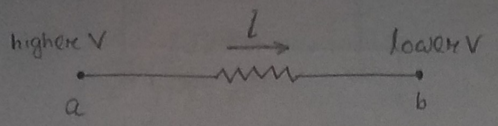
$$\Rightarrow V_1 - V_2 = i(R_1 + R_2 + R_3)$$

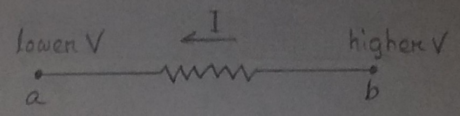
$$\Rightarrow i = \frac{V_1 - V_2}{R_1 + R_2 + R_3}$$

We consider that the voltage drop as positive as when current flows from positive to negative potential. Hence V_1 is negative while V_2 is positive in the first step of equation.

V. Sign Convention –

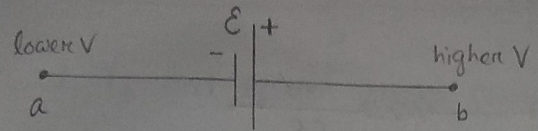
Travel direction is always from left to right.

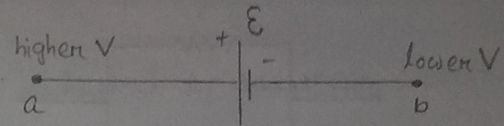




$$\Delta V = V_b - V_a = -IR$$

$$\Delta V = V_b - V_a = IR$$





$$\Delta V = V_b - V_a = +\mathcal{E}$$

$$\Delta V = V_b - V_a = -\mathcal{E}$$

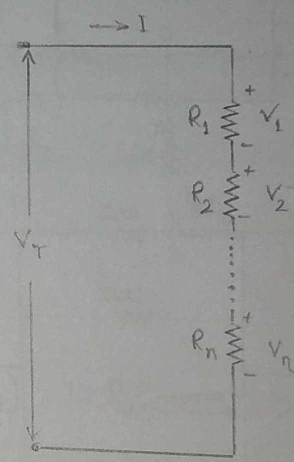
Voltage division and current division

A set of two or more series connected resistors is frequently referred to as a voltage divider.

From Ohm's law,

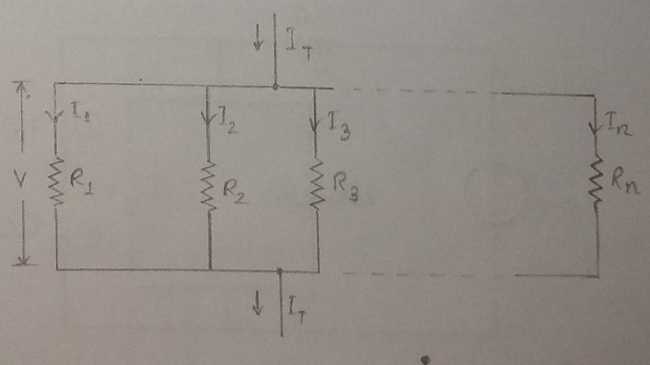
$$\frac{V_j}{V_K} = \frac{IR_j}{IR_K} = \frac{R_j}{R_K}$$

$$\frac{P_j}{P_K} = \frac{I^2R_j}{I^2R_K} = \frac{R_j}{R_K}$$



Total voltage V_T and total absorbed power P_T are divided in the ratio of the resistances

Two or more resistors on a parallel connection will devide the total current I_T and the total absorbed power P_T in the inverse ratio of the resistances.



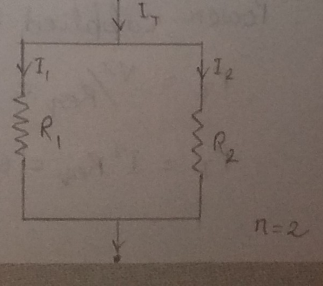
$$\frac{I_j}{I_K} = \frac{V/R_j}{V/R_K} = \frac{R_K}{R_J}$$

$$\frac{P_j}{P_K} = \frac{V^2/R_J}{V^2/R_K} = \frac{R_K}{R_J}$$

In Particular for $n = 2$

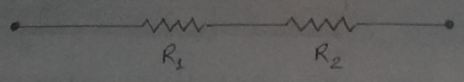
$$I_1 = \frac{R_2}{R_1 + R_2} I_T, \quad P_1 = \frac{R_2}{R_1+R_2} P_T$$

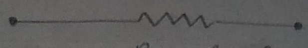
$$I_2 = \frac{R_1}{R_1+R_2} I_T, \quad P_2 = \frac{R_1}{R_1+R_2} P_T$$



Series - Parallel network reduction -

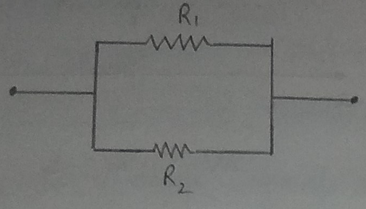
* For two series circuit elements :

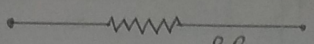




$R_{eq} = R_1 + R_2$

* For two parallel circuit elements :

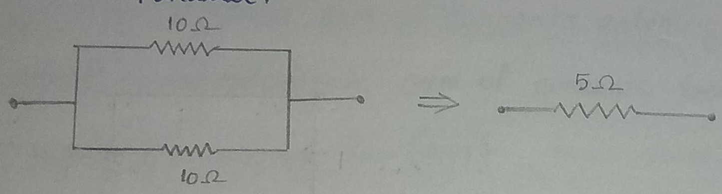




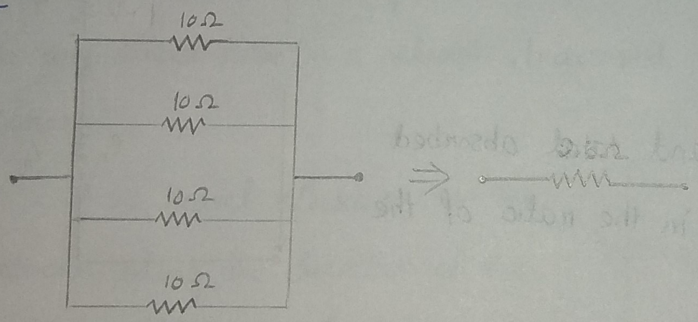
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

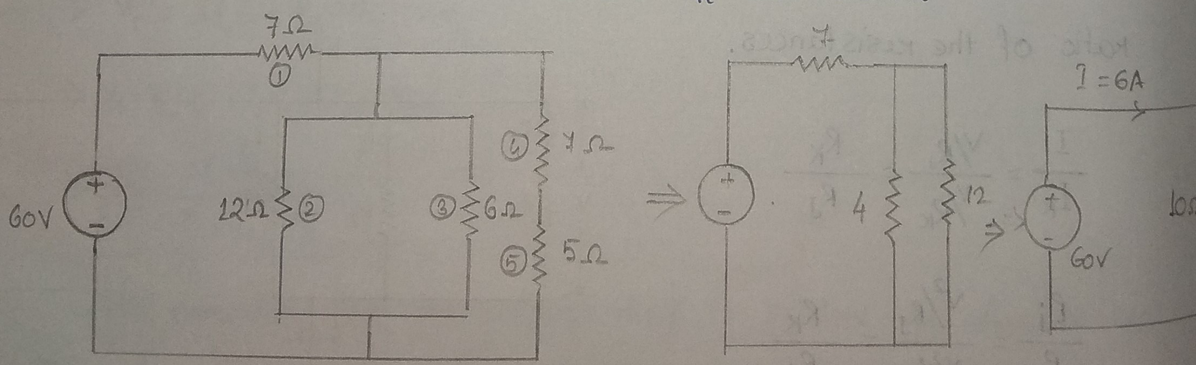
Ex 1 - What is the equivalent resistance if two 10Ω resistors connected in parallel.



Ex 2 -



Ex 3 - Obtain the total power supplied by ~~1~~60V source and the power absorbed in each resistor ~~in~~ the network.



Power supplied by the source

$P_T = V^2/R_{eq} = \frac{60^2}{10} = 360W$

$P_T = I^2 R_{eq} = 6^2 \times 10 = 360 W$

① ⇒ $7 \times 6^2 = 252 W$

② ⇒ $18^2/12 = 27 W$

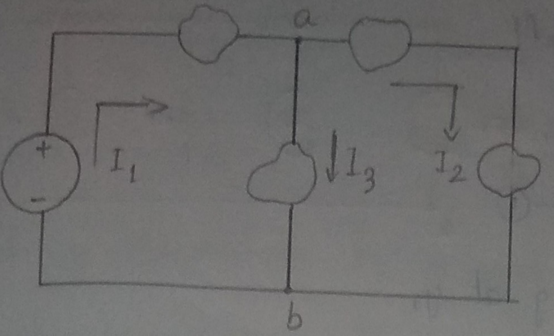
③ ⇒ $18^2/6 = 54 W$

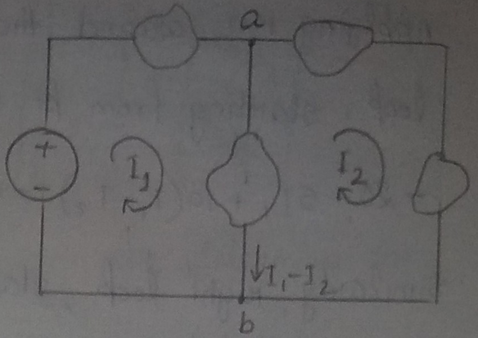
④ ⇒ $7 \times 1.5^2 = 15.75 W$

⑤ ⇒ $5 \times 1.5^2 = 11.25 W$

Power absorbed in each resistors.

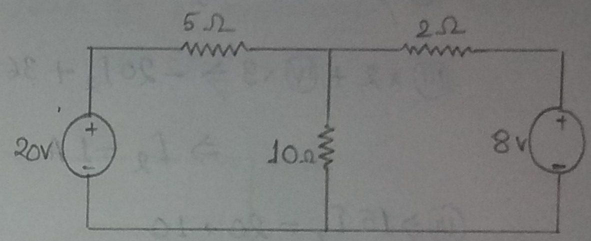
DC ~~machines~~ meshes and node analysis



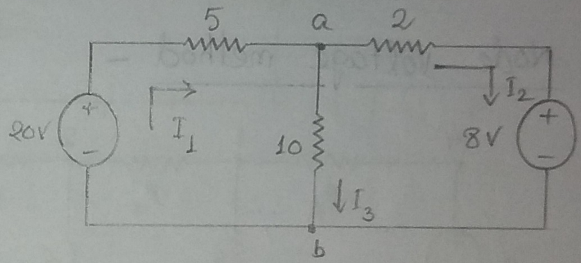


I_1, I_2 and I_3 are branch current $\qquad$ I_1 and I_2 are mesh current.

Ex - Solve the network by the (a) branch current and (b) Mesh current method.



Solⁿ a) Branch current method



Applying KCL at node 'a'

$$I_1 = I_2 + I_3 \quad \text{—— (I)}$$

Applying KVL to the left loop and right loop wrt voltage, V_{ab}

$$20 - 5I_1 = 10I_3 \quad \text{—— (II)} \Rightarrow 20 - 5(I_2 + I_3) = 10I_3 \Rightarrow 20 - 5I_2 = 15I_3 \quad \text{—— (II)}$$

$$20 - 5I_1 = 2I_2 + 8 \quad \text{—— (III)} \Rightarrow 20 - 5(I_2 + I_3) = 2I_2 + 8 \Rightarrow 12 - 7I_2 = +5I_3 \quad \text{—— (III)}$$

~~Now, (III) − (II) ⇒ $2I_2 + 8 = 10I_3$~~

$$\text{(II)} - 3\text{(III)} \Rightarrow 20 - 5I_2 - 36 + 21I_2 = 0$$

$$\Rightarrow 16I_2 = 16$$

$$\Rightarrow I_2 = 1$$

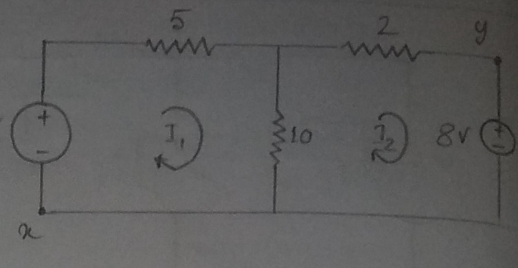
$$\text{(III)} \Rightarrow 5I_3 = 12 - 7 \times 1$$

$$\Rightarrow I_3 = \cancel{5}\ 1$$

$$\text{(I)} \Rightarrow I_1 = 1 + 1 = 2$$

$\therefore$ $I_1 = 2A$, $I_2 = 1A$, $I_3 = 1A$

b) Mesh current method



Applying KVL around the left loop, starting from pt x.

$$-20 + 5I_1 + 10(I_1 - I_2) = 0 \quad \text{—(I)}$$

Similarly, right loop, starting at y,

$$8 + 10(I_2 - I_1) + 2I_2 = 0 \quad \text{—(II)}$$

$$\text{(I)} \Rightarrow 15I_1 - 10I_2 = 20 \quad \text{—(III)} \qquad 8 - 10I_1 + \cancel{12I_2}\ 12I_2 + 8 = 0 \quad \text{—(IV)}$$

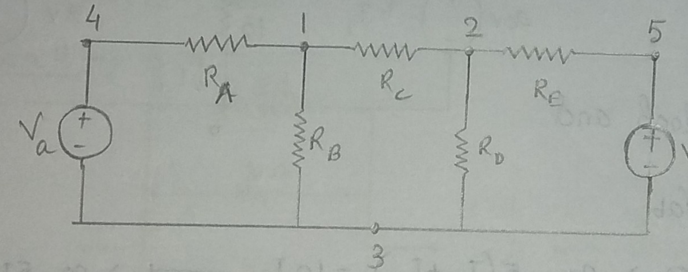
$$\text{(III)} \times 2 + \text{(IV)} \times 3 \Rightarrow -20I_2 + 36I_2 = 40 - 24$$

$$\Rightarrow I_2 = 1A$$

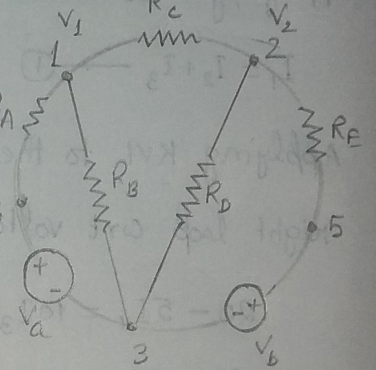
$$\text{(III)} \Rightarrow 15I_1 = 20 + 10$$

$$\Rightarrow I_1 = 2A$$

Node voltage method -



$\Rightarrow$



* Five nodes : 1, 2 and 4 are principal nodes
  : 4, and 5 are simple nodes
  : One of the principal nodes (node 3) is selected as reference node.

* Voltages are assigned ~~as~~ at principal nodes except reference node and voltages are considered w.r.t reference node.

* Applying KCL at principal nodes we have

At node 1: $$\frac{V_1 - V_a}{R_A} + \frac{V_1}{R_B} + \frac{V_1 - V_2}{R_C} = 0 \quad \text{—(I)}$$

At node 2: $$\frac{V_2 - V_1}{R_C} + \frac{V_2}{R_D} + \frac{V_2 - V_b}{R_E} = 0 \quad \text{—(II)}$$

where V_1 and V_2 are two unknowns.

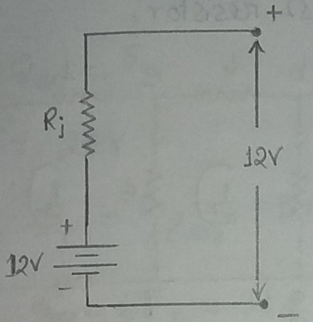
In matrix form:

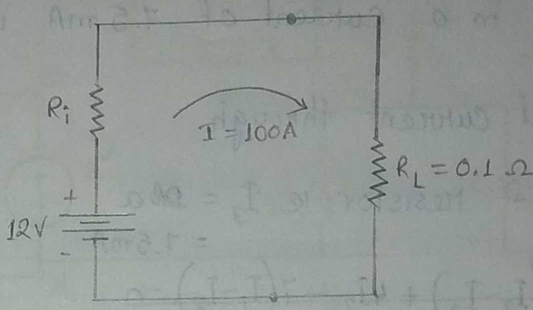
$$\begin{bmatrix} \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C} + \frac{1}{R_D} + \frac{1}{R_E} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_a/R_A \\ V_b/R_E \end{bmatrix}$$

Solving the matrix for V_1 and V_2, we can determine the ~~value~~ branch currents.

Ex 1 - Calculate the internal resistance of a battery which has an open circuit voltage 12V and delivers 100A to a resistance of 0.10Ω

Sol^n: It is a practical voltage source.





$$I = 100A = \frac{12}{0.1 + R_i} \Rightarrow R_i = 0.02\ \Omega$$

Ex 2 Measurements made on a practical DC source shows a terminal voltage of 100V for a load resistance of 100Ω and 105V for a resistance of 210Ω. Calculate the source voltage and internal resistance.

Sol^n Given $\mathcal{E}_1 = 100V$, $R_1 = 100\ \Omega$

$\mathcal{E}_2 = 105V$, $R_2 = 210\ \Omega$

Let the internal resistance be r.

From ohm's law, we say, $\mathcal{E} = I(R+r)$

$$100 = I(100 + r) \quad \text{—(i)}$$

$$105 = I(210 + r) \quad \text{—(ii)}$$

Now, $\frac{(i)}{(ii)} \Rightarrow \frac{100}{105} = \frac{100+r}{210+r}$

$\Rightarrow 21000 + 100r = 10500 + 105r$

$\Rightarrow 10500 = 5r \Rightarrow r =$

$$R_{eq} = \frac{Rr}{R+r}$$

$\mathcal{E} = I R_{eq}$

$\therefore 100 = I \cdot \frac{100r}{100+r}$ —(i)

$105 = I \cdot \frac{210r}{210+r}$ —(ii)

Now, $\frac{(i)}{(ii)} \Rightarrow \frac{100}{105} = \frac{100r(210+r)}{210r(100+r)}$ (with 2 written under 210)

$\Rightarrow 200 + 2r = 210 + r$

$\Rightarrow r = \cancel{2}\ 10$

$\therefore \cancel{V = I \frac{100 \times 10}{100+10}}$ $\quad \cancel{\therefore V = I(R+r)}$

$\cancel{= 100I +}$

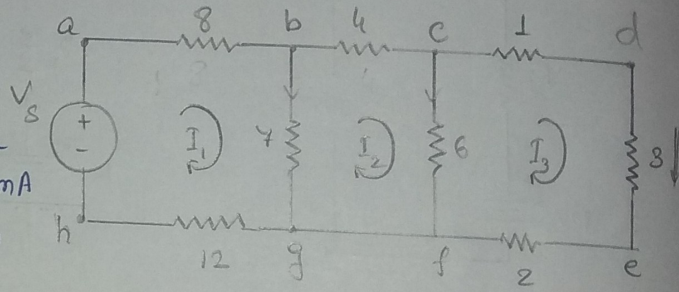
$\cancel{\Rightarrow V = \frac{100}{11} I}$

$\mathcal{E} = I R_{eq}$

$\Rightarrow I = \mathcal{E}/R_{eq} = \frac{100}{\frac{100}{11}} = \cancel{\frac{231}{20}} \cancel{A}\ 11\,A$

$V = \cancel{IR}\ Ir = 11 \times 10 = 110\,V$

Ex 3 - In the network find the source voltage V_s which results in a current of 7.5 mA in the 3 Ω resistor.



Sol^n Let current through 3 Ω resistor ie $I_3 = \cancel{}\ a$
$= 7.5\,mA$

$\cancel{6(I_3 - I_2) + 4I_2 - 7(I_1 - I_2) = 0}$

~~By applying KCL on loop bcfg, we get~~

$\cancel{6(I_3 - I_2) + 4I_2 - 7(I_1 - I_2) = 0}$

$\cancel{\Rightarrow 6I_3 + 5I_2 - 7I_1 = 0}$

$\cancel{\Rightarrow 7I_1 - 5I_2 = 6 \quad (1)}$

~~By applying KCL on loop cdef, we get~~

By applying KCL on loop cdef, we get

$2I_3 + 3I_3 + I_3 = 6(I_2 - I_3)$

$\Rightarrow 12I_3 = 6I_2$

$\Rightarrow I_2 = 2I_3 = 2a$

By applying KCL on loop bcfg, we get

$6(I_2 - I_3) + 4I_2 = 7(I_1 - I_2)$

$\Rightarrow 17I_2 = 7I_1 + 6I_3$

$\Rightarrow 34a - 6a = 7I_1 \Rightarrow I_1 = \frac{28}{7} a = 4a$

Applying KVL on loop abgh, we get,

$$12I_1 + 7(I_1 - I_2) + 8I_1 = V_S$$

$$\Rightarrow V_S = 27I_1 - 7I_2$$

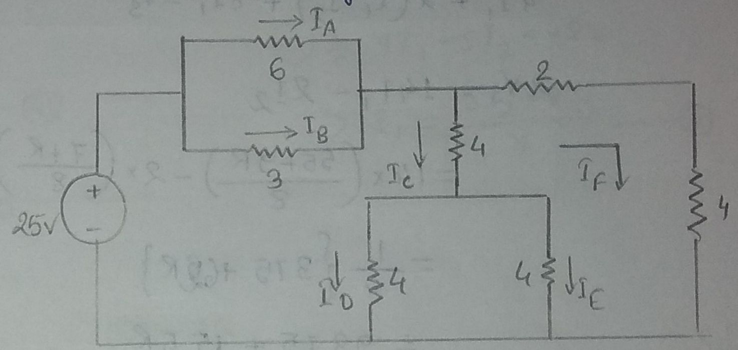
$$= 108a - 14a$$

$$= 94a$$

$$= 94 \times 7.5 \times 10^{-3}\ V$$

$$= 0.705\ V.$$

Ex 4 - Obtain the current in each resistor using network reduction method.



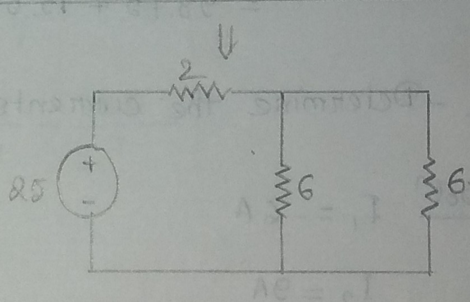
Sol^n $R_{eq} = 5$

$V = 25V$

$\therefore I_T = V/R_{eq} = 5A$

$6I_A = 3I_B$

$\Rightarrow I_B = 2I_A$



$\therefore I_B = \frac{5}{3}\ A$

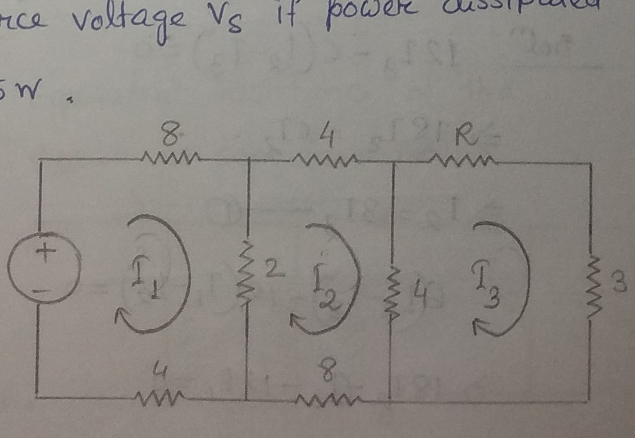
$I_A = \frac{10}{3}\ A$

$I_C = I_F = \frac{5A}{2} = 2.5A$

$I_D = I_E = \frac{5}{4}\ A = 1.25\ A.$

Ex-5 - Find the value of the source voltage V_S if power dissipated in the 3Ω resistor is 0.75 W.

Doubt.



Sol^n A/Q $3I_3^2 = P_{3\Omega} = 0.75\ W$

$$\Rightarrow 3I_3^2 = 0.75$$

$$\Rightarrow I_3^2 = 0.25$$

$$\Rightarrow I_3 = 0.5\ A = \frac{1}{2}\ A$$

~~We will~~ By applying Kirchhoff's law on the ckt, we get

$$3I_3 + RI_3 = 4(I_2 - I_3)$$

$$\Rightarrow (7 + R)I_3 = 4I_2 \Rightarrow I_2 = \left(\frac{7+R}{8}\right)A$$

$$8I_2 + 4(I_2 - I_3) + 4I_2 = 2(I_1 - I_2)$$

$$\Rightarrow 18I_2 - 4I_3 = 2I_1$$

$$\Rightarrow 2I_1 = \frac{7+R}{8} \times 18 - 4 \times \frac{1}{2}$$

$$\Rightarrow 2I_1 = \frac{9(7+R)}{4} - 2$$

$$\Rightarrow 2I_1 = \frac{55+9R}{4}$$

$$\Rightarrow I_1 = \left(\frac{55+9R}{8}\right) A$$

And,

$$4I_1 + 2(I_1 - I_2) + 8I_1 = V_s$$

$$\Rightarrow V_s = 14I_1 - 2I_2$$

$$= 14 \times \left(\frac{55+9R}{8}\right) - 2 \times \left(\frac{7+R}{8}\right)$$

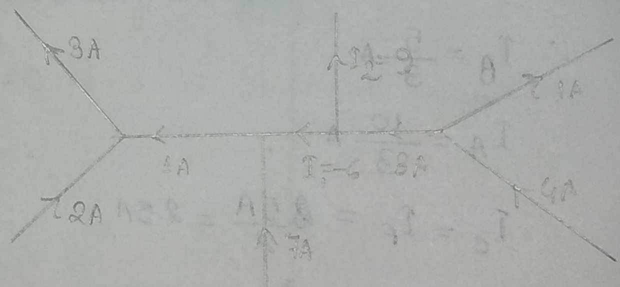
$$= \frac{1}{4}\{375 + 62R\}$$

$$\doteq 93.75 + 15.5R$$

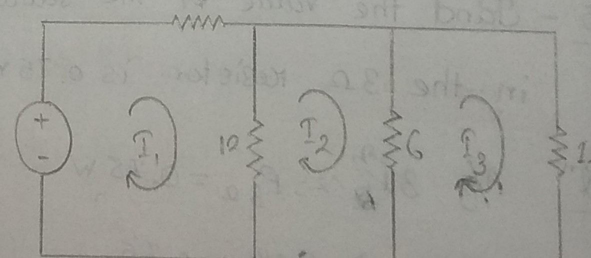
Ex-6 - Determine the currents I_1 and I_2

Sol^n $I_1 = -6A$

$I_2 = 9A$



Ex 7 - Solve the network using mesh current method



Sol^n $12I_3 - 6(I_2 - I_3) \doteq 0$

$$\Rightarrow 18I_3 = 6I_2$$

$$\Rightarrow I_2 = 3I_3 \quad \text{— (i)}$$

$$6(I_2 - I_3) - 12(I_1 - I_2) = 0$$

$$\Rightarrow 18I_2 - 6I_3 - 12I_1 = 0$$

$$\Rightarrow 12I_1 = 54I_3 - 6I_3$$

$$\Rightarrow 12I_1 = 48I_3$$

$$\Rightarrow I_1 = 4I_3 \quad \text{— (ii)}$$

$$12(I_1 - I_2) + 7I_1 = 60$$

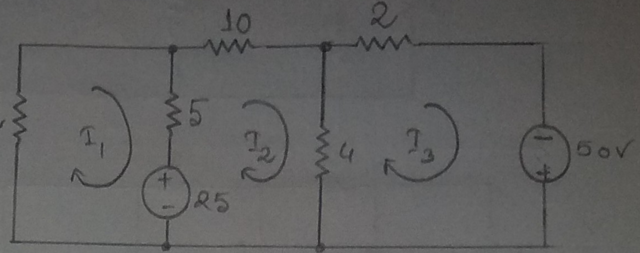
$$\Rightarrow 19 \times 4I_3 - 12 \times 3I_3 = 60$$

$$\Rightarrow 40I_3 = 60$$

$$\Rightarrow I_3 = \frac{3}{2} = 1.5A$$

$$\therefore I_1 = 4 \times \frac{3}{2} = 6A \ \& \ I_2 = 3 \times \frac{3}{2} = 4.5A$$

Ex-8 Solve the network using (a) mesh current method
(b) Node-voltage method.



Sol^n a) Mesh current method

$$-50+4(I_3-I_2)+2I_3=0$$

$$\Rightarrow 6I_3-4I_2=50 \quad \text{— (I)}$$

$$10I_2+4(I_2-I_3)+5(I_2-I_1)=25$$

$$\Rightarrow 19I_2-4I_3-5I_1=25 \quad \text{— (II)}$$

$$5(I_1-I_2)+25+2I_1=0$$

$$\Rightarrow 7I_1-5I_2=-25 \quad \text{— (III)}$$

$$\text{(I)} \Rightarrow 3I_3=25+2I_2$$

$$\Rightarrow I_3=\frac{25+2I_2}{3} \quad \text{— (IV)}$$

$$\text{(III)} \Rightarrow 7I_1-5I_2=-25$$

$$\Rightarrow I_1=\frac{5I_2-25}{7} \quad \text{— (V)}$$

~~(I) ⇒ 2I₂ = 25 3I₃ − 25~~

~~⇒ I₂ =~~

$$\text{(II)} \Rightarrow 19I_2-4\times\frac{25+2I_2}{3}-5\times\frac{5I_2-25}{7}=25$$

$$\Rightarrow\left(19-\frac{8}{3}-\frac{25}{7}\right)I_2=25+\frac{100}{3}-\frac{125}{7}$$

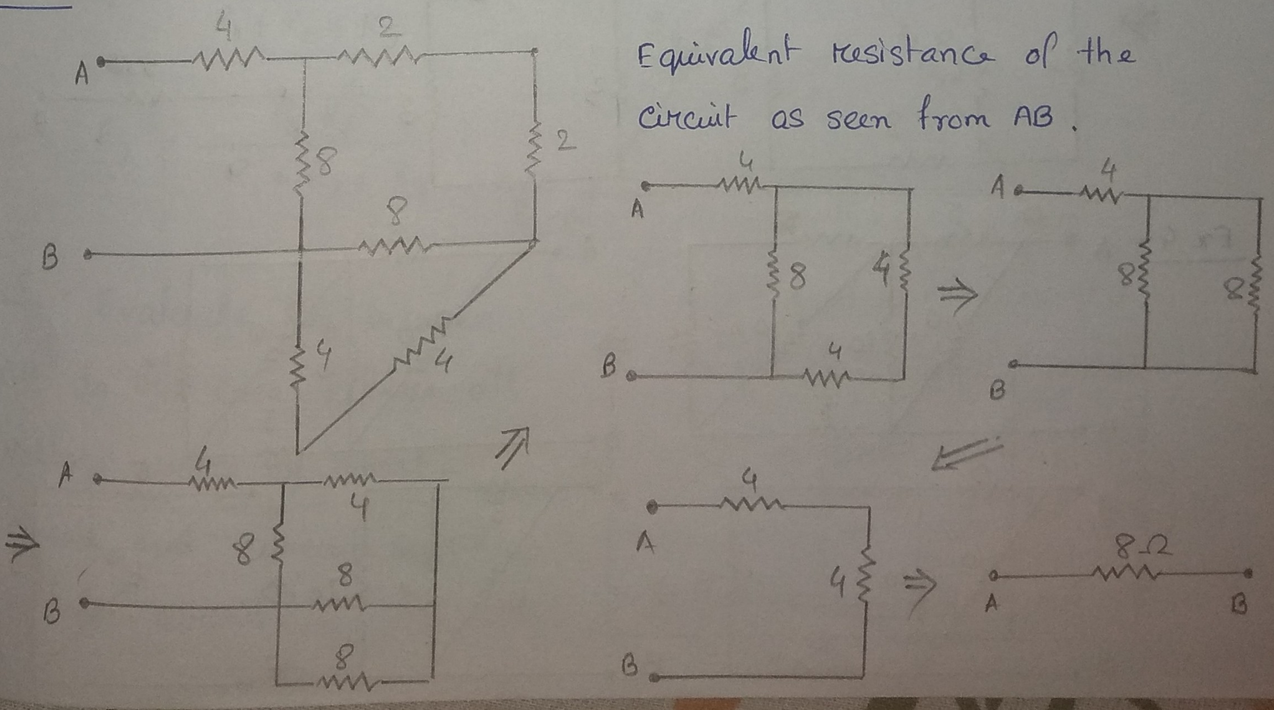
$$\Rightarrow I_2=\frac{40.4762}{12.7619}=3.17\text{ A}$$

$I_1=-1.31$ A $\qquad I_2=3.17$ A $\qquad I_3=10.45$ A.

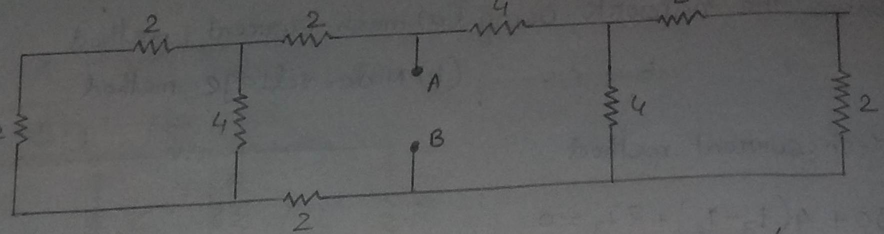
Equivalent Resistance

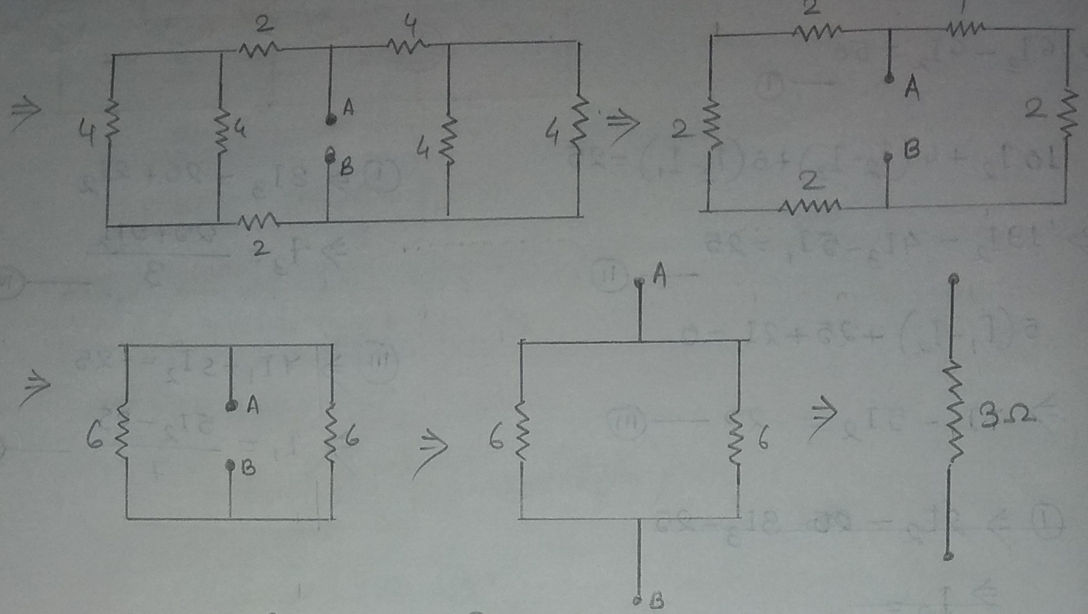
Ex 1

Equivalent resistance of the circuit as seen from AB.

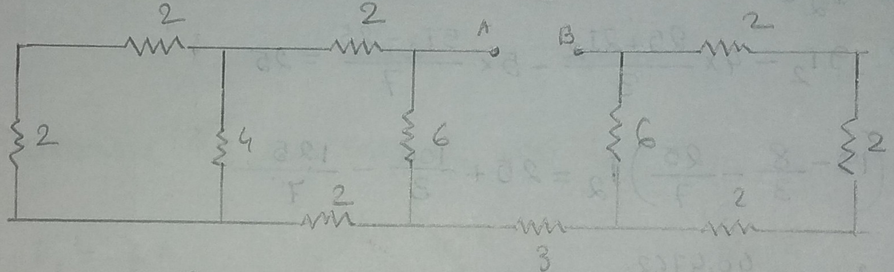


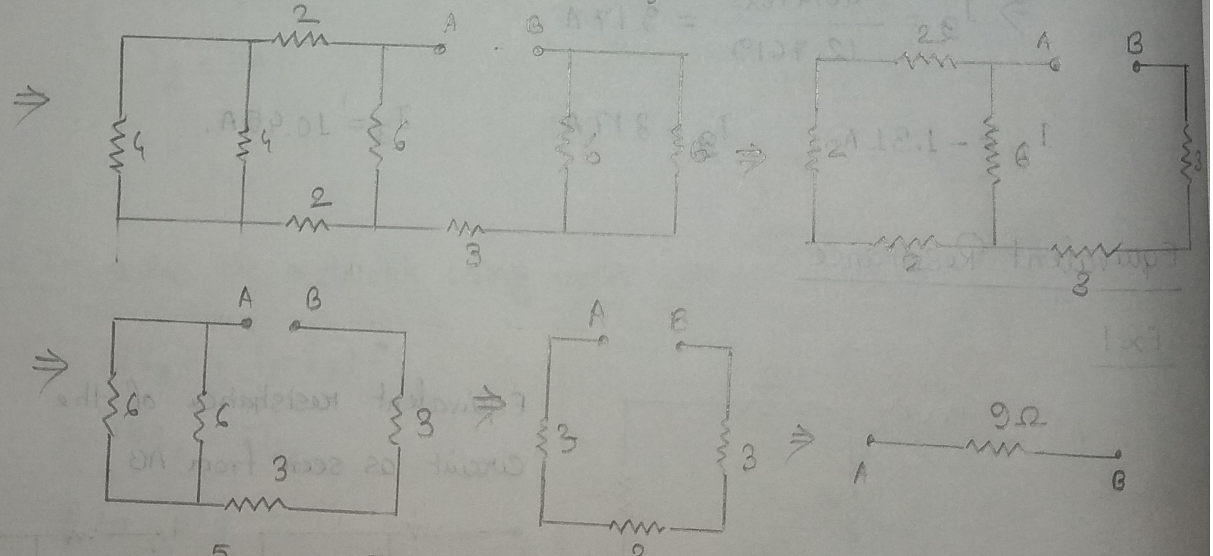
Ex 2 –



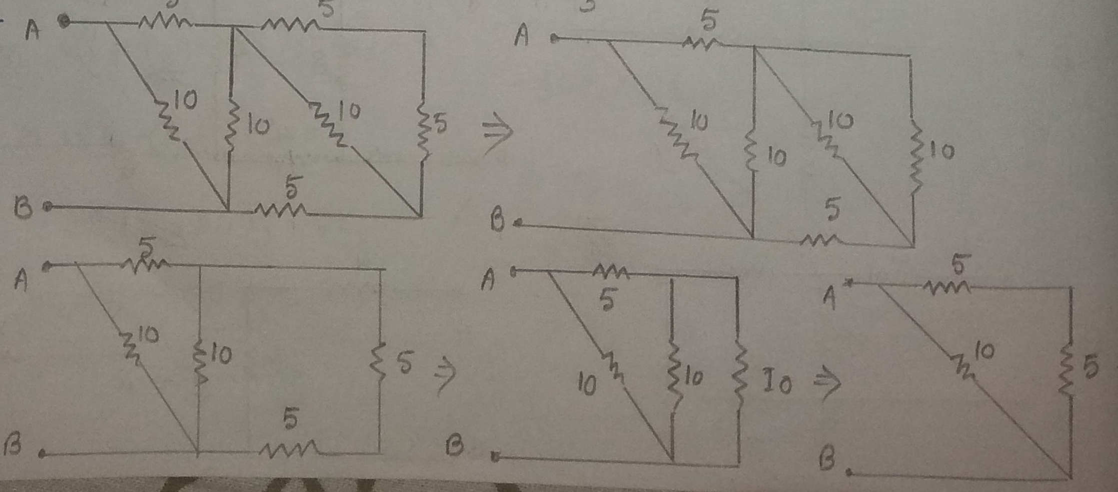


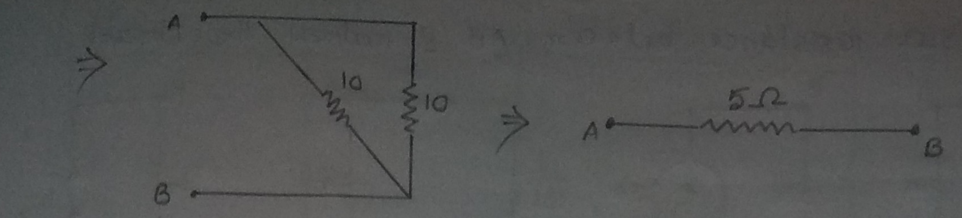
Ex 3 –



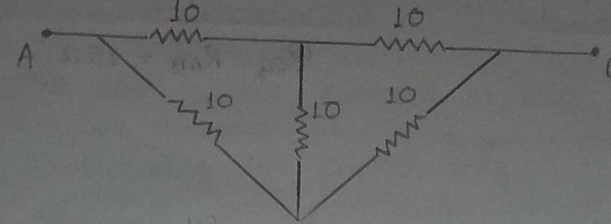


Ex 4 –

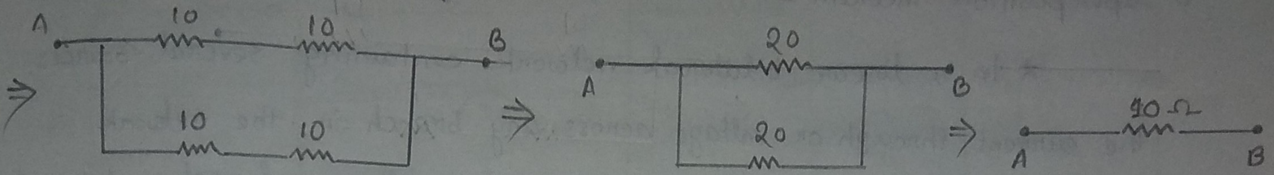




Ex-5



wheastone bridge



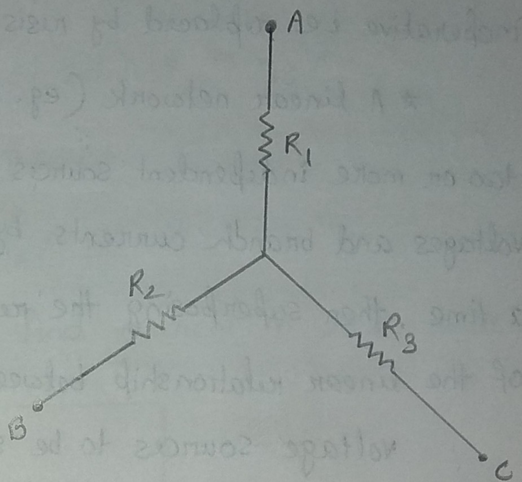
Star-Delta conversion -

Y - Δ Transformation -

$$R_1' = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_2' = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_3' = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

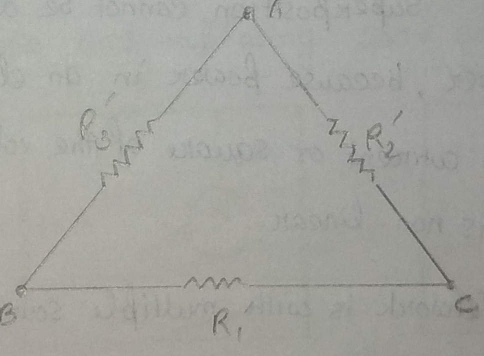


Δ - Y Transformation -

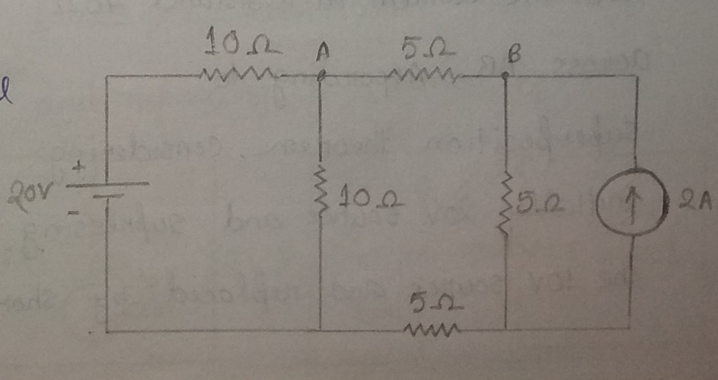
$$R_1 = \frac{R_2' R_3'}{R_1' + R_2' + R_3'}$$

$$R_2 = \frac{R_1' R_3'}{R_1 + R_2 + R_3}$$

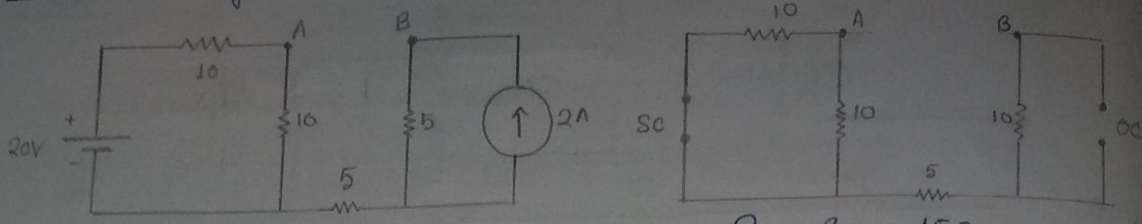
$$R_3 = \frac{R_1' R_2'}{R_1 + R_2 + R_3}$$



Ex 6 - Evaluate Req between AB. In this case, replace all voltage source by 'short-circuit' and circuit source by 'open-circuit.'



Sol^n Removing 5Ω resistance between AB & redraw the circuit.



$R_{eq} = R_{AB} = 15\Omega$

Network Theorems -

1) Superposition theorem -

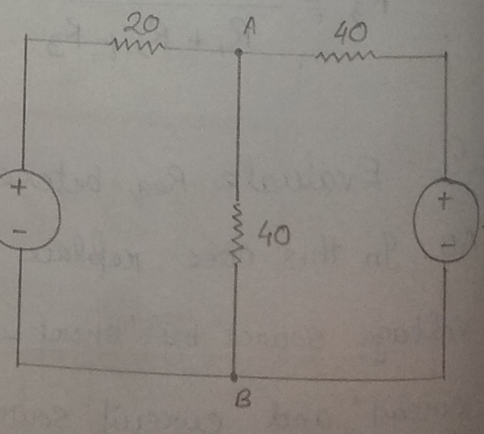
★ In a linear bilateral network containing several sources, the current through or voltage across any branch in the network equals the algebric sum of the currents or voltages of each individual source considered seperately with all other sources made ~~inappropriative~~ inoperative i.e replaced by resistances equal to their internal resistances.

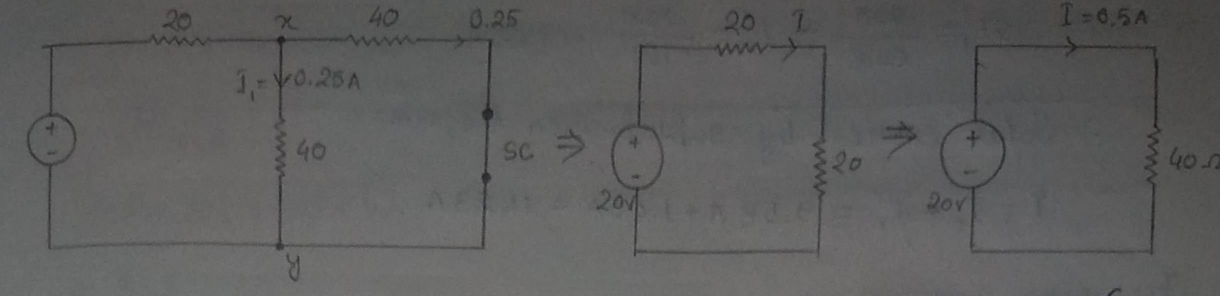
★ A linear network (eg. a DC resistive network) which contains two or more independent sources can be analyzed to obtain the various voltages and branch currents by allowing the sources to act one at a time, then superposing the results. This principal applies because of the linear relationship between current and voltage.

Voltage sources to be suppressed while a single source acts are replaced by short circuits; Current sources are replaced by open circuits.

Superposition cannot be directly applied to the computation of power, because power in an element is proportional to the square of the current or square of the voltage. (ie Power $= i^2R$ or V^2/R), which is non-linear.

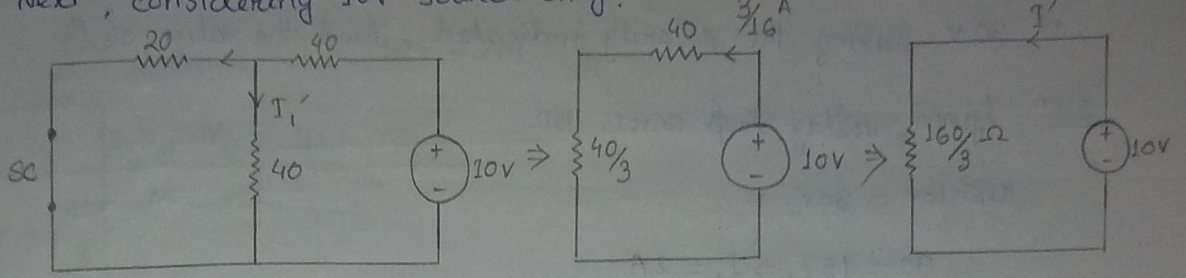
Ex- Network is with multiple sources. Determine current in resistance 40Ω across AB. According to superposition theorem, considering initially 20V source and supressing the 10V source and replaced by short circuit.





So, due to 20V source current in 40 Ω resistor is $I_1 = 0.25A$ (x to y)

Next, considering 10V source only.



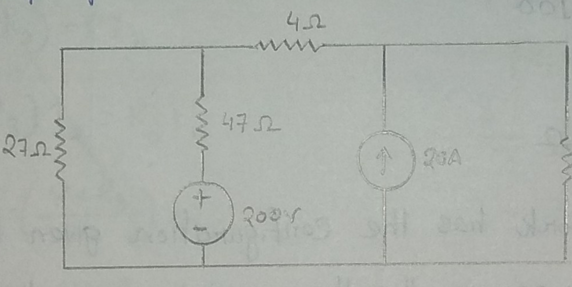
$$I' = \frac{10}{160/3} = \frac{30}{160} = \frac{3}{16}A \quad , \quad I_1' = \frac{20}{20+40} \times \frac{3}{16} = \frac{20}{60} \times \frac{3}{16} = \frac{1}{16}A$$

∴ Current in 40 Ω when 10V is considered is $\frac{1}{16}A$

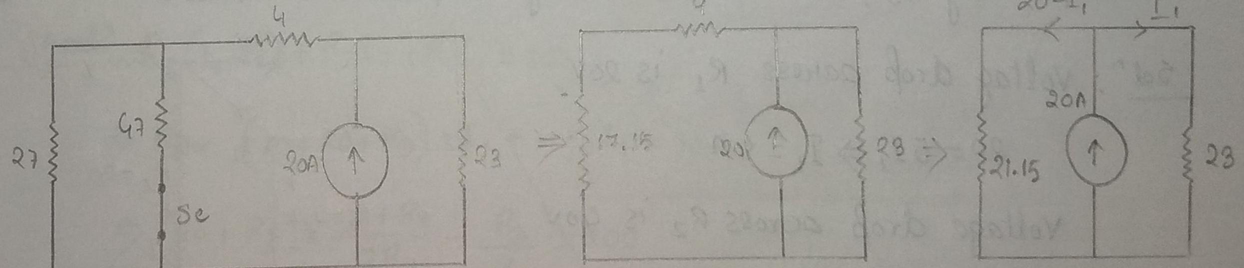
∴ Total current by superposition theorem

$$I = I_1 + I_1' = \frac{1}{4} + \frac{1}{16} = \frac{4+1}{16} = \frac{5}{16}A$$

Ex Apply superposition theorem to find current in 23 Ω resistor.

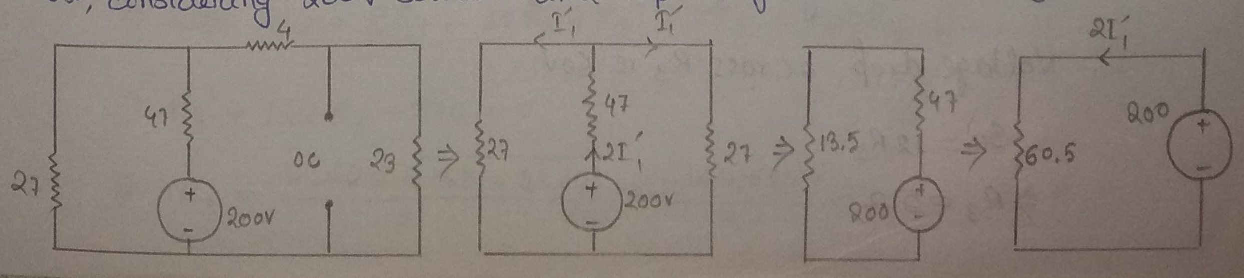


Soln Considering 20A current source and replacing 200V source by SC.



$$21.15(20 - I_1) = 23I_1 \Rightarrow 44.15 I_1 = 423 \Rightarrow I_1 = \cancel{9.64}\ 9.58A$$

Now, considering 200V source and replacing 20A source by OC.



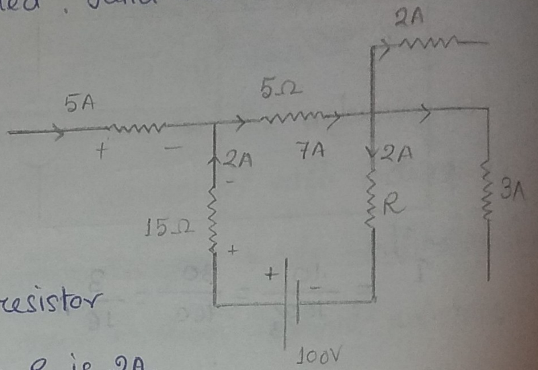
$$2I_1' = \frac{200}{60.5} \Rightarrow I_1' = \frac{200}{121} = 1.65A$$

$\therefore$ Total current by superposition theorem

$$I = I_1 + I_1' = 9.58A + 1.65A = 11.23A.$$

Few more exercise →

1) The voltage drop across the 15Ω resistance in the circuit of is 30 V having the polarity indicated. Find the value of R.



Sol^n Given, voltag drop across 15Ω resistor = 30V

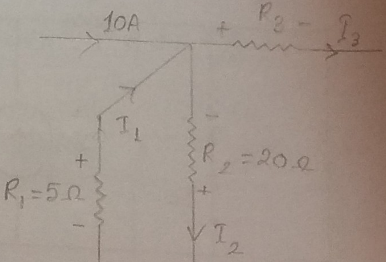
$\therefore 30 = 15I_1 \Rightarrow I_1 = 2A$

$\therefore$ Current passing through 5Ω resistor is 7A and through Resistor R is 2A.

By applying KVL, we get

$$15\times2 + 5\times7 + 2R = 100$$
$$\Rightarrow 30 + 35 + 2R = 100$$
$$\Rightarrow 65 + 2R = 100$$
$$\Rightarrow 2R = 35$$
$$\Rightarrow R = 17.5\Omega$$

2) A portion ~~hof~~ a network has the configuration given below. The voltage drops existing across the three resistors are known to be respectively 20, 40 and 60 V having the polarities indicated. Find R_3



Sol^n: Voltag drop across R_1 is 20V

$$20 = 5I_1 \Rightarrow I_1 = 4A$$

Voltage drop across R_2 is 40V

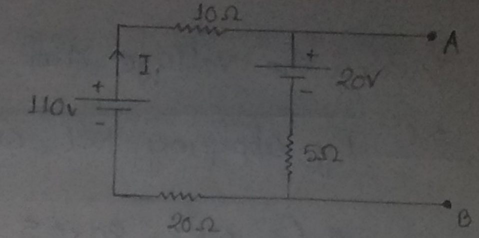
$$40 = 20I_2 \Rightarrow I_2 = 2A$$

Current through R_3 ~~is 60~~ is $I_3 = 10 + 4 - 2 = 12A$

Voltage drop across R_3 is 60V

$$60 = 12R_3$$
$$\Rightarrow R_3 = 5\Omega$$

3) Find the voltage across AB



Solⁿ: Applying KVL, we get 2 eqⁿs.

$$20 + 10I + 20I + 5I = 110$$

$$\Rightarrow 35I = 90$$

$$\Rightarrow I = 18/7 \text{ A}$$

And, $V_A - 10I + 110V - 20I = V_B$

$$\Rightarrow V_A - V_B = 30I - 110$$

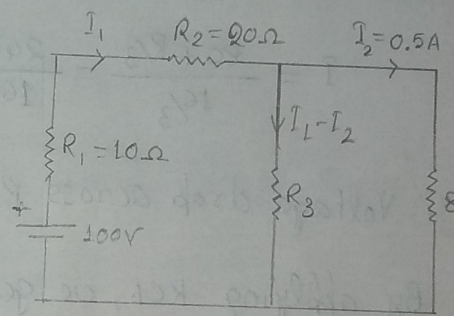
$$= \frac{540}{7} - 110$$

$$= \frac{-770 + 540}{7}$$

$$= \frac{230}{7} = -32.86 \text{ V}$$

∴ The voltage across AB is 32.86 V.

4) Find the voltage drop across R_1 and R_2. The resistance R_3 is not specified.



Solⁿ $R_3(I_1 - I_2) = 8I_2$

$$\Rightarrow (8 + R_3) I_2 = R_3 I_1$$

$$\Rightarrow I_1 = \left(\frac{8 + R_3}{R_3}\right) \times 0.5$$

$$= \frac{8 + R_3}{2R_3}$$

And, $R_3(I_1 - I_2) + R_2 I_1 + R_1 I_1 = 100$

$$\Rightarrow (10 + 20 + R_3) \times I_1 - R_3 I_2 = 100$$

$$\Rightarrow (30 + R_3) \times \frac{8 + R_3}{2R_3} - \frac{R_3}{2} = 100$$

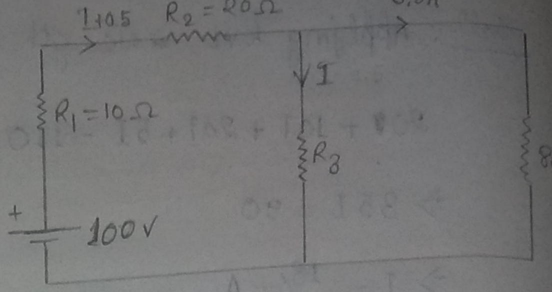
$$\Rightarrow (30 + R_3)(8 + R_3) - R_3^2 = 200R_3$$

$$\Rightarrow 240 + 8R_3 + 30R_3 + R_3^2 - R_3^2 = 200R_3$$

$$\Rightarrow R_3^2 + 162R_3 - 240 = 0$$

$$\Rightarrow R_3 = \frac{-162 \pm \sqrt{162^2 - 4 \times 240}}{2} = \frac{-162 + 159}{2} =$$

4) Find the voltage drop across R_1 & R_2. The resistance R_3 not specified.



Sol^n By applying KCL, we get

$$R_3(I-0.5) = 80\times 0.5$$

$$\Rightarrow R_3 I - R_3/2 = 40$$

$$\Rightarrow I = \frac{40 + R_3/2}{R_3} = \frac{80+R_3}{2R_3}$$

By applying KVL, we get

$$R_3(I-0.5) + 20I + 10I = 100$$

$$\Rightarrow R_3 \times \frac{80+R_3}{2R_3} - \frac{R_3}{2} + 30\times\frac{80+R_3}{2R_3} = 100$$

$$\Rightarrow 80R_3 + \cancel{R_3^2} - \cancel{R_3^2} + 240 + 30R_3 = 200R_3$$

$$\Rightarrow 90R_3 = 240$$

$$\Rightarrow R_3 = \frac{8}{3} = 2.66\,\Omega$$

$$I = \frac{80+8/3}{16/3} = \frac{248}{16} = 15.5\text{ A}$$

$\therefore$ Voltage drop across R_1 is $10\times 15.5 = 155\text{ V}$

By applying KCL, we get

$$R_3(I) = 80\times 0.5$$

$$\Rightarrow IR_3 = 40 \quad \text{——} \quad (1)$$

By applying KVL, we get

$$IR_3 + 20I + 10I = 100$$

$$\Rightarrow 40 + 30I = 100$$

$$\Rightarrow 30I = 60$$

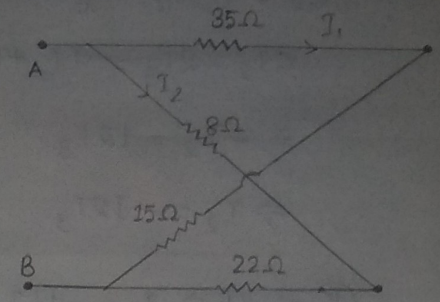
$$\Rightarrow I = 2A$$

$$\therefore R_3 = \frac{40}{2} = 20\,\Omega$$

$\therefore$ Voltage drop across R_1 is $10\times 2 = 20\text{V}$

Voltage drop across R_2 is $20\times 2 = 40\text{V}$

5) Determine equivalent resistance between terminals A and B in the following circuit.



Sol^n By applying KVL, we get

$V_B - 15I_1 - 35I_1 = V_A$

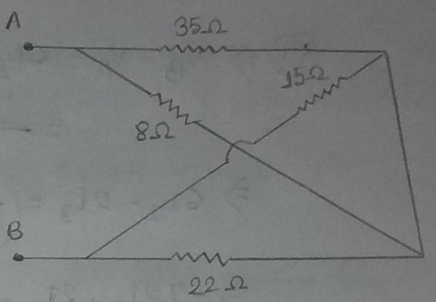
$\Rightarrow V_B - V_A = 50I_1$ — (I)

And, $V_B - 22I_2 - 8I_2 = V_A$

$\Rightarrow V_B - V_A = 30I_2$ — (II)

From (I) & (II) $\Rightarrow 50I_1 = 30I_2$

$\Rightarrow I_1 = 3/5\, I_2$



Now, $V = IR$

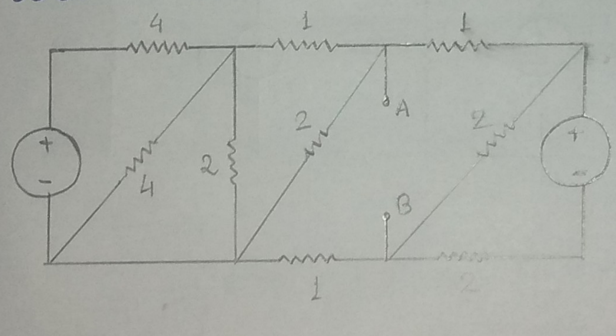
$V_B - V_A = (I_1 + I_2)R$

$\Rightarrow 30I_2 = (3/5\, I_2 + I_2)R$

$\Rightarrow 30 = 8/5\, R$

$\Rightarrow R = 18.75\,\Omega$

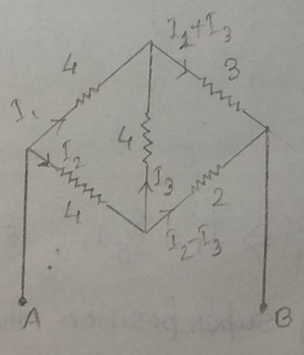
6) Determine equivalent resistance between AB



By taking voltage sources as short circuit, we get

Req between AB = 1 Ω

Sol^n By applying KCL, we get



$4I_2 + 4I_3 = 4I_1$

$\Rightarrow I_1 = I_2 + I_3$ — (I)

$3(I_1 + I_3) + 4I_3 = 2(I_2 - I_3)$

$\Rightarrow 9I_3 + 3I_1 = 2I_2$

$\Rightarrow 9I_3 + 3I_2 + 3I_3 = 2I_2$

$\Rightarrow 12I_3 + I_2 = 0$ — (II)

$V_A + 4I_2 + 2(I_2 - I_3) = V_B$

$\Rightarrow V_B - V_A = 6I_2 - 2I_3$ — (III)

$V_A + 4I_1 + 3(I_1 + I_3) = V_B$

$\Rightarrow V_B - V_A = 7I_2 + 10I_3$ — (IV)

From (III) & (IV), we get

$6I_2 - 2I_3 = 7I_2 + 10I_3$

$\Rightarrow -I_2 = 12I_3$

$\Rightarrow I_2 = -12I_3$

(X) $\Rightarrow$ (I) $\Rightarrow I_1 = -11I_3$

(XII) $\Rightarrow V_B - V_A = \cancel{6I_2 - 2I_3}\,(I_1 + I_2)R$

$= \cancel{-72I_3}$

$\Rightarrow 6I_2 - 2I_3 = (I_1 + I_2)R$

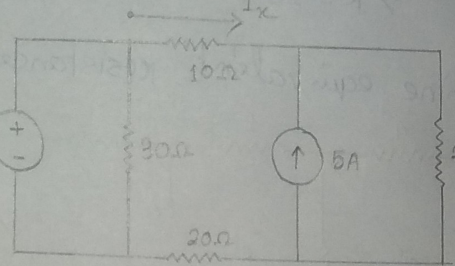
$\Rightarrow -72I_3 - 2I_3 = (-12 - 11)I_3 R$

$\Rightarrow 74 = 33R$

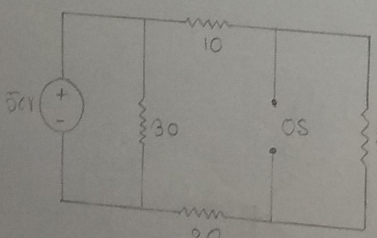
$\Rightarrow R = 74/33$

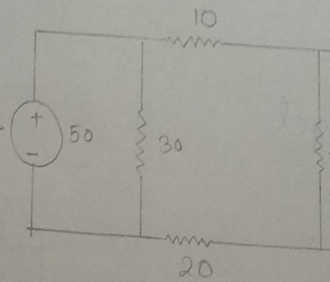
$\therefore$ The equivalent resistance is $74/33$

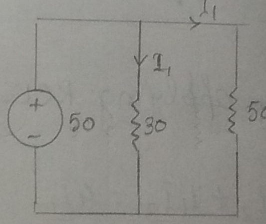
7) obtain the current I_x in the $10\,\Omega$ resistor using superposition theorem.



Soln By using superposition theorem ~~on 50V~~ 1st we take 50V source and 5A will become open circuit.

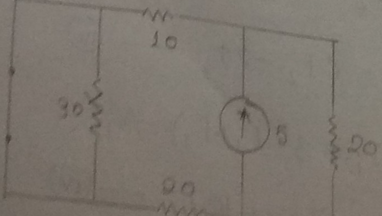


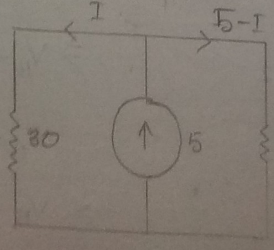




$30I_1 = 50I_1' \Rightarrow I_1' = \frac{3}{5}I_1$, $50I_1' = 50 \Rightarrow I_1' = 1A$

By using superposition theorem on circuit, we take 5A source and 50V source will become ~~of~~ short circuit. (Due to SC, current through 30Ω will be 0)





$20(5 - I) = \cancel{30I}\ 30I$

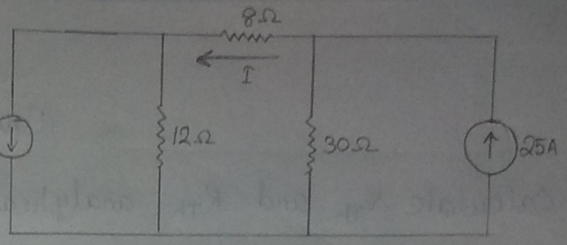
$\Rightarrow 100 = \cancel{100I}\ 50I$

$\Rightarrow I = 2A$

By taking sign convension,

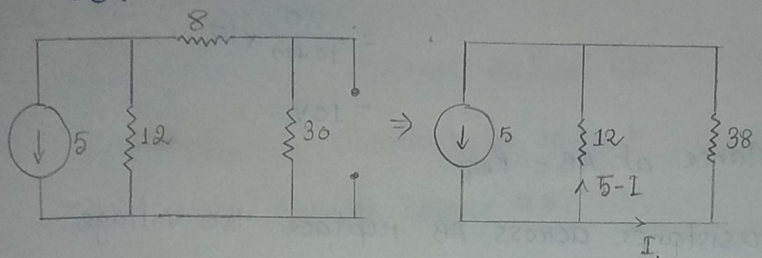
$$I_x = I_1' - I$$
$$= 1A - 2A$$
$$= -1A.$$

8) Use superposition theorem and find current I.



Solⁿ Using superposition theorem,

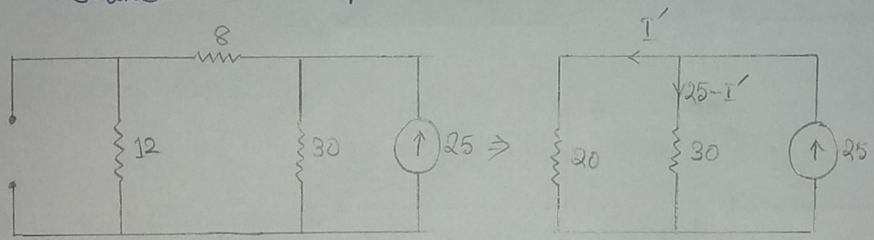
1st we take 5A current source and 25A source become OC.



~~$5 = 38I' \Rightarrow I' = 5/38$~~

$$38I_1 = 12(5 - I_1)$$
$$\Rightarrow 50I_1 = 60$$
$$\Rightarrow I_1 = 6/5\ A$$

Using superposition theorem, we take 25A current source and 5A source become open circuit.



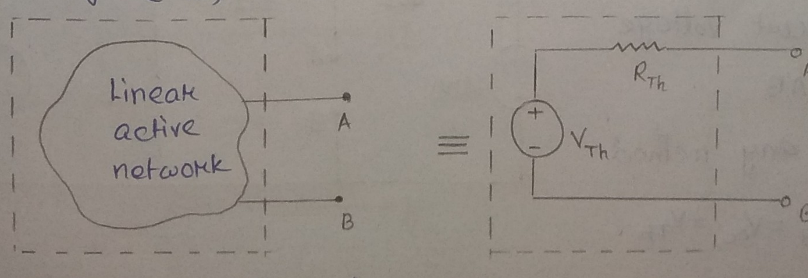
$$30(25 - I') = 20I'$$
$$\Rightarrow 750 = 50I'$$
$$\Rightarrow I' = 15\ A$$

∴ After taking sign convension

$$I = I_1 + I'$$
$$= 6/5 + 15 = 1.2 + 15 = 16.2\ A$$

Thevenin's Theorem –

A linear active resistive network which contains one or more voltage or current sources can be replaced by a single voltage and a series resistance. The voltage is called the Thevenin's equivalent voltage (V_{Th}) and the resistance is Thevenin's equivalent resistance (R_{Th})

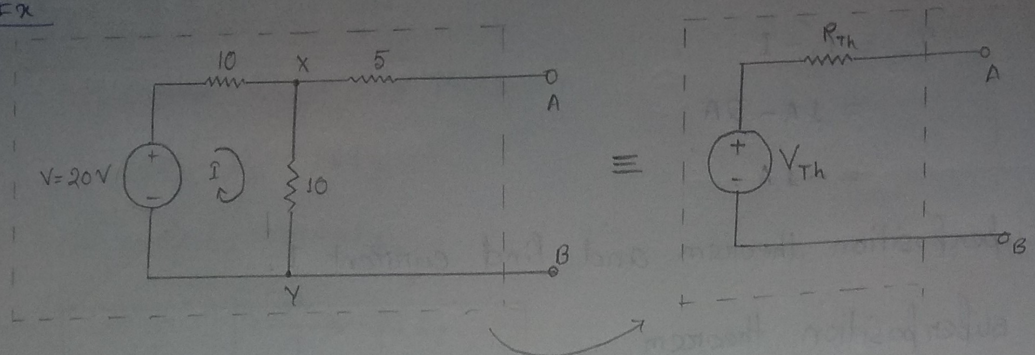


V_{Th} = Open circuit Voltage at terminals AB

R_{Th} = Equivalent resistance of the network as seen from AB.

Equivalent circuit.

Ex



Equivalent circuit.

* Calculate V_{Th} and R_{Th} analytically

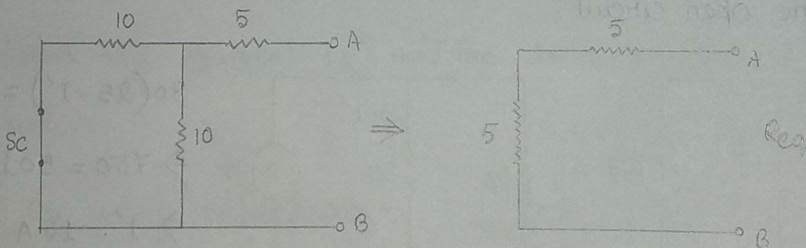
V_{Th} = open circuit voltage across AB = V_{xy} $= I \times 10$

$$= \frac{20}{10+10} \times 10$$

$$= 10V$$

R_{Th} = equivalent resistance at AB = R_{eq}

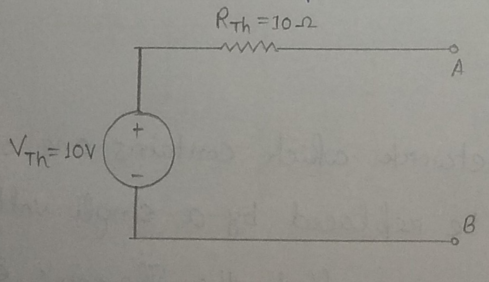
To calculate equivalent resistance across AB replace the voltage source by short circuit. The circuit becomes



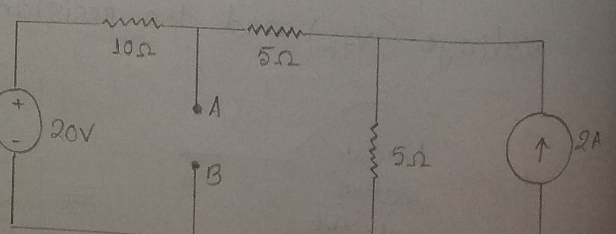
$\therefore R_{eq} = 10\Omega$

$\therefore R_{Th} = 10\Omega$

So the Thevenin's equivalent circuit will be



Ex Draw Thevenin's equivalent circuit across AB

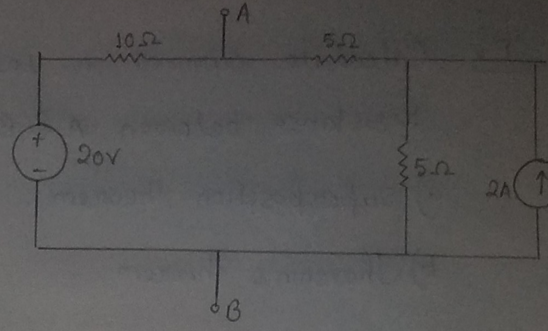


Sol^n V_{Th} = open circuit voltage across AB

We can apply any method to find out $V_{AB} = V_{oc} = V_{Th}$.

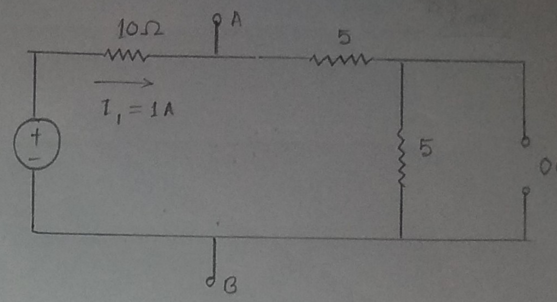
~~Let's apply~~

Let's apply superposition theorem.



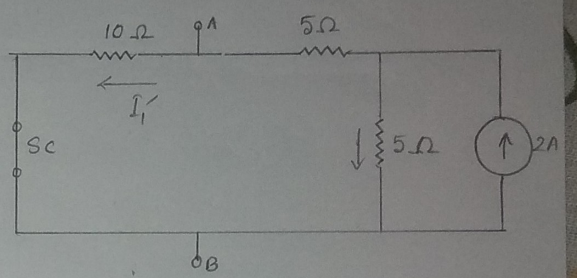
Consider 20 V source alone and calculate current in 10Ω branch (make the current source open)

$I_1 = \frac{20}{20} = 1A.$



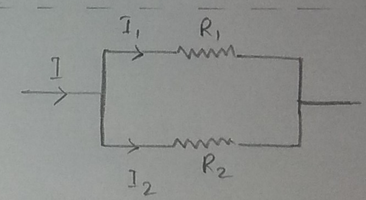
Now, consider current source alone and calculate current in 10Ω resistance (make the voltage source short circuit)

~~Now, consider Ctt~~



Applying current division method we can calculate current in 10 Ω resistance due to source 2 A.

$I_1' = \frac{5}{10+5+5} \times 2A = \frac{1}{2} A.$



$I_1 = \frac{R_2}{R_1+R_2} \times I$

$I_2 = \frac{R_1}{R_1+R_2} \times I$

∴ Resultant current due to both sources at 10Ω resistance is

$I = I_1 - I_1' = 1 - \frac{1}{2} = \frac{1}{2} A$ (from L to R)

Now, considering the original circuit

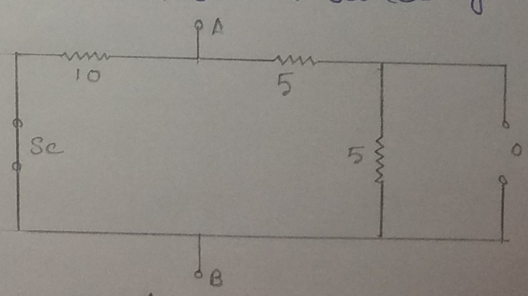
Voltage across AB, $V_{AB} = 20V - \frac{1}{2} \times 10V$

$= 15V$

$V_{Th} = 15 V$

Calculation of R_{Th} –

Replace voltage source by short circuit and current source by open circuit.



Thus Thevenin's circuit will be

⇓