6 feb

Discrete

Convolution

Date : ....

Z- Transform :- Basic Tool

Convol^n of a signal $x(n)$ & $h(n)$ is given as

$$y(n) = x(n) * h(n)$$

$$= \sum_{K=-\infty}^{\infty} x(n)\, h(n-K)$$

K = arbitrary const

Step 1

n is replaced by K

$$\therefore \underbrace{y(n)}_{\text{Fixed}} = x(K) * \underbrace{h(K)}_{h(-K)}$$

Step 2 Time reversal of $h(K)$ to $h(-K)$

Step 3 Sliding over $x(K)$

$h(n-K)$

$\downarrow$ multiplication

$x(K)\, h(n-K) \longrightarrow$ Addition.

Linear & Circular ?

LC is used for aperiodic signal
CC " " periodic "

Q. Find the conv. of $x(n)$

$$x(n) = \{\overset{n=0}{\underset{\uparrow}{1}}, \overset{1}{2}, \overset{2}{3}, \overset{3}{4}\}$$

$\uparrow$ = initialiser

$$h(n) = \{\overset{n=0}{\underset{\uparrow}{4}}, \overset{1}{4}, \overset{2}{3}, \overset{3}{2}\}$$

No of samples N_1 for $x(n)$ $\quad n_1$
" " " N_2 " $h(n)$ $\quad n_2$

$\Big\}$ Initial index Value

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☆ The length of the Convoluted output is : (for $x(n)$ & $h(n)$)

$$\boxed{N_1 + N_2 - 1}$$

☆ The initial starting sequence of the convoluted output is

$$\boxed{n_1 + n_2}$$

A/Q

$n_1 = 0, \quad n_2 = 0$

$\therefore \; n_1 + n_2 = 0$

☆ The final sequence is given by:

$$\underset{0}{n_1} + \underset{0}{n_2} + \underset{4}{N_1} + \underset{4}{N_2} - 2$$

$= 6$

$$y(n) = \left\{ \underset{n=0}{4} \quad \underset{1}{12} \quad \underset{2}{23} \quad \underset{3}{36} \quad \underset{4}{18} \quad \underset{5}{8} \quad \underset{6}{} \right\}$$

Shortcut Method. multiply row & column.

$x(n)$ \ $h(n)$	4	4	3	2
1	4	4	3	2
2	8	8	6	4
3	12	12	9	6
4	16	16	12	8

add Them groupwise and write in The $y(n)$

Convolution is also called Convoluted Sum.

Date : ....

Role of initializer

$$x(n) = \{ \underset{-1}{1} \quad \underset{\uparrow\, n=0}{2} \quad \underset{1}{3} \quad \underset{2}{4} \}$$

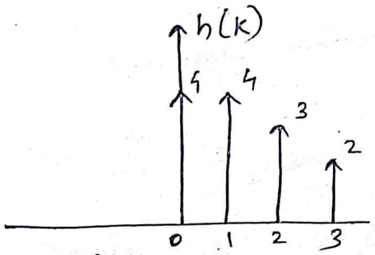
$$h(n) = \{ \underset{-2}{4} \quad \underset{-1}{4} \quad \underset{\uparrow\, n=0}{3} \quad \underset{1}{2} \}$$

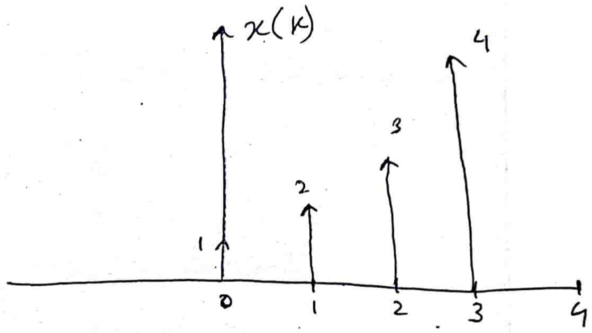
$$\therefore \quad n_1 + n_2 + N_1 + N_2 - 2 = 3$$

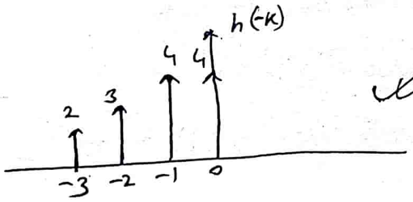
$$y(n) = \{ \underset{n=0}{4} \quad \underset{1}{12} \quad \underset{2}{23} \quad \underset{\uparrow 3}{36} \quad \underset{4}{29} \quad \underset{5}{18} \quad \underset{6}{8} \}$$

Graphical Method

Dont use matrix method







Shifting is done on This!!

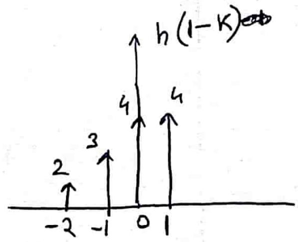
$n - K = 0 \implies -K = -n$

No ustadi

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$$

when $n=0$ $\quad y(0) = \sum_{K=-\infty}^{\infty} x(K) h(0-K) = 4$

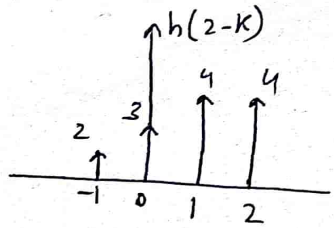
$n=1$ $\quad y(1) = \sum_{K=-\infty}^{\infty} x(K)\, h(1-K) = 4 + 8 = 12$



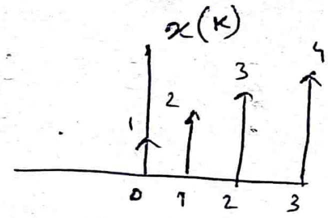
when $n=1$, The time shifting we perform on $h(-K)$

Teacher's Signature ....

For $n=2$



$\times$



$= 3 + 8 + 12 = 23$

From Monday

Pesonality Development & Aptitude

Volunteer to browse through aptitude QP.

10Q - 30min, 20min, 10min

Room Safa
Glass Safa
Go ask bearer.

শিক্ষাৰ অধিকাৰ

Date: Trick From monday. -

IIT Madras - Personality Grooming Course.

Next 15 min CV writing   we should sit for achievement

Find a Psycologist: - Psycometric Test - Ankur & Mukut* (Hold on)
End of march

Read sampling from Simon Hykin and for interviews. - Auriphinilis. & Concept & Question.

in Sampling & Aliasing - real mod ~~log~~ function.

DSP → no no to Mitra

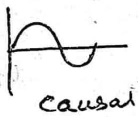
2 → Opena Sepheer

3 → Proakis. — ₹300

Sharma / Nagur - Indian for problem solving.

Signal Causality

non causal
-ve have values



causal
only +ve values

System Causality

we know it
past present etc.

We do not use non causal system as ~~i/p~~ o/p depend on future i/p.

Clear out stability   we get a measurable o/p.

Parameter of system in time domain are the constants.

like in convolution, $y[n] = \sum_{m=0}^{N-1} h[n] \, x[n-m]$

Imp of TF = freq^n domain   It gives the system ch in time domain.

Fourier Series = ?

Only applicable to Periodic signal $E = \frac{6}{2} P \times t.$

Fourier Transform :- applicable to aperiodic signal.

Laplace deals with Continuous time ~~&~~ signal

Z-Trans. " " discrete " "

System stability cannot be analysed with impulse f^n

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Date : ....

For eg:- $h[n] = [\underset{\uparrow}{-1}\ 2\ 1]$

BIBO stability & Parseval Theorem.

Add the elements in the [ ] and take | |

It should be $< \infty$

<u>Constraint</u> :- Input should be bounded.

<u>Recursive & Non recursive system</u>

Say ~~Accumulator~~ to add sum of n numbers

1 2 3 4 5 6

add like this

The Accumulator keeps the result of previous two ~~result~~ operation
Then we add the next

∴ System eqn $y[5] = y[4] + x[5]$

Generalised exp = $y[n] = y[n-1] + x[n]$ — <u>recursive</u>

<u>Non recursive method</u> (not optimised)

$$y[n] = \sum_{m=0}^{n-1} x[m]$$

<u>Finite Impulse Response (FIR) & IIR</u>

FIR → recursive

IIR → non recursive

do not consider the previous inputs.

Date : 13/02/23

Properties of Linear Convolution

i) Commutative
ii) Associative
iii) Distributive

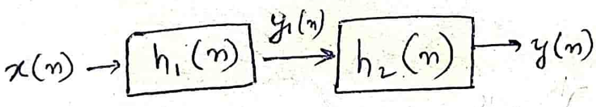
I) Com $\quad x_1(n) * x_2(n) = x_2(n) * x_1(n)$

II) Asso $\quad [x_1(n) * x_2(n)] * x_3(n) = x_1(n) * [x_2(n) * x_3(n)]$

III) Distri :- $\quad x_1(n) * [x_2(n) + x_3(n)] = x_1(n) * x_2(n) + x_1(n) * x_3(n)$

Interconnection

Cascade

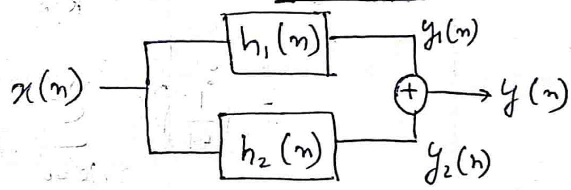


$\Downarrow \quad y_1(n) = x(n) * h(n)$

$$y(n) = [x(n) * h(n)] * h_2(n)$$
$$= x(n) * \underbrace{[h_1(n) * h_2(n)]}_{h(n)}$$

(h(n) is the ~~causality~~ impulse response)

Parallel



$$y(n) = y_1(n) + y_2(n)$$
$$= x(n) * h(n) + x(n) * h_2(n)$$
$$= x(n) * \underbrace{[h_1(n) + h_2(n)]}_{h(n)}$$

Q) Determine the impulse response for cascade of two LTI systems

$h_1(n) = \left(\frac{1}{2}\right)^n u(n) \qquad h_2(n) = \left(\frac{1}{4}\right)^n u(n) \quad \Big\} \; n \geq 0$

Ans ① Determination of given signal

Both the given signal $h_1(n)$ & $h_2(n)$ exist for $n \geq 0$

Find $h(n)$ (Impulse response)

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Date : ....

$$h(n) = h_1(n) * h_2(n) \quad \text{[cascade]}$$

$$= \sum_{k=0}^{n} h_1(k)\, h_2(n-k) \quad \text{(Conv}^n \text{ sum formula)}$$

Replace 'n' in both the equations by K & (n-K) respectively.

$$\therefore \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} \left(\frac{1}{4}\right)^{(n-k)}$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} \underbrace{\left(\frac{1}{4}\right)^{n}}_{\text{constant}} \left(\frac{1}{4}\right)^{-k}$$

Finite Geometric Series

$$\sum_{n=0}^{N} C^{n} = \frac{C^{N+1}-1}{C-1}$$

$$= \left(\frac{1}{4}\right)^{n} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} \left(\frac{1}{4}\right)^{-k}$$

$$= \left(\frac{1}{4}\right)^{n} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} 4^{k}$$

$$= \left(\frac{1}{4}\right)^{n} \sum_{k=0}^{n} 2^{k}$$

Using finite geometric series sum formula.

$$= \left(\frac{1}{4}\right)^{n} \frac{2^{n+1}-1}{2-1}$$

$$= \left(\frac{1}{4}\right)^{n} \left(2^{n+1}-1\right) u(n)$$

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সকলোৱে পঢ়ো সকলোৱে আগবাঢ়ো

Circular Convolution :-

The CC of two periodic discrete time sequences $x_1(n)$ & $x_2(n)$ with periodicity of N samples is defined as

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m) \cdot x_2((n-m))_N$$

or $$\sum_{m=0}^{N-1} x_2(m) \cdot x_1((n-m))_N$$

To perform circular Conv, any of the two sequences $x_1(n)$ or $x_2(n)$ needs to be periodic. Shifting is done in periodic signal only. (out of $x_1(n)$ or $x_2(n)$ any of the two is periodic)

The CC of finite duration sequences can be performed only if both the sequences consists of same no of samples. If the sequences have diff. no. of samples, then convert the smaller size seq^n to the length of the longer size seq^n by appending zeros (padding)

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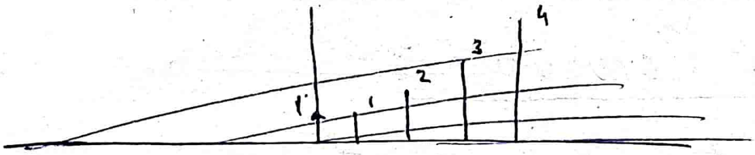
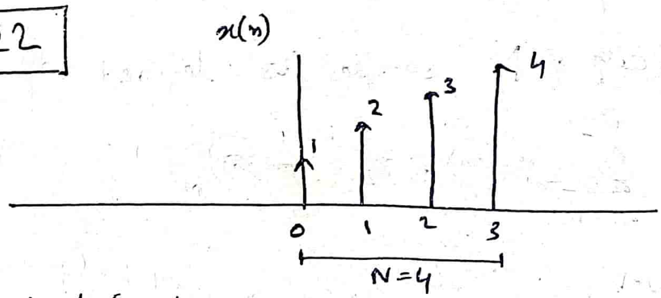
20 — Pass.

Date : ....

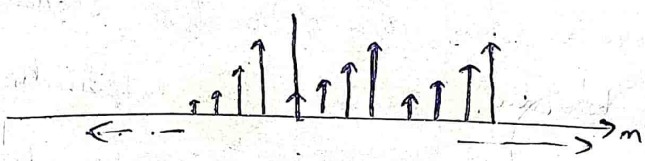


20/Feb/22



If This signal is periodic, how will we ~~do~~ check?

After n=4, The same signal will repeat (also before 0)



☆ Periodic signals are of infinite length. $(-\infty$ to $+\infty)$

Q Let $x_1(n) = \{2, 1, 2, 1\}$

$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Perform circular convolution.

ans

Method 1 :- Matrix Method

Fix $x_1(n)$ and rotate x_2 (shifting)

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 3 \\ 2 & 4 \end{bmatrix} \quad \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix} \quad \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ \\ \end{bmatrix}$$

1+

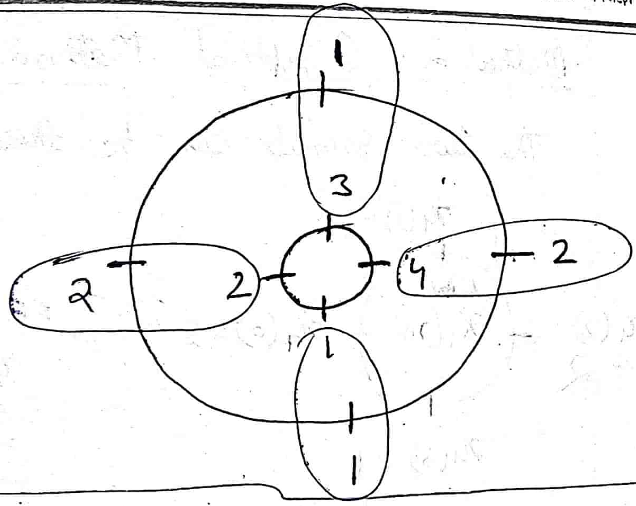
N.B:- To check whether you have done it correctly or not, see the element of 1st row 1st colm

Date : ....

Put $n = 3$

$3\times1 + 4\times2 + 1\times1 + 2\times2$

$= 14$



ans:-

Method 1 :- Matrix Method (don't do in exam)

Make two matrices of $x_1(n)$ and $x_2(n)$ such that one is fixed and the other is kept shifting. Let us keep $x_1(n)$ fixed.

If $x_1(n) = \{x_1(0),\ x_1(1),\ x_1(2),\ x_1(3)\}$ —— Fix

and $x_2(n) = \{x_2(0),\ x_2(1),\ x_2(2),\ x_2(3)\}$ —— Rotating.

Then, the matrices are

$$\begin{bmatrix} x_2(0) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(0) \end{bmatrix}_{4\times4} \times \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix}_{4\times1}$$

Matrix multiply and we will get the result.

N.B:- To check whether you have done it correctly or not, see the element of 1st row 1st col^m should be equal to the element of Last row last column

A/Q $x_1(0) = 2,\quad x_1(1) = 1,\quad x_1(2) = 2,\quad x_1(3) = 1$

$x_2(0) = 1,\quad x_2(1) = 2,\quad x_2(2) = 3,\quad x_2(3) = 4$

$\therefore$ Convoluted result by matrix multiplication method is given by:

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

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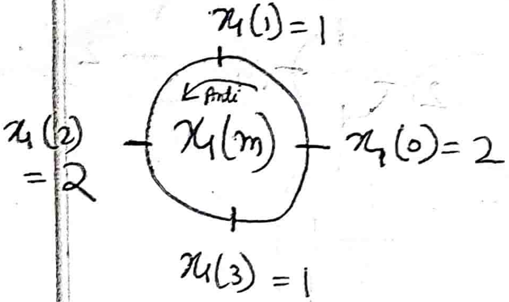
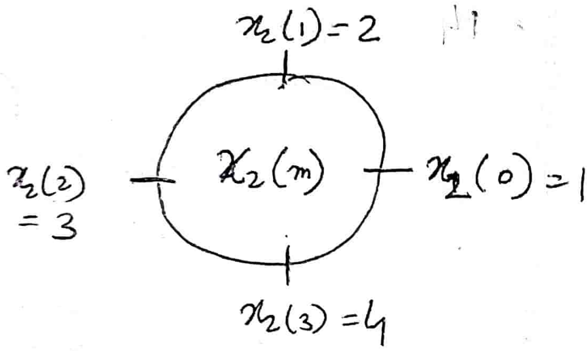
Date : ....

Method 2:- Graphical Method:- (Do in exam)

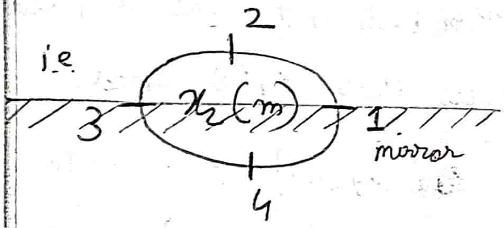
The two signals can be drawn as follows



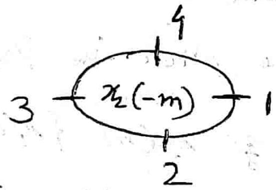


Rotate $x_2(m)$ about $x_2(0)$ i.e. initial point and get $x_2(-m)$

i.e



$\Rightarrow$ mirroring



Now, to determine circular convoluted o/p, $x_3(m)$, we will use

$$x_3(m) = \sum_{n=0}^{N-1} x_1(m)\, x_2(n-m)\Big|_N$$

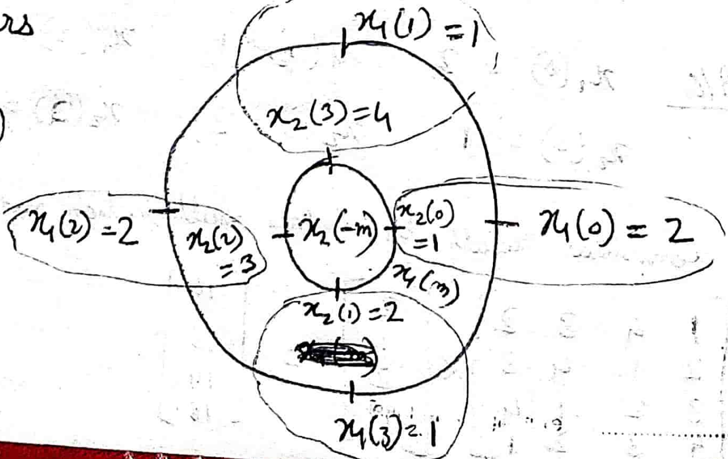
Here $N=4$, $\therefore$ $N-1 = 3$. $\therefore$ we will go from 0 to 3

i.e. rotate 3 times.

For $n=0$, Superimpose $x_1(m)$ over $x_2(-m)$ and multiply the adjecent numbers

$(1\times4)+(2\times1)+(1\times2)+(2\times3)$

$= 14$



Date : ....

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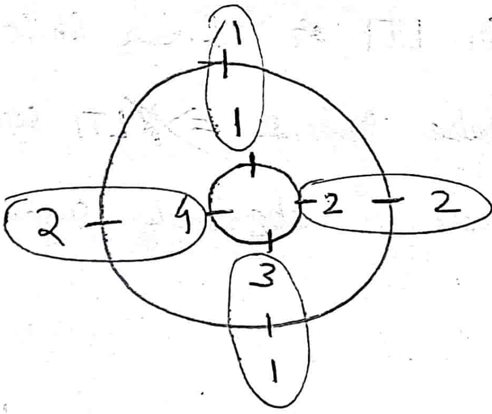
সকলোৱে পঢ়ো সকলোৱে আগবাঢ়ো

For $n=1$, x_2 will take a form of $x_2(n-m+1)$

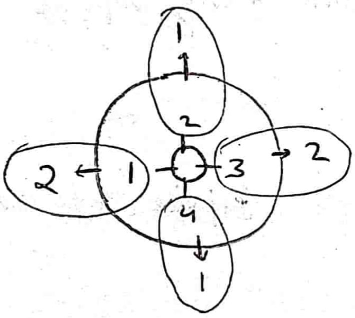
So, rotate the 2nd sequence i.e. x_2 in anticlockwise directi



$= (1\times 1) + (2\times 2) + (3\times 1) + (2\times 4)$

$= 16$

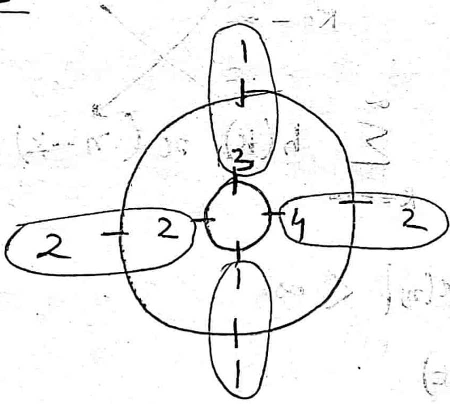
For $n=2$



$= (1\times 2) + (2\times 3) + (1\times 4) + (1\times 2)$

$= 14$

For $n=3$



$= (1\times 3) + (2\times 4) + (1\times 1) + (2\times 2)$

$= 16$

outer circle is fixed. Inner circle rotates anticlockwise

Teacher's Signature ....

Stability of a System

Date: ....

A discrete time system is stable is bounded input produces bounded output. "Bounded" means: ~~bounded~~ BIBO

Stability criteria for LTI or Linear Shift invarient (LSI) in terms of unit impulse response $\Rightarrow$ LTI system is stable if its impulse response is absolutely summable — Derive (3m)

Proof :-

Let us consider an LTI system having impulse response $h(n)$

$$Y(s) = H(s)\,X(s)$$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)}$$ Impulse response = ILT of $H(s) = h($

Let $x(n)$ be the i/p applied to the system.

As per convolution, $$y(n) = \sum_{K=-\infty}^{\infty} x(K)\,h(n-K) \quad \text{——— (1)}$$

we can interchange $$\Rightarrow y(n) = \sum_{K=-\infty}^{\infty} h(K)\,x(n-K) \quad \text{——— (2)}$$

$\because$ $x(n)$ is bounded, Then $|x(n)| < \infty$

$\therefore |x(n)| \leq M_x$ (ut)

$\Rightarrow |x(n)| \leq M_x < \infty$

$$② \Rightarrow |y(n)| = \left| \sum_{K=-\infty}^{\infty} h(k)\, x(n-k) \right| \quad \text{—— ③}$$

$$\therefore \left| \sum_{K=-\infty}^{\infty} h(k)\, x(n-k) \right| \leq \sum_{K=-\infty}^{\infty} |h(k)|\, |x(n-k)| \quad \text{—— ④}$$

Always absolute value of sum of terms is less than or equal to the sum of absolute value of terms.

From eq ③ & ④, we can write

$$|y(n)| \leq \sum_{n=-\infty}^{\infty} |h(k)|\, |x(n-k)| \quad \text{—— ⑤}$$

Since $|x(n)| \leq M_x$ (M_x = constant)

$$\Rightarrow |x(n-k)| \leq M_x$$

$$⑤ \Rightarrow |y(n)| \leq \sum_{K=-\infty}^{\infty} |h(n)|\, M_x$$

$$\Rightarrow |y(n)| \leq M_x \sum_{K=-\infty}^{\infty} |h(k)|$$

Since, the output of a system to be bounded, $\therefore |y(n)| < \infty$.

This implies, $M_x \sum_{K=-\infty}^{\infty} |h(k)| < \infty$

Replacing 'K' by 'n', $M_x \sum_{K=-\infty}^{\infty} |h(n)| < \infty$

or $\int_{-\infty}^{\infty} h(t)\, dt < \infty$ for time domain.

Say $y(t) = \sin \omega t$ → Its bounded from -1 to 1

Tan is not

Date: Causality of LTI System

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A discrete time system is said to be Causal.

If output of the system depends only on present and the past value of the input and not on the future

Causal — Present + Past

Non Causal — Present + Past + Future

Anti causal — Future.

☆ LTI System is causal if its impulse response $= 0$ for $n < 0$

$$\therefore \boxed{h(n) = 0 \ ; \ n < 0}$$

A/ Linear convolution. Let, $x(n) =$ i/p, $y(n) =$ o/p, $h(n) =$ imp res

$$y(n) = \sum_{n=-\infty}^{\infty} x(k)\, h(n-k)$$

$$\Rightarrow y(n) = \sum_{n=-\infty}^{\infty} h(k)\, x(n-k)$$

$$\Rightarrow y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k) + \sum_{k=-\infty}^{-1} h(k)\, x(n-k)$$

$\cancel{[h(0)\, x(0-k) + h(1)\, x(1-k) + h(2)\, x(2-k) \ldots}$

$\cancel{+ [h(-1)\, x(}$ $]$

$[h(0)\, x(n) + h(1)\, x(1-k) + h(2)\, x(2-k) \ldots$

$+ h(-1)\, x(n+1) + \cancel{h(-2)\, x(-2-k)}\ h(-2)\, x(n+2) \ldots$

(Put $k = 0, -1$)

Date: ....

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If we consider $n=0$ (present input) then $x(n-1), x(n-2)....$ are delayed i/p

If we consider $n=-1$ (past i/p), then $x(n+1), x(n+2)$ .... are advanced i/p

But $x(n+1), x(n+2)....$ can't be used (as they are future i/p)

$$\therefore y(n) = \underbrace{\sum_{K=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{K=-\infty}^{\infty} h(k)\, x(n-k)}$$

$$\Rightarrow \sum_{K=0}^{\infty} x(k)\, h(n-k) = \sum_{K=0}^{\infty} h(k)\, x(n-k)$$

We are not allowed to consider $n<0$ as we are considering only ~~future~~ present i/p $\rightarrow$ Causality criterion

$\boxed{h(n) = 0 \text{ if } n<0}$ will be required in FIR & IIR

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Finite Impulse Response (FIR)

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Date: 22/02/22

We know that, $y(n) = \sum_{K=-M}^{M} h(n)\, x(n-k)$ → Non Causal

$\therefore y(n) = \sum_{K=0}^{M} h(k)\, x(n-k)$ → causal

expanding $\Rightarrow y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \cdots + h(M)x(n-M)$ (M is a finite number)

−ve value of K will give us the future i/p

M is a memory element. For 8 bit K range from 0 to 7

It has a finite storage capacity.

Used in FIR filters.

FIR has finite memory requirement

IIR Infinite ~~Impulse~~ Response

$y(n) = \sum_{K=-\infty}^{\infty} h(k)\, x(n-k)$ → Non causal

$y(n) = \sum_{K=0}^{\infty} h(k)\, x(n-k)$ — Causal

$\Rightarrow y(n) = h(0)\underbrace{x(n)}_{\text{Present i/p}} + h(1)\underbrace{x(n-1)}_{\text{past i/p}} + \cdots + h(\infty)\, x(n-\infty)$

IIR has infinite memory requirement. (drawback)

can't be realised in practise.

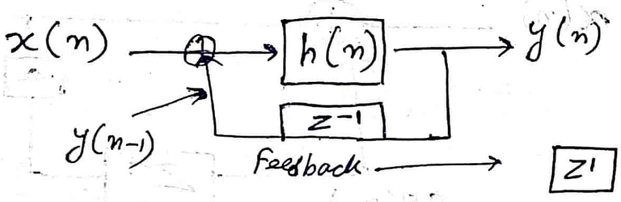
Both take in account - Causality

Depending upon the output of the system, the discrete time domain system can be classified as <u>non Recursive</u> & <u>Recursive System</u>

Non Recursive — without feedback

<u>Recursive</u> — with feedback

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$



Feedback → $\boxed{Z^{1}}$ or $\boxed{Z^{-1}}$

Advance Delay

$$\therefore \underbrace{y(n)}_{\text{present o/p}} = \underbrace{x(n)}_{\text{i/p}} + \underbrace{y(n-1)}_{\text{previous value of o/p}}$$

<u>Non recursive</u>

This system does not require any ~~past~~ past o/p sample to calculate the present o/p

Let us consider a causal LTI system as:-

$$y(n) = \sum_{K=0}^{m} x(K)\,h(n-K) \quad \times$$

In DSP Input is varied System is fixed

$$\text{or } y(n) = \sum_{K=0}^{m} h(K)\,x(n-K) \quad \checkmark$$

Property of convolution > we can ~~ch~~ interchange x and h

we need $x(n)$ for causality.

$$= h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) \pm \ldots + h(m)x(n-m)$$

All non recursive systems are FIR

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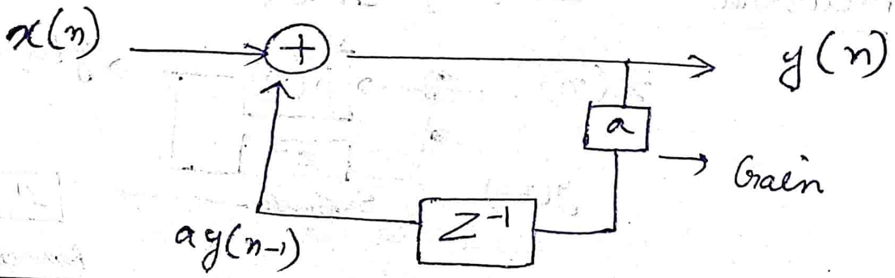
Recursive System

A discrete time system in which the o/p $y(n)$ depends on present, past ~~as well as~~ i/p as well as previous outputs.



$$\boxed{y(n) = x(n) + ay(n-1)}$$

$n=0 \Rightarrow \underbrace{y(0)}_{\text{present o/p}} = \underbrace{x(0)}_{\text{present i/p}}$ ~~+ a(y)~~ $\underbrace{ay(-1)}_{\text{past o/p}}$

Z-Transform

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$x(n) \rightarrow$ Discrete time signal

$\downarrow$ Z - transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

Formula :-

① $\delta(n) \xrightarrow{ZT} 1$

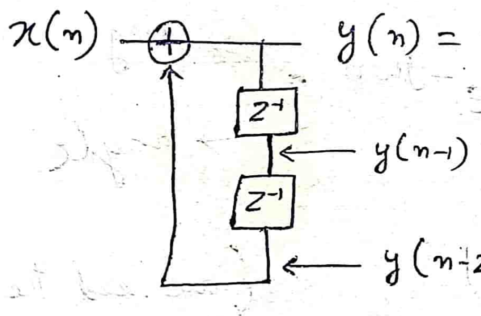
② $\delta(n-1) \xrightarrow{ZT} z^{-1}$

③ $\delta(n-2) \xrightarrow{ZT} z^{-2}$

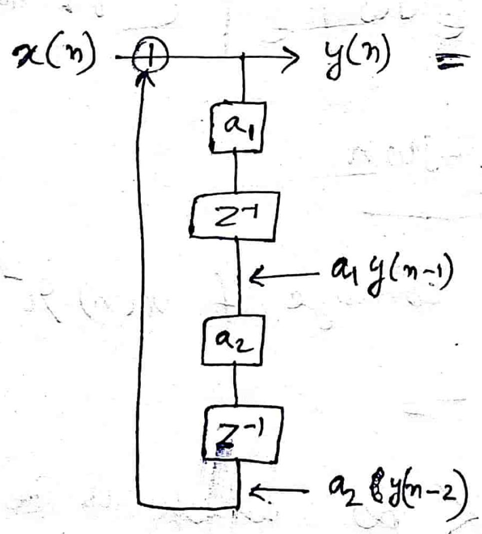
④ $\delta(n+1) \xrightarrow{ZT} z^{1}$

⑤ $u(n) \xrightarrow{ZT} \dfrac{z}{z-1} = \dfrac{1}{1-\frac{1}{z}}$

⑥ $a^n u(n) \xrightarrow{ZT} \dfrac{z}{z-a} = \dfrac{1}{1-\frac{a}{z}}$



$y(n) = x(n) + y(n-1) + y(n-2)$



$y(n) = x(n) + a_1 y(n-1) + a_2 y(n-2)$

This is a difference equation.

$\dfrac{d^2 x(t)}{dt^2} + \dfrac{2\, dx(t)}{dt} + 3$ (for eg)

is a differential equation

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$$X[z] = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

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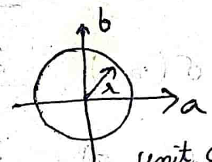
$\underbrace{n=-\infty \text{ to } -1}_{\text{Left signal signal}}$ $\underbrace{n=0 \text{ to } \infty}_{\text{Right sided signal}}$

Region of Convergence

We know, $X[z] = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$

$$= \sum_{n=-\infty}^{\infty} x(n) \left(re^{j\omega}\right)^{-n}$$

where, $z = re^{j\omega} = a + ib$



unit circle.

ROC for		
Z transform	$\rightarrow$	circle
Laplace "	$\rightarrow$	plane

$$\sum_{n=-\infty}^{\infty} \left\{x(n)\, r^{-n}\right\} e^{-j\omega n}$$

$\rightarrow$ mag

$\rightarrow$ angle

when $r=1$, then $|z|=1$

$$|z| = r|e^{j\omega}| = |e^{j\omega}| = 1$$

(we need the magnitude of Z transform to be

Hence $\sum_{n=-\infty}^{\infty} \left\{x(n)\, r^{-n}\right\} e^{-j\omega n}$

This particular term will converge if $x(n)\, r^{-n}$ is absolutely summable.

$\therefore \sum_{n=-\infty}^{\infty} x(n)\, r^{-n} < \infty$ which is the condition of existance of "Z-transform

Infinite Series formula

$$\sum_{n=0}^{\infty} c^n = \frac{1}{1-c}$$

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Q Determine the Z-transform and ROC for the following signal

$$x(n) = a^n \quad \text{for} \quad n \geq 0$$
$$= 0 \quad \text{for} \quad n < 0$$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} = \sum_{n=0}^{\infty} x(n)\, z^{-n} = \sum_{n=0}^{\infty} a^n z^{-n}$$

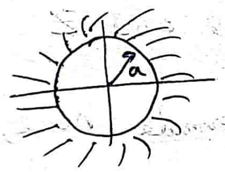
$$= \underbrace{\sum_{n=0}^{\infty} \left(a z^{-1}\right)^n}_{} = \frac{1}{1 - a z^{-1}}$$

convert to infinite series expression "c^n"

$$= \frac{1}{1 - \frac{a}{z}} = \frac{z}{z-a}$$ Ans

To calculate the ROC, always look for

For ROC to be valid, $\frac{1}{1-\frac{a}{z}} \neq \infty$



ROC

$\therefore z \neq 0$ or $\frac{z}{z-a} \neq \infty$

or $\left|\frac{a}{z}\right| < 1 \Rightarrow |z| > a$

we can't take "less than" condition too as $n \geq 0$, $0; n < 0$

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Q $x(n) = 0.8^n u(-n-1)$. Determine the Z transform & ROC

It is a left sided signal

$-n-1=0$

$\Rightarrow n=-1$

$\therefore$ The range is $-\infty$ to -1

$$\therefore X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n} = \sum_{n=-\infty}^{-1} x(n) z^{-n}$$

$$= \sum_{n=-\infty}^{-1} 0.8^n Z^{-n}$$

$$= \sum_{n=1}^{\infty} 0.8^{-n} Z^n \quad !!$$

To interchange the limit of summation, we need to change the signs of the powers of 0.8 and Z

Imp We can't apply infinite sum series formula here, as the sum range is $\sum_{n=1}^{\infty}$ and not $\sum_{n=0}^{\infty}$

$\therefore$ Change !
$$\sum_{n=0}^{\infty} 0.8^{-n} Z^n - 0.8^0 Z^{-0}$$

$$= \sum_{n=0}^{\infty} 0.8^{-n} Z^n - 1$$

$$= \sum (0.8^{-1} Z)^n - 1$$

$$= \frac{1}{1-0.8^{-1}Z} - 1$$

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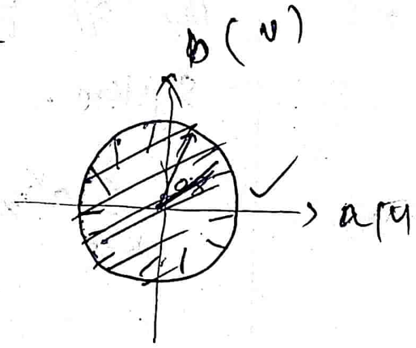
$$\frac{1}{1-\frac{Z}{0.8}} = \frac{0.8}{0.8-Z} \longrightarrow \underline{Ans}$$



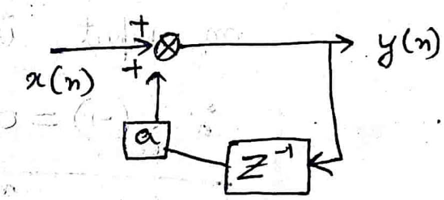
$$0.8 - Z = 0 \Rightarrow Z = 0.8$$

$$|Z| < 0.8$$

23/feb LTI system characterised by Constant coefficient difference Eq^n

Let us ~~cond~~ consider a recursive system

$$y(n) = x(n) + ay(n-1) \quad \text{—} (1)$$



for $n=0 \Rightarrow y(n) = x(0) + ay(-1)$ — (2)

for $n=1 \Rightarrow y(1) = x(1) + ay(0)$

$$= x(1) + a[x(0) + ay(-1)]$$

$$= x(1) + ax(0) + a^2 y(-1)$$

$$\therefore y(1) = a^2 y(-1) + aX(0) + x(1) \quad \text{—} (3)$$

for $n=2 \Rightarrow y(2) = a^3 y(-1) + a^2 x(0) + ax(1) + x(2)$ — (4)

In general,

$$y(n) = a^{n+1} y(-1) + a^n x(0) + a^{n-1} x(1) + \dots + ax(n-1) + x(n)$$

$$\boxed{\Rightarrow y(n) = a^{n+1} y(-1) + \sum_{K=0}^{n} a^K x(n-K)}$$

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This o/p $y(n)$ is defined for $n \geq 0$ ie for Causal System. $y(-1)$ is the initial condition of the system. (Feedback)

$\therefore \sum_{k=0}^{n} a^k x(n-k)$ is The response of The system to The input $x(n)$.

Zero State response or Forced Response

If $y(-1) = 0$ denoted by $y_{zs}(n)$

$\because$ The system is forced to the input ie. completely depends on input is called forced response.

$\because y(-1) = 0$ it is ~~as~~ also called as Relaxed System.

$$\boxed{y(n) = \sum_{k=0}^{n} a^k x(n-k)}$$

Zero input response or Natural Response

Input is not applied to The system & initial condition is not equals to zero. Thus, This kind of o/p without applying The i/p $x(n)$ is called ZIR or NR

Since, $\underbrace{|y(-n)|}_{\text{generalised initial condition.}} \neq 0$ is ~~cal~~ called as non relaxed initial Condition.

$$\boxed{\therefore y(n) = y_{zi}(n) = a^{n+1} y(-1)}$$

$\therefore$ overall response of The system is

$$y(n) = y_{zi}(n) + y_{zs}(n)$$

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$$\Rightarrow y(n) = -\sum_{K=1}^{N} a_K y(n-K) + \sum_{K=0}^{M} b_K x(n-K)$$

initial response — zero state response. Memorise

Linear Constant coefficient difference eq^n

$$y(n) = -\sum_{K=1}^{N} a_K y(n-K) + \sum_{K=0}^{M} b_K x(n-K)$$

Where a_K & b_K are coefficients ~~are~~ & N is the order of the system (Numbers ~~Laplace~~ of past ~~first~~ o/p, present i/p and M is the no. of past i/p where

~~$N<M$~~ $M \leq N$

(*) -ve constant or coefficients are inserted for o/p signals because o/p signals are feedback from o/p to i/p the constant are inserted for i/p signals.

because i/p signals are fed forward from i/p to o/p

The general diff eq^n governing first order (difference) ~~first or~~ discrete time LTI system is ~~y(n)=~~

$y(n) = a_1 y(n-1) + b_0 x(n) + b_1 x(n-1)$. sol^n of this expression can be obtained by homogeneous complementary or by implementing particular sol^n

Q. Construct the block diagram & SFG of the discrete time system whose i/p & o/p relationships whose i/p & o/p are described by the difference eq^n.

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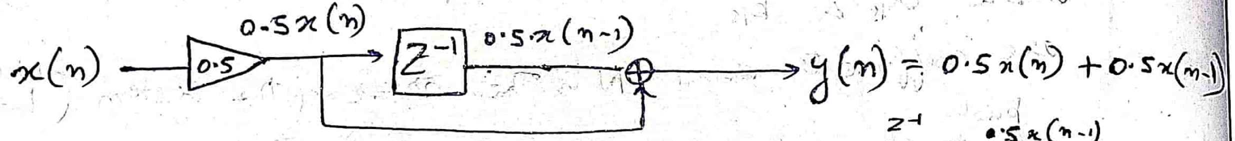
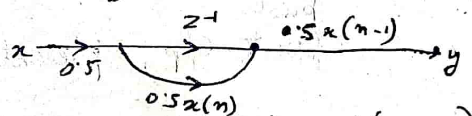
<u>materials required</u> :-

i/p, o/p, adder, multiplier, Delay element, Advanced elmt

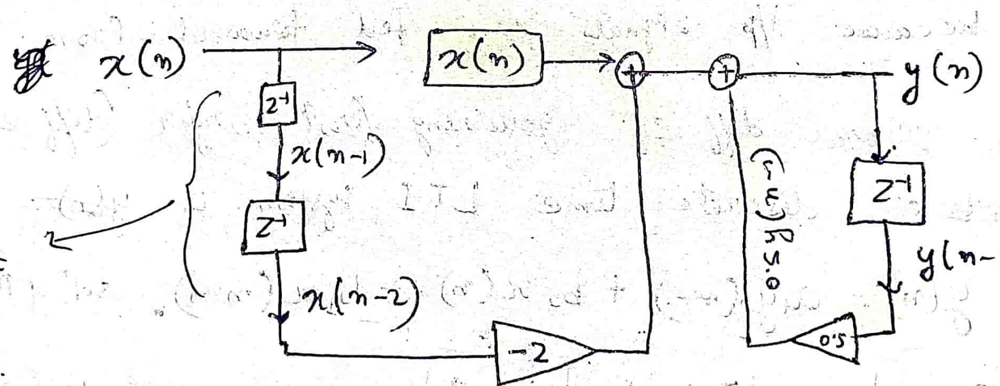
ⓐ $y(n) = 0.5x(n) + 0.5x\underbrace{(n-1)}_{\text{delay}}$ $\quad x(n+1)$ = Advanced



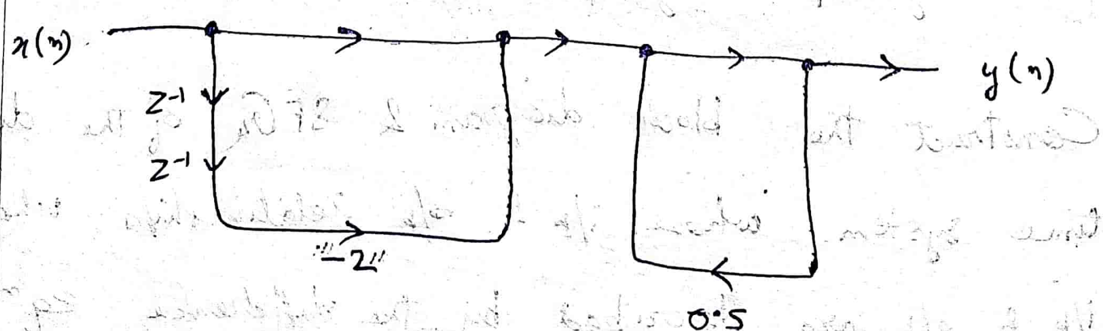


Always introduce a delay ~~block~~ element when There is a term $(n-k)$
and " " advance element " " " " " $(n+k)$

ⓑ $y(n) = 0.5y(n-1) + x(n) - 2x(n-2)$

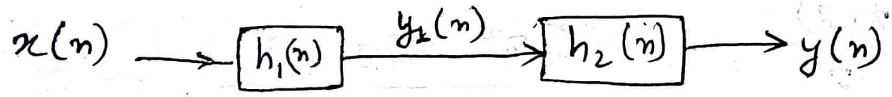


Don't put $\boxed{Z^{-2}}$

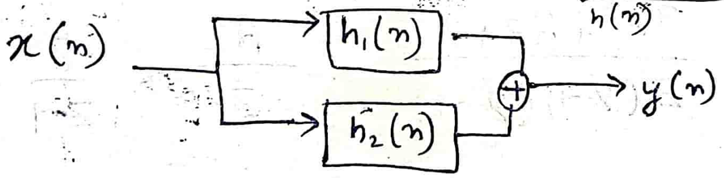


Interconnection of discrete time system

① Cascaded Connection

ⓐ 

$$y(n) = x(n) * \underbrace{[h_1(n) * h_2(n)]}_{h(n)}$$

ⓑ 

$$y(n) = x(n) * \underbrace{[h_1(n) + h_2(n)]}_{h(n)}$$

To find the system response, Find $h(n)$.

Then $h(n) = \dfrac{y(n)}{x(n)}$

$$H(z) = \frac{Y(z)}{X(z)}$$

Q Determine inverse Z transform for $X(z) = \dfrac{3 + 2z^{-1} + z^{-2}}{1 - 3z^{-1} + 2z^{-2}}$

(Find $\xrightarrow[Z^T]{Inv} x(n)$)

-2 is the highest power here.

$$X(z) = \frac{\cancel{z^{-2}}\{3/z^{-2} + 2z + 1\}}{\cancel{z^{-2}}\{1/z^{-2} - 3z + 2\}}$$

$$= \frac{3z^2 + 2z + 1}{z^2 - 3z + 2}$$

$$x(z) = \frac{3z^2 + 2z + 1}{\cancel{z-1}\,(z-1)(z-2)}$$

$$\frac{x(z)}{z} = \frac{3z^2 + 2z + 1}{z(z-1)(z-2)}$$

$$\therefore \frac{x(z)}{(z)} = \frac{3z^2 + 2z + 1}{z(z-1)(z-2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2}$$

$$3z^2 + 2z + 1 = A(z-1)(z-2) + B(z)(z-2) + C(z)(z-1)$$

<u>when $z = 0$</u>

$\cancel{A(-1) = 1}$ $\quad 1 = A(-1)(-2)$

$A = \frac{1}{2}$

<u>when $z = 1$</u>

$B(1)(1-2) = 3 \times 1^2 + 2 \times 1 + 1$

$\Rightarrow -B = 6$

$\Rightarrow B = -6$

<u>when $z = 2$</u>

$C(2)(2-1) = 3 \times 2^2 + 2 \times 2 + 1$

$\Rightarrow 2C = 17$

$\Rightarrow C = \frac{17}{2} = 8.5$

$$\begin{array}{r} 12 \\ 4 \\ \hline 16 \\ 1 \\ \hline 17 \end{array}$$

LTI
alw..
Cau..
$\therefore$ T..
m..

$$\frac{X(z)}{z} = \frac{1/2}{z} + \frac{-6}{z-1} + \frac{8.5}{z-2}$$

$$\Rightarrow X(z) = \frac{1}{2} - 6\frac{z}{z-1} + 8.5\frac{z}{z-2}$$

Properties

$$Z\{\delta(n)\} = 1$$

$$Z\{u(n)\} = \frac{z}{z-1}$$

$$Z\{a^n u(n)\} = \frac{z}{z-a}$$

Taking inverse Z transform

$$x(n) = \frac{1}{2}\delta(n) - 6u(n) + 8.5[2^n u(n)]$$

LTI is always causal

$\therefore$ Take $n \geq 0$

If $n=0$, $x(n) = \frac{1}{2} - 6 + 8.5$

$= 3$

$2^0 = 1$
$u(n) = 1$
$\delta(n) = 1$

If $n=1$

$x(n) = 0 - 6 + 8.5 \times 2$

$= 11$

$\delta(n) = 0$

$u(n) = 1$

$2^1 = 1$

If $n=2$

$x(n) = 0 - 6 + 8.5 \times 4$

$= 28$

$$\begin{array}{r} 34 \\ -6 \\ \hline 28 \end{array}$$

$$\therefore x(n) = \{\underset{\uparrow}{3}, 11, 28, \ldots\}$$

Memorise Z transform formulae

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Transfer f^n of LTI discrete time system

Imp

Let $x(n)$ and $y(n)$ be the input and o/p of discrete time LTI system. we know
$$y(n) = -\sum_{m=1}^{N} a_m\, y(n-m) + \sum_{m=0}^{M} b_m\, x(n-m) \quad \text{--- (1)}$$

Looks like CS but in discrete domain

Considering z Transform for eq (1) with zero initial condition i.e.
$$y(n) = 0 \text{ for } n<0 \text{ and}$$
$$x(n) = 0 \text{ for } n<0$$

Z transform

$$\therefore Z\{y(n)\} = Z\left\{ -\sum_{m=1}^{N} a_m\, y(n-m) + \sum_{m=0}^{M} b_m\, x(n-m) \right\}$$

$$\Rightarrow Y(z) = Z\left\{ -\sum_{m=1}^{N} a_m\, y(n-m) \right\} + Z\left\{ \sum_{m=0}^{M} b_m\, x(n-m) \right\}$$

$$\Rightarrow Y(z) = -\sum_{m=1}^{N} a_m\, Z\{y(n-m)\} + \sum_{m=0}^{M} b_m\, Z\{x(n-m)\}$$

$$\Rightarrow Y(z) = -\sum_{m=1}^{N} a_m\, z^{-m}\, Y(z) + \sum_{m=0}^{M} b_m\, z^{-m}\, X(z)$$

$$\Rightarrow Y(z)\left[1 + \sum_{m=1}^{N} a_m\, z^{-m}\right] = \sum_{m=0}^{M} b_m\, z^{-m}\, X(z)$$

$$\boxed{\frac{Y(z)}{X(z)} = \frac{\sum_{m=0}^{M} b_m\, z^{-m}}{1 + \sum_{m=1}^{N} a_m\, z^{-m}}}$$

limit { - Feedback starts from 0
i/p starts from 1

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On expanding

$$\frac{y(z)}{x(z)} = \frac{b_0 + b_1 z^{-1} + b_2 Z^{-2} + \dots b_m z^{-m}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots a_N z^{-N}}$$

Looks like CS but in discrete domain.

$$\therefore \frac{y(z)}{x(z)} = G \frac{(z-z_1)(z-z_2)\dots(z-z_m)}{(z-p_1)(z-p_2)\dots(z-p_N)}$$

zeroes

Poles.

Impulse Response and Transfer Function

$x(n)$, $y(n)$, $h(n)$ → we need these.

We know: $y(n) = x(n) * h(n)$

$$y(n) = \sum_{m=-\infty}^{\infty} x(m)\, h(n-m)$$

$$\Rightarrow Z\{y(n)\} = Z\left\{\sum_{m=-\infty}^{\infty} x(m)\, h(n-m)\right\}$$

$$\Rightarrow y(z) = \cancel{H(z)}\ X(z)\, H(z)$$

$$H(z) = \frac{y(z)}{x(z)} = TF$$

If $Z\{x(m)\} = X(z)$ and $Z(h(n)) = H(z)$ then,

$Z\{x(n) * h(n)\} = X(z)\, H(z)$

Convolution property

$$\frac{o/p}{i/p} = \text{profit/loss}$$

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