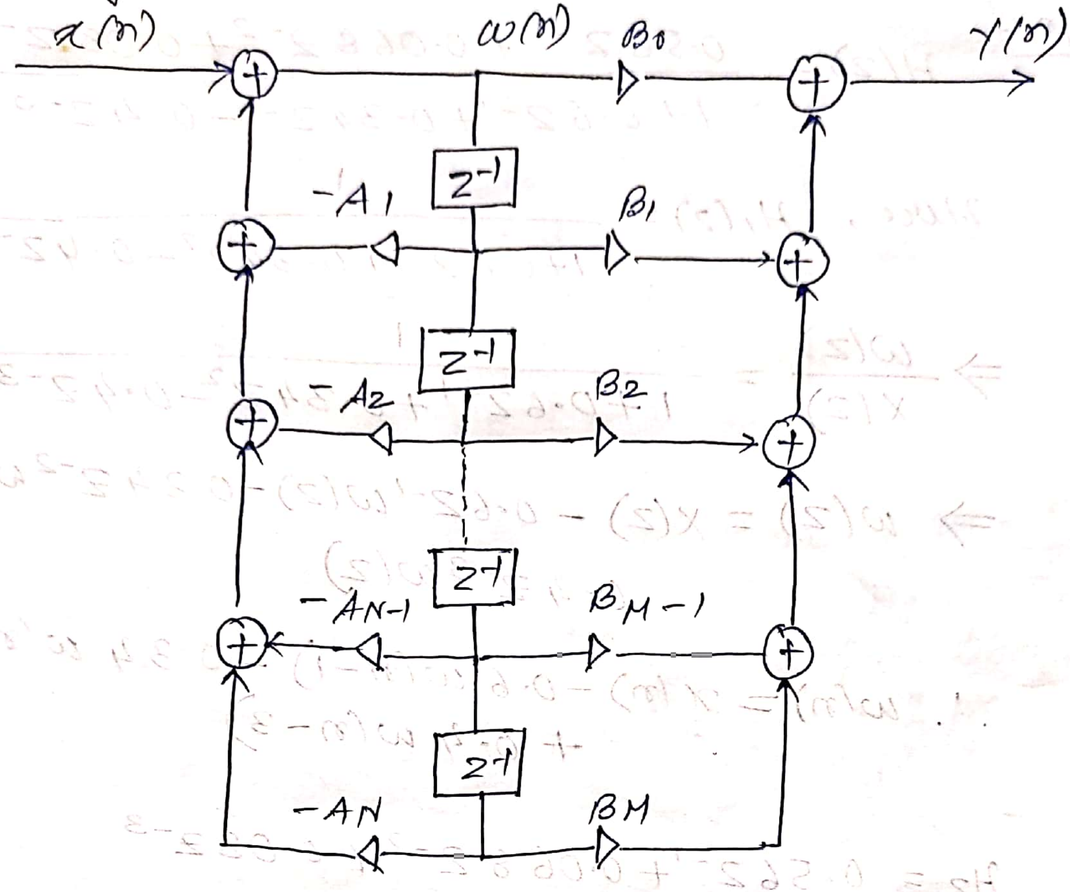
The direct form I structure thus obtained by cascading $H_1(z)$ and $H_2(z)$



★★ **Important point :**

The direct form II structure has reduce no of delay elements compared to direct form I structure this means that memory locations are reduced since the no of delay elements in direct form II is same as that of the order of difference eqn. it is also called as canonical structure.

Q. $H(z) = \dfrac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.02}$

Soln:- $H(z) = \dfrac{0.56z^{-1} + 0.068z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$

Here, $H_1(z) = \dfrac{1}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$

$\Rightarrow \dfrac{W(z)}{X(z)} = \dfrac{1}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$

$\Rightarrow W(z) = X(z) - 0.6z^{-1}W(z) - 0.34z^{-2}W(z) + 0.4z^{-3}W(z)$

$\therefore w(n) = x(n) - 0.6\,w(n-1) - 0.34\,w(n-2) + 0.4\,w(n-3)$

$H_2 = 0.56z^{-1} + 0.068z^{-2} + 0.08z^{-3}$

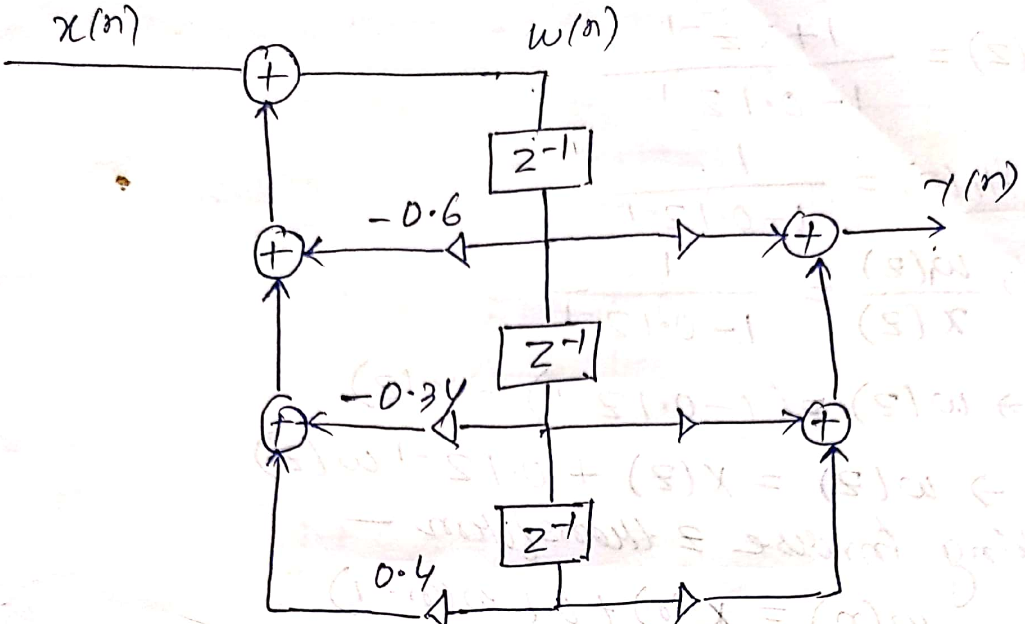
$\Rightarrow \dfrac{Y(z)}{W(z)} = 0.56z^{-1} + 0.068z^{-2} + 0.08z^{-3}$

$\Rightarrow Y(z) = 0.56z^{-1}W(z) + 0.068z^{-2}W(z) + 0.08z^{-3}W(z)$

Taking inverse z transform

~~$y(n) = 0.56\,w(n) + 0.068\,w(n-1) + 0.08(w(n-$~~

$y(n) = 0.56\,w(n-1) + 0.068\,w(n-2) + 0.08\,w(n-3)$



Cascade form structure realisation of IIR Filter:-

The cascade form is obtained by cascading the substransfer function or sub system functions which are realised by using the direct form I or II structure.

Q. $H(z) = \dfrac{2(z+2)}{z(z-0.1)(z+0.5)(z+0.4)}$

cascade realisation

Soln:-

$$H(z) = \frac{2(z+2)}{z(z-0.1)(z+0.5)(z+0.4)}$$

$$= \frac{2\cancel{z}(1+2z^{-1})}{z\cdot\cancel{z}(1-0.1z^{-1})\,z(1+0.5z^{-1})z(1+0.4z^{-1})}$$

$$= \frac{2z^{-3}(1+2z^{-1})}{(1-0.1z^{-1})(1+0.5z^{-1})(1+0.4z^{-1})}$$

$$H(z) = 2z^{-3}.H_A(z)\,H_B(z)\,H_C(z)$$

$$H_A(z) = \frac{1+2z^{-1}}{1-0.1z^{-1}}$$

$$H_1(z) = \frac{1}{1-0.1z^{-1}}$$

$$\Rightarrow \frac{w_1(z)}{x(z)} = \frac{1}{1-0.1z^{-1}}$$

$$\Rightarrow w(z)(1-0.1z^{-1}) = x(z).$$

$$\Rightarrow w(z) = x(z) + 0.1z^{-1}w(z)$$

Taking inverse z transform —

$$w(n) = x(n) + 0.1\, w(n-1)$$

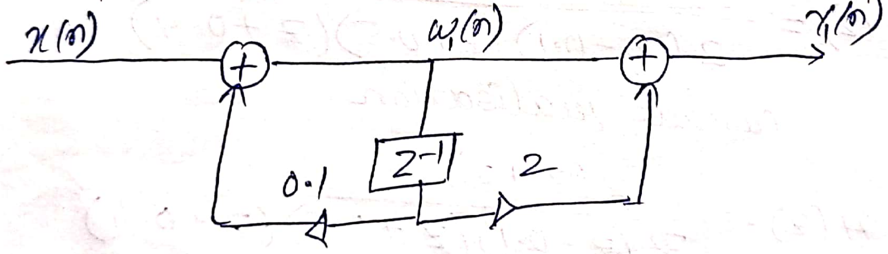
$$H(z) = 1+2z^{-1}$$

$$\Rightarrow \frac{y_1(z)}{w_1(z)} = 1+2z^{-1}$$

$$\Rightarrow y_1(z) = w_1(z) + 2z^{-1}w_1(z)$$

Taking inverse z transform,

$$y_1(n) = w_1(n) + 2\,w_1(n-1)$$



$$H_B(z) = \frac{1}{1+0.5z^{-1}}$$

$$H_1(z) = \frac{1}{1+0.5z^{-1}}$$

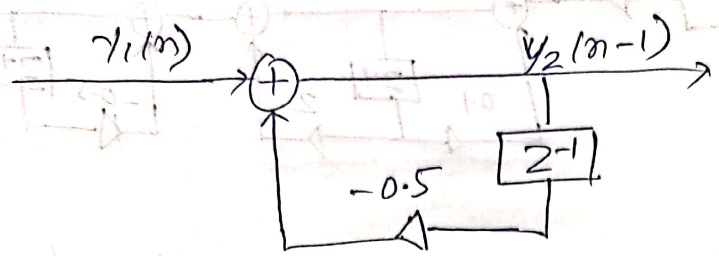
$$\frac{y_2(z)}{y_1(z)} = \frac{1}{1+0.5z^{-1}}$$

$$\Rightarrow y_2(z)(1+0.5z^{-1}) = y_1(z)$$

$$\Rightarrow y_2(z) = y_1(z) - 0.5z^{-1}y_2(z)$$

Taking inverse ~~laplace~~ z- transform,

$$y_2(n) = y_1(n) - 0.5\, y_2(n-1)$$



$$H_c(z) = \frac{1}{1+0.4z^{-1}}$$

$$H_1(z) = \frac{1}{1+0.4z^{-1}}$$

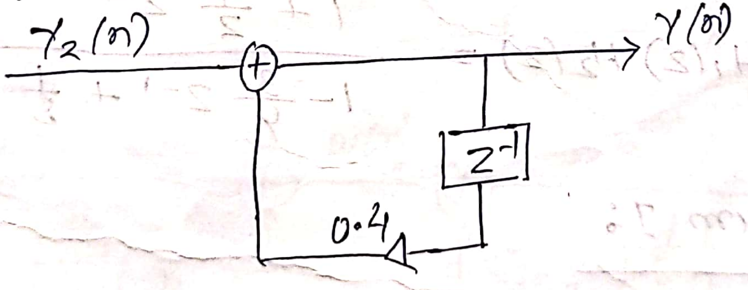
$$\Rightarrow \frac{Y(z)}{Y_2(z)} = \frac{1}{1+0.4z^{-1}}$$

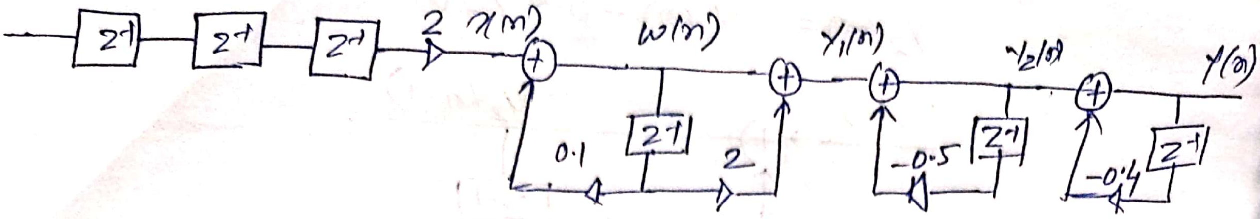
$$\Rightarrow Y(z)\,[1+0.4z^{-1}] = Y_2(z)$$

$$\Rightarrow Y(z) = Y_2(z) - 0.4z^{-1}\,Y(z)$$

Taking inverse z transform,

$$y(n) = y_2(n) - 0.4z^{-1}y(n)$$





H.W $y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n) + \frac{1}{2} x(n-1)$

Solution: DFI and DFII, Cascade form

Taking z transform on both sides of the given difference equation and neglecting initial conditions, we get

$$Y(z) - \frac{3}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z) = X(z) + \frac{1}{2} z^{-1} X(z)$$

The transfer function of the given IIR system is,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$H(z) = H_1(z) H_2(z) = \frac{1 + \frac{1}{2} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

Direct form I:

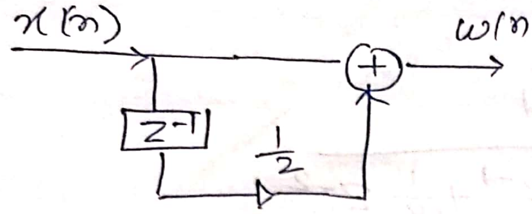
$$H_1 = 1 + \frac{1}{2} z^{-1}$$

$$\Rightarrow \frac{W(z)}{X(z)} = 1 + \frac{1}{2} z^{-1}$$

$$\Rightarrow W(z) = X(z) + \frac{1}{2} z^{-1} X(z)$$

Taking inverse z transform,

$$w(n) = x(n) + \frac{1}{2} x(n-1)$$



$$H_2(z) = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

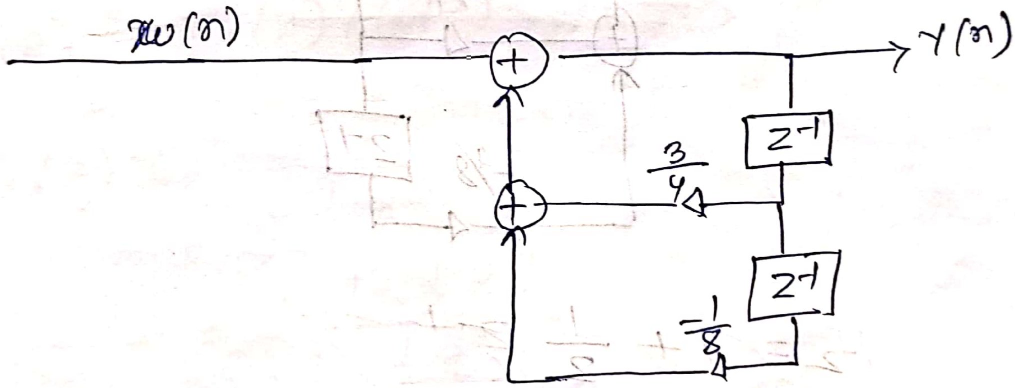
$$\frac{Y(z)}{W(z)} = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

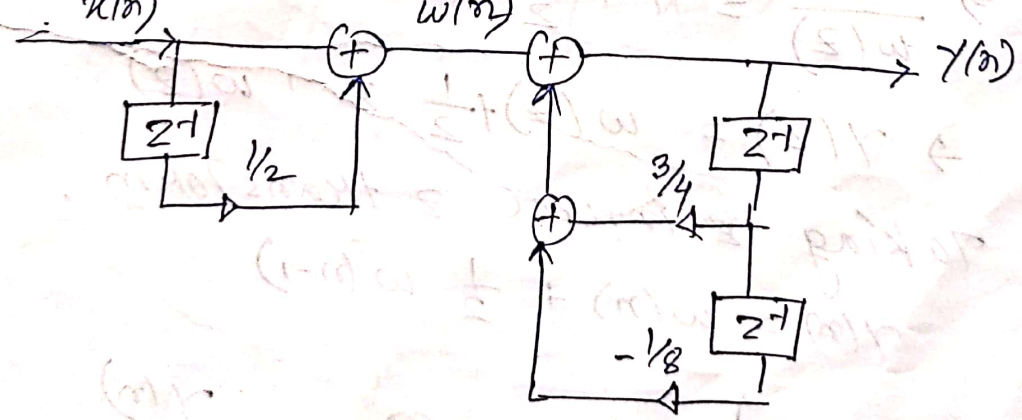
$$\Rightarrow Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = W(z)$$

$$\Rightarrow Y(z) = W(z) + \frac{3}{4}z^{-1}Y(z) - \frac{1}{8}z^{-2}Y(z)$$

Taking inverse Z transform,

$$y(n) = w(n) + \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2)$$





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Direct form II:

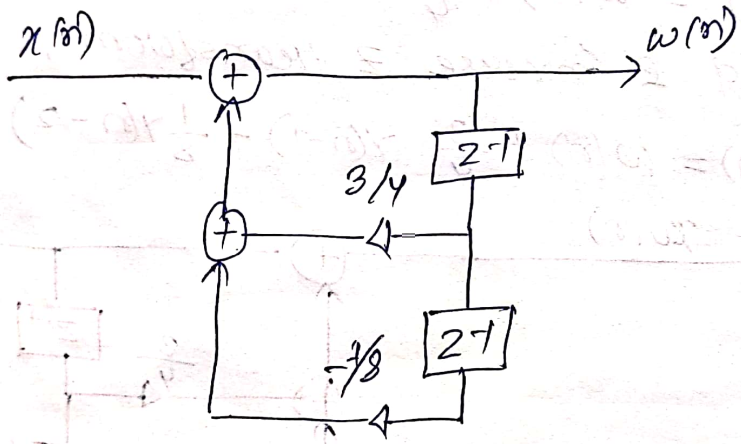
$$H_1(z) = \frac{1}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$\Rightarrow W(z) = X(z) + \frac{3}{4} z^{-1} W(z) - \frac{1}{8} z^{-2} W(z)$$

Taking inverse z transform

$$w(n) = x(n) + \frac{3}{4} w(n-1) - \frac{1}{8} w(n-2)$$



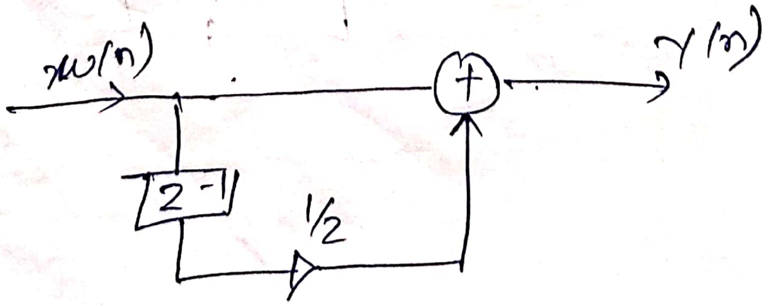
$$H_2 = 1 + \frac{1}{2} z^{-1}$$

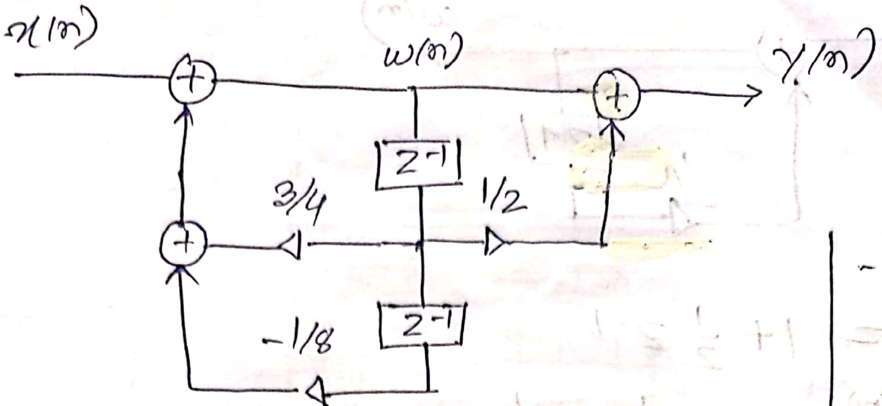
$$\Rightarrow \frac{Y(z)}{W(z)} = 1 + \frac{1}{2} z^{-1}$$

$$\Rightarrow Y(z) = W(z) + \frac{1}{2} z^{-1} W(z)$$

Taking inverse z transform,

$$y(n) = w(n) + \frac{1}{2} w(n-1)$$





$$H(z) = \frac{1+\frac{1}{2}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

$$= \frac{1+\frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{\frac{3}{4} \pm \sqrt{\frac{9}{16}-4\times\frac{1}{8}}}{\frac{1}{4}}$$

$$= \frac{\frac{3}{4}\sqrt{\frac{9}{16}-\frac{1}{2}}}{\frac{1}{4}}$$

$$= 3\sqrt{\frac{9-8}{16}}$$

$$= \pm\frac{3}{4}$$

$$H_0(z) = H_A(z)\,H_B(z)$$

$$H_A(z) = \frac{1+\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$\Rightarrow \cancel{H_1 = \frac{w(n)}{x(n)} = \frac{1+\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}}$$

$$\cancel{\Rightarrow w(n)\left[1-\frac{1}{2}z^{-1}\right] = x(n)+\frac{1}{2}z^{-1}x(n)}$$

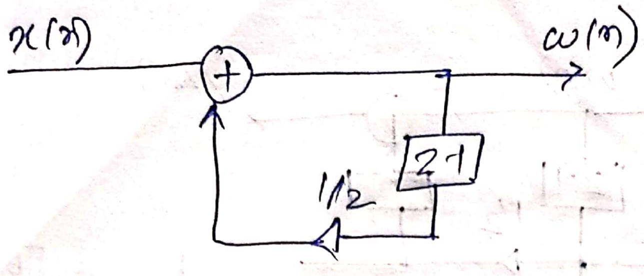
$$H_1 = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$\Rightarrow \frac{w(z)}{x(z)} = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$\Rightarrow w(z) = x(z) + \frac{1}{2}z^{-1}w(z)$$

Taking inverse z transform,

$$w(n) = x(n) + \frac{1}{2}w(n-1)$$



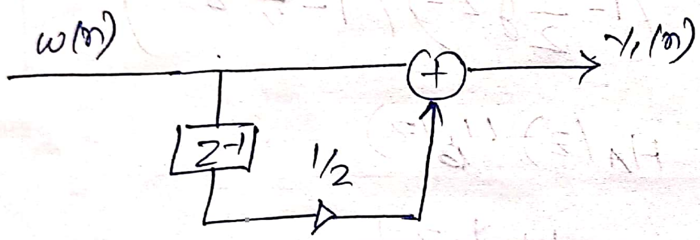
$$H_2 = 1 + \frac{1}{2} z^{-1}$$

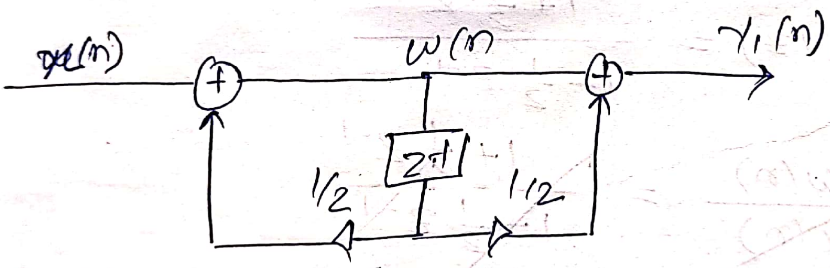
$$\Rightarrow \frac{Y_1(z)}{W(z)} = 1 + \frac{1}{2} z^{-1}$$

$$\Rightarrow Y_1(z) = W(z) + \frac{1}{2} z^{-1} W(z)$$

Taking inverse - z transform

$$\Rightarrow y_1(n) = w(n) + \frac{1}{2} w(n-1)$$





$$H_B(z) = \frac{1}{1 - \frac{1}{4} z^{-1}}$$

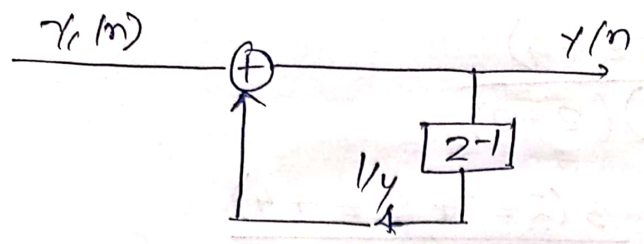
$$\Rightarrow H_1 = \frac{1}{1 - \frac{1}{4} z^{-1}}$$

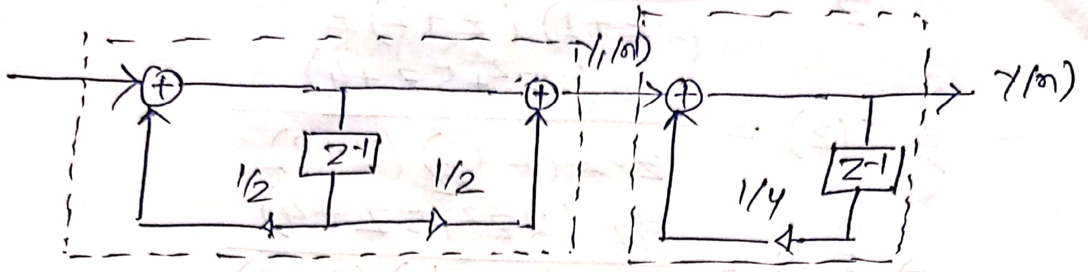
$$\Rightarrow \frac{Y_0(z)}{Y_1(z)} = \frac{1}{1 - \frac{1}{4} z^{-1}}$$

$$\Rightarrow Y(z) = Y_1(z) + \frac{1}{4} z^{-1} Y(z)$$

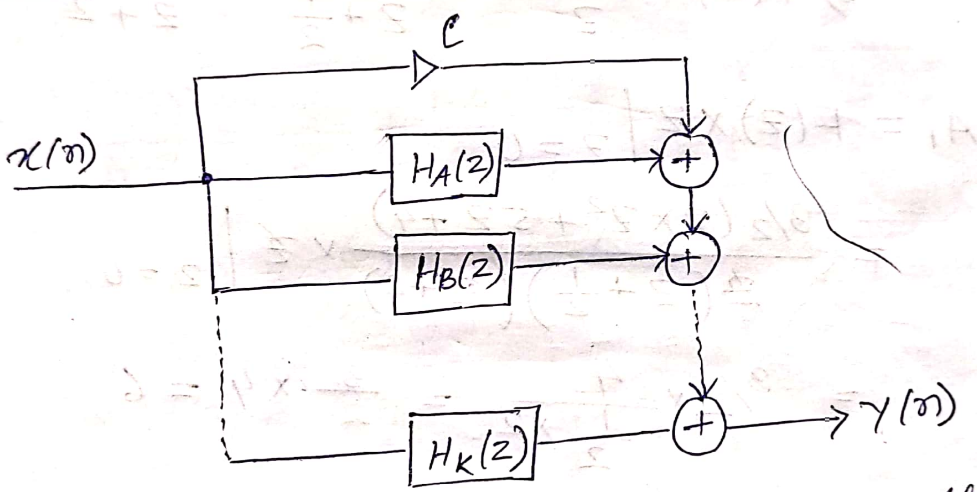
Taking inverse z transform,

$$y(n) = y_1(n) + \frac{1}{4} y(n-1)$$





★ Parallel form structure for IIR filter :-



Parallel form structure is the parallel connection of substansfer function or sub system function. To determine the parallel realisation we shall first determine the partial ~~fun~~ fraction expansion of $H(z)/z$. Then the overall transfer function $H(z)$ is expressed as $H(z) = C + H_A(z) + H_B(z) + \cdots + H_K(z)$

Here C is a constant.

Q. $H(z) = \dfrac{3(2z^2+5z+4)}{(2z+1)(z+2)}$.

Sol^n:

$$\frac{H(z)}{z} = \frac{3(2z^2+5z+4)}{(2z+1)(z+2)z}$$

$$\Rightarrow F(z) = \frac{3(2z^2+5z+4)}{z\cdot 2(z+\frac{1}{2})(z+2)}$$

$$\Rightarrow F(z) = \frac{3/2\,(2z^2+5z+4)}{z(z+\frac{1}{2})(z+2)}$$

$$\Rightarrow F(z) = \frac{A_1}{z} + \frac{A_2}{z+\frac{1}{2}} + \frac{A_3}{z+2}$$

$$A_1 = F(z)\times z\Big|_{z=0}$$

$$= \frac{3/2\,(2\times z^2+5z+4)}{\cancel{z}(z+\frac{1}{2})(z+2)}\times \cancel{z}\Big|_{z=0}$$

$$= 3/2 \times \frac{4}{\frac{1}{2}\times 2} = \frac{3}{2}\times 4 = 6$$

$$A_2 = F(z)\times\left(z+\frac{1}{2}\right)\Big|_{z=-\frac{1}{2}}$$

$$= \frac{3/2\,(2z^2+5z+4)}{z\,\cancel{(z+\frac{1}{2})}(z+2)}\times \cancel{(z+\frac{1}{2})}\Big|_{z=-\frac{1}{2}}$$

$$= \frac{3/\cancel{2}\left(2\times(-\frac{1}{2})^2+5\times(\frac{-1}{2})+4\right)}{-\frac{1}{\cancel{2}}\left(-\frac{1}{2}+2\right)}$$

$$= -\cancel{3}\frac{\left(2\times\frac{1}{4} - \frac{5}{2}+4\right)}{\frac{\cancel{3}}{2}}$$

$$= -2\left(\frac{1}{2}-\frac{5}{2}+4\right)$$

$$= -2\left(\frac{1-5+8}{2}\right)$$

$$= -\cancel{2}\times \frac{4}{\cancel{2}}$$

$$A_3 = F(z) \times (z+2) \Big|_{z=-2}$$

$$= \frac{3/2\,(2z^2+5z+4)}{z(z+\frac{1}{2})\cancel{(z+2)}} \times \cancel{(z+2)} \Big|_{z=-2}$$

$$= \frac{3/2\,(2\times(-2)^2+5\times(-2)+4}{-2\,(-2+\frac{1}{2})}$$

$$= \frac{3/2\,(8-10+4)}{-\cancel{2}\times\frac{-8}{\cancel{2}}}$$

$$= \frac{1}{2}\times 2 = 1$$

$$\therefore \frac{H(z)}{z} = \frac{6}{z} + \frac{(-4)}{z+\frac{1}{2}} + \frac{1}{z+2}$$

$$\Rightarrow H(z) = \frac{6}{z}\times z + \frac{(-4z)}{z+\frac{1}{2}} + \frac{z}{z+2}$$

$$\Rightarrow H(z) = 6 + \frac{-4}{1+\frac{1}{2}z^{-1}} + \frac{1}{1+2z^{-1}}$$

$$H_A(z) = \frac{-4}{1+\frac{1}{2}z^{-1}}$$

$$H_{B1} = \frac{1}{1+\frac{1}{2}z^{-1}}$$

Let $$\frac{W(z)}{X(z)} = \frac{1}{1+\frac{1}{2}z^{-1}}$$

$$\Rightarrow W(z) = X(z) - \frac{1}{2}z^{-1}W(z)$$

Taking inverse-z transform

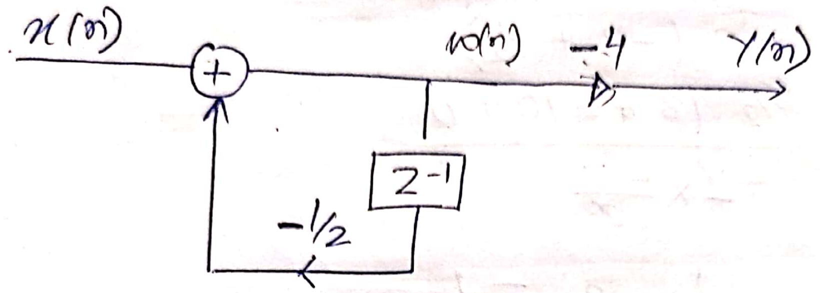
$$w(n) = x(n) - \frac{1}{2}w(n-1)$$

$H_2 = -4$

$$\frac{Y(z)}{W(z)} = -4 \Rightarrow Y(z) = -W(z) \cdot 4$$

Taking inverse z transform,

$$\Rightarrow Y(n) = -4\, w(n)$$



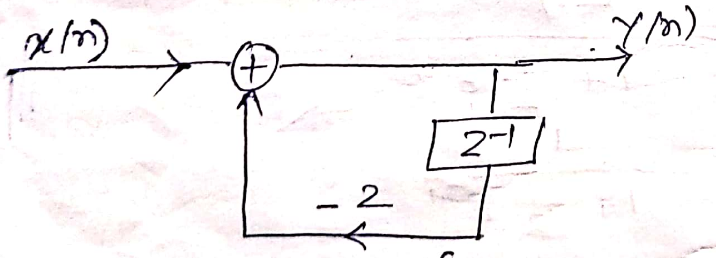
$$H_B(z) = \frac{1}{1+2z^{-1}}$$

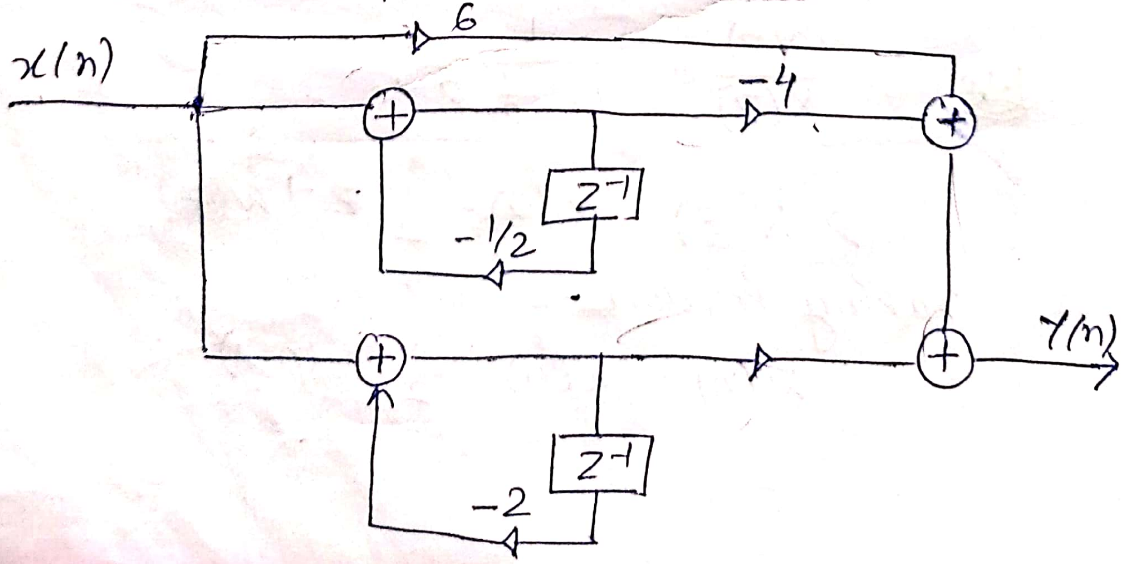
$$\frac{Y(z)}{X(z)} = \frac{1}{1+2z^{-1}}$$

$$\Rightarrow Y(z) = X(z) - 2z^{-1}Y(z)$$

Taking inverse z transform

$$Y(n) = x(n) - 2\,Y(n-1)$$





Q. $H(z) = \dfrac{3z(5z-2)}{\left(z+\frac{1}{2}\right)(3z-1)}$

Soln: $\dfrac{H(z)}{z} = \dfrac{3(5z-2)}{\left(z+\frac{1}{2}\right)(3z-1)}$

$$\Rightarrow F(z) = \frac{3(5z-2)}{\left(z+\frac{1}{2}\right)3\cdot\left(z-\frac{1}{3}\right)}$$

$$\Rightarrow F(z) = \frac{5z-2}{\left(z+\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$$

$$\Rightarrow F(z) = \frac{A_1}{z+\frac{1}{2}} + \frac{A_2}{z-\frac{1}{3}}$$

$$A_1 = F(z)\times\left(z+\frac{1}{2}\right)\Big|_{z=-\frac{1}{2}}$$

$$= \frac{(5z-2)\times\cancel{\left(z+\frac{1}{2}\right)}}{\cancel{\left(z+\frac{1}{2}\right)}\left(z-\frac{1}{3}\right)}\Bigg|_{z=-\frac{1}{2}}$$

$$= \frac{5z-2}{z-\frac{1}{3}}$$

$$= \frac{5\times\left(-\frac{1}{2}\right)-2}{-\frac{1}{2}-\frac{1}{3}} = \frac{\frac{-5-4}{2}}{\frac{-3-2}{6}} = \frac{+9\times\cancel{6}^{3}}{\cancel{2}\times+5} = \frac{27}{5}$$

$$A_2 = F(z)\times\left(z-\frac{1}{3}\right)\Big|_{z=\frac{1}{3}}$$

$$= \frac{5z-2}{\left(z+\frac{1}{2}\right)\cancel{\left(z-\frac{1}{3}\right)}}\times\cancel{\left(z-\frac{1}{3}\right)}\Bigg|_{z=\frac{1}{3}}$$

$$= \frac{5\times\frac{1}{3}-2}{\frac{1}{3}+\frac{1}{2}} = \frac{\frac{5-6}{3}}{\frac{2+3}{6}}$$

$$= \frac{-1\times\cancel{6}^{2}}{\cancel{3}\times5}$$

$$= \frac{-2}{5}$$

$$\frac{H(z)}{z} = \frac{27/5}{z+\frac{1}{2}} + \frac{(-2/5)}{z-\frac{1}{3}}$$

$$\Rightarrow H(z) = \frac{27/5 \cdot z}{z+\frac{1}{2}} + \frac{-2/5 \cdot z}{z-\frac{1}{3}}$$

$$\Rightarrow H(z) = \frac{27/5}{1+\frac{1}{2}z^{-1}} + \frac{-2/5}{1-\frac{1}{3}z^{-1}}$$

$$H_A = \frac{27/5}{1+\frac{1}{2}z^{-1}}$$

$$H_1 = \frac{1}{1+\frac{1}{2}z^{-1}}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1+\frac{1}{2}z^{-1}}$$

$$\Rightarrow W(z) = X(z) - \frac{1}{2}z^{-1}W(z)$$

Taking inverse z transform,

$$\Rightarrow w(n) = x(n) - \frac{1}{2}w(n-1)$$

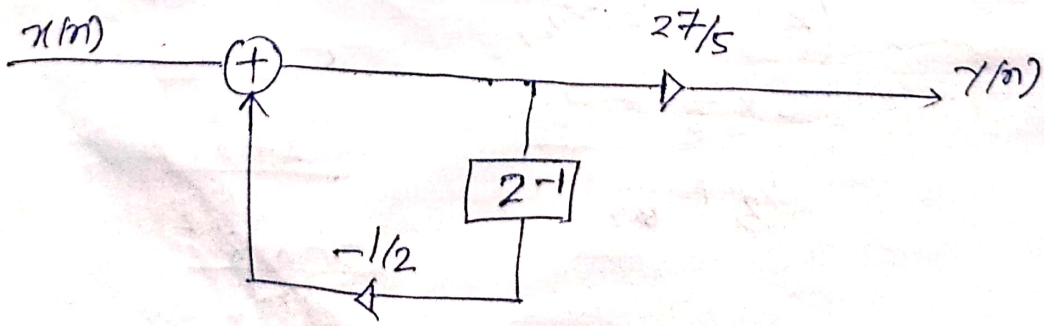
$$H_2 = \frac{27}{5}$$

$$\Rightarrow \frac{Y(z)}{W(z)} = \frac{27}{5}$$

$$\Rightarrow Y(z) = \frac{27}{5}W(z)$$

Taking inverse z transform

$$y(n) = \frac{27}{5}w(n)$$



$$H_B(z) = \frac{-2/5}{1-\frac{1}{3}z^{-1}}$$

$$H_1 = \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$\Rightarrow W(z) = X(z) + \frac{1}{3}z^{-1}W(z)$$

Taking inverse z transform

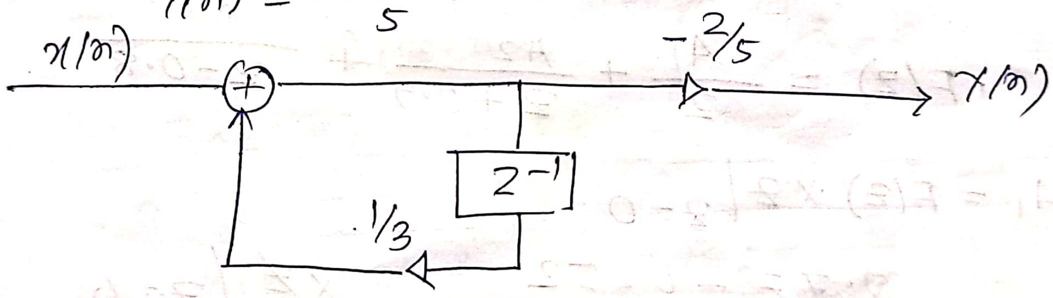
$$w(n) = x(n) + \frac{1}{3}w(n-1)$$

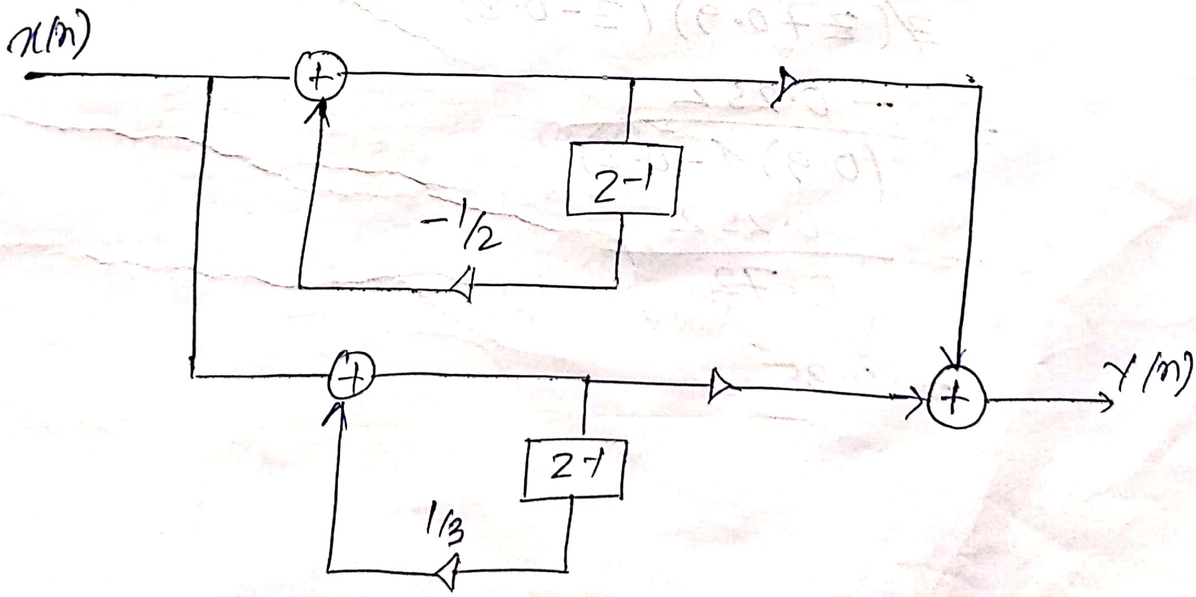
$$H_2 = \frac{-2}{5}$$

$$\Rightarrow \frac{Y(z)}{W(z)} = -\frac{2}{5} \Rightarrow Y(z) = -\frac{2}{5}W(z)$$

Taking inverse z transform

$$y(n) = -\frac{2}{5}w(n)$$





* Q. $y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7\, x(n) - 0.252\, x(n-2)$

Soln: $y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7\, x(n) - 0.252\, x(n-2)$

Taking inverse z transform

$$\Rightarrow Y(z) + 0.1 z^{-1} Y(z) - 0.72 z^{-2} Y(z) = 0.7 X(z) - 0.252 z^{-2} X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

$$\Rightarrow H(z) = \frac{0.7 - 0.252 z^{-2}}{(1+0.9z^{-1})(1-0.8z^{-1})}$$

Now,

$$\frac{H(z)}{z} = \frac{0.7z^2 - 0.252}{z(z+0.9)(z-0.8)}$$

$$F(z) = \frac{A_1}{z} + \frac{A_2}{z+0.9} + \frac{A_3}{z-0.8}$$

$$A_1 = F(z) \times z \Big|_{z=0}$$

$$= \frac{0.7z^2 - 0.252}{\cancel{z}(z+0.9)(z-0.8)} \times \cancel{z} \Big|_{z=0}$$

$$= \frac{-0.252}{(0.9)(-0.8)}$$

$$= \frac{0.252}{0.72}$$

$$= 0.35$$

$$A_2 = F(z) \times (z+0.9)$$

$$= \frac{0.7z^2 - 0.252}{z\cancel{(z+0.9)}(z-0.8)} \times \cancel{(z+0.9)} \Big|_{z=-0.9}$$

$$= \frac{0.7z^2 - 0.252}{z(z-0.8} \Big|_{z=-0.9}$$

$$= \frac{0.7(-0.9)^2 - 0.252}{(-0.9)(-0.9-0.8)}$$

$$= \frac{0.567 - 0.252}{1.53}$$

$$= 0.206$$

$$A_3 = F(z) \times (z-0.8) \Big|_{z=0.8}$$

$$= \frac{0.7z^2 - 0.252}{z(z+0.9)\cancel{(z-0.8)}} \times \cancel{(z-0.8)} \Big|_{z=0.8}$$

$$= \frac{0.7 \times (0.8)^2 - 0.252}{0.8(0.8+0.9)}$$

$$= \frac{0.448 - 0.252}{1.36} = \frac{0.196}{1.36} = 0.144$$

$$\therefore \frac{H(z)}{z} = \frac{0.35}{z} + \frac{0.206}{z+0.9} + \frac{0.144}{z-0.8}$$

$$H(z) = 0.35 + \frac{0.206}{1+0.9z^{-1}} + \frac{0.144}{1-0.8z^{-1}}$$

$$H_A = \frac{0.026}{1+0.9z^{-1}}$$

$$H_1 = \frac{1}{1+0.9z^{-1}}$$

$$\frac{w(z)}{X(z)} = \frac{1}{1+0.9z^{-1}}$$

$\Rightarrow w(z) = X(z) - 0.9z^{-1}w(z)$

Taking inverse z transform

$w(n) = x(n) - 0.9\,w(n-1)$

$H_2 = 0.026$

$$\frac{Y(z)}{w(X)} = 0.026$$

$\Rightarrow Y(z) = 0.026\,w(z)$

Taking inverse z transform

$y(n) = 0.026\,w(n)$

$$H_C = \frac{0.144}{1-0.8z^{-1}}$$

$$H_1 = \frac{1}{1-0.8z^{-1}}$$

$$\frac{w(z)}{X(z)} = \frac{1}{1-0.8z^{-1}}$$

$\Rightarrow w(z) = X(z) + 0.8z^{-1}w(z)$

Taking inverse z transform

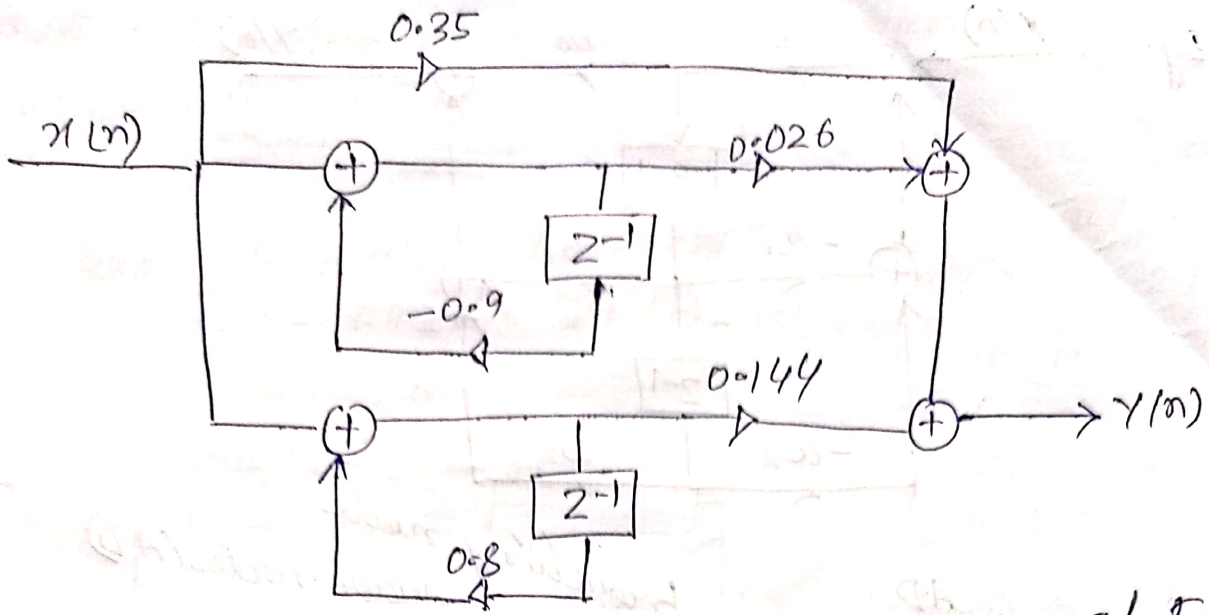
$w(n) = x(n) + 0.8\,w(n-1)$

$H_2 = 0.144$

$\frac{Y(z)}{w(z)} = 0.144 \Rightarrow Y(z) = 0.144\,w(z)$

Taking inverse z transform

$y(n) = 0.144\,w(n)$



★ Representation of Structure using Signal Flow Graph :–

Basically SFG is a graphical representation of the block diagram structure. Both the signal flow graph and block diagram structure provide the same information.

Steps to draw a signal flow graph –

① Replace all adders by adder nodes.

② Whenever there are different branches draw the branching nodes.

③ Keep the directions of arrows and the corresponding coefficient as it is.

④ Replace every delay element by simple transmittance branch and for the branch write z^{-1} to indicate the delay operation.

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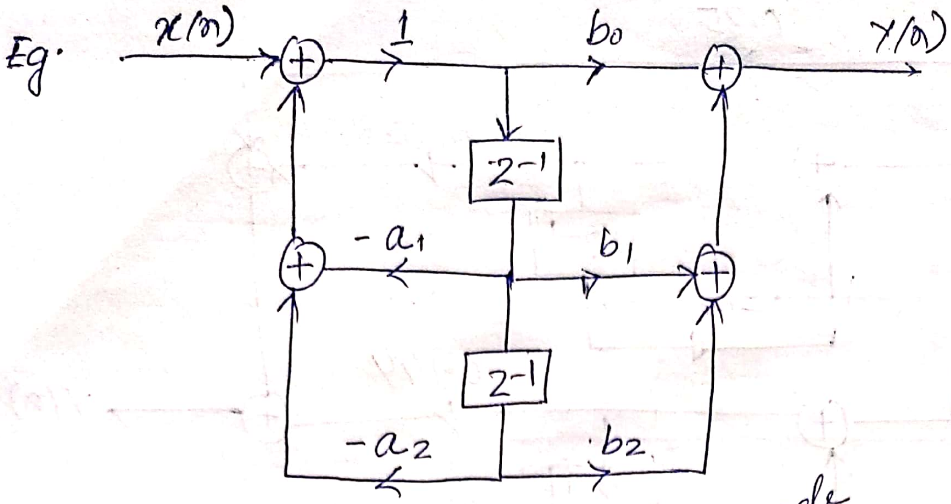
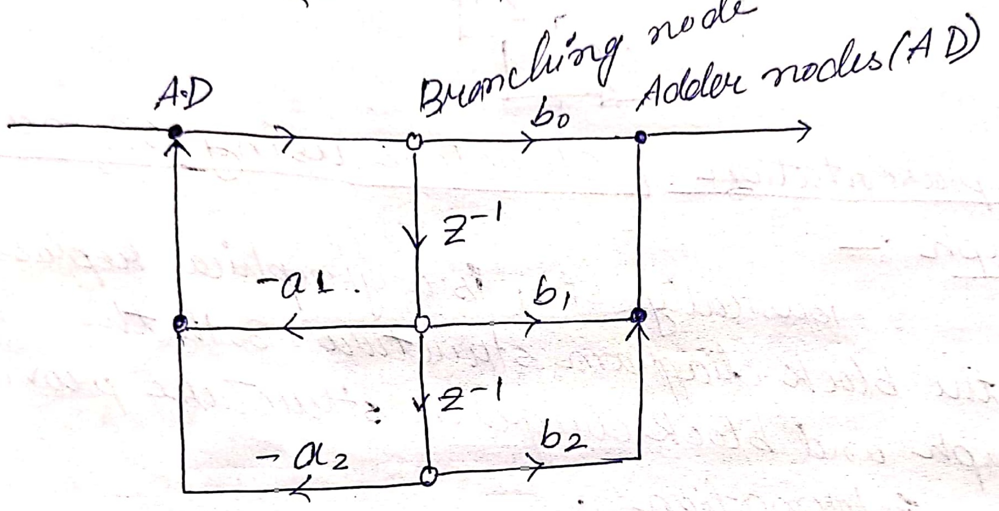
Eg.





FIR filter structure :-

The difference eqn of an FIR filter is given by $y(n) = \sum_{k=0}^{M-1} B_k\, x(n-k)$

(i) Direct form structure of FIR digital structures:-

The direct form realisation of FIR filter may be obtained by using the eqn of linear convolution. i.e $y(n) = x(n) * h(n)$

$$= h(n) * x(n)$$

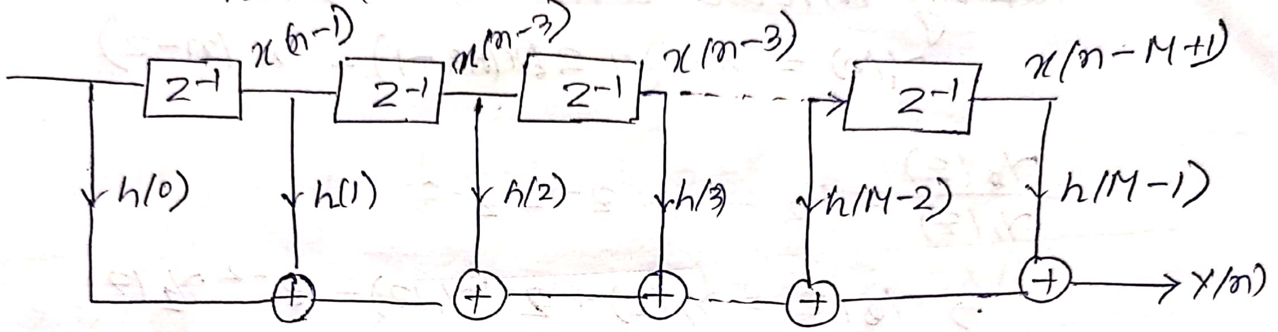
$$= \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k) \quad \text{— (i)}$$

Let us consider that there are M sample, thus eqn (1) becomes -

$$y(n) = \sum_{k=0}^{M-1} h(k)\,x(n-k)$$

expanding the above expression we get,

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + h(2)x(n-2) + \cdots + h(M-1)\,x(n-M+1)$$



Tapped Structure

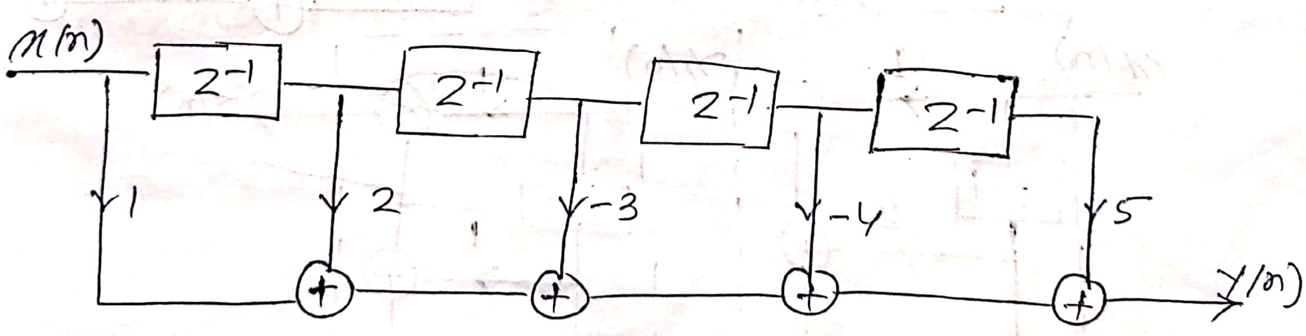
Q. $H(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

Soln: $\dfrac{Y(z)}{X(z)} = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

$$\Rightarrow Y(z) = X(z) + 2X(z)z^{-1} - 3z^{-2}X(z) - 4z^{-3}X(z) + 5z^{-4}X(z)$$

Taking inverse z transform

$$y(n) = x(n) + 2x(n-1) - 3x(n-2) - 4x(n-3) + 5x(n-4)$$



Q. $H(z) = (1+2z^{-1}-z^{-2})\cdot(1+z^{-1}-z^{-2})$

Soln: $H_1 = (1+2z^{-1}-z^{-2})$

$\Rightarrow \frac{Y_1(z)}{X(z)} = 1+2z^{-1}-z^{-2}$

$\Rightarrow Y_1(z) = X(z)+2z^{-1}X(z)-z^{-2}X(z)$

Taking inverse z transform,

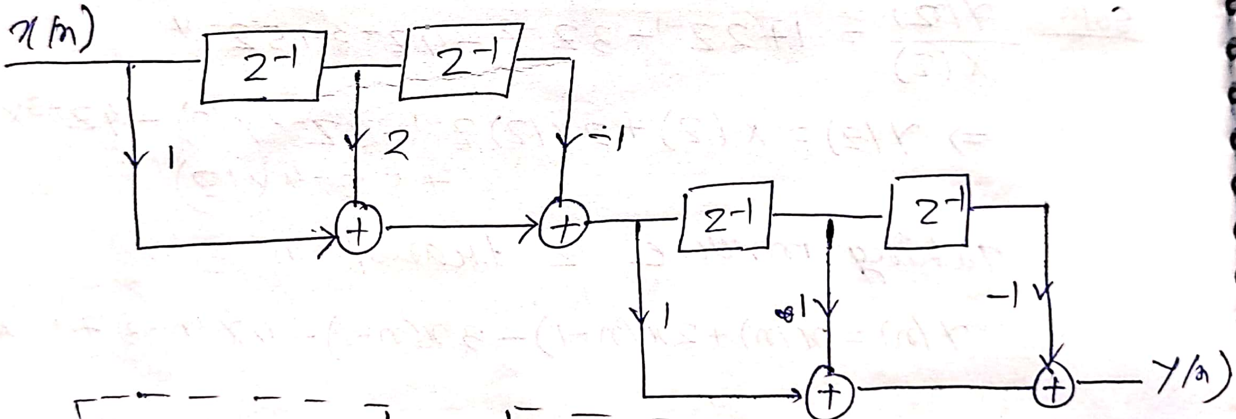
$y_1(n) = x(n)+2x(n-1)-x(n-2)$

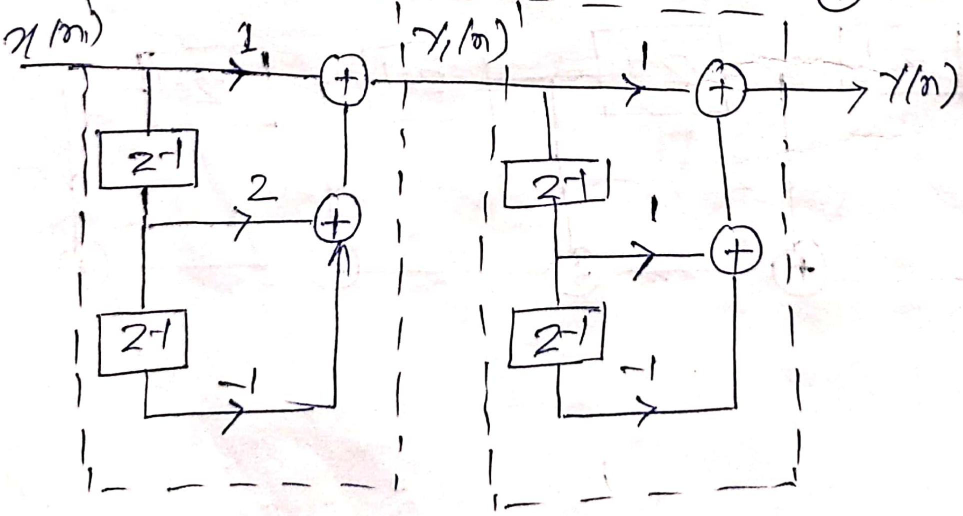
$\frac{Y_2(z)}{Y_1(z)} = H_2 = 1+z^{-1}-z^{-2}$

$\Rightarrow Y_2(z) = Y_1(z)+z^{-1}Y_1(z)-z^{-2}Y_1(z)$

Taking inverse z transform.

$y_2(n) = y_1(n)+y_1(n-1)-y_1(n-2)$





Q. $H(z) = (1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2})(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2})$

cascade and direct form of FIR

Soln:-

Cascade

$$H(z) = (1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2})(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2})$$

$$H_1 = (1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2})$$

$$\frac{Y_1(z)}{X(z)} = 1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}$$

$$\Rightarrow Y_1(z) = X(z) - \frac{1}{4}z^{-1}X(z) + \frac{3}{8}z^{-2}X(z)$$

Taking inverse z transform,

$$y_1(n) = x(n) - \frac{1}{4}x(n-1) + \frac{3}{8}x(n-2)$$

$$H_2 = 1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}$$

$$\frac{Y(z)}{Y_1(z)} = 1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}$$

$$Y(z) = Y_1(z) - \frac{1}{8}z^{-1}Y_1(z) - \frac{1}{2}z^{-2}Y_1(z)$$

Taking inverse z transform.

$$y(n) = y_1(n) - \frac{1}{8}y_1(n-1) - \frac{1}{2}y_1(n-2)$$

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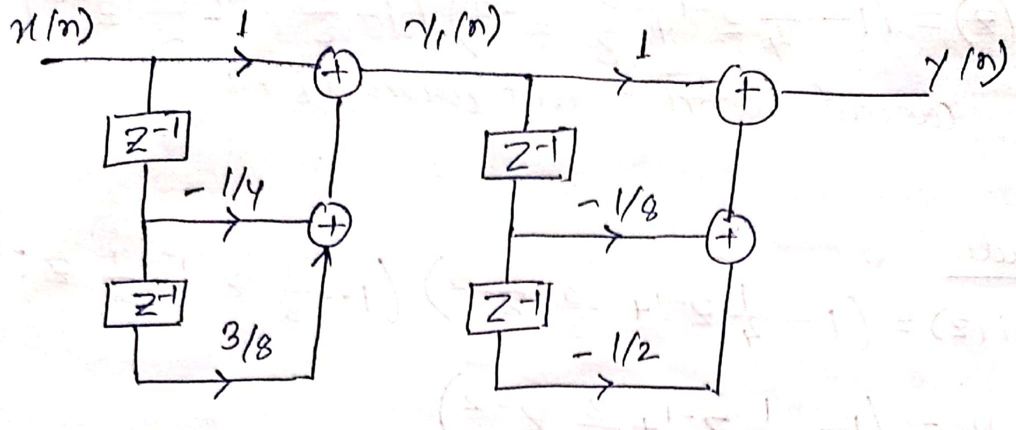


Direct form :

$$H(z) = \left(1-\frac{1}{4}z^{-1}+\frac{3}{8}z^{-2}\right)\left(1-\frac{1}{8}z^{-1}-\frac{1}{2}z^{-2}\right)$$

$$= 1-\frac{1}{8}z^{-1}-\frac{1}{2}z^{-2}-\frac{1}{4}z^{-1}+\frac{1}{32}z^{-2}+\frac{1}{8}z^{-3}$$
$$+\frac{3}{8}z^{-2}-\frac{3}{64}z^{-3}-\frac{3}{16}z^{-4}$$

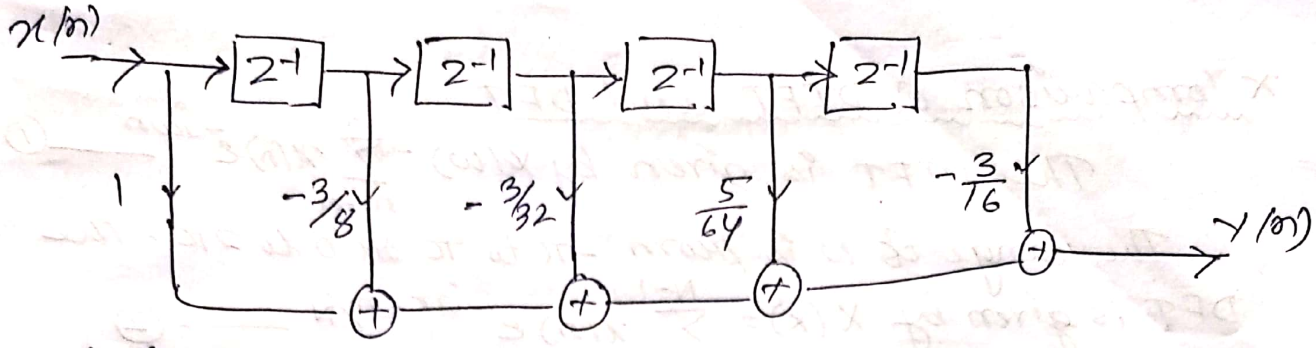
$$= 1-\left(\frac{1}{8}z^{-1}+\frac{1}{4}z^{-1}\right)+\left(\frac{1}{32}z^{-2}-\frac{1}{2}z^{-2}+\frac{3}{8}z^{-2}\right)+\left(\frac{1}{8}z^{-3}-\frac{3}{64}z^{-3}\right)$$
$$-\frac{3}{16}z^{-4}$$

$$\frac{Y(z)}{X(z)} = 1-\frac{3}{8}z^{-1}-\frac{3}{32}z^{-2}+\frac{5}{64}z^{-3}-\frac{3}{16}z^{-4}$$

$$\Rightarrow Y(z) = X(z)-\frac{3}{8}z^{-1}X(z)-\frac{3}{32}z^{-2}X(z)+\frac{5}{64}z^{-3}X(z)$$
$$-\frac{3}{16}z^{-4}X(z)$$

Taking inverse z transform

$$y(n) = x(n)-\frac{3}{8}x(n-1)-\frac{3}{32}x(n-2)+\frac{5}{64}x(n-3)$$
$$-\frac{3}{16}x(n-4)$$



Module 4 :

Discrete Fourier Transform :-

It is a finite duration discrete frequency sequence which is obtained by sampling one period of fourier transform. Sampling is done at N equally spaced points over the period extending from $\omega = 0$ to $\omega = 2\pi$

The discrete fourier transform of discrete time sequence $x(n)$ is denoted by $X(K)$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi K n/N}$$

Here, $K = 0, 1, 2, 3, \ldots, N-1$

Since this summation is taken for N points, it is called as N point DFT. The inverse discrete fourier transform (IDFT) is given by.

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j2\pi K n/N}$$

Here, $n = 0, 1, 2, 3, \ldots, N-1$

This called as N point IDFT.

Twiddle factor :

The twiddle factor is given by $W_N = e^{-j2\pi/N}$. The twiddle factor makes the computation of DFT a bit easy and fast. Using twiddle factor we can write eq^n of DFT as

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{Kn}$$

eq^n of IDFT is

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, W_N^{-Kn}$$

* Comparison of DTFT and DFT

The DTFT is given by $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ —— ①

The range of ω is from $-\pi$ to π or 0 to 2π. The DFT is given by $X(K) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi Kn/N}$ —— ②

Comparing eqn ① and ② we can say that DFT is obtained from DTFT by substituting $\omega = 2\pi \frac{K}{N}$. Hence $X(K) = X(\omega)\big|_{\omega = 2\pi K/N}$

By comparing DFT and DTFT we can write -

① The continuous frequency spectrum $X(\omega)$ is replaced by discrete fourier spectrum $X(K)$

② Infinite summation in DTFT is replaced by finite summation in DFT.

③ The continuous frequency variable is replaced by finite number of frequencies located as $2\pi K/nT_s$ where T_s is called sampling time.

Q. Obtained DFT of unit impulse $\delta(n)$

Soln: $X(K) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi/N\, kn}$

$$= \sum_{n=0}^{N-1} \delta(n) e^{-j2\pi/N\, kn}$$

$$= \cancel{0}\ 1$$

$$\delta(n) \xleftrightarrow{DFT} 1$$

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

Q. Obtain DFT of delayed unit impulse $\delta(n-n_0)$

Answer:

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi Kn/N}$$

$$= \sum_{n=0}^{N-1} \delta(n-n_0) e^{-j2\pi Kn/N}$$

But, $\delta(n-n_0)$ exist only at $n = n_0$

$$\therefore X(K) = e^{-j\frac{2\pi}{N}Kn_0}$$

$$\therefore \delta(n-n_0) \xleftrightarrow{DFT} e^{-j\frac{2\pi}{N}Kn_0}$$

And $\delta(n+n_0) \xleftrightarrow{DFT} e^{j\frac{2\pi}{N}Kn_0}$

Q. Compute N-point DFT of $x(n) = a^n u(n)$ for $0 \leq n \leq N-1$.

Answer: $X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$

$$X(K) = \sum_{n=0}^{N-1} a^n u(n) e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} a^n e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} \left(ae^{-j\frac{2\pi}{N}k}\right)^n$$

Let us use the following summation expression

$$\sum_{K=N_1}^{N_2} A^K = \frac{A^{N_1} - A^{N_2+1}}{1-A}$$

$$\therefore \sum_{n=0}^{N-1} \left(ae^{-j2\pi/N\, k}\right)^n = \frac{\left(ae^{-j\frac{2\pi}{N}k}\right)^0 - \left(ae^{-j\frac{2\pi}{N}k}\right)^N}{1-ae^{-j\frac{2\pi}{N}k}}$$

$$= \frac{1 - a^N e^{-j\frac{2\pi}{N}kN}}{1-ae^{-j\frac{2\pi}{N}k}}$$

$$= \frac{1-a^N e^{-j2\pi K}}{1-ae^{-j\frac{2\pi}{N}K}}$$

Now, $e^{-2j\pi K} = \cos 2\pi K - j\sin 2\pi K = 1$

$$X(K) = \frac{1-a^N}{1-ae^{-j\frac{2\pi}{K}K}}$$

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Q. Compute the DFT of $w(n) = u(n) - u(n-4)$

Soln: $X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$

$$= \sum_{n=0}^{3} \cancel{x(n)\, e}\ 1 \cdot e^{-j\frac{2\pi}{N}kn}$$

$$X(0) = \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}0\cdot n}$$

$$= \cancel{\sum_{n}} e^{-j\frac{2\pi}{N}0\cdot 0} + e^{-j\frac{2\pi}{N}\cdot 0\cdot 1} + e^{-j\frac{2\pi}{N}\cdot 0\cdot 2} + e^{-j\frac{2\pi}{N}\cdot 0\cdot 3}$$

$$= 1 + 1 + 1 + 1 = 4$$

$$X(1) = \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}n}$$

$$= \cancel{\sum} e^{-j\frac{2\pi}{4}\cdot 0} + e^{-j\frac{2\pi}{4}\cdot 1} + e^{-j\frac{2\pi}{4}\cdot 2} + e^{j\frac{2\pi}{4}\cdot 3}$$

$$= 1 - j - 1 + j = 0$$

$$X(2) = \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{4}\cdot 2\cdot n}$$

$$= e^{-j\frac{2\pi}{4}\cdot 2\cdot 0} + e^{-j\frac{2\pi}{4}\cdot 2\cdot 1} + e^{-j\frac{2\pi}{4}\cdot 2\cdot 2} + e^{-j\frac{2\pi}{4}\cdot 2\cdot 3}$$

$$= 1 - 1 + 1 - 1 = 0$$

$$X(3) = \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{4}\cdot 3\cdot n}$$

$$= e^{-j\frac{2\pi}{4}\cdot 3\cdot 0} + e^{-j\frac{2\pi}{4}\cdot 3\cdot 1} + e^{-j\frac{2\pi}{4}\cdot 3\cdot 2} + e^{-j\frac{2\pi}{4}\cdot 3\cdot 3}$$

$$= 1 + j - 1 - j = 0$$

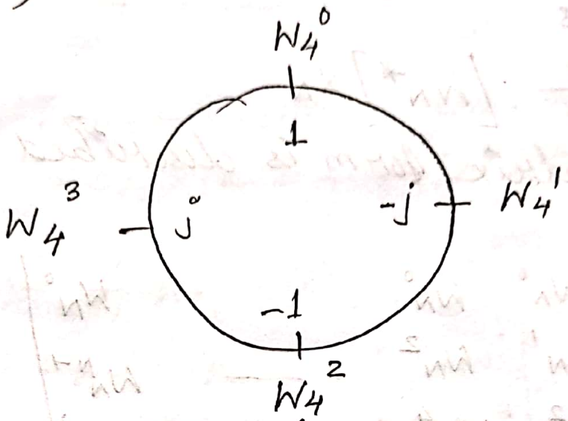
$$X(K) = \{4, 0, 0, 0\}$$

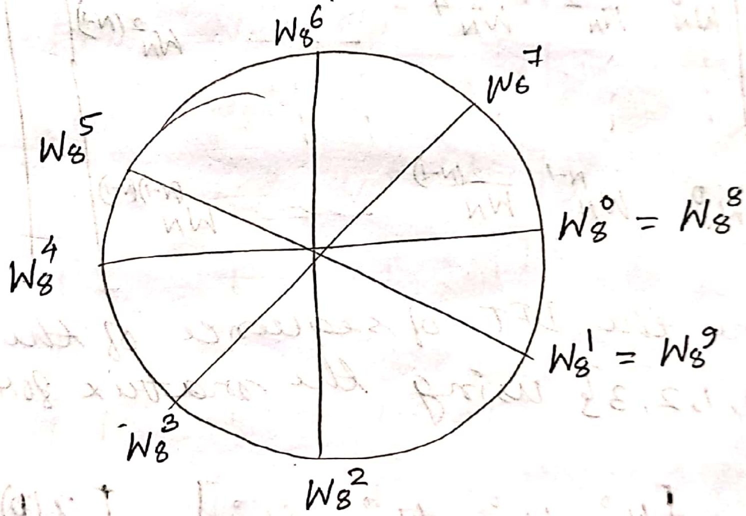
* Cyclic property of twiddle factor:

$W_4^0 = 1$, $W_4^1 = \left(e^{-j\frac{2\pi}{4}}\right)^1 = -j$, $W_4^2 = \left(e^{-j\frac{2\pi}{4}}\right)^2 = -1$

$W_4^3 = \left(e^{-j\frac{2\pi}{4}}\right)^3 = j$, $W_4^4 = \left(e^{-j\frac{2\pi}{4}}\right)^4 = 1$, $W_4^5 = \left(e^{-j\frac{2\pi}{4}}\right)^5 = -j$

$W_4^6 = \left(e^{-j\frac{2\pi}{4}}\right)^6 = -1$, $W_4^7 = \left(e^{-j\frac{2\pi}{4}}\right)^7 = j$





$W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -j0.707$

$W_8^4 = -1$

$W_8^5 = -0.707 + j0.707$

$W_8^6 = +j$

$W_8^7 = 0.707 + j0.707$

Matrix Formulation of DFT

The DFT can be represented in the Matrix form as

$X(k) = [W_N] x_N$

Similarly the IDFT can be expressed in a matrix form as

$$x_n = \frac{1}{N} \cdot [W_N^*] X_N$$

The full matrix form is described as

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{bmatrix} = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & \cdots & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & \cdots & W_N^{N-1} \\ W_N^0 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ W_N^0 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{bmatrix}$$

Q. Determine the DFT of sequence of the sequence $x(n) = \{0, 1, 2, 3\}$ using the matrix form

Soln:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$X(K) = \{6, -2+2j, -2, -2-2j\}$$

Q - Compute the four length sequence from its IDFT which is given by $X(K) = \{4, 1-j, -2, 1+j\}$

Sol^n :- $x(n) = \frac{1}{N} W_N^* X(K)$ $N = 4$

$$W_N = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix}$$

$$W_N^n = e^{-j\frac{2n\pi}{N}}$$

$$W_N^{n*} = e^{j\frac{2n\pi}{N}}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\therefore W_N^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$x(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 4 + 1-j-2+1+j \\ 4 + j(1-j) - 1(-2) - j(1+j) \\ 4 - 1(1-j) + 1(-2) - 1(1+1) \\ 4 - j(1-j) - 1(-2) + j(1+j) \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4 \\ 8 \\ 0 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Properties of DFT :-

(1) **Periodicity** :- This property states that if a discrete time signal is periodic then its DFT is also be periodic.

If DFT of $x(n)$ is $X(K)$ i.e

$$x(n) \xleftrightarrow{DFT} X(K)$$ and $x(n)$ is periodic with period N

$$x(n) = x(n+N)$$ for all n then $x(K)$ is also periodic with period N.

i.e $X(K) = X(K+N)$

Proof :-

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$X(K+N) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}(K+N)n}$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}Kn} \cdot e^{-j\frac{2\pi}{N}Nn}$$

$$= \sum_{n=0}^{N-1} x(n) W_N^{Kn} \cdot e^{-j2\pi n}$$

Now, $W_N = e^{-j\frac{2\pi}{N}}$

$$W_N^{kNn} = e^{-j\frac{2\pi}{N}N.n} = e^{-j2\pi n}$$

$$e^{-j2\pi n} = \cos 2\pi n - j\sin 2\pi n = 1 - 0 = 1$$

$$X(K+N) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{kn} \cdot 1$$

$$= X(K)$$

Hence $X(K+N) = X(K)$ //

2. Linearity :-

Linearity property states that DFT of linear combination of two or more signals is equal to the sum of linear combination of DFT of individual signals.

Mathematically,

If $x_1(n) \xleftrightarrow{DFT} X_1(K)$ and

$x_2(n) \xleftrightarrow{DTF} X_2(K)$

then $ax_1(n) + bx_2(n) \xleftrightarrow{DFT} aX_1(K) + bX_2(K)$

Here a and b are some constants

Proof : $X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

$$X_1(K) = \sum_{n=0}^{N-1} x_1(n) W_N^{kn}$$

$$X_2(K) = \sum_{n=0}^{N-1} x_2(n) W_N^{kn}$$

Let $ax_1(n) + bx_2(n) \xleftrightarrow{DFT} X(K')$

$$X(K') = \sum_{n=0}^{N-1} \{ax_1(n) + bx_2(n)\} W_N^{kn}$$

$$= \sum_{n=0}^{N-1} ax_1(n) W_N^{kn} + \sum_{n=0}^{N-1} bx_2(n) W_N^{kn}$$

$$= aX_1(K) + bX_2(K)$$

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Hence, $a x_1(n) + b x_2(n) \xleftrightarrow{DFT} a X_1(K) + b X_2(K)$

★ Circular time shift of a sequence :-

This property states that shifting the sequence in time domain by m samples is equivalent to multiplying the sequence in frequency domain by $e^{-j\frac{2\pi k m}{m}}$

Mathematically, if $x(n) \xleftrightarrow{DFT} X(K)$

Then, $x(n-m) \xleftrightarrow{DFT} e^{-j\frac{2\pi K m}{N}} X(K)$

or $x(n-m) \longleftrightarrow W_N^{Km} X(K)$

Proof :

$$DFT[x(n)] = X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}.$$

$$DFT[x(n-m)] = \sum_{n=0}^{N-1} x(n-m) e^{-j\frac{2\pi}{N}kn}$$

$$= \underbrace{\sum_{n=0}^{m-1} x(n-m) e^{-j\frac{2\pi}{N}kn}}_{A} + \underbrace{\sum_{n=m}^{N-1} x(n-m) e^{-j\frac{2\pi}{N}kn}}_{B}$$

if $x(n) = x(n+N)$

then $x(n-m) = x(n-m+N)$

For A,

$$\sum_{n=0}^{m-1} x(n-m) e^{-j\frac{2\pi}{N}kn}$$

$$A = \sum_{n=0}^{m-1} x(n-m+N) e^{-j\frac{2\pi}{N}kn}$$

Let, $N - m + n = l$

$\Rightarrow n = l + m - N$

when, $n=0$, $l=N-m$

when, $n=m-1$, $l=N-1$

$$A=\sum_{l=N-m}^{N-1} x(l)e^{-j\frac{2\pi}{N}k(l+m-N)}$$

$$A=\sum_{l=N-m}^{N-1} x(l)e^{-j\frac{2\pi}{N}k.l}\cdot e^{-j\frac{2\pi}{N}km}\cdot 1$$

$$=\sum_{l=N-m}^{N-1} x(l)\cdot e^{-j\frac{2\pi}{N}k(l+m)} \quad \text{—— (i)}$$

Simillarly for B,

let, $n-m+N=l$

$\Rightarrow n=l+m-N$

when, $n=m$, $l=N$

when, $n=N-1$, $l=2N-m-1$

$$\sum_{n=m}^{N-1} x(n-m)e^{-j\frac{2\pi}{N}kn}=\sum_{n=m}^{N-1} x(n-m+N)e^{-j\frac{2\pi}{N}kn}$$

$$B=\sum_{l=N}^{2N-m-1} x(l)e^{-j\frac{2\pi}{N}k(l+m)} \quad \text{—— (ii)}$$

$$\therefore DFT\{x(n-m)\}=\sum_{l=N-m}^{N-1} x(l)e^{-j\frac{2\pi}{N}k(l+m)}+\sum_{l=N}^{2N-m-1} x(l)e^{-j\frac{2\pi}{N}k(l+m)}$$

Let the summation runs from $l=0$ to $N-1$

$$\therefore DFT\{x(n-m)\}=\sum_{x=0}^{N-1} x(l)e^{-j\frac{2\pi}{N}kl}\cdot e^{-j\frac{2\pi}{N}km}$$

$$=e^{-j\frac{2\pi}{N}km}\cdot\sum_{x=0}^{N-1} x(l)e^{-j\frac{2\pi}{N}kl}$$

Let $l=n$,

$$\therefore DFT\{x(n-m)\}=e^{-j\frac{2\pi}{N}km}\cdot X(K)$$

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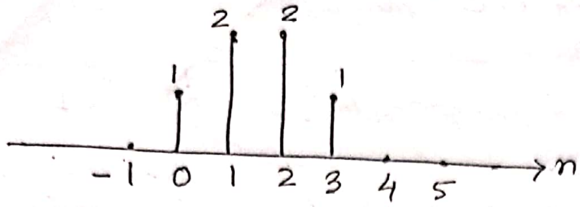
Q. Consider the finite length sequence $x(n)$ in figure below, the 5 point DFT of $x(n)$ is denoted by $X(K)$ plot the sequence post DFT is $Y(K) = e^{-j4\pi K/5}\, X(K)$



Soln: $\text{DFT}\left[x(n-m)_N\right] = e^{-j\frac{2\pi K m}{N}} X(K)$

$\text{DFT}\left[x(n-m)_5\right] = e^{-j\frac{2\pi K(2)}{5}}\, x(K)$

$\text{DFT}\left[y(n)\right] = Y(K)$

Here, $m = 2$

$y(n) = x(n-m)_5$

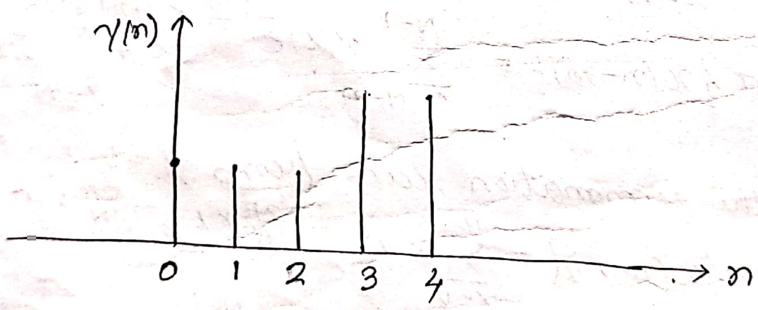
$y(0) = x(0-2)_5 = x(-2)_5 = x(-2+5)_5 = x(3)_5 = 1$

$y(1) = x(1-2)_5 = x(-1)_5 = x(-1+5)_5 = x(4)_5 = 1$

$y(2) = x(2-2)_5 = x(0) = 1$

$y(3) = x(3-2)_5 = x(1) = 2$

$y(4) = x(4-2)_5 = x(2) = 2$



Q. If the DFT of the sequence $x(n) = \{1, 2, 1, 1, 2, -1\}$ is $X(K)$ plot the sequence whose DFT is $Y(K) = e^{-j\pi K} X(K)$

Soln: $DFT[x(n-m)]_N = e^{-j\frac{2\pi}{N}km} X(K)$

$DFT[x(n-m)_6] = e^{-j\frac{2\pi K}{6} 3} X(K)$

$DFT[y(n)] = Y(K)$

Here, $m = 3$

$y(n) = x(n-m)$

$y(0) = x(0-3) = x(-3) = x(-3+6) = x(3) = 1$

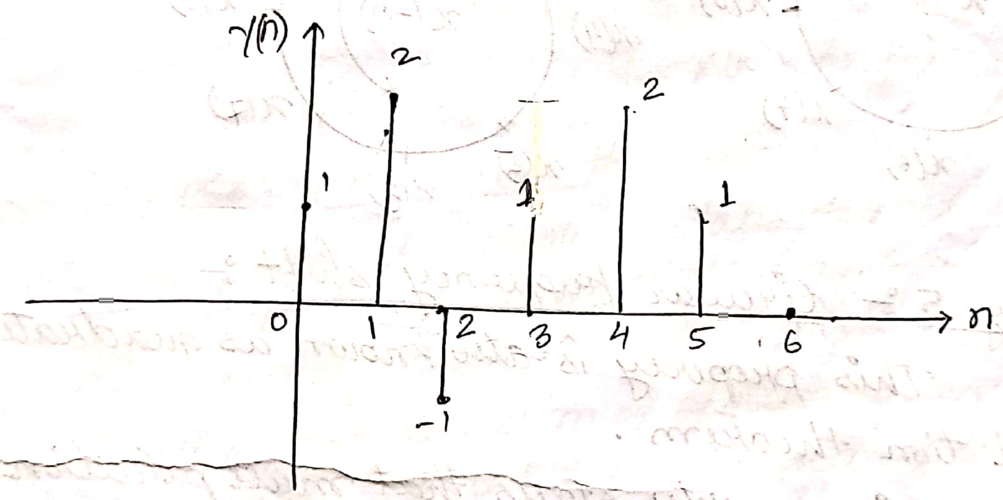
$y(1) = x(1-3) = x(-2) = x(-2+6) = x(4) = 2$

$y(2) = x(2-3) = x(-1) = x(-1+6) = x(5) = -1$

$y(3) = x(3-3) = x(0) = 1$

$y(4) = x(4-3) = x(1) = 2$

$y(5) = x(5-2) = x(2) = 1$



Property 4: Time reversal of a sequence:-

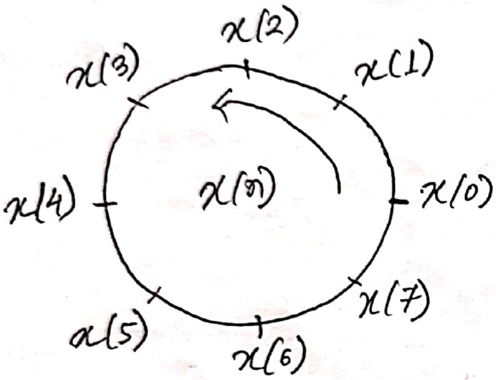
This property states that if a sequence is circularly folded, its DFT is also circularly folded. The time reversal of a N point sequence $x(n)$ is obtained by wrapping the sequence $x(n)$ around the circle in the clockwise direction. It is denoted by $x(-n)_N$ and $x(-n)_N = x(N-n)$; $0 \leq n \leq N-1$

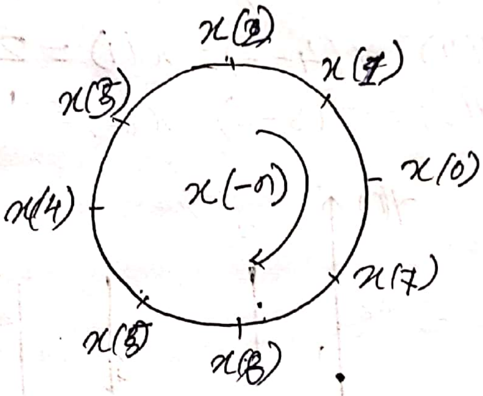
Mathematically; if $DFT[x(n)] = X(K)$, then

$$DFT[x(-n)_N] = DFT[x(N-n)]$$

$$= X(-K)_N$$

$$= X(N-K)$$





Property 5:- Circular frequency shift:-

This property is also known as quadrature modulation theorem.

This property states that multiplication of a sequence $x(n)$ by $e^{\pm j\frac{2\pi K l}{N}}$ is equivalent to the circular shift of DFT in time domain by l samples.

Mathematically, if $DFT[x(n)] = X(K)$, then

$$DFT\left[x(n)e^{j\frac{2\pi K l}{N}}\right] = X(K-l)_N$$

or $DFT\left[x(n)e^{-j\frac{2\pi K l}{N}}\right] = X(K+l)_N$

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Property 6: Complex conjugate property:

The DFT of complex conjugate of the sequence is equal to the complex conjugate of the DFT of that sequence with the sequence delayed by K samples in the frequency domain.

Mathematically, if

$$DFT[x(n)] = X(K) \text{ then}$$

$$DFT[x^*(n)] = X^*(N-K)$$

$$= X^*(-K)_N$$

Property 7: Circular convolution:-

This property states that the multiplication of two DFT's is equivalent to circular convolution of their sequences in time domain.

Mathematically if

$$DFT[x_1(n)] = X_1(K) \text{ and}$$

$$DFT[x_2(n)] = X_2(K) \text{ then}$$

$$DFT[x_1(n) \circledast x_2(n)] = X_1(K) \cdot X_2(K)$$

Property 8: Parseval's theorem:-

The perseval's theorem states that the DFT is an energy conserving transformation and allows us to find the signal energy either from the signal or its spectrum. This implies that the sum of ~~sequen~~ squares of the signal samples is related to the sum of squares of the magnitude of the DFT samples.

Mathematically if,

$$DFT[x_1(n)] = X_1(K) \text{ and}$$

$$DFT[x_2(n)] = X_2(K) \text{ then}$$

$$\sum_{n=0}^{N-1} x_1(n)\, x_2^*(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, X_2^*(K)$$

if $x_1(n) = x_2(n)$ then

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{K=0}^{N-1} |X(K)|^2$$

Property 9: Circular correlation :-

Circular cross correlation of two sequences in time domain is equivalent to the multiplication of DFT of one sequence with the complex conjugate DFT of other sequence.

Mathematically if

$$DFT[x(n)] = X(K) \text{ and}$$
$$DFT[Y(n)] = Y(K) \text{ then}$$

$$DFT[r_{xy}(l)] = X(K)\,Y^*(K)$$

Property 10: Multiplication of two sequence

The multiplication of two sequence in time domain is equivalent to its circular convolution in the frequency domain.

Mathematically, if

$$DFT[x_1(n)] = X_1(K) \text{ and}$$
$$DFT[x_2(n)] = X_2(K) \text{ then}$$
$$DFT[x_1(n) \cdot x_2(n)] = \frac{1}{N}\left[X_1(K) \, Ⓝ \, X_2(K)\right]$$

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Circular convolution:

The methods used ~~in~~ to find circular convolution of two sequence are —

① Concentric circle method
② Matrix multiplication method.

① Concentric circle method:

Given two sequences $x1(n)$ and $x2(n)$, the circular convolution of this two sequences represented as- $y(n) = x_1(n) \circledN x_2(n)$, can be found using the following steps —

(i) Plot N samples of $x_1(n)$ as equally spaced points around an outer circle in anticlockwise direction.

(ii) Start at the same point as $x_1(n)$ plot N samples of $x_2(n)$ as equally spaced points around an inner circle in clockwise direction

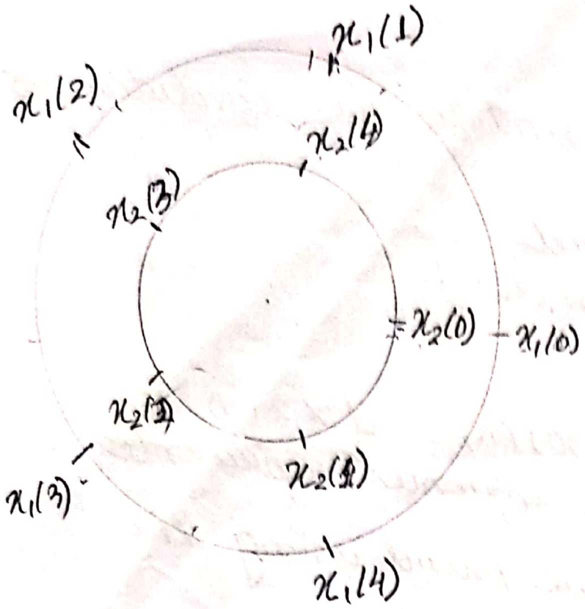
(iii) Multiply corresponding samples on the two circles and sum the products to produce output.

(iv) Rotate the inner circle. 1 sample at a time in anticlockwise direction and go to step no 3 to obtain the next value of output.

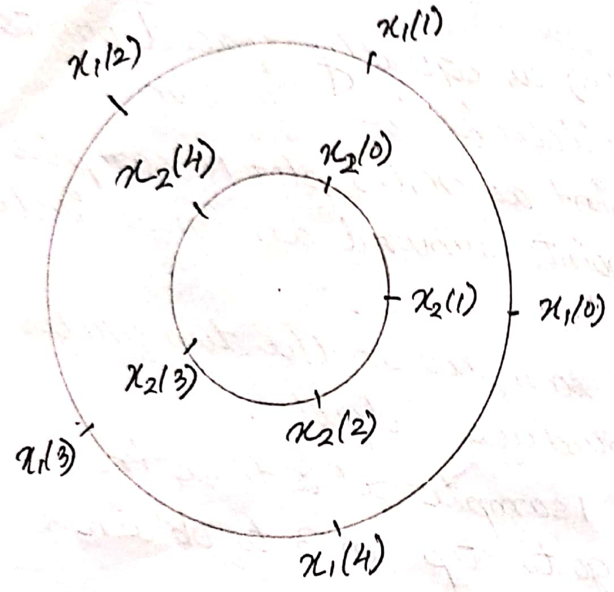
(v) Repeat step no 4 untill the inner circle 1st sample lines up with the first sample of the exterior circle once again.

Q. $x_1(n) = \{1, -1, -2, 3, -1\}$
$x_2(n) = \{1, 2, 3\}$

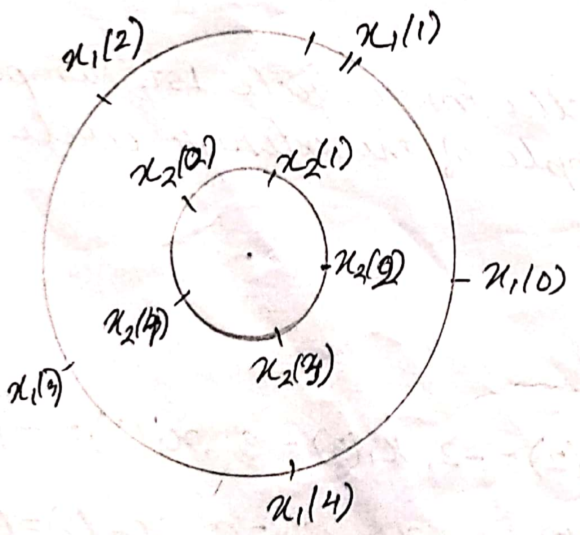
Ans: $x_1(0) = 1$, $x_1(1) = -1$, $x_1(2) = -2$, $x_1(3) = 3$, $x_1(4) = -1$
$x_2(0) = 1$, $x_2(1) = 2$, $x_2(2) = 3$, $x_2(3) = 0$, $x_2(4) = 0$



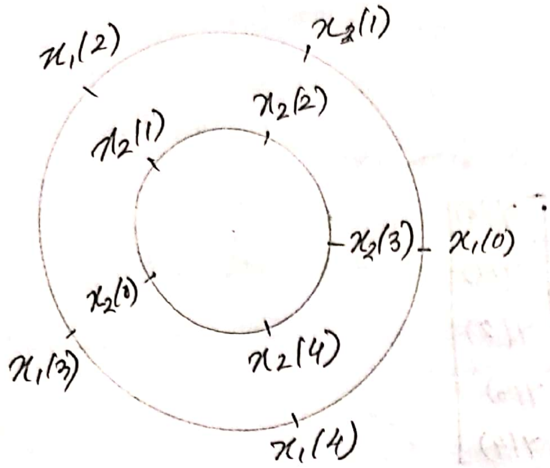
$$y(0) = 1\times1+(-1)\times0+(-2)\times0+3\times3+(-1)\times2$$
$$= 1+9-2$$
$$= 8$$



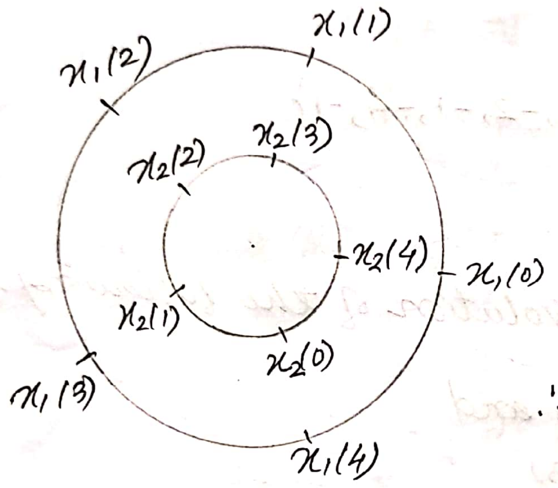
$$y(1) = 1\times2+(-1)\times1+(-2)\times0+3\times0+(-1)\times3$$
$$= 2-1-3$$
$$= -2$$



$$y(2) = 1\times3+(-1)\times2+(-2)\times1+3\times0+(-1)\times0$$
$$= 3-2-2$$
$$= -1$$



$$Y(3) = 1\times0 + (-1)\times3 + (-2)\times2 + 3\times1 + (-1)\times0$$
$$= 0-3-4+3$$
$$= -4$$



$$Y(4) = 1\times0 + (-1)\times0 + (-2)\times3 + 3\times2 + (-1)\times1$$
$$= 0+0-6+6-1 = -1$$

$$\therefore Y(n) = \{8, -2, -1, -4, -1\}$$

2) Matrix multiplication method :-

In the matrix multiplication method the sequence $x_1(n)$ is represented as column matrix. The sequence $x_2(n)$ is repeated via circular shift of samples and represented in N/N matrix form

$$\begin{bmatrix} x_2(0) & x_2(N-1) & x_2(N-2) & \cdots & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(N-1) & \cdots & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & \cdots & x(4) & x_2(3) \\ \vdots & & & & & \vdots \\ x_2(N-2) & x_2(N-3) & x_2(N-4) & \cdots & x_2(0) & x(N-1) \\ x_2(N-1) & x_2(N-2) & x_2(N-3) & \cdots & x_2(1) & x(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ \vdots \\ x_1(N-2) \\ x_1(N-1) \end{bmatrix} = \begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ \vdots \\ Y(N-2) \\ Y(N-1) \end{bmatrix}$$

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Q. $x_1(n) = \{1, -1, -2, 3, -1\}$

$x_2(n) = \{1, 2, 3\}$

Soln:

$$\begin{bmatrix} 1 & -1 & 3 & -2 & -1 \\ -1 & 1 & -1 & 3 & -2 \\ -2 & -1 & 1 & -1 & 3 \\ 3 & -2 & -1 & 1 & -1 \\ -1 & 3 & -2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 \\ -2 \\ -1 \\ -4 \\ -1 \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \end{bmatrix}$$

$y(n) = \{8, -2, -1, -4, -1\}$

Q. Find the 4 point circular convolution of the following sequence -

$x(n) = \delta(n) + 2\delta(n-2) + \delta(n-3)$ and

$h(n) = \delta(n) + \delta(n-2) + 2\delta(n-3)$

Soln: $x(0) = 1,\ x(1) = 0,\ x(2) = 2,\ x(3) = 1$

$h(0) = 1\ \ h(1) = 0,\ h(2) = 1,\ h(3) = 2$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 \\ 1 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix}$$

$y(0) = 1\times1 + 1\times0 + 2\times1 + 0\times2 = 3$

$y(1) = 0\times1 + 1\times0 + 1\times1 + 2\times2 = 5$

$y(2) = 2\times1 + 0\times0 + 1\times1 + 1\times2 = 5$

$y(3) = 1\times1 + 2\times0 + 0\times1 + 1\times2 = 3$

$\therefore y(n) = \{3, 5, 5, 3\}$

Q. DFT of a sequence $x(n)$ is given by $X(K) = \{6, 0, -2, 0\}$

(i) Determine $x(n)$

(ii) plot $x_1(n)$ if $X_1(K) = X(K)\, e^{-j2\pi K/2}$

(iii) Determine circular auto correlation of $x(n)$ using DFT and IDFT only

Soln:-

(i) $x(n) = \frac{1}{N} W_N^* X(K)$

$$W_N^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$x(N) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ -2 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 6-2 \\ 6+2 \\ 6-2 \\ 6+2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

(ii) $X_1(K) = x_1(n)\, e^{-j2\pi K/2}$

$$DFT\,[x(n-m)_N] = x(n)\, e^{-j\frac{2\pi K m}{N}}$$

$$= x(n)\, e^{-j\frac{2\pi K \cdot 2}{4}}$$

Here $m = 2$

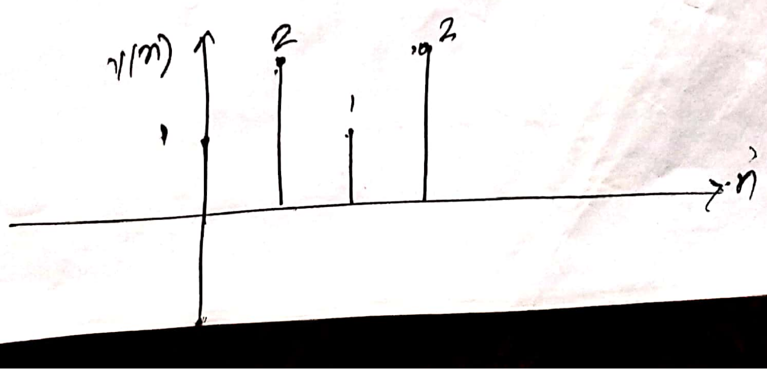
$N = 4$

$y(0) = x(0-2)_4 = x(-2) = x(-2+4) = x(2) = 1$

$y(1) = x(1-2)_4 = x(-1) = x(-1+4) = x(3) = 2$

$y(2) = x(2-2)_4 = x(0) = 1$

$y(3) = x(3-2)_4 = x(1) = 2$



(c) $DFT[r_{xx}(l)] = X(K)\cdot X^{*}(K)$

$X(K) = \{6, 0, -2, 0\}$

$X^{*}(K) = \{6, 0, -2, 0\}$

$\therefore DFT[r_{xx}(l)] = \{36, 0, 4, 0\}$

$$r_{xx}(l) = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 36 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 36+4 \\ 36-4 \\ 36+4 \\ 36-4 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 40 \\ 32 \\ 40 \\ 32 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 10 \\ 8 \end{bmatrix}$$

$r_{xx}(l) = \{10, 8, 10, 8\}$