Electromagnetic Waves

Date:- 03/02/2023

Books:-
1. John D. Krauss
2. R.K. Shevgaonkar

Physical World

- Classical Mechanics (Newton)
- Quantum Mechanics (Schrodinger, Bohr, Heisenberg)
- Special Relativity (Einstein)
- Quantum Electrodynamics (Feynman, Dirac)

Mechanics: Study of a system under different forces

Forces

(1) → Strong force (proton-proton, proton-neutron)
(3) → Weak force (Radiation)
(4) → Gravitational force
(2) → Electromagnetic force

→ EM Waves (Electromagnetic force theory) was mostly developed by Faraday & Maxwell

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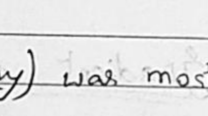
Vector Analysis / Calculus:

Any quantity which has magnitude & dir^n.



Properties of vectors:

(i) Addition of two vector. (a) $\vec{A} + \bar{B} = \vec{B} + \vec{A}$

(b) $(\vec{A} + \vec{B}) + \bar{C} = \bar{A} + (\bar{B} + \bar{C})$

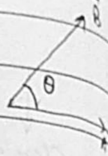
(c) $(\bar{A} - \bar{B}) = \bar{A} + (-\bar{B})$

(ii) Multiplication of vectors by scalar.

(a) $\alpha(\vec{A}+\vec{B}) = \alpha\vec{A} + \alpha\vec{B}$

(iii) Dot Product.

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$



(iv) Cross Product.

(a) $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

(b) $\vec{A}\times(\vec{B}+\vec{C}) = (\vec{A}\times\vec{B}) + (\vec{A}\times\vec{C})$

Scalar Triple Product :

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

Vector Triple Product :

$$\vec{A}\times(\vec{B}\times\vec{C}) = \cancel{\vec{B}\cdot(\vec{C}\cdot\vec{A}) - \vec{C}(\vec{A}\cdot\vec{B})}$$

$$\vec{B}(\vec{C}\cdot\vec{A}) - \vec{C}(\vec{A}\cdot\vec{B})$$

Gradient of a Vector :-

T is a function of (x,y,z) ; $T(x,y,z)$

$$dT = \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz$$

$$= \left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right)\cdot\left(\hat{x}dx + \hat{y}dy + \hat{z}dz\right)$$

$$= (\vec{\nabla}T\cdot d\vec{l})$$, where

$\vec{\nabla}T \rightarrow$ Gradient of T

$$\vec{\nabla}T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$$

Dot Product of Gradient:

$$\vec{\nabla}\cdot\vec{T} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\vec{T}$$

$$\Rightarrow \vec{\nabla}\cdot\vec{T} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\left(T_x\hat{x} + T_y\hat{y} + T_z\hat{z}\right)$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{T} = \frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + \frac{\partial T_z}{\partial z}}$$

Cross Product of a Gradient:

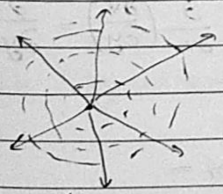
$$\vec{\nabla}\times\vec{T} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ T_x & T_y & T_z \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial T_z}{\partial y} - \frac{\partial T_y}{\partial z}\right) - \hat{y}\left(\frac{\partial T_z}{\partial x} - \frac{\partial T_x}{\partial z}\right) + \hat{z}\left(\frac{\partial T_y}{\partial x} - \frac{\partial T_x}{\partial y}\right)$$

Physical Significance:

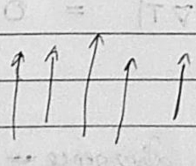
Dot Product $\rightarrow$ Dot product signifies divergence i.e. how much two vectors diverge from each other.

Eg $\rightarrow$ (i) waves generated are circular and diverge, when a stone is thrown at a pond.



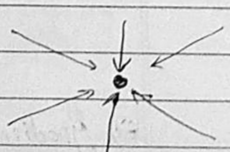
circular waves

Very high value



Parallel flow

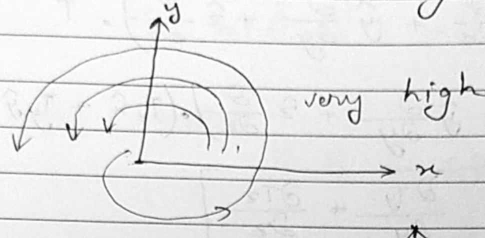
0-magnitude

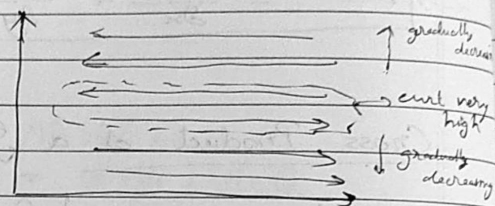


water flowing down sink

(high value)

Cross Product $\longrightarrow$ Cross product of two vectors give curl of a vector





Second Derivative of Gradient

(i) Divergence of a Gradient =

(I) $\nabla \cdot (\nabla T) = \nabla^2 T$ (Divergence of a scalar)

$$= \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right), \text{ where}$$

$T \rightarrow$ scalar variable

$V \rightarrow$ vector quantity

$\nabla^2 \rightarrow$ Laplacian Operator

(II) $(\nabla \cdot \nabla) V = \nabla^2 V$ (Divergence of a vector)

(ii) Curl of a Gradient =

$$\nabla \times (\nabla T) = 0 \qquad \left\{ \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial y}\right) = \frac{\partial}{\partial y}\left[\frac{\partial v}{\partial x}\right] \right\}$$

(iii) Gradient of a Divergence of a vector =

$$\nabla(\nabla \cdot V) \neq (\nabla \cdot \nabla) V$$

(iv) Divergence of a curl –

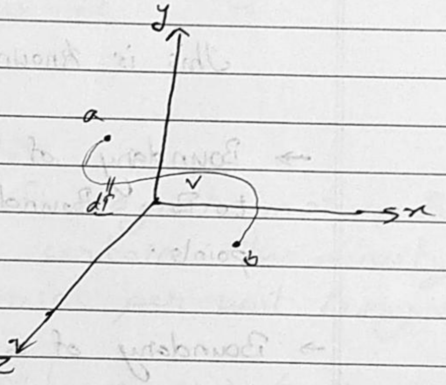
$$\nabla \cdot (\nabla \times \vec{V}) = 0$$

(v) Curl of a curl –

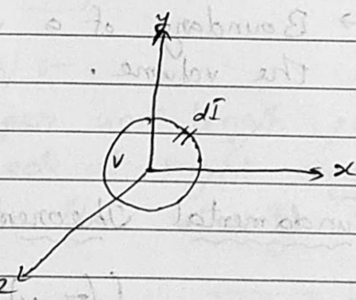
$$\nabla \times (\nabla \times V) = \nabla(\nabla \cdot V) - V(\nabla \cdot \nabla)$$
$$= \nabla(\nabla \cdot V) - \nabla^2 V$$

Integral Calculus :–

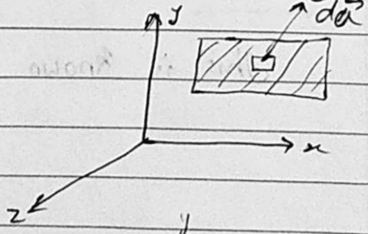
(a) Line Integral :– $\int_a^b V dI$



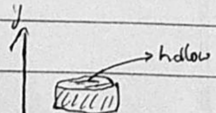
$\oint V \cdot dI$ (for closed loop)



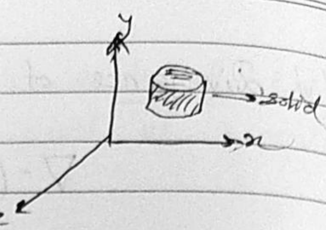
(b) Surface Integral :– $\int_S V \cdot d\vec{a}$



$\oint V \cdot d\vec{a}$



(c) Volume Integral $= \int_V T \cdot d\tau$



$$d\tau = dx\, dy\, dz$$

$$\int_V v\, d\tau = \int_V (v_x \hat{x} + v_y \hat{y} + v_z \hat{z})\, d\tau$$

$$= \int v_x\, dx + \int v_y\, dy + \int v_z\, dz$$

Fundamental Theorem of Divergence :-

$$\int_V (\nabla \cdot \vec{V})\, d\tau = \oint_S \vec{v} \cdot da$$

This is known as Green's Theorem / Gauss's Theorem

→ Boundary of a line of a string starting from A to B. Boundary of a line is just the two end points.

→ Boundary of a surface is its perimeter.

→ Boundary of a volume is the surface enclosing the volume.

Fundamental Theorem of Curl :-

$$\int_S (\nabla \times v) \cdot d\vec{a} = \oint \vec{v} \cdot d\vec{l} = \oint \vec{v} \cdot d\vec{l}$$

This is known as Stokes' Theorem

Basic Quantities of Electromagnetic-Waves :-

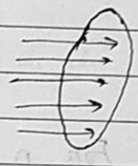
(1) Electric Field Intensity, (E) :-

It is the force experienced by a unit positive charge placed at that point. i.e.

$$\boxed{E = F/q}$$

(2) Electric Flux Density, (D) :-

It is the electric flux passing through a unit area perpendicular to the direction of the flux. i.e.



$$\boxed{D = \varepsilon E}$$

(3) Magnetic Field Intensity, (H) :-

It is the ratio of magnetic motive force (MMF) needed to create a certain flux density within a particular material per unit length. i.e.

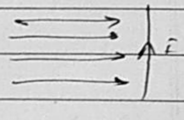
$$\boxed{H = B/\mu - M} \rightarrow \text{Magnetisation (mostly 0).}$$

(4) Magnetic Flux Density, (B) :-

The force acting per unit length per unit current on a wire placed at right angle to the magnetic field is the magnetic flux density. i.e.

$$F = qvB\sin\theta$$

$$\Rightarrow \boxed{B = \frac{F}{qv\sin\theta}}$$



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$C = 3\times10^{8}\ m/s$

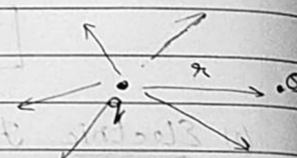
$\varepsilon_0 = 8.854\times10^{-12}\ F/m$

$\mu_0 = 4\pi\times10^{-7}\ H/m$

Basic Laws of Electro Magnetics :-

Coulomb's Law:

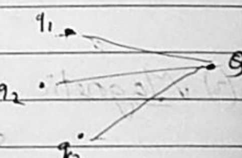
$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\hat{r}},$$



where Q is a test charge in elc. field of q

For n charges in electric field of q,

$$\boxed{\text{Force, } F = \frac{Q}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i}$$

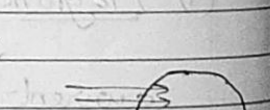


Electric Field, $E = F/Q$

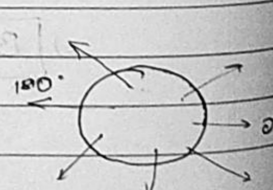
$$\Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cdot\hat{r}}$$

→ **Electric Flux for a sphere :-**

Electric Flux through the sphere is given by,



$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$



$$\Rightarrow \Phi_E = \oint E\,da\cos\theta$$

$$= \oint E\,da$$

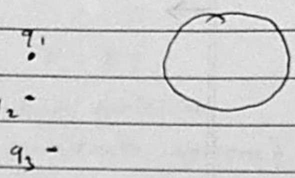
$$= E\oint da$$

$$= E\times 4\pi r^2$$

$$= 4\pi r^2 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1^2}$$

$$\Rightarrow \boxed{\Phi_E = \frac{q}{\varepsilon_0}}, \text{ for } r = r_1 \quad (\text{Gauss's Law})$$

For n charges around a sphere,



Flux, $\Phi_E = \sum \frac{q_i}{\varepsilon_0}$

$\Rightarrow \boxed{\Phi_E = \frac{Q_{enclosed}}{\varepsilon_0}}$, where $Q_{enclosed} = \sum q_i$

For multiple point sources,

From the principle of superposition,

$$\text{Elc. Field, } E = \sum_{i=1}^{n} E_i$$

$\therefore$ Flux through a sphere $\oint \vec{E}\cdot d\vec{A}\,(\Phi_E) = \sum_{i=1}^{n} \oint \vec{E_i}\cdot d\vec{a}$

Gauss's Law in Differential Form:

We know

$$\oint_S \vec{E}\cdot d\vec{a} = \int_V (\nabla\cdot\vec{E})\, d\tau \quad \text{[From Divergence Theorem]}$$

Rewriting the charge enclosed (Q_{enc}),

$Q_{enc} = \int_V \rho\, d\tau$, where $\rho \to$ charge density

$\because \oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$

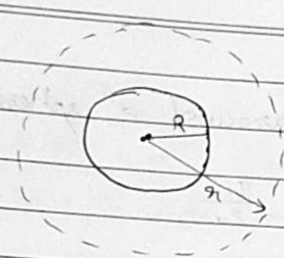
$d\tau \to dx\,dy\,dz$ ↑ cartesian co-ordinate system

$\Rightarrow \int_V (\nabla\cdot\vec{E})\, d\tau = \frac{1}{\varepsilon_0}\int_V \rho\, d\tau$

$\Rightarrow \boxed{\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}}$, This is the Gauss's Law in differential form

Applications of Gauss's Law:

Ex:(i) Find the electric field outside a uniformly charged solid sphere of radius R and total charge, q.



$r \rightarrow$ Gaussian radius

From Gauss's Law,

$$\phi_E = \oint_S \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \phi_s = \frac{q}{\varepsilon_0} \longrightarrow (1)$$

Further,

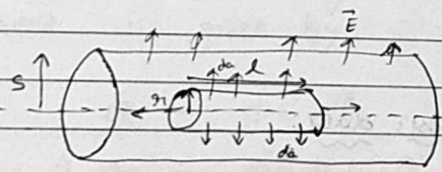
$$\oint_S \vec{E}\cdot d\vec{a} = \oint E da \cos\theta$$
$$= \oint E da \cos 0^\circ$$
$$= \oint E da$$
$$= E \oint da \quad (\because E \text{ is uniformly distributed})$$
$$= E \cdot 4\pi r^2$$

Putting this in eq^n (1),

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \boxed{E = \frac{1}{4\pi r^2}\frac{q}{\varepsilon_0}} \longrightarrow (2) \text{ \{Coulombs Law\}}$$

Ex:- (2) A long cylinder carries a charge density that is proportional to the distance from the axis $\rho = ks$, for some constant k. Find the electric field inside the cylinder.



$s \rightarrow$ radius

Applying Gauss's Law,

$$\oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0} \longrightarrow (1)$$

Now,

$$Q_{enc} = \int_v \rho \, d\tau \quad , \text{ where } \rho = ks$$

$$= \int_v (ks)(dx\,dy\,dz) \quad \{dx\,dy\,dz \text{ in cartesian co-ordinate system}\}$$

$$= \int_v (ks)(s\,ds\,d\phi\,dl) \quad \{s\,ds\,d\phi\,dl \text{ in cylindrical co-ordinate system}\}$$

$$\boxed{dx\,dy\,dz = r\,dr\,d\phi\,dh}$$

$r \rightarrow$ radius

$h \rightarrow$ height

$\phi \rightarrow$ circular angle

$$\Rightarrow Q_{enc} = k\int_v s^2 ds\, d\phi\, dl$$

$$= k\int_0^s s^2 ds \int_0^{2\pi} d\phi \int_0^l dl$$

$$= 2\pi l k \int s^2 ds$$

$$= \frac{2\pi l k s^3}{3}$$

$$(1) \Rightarrow \oint \vec{E}\cdot d\vec{a} = \oint E\,da$$

$$= E\int da \text{ (for curved surface)}$$

$$= E \times 2\pi s l, \quad s \rightarrow \text{radius}$$

$$\Rightarrow E \times 2\pi s l = \frac{Q_{enc}}{\varepsilon_0}$$

$$\Rightarrow E \times \cancel{2\pi s l} = \frac{2\pi l k s^{\cancel{3}\,2}}{3}$$

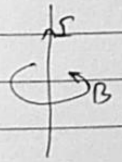
$$\Rightarrow \boxed{E = \frac{ks^2}{3\varepsilon_0}} \rightarrow (2)$$ is the reqd. elc. field inside the cylinder

Ampere's Law

→ $\nabla \times H = \frac{\partial D}{\partial t} + J$; where $H \rightarrow$ Magnetic field intensity

$D \rightarrow$ electric flux density

$J \rightarrow$ flowing electric current



Ampere's law in differential form

Ampere's Law in Integral Form:

$$\oint H \cdot dl = I_{enclosed}$$

$$\Rightarrow \oint \mu_0 H\, dl = \mu_0 I_{enclosed}$$

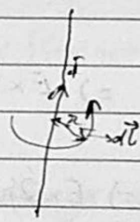
$$\Rightarrow \oint B \cdot dl = \mu_0 I_{enc} \quad ; \quad B \rightarrow \text{Magnetic field.}$$

Defⁿ:- The magnetic field enclosed by an electric current is proportional to the size of that electric current with a constant of proportionality, which is permeability (i.e. $\mu_0 I$).

Application :-

(i) Given a infinite wire carrying current $\vec{I}$, the magnetic field around the wire is given by :-

→ By Ampere's Law,



$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc} \rightarrow (1)$$

$\vec{dl}$ = area

$$\Rightarrow \oint \vec{B} \cdot \vec{dl} = \oint B\,dl \cos\theta$$

$$= \oint B\,dl \cos 0^\circ \quad [\because \theta = 0^\circ]$$

$$= \oint B\,dl$$

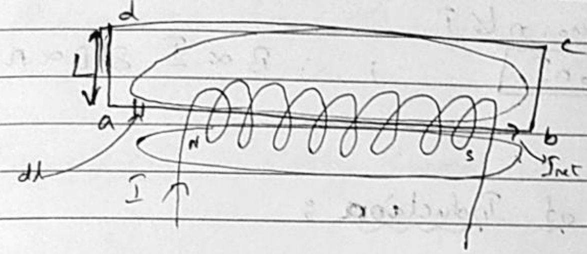
$$= B \oint dl$$

$$= B \times 2\pi r$$

$$\therefore (1) \Rightarrow B \times 2\pi r = \mu_0 I_{enc}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I_{enc}}{2\pi r} = \frac{\mu_0 \times 2 I_{enc}}{4\pi r}}$$

(ii) Solenoid :-



For ab,
$$\oint_a^b B\cdot dl = \mu_0 I$$

$$\Rightarrow \int_a^b B\,dl\cos\theta = \mu_0 I$$

$$\Rightarrow \int_a^b B\,dl = \mu_0 I \qquad [\because \cos\theta = \cos 0^\circ = 1]$$

$$\Rightarrow B\int_a^b dl = \mu_0 I$$

$$\Rightarrow BL = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{L}$$

$$\oint_{\text{whole area}} \vec{B}\cdot d\vec{l} = \oint_a^b B\cdot dl + \cancel{\oint_b^c B\,dl}^{\,0} + \oint_c^d B\,dl + \cancel{\oint_d^a B\,dl}^{\,0}$$

$$\Rightarrow \oint_{\text{whole area}} \vec{B}\cdot d\vec{l} = \oint_a^b B\,dl + \cancel{\oint_c^d B\,dl}^{\,0} \qquad \left[\because \text{for cd side, } B \approx 0\right]$$

$$\Rightarrow \oint_{\text{whole area}} \vec{B}\cdot d\vec{l} = BL$$

$\therefore$ from Ampere's Law, $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{net}$, $I_{net} \rightarrow$ induced current

$$\Rightarrow BL = \mu_0 I_{net} \longrightarrow (1)$$

* $I_{net} = nLI$,
$L \rightarrow$ ~~inductance~~ length
$n \rightarrow$ no. of ~~turns~~ coils/length
$I \rightarrow$ current

$\therefore (1) \Rightarrow BL = \mu_0 I_{net}$

$\Rightarrow B\cancel{L} = \mu_0 . n \cancel{L} I$

$\Rightarrow \boxed{B = \mu_0 n I}$; $\therefore B \propto I$ & $B \propto n$

Faraday's Law of Induction :

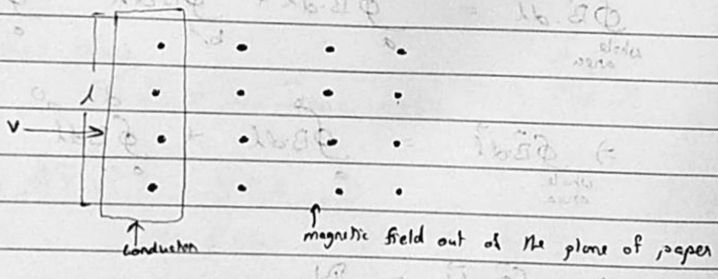
A change in magnetic flux can induce an emf and this theorem is called Faraday's Law of Induction.

for single loop, $\boxed{e = -\frac{\Delta \Phi_B}{\Delta t}} = -\frac{\partial \Phi_B}{\partial t}$; $\Phi_B \rightarrow$ magnetic flux
Lenz's Law ; $t \rightarrow$ time

for N number of loops, emf, $\boxed{e = -N \frac{\Delta \Phi_B}{\Delta t}}$; $\Phi_B = BA \cos\theta$

$$\therefore -\frac{\partial \Phi_B}{\partial t} = -\frac{\partial (BA\cos\theta)}{\partial t}$$

Example:- 1



Emf generated due to movement of conductor,

$$e = -\frac{\Delta \Phi_B}{\Delta t}$$

$\Delta A \rightarrow$ change in area

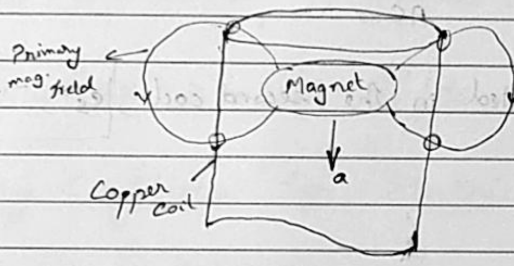
$$= -\frac{B \Delta A}{\Delta t}$$

$$= -\frac{B v \Delta t . l}{\Delta t}$$

$$\Rightarrow \boxed{e = -Bvl}$$

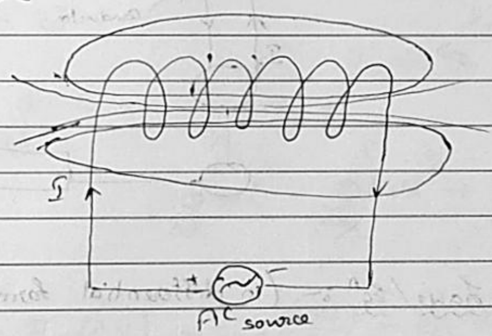
The induced current is in a dirⁿ that tends to slow the moving bar. It takes an ext. force to keep it moving.

Example :- 2 Eddy Current



The current induced in a bulk piece of conductor as shown in the expt. is the eddy current.

Self Induction :-



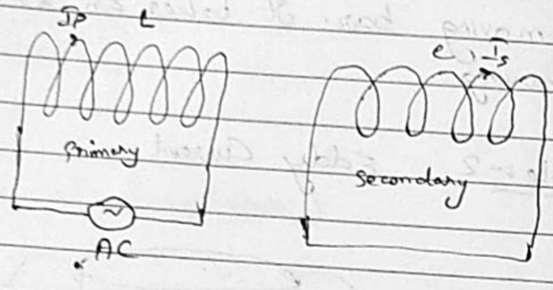
The effect of changing current in a circuit induces an emf in the same circuit and this is called self induction.

$$e = -L \frac{\Delta I}{\Delta t}$$ $L \rightarrow$ self induction

Mutual Induction :-

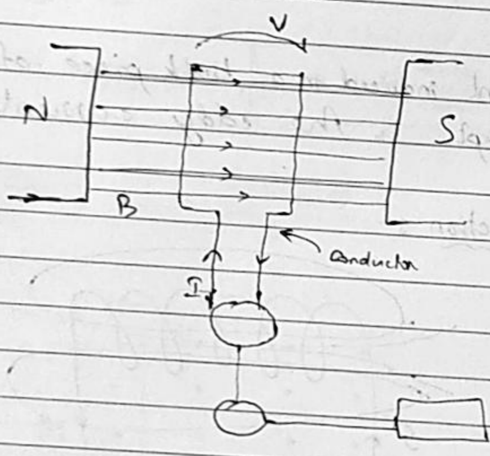
I_p → current in primary

I_s → current gen. in secondary



Emf generated in the second coil → $\boxed{e_s = -\frac{M \Delta I_p}{\Delta t}}$

Generator :-



Maxwell's Laws / Eq^n :- (in differential form)

(i) $\nabla \cdot D = \rho$ (Gauss's Law)

(ii) $\nabla \cdot B = 0$ {Gauss's Law for magnetic field}

(→ This implies magnetic field never diverges)

(→ Hence, there is no magnetic monopole)

(iii) $\nabla \times E = -\frac{\partial B}{\partial t}$ (Faraday's Law)

(iv) $\nabla \times H = \left(J + \frac{\partial D}{\partial t}\right)$ (Ampere's Law)

Maxwell's Laws (in Integral form):-

(i) $$\oiint_S \vec{D}\cdot ds = \iiint_V \rho\, dV$$

(ii) $$\oiint_S \vec{B}\cdot ds = 0$$

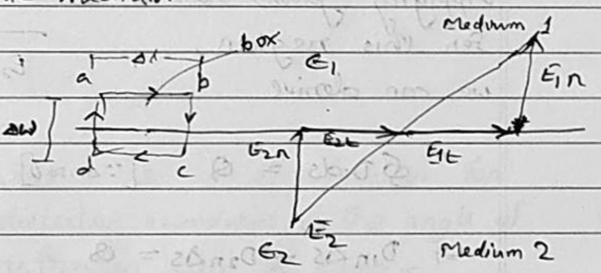
(iii) $$\oint \vec{E}\cdot d\vec{l} = \iint \frac{\partial \vec{B}}{\partial t}\cdot ds$$

(iv) $$\oint H\cdot dl = \iint \left(J + \frac{\partial D}{\partial t}\right) ds$$

Boundary Conditions for Electric Field :-

1> Dielectric - dielectric medium -

We have,



$$\oint \vec{E}\cdot d\vec{l} = 0$$

$$\Rightarrow \oint_{abcda} E dl = 0$$

$$\Rightarrow \oint_{ab} E dl + \oint_{bc/2} E dl + \oint_{bc/2} E dl + \oint_{cd} E dl + \oint_{da/2} E dl + \oint_{da/2} E dl = 0$$

$$\Rightarrow E_{1t}\Delta l - \cancel{E_{1n}\left(\frac{\Delta w}{2}\right)} - \cancel{E_{2n}\left(\frac{\Delta w}{2}\right)} - E_{2t}\Delta l + \cancel{E_{2n}\left(\frac{\Delta w}{2}\right)} + \cancel{E_{1n}\left(\frac{\Delta w}{2}\right)} = 0$$

$$\Rightarrow E_{1t}\Delta l - E_{2t}\Delta l = 0$$

$$\Rightarrow \boxed{E_{1t} = E_{2t}}$$

$E_n \rightarrow$ Normal component of E

$E_t \rightarrow$ Tangential component of E

$\Rightarrow$ Tangential Components of $\vec{E}$ are equal at the boundary. i.e. E_t is continuous and undergoes no change at dielectric boundaries.

$$\because \quad E_{1t} = E_{2t}$$

$$\Rightarrow \cancel{\frac{E_{1t}}{\epsilon_1}} \quad \cancel{\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}}$$

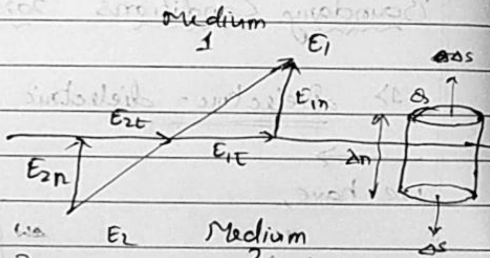
$$\Rightarrow \frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}$$

$$\Rightarrow \boxed{\epsilon_2 D_{1t} = \epsilon_1 D_{2t}} \quad \because \epsilon_1 \neq \epsilon_2$$

Hence, D_t (tangential component of D) is discontinuous across dielectric boundary.

$\longrightarrow$

Applying Gauss's Law for this region, we can derive



$$\oint D \cdot ds = Q \quad [\because \Delta n \approx 0]$$

$$\Rightarrow D_{1n} \Delta s - D_{2n} \Delta s = Q$$

$$\Rightarrow D_{1n} \cancel{\Delta s} - D_{2n} \cancel{\Delta s} = \rho_s \cancel{\Delta s} \quad ; \quad \rho_s \rightarrow \text{Surface Charge density}$$

$$\Rightarrow \boxed{D_{1n} - D_{2n} = \rho_s}$$

If there are no free charge present in the region, then

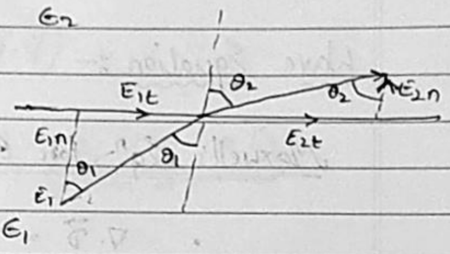
$$D_{1n} - D_{2n} = 0$$

$$\Rightarrow \boxed{D_{1n} = D_{2n}}$$, for regions where there are no free charge

$$\Rightarrow \boxed{\epsilon_1 E_{1n} = \epsilon_2 E_{2n}}$$

Normal component of D are equal at the boundary for no free charge. E is discontinuous for normal component and is given by the above eq^n.

→ We have,



$$E_{1t} = E_{2t}$$

$$\Rightarrow E_1 \sin\theta_1 = E_2 \sin\theta_2 \longrightarrow (1)$$

$$D_{1n} = D_{2n}$$

$$\Rightarrow \epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

$$\Rightarrow \epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2 \longrightarrow (2)$$

$$(1) \div (2) \Rightarrow \frac{\cancel{E_1} \sin\theta_1}{\epsilon_1 \cancel{E_1} \cos\theta_1} = \frac{\cancel{E_2} \sin\theta_2}{\epsilon_2 \cancel{E_2} \cos\theta_2}$$

$$\Rightarrow \frac{1}{\epsilon_1} \tan\theta_1 = \frac{1}{\epsilon_2} \tan\theta_2$$

$$\Rightarrow \boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2}}$$

This is the law of refraction for dielectric boundaries & the angle of refraction can be found.

ASSIGNMENT - I

Date:- 17/02/23

1> Find the magnetic field of a toroid & mention the differences between a toroid & a solenoid.

2> Evaluate the boundary condition for conductor-dielectric boundary & finally show the special condition of conductor free space boundary.

Due:- 23/02/23

Date:- 17/02/03

Wave Equation :-

Maxwells Eq^n for lossless or non conducting medium :-

- $\nabla \cdot \vec{D} = 0 \quad [\because \rho = 0] \longrightarrow (a)$
- $\nabla \cdot \vec{B} = 0 \longrightarrow (b)$
- $\nabla \times E = -\frac{\partial B}{\partial t} \longrightarrow (c)$
- $\nabla \times H = \frac{\partial D}{\partial t} \quad [\text{for } \vec{J} = 0] \longrightarrow (d)$

Wave Eq^n for lossless or non-conducting medium :-

From (d) $\Rightarrow \quad \nabla \times H = \frac{\partial D}{\partial t}$

$$\Rightarrow \frac{\partial}{\partial t}(\nabla \times H) = \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t}\right)$$

$$\Rightarrow \left(\nabla \times \frac{\partial H}{\partial t}\right) = \epsilon \frac{\partial^2 E}{\partial t^2} \quad [\text{from property}] \longrightarrow (1)$$

Now,

$$(c) \Rightarrow \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\Rightarrow \nabla \times (\nabla \times E) = -\nabla \times \frac{\partial B}{\partial t}$$

$$\Rightarrow \nabla \times (\nabla \times E) = -\mu\left(\nabla \times \frac{\partial H}{\partial t}\right) \quad [\because B = \mu H]$$

From (1), we get

$$\nabla \times (\nabla \times E) = -\mu \epsilon \frac{\partial^2 E}{\partial t^2} \longrightarrow (2)$$

We know,

$$(\vec{A} \times \vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

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$$\therefore \nabla\times(\nabla\times E) = \nabla(\nabla\cdot\vec{E}) - \vec{E}(\vec{\nabla}\cdot\vec{\nabla})$$

$$\Rightarrow \nabla\times(\nabla\times E) = \nabla(\nabla\cdot\vec{E}) - \underbrace{\vec{\nabla}^2 E}_{\text{laplacion of } E} \longrightarrow (3)$$

Again,

$$\nabla\cdot\vec{D} = 0 \quad [\rho = 0]$$

$$\text{or, } \nabla\vec{E} = 0 \quad [\because D = \epsilon E]$$

$$\therefore (3) \Rightarrow \nabla\times(\nabla\times E) = -\nabla^2 E \longrightarrow (4) \quad [\because \nabla\cdot\vec{E} = 0]$$

$$\therefore (2) \Rightarrow -\nabla^2 E = -\mu\epsilon\frac{\partial^2 E}{\partial t^2} \quad [\text{From eq}^n\ (4)]$$

$$\Rightarrow \boxed{\nabla^2 E = \mu\epsilon\frac{\partial^2 E}{\partial t^2}}$$

This eq^n is called the Wave Eq^n in free space & in terms of $\vec{E}$

We have,

$$\nabla\times E = -\frac{\partial B}{\partial t} \quad [\text{eq}^n\ (c)]$$

$$\& \quad \nabla\times H = \frac{\partial D}{\partial t} \quad [\text{eq}^n\ (d)]$$

$$(c) \Rightarrow \nabla\times E = -\frac{\partial B}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t}(\nabla\times E) = -\frac{\partial^2 B}{\partial t^2}$$

$$\Rightarrow \left(\nabla\times\frac{\partial E}{\partial t}\right) = -\mu\frac{\partial^2 H}{\partial t^2} \longrightarrow (1)$$

$$(d) \Rightarrow \nabla\times H = \frac{\partial D}{\partial t}$$

$$\Rightarrow \nabla\times\nabla\times H = \nabla\times\frac{\partial D}{\partial t} = \epsilon\left(\nabla\times\frac{\partial E}{\partial t}\right)$$

$$\Rightarrow \nabla\times\nabla\times H = -\mu\epsilon\frac{\partial^2 H}{\partial t^2} \longrightarrow (2) \quad [\text{from } (1)]$$

$$\Rightarrow \nabla \times \nabla \times H = \nabla(\nabla . H) - H(\nabla . \nabla)$$

$$= \nabla(\nabla . H) - \nabla^2 H$$

Again,

$$\nabla . B = 0 \quad \text{[from Maxwell eq}^n\text{]}$$

$$\Rightarrow \nabla . H = 0$$

$$\therefore \nabla \times \nabla \times H = -\nabla^2 H \longrightarrow (3)$$

$$\therefore (2) \Rightarrow -\nabla^2 H = -\mu \epsilon \frac{\partial^2 H}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 H = \mu \epsilon \frac{\partial^2 H}{\partial t^2}} \rightarrow \text{This is wave eq}^n \text{ in terms of } \vec{H}$$

In rectangular co-ordinate system, we can write as follows :—

$$\nabla^2 E_x = \mu \epsilon \frac{\partial^2 E_x}{\partial t^2}$$

$$\nabla^2 E_y = \mu \epsilon \frac{\partial^2 E_y}{\partial t^2}$$

$$\nabla^2 E_z = \mu \epsilon \frac{\partial^2 E_z}{\partial t^2}$$; similarly for D, H & B

If the electric field intensity is varying harmonically with time, then

$$\vec{E} = E_s e^{j\omega t}$$

$$\Rightarrow \frac{\partial \vec{E}}{\partial t} = E_s \frac{\partial}{\partial t} e^{j\omega t}$$

$$= E_s (j\omega) e^{j\omega t}$$

$$= j\omega E_s e^{j\omega t}$$

$$= j\omega \vec{E} \quad ; \quad \vec{E} = E_s e^{j\omega t}$$

$$\Rightarrow \frac{\partial^2 \vec{E}}{\partial t^2} = j\omega \left(\frac{\partial \vec{E}}{\partial t}\right)$$

$$= (j\omega) \cdot \left(\frac{\partial}{\partial t} E_s e^{j\omega t}\right)$$

$$= j\omega E_s \left(j\omega e^{j\omega t}\right)$$

$$\Rightarrow \frac{\partial^2 \vec{E}}{\partial t^2} = j^2 \omega^2 \vec{E}$$

$$\Rightarrow \boxed{\frac{\partial^2 \vec{E}}{\partial t^2} = -\omega^2 \vec{E}}$$

We have,

wave eq^n for electric field,

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\therefore \boxed{\nabla^2 \vec{E} = -\mu \epsilon \omega^2 \vec{E}} \rightarrow \text{This is wave } eq^n \text{ when it is time harmonic}$$

Similarly, $\nabla^2 \vec{H} = -\mu \epsilon \omega^2 \vec{H}$

$$\nabla^2 \vec{B} = -\mu \epsilon \omega^2 \vec{B}$$

$$\nabla^2 \vec{D} = -\mu \epsilon \omega^2 \vec{D}$$

This eq^n's are known as Homogeneous Vector Wave eq^n in complex time harmonic form for free space

Date: 24/02

$$\Rightarrow \nabla^2 E = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Also, $v = \frac{1}{\sqrt{\mu \epsilon}}$

$$\Rightarrow v^2 = \frac{1}{\mu \epsilon}$$

$$\therefore \boxed{\nabla^2 E = \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2}}$$

Wave Eq^n in Conducting/lossy medium:

$$\rightarrow \quad \nabla \times H = J + \frac{\partial D}{\partial t}$$

$$= \sigma \vec{E} + \frac{\partial \epsilon \vec{E}}{\partial t} \quad [\because J = \sigma \vec{E},\ \sigma = \text{Conductivity}]$$

$$= \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t}(\nabla \times H) = \sigma \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2}$$

$$\Rightarrow \left(\nabla \times \frac{\partial H}{\partial t}\right) = \sigma \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2} \longrightarrow (1)$$

From Faraday's law,

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

$$\Rightarrow \nabla \times (\nabla \times E) = \nabla \times \left(-\frac{\partial B}{\partial t}\right)$$

$$\Rightarrow \nabla \times \nabla \times E = -\mu \left(\nabla \times \frac{\partial H}{\partial t}\right) \quad [\because B = \mu H]$$

$$\Rightarrow \nabla \times \nabla \times E = -\mu \left(\sigma \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2}\right) \rightarrow (2) \quad [\text{From eq}^n\ (1)]$$

Now,

$$\nabla \times \nabla \times E = \nabla(\nabla . E) - E(\nabla . \nabla)$$

$$= \nabla(\nabla . E) - \nabla^2 E \longrightarrow (3)$$

$$\therefore (2) \Rightarrow \nabla(\nabla . E) - \nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} \rightarrow (4)$$

For a conductor, there is no charge within it. Hence, we can write :- $\nabla D = 0 \quad [\because \rho = 0]$

$$\Rightarrow \nabla E = 0$$

$\therefore (4) \Rightarrow -\nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2}$

$\Rightarrow \boxed{\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2}}$ $\longrightarrow$ This is the wave eq^n for conducting medium in terms of elc. field, E.

Wave Eq^n for conducting medium in terms of H:

We have,

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$= \sigma E + \epsilon \frac{\partial E}{\partial t}$$

$$\Rightarrow \nabla \times \nabla \times H = \sigma (\nabla \times E) + \epsilon \left(\nabla \times \frac{\partial E}{\partial t}\right) \longrightarrow (1)$$

From Maxwell's eq^n, $\nabla \times E = -\frac{\partial B}{\partial t}$

$$\Rightarrow \nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t}(\nabla \times E) = -\mu \frac{\partial^2 H}{\partial t^2}$$

$$\Rightarrow \nabla \times \frac{\partial E}{\partial t} = -\mu \frac{\partial^2 H}{\partial t^2} \longrightarrow (2)$$

$$\therefore (1) \Rightarrow \nabla \times \nabla \times H = \sigma (\nabla \times E) + \epsilon \left(-\mu \frac{\partial^2 H}{\partial t^2}\right)$$

$$= \sigma \left(-\frac{\partial B}{\partial t}\right) - \mu\epsilon \frac{\partial^2 H}{\partial t^2}$$

$$= -\mu\sigma \frac{\partial H}{\partial t} - \mu\epsilon \frac{\partial^2 H}{\partial t^2} \quad [\because B = \mu H] \longrightarrow (3)$$

$$\Rightarrow \nabla \times \nabla \times H = \nabla(\nabla \cdot H) - H(\nabla \cdot \nabla)$$

$$= \nabla(\nabla \cdot H) - \nabla^2 H$$

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$$\therefore (3) \Rightarrow \nabla\cancel{(\nabla\cdot H)}^{0} - \nabla^2 H = -\mu\sigma\frac{\partial H}{\partial t} - \mu\epsilon\frac{\partial^2 H}{\partial t^2}$$

[$\because \nabla B = 0$ by Maxwell's Eqⁿ]

$$\Rightarrow -\nabla^2 H = -\mu\sigma\frac{\partial H}{\partial t} - \mu\epsilon\frac{\partial^2 H}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 H = \mu\sigma\frac{\partial H}{\partial t} + \mu\epsilon\frac{\partial^2 H}{\partial t^2}}$$ → This is the wave eqⁿ for conducting medium in terms of H.

$E = E_s e^{(\sigma + j\omega)t}$;

$H = H_s e^{(\sigma + j\omega)t}$

Putting this in the wave eqⁿ, we get

$$\boxed{\nabla^2 E = j\omega\mu\sigma E - \omega^2\mu\epsilon E}$$

Let $\gamma^2 = j\omega\mu\sigma - \omega^2\mu\epsilon$

$\therefore$ The wave eqⁿ :- $\boxed{\nabla^2\vec{E} = \gamma^2\vec{E}}$ → This is the wave eqⁿ for time harmonic signal.

Characteristics of Uniform Plane Wave :-

(i) EM Waves generateds from a point source and spreads out uniformly, which is called spherical waveform.

(ii) An observer at a large distance observes the EM Wave as a plane wave.

(iii) ~~E&H~~ $\mathcal{E}$ & H are perpendicular to each other & to the dir^n of propagation. Hence, they are called transverse EM Wave.

(iv) A uniform plane wave is one in which $\mathcal{E}$ & H lie in a plane and ~~the~~ have the same value everywhere in that plane.

(v) For a uniform plane wave travelling in z-direction, the space variations of $\mathcal{E}$ & H are zero ~~o~~ over Z = ~~c~~ K (constant) ~~plane~~.