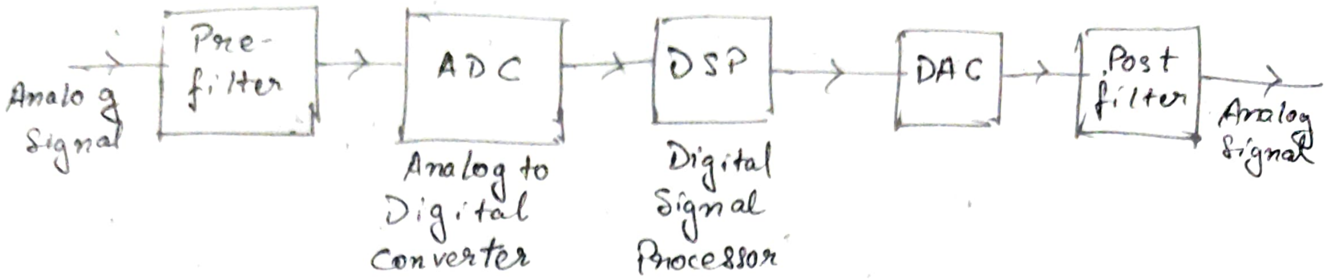
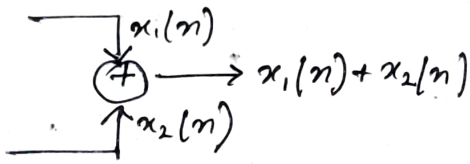
DSP



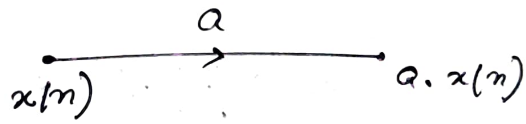
Digital Systems → Digital filters → Linear time Invariant Systems - (LTI).

Blocks in DSP

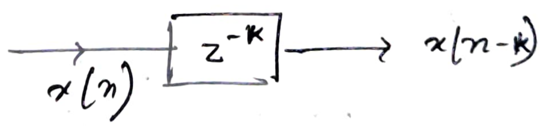
1) Adder



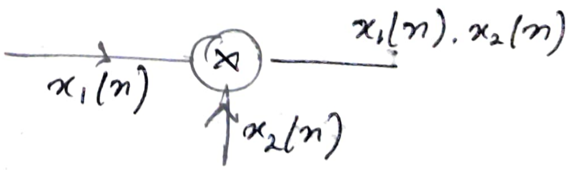
2) Const Multiplier



3) Delay element



4) Signal Multiplier



5) Time Advance Element

Realization of IIR & FIR filters

LTI system Difference eq^n $\Rightarrow$ $y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \underbrace{\sum_{k=0}^{M} b_k x(n-k)}_{\text{FIR filter}}$, $M \leq N$

b_k = Delay dependent feed forward co-efficient

a_k = Delay dependent feedback co-efficient.

Taking z-transform.

$$\sum_{n=-\infty}^{\infty} y(n) z^{-n} = \sum_{n=-\infty}^{\infty} \left[-\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \right] z^{-n}$$

$$\Rightarrow Y(z) = -\sum_{k=1}^{N} a_k \left[\sum_{n=-\infty}^{\infty} y(n-k) z^{-n} \right] + \sum_{k=0}^{M} b_k \left[\sum_{n=-\infty}^{\infty} x(n-k) z^{-n} \right]$$

Property of z-transform

$$\Rightarrow Y(z) = -\sum_{k=1}^{N} a_k \, Y(z) z^{-k} + \sum_{k=0}^{M} b_k \, X(z) z^{-1}$$

$$\Rightarrow Y(z) \left\{ 1 + \sum_{k=1}^{N} a_k z^{-k} \right\} = X(z) \sum_{k=0}^{M} b_k z^{-k}$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$\boxed{H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}}$$

$\rightarrow$ Transf^n f^n for Digital filters

IIR filter

Numerator + Denominator

Only Denominator

~~pole zeroes~~

$$\frac{\text{zeroes}}{\text{poles}}$$

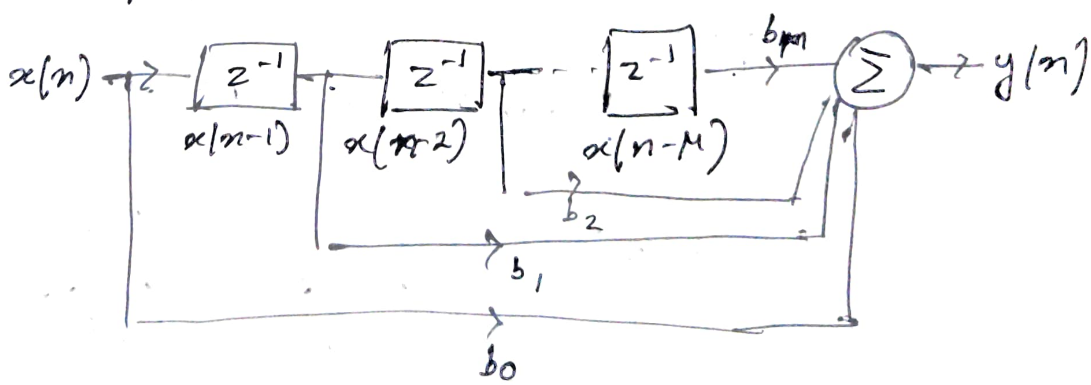
FIR

$$\boxed{H(z) = \sum_{k=0}^{M} b_k z^{-k}}$$ ← Only numerator.

FIR filter	IIR filter
1. Finite Impulse response $h(n) = \{1, 2, 3, 4\}$	1) $h(n) = \{1, 2, 3, \dots\}$
2) All zero filter	2) All pole or pole-zero filter.
3) Inherently stable.	3) Stable or unstable (position of poles.
4) Implement → Convolution	4) Difference eq^n
5) Non-Recursive (no feedback)	5) Recursive.

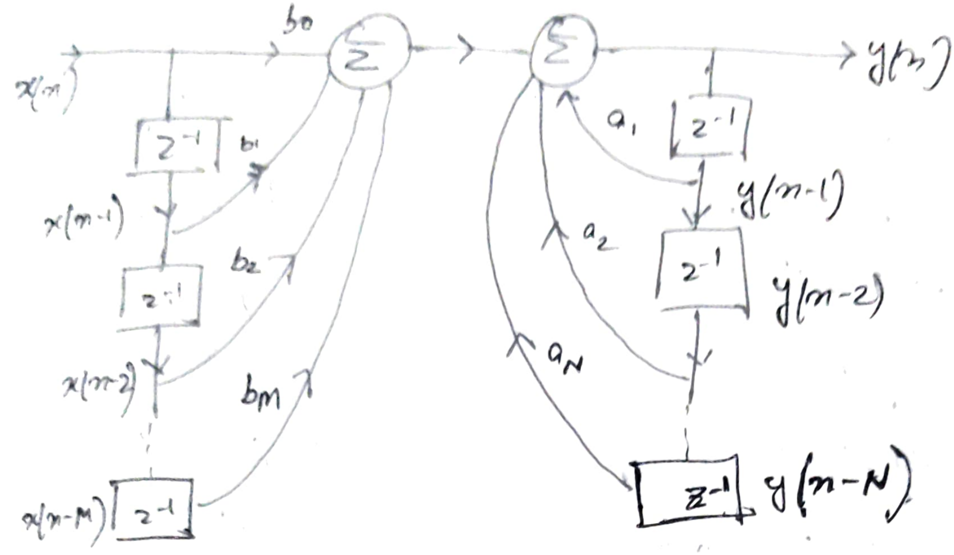
Basic Design of FIR filter

$$y(n) = \sum_{k=0}^{M} b_k x(n-k).$$



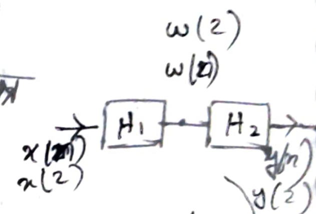
Basic Design of IIR filter

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$



Direct form - I Realization of IIR Filter

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}$$

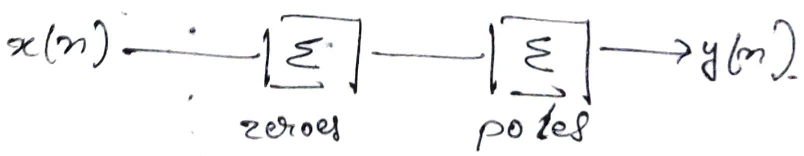


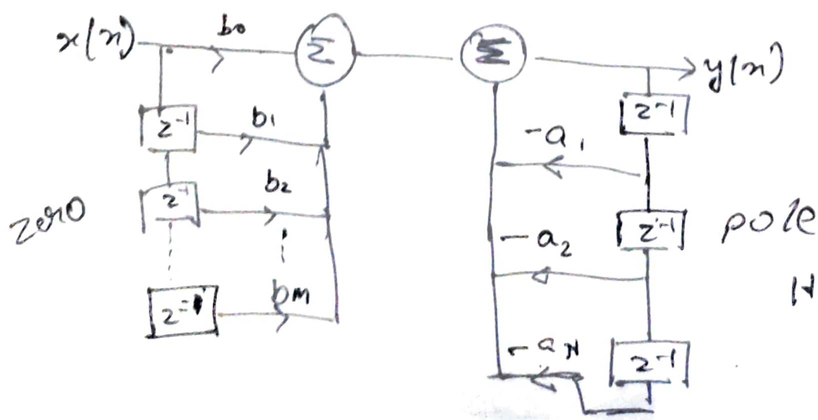
DF I → $H_1(z)$ → All zero part] → cascade

DF I → $H_2(z)$ → All pole part]

$H_1(z) = \frac{W(z)}{X(z)}$

$H_2(z) = \frac{Y(z)}{W(z)}$





$H(z) = H_1(z) \cdot H_2(z)$

zero ↓ pole

Q. $H(z) = 3 + \dfrac{4z}{z-\frac{1}{2}} - \dfrac{2}{z-\frac{1}{4}}$

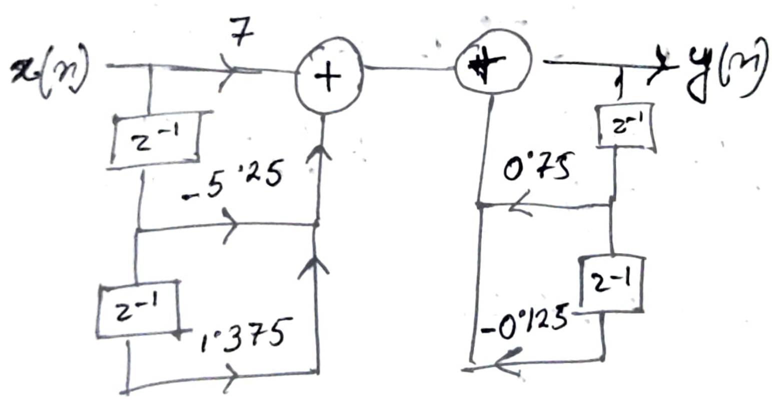
$\Rightarrow H(z) = 3 + \dfrac{4z}{z-0.5} - \dfrac{2}{z-0.25}$

$= \dfrac{3(z-0.5)(z-0.25) + 4z(z-0.25) - 2(z-0.5)}{(z-0.5)(z-0.25)}$

$= \dfrac{7 - 5.25z^{-1} + 1.375z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$

$b_0 = 7 \qquad b_1 = -5.25 \qquad b_2 = 1.375$

$a_1 = -0.75 \qquad a_2 = 0.125$



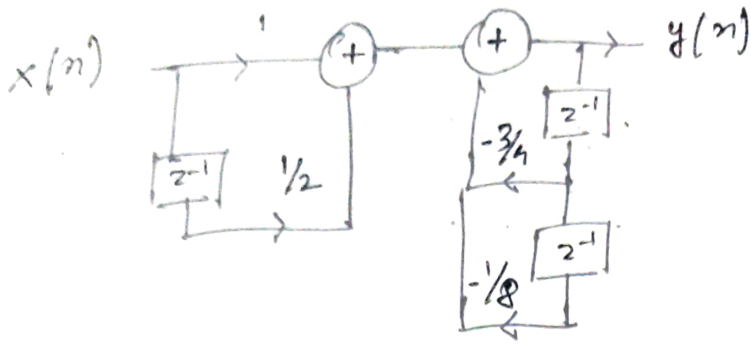
Q. $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}x(n-1)$

Taking z-transform

$Y(z) - \frac{3}{4}z^{-1}(z) + \frac{1}{8}z^{-2}Y(z) = X(z) + \frac{1}{2}z^{-1}X(z)$

$\Rightarrow Y(z)\left\{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right\} = X(z)\left\{1 + \frac{1}{2}z^{-1}\right.$

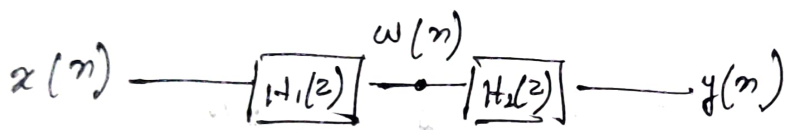
$$H(z) = \frac{1+\frac{1}{2}z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$



Direct Form - II of IIR filters

(Poles are realized first)

$$H(z) = \frac{Y(z)}{X(z)}.$$



$$H_1(z) = \frac{W(z)}{X(z)} \qquad H_2(z) = \frac{Y(z)}{W(z)}.$$

$$H(z) = \underbrace{H_1(z)}_{\text{poles}} \cdot \underbrace{H_2(z)}_{\text{zeroes.}}$$

$$H_1(z) = \frac{W(z)}{X(z)} = \frac{1}{1+\sum_{K=1}^{N} a_K z^{-K}}$$

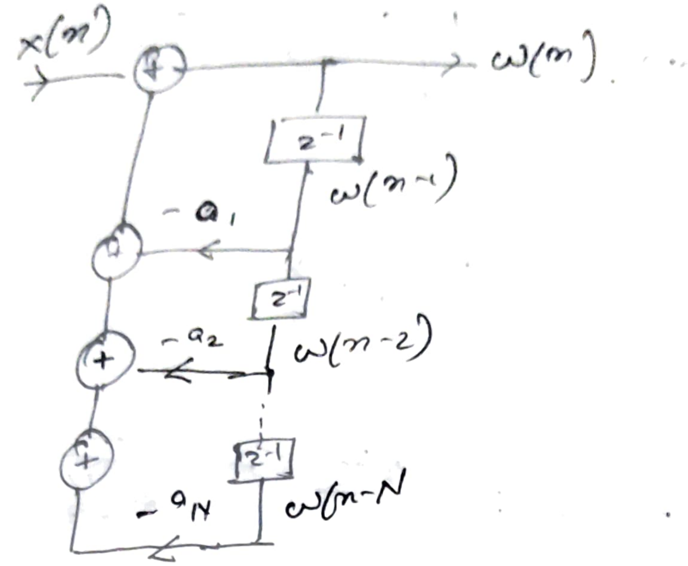
$$\Rightarrow W(z) + \sum_{K=1}^{N} a_K z^{-K} W(z) = X(z)$$

$$\Rightarrow W(z) = X(z) - \sum_{K=1}^{K} a_K z^{-K} W(z).$$

$$\Rightarrow W(z) = X(z) - a_1 z^{-1} W(z) - a_2 z^{-2} W(z) - \dots a_N z^{-N} W(z)$$

Taking inverse laplace.

$$\omega(n) = x(n) - a_1 \omega(n-1) - a_2 \omega(n-2) \dots - a_N \omega(n-N)$$

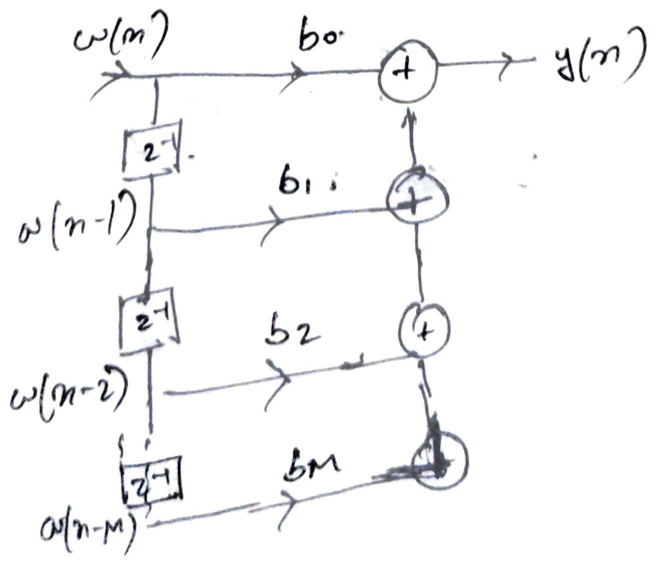


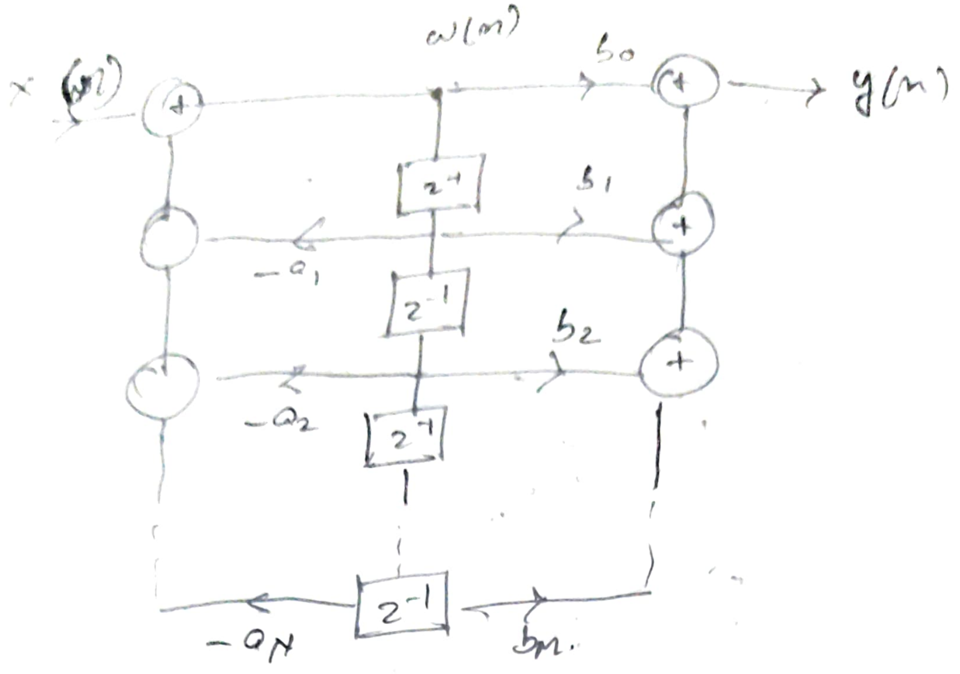
$$H_2(z) = \frac{Y(z)}{\omega(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

$$\Rightarrow \cancel{\omega(z)}\, Y(z) = b_0 \omega(z) + b_1 z^{-1} \omega(z) + \dots + b_M z^{-M} \omega(z)$$

I L.

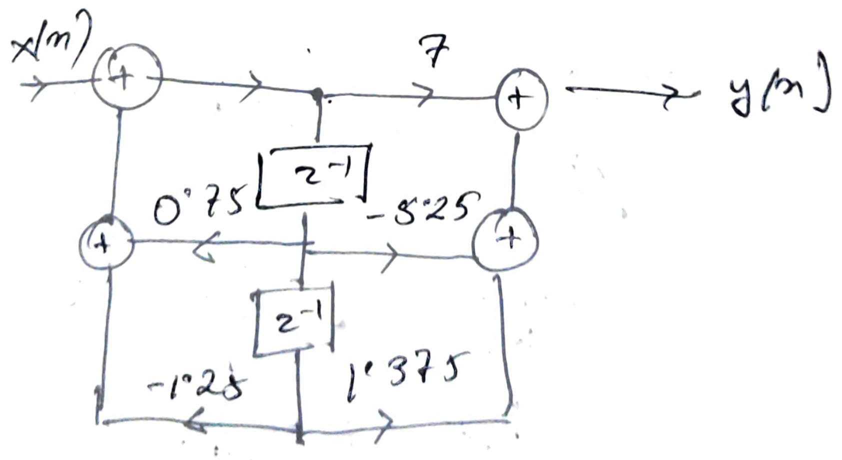
$$y(n) = b_0 \omega(n) + b_1 \omega(n-1) + \dots + b_M \omega(z-M)$$





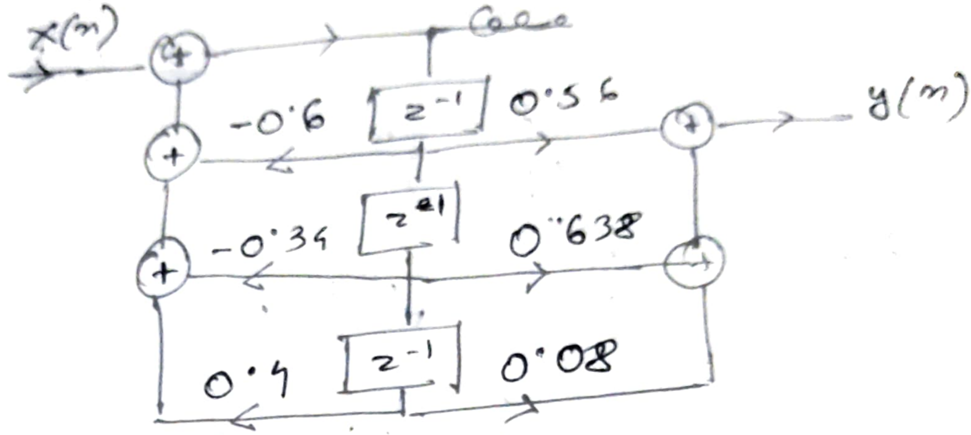
Q. $$H(z) = 8 + \frac{4z}{z - 1/2} - \frac{2}{z - 1/4}$$

$$= \frac{7 - 5.25z^{-1} + 1.375z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$



Q. $H(z) = \dfrac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2}$

$$= \frac{0.56z^{-1} + 0.638z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$$



Q. Obtain DF-I & DF-II for following

$$y(n) = -0.1y(n-1) - 0.72y(n-2) + 0.7x(n) - 0.2x(n-2)$$

Cascade Realization - IIR Filter

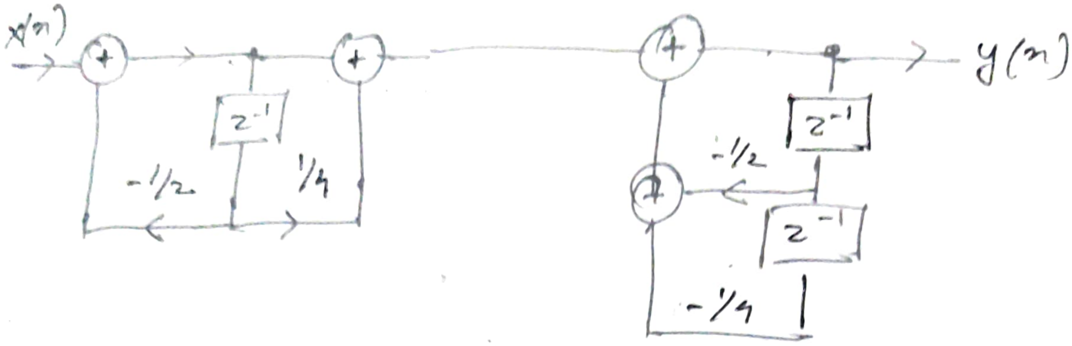
$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} = H_1(z) \cdot H_2(z) \text{ --- } H_n(z)$$

⇒ 2nd order polynomials or less

⇒ Direct form - II

Q. $H(z) = \dfrac{1 + \frac{1}{4}z^{-1}}{(1 + \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2})}$

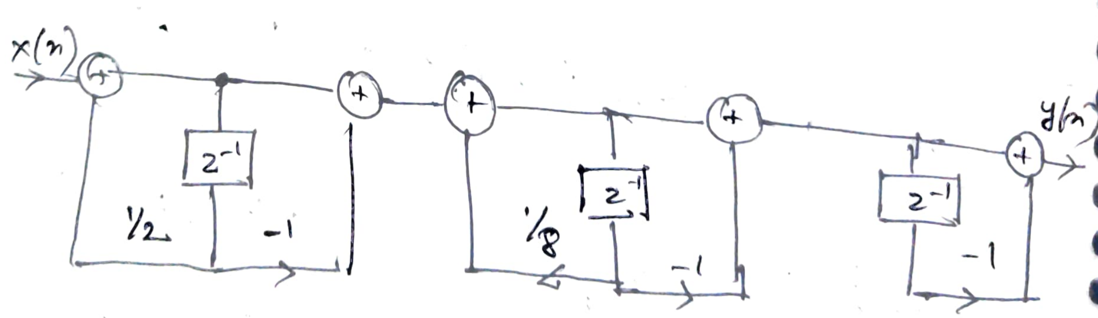
$$H_1(z) = \frac{1+\frac{1}{4}z^{-1}}{1+\frac{1}{2}z^{-1}} \qquad H_2(z) = \frac{1}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$



Q. $$H(z) = \frac{(1-z^{-1})^3}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{8}z^{-1}\right)}$$

$$H(z) = \frac{(1-z^{-1})(1-z^{-1})(1-z^{-1})}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{8}z^{-1}\right)}$$

$$H_1(z) = \frac{1-z^{-1}}{1-\frac{1}{2}z^{-1}} \qquad H_2(z) = \frac{1-z^{-1}}{1-\frac{1}{8}z^{-1}} \qquad H_3(z) = 1-z^{-1}$$



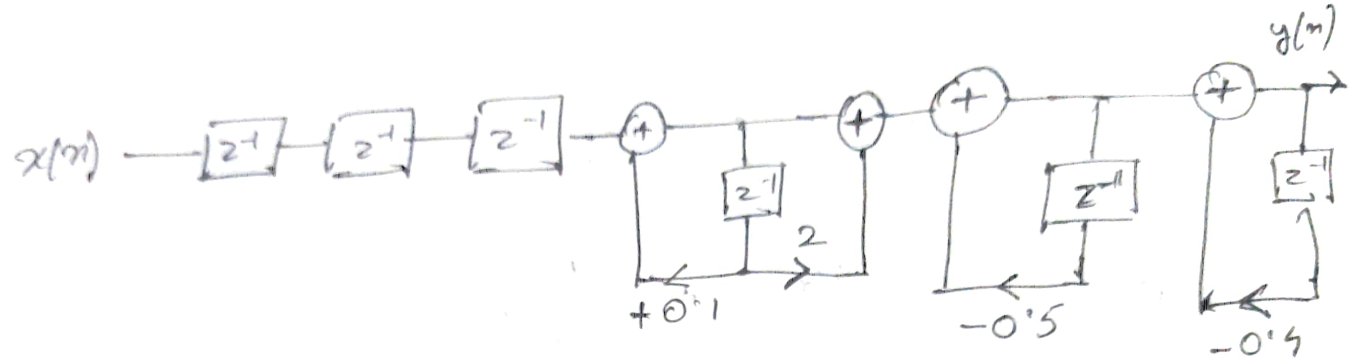
Q. $$H(z) = \frac{2(z+2)}{z(z-0.1)(z+0.5)(z+0.4)}$$

$$= \frac{2z^{-3}(1+2z^{-1})}{1(1-0.1z^{-1})(1+0.5z^{-1})(1+0.4z^{-1})}$$

$$= \cancel{2z^{-1} z^{-1} z^{-1} H_1(z) H_2(z) H_3(z)}$$

$$= 2z^{-1} z^{-1} z^{-1} H_1(z) H_2(z) . H_3(z)$$

$$H_1(z) = \frac{1+2z^{-1}}{1-0.1z^{-1}} \qquad H_2(z) = \frac{1}{1+0.5z^{-1}} \qquad H_3(z) = \frac{1}{1+0.4z^{-1}}$$



Parallel form Realization.

IIR filter

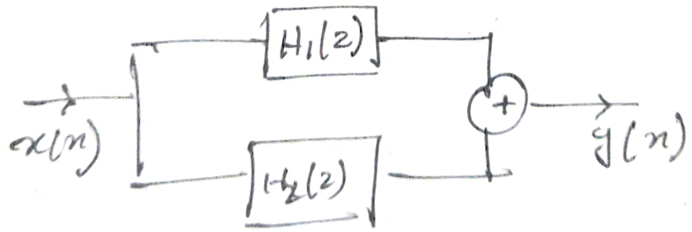
$$H(z) = \frac{\sum_{k=0}^{m} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

→ Every input signal is processed by diffⁿ subsystems

→ Overall $H(z) = H_1(z) + H_2(z) + \ldots$

or

$C + H_1(z) + H_2(z)$



Q. $$H(z) = \frac{1-z^{-1}}{1-0.2z^{-1}-0.15z^{-2}}$$

$$= \frac{z^2(1-z^{-1})}{z^2-0.2z-0.15}$$

$$= \frac{z^2-z}{z^2-0.2z-0.15}$$

$$H(z) = \frac{z(z-1)}{z^2 - 0{\cdot}2z - 0{\cdot}15}$$

$$\frac{H(z)}{z} = \frac{z-1}{z^2 - 0{\cdot}2z - 0{\cdot}15}$$

$$= \frac{z-1}{(z-0{\cdot}5)(z+0{\cdot}3)}$$

$$\frac{H(z)}{z} = \frac{A}{(z-0{\cdot}5)} + \frac{B}{(z+0{\cdot}3)}$$

$$A = (z-0{\cdot}5)\left.\frac{H(z)}{z}\right|_{z=0{\cdot}5}$$

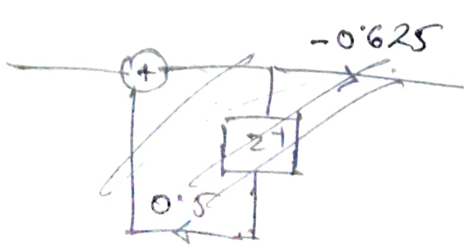
$$= \cancel{(z-0{\cdot}5)}\frac{(z-1)}{\cancel{(z-0{\cdot}5)}(z+0{\cdot}3)}$$

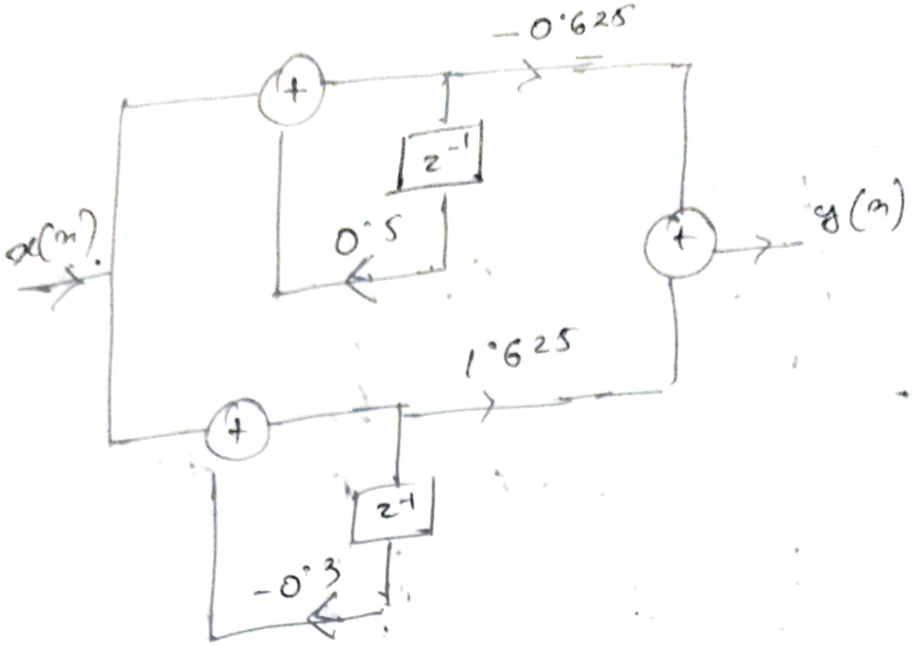
$$= -0{\cdot}625$$

$$B = 1{\cdot}625$$

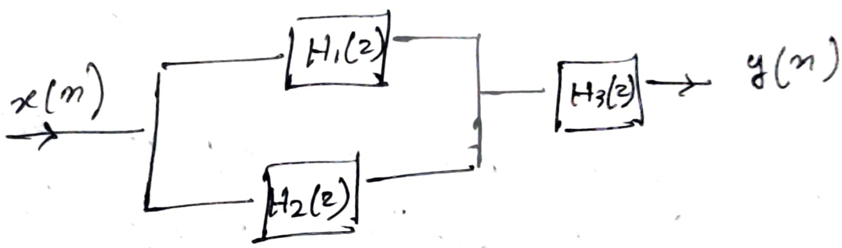
$$\frac{H(z)}{z} = \frac{-0{\cdot}625}{z-0{\cdot}5} + \frac{1{\cdot}625}{z+0{\cdot}3}$$

$$H(z) = \frac{-0{\cdot}625}{1-0{\cdot}5z^{-1}} + \frac{1{\cdot}625}{1+0{\cdot}3z^{-1}}$$





Q. A system has a transfer f^n as shown:



$H_1(z)$ & $H_2(z)$ has poles at $+1$ & -1

$H_1(z)$ has 2 zeroes at origin.

$H_2(z)$ has 1 zero at origin.

~~$H_3(z)$ has 1 zero at origin.~~

$H_3(z) = 1 + 1.5z^{-1} - 1.5z^{-2} - z^{-3}$

Realise this circuit?

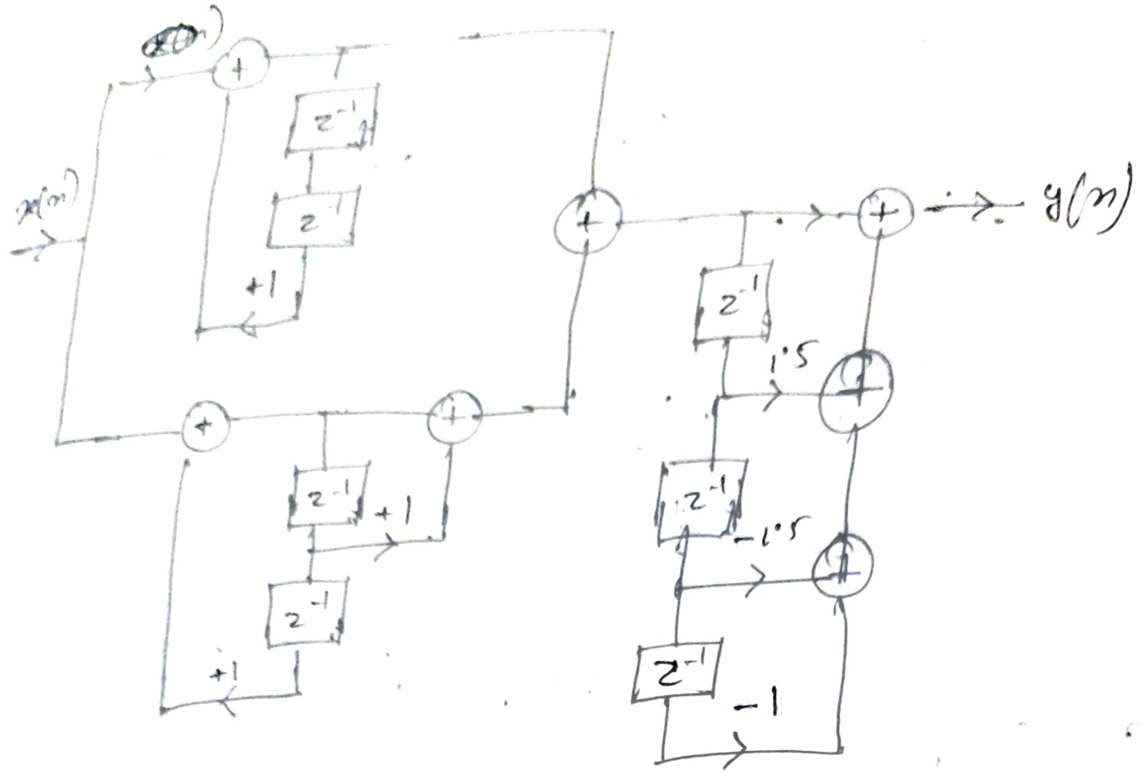
→ $H_1(z) = \frac{z^2}{(z-1)(z+1)}$

$= \frac{z^2}{z^2-1}$

$= \frac{1}{1-z^{-2}}$

$H_2(z) = \frac{z}{(z-1)(z+1)}$

$= \frac{z^{-1}}{1-z^{-2}}$

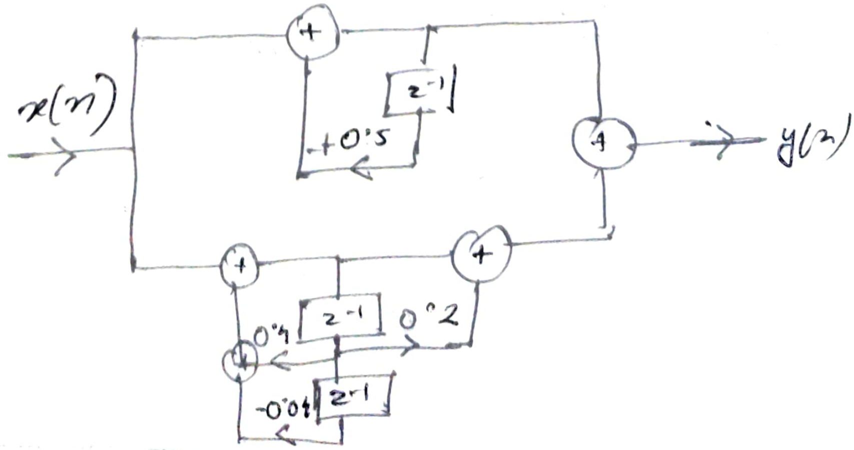


Q. A system has an impulse response
$h(n) = (0.5)^n u(n) + n(0.2)^n u(n)$.
Realize the system using parallel form.

$$\rightarrow \quad H(z) = \frac{z}{z-0.5} + \frac{0.2z}{(z-0.2)^2}$$

$$= \frac{1}{1-0.5z^{-1}} + \frac{0.2z}{z^2+0.04-0.4z}$$

$$= \frac{1}{1-0.5z^{-1}} + \frac{0.2z^{-1}}{1-0.4z^{-1}+0.04z^{-2}}$$



Q. $H(z) = \dfrac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$

Ladder Structure.

Advantage – Small changes in filter parameters have little effect on filter performance.

$$\Rightarrow H(z) = \frac{a_N z^{-N} + a_{N-1} z^{-N+1} + \cdots a_1 z^{-1} + a_0}{b_N z^{-N} + b_{N-1} z^{-N+1} \cdots + b_1 z^{-1} + b_0}$$

(I) Routh Table

(II) Continued fraction.

$$H(z) = \alpha_0 + \cfrac{1}{\beta_1 z^{-1} + \cfrac{1}{\alpha_1 + \cfrac{1}{\beta_2 z^{-1} + \cdots + \cfrac{1}{\alpha_n}}}}$$

$\alpha_0 = \dfrac{a_N}{b_N}$ $\quad \beta_1 = \dfrac{b_N}{C_{N-1}}$ $\quad \alpha_1 = \dfrac{C_{N-1}}{d_{N-1}}$

Q. $H(z) = \dfrac{z^{-2} + 2z^{-1} + 1}{3z^{-2} + 3z^{-1} + 1}$

z^{-2}	1	2	1
z^{-2}	3	3	1
z^{-1}	$\frac{3\times2-3\times1}{3} = 1$	$\frac{3\times1-1\times1}{3} = \frac{2}{3}$	0
z^{-1}	$\frac{1\times3-3\times\frac{2}{3}}{1} = 1$	1	0
z^{0}	$\frac{1\times\frac{2}{3}-1}{1} = -\frac{1}{3}$	0	0
z^{0}	$\frac{-\frac{1}{3}\times1}{-\frac{1}{3}} = 1$		

z^{-2}	1	2	1
z^{-2}	3	3	1
z^{-1}	1	2/3	0
z^{-1}	1	1	0
1	$-1/3$	0	0
1	1	0	0

z^{-N} a_N a_{N-1} - - - a_1 a_0

z^{-N} b_N b_{N-1} - - - b_1 b_0

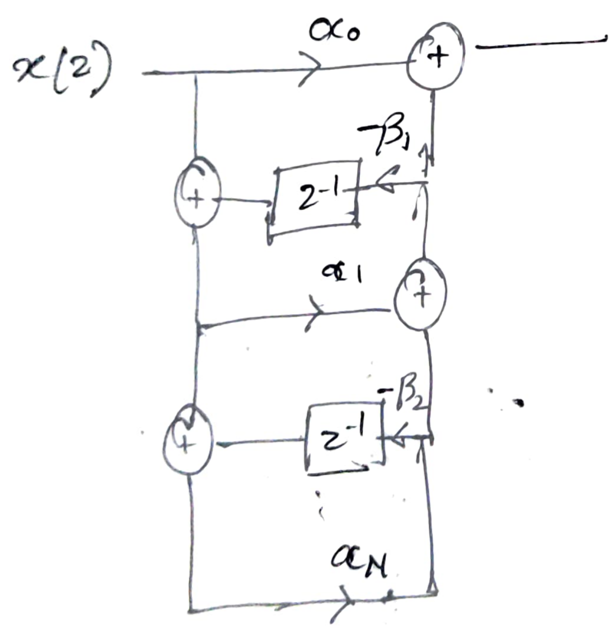
z^{-N+1} c_{N-1} c_{N-2} - - - c_0

z^{-N+1} d_{N-1} d_{N-2} - - - d_0

z^{-N-2} e_{N-2} e_{N-3} - - - e_0

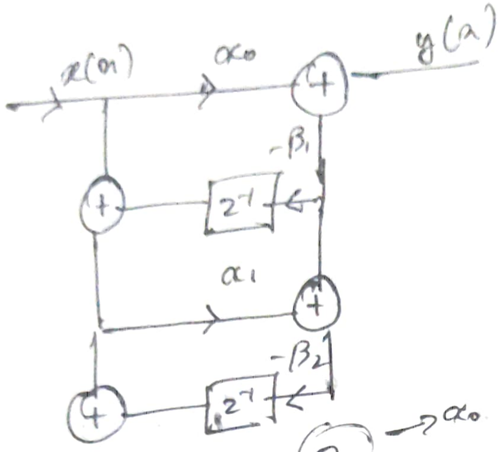
z^{-N-2} f_{N-2} f_{N-3} - - - f_0

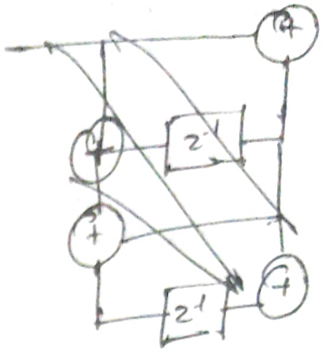
$\alpha_0 = 1/3$ $\beta_2 = 3$ $\beta_1 = 3$. $\alpha_2 = -1/3$ $\alpha_1 = 1$



(11)

Q) $H(z) = \dfrac{2z^{-2}+2z^{-1}+1}{z^{-2}+4z^{-1}+2}$





$2 \rightarrow \alpha_0$

$z^{-2}+4z^{-1}+2 \,\big)\, 2z^{-2}+2z^{-1}+1$

$2z^{-2}+8z^{-1}+4$

$-6z^{-1}-3 \,\big)\, z^{-2}+4z^{-1}+2$ $\qquad -\frac{1}{6}z^{-1} \rightarrow \beta_1$

$z^{-2}+\frac{1}{2}z^{-1}$

$\frac{7}{2}z^{-1}+2 \,\big)\, -6z^{-1}-3$ $\qquad -\frac{12}{7} \rightarrow \alpha_1$

$-6z^{-1}-\frac{24}{7}$ $\qquad \frac{49}{6}z^{-1}$ $\qquad \beta_2$

$\frac{3}{7} \,\big)\, \frac{7}{2}z^{-1}+2$

$\frac{7}{2}z^{-1}$

$\frac{3}{14}$

$2 \,\big)\, \frac{3}{7}$

$\frac{3}{7}$

$\times$

Q. $H(z) = \dfrac{2+8z^{-1}+6z^{-2}}{1+8z^{-1}+12z^{-2}}$

Two port Ladder network

$$H(z) = \frac{a_0}{a_N z^{-N} + a_{N-1} z^{-N+1} + \dots + a_1 z^{-1} + a_0}$$

Ex $N = 5$

$$H(z) = \frac{a_0}{a_5 z^{-5} + a_4 z^{-4} + \dots + a_0}$$

z^{-5}	a_5	a_3	a_1
z^{-4}	a_4	a_2	a_0
z^{-3}	b_3	b_1	—
z^{-2}	b_2	b_0	—
z^{-1}			
1			

$$b_3 = \frac{a_4 a_3 - a_5 a_2}{a_4}$$

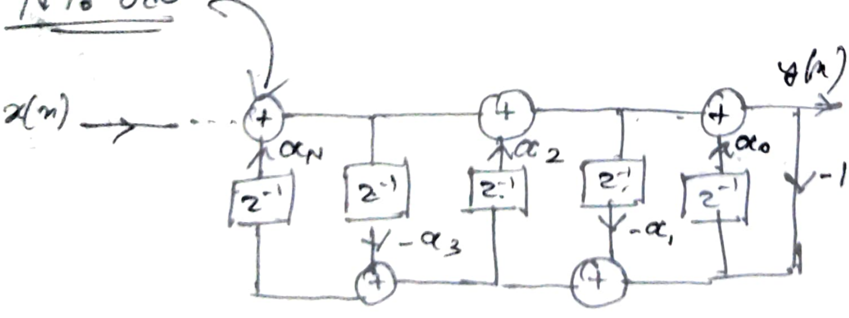
$$b_1 = \frac{a_4 a_1 - a_5 a_0}{a_4}$$

$$b_2 = \frac{b_3 a_2 - b_1 a_4}{b_3}$$

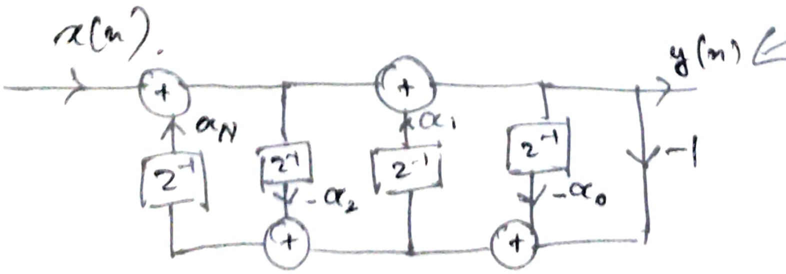
$$b_0 = \frac{b_3 a_0}{b_3}$$

$$\alpha_0 = \frac{a_5}{a_4} \quad \alpha_1 = \frac{a_4}{b_3} \quad \alpha_2 = \frac{b_3}{b_2}$$

N is Odd



N is even

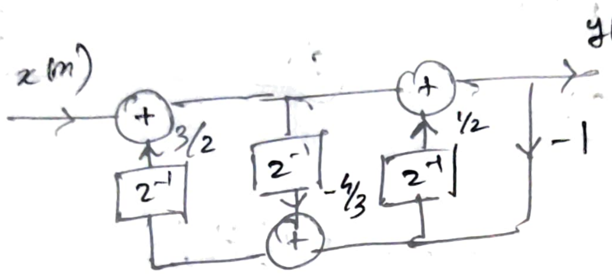


Q. $H(z) = \dfrac{1}{z^{-3} + 2z^{-2} + 2z^{-1} + 1}$, $\underline{N = 3}$

z^{-3}	1	2
z^{-2}	2	1
z^{-1}	$3/2$	0
1	1	

$\alpha_0 = \frac{1}{2}$ $\alpha_1 = \frac{4}{3}$

$\alpha_2 = \frac{3}{2}$



FIR filter Realization.

Basic diff^n eq^n of LTI system.

$$\Rightarrow y(n) = -\sum_{K=1}^{N} a_K\, y(n-k) + \sum_{K=0}^{M-1} b_K\, x(n-K)$$

FIR → finite impulse response.
→ no feedback. ∴ $y(n-k) = 0$

$$\therefore \boxed{y(n) = \sum_{K=0}^{M-1} b_K\, x(n-K)}$$

$(M-1)$ poles ⇒ $z = 0$
$(M-1)$ zeroes ⇒ Anywhere within z-plane.

$$\boxed{Y(z) = \sum_{K=0}^{M-1} b_K\, z^{-K} X(z)}$$

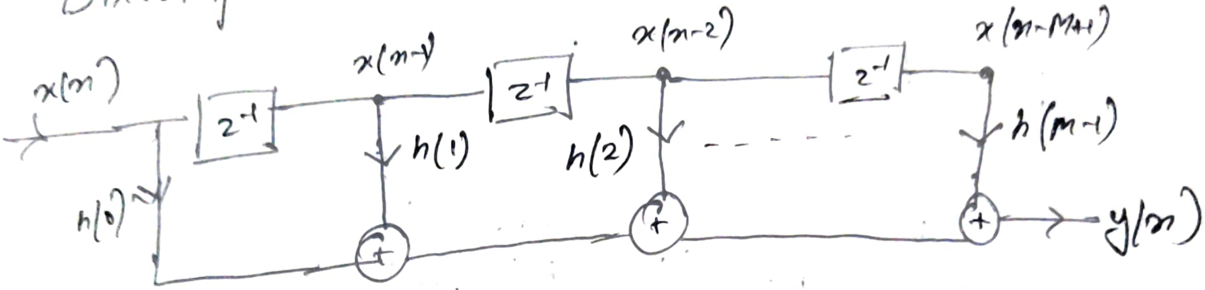
$$\boxed{H(z) = \sum_{K=0}^{M-1} b_K\, z^{-K}}$$

$h(n) = \begin{cases} b_n & \forall\ 0 \le n < M-1 \\ 0, & \text{otherwise} \end{cases}$

$$y(n) = \sum_{K=0}^{M-1} h(K)\, x(n-K)$$

$$y(n) = h(0)\, x(n) + h(1)\, x(n-1) + h(2)\, x(n-2) + \cdots + h(M-1)\, x(n-M+1)$$

Direct form realization.



Cascade form $\quad H(z) = H_1(z)\, H_2(z)$

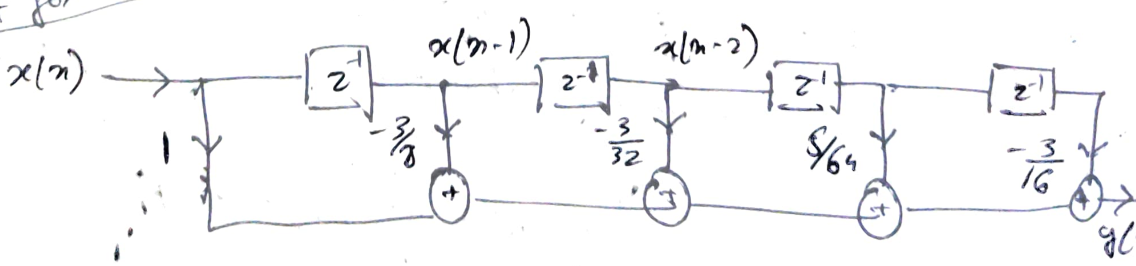
Direct form II

$$H(z) = \underbrace{\left(1 - \frac{1}{4} z^{-1} + \frac{3}{8} z^{-2}\right)}_{H_1(z)} \underbrace{\left(1 - \frac{1}{8} z^{-1} - \frac{1}{2} z^{-2}\right)}_{H_2(z)}$$

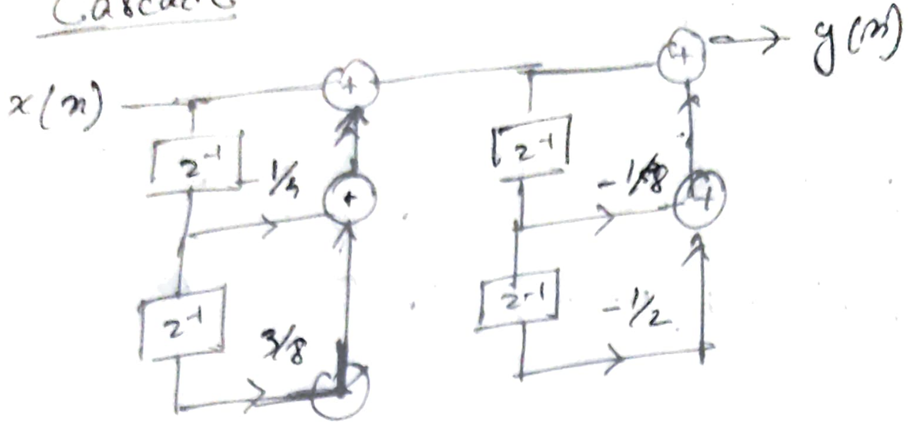
Q.

$$= 1 - \frac{3}{8} z^{-1} + \frac{3}{32} z^{-2} + \frac{5}{64} z^{-3} - \frac{3}{16} z^{-4}$$

Direct form



Cascade



Q. $H(z) = 1+\frac{5}{2}z^{-1}+2z^{-2}+2z^{-3}$ (cascade)

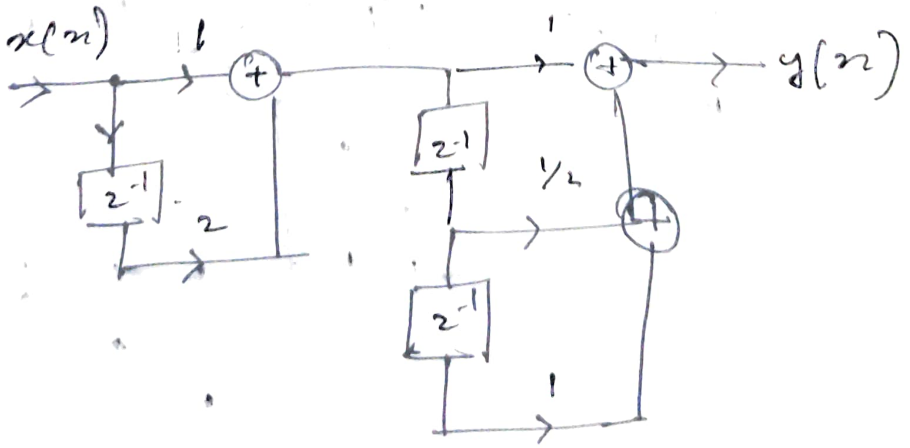
$$H(z) = 1+\frac{5}{2z}+\frac{2}{z^2}+\frac{2}{z^3}$$

$$= \frac{z^3+\frac{5}{2}z^2+2z+2}{z^3}$$

$$= \frac{(z+2)(z^2+\frac{1}{2}z+1)}{z \cdot z^2}$$

$$= \left(\frac{z+2}{z}\right)\left(\frac{z^2+\frac{1}{2}z+1}{z^2}\right)$$

$$= (1+2z^{-1})\left(1+\frac{1}{2}z^{-1}+z^{-2}\right)$$



Linear Phase FIR filter

→ Frequency response is symmetric about reference.

(Symmetric)

$$\boxed{h(n) = h(M-1-n)} \quad \forall\ 0 \le n \le M-1$$

Ex

Symmetric

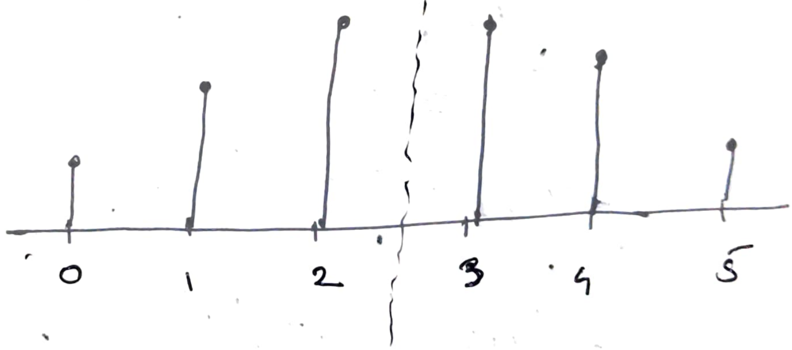
$M = 6 \qquad 0 \le n \le 5$

$h(0) = h(6-1-0) = h(5)$

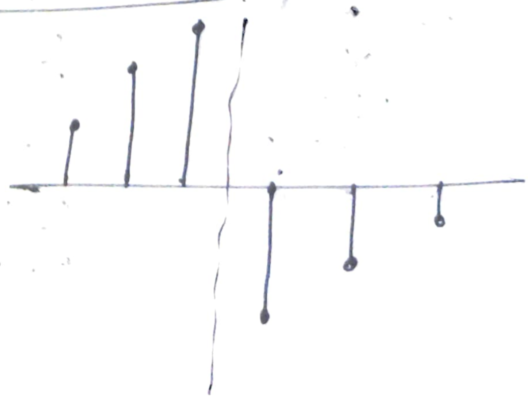
$h(1) = h(6-1-1) = h(4)$

$h(2) = h(6-1-2) = h(3)$

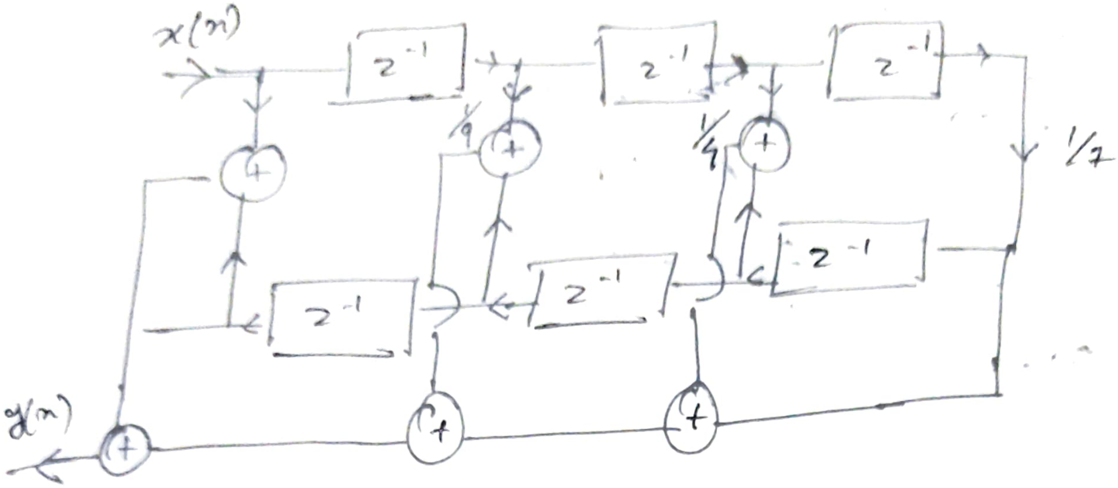
$h(3) = h(2)$.



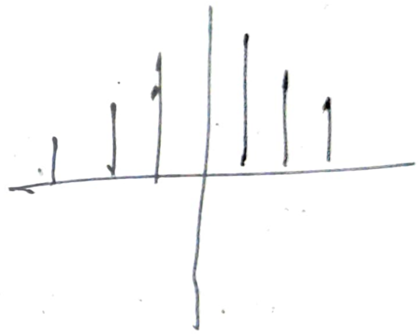
$$\boxed{h(n) = -h(M-1-n)}$$ (Antisymmetry)



Q. $H(z) = 1 + \frac{1}{9} z^{-1} + \frac{1}{4} z^{-2} + \frac{1}{7} z^{-3} + \frac{1}{4} z^{-4} + \frac{1}{9} z^{-5} + z^{-6}$



M - even.



$0 \rightarrow M-1$

$0 \rightarrow \frac{M}{2} - 1$ $\quad \frac{M}{2} \rightarrow M-1$

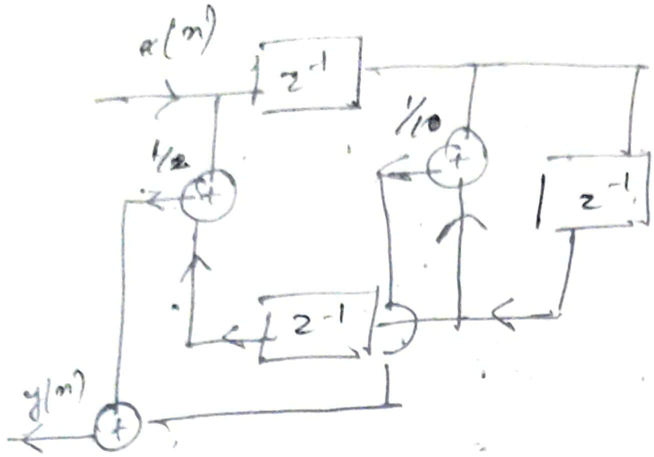
$\frac{M-1}{2}$

$$\boxed{\sum_{0}^{M-1} = \sum_{0}^{\frac{M}{2}-1} + \sum_{M/2}^{M-1}}$$

$\frac{M}{2} - \frac{1}{2} - \frac{1}{2} = \frac{M}{2} - 1$

$\frac{M}{2} - \frac{1}{2} + \frac{1}{2} = \frac{M}{2}$

Q. $H(z) = \frac{1}{2} + \frac{1}{10} z^{-1} + \frac{1}{10} z^{-2} + \frac{1}{2} z^{-3}$



Q. $H(z) = \frac{2}{3} z + 1 + \frac{2}{3} z^{-1}$

$$\frac{Y(z)}{X(z)} = 1 + \frac{2}{3}(z + z^{-1})$$

$$Y(z) = X(z) + \frac{2}{3}(z + z^{-1}) X(z).$$

$$y(n) = x(n) + \frac{2}{3} x(n+1) + \frac{2}{3} x(n-1).$$