5

D.C. Motors

INTRODUCTION

D. C. motors are seldom used in ordinary applications because all electric supply companies furnish alternating current. However, for special applications such as in steel mills, mines and electric trains, it is advantageous to convert alternating current into direct current in order to use d.c. motors. The reason is that speed/torque characteristics of d.c. motors are much more superior to that of a.c. motors. Therefore, it is not surprising to note that for industrial drives, d.c. motors are as popular as 3-phase induction motors. Like d.c. generators, d.c. motors are also of three types *viz.*, series-wound, shunt-wound and compound-wound. The use of a particular motor depends upon the mechanical load it has to drive.

5.1 D.C. MOTOR PRINCIPLE

A machine that converts d.c. power into mechanical power is known as a d.c. motor. Its operation is based on the principle that when a current carrying conductor is palced in a magnetic field, the conductor experiences a mechanical force. The direction of this force is given by Fleming's left hand rule and magnitude is given by ;

$$F = B I l \text{ newtons}$$

Basically, there is no constructional difference between a d.c. motor and a d.c. generator. The same d.c. machine can be run as a generator or motor.

5.2 WORKING OF D.C. MOTOR

Consider a part of a multipolar d.c. motor as shown in Fig. 5.1. When the terminals of the motor are connected to an external source of d.c. supply :

(*i*) the field magnets are excited developing alternate *N* and *S* poles.

(*ii*) the armature conductors carry *currents. All conductors under *N*-pole carry currents in one direction while all the conductors under *S*-pole carry currents in the opposite direction.

Suppose the conductors under *N*-pole carry currents into the plane of the paper and those under *S*-pole carry currents out of the plane of the paper as shown in Fig. 5.1. Since each armature conductor is carrying current and is placed in the magnetic field, mechanical force acts on it. Referring to Fig. 5.1 and applying Fleming's left hand rule, it is clear that force on each conductor is tending to rotate the armature in anticlockwise direction. All these forces add together to produce a driving torque which sets the armature rotating. When the conductor moves from one side of a brush to the other, the current in that conductor is reversed and at the same time it comes under the influence of next pole which is of opposite polarity. Consequently, the direction of force on the conductor remains the same.

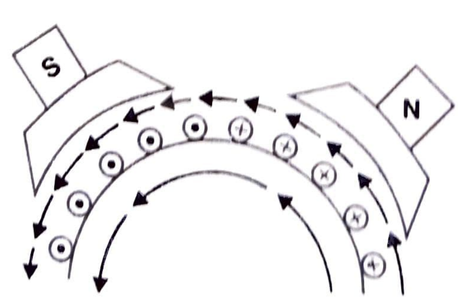


Fig. 5.1

* Since the armature of a motor is the same as that of generator, the current from the supply line must divide and pass through several paths of the armature—2 for wave wound armature and *P* for lap wound armature. As in a generator, currents through all conductors under a pole will be in the same direction.

5.3 BACK OR COUNTER E.M.F.

When the armature of a d.c. motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence e.m.f. is induced in them as in a generator. The induced e.m.f. acts in opposite direction to the applied voltage V (*Lenz's law) and is known as *back or counter e.m.f.* E_b. The back e.m.f. $E_b (= P \phi Z N/60 A)$ is *always* less than the applied voltage V although this difference is small when the motor is running under normal conditions.

Consider a shunt wound motor shown in Fig. 5.2. When d.c. voltage V is applied across the motor terminals, the field magnets are excited and armature conductors are supplied with current. Therefore, driving torque acts on the armature which begins to rotate. As the armature rotates, back e.m.f. E_b is induced which opposes the applied voltage V. The applied voltage V has to force current through the armature against the back e.m.f. E_b. The electric work done in overcoming and causing the current to flow against E_b is converted into mechanical energy developed in the armature. It follows, therefore, that energy conversion in a d.c. motor is only possible due to the production of back e.m.f. E_b.

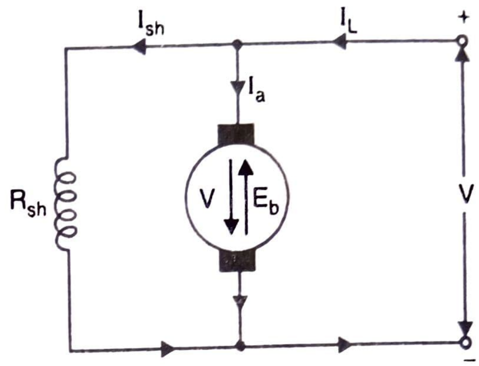


Fig. 5.2

Net voltage across armature circuit $= V - E_b$

If R_a is the armature circuit resistance, then, $I_a = \dfrac{V - E_b}{R_a}$

Since V and R_a are usually fixed, the value of E_b will determine the current drawn by the motor. If the speed of the motor is high, then back e.m.f. $E_b (= P \phi Z N/60 A)$ is large and hence the motor will draw less armature current and *vice-versa*.

5.4 SIGNIFICANCE OF BACK E.M.F.

The presence of back e.m.f. makes the d.c. motor a *self-regulating* machine *i.e.*, it makes the motor to draw as much armature current as is just sufficient to develop the torque required by the load.

$$\text{Armature current, } I_a = \frac{V - E_b}{R_a}$$

(*i*) When the motor is running on no load, small torque is required to overcome the friction and windage losses. Therefore, the armature current I_a is small and the back e.m.f. is nearly equal to the applied voltage.

(*ii*) If the motor is suddenly loaded, the first effect is to cause the armature to slow down. Therefore, the speed at which the armature conductors move through the field is reduced and hence the back e.m.f. E_b falls. The decreased back e.m.f. allows a larger current to flow through the armature and larger current means increased driving torque. Thus, the driving torque increases as the motor slows down. The motor will stop slowing down when the armature current is just sufficient to produce the increased torque required by the load.

(*iii*) If the load on the motor is decreased, the driving torque is momentarily in excess of the requirement so that armature is accelerated. As the armature speed increases, the back e.m.f. E_b also increases and causes the armature current I_a to decrease. The motor will stop accelerating when the armature current is just sufficient to produce the reduced torque required by the load.

* According to Lenz's law, the direction of induced e.m.f. is such that it opposes the cause producing it. The cause producing the back e.m.f. E_b is the applied voltage V. Hence E_b opposes the applied voltage V.

It follows, therefore, that back e.m.f. in a d.c. motor regulates the flow of armature current i.e., it automatically changes the armature current to meet the load requirement.

5.5 VOLTAGE EQUATION OF D.C. MOTOR

Let in a d.c. motor (See Fig. 5.3),

V = applied voltage

E_b = back e.m.f.

R_a = armature resistance

I_a = armature current

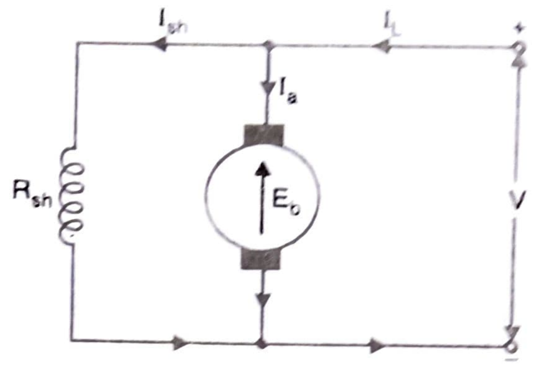


Fig. 5.3

Since back e.m.f. E_b acts in opposition to the applied voltage V, the net voltage across the armature circuit is $V - E_b$. The armature current I_a is given by ;

$$I_a = \frac{V - E_b}{R_a}$$

or

$$V = E_b + I_a R_a \qquad ...(i)$$

This is known as voltage equation of the d.c. motor.

5.6 POWER EQUATION

If eq. (*i*) above is multiplied by I_a throughout, we get,

$$V I_a = E_b I_a + I_a^2 R_a$$

This is known as power equation of the d.c. motor.

$V I_a$ = electric power supplied to armature (*armature input*)

$E_b I_a$ = *power developed by armature (*armature output*)

$I_a^2 R_a$ = electric power wasted in armature (*armature Cu loss*)

Thus out of the armature input, a small portion (about 5%) is wasted as $I_a^2 R_a$ and the remaining portion $E_b I_a$ is converted into mechanical power within the armature.

5.7 CONDITION FOR MAXIMUM POWER

The mechanical power developed by the motor is $P_m = E_b I_a$

Now,
$$P_m = V I_a - I_a^2 R_a$$

Since, V and R_a are fixed, power developed by the motor depends upon armature current. For maximum power, dP_m/dI_a should be zero.

$$\therefore \quad \frac{dP_m}{dI_a} = V - 2 I_a R_a = 0$$

or
$$I_a R_a = V/2$$

Now,
$$V = E_b + I_a R_a = E_b + V/2 \qquad [\because I_a R_a = V/2]$$

$$\therefore \quad E_b = V/2$$

Hence mechanical power developed by the motor is maximum when back e.m.f. is equal to half the applied voltage.

Limitations : In practice, we never aim at achieving maximum power due to the following reasons :

(*i*) The armature current under this condition is very large — much excess of rated current of the machine.

* This will be the mechanical power output in watts.

(*ii*) Half of the input power is wasted in the armature circuit. In fact, if we take into account other losses (iron and mechanical), the efficiency will be well below 50%.

5.8 TYPES OF D.C. MOTORS

Like generators, there are three types of d.c. motors characterised by the connections of field winding in relation to the armature *viz.* :

(*i*) **Shunt-wound motor** in which the field winding is connected in parallel with the armature [See Fig. 5.4]. The current through the shunt field winding is not the same as the armature current. Shunt field windings are designed to produce the necessary m.m.f. by means of a relatively large number of turns of wire having high resistance. Therefore, shunt field current is relatively small compared with the armature current.

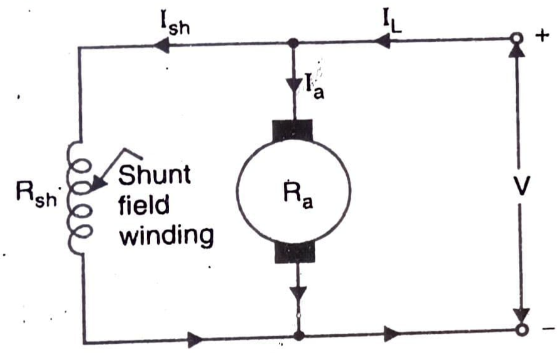


Fig. 5.4

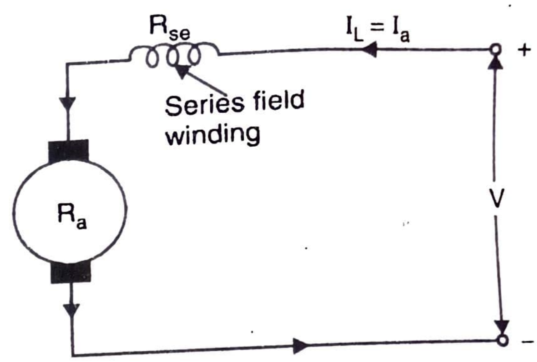
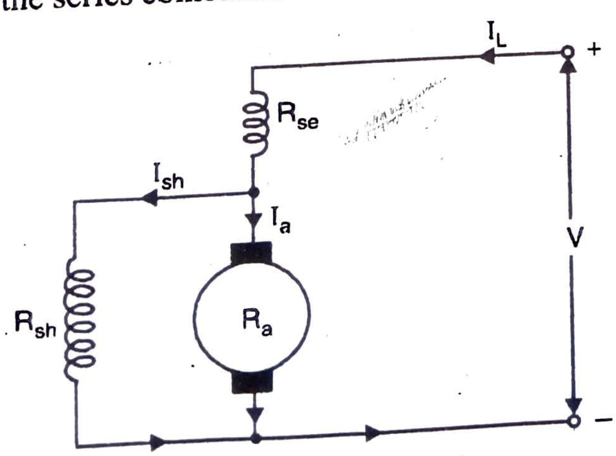


Fig. 5.5

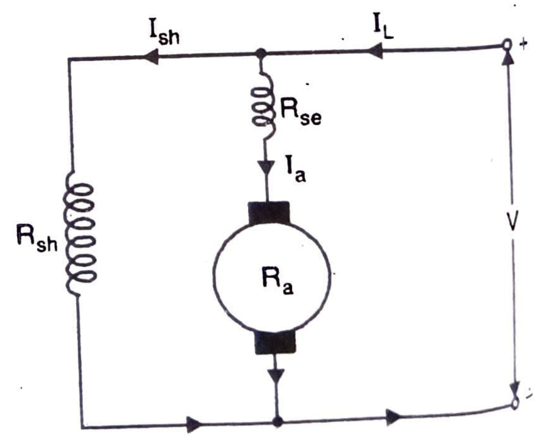
(*ii*) **Series-wound motor** in which the field winding is connected in series with the armature [See Fig. 5.5]. Therefore, series field winding carries the armature current. Since the current passing through a series field winding is the same as the armature current, series field windings must be designed with much fewer turns than shunt field windings for the same m.m.f. Therefore, a series field winding has a relatively small number of turns of thick wire and, therefore, will possess a low resistance.

(*iii*) **Compound-wound motor** which has two field windings ; one connected in parallel with the armature and the other in series with it. There are two types of compound motor connections (like generators). When the shunt field winding is directly connected across the armature terminals [See Fig. 5.6], it is called *short-shunt connection*. When the shunt winding is so connected that it shunts the series combination of armature and series field [See Fig. 5.7], it is called *long-shunt connection*.



Short-shunt connection

Fig. 5.6



Long-shunt connection

Fig. 5.7

The compound machines (generators or motors) are always designed so that the flux produced by shunt field winding is considerably larger than the flux produced by the series field winding. Therefore, shunt field in compound machines is the basic dominant factor in the production of the magnetic field in the machine.

Example 5.1. *A 250 V shunt motor takes a total current of 20 A. The shunt field and armature resistances are 200 Ω and 0·3 Ω respectively. Determine (i) value of back e.m.f. (ii) gross mechanical power in the armature.*

Solution.

(i)

$$\text{Shunt current, } I_{sh} = 250/200 = 1{\cdot}25 \text{ A}$$

$$\text{Armature current, } I_a = 20 - 1{\cdot}25 = 18{\cdot}75 \text{ A}$$

$$\text{Back e.m.f., } E_b = V - I_a R_a = 250 - 18{\cdot}75 \times 0{\cdot}3 = \mathbf{244{\cdot}4 \text{ V}}$$

(ii)

$$\text{Mechanical power developed} = E_b I_a = 244{\cdot}4 \times 18{\cdot}75 = \mathbf{4582{\cdot}5 \text{ W}}$$

Example 5.2. *A 230 V motor has an armature circuit resistance of 0·6 Ω. If the full-load armature current is 30 A and no-load armature current is 4A, find the change in back e.m.f. from no load to full-load.*

Solution.

$$E_b = V - I_a R_a$$

$$\text{At no-load, } E_b = 230 - 4 \times 0{\cdot}6 = 227{\cdot}6 \text{ V}$$

$$\text{At full-load, } E_b = 230 - 30 \times 0{\cdot}6 = 212 \text{ V}$$

$$\text{Change in back e.m.f.} = 227{\cdot}6 - 212 = \mathbf{15{\cdot}6 \text{ V}}$$

Example 5.3. *A 4-pole, 500 V shunt motor has 720 wave-connected conductors in the armature. The full-load armature current is 60 A and the flux per pole is 0·03 Wb. The armature resistance is 0·2 Ω and the contact drop is 1 V per brush. Calculate the full-load speed of the motor.*

Solution.

$$V = E_b + I_a R_a + \text{Brush drop}$$

$$500 = E_b + 60 \times 0{\cdot}2 + 2 \times 1$$

$$\therefore \quad E_b = 500 - 60 \times 0{\cdot}2 - 2 \times 1 = 486 \text{ V}$$

Now

$$E_b = \frac{P \phi Z N}{60 A}$$

$$\therefore \quad N = \frac{E_b \times 60 A}{P \phi Z} = \frac{486 \times 60 \times 2}{4 \times 0{\cdot}03 \times 720} = \mathbf{675 \text{ r.p.m.}}$$

Example 5.4. *The counter e.m.f. of a shunt motor is 227 V, the field resistance is 160 Ω and field current is 1·5 A. If the line current is 39·5 A, find the armature resistance. Also find the armature current when the motor is stationary.*

Solution.

$$\text{Applied voltage, } V = I_{sh} R_{sh} = 1{\cdot}5 \times 160 = 240 \text{ volts}$$

$$\text{Armature current, } I_a = I_L - I_{sh} = 39{\cdot}5 - 1{\cdot}5 = 38 \text{ A}$$

$$V = E_b + I_a R_a$$

$$\therefore \quad R_a = \frac{V - E_b}{I_a} = \frac{240 - 227}{38} = \mathbf{0{\cdot}342 \ \Omega}$$

At the moment of start-up, the armature is stationary so that $E_b = 0$.

$$\therefore \quad I_a = \frac{V}{R_a} = \frac{240}{0{\cdot}342} = \mathbf{701{\cdot}5 \text{ A}}$$

Example 5.5. *A 440 V shunt motor has armature resistance of 0·8 Ω and field resistance of 200 Ω. Determine the back e.m.f. when giving an output of 7·46 kW at 85% efficiency.*

Solution.

$$\text{Motor input power, } P_i = \frac{7{\cdot}46 \times 10^3}{\eta} = \frac{7{\cdot}46 \times 10^3}{0{\cdot}85} \text{ W}$$

$$\text{Motor line current, } I_L = \frac{P_i}{V} = \frac{7{\cdot}46 \times 10^3}{0{\cdot}85 \times 440} = 19.95 \text{ A}$$

$$\text{Shunt field current, } I_{sh} = \frac{V}{R_{sh}} = \frac{440}{200} = 2{\cdot}2 \text{ A}$$

$$\text{Armature current, } I_a = I_L - I_{sh} = 19{\cdot}95 - 2{\cdot}2 = 17{\cdot}75 \text{ A}$$

Now,

$$E_b = V - I_a R_a = 440 - 17{\cdot}75 \times 0{\cdot}8 = \mathbf{425{\cdot}8 \text{ V}}$$

Example 5.6. *A 20 kW, 250V d.c. shunt generator has armature and field resistances of 0·1 Ω and 125 Ω respectively. Calculate the total armature power developed when running (i) as a generator delivering 20 kW output (ii) as a motor taking 20 kW input.*

Solution.

(*i*) **As a generator.** Fig. 5.8 (*i*) shows the connections of shunt generator.

$$I_L = 20 \times 10^3/250 = 80 \text{ A}$$
$$I_{sh} = 250/125 = 2 \text{ A}$$
$$I_a = I_L + I_{sh} = 80 + 2 = 82 \text{ A}$$
$$E_g = V + I_a R_a = 250 + 82 \times 0{\cdot}1 = 258{\cdot}2 \text{ V}$$

∴ Power developed in the armature

$$= E_g I_a = 258{\cdot}2 \times 82 = 21{\cdot}17 \times 10^3 \text{ W} = \mathbf{21{\cdot}17 \text{ kW}}$$

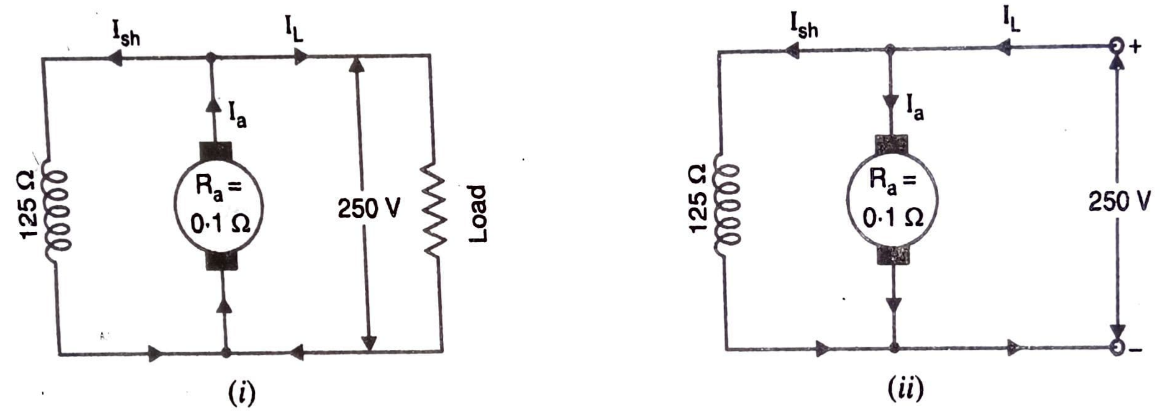


Fig. 5.8

(*ii*) **As a motor.** Fig. 5.8 (*ii*) shows the connections of shunt motor.

$$I_L = 20 \times 10^3/250 = 80 \text{ A}$$
$$I_{sh} = 250/125 = 2 \text{ A}$$
$$I_a = I_L - I_{sh} = 80 - 2 = 78 \text{ A}$$
$$E_b = V - I_a R_a = 250 - 78 \times 0{\cdot}1 = 242{\cdot}2 \text{ V}$$

$$\text{Power developed in the armature} = E_b I_a = 242{\cdot}2 \times 78 = 18{\cdot}9 \times 10^3 \text{ W} = \mathbf{18{\cdot}9 \text{ kW}}$$

Example 5.7. *Find the useful flux per pole on no-load of 250 V, 6-pole shunt motor having a wave-connected armature winding with 110 turns. The armature resistance is 0.2 Ω. The armature current is 13·3 A at the no-load speed of 908 r.p.m.*

Solution.

$$E_b = V - I_a R_a = 250 - 13.3 \times 0.2 = 247{\cdot}34 \text{ V}$$

Now

$$E_b = \frac{P \phi Z N}{60\, A}$$

∴

$$\text{Flux/pole, } \phi = \frac{E_b \times 60\, A}{P Z N}$$

Here

$$E_b = 247{\cdot}34 \text{ V}; A = 2; P = 6; Z = 110 \times 2 = 220; N = 908 \text{ r.p.m.}$$

∴

$$\phi = \frac{247{\cdot}34 \times 60 \times 2}{6 \times 220 \times 908} = 24{\cdot}8 \times 10^{-3} \text{ Wb} = \mathbf{24.8 \text{ mWb}}$$

Example 5.8. *A 250 V shunt motor on no-load runs at 1000 r.p.m. and takes 5A. The total armature and shunt field resistances are 0.2 Ω and 250 Ω respectively. Calculate the speed when loaded and taking current of 50 A if armature reaction weakens the field by 3%.*

Solution.

At no-load. At no-load (See Fig. 5.9),

$$I_{sh} = \frac{250}{250} = 1\text{ A}; I_{a1} = 5-1 = 4\text{ A}; N_1 = 1000\text{ r.p.m.}$$

$\therefore$ Back e.m.f. at no-load, $E_{b1} = V - I_{a1}R_a = 250 - 4\times 0{\cdot}2 = 249{\cdot}2\text{V}$

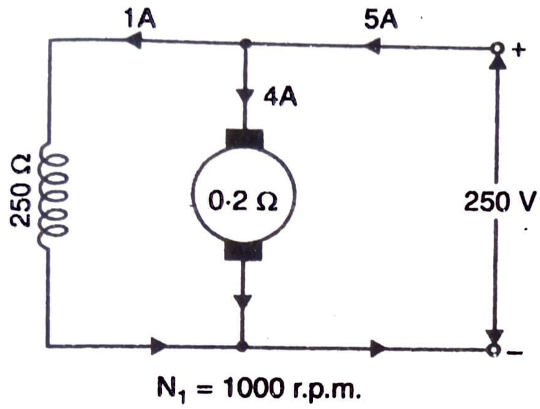


Fig. 5.9

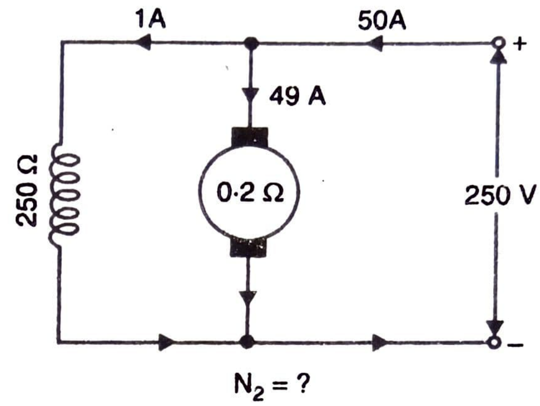


Fig. 5.10

At load. At load (See Fig. 5.10),

$$I_{sh} = \frac{250}{250} = 1\text{ A}; I_{a2} = 50-1 = 49\text{ A}; N_2 = ?$$

$\therefore$ Back e.m.f. at load, $E_{b2} = V - I_{a2}R_a = 250 - 49\times 0{\cdot}2 = 240{\cdot}2\text{V}$

Now $$\frac{E_{b2}}{E_{b1}} = \frac{N_2\,\phi_2}{N_1\,\phi_1} \qquad (\because E_b \propto \phi N)$$

$\therefore$ $$N_2 = \frac{E_{b2}}{E_{b1}} \times \frac{N_1\,\phi_1}{\phi_2}$$

$$= \frac{240{\cdot}2}{249{\cdot}2}\times 1000 \times \frac{1}{0{\cdot}97} = \mathbf{994\ r.p.m.} \qquad (\because \phi_2 = 0{\cdot}97\phi_1)$$

TUTORIAL PROBLEMS

1. A 220 V shunt motor takes a total current of 20 A. The shunt field resistance is 250 Ω while armature resistance is 0·3 Ω. Calculate the back e.m.f. **[214.26V]**
2. The armature of a d.c. machine has a resistance of 0·1 Ω and is connected to a 230 V supply. Calculate the back e.m.f. when it is running (*i*) as a generator giving 80 A (*ii*) as a motor taking 80 A. **[(*i*) 238 V (*ii*) 222 V]**
3. A 4-pole d.c. motor is connected to a 500 V d.c. supply and takes an armature current of 80 A. The resistance of the armature circuit is 0·4 Ω. The armature is wave wound with 522 conductors and useful flux per pole is 0·025 Wb. Determine the speed of the motor. **[1075 r.p.m.]**
4. A 4-pole motor is fed at 440 V and takes an armature current of 50 A. The resistance of the armature circuit is 0·28 Ω. The armature winding is wave-connected with 888 conductors and useful flux per pole is 0·023 Wb. Calculate the speed of the motor. **[626 r.p.m.]**
5. A 220 V d.c. machine has an armature resistance of 0·5 Ω. If the full-load armature current is 20 A, find the induced e.m.f. when the machine acts as a generator and as a motor. **[230 V ; 210V]**

5.9 ARMATURE TORQUE OF D.C. MOTOR

Torque is the turning moment of a force about an axis and is measured by the product of force (*F*) and radius (*r*) at right angle to which the force acts *i.e.*

$$T = F \times r$$

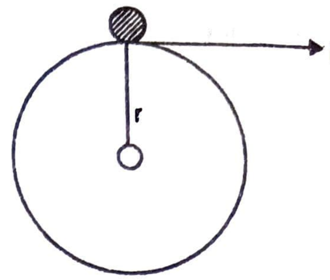


Fig. 5.11

In a d.c. motor, each conductor is acted upon by a circumferential force F at a distance r, the radius of the armature (Fig. 5·11). Therefore, each conductor exerts a torque, tending to rotate the armature. The sum of the torques due to all armature conductors is known as *gross or armature torque* (T_a).

Let in a d.c. motor

r = average radius of armature in m
l = effective length of each conductor in m
Z = total number of armature conductors
A = number of parallel paths
i = current in each conductor = I_a/A
B = average flux density in Wb/m^2
ϕ = flux per pole in Wb
P = number of poles

Force on each conductor, $F = B\,i\,l$ newtons
Torque due to one conductor $= F \times r$ newton- metre
Total armature torque, $T_a = Z\,F\,r$ newton-metre
$= Z\,B\,i\,l\,r$

Now $i = I_a/A$, $B = \phi/a$ where a is the x-sectional area of flux path per pole at radius r. Clearly, $a = 2\pi\,r\,l/P$.

$$\therefore \quad T_a = Z \times (\phi/a) \times (I_a/A) \times l \times r$$

$$= Z \times \frac{\phi}{2\pi\,r\,l/P} \times \frac{I_a}{A} \times l \times r$$

$$= \frac{Z\phi I_a P}{2\pi A} \text{ N-m}$$

or $$T_a = 0{\cdot}159\, Z\,\phi\, I_a\,(P/A) \text{ N-m} \quad ...(i)$$

Since Z, P and A are fixed for a given machine,

$$\therefore \quad T_a \propto \phi I_a$$

Hence *torque in a d.c. motor is directly proportional to flux per pole and armature current.*

(*i*) For a *shunt motor*, flux ϕ is practically constant.

$$\therefore \quad T_a \propto I_a$$

(*ii*) For a *series motor*, flux ϕ is directly proportional to armature current I_a provided magnetic saturation does not take place.

$$\therefore \quad T_a \propto I_a^2 \quad \text{...upto magnetic saturation}$$

Alternative expression for T_a

$$E_b = \frac{P\phi Z N}{60\,A}$$

$$\therefore \quad \frac{P\phi Z}{A} = \frac{60 \times E_b}{N}$$

From eq. (*i*), we get the expression of T_a as :

$$T_a = 0{\cdot}159 \times \left(\frac{60 \times E_b}{N}\right) \times I_a$$

or $$T_a = 9.55 \times \frac{E_b\, I_a}{N} \text{ N-m}$$

Note that developed torque or gross torque means armature torque T_a.

5.10 SHAFT TORQUE (T_{sh})

The torque which is available at the motor shaft for doing useful work is known as *shaft torque*. It is represented by T_{sh}. Fig. 5.12 illustrates the concept of shaft torque.

The total or gross torque T_a developed in the armature of a motor is not available at the shaft because a part of it is lost in overcoming the iron and frictional losses in the motor. Therefore, shaft torque T_{sh} is somewhat less than the armature torque T_a. The difference $T_a - T_{sh}$ is called lost torque.

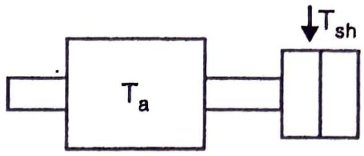


Fig. 5.12

Clearly,

$$T_a - T_{sh} = 9{\cdot}55 \times \frac{\text{Iron and frictional losses}}{N}$$

For example, if the iron and frictional losses in a motor are 1600 W and the motor runs at 800 r.p.m., then,

$$T_a - T_{sh} = 9{\cdot}55 \times \frac{1600}{800} = 19{\cdot}1 \text{ N-m}$$

As stated above, it is the shaft torque T_{sh} that produces the useful output. If the speed of the motor is N r.p.m., then,

$$\text{Output in watts} = \frac{2\pi N T_{sh}}{60}$$

or

$$T_{sh} = \frac{\text{Output in watts}}{2\pi N/60} \text{ N-m}$$

or

$$T_{sh} = 9{\cdot}55 \times \frac{\text{Output in watts}}{N} \text{ N-m} \qquad (\because 60/2\pi = 9{\cdot}55)$$

5.11 BRAKE HORSE POWER (b.h.p.).

The horse power developed by the shaft torque is known as *brake horse power* (*b.h.p.*). If the motor is running at N r.p.m. and the shaft torque is T_{sh} newton-metres, then,

$$\text{W.D./revolution} = \text{force} \times \text{distance moved in 1 revolution}$$

$$= F \times 2\pi r = 2\pi \times T_{sh} \text{ J}$$

$$\text{W.D./minute} = 2\pi N T_{sh} \text{ J}$$

$$\text{W.D./sec.} = \frac{2\pi N T_{sh}}{60} \text{ Js}^{-1} \text{ or watts} = \frac{2\pi N T_{sh}}{60 \times 746} \text{ h.p.}$$

$$\therefore \quad \text{Useful output power} = \frac{2\pi N T_{sh}}{60 \times 746} \text{ h.p.}$$

or

$$\text{b.h.p.} = \frac{2\pi N T_{sh}}{60 \times 746}$$

Example 5.9. *Calculate the value of torque established by the armature of a 4-pole motor having 774 conductors, two paths in parallel, 24 mWb flux per pole, when the total armature current is 50 A.*

Solution.

$$\text{Armature torque, } T_a = 0{\cdot}159\, Z \phi I_a (P/A)$$

Here,

$$Z = 774\,;\ \phi = 24 \times 10^{-3} \text{ Wb}\,;\ I_a = 50 \text{ A}\,;\ P = 4\,;\ A = 2$$

$$\therefore \quad T_a = 0{\cdot}159 \times 774 \times 24 \times 10^{-3} \times 50 \times (4/2) = \mathbf{295{\cdot}35 \text{ N-m}}$$

Example 5.10. *An armature of a 6-pole machine 75cm in diameter has 664 conductors each having an effective length of 30 cm and carrying a current of 100 A. If 70% of total conductors lie simultaneously in the field of average flux density 0·85 Wb/m², calculate (i) armature torque (ii) horse power output at 250 r.p.m.*

Solution.

(*i*) Force on one conductor, $F = B\,i\,l = 0{\cdot}85 \times 100 \times 0{\cdot}3 = 25{\cdot}5$ N

Torque due to one conductor $= F \times r = 25{\cdot}5 \times 0{\cdot}375 = 9{\cdot}56$ N-m

No. of effective conductors, $Z = 664 \times 70/100 = 465$

∴ Total armature torque, $T_a = 9{\cdot}56 \times 465 =$ **4445·4 N-m**

(*ii*) Power output $= \dfrac{2\pi N T_a^*}{60 \times 746}$ h.p.

$$= \frac{2\,\pi \times 250 \times 4445{\cdot}4}{60 \times 746} = \mathbf{156\ h.p.}$$

Example 5.11. *A 230 V d.c. shunt motor takes a current of 40 A and runs at 1100 r.p.m. If armature and shunt field resistances are 0·25 Ω and 230 Ω respectively, find the torque developed by the armature.*

Solution.

Shunt field current, $I_{sh} = 230/230 = 1$A

Armature current, $I_a = 40 - 1 = 39$ A

Back e.m.f., $E_b = V - I_a R_a = 230 - 39 \times 0{\cdot}25 = 220{\cdot}25$ V

Now $T_a = 9{\cdot}55 \times E_b I_a/N$ ...See Art. 5·9

$= 9{\cdot}55 \times (220{\cdot}25 \times 39/1100)$ N-m $=$ **74·66 N-m**

Example 5.12. *A d.c. motor takes an armature current of 110 A at 480 V. The armature circuit resistance is 0·2 Ω. The machine has 6 poles and the armature is lap-connected with 864 conductors. The flux per pole is 0·05 Wb. Calculate (i) the speed and (ii) the gross torque developed by the motor.*

Solution.

(*i*) $E_b = V - I_a R_a = 480 - 110 \times 0{\cdot}2 = 458$ volts

Now $E_b = \dfrac{P\,\phi\,Z\,N}{60\,A}$

or $458 = \dfrac{6 \times 0{\cdot}05 \times 864 \times N}{60 \times 6}$

∴ $N =$ **636 r.p.m.**

(*ii*) Gross torque means armature torque T_a.

$$T_a = 0{\cdot}159\,\phi\,Z\,I_a \times \left(\frac{P}{A}\right) = 0{\cdot}159 \times 0{\cdot}05 \times 864 \times 110 \times \left(\frac{6}{6}\right) = \mathbf{756{\cdot}3\ N\text{-}m}$$

Example 5.13. *A 100 h.p., 500 V shunt motor has 4 poles and a 2 circuit wave winding with 492 armature conductors. The flux is 50 mWb per pole and the full-load efficiency 92%. The armature and commutating field windings have a total resistance of 0·1 Ω. The shunt field resistance is 250 Ω. Calculate for full load (i) the speed (ii) the useful torque.*

Solution.

Fig. 5.13 shows the shunt motor connections. The commutating field winding is connected in series with the armature winding so that it carries the armature current.

(*i*) Motor input power $= \dfrac{100 \times 746}{0{\cdot}92} = 8{\cdot}1 \times 10^4$ W

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* Iron and frictional losses are neglected so that $T_a = T_{sh}$.

$$\text{Line current, } I_L = \frac{8\cdot1\times10^4}{500} = 162\text{ A}$$

$$I_{sh} = \frac{500}{250} = 2\text{ A}$$

$$I_a = I_L - I_{sh}$$

$$= 162 - 2 = 160\text{ A}$$

Now $$E_b = V - I_a R_a$$

$$= 500 - 160\times0\cdot1$$

$$= \mathbf{484\ volts}$$

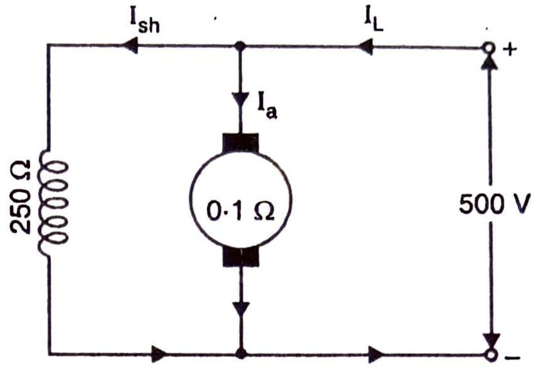

Fig. 5.13

Also $$E_b = \frac{P\phi Z N}{60\,A}$$

or $$484 = \frac{4\times50\times10^{-3}\times492\times N}{60\times2}$$

$\therefore$ $$N = \mathbf{590\ r.p.m.}$$

(*ii*) $$\text{Output power in watts} = \frac{2\pi N T_{sh}}{60}$$

or $$100\times746 = \frac{2\pi\times590\times T_{sh}}{60}$$

$\therefore$ $$T_{sh} = \frac{100\times746\times60}{590\times2\pi} = \mathbf{1200\ N\text{-}m}$$

Example 5.14. *A 220 V d.c. shunt motor runs at 500 r.p.m. when the armature current is 50 A. Calculate the speed if the torque is doubled. The armature resistance of the motor is 0·2 Ω.*

Solution.

$$T \propto \phi I_a.\ \text{Since } \phi \text{ is constant, } T \propto I_a.$$

Therefore, when torque is doubled, armature current drawn by the motor is also doubled.

$\therefore$ $$I_{a1} = 50\text{ A and } I_{a2} = 2\times50 = 100\text{ A}$$

Now $$E_{b1} = V - I_{a1}R_a = 220 - 50\times0\cdot2 = 210\text{ volts}$$

and $$E_{b2} = V - I_{a2}R_a = 220 - 100\times0\cdot2 = 200\text{ volts}$$

Now $$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1} \qquad (\because \phi \text{ is constant})$$

$\therefore$ $$N_2 = \frac{E_{b2}}{E_{b1}}\times N_1 = \frac{200}{210}\times500 = \mathbf{476\ r.p.m.}$$

Example 5.15. *A 240 V, 4-pole shunt motor running at 1000 r.p.m. gives 15 h.p. with an armature current of 50 A and field current of 1 A. The armature winding is wave-connected and has 540 conductors. The armature resistance is 0·1 Ω and the drop at each brush is 1 V. Find (i) the useful torque (ii) the total torque (iii) useful flux per pole and (iv) iron and frictional losses.*

Solution. Fig. 5.14 shows the shunt motor connections.

(*i*) $$\text{Useful power} = 15\times746 = 11190\text{ W}$$

Now $$\frac{2\pi N T_{sh}}{60} = 11190$$

$$T_{sh} = \frac{11190\times60}{2\pi N}$$

$$= \frac{11190\times60}{2\pi\times1000} = \mathbf{106\ N\text{-}m}$$

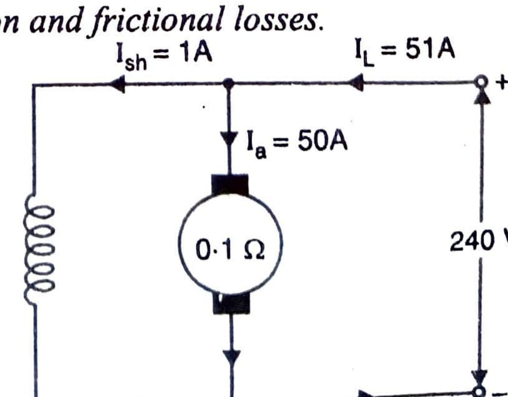

Fig. 5.14

(*ii*) $$\text{Back e.m.f., } E_b = V - I_a R_a - \text{Brush drop}$$