Physical World

Classical mechanics	Quantum	Special Relativity	Quantum Electrodynamics
Newton	Schrodinger, Bohr, Heisenburg	Einstein	Feynmann, Dirac

Date: ....

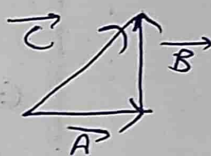
Mechanics :- Study of system under different forces.

Forces :-
- Strong
- weak
- Gravi
- Electromagnetic

EMW was mostly developed by Faraday & Maxwell.

Vector Analysis / Calculus

Any quantity with magnitude and direction.



Properties :-

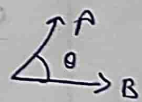
(i) Addition :- a) $\vec{A}+\vec{B} = \vec{B}+\vec{A}$

b) $(\vec{A}+\vec{B})+\vec{C} = \vec{A}+(\vec{B}+\vec{C})$

c) $\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$

(ii) Multiplication (a) $\alpha(\vec{A}+\vec{B}) = \alpha\vec{A}+\alpha\vec{B}$

(iii) Dot product $\vec{A}\cdot\vec{B} = AB\cos\theta$



(iv) Cross product a) $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

b) $\vec{A}\times(\vec{B}+\vec{C}) = (\vec{A}\times\vec{B})+(\vec{A}\times\vec{C})$

Scalar Triple Product

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}(\vec{A}\times\vec{B})$$

vector Triple product

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{C}\cdot\vec{A}) - \vec{C}(\vec{A}\cdot\vec{B})$$

Teacher's Signature ....

Gradient of a vector

T is a f^n of (x,y,z) ; $T(x,y,z)$

$$dT = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz$$

$$= \left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right).\left(\hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz\right)$$

$$= \vec{\nabla}T.\,d\vec{l} \qquad (\vec{\nabla}T = \text{gradient of } T)$$

$$\therefore \vec{\nabla}T = \frac{\partial T}{\partial x}.\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$$

~~Cross product~~

Dot product of gradient :-

$$\vec{\nabla}.T = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial x}\right).\vec{T}$$

$$\vec{\nabla}.\vec{T} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right).\left(T_x\hat{x} + T_y\hat{y} + T_z\hat{z}\right)$$

$$\vec{\nabla}.\vec{T} = \frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + \frac{\partial T_z}{\partial z}$$

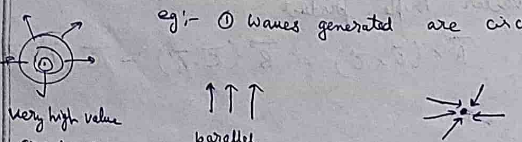
Cross product of a gradient :-

$$\vec{\nabla}\times T = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ T_x & T_y & T_z \end{vmatrix} = \hat{x}\left(\frac{\partial T_z}{\partial y} - \frac{\partial T_y}{\partial z}\right) - \hat{y}\left(\frac{\partial T_z}{\partial x} - \frac{\partial T_x}{\partial z}\right) + \hat{z}\left(\frac{\partial T_y}{\partial x} - \frac{\partial T_x}{\partial y}\right)$$

Physical Significance :-

Dot Product → Dot product signifies divergence i.e how much two vectors diverge from each other

eg:- ① Waves generated are circular & diverge (stone in pond)



very high value circular wave

parallel zero magnitude

Second Derivative of Gradient

Gradient ∇T
Divergence $\nabla.\vec{V}$
Curl $\nabla\times\vec{V}$

T = Scalar
V = vector

① Divergence of a Gradient

$$\nabla.(\nabla T) = \nabla^2 T$$

$$\text{or } (\nabla.\nabla)T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

∇ = Del operator

∇^2 = Laplacian operator

also $(\nabla.\nabla)v = \nabla^2 v$

(ii) Curl of a Gradient

$$\nabla\times(\nabla T) = 0$$

$\because \nabla\times\nabla = 0$

assumption $\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x}\right)$

(iii) Gradient of a Divergence

$$\nabla(\nabla.V) \neq (\nabla.\nabla)V$$

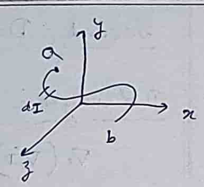
(iv) ~~Curl of~~ Divergence of a Curl

$$\nabla.(\nabla\times\vec{V}) = 0$$

(v) Curl of a Curl

$$\nabla\times(\nabla\times V) = \nabla(\nabla.V) - V(\nabla.\nabla)$$
$$= \nabla(\nabla.V) - \Delta^2 V$$

Integral Calculus

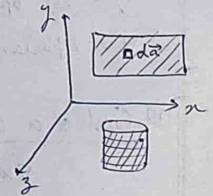


(i) Line Integral

$\int_a^b V.dI$

For closed loop, it is $\oint_a^b V.dI$

(ii) Surface Integral



$\int_S V.da$ or $\oint_S V.da$

contour surface

$da = dxdy$

(iii) Volume Integral

$\int_V T d\tau$ $\quad d\tau = dxdydz$

or $\int_V V d\tau = \int_V (V_x \hat{x} + V_y \hat{y} + V_z \hat{z})\, d\tau$

$= \int V_x dx + \int V_y dy + \int V_z dz$

Fundamental Theorem of Divergence

$\int_V (\nabla . \vec{V})\, d\tau = \oint_S \vec{V}.da$

at boundary.

area surrounding the vol^m are the boundaries

Green's Theorem / Gauss's Theorem

Gauss Theorem

1D → 0D	2D → 1D	3D → 2D

→ Boundary of a line are the starting and end points. a & b

→ " " " surface is the perimeter of the surface

→ " " " vol^m " " surface enclosing the vol^m

Fundamental Theorem of Curl (Stoke's Theorem)

$\int_S (\nabla \times V) . d\vec{a}$

$\int_S (\nabla \times V). da = \oint \vec{V} . d\vec{I}$

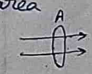
Basic Quantities of EMW

(1) Electric Field Intensity

$\vec{E}$ is the force experienced by a unit +ve charge placed at that point $\quad E = \frac{F}{q}$

(2) Electric Flux Density

$\vec{D}$ is the electric flux passing through a unit area $\perp$ to the direction of the flux.



$D = \varepsilon E$

(3) Magnetic Field Intensity H

It is the ratio of MMF needed to create a certain flux density within a particular material/unit length

$H = \frac{B}{\mu} - M$

magnetic motive force − MMF

$MMF = NI$ N-turns, I-current

$H = \frac{NI}{L}$, $B = \frac{\Phi}{A}$, $B = \mu H$

(4) Magnetic Flux density B

The force acting / unit current / unit length on a wire placed at right angle to the magnetic field is the magnetic flux density.

$F = qvB\sin\theta$ or $V \times B$ $\quad B = \frac{F}{qv\sin\theta}$

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Date: 8/02/2022

Constants:- $C = 3\times10^{8}\ m/s$ Speed of light

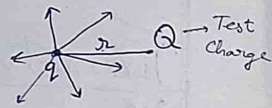
$\varepsilon_0 = 8.85\times10^{-12}$ F/m ~~Permiability~~ Primitivity

$\mu_0 = 4\pi\times10^{-7}$ H/m Permiability.

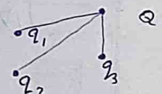
Basic Laws of EM

① Cōlamb's Law

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\hat{r}}$$ (for test charge)



$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i}$$ (for multiple charges)

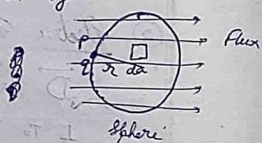


$$\boxed{E = \frac{F}{Q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}}$$

Electric flux for a sphere (Gauss's Law)

Electric flux through the sphere is given by

$$\Phi = \Phi_E = \oint \vec{E}\cdot\vec{da}$$



$$= \oint E\,da\cos\theta$$

$$= E\oint da \quad \left(\text{For max, } \cos\theta = 1 \text{ (perpendicular)}\right)$$

$$= E\,4\pi r^2$$

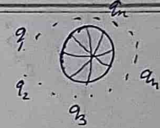
$$\therefore \quad = 4\pi r^2 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\boxed{\Phi_E = \frac{q}{\varepsilon_0}}$$ Gauss's Law

Date:

For multiple charge

$$\Phi_E = \sum_{i=1}^{n}\frac{q_i}{\varepsilon_0}$$



$$\Phi_E = \frac{Q_{enclosed}}{\varepsilon_0} \quad \text{where } Q_{enc} = \sum q_i$$

From the principle of superposition, $E = \sum_{i=1}^{n} E_i$

$$\Phi_E = \oint \vec{E}\cdot\vec{da} = \sum_{i=1}^{n}\oint \vec{E_i}\cdot\vec{da}$$

Gauss Law in Differential Form

We know $\oint_S \vec{E}\cdot\vec{da} = \int_V \nabla\cdot\vec{E}\,d\tau$. (Divergence Theorem)

Rewriting Qenc

$$Q_{enc} = \int_V \rho\,d\tau \quad \left(\rho \rightarrow \text{charge density},\ d\tau = dx\,dy\,dz,\ \frac{Q}{V}\right)$$

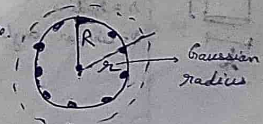
$$\because \oint \vec{E}\cdot\vec{da} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \int(\nabla\cdot\vec{E})\,d\tau = \frac{1}{\varepsilon_0}\int_V \rho\,d\tau$$

$$\boxed{\Rightarrow \nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}}$$

Applications of Gauss Law

Q1) Find the electric field outside a uniformly charged solid sphere of radius R and total charge 'q'. So basically in this example, we define another sphere outside the given sphere.



ans:- From Gauss Law, $\oint_S \vec{E}\cdot\vec{da} = \frac{Q_{enc}}{\varepsilon_0}$

$$= \frac{q}{\varepsilon_0}$$

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$$\oint_s \vec{E}\cdot\vec{da} = \oint E\,da\cos\theta$$ (E & da are aligned 0°)

$$= \oint E\,da$$

$$= E\oint da$$

$$= \cancel{4E\pi r^2}\; E\cdot 4\pi r^2 \quad \text{(ii)}$$

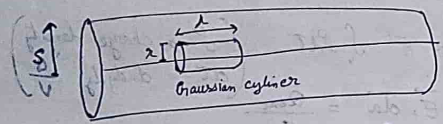
$\therefore$ ① $\Rightarrow E\cdot 4\pi r^2$

$\therefore E = \frac{1}{4\pi r^2}\, q/\varepsilon_0$ (Coulomb's Law using Gauss Law)

Ex-2 A long cylinder carries a charge density that is ~~ρ~~ proportional to the distance from the axis $\rho = ks$ for some constant k. Find the electric field inside the cylinder.

s = distance from axis

ans:-



using Gauss law, $\oint E\cdot da = \frac{Q_{enc}}{\varepsilon_0}$

Now, $Q_{enc} = \int_v \rho\, d\tau = \int_v (ks)\, dx\,dy\,dz \rightarrow$ cartesian coordinate

$$= \int_v (ks)\,(s\,ds\,d\phi\,dl)$$

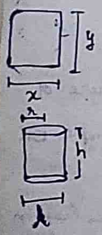
$$= k\int_v s^2 ds\, d\phi\, dl$$

$$= k\int_0^s s^2 ds \int_0^{2\pi} d\phi \int_0^l dl$$

$$= 2\pi l k \int s^2 ds$$

$$= \frac{2\pi l k s^3}{3}$$

dx	dy	dz
$= r\,dr$	$d\phi$	dh



h = height
r = radius
ϕ = circular angle

$$\oint \vec{E}\cdot\vec{da} = \oint E\,da$$

$$= E\int_{\text{curved surface}} da$$

$$= E\times 2\pi r h \qquad \begin{bmatrix} r = s \\ h = l \end{bmatrix}$$

$$= E\times 2\pi s l$$

$$\therefore E\times 2\pi s l = \frac{2\pi l k s^3}{3\varepsilon_0}$$

$$\Rightarrow E = \frac{ks^2}{3\varepsilon_0}$$

$\left(\oint E\,da = \frac{Q}{\varepsilon_0}\right.$

$\left.\therefore \varepsilon_0 \oint E\,da = Q = \frac{2\pi l k s^3}{3}\right)$

13/2/23

13/02/23

Faraday's Law of Induction

$$e_{ind} = \frac{-\Delta\phi_B}{\Delta t} = \frac{\partial\phi_B}{\partial t} = \frac{\partial\, BA\cos\theta}{\partial t}$$

Lenz

$$\left(\phi_B = \int B\cdot da = BA\cos\theta\right)$$

Q1 a rectangular conductor is pushed through a magnetic field coming out of plane in left → right direction. Find $e_{induced}$

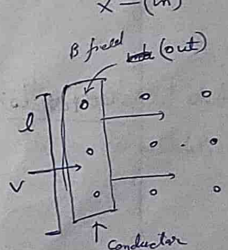
$$e = \frac{\Delta\phi_B}{\Delta t}$$

$$= \frac{B\Delta A}{\Delta t}$$

ΔA = change in area

$$= \frac{B\cdot(V\Delta t)\,l}{\Delta t}$$

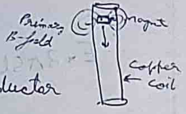
change in distance



$$\boxed{e = BVl}$$

The induced current is in a direction that tends to slow the moving bar. It takes an external force to keep it moving. (Bullet Train)

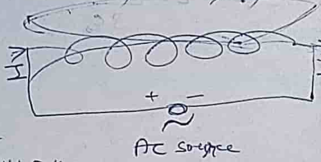
Example 2 **Eddy current**



Magnet will slow down.

The current induced in a hollow piece of conductor as shown in the exp. is the eddy current.

Ex-3 **Self induction**



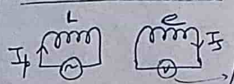
The effect of changing current in a circuit induces an emf in the same ckt and this is called ~~self~~ self induction

$$e = -L \frac{\Delta I}{\Delta t}$$

Self induction

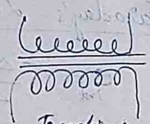
Henry

Ex 4 **Mutual Induction**

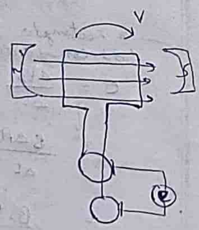


$$\boxed{e_s = \frac{-M \Delta I_p}{\Delta t}}$$

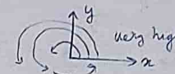
mutual inductance

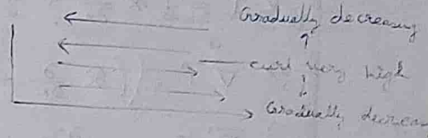


Generator :-



Cross Product → Cross Product of two vectors give curl of a vector





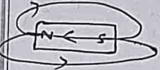
11/02/23 **Ampere's Law**

B-field can be produced by (i) ~~change~~ running current through a wire
(ii) Change in Electric flux density over time

$$\boxed{\nabla \times H = \frac{\partial D}{\partial t} + J}$$

Flowing electric current.

curl of mag. field. Ampere's law in diff. form.



B-field is circular in nature. Hence we use curl.

Ampere's law in integral form.

$$\boxed{\oint H dl = I_{enc}}$$

$$\Rightarrow \oint \mu_0 H dl = \mu_0 I_{enc}$$

$$\boxed{\Rightarrow \oint B.dl = \mu_0 I_{enc}}$$

The B-field enclosed by an electric current is prop. to the size of that electric current with a constant of proportionality which is μ_0 = permeability

Application:- $\oint B.dl = \mu_0 I_{enc}$ —(1)

Date: Current carrying infinite conductor

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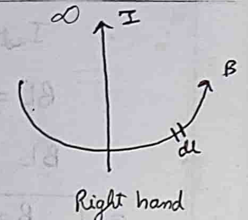
$$\oint \vec{B}\cdot\vec{dl} = \oint B.dl \cos\theta^\circ$$
$$= \oint B.dl \; [\because \theta = 0^\circ]$$
$$= B\oint dl$$
$$= B\times 2\pi r$$

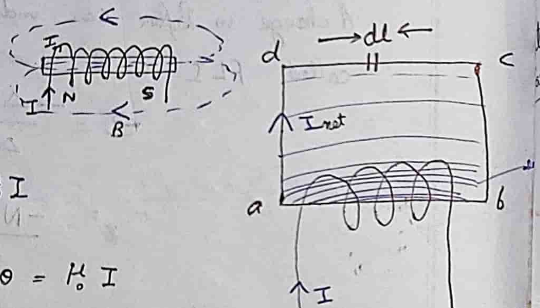


Right hand

① ⇒ $B\times 2\pi r = \mu_0 I_{enc}$

$$\boxed{B = \frac{\mu_0 I_{enc}}{2\pi r}} = \frac{\mu_0 2 I_{enc}}{4\pi r}$$

Application :- Solenoid



For ab,
$$\int_a^b B.dl = \mu_0 I$$
$$= \int_a^b B dl \cos\theta = \mu_0 I$$
$$= B\oint dl = \mu_0 I$$
$$= B\times L = \mu_0 I$$

L = length of square

For whole area

$$\oint B.dl = \int_a^b \vec{B}\vec{dl} + \cancel{\int_b^c \vec{B}\vec{dl}} + \int_c^d \vec{B}.\vec{dl} + \cancel{\int_d^a \vec{B}.\vec{dl}}$$
$$= \int_a^b B\times L = \mu_0 I + 0$$

(far away from solenoid)

$$= BL$$

∴ From ampere's cktal law

$\oint B.dl = \mu_0 I_{net}$ ∵ $I_{net} = nLI$

$BL = \mu_0 I_{net}$ ∴ $BL = \mu_0 nLI$ ∴ $B = \mu_0 nI$

Date: 15/02/23

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① $\nabla . D = \rho$ ⟶ Gauss law (for electric field)

② $\nabla . B = 0$ ⟶ Gauss law for magnetic field

⟶ Hence the is no magnetic magnetic monopoles.

③ $\nabla\times E = \frac{\partial B}{\partial t}$ ⟶ Faraday's law.

④ $\nabla\times H = \left(J + \frac{\partial D}{\partial t}\right)$ ⟶ Ampere's law

↳ current density.

Maxwell's eqn in differential form

① $\oint_S \vec{D}ds = \iiint \rho dv$

② $\oint_S \vec{B}.ds = 0$

③ $\oint E dl = \iint \frac{\partial \vec{B}}{\partial t}.ds$

④ $\oint H.dl = \iint \left(J + \frac{\partial D}{\partial t}\right)ds$

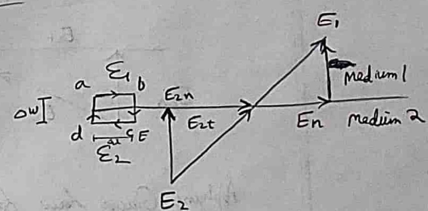
Integral form

Boundary Conditions :for E-field :-

① Dielectric - Dielectric medium.

$\oint E dl = 0$

$\Rightarrow \oint_{abcda} E dl = 0$



$\Rightarrow \cancel{\oint_{ab} E dl + \oint_{\frac{bc}{2}} E dl + \oint_{cd} E dl + \int}$

$$\Rightarrow \int_{ab} E dl + \int_{\frac{bc}{2}} E dl + \int_{\frac{bc}{2}} E dl + \int_{cd} E dl + \int_{\frac{da}{2}} E dl + \int_{\frac{da}{2}} E dl = 0$$

$$\Rightarrow E_{1t}\Delta l - E_{1n}\frac{\Delta w}{2} - E_{2n}\frac{\Delta w}{2} - E_{2t}\Delta l + E_{2n}\frac{\Delta w}{2} + E_{1n}\frac{\Delta w}{2}$$

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$\Rightarrow E_{1t}\Delta l - E_{2t}\Delta l = 0$

$\Rightarrow E_{1t} = E_{2t}$

Tangential Components of E are equal at the boundary.

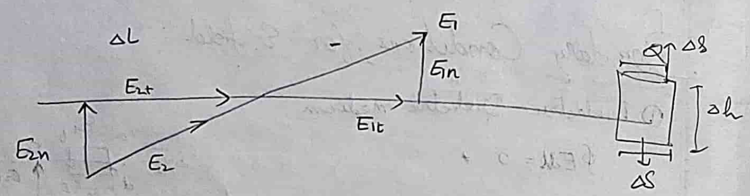
i.e. E_t is continuous and undergoes no change at dielectric boundaries

$\Rightarrow E_{1t} = E_{2t}$.

$\Rightarrow \dfrac{D_{1t}}{\varepsilon_1} = \dfrac{D_{2t}}{\varepsilon_2}$ $\left(D = \varepsilon E,\ \varepsilon_1 \neq \varepsilon_2 \right)$

$\Rightarrow \boxed{D_{1t}\,\varepsilon_2 = D_{2t}\,\varepsilon_1}$

Hence D_t is discontinuous accross dielectric boundary.



Applying Gauss law of we can write as :-

$\oint D\cdot ds = Q$ $[\because \Delta h = 0]$ Q = charge enclosed within this region

$D_{1n}\Delta S - D_{2n}\Delta S = Q$

$= \rho_s \Delta S$

$\boxed{D_{1n} - D_{2n} = \rho_s}$

If there are no free charge present in the region then.

$D_{1n} - D_{2n} = 0$.

$\Rightarrow \boxed{\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}}$

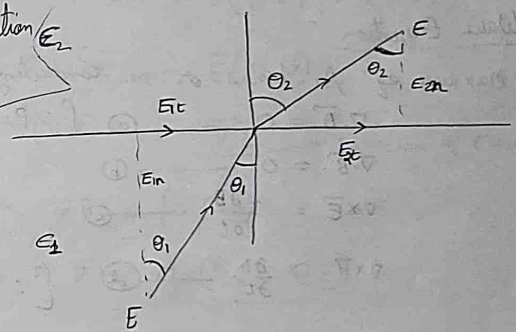
Remember This Only

Rest all ulta.

$\Rightarrow$ Normal component of D are equal at the boundary for no free charge.

$\Rightarrow$ E is discontinuous for normal component and is given by the above eqn.

Law of diffraction for dielectric boundaries



$E_{1t} = E_{2t}$

$E_1 \sin\theta = E_2 \sin\theta_2$ —— (i)

$D_{1n} = D_{2n}$

$\Rightarrow \varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$

$\Rightarrow \varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2$ —— (ii)

$\dfrac{(i)}{(ii)} \Rightarrow \dfrac{\cancel{E_1}\sin\theta_1}{\varepsilon_1 \cancel{E_1}\cos\theta_1} = \dfrac{\cancel{E_2}\sin\theta_2}{\varepsilon_2 \cancel{E_2}\cos\theta_2}$

$\dfrac{\tan\theta_1}{\varepsilon_1} = \dfrac{\tan\theta_2}{\varepsilon_2}$

$\boxed{\Rightarrow \dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\varepsilon_1}{\varepsilon_2}}$

Date: 17/2/2023

23/2/23

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Assignment 1:- Submission – 23/2/23

① Find the ~~magnitude~~ magnetic field for a torroid and mention the differences bet^n torroid and solenoid

② Evaluate the boundary condition for conductor - dielectric boundary and finally show the special condition of conductor – free space boundary.

Wave Equation :-

Maxwell eq^n for lossless or non conducting medium.

$$\nabla \cdot \vec{D} = 0 \quad \text{——— ⓐ} \quad [\because \rho = 0 \text{ for lossless}]$$

$$\nabla \cdot \vec{B} = 0 \quad \text{——— ⓑ}$$

$$\nabla \times \vec{E} = \frac{-\partial B}{\partial t} \quad \text{——— ⓒ}$$

$$\nabla \times \vec{H} = \frac{\partial D}{\partial t} \quad \text{——— ⓓ} \quad [\because \vec{J} = 0 \text{ for lossless}]$$

∴ Wave Equation for lossless or non conducting medium is

ⓓ $\Rightarrow \nabla \times H = \frac{\partial D}{\partial t}$

$$\frac{\partial}{\partial t}(\nabla \times H) = \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t}\right) \quad \left(\text{multiply } \frac{\partial}{\partial t} \text{ both side}\right)$$

$$\Rightarrow \nabla \times \frac{\partial H}{\partial t} = \frac{\partial^2 D}{\partial t^2}$$

$$\Rightarrow \nabla \times \frac{\partial H}{\partial t} = \varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{——— ①}$$

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Now, ⓒ $\Rightarrow \nabla \times \vec{E} = \frac{-\partial B}{\partial t}$

$$\Rightarrow \nabla \times (\nabla \times E) = -\nabla \times \left(\frac{\partial B}{\partial t}\right) \quad \left(\text{Taking curl on both side}\right)$$

$$\Rightarrow \nabla \times \nabla \times E = -\mu \, \nabla \times \frac{\partial H}{\partial t} \quad (B = \mu H)$$

$$\Rightarrow \nabla \times \nabla \times E = -\mu \varepsilon \frac{\partial^2 E}{\partial t^2} \quad (\text{using ①})$$

We know, $\vec{A} \times \vec{B} \times \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

$$\therefore \nabla \times \nabla \times E = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{E}(\vec{\nabla} \cdot \vec{\nabla})$$

$$= \nabla(\nabla \cdot \vec{E}) - \vec{\nabla}^2 E \quad \text{——— ③}$$

$\vec{\nabla}^2 E \rightarrow$ Laplacian of Electric field

Again, we know, $\vec{\nabla} \cdot \vec{D} = 0 \quad (\because \rho = 0)$

$$\therefore \vec{\nabla} \cdot \vec{E} = 0 \quad (D = \varepsilon E)$$

③ $\Rightarrow \nabla \times \nabla \times E = -\nabla^2 E$ ——— ④

② $\Rightarrow -\nabla^2 E = -\mu\varepsilon \frac{\partial^2 E}{\partial^2 t}$ (from ④)

$$\boxed{\therefore \nabla^2 E = \mu\varepsilon \frac{\partial^2 E}{\partial^2 t}}$$

This eq^n is called the wave eq^n in free space and in terms of electric field E for lossless medium.

To find ~~Befield~~ in terms of magnetic field look]

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We know, $\nabla\times E = \dfrac{-\partial B}{\partial t}$ — (c)

$\nabla\times H = \dfrac{\partial D}{\partial t}$ — (d)

(c) $\Rightarrow$ $\nabla\times E = -\dfrac{\partial B}{\partial t}$

$\dfrac{\partial}{\partial t}(\nabla\times E) = \dfrac{-\partial^2 B}{\partial t^2}$ (Taking div)

$\Rightarrow \left(\nabla\times\dfrac{\partial E}{\partial t}\right) = -\mu\dfrac{\partial^2 H}{\partial t^2}$ — (1)

(d) $\Rightarrow$ $\nabla\times H = \dfrac{\partial D}{\partial t}$

$\Rightarrow \nabla\times\nabla\times H = \nabla\times\dfrac{\partial D}{\partial t}$ (Taking curl)

$= \varepsilon\left(\nabla\times\dfrac{\partial E}{\partial t}\right)$ $(D = \varepsilon E)$

using (1)

$\nabla\times\nabla\times H = -\mu\varepsilon\dfrac{\partial^2 H}{\partial t^2}$ — (2)

$\nabla\times\nabla\times H = \nabla(\nabla\cdot H) - H(\nabla\nabla)$

$= \nabla(\nabla H) - \underline{\nabla^2 H}$ Laplacian

We know,

$\nabla\cdot B = 0$

$\nabla\cdot H = 0$ $(\because B = \mu H)$

$\therefore \nabla\times\nabla\times H = -\nabla^2 H$ — (3)

(2) $\Rightarrow$ $\cancel{-}\mu\varepsilon\dfrac{\partial^2 H}{\partial t^2} = \cancel{-}\nabla^2 H$

$$\boxed{\nabla^2 H = \mu\varepsilon\frac{\partial^2 H}{\partial t^2}}$$ wave eq^n interms of H

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Wave eq^n_s

In terms of E	$\nabla^2 E = \mu\varepsilon\dfrac{\partial^2 E}{\partial t^2}$
In terms of H	$\nabla^2 H = \mu\varepsilon\dfrac{\partial^2 H}{\partial t^2}$

In rectangular coordinate system, we can write as follows.

$\nabla^2 E_x = \mu\varepsilon\dfrac{\partial^2 E_x}{\partial t^2}$

$\nabla^2 E_y = \mu\varepsilon\dfrac{\partial^2 E_y}{\partial t^2}$

$\nabla^2 E_z = \mu\varepsilon\dfrac{\partial^2 E_z}{\partial t^2}$

Similarly for D, H & B

~~Sometime~~ If the electric field intensity is varying harmonically with time then, $\vec{E} = E_s e^{j\omega t}$

$\dfrac{\partial\vec{E}}{\partial t} = E_s\dfrac{\partial e^{j\omega t}}{\partial t}$

$= E_s(j\omega)e^{j\omega t}$

$= j\omega\,\underbrace{E_s e^{j\omega t}}$

$= j\omega\vec{E}$

$\Rightarrow \dfrac{\partial^2\vec{E}}{\partial t^2} = j\omega\left(\dfrac{\partial}{\partial t}\vec{E}\right)$

$= j\omega\left(\dfrac{\partial}{\partial t}E_s e^{j\omega t}\right)$

$j\omega E_s(j\omega e^{j\omega t})$

$\Rightarrow \dfrac{\partial^2 E}{\partial t^2} = j^2\omega^2\vec{E}$

$$\boxed{\frac{\partial^2 E}{\partial t^2} = -\omega^2\vec{E}}$$

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$\vec{E} = E_s e^{j\omega t}$ Spiral

$$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$= \mu\varepsilon(-\omega^2 \vec{E})$$

$\nabla^2 \vec{E} = -\mu\varepsilon\omega^2 \vec{E}$
Ily $\nabla^2 H = \mu\varepsilon\omega^2 \vec{H}$
Ily $\nabla^2 \vec{B} = \mu\varepsilon\omega^2 \vec{B}$
Ily $\nabla^2 D = -\mu\varepsilon\omega^2 D$

These eq^n are known as homogeneous vectors wave eq^n in complex time harmonic form. for free space.

24/2/23 Wave eq^n is given by $\nabla^2 E = \mu\varepsilon \frac{\partial \vec{E}}{\partial t^2}$

and $V = \frac{1}{\sqrt{\mu\varepsilon}}$ $\varepsilon \longrightarrow$ epsilon

$$V^2 = \frac{1}{\mu\varepsilon}$$

$$\boxed{\Rightarrow \nabla^2 \vec{E} = \frac{1}{V^2}\frac{\partial \vec{E}}{\partial t^2}}$$

Wave eq^n in Conducting medium / lossy medium

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$= a\vec{E} + \frac{\partial \varepsilon \vec{E}}{\partial t} \quad [\because J = aE]$$

a = conductivity (Sigma)

$$= a\vec{E} + \varepsilon \frac{\partial E}{\partial t}$$

Sir ka 'a' aur 'σ' ek jaisa dikhta hai

$$\frac{\partial}{\partial t}(\nabla \times H) = \alpha \frac{\partial E}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla \times \frac{\partial H}{\partial t} = \alpha \frac{\partial E}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{—— ①}$$

$$\nabla \times E = \frac{-\partial B}{\partial t}$$

$$\nabla \times (\nabla \times E) = \nabla \times \left(\frac{-\partial B}{\partial t}\right)$$

$$\nabla \times \nabla \times E = -\mu\left(\nabla \times \frac{\partial H}{\partial t}\right)$$

$$\nabla \times \nabla \times E = -\mu\left(a\frac{\partial E}{dt} + \varepsilon \frac{\partial^2 E}{\partial t^2}\right) \quad (\text{From ①})$$

Now,

$$\nabla \times \nabla \times E = \nabla(\nabla E) - E(\nabla \cdot \nabla) \quad \text{—— ②}$$

$$= \nabla(\nabla E) - \nabla^2 E \quad \text{—— ③}$$

$$② \Rightarrow \nabla(\nabla E) = \nabla^2 E = -\mu a \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{— ④}$$

For a conductor, there is no charge within it.

Hence, $\nabla D = 0$ $(\because \rho = 0)$

$\nabla E = 0$

$$④ \Rightarrow -\nabla^2 E = -\mu a \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

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$$\boxed{\Rightarrow \nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t}}$$

This is the ~~in term~~ wave eq^n for conducting medium in terms of E

Ily, $\nabla \times H = J + \frac{\partial D}{\partial t}$

$= aE + \varepsilon \frac{\partial E}{\partial t}$

$\nabla \times \nabla \times H = a(\nabla \times E) + \varepsilon\left(\nabla \times \frac{\partial E}{\partial t}\right)$ —— ①

$\nabla \times E = \frac{-\partial B}{\partial t}$

$\nabla \times E = -\mu \frac{\partial H}{\partial t}$

$\frac{\partial}{\partial t}(\nabla \times E) = -\mu \frac{\partial^2 H}{\partial t^2}$

$\Rightarrow \nabla \times \frac{\partial E}{\partial t} = -\mu \frac{\partial^2 H}{\partial t^2}$

① $\Rightarrow \nabla \times \nabla \times H = a(\nabla \times E) + \varepsilon\left(-\mu \frac{\partial^2 H}{\partial t^2}\right)$

$= a\left(\frac{-\partial B}{\partial t}\right) - \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$

$= -\mu a \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$ —— ②

$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - H(\nabla \cdot \nabla)$

$= \nabla(\nabla \cdot H) - \nabla^2 H$ —— ③

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$(\nabla \cdot B = 0)$
$\therefore (\nabla \cdot H = 0)$

② $\Rightarrow \nabla(\cancel{\nabla \cdot H}) - \nabla^2 H = -\mu a \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$

$\Rightarrow -\nabla^2 H = -\mu a \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$

$$\boxed{\Rightarrow \nabla^2 H = \mu a \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}}$$

This is the wave eq^n for conducting medium in terms of H.

For non conducting $a = 0$. ∴ we get the wave eq^n for non-conducting medias too.

We know,

$E = E_s e^{(\sigma + j\omega)t}$ → angular ~~velocity~~ freq^n (conductivity)

$H = H_s e^{(\sigma + j\omega)t}$

} Time harmonics

For time harmonics.

$$\boxed{\nabla^2 E = j\omega\mu\sigma E - \omega^2 \mu\varepsilon E}$$

Let $\gamma^2 = j\omega\mu\sigma - \omega^2\mu\varepsilon$

Replacing in the above eq^n

$$\boxed{\nabla^2 \vec{E} = \gamma^2 \vec{E}}$$

At a distance, a spherical/circular wave becomes plane wave

Characteristics of uniform plane wave :-

- EM wave generated from a point source & spreads out uniformly which is called spherical wave front
- An observer at a large distance observes the EMW as a plane wave.

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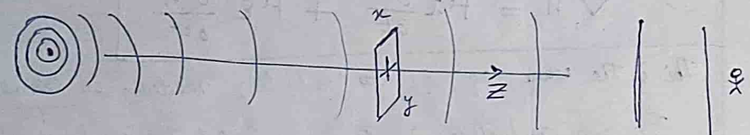
- E and H are perpendicular to each other and to the direction of propagation. Hence they are called Transverse EM wave (TEM)

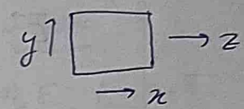
- A uniform plane wave is one in which E & H lie in a plane and have the same value everywhere in that plane.



- For a uniform plane wave travelling in z direction the space variation of E & H are zero over z = constant plane

22/3/23

Transverse Electric & Magnetic waves are the one which we are dealing with. Not TE, TM



For H

From maxwell eq^n for non conducting and source free medium, we can write

$$\nabla \times E = -j\omega\mu H$$

$$\Rightarrow H = \frac{1}{-j\omega\mu}(\nabla \times E)$$

$$= \frac{-1}{j\omega\mu}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \text{ ——— (1)}$$

H_y
$\hat{z}$
E_x

Again, $\vec{E} = \vec{E_0}\, e^{-jK\vec{r}}$

$$= (E_{0x}\hat{x} + E_{0y}\hat{y} + E_{0z}\hat{z})\, e^{-jK\vec{r}}$$

$$\frac{\partial}{\partial x}\vec{E} = -jK_x\vec{E}$$

$$\frac{\partial}{\partial x} = -jK_x, \quad \text{Ily } \frac{\partial}{\partial y} = -jK_y, \quad \frac{\partial}{\partial z} = -jK_z$$

$$\text{(1)} \Rightarrow H = \frac{-1}{j\omega\mu}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -jK_x & -jK_y & -jK_z \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \frac{-1}{j\omega\mu}(-j)\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ K_x & K_y & K_z \\ E_x & E_y & E_z \end{vmatrix}$$

$$H = \frac{1}{\omega\mu}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ K_x & K_y & K_z \\ E_x & E_y & E_z \end{vmatrix} = \frac{1}{\omega\mu}(K \times E) \text{ curl.}$$

Phase velocity in any direction (in z for this case)

$$\vec{E} = \vec{E_0}\, e^{-jK\vec{r}}$$

$$= \vec{E_0}\, e^{-j(\beta\cos\phi_x \hat{x} + \beta\cos\phi_y \hat{y} + \beta\cos\phi_z \hat{z})}$$

$$= \underbrace{\vec{E_0}\, e^{-j\beta(\cos\phi_x \hat{x} + \cos\phi_y \hat{y})}}_{\text{take it as an extended version of } \vec{E_0}\text{. Let it be represented as } \vec{P_0}} \cdot \underbrace{e^{j\beta\cos\phi_z \hat{z}}}_{\text{separate}}$$

$$= \vec{P_0}\, e^{-j\beta\cos\phi_z \hat{z}}$$

$\beta\cos\phi_z \longrightarrow$ Phase constant

∴ The wave phase constant in z direction is given by

$$\beta_z = \beta\cos\phi_z$$

$$\therefore v_{pz} = \frac{\omega}{\beta_z} = \frac{\omega}{\beta\cos\phi_z} = \frac{v_0}{\cos\phi_z}$$

(Where v_0 is the velocity of wave in direction $\perp$ to constant phase front)

Similarly, $v_{px} = \frac{v_0}{\cos\phi_x}$ & $v_{py} = \frac{v_0}{\cos\phi_y}$

∵ $|\cos\phi_x|$ range from 0 to 1

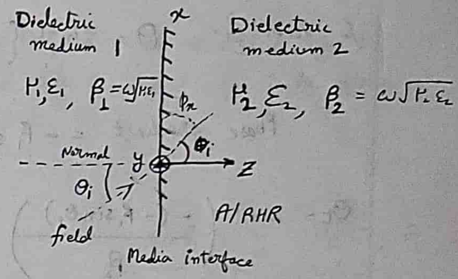
$$\therefore \infty \le v_{px} \le v_0$$

Energy propagation can never by ∞.

Plane Wave at Dielectric interface

$$\vec{F} = \vec{F_0}\, e^{-jKr}$$

$$= \vec{F_0}\, e^{-j\beta_1(\cos\phi_x x + \cos\phi_y y + \cos\phi_z z)} \quad \text{—— ①}$$



$\phi_x = \frac{\pi}{2} - \theta_i$

$\phi_y = \pi/2$

$\phi_z = \theta_i$

~~① ⇒ F_0~~

$\cos\phi_x = \cos\left(\frac{\pi}{2} - \theta_i\right) = \sin\theta_i$

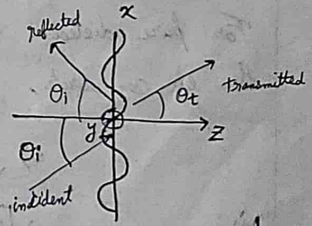
$\cos\phi_y = \cos\left(\frac{\pi}{2}\right) = 0$

$\cos\phi_z = \cos\theta_i$

$$\text{①} \Rightarrow F = F_0\, e^{-j\beta_1(x\sin\theta_i + 0 + z\cos\theta_i)} \quad \text{—— ②}$$

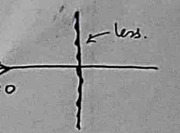
To see the amplitude variation of field in xy plane, we can write $Re\{\vec{F_i}\} = \vec{F_0}\cos(x\beta_1\sin\theta_i)$ · In xy plane, $z = 0$

So, at boundary condition.



The ~~Spacial~~ Spatial phase ~~variation~~ gradient in x direction is

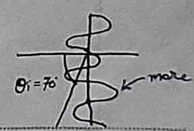
→ Phase gradient $= -\beta_1 \sin\theta_i$



Phase const per unit dist x

Phase constant $= -\beta_1 x \sin\theta_i$

The incident wave, the tx wave, & the reflected wave lie on the same plane of incidence which shows our first law of reflection



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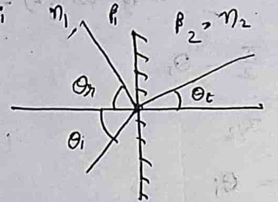
Now, the angle which the wave makes with its normal is given by $\theta = \sin^{-1}\left\{\dfrac{-\text{phase gradient}}{\text{phase constant}}\right\}$

Phase gradient $= -\beta_1 \sin\theta_i$



$\theta_t = \sin^{-1}\left(\dfrac{-(-\beta_1 \sin\theta_i)}{\beta_2}\right)$

$= \sin^{-1}\left(\dfrac{\beta_1 \sin\theta_i}{\beta_2}\right)$

$\theta_r = \sin^{-1}\left(\dfrac{-(-\beta \sin\theta_i)}{\beta_1}\right)$

$\theta_r = \theta_i$

$\boxed{\beta_1 \sin\theta_i = \beta_2 \sin\theta_t}$

$\Rightarrow \cancel{\omega}\sqrt{\mu_1\varepsilon_1}\,\sin\theta_i = \cancel{\omega}\sqrt{\mu_1\varepsilon_2}\,\sin\theta_t$

$\Rightarrow \sqrt{\mu\varepsilon_1}\,\sin\theta_i = \sqrt{\mu_1\varepsilon_2}\,\sin\theta_t$ Snell's law (generalised form)

For pure dielectric medium, we have $\mu_1 = \mu_2 = \mu$,
then, we can write, $\sqrt{\varepsilon_1}\,\sin\theta_i = \sqrt{\varepsilon_2}\,\sin\theta_t$

$\Rightarrow \sqrt{\cancel{\varepsilon_0}\varepsilon_{r_1}}\,\sin\theta_i = \sqrt{\cancel{\varepsilon_0}\varepsilon_{r_2}}\,\sin\theta_t$

$\Rightarrow \boxed{\eta_1 \sin\theta_i = \eta_2 \sin\theta_t}$

Special case of Snell's Law.

Reflection coefficient $\Gamma = \dfrac{E_r}{E_i}$, Transmission coefficient $\tau = \dfrac{E_t}{E_i}$

$\eta_1 = \sqrt{\dfrac{\mu_1}{\varepsilon_1}}$

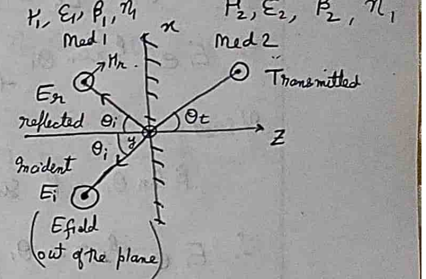
$\eta_2 = \sqrt{\dfrac{\mu_2}{\varepsilon_2}}$

$\eta_1 = \dfrac{E_i}{H_i}$

The arbitrary direction of E & H can be decomposed into a $\perp^r$ and $\parallel^l$ to the plane of incidence

$\dfrac{E_r}{H_r} = \eta_1$, $\dfrac{E_t}{H_t} = \eta_2$

For $\perp$ polarisation in terms of E-field.



For incident wave $\vec{E_i} =$

// $\vec{F_i} = \vec{F_{0i}}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$

$\therefore \vec{E_i} = E_i\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$

For reflected wave, $\vec{E_r} = E_r e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$

For transmitted wave $\vec{E_t} = E_t\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$

// At boundary, tangential component are equal

$\therefore$ At boundary, electric field component (Z component) is 0

$\therefore \vec{E_i} = E_i\, e^{-j\beta_1(x\sin\theta_i)}$ —— (i)

$\vec{E_r} = E_r\, e^{-j\beta_1 x\sin\theta_i}$ —— (ii)

$\vec{E_t} = E_t\, e^{-j\beta_2 x\sin\theta_t}$ —— (iii)

We know the tangential component of E should be continuous across the dielectric - dielectric boundary.

At interface i.e. $z=0$ plane, the electric fields are given by the above eq^{n}s ①, ⑪ & ⑪i

Hence, $\vec{E_i} + \vec{E_r} = \vec{E_t}$

$\Rightarrow E_i e^{-j\beta_1 x \sin\theta_i} + E_r e^{-j\beta_1 x \sin\theta_i} = E_t e^{-j\beta_2 (x \sin\theta_t)}$

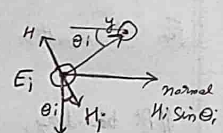
A/ Snell's law,

$\beta_1 \sin\theta_1 = \beta_2 \sin\theta_t$

$\therefore E_i e^{-j\beta_1 x \sin\theta_i} + E_r e^{-j\beta_1 x \sin\theta_i} = E_t e^{-j\beta_1 x \sin\theta_i}$

$\Rightarrow (E_i + E_r)^{-j\beta_1 x \sin\theta_i} = (E_t) e^{-j\beta_1 x \sin\theta_i}$

$\therefore E_i + E_r = E_t$

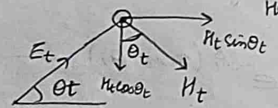


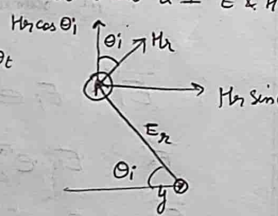
A/to Poynting vector

$\vec{E} \times \vec{H}$

(follows RHR)

Energy flow is ⊥ E & H





∵ There are no surface currents, so Tangential component of H is also continuous.

γ

due to opposite direction of $\cos\theta_i$

$\therefore H_i \cos\theta_i - H_r \cos\theta_i = H_t \cos\theta_t$

electric field intensity

$H = \frac{E}{\eta}$

$\therefore \frac{E_i}{\eta_1}\cos\theta_i - \frac{E_r}{\eta_1}\cos\theta_i = \frac{E_t}{\eta_2}\cos\theta_t$ ——— ⑥

Intrinsic impedance formula.

$\frac{E_r}{E_i}$ = ~~Transmission~~ Reflection coefficient Γ

$\frac{E_t}{E_i}$ = Tx coefficient τ

Reflection coefficient $\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$

Gamma

Transmission coefficient $\tau_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$

tau

Dividing eq 5 by E_i

$\Rightarrow \frac{E_i}{E_i} + \frac{E_r}{E_i} = \frac{E_t}{E_i}$

$\Rightarrow 1 + \Gamma_\perp = \tau_1$

$\therefore |\Gamma_\perp| \leq 1$ & $|\tau_\perp| \geq 1$

$\Rightarrow E_t > E_i$ violating conservation of energy?? No!!!!

∵ H is also there! Importance of pointing vector.

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Date : ....

We know

For perpendicular polarisation

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\tau_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

P

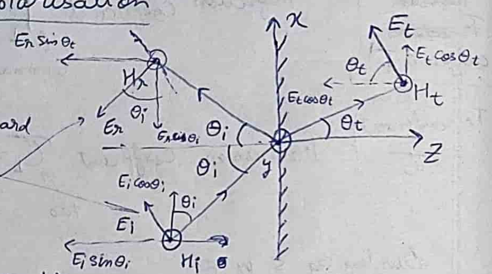
Now, for parallel polarisation

As H is ⊥ E.

Let H be coming out of the board

Using pointing vector

E_i is like this



Starting with E, then go to H

Now, follow the same steps we will find $H_i + H_r = H_t$

$$\Rightarrow \frac{E_i}{\eta_1} + \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2} \quad \text{—— (I)}$$

Dividing this eq by E_i

$$\Rightarrow \frac{1}{\eta_1} + \frac{1}{\eta_1}\frac{E_r}{E_i} = \frac{1}{\eta_2}\frac{E_t}{E_i}$$

$$\Rightarrow \frac{1}{\eta_1}\left(1 + \Gamma_{\parallel}\right) = \frac{1}{\eta_2}\tau_{\parallel}$$

$$\boxed{\Rightarrow 1 + \Gamma_{\parallel} = \frac{\eta_1}{\eta_2}\tau_{\parallel}}$$

Date : ....

Using boundary condition (⊥ components are equal)

$$E_i \cos\theta_i - E_r \cos\theta_i = E_t \cos\theta_t \quad \text{—— (II)}$$

Using (I) & (II) we get

$$\boxed{\Gamma_{\parallel} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}}$$

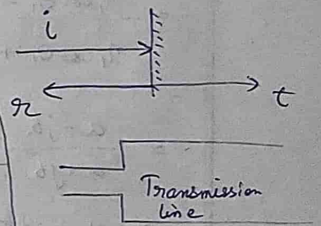
$$\boxed{\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}}$$

2 becomes 1
& 1 → 2
in $\parallel^{el}$ field

For normal incidence

$$\theta_i = \theta_r = \theta_t = 0^\circ$$

$\therefore \Gamma_\perp = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$	$\Gamma_{\parallel} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$
$\tau_\perp = \frac{2\eta_2}{\eta_2 + \eta_1}$	$\tau_{\parallel} = \frac{2\eta_1}{\eta_1 + \eta_2}$



If $\eta_1 > \eta_2$, $\Gamma_1 = -ve$

E field is out of phase

Mag remains same !

TIR :-

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CAT-2

Date :

Total Internal Reflection

$\beta_1 \sin\theta_i = \beta \sin\theta_t$ ≫ Snell's Law

$\Rightarrow \sin\theta_t = \frac{\beta_1}{\beta_2} \sin\theta_i$

For $\theta_t = 1$, $\sin\theta_t = 1$ ≫ Condition for TIR

Now, $\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i}$

$\Gamma_{\parallel} = \frac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}$ $\quad = j\sqrt{\frac{\beta_1^2}{\beta_2^2} - 1}$

$= \frac{\eta_1\cos\theta_i - j\eta_2\sqrt{\beta_1^2/\beta_2^2 \sin^2\theta_i - 1}}{\eta_1\cos\theta_i + j\eta_2\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}$

$= \frac{a-jb}{a+jb}$ ⟶ No Transmission → TIR.

Magnitude $|\Gamma_{\parallel}| = 1$ (energy incident = " reflected ~~Transmitted~~)

Phase $\angle\Gamma_{\parallel} = (-2\tan^{-1}\frac{b}{a})$

For ⊥ components

η_1 & η_2 will interchange

$\Gamma_{\perp} = 1, \quad \angle\Gamma_{\parallel} = (-2\tan^{-1}\frac{b'}{a'})$

Date : 01/04/23

$\frac{\beta_1}{\beta_2}\sin\theta_i > 1$; Condition for TIR

$\Rightarrow \frac{\cancel{\omega}\sqrt{\mu_1\varepsilon_1}}{\cancel{\omega}\sqrt{\mu_2\varepsilon_2}}\sin\theta_i > 1$

$\Rightarrow \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin\theta_i > 1$

For non magnetic media, $\mu_1 = \mu_2$

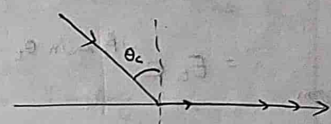
$\therefore \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\sin\theta_i > 1$

$\Rightarrow \frac{\eta_1}{\eta_2}\sin\theta_i > 1$

Since $\sin\theta_i\,(max) = 1$

$\therefore \frac{\eta_1}{\eta_2} > 1 \Rightarrow \eta_1 > \eta_2$

For critical angle



$\frac{\beta_1}{\beta_2}\sin\theta_i = 1$

$\Rightarrow \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\sin\theta_i = 1$

$\Rightarrow \frac{\eta_1}{\eta_2}\sin\theta_c = 1$

$\Rightarrow \theta_c = \sin^{-1}\left(\frac{\eta_2}{\eta_1}\right)$

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Date : ....

(2) Wave undergoes a ~~π~~ phase change at TIR and the phase change is different for $\parallel^{el}$ and $\perp$ar polarisation.

$$\phi_{\parallel} = -2\tan^{-1} \frac{\eta_2 \sqrt{\frac{\beta_2^2}{\beta_1^2}\sin^2\theta_i - 1}}{\eta_1 \cos\theta_i}$$

$$\phi_{\perp} = 2\tan^{-1} \frac{\eta_1 \sqrt{\frac{\beta_2^2}{\beta_1}\sin^2\theta_i - 1}}{\eta_2 \cos\theta_i}$$

(3) Fields in medium 2

$$\vec{E_t} = E_t\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$= E_t\, e^{-j\beta_2\left(x\sin\theta_t \pm jz\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin\theta_i - 1}\right)}$$ [Look ~~for~~ previous classes]

$$= E_t\, e^{-j\beta_2 x\sin\theta_t \pm z\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}$$ $\left(\cancel{-j\times j} \quad -j\times \pm j = \pm 1\right)$

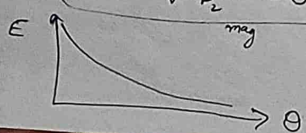
without j term it is magnitude

with j term it is phase

$$= \underbrace{E_t\, e^{\pm z\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}}_{\text{magnitude}} \cdot \underbrace{e^{-j\beta_2 x\sin\theta_t}}_{\text{phase}}$$

(remove + sign)

$$= \underbrace{E_t\, e^{-z\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}}_{\text{mag}} \cdot \underbrace{e^{-j\beta_2 x\sin\theta_t}}_{\text{phase}}$$



Magnitude always exponentially decreases

Date : ....

At $\theta_i = \theta_c$

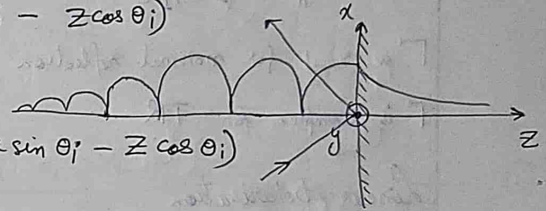
$E_t \times$ (phase term)

(4) Fields in medium 1

$$\vec{E_i} = E_i\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

we have both incident & reflected ray in med 1.

$$\vec{E_r} = E_r\, e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$



~~$\vec{E_{med1}} = \vec{E_i} + \vec{E_r}$~~

$$= E_i\, e^{j\phi}\, e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

$$= E_{med} = \vec{E_i} + \vec{E_r}$$

$$= \underbrace{E_i\, e^{-j\beta_1 x\sin\theta_i}}_{\text{Travelling wave in } x\text{-direction}} \underbrace{\left(e^{-j\beta_1 z\cos\theta_i} + e^{-j\theta}\, e^{j\beta_1 z\cos\theta_i}\right)}_{\text{Standing wave in } z\text{-direction}}$$

In med 1 - near normal → Standing wave

far from " → mostly travelling wave

In med 2 - Exponentially decaying

Not to lose any energy in medium 2, we use thicker cladding in optical fibers.

Decaying field is also called Evanescent field

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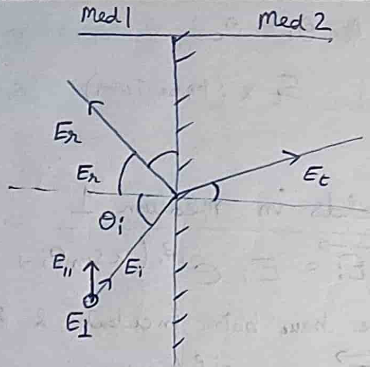
Polarisation at media interface

Date: 5th April

$$E_r = E_{r\parallel} + E_{r\perp} \underbrace{e^{j\phi}}_{\text{phase difference}}$$

$$= \Gamma_{\parallel} E_{i\parallel} + \Gamma_{\perp} E_{i\perp}$$



$$E_t = E_{t\parallel} + E_{t\perp}$$

$$= \tau_{\parallel} E_{i\parallel} + \tau_{\perp} E_{i\perp} e^{j\phi}$$

Γ is real for normal reflection

Γ is complex for TIR

Linear polarisation

$\phi = 0^\circ$

(a) Normal refl^n → $\Gamma_\perp$, $\Gamma_\parallel$ are real ~~real~~ $(\theta_i < \theta_c)$

$\Gamma_\perp \neq \Gamma_\parallel$

Then its said to be ~~so~~ linearly polarised. (Direction might change)

(b) TIR

~~$\phi = 90°$~~

$\Gamma_\perp$, $\Gamma_\parallel$ are complex

$$|\Gamma_\perp| = |\Gamma_\parallel| = 1$$

Then it is said to be Elliptical polarisation.

Date: ....

For parallel polarisation

$$\Gamma_\parallel = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$\therefore$ $\Gamma_\parallel = 0$ when $\eta_1 \cos\theta_i = \eta_2 \cos\theta_t$

For perpendicular polarisation

Similarly for $\Gamma_\perp = 0$ when $\eta_2 \cos\theta_i = \eta_1 \cos\theta_t$

$$\therefore \tan\theta_i = \frac{\beta_2}{\eta_1}\left\{\frac{\eta_2 - \eta_1}{\beta_2 - \beta_1}\right\}^{1/2}$$

$$\tan\theta_{B\perp} = \frac{\beta_2}{\eta_1}\left\{\frac{\eta_2 - \eta_1}{\beta_2 - \beta_1}\right\}^{1/2} \quad \text{(perp comp)} \text{ —— (1)}$$

$\theta_{B\perp}$ → $\perp^r$ Brewster angle

$$\tan\theta_{B\parallel} = \frac{\beta_2}{\eta_2}\left\{\frac{\eta_1 - \eta_2}{\beta_2 - \beta_1}\right\}^{1/2} \quad \text{(para comp)} \text{ —— (11)}$$

If $\Gamma_\parallel = 0$, Then, The angle is The Brewsters angle for parallel polarisation.

If $\Gamma_\perp = 0$, Then The angle is the " " " $\perp$ polarisation.

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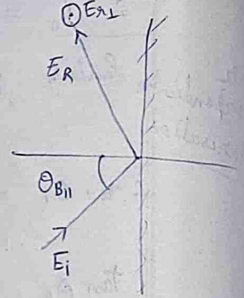
Date : ....

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}, \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} \qquad \beta_1 = \omega\sqrt{\mu_1\varepsilon_1}, \quad \beta_2\,\omega\sqrt{\mu_2\varepsilon_2}$$

$$\theta_{B\perp} = \tan^{-1}\left\{\sqrt{\frac{\mu_1}{\mu_2}}\left[\frac{\mu_2\varepsilon_1 - \mu_2\varepsilon_2}{\mu_2\varepsilon_2 - \mu_1\varepsilon_1}\right]^{1/2}\right\}$$

No $E_{R\parallel}$
No $E_{i\perp}$



$$\theta_{B\parallel} = \tan^{-1}\left\{\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\left[\frac{\mu_1\varepsilon_2 - \mu_2\varepsilon_1}{\mu_2\varepsilon_2 - \mu_1\varepsilon_1}\right]^{1/2}\right\}$$

If $\mu_1 = \mu_2$

$$\theta_{B\perp} = \tan^{-1}\left\{\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\left(\text{Some }-\text{ve numbers}\right)\right\}$$

$\therefore$ $\theta_{B\perp}$ is imaginary angle. We can never find as $-\infty < \tan < \infty$

$$\theta_{B\parallel} = \tan^{-1}\left\{\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\left[\frac{1}{1}\right]^{1/2}\right\}$$

$$= \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$= \tan^{-1}\frac{\eta_2}{\eta_1}$$

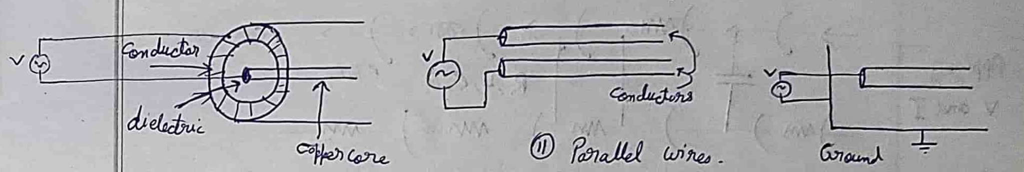
$\left(\eta = \sqrt{\mu\varepsilon}\right) \rightarrow$ RI

Mostly polarizers are linear polarisers.

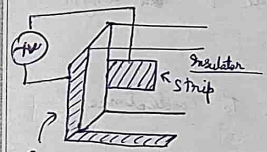
Unit 2 → Transmission line

Date : 26/6/23

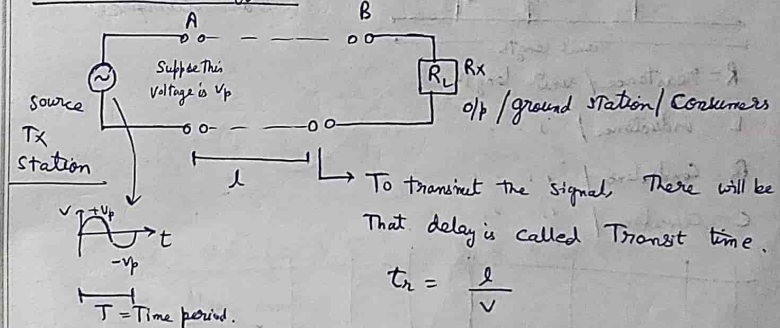
Transmission line is time varying voltage and current.



We need to transmit power, For that we apply voltage



Transit Time effect



To transmit the signal, There will be some delay That delay is called Transit time.

$$t_r = \frac{l}{v}$$

When we find V_p at point B, voltage at A will change in The meantime.

There is a voltage difference That causes loss

$\therefore$ We need to reduce transit time. (For large circuit)

$$T >> t_r \quad \text{or} \quad \frac{1}{f} >> \frac{l}{v} \quad \frac{v}{f} >> l \Rightarrow \boxed{\lambda >> l}$$

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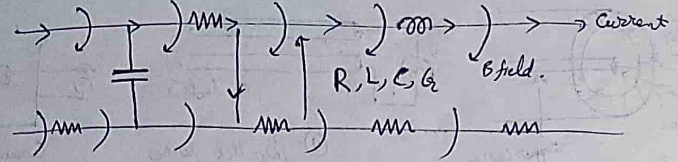
Date:

~~We distribute the line into small equal sections to reduce T_x~~

We represent all the parameters in per unit length.

Applying V and I



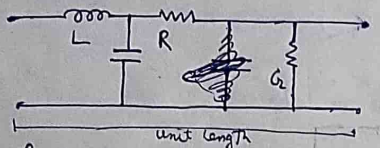
There will be mutual inductance L in T_x line

The T_x line itself behave as inductor

There will also be some resistance due to ~~len~~ length.

and capacitance betⁿ the line

Currents ~~&~~ may be current flow due to conductance



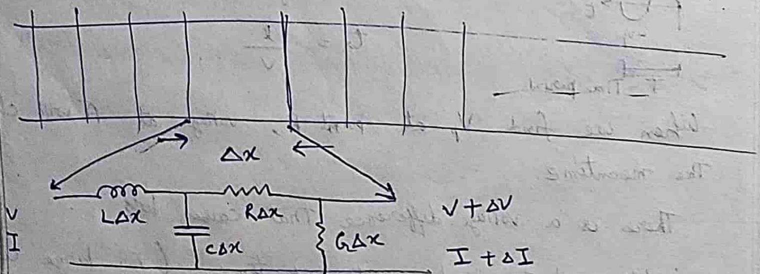
Distributed circuits
(Not lumped)

R = resistance / unit length

L = inductance / " "

G = Conductance / " "

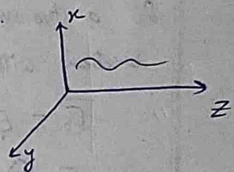
C = Capacitance / " "



We get some change at the ~~o/p~~ output.

Date: 13/3/23

> Bounded medium > Unbounded Medium.

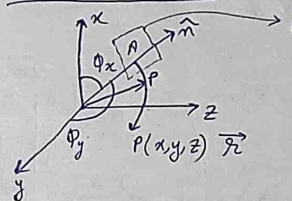


In unbounded, we can take coordinate axis is any direction.

For bounded medium, x, y, z are in fixed directions.

This is because in unbounded medium, waveform is symmetrical in all direction.

Bounded medium



Plane wave

direction of propagation $\hat{n}$

Plane wave in arbitrary direction —
(for bounded medium)

$\hat{n}$ = unit vector along direcⁿ of prop

$\hat{n} = \cos\phi_x \hat{x} + \cos\phi_y \hat{y} + \cos\phi_z \hat{z}$

$\vec{OP} = x\hat{x} + y\hat{y} + z\hat{z}$

Eqⁿ of constant phase plane is given by

$\vec{r}\cdot\hat{n} = \vec{OA} = \text{constant}$

$\Rightarrow (x\hat{x} + y\hat{y} + z\hat{z})\cdot(\cos\phi_x \hat{x} + \cos\phi_y \hat{y} + \cos\phi_z \hat{z}) = \text{Constant}$

Let β = phase constant

Phase at A point,

$\beta \times \vec{OA} = \beta(\hat{n}\cdot\vec{r})$

$$\boxed{\vec{E} = \vec{E_0}\, e^{j\beta(\hat{n}\cdot\vec{r})}} \longrightarrow (1)$$

So, a transverse em wave travelling in some arbitrary direction with unit vector $\hat{n}$ is given by the above eqn ①

$$\vec{E_0} = E_{0x}\hat{x} + E_{0y}\hat{y} + E_{0z}\hat{z}$$

$$E_0 \cdot \hat{n} = 0$$

$$E_0 \perp \hat{n}$$

Wave vector / wave number (K)

$$K = \beta\hat{n}$$

eq ① $\Rightarrow \vec{E} = E_0 e^{-j\vec{K}\cdot\vec{x}}$

To verify

Let $\hat{n} = \hat{z}$

$$K = \beta\hat{n} = \beta\hat{z}$$

$$\hat{n} = \cos\phi_x\hat{x} + \cos\phi_y\hat{y} + \cos\phi_z\hat{z}$$

when $\hat{n} = \hat{z}$

$$\hat{n} = \hat{z} = 0 + 0 + \cos\phi_z\hat{z}$$

$$\vec{E} = E_0 e^{-j\beta(\hat{z}\cdot\vec{x})}$$

$$\vec{x} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\boxed{\therefore \vec{E_0}\, e^{-j\beta z}}$$

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Now, $\frac{\Delta V}{I} = -(R\Delta x + j\omega L\Delta x)$

$$\boxed{\Rightarrow \Delta V = -I(R\Delta x + j\omega L\Delta x)} \quad \text{——— ①}$$

Similarly $\frac{\Delta I}{V} = -(G\Delta x + j\omega C\Delta x)$

$$\boxed{\Rightarrow \Delta I = -V(G\Delta x + j\omega C\Delta x)} \quad \text{——— (ii)}$$

① $\Rightarrow \lim_{\Delta x\to 0} \frac{\Delta V}{\Delta x} = -(R + j\omega L)\, I$ ——— (iii)

Limit is taken to reduce TTE.

(ii) $\Rightarrow \lim_{\Delta x\to 0} \frac{\Delta I}{\Delta x} = -(G + j\omega C)\, V$ ——— (iv)

(iii) $\Rightarrow \frac{dV}{dx} = -(R + j\omega L)\, I \longrightarrow \frac{d^2V}{dx^2} = -(R+j\omega L)\frac{dI}{dx}$

(iv) $\Rightarrow \frac{dI}{dx} = -(G + j\omega C)\, V$

$$= -(R+j\omega L)(-G+j\omega C)V$$

(using (iv)) wave eqn (whole network circuit)

$$\therefore \frac{d^2V}{dx^2} = \gamma^2 V$$

where $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$ ~~wave~~

Similarly $\frac{d^2I}{dx^2} = \gamma^2 I$

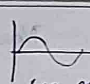
Soln: $V(x) = V^+e^{-\gamma x} + V^-e^{+\gamma x}$

of 2nd order diff eqn

Average or RMS not instantaneous.

Date :

$$= V^+ e^{-\gamma x} e^{j\omega t} + V^- e^{\gamma x} e^{j\omega t}$$



we multiply $e^{j\omega t}$ for sinusoids

we know $\gamma = \alpha + j\beta$ → phase const. ; α → attenuation

$$= \underbrace{V^+ e^{-\alpha x} e^{j(\omega t - \beta x)}}_{\text{1st term.}} + \underbrace{V^- e^{+\alpha x} e^{j(\omega t + \beta x)}}_{\text{2nd term}}$$

In most cases $\alpha = 0$; V is real.

For 1st term

we take $Re\left\{V^+ e^{j(\omega t - \beta x)}\right\}$

$\left\{V^+ \cos(\omega t - \beta x)\right\}$ (j sin was ing)

$e^{j\theta} = \cos\theta + \cancel{j\sin\theta}$ → 0

Similar with 2nd term

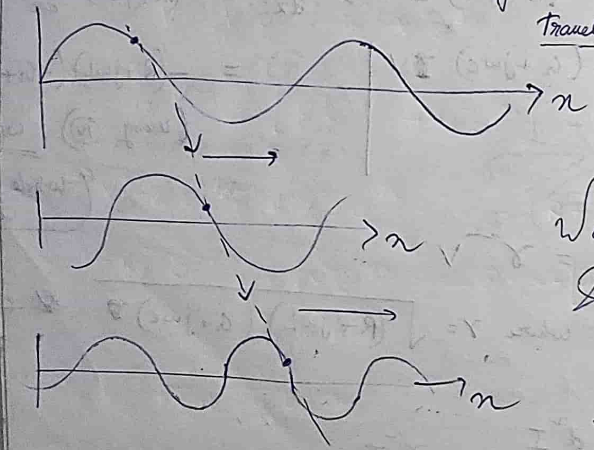
Trav wave prop at -ve x direction

→ Travelling wave at any t, Travelling in +ve x direction

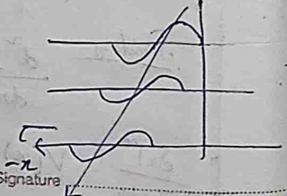
$t_1 = 0$

$t_2 > t_1$

$t_3 > t_2$



Waves of voltage & current



Teacher's Signature

Date : 3rd May 2023 Wednesday

Basics about $\gamma, \beta, Z_0, \Gamma$

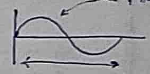
γ = prop constant
β = phase const
Z_0 = impedence
Γ = reflection coefficient

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= \alpha + j\beta$$

$$V(t) = \underbrace{V^+ e^{-\gamma x}}_{\text{1st term}} + \underbrace{V^- e^{\gamma x}}_{\text{2nd term}}$$

$$V^+ e^{-\gamma x} = V^+ e^{-(\alpha + j\beta)x}$$
$$= \underbrace{V^+ e^{-\alpha x}}_{\text{Amplitude}} e^{-j\beta x}$$

Space phase $= -\beta x$ = total phase



Phase is changing

Repeats after 2π

we know, β = phase/unit length

λ phase change at 2π distance (for sine)

For unit phase ⟶ $\frac{2\pi}{\lambda}$

$\therefore \beta = \frac{2\pi}{\lambda}$

$$V(x) = \underbrace{V^+ e^{-\gamma x}}_{\text{Forward}} + \underbrace{V^- e^{-\gamma x}}_{\text{Backward}}$$

α = attenuation constant

Effective Travel distance of any wave $= \frac{1}{\alpha}$; ∞ for $\alpha = 0$

Unit of α = nepers/meter

in dB ⟶ $-20\log\left(e^{-\alpha x}\right)$

$\alpha = 1$ Nepers/m $x = 1m$
$-20\log e^{-1} = 8.68$ dB/m
Imp for Gate

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Date :

If $R = 0.5\ \Omega/m$, $L = 0.2\ \mu H/m$, $C = 100\ pF/m$, $G = 0.1\ ℧/m$

$f = 1\,GHz$, what is the attenuation and phase constant for the Tx line or line

ans:-

$\alpha = ?\quad \beta = ?$

$f = 1\times 10^9\ Hz$, $\quad \omega = \dfrac{\cancel{\quad}}{2\pi 10^9} = 2\pi 10^9\ rad/sec$

$$\gamma = \sqrt{(R+j\omega C)(G+j\omega C)} =$$

$$= \sqrt{(0.5 + j\omega \times 0.2\times 10^{-6})(0.1 + j\omega \times 100\times 10^{-12})}$$

$$= 2.23 + j282$$

$\therefore\ \alpha = 2.23\ n/m \qquad \beta = 282\ rad/m$

8/5/23

We saw; $\dfrac{dV}{dx} = -(R+j\omega L)\, I$ —— (1)

$\therefore\ V = V^+e^{-\gamma x} + V^-e^{\gamma x}$ ——→ It's not a complete sol^n, we have to put this in (1) & get complete sol^n.

Now

$\therefore (1) \Rightarrow \dfrac{d}{dx}V = \dfrac{d}{dx}\left[V^+e^{-\gamma x} + V^-e^{\gamma x}\right] = -(R+j\omega L)\, I$

$\Rightarrow V^+(-\gamma)e^{-\gamma x} + V^-\gamma e^{\gamma x} = -(R+j\omega L)\, I$

Complete Sol^n

$$\Rightarrow \underbrace{-\gamma V^+e^{-\gamma x}}_{\text{Forward Travelling wave}} + \underbrace{\gamma V^-e^{\gamma x}}_{\text{BW FTW}} = -(R+j\omega L)\left[\underbrace{I^+e^{-\gamma x}}_{\text{FTW}} + \underbrace{I^-e^{\gamma x}}_{\text{BW TW}}\right]$$

equating FTW & BW TW

$\therefore\ \cancel{-}\gamma\cdot V^+\cancel{e^{-\gamma x}} = \cancel{-}(R+j\omega L)\, I^+\cancel{e^{-\gamma x}}$ —— (ii)

$+\gamma V^-\cancel{e^{\gamma x}} = -(R+j\omega L)\, I^-\cancel{e^{\gamma x}}$ —— (iii)

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Date :

From eq (ii), we get $\quad \gamma V^+ = (R+j\omega L)\, I^+$

$$\therefore\ \frac{V^+}{I^+} = \frac{R+j\omega L}{\gamma}$$

$$= \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}} \longleftarrow \text{This is } \gamma.$$

$$= \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad \text{—— (A)}$$

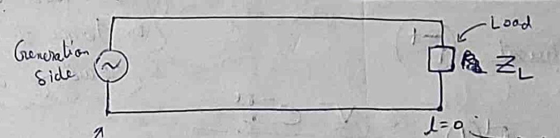
From eq (iii), we get

$$\gamma V^- = -(R+j\omega L)\, I^-$$

$$\frac{V^-}{I^-} = \frac{-(R+j\omega L)}{\gamma} = -\sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad \text{—— (B)}$$

(A) & (B) are the characteristics impedence denoted by $Z_0 \left(= \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}\right)$

(A) $\Rightarrow \dfrac{V^+}{I^+} = +Z_0$ (Forward) $\quad$ & (B) $= \dfrac{V^-}{I^-} = -Z_0$ (Backward)



$l = 0$ ⟶ we take origin at load / consumer

It will be some negative distance

For any distance from load ~~Towards~~ towards the ~~load~~ source side

$$l = -x$$

We $\quad V = V^+e^{\gamma l} + V^-e^{-\gamma l}$ —— (C) $\qquad$ (Changing $l = -x$)

Know $\quad I = I^+e^{\gamma l} + I^-e^{-\gamma l}$ —— (D)

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Date : ....

Using Boundary condition

$l = 0$

$Z = Z_L$

$\cancel{\frac{C}{D} = \frac{V}{I} = \frac{V^+}{I^-} + \frac{V^-}{I^-}}$

$$\frac{C}{D} = \frac{V}{I} = \frac{V^+ e^{\gamma l} + V^- e^{-\gamma l}}{I^+ e^{\gamma l} + I^- e^{-\gamma l}}$$

$$Z_L = \frac{V^+ + V^-}{I^+ + I^-} \quad (\text{at } l = 0).$$

$$= \frac{V^+ + V^-}{\frac{V^+}{Z_0} - \frac{V^-}{Z_0}} \quad (\text{using previous eqs of } Z_0)$$

$$\boxed{Z_L = Z_0 \frac{V^+ + V^-}{V^+ - V^-}}$$

~~Z_0 ch imp~~

Z_0 = characteristic impedance

Z_L = load impedence in terms of forward & backward travelling wave

Reflection Coefficient (Γ)

$$\Gamma(l) = \frac{V^- e^{-\gamma l}}{V^+ e^{\gamma l}}$$

$$\Gamma(0) = \frac{V^-}{V^+}$$

Now

$$Z_L = Z_0 \frac{V^+ + V^-}{V^+ - V^-}$$

$$= Z_0 \frac{1 + V^-/V^+}{1 - V^-/V^+}$$

$$\boxed{Z_L = Z_0 \frac{1+\Gamma(0)}{1-\Gamma(0)}} \longrightarrow Z_L \text{ in terms of reflection coefficient}$$

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Date : 10/5/23

In last class, we found $Z_L = Z_0 \frac{V^+ + V^-}{V^+ - V^-}$

$$= Z_0 \frac{1 + V^-/V^+}{1 - V^-/V^+}$$

$$\boxed{Z_L = Z_0 \frac{1+\Gamma(0)}{1-\Gamma(0)}}$$

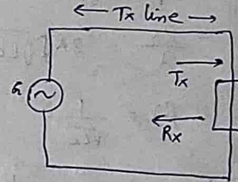
$$\boxed{\therefore \Gamma(0) = \frac{Z_L - Z_0}{Z_L + Z_0}} \boxed{= \frac{V^-}{V^+}}$$ (For reflectance in terms of Z_0 and Z_L)

When $Z_L = Z_0$

Γ at load or at 0 is $\therefore \Gamma_L = \Gamma(0) = \frac{V^-}{V^+} = 0$

If reflectance comes To Generator, it is a loss.

So, when $\Gamma(0) = 0$, it is no loss



Voltage at any location l

$$V(l) = \cancel{V^+ e^{\gamma l}[1+\Gamma(l)]}\ V^+ e^{\gamma l} + V^- e^{-\gamma l}$$

$$= V^+ e^{\gamma l}[1+\Gamma(l)]$$

$$I(l) = I^+ e^{\gamma l}[1-\Gamma(l)]$$

$$= \frac{V^+}{Z_0} e^{\gamma l}[1-\Gamma(l)]$$

$$\boxed{Z(l) = \frac{V(l)}{I(l)} = Z_0\left[\frac{1+\Gamma(l)}{1-\Gamma(l)}\right]}$$

$$\boxed{\Gamma(l) = \frac{Z(l) - Z_0}{Z(l) + Z_0}}$$

Imp

$$Z(l) = Z_0 \frac{Z_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_L \sinh \gamma l}$$

h = hyperbolic

γ = gamma

Formula

$$\cosh(\gamma l) = \frac{e^{\gamma l} + e^{-\gamma l}}{2}$$

$$\sinh(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{2}$$

$\equiv$ equivalent

For lossless Tx line

$$R \equiv 0 ; \quad G \equiv 0$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{j\omega L \times j\omega C} \qquad = j\omega\sqrt{LC}$$

$$\gamma = \underbrace{\alpha}_{\text{attenuation}} + j\beta$$

$$\therefore \alpha + j\beta = j\omega\sqrt{LC}$$

$$\therefore \alpha = 0 \qquad \beta = \omega\sqrt{LC}$$

attenuation is 0

$$\beta = \omega\sqrt{LC}$$

$$\Rightarrow \frac{2\pi}{\lambda} = 2\pi f\sqrt{LC}$$

$$\Rightarrow f\lambda = \frac{1}{\sqrt{LC}}$$

Impedence is purely real

$V = f\lambda$

$$\boxed{\text{Velocity } v = \frac{1}{\sqrt{LC}}}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} = \text{Real}$$

For Low Loss Tx line

$$R \ll \omega L \; ; \; G \ll \omega C$$

Using series expansion and neglecting higher terms.

$$\gamma = \underbrace{j\omega\sqrt{LC}}_{\beta} + \underbrace{\frac{1}{2}R\sqrt{\frac{C}{L}} + \frac{1}{2}G\sqrt{\frac{L}{C}}}_{\alpha}$$

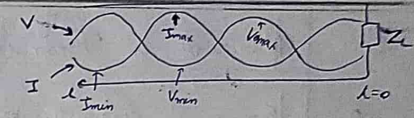
$\therefore$ & $\beta = \omega\sqrt{LC}$ is same as lossless

But α has some loss (attenuation)

$$Z_0 = \sqrt{\frac{L}{C}}\left\{1 - j\frac{R}{2\omega L} + j\frac{G}{2\omega L}\right\}$$

It is almost real

In Transmission line, current and Voltage are fluctuating.



$\therefore$ There will be I_{max}, I_{min}, V_{min}, V_{max}.

$$V_{max} = V^+ (1 + \Gamma_L)$$

$$V_{min} = V^+ (1 - \Gamma_L)$$

$$I_{max} = \frac{V^+}{Z_0}(1 + \Gamma_L) = \frac{V_{max}}{Z_0}$$

$$I_{min} = \frac{V^+}{Z_0}(1 - \Gamma_L) = \frac{V_{min}}{Z_0}$$

voltage standing wave ~~equations~~ ratio (SWR)

$$\rho = \frac{V_{max}}{V_{min}} = \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

$$\Rightarrow \Gamma_L = \frac{\rho - 1}{\rho + 1}$$

There is a range of Z_L for being Lossless or low loss

Imp

$$Z_{L\,max} = \frac{V_{max}}{I_{min}}$$

numerator max
denominator min

$$= Z_0\left(\frac{1 + \Gamma_L}{1 - \Gamma_L}\right) = Z_0\rho$$

$$Z_{L\,min} = \frac{V_{min}}{I_{max}} = Z_0\left(\frac{1 - \Gamma_L}{1 + \Gamma_L}\right) = \frac{Z_0}{\rho}$$

Date: 12/May/23 (Online)

Numericals on Transmission Line and Smith Chart

Q For a Tx line, the per unit length parameters are $0.1\,\Omega/m$, $0.2\,\mu H/m$, $10\,pF/m$ and $0.02\,\mho/m$. Find the complex prop const.

ans a) γ (comp prop const) $= \sqrt{(R+j\omega L)(G+j\omega C)}$

$R = 0.1\,\Omega/m \qquad L = 0.2 \times 10^{-6}\,H/m$

$G = 0.2\,\mho/m \qquad C = 10 \times 10^{-12}\,F/m$

$\omega = 2\pi f = 2\pi \times 10^{6}\,rad/s$

$$\gamma = \sqrt{(0.1 + j2\pi 10^{6} \cdot 0.2 \times 10^{-6})(0.2 + j2\pi 10^{6} \cdot 10 \times 10^{-12})}$$

$= 0.117 + j0.108\,/m$

b) At 1GHz, $\omega = 2\pi 10^{9}\,rad/s$

$\therefore \gamma = 1.4 + j9/m$

Q 2.6 A tx ~~line~~ line has primary constants $R = 0.1\,\Omega/m$, $G = 0.01\,\mho/m$, $L = 0.01\,\mu H/m$, $C = 100\,pF/m$. Find the characteristic impedence of the line at 2GHz

sol $\omega = 2\pi \times 2 \times 10^{9} = 4\pi \times 10^{9}$ rad/sec

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{0.1 + j4\pi 10^{9} \cdot 0.01 \times 10^{-6}}{0.01 + j4\pi 10^{9} \cdot 100 \times 10^{-12}}}$$

$= 10 + j0.0358\,\Omega$

Date:

Eg 2.8 The Transmission line in eg 2.6 is connected to a load impedence $10 + j20\Omega$ at 2GHz. Find the reflection coefficient (i) at the load-end of the line (ii) at a dist 20 cm $= 0.2\,m$ from the load.

Sol (i) From Eg 2.6 we have $Z_0 = 10 + j0.0358\,\Omega$

The reflection coefficient at the load-end of the line is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(10+j20) - (10+j0.0358)}{(10+j20) + (10+j0.0358)} = 0.499 + 0.498j$$

(ii) As solved in example 2.7. $\gamma = 0.055 + j12.566\,/m$.

$\therefore \Gamma(l = 20\,cm) = \Gamma_L e^{-2\gamma l}$

$= (0.499 + j0.498)e^{-2 \times 0.055 + j12.566}$

2.9 A tx line has the prop. const $\gamma = 0.1 + j10/m$ and ch. impedence $Z_0 = 50 + j5\Omega$. The line is terminal in an impedence $100 - j30\Omega$. Find the impedence at a ditance 1.5m from the load.

ans The impedence can be obtained by $Z(l) = Z_0 \left[\frac{Z_L \cosh\gamma l + Z_0 \sinh\gamma l}{Z_L \sinh\gamma l + Z_0 \cosh\gamma l}\right]$

$\cosh\gamma l = \cosh[(0.1 + j10)1.5] = -0.7683 + 0.0979j$

$\sinh\gamma l = \sinh[(0.1 + j10)1.5] = -0.1144 + 0.6576j$

$\therefore Z(l) = Z_0\left(\quad\right) = 57.3175 + 38.6619j\,\Omega$

Date : ....

2.11 A transmission line has $L = 0.25\,\mu H/m$. $C = 100\,pF/m$ $G = 0$. What should be the value of R for the line so that the line can be treated as low loss line? The freq^n of operation is $\underline{100\,MHz} = f$

Ans: ~~Sol~~ The phase constant of the low loss line is

$$\beta \approx \omega\sqrt{LC} = 2\pi\sqrt{0.25\times10^{-6}\times100\times10^{-12}} \times 100\times10^{6}$$
$$= \pi$$

The attenuation const^t is $\alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$ $(G=0)$

$$= \frac{R}{2}\sqrt{\frac{10^{-10}}{0.25\times10^{-6}}} = 0.01\,R \text{ neper/m}$$

For low-loss we should have $\beta \gg \alpha$. Taking $\alpha = 1\%$ of β we get

$$0.01R < \frac{\beta}{100}$$
$$\Rightarrow R < \pi = 3.14\ \Omega/m$$

2.13 A 50Ω transmission line is connected to a parallel combination of a 100Ω resistance and a ~~1pF~~ 1nF capacitance. Find the VSWR on the line at a freq^n of 2MHz. Also find the max & min resistance seen on the line.

Sol^n Load Impedence Z_L is a parallel combination of $R = 100\Omega$ and $C = 1nF$

$$\therefore Z_L = \frac{R(1/j\omega c)}{R + (1/j\omega c)} = \frac{R}{1 + j\omega RC} = 38.77 - j48.72$$

The reflection coefficient at the load-end of the line is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.134 - j0.475$$

$$|\Gamma_L| = 0.494$$

The VSWR $\rho = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = 2.95$

$\therefore R_{max} = \rho Z_0 = 147.53\,K\Omega$

$R_{min} = Z_0/\rho = 16.94\,\Omega$

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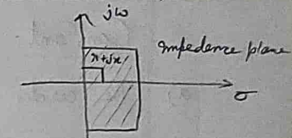
Date : ....

Complex Impedence Plane



An impedance $Z = R + jX$ when normalized with the characteristics impedance Z_0 is denoded by $\bar{Z} = r + jx$

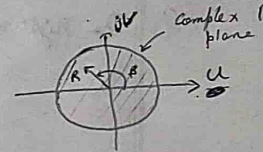
where $\bar{Z} = \frac{Z}{Z_0}$, $r = \frac{R}{Z_0}$ & $x = X/Z_0$

The ~~Relat~~ reflection coefficient for a normalised impedance $\bar{Z}$ is then given as

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{\bar{Z} - 1}{\bar{Z} + 1}$$

$$= \frac{r + jx - 1}{r + jx + 1} = \frac{(r-1) + jx}{(r+1) + jx}$$

Complex Reflection Coefficient Plane



The semi infinite impedence plane is mapped to the area within the unit circle in the Γ plane with one to one correspondance bet^n the points in the two planes. One can show that the transformation from $\bar{Z}$-plane to Γ-plane is a conformal transformation. Let us map a point $\bar{Z} = r + jx$ onto the $\Gamma = u + jv$ plane.

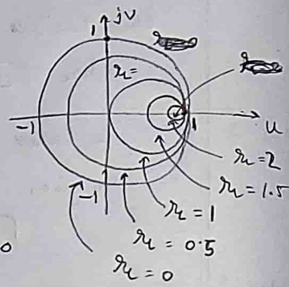
$$\therefore Z = \frac{1+\Gamma}{1-\Gamma}$$

$$\Rightarrow r + jx = \frac{1 + (u + jv)}{1 - (u - jv)}$$

Constant Resistance Circles

a) The circles always have ~~common~~ center on the real Γ axis (u axis)

b) All circles pass through point $(1,0)$



c) For $r_L = 0$, the center lies on origin of Γ plane and shifts to right as r increases

d) As r_L increases, the radius of circles reduces.

as $r_L \to \infty$, circle becomes a point as radius $\to 0$

e) The outermost circle with center $(0,0)$ and radius unity corresponds to $r_L = 0$ or in other words represents reactive loads only.

f) The right most point on the unit circle $(1,0)$ represents $r_L = 0$ as well as $r_L = \infty$

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Date : ....