CONTROL SYSTEM

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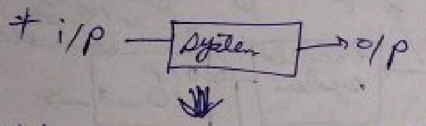
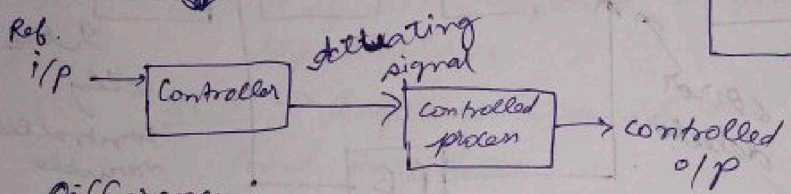
Classification of Control system -

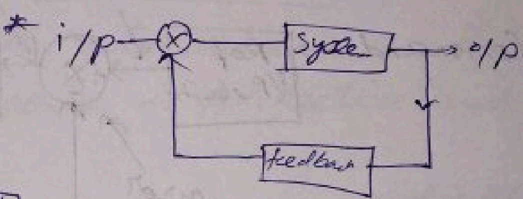
(1) Open loop CS and close loop CS

OPCS

* 



CLCS

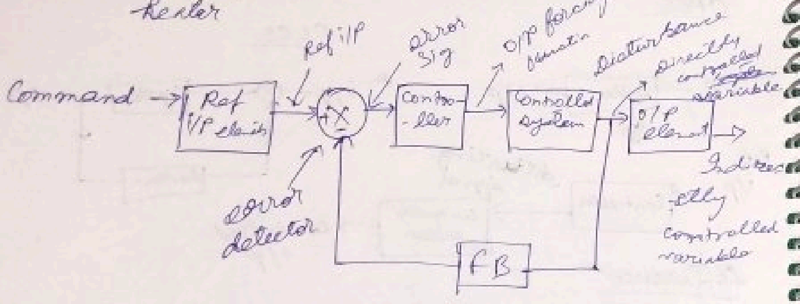
* 

Difference

OPCS	CLCS
* There is no feedback	* There is a f/b betⁿ Ref. i/p and the o/p or desired o/p
* They are generally stable	* They are unstable under certain (examples) condition
* Accuracy is determined by the calibration of their elements. Simple to develop and chip	* More complex, complicated to construct and costly.
* Maintance is simple	* Maintance is complecated.

* Affected by non-lineari-ties
eg: Washing machine fixed time control system, room heater

* Adjust to the effects of non-linearity present the system
eg: servo motor control



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② Linear and non-linear control system
③ Time invariant and time variant
④ Continuous and discrete
→ Constant coefficient } CTS Laplace
• Differential eqⁿ

DTS → Difference eqⁿ } Z-transform

$$y(n) = \sum_{n=1}^{N} y(n-m) + \sum_{n=1}^{M} x(n-m)$$

$$y(n) = y(n-1) + x(n-1) + x(n)$$

⑤ SISO & MIMO (Single i/p Single o/p) (Multiple)

⑥ lumped parameter and distributed parameter CS
LP → CS which can be described by ordinary differential eqⁿ whereas distributed CS is can be described by partial DE

⑦ Determinastic and stocastic
DCS → if the response is predective and repeatative
SCS → Response can't be unpredicatable and random

⑧ Static and dynamic
STS → .

Modelling of a control system (Mathmetical modelling)

(TF is possible for linear system)

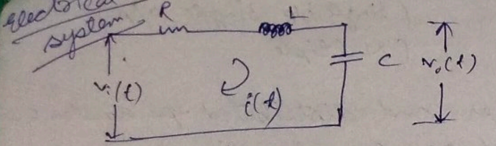
① Depends on transfer function

$$TF = \frac{Y(S) \rightarrow \text{zeros}}{X(S) \rightarrow \text{poles}}$$ provided initial response is zero

↳ It doesn't address
if poles in right → unstable } integrate
left → stable

zero → differentiation

electrical system



When mesh analy variable - current.
when node → voltage

$$v_i(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$

LT $\boxed{V_i(S) = RI(S) + LS\,I(S) + \frac{1}{CS}I(S)} \to ①$

(If there is a differ than at a S is added up co in multi LT and if integrator divided by S)

$$\Rightarrow v_o(t) = \frac{1}{C}\int i(t)dt$$

LT $\boxed{V_o(t) = \frac{1}{CS}I(S)} \to ②$ (o/p across capacitor)

②/① $\frac{V_o(t)}{V_i(t)} = \frac{\frac{1}{CS}I(S)}{RI(S)+LS\,I(S)+\frac{1}{CS}I(S)}$

$$= \frac{1/CS}{R+LS+\frac{1}{CS}}$$

$$= \frac{1}{CS\left(R+LS+\frac{1}{CS}\right)}$$

$$= \frac{1}{\frac{CS^2R+LS^2C+CS}{CS}}$$

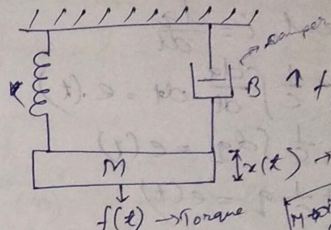
$$= \frac{CS}{C^2S^2R+LS^2C+CS}$$

$$= \frac{1}{CSR+LS+1}$$

Mechanical system

* Translational and rotational system
(Displacement is linear) (angular displacement)

Translational mechanical system



$f_2 = kx$

$f_3 = B\dot{x}$

$f_1 = m\ddot{x}$

$x \to$ displacement

$\dot{x} \to$ vel $\frac{dx}{dt}$ v

$\ddot{x}$ - accl $\frac{d^2x}{dt^2} = a$

$\boxed{\text{M will be replaced by moment of inertia}}$

$\boxed{\text{Torque} \to \text{moment of force, Nm}}$

$$f(t) = f_1 + f_2 + f_3$$

$$\Rightarrow f(t) = m\frac{d^2x(t)}{dt^2} + Kx(t) + \frac{B\,dx(t)}{dt}$$

LS $F(S) = mS^2X(S) + KX(S) + BSX(S)$

$$\Rightarrow \frac{F(S)}{X(S)} = mS^2 + K + BS$$

$$\Rightarrow \frac{X(S)}{F(S)} = \frac{1}{mS^2+K+BS}$$

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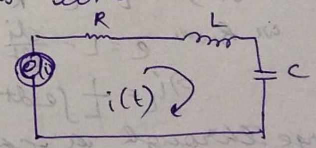
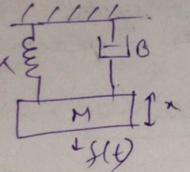
Force Voltage Analog (always series)

Let us consider a series LCR ckt



$$\to L\frac{di}{dt} + Ri + \frac{1}{C}\int i(t)dt = e(t)$$

And a translational mechanical system,



$$\rightarrow M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + K(x) = f(t)$$

We know a current $i = \frac{dq}{dt}$

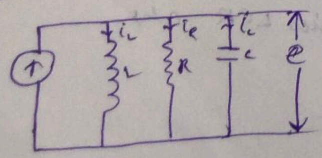
$$\therefore L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}\int\frac{dq}{dt}\,dt = e(t)$$

$$\Rightarrow L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}\int dq = e(t)$$

$$\Rightarrow L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = e(t)$$

Force		Voltage
x (displacement)	$\longrightarrow$	q (charge)
force (F)	$\longrightarrow$	voltage (V)
Mass (M)	$\longrightarrow$	Inductor (L)
B	$\longrightarrow$	R
K	$\longrightarrow$	$\frac{1}{C}$
velocity (v) $(\frac{dx}{dt})$	$\longrightarrow$	current (i) $(\frac{dq}{dt})$

Force current Analogy (alway parallel)



$i(t) = i_L + i_C + i_R$

we know, $e = L\frac{di}{dt}$

$$\Rightarrow i_L = \frac{1}{L}\int e\,dt$$

charge through a capacitor,

$$q = \frac{i}{C}\int i_C\,dt$$

$$\Rightarrow i_C = C\frac{de}{dt}$$

$$i_R R = e \Rightarrow i_R = \frac{1}{R}e$$

Putting them,

$$i(t) = \frac{1}{L}\int e\,dt + C\frac{de}{dt} + \frac{1}{R}e$$

Flux linkage φ can be represented as, $\boxed{e = \frac{d\varphi}{dt}}$

$$\therefore i(t) = \frac{1}{L}\varphi + C\frac{d^2\varphi}{dt^2} + \frac{1}{R}\frac{d\varphi}{dt}$$

Force		Current
Force (F)	$\longrightarrow$	Current (i)
Displacement (x)	$\longrightarrow$	flux linkage (φ)
Mass (M)	$\longrightarrow$	capacitance (C)
B	$\longrightarrow$	$\frac{1}{R}$ (conductance) (B $\rightarrow$ damping)
K (spring constant)	$\longrightarrow$	$\frac{1}{L}$ (reciprocal of inductor)
velocity $(\frac{dx}{dt})$	$\longrightarrow$	voltage $(\frac{d\varphi}{dt})$

Q. Show that the ckt shown in fig B is the analogous ~~network~~ electrical network for the system shown in fig A using force voltage analogy

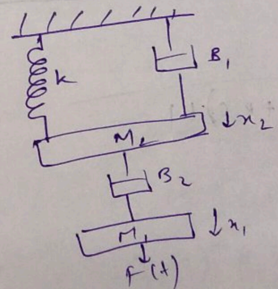


fig (A)

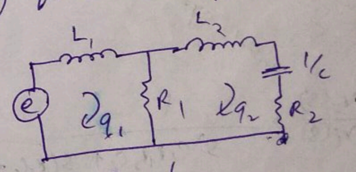
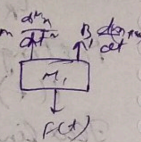


Fig b

Sol^n

Force relationship for ~~no~~ Mass 1

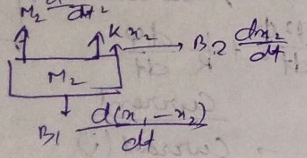
FBD of M_1



from FBD,

$$f(t) = M_1\frac{d^2x_1}{dt^2} + B_1\frac{d(x_1-x_2)}{dt^2} \rightarrow ①$$ (b/c B is common bet^n M_2 & M_1)

FBD for M_2



$f(t)$

$$B_1\frac{d}{dt}(x_1-x_2) = M_2\frac{dx_2^2}{dt^2} + B_2\frac{dx_2}{dt} + Kx_2 \longrightarrow ②$$

Using force voltage analogy

$$① \rightarrow e(t) = L_1\frac{d^2q_1}{dt^2} + R_1\frac{d(q_1-q_2)}{dt} \longrightarrow ③$$

$$② \Rightarrow R\frac{d}{dt}(q_1-q_2) = L_2\frac{dq^2}{dt^2} + R_2\frac{dq_2}{dt} + \frac{1}{c}q_2 \longrightarrow ④$$

$$i = \frac{dq}{dt} \Rightarrow dq = i\,dt \Rightarrow q = \int i\,dt$$

$$③ \Rightarrow e(t) = L_1\frac{di_1}{dt} + R_1(i_1 - i_2)$$

$$L_1\frac{d^2q_1}{dt^2} = L_1\frac{d}{dt}\left(\frac{dq}{dt}\right) = L\frac{di}{dt}$$

2nd syed

$$① \quad \frac{V_c(s)}{V_i(s)} = \frac{}{s^2C_1R_1R_2C_2 + s(R_1C_1 + R_1C_2 + R_2C_2) + 1}$$

$$② \quad \frac{X(s)}{F(s)} = M_1s^2 + B_1s + B_2s + k + k_1$$

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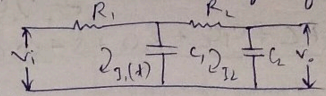
Q. Derive the transfer fun^n of the n/w shown in fig below



Sol^n

$$V_i = R_1I_1 + \frac{1}{C_1}\int(I_1 - I_2)\,dt$$

LT $$V_i(s) = R_1I_1(s) + \frac{1}{sC_1}(I_1(s) - I_2(s)]$$

$$\Rightarrow V_i(s) = R_1I_1(s) + \frac{1}{sC_1}I_1(s) - \frac{1}{sC_1}I_2(s)$$

$$\Rightarrow V_i(s) = \left(R_1 + \frac{1}{sC_1}\right)I_1(s) - \frac{1}{sC_1}I_2(s) \longrightarrow ①$$

2nd loop,

$$\frac{1}{C_1}\int(I_2 - I_1)\,dt + R_2I_2 + \frac{1}{C_2}\int I_2\,dt = 0$$

LT,

$$\frac{1}{sC_1}I_2(s) - \frac{1}{sC_1}I_1(s) + R_2I_2(s) + \frac{1}{sC_2}I_2(s) = 0$$

$$\Rightarrow \frac{1}{sC_1}I_1(s) = \left(\frac{1}{sC_1} + R_2 + \frac{1}{sC_2}\right)I_2(s)$$

$$\Rightarrow I_1(s) = sC_1\left[\frac{1}{sC_1} + R_2 + \frac{1}{sC_2}\right]I_2(s) \longrightarrow ②$$

$$V_o(i) = \frac{1}{C_2}\int I_2(s)$$

$$V_o(s) = \frac{1}{sC_2}I_2(s) \longrightarrow ③$$

Substituting ② in ①,

$$V_i(s) = \left(R_1 + \frac{1}{sC_1}\right)\left(sC_1\left[\frac{1}{sC_1} + R_2 + \frac{1}{sC_2}\right]\right)I_2(s) - \frac{1}{sC_1}I_2(s)$$

$$= I_2(s)\left[\left(R_1 + \frac{1}{sC_1}\right)\left(sC_1\left(\frac{1}{sC_1} + R_2 + \frac{1}{sC_2}\right) - \frac{1}{sC_1}\right]$$

$$= \cancel{I_2(s)\left[\left(R_1 + \frac{1}{sC}\right)\right(}$$

$$= I_2(s)\left[\left(R_1 + \frac{1}{sC}\right)\left(1 + sC_1R_2 + \frac{C_1}{C_2}\right)\right] - \frac{1}{sC_1}I_2(s)$$

$$= I_2(s)\left[R_1 + R_1sC_1R_2 + \frac{RC_1}{C_2} + \frac{1}{sC} + \frac{C_1R_2}{C} + \frac{C_1}{sCC_2}\right] - \frac{1}{sC_1}I_2(s)$$

$$= I_2(S)\left[\frac{SC_2R_1 + \cancel{R_2}R_1R_2 + RC_1SC + C_2 + C_1R_2SC_2 + \cancel{C_1}}{SC_2\cancel{C}}\right] - \frac{1}{SC_1} I_2(S)$$

$$= I_2(S)\left[\frac{CC_2R_1C_1 + R_1R_2C_1 + RC + C_1C_2 + R_2C_2 + S + CC_1C_2}{S\cancel{C_2}C_1C}\right]$$

$$=$$

$$\frac{V_o}{V_i} = \frac{1}{\cancel{S}C_2} \cdot \frac{I_2(S)\, \cancel{S}C_2C_1}{(C_2R_1C_1 + R_1R_2C_1 + RC + C_1C_2 + R_2C_2 + S + CC_1C_2) I_2(S)}$$

$$= \frac{C_1}{C_1R_1C_2 + R_1R_2C_2 + RC + C_1C_2 + R_2C_1 + S + CC_1C_2}$$

$$V_i(S) = \left(R_1 + \frac{1}{SC_1}\right)\left(1 + SC_1R_2 + \frac{C_1}{C_2}\right) I_2(S) - \frac{1}{SC_1} I_2(S)$$

$$= \left(R_1 + SC_1R_1R_2 + \frac{R_1C_1}{C_2} + \cancel{\frac{1}{SC_1}} + R_2 + \frac{1}{SC_2} - \cancel{\frac{1}{SC_1}}\right) I_2(S)$$

$$= R_1 + SC_1R_1R_2 + \frac{R_1C_1}{C_2} + R_2 + \frac{1}{SC_2}$$

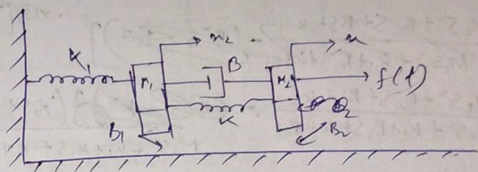
$$\frac{V_o(S)}{V_i(S)} = \frac{\frac{1}{SC_2} \cancel{I_2(S)}}{\left(R_1 + SC_1R_1R_2 + \frac{R_1C_1}{C_2} + R_2 + \frac{1}{SC_2}\right) \cancel{I_2(S)}}$$

$$= \frac{1}{SC_2} \times \frac{1}{\frac{SC_2R_1 + S^2C_1C_2R_1R_2 + SR_1C_1 + SC_2R_2 + 1}{SC_2}}$$

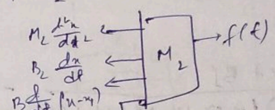
$$= \frac{1}{1 + SC_2R_1 + S^2C_1C_2R_1R_2 + SR_1C_1 + SC_2R_2}$$

$$= \frac{1}{1 + S^2C_1C_2R_1R_2 + S(R_1C_1 + R_2C_2 + R_1C_2)}$$

Q. Obtain the transfer function of the mechanical system shown in fig and draw its analogous ckt.



Sol^n FBD of M_2



$$f(t) = M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + K(x - x_1)$$

LT,

$$F(S) = M_2S^2X(S) + B_2SX(S) + BSX(S) - BSX_1(S) + KX(S) - KX_1(S)$$

$$\Rightarrow F(S) = (M_2S^2 + B_2S + BS + K)X(S) - (BS + K)X_1(S)$$

for M_1

$$M_1 \frac{d^2x_1}{dt^2} + K_1x_1 + B_1 \frac{dx_1}{dt} = B \frac{d}{dt}(x - x_1) + K(x - x_1)$$

LT,

$$M_1S^2X_1(S) + K_1X_1(S) + B_1SX_1(S) = BSX(S) - BSX_1(S) + KX(S) + KX_1(S)$$

$$\Rightarrow X_1(S)\left[M_1S^2 + K_1 + B_1S + K + BS\right] = X(S)\left[BS - BK\right]$$

$$\Rightarrow X(S) = \frac{X_1(S)\left[M_1S^2 + K_1 + B_1S + K + BS\right]}{BS - K}$$

$$\Rightarrow X_1(S) = \frac{X(S)\left[BS + K\right]}{\left[M_1S^2 + K_1 + B_1S + K + BS\right]}$$

$$F(S) = \left(M_2S^2 + B_2S + BS + K\right)X(S) - \frac{(BS+K)}{M_1S^2 + K_1 + B_1S + K + BS}$$

$$= X(S)\left[\frac{M_2S^2 + B_2S + BS + K}{M_1S^2 + K_1 + B_1S + K + BS} - \left(B^2S^2 - K^2\right)\right] \cdot X(S)\left(BS+K\right)$$

$$= X(S)\left[\frac{M_2S^2 + B_2S + BS + K}{M_1S^2 + K_1 + B_1S + K + BS} - \frac{(BS+K)^2}{M_1S^2 + K_1 + B_1S + K + BS}\right]$$

$$= X(S)\left[\frac{M_1M_2S^4 +}{M_1S^2 + K_1 + B_1S + K + BS}\right]$$

$$= X(S)$$

$$\frac{X(S)}{F(S)} = \frac{M_1S^2 + K_1 B_1S + K + BS}{(M_2S^2 + B_2S + BS + K)(M_1S^2 + K_1 + B_1S + K + BS) - (BS+K)^2}$$

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$$f(t) = M_2\ddot{x} + B_2\dot{x} + B(\dot{x} - \dot{x}_1) + K(x - x_1)$$

$$M_1\ddot{x}_1 + B_1\dot{x}_1 + K_1x_1 = B(\dot{x} - \dot{x}_1) + K(x - x_1)$$

$M_2 \to L_2$

$B_2 \to R_2$

$B \to R$

$K \to \frac{1}{C}$

$$① \Rightarrow f(t) = M_2\frac{d^2x}{dt^2} + B_2\frac{dx}{dt} + B\frac{d}{dt}(x - x_1) + K(x - x_1)$$

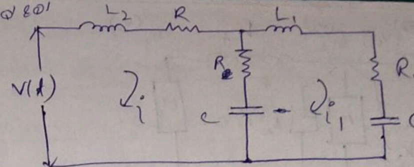
$$\Rightarrow V = L_2\frac{d^2q}{dt^2} + R_2\frac{dq}{dt} + R\frac{d}{dt}(q - q_1) + \frac{1}{C}(q - q_1)$$

$$= L_2\frac{di}{dt} + R_2 i + R(i - i_1) + \frac{1}{C}\left(\int i\,dt - \int i_1\,dt\right) \to ①'$$

$$② \Rightarrow R\left(\frac{dq}{dt} - \frac{dq_1}{dt}\right) + \frac{1}{C}(q - q_1) = L_1\frac{d^2q_1}{dt^2} + R_1\frac{dq_1}{dt} + \frac{1}{C_1}q_1$$

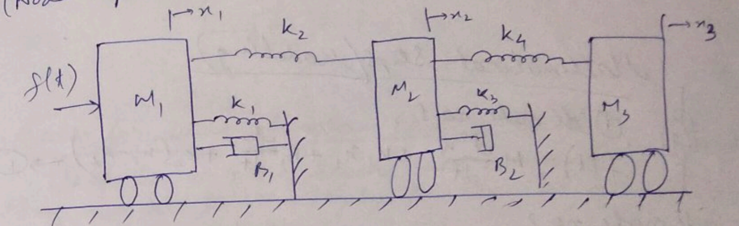
$$\Rightarrow R(i - i_1) + \frac{1}{C}\left[\int i\,dt - \int i_1\,dt\right] = L_1\frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1\,dt \to ②'$$

from ①' & ②'



(from eq. ①' & ②' series এ আছে)

Q. Draw the network diagram for the system and also draw its force current analogy. (Nodal representation method)



Sol^n

(Nodal analysis → force-current analogy)

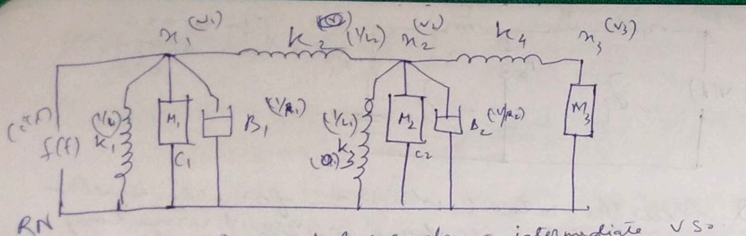
① Total no. of nodes = Total no. of displacement
One reference node is taken in addition therefore total no. of nodes $(n+1)$ (4)

② M_1 is connected between x_1 and the reference

$M_2 \to x_2$ & reference

$M_3 \to x_3$ & reference

③



n (d10 φ 225 but f-c analogy ⊙ intermediate v s.
v (n)

Mathematical Steps (Modelling)

(1) At node no. 1

$$f(t) = M_1\frac{d^2x_1}{dt^2} + k_1x_1 + B_1\frac{dx_1}{dt} + k_2(x_1 - x_2) \rightarrow (1)$$

(2) At node no. 2.

$$k_2(x_1 - x_2) = M_2\frac{d^2x_2}{dt^2} + k_3 x_2 + B_2\frac{dx_2}{dt} + k_4(x_2 - x_3) \rightarrow (ii)$$

(3) At node no. (3)

$$\varepsilon = -\frac{d\varphi}{dt}$$

$$K_4(x_2 - x_3) = M_3\frac{d^2x_3}{dt^2} \longrightarrow (iii)$$

Using force - current analogy,

$$(1) \Rightarrow i(t) = C_1\frac{d^2\varphi}{dt^2} + \frac{1}{L_1}\varphi_1 + \frac{1}{R_1}\frac{d\varphi_1}{dt} + K(\varphi_1 - \varphi_2) \rightarrow (1)'$$

$$(2) \Rightarrow \frac{1}{L_1}\left(\frac{d\varphi_1}{dt} - \frac{d\varphi_2}{dt}\right) = C_1\frac{d^2\varphi_2}{dt^2} + \frac{1}{L_3}\varphi_2 + \frac{1}{R_1}\frac{d\varphi_2}{dt} + \frac{1}{L_4}(\varphi_2 - \varphi_3)$$

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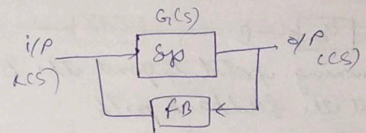
Block Diagram Reduction Technique

Transfer function

State space analysis



$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)}$$

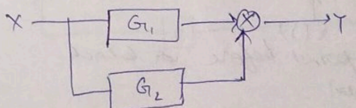
Rules for block diagram redn method

(1) If two or more blocks are connected in series then we are to multiply the transfer functions and put as one block

$$X \rightarrow \boxed{G_1} \rightarrow \boxed{G_2} \rightarrow Y$$

$$\Rightarrow X \rightarrow \boxed{G_1G_2} \rightarrow Y$$

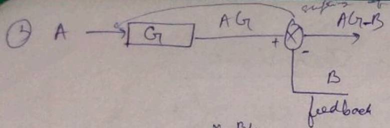
(2) When two blocks are connected in parallel the transfer function are to be added.

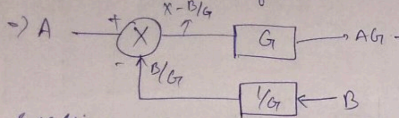


$$\Rightarrow X \rightarrow \boxed{G_1 \pm G_2} \rightarrow Y$$

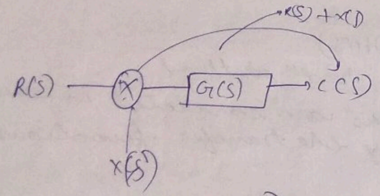
(3) When shifting the summing point ahead i.e. before a block. In this shifting the original i/p, o/p and feedback quantities must not change. A block 1/G has to be introduced in the f/b path to maintain the i/p, o/p and f/b signals unaltered.

③ shifting of summing pt ahead of B

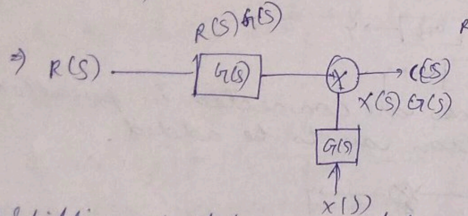




④ Shifting the summing point beyond the block
then we multiply G at the feedback path

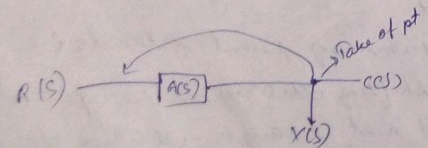


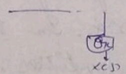
$= (R(S)+X(S))\,G(S)$

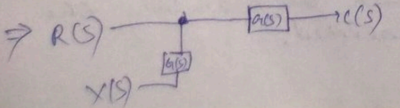


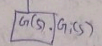
$R(S)G(S)+X(S)G(S)$
$= [R(S)+X(S)]\,G(S)$

⑤ Shifting of take of point before a block
(node)
(branching junction)

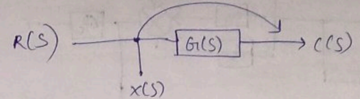


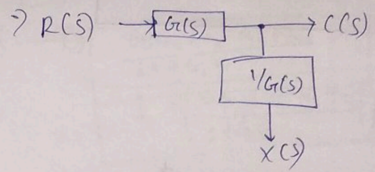






⑥ Shifting a take of pt behind the block





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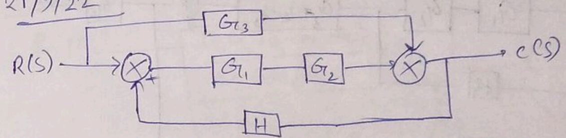
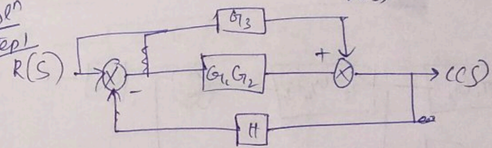
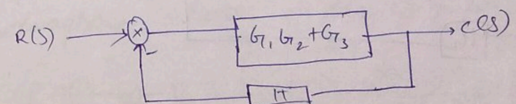


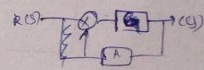
Determine the ratio $\frac{C(S)}{R(S)}$ shown in the block diagram

Sol^n
step1



Step2

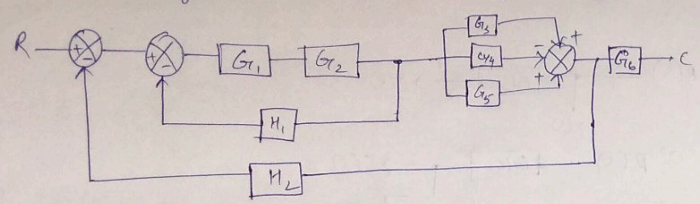




$\frac{C(S)}{R(S)} = \frac{G}{1+GH}$

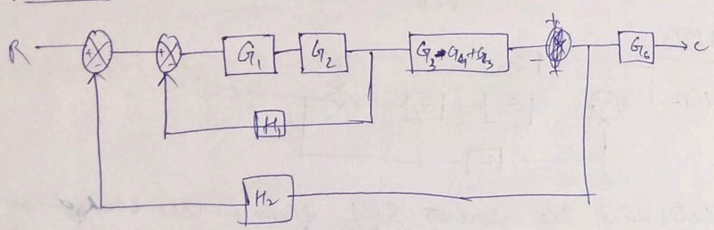
$$\frac{C(S)}{R(S)} = \frac{G_1G_2+G_3}{1+(G_1G_2+G_3)H}$$

Q. Find the single block equivalent for the figure

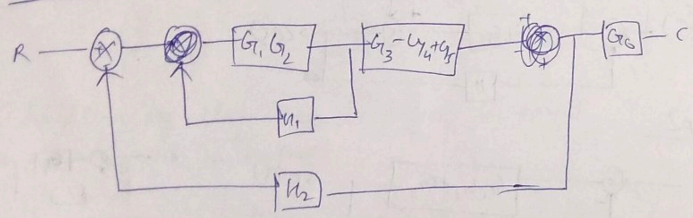


Sol^n

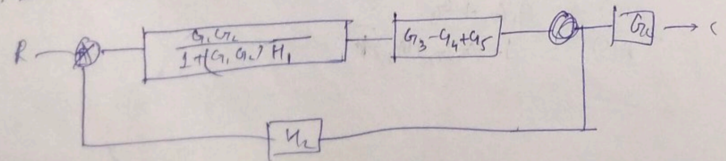
Step 1



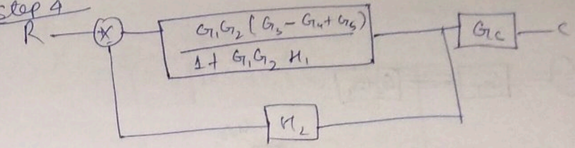
Step 2



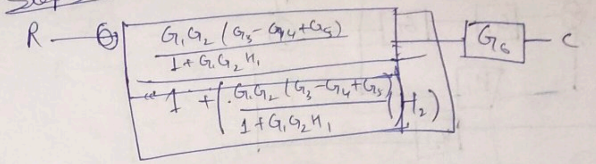
Step 3



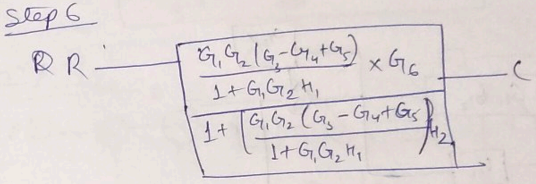
Step 4



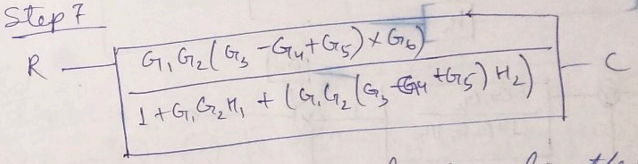
Step 5



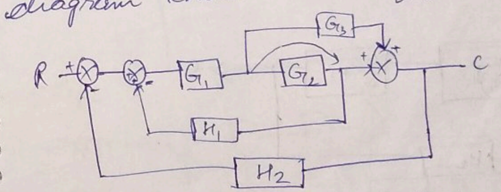
Step 6



Step 7

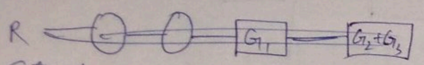


Q. Obtain the transfer function for the block diagram shown in the figure

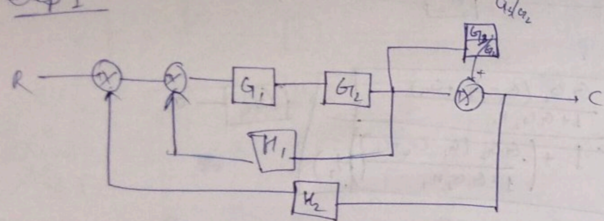


(Shifting the G_3 block behind or after G_3)

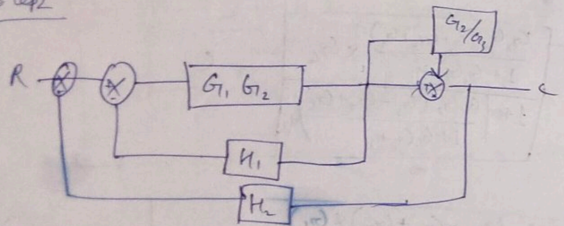
Sol^n



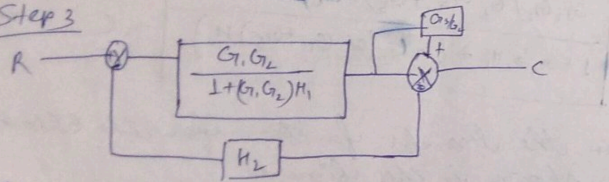
Step 1



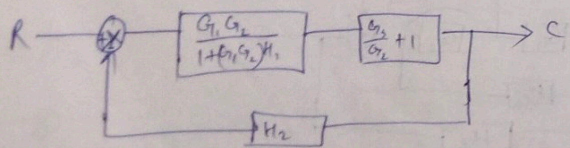
Step2



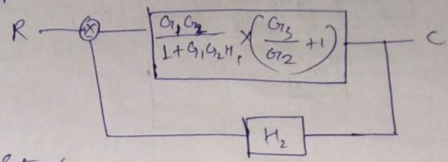
Step 3



Step4



Step 5



Step6

$$R \rightarrow \boxed{\frac{\frac{G_1G_2}{1+G_1G_2H_1}\times\left(\frac{G_3}{G_2}+1\right)}{1+\left\{\frac{G_1G_2}{1+G_1G_2H_1}\times\left(\frac{G_3}{G_2}+1\right)\right\}H_2}} \rightarrow C$$

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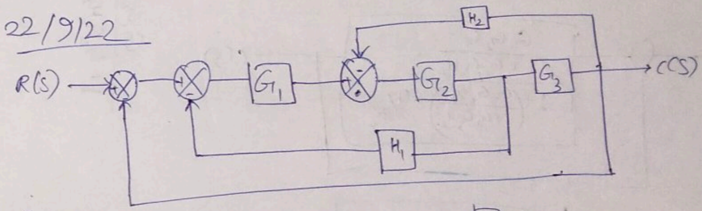
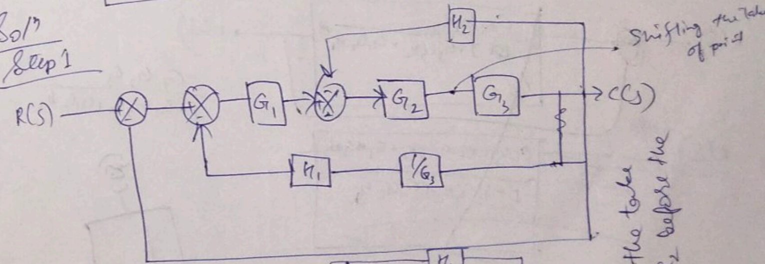
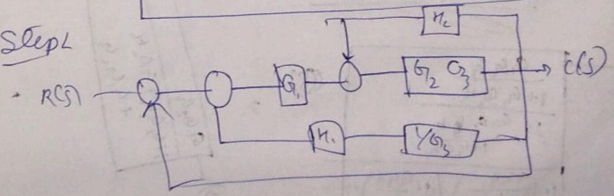


Sol^n

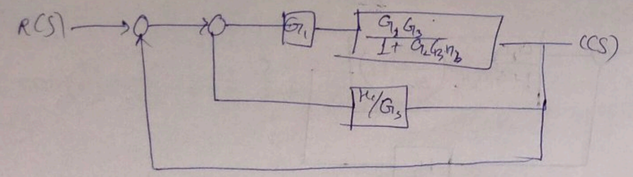
Step 1

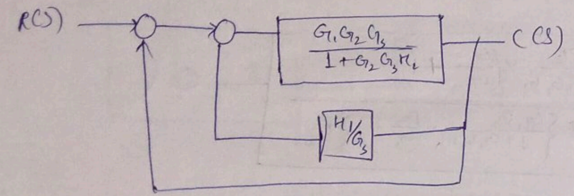


Step2

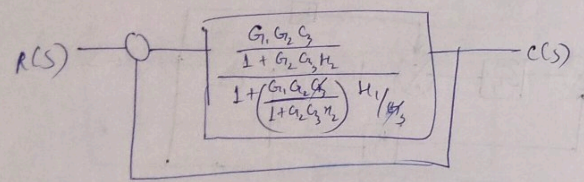


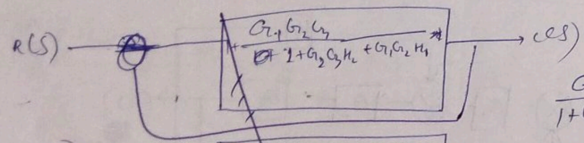
(H.W. Shifting the take of point of H_2 before the G_3)



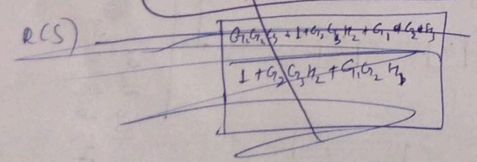


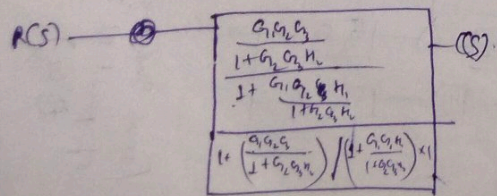
$$\frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1+G_1G_2G_3}$$

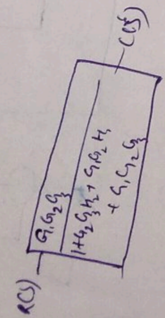




$$\frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1}$$



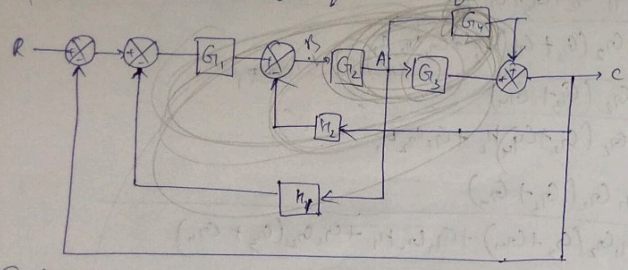




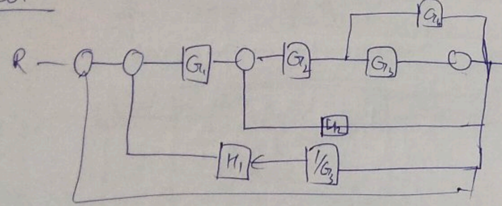
$$\frac{\dfrac{G_1G_2(G_3+G_4)}{1+G_2(G_3+G_4)H_2+G_1G_2H_1}}{1+\dfrac{G_1G_2(G_3+G_4)}{1+G_2(G_3+G_4)H_2+G_1G_2H_1}}$$

$$= \frac{G_1G_2(G_3+G_4)}{1+G_1G_2(G_3+G_4)+G_1G_2H_1+G_1G_2(G_3+G_4)}$$

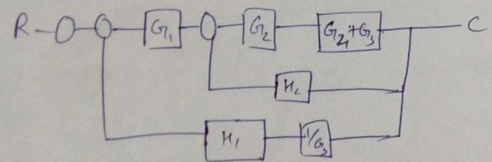
Q. Obtain the transfer function for the block diagram

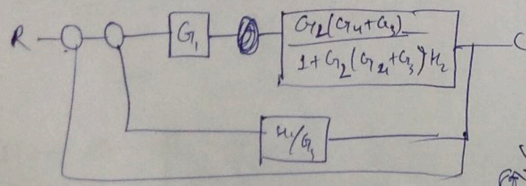


Soln

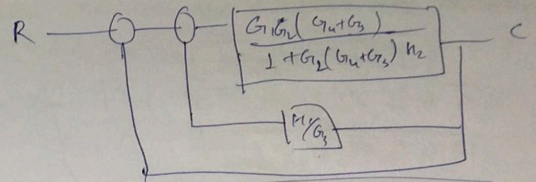


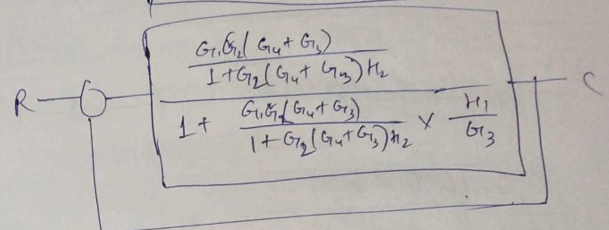
$$\frac{G_2(G_4+G_3)}{1+G_2(G_4+G_3)H_2}$$





$$\frac{G_1G_2(G_3+G_4)\,G_3}{1+G_2(G_3+G_4)(H_2+G_1)+H_1G_1G_2}$$





$$\frac{\dfrac{G_1G_2(G_4+G_3)}{1+G_2H_2(G_4+G_3)}}{\dfrac{1+G_2(G_4+G_3)H_2+G_1G_2(G_4+G_3)H_1}{(1+G_1(G_4+G_3)H_2)G_3}}$$

$$= \frac{G_1G_2G_3(G_4+G_3)}{1+G_2(G_3+G_4)(H_2+G_1)+H_1G_1G_2}$$

$$\frac{G_1G_2(G_3+G_4)}{1+G_2(G_3+G_4)(H_2+G_1)+H_1G_1G_2}$$

$$1+G_2G_3H_2+G_2G_3G_4+G_2G_4H_2+G_1G_2G_4G_1+H_1G_1G_2$$

$$G_2G_4G_3 \quad \frac{(G_2G_4+G_3)G_2}{1+(G_2G_3+G_3)G_2H_2}$$

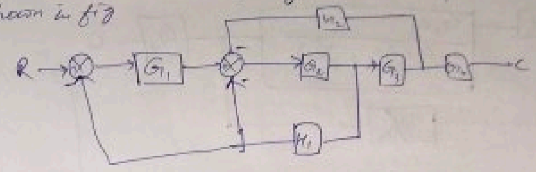
$$\frac{G_2G_1(G_2G_4+G_3)}{1+(G_2G_4+G_3)G_2H_2}$$

$$1+\frac{G_2G_1(G_2G_4+G_3)}{1+(G_2G_4+G_3)G_2H_2}H_1$$

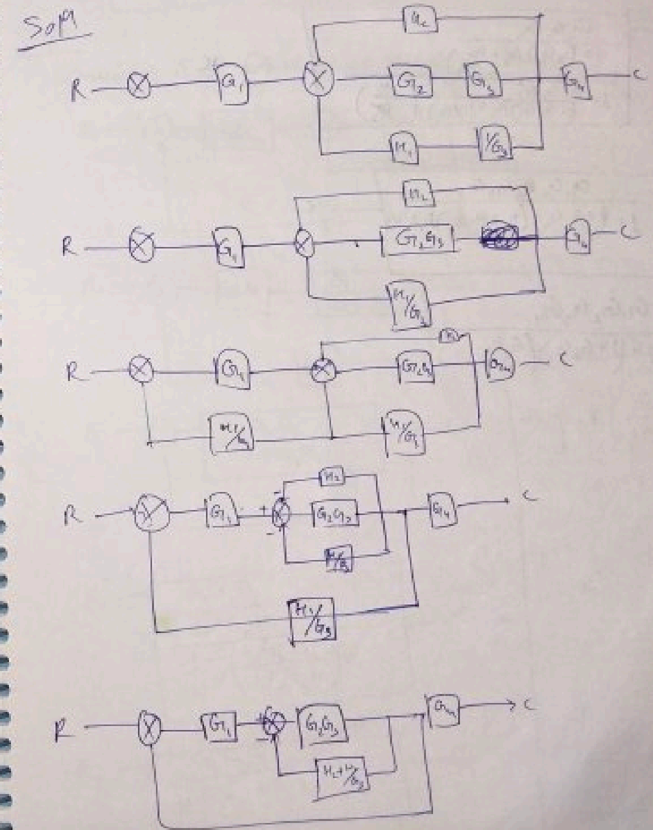
$$=\frac{G_2G_1(G_2G_4+G_3)}{1+G_2G_3(G_2G_4+G_3)H_1}$$

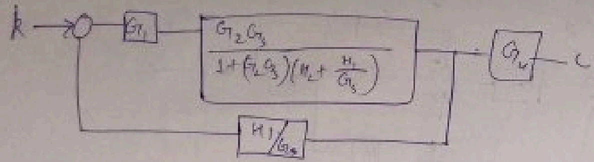
$$=\frac{G_2G_1(G_2G_4+G_3)+1+G_2G_3(G_2G_4+G_3)H_1}{1+G_2G_4(G_2G_4+G_3)H_1}$$

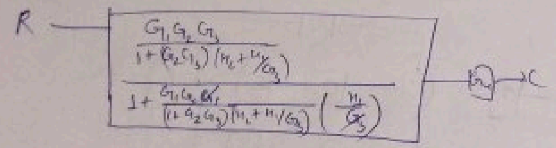
Q. Obtain the transfer function of the block diagram shown in fig

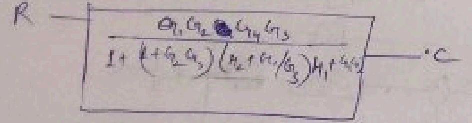


Soln



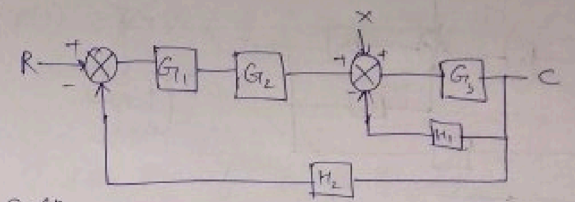






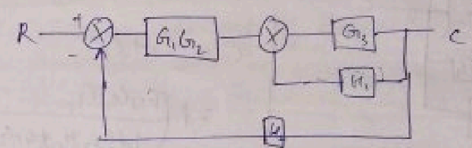
$$\frac{G_1G_2G_3G_4}{1+(1+G_2G_3)(G_3H_2}$$

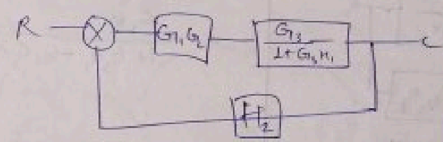
Q. Find the o/p of the system shown in fig below

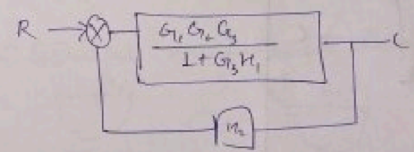


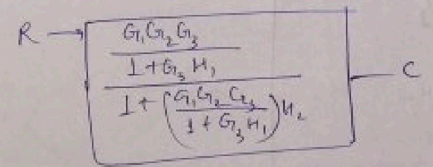
Soln

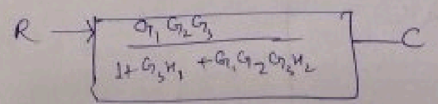
Consider R as input and X = 0









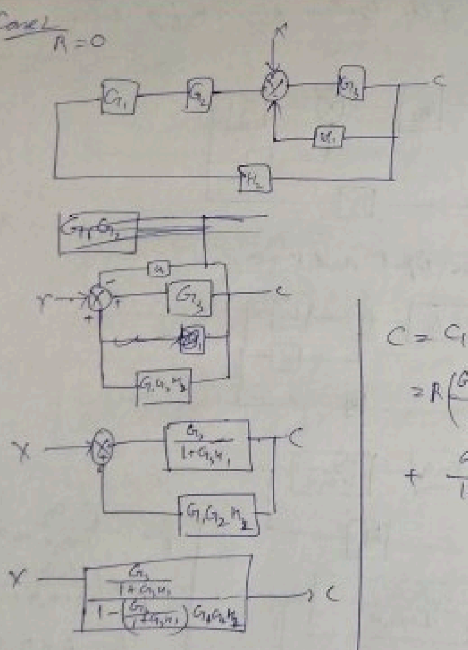


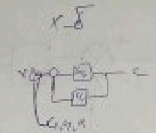
$$\frac{C}{R} = \frac{G_1G_2G_3}{1+G_3H_1+G_1G_2G_3H_2}$$

$$\Rightarrow C = R\left(\frac{G_1G_2G_3}{1+G_3H_1+G_1G_2G_3H_2}\right)$$

for $R \neq 0, X = 0$

Case 2
R = 0





$$\frac{G_3}{1+G_3H_1-G_1G_2G_3H_2}$$

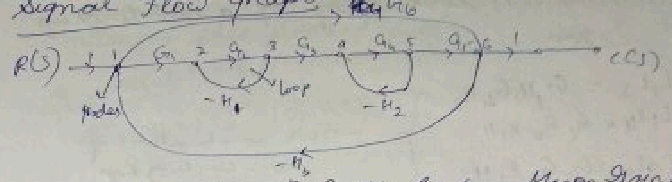
$$\frac{C_1}{X} = \frac{G_3}{1+G_3H_1-G_1G_2G_3H_2}$$

$$\rightarrow C_2 = \frac{G_3X}{1+G_3H_1-G_1G_2G_3H_2}$$

$$C = C_1 + C_2$$

$$= R\left(\frac{G_1G_2G_3}{1+G_3H_1+G_1G_2G_3H_2}\right) + \frac{G_3X}{1+G_3H_1-G_1G_2G_3H_2}$$

Signal Flow Graph



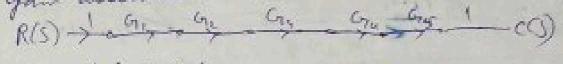
Find the transfer f^n for the fig by Mason gain formula

(forward path → connects i/p & o/p directly)

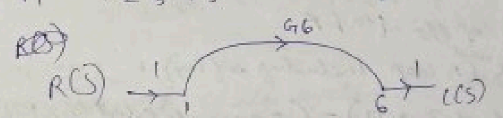
S.G.

Rule

① Determine the no of forward path and find the gain associates with it.

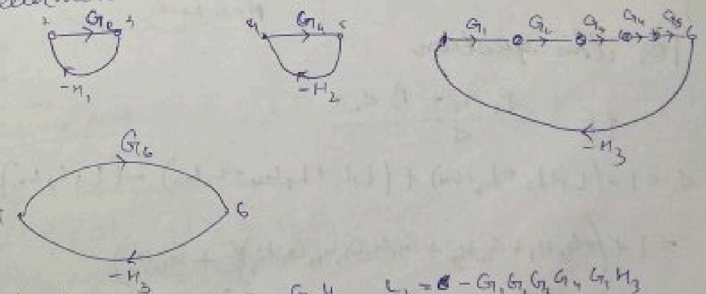


$P_1 = G_1G_2G_3G_4G_5$



$P_2 = G_6$

② Determine individual loops and also gains.



Loop 1 → $L_1 = -G_2H_1$, $L_2 = -G_4H_2$, $L_3 = -G_1G_2G_3G_4G_5H_3$

$L_4 = -G_6H_3$

③ Determine the combination of pairs of non-touching loops & gain

$L_1L_2 = G_2H_1G_4H_2$
$L_1L_4 = G_2G_6H_1H_3$
$L_2L_4 = G_4G_6H_2H_3$ } → Two combination of non-touching loop

$L_1L_2L_4 = -G_2G_4G_6H_1H_2H_3$ → Three combination of non-touching loop

There are no higher order non-touching loop.

Mason's Gain formula

$$TF = \frac{C(S)}{R(S)} = \frac{\sum_{i=1}^{N} P_i\Delta_i}{\Delta}$$ self node

Where N = Total no.s of FP
P_i = Gain of the i^{th} FP

$\Delta = 1 - (\Sigma$ all individual FB loop including self loop$)$ + $(\Sigma$ gain product 2 NTL$) - (\Sigma$ gain product of 3 NTL$)$ + $(\Sigma$ gain product 4 NTL$)$ − ......

N=2 here

For this question

$$TF = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\Delta = 1 - (L_1+L_2+L_3+L_4) + (L_1L_2 + L_1L_4 + L_2L_4) - (L_1L_2L_4)$$

$$= 1 + (G_2H_1 + G_4H_2 + G_1G_2G_3G_4G_5H_3 + G_6H_3)$$

$$+ (G_2H_1G_4H_2 + G_2G_6H_1H_3 + G_4G_6H_2H_3) + G_2G_4G_6H_1H_2H_3$$

Δ_i = Value of Δ after illuminating all loops that touche each forward path.

Δ_i associated with P_i

$$\Delta_1 = 1 - (0) + (0) - (0)$$
$$= 1$$

Δ_2 associated with P_2

$$\Delta_2 = 1 - (L_1 + L_2) + (L_1L_2) - (0)$$
$$= 1 + (G_2H_1 + G_4H_2) + (G_2G_4H_1H_2)$$

$$TF = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_6(1 + G_2H_1 + G_4H_2 + G_2G_4H_1H_2)}{1 + G_2H_1 + G_4H_2 + G_1G_2G_3G_4G_5H_3 + G_6H_3 + G_2H_1G_4H_2 + G_2G_6H_1H_3 + G_4G_6H_2H_3 + G_2G_4G_6H_1H_2H_3}$$

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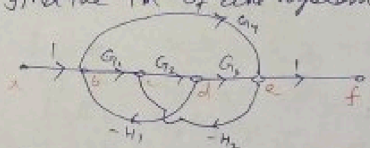
Q Find the TF of the system as shown in fig.



Solⁿ

Forward path.

① $P_1 = abcdef = G_1G_2G_3$.

② $P_2 = abef = G_4$

(1) $L_1 = bcdb = -G_1G_2H_1$

$L_2 = cdec = -G_2G_3H_2$

There is no higher order non touching loop

$$TF = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{N} P_i \Delta_i}{\Delta}$$

$$= \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$\Delta = 1-(L_1+L_2)$

$= 1-(-G_1G_2H_1 - G_2G_3H_2)$

$= 1+G_1G_2H_1+G_2G_3H_2$

$\Delta_1 = 1-(0)+(0) = 1$

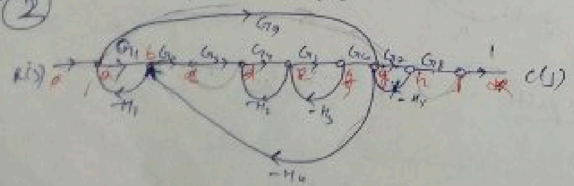
$\Delta_2 = 1-(0)+(0) = 1$

$$\therefore TF = \frac{G_1G_2G_3+G_4}{1+G_1G_2H_1+G_2G_3H_2}$$

$1-(L_4+L_5)$

$+(L_3L_5)$

(2)



(1) $P_1 = abcdefghi$

$= G_1G_2G_3G_4G_5G_6G_7G_8$

$P_2 = \cancel{a b h i k}\ ag\,hi$

$= \cancel{G_1}\ G_7G_8G_9$

(2) $L_1 = aba = -G_1H_1$

$L_2 = ded = -G_4H_2$

$L_3 = efe = -G_5H_3$

$L_4 = ghg = -G_7H_5$

$L_5 = bcdefgb = -G_2G_3G_4G_5G_6H_4$

$L_6 = agba$

$= G_9H_1H_4$

(3) $L_1L_2 = G_1G_4H_1H_2$

$L_1L_3 = G_1G_5H_1H_3$

$L_1L_4 = G_1G_7H_1H_5$

$L_2L_4 = \cancel{G_4G_7}\ G_4G_7H_2H_5$

$L_2L_6 = -G_4G_9H_1H_2H_4$

$L_3L_4 = G_5G_7H_3H_5$

$L_3L_6 = -G_5G_9H_1H_3H_4$

(3) $L_1L_2L_4 = -G_1G_4G_7H_1H_2H_5$

$L_1L_3L_4 = -G_1G_5G_7H_1H_3H_5$

$\Delta = 1-(L_1+L_2+L_3+L_4+L_5+L_6)+(L_1L_2+L_1L_3+L_1L_4+L_2L_4$

$L_2L_6+L_3L_4+L_3L_6)-(L_1L_2L_4+L_1L_3L_4)$

$\Delta_i = 1 + (G_1H_1 + G_4H_2 + G_5H_3 + G_7H_5 + G_2G_3G_4\ G_6G_4)$

$\Delta_1 = 1a = 1$

$\Delta_2 = 1 + (L_1 + L_2)$

$= 1 - (-G_4H_2 + G_5H_3) = 1 + G_4H_2 + G_5H_3$

$TF = \dfrac{C(s)}{R(s)} = \dfrac{\sum_{i=1}^{N} P_i\Delta_i}{\Delta}$

$= \dfrac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

$= \dfrac{G_1G_3G_4G_5G_6G_7G_3 + G_7G_8G_9}{\Delta}$

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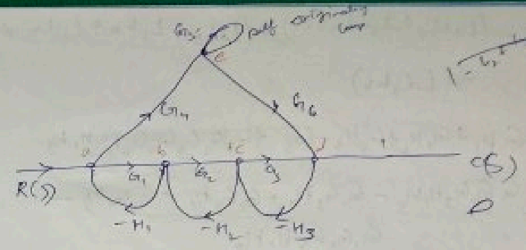
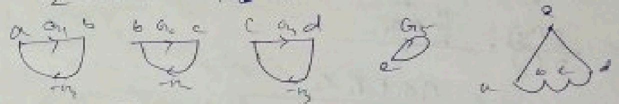
Q. Find transfer function of the system shown in fig below.



Sol^n

FP, $P_1 = abcd = G_1G_2G_3$

$P_2 = aed = G_4\ G_6$



$L_1 = -G_1H_1 = aba$

$L_2 = -G_2H_2 = bcb$

$L_3 = -G_3H_3 = cdc$

$L_4 = G_5 = ee$

$L_5 = aed\ cba = -G_4G_6H_3H_2H_1G_3G_2G_1$

NTL,

for 2 loop,

$L_1L_3 = G_1G_3H_1H_3$

$L_1L_4 = -G_1G_5H_1$

$L_2L_4 = -G_2G_5H_2$

$L_3L_4 = -G_3G_5H_3$

for 3 loop,

$L_1L_3L_4 = G_1G_2G_5H_1H_3$

In determining FP or feedback path the self loop should not be taken into account

$$\Delta = 1 - (L_1+L_2+L_3+L_4+L_5) + (L_1L_3 + L_1L_4 + L_2L_4 + L_3L_4)$$
$$- (L_1L_3L_4)$$
$$= 1 + G_1H_1 + G_2H_2 + G_3H_3 - G_5 + G_1G_2G_3G_4G_6H_1H_2H_3$$
$$+ G_1G_3H_1H_3 - G_1G_5H_1 - G_2G_5H_2 - G_3G_5H_3 -$$
$$G_1G_2G_5H_1H_3$$

$\Delta_1 = 1 - L_4 = 1 - G_5$

$\Delta_2 = 1 - L_2 = 1 + G_2H_2$

$$TF = \frac{C(S)}{R(S)} = \frac{\sum_{i=1}^{N} P_i\Delta_i}{\Delta}$$
$$= \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$
$$= \frac{G_1G_2G_3(1-G_5) + G_4G_6(1+G_2H_2)}{\Delta}$$

Q. Construct the signal flow graph for the following equation.

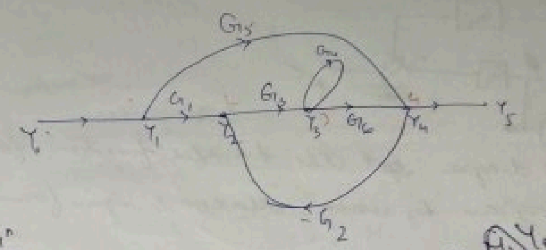
$Y_2 = G_1Y_1 - G_2Y_4$

$Y_3 = G_3Y_2 + G_4Y_3$

$Y_4 = G_5Y_1 + G_6Y_3$

where Y_4 is o/p

Using Mason gain formula calculate transfer fn



$Y_2\ Y_3\ Y_4\ Y_2$

$Y_3\ Y_3$

Sol^n

$P_1 = G_1G_3G_6$

$P_2 = G_5$

$L_1 = Y_3Y_3 = G_4$

$L_2 = 2\,3\,4\,2 = -G_3G_6G_2$

No higher order NTL

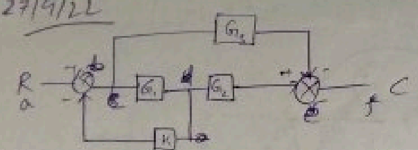
$\Delta = 1 - (L_1 + L_2) = G_4 - G_3G_6G_2$

$\Delta_1 = 1$

$\Delta_2 = 1 - L_1 = 1 - G_4$

$$TF = \frac{C(S)}{R(S)} = \frac{\sum_{i=1}^{N} P_i\Delta_i}{\Delta} = \frac{G_1G_3G_6 + G_5(1-G_4)}{G_4 - G_3G_6G_2}$$
$$= \frac{G_1G_3G_6 + G_5 - G_4G_5}{G_4 - G_3G_6G_2}$$

27/9/22



Solve the block diagram get the transfer function for this block diagram by using Mason's gain formula

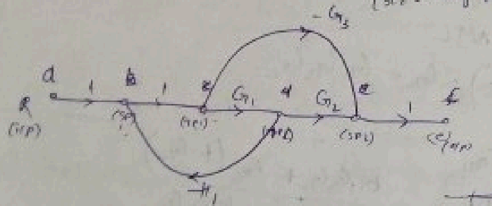
Sol^n

FP,

$P_1 = Rabc = G_1 G_2$ (all the nodes, summing points, take of points are nodes)

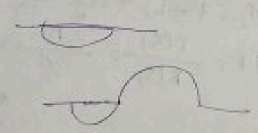
$P_2 - Rac = G_3$

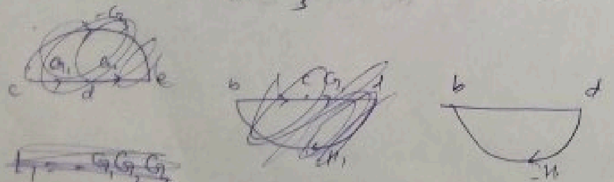
(1st summing point a -ve sign will be)



$P_1 = abdef = G_1 G_2$

$P_2 = abcef = -G_3$





~~$L_1 = G_1 G_2 G_3$~~

$L_1 = -G_1 H_1$

$\Delta_p = 1 + G_1 H_1$

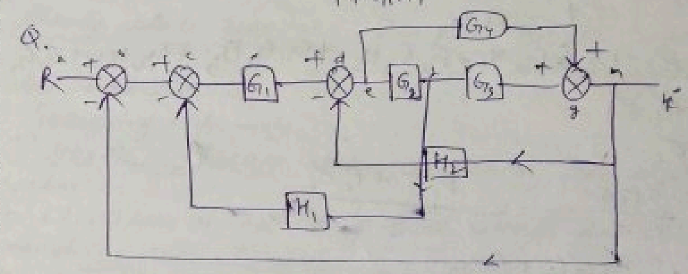
$\Delta_1 = 1 \qquad \Delta_2 = 1$

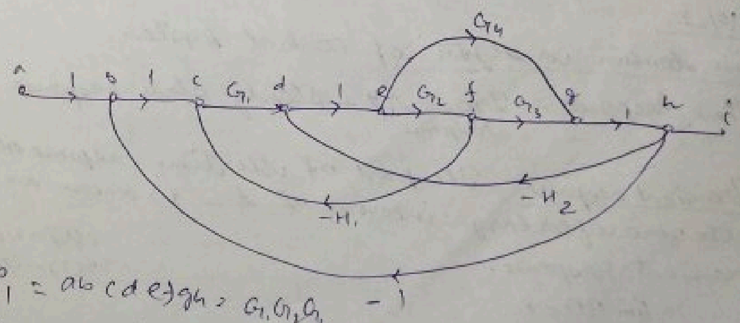
$TF = \frac{C(S)}{R(S)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$

$= \frac{G_1 G_2 1 - G_3 \times 1}{1 + G_1 H_1}$

$= \frac{G_1 G_2 - G_3}{1 + G_1 H_1}$

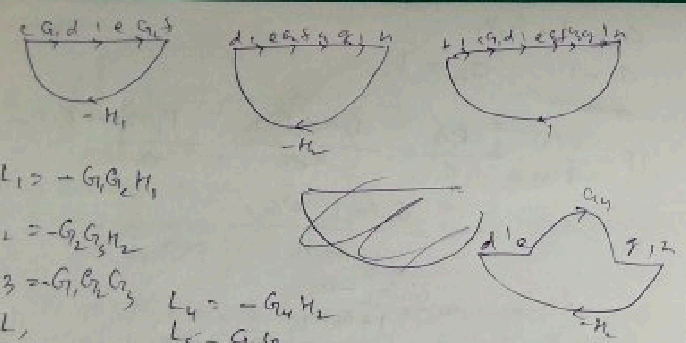
Q.





$P_1 = abcdefgh = G_1 G_2 G_3$

$P_2 = abcdegh = G_1 G_4$



$L_1 = -G_1G_2H_1$

$L_2 = -G_2G_3H_2$

$L_3 = -G_1G_2G_3$

$L_4 = -G_4H_2$

$L_5 = -G_1G_4$

NTL,

$\Delta = 1 + G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3 + G_4H_2 + G_1G_4$

$\Delta_1 = 1$

$\Delta_2 = 1$

$$TF = \frac{\sum_{i=1}^{N} P_i\Delta_i}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3 + G_4H_2 + G_1G_4}$$

12/10/22

Time domain analysis of control system

Time response = Transient response + Steady state response

Transient response: The part of the time response which goes to zero after large interval of time is known as transient response.

$\lim_{t\to\infty} c(t) = 0$

$c(t) \to o/p$ $C(s)$
$r(t) \to i/p$ $R(s)$

Properties

① Transient response gives the following info —

① The time interval after which the system response taking the instant of application of excitation as reference.

② The total time it takes to achieve the output for the first time.

③ Wheather or not o/p suits beyond the desired value and how much.

④ Wheather or not the o/p oscillates about the final value.

⑤ The time it takes to settle down to the final value.

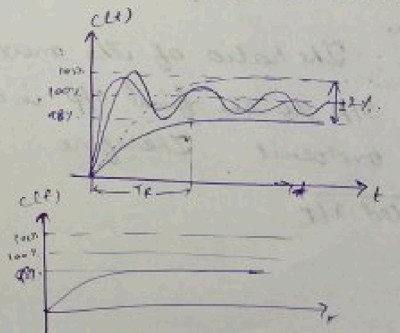
Steady state response: The part of the system response that remain even after the transients have died out.

Following info —

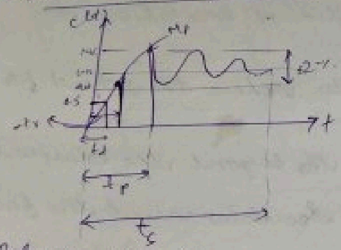
① The time that o/p takes to reach the steady state value.

② Wheather or not any error exists betⁿ the desired or actual value.

③ Wheather this error is constant, zero or infinite.



System time response



$t_d \rightarrow$ delay time
$t_r \rightarrow$ Rise time
$t_p \rightarrow$ peak time
$t_s \rightarrow$ settling time

Delay time: The time that a system o/p takes to reach 50% of its final value is called delay time

Rise time: The time that a system response takes to reach from 10% to 90% of its final value

Settling time: The time that the to settle down and stays within ±2 of its final value

Peak time: The time taken by the system response to reach first max^m or the peak value is called the peak time

Mp (Max^m overshoot): The ratio of the max^m value of ~~state~~ step excited O/P to the final o/p is called the overshoot or max^m overshoot. Therefore

$$\%\ \text{overshoot} = \frac{\text{final o/p}}{\text{step excited i/p}}$$

12/10/22

Time domain analysis of control system:

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$$\lim_{t\to\infty} c(t) = 0$$

$c(t) \to o/p \xrightarrow{LT} C(S)$

$r(t) \to i/p \xrightarrow{LT} R(S)$

Properties:

Transient response gives the following information

① The time interval after which the system response taking the instant of application of excitation as reference.

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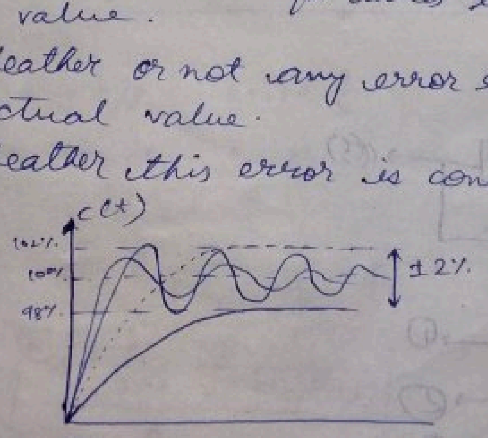
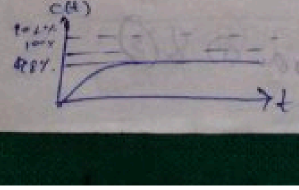
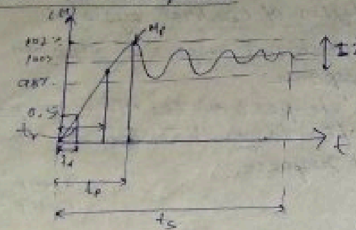




System time response



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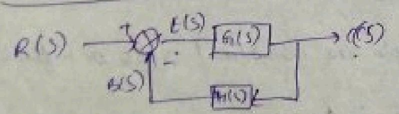
$$\% \text{ overshoot} = \frac{\text{final o/p}}{\text{step excited o/p}}$$

13/10/22

Analysis of SSE



From the figure

$E(S) = R(S) - B(S) \longrightarrow ①$

$C(S) = G(S)E(S) \longrightarrow ②$

$B(S) = C(S)H(S) \longrightarrow ③$

Now, $E(S) = R(S) - C(S)H(S)$ [from ① & ③]

$\Rightarrow E(S) = R(S) - G(S)E(S)H(S)$ [using ②]

$\Rightarrow E(S) + G(S)E(S)H(S) = R(S)$

$\Rightarrow E(S)[1 + G(S)H(S)] = R(S)$

$\Rightarrow E(S) = \frac{R(S)}{1+G(S)H(S)}$

$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$

Now, corresponding error in time domain will be $e(t)$. Now, during the steady state $t \to \infty$ therefore

$e(t)$ $\boxed{e_{ss} = \lim_{t\to\infty} e(t)}$

$e_{ss} = \lim$

By using final value theorem

$\Rightarrow e_{ss} = \lim_{s\to 0} sE(S)$

$= \lim_{s\to 0} s \frac{R(S)}{1+G(S)H(S)}$

$\Rightarrow \boxed{e_{ss} = \lim_{s\to 0} s \frac{R(S)}{1+G(S)H(S)}}$

From the expression of steady state error it is found that it depends on,

① Type and magnitude of $R(S)$

② It depends on the open loop transfer function

③ Presence of any dominant non linearities

Type of inputs & steady state errors

Step I/P

For step I/P $R(S) = \frac{1}{s}$

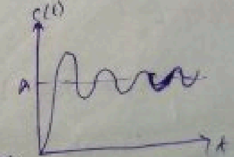
Let us consider $Au(t)$ as the signal

$\therefore R(S) = \frac{A}{s}$

$\therefore e_{ss} = \lim_{s\to 0} s \frac{E(S)}{1+G(S)H(S)} = \lim_{s\to 0} s \frac{A/s}{1+G(S)H(S)}$

$\boxed{e_{ss} = \lim_{s\to 0} \frac{A}{1+G(S)H(S)}}$

$= \frac{A}{1 + \lim_{s\to 0} G(S)H(S)}$ $\rightarrow K_p$ position error



For system $\boxed{e_{ss} = \frac{A}{1+K_p}}$

$\boxed{\begin{array}{l} \text{Step} \rightarrow \text{position} \\ \text{Ramp} \rightarrow \text{velocity} \\ \text{Parabolic} \rightarrow \text{accl}^n \end{array}}$

For Ramp I/P

For Ramp i/p $R(S) = \frac{1}{S^2}$

integral ऐ pole add 2nd (integral) differential ऐ zero add

Let us consider $A\,t\,u(t)$ as the signal

$\therefore R(S) = A/S^2$

$e_{ss} = \lim_{S \to 0} S \frac{R(S)}{1+G(S)H(S)} = \lim_{S\to 0} S \frac{A/S^2}{1+G(S)H(S)}$

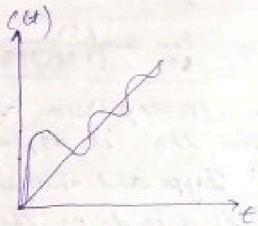
$e_{ss} = \lim_{S\to 0} \frac{A}{S(1+G(S)H(S))}$

$\boxed{L(t^n) = \frac{\lfloor n}{S^{n+1}}}$

$= \lim_{S\to 0} \frac{A}{\cancel{S} + S\,G(S)H(S)}$

$= \frac{A}{\underbrace{\lim_{S\to 0} S\,G(S)H(S)}_{K_v}}$

$= \frac{A}{K_v}$



$R(S) = \frac{1}{S^3}$

Let us consider $A t^2 u(t)$ as the signal

$\therefore R(S) = A/S^3$

$e_{ss} = \lim_{S\to 0} S \frac{R(S)}{1+G(S)H(S)} = \lim_{S\to 0} S \frac{A/S^3}{1+G(S)H(S)}$

$e_{ss} = \lim_{S\to 0} \frac{A}{S^2(1+G(S)H(S))} = \lim_{S\to 0} \frac{A}{S^2 + S^2 G(S)H(S)}$

$= \frac{A}{\lim_{S\to 0} S^2 G(S)H(S)}$

$= \frac{A}{K_a}$ $\quad K_a = \lim_{S\to 0} S^2 G(S)H(S)$

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Errors for step i/p

The error for a step i/p of value A is given by

$e_{ss} = \frac{A}{1+K_p}$

where $K_p = \lim_{S\to 0} G(S)H(S)$

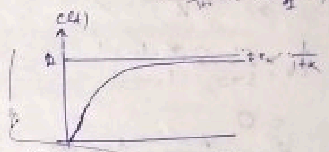
Now for type 0 system

$$\boxed{G(S)H(S) = \frac{K(1+T_1S)(1+T_2S)\cdots(1+T_nS)}{S^N(1+T_aS)(1+T_bS)\cdots(1+T_mS)}}$$

For 0 system, N = 0

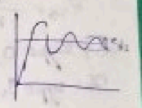
$\therefore K_p = \lim_{S\to 0} \frac{K.1\ldots1}{1\ldots1}$

$\Rightarrow \boxed{K_p = K}$ $\quad \therefore e_{ss} = \frac{A}{1+K}$



Type 1

$G(S)H(S) = \frac{K(1+T_1S)(1+T_2S)\ldots(1+T_nS)}{S^N(1+T_aS)(1+T_bS)\ldots(1+T_mS)}$



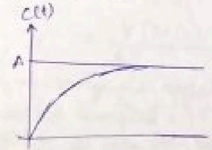
For type 1, N = 1

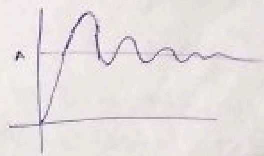
$\therefore K_p = \lim_{S\to 0} \frac{K(1+T_1S)(1+T_2S)\ldots(1+T_nS)}{S^N(1+T_aS)(1+T_bS)\ldots(1+T_mS)}$

$= \lim_{S\to 0} \frac{K}{S}$

$= \infty$

$\therefore e_{ss} = \frac{A}{1+\infty} = 0$





Conclusion

For step i/p

$e_{ss} = \frac{A}{1+K}$ for type 0

For type 1 or greater than 1 $e_{ss} = 0$

Errors for Ramp signal

$e_{ss} = \frac{A}{K_v}$, $K_v = \lim_{s\to0} s\,G(s)H(s)$

Now for type 0, $\cancel{A}=0$

$G(s)H(s) = \frac{K(1+T_1s)(1+T_2s)\ldots(1+T_ns)}{(1+T_a s)(1+T_b s)\ldots(1+T_m s)}$

$K_p = \lim_{s\to0} \frac{sK.1\ldots1}{1\ldots1}$

$= K$

$\therefore e_{ss} = \frac{sA}{K}$

$K_v = \lim_{s\to0} s\,\frac{K.1\ldots1}{1\ldots1}$

$= 0$

$\therefore e_{ss} = \frac{A}{K_v} = \frac{A}{0} = \infty$

For type 0, $e_{ss} = \infty$

For type 1, $K_v = K$

$\therefore e_{ss} = \frac{A}{K}$

For type 2, $K_v = \infty$

$\therefore e_{ss} = 0$

So for

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Q. Find K_p, K_v, K_A for the following systems whose

$G(s) = \frac{10}{s^2}$ and $H(s) = 0.7$

$\Rightarrow$ $K_p = \lim_{s\to0} G(s)H(s) = \lim_{s\to0} \frac{10}{s^2}\times0.7 = \infty$

$K_v = \lim_{s\to0} sG(s)H(s) = \lim_{s\to0} s\frac{10}{s^2}\times0.7 = \infty$

$K_A = \lim_{s\to0} s^2G(s)H(s) = \lim_{s\to0} s^2\frac{10}{s^2}\times0.7 = 7$

Q. Find K_p, K_v, K_A. $G(s) = \frac{5}{s^2+3s+5}$, $H(s) = 0.6$

$\Rightarrow$

$K_p = \lim_{s\to0} G(s)H(s) = \lim_{s\to0} \frac{5}{s^2+3s+5}(0.6) = 0.6$

$K_v = \lim_{s\to0} sG(s)H(s) = \lim_{s\to0} \frac{5s}{s^2+3s+5}\times0.6 = 0$

$K_A = \lim_{s\to0} s^2G(s)H(s) = \lim_{s\to0} s^2\frac{5}{s^2+3s+5}\times0.6 = 0$

Q. For the system having $G(s)H(s) = \frac{K(s+4)}{s(s^3+5s^2+6s)}$ find

(i) type of the system

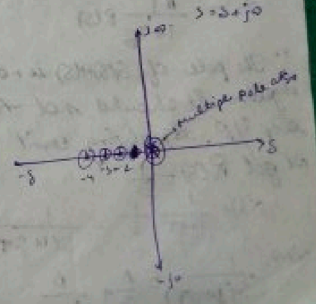
(ii) Error coefficient

(iii) Error due i/p $\frac{At^2}{2}$

$\Rightarrow$ $G(s)H(s) = \frac{K(s+4)}{s^2(s^2+5s+6)} = \frac{K(s+4)}{s^2(s+2)(s+3)}$

$= \frac{K(s+4)}{s}$

[when pole origin is more than oscillate or marginal & less stable. If more poles left side of origin & for more stability poles should be far away from imaginary axis]



$= \frac{K(s+4)}{s^2(s+2)(s+3)}$

$= \frac{K(s+4)}{}$

$$= \frac{K4(S/4+1)}{S^2 \times 2(S/2+1) \cdot 3(S/3+1)}$$

$$= \frac{4K(S/4+1)}{6S^2(1+S/2)(1+S/3)}$$

$$K_P = \lim_{S\to 0} G(S)H(S) = \lim_{S\to 0} \frac{2K(1+S/4)}{3S^2(1+S/2)(1+S/3)}$$

$$= \infty$$

$$K_V = \lim_{S\to 0} S\,G(S)H(S) = \lim_{S\to 0} \frac{2K(1+S/4)S}{\ldots}$$

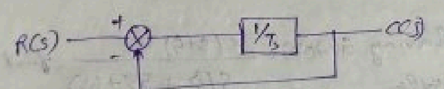
$$= \infty$$

$$K_A = \lim_{S\to 0} S^2 G(S)H(S) = \lim_{S\to 0} S^2 \frac{2K(1+S/4)}{3S^2(1+S/2)(1+S/3)}$$

$$= \frac{2K}{3}$$

(iii) $e_{SS} = \frac{A}{K_A} = \frac{A}{\frac{2K}{3}} = \frac{3A}{2K}$

Analysis of 1st order system



$G(S)H(S) \to$ open loop transfer function

$G(S)H(S) = \frac{1}{Ts}$

Here, $G(S) = \frac{1}{T_S}$ & $H(S) = 1$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{G(S)H(S)+1} = \frac{1/T_S}{1+1/T_S}$$

$$\frac{C(S)}{R(S)} = \frac{1}{1+T_S}$$

$$C(S) = \frac{1}{1+T_S} R(S)$$

∵ The pole of $G(S)H(S)$ is not at $S=0$, it is a type 1 system. It should not have steady state error for step i/p. Applying unit step signal as an i/p we get $R(S) = \frac{1}{S}$

$$C(S) = \frac{1}{1+ST} \cdot \frac{1}{S} = \frac{1}{S(1+ST)}$$

Now, $\frac{1}{S(1+ST)} = \frac{A}{S} + \frac{B}{1+ST}$

$$S(1+ST) = A(1+ST) + BS$$

$$\Rightarrow S + S^2T = A + AST + BS$$

$S^2T + S = S(AT+B) + A$

$A + B = 1$

$A = $

If $S = 0$

$1 = A$

$S = -1/T$, $1 = B(-1/T)$

$$\frac{1}{S(1+ST)} = \frac{1}{S} + \frac{-T}{1+ST} = \frac{1}{S} - \frac{T}{1+ST}$$

$$= \frac{1}{S} - \frac{1}{1/T + S} \quad (\div \text{ by } T)$$

Taking inverse laplace transform

~~$c(t) = (1 - e^{-t/T})u(t)$~~

$$c(t) = (1 - e^{-t/T})u(t)$$

For $t = 0$ $(T=1)$

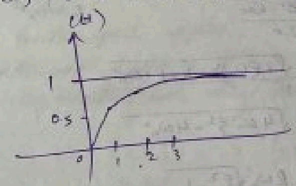
~~$c(t) = 1 - e^{-1}$~~

$c(t) = 1 - e^0 = 0$

At $t=1$, $c(t) = 1 - e^{-1} = 0.62$ $(T=1)$

$t=2$, $c(t) = 1 - e^{-2} = 0.86$ $(T=1)$

$t=3$, $c(t) = 1 - e^{-3} = 0.95$ $(T=1)$



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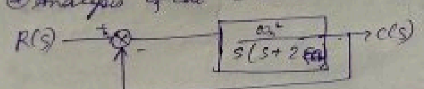
Analysis of the 2nd order system



$$G(S) = \frac{\omega_n^2}{S(S+2\xi\omega_n)}$$

$$G(S)H(S) = \frac{\omega_n^2}{S(S+2\xi\omega_n)} \qquad (\because H(S) = 1)$$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)} = \frac{\frac{\omega_n^2}{S(S+2\xi\omega_n)}}{1+\frac{\omega_n^2}{S(S+2\xi\omega_n)}}$$

$$= \frac{\omega_n^2}{S(S+2\xi\omega_n)+\omega_n^2}$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}}$$ 2nd order transfer function

Considering denominator

$$S^2+2\xi\omega_n S+\omega_n^2 = 0$$

$a=1$, $b=2\xi\omega_n$, $c=\omega_n^2$

$$\therefore \text{Roots} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2-4\times1\times\omega_n^2}}{2\cdot1}$$

$$= \frac{-2\xi\omega_n \pm \sqrt{4\omega_n^2\xi^2-4\omega_n^2}}{2}$$

$$= \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2-1}}{2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$$\therefore S_1 = -\xi\omega_n + \omega_n\sqrt{\xi^2-1}$$

$$S_2 = -\xi\omega_n - \omega_n\sqrt{\xi^2-1}$$

1. The poles are real and unequal if $\sqrt{\xi^2-1} > 0$
   $\sqrt{\xi^2-1} > 0 \Rightarrow \xi > 1$
2. The poles are real and equal if $\sqrt{\xi^2-1} = 0$
   $\Rightarrow \xi = 1$
3. The poles are ~~real and~~ complex conjugate if
   $\sqrt{\xi^2-1} < 0 \Rightarrow \xi < 1$

Here ξ is damping coefficient or damping factor

Effect of ξ on 2nd order system

Case 1: $0 < \xi < 1$

~~System is~~ This 2nd order system is known as under ~~mode~~ damped system. Poles are located on the 2nd & 3rd coordinates due to the existance of both real & imaginary part.

Case 2: $\xi = 1$

This 2nd order system is known as critically damped system. The poles are real and equal. They lie on the $-ve$ σ axis.

Case 3: $\xi < 1$

This 2nd order system is known as over damped system. The poles are real and unequal. Since there are no imaginary terms, the poles lie on the $-ve$ σ axis and at unequal places.

Case 4: $\xi = 0$

This is undamped system.

For undamped system poles are complex and conjugate to each other and they lie on $j\omega$ axis

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Delay time $T_d = \frac{1+0.7\xi}{\omega_n}$

Rise time $T_r = \frac{\pi-\alpha}{\omega_d}$; where $\alpha = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

or $\cos\alpha = \xi$

or $\sin\alpha = \sqrt{1-\xi^2}$

Peak time:

$$T_p = \frac{n\pi}{\omega_d}$$

For 1st overshoot $n=1$

2nd overshoot $n=2\ldots$

3) Settling time

$T_s = \frac{4}{\xi\omega_n}$, where $T = \frac{1}{\xi\omega_n}$ = Time const.

4) Max^n overshoot

$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$

$\% M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%$

Q.1 For a system having $G(s) = \frac{25}{s(s+10)}$ $H(s) = 1$, determine (i) ω_n, (ii) ξ, (iii) ω_d, (iv) T_p & (v) M_p

Solution

$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

$= \frac{25/s(s+10)}{1+\frac{25}{s(s+10)}\cdot 1}$

$\frac{C(s)}{R(s)} = \frac{25}{s(s+10)+25} = \frac{25}{s^2+10s+25} \longrightarrow (1)$

2nd order system,

$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \longrightarrow (2)$

Comparing (1) & (2),

$\omega_n^2 = 25$

(i) $\Rightarrow \omega_n = 5$

& $2\xi\omega_n = 10 \Rightarrow 2\xi \times 5 = 10 \Rightarrow \xi = 1$

(ii) $\xi = 1$

$\omega_d = \sqrt{1-\xi^2} = \sqrt{1-1^2} = 0$

$\therefore$ (iii) $\omega_d = 0$

(iv) $T_p = \frac{\pi}{\omega_d}$

$= \frac{\pi}{0}$

$= \infty$

(v) $M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = e^{-\infty} = \frac{1}{e^\infty} = 0$

($\because T_p$ infinite, we don't have the peak, so M_p should be zero, must)

Q.2: Find the value of K to limit the error of the system for i/p $1+8t+9t^2$ is 0.8 having ~~G(s)H(s)~~

$G(s)H(s) = \frac{K}{s^2(s+1)(s+4)}$

Sol^n

The system is 2nd order

$\therefore e_{ss} = \lim_{s\to 0} s^2 G(s)H(s)$

$= \lim_{s\to 0} s^2 \frac{K}{s^2(s+1)(s+4)}$

$= \lim_{s\to 0} \frac{K}{(s+1)(s+4)}$

$= \frac{K}{4}$

$r(t) = 1+8t+9t^2$

$\therefore R(s) = \frac{1}{s}+\frac{8}{s^2}+\frac{9\times 2}{s^3}$

$= \frac{1}{s}+\frac{8}{s^2}+\frac{18}{s^3}$

($\because$ G(s)R(s) = Laplace)

$= \frac{s^2+8s+18}{s^3}$

$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)R(s)}$

$= \lim_{s\to 0} \frac{s\cdot\frac{s^2+8s+18}{s^3}}{1+\frac{K}{s^2(s+1)(s+4)}}$

$= \lim_{s\to 0} \frac{\frac{s^2+8s+18}{s^2}}{1+\frac{K}{s^2(s+1)(s+4)}}$

$= \lim_{s\to 0} \frac{(s^2+8s+18)(s+1)(s+4)}{s^2(s+1)(s+4)+K}$

$= \lim_{s\to 0} \frac{18\times 4}{K}$

$0.8 = \frac{18\times 4}{K}$

$\Rightarrow K = \frac{18\times 4}{0.8}$

$= 90$

$0.8 = \frac{18\times 4}{K}$

$\Rightarrow K = 90$

Q.3 A system has $G(s) = \frac{K}{s(1+sT)}$ with unity feedback when K & T are const. Determine the factor by which K should be multiplied to reduce overshoot 85% to 95%.