~~Butterworth Filter~~

Discrete Fourier Transform.

DTFT $\Rightarrow X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

$\omega \rightarrow$ continuous $(0 \rightarrow 2\pi)$

$\omega \rightarrow \omega_k = \frac{2\pi}{N} \cdot k$, $k = 0, 1, 2, \ldots N-1$

$$\boxed{\text{DFT} \Rightarrow X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} k \cdot n}}$$

$L \rightarrow$ Length of sequence $x(n)$

$\boxed{N \geq L}$
- → Decides the limit of 'k & n'
- → Decides the amount of processing time.
- → N is required to be power of 2.

DTFT → Double sided → No Digital Processing

DFT → Single sided → Digital Processing allowed

Q. $\delta(n)$ $\qquad\qquad \delta(n) = 1, n=0$

$X(k) = \sum_{n=0}^{N-1} \delta(n) e^{-j\frac{2\pi}{N} k \cdot n}$

$= 1 e^{-j\frac{2\pi}{N} k \cdot 0} = 1.$

Q. $\delta(n-n_0)$ $\quad = 1 \cdot e^{-j\frac{2\pi}{N} k \cdot n_0} = e^{-j\frac{2\pi}{N} k n_0}$

Q. $x(n) = a^n$

Digital commu... Design speech ... S.I.T. ...

$$X(K) = \sum_{n=0}^{N-1} a^n \left(e^{-j\frac{2\pi}{N}K}\right)^n$$

$$= \sum_{n=0}^{N-1} \left(a e^{-j\frac{2\pi}{N}K}\right)^n$$

$$\sum_{K=N_1}^{N_2} A^K = \frac{A^{N_1} - A^{N_2+1}}{1-A}$$

$$\boxed{x(n) = \frac{1-a^N}{1-a e^{-j\frac{2\pi}{N}K}}}$$

Q. $x(n) = u(n) - u(n-N)$.

Q. $x(n) = \cos\left(\frac{2\pi}{N} \cdot K_0 n\right)$.

DFT - As linear Transformation

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}K \cdot n}$$

$$e^{-j\cdot\frac{2\pi}{N}} \Rightarrow \omega_N$$

$$X(K) = \sum_{n=0}^{N-1} x(n) (\omega_N)^{K \cdot n}$$

$$X(K) = x(0) \cdot 1 + x(1)(\omega_N)^K + x(2)(\omega_N)^{2K} + \ldots + x(N-1)(\omega_N)^{(N-1)K}$$

$K=0$ $X(0) = x(0) \cdot 1 + x(1)(\omega_N)^0 + x(2)(\omega_N)^{2 \cdot 0} + \ldots$

$K=1$ $X(1) = x(0) \cdot 1 + x(1)(\omega_N)^1 + x(2)(\omega_N)^{2 \cdot 1} + \ldots$

$K=2$ $X(2) = x(0) \cdot 1 + x(1)(\omega_N)^2 + x(2)(\omega_N)^4 + \ldots$

⋮

$$\begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ \vdots \\ X_{N-1} \end{bmatrix} = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & \cdots & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & \cdots & W_N^{N-1} \\ W_N^0 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & & & & \\ W_N^0 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}_{N\times N} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}$$

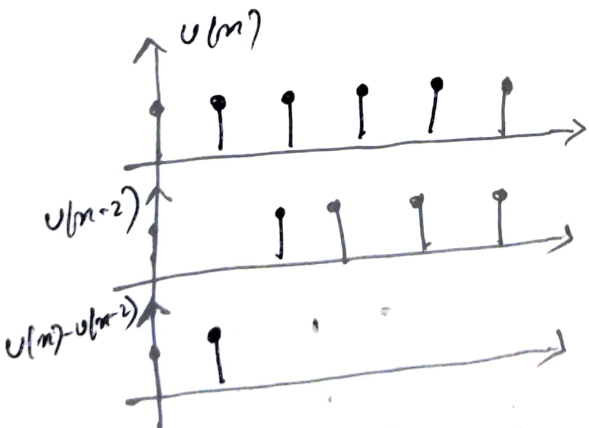
$$\boxed{X_N = [W_N]\, x(n)} \qquad \boxed{x_N = \frac{1}{N}[W_N^*] X_N}$$

multiply by W_N^{-1}

$$\Rightarrow \boxed{W_N^{-1} X_N = (W_N^{-1})(W_N)^{-1} x(n)}$$

Periodicity of W_N $\quad \boxed{W_N^K = W_N^{K+N}}$

Q. Obtain 4 point DFT of $x(n) = u(n) - u(n-2)$.



$$x(n) = \{1, 1, 0, 0\}.$$

$$W_N^{Kn} = e^{-j\frac{2\pi}{N}K\cdot n}$$

$N = 4$

$$\left[W_4^{kn}\right] = \begin{array}{c} \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{array}{l} k=0 \\ k=1 \\ k=2 \\ k=3 \end{array} \\ \begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \end{array} \end{array}$$

$$\left(W_N\right)^{k} = W_N^{k+N} \qquad \left(W_N\right)^{kn} = e^{-j\frac{2\pi}{N}kn}$$

$$\left(W_4\right)^{kn} = e^{-j\frac{\pi}{2}k.n.}$$

$(W_4)^0 = 1$

$(W_4)^1 = -j$

$(W_4)^2 = -1$

$(W_4)^3 = +j$

$(W_4)^0 = (W_4)^{0+4} = W_4^4 = 1$

$W_4^2 = W_4^6 = -1$

$W_4^1 = W_4^5 = W_4^9 = -j$

$kn = 0.$

$(W_4)^0 = 1.$

$kn = 1$

$(W_4)^1 = e^{-j\pi/2}$

$= \cos\pi/2 - j\sin\pi/2 = -j$

$kn = 2$

$(W_4)^2 = e^{-j\pi}$

$= \cos\pi - j\sin\pi = -1$

$$\left[W_N^{kn}\right] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$x_N = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$X_N = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$X_4 = \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

Properties of DFT.

1) Linearity

$$x_1(n) \xleftrightarrow{DFT} X_1(k)$$

$$x_2(n) \longleftrightarrow X_2(k)$$

$$a_1 x_1(n) + a_2 x_2(n) \longleftrightarrow a_1 X_1(k) + a_2 X_2(k)$$

2) Periodicity

$$x(n) \longleftrightarrow X(k)$$

$\Rightarrow$ if $x(n+N) = x(n)$

$$X(k+N) = X(k)$$

Proof:-

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

$$X(k+N) = \sum_{n=0}^{N-1} x(n)\, W_N^{(k+N)n}$$

$$= \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \cdot W_N^{Nn}$$

$$= \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

$$W_N^{Nn} = e^{-j\frac{2\pi}{N} Nn} = e^{-j2\pi n} = \cos(2\pi n) - \cancel{j\sin(2\pi n)} = 1$$

3) Symmetry for real $x(n)$.

$$x(n) \longleftrightarrow X(k)$$

$$\boxed{X(N-k) = X^*(k) = X(-k)}$$

Proof:

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{Kn}$$

$$X(N-K) = \sum_{n=0}^{N-1} x(n)\, W_N^{(N-K)n}$$

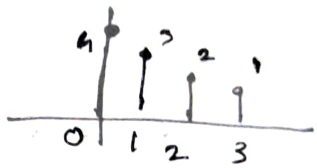
$$= \sum_{n=0}^{N-1} x(n)\, W_N^{-Kn}\, \cancel{W_N^{Nn}} \to 1$$

$$= \sum_{n=0}^{N-1} x(n)\, W_N^{(-Kn)}$$

$$= X(-K).$$

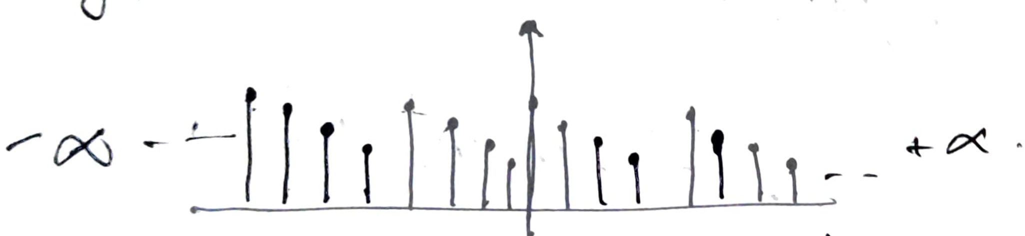
4) Circular Symmetry.

$$x(n) \xleftrightarrow{DFT} X(K).$$



$x(n) = \{4, 3, 2, 1\}$

Taking periodic repetition of $x(n) \to x_p(n)$.



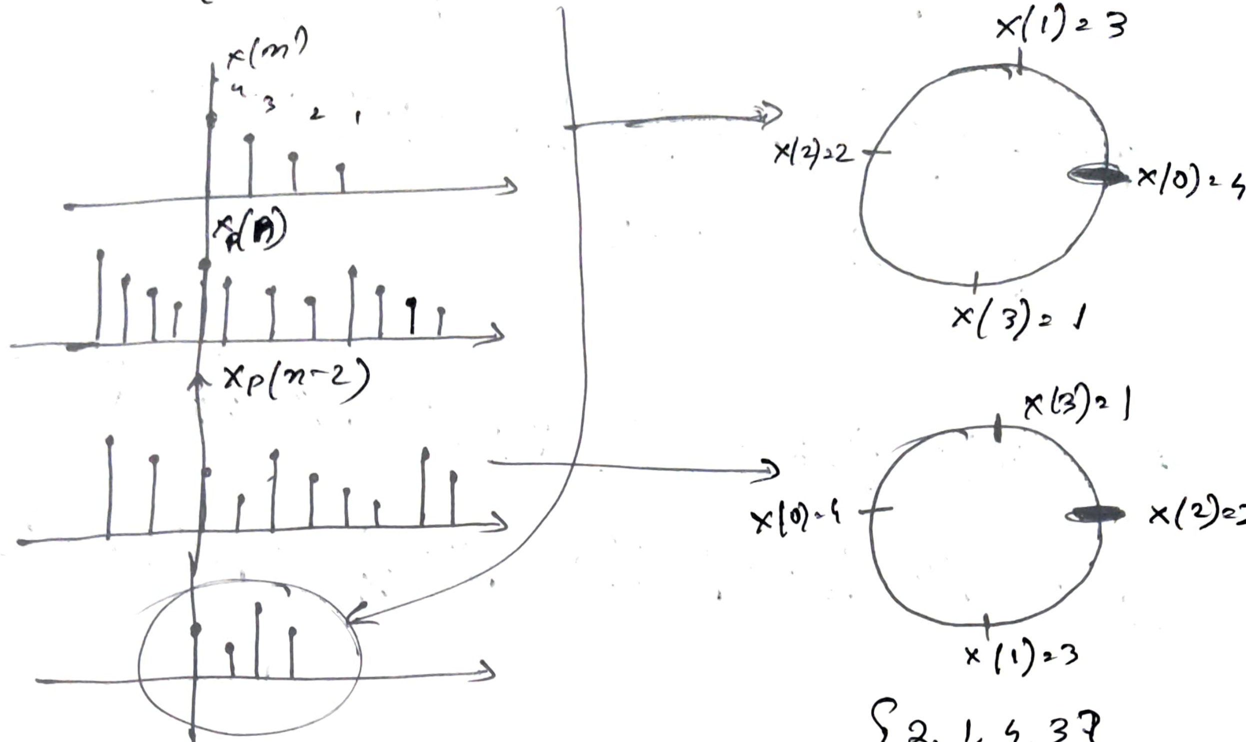
$$x_p(n) \xleftrightarrow{DFT} X(K)$$

$$\boxed{x_p(n) = \sum_{l=-\infty}^{\infty} x(n-lN)}$$

$$x_p'(n) = \sum_{l=-\infty}^{\infty} x(n-K-lN)$$ ← linear shift

$$\boxed{x'(n) = x((n-N))_N}$$ → circular shift (modulo

$x(n) = \{4, 3, 2, 1\} \quad 0 \le x \le 3$



$\{2, 1, 4, 3\}$.

* Circular evensift

$$x(N-n) = x(n)$$

* Circular odd shift

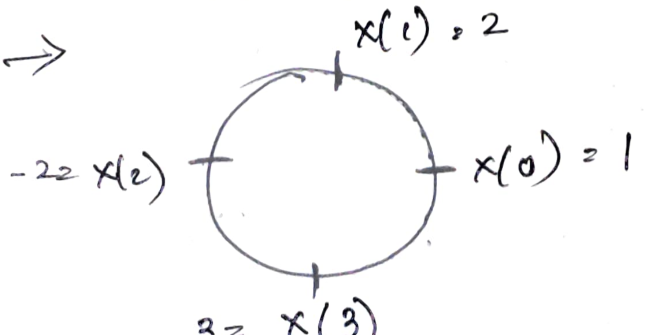
$$x(N-n) = -x(n)$$

**** Circular folded shift

$$x((-n))_N = x(N-n)$$

Q. $x(n) = \{1, 2, -2, 3\}$ $\quad y(n) = x((n+2))_4$ $\quad y(n) = ?$

$\rightarrow$



$n = 0, 1, 2, 3$

$n = 0, \quad y(0) = x((0+2))_4$

$= x((2))_4$

$= x(2) = -2$

$y(1) = x((1+2)) = x((3))$

$= x(3) = 3$

$y(2) = x((2+2))_4 = x((4))_4 \Rightarrow x((0+4))_4 \to x((0))_4 = x(0) = 1$ (periodicity)

$y(3) = x((3+2))_4 = x((5+4))_4 \to x((1))_4 \Rightarrow x(1) = 2$

$y(n) = \{-2, 3, 1, 2\}$

Q. $x(n) = \{1, 2, -2, 3\}$, $y(n) = x((n-1))_4$, $y(n) = ?$

$y(0) = x((0-1))_4 = x((-1))_4 = x((4-1))_4 = x((3))_4 = x(3) = 3$

Discrete Time Fourier Transform.

$x(n) \to$ periodic finite energy.

$$\boxed{X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}}$$

$x(n) \xrightarrow{DTFT} X(\omega)$

IDFT

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega} d\omega$$

$X(\omega) \to -\pi$ to π

$0 \to 2\pi$

$$X(\omega + 2\pi k) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn(\omega+2\pi k)}$$

$$= \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega} \cdot e^{-j2\pi nk}$$

$$= \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega} (\cos 2\pi nk - j \sin 2\pi nk)$$

$$= \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega}$$

$\therefore X(\omega) \to$ periodic

$$\boxed{\sum_{n=-\infty}^{\infty} |x(n)| < \infty} \rightarrow \text{FT will exist.}$$

Q. $x(n) = \delta(n) \xleftrightarrow{DTFT} 1$

$$X(\omega) = \sum_{n=-\infty}^{\infty} \delta(n) e^{-j\omega n}$$

$$X(\omega) = e^{-j\omega 0} = 1$$

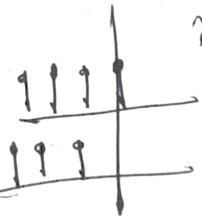
Q. $x(n) = a^n u(n)$.

$$X(\omega) = \sum_{n=-\infty}^{\infty} a^n u(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(a e^{-j\omega}\right)^n$$

$$= \ldots$$

Q. $x(n) = a^{-n} u(-n-1)$.



$$X(\omega) = \sum_{n=-\infty}^{-1} a^{-n} e^{-j\omega n}$$

let $n = -n$

$$= \sum_{1}^{\infty} \left(a e^{j\omega}\right)^n = \ldots$$

Q. $x(n) = u(n)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} u(n) e^{-j\omega n}$$

$$= \sum_{0}^{\infty} 1 \left(e^{-j\omega}\right)^n$$

$$= 1 + e^{-j\omega} + e^{-2j\omega} + \ldots \infty.$$

$$= \frac{1}{1 - e^{-j\omega}}$$

$\forall \omega = 0 = \infty$ (x)

$$X(\omega) = \frac{1}{e^{j\omega/2} e^{-j\omega/2} - e^{-j\omega/2} e^{-j\omega/2}}$$

$$= \frac{1}{2j e^{-j\omega/2} \left[\frac{e^{j\omega/2} - e^{-j\omega/2}}{2j}\right]}$$

$$= \frac{e^{j\omega/2}}{2j \sin \omega/2}$$

$$u(n) \xleftrightarrow{DTFT} \frac{e^{j\omega/2}}{2j \sin \omega/2}$$

Q. Find DTFT of rectangular pulse.

$x(n) = A \quad \forall \quad 0 \le n \le L-1$

$= 0 \quad$ otherwise..

$\rightarrow$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} A\, e^{-j\omega n} = A \sum_{n=0}^{L-1} \left(e^{-j\omega}\right)^n$$

$$\sum_{K=N_1}^{N_2} a^K = \frac{a^{N_1} - a^{n_2+1}}{1-a}$$

$n_1 = 0$

$n_2 = L-1$

$$= A\left[\frac{\left(e^{-j\omega}\right)^0 - \left(e^{-j\omega}\right)^{L-1+1}}{1-e^{-j\omega}}\right]$$

$$= A\left[\frac{1-e^{-j\omega L}}{1-e^{-j\omega}}\right]$$

$$= A\left[\frac{e^{j\omega L/2}\, e^{-j\omega L/2} - e^{-j\omega L/2}\, e^{-j\omega L/2}}{e^{-j\omega/2}\, e^{j\omega/2} - e^{-j\omega/2}\, e^{-j\omega/2}}\right]$$

$$= A\, \frac{e^{-j\omega L/2}}{e^{-j\omega/2}}\left[\frac{\left(e^{j\omega L/2} - e^{-j\omega L/2}\right)/2j}{\left(e^{j\omega/2} - e^{-j\omega/2}\right)/2j}\right]$$

$$= A\, e^{-j\omega/2 (L-1)}\, \frac{2 \sin \omega L/2}{2 \sin \omega/2}$$

$$X(\omega) = A e^{-\frac{j\omega}{2}(L-1)}\, \frac{\sin \omega L/2}{\sin \omega/2}$$

Q. $x(n) = a^n \cos \omega_0 n \;\;\;\; a^n \sin \omega_0 n$

$\rightarrow$ Euler's $- e^{j\theta}$

Properties

$$x_1(n) \longleftrightarrow X_1(\omega)$$
$$x_2(n) \longleftrightarrow X_2(\omega)$$

1) $a_1 x_1(n) + a_2 x_2(n) \xleftrightarrow{DTFT} a_1 X_1(\omega) + a_2 X_2(\omega)$

$x_1(n) * x_2(n) \longleftrightarrow X_1(\omega) X_2(\omega)$

2) $x(n) \xleftrightarrow{DTFT} X(\omega)$

$x(n-k) \longleftrightarrow e^{-j\omega k} X(\omega)$

3) $e^{j\omega_0 n} x(n) \longleftrightarrow X(\omega - \omega_0)$

4) Time Reversal $\;\; x(-n) \longleftrightarrow X(-\omega)$

5) $x(n) \longleftrightarrow X(\omega)$

$n \cdot x(n) \longleftrightarrow j \cdot \frac{\partial}{\partial \omega} X(\omega)$

Time domain $x(n)$	Freq. domain $X(\omega)$
Real & Even	Real & Even
Real & Odd	Odd & Imaginary
Purely Imaginary	Odd & Real
Imaginary & Even	Imaginary & Even

Q. Consider a causal LTI system with freq. Response $H(j\omega) = \frac{1}{j\omega+2}$. For a given i/p $x(t)$, the o/p is

$y(t) = e^{-2t}u(t) - e^{-3t}u(t)$. Find $X(\omega)$

$\rightarrow$ $y(t) = e^{-2t}u(t) - e^{-3t}u(t)$ | $e^{-at} = \frac{1}{s+a}$

DTFT

$Y(j\omega) = \frac{1}{j\omega+2} - \frac{1}{j\omega+3}$

$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$

$X(j\omega) = \left(\frac{1}{j\omega+2} - \frac{1}{j\omega+2}\right)(j\omega+2)$

$X(j\omega) = 1 - \frac{j\omega+2}{j\omega+3}$

$x(n) = IDFT[X(\omega)]$

$= \cancel{IDFT[1]} + IDFT\left[\frac{j\omega+2}{j\omega+3}\right]$

0

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} 1 . e^{j\omega} d\omega$

$= \frac{1}{2\pi} \left.\frac{e^{j\omega}}{j}\right|_{-\pi}^{\pi}$

$= \frac{1}{2\pi j}\left[e^{\pi} - e^{-\pi}\right]$

$= \frac{1}{\pi}\sin\pi = 0$

$$x(\omega) = -\frac{j\omega+2}{j\omega+3}$$

$$\frac{x(\omega)}{j\omega} = \frac{-j\omega+2}{(j\omega)(j\omega+3)} = \frac{A_1}{j\omega} + \frac{A_2}{j\omega+3}$$

$$\frac{x(\omega)}{j\omega} = \frac{2}{3 j\omega} - \frac{1}{j\omega+3}$$

$$x(\omega) = \frac{2}{3} - \frac{j\omega}{j\omega+3}$$

$$x(n) = \frac{2}{3}\delta(n) - (3)^n u(n)$$

Q. Find the convolution using Fourier transform.

$$x_1(n) = \left(\frac{1}{2}\right)^n u(n) \qquad x_2(n) = \left(\frac{1}{3}\right)^n u(n).$$

$$x_1(n) * x_2(n) \longleftrightarrow x_1(\omega) x_2(\omega).$$

$$x_1(\omega) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} \qquad x_2(\omega) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$x_1(\omega) x_2(\omega) = \frac{1}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{3}e^{-j\omega}\right)}$$

$$= \frac{e^{j\omega}}{\left(e^{j\omega}-\frac{1}{2}\right)\left(e^{j\omega}-\frac{1}{3}\right)}$$

$$\frac{x(\omega)}{e^{j\omega}} = \frac{A_1}{e^{j\omega}-\frac{1}{2}} + \frac{A_2}{e^{j\omega}-\frac{1}{3}}$$

$$X(\omega) = \frac{3e^{j\omega}}{e^{j\omega}-\frac{1}{2}} - \frac{2e^{j\omega}}{e^{j\omega}-\frac{1}{3}}$$

$$X(\omega) = \frac{3}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{3}e^{-j\omega}}$$

$$= 3\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{3}\right)^n u(n)$$

Q. Find the frequency response of I order system described by difference eq^n

$$y(n) = ay(n-1) + x(n).$$

Find the impulse response as well

$$\Rightarrow \quad Y(j\omega) = ae^{-j\omega}Y(j\omega) + X(j\omega)$$

$$Y(j\omega)\left[1-ae^{-j\omega}\right] = X(j\omega)$$

$$H(j\omega) = \frac{1}{1-ae^{-j\omega}} \rightarrow h(n) = a^n u(n)$$

Parseval's Theoram.

Utility

It relates the energy and power in time domain to the energy and power in frequency domain

Parseval's Energy Theoram.—

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} x(t).x^*(t)\,dt$$

$x(t) = IFT\ X(\omega)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

$$x^*(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)e^{-j\omega t}d\omega$$

$|x| = \sqrt{\alpha^2+\beta^2}$

$x = \alpha + j\beta$

$x^* = \alpha - j\beta$

$x.x^* = (\alpha+j\beta)(\alpha-j\beta)$

$= \alpha^2+\beta^2$

$x.x^* = |x|^2$

$x \rightarrow$ Real valued signal

$x = x^*$ $\quad j \rightarrow 0$

$$E = \int_{-\infty}^{\infty} x(t)\frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)e^{-j\omega t}d\omega$$

$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t)e^{-j\omega t}X^*(\omega)\,d\omega\,dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)\underbrace{\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt}_{FT\ of\ x(\omega)}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(\omega)X(\omega)d\omega$$

$$\boxed{E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega = \int_{-\infty}^{\infty} |x(t)|^2 \, dt}$$

$|X(\omega)|^2 \rightarrow S_x(\omega)$ or $S_e(\omega) \rightarrow$ Energy spectral Density.

<u>System.</u>

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t).$$

$$y(t) = x(t) * h(t)$$

FT, $Y(\omega) = X(\omega) \cdot H(\omega)$

$$\boxed{|Y(\omega)|^2 = |X(\omega)|^2 \cdot |H(\omega)|^2}$$

o/p energy spectral density   i/p energy spectral density

Q. $x(t) = e^{-at}u(t) \rightarrow$ Verify Parseval's Th^m

$\rightarrow$ $$E_T = \int_{-\infty}^{\infty} \left(e^{-at} u(t)\right)^2 dt = \int_{-\infty}^{\infty} e^{-2at} \, dt = \frac{1}{2a} e^{-at} \Big|_{-\infty}^{\infty} = \frac{1}{2a}$$

$$X(\omega) = FT\left[e^{-at} u(t)\right] = \frac{1}{j\omega + a}$$

$$|X(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \Rightarrow |X(\omega)|^2 = \frac{1}{\omega^2+a^2}.$$

$$E_F = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{\omega^2+a^2} \, d\omega = \frac{1}{2\pi a} \tan^{-1}\left(\frac{\omega}{a}\right)\Big|_{-\infty}^{\infty}$$

$$= \frac{1}{2a} \quad //$$

Parseval's Power Theorem

Power signals → Periodic Signals → Can be represented as exponential fourier series.

$$x(t) = \sum_{n=-\alpha}^{\alpha} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$P = \frac{1}{T}\int_{-T/2}^{T/2} [x(t)]^2 dt$$

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |C_n|^2 \underbrace{|e^{jn\omega_0 t}|^2}_{1} dt$$

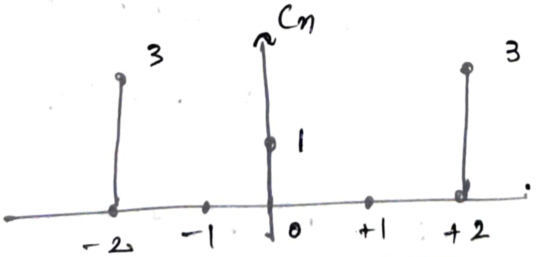
$$P = \sum_{n=-\alpha}^{\alpha} C_n^2 \frac{1}{T}\int_{-T/2}^{T/2} 1\, dt$$

$$\boxed{P = \sum_{n=-\alpha}^{\alpha} |C_n|^2}$$

$e^{j\theta} = \cos\theta + j\sin\theta$

$|e^{j\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$

Q.



Power = ?

$= \sum C_n e^{jn\omega_0 t}$

$= C_{-2} e^{-2j\omega_0 t} + 0 + C_0 + 0 + C_2 e^{2j\omega_0 t}$

$= 3e^{-2j\omega_0 t} + 1 + 3e^{2j\omega_0 t}$

$= 1 + 2\times 3\left(\frac{e^{2j\omega_0 t} + e^{-2j\omega_0 t}}{2}\right)$

Avg power $A^2/2$

$= 1 + 6\cos 2\omega_0 t \quad = 1 + 18 = 19$

$$P = \sum_{-\alpha}^{\alpha} |c_n|^2$$

$$= |C_{-2}|^2 + |C_{-1}|^2 + |C_0|^2 + |C_1|^2 + |C_2|^2$$

$$= 3^2 + 0 + 1^2 + 0 + 3^2$$

$$= 1.9$$

~~Correlation - Cross-Correlation & Auto Correlation~~

Convolution - Discrete time Sequence.

Method - 1

Step - 1 Choose an initial value 'n'

we have 2 sequence $x(n)$ & $h(n)$

Initial values - n_1 & n_2

Initial value of $y(n) = x(n) * h(n)$ will be

$$\boxed{n = n_1 + n_2}$$

Step - 2 Express $x(n)$ & $h(n)$ in terms of 'k' as $x(k)$ & $h(k)$.

Step - 3 Fold $h(k)$ about $k = 0$ & find $h(-k)$.

Step - 4 If 'n' is +ve shift to right side

If 'n' is -ve " " left side

Step-5 Multiply $x(k)$ & $h(n-k)$ element by element & sum the product to obtain $y(n)$.

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)}$$

Step-6 Repeat the above step till the sum of products is zero for remaining value of n

Q. $x(n) = \begin{cases} n/3 & \forall\ 0 \le n \le 6 \\ 0 & \text{otherwise} \end{cases}$

$h(n) = \begin{cases} 1 & \forall\ -2 \le n \le 2 \\ 0 & \text{otherwise} \end{cases}$

Method   Matrix / Tabular method

$y \rightarrow n = n_1 + n_2 - 1$

$n_1 \downarrow\ x(n)$, $n_2 \downarrow\ h(n)$.

Ex → See video simple

Properties

1) $u[n] * u[n] = (n+1)\, u[n]$

2) $u[n-N_1] * u[n-N_2] = (n - N_1 - N_2 + 1)\, u[n - N_1 - N_2]$

3) $u[n] * r[n] = \frac{n(n+1)}{2} u[n]$

4) $u[n] * a^n u[n] = \frac{1-a^{n+1}}{1-a} u[n]$

5) $x[n] * \delta[n] = x[n]$

6) $x[n-N_1] * \delta[n-N_2] = x[n-N_1-N_2]$

7) $x[n] * \delta[n-N_0] = x[n-N_0]$

8) $\delta[n-1] * \delta[n+5] = \delta[n+4]$

9) $u[n-1] * \delta[n+5] = u[n+4]$

*10) $a^n u(n-2) * \delta(n-2) = a^{n-2} u(n-4)$

Q. $x_1(n) = \delta(n+1) + 2\delta(n) + \delta(n-1)$ Find $x_1(n) * x_2(n)$

$x_2(n) = 3\delta(n+1) + 5\delta(n) + 3\delta(n-1)$

$x_1(n) = \{1, \underset{\uparrow}{2}, 1\}$ $x_2 = \{3, \underset{\uparrow}{5}, 3\}$

Q. $y(n) = \sum_{k=-\infty}^{\infty} x(k)\, g(n-2k)$ where $g(n) = u(n) - u(n-4)$

Find $y(n)$ when $x(n) = \delta(n-2)$

$\rightarrow$ $\delta(n-2) = \begin{cases} 1, & n=2 \\ 0, & \text{otherwise} \end{cases}$

$\therefore$ value exist only at $k=2$.

Now, $\sum_{k=-\infty}^{\infty} \delta(n-2)\, g(n-2k)$

$\underline{y(n)} \leftarrow g(n-4) = u(n-4) - u(n-8)$

Q. $x(n) = u(n) \qquad h(n) = 2^n u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(n)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} u(k)\, 2^{n-k} u(n-k)$$

$$= \sum_{0}^{k} 2^{n-k}$$

$$= 2^n \sum_{0}^{k} \frac{1}{2^k}$$

$$= 2^n \left[1 + \frac{1}{2} + \frac{1}{2^2} + \cdots \right]$$

Stability

$$\sum_{-\infty}^{\infty} |h(n)| < \infty$$

Q. $x(n) = u(n) - u(n-7)$

$h(n) = u(n-1) - u(n-4)$

Circular Convolution Property - DFT

1) If $x_1(n) \xleftrightarrow{DFT} X_1(K)$

$x_2(n) \longleftrightarrow X_2(K)$

$$X_1(K) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$X_2(K) = \sum_{l=0}^{N-1} x_2(l)\, e^{-j\frac{2\pi}{N}kl} \quad K = 0, 1, \ldots N-$$

then $x_1(n) \circledast x_2(n) \xleftrightarrow{DFT} X_1(K) \cdot X_2(K)$

$\Downarrow$

$x_3(m) \xleftrightarrow{DFT} X_3(K)$

$\therefore X_3(K) = X_1(K) \cdot X_2(K)$

Inverse DFT

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1} X_3(k)\, e^{j\frac{2\pi}{N}k m}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X_1(k)\, X_2(k)\, e^{j\frac{2\pi}{N}k.m}$$

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1}\left\{\sum_{n=0}^{N-1} x_1(n)\, e^{-j\frac{2\pi}{N}k.n} \cdot \sum_{l=0}^{N-1} x_2(l)\, e^{-j\frac{2\pi}{N}k.l}\right\} e^{j\frac{2\pi}{N}km}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l)\left[\sum_{k=0}^{N-1} e^{-j\frac{2\pi}{N}k.n}\, e^{-j\frac{2\pi}{N}kl} \cdot e^{j\frac{2\pi}{N}km}\right]$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l)\left\{\sum_{k=0}^{N-1} \underbrace{e^{j\frac{2\pi}{N}(m-n-l)k}}_{=a}\right\}$$

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N, & a=1 \\ \frac{1-a^N}{1-a}, & a \neq 1 \end{cases}$$

$(m-n-l) \to$ integral multiple of N.
$= N, 2N, 3N, \ldots$

$(m-n-l) \to IN$

$$\sum_{k=0}^{N-1} \frac{1 - e^{j\frac{2\pi}{N}(m-n-l)N}}{1 - e^{j\frac{2\pi}{N}(m-n-l)}} = 0$$

$$x_3(m) = \frac{1}{\cancel{N}}\sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l) \cdot \cancel{N}$$

$$= \sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l) \quad \Rightarrow m-n-l \approx N.$$

$$\boxed{m-n-l = \pm pN}$$

$$l = m-n \pm pN$$

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(m-n \pm pN).$$

$$= \sum_{n=0}^{N-1} x_1(n)\, x_2(m-n \pm pN).$$

$$x_2(m-n \pm pN) = x_2(m-n, \text{ modulo } N)$$

$$= x_2((m-N))_N$$

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)\, x_2((m-n))_N$$

$$\boxed{x_3(m) = x_1(n) \circledN x_2(n) = \sum_{n=0}^{N-1} x_1(n)\, x_2((m-n))_N}$$

Circular Convolution.
- → Graphical method
- → Using matrix
- → Using DFT & IDFT.

<u>zero padding.</u>

$x_1(n) = 4 = \{a, b, c, d\}$

$x_2(n) = 3 + \text{zero} = \{f, g, h\} = \{f, g, h, 0\}$

<u>Matrix Method</u>

Q. $x(n) = \{1, 2, 3, 1\}$ $h(n) = \{4, 3, 2, 2\}$

$y(m) = x_1(n) \circledast x_2(n)$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} h(0) & h(3) & h(2) & h(1) \\ h(1) & h(0) & h(3) & h(2) \\ h(2) & h(1) & h(0) & h(3) \\ h(3) & h(2) & h(1) & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 & 3 \\ 3 & 4 & 2 & 2 \\ 2 & 3 & 4 & 2 \\ 2 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix} \quad //$$

ii) Graphical method.

$y(n) = x_1(n) \circledast x_2(n)$

Step-1 Plot $x_1(n)$ anticlockwise on inner circle.

Step-2 Plot $x_2(n)$ clockwise on ~~outher~~ outer circle.

Step-3 Multiply point to point samples on 2 circle

Step-4 Add all multiplications to get $y(0)$.

Step-5 For next value, shift outer circle anticlockwise by one sample.

Step-6 Repeat Step 3 to Step-5 till all values are calculated.

Examples Video 31

Circular Convolution Using DFT & IDFT

Q. $x_1(n) = \{1, 2, 3, 1\}$ $\quad$ $x_2(n) = \{4, 3, 2, 2\}$

$x_1(n) \xrightarrow{DFT} X_1(K)$ → Linear transformation

$x_2(n) \xrightarrow{DFT} X_2(K) \xrightarrow{\text{multiply}} X_1(K)\,X_2(K) = X_3(K)$

$X_3(K) \xrightarrow{IDFT} x_3(n)$

$x_1(n) \circledast x_2(n) \leftarrow x_3(n)$

$$X_1(K) = [W_4]\, x_1(n) \qquad W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$X_1(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix}$$

$$X_2(K) = \begin{bmatrix} & .. & \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \\ 2-j \\ 1 \\ 2+j \end{bmatrix}$$

$$X_N = X_1(K) \cdot X_2(K) = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix} \begin{bmatrix} 11 \\ 2-j \\ 1 \\ 2+j \end{bmatrix} = \begin{bmatrix} 77 \\ -5 \\ 1 \\ -5 \end{bmatrix}$$

IDFT

$$\boxed{y(m) = \frac{1}{N}\left[W_N^*\right] \cdot X_N}$$

In this matrix first revert the signs of complex values in W_N

$$y(m) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

FFT- Algorithms

Conventional FFT

Applications of DFT
→ Filtering
→ Correlation
→ Spectrum Analysis,
etc, - -

Computation of FFT
- → Divide & Conquer Approach
- → Linear filtering
  - → Chirp-z Algo
  - → Goertzel Algo

Divide & conquer Approach
- → Radix-2 FFT
- → Radix-4 FFT
- → Split Radix FFT

→ Decimation in Time (DIT)
→ DIF

$x(n) \longrightarrow X(k)$
Input   Output

Radix-2 $\boxed{N = 2^{v}}$

8 point DFT $\Rightarrow 8 = 2^{v} = 2^{3}$ 3 stage
4 point DFT $\Rightarrow 4 = 2^{v} = 2^{2}$ 2 stage
2 point $\longrightarrow$ 1 stage.

Properties of Twiddle Factor

1) $W_N^K = W_N^{K+N}$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$W_N^{K+N} = e^{\left(-j\frac{2\pi}{N}\right)(K+N)}$$

$$= e^{-j\frac{2\pi}{N}K} \cdot e^{-j\frac{2\pi}{N}N}$$

$$= \left(e^{-j\frac{2\pi}{N}}\right)^K$$

2) $W_N^{K+N/2} = -W_N^K$

3) $W_N^2 = W_{N/2}$

$$W_N = e^{-j2\pi/N}$$

$$W_{N/2} = e^{-j\frac{2\pi}{N/2}}$$

$$= \left(e^{-j\frac{2\pi}{N}}\right)^2$$

N-point sequence → $N/2$ (even), $N/2$ (odd)

Radix-2 Decimation in Time (DIT) Algorithm

Decimation. $\Rightarrow$ Break into ~~for~~ parts.

8 parts $\Rightarrow 2^3 \Rightarrow$ 3 stages.

$x(N) \longrightarrow$ has 'N' sequence

$$x(n) = \underset{\text{Even}}{f_1(m)} + \underset{\text{Odd}}{f_2(m)}$$

$f_1(m) = x(2m)$ $\qquad$ $f_2(m) = x(2m+1)$

$m = 0, 1, \ldots \frac{N}{2}-1$ $\qquad$ $m = 0, 1, \ldots \frac{N}{2}-1$

If we have started with $N = 8$ then now $f_1(m)$ & $f_2(m)$ are 4 point sequence.

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{Kn}$$

$$X(K) = \sum_{\text{Even}} x(n)\, W_N^{kn} + \sum_{\text{odd}} x(n)\, W_N^{kn}$$

$$X(K) = \sum_{m=0}^{N/2-1} x(2m)\, W_N^{K(2m)} + \sum_{m=0}^{N/2-1} x(2m+1)\, W_N^{K(2m+1)}$$

$$X(K) = \sum_{m=0}^{N/2-1} f_1(m)\, (W_N^2)^{Km} + W_N^K \sum_{m=0}^{N/2-1} f_2(m)\, (W_N^2)^{km}$$

$$X(K) = \sum_{m=0}^{N/2-1} f_1(m)\, W_{N/2}^{km} + W_N^K \sum_{m=0}^{N/2-1} f_2(m)\, W_{N/2}^{km}$$

(By property)

From basic DFT defination

$$\boxed{X(K) = F_1(K) + W_N^K F_2(K)} \quad \text{---} \; (\alpha)$$

They are now 4-point DFTs

Periodic with period N

They are periodic with $N/2$ period

$$\therefore F_1(K + N/2) = F_1(K)$$

$$F_2(K + N/2) = F_2(K)$$

Now in eq^n (α) replace K by $(K + N/2)$

$$X(K + N/2) = F_1(K + N/2) + W_N^{K+N/2} F_2(K + N/2)$$

$$\boxed{X(K + N/2) = F_1(K) - W_N^K F_2(K)} \quad \text{---} \; (\beta)$$

In eq^n (α) $K = 0, 1, 2, 3$ (As it is a 4-point DFT)

$$X(0) = F_1(0) + W_N^0 F_2(0)$$
$$X(1) = F_1(1) + W_N^1 F_2(1)$$
$$X(2) = F_1(2) + W_N^2 F_2(2)$$
$$X(3) = F_1(3) + W_N^3 F_2(3)$$

In eq^n (β) $\underline{N=8}$ if we put $K = 0, 1, 2, 3$ we get

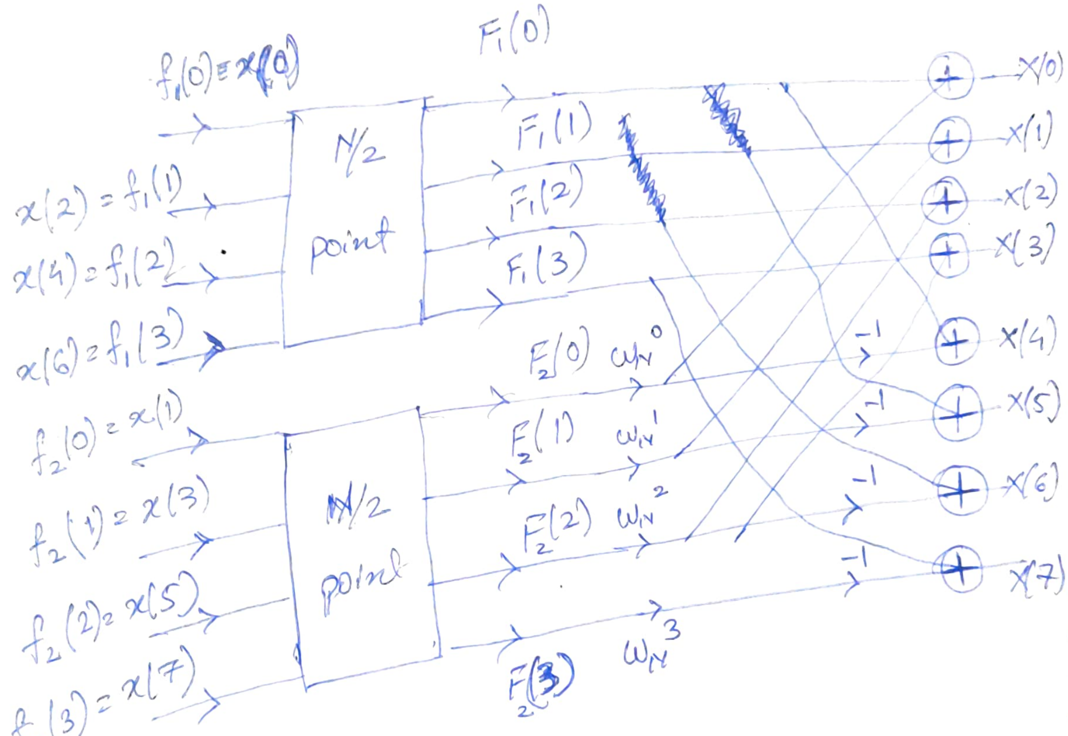
$$X(4) = F_1(0) - W_N^0 F_2(0)$$
$$X(5) = F_1(1) - W_N^1 F_2(1)$$
$$X(6) = F_1(2) - W_N^2 F_2(2)$$
$$X(7) = F_1(3) - W_N^3 F_2(3)$$

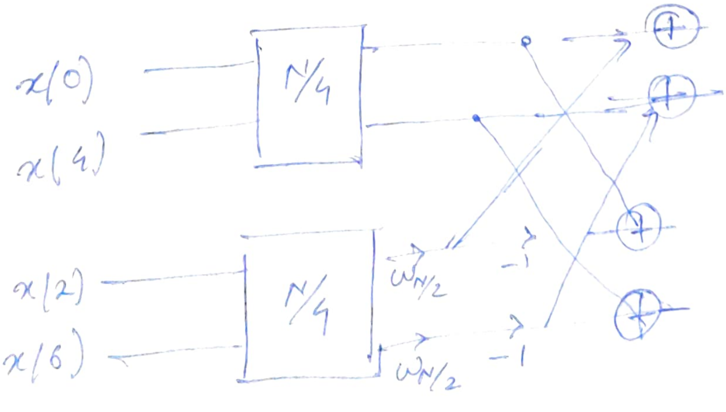
$f_1(m) = x(2m)$

$f_2(m) = x(2m+1)$

$x(0),\ x(1),\ \ldots\ldots,\ x(7)$

$f_1(m) = \begin{cases} \to x(0) \\ \to x(2) \\ \to x(4) \\ \to x(6) \end{cases}$ $\qquad$ $f_2(m) = \begin{cases} \to x(1) \\ \to x(3) \\ \to x(5) \\ \to x(7) \end{cases}$

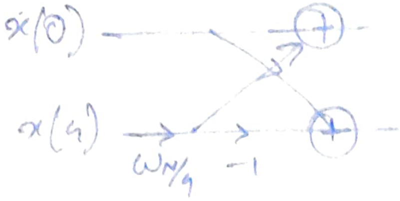


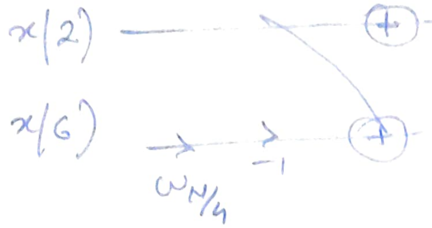


Similarly

$x(1)$ $\qquad$ $x(3)$

$x(5)$ $\qquad$ $x(7)$



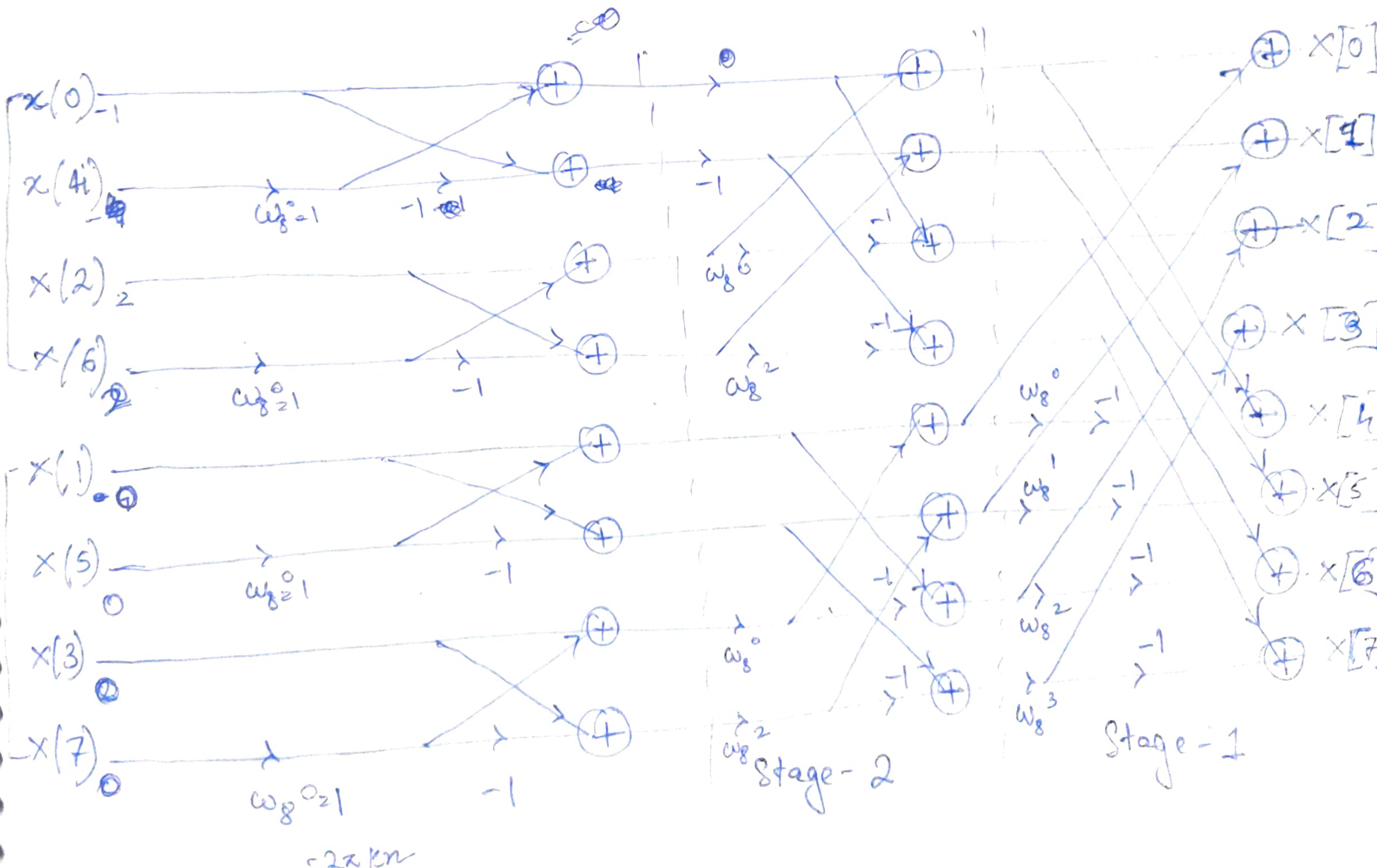


Similarly

$x(1)$ $\quad$ $x(3)$

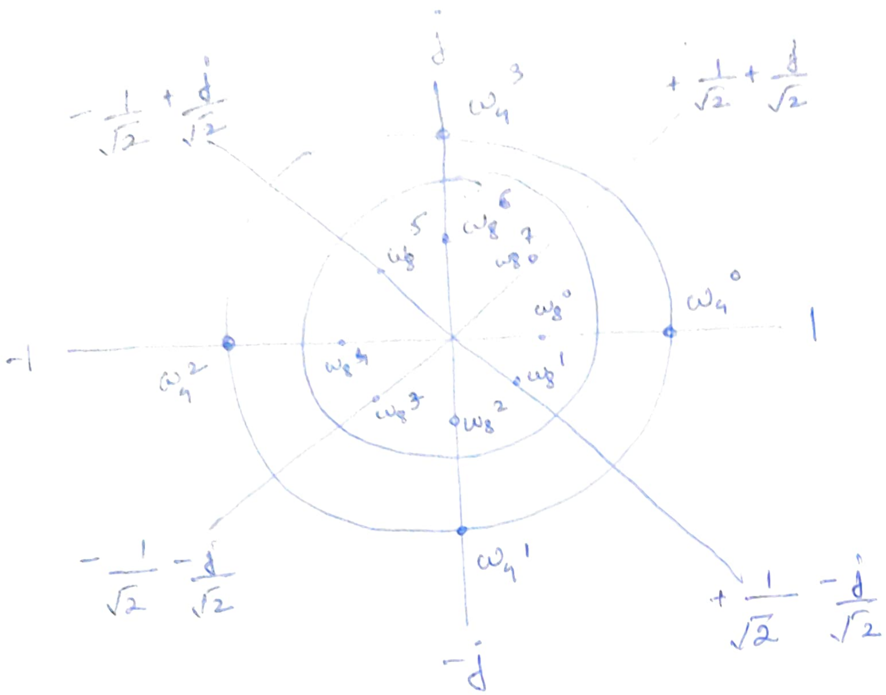
$x(5)$ $\quad$ $x(7)$.

Q. $x(n) = \{-1, 0, 2, 0, -4, 0, 2, 0\}$ Find 8-point DFT using FFT Algorithm.

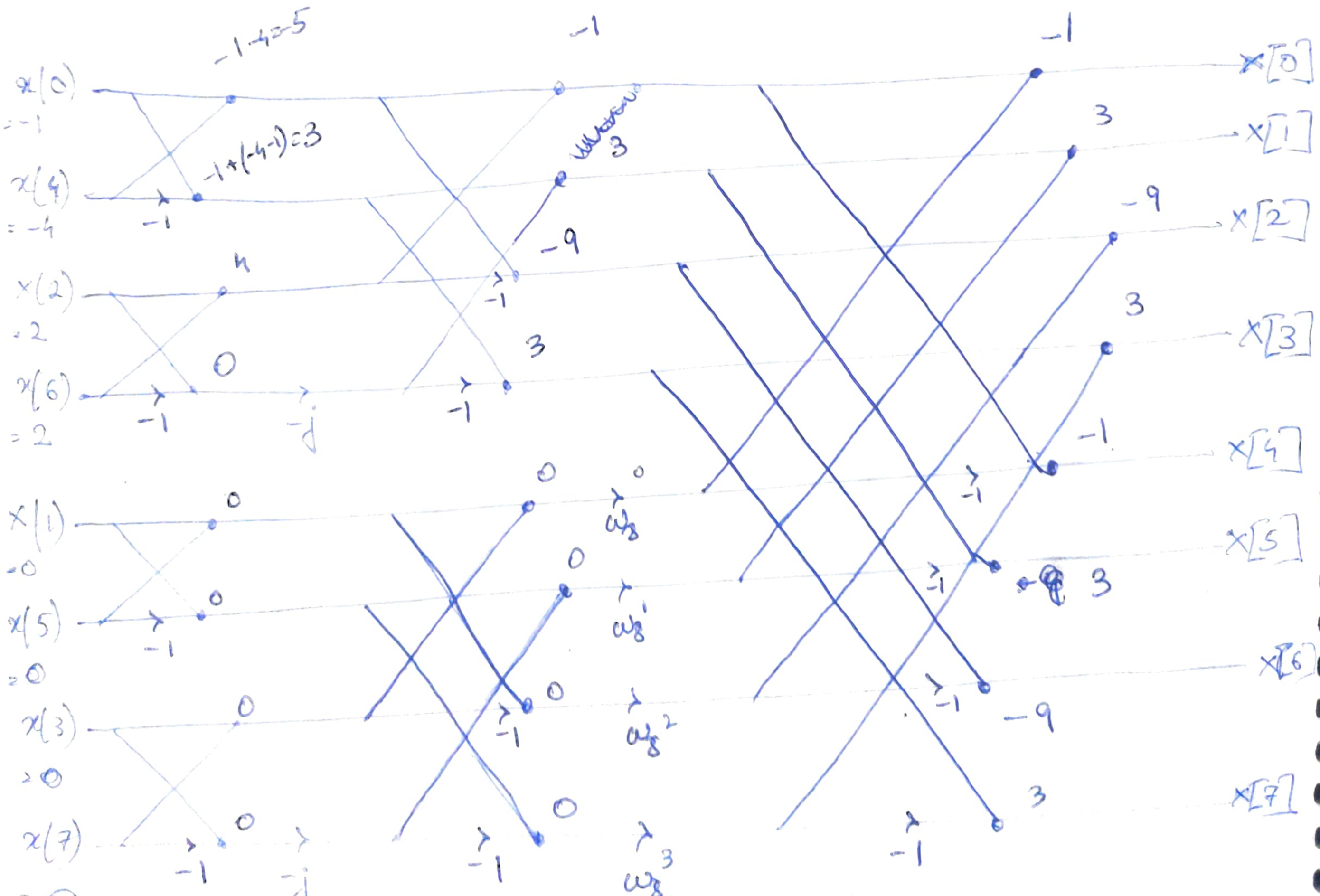


$W_N^{kn} = e^{-j\frac{2\pi}{N}kn}$

Tricks - DITFFT & DIFFT.



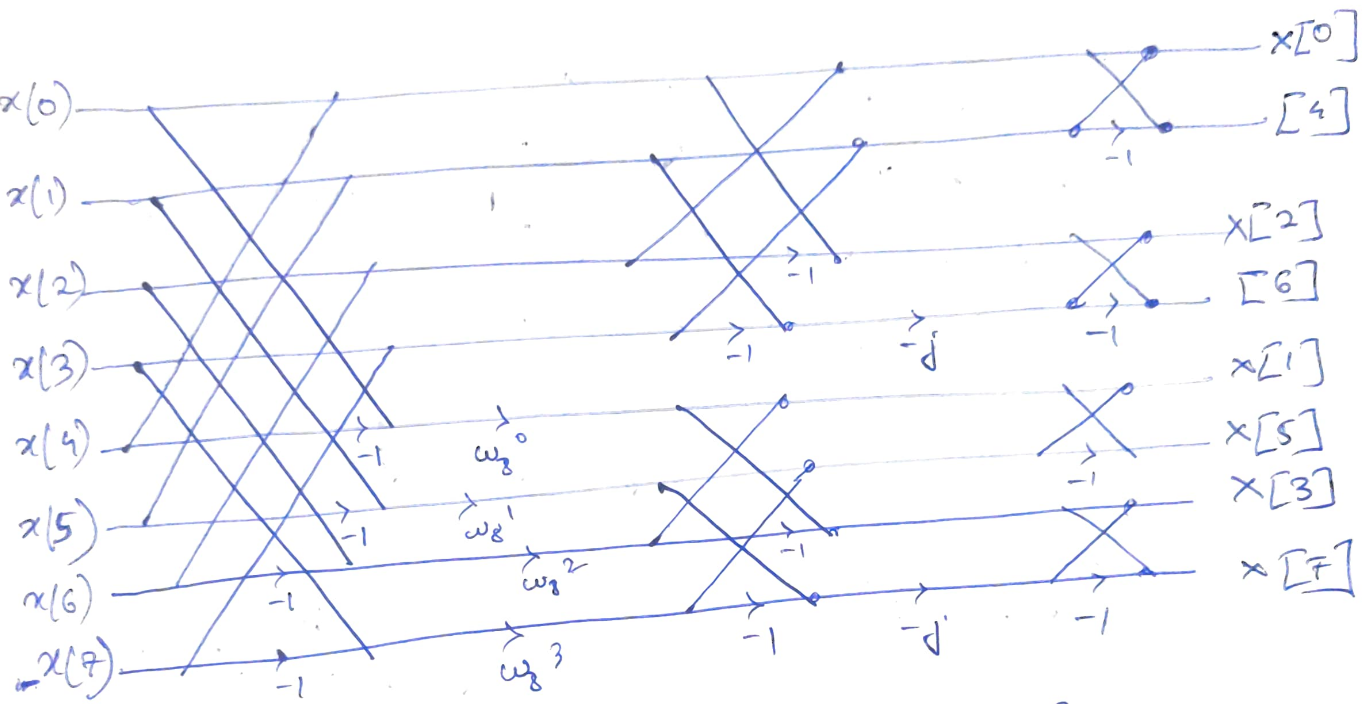
DITFFT



$X(K) = \{-1, 3, -9, 3, -1, 3, -9, 3\}$

DIFFT

$x(n) \longrightarrow X[k]$



$x(n) = \{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0\}$

$X(k) = \{2,\ 0.5 - j1.207,\ 0,\ 0.5 - j0.207,\ 0,\ 0.5 + j0.207,$
$0,\ 0.5 + j1.21\}$

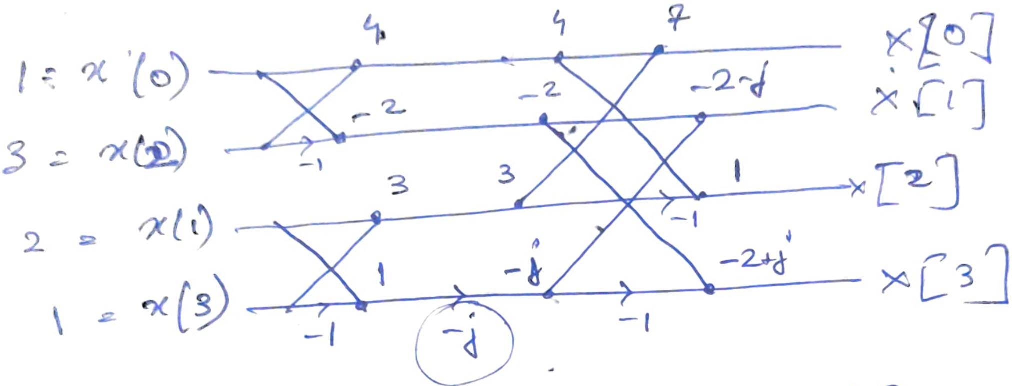
DIT-IFFT

FFT $\rightarrow$ $X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$

Inverse $\rightarrow$ $x(n) = \frac{1}{N} \sum_{n=0}^{K-1} X(k)\, W_N^{-kn}$

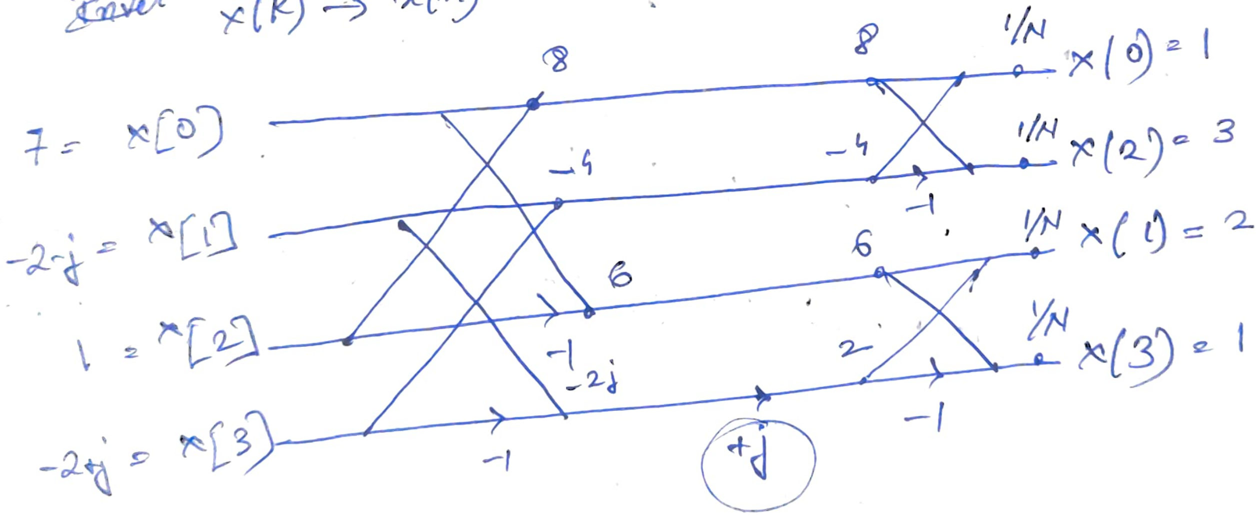
Q. N = 4, $x(n) = \{1, 2, 3, 1\} \xrightarrow{FFT} X(K)$ DIT-FFT Hence $\xrightarrow{IFFT} x(n)$

→ DIT $x(n) \rightarrow$ Decimation.



$X(K) = \{7, -2-j, 1, -2+j\}$

Inverse $X(K) \rightarrow x(n)$.



DIF-IFFT $X[K] \rightarrow$ Decimation

Correlation - Cross - Correlation & Auto Correlation

Cross-Correlation

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\, y(t-\tau)\, dt$$

If $y(t)$ is a Real signal

$$y^*(t) = y(t). \qquad R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\, y(t-\tau)\, dt$$

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\, y^*(t-\tau)\, dt = \int_{-\infty}^{\infty} x(t+\tau)\, y^*(t)\, dt$$

$$x(t) \xleftrightarrow{FT} X(\omega) \qquad y(t) \xleftrightarrow{FT} Y(\omega).$$

$y(t) =$ IFT of $Y(\omega)$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)\, e^{j\omega t}\, d\omega \Rightarrow y(t-\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)\, e^{j\omega(t-\tau)}\, d\omega$$

$$\Rightarrow \cancel{y(t-\tau) = \int_{-\infty}^{\infty}}$$

$$Y(t-\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)\, e^{j\omega t}\, e^{-j\omega\tau}\, d\omega.$$

$$y^*(t-\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y^*(\omega)\, e^{-j\omega t}\, e^{j\omega\tau}\, d\omega$$

$$R_{xy}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t) \int_{-\infty}^{\infty} Y^*(\omega)\, e^{-j\omega t}\, e^{j\omega t}\, d\omega\, dt$$

$$R_{xy}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y^*(\omega) \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt\; e^{j\omega\tau}\, d\omega$$

$$\boxed{R_{xy}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, Y^*(\omega)\, e^{j\omega\tau}\, d\omega}$$

Let, $Z(\omega) = X(\omega)\, Y^*(\omega)$

$$R_{xy}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z(\omega)\, e^{j\omega\tau}\, d\omega$$

Auto-correlation

$$R_{xx}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, X^*(\omega)\, e^{j\omega\tau}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, e^{j\omega\tau}\, d\omega$$

$|X(\omega)|^2 = S_n(\omega) = S_e(\omega)$ — Energy ~~spectr~~ spectral density

Energy signal → $$\boxed{R_{xx}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_n(\omega)\, e^{j\omega\tau}\, d\omega}$$

Energy Spectral is the IFT of $R_{xx}(\tau)$

$$\therefore\ \boxed{R_{xx}(\tau) \xleftrightarrow{FT} S_n(\omega)}$$

Power Signal → $R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x^*(t-\tau)\, dt$

Or

when period is given

$$R_{xx}(\tau) = \frac{1}{2T_0} \int_{-T_0/2}^{T_0/2} x(t)\, x^*(t-\tau)\, dt$$

$$\boxed{R_{xx}(\tau) \xleftrightarrow{FT} \text{Power Spectral Density}}$$

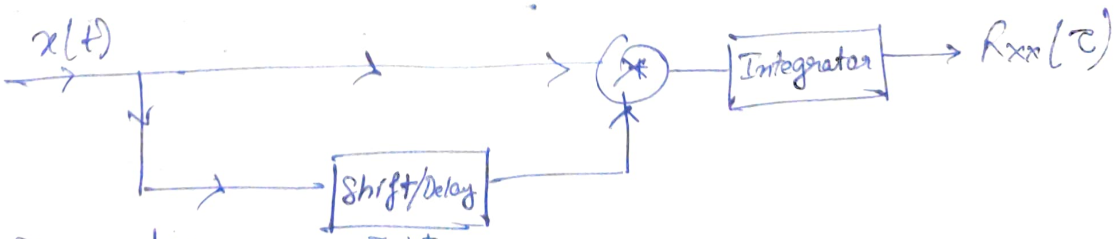
In Energy signal

$$R_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |x(\omega)|^2 e^{j\omega\tau}\, d\omega$$

If $\tau = 0$.

$$\boxed{R_{xx}(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |x(\omega)|^2 d\omega}$$ ← Parseval's Energy Thm.

$$\boxed{\begin{array}{l} R_{xx}(0) = \text{Energy of Signal} \\ \qquad\quad\;\, \text{Power of Signal} \end{array}}$$



Property

$$\boxed{x(\tau) * x(-\tau) = R_{xx}(\tau)}$$

$$\boxed{R_{xx}(-\tau) = x(-\tau) * x(\tau)}$$

$\therefore$ $\boxed{R_{xx}(-\tau) = R_{xx}(\tau)}$ $\forall$ Real values

$\therefore$ Auto-Correlation f^n is an even f^n

Q. *** Find auto-correlation for $x(t) = \sin \omega_0 t$ $\Rightarrow$ Power Signal

$$R_{xx}(\tau) = \frac{1}{2} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} 2 \sin \omega_0 t \sin \omega_0 (t-\tau) dt$$

$=$

~~To find Power ~~septr~~ spectral density Put $\tau = 0$ in $R_{xx}(\tau)$~~

$$= \frac{1}{2T_0} \left[\cos(\omega_0 \tau) - \cos \omega_0 (2t - \tau) \right]_{-T_0/2}^{T_0/2}$$

$$= \frac{1}{2T_0} \left[\left\{ \cos(\omega_0 \tau) - \cos(T_0 - \tau) \right\} - \left\{ \cos(\omega_0 \tau) - \cos \omega_0 (-T_0 - \tau) \right\} \right]$$

$$= \frac{1}{2T_0} \left[\cos \omega_0 (T_0 + \tau) - \cos(T_0 - \tau) \right]$$