29-03-22

Module 1:

• Signals play a major role in our life. In general a signal can be function of time, distance, position temperature, pressure etc and it represents some variable of interest associated with a system.

• For example, in an electrical system the associated signals are electric current and voltage. In a mechanical system, the associated signals may be force torque etc, In addition to these some other examples of signals are speech, voice, music, picture and video signals. In general a signal carries information and objective of signal processing is to extract these information.

• Digital signal processing is processing of signals by digital means signals are processed for extracting useful information.

Signal processing may be defined as an operation for extracting, enhancing, storing and transmitting information. e.g- enhancing frequency component in a filter separation of noise and interference component from the desired component, separating two signals amplifiers for enhancing voltage or power etc.

• A system may be defined as an integrated unit composed of diverse interacting structures to perform a desired task. The task may vary such as filtering of noise in a communication receiver, detection of range. target in a radar system or monitoring the steam pressure in a boiler.

• The function of a system is to process a given input sequence to generate and output sequence.

★ Growth of Digital signal Processing (DSP):-

Due to advent of high speed digital computer, VLSI technology. Signals are processed by digital hardware containing adders, multipliers, delay units. For digit signal processing we need to convert an analog signal to digital signal.

Advantages of DSP:-

(i) Ease of data Storage :- Digital signals can be readily stored without ~~loss~~ loss of fidelity or any loss of info for processing offline in a remote analytical laboratory. e.g:- Images.

(ii) Time sharing: Digital processing allow the sharing of a given processor among a no. of signals by time sharing thus reducing the cost of processing per signals.

(iii) Greater flexibility :- DSP system can be programmed and reprogrammed to perform a variety of functions without modifying the hardware.

(iv) Perfect reproducibility :- Identical performance from unit to unit is obtained since there is no variation due to component tolarences.

(v) No drift in performance :- Unlike analog circuits the operation of digital circuits does not depend upon precise value of the digital signal. As a result a digital system is less sensitive to tolerances to component values and it fairly independently of temperature, ageing and most other external parameters. In analog systems components values drift with time and are prone to thermal effects.

(vi) Cost, size and reliability :- The software control algorithm can be complex but it can be implemented accurately with less effort. The DSP system are small in size, more reliable and less expensive compared to analog systems.

(vii) Accuracy :- Accuracy of DSP systems is decided by resolution of analog-digital converter, number of bits to represent digital data, floating/fixed point arithmatic etc. Accuracy of DSP system is much higher than analog systems. The analog systems suffer from component tolerance breakdown etc. Hence it is dificult to attain high accuracy in analog system.

★ Disadvantages of DSP :-

(i) When the analog signals have wide bandwidth then high speed analog to digital comparators are required. Such high speeds of AD conversion are difficult to achieve for same signals. For such applications analog system must be used.

(ii) The DSP systems are expensive for small applicati-ons. Hence the selection is done on the basis of cost complexity and performance.

★ Applications of DSP :-

(i) Speech processing :- speech enhanchment, speech compression, speech recognition, text to speech conversion.

(ii) Image processing :- Image enhancement, image compression, pattern recognition etc.

(iii) Telecommunication :- Video conferencing, channel multiplexing, cellular phone, fax etc.

(iv) Instrumentation Engineering :- Digital filtering, Noise reduction, spectrum analysis, Siesmic signal processing.

(v) Control application :- servo control, engine control, Laser-printer control, Robot control etc.

(vi) Biomedical Engineering :- ECG analysis, ultrasound equipment, EEG analysis, CT scan equipment, patient monitoring system etc.

(vii) Military / Defence applications : Radar signals processing, SONAR signal processing, Missle guidance etc.

(viii) Consumers applications :- electronic toys, music system, television.

(ix) Industrial applications : Robotics, Power time monitor, security access.

(x) Automotive :- Noise cancellation, vibration analysis, Digital Radio.

(xi) Commercial/entertainment applications: CD/ VCD/ DVD player, mp3, music, speed recognition, phone communication, sound and video compression.

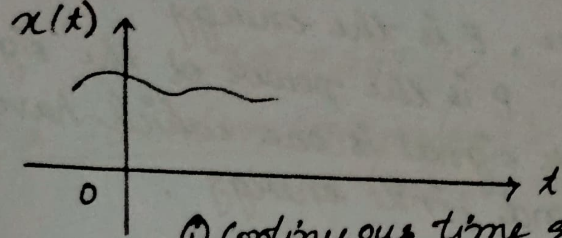
1-04-2022

Discreate time signals:-

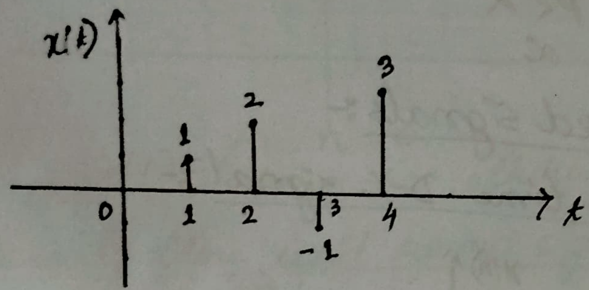
The signals may be broadly classified as -

(i) continuous time signal

(ii) Discreate time signal



(i) continuous time signal



(ii) Discreate time signal.

→ Both continuous and discreate time signal further classified as -

(i) Deterministic and non-deterministic signal.

(ii) Periodic and non periodic

Periodic → $x(n) = x(n+N)$

Non periodic → $x(n) \neq x(n+N)$

(iii) Symmetric (even) or asymmetric (odd) signals

$x(n) = x(-n)$ → even

$x(n) = -x(-n)$ → odd

(14) Energy and Power signals –

* The energy signal is one which has finite energy and zero average power.

$$0 < E < \alpha$$
$$P = 0$$

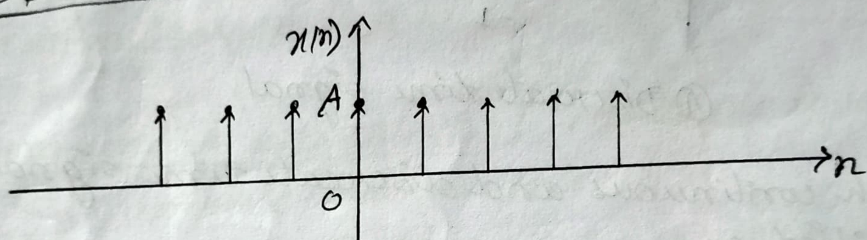
where, E is the energy
P is the power of the signal

* The power signal is one which have finite average power and finite energy.

$$0 < P < \alpha$$
$$E = \alpha$$

* Some standard signals :-

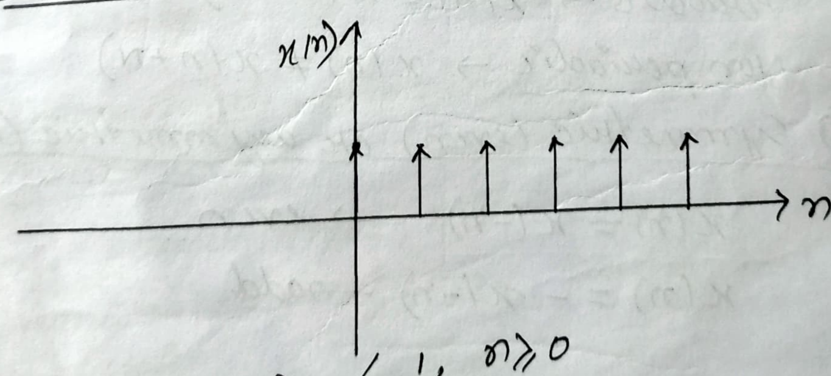
① Discreate time D-C signal :-



Mathematically,

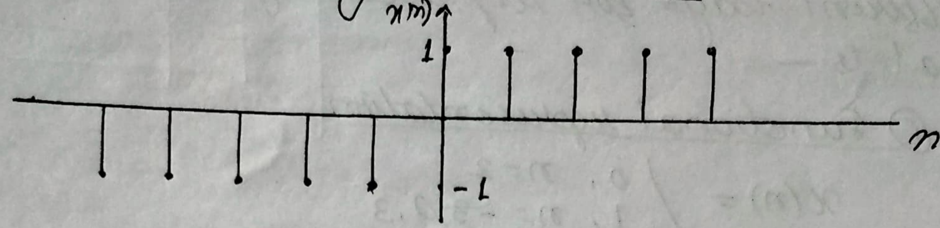
$$x(n) = A, \quad -\alpha < n < \alpha$$

② Distreate time unit step signal :-



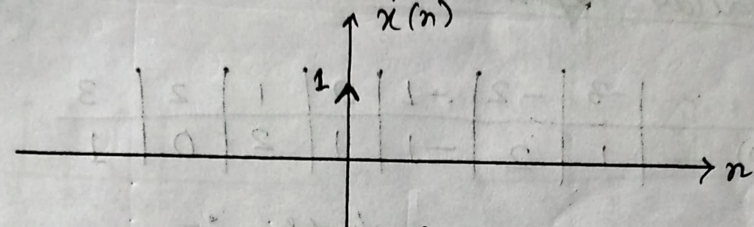
$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

③ Discreate time signum function :-



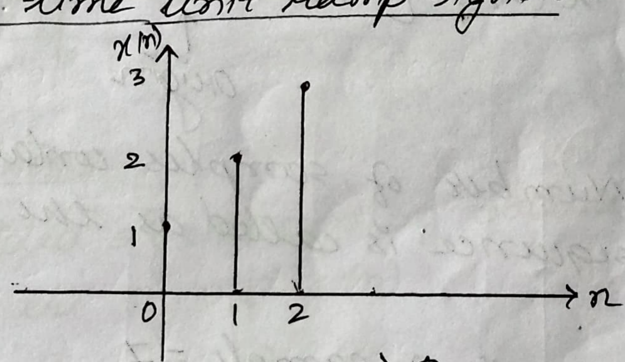
$$x(n) = sgn(n)$$

④ Delta function (Impulse) :-



$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \text{ . otherwise} \end{cases}$$

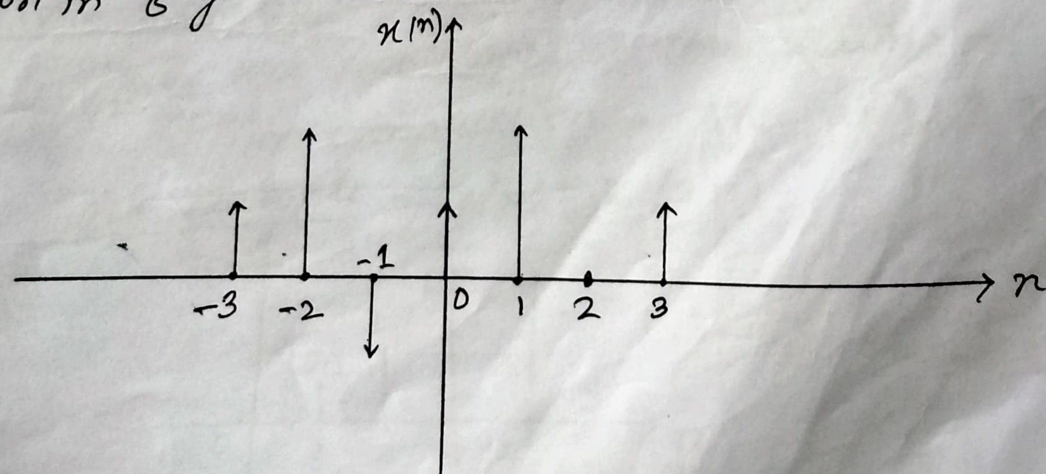
⑤ Discreat time unit ramp signal :-



$$r(n) = \begin{cases} n ; & n \geq 0 \\ 0 , & \text{otherwise} \end{cases}$$

Representation of Discreate time sequences :-

The discreate time sequence is denoted by $x(n)$. Let us consider a discreate time signal as shown in figure .



The different ways for representing discreat time signal is —

① Functional representation

$$x(n) = \begin{cases} 0, & n=2 \\ 1, & n=-3,0,3 \\ 2, & n=-2,1 \\ -1, & n=-1 \end{cases}$$

(ii) Tabular Representation —

n	...	-3	-2	-1	0	1	2	3
$x(n)$	--	1	2	-1	1	2	0	1

(iii) Sequence representation:-

$$x(n) = \{1, 2, -1, \underset{\uparrow}{1}, 2, 0, 1\}$$

origin

Number of samples contained in the given sequence is called as the length of the sample.

Length of sample = 7

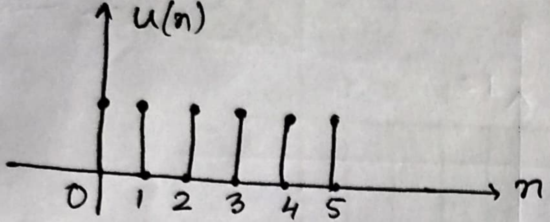
To adjust the length of sequence we can add any no. of zeros. This is called as zero padding.

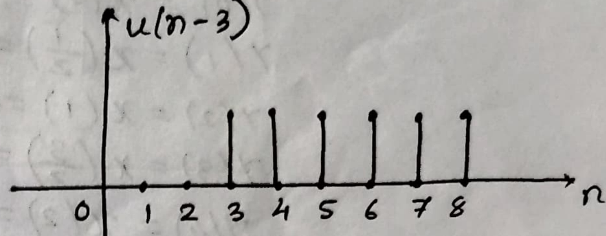
Operations on discreate time signal :-

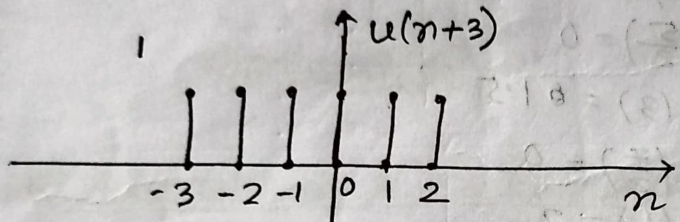
① Delay / advance
② Time folding
③ Time scaling

1. Delay / Advance

(i) Delay means shifting right
(ii) Advance means shifting left





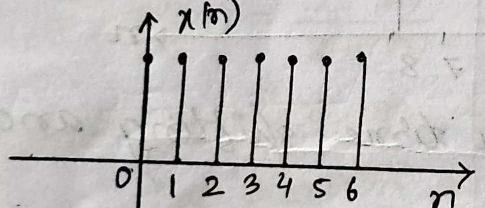


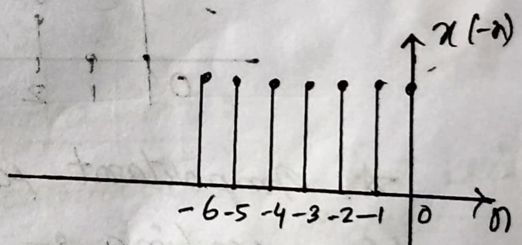
$u[n-(-3)]$

2. Time folding :

The folded signal can be obtained by replacing n by -n of the signal x(n).

$$y(n) = x(-n)$$



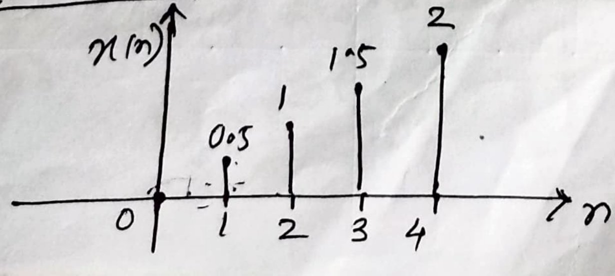


3. Time Scaling :-

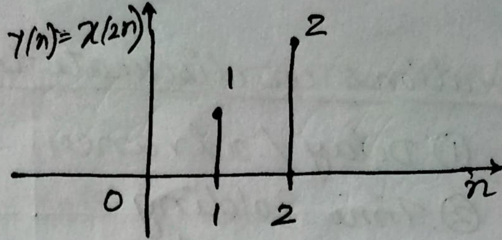
There are two types of time scaling -

① Time compression → Time axis is compress

for example - $y(n) = x(2n)$



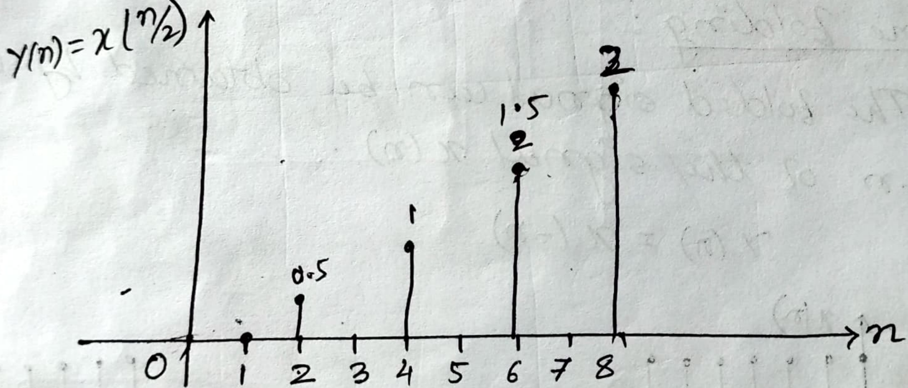
$y(0) = x(0) = 0$
$y(1) = x(2) = 1$
$y(2) = x(4) = 2$



(ii) Time expansion – The time axis is expanded.

eg: $y(n) = x(n/2)$

$y(0) = x(0) = 0$
$y(1) = x(\frac{1}{2}) = 0$
$y(2) = x(1) = 0.5$
$y(3) = x(\frac{3}{2}) = 0$
$y(4) = x(2) = 1$
$y(5) = x(\frac{5}{2}) = 0$
$y(6) = x(3) = 1.5$
$y(7) = x(\frac{7}{2}) = 0$
$y(8) = x(4) = 2$



* Important rules for time shifting and time scaling :-

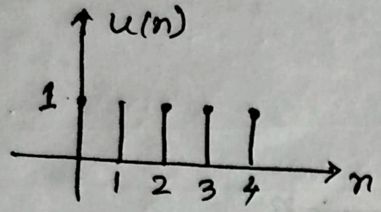
<u>example</u>
$y(n) = x(-2n+3)$

(i) ~~Alwy~~ Always do the shifting operation first and do the time scaling.

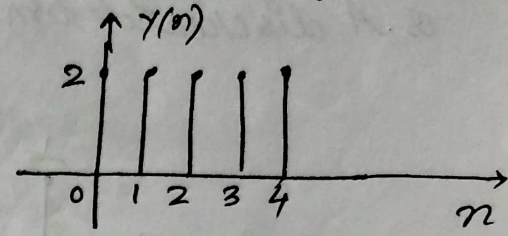
* Transformation on amplitude of the signal:

① Amplitude Scaling:-

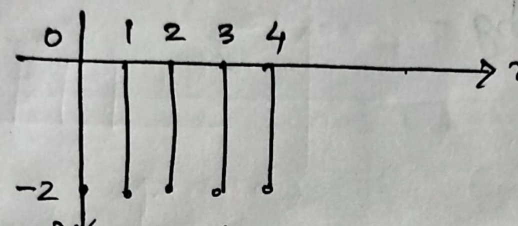
Let us consider the unit step signal $u(n)$



Let $y(n) = 2u(n)$



* consider $y(n) = -2u(n)$



$y(n) = -2u(n)$

② Addition & substraction

Let $x_1(n)$ and $x_2(n)$ be two discreate time signals. The addition is given as $y(n) = x_1(n) + x_2(n)$ and the substraction is given as $y(n) = x_1(n) - x_2(n)$

③ Multiplication & Division:-

$y(n) = x_1(n) \cdot x_2(n) \rightarrow$ multiplication

$y(n) = \frac{x_1(n)}{x_2(n)} \longrightarrow$ division.

④ Differentiation and integration:-

Integration and differentiation do not exist directly for discreate time signal. But difference and accumulation operations exist

and accumulation operations exist

Eg: The difference operation is given as

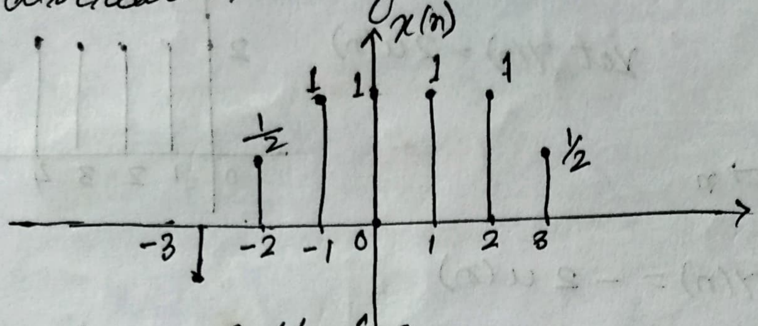
$$y(n) = x(n) - x(n-1)$$

Similarly the accumulation operation is given as

$$y(n) = \sum_{k=-\infty}^{\infty} x(n)$$

These operations do not represents differentiation & integration but they are used simillarly.

Q. A discreate time signal is shown in figure —



sketch the following —

(i) $x(n-3)$

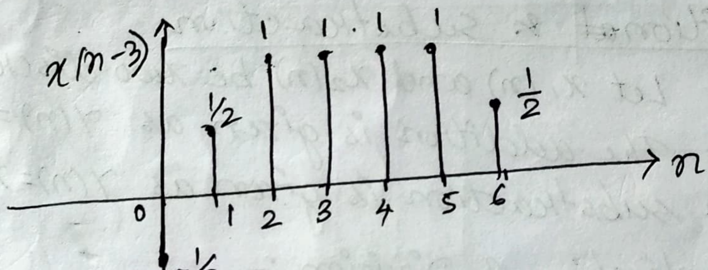
(ii) $x(3-n)$

(iii) $x(2n)$

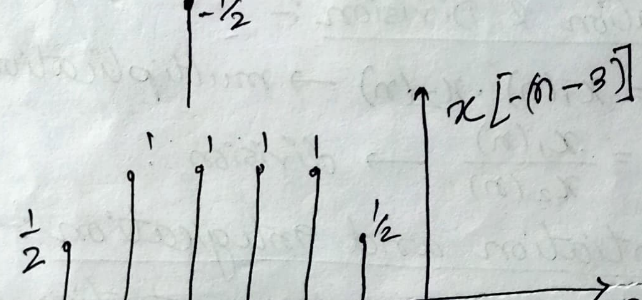
(iv) $x(n) \cdot u(3-n)$

(v) $x[(n-1)^2]$

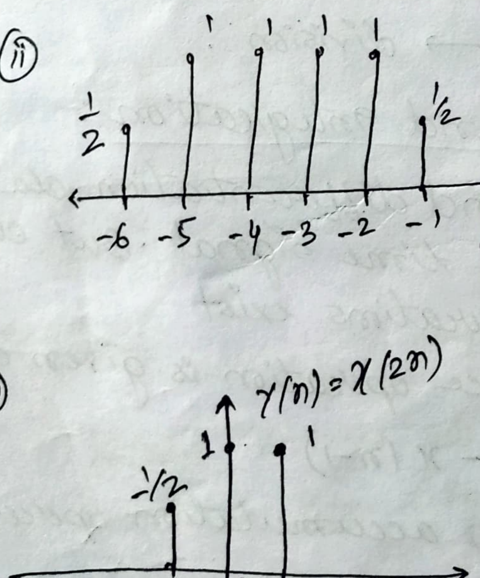
Ans:- (i)



(ii)



(iii)



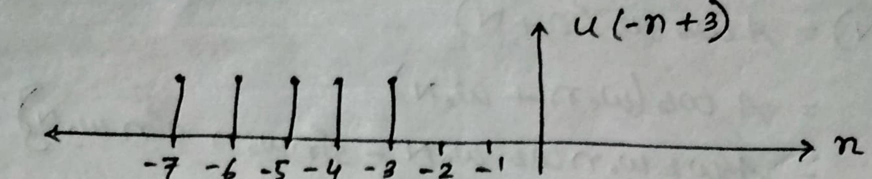
$y(n) = x(2n)$

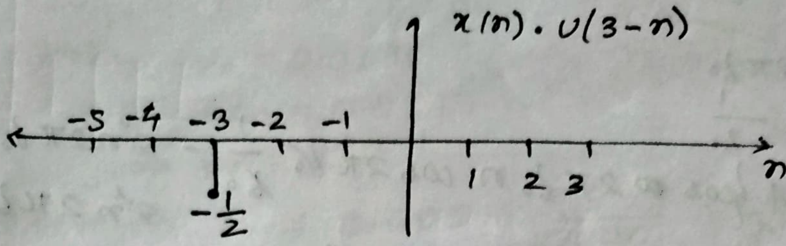
$\Rightarrow y(0) = x(0) = 1$

$\Rightarrow y(1) = x(2n) = 1$

$\Rightarrow y(-1) = x(-2) = \frac{1}{2}$

(iv)





(v) $x[(n-1)^2]$

* Important condition for periodicity of a discrete time signal :-

A discrete time signal is periodic if its frequency f_0 is rational -

$$f_0 = \frac{k}{N}$$

where k and N both are integers.

Standard expression —

$$x(n) = x(n+N)$$

$$x(n) = A \cos \omega_0 n$$

$$= A \cos 2\pi f_0 n$$

$$x(n+N) = A \cos \omega_0 (n+N)$$

$$x(n+N) = A\cos \omega_0 (n+N)$$
$$= A\cos(\omega_0 n + \omega_0 N)$$
$$= A\{\cos \omega_0 n \cos \omega_0 N - \sin \omega_0 n \sin \omega_0 N\}$$

Here, $\omega = 2\pi f_0$

$$N = \frac{1}{f_0}$$
$$= A\{\cos 2\pi f_0 n \cos 2\pi f_0 \frac{1}{f_0} - \sin 2\pi f_0 n \sin 2\pi f_0 \frac{1}{f_0}\}$$
$$= A(\cos 2\pi f_0 n - 0)$$
$$= A\cos \omega_0 n$$
$$= x(n)$$

Hence prove.

Q. Prove that A sin wave is a periodic signal

<u>Ans</u>: $x(n) = A\sin \omega_0 n$

$$x(n+N) = A\sin 2\pi f_0 (n+N)$$
$$= A\sin(2\pi f_0 n + 2\pi f_0 N)$$
$$= A(\sin 2\pi f_0 n \cos 2\pi f_0 N + \cos 2\pi f_0 n \sin 2\pi f_0 N)$$
$$= A(\sin 2\pi f_0 n \cos 2\pi + \cos 2\pi f_0 n \cdot \sin 2\pi)$$
$$= A\sin \omega_0 n + 0$$
$$= x(n)$$

Hence sin wave is periodic signal

Q. Determine wheather the following discrete time signals are periodic or not. If periodic determine fundamental period.

(a) $\cos(0.01\pi n)$

Soln: $\cos(0.01\pi n)$

Here, $\omega_0 = 0.01\pi$

$\pi 2 f_0 = 0.01\pi$

$\Rightarrow f_0 = \frac{0.01}{2}$

$\Rightarrow f_0 = 0.005 = \frac{5}{1000} = \frac{K}{N} = \frac{1}{200}$

Hence periodic.

$N = 200$

(b) $\cos(3\pi n)$

Soln: Here $\omega_0 = 3\pi$

$2\pi f_0 = 3\pi$

$\Rightarrow f_0 = \frac{3}{2} = \frac{K}{N}$

Hence periodic.

(c) $\sin 3n$

Soln: Here, $\omega_0 = 3$

$\Rightarrow 2\pi f_0 = 3$

$\Rightarrow f_0 = \frac{3}{2\pi}$

Hence Aperiodic.

(d) $\cos 2\pi n/5 + \cos 2\pi n/7$

Soln: Here, $\omega_{01} = 2\pi/5$

$\Rightarrow 2\pi f_0 = \frac{2\pi}{5}$

$\Rightarrow f_{0_1} = \frac{1}{5} = \frac{K}{N_1}$

$N_1 = 5$

$\omega_{02} = \frac{2\pi}{7}$

$\Rightarrow f_{02} = \frac{1}{7}$

$N_2 = 7$

$\frac{N_1}{N_2} = \frac{5}{7}$

$N = 35 = LCM(7,5)$

Hence periodic.

(e) $\sin(\pi + 0.2n)$

Soln: ~~$2\pi f_0 = 0.2$~~

$\Rightarrow \sin(\pi+0.2n) = \sin(0.2n)$

$2\pi f_0 = 0.2$

$\Rightarrow f_0 = \frac{0.1}{\pi}$

Hence non period.

(f) $e^{(j\pi/4)n}$

Soln: $\cos\frac{\pi n}{4} + j\sin\frac{\pi n}{4}$ $\quad [e^{j\theta} = \cos\theta + j\sin\theta]$

$N = \frac{2\pi}{\pi/4} = 8$ $\quad 2\pi f_0 = \frac{\pi}{4}$

$f_0 = \frac{1}{8}$ periodic $\quad f_0 = \frac{1}{4\times 2}$

$N = 8$

———o—o———

* Important points to check wheather a signal is a energy signal or power signal

(1) The energy of a discrete time signal is given as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

and power is given as $P = \lim_{N\to\infty} \frac{1}{N} \sum_{n=-\frac{N}{2}}^{N/2} |x(n)|^2$

(2) First we observe the signal carefully if it is periodic and of infinite duration then it can be power signal. Hence we calculate power directly.

(3) If the signal is periodic and of finite duration then it can be energy signal. Hence we calculate its energy directly.

(4) If the signal is non periodic then it can be energy signal. Hence we calculate energy directly.

(5) $x(n)$ is an energy signal if $0 < E < \infty$ and $P = 0$

$x(n)$ is a power signal if $0 < P < \infty$ and $E = \infty$

⑥ A signal that does not satisfy any of the above two conditions is neither an energy signal nor a power signal.

Q. ① $x(n) = \left(\frac{1}{2}\right)^n u(n)$ } $n \geq 0$ $(4/3)$

ⓘⓘ $x(n) = u(n) \rightarrow$ unit step (periodic) $\rightarrow$ power.

Soln: ① $x(n) = \left(\frac{1}{2}\right)^n u(n)$

$x(n) = \left(\frac{1}{2}\right)^n \quad u(n) = 1$

exponential signal is non periodic, so we can calculate Energy signal

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} \left|\left(\frac{1}{2}\right)^n\right|^2$$

$$1 + A + A^2 + A^3 + \dots = \frac{1}{1-A} \quad \text{for } |A| < 1$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$\sum_{n=0}^{\infty} = \frac{1}{1-A} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3} \text{ // Joule}$$

$$\text{Power, } P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$= Lt \frac{1}{2N+1} \left[\frac{1-\left(\frac{1}{4}\right)^{N+1}}{1-1/4}\right]$$

$$= 0$$

(ii) $x(n) = u(n)$

Soln: The signal is unit step. Unit step signals are periodic. Hence it may be power signal,

$$P = \lim_{N\to\alpha} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{n\to\alpha} \frac{1}{2N+1} \sum_{n=0}^{N} 1^2$$

Here, $\sum_{n=0}^{N} 1^2 = 1+1+1+1 \ldots +(N+1)$

$$P = \lim_{N\to\alpha} \frac{1}{2N+1} (N+1)$$

$$= \lim_{N\to\alpha} \frac{N+1}{2N+1}$$

$$= \lim_{n\to\alpha} \frac{1+\frac{1}{N}}{2+\frac{1}{N}} = \frac{1}{2}$$ //

** Important condition for periodicity of a discrete time signal:

A discrete time sinusoidal signal is periodic only if its frequency, f_0 is rational.

$$f_0 = K/N$$

K and N both are integer.

Proof:- $x(n+N) = x(n)$

$x(n) = A\cos(2\pi f_0 n + \theta)$

A = Amplitude, θ = phase shift

$x(n+N) = A\cos[2\pi f_0 (n+N) + \theta]$

$\therefore x(n) = x(n+N)$

$\Rightarrow A\cos(2\pi f_0 n + \theta) = A\cos[2\pi f_0 (n+N) + \theta]$

$\Rightarrow A\cos(2\pi f_0 n + \theta) = A\cos[2\pi f_0 n + 2\pi f_0 N + \theta]$

$= 2\pi f_0 N = 2\pi k$

$\Rightarrow f_0 = k/N$

Odd and Even part of a signal :-

- A signal is said to be an even signal if $x(n) = x(-n)$
- A signal is said to be an odd or antisymmetric signal if $x(n) = -x(n)$
- Any continuous or discrete time signal can be expressed as the summation of even part or odd part as

$$x(n) = x_e(n) + x_o(n)$$

where $x_e(n) \rightarrow$ even component of signal $x(n)$
and $x_o(n) \rightarrow$ is the odd component of the signal $x(n)$

- The even component can be expressed as

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

- The odd component can be expressed as

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

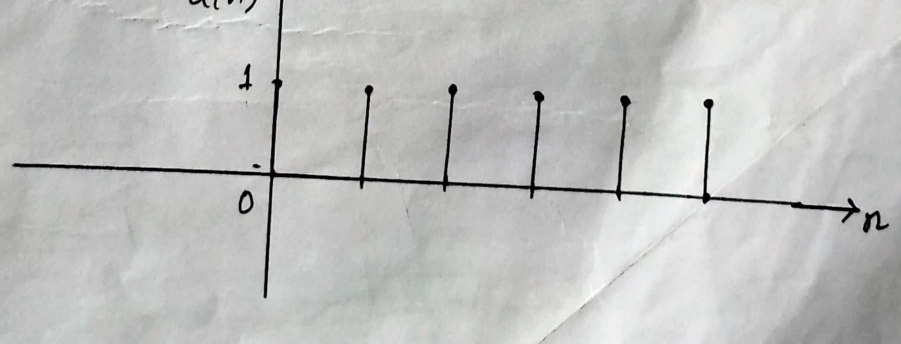
Q. Find the even and odd part of the signal.

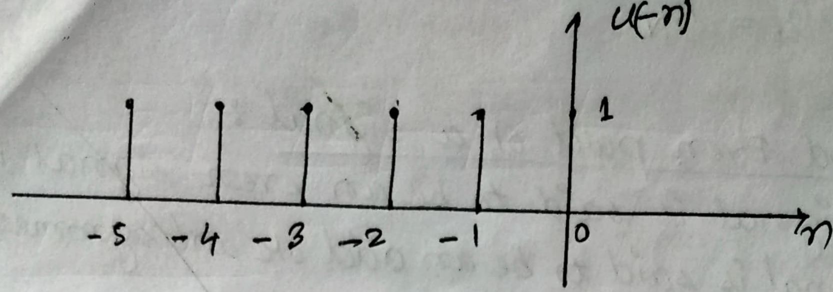
$$x(n) = u(n)$$

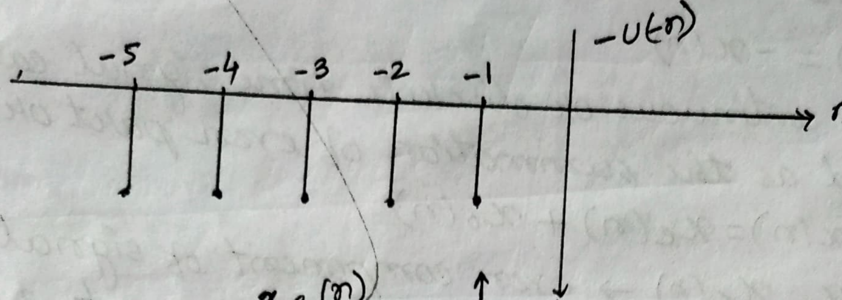
also sketch the signal.

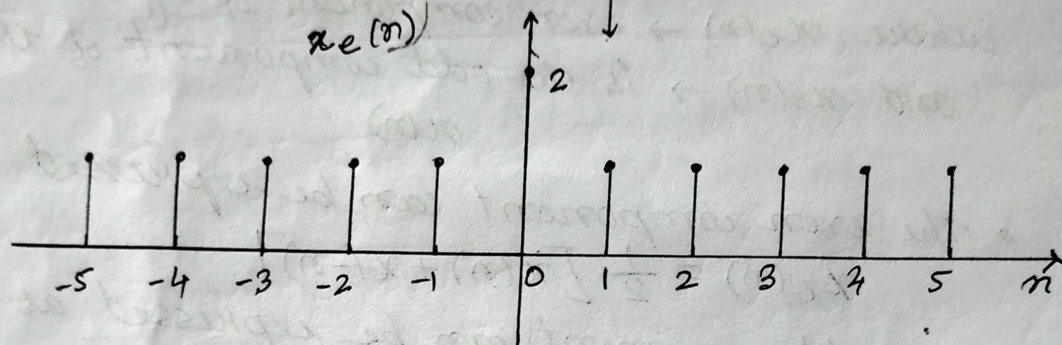
Ans: $x_e(n) = \frac{1}{2}[u(n) + u(-n)]$

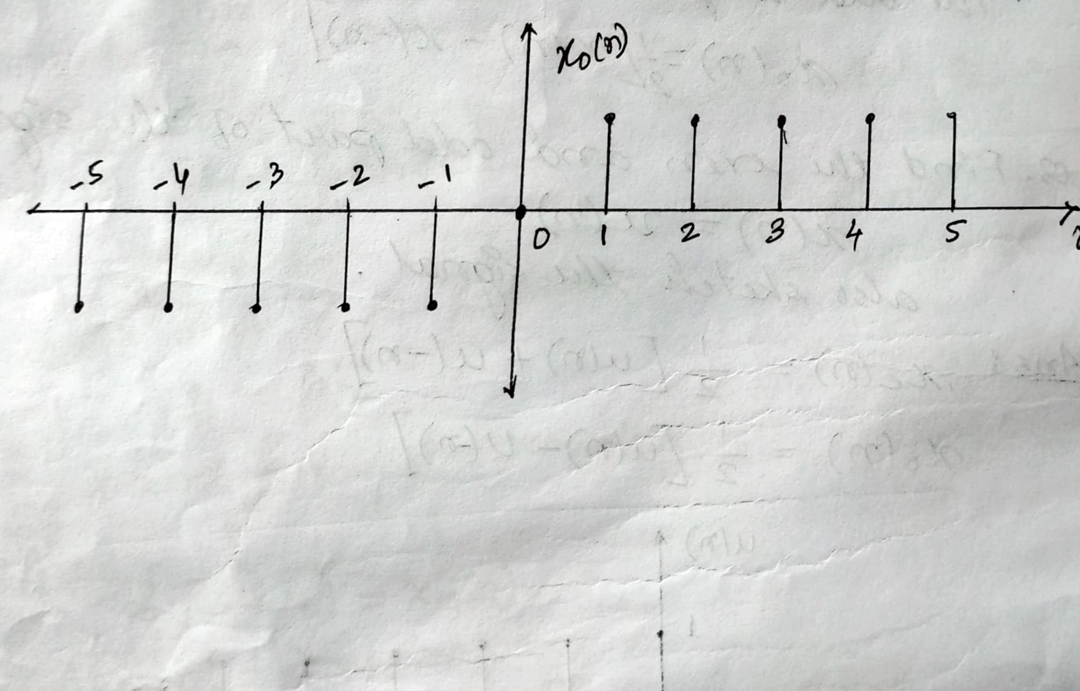
$x_o(n) = \frac{1}{2}[u(n) - u(-n)]$




u(-n)
1
-5 -4 -3 -2 -1 0
n

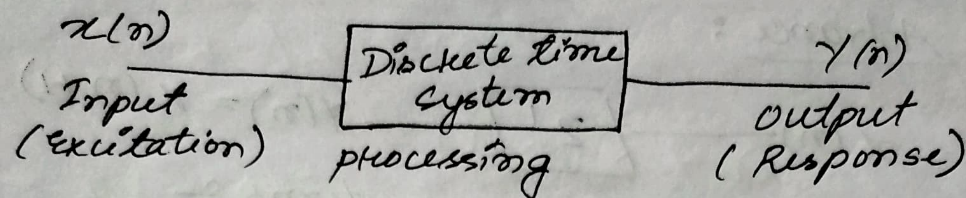

-u(-n)
-5 -4 -3 -2 -1
n


xe(n)
2
-5 -4 -3 -2 -1 0 1 2 3 4 5
n


xo(n)
-5 -4 -3 -2 -1
0 1 2 3 4 5
n

Discrete time ~~signals~~ & system :-

A discrete time system is a physical device which perform required operation on a discrete time signal. It is represented as



When an input signal is passed through the system then it is represent by following notation

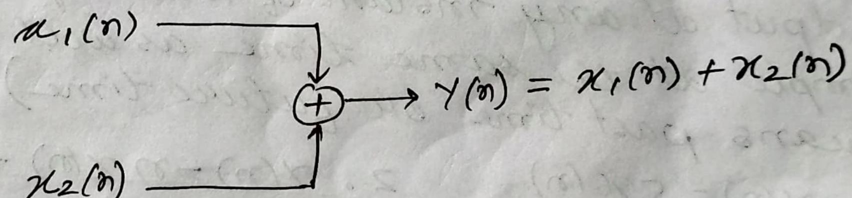
$$x(n) \xrightarrow{T} y(n)$$

or $y(n) = T[x(n)]$

Here "T" is called as transformation operation.

Symbols used for Discrete time system :-

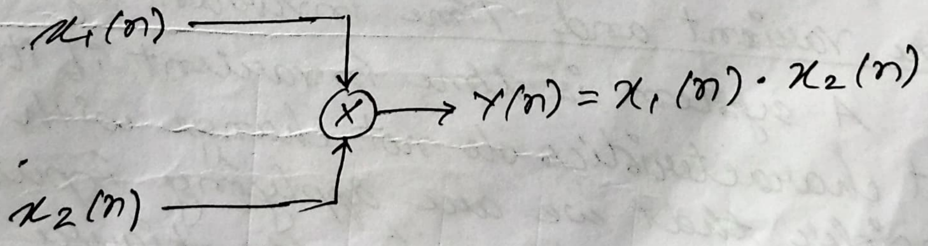
1. Adder :



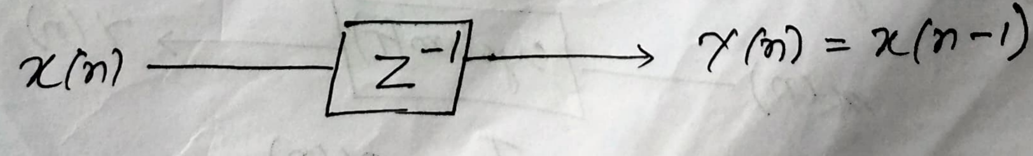
2. Constant multiplier :-

$$x(n) \xrightarrow{a} y(n) = ax(n)$$

3. Signal multiplier :



4. Unit Delay :-



$$y(n) = x(n-3)$$

$$x(n) \longrightarrow \boxed{z^{-1}} \longrightarrow \boxed{z^{-1}} \longrightarrow \boxed{z^{-1}} \longrightarrow y(n) = x(n-3)$$

5. Unit Advance :

$$x(n) \longrightarrow \boxed{z^{1}} \longrightarrow y(n) = x(n+1)$$

* Classification of Discrete time systems :

① Dynamic or static system

Static system is a system in which output at any instant of time depends on input sample at athe same time.

Dynamic system is a system in which output at any instant of time depends on input sample at the same time as well as other times (means past time or future time)

1. $y(n) = 5x(n)$
→ static system

2. $y(n) = n\,x(n) + b x^2(n)$
→ static

3. $y(n) = x(n) + 5x(n-1)$
→ Dynamic

4. $y(n) = x(n^2)$
→ Dynamic

② Time variant and Time invariant system :-

A system is time invarient if its input output characteristics do not change with time. Let us consider that we are applying input signal $x(n)$ to the system as shown in the figure.

$$x(n) \longrightarrow \boxed{\text{system}(T)} \longrightarrow y(n)$$

$$x(n) \xrightarrow{T} y(n)$$

Now delay input by k samples this means that input becomes $x(n-k)$. Now we apply this delayed input to the same system as shown below —

$$x(n-k) \longrightarrow \boxed{\begin{matrix}\text{system}\\ (T)\end{matrix}} \longrightarrow y(n-k)$$

Let us say the system gives output $y(n-k)$ then the system is called as time invarient system. Thus a system is time invarient if and only if $x(n) \xrightarrow{T} y(n)$ implies that $x(n-k) \xrightarrow{T} y(n-k)$

* Steps to determine wheather a system is Time invarient or time varient —

① Delay input $x(n)$ by k units i.e $x(n-k)$. Denote the corresponding output by $y(n,k)$.

② In the given expression of system $y(n)$, replace n by $(n-k)$ through out. This output is $y(n-k)$

③ If $y(n,k)$ is equal to $y(n-k)$ then system is time invarient and if $y(n,k)$ is not equal to $y(n-k)$ then the system is time varient

Q. 1. $y(n) = x(n) - x(n-1)$

$\Rightarrow$ $x(n-k)$ in ~~x(n)~~

$$y(n,k) = x(n-k) - x(n-k-1)$$

$$y(n-k) = x(n-k) - x(n-k-1)$$

$$\because y(n,k) = y(n-k)$$

Time invarient.

2. $y(n) = y(n-4) + x(n-4)$

Sol^n:

$$y(n,k) = y(n-4) + x(n-k-4)$$

$$y(n-k) = y(n-k-4) + x(n-k-4)$$

$$y(n,k) \neq y(n-k)$$

Hence system is time variant.

★ Linear or Non Linear system:-

A system is said to be linear if the combined response of $a_1x_1(n)$ and $a_2x_2(n)$ is equal to the addition of individual responses.

i.e $T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$ — (i)

This is called as superposition theorem.

★ Steps to check wheather the system is linear or non linear:

1. First we apply the zero input and check the output. If output is zero then the system is linear. If this step is satisfied then followed the remaining steps.

2. Then we apply individual input to the system and note down corresponding outputs. Then we add all outputs. We denote this addition by $y'(n)$. This is RHS of equation (i).

3. Next we combine all inputs, apply it to the system and find out output $y''(n)$. This is LHS of eq^n (i)

4. If $y'(n) = y''(n)$ then the system is linear otherwise the system is non linear.

Q 1. $y(n) = n\,x(n)$

Sol^n: • When input $x(n)$ is zero, then output $y(n) = n \cdot 0 \cdot 0$

Thus the system is linear.

• Let there are two inputs $x_1(n)$ and $x_2(n)$ given eq^n of output is

$$y(n) = n\,x(n)$$

$x(n) \longrightarrow n\,x(n)$, Hence, the function system is to multiply input $x(n)$ by 'n'. Now for two inputs $x_1(n)$ and $x_2(n)$, the corresponding outputs $y_1(n)$ and $y_2(n)$.

$$x_1(n) \xrightarrow{T} y_1(n) = n\,x_1(n)$$

$$x_2(n) \xrightarrow{T} y_2(n) = n\,x_2(n)$$

add this output to get $y'(n)$

$$y'(n) = y_1(n) + y_2(n) = n\,x_1(n) + n\,x_2(n)$$

$$y'(n) = n\,[x_1(n) + x_2(n)] \longrightarrow \text{(i)}$$

• Now we add input $x_1(n)$ and $x_2(n)$ and then pass it through the system

Thus, $x_1(n) + x_2(n) \xrightarrow{T} y''(n)$

$$= T\,[x_1(n) + x_2(n)]$$

$$= n\,[x_1(n) + x_2(n)]$$

The function of system is to multiply input by 'n'
Here $[x_1(n) + x_2(n)]$ acts as one input to the system.
Hence, the corresponding output is,

$$y''(n) = n\,[x_1(n) + x_2(n)] \quad \text{--- (ii)}$$

compare eqn (i) and (ii)

$$y'(n) = y''(n)$$

The system is linear.

(ii) $T[x(n)] = e^{x(n)}$

Soln: If input $x(n)$ is zero then output $y(n) = e^0 = 1$.

The system is non linear.

Let there are two inputs $x_1(n)$ and $x_2(n)$

Given eqn, $T[x(n)] = e^{x(n)}$

Thus, $x_1(n) \xrightarrow{T} y_1(n) = e^{x_1(n)}$

$x_2(n) \longrightarrow y_2(n) = e^{x_2(n)}$.

add this outputs to get $y'(n)$.

$$y'(n) = e^{x_1(n)} + e^{x_2(n)} \quad \text{--- (i)}$$

Now add inputs $x_1(n)$ and $x_2(n)$ and then we apply it to the system.

$$[x_1(n) + x_2(n)] \xrightarrow{T} y''(n) = e^{[x_1(n) + x_2(n)]}$$

$$y''(n) = e^{x_1(n)} \cdot e^{x_2(n)} \quad \text{--- (ii)}$$

compare (i) and (ii).

$$y'(n) \neq y''(n)$$

The system is non linear.

Causal and Non Causal System

A system is said to be causal system if output at any instant of time depends only on present and past input

ex.:- $y(n) = x(n) + x(n-1)$

Since causal system does not include future input samples hence such system is practically reliable. Generally all real time system are causal system because in ~~b~~ real time application all the present and past samples are present.

Anti Causal or Non Causal system

Here the output depends not only on present and past inputs but also on future inputs.

$$y(n) = x(n) + x(n+1)$$

Such systems are practically not ~~reali~~ realisable.

Q. $y(n) = x(n^2)$

Ans = ~~AntCausal~~ Non Causal

Q. $T[x(n)] = ax(n) + b$

Ans:- Causal

Stable and Unstable :

To define stability of a system, the term BIBO is used. It stands for Bounded input Bounded output. The meaning of Bounded is some finite value. Hence bounded input means input signal is having some finite value. Similarly bounded output means the output signal attains some finite value.

A system is BIBO stable if and only if every bounded input produces bounded output. This is mathamatically represented as

$$|x(n)| < M_x < \alpha$$

similarly,

$$|y(n)| < M_y < \alpha$$

A system is unstable if bounded input produces unbounded outputs.

Ex. $T[x(n)] = e^{x(n)}$

→ stable

We have to check the stability of the system by applying bounded ~~system~~ input. If $x(n)$ is bounded then $y(n)$ is also bounded.

Q. $y(n) = \sum_{k=-\alpha}^{n} x(k)$

→ Here $x(k)$ is the input signal to check the stability of a system we have to apply bounded input signal. Let us assume that we are applying unit step signal as an input signal.

i.e $x(k) = u(k)$

We know that unit step is present from $n=0$ to $n=\alpha$ and its magnitude is always 1. so its value is bounded. Mathematically it is expressed as

$$u(k) = \begin{cases} 1, & k \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Therefore the above eqⁿ becomes,

$$y(n) = \sum_{k=0}^{n} u(k)$$

Let us calculate $y(n)$ for few values of n,

for $n=2$,

$$y(n) = \sum_{k=0}^{2} u(k)$$

$$= u(0) + u(1) + u(2)$$

$$y(2) = 3$$

$$y(n) = \sum_{k=0}^{5} u(k)$$

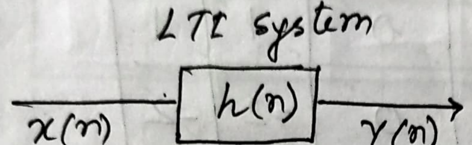
$$= 5$$

So it is clear that as $n \to \alpha$, $y(n) \to \alpha$ even if $u(k)$ is bounded. Thus the system is unstable.

Linear time Invarient System —

A particularly important class of system consist of those that are linear and time invarient. This two properties in combination lead to specially convenient representation for such systems. Most importantly this class of system has significant signal processing applications.

LTI system is also known as LSI system (Linear shift invarient system)



Here, $x(n) \rightarrow$ input sequence

$h(n) \rightarrow$ impulse response of a system

$y(n) \rightarrow$ output sequence.

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\delta(n-2) = \begin{cases} 1, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

$$\delta(n+2) = \begin{cases} 1, & n = -2 \\ 0, & \text{otherwise} \end{cases}$$

Linear Convolution :-

The different methods used for the computation of linear convolution are —

1. Tabulation method.
2. Graphical method.
3. Using mathematical eq^n of convolution
4. Multiplication method.

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{K=-\infty}^{\infty} x(K) \cdot h(n-K)$$

1. Tabulation method:-

Compute the convolution between the two sequence $x(n) = \{1, 1, 0, 1, 1\}$

$h(n) = \{1, -2, -3, 4\}$

$h(n) = \{1, -2, -3, 4, 0\}$ ← Padding

$x(n)$

	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
4	4	4	0	4	4
0	0	0	0	0	0

$y(n) = \{1, -1, -5, 2, 3, -5, 1, 4, 0\}$

$x(n) = \{1, 2, 1, 2\}$

$h(n) = \{1, 1, 1\}$

$h(n) = \{1, 1, 1, 0\}$

$x(n)$

$h(n)$	1	2	1	2
1	1	2	1	2
1	1	2	1	2
1	1	2	1	2
0	0	0	0	0

$y(n) = \{1, 3, 4, 5, 3, 2, 0\}$

Q. $x(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3)$

$h(n) = \delta(n-4) + \delta(n-3) + \delta(n-1) + \delta(n-2)$

$x(n) = \{1, 1, 1, 1\}$

$h(n) = \{1, 1, 1, 1\}$

	$x(n)$: 1	1	1	1
$h(n)$: 1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

$y(n) = \{1, 2, 3, 4, 3, 2, 1\}$

Q. Complete the convolution $(y(n) = x(n) * h(n))$ where, $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$ and $h(n) = \{1, -2, -3, \underset{\uparrow}{4}\}$

<u>Soln:</u> Let, x_l = lowest range of $x(n)$

x_h = highest range of $x(h)$

h_l = lowest range of ~~x~~ $h(n)$

h_h = highest range of $h(h)$

Now we used the following formula range of $y(n)$

y_l = lowest range of $y(n) = x_l + h_l = -2 + (-3) = -5$

y_h = highest range of $y(n) = x_h + h_h = 2 + 1 = 3$

$y(n)$ varies from $y(-5)$ to $y(3)$

using tabular method,

	1	-2	-3	4	0
1	1	-2	-3	4	0
1	1	-2	-3	4	0
0	0	0	0	0	0
1	1	-2	-3	4	0
1	1	-2	-3	4	0

$y(n) = \{1, -1, -5, 2, 3, -5, 1, 4, 0\}$

-5 is origin

<u>Graphical method for performing convolution:-</u>

Obtain linear convolution of following sequences.

$x(n) = \{1, 2, 1, 2\}$

$h(n) = \{1, 1, 1\}$

The eq^n of convolution is,

$$y(n) = \sum_{k=-\alpha}^{\alpha} x(k)\, h(n-k)$$

Now, let us decide the range of n and k

Range of n for $y(n)$

$y_l = x_l + h_l$

$= 0 + 0 = 0$

$y_h = x_h + h_h$

$= 3 + 2 = 5$

The range of $y(n)$ is from $y(0)$ to $y(5)$

2nd list

Range of k:

The value of k in the summation sign will be always same as the sequence $x(k)$ i.e $x(n)$. Thus the range of k is from 0 to 3.

The range of k is —

$$y(n) = \sum_{k=0}^{3} x(k)\, h(n-k)$$

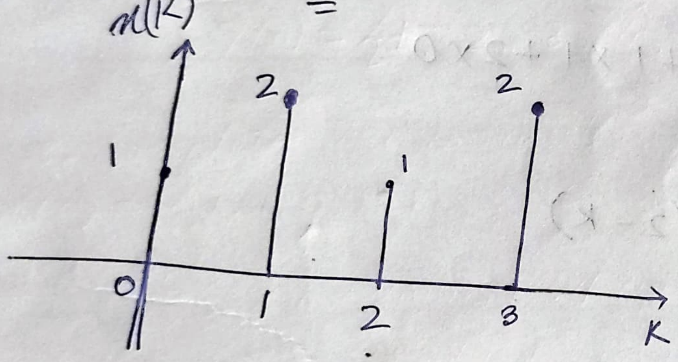
Here, $x(k) = \{1, 2, 1, 2\}$

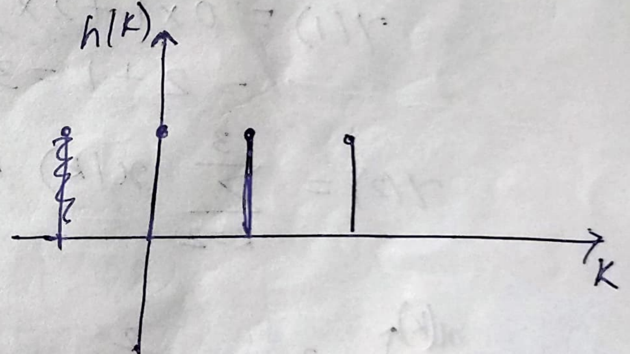
$h(k) = \{1, 1, 1\}$

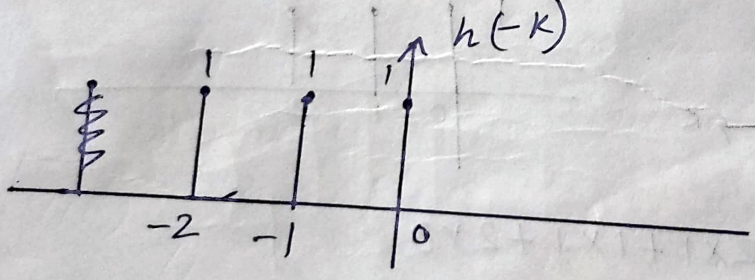
$$y(0) = \sum_{k=0}^{3} x(k)\, h(0-k)$$

$$= \sum_{k=0}^{3} x(k)\, h(-k) \rightarrow \text{folding operator}$$

=



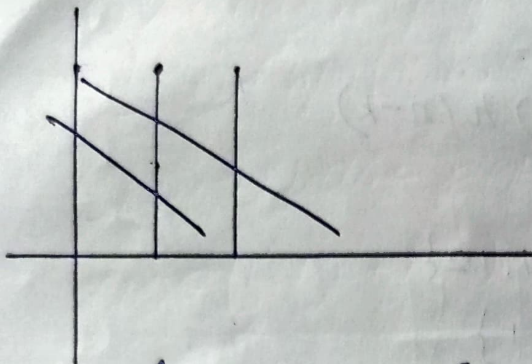


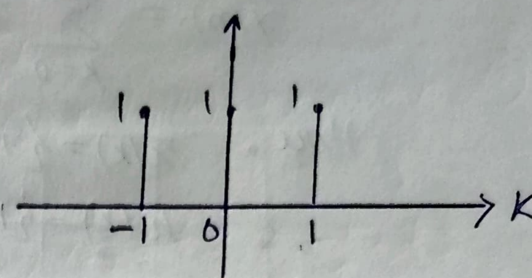


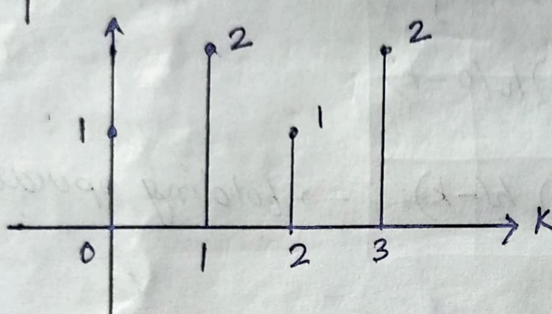
$y(0) = 0\times1 + 0\times1 + 1\times1 + 2\times0 + 1\times0 + 2\times0$

$= 1$

$y(1) = \sum_{k=0}^{3} x(k)\, h(1-k)$



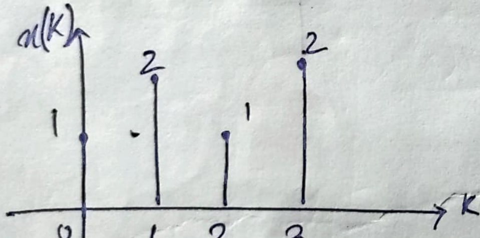


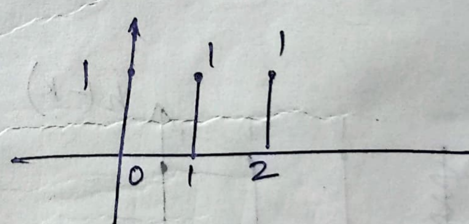


$y(1) = 0\times1 + 2\times1 + 1\times1 + 2\times0$

$= 2 + 1 = 3$

$y(2) = \sum_{k=2}^{3} x(k)\, h(2-k)$





$y(2) = 1\times1 + 2\times1 + 1\times1 + 2\times0$

$= 1 + 2 + 1 = 4$

$y(3) = 5, \quad y(4) = 3, \quad y(5) = 2$

$y(n) = \{y(0), y(1), y(2), y(3), y(4), y(5)\}$

$= \{1, 3, 4, 5, 3, 2\}$

Multiplication method for linear convolution :-

$$x(n) = \{1, 2, \underset{\uparrow}{1}, 2\}$$

$$h(n) = \{2, \underset{\uparrow}{2}, -1, 1\}$$

Range of : $y(l) = x_l + h_l$, $y(h) = x_h + h_h$

$= -1 + (-2)$ $\qquad = 2+1$

$= -3$ $\qquad\qquad = 3$

The range of $y(n)$ is from $y(-3)$ to $y(3)$

$x(n)$:				1	2	1	2
$h(n)$:				2	2	-1	1
				1	2	1	2
			-1	-2	-1	-2	
		2	4	2	4		
	2	4	2	4			
	2	6	5	5	5	-1	2

	2	2	-1	1
1	2	2	-1	1
2	4	4	-2	2
1	2	2	-1	1
2	4	4	-2	2

$$y(n) = \{2, 6, 5, 5, 5, -1, 2\}$$

Properties of linear convolution:

(i) Commulative property

$$x(n) * h(n) = h(n) * x(n)$$

(ii) Associative property

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

(iii) Distributive property

$$x(n) * (h_1(n) + h_2(n)) = x(n) * h_1(n) + x(n) * h_2(n)$$

H.W: $x(n) = \{1, \underset{\uparrow}{2}, 1, 2\}$, $h(n) = \{2, 2, \underset{\uparrow}{-1}, 1\}$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Range of n

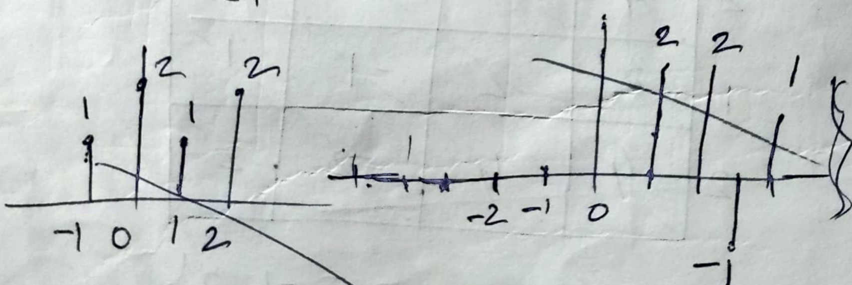
$y_L = x_L + h_L = -1 - 2 = -3$ $\quad y_h = x_h + h_h = 2 + 1 = 3$

Range of k is -1 to 2

$$y(n) = \sum_{-1}^{2} x(k)\, h(n-k)$$

$$y(-3) = \sum_{-1}^{2} x(k)\, h(-3-k)$$

$$= \sum_{-1}^{2} x(k) \{h - (k+3)\}$$



$$y(-3) = 1 \times 2 + 2 \times 2 = 6$$

Module 3: Realization of Digital systems

Classification of digital filters :-

1. Recursive or infinite impulse response (IIR) digital filter :-

When a filter produces an impulse response $h(n)$ that has infinite duration. That is called as IIR filter.

2. Non recursive or finite impulse response (FIR) digital filter :

If a filter has an impulse response that has finite duration is called as finite impulse response or FIR filter.

Digital filter classification can also be based on filter structure. In general, the output of a filter can be a function of future, current and past input values as well as past output values. If the present output is a function of past output then a feedback or recursive mechanism from the output must exist. Hence such a filter is reffered to as recursive filter. IIR filter is a recursive filter because the current output $y(n)$ is a function of past output, past input and present input. The term IIR and recursive are interchangebly used.

If the current output is a function of only past and present input then it is called as non recursive or FIR filter. Thus in case of FIR filter the feedback path does not exist. The term FIR and non recursive are interchange-bly used.

2. Canonic & non canonic filter:—

If the no of delays in the basic realisation block diagram is equal to the order of the difference eqn or the order of the transfer function $H(z)$ of a digital filter then the realisation structure is known as canonic. On the other hand if delays on the structure are not same as the order then this called non canonic.

Realisation of digital filter:—

(i) Block diagram representation
(ii) Signal flow graph

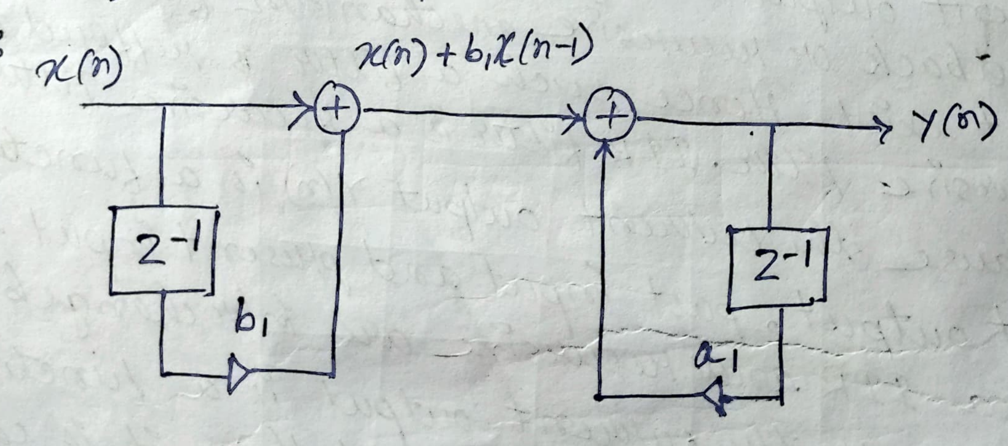
(i) Block diagram representation:—

Building blocks of block diagram representation —

(i) Adder (ii) Multiplier (iii) Delay unit.

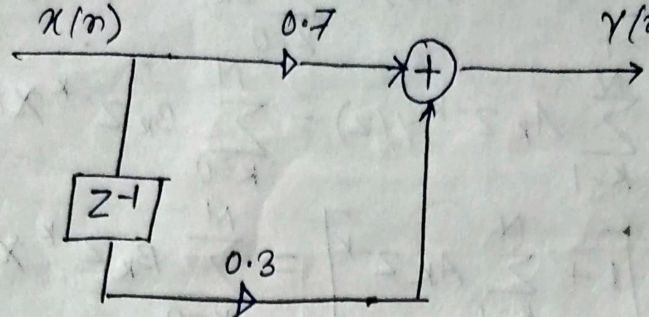
Q. $y(n) = a_1 y(n-1) + x(n) + b_1 x(n-1)$

Ans:



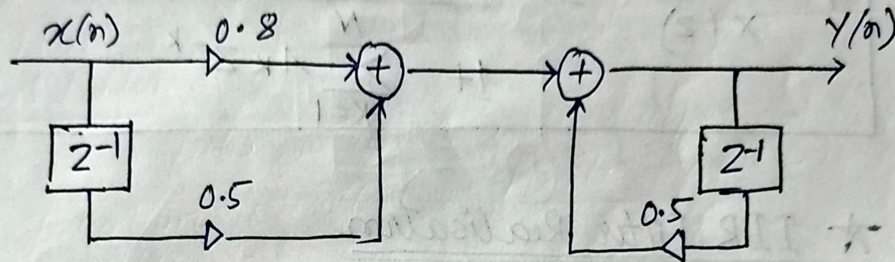
Q. $y(n) = 0.7\,x(n) + 0.3\,x(n-1)$

Ans:



Q. $y(n) = 0.5\,y(n-1) + 0.8\,x(n) + 0.4\,x(n-1)$

Ans:



* **Transfer function of a finite order LTI system:**

We know that the output of a finite order LTI system at time n may be expressed as a linear combination of the inputs and outputs i.e

$$y(n) = -\sum_{k=1}^{N} A_k\, y(n-k) + \sum_{k=0}^{M} B_k\, x(n-k)$$

Here A_k and B_k are constants. taking z transform of the output sequence $y(n)$

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty}\left[-\sum_{k=1}^{N} A_k\, y(n-k) + \sum_{k=0}^{M} B_k\, x(n-k)\right] z^{-n}$$

Changing the order of summation of the above eqn we may write

$$= \sum_{k=1}^{N} A_k\left[-\sum_{n=-\infty}^{\infty} y(n-k)\, z^{-n}\right] + \sum_{k=0}^{M} B_k\left[\sum_{n=-\infty}^{\infty} x(n-k)\, z^{-n}\right]$$

$$Y(z) = -\sum_{k=1}^{N} A_k z^{-k} Y(z) + \sum_{k=0}^{M} B_k z^{-k} X(z)$$

$$\Rightarrow Y(z) + \sum_{k=1}^{N} A_k z^{-k} Y(z) = \sum_{k=0}^{M} B_k z^{-k} X(z)$$

$$\Rightarrow Y(z) \left[1 + \sum_{k=1}^{N} A_k z^{-k}\right] = \sum_{k=0}^{M} B_k z^{-k} X(z)$$

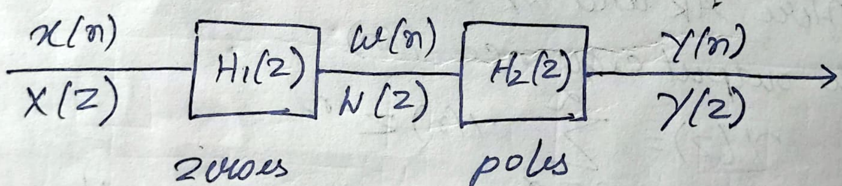
$$\boxed{\Rightarrow \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} B_k z^{-k}}{1 + \sum_{k=1}^{N} A_k z^{-k}}}$$

★ IIR Filter Realisation

1. Direct form I realisation

$$H(z) = \frac{\sum_{k=0}^{M} B_k z^{-k}}{1 + \sum_{k=1}^{N} A_k z^{-k}} = \frac{H_1(z) \times H_2(z)}{\cancel{H_2(z)}}$$

In direct form I zeroes are represented first followed by poles —



$$H_1(z) = \sum_{k=0}^{M} B_k z^{-k}$$

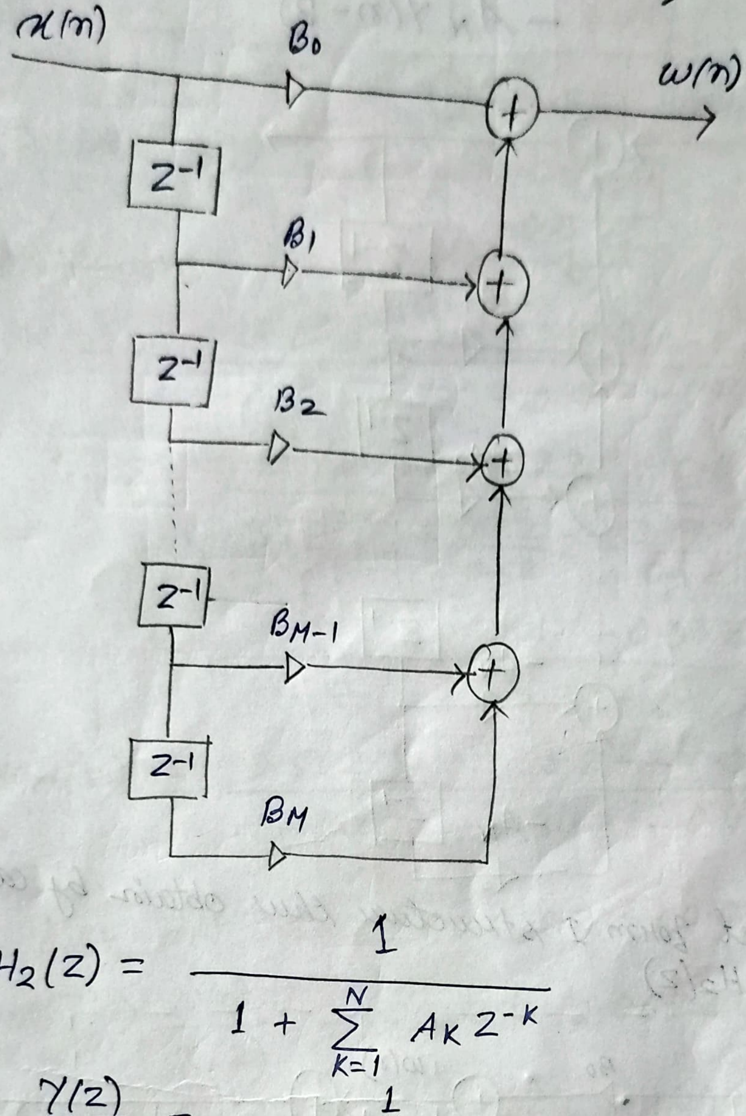
$$\Rightarrow \frac{W(z)}{X(z)} = \sum_{k=0}^{M} B_k z^{-k}$$

$$\Rightarrow \frac{W(z)}{X(z)} = B_0 z^0 + B_1 z^{-1} + B_2 z^{-2} + \cdots + B_M z^{-M}$$

$$\Rightarrow W(z) = B_0 X(z) + B_1 z^{-1} X(z) + B_2 z^{-2} X(z) + \cdots + B_M z^{-M} X(z)$$

Taking inverse z-transform

$$w(n) = B_0 x(n) + B_1 x(n-1) + B_2 x(n-2) + \cdots + B_M x(n-M)$$



$$H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} A_k z^{-k}}$$

$$\Rightarrow \frac{Y(z)}{W(z)} = \frac{1}{1 + \sum_{k=1}^{N} A_k z^{-k}}$$

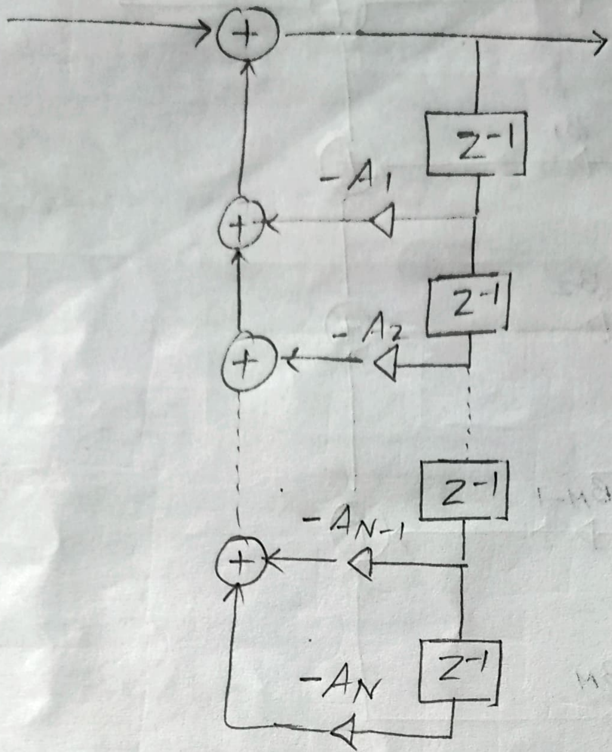
$$\Rightarrow Y(z) \left[1 + \sum_{k=1}^{N} A_k z^{-k}\right] = W(z)$$

$$\Rightarrow Y(z) = W(z) - \sum_{k=1}^{N} A_k z^{-k} Y(z)$$

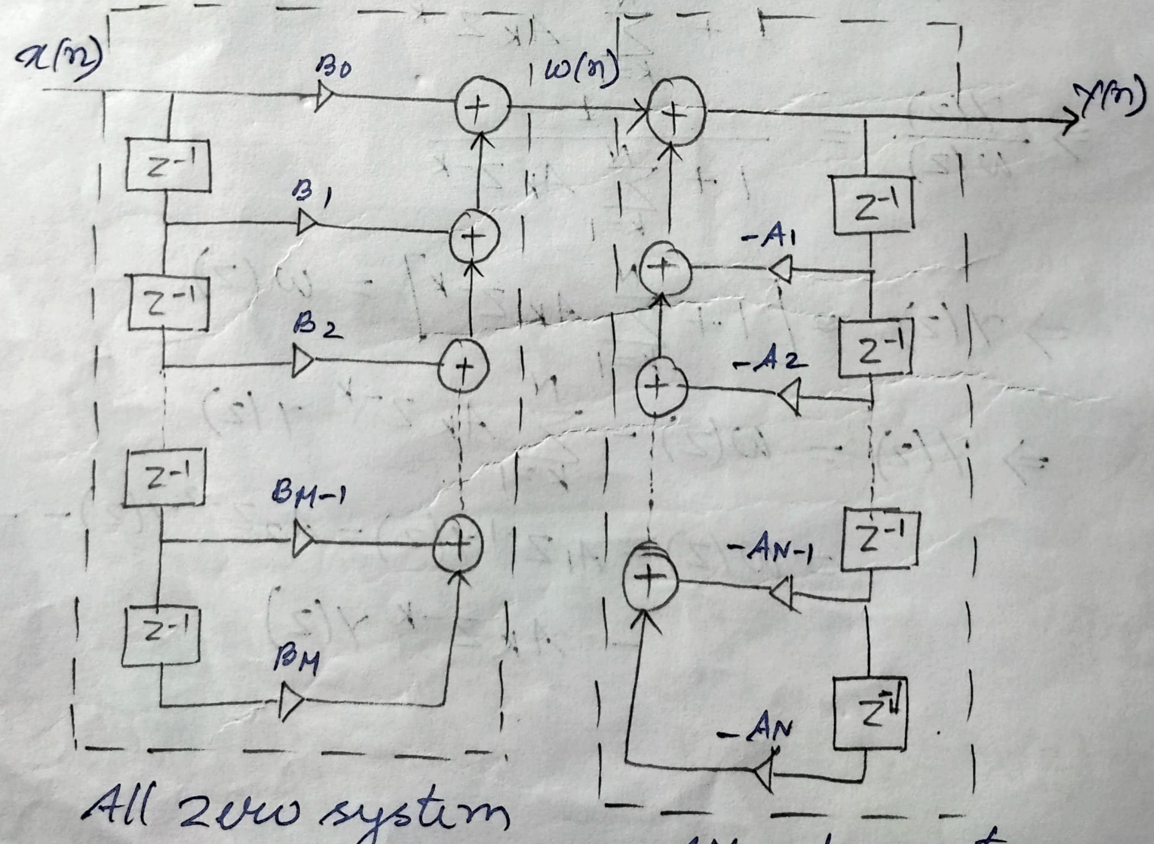
$$= W(z) - A_1 z^{-1} Y(z) - A_2 z^{-2} Y(z) - \cdots - A_k z^{-k} Y(z)$$

Taking inverse Z-transform,

$$Y(n) = W(n) - A_1 Y(n-1) - A_2 Y(n-2) - \cdots - A_N Y(n-B)$$

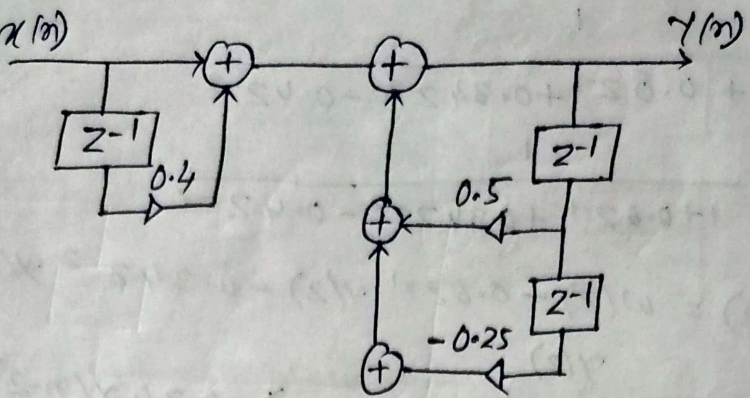


The direct form I structure thus obtain by cascading $H_1(Z)$ and $H_2(Z)$



Q. $y(n) = 0.5\,y(n-1) - 0.25\,y(n-2) + x(n) + 0.4\,x(n-1)$

Ans:



Q. $H(z) = \dfrac{0.28z^2 + 0.319z + 0.04}{0.52z^3 + 0.32z^2 + 0.17z - 0.02}$

Ans:

$$H(z) = \frac{z^2(0.28 + 0.319z^{-1} + 0.04z^{-2})}{0.5z^3\left(1 + \frac{0.32}{0.52}z^{-1} + \frac{0.172}{0.52}z^{-2} - \frac{0.02}{0.52}\right)}$$

$$= \frac{0.28 + 0.319z^{-1} + 0.04z^{-2}}{0.5z(1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3})}$$

$$H_1(z)\cdot H_2 = H(z) = \frac{0.56z^{-1} + 0.068z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{2} - 0.4z^{-3}}$$

Here $H_1(z) = 0.56z^{-1} + 0.068z^{-2} + 0.08z^{-3}$

$$\Rightarrow \frac{W(z)}{X(z)} = 0.56z^{-1} + 0.068z^{-2} + 0.08z^{-3}$$

$$\Rightarrow W(z) = X(z)\,0.56z^{-1} + X(z)\,0.068z^{-2} + 0.08z^{-1}X(z)$$

taking inverse z-transform

$w(n) = \cancel{0.56x(n-1) + 0.068x(n-2)} + 0.08x(n)$

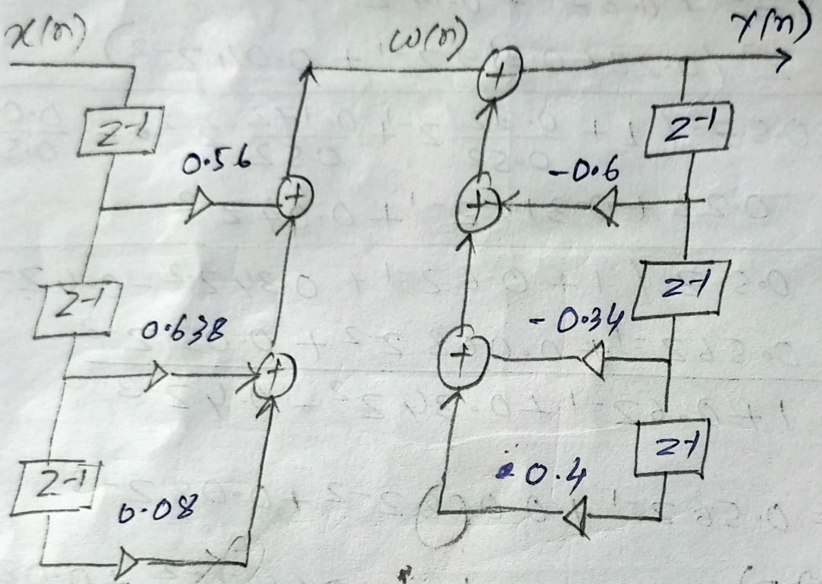
$0.56x(n-1) + 0.068x(n-2) + 0.08x(n-2)$

$$H_2 = \frac{1}{1+0.6z^{-1}+0.34z^{-2}-0.4z^{-3}}$$

$$\frac{Y(z)}{W(z)} = \frac{1}{1+0.6z^{-1}+0.34z^{-2}-0.4z^{-3}}$$

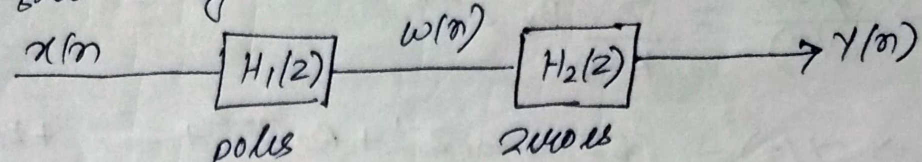
$$\Rightarrow Y(z) = W(z) - 0.6z^{-1}Y(z) - 0.34z^{-2}Y(z) + 0.4z^{-3}Y(z)$$

$$\therefore Y(n) = W(n) - 0.6Y(n-1) - 0.34Y(n-2) + 0.4Y(n-3)$$



Direct form II structure :

In this structure the poles are ~~respo~~ represented first followed by zeroes.



$$H_1(z) = \frac{1}{1+\sum_{k=1}^{N} A_k z^{-k}}$$

$$\Rightarrow \frac{W(z)}{X(z)} = \frac{1}{1+\sum_{k=1}^{N} A_k z^{-k}}$$

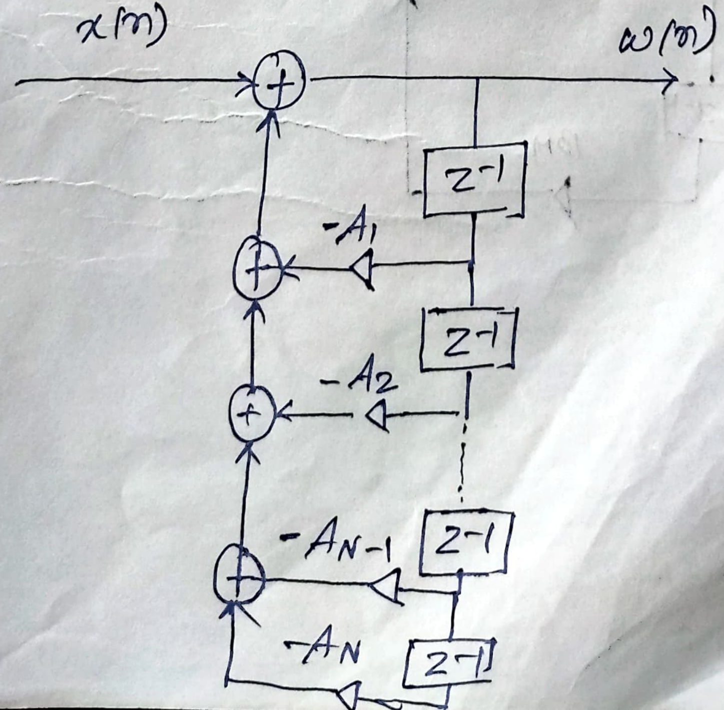
$$\Rightarrow W(z)\left[1+\sum_{k=1}^{N} A_k z^{-k}\right] = X(z)$$

$$\Rightarrow W(z) = X(z) - \sum_{k=1}^{N} A_k z^{-k} W(z)$$

$$\Rightarrow W(z) = X(z) - A_1 z^{-1} W(z) - A_2 z^{-2} W(z) - \cdots - A_N z^{-N} W(z)$$

Taking inverse z-transform

$$w(n) = x(n) - A_1 w(n-1) - A_2 w(n-2) - \cdots - A_N w(n-N)$$



$$H_2(z) = \sum_{k=0}^{M} B_k z^{-k}$$

$$\Rightarrow \frac{Y(z)}{W(z)} = \sum_{k=0}^{M} B_k z^{-k}$$

$$\Rightarrow \frac{Y(z)}{W(z)} = B_0 z^0 + B_1 z^{-1} + B_2 z^{-2} + \cdots + B_M z^{-M}$$

$$\Rightarrow Y(z) = B_0 W(z) + B_1 z^{-1} W(z) + B_2 z^{-2} W(z) + \cdots + B_M z^{-M} W(z)$$

Taking inverse z-transform

$$y(n) = B_0 w(n) + B_1 w(n-1) + B_2 w(n-2) + \cdots + B_M w(n-M)$$