

MAIN

PHYSICS 101

PAPER 181101

MODULE 2 – OPTICS

Aberration in lenses, Spherical and Chromatic Aberration, methods of minimization.

Interference of light by division of wavefront (brief discussion) and division of amplitude. Interference due to reflected light in plane parallel film. Interference in variable thickness (wedge shaped) film. Newton's Ring.

Books:

1. Principles of Optics. By B. K. Mathur.
2. A text book on Light for Degree Students. By K. G. Mazumdar.
3. Applied Physics for Engineers. by Neeraj Mehta.
4. Optics. by Ajoy Ghatak.
5. Engineering Physics. by Gaur & Gupta.

Introduction: The branch of physics which deals with the phenomena associated with light is known as **OPTICS**.

Optics is one of the branches of physics in which the nature and properties of light are studied. It is divided into 3 different branches.

1. Geometrical Optics.

2. Physiological Optics.

3. Physical Optics.

Different terms – Ray,

beam-parallel, converging, diverging beam.

Optical medium,

Luminous and Non-Luminous bodies.

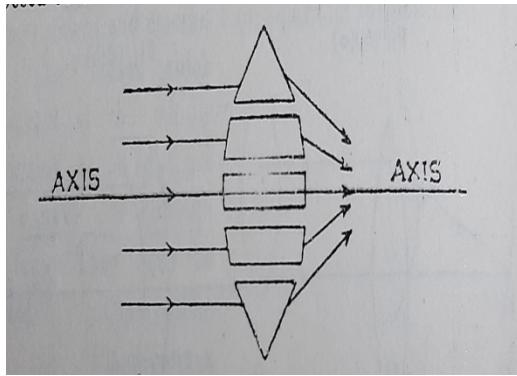
Transparent and translucent opaque bodies etc.

Object and image- real and virtual

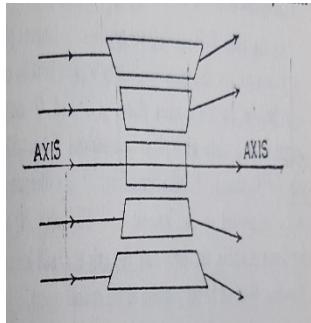
Principle of rectilinear propagation of light.

Laws of reflection and refraction etc.

Convex lens is converging.



Concave lens is diverging :



Defect of Image and their remedy.

The simple formulae for mirrors and lenses are true only when the rays are incident very close to the axis, however the rays are incident at a large angle of incidence at a point far away from the axis and hence the simple formulae are inapplicable there.

Thus, the image formed by reflection or refraction is defective. This defect of the image arising out of the wide angles of incidence of rays of a particular wavelength is called **Monochromatic Aberration**.

There are 5 such monochromatic aberrations which are known as

Seidal Aberrations.

They are

- 1) Spherical aberration.
- 2) Coma.
- 3) Astigmatism.
- 4) Curvature.
- 5) Distortion.

Again, when the image is formed by refraction of white light, they become colored due to dispersion of white light and this defect of image is called **Chromatic Aberration**.

Monochromatic Spherical Aberration:

A wide beam of parallel light is made incident on the refracting surface.

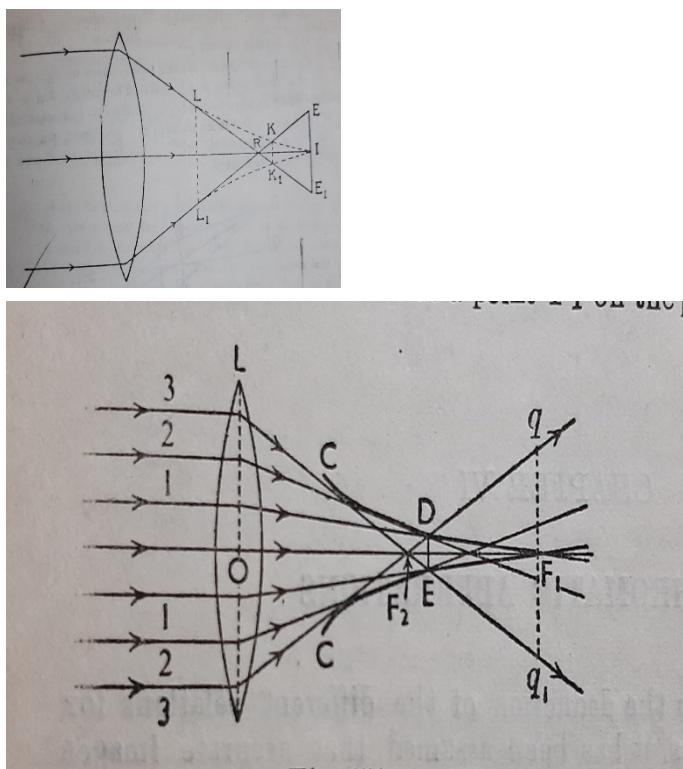
The rays which are incident very close to the axis (paraxial rays) after reflection (in case of mirror) or refraction pass through(I).

The rays which are incident on the peripheral region, after reflection or refraction passes through(R).

Thus, there is no sharp point image for an object at infinity. This defect of the image, in which the central and peripheral incident rays form images at different points on the axis is called **spherical aberration**.

If a screen is placed perpendicular to the axis at LL₁, a bright circular edge and a faint center is appears. And on putting a screen at EE₁, a bright center with faint edge will appear. But when the screen is at KK₁, a circular patch of almost uniform illumination appears. This circle is called circle of least confusion. The distance RI is a measure of **Longitudinal Spherical Aberration** while IE measures **Lateral Spherical Aberration**.

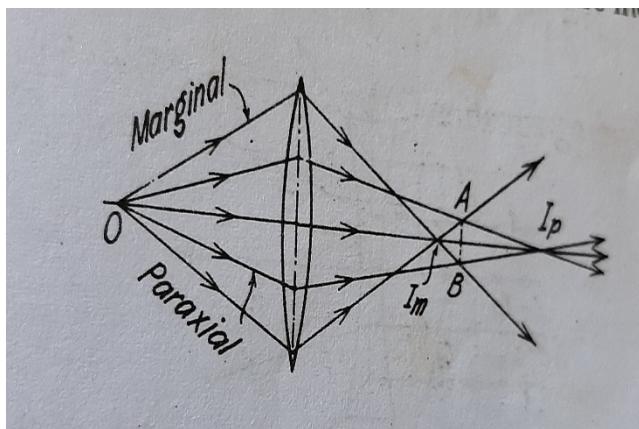
Fig 1



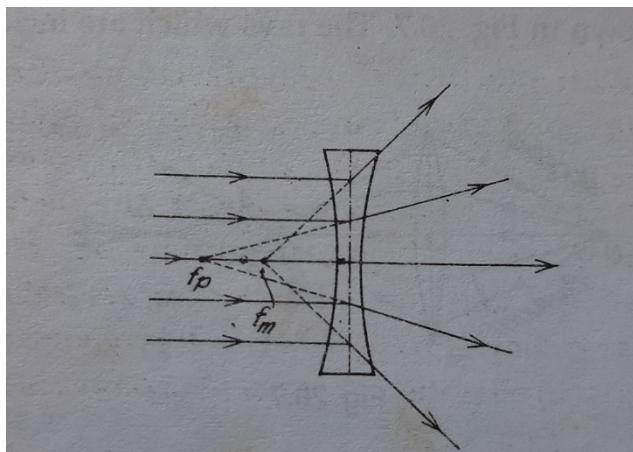
Spherical Aberration:

A point source **O** of monochromatic light placed on the axis of a large aperture convex lens fig(i). The rays which are incident near the axis called paraxial rays come to focus at **I** while the ray incident near the rim of the lens called the marginal or peripheral rays come to focus at **I'**.

The intermediate rays are brought to focus between **I** and **I'**. It is clear from the figure that the paraxial rays form the image at a point distance than the marginal rays. Thus, the image is not sharp at any point on the axis. Thus, the failure or inability of the lens to form a point image of an axial point object is called **Spherical Aberrations**.



Fig(i)



Fig(ii)

The reason of spherical aberration is as follows:

Let the lens be divided into circular zones. It can be proved mathematically that the focal lengths slightly vary with the radius of the zones. i.e, different zones have different focal lengths. The focal length of marginal zone is lesser than the paraxial zone. Hence the marginal rays are focused first.

The spherical aberration can also be explained by saying that the marginal rays suffer greater deviation than the paraxial rays because they are incident at a greater height than the later.

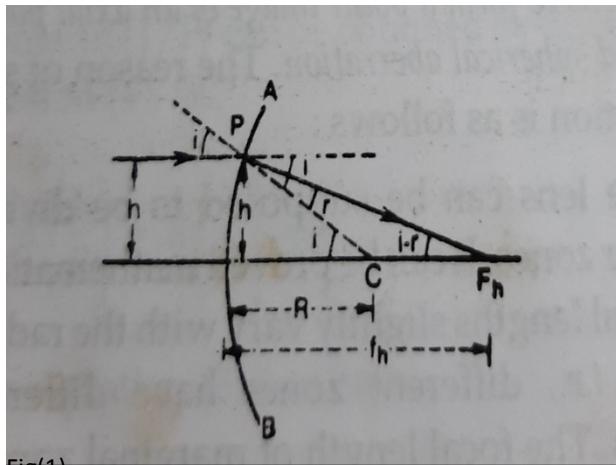
The distance between f_m and f_p is the measure of **longitudinal or axial spherical aberration**.

If a screen is placed normal to the principal axis at f_p , then the image on the screen consists of circular disc. The disc would be sharply focused at the center but diffused near the outer edge. A disc image would again be obtained if the screen is placed at f_m . If the screen is moved between f_m and f_p then the size of the disc is minimum in position AB where the paraxial and marginal rays cross.

Due to the smallest cross section, this is known as **circle of least confusion**. This is the nearest approach to a point image. i.e. if the screen is placed in the position AB, then it is the best possible image of uniform sharpness. The radius of the circle of least confusion is called the **lateral spherical aberration**.

The spherical aberration produced by a lens is negative while that produced by a spherical surface is positive.

Spherical aberration due to spherical surface:



Fig(1)

AB is a spherical surface of radius of curvature R. Let an incident ray parallel to the principal axis meet the spherical surface at a height h from the axis.

The refracted ray intersects the axis at a point

Therefore $O =$ for rays in zone h .

From fig 1

$$= R +$$

From CP

==

C=

$$= ,$$

$$=$$

$$=$$

Since,

$$= R +$$

$$= R [1 +]$$

For paraxial rays, when h tends to 0

and tends to 1.

Therefore,

$$=$$

focal length of paraxial rays.

Now, the change of focal length for h zone as compared to axial zone is given by,

$$= -R [1+]$$

$$= R [-]$$

This represents the **Longitudinal Spherical Aberration**.

The approximate value of spherical aberration can be calculated as follows:

Since $\sin i =$ from fig1

Or $\sin r = , \text{ since } = \mu$

$$\text{Now, } i) = (1 -)$$

$$(1 -)$$

And

$$\cos r = (1 - r) = (1 -)$$

$$(1 -)$$

Therefore,

$$= R[-]$$

Solving,

$$=$$

Where

$$=$$

This gives the approximate value of the spherical aberration due to a spherical surface.

Deviation produced by a thin lens:

The angle between the incident light ray and the corresponding refracted ray from a thin lens is known as angle of deviation.

From fig L is the converging lens, O is a point object, LO=u. A ray OX meets the lens at a height h above the principal axis. A deviation is produced and meets the principal axis at a point I at a distance LI=v, it is the real image of point object O.

Let angle XOL= α & angle XIL= β

Therefore,

$$= \alpha + \beta$$

For paraxial rays the angles α & β are very small.

In right angle triangle

ΔXLO & ΔXLI

$$\alpha = \tan \alpha =$$

$$\beta = \tan \beta =$$

therefore

$$=$$

Or,

Lens formula for thin lens is

Comparing, we get, =

this is the expression for deviation produced by a thin lens.

Minimization of Spherical Aberration:

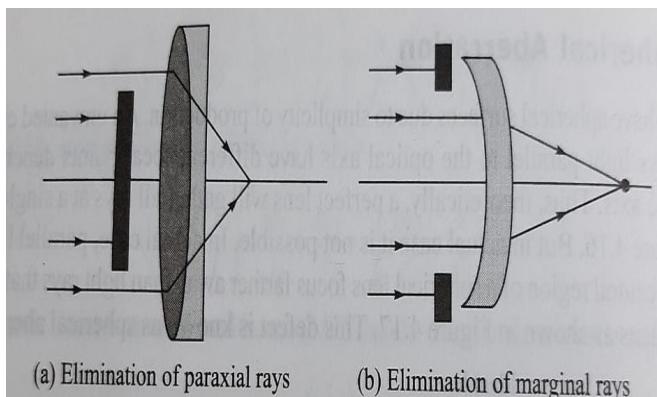
The different methods are

- 1) By means of stop
- 2) By the use of planoconvex lens or by dividing the deviation equally.
- 3) By using two suitable lenses in contact
- 4) By using crossed lens
- 5) By using two planoconvex lenses separated by a distance.

By means of stop:

In this method either the marginal or paraxial rays are cut off by using stop method. Then the rest rays converge practically to a common focal point as shown in fig (1).

In telescope objectives this method is used to cut off paraxial light rays while in case of camera lenses this method is used to cut off marginal light rays.



(2) By using planoconvex lens or by dividing the deviation equally:

The spherical aberration may be

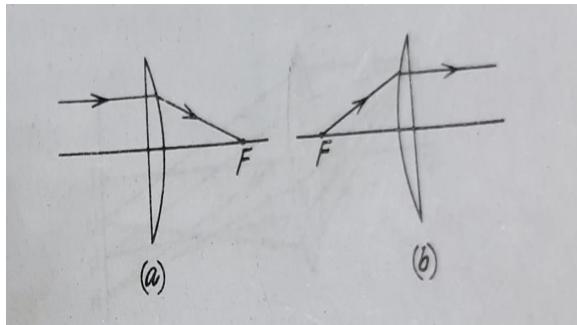
minimized by using planoconvex lens. It has been observed that the spherical aberration is proportional to the square of the total deviation produced by the lens.

If δ be the total deviation produced by the lens and δ_1 and δ_2 be the deviation produced at the two surfaces of the lens, then spherical aberration

+ 4

It is obviously minimum when i.e. the deviation is equally divided at the two surfaces.

It is important to note that if a ray parallel to the principal axis is incident on the plan surface of a Plano Convex Lens fig(a) then there will be no deviation at plane surface and the whole deviation will be on the 2nd surface i.e. the convex surface. so, there are no question of equal deviation at the two surfaces.

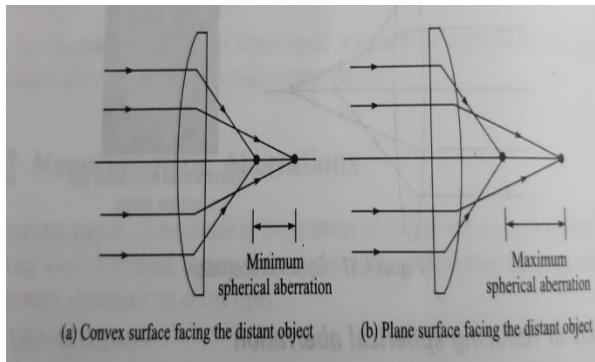


Similarly, when the object is at focus towards the curve surface fig(b). Therefore, to minimize the spherical aberration how a plano-convex lens is placed is important.

Convex side facing the object fig(I)

If the plane side is placed towards the object, the deviation would be only at the 2nd surface. i.e, the convex lens. so the longitudinal spherical aberration will be as fig (II). Spherical aberration is maximum in the 2nd case.

Fig(I) & Fig (II)



The general rule is to minimize the spherical aberration in a plano-convex lens is that the convex side should be face the incident or emergent beam whichever is more parallel to the axis.

(3) By using two suitable lenses in contact:

In case of a convex lens, the marginal image lies towards the left of paraxial image while in case of a concave lens, the marginal image lies towards the right of paraxial image

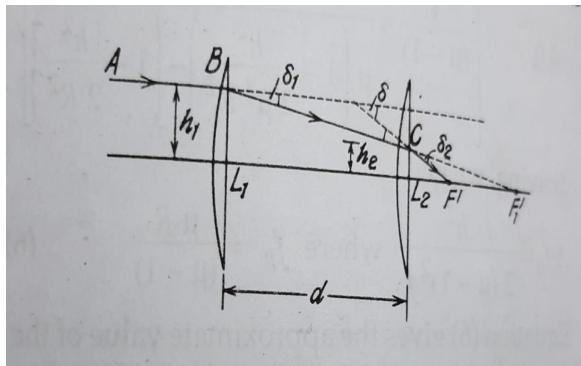
Thus, by a suitable combination of two lenses the spherical aberration may be minimized.

(4) by using crossed lens:

(5)By using two planoconvex lenses separated by a distance:

The spherical aberration can be minimized by using two planoconvex lenses of the same material placed at a distance equal to the difference of their focal lengths.

In this case the deviation produced in a ray is spread over four surfaces and is shared equally by two lenses.



fig(1)

Let, and be two planoconvex lenses of focal lengths and separated by a distance d as shown in fig (1).

Let a ray of light AB parallel to the axis meet the first lens at height and suffer a deviation

$$= .$$

This is directed towards , and 2nd focal point of , the refracted ray BC strikes the lens , at height and suffers a deviation

=

The emergent ray meets the axis at

The 2nd focal point of the combination.

For minimum spherical aberration

=

Therefore, = =

From similar triangles

and

= ==

Since,

=

Therefore,

=

[=

=[

This is the required condition for minimizing spherical aberration.

Deviation produced by a thin lens:

The angle between the light ray and the corresponding refracted ray from a thin lens is known as angle of deviation or simply deviation.

Let a thin converging lens L, O is a point object placed at a distance LO=u from the lens, the light ray OX meets the lens at a height h above the principle axis. It suffers a deviation and after emerging meets the principal axis at a point I at a distance LI=v, which is the real image of point object O.

Let angle XOL= and angle XIL= β

From fig,

= + β

For paraxial rays β are small.

From right angles

XLO and XLI

= tan = =

β = tan β = =

therefore,

$$= +\beta$$

$$= +$$

$$= h []$$

From, [] =

therefore

$$=$$

It is the deviation produced by a thin lens.

Reduction of spherical aberration:

If the plane side is turned towards the object the spherical aberration is very large. It is obvious that if the deviation of marginal rays is made minimum then spherical aberration will be least. like prism the deviation is minimum when the incident and emergent rays make equal angles with the focus.

Similarly, in case of a lens the deviation of marginal rays will be minimum when they enter the 1st lens surface and leave the 2nd surface at more or less equal angles. Thus, in general spherical aberration will be minimized.

Dispersion and dispersive power:

Let PQ be a ray of light (white) incident on the principal section ABC of a prism. The ray after emergence from the prism will be deviated and its constituent colors will be separated or dispersed. The less refrangible red ray proceeds along LR while the most refrangible violet light proceed along MV.

Let, the red and violet rays be produced backward to meet at O. Then the angular separation between red and violet rays, angle ROV is called **Dispersion** between red and violet rays.

Let be a straight line through O drawn parallel to PQ, the direction of the incident ray. Then angle XOR= is the deviation for red light while the angle XOV= is the deviation for violet rays.

Thus, Dispersion=angle ROV=()

If the prism be thin, then

$$= ()A$$

$$()A$$

Therefore Dispersion =()

$$=-)A$$

Again if and

Be the refractive indices of a material for the two given colors say red and violet then there is an intermediate color between the two given colors so the refractive index of the material for that intermediate color will be equal to

This intermediate color is called the mean color between the two given colors.

The dispersive power of the material of the prism is defined as

Dispersive Power :

=

=

=

=

=

=

Where,

Change of refractive index of the material when the color changes from violet to red.

It should be remembered that the dispersive power of a material is different from its refractive power. The refracting power is greater when the refractive index is higher.

Thus, for example diamond has low dispersive power though its refracting power is higher due to high refractive index.

Chromatic Aberration in lenses:

A lens may be looked upon as a number of thin prisms. so when a beam of white light is incident on any zone of a lens in a direction parallel to the principal axis of the lens it is not brought to a point focus after refraction because each of the rays will be dispersed by the lens into its constituent colors.

The red components being least refrangible will be focused at on the principal axis. This is the principal focus for the red rays and the violet components will be more deviated and the corresponding principal focus for the violet rays will be at a point which is much closer to the lens surface than .

This wandering of the principal focus along the axis of the lens due to the composite nature of the incident beam is known as the **axial or longitudinal chromatic aberration** of the lens.

If the screen is placed at the center of the colored patch will be intensely violet with the outer edge colored red. and if the screen is shifted on to the position the center of the colored patch will be intensely red and with its outer edge colored violet. If the screen is placed at CD the patch will be more or less uniformly illuminated and color effect will be minimum there. this patch is known as the **circle of least confusion** or of least chromatic aberration. And diameter of this circular patch at CD is a measure of the **lateral or transverse chromatic aberration**.

The Lateral chromatic aberration is actually the difference in magnification of the two extreme colored images formed due to the chromatic error in a lens.

The longitudinal chromatic aberration is measured by the difference in the focal lengths of the two extreme colored components and is initially associated with the position of the images formed by the lens while the lateral chromatic aberration is associated with the question of magnification of the images. It is due to chromatic aberration that a real image produced by an uncorrected single lens is found to be fringed with the colors of the spectrum.

Calculation of chromatic aberration:

(a) Longitudinal chromatic aberration:

The longitudinal chromatic aberration is measured by the difference in the focal lengths of the two extreme colored components of the incident composite beam. i.e. the longitudinal chromatic aberration is Δf .

Since, $\Delta f = (\frac{1}{f_1} - \frac{1}{f_2})[n_2 - n_1]$

f is the mean focal length.

$f = (\frac{f_1 f_2}{f_1 + f_2})$ and

$\Delta f = (\frac{f_1 f_2}{f_1 + f_2})[n_2 - n_1]$

Therefore,

$\Delta f = (\frac{f_1 f_2}{f_1 + f_2})[n_2 - n_1]$

$= (\frac{f^2}{f_1 + f_2})[n_2 - n_1]$

Therefore,

$=$

But

$\Delta f = \frac{f^2}{f_1 + f_2}[n_2 - n_1]$ for all practical purposes.

$=$

-----(1)

This is the expression for the dispersive power of a lens. and

$\lambda =$

From eq (1)

λ is the longitudinal chromatic aberration which is equal to $\lambda C D$.

(b) Lateral or Transverse Chromatic Aberration

The lateral chromatic aberration is measured by the diameter of the circle of least chromatic aberration. i.e, by CD.

To deduce an expression for CD in terms of the constants of the lens, let us from fig(1),

where the similar triangle,

and

$\lambda_1 = \frac{\lambda}{n_1}$ -----(i)

Again, the similar triangles

and

$\lambda_2 = \frac{\lambda}{n_2}$ -----(ii)

From (i) and (ii)

=

=

Here,

) =

And

) = $2f$ (approx..)

And = d the effective aperture of the lens. Therefore,

=

CD = ----- (iii)

This is the expression for the lateral chromatic aberration of a lens.

Achromatic system:

An ideal case of achromatism in a system of lenses is that in which all the colored images are of the same size (when) and at the same place (when).

By the superposition of different colors the coloring effect of the image is destroyed. In this ideal case, the overlapping of different colored image should occur not for one position of the object but for its every position. This ideal condition is very difficult to attain.

In practice, partial achromatism is obtained for different colored images either they have different magnification but of same image distance or of equal magnification but of different image distances. Again, when partial achromatism is obtained for one distance of the object, it will not remain same for other distances of the object.

Thus, for achromatism w.r.t size / or magnification m of the image, should have

And for achromatism w.r.t position or image distance v, then $dv=0$.

Achromatism in lenses:

(a) Single lens:

In a single lens of mean focal length f , the chromatic aberration is given by so the lens to make achromatic, the principal focus for the red rays must be made coincident with the principal focus for the violet rays. For achromatism $(\Delta)=0$

Since,

[=

The condition of achromatism demands the (Δ) should also be zero.

According to the convention of calculus, the condition for achromatism as $d(\Delta)=0$,

The differentiation w.r.to λ .

Therefore,

$= (\mu - 1) []$

$d(\Delta) = d\mu []$

$(\mu - 1) []$

Since, d is the dispersive power.

For achromatism of a single lens

$d=0$ but therefore the condition demands that f . So no single lens of finite focal length can be made achromatic.

(b) **Two lenses in contact:**

when two lenses are in contact, the equivalent focal length is given by F , where ----- (1)

here d_1 and d_2 are the focal lengths of component lenses.

Due to the chromatic aberration the principle foci for the red and the violet components will be different and for achromatism of the system the difference between the focal lengths of the red and the violet components must vanish.

Therefore, the condition of achromatism is that

Again, $d = d_1 + d_2$

The condition of achromatism demands that,

$$d=0$$

but, $d=d_1+d_2$

since

$$d_1 = d_2$$

$$=$$

$$=$$

where d is the dispersive power of the lens C of focal length .

Similarly,

$$d_1 =$$

is the dispersive power of the lens of focal length .

Therefore

$$d_1 = +$$

for achromatism of the combination

$$d_1 = 0$$

$$[+] = 0 \quad \text{----- (1)}$$

When relation (1) is satisfied for a combination of two lenses in contact, it becomes achromatic for the two extreme colors with reference to which and have been defined from (1) we have,

Here, and are positive. Hence, must be of opposite signs.

If refers to a convex lens and would refer a concave lens and let both the lenses are of same material numerically. (since $=$).

This would make $=0$

So the purpose of the lens combination is not served here.

So, .

If refers to a crown glass and refers to a flint glass, being greater than so must be greater than .

Thus, in order to make a conveying achromatic combination of two lenses in contact.

A convex lens of crown glass of smaller focal length is to be placed in contact with a concave lens of flint glass of greater focal length so that the above relation (1) satisfied.

In **achromatic doublet** generally the flint glass lens is made planoconcave, the concave surface of which is placed in contact with a suitable convex lens of crown glass. If the convex surface face the incident ray, this achromatic

doublet also sufficient reduce the spherical aberration.

If a number of lenses are put in contact to form an achromatic combination, the condition is given by

.

Where summation () extends over all component lenses.

Two lenses separated by a distance:

the equivalent focal length F in this case is given by
is the separation between the two component lenses.

For achromatism $d()=0$

The differentiation being w.r.t .

But $d()=d() + d() + a()d() + a()d()$

And $d() =()$

$d() =()$

substituting in the expression for $d()$, we have

$d() = () + () + () + ()$

Therefore, for achromatism,

$$() + () + () = 0 \quad \text{-----}(\alpha)$$

If the lenses are of the same material

=

Therefore,

$$(\alpha) + (\beta) = 0 \quad \text{---} \quad (\gamma)$$

(for achromatism).

If the component lenses are convex the condition would be

=

=

$$\text{---} \quad (\gamma)$$

Thus, the relation (α) gives the condition of achromatism for two lenses separated by a distance when the lens materials are not necessarily the same.

The relation (γ) gives the condition of achromatism for the two lenses of the same material separated by a distance. This principle is utilized in the construction of the Huygens eyepiece.

In relation (γ) being absent this condition of achromatism is valid for all colors but it must be remembered that the equivalent focal length F is used here indicates that the achromatism of magnification is only ensured and not necessarily that of position. It can be easily shown that for complete achromatization of the system both for the magnification as well as for the position, each of the component lenses must itself be achromatic.

Wave theory of light

Theories of light:

Various theories have so far been put forward by different scientists to explain the nature of light.

- (a) Newton's corpuscular theory (1642-1727)
- (b) Wave theory--1678
- (c) electromagnetic theory –1873
- (d) Quantum theory of light – 1900

Wave theory: Huygens in 1678 suggested that light is propagated as a wave motion in a subtle (difficult to describe or analyse), imponderable(impossible to estimate or assess), elastic medium called ether, which permeates all matter and space.

The supposition of such a medium was necessary as no wave can propagate without a medium.

When these waves strike our eyes we feel the sensation of vision.

He supposed that the light waves are longitudinal and the phenomena of reflection, refraction, interference and diffraction were successfully explained.

Origin of wave theory:

Wave motion- A wave motion is produced in a medium due to the periodic motion or vibration of its particles.

Wavefront and Ray- A wavefront is defined as the envelope or trace drawn through all the points on a wave which are exactly in the same condition as regards displacement and direction of motion, i.e. in the same phase. Thus, a surface drawn along the crests of a water wave is a wave front and also a surface drawn along the troughs would be another wavefront.

A normal drawn to the wavefront at any point indicates the direction of propagation of the wave and is known as ray.

Interference of light:

When two light waves of same frequency and of nearly the same amplitude travel through the same region of space at the same time then due to their superposition the resultant intensity at different points of the medium undergoes some change from point to point. The change in the intensity of light in a medium

from point to point is called interference of light.

Thus, the modification or change in the uniform distribution of light intensity in a medium due to superposition of two light waves is called interference of light.

Types of interference:

- (1) Constructive and (2) Destructive.

When the resultant amplitude is the sum of the amplitude due to two waves, the interference is known as constructive interference and when the resultant amplitude is equal to the difference of two amplitudes, the interference is known as destructive interference.

INTERFERENCE

Interference of light waves was observed by Newton (1676) in the phenomenon of colors of thin films and Newton's Rings. Newton however could not explain the above phenomena of light.

Grimaldi a contemporary of Newton, investigate the true nature of light, observed that two pencils of light rays when superposed tends to extinguish each other in certain cases.

Condition for sustained Interference:

In order to obtain a phenomenon of continuous or sustained interference of light following conditions are to be satisfied.

- (i)The two sources should continuously emit waves of same wavelength.
- (ii)The phase relation between the two sets of waves coming from the two sources must remain the same for all times i.e. if there is any change of phase in one wave, there must be an identical change of phase in the other. In other words, the sources of light waves must be coherent.
- (iii) The amplitudes of the interfering waves must be equal or very nearly equal.
- (iv)In order to obtain an observable interference pattern the distance between the two coherent sources must be as small as possible.
- (v)The two light waves must be on the same state of polarization.

Methods of obtaining Interference effects:

All interference effects can be broadly classified into two groups.

- (1) Methods involving **Division of wavefront** i.e. methods requiring a point or line source from which light waves spread out as spherical or cylindrical waves. Any two points on such a spherical or cylindrical wavefront are taken as the interfering sources, which are evidently coherent. Fresnel's Biprism and Bimirror, Loyld's Mirror, Billet's Split lens, Rayleigh interferometer, Diffraction Gratings... etc. are some examples of this class.
- (2) Methods of involving **Division of amplitude** i.e. methods requiring an extended source. Here the interference takes place between a ray and its reflected or refracted component of the same ray. Some of the examples of this class are the interference effects in thin films, Jamin's interferometer, Michelson's interferometer, Febry Perot interferometer, Lummer-Gehrcke plate and Newton's Rings.

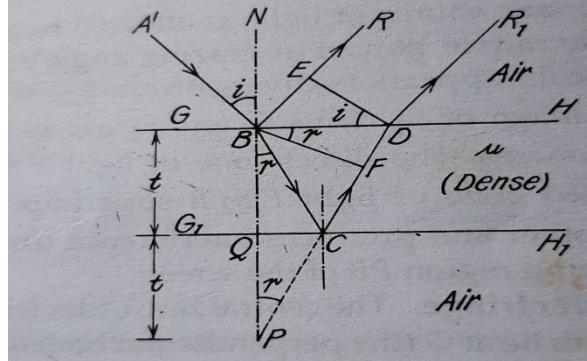
Coherent Sources:

The two sources of light whose frequencies are same and the phase different between the waves emitted by them remains constant with respect to time, are defined as coherent sources.

Interference by Division of Amplitudes:

If a plane wave falls on a thin film then the wave reflected from the upper surface interfere with the wave reflected from the lower surface. There are many practical applications and also explain phenomena like the formation of beautiful colours produced by a soap film illuminated by white light.

Interference in the films: (due to the reflected light)



Let GH and G_1H_1 be the two surfaces of a transparent film of uniform thickness t and refractive index μ . Let AB, a monochromatic light be incident on its upper surface and is partly reflected along BR and refracted along BC.

After one internal reflection at C the ray CD after refraction at D the ray finally emerges out along D in air. Obviously, Dis parallel to BR. One aim is to find out the effective path difference between the ray BR and D.

Let us draw a normal DE on BR and BF on CD. Produce DC in the backward direction to meet BQ at P.

From fig

$$\text{Angle } ABN = i \quad \text{and} \quad \text{Angle } QBC = r$$

Therefore, from geometry,

$$\text{Angle } BDE = i$$

$$\text{Angle } QPC = r$$

The optical path difference between the two reflected light rays BR and D is given by.

$$= \text{Path } (BC+CD) \text{ in film} - \text{Path } (BE) \text{ in air}$$

$$= (BC+CD) - BE \quad \dots \dots \dots (1)$$

Since

$$==== =$$

$$BE = \dots \dots \dots \dots \dots (2)$$

From (1) & (2)

$$(BC + CD) -$$

$$= (BC + CF + FD) -$$

$$= (BC + CF)$$

$$= (PC + CF), \text{ since } BC = CP$$

$$= (PF) \quad \dots \dots \dots \dots \dots (3)$$

From

$$\cos r =$$

$$=PF = BP \cos r = 2t \cos r \quad \text{-----(4)}$$

From (3)

$$\text{-----(5)}$$

It should be remembered that a ray reflected at a surface backed by a denser medium suffer an abrupt phase change of which is equivalent to a path difference .

Thus, the effective path difference between the two reflected rays is

$$2\mu t \cos r \pm$$

Since the maxima occurs when effective path difference is $\Delta=n\lambda$

Since

$$2\mu t \cos r \pm = n\lambda \quad (\text{maxima})$$

$$2\mu t \cos r = (2n \pm 1) \quad \text{-----(A)}$$

If this condition is fulfilled, the film will appear bright in the reflected light and The minimum occurs when the effective path difference is $=(2n \pm 1)$

$$\text{i.e. } 2\mu t \cos r \pm = (2n \pm 1) \quad \text{-----(minima)}$$

$$2\mu t \cos r = n\lambda \quad \text{-----(B)}$$

Because $(n=1)$ or $(n-1)$ can also be taken. as integer. When n is a integer, here $n= 0, 1, 2, \dots$ etc. When this condition is fulfilled the film will appear dark in the reflected light.

INTERFERENCE DUE TO TRANSMITTED LIGHT:

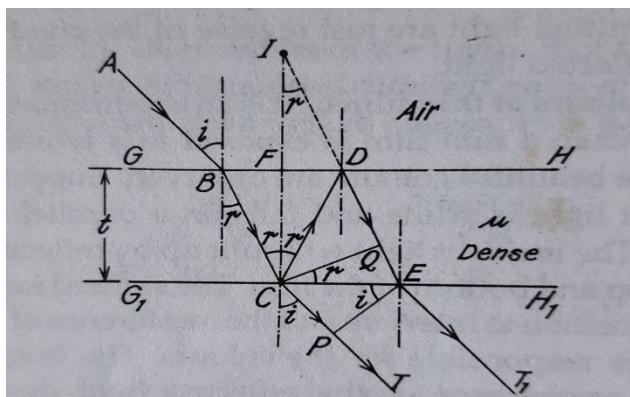


Fig shows the geometry of the transmitted light. Due to simultaneous reflection and refraction transmitted ray CT and ET' are produced. These rays have originated from the same point source. Hence they have a constant phase difference and are in a position to produce sustained interference when combined. In order to calculate the path difference between the two transmitted rays. CQ and EP are normal on DE and CT respectively. Produce ED in the backward direction which meets CF at I. the effective path difference is given by

$$\Delta = \mu(CD + DE) - CP \quad \text{-----(1)}$$

$\mu = \dots$

$$CP = \mu QE \dots \text{---(2)}$$

From equation (1) and (2)

$$\Delta = \mu(CD + DQ + QE) - \mu(QE) \quad \text{since } CP = \mu(QE)$$

$$= \mu(CD + DQ)$$

$$= \mu(Cl)$$

$$= 2\mu t \cos r$$

Here, inside the film, reflection at different points takes place at the surface backed by rarer medium (air). Thus, no abrupt change of takes place in this case.

The maxima occur when effective path difference $\Delta = n\lambda$.

$$\text{i.e., } 2\mu t \cos r = n\lambda. \text{ ---(a)}$$

if this condition is fulfilled, the film will appear bright in transmitted light.

The minima occurs when the effective path difference is $(2n \pm 1)$ ---(b)

When $n=1, 2, 3, \dots$ This condition if fulfilled. the film will appear dark.

Thus the conditions of maxima and minima in transmitted light are reverse of the conditions for reflected light.----

COLOURS OF THIN FILM

It is an experimental mental fact that when a thin film is exposed to a white light source beautiful colours are observed. Let white incident light falls on a parallel sided film then the incident light will split up by reflection at top and bottom of the film . The splitted light rays arein a position to interfere and interference these ray is responsible for the colours . The bright or dark appearance of the reflected light depends upon μ , t and r .

In case of white light even t and r made constant, μ varies wavelength. At a particular point of the film and for a particular position of the eye the interfereing rays only contain wavelength will have a path difference satisfying the conditions of fringe with bright.

Hence, only such wavelength(colours) will be present there. Other wavelengths will be present with diminished intensity. The colours for which the condition for minimum is satisfied are absent.

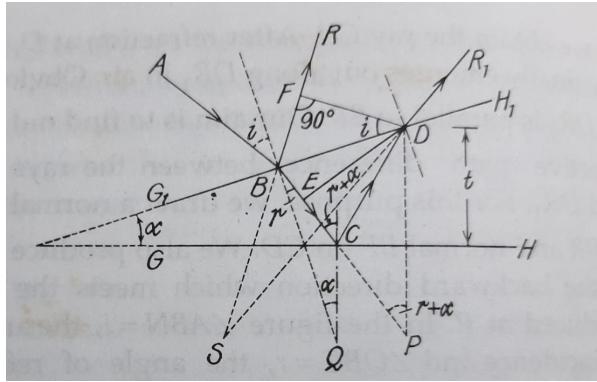
In a similar way, if the same point on the film observed with eye in different positions or different points of the film are observed with eye in same position , a different set of colours is observed each time for maxima and minima in transmitted light are reversed that of reflected light .

Hence the colours suppressed or absent in reflected light appear as intense colours in transmitted light or colours if reflected and transmitted light are complementary.

WEDGE SHAPED FILM:

Let us consider two plane surfaces GH and inclined at an angle α and enclosing a wedge shape air film. The thickness of air film increases from G to H.Let μ be the refractive index of the material of the film .When this is illuminated by sodium light then the interference between two system of rays , one reflected from front surface and the other obtained by internal reflection at the back surface and consequent transmission at the first surface

takes place. It is observed from the figure that interfering waves BR and are not parallel. But to diverge from a point S. Thus the interference takes place at S which is virtual. Fig below.



In order to consider the interference between these two waves, let us calculate the path difference between them. Clearly path difference Δ is

$$\Delta = \mu(BC + CD) - BF$$

$$= \mu(BE + EC + CD) - \mu BE \text{ since, } BF = \mu BE$$

$$= \mu(EC + CD)$$

$$= \mu(EP)$$

$$\Delta = 2\mu t \cos(r+\alpha) \quad \dots \dots \dots (1)$$

Due to reflection and additional phase change of π is introduced. Hence

$$\Delta = 2\mu t \cos(r+\alpha) \pm \quad \dots \dots \dots (2)$$

For constructive interference $\Delta = n\lambda$

$$2\mu t \cos(r+\alpha) \pm = n\lambda$$

$$2\mu t \cos(r+\alpha) = (2n \pm 1) \quad \dots \dots \dots (3)$$

For destructive interference

$$\Delta = (2n \pm 1)$$

$$2\mu t \cos(r+\alpha) = n\lambda \quad \dots \dots \dots (4),$$

$$n = 1, 2, 3, \dots, ;:$$

NATURE OF INTERFERENCE PATTERN:

If the light illuminating the film is parallel then I is constant everywhere and so is r , the angle of refraction. In addition, if the light is used as monochromatic, the path change will occur only due to t . in this case the fringes will be characteristic of equal optical thickness.

Since in the case of a wedge shaped film, t remains constant only in direction parallel to the thin edge of the wedges, hence the straight fringes parallel to the edge of the wedge are obtained.

Thus bright or dark fringes are obtained here as the condition for the thickness t is satisfied according to equation (3) and (4) respectively.

Spacing between the consecutive bright bands:

For nth maxima, we have

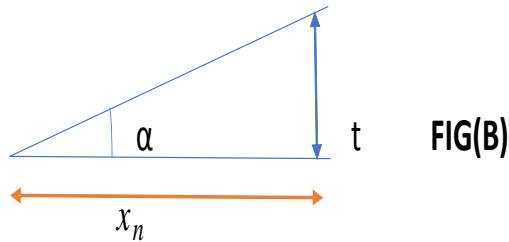
$$2\mu t \cos(r+\alpha) = (2n \pm 1)$$

For normal incidence and air film $r=0$, $\mu=1$

$$2\mu t \cos\alpha = (2n \pm 1) \quad \text{-----}(1)$$

Let this band be obtained at a distance

From the thin edge as showing in fig (B)



From figure, $t = x_n \tan\alpha \quad \text{-----}(2)$

From (1)&(2)

$$2x_n \tan\alpha \cos\alpha = (2n+1) \frac{\lambda}{2}$$

$$2x_n \sin\alpha = (2n+1) \frac{\lambda}{2} \quad \text{-----}(3)$$

if $(n+1)$ th maxima is obtained at a distance from the edge, then

$$2 \sin\alpha = [2(n+1) + 1]$$

$$= (2n+3) \quad \text{-----}(4)$$

Subtracting (3) from (4)

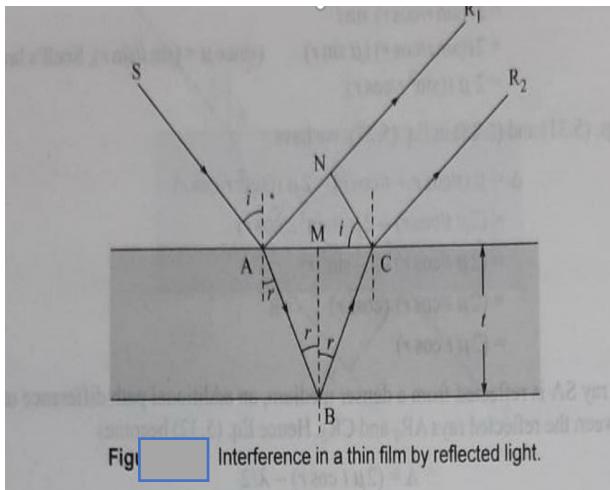
$$2(\beta) \sin\alpha = \lambda$$

Spacing $\beta = ()$

=

$\beta =$

where α is small and measured in radians.



Interference in a Thin Film by Reflected Light:

Fig. [] Interference in a thin film by reflected light.

Let us consider a thin transparent film of thickness t and refractive index μ (Fig 1). When a light ray SA is incident on the film, it is partly reflected along AN and partly refracted along AB . Point B of lower surface of thin film, the light ray AB is reflected along BC . Finally, it emerges along R_2 . Since the reflected rays are derived from the same incident ray SA they act as coherent light rays and produce an interference pattern. Let i and r be the angle of incidence and refraction.

Let us draw two normal BM and CN on AC and BC . The path difference between the reflected rays will be

$$\Delta = \text{path } ABC \text{ in thin film} - \text{path } AN \text{ in air}$$

$$\Delta = \mu(AB + BC) - AN \quad \dots \dots \dots (1)$$

In the right angled triangles AMB and CMB , we have

$$\cos r =$$

$$AB = \quad \& \quad BC =$$

$$AB = \quad \& \quad BC = \quad \dots \dots \dots (2) \quad \text{since } BM = r$$

Also right angled triangle ANC ,

$$\sin i =$$

$$AN = AC \sin i$$

$$AN = (AM + MC) \sin i \quad \dots \dots \dots (3)$$

Again from right-angle triangles AMB & CMB ,

$$\tan r = \quad \& \quad \tan r =$$

$$AM = MB \tan r \quad \& \quad CM = MB \tan r$$

$$AM = t \tan r \quad \& \quad CM = t \tan r$$

Therefore,

$$AN = (t \tan r + t \tan r) \sin i$$

$$AN = (2t \tan r) \sin i$$

$$AN = 2t (\mu \sin r)$$

= $2t (\mu \sin r)$ (Snell's law)

$$=2\mu t (\mu)$$

From equations **ttt**

$$\Delta = \mu (\mu) - 2\mu t (\mu)$$

$$= [2\mu t (\mu)$$

=

= [

$$=2\mu t \cos r$$

Since the ray SA is reflected from a denser medium an additional path difference of is produced between the reflected rays and

Hence equation () becomes

$$\Delta = (2\mu t \cos r) - .$$

This is the actual path difference produced between the interfering reflected rays

Condition for constructive interference and destructive interference:

For constructive interference ,the path difference will be an even multiple of .

$$\Delta = 2n(\lambda), \quad \text{where } n=0,1,2,3....$$

$$\text{Or } (2\mu t \cos r) - = 2n(\lambda)$$

$$\text{Or } (2\mu t \cos r) = 2n(\lambda) +$$

$$\text{Or } 2\mu t \cos r = (2n+1)\lambda$$

This is the required condition for constructive interference. In this case the film will appear bright.

For destructive interference the path difference will be an odd multiple of

$$\Delta = (2n-1)(\lambda), \quad \text{where } n=1,2,3....$$

$$\text{Or } (2\mu t \cos r) - = (2n-1)(\lambda)$$

$$\text{Or } (2\mu t \cos r) = n\lambda$$

This is the required condition for destructive interference. In this case the film will appear dark.

INTERFERENCE IN A THIN FILM BY TRANSMITTED LIGHT:

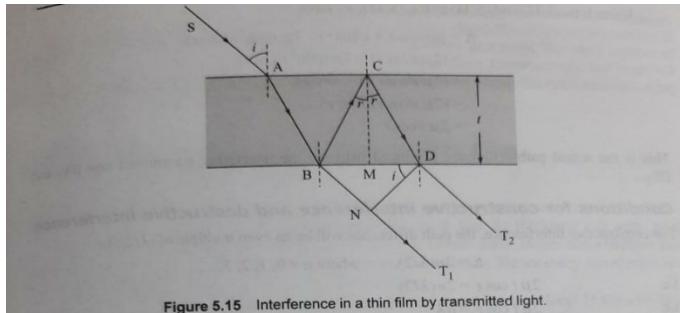


Figure 5.15 Interference in a thin film by transmitted light.

Let us consider a thin transparent film of uniform thickness t and refractive index μ . When a light ray SA is incident on the film, it is partly transmitted along AB. At point B of lower surface of thin film the light ray is partly reflected along BC and partly transmitted along . At point C of upper surface of thin film, the light ray BC is partly reflected along CD. Finally, it emerges along . Since the transmitted rays and are derived from the same incident ray SA, and they act as coherent light rays and produce interference pattern. Let i and r be the angle of incidence and refraction respectively.

Let us draw two normals CM and DN on BD and from C & D respectively. The path difference between the transmitted rays and , is

$$\Delta = \text{path BCD in thin film} - \text{path BN in air}$$

$$= \mu(BC + CD) - BN \quad \dots \dots \dots (1)$$

In right angle triangles CMB and CMD,

$$\cos r = \quad \& \quad \cos r =$$

$$BC = \quad \& \quad CD =$$

$$BC = \quad \& \quad CD =$$

$$\text{since } CM = t \quad \dots \dots \dots (2)$$

Also, in the right- angle triangle BND,

$$\sin i =$$

$$BN = BD \sin i$$

$$= (BM + MD) \sin i$$

Now again in right angle triangles CMB and CMD

$$\tan r = \quad \& \quad \tan r =$$

$$BM = CM \tan r \quad \& \quad MD = CM \tan r$$

$$BM = t \tan r \quad \& \quad MD = t \tan r \quad \dots \dots \dots ()$$

Using equation () in equation ()

$$BN = (t \tan r + t \tan r) \sin i$$

$$= (2t \tan r) \sin i$$

$$= (2t) (\mu \sin r)$$

since, Snell's law $\mu =$

$$= 2\mu t$$

Using equation() & () in equation ()

Path difference Δ will be

$$\Delta = \mu() - 2\mu t$$

$$= -2\mu t$$

$$=[1 -]$$

$$= []$$

=

This is the actual path difference produced between the interfering transmitted rays and .

CONDITIONS FOR CONSTRUCTIVE INTERFERENCE AND DESTRUCTIVE INTERFERENCE:

For constructive interference, the path difference will be an even multiple of (),

$$\Delta=2n(), \text{ where } n=0,1,2,3....$$

$$2\mu t \cos r = 2n()$$

$$2\mu t \cos r = n\lambda$$

This is the required condition for constructive interference. In this case the film will appear bright.

For destructive interference , the path difference will be an odd multiple of (),

$$\Delta=(2n-1)(), \text{ where } n=1,2,3....$$

$$2\mu t \cos r = (2n-1)()$$

This is the required condition for destructive interference. In this case the film will appear dark.

Difference between the interference patterns produced by reflection and refraction in a Thin Film:

In a reflected system, there is an additional path difference of () between the rays producing interference as one of the rays suffers reflection at a denser medium, while in the transmitted system it is not so. Thus for the same path difference in the reflected system a bright band will correspond to a dark band in the transmitted system and vice versa. Thus, the two systems are complementary.

Interference in a wedge-shaped Thin Film by Reflected Light:



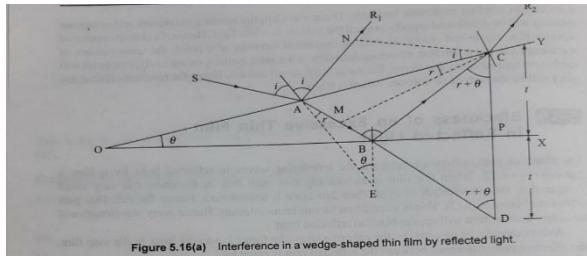


Figure 5.16(a) Interference in a wedge-shaped thin film by reflected light.

Let us consider a thin transparent wedge-shaped film of varying thickness and refractive index μ as shown in fig (1). When a light ray SA is incident on the thin film it is partly reflected along AN and partly refracted along AB . At point B of lower surface of thin film, the light ray AB is reflected along BC finally it emerges along . Since the reflected rays and are derived from the same incident ray SA, they act as coherent light rays and produce an interference pattern.

Let us draw two normals CM & CN on AB & respectively from C. The path difference between the reflected rays and is given by

$$\Delta = \text{path ABC in thin film} - \text{path AN in air}$$

$$= \mu(AB+BC)$$

$$= \mu(AM+MB+BC)-AN \quad \dots\dots\dots(1)$$

Let i and r be the angles of incidence and refraction. From Snell's Law,

$$\mu = \frac{\sin i}{\sin r}$$

$$AN = \mu AM \quad \dots\dots\dots(2)$$

Using equation (1) & (2)

$$\Delta = \mu(AM+MB+BC) - (\mu AM)$$

$$= \mu(MB+BC) \quad \dots\dots\dots(3)$$

Let us draw a normal CP on lower surface OX, and extending CP & AB, they meet at a point D.

In right angle triangles CPB & DPB,

$$CP = PD = t, BC = BD$$

$$\& \text{angle } BPD = \text{angle } BPC =$$

Hence from geometry,

$$\text{angle } BDP = \text{angle } BCP = (r+\theta)$$

Then equation () will be

$$\Delta = \mu(MB+BD)$$

$$= \mu(MD) \quad \dots\dots\dots()$$

In right angled triangle CMD, we have

$$\cos(r+\theta) =$$

$$MD = CD \cos(r+\theta)$$

$$=2t \cos(r+\theta) \quad \dots \dots \dots (1)$$

Eliminating MD from equation(1) & (2)

$$\Delta = 2\mu t \cos(r+\theta) \quad \dots \dots \dots (2)$$

Condition for constructive interference and destructive interference:

For constructive interference, the path difference will be an even multiple of .

$$\Delta = 2n, \quad \text{where } n=0,1,2,3\dots$$

$$\text{Or } 2\mu t \cos(r+\theta) - = 2n \quad (1)$$

$$\text{Or } 2\mu t \cos(r+\theta) = 2n + \quad (2)$$

$$\text{Or } 2\mu t \cos(r+\theta) = (2n+1) \quad (3)$$

This is the required condition for constructive interference. In this case the film will appear bright.

For destructive interference the path difference will be an odd multiple of

$$\Delta = (2n-1), \quad \text{where } n=1,2,3\dots$$

$$\text{Or } 2\mu t \cos(r+\theta) - = (2n-1) \quad (1)$$

$$\text{Or } 2\mu t \cos(r+\theta) = n\lambda$$

This is the required condition for destructive interference. In this case the film will appear dark.

Fringe-width in a wedge-shaped film:

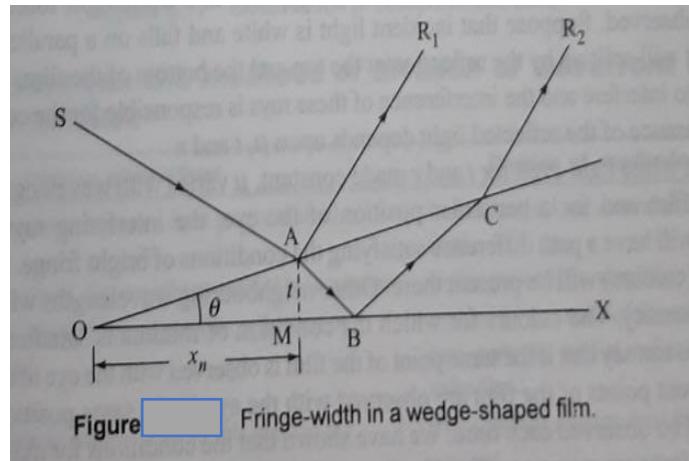


Figure [] Fringe-width in a wedge-shaped film.

Let us consider a thin transparent wedge-shaped film of varying thickness and refractive index μ as shown in fig(1). Suppose R_1 & R_2 are the light rays formed by the reflection of an incident ray SA from the upper and the lower surface of the film. Let AM be the normal on OX & $AM=t$.

The condition for dark fringe (destructive interference) at A is given by $2\mu t \cos(r+\theta) = n\lambda \quad \dots \dots \dots (1)$

Let the dark ring be located at M at a distance x_n from the wedge O of the film. Then in ΔAMO , we have

$$\tan \theta =, t = \tan \theta \quad \dots \dots \dots (2)$$

eliminating t from equation()&()

$$2\mu \tan \theta \cos (r+\theta) = n\lambda \quad \dots \dots \dots \quad (8)$$

Similarly, if the $(n+1)$ th dark ring is located at a distance from the wedge O of the film then equation (),

$$2\mu \tan \theta \cos (r+\theta) = (n+1)\lambda \quad \dots \dots \dots \quad (9)$$

Subtracting equation() from ()

$$2\mu - \tan \theta \cos (r+\theta) = (n+1)\lambda - n\lambda$$

$$2\mu - \tan \theta \cos (r+\theta) = \lambda$$

Hence the separation between the two successive dark fringes, or the fringe width will be $W = \frac{\lambda}{2\mu}$.

This is independent of n. It shows that all the bright fringes have the same fringe width. Similarly all the dark fringes have the same fringe width. Let the dark ring be located at M at a distance from the wedge O of the film. Then in ΔAMO , we have

$$\tan \theta =$$

$$t = \tan \theta \quad \dots \dots \dots \quad (9)$$

eliminating t from equation (8) & (9)

$$2\mu \tan \theta \cos (r+\theta) = n\lambda \quad \dots \dots \dots \quad (10)$$

Similarly, if the $(n+1)$ th dark ring is located at a distance from the wedge O of the film then equation (10),

$$2\mu \tan \theta \cos (r+\theta) = (n+1)\lambda \quad \dots \dots \dots \quad (11)$$

Subtracting equation(10) from (11)

$$2\mu - \tan \theta \cos (r+\theta) = (n+1)\lambda - n\lambda$$

$$2\mu - \tan \theta \cos (r+\theta) = \lambda$$

$=$

NEWTON'S RINGS:

When a plano-convex lens of long focal length is placed on a glass plate, a thin film of air is enclosed between the lower surface of the lens and the upper surface of the plate. The thickness of the air film is negligible at the point of contact and it gradually increases from the center towards the periphery. When the film is illuminated by a monochromatic light, the concentric dark and bright circular fringes with dark center are observed in the interference pattern. Since Newton was the first to obtain experimentally these circular fringes, these are known as Newton's rings.

???

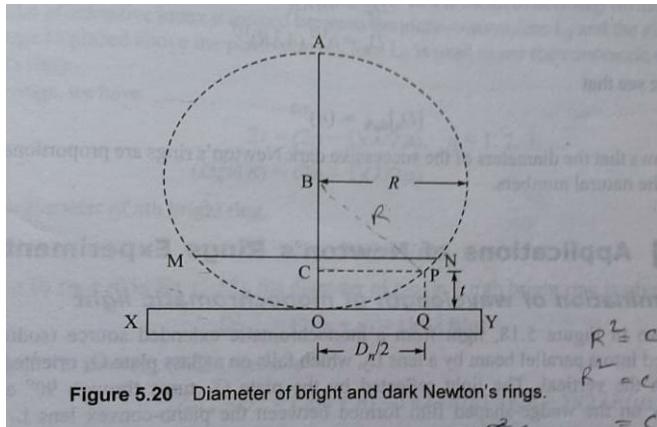


Figure 5.20 Diameter of bright and dark Newton's rings.

from geometry, =

=

=

= [

=

=

=4

Since, $t \ll R$

$= 8Rt$

$2t = ()$

For bright rings, $2t = [2n-1]()$ $n=1, 2, 3, \dots$

$= () = [2n-1]()$

i.e., =

thus, for bright fringe

The diameters of the successive bright Newton's rings are proportional to the square root of the odd natural numbers. For dark rings $2t = n\lambda$ ($n=0, 1, 2, \dots$)

$= () = n\lambda$

$= 4Rn\lambda$

=

Thus, for dark fringe, this shows that the diameters of the successive dark Newton's rings are proportional to the square root of the natural numbers.
