3/2/2023

Physical Word

Classification Mechanics (Newdon)

Quantum Mechanics

Special Relativity (Einstein)

Quantum Electrodynamics.

Mechanics → how a system behave.
→ behavioor of a system in different forces.

Forces :-

① → Strongg force (eg:- neutron $\frac{TG}{SG}$ Protons)

M.K. MK@123

③ → Weak force (eg: radiation)

④ → 'Gravity'

⑤ → Electromagnetic force

Em mostly developed by measu-

4/2/2023

Vector Analysis / calculus :-

If a quantity has dirn & mag with magnitude is vector.

Properties of Vector operations :-

(1) Addition of 2 Vectors

$\vec{A}+\vec{B} = \vec{B}+\vec{A}$

$(\vec{A}+\vec{B})+\vec{C} = \vec{A}+(\vec{B}+\vec{C})$

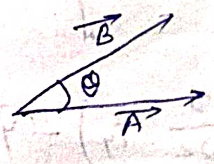
$(\vec{A}-\vec{B}) = \vec{A}+(-\vec{B})$

2) Multiplication of Vector by Scalar.

$\alpha(\vec{A}+\vec{B}) = \alpha\vec{A}+\alpha\vec{B}$

3) Dot Product.

$\vec{A}\cdot\vec{B} = AB\cos\theta$



4) Cross Product.

$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$.

$\vec{A}\times(\vec{B}+\vec{C}) = (\vec{A}\times\vec{B})+(\vec{A}\times\vec{C})$

(a) Scalar tripple Product.

$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}(\vec{C}\times\vec{A}) = \vec{C}(\vec{A}\times\vec{B})$

(b) Vector tripple Product

$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{C}\cdot\vec{A}) - \vec{C}(\vec{A}\cdot\vec{B})$

Gradient of a Vector :-

$T(x, y, z)$

$$dT = \frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial z} dz$$

$$= \left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right) \cdot \left(\hat{x}dx + \hat{y}dy + \hat{z}dz\right)$$

$$= (\vec{\nabla} T \cdot d\vec{l})$$

$$\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial x}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$$

Dot Product :-

$$\vec{\nabla} \cdot \vec{T} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\vec{T}$$

$$= \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\left(T_x\hat{x} + T_y\hat{y} + T_z \cdot \hat{z}\right)$$

$$= \frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + \frac{\partial T_z}{\partial z}.$$

Cross Product :-

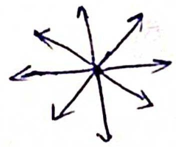
$$\vec{\nabla} \times \vec{T} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \times \vec{T}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ T_x & T_y & T_z \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial}{\partial y}T_z - \frac{\partial}{\partial z}T_y\right) - \hat{y}\left(\frac{\partial}{\partial x}T_z - \frac{\partial}{\partial z}T_x\right)$$

$$+ \hat{x}\left(\frac{\partial}{\partial x}T_y - \frac{\partial}{\partial y}T_x\right).$$

Significance of Dot Product:-



dot product signifies divergence

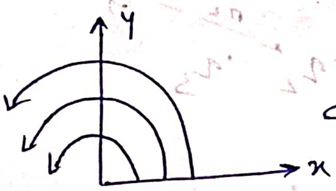
(very high)

↑↑↑ ↑ — parallel so zero

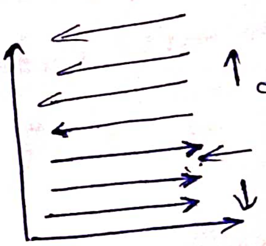
convergent so negative value.

Significance of Cross product:-

Cross product gives curl of a vector.



curl of a vector gives very high value.



↑ div

curl value high.

zero value.

8/2/23

$C = 3\times10^{8}\ m/s$

$\epsilon_0 = 8.854\times10^{-12}\ F/m$

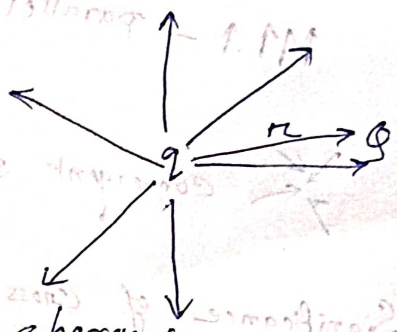
$\mu_0 = 4\pi\times10^{-7}\ H/m$

Basic law of Electromagnetic :

~~Colombs law~~

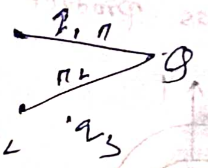
Collomb's Law:-

$$\boxed{F = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{r}}$$



Coulomb's Law for multiple charges:-

$$\boxed{F = \frac{Q}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i}$$



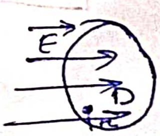
$$E = \frac{F}{q}$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$$

Electric flux for a sphere :-

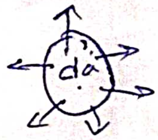
Electric flux through the sphere is given by,

$$\boxed{\phi_E = \oint \vec{E}\cdot\vec{da}}$$



q
P(point charge)
How many Electric field line passing through the ~~A~~ charge. sphere.

da → Small area



$= \oint E\,da\cos\theta$

$= \oint E da$

$= E\oint da$

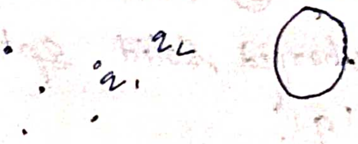
$= E\,\cancel{4}4\pi r^2$

$= \cancel{4\pi r^2} \times \frac{1}{\cancel{4\pi}\epsilon_0}\frac{q}{\cancel{r^2}}$

~~Electric~~ dirn of the electric field coming out. then $\cos\theta$ will be aline so zero

$$\boxed{\phi_E = \frac{q}{\epsilon_0}}$$ — Gauss' Law.

$$\phi_E = \sum \frac{q_i}{\epsilon_0}$$
~~Qenc~~



$$\boxed{\phi_E = \frac{Q_{enc}}{\epsilon_0}}$$

$$Q_{enc} = \Sigma q_i$$

For multiple pt sources from the principle of superposition :-

$$E = \sum_{i=1}^{n} E_i$$

$$\phi_E = \oint \vec{E}\cdot\vec{da} = \sum_{i=1}^{n} \oint \vec{E_i}\,\vec{da}.$$

Gauss law in differential form :—

{ Stoke's Law - sphere surface
divergence — volume.

We know, (from divergence theorem)

$$\oint_s \vec{E}\cdot\vec{da} = \int_v (\nabla\cdot\vec{E})\, d\tau$$

Rewriting Qenc

$$Q_{enc} = \int_v \rho\, dv$$ (ρ → is charge density)
($d\tau$ ⇒ $dx\,dy\,dz$)

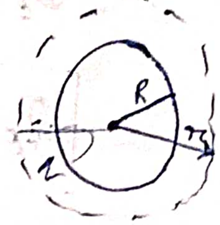
$$\therefore \oint \vec{E}\,\vec{da} = \frac{Q_{enc}}{\epsilon_0}$$

$$\int_v (\nabla\vec{E})\, d\tau = \frac{1}{\epsilon_0}\int_v \rho\, d\tau$$
$$\left(\rho = \frac{Q}{V}\right)$$

$$\boxed{\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}}$$

Applications of Gauss law :-

Ex ① ① Find the electric field outside a uniformly charged solid sphere of radius 'R' and total charge 'q'.



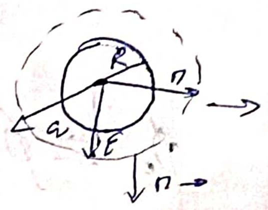
Soln

From Gauss Law,

$$\phi_s = \oint_s \vec{E}\,\vec{da} = \frac{q_{enc}}{\epsilon_0}$$

$$\phi_s = \frac{q}{\epsilon_0} \quad \text{—— (1)}$$

Further



$$\oint_s \vec{E}\,\vec{da} = \oint E\,da\cos\theta$$

$$= \oint E\,da\cos 0^\circ$$

$$= \oint E\,da$$

$$= E\oint da$$

$$= E \times 4\pi R^2$$

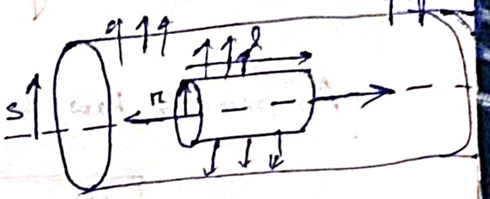
$$1 \Rightarrow E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\boxed{E = \frac{1}{4\pi R^2} \times \frac{q}{\epsilon_0}}$$

~~Cobln~~ Coulomb's Law.

Ex ② A long cylinder carries a charge density that is proportional to the distance form the axis. $\rho = ks$, for some constant k. Find the electric field inside the cylinder.

Soln From Gauss's law,



$$\oint E\,da = \frac{Q_{enc}}{\epsilon_0}$$

Now $Q_{enc} = \int_V \rho\, d\tau$

$$= \int_V (ks)\,(dx\,dy\,dz)$$

dxdydz = rdrdϕ dl

$$= \int_V (ks)\,(s\,ds\,d\phi\,dl)$$

dϕ — circular angle

$$= k\int_V s^2 ds\, d\phi\, dl$$

dl — Height of the cylinder.

$$= k\int_0^s s^2 ds \int_0^{2\pi} d\phi \int_0^l dl$$

$$= k\left[\frac{s^3}{3}\right]_0^s \left[\phi\right]_0^{2\pi} \left[l\right]_0^l$$

$$= k\,\frac{s^3}{3}\,2\pi l$$

$$= \frac{2\pi l k s^3}{3}$$

$$\oint \vec{E}\cdot\vec{da} = \oint E\,da$$

$$= E\int_{\text{curved surface}} da$$

$$= E \times 2\pi s l$$

$$\oint E\,da = \frac{Q_{enc}}{\epsilon_0}$$

$$\Rightarrow E \times \cancel{2\pi} \cancel{s} \cancel{l} = \frac{\cancel{2\pi} \cancel{l} k s^{\cancel{3}}}{3\epsilon_0}$$

$$\boxed{E = \frac{ks^2}{3\epsilon_0}}$$

Ampere's law:—

$$\boxed{\nabla \times H = \frac{\partial D}{\partial t} + J}$$ → Ampere's law in differential form.

J – flowing electric current

Ampere's law in ~~critical~~ Integral form:—

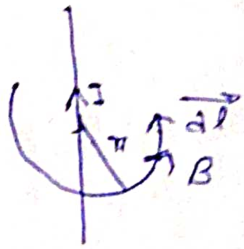
$$\boxed{\oint H dl = I_{enc}}$$

$$\Rightarrow \oint \mu_0 H dl = \mu_0 I_{enc}$$

$$\Rightarrow \boxed{\oint B dl = \mu_0 I_{enc}}$$

The magnetic field enclosed by a electric current is porportional to the size of that electric current with a constant of porportionality which is permeability.

Application ① : current carrying infinite conductor:—



$$\oint \vec{B}\,\vec{dl} = \mu_0 I_{enc} \longrightarrow ①$$

$$\oint \vec{B}.\vec{dl} = \oint B dl \cos 0^\circ$$

$$= \oint B dl \; [\because \theta = 0^\circ]$$

$$= B \oint dl$$

$$= B \times 2\pi r$$

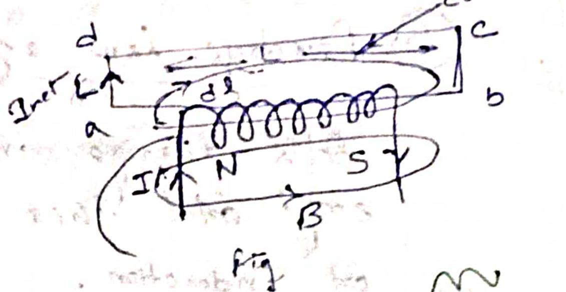
$$① \Rightarrow B \times 2\pi r = \mu_0 I_{enc}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I_{enc}}{2\pi r}}$$

$$\text{or } \boxed{B = \frac{\mu_0 2 I_{enc}}{4\pi r}}$$ current carrying infinite conductor.

② Solenoid

Incase of conductor:-
abcd is a conductor.
length — L



For ab

$$\oint_a^b B\,dl = \mu_0 I$$

$$\Rightarrow \oint_a^b B\,dl \cos 0^\circ = \mu_0 I$$

$$\Rightarrow \oint B\,dl = \mu_0 I$$

$$\Rightarrow B\oint dl = \mu_0 I$$

$$\Rightarrow B \times L = \mu_0 I$$

For whole area :-

$$\oint_{\text{whole area}} \vec{B}\,\vec{dl} = \oint_a^b B\,dl + \int_b^c \vec{B\,dl}^{\,0} + \int_c^d B\,dl + \int_d^a \vec{B\,dl}^{\,0}$$

$$\oint \vec{B}\,\vec{dl} = \int_a^b B\,dl + \int_c^d \vec{B\,dl}^{\,0} \quad (\text{since for cd } \& \text{ da } B \approx 0)$$

$$= BL$$

Therefore Ampere circuital (integration law) law or Ampere's law -

$$\boxed{\oint B\,dl = \mu_0 I_{net}}$$

$$\boxed{BL = \mu_0 I_{net}} \quad \text{①}$$ {Inet is induce current from changing magnetic field.}

$I_{net} = nLI$
↑ no of turns.

$$\text{①} \Rightarrow B\cancel{L} = \mu_0 n \cancel{L} I$$
$$BL = \mu_0 I_{net}$$
$$\Rightarrow B\cancel{L} = \mu_0 n \cancel{L} I$$
$$\boxed{B = \mu_0 n I}$$

$B \propto n$
$B \propto I$

(3) Faradays Law of induction :-

It's change in ~~electric~~ magnetic flux and can induce a emf and this theorem is called Faraday's law of induction.

$$\boxed{e = -\frac{\Delta \phi_B}{\Delta t}}$$

B

$$\boxed{e = -N \frac{\Delta \phi_B}{\Delta t}}$$

$$\boxed{e = -\frac{\Delta \phi_B}{\Delta t}}$$
— Lenz Law

~~9/1~~

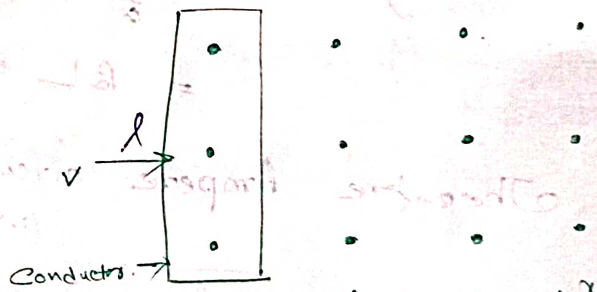
Application :- $e = -\frac{\Delta \phi_B}{\Delta t} = -\frac{\partial \phi_B}{\partial t}$ $\phi_B = \int B\,da$

Faraday. $= -\frac{\partial (BA\cos\theta)}{\partial t}$ $= BA\cos\theta$

12/2

Example of Faradays Law :-

(1)



(magnetic field coming out (•)
" " goining into (×))

$$e = -\frac{\Delta \phi_B}{\Delta t}$$

$$= -\frac{B \Delta A}{\Delta t}$$

$\Delta A \rightarrow$ change in area

$v\Delta t \rightarrow$ change in distance with change in length (l)

$$= \frac{B \cancel{v \Delta t}\, l}{\cancel{\Delta t}}$$

$$= Bvl$$

$$\boxed{e = -Bvl}$$

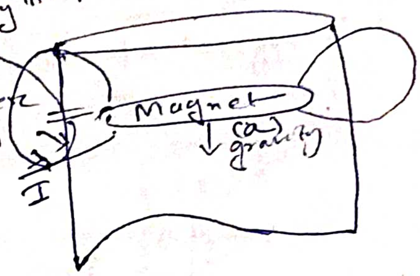
Push the conductor with velocity 'v' — area changing & velocity chang then emf will be induced.

The induced current is in a direction that dreands to slow the moving choosen the bar. It takes an external force keeping it moving.

eg: Bullet.

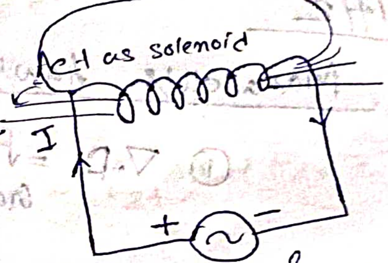
Ex-2 — Eddy Current :-

The current induced in a bulk piece of conductor as shown in the ckt is eddy current.



ex-3 Self Induction :-

The effect of changing current in a circuit induces an emf in the same circuit and this is called Self induction.



AC Source

$$\boxed{e = -L \frac{\Delta I}{\Delta t}}$$

$L \rightarrow$ Self induction. (Henry)

ex-4 — Mutual Induction

Fig-3
$$\boxed{e_s = -\frac{M \Delta I_P}{\Delta t}}$$

M — Mutual Inductance.

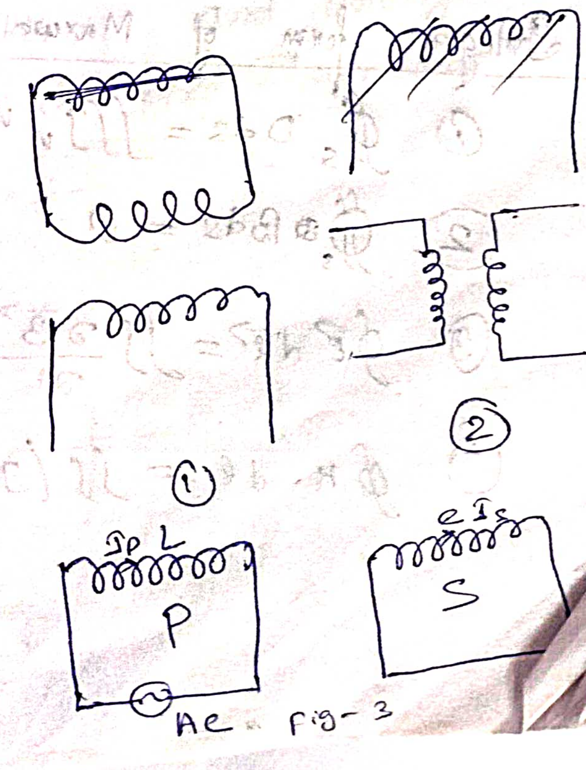
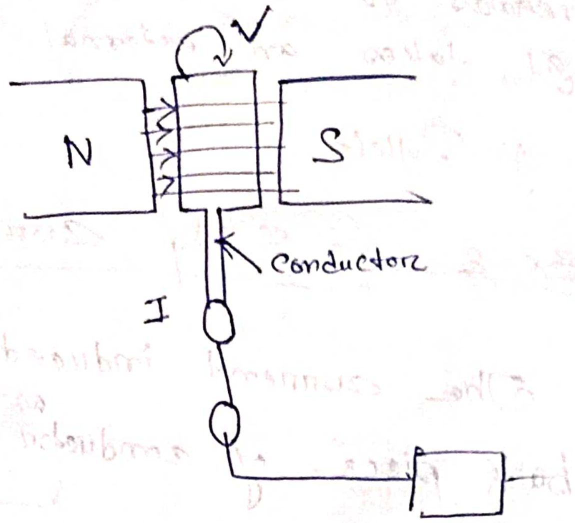


fig-3

Ex—5 Generator



14/2

Maxwell's Law's / Equⁿ :— (Differential form)

① $\nabla . D = \rho$
Gauess' Law.

② $\nabla . B = 0$ → Gauess law for magnetic field.

There is no magnetic monopole. → (Single N-pole / S-pole)

③ $\nabla \times E = -\frac{\partial B}{\partial t}$, Faradays law.

④ Ampere's law ⇒ $\nabla \times H = \left(J + \frac{\partial D}{\partial t}\right)$

Integral form of Maxwell Equation :—

① $\oint_S D ds = \iiint_V \rho dv$

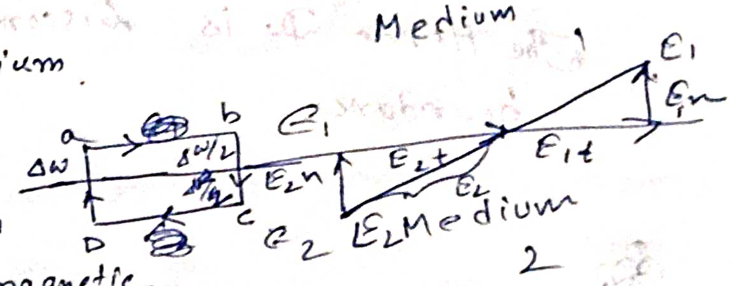
② $\oint_S B ds = 0$

③ $\oint \vec{E} d\vec{l} = \iint \frac{\partial \vec{B}}{\partial t} . ds$

④ $\oint H . dl = \iint \left(J + \frac{\partial D}{\partial t}\right) ds$

Boundary Condition for E-field:-

(1) Dielectric – diectric Medium.



$\oint E\,dl = 0$ (if there is no change in magnetic field)

t – tangential
n – normal

$\Rightarrow \oint_{abcd} E\,dl = 0$

$\Rightarrow \int_{ab} E\,dl + \int_{b}^{c/2} E\,dl + \int_{b\,c/2} E\,dl + \int_{cd} E\,dl + \int_{d}^{a/2} E\,dl + \int_{d\,a/2} E\,dl = 0$

$\Rightarrow E_{1t}\Delta l - \cancel{E_{1n}\frac{\Delta w}{2}} - \cancel{E_{2n}\frac{\Delta w}{2}} - E_{2t}\Delta l + \cancel{E_{2n}\frac{\Delta w}{2}} + \cancel{E_{1n}\frac{\Delta w}{2}} = 0$

$\Rightarrow E_{1t}\cancel{\Delta l} - E_{2t}\cancel{\Delta l} = 0$

$\Rightarrow E_{1t} = E_{2t}$

Tangential Components of 'E' are equal at the boundary. i.e E_t is continuous and undergoes no change at ~~die~~ ~~dielectric~~ dielectric boundary.

$\therefore E_{1t} = E_{2t}$

$\Rightarrow \frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}$

$$\boxed{\epsilon_2 D_{1t} = \epsilon_1 D_{2t}}$$

Tangential of first medium

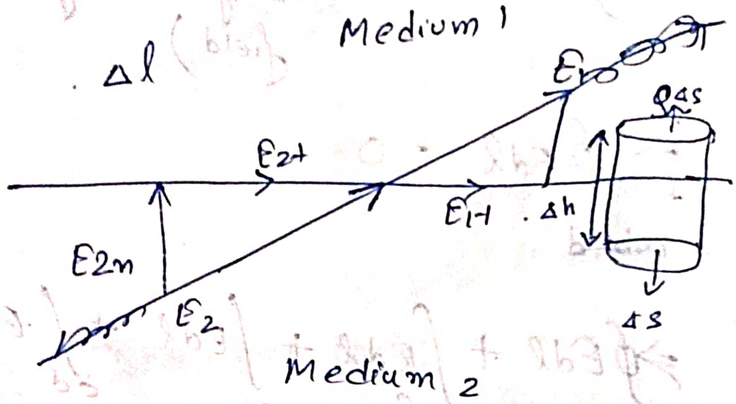
Tang.

D → Electric flux density.

~~The~~ Hence D_t is Discontinous across dielectric boundaries.

For Normal

Applying Gauss's law for this region we can write as —



$$\oint D \cdot ds = \rho.$$

$$\Rightarrow D_{1n} \cancel{\Delta S} - D_{2n} \cancel{\Delta S} = \rho \left[\because \Delta h = 0 \right]$$

$$= \rho_s \cancel{\Delta S}$$

$$\Rightarrow \boxed{D_{1n} - D_{2n} = \rho_s}$$ → Both are for normal component

If there ~~is~~ are no free charge present in ~~normal~~ the region, then we can write —

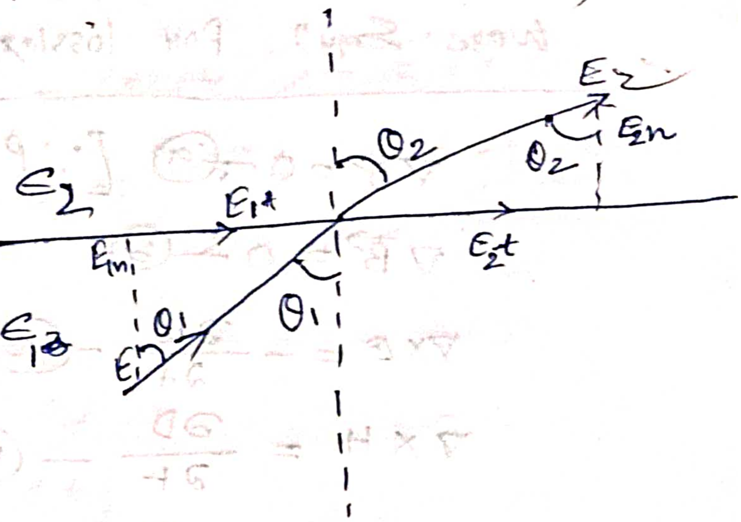
$$D_{1n} - D_{2n} = 0$$

$$\Rightarrow \boxed{D_{1n} = D_{2n}}$$

$$\Rightarrow \boxed{\epsilon_1 E_{1n} = \epsilon_2 E_{2n}}$$

* Normal Component of D are equal at the boundary for no free charge.
* E is Discontinous for normal Component and is given by the above Equation.

Angle of Refraction :- (For dielectric boundary)



$$E_{1t} = E_{2t}$$

$$\Rightarrow E_1 \sin\theta_1 = E_2 \sin\theta_2 \longrightarrow (1)$$

$$D_{1n} = D_{2n}$$

$$\Rightarrow \epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

$$\Rightarrow \epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2 \longrightarrow (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\cancel{E_1}\sin\theta_1}{\epsilon_1 \cancel{E_1}\cos\theta_1} = \frac{\cancel{E_2}\sin\theta_2}{\epsilon_2 \cancel{E_2}\cos\theta_2}$$

$$\Rightarrow \frac{1}{\epsilon_1}\tan\theta_1 = \frac{1}{\epsilon_2}\tan\theta_2$$

$$\Rightarrow \boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2}}$$

17/2

Wave Equn for lossless medium:-

$$\nabla \vec{D} = 0 \quad \text{--- (a)} \quad [\because \rho = 0]$$

$$\nabla \vec{B} = 0 \quad \text{--- (b)}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \text{--- (c)}$$

$$\nabla \times H = \frac{\partial D}{\partial t} \quad \text{--- (D)}$$

$$(d) \Rightarrow \nabla \times H = \frac{\partial D}{\partial t}$$

$$\frac{\partial}{\partial t}(\nabla \times H) = \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t}\right)$$

$$\Rightarrow \left(\nabla \times \frac{\partial H}{\partial t}\right) = \epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{--- (1)}$$

$$(c) \quad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times (\nabla \times E) = -\nabla \times \frac{\partial B}{\partial t}$$

$$\Rightarrow \nabla \times \nabla \times E = -\mu \nabla \times \frac{\partial H}{\partial t} \quad [\because B = \mu H]$$

$$(d) \Rightarrow \nabla \times H = \frac{\partial D}{\partial t}$$

$$\cancel{\frac{\partial}{\partial t}(\nabla \times H) = \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t}\right)}$$

$$\cancel{\Rightarrow \partial\left(\nabla \times \frac{\partial H}{\partial t}\right) = \epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{--- (1)}}$$

$$\Rightarrow \nabla \times \nabla \times E = -\mu\epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{--- (2)}$$

We know, $\vec{A} \times \vec{B} \times \vec{C} = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$

So, $\nabla \times \nabla \times E = \nabla(\nabla \vec{E}) - \vec{E}(\vec{\nabla}\vec{\nabla})$

$= \nabla(\nabla \vec{E}) - \vec{\nabla}^2 E$ — (3)

Again, $\nabla \vec{D} = 0 \quad [\rho = 0]$

$\Rightarrow \nabla \vec{E} = 0 \quad [D = \epsilon E]$

(3) $\Rightarrow \nabla \times \nabla \times E = -\nabla^2 E$ — (4)

(2) $\Rightarrow \cancel{-}\nabla^2 E = \cancel{-}\mu\epsilon \dfrac{\partial^2 E}{\partial t^2}$ [form eqn (4)]

$$\boxed{\nabla^2 E = \mu\epsilon \frac{\partial^2 E}{\partial t^2}}$$ — Laplacian of Wave Equation

This equn is called the wave equn in terms of 'E' for ~~travels~~ loss less free space and medium.

Wave Equn for magnetic field :—

$\nabla \times E = -\dfrac{\partial B}{\partial t}$ — (c)

$\nabla \times H = \dfrac{\partial D}{\partial t}$ — (d)

(c) $\Rightarrow \nabla \times E = -\dfrac{\partial B}{\partial t}$

$\Rightarrow \dfrac{\partial}{\partial t}(\nabla \times E) = -\dfrac{\partial B^2}{\partial t^2}$

$\Rightarrow \left(\nabla \times \dfrac{\partial E}{\partial t}\right) = -\mu \dfrac{\partial^2 H}{\partial t^2}$ — (1)

(d) $\Rightarrow \nabla \times H = \dfrac{\partial D}{\partial t}$

$\nabla \times \nabla \times H = \nabla \times \dfrac{\partial D}{\partial t}$

$= \epsilon\left(\nabla \times \dfrac{\partial E}{\partial t}\right)$ — (2)

$\nabla \times \nabla \times H = -\mu\epsilon \dfrac{\partial^2 H}{\partial t^2}$

$$\nabla \times \nabla \times H = \nabla(\nabla H) - H(\nabla \nabla)$$

$$= \nabla(\nabla H) - \nabla^2 H$$

Again,

$$\nabla \cdot B = 0$$

$$\Rightarrow \nabla \cdot H = 0$$

$$\therefore \nabla \times \nabla \times H = -\nabla^2 H \quad \text{--- (3)}$$

Equn (2) $\Rightarrow \cancel{-}\nabla^2 H = \cancel{-}\mu\epsilon \frac{\partial^2 H}{\partial t^2}$

$\Rightarrow \boxed{\nabla^2 H = \mu\epsilon \frac{\partial^2 H}{\partial t^2}}$ — Wave Equn for magnetic field.

পানীত stone doliyale heitu nature Equn হয়.

* Wave Equn always variation in magnetic field & time in second (or Double) differentation.

In Rectangular cordinate system we can write as follow:—

$$\nabla^2 E_x = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}$$

$$\nabla^2 E_y = \mu\epsilon \frac{\partial^2 E_y}{\partial t^2}$$

$$\nabla^2 E_z = \mu\epsilon \frac{\partial^2 E_z}{\partial t^2}$$

& Similarly for D, H & B

If the Electric field intensity is varying harmonically with time then you can write it as —

If Electri.

$$\vec{E} = E_s e^{j\omega t}$$

$$\frac{\partial \vec{E}}{\partial t} = E_s \frac{\partial e^{j\omega t}}{\partial t}$$

$$= E_s (j\omega) e^{j\omega t}$$

$$= j\omega \underbrace{E_s e^{j\omega t}}$$

$$\partial t = j\omega \vec{E}$$

$$\Rightarrow \frac{\partial^2 \vec{E}}{\partial t^2} = j\omega \left(\frac{\partial \vec{E}}{\partial t}\right)$$

$$= j\omega \left(\frac{\partial E_s}{\partial t} e^{j\omega t}\right)$$

$$= j\omega E_s (j\omega e^{j\omega t})$$

$$\Rightarrow \frac{\partial^2 E}{\partial t^2} = j^2 \omega^2 \vec{E}$$

$$\boxed{\frac{\partial^2 E}{\partial t^2} = -\omega^2 \vec{E}}$$

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 E}{\partial t^2}$$

$$= \mu \epsilon (-\omega^2 \vec{E})$$

$$\boxed{\nabla^2 \vec{E} = -\mu \epsilon \omega^2 \vec{E}}$$ — wave equn for Electric field with resp to harmonic field.

$$\boxed{\nabla^2 \vec{H} = -\mu \epsilon \omega^2 \vec{H}}$$

$$\boxed{\nabla^2 \vec{B} = -\mu \epsilon \omega^2 \vec{B}}$$

$$\boxed{\nabla^2 D = -\mu \epsilon \omega^2 D}$$

This equn are known as homogenous vector wave Equn, In complex time harmonic form for free Space.

24/2 Friday

$$\nabla^2 E = \mu \epsilon \frac{\partial \vec{E}}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\mu \epsilon}}$$

$$\boxed{\nabla^2 E = \frac{1}{v^2} \frac{\partial \vec{E}}{\partial t^2}}$$

Wave Equn in conducting medium / lossy medium.

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$= \sigma \vec{E} + \frac{\partial \epsilon \vec{E}}{\partial t} \quad [\because J = \sigma E]$$

$\sigma \rightarrow$ conductivity.

$$= \sigma E + \epsilon \frac{\partial E}{\partial t}$$

$$\frac{\partial}{\partial t}(\nabla \times H) = \sigma \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{--- (1)}$$

$$\Rightarrow \left(\nabla \times \frac{\partial H}{\partial t}\right) = \sigma \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{--- (1)}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}.$$

$$\Rightarrow \nabla \times (\nabla \times E) = \nabla \times \left(-\frac{\partial B}{\partial t}\right)$$

$$\Rightarrow \nabla \times \nabla \times E = -\mu \left(\nabla \times \frac{\partial H}{\partial t}\right)$$

$$\Rightarrow \nabla \times \nabla \times E = -\mu \left(\sigma \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2}\right) \quad [\text{From (1)}]$$

Now

$$\nabla \times \nabla \times E = \nabla(\nabla E) - E(\nabla \cdot \nabla)$$

$$= \nabla(\nabla E) - \nabla^2 E \quad \text{--- (3)}$$

$$(2)\quad \nabla(\nabla E) - \nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \mu \frac{\partial^2 E}{\partial t^2} \quad \text{--- (4)}$$

For a conductor there is no charge within it.

Hence we can write it as.

$$\nabla D = 0 \quad [\because \rho = 0]$$

$$\Rightarrow \nabla E = 0$$

From Equn (4) $\Rightarrow \cancel{-}\nabla^2 E = \cancel{-}\mu\sigma \frac{\partial E}{\partial t} \cancel{-} \mu\epsilon \frac{\partial^2 E}{\partial t^2}$

$$\Rightarrow \boxed{\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2}}$$

This is the Wave Equn for Conducting medium in term of E

In terms of H :—

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

$$\nabla \times \nabla \times H = \sigma(\nabla \times E) + \epsilon\left(\nabla \times \frac{\partial E}{\partial t}\right) \quad \text{--- (1)}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\Rightarrow \nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\frac{\partial}{\partial t}(\nabla \times E) = -\mu \frac{\partial^2 H}{\partial t^2}$$

$$\Rightarrow \nabla \times \frac{\partial E}{\partial t} = -\mu \frac{\partial^2 H}{\partial t^2} \quad \text{--- (2)}$$

$$(1) \Rightarrow \nabla \times \nabla \times H = \sigma(\nabla \times E) + \epsilon\left(-\mu \frac{\partial^2 H}{\partial t^2}\right)$$

$$= \sigma\left(-\frac{\partial B}{\partial t}\right) - \mu\epsilon \frac{\partial^2 H}{\partial t^2}$$

$$= -\mu\sigma \frac{\partial H}{\partial t} - \mu\epsilon \frac{\partial^2 H}{\partial t^2} \quad \text{--- (3)}$$

$$\nabla \times \nabla \times H = \nabla(\nabla H) - H(\nabla \nabla)$$

$$= \nabla(\nabla H) - \nabla^2 H$$

$(\nabla . B = 0)$

$$③ \Rightarrow \nabla(\nabla H)^{\nearrow 0} - \nabla^2 H = -\mu a \frac{\partial H}{\partial t} - \mu \epsilon \frac{\partial^2 H}{\partial t^2}$$

$$\nabla \cdot B = 0$$

$$\nabla H = 0$$

$$\Rightarrow \cancel{-} \nabla^2 H = \cancel{-} \mu a \frac{\partial H}{\partial t} \cancel{-} \mu \epsilon \frac{\partial^2 H}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 H = \mu a \frac{\partial H}{\partial t} + \mu \epsilon \frac{\partial^2 H}{\partial t^2}} \quad -⑧$$

This the Wave equn for conducting medium in terms of H

$$E = E_s e^{(\sigma + j\omega)t} \qquad \sigma - \text{conductivity}$$

$$H = H_s e^{(\sigma + j\omega)t}.$$

For type Harmonic after Replacing with equn (E).

$$③ \Rightarrow \boxed{\nabla^2 E = j\omega \mu \sigma E - \omega^2 \mu \epsilon E}$$

Let

$$\gamma^2 = j\omega \mu a - \omega^2 \mu \epsilon.$$

Replacing with wave eqn.

$$\boxed{\nabla^2 \vec{E} = \gamma^2 \vec{E}}$$

These is the wave equn of type of harmonics

Characteristics of uniform plane wave :-

① EM Wave generate form a point source and Spread, out uniformly. which is called Spherical Waveform.

② An observer at a large distance observe the EM waves as a plane wave.

③ E & H are ⊥r to each and to the dirn of propagation. Hence they are called transverse EM wave (TEM)

④ A uniform plane wave is one in which E & H lying in a plane and Hence the same value everywhere in that plane.

⑤ For a uniform plane wave travelling in z-dirn of E & H are zero over the space variation ~~E&H~~ E&H are zero over the space variation of

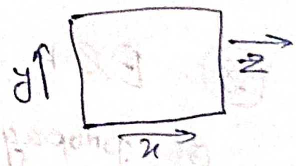
$Z =$ constant plane.

22/05/2023

$$\vec{E} = \vec{E_0}\, e^{-j\vec{k}\cdot\vec{r}}$$

T.E.M (Transverse Electric & Magnetic)

For H



From maxwell equn for non coducting and source free medium we can write.

$$\nabla \times E = -j\omega\mu H$$

$$\Rightarrow H = -\frac{1}{j\omega\mu}(\nabla \times E)$$

$$= -\frac{1}{j\omega\mu}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \quad \text{--- (1)}$$

Again,

$$\vec{E} = \vec{E_0}\, e^{-j\vec{k}\cdot\vec{r}}$$

$$= (E_{0x}\hat{x} + E_{0y}\hat{y} + E_{0z}\hat{z})\, e^{-jkr}$$

$$\left(\frac{\partial}{\partial x}\vec{E}\right) = -jk_x \vec{E}$$

$$\frac{\partial}{\partial x} = -jk_x$$

$$\frac{\partial}{\partial y} = -jk_y$$

$$\frac{\partial}{\partial z} = -jk_z$$

$$① \Rightarrow H = -\frac{1}{j\omega\mu} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -jk_x & -jk_y & -jk_z \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \cancel{+}\frac{1}{\cancel{j}\omega\mu}\cancel{(+j)} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ k_x & k_y & k_z \\ E_x & E_y & E_z \end{vmatrix}$$

$$H = \frac{1}{\omega\mu} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ k_x & k_y & k_z \\ E_x & E_y & E_z \end{vmatrix}$$

$$\vec{H} = \frac{1}{\omega\mu}(k \times E) \longrightarrow ②$$

Magnetic field for TEM

Phase Velocity in any dir^n:- (z direction) :—

$$\vec{E} = \vec{E_0}\, e^{-j\vec{k}\cdot\vec{r}}$$

$$= \vec{E_0}\, e^{-j(\beta\cos\theta_x \hat{x} + \beta\cos\theta_y \hat{y} + \beta\cos\theta_z \hat{z})}$$

$$= \vec{E_0}\, e^{-j\beta(\cos\theta_x \hat{x} + \beta\cos\theta_y)}\, e^{-j\beta\cos\theta_z \hat{z}}$$

$$= \vec{P_0}\, e^{-j\beta\cos\theta_z \hat{z}}$$

$\therefore$ the wave phase constant in z-direction is given by $\beta_z = \beta\cos\theta_z$

$$\therefore V_{pz} = \frac{\omega}{\beta_z} = \frac{\omega}{\beta\cos\theta_z} = \frac{v_0}{\cos\theta_z}$$

Where v_0 is the velocity of wave in dirn perpendicular v_0 constant ~~phase~~ phase front.

$$\boxed{V_{px} = \frac{v_0}{\cos\theta_x}}$$

$0 \leq |\cos\theta_x| \leq 1$

$\infty \leq v_{px} \leq v_0$

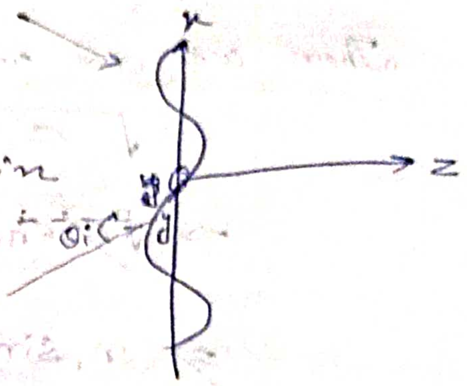
$$\boxed{V_{py} = \frac{v_0}{\cos\theta_y}}$$

24/3/23

Plane wave at Dielectric interface:-

To see amplitude variation in xy plane we can write

$$Re\{\vec{F_i}\} = \vec{F_0}\cos(x\beta_1 \sin\theta_i)$$



The special phase gradient in x-direction is ~~or~~

~~Gradient = $\beta_1 \sin$~~

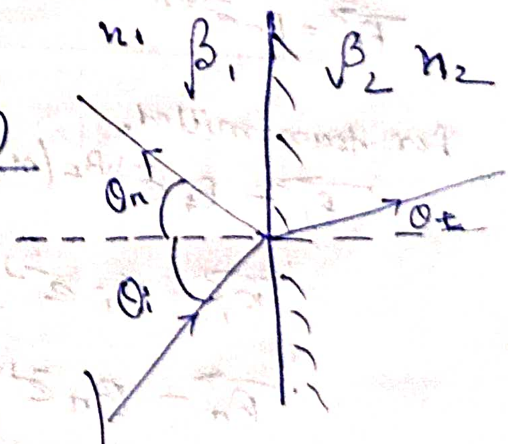
Gradient $= -\beta_1 \sin\theta_i$

Constant $= -\beta_1 x \sin\theta_i$

Hence we can say that the ~~incident~~ incident wave, transmitted wave, reflected on the same plane which shows our first law of reflection.

Now the angle ~~through~~ which the wave makes which is normal is given by,

$$\boxed{\theta = \sin^{-1}\left\{\frac{-\text{Phase Gradient}}{\text{phase constant}}\right\}}$$



Phase Gradient =

$$\theta_r = \sin^{-1}\left(\frac{+(+\cancel{\beta_1}\sin\theta_i)}{\cancel{\beta_1}}\right)$$

$$\boxed{\theta_r = \theta_i}$$

$$\theta_t = \sin^{-1}\left(\frac{-(-\beta_1\sin\theta_i)}{\beta_2}\right)$$

$$= \sin^{-1}\left(\frac{\beta_1\sin\theta_i}{\beta_2}\right)$$

$$\Rightarrow \beta_1\sin\theta_i = \beta_2\sin\theta_t$$

$$\Rightarrow \cancel{\omega}\sqrt{\mu_1\epsilon_1}\sin\theta_i = \cancel{\omega}\sqrt{\mu_2\epsilon_2}\sin\theta_t$$

$$\boxed{\sqrt{\mu_1 \epsilon_1}\, \sin\theta_i = \sqrt{\mu_2 \epsilon_2}\, \sin\theta_t} \rightarrow \text{Snell's law.}$$

For pure dielectric medium we have,

$$\mu_1 = \mu_2 = \mu$$

Then we can write –

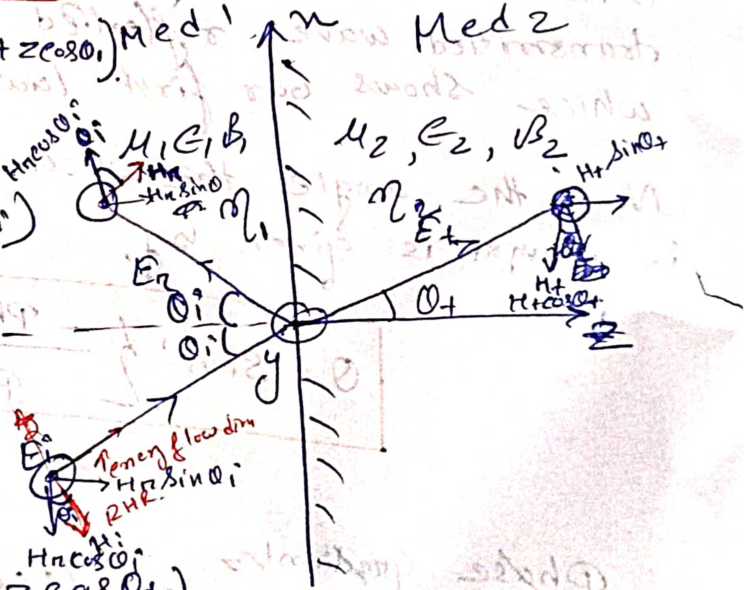
$$\sqrt{\epsilon_1}\, \sin\theta_i = \sqrt{\epsilon_2}\, \sin\theta_t$$

$$\Rightarrow \sqrt{\cancel{\epsilon_0}\epsilon_{r_1}}\, \sin\theta_i = \sqrt{\cancel{\epsilon_0}\epsilon_{r_2}}\, \sin\theta_t$$

$$\Rightarrow n_1 \sin\theta_i = n_2 \sin\theta_t$$

For ⊥r polarization

$$\vec{F_1} = \vec{F_{0i}}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$



For incident wave,

$$\vec{E_i} = E_i\, e^{-j\beta(x\sin\theta_i + z\cos\theta_i)}$$

For reflected wave,

$$\vec{E_r} = E_r\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

For transmitted,

$$\vec{E_t} = E_t\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\vec{E_i} = E_i\, e^{-j\beta_1 x\sin\theta_i} \longrightarrow (1)$$

$$\vec{E_r} = E_r\, e^{-j\beta_1 x\sin\theta_i} \longrightarrow (2)$$

$$\vec{E_t} = E_t\, e^{-j\beta_2 x\sin\theta_t} \longrightarrow (3)$$

We know, the tagential component of + Should be continous across the diaelectric - dielectric boundary. At ~~intfer~~ interface $z =$ zero plane the electric fields are given by ~~the~~ the above equn (1), (2) & (3)

Hence, $\boxed{\vec{E_i} + \vec{E_r} = \vec{E_t}}$

$$\Rightarrow E_i e^{-j\beta_1 x \sin\theta_i} + E_r e^{-j\beta_1 x \sin\theta_i'} = E_t e^{-j\beta_2 x \sin\theta_2} \quad \text{—— (4)}$$

Snell's law,

$$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

$\therefore$ ~~$E_t \sin\theta_i =$~~

(4) $\Rightarrow$ $E_i + E_r = E_t$ —— (5) (Electric-field equn)

Pointing Vectors-

$\vec{E} \times \vec{H}$ }- Energy flow of EM wave

Since there are no surface currents so tangential component of 'H' is also ~~got~~ continous.

(Bcz both tagential componet are opposite dirn)

$$\therefore \boxed{H_i \cos\theta_i - H_r \cos\theta_i' = H_t \cos\theta_t} \quad \text{—— (6)}$$

$$\eta = \frac{E}{H}$$

$$\boxed{\frac{E_i}{\eta_1} \cos\theta_i - \frac{E_r}{\eta_1} \cos\theta_i' = \frac{E_t}{\eta_2} \cos\theta_t} \quad \text{—— (6)}$$

Relflection co-efficent.

dividing the eqn (6) by E_r

$$\boxed{\Gamma = \frac{\cancel{\eta}\cdot\eta_2 \cos\theta_i \;\; \eta_1 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

Transmitted cot-efficent,

$$\boxed{\tau = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos t}}$$

$$(4) \Rightarrow E_i + E_r = E_t$$

$$\Rightarrow 1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$\Rightarrow 1 + \Gamma_1 = \tau_1$$

$$|\Gamma_1| < 1$$

$$|\tau_{\cancel{1}}| \overset{\text{proper}}{\geq} 1 \equiv E_t > E_i$$

For parallel polarization :-

29/3/2022

First - electric field
second - Magnetic field.

For Parallel Polarization :

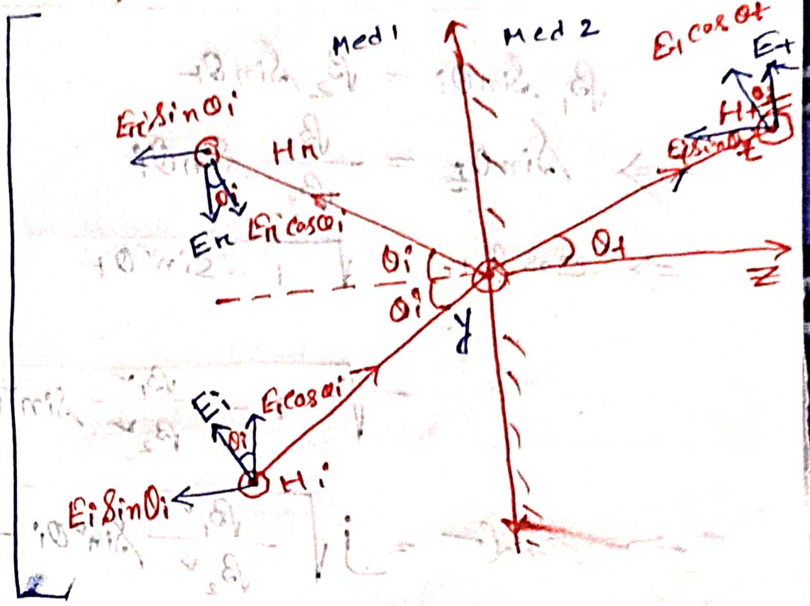
$$H_i + H_r = H_t$$

$$\Rightarrow \frac{E_i}{\eta_1} + \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2} \rightarrow (1)$$

$$\Rightarrow \frac{1}{\eta_1} + \frac{1}{\eta_1} \times \frac{E_r}{E_i} = \frac{1}{\eta_2} \frac{E_t}{E_i}$$

$$\Rightarrow 1 + \Gamma_{11} = \frac{\eta_1}{\eta_2} \tau_{11}$$

$$E_i \cos\theta_i - E_r \cos\theta_i = E_t \cos t \rightarrow (2)$$



$$\boxed{\Gamma_{11} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}} \rightarrow \text{Reflection coff.}$$

$\frac{E_r}{E_i}$ } Reflection co-effn

$\frac{E_t}{E_i}$ } Transmf co-effn.

$$\boxed{\tau_{11} = \frac{2\eta_2}{\eta_1 \cos\theta_i + \eta_2 \cos t}} \rightarrow \text{Transmission Coff.}$$

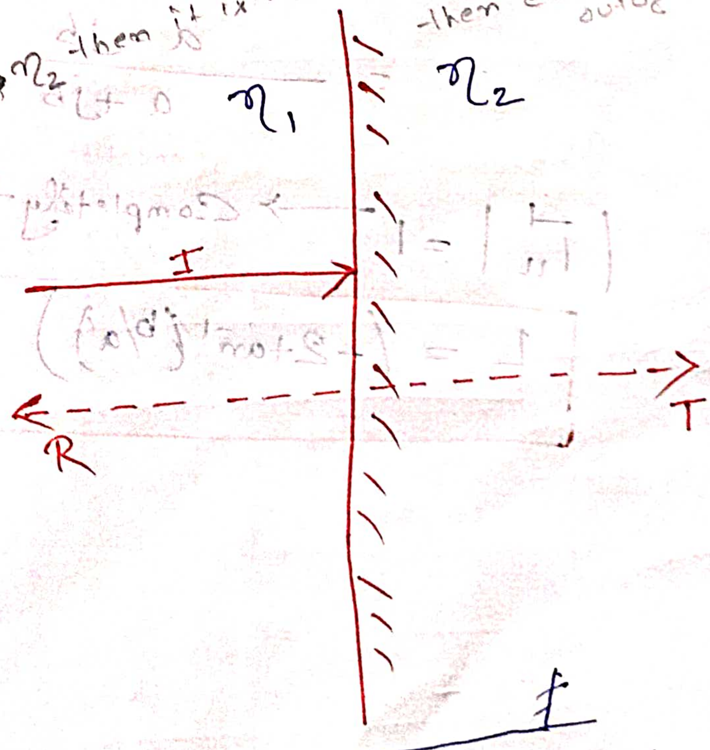
For Normal incidence :

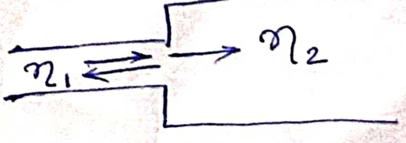
$$\theta_i = \theta_r = \theta_t = 0^\circ$$

$$\Gamma_1 = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

if $\eta_1 > \eta_2$ then it is -ve Refl. coff. but magnitude is same then electric field out of phase.

$$\tau_1 = \frac{2\eta_2}{\eta_2 + \eta_1}$$





Total Internal Reflection :-

If $\frac{\beta_1}{\beta_2} \sin\theta_i$ is > 1 then there is no angle so it is TIR.

$$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

$$\Rightarrow \sin\theta_t = \frac{\beta_1}{\beta_2} \sin\theta_i$$

$$\Rightarrow \cos\theta_t = \sqrt{1 - \sin^2\theta_t}$$

$$= \sqrt{1 - \frac{\beta_1^2}{\beta_2^2} \sin^2\theta_i}$$

$$\Rightarrow \cos\theta_t = j\sqrt{\frac{\beta_1^2}{\beta_2^2} \sin^2\theta_i - 1}$$

For parallel polarization,

$$\Gamma_{\parallel} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$$= \frac{\eta_1 \cos\theta_i - j\eta_2 \sqrt{\frac{\beta_1^2}{\beta_2^2} \sin^2\theta_t - 1}}{\eta_1 \cos\theta_i + j\eta_2 \sqrt{\frac{\beta_1^2}{\beta_2^2} \sin^2\theta_i - 1}}$$

$$= \frac{a - jb}{a + jb}$$

amount of incidence and amount of Reflection.

$|\Gamma_{\parallel}| = 1 \longrightarrow$ Completely transmitted.

$$\boxed{\angle = \left(-2\tan^{-1}\left(b/a\right)\right)}$$

— No transmission.

Wave ungergo ~~fa~~ phase change at TIR and the phase change is different for parallel and perpendicular polarization,

② $$\theta_{11} = -2\tan^{-1} \frac{\eta_2 \sqrt{\frac{\beta_2}{\beta_1} \sin^2\theta_i - 1}}{\eta_1 \cos\theta_i}$$

$$\theta_1 = -2\tan^{-1} \frac{\eta_1 \sqrt{\beta_2^2/\beta_1^2 \sin^2\theta_i - 1}}{\eta_2 \cos\theta_i}$$

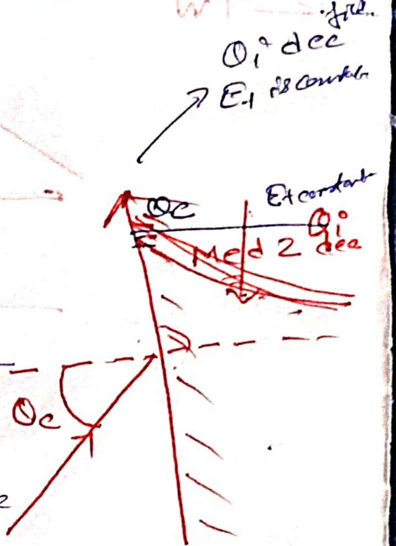
③ Fields ~~for~~, Med-2

$$\vec{E_t} = E_t e^{-j\beta_2 (x \sin\theta_t + z\cos\theta_t)}$$

$$= E_t e^{-j\beta_2 \left(x \sin\theta_t \pm jz \sqrt{\frac{\beta_1}{\beta_2} \sin^2\theta_i - 1}\right)}$$

$$= E_t e^{-j\beta_2 x \sin\theta_t} \pm z \sqrt{\frac{\beta_1^2}{\beta_2^2} \sin^2\theta_i - 1}$$

$$= E_t \underbrace{e^{\pm z \sqrt{\frac{\beta_1}{\beta_2^2} \cancel{\sin^2\theta} - 1}}}_{\text{Mag. exp. decaying in the direction } z \text{ so } -z} \underbrace{e^{-j\beta_2 x \sin\theta_t}}_{\text{Phase}} \rightarrow \text{phase}$$



At

$$\theta_i = \theta_c \quad (\text{constant field})$$

$$= E_t / (\text{phase term})$$

④ Field for Med-1

$$\vec{E_i} = E_i e^{-j\beta_1 (x \sin\theta_i - z\cos\theta_i)}$$

$$e^{-j\beta_1 (x\sin\theta_i - z\cos\theta_i)} \Rightarrow E_i e^{j\theta} e^{-j\beta_1 (x\sin\theta_i - z\cos\theta_i)}$$

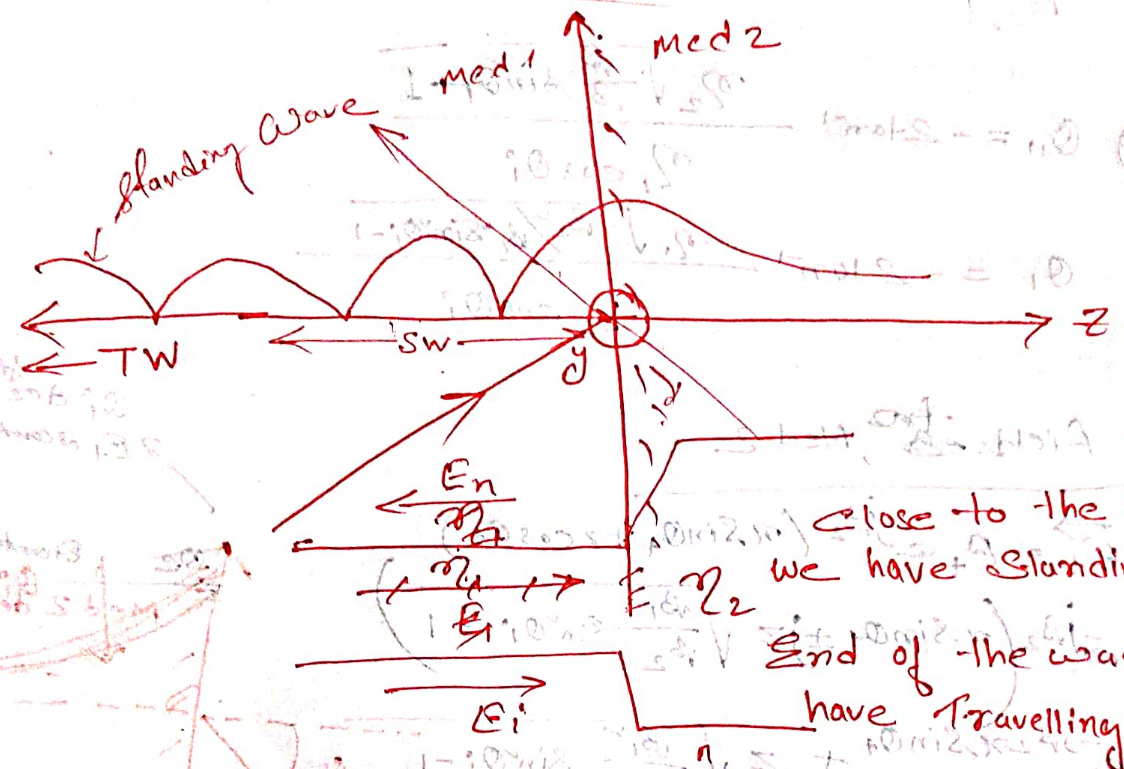
$$= \vec{E_r} - E_r e$$

$$\vec{E}_{med1} = \vec{E_i} + \vec{E_r}$$

$$= \underbrace{E_i e^{-j\beta_1 x \sin\theta_i}}_{\text{Travelling wave in } x \text{ dirn}} \underbrace{\left(e^{-j\beta_1 z\cos\theta_i} + e^{j\theta} e^{-j\beta_1 z\cos\theta_i}\right)}_{\text{Standing wave in } z \text{ dirn}}$$

Total field in med 1 & med 2

Condition for TIR



Close to the boundary we have Standing wave

End of the wave we have Travelling Wave

Decaying field is called Evanescent field.