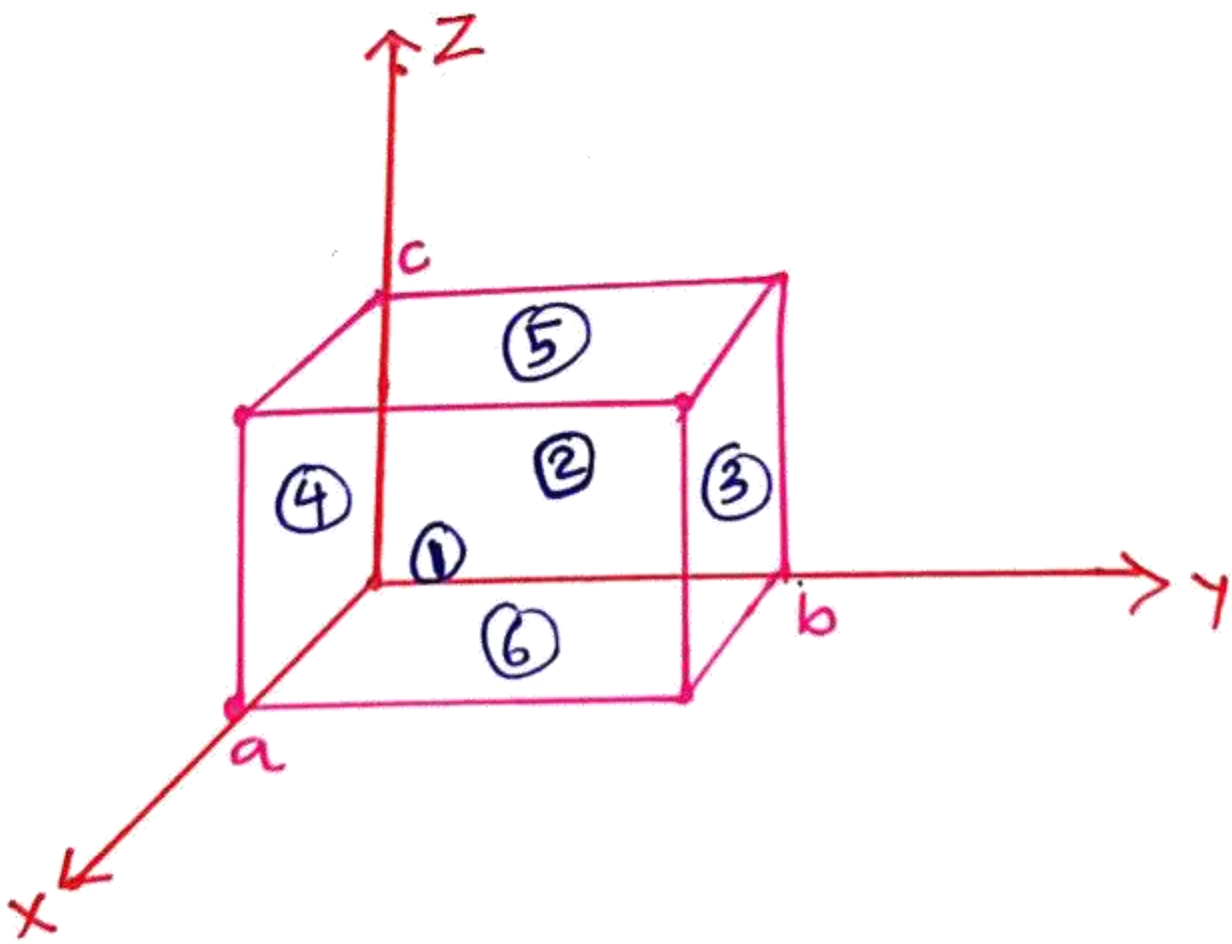
Electro - Magnetics.

- If $\vec{F} = xy\,\hat{i}_x + yz\,\hat{i}_y + zx\,\hat{i}_z$
  $a = 1$, $b = 3$, $c = 2$ then

1] Determine ✓ $\int \vec{\nabla}\cdot\vec{F}\,dv = ?$ (18)

2] Determine $\oint \vec{F}\cdot d\vec{s} = ?$ (18)

3] Verify Divergence theorm



$$\vec{\nabla}\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$= \frac{\partial (xy)}{\partial x} + \frac{\partial (yz)}{\partial y} + \frac{\partial (zx)}{\partial z}$$

$$= y + z + x$$

$\rightarrow$ $0 < x < 1$

$0 < y < 3$

$0 < z < 2$

$$\rightarrow \int \vec{\nabla}\cdot\vec{F}\, dV = \int_0^2\int_0^3\int_0^1 y + z + x \; dx\, dy\, dz$$

$$= \int_0^2\int_0^3 \left(yx + zx + x^2/2\right)_0^1 dy\, dz$$

$$= \int_0^2\int_0^3 (y + z + 1/2)\; dy\, dz$$

$$= \int_0^2 \left(y^2/2 + yz + y/2\right)_0^3 dz$$

$$= \int_0^2 9/2 + 3z + 3/2 \; dz$$

$$= \int_0^2 3z + 6 \; dz$$

$$= \left(3z^2/2 + 6z\right)_0^2$$

$$= 6 + 12 = 18$$

1 $\Rightarrow$ $x = a = 1$, $dx = 0$

$y \in (0,3)$

$z \in (0,2)$ $\quad d\vec{s} = dy\, dz\, \hat{i}_x$

$$- \int \vec{F}\cdot d\vec{s} = \int_0^2\int_0^3 xy \; dy\, dz$$

$$= (1)\left(y^2/2\right)_0^3 (z)_0^2 = 9$$

2 $\Rightarrow$ $x = a = 0$, $dx = 0$. $\int \vec{F}\cdot d\vec{s} = 0$.

3 $\Rightarrow$ $y = b = 3$, $dy = 0$

$x \in (0,1)$

$z \in (0,2)$ $\quad d\vec{s} = dx\, dz\, \hat{i}_y$.

$$- \int \vec{F}\cdot d\vec{s} = \int_0^2\int_0^1 yz \; dx\, dz.$$

$$= 3 \times \left(z^2/2\right)_0^2 (x)_0^1 = 6$$

4 $\Rightarrow$ $y = 0$, $dy = 0$, $\int \vec{F}\cdot d\vec{s} = 0$

5 $\Rightarrow$ $z = c = 2$, $dz = 0$.

$x \in (0,1)$

$y \in (0,3)$, $\quad d\vec{s} = dx\, dy\, \hat{i}_z$

$$- \int \vec{F}\cdot d\vec{s} = \int_0^3\int_0^1 zx \; dx\, dy$$

$$= 2 \times \left(x^2/2\right)_0^1 \times (y)_0^3 = 3$$

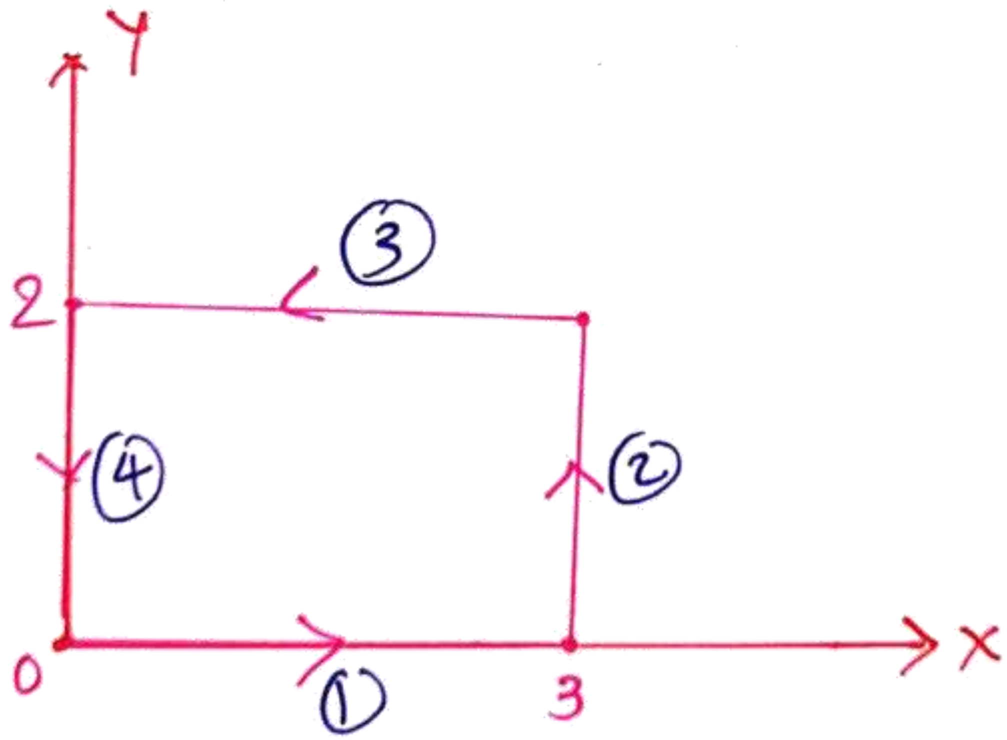
6 $\Rightarrow$ $z = 0$, $dz = 0$, $\int \vec{F}\cdot d\vec{s} = 0$

$$\rightarrow \int \vec{F}\, d\vec{s} = 9 + 0 + 6 + 0 + 3 + 0 = 18$$

Electro - Magnetics

- If $\vec{F} = x^2 y \, \hat{i}_x + (x+y^2) \, \hat{i}_y + xy \, \hat{i}_z$ then

1) $\int (\vec{\nabla} \times \vec{F}) \cdot d\vec{s} = ?$ ✓ (-12)
2) $\oint \vec{F} \cdot d\vec{l} = ?$ ✓ -12
3) Verify stokes theorm



$\rightarrow \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & x+y^2 & xy \end{vmatrix}$

$= \hat{i}\left(\frac{\partial(xy)}{\partial y} - \frac{\partial(x+y^2)}{\partial z}\right)$

$- \hat{j}\left(\frac{\partial(xy)}{\partial x} - \frac{\partial(x^2y)}{\partial z}\right)$

$+ \hat{k}\left(\frac{\partial(x+y^2)}{\partial x} - \frac{\partial(x^2y)}{\partial y}\right)$

$= \hat{i}(x) - \hat{j}(y) + \hat{k}(1-x^2)$

$\rightarrow (\vec{\nabla} \times \vec{F}) \cdot d\vec{s} = 1 - x^2$

$\rightarrow x \rightarrow 0$ to 3

$y \rightarrow 0$ to 2

$\rightarrow \int \vec{\nabla} \times \vec{F} \cdot d\vec{s} = \int_0^2 \int_0^3 (1-x^2)\, dx\, dy$

$= \int_0^2 \left(x - \frac{x^3}{3}\right)_0^3 dy$

$= \int_0^2 (3-9)\, dy$

$= -6\,[y]_0^2 = -12$

$\rightarrow$ Stoke $\int \vec{F} \cdot d\vec{l} = 0 + \frac{26}{3} + (-18) - \frac{8}{3}$

$= -12$

① $y = 0, \quad dy = 0, \quad 0 < x < 3$

$\rightarrow d\vec{l} = dx\, \hat{i}_x + dy\, \hat{i}_y + dz\, \hat{i}_z$

$d\vec{l} = dx\, \hat{i}_x$

$\rightarrow \int \vec{F} \cdot d\vec{l} = \int_0^3 x^2 y\, dx = 0$

② $x = 3, \quad dx = 0, \quad 0 < y < 2$

$\rightarrow d\vec{l} = dy\, \hat{i}_y$

$\rightarrow \int \vec{F} \cdot d\vec{l} = \int_0^2 (x + y^2)\, dy$

$= \int_0^2 3 + y^2\, dy = \left[3y + \frac{y^3}{3}\right]_0^2$

$= 6 + \frac{8}{3} = \frac{26}{3}$

③ $y = 2, \quad dy = 0, \quad 3 < x < 0$

$\rightarrow d\vec{l} = dx\, \hat{i}_x$

$\rightarrow \int \vec{F} \cdot d\vec{l} = \int_3^0 x^2 y\, dx = \int_3^0 2x^2\, dx$

$= 2\left[\frac{x^3}{3}\right]_3^0 = 2[-9] = -18$

④ $x = 0, \quad dx = 0, \quad 2 < y < 0$

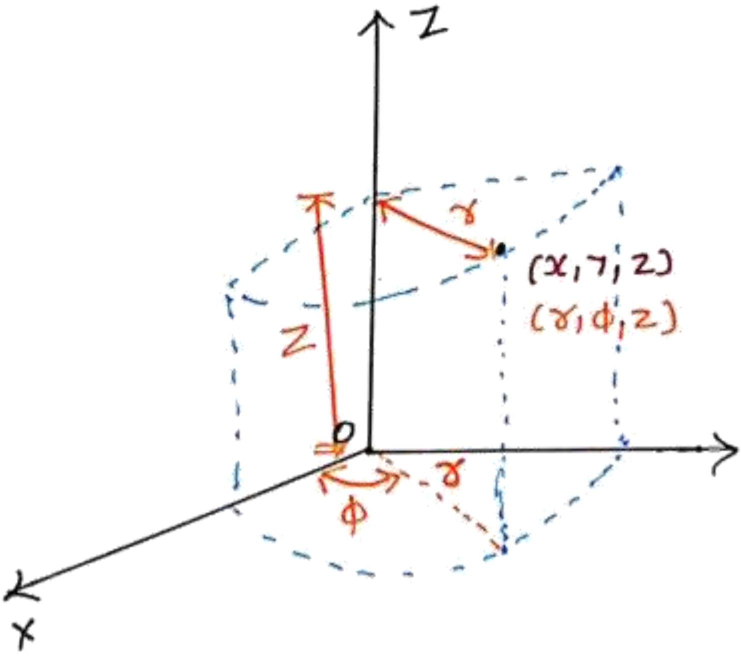
$\rightarrow d\vec{l} = dy\, \hat{i}_y$

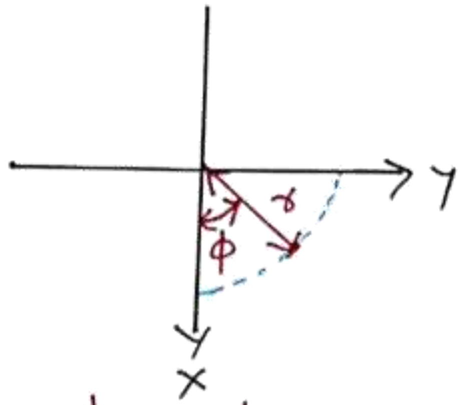
$\rightarrow \int \vec{F} \cdot d\vec{l} = \int_2^0 x + y^2\, dy = \int_2^0 y^2\, dy = \left[\frac{y^3}{3}\right]_2^0$

$= -\frac{8}{3}$

Cylindrical Co-ordinate System $[r, \phi, z]$

- r — Radius of Cylinder
- ϕ — Angle w.r.t x axis on xy plane.
- z — Length of Cylinder.





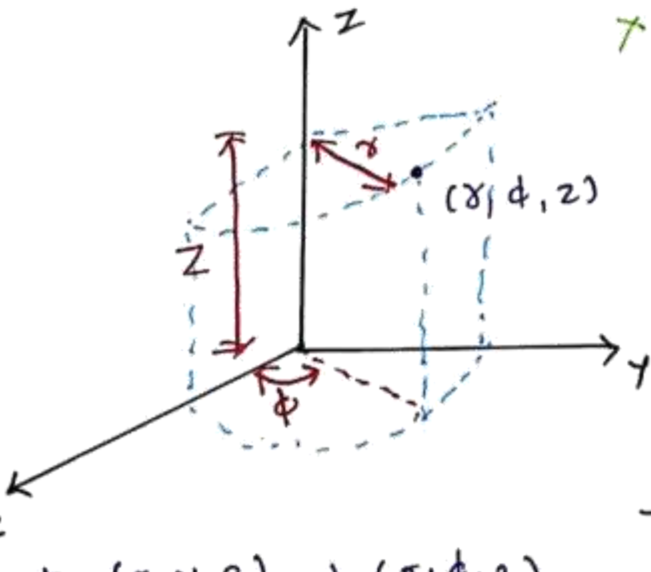
- Conversion of Cylindrical Co-ordinates into Cartesian Co-ordinates.

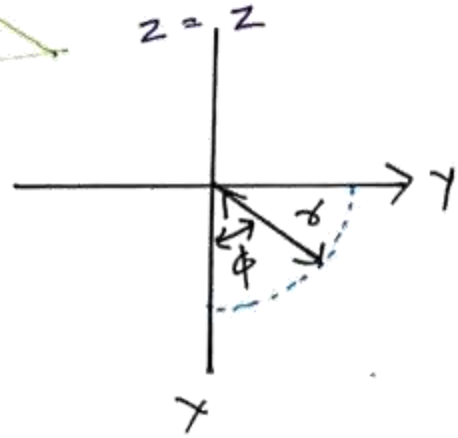
$(r, \phi, z) \rightarrow (x, y, z)$

$x = r\cos\phi$

$y = r\sin\phi$

$z = z$





Convert $(x, y, z) \rightarrow (r, \phi, z)$

$r = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}(y/x)$

$z = z$

Point $(1, 1, 2) \rightarrow (r, \phi, z)$

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\phi = \tan^{-1}(1/1) = 45^\circ$

$z = 2$

$(r, \phi, z) = (\sqrt{2}, 45^\circ, 2)$.

- $(5, 60^\circ, 2) \rightarrow (x, y, z)$

$x = r\cos\phi = 5\cos 60 = 5/2$

$y = r\sin\phi = 5\sin 60 = 5\sqrt{3}/2$

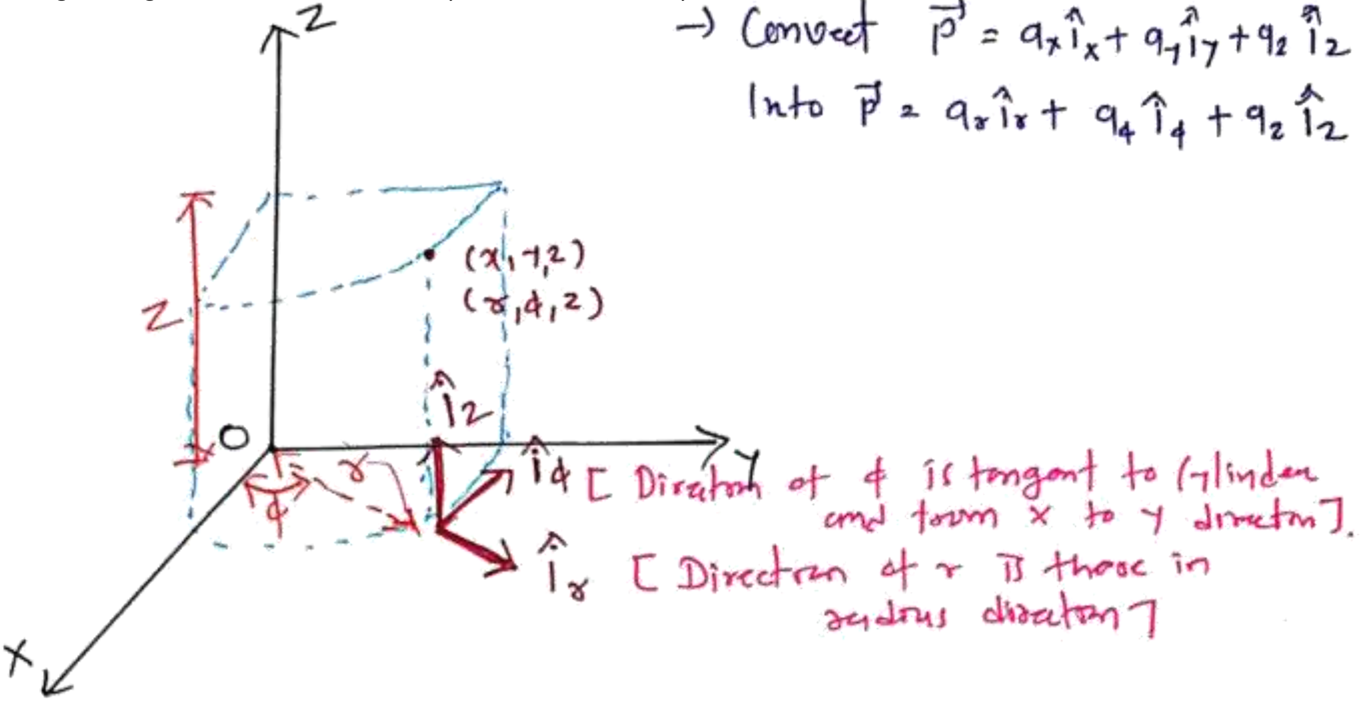
$z = 2$

- $(x, y, z) = (5/2, 5\sqrt{3}/2, 2)$.

Cartezian Vector to Cylindrical Vector Conversion

→ Convert $\vec{P} = a_x\hat{i}_x + a_y\hat{i}_y + a_z\hat{i}_z$

Into $\vec{P} = a_r\hat{i}_r + a_\phi\hat{i}_\phi + a_z\hat{i}_z$



→ $a_r = \vec{P}\cdot\hat{i}_r = (a_x\hat{i}_x + a_y\hat{i}_y + a_z\hat{i}_z)\cdot\hat{i}_r = a_x\underbrace{(\hat{i}_x\cdot\hat{i}_r)}_{\phi} + a_y\underbrace{(\hat{i}_y\cdot\hat{i}_r)}_{90-\phi} + a_z\underbrace{(\hat{i}_z\cdot\hat{i}_r)}_{90}$

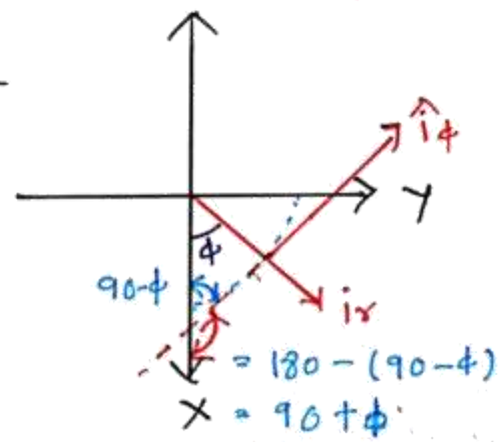
$= a_x\cos\phi + a_y\sin\phi$

→ $a_\phi = \vec{P}\cdot\hat{i}_\phi = (a_x\hat{i}_x + a_y\hat{i}_y + a_z\hat{i}_z)\cdot\hat{i}_\phi = a_x\underbrace{(\hat{i}_x\cdot\hat{i}_\phi)}_{90+\phi} + a_y\underbrace{(\hat{i}_y\cdot\hat{i}_\phi)}_{\phi} + a_z\underbrace{(\hat{i}_z\cdot\hat{i}_\phi)}_{90}$

$= -a_x\sin\phi + a_y\cos\phi$

→ $a_z = \vec{P}\cdot\hat{i}_z = (a_x\hat{i}_x + a_y\hat{i}_y + a_z\hat{i}_z)\cdot\hat{i}_z$

$= a_z$



$$\boxed{\begin{aligned} a_r &= a_x\cos\phi + a_y\sin\phi \\ a_\phi &= -a_x\sin\phi + a_y\cos\phi \\ a_z &= a_z \end{aligned}}$$

→ $\vec{F} = 2\sqrt{3}\,\hat{i}_x + 2\hat{i}_y + 4\hat{i}_z$ Convert into Cylindrical Vector.

→ $\phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{2}{2\sqrt{3}}\right) = 30^\circ$

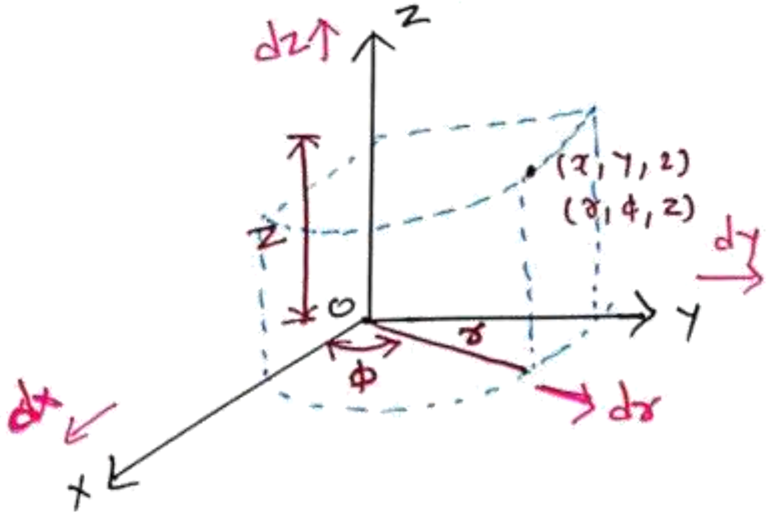
→ $a_r = a_x\cos\phi + a_y\sin\phi = 2\sqrt{3}\cos 30 + 2\sin 30 = 4$

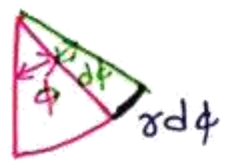
→ $a_\phi = -a_x\sin\phi + a_y\cos\phi = -2\sqrt{3}\sin 30 + 2\cos 30 = 0$

→ $a_z = 4$

→ $\boxed{\vec{F} = 4\hat{i}_r + 0\hat{i}_\phi + 4\hat{i}_z}$

* Cylindrical Co-ordinate System for line, Surface, Volume, Gradient, Divergence and Curl Calculation





→ Cartezian Co-ordinate System

→ **line**

$\rightarrow d\vec{l} = dx\,\hat{i}_x + dy\,\hat{i}_y + dz\,\hat{i}_z$

→ **Surface**

$\rightarrow d\vec{s} = dx\,dy\;\hat{i}_z$

$= dx\,dz\;\hat{i}_y$

$= dy\,dz\;\hat{i}_x$

→ **Vol.^m**

$\rightarrow dV = dx\,dy\,dz$

→ **Gradient**

$$\rightarrow \vec{\nabla} = \frac{\partial}{\partial x}\hat{i}_x + \frac{\partial}{\partial y}\hat{i}_y + \frac{\partial}{\partial z}\hat{i}_z$$

→ **Curl** $(\vec{\nabla}\times\vec{F})$

$\vec{F} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix}$$

→ Divergence

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

→ Cylindrical Co-ordinate System

$\rightarrow d\vec{l} = dr\,\hat{i}_r + r\,d\phi\,\hat{i}_\phi + dz\,\hat{i}_z$

$\rightarrow d\vec{s} = r\,dr\,d\phi\;\hat{i}_z$

$= dr\,dz\;\hat{i}_\phi$

$= r\,d\phi\,dz\;\hat{i}_r$

$\rightarrow dV = r\,d\phi\,dr\,dz$

$$\rightarrow \vec{\nabla} = \frac{\partial}{\partial r}\hat{i}_r + \frac{\partial}{r\,\partial\phi}\hat{i}_\phi + \frac{\partial}{\partial z}\hat{i}_z$$

$\rightarrow \vec{F} = F_r\hat{i}_r + F_\phi\hat{i}_\phi + F_z\hat{i}_z$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i}_r & \hat{i}_\phi & \hat{i}_z \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \\ F_r & F_\phi & F_z \end{vmatrix}$$

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial(rF_r)}{r\,\partial r} + \frac{\partial F_\phi}{r\,\partial\phi} + \frac{\partial F_z}{\partial z}$$

Spherical Co-ordinate System $[r, \theta, \phi]$

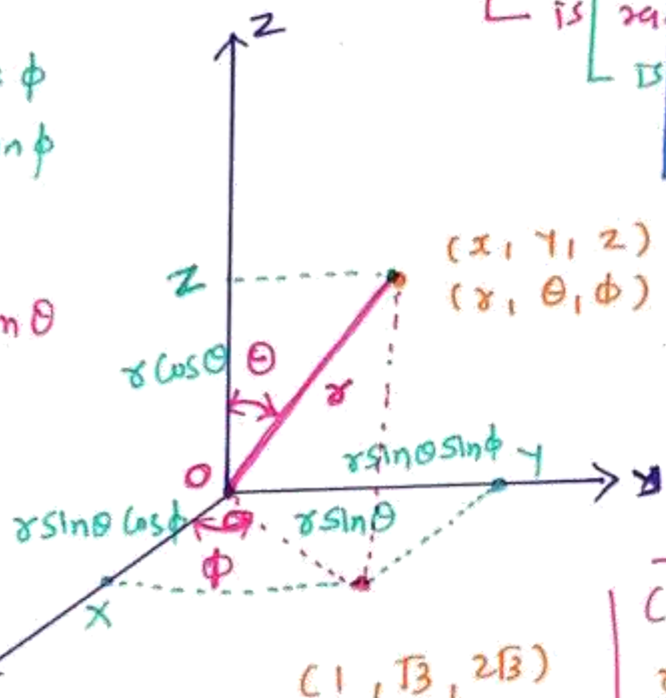
- r is radius w.r.t origin
- θ is angle of line w.r.t z axis.
- ϕ is angle w.r.t x axis in xy plane.

$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$

$\sqrt{x^2+y^2} = r \sin\theta$



$r = \sqrt{x^2+y^2+z^2}$

$\phi = \tan^{-1}(y/x)$

$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$Sp(4, 30^\circ, 60^\circ) \rightarrow Ca(x, y, z)$ $(1, \sqrt{3}, 2\sqrt{3})$

$r = 4, \quad \theta = 30^\circ, \quad \phi = 60^\circ$

$x = r\sin\theta\cos\phi = 4 \sin 30 \cos 60 = 1$

$y = r\sin\theta\sin\phi = 4\sin 30 \sin 60 = \sqrt{3}$

$z = r\cos\theta = 4\cos 30 = 2\sqrt{3}$

$Ca(1, 2, 3) \rightarrow Sp(r, \theta, \phi)$ $(\sqrt{14}, \tan^{-1}\frac{\sqrt{5}}{3}, \tan^{-1} 2)$

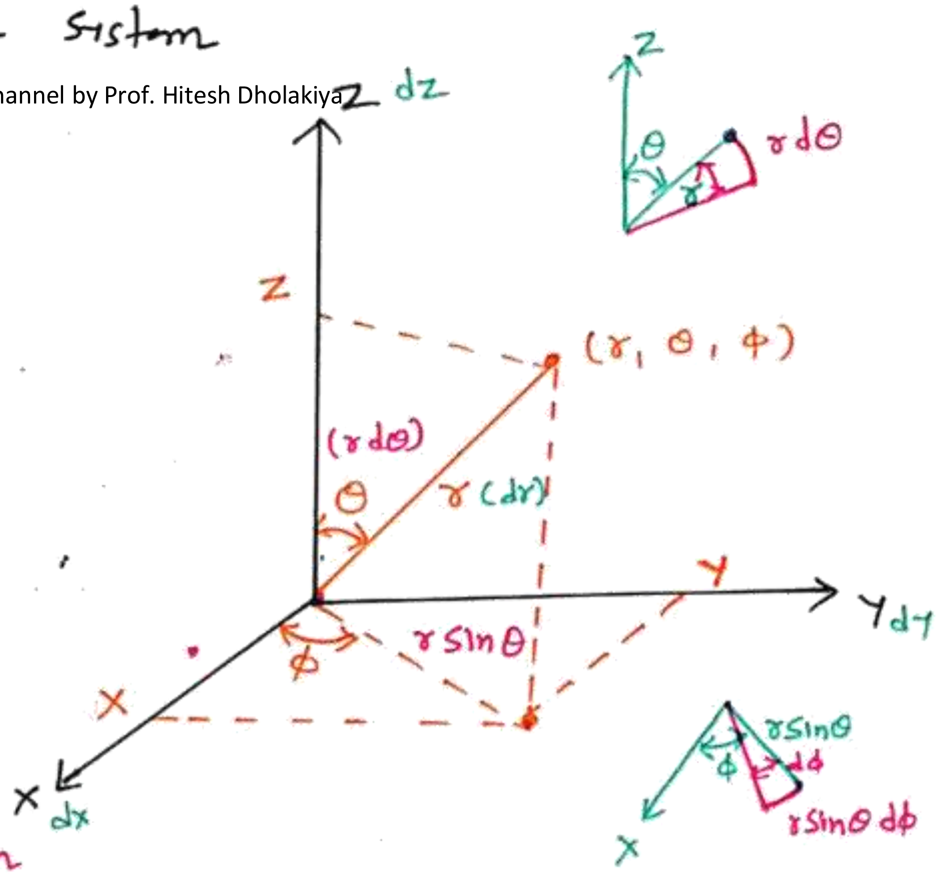
$r = \sqrt{1+4+9} = \sqrt{14}$

$\theta = \tan^{-1}\frac{\sqrt{1+4}}{3} = \tan^{-1}\left(\frac{\sqrt{5}}{3}\right)$

$\phi = \tan^{-1}\left(\frac{2}{1}\right) = \tan^{-1} 2$

Spherical Co-ordinate System

- line
- Surface
- Volume.
- Gradient
- Divergence
- Curl



- For line Integration

$$d\vec{l} = dr\,\hat{i}_r + r\,d\theta\,\hat{i}_\theta + r\sin\theta\,d\phi\,\hat{i}_\phi$$

- For Surface Integration

$$d\vec{S} = r\,dr\,d\theta\,\hat{i}_\phi$$
$$= r\sin\theta\,dr\,d\phi\,\hat{i}_\theta$$
$$= r^2\sin\theta\,d\theta\,d\phi\,\hat{i}_r.$$

- For vol.^{m} Integration

$$dV = r^2\sin\theta\,dr\,d\theta\,d\phi$$

- For del Operator.

$$\vec{\nabla} = \frac{\partial}{\partial r}\hat{i}_r + \frac{\partial}{r\,\partial\theta}\hat{i}_\theta + \frac{\partial}{r\sin\theta\,\partial\phi}\hat{i}_\phi$$

- For Curl $(\vec{F} = a_r\hat{i}_r + a_\theta\hat{i}_\theta + a_\phi\hat{i}_\phi)$.

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i}_r & \hat{i}_\theta & \hat{i}_\phi \\ \frac{\partial}{\partial r} & \frac{1}{r}\frac{\partial}{\partial\theta} & \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} \\ a_r & a_\theta & a_\phi \end{vmatrix}$$

- For divergence.

$$\vec{\nabla}\cdot\vec{F} = \frac{1}{r^2}\frac{\partial(r^2 a_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(a_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial a_\phi}{\partial\phi}$$

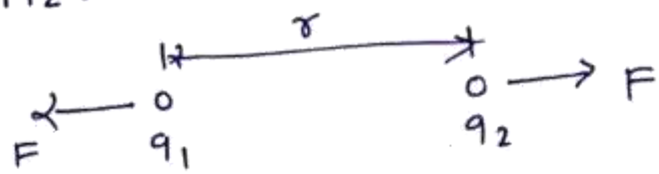
Coulomb's Law

- Force acting between two charges is directly proportional to charges and inversely proportional to square of distance bet.^{n} them.

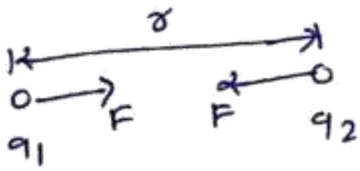
$$\Rightarrow F \propto \frac{q_1 q_2}{r^2}$$

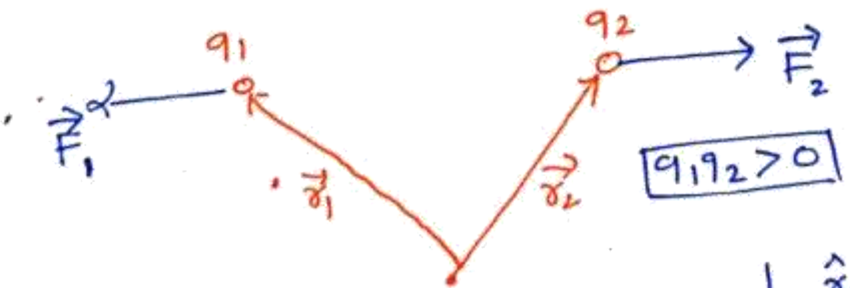
$$\Rightarrow \boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}$$

Case - I $\quad q_1 q_2 > 0$



Case - II $\quad q_1 q_2 < 0$

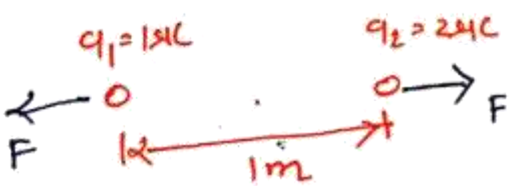




$$\Rightarrow \vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_1 \qquad \Big| \qquad \hat{r}_1 = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}_1 - \vec{r}_2}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\left(\frac{\vec{r}}{r}\right)$$

$$\Rightarrow \vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\vec{r} \qquad \Big| \qquad \vec{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}(\vec{r}_2 - \vec{r}_1)$$



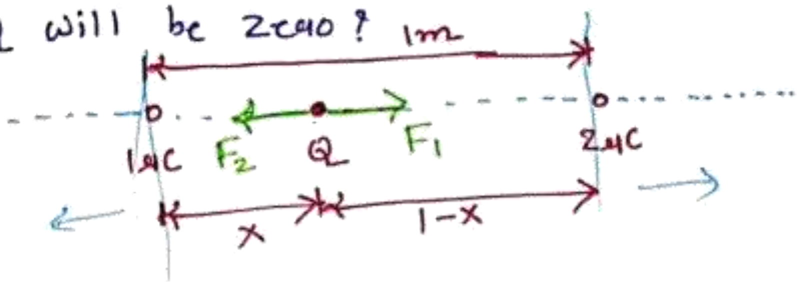
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$= \frac{9\times10^9 \times 10^{-6} \times 2\times10^{-6}}{1^2}$$

$$= 18 \times 10^{-3}\ N$$

Examples on Coulomb's law

1] If two charges 1μC and 2 μC are seperated by 1m, then at what distance from 1μC charge, force on charge Q will be zero?



$$\Rightarrow F_1 = F_2$$

$$\Rightarrow \frac{\cancel{1}}{\cancel{4\pi\varepsilon_0}} \frac{\cancel{Q}(1\times10^{\cancel{-6}})}{x^2} = \frac{\cancel{1}}{\cancel{4\pi\varepsilon_0}} \frac{\cancel{Q}(2\times10^{\cancel{-6}})}{(1-x)^2}$$

$$\Rightarrow (1-x)^2 = 2x^2$$

$$\Rightarrow x^2 - 2x + 1 = 2x^2$$

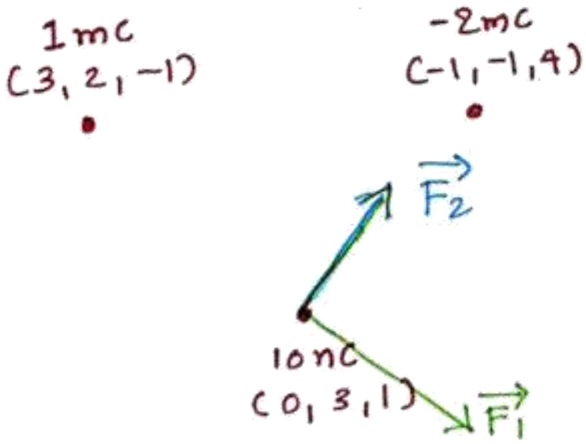
$$\Rightarrow x^2 + 2x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4+4}}{2}$$

$$= \frac{-2 \pm \sqrt{8}}{2} = \boxed{0.41} \checkmark \quad \text{or} \quad \boxed{-2.41} \times$$

2] Point charges 1mC and -2mC are located at (3, 2, -1) and (-1, -1, 4) respectively. Calculate the electric force on 10 nC charge located at (0, 3, 1).



$$\vec{F_1} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_1^2} \left(\frac{\vec{r_1}}{r_1}\right)$$

$$\rightarrow \vec{r_1} = (0, 3, 1) - (3, 2, -1)$$

$$= (-3, 1, 2)$$

$$\rightarrow r_1 = \sqrt{9+1+4} = \sqrt{14}$$

$$\vec{F_1} = 9\times10^9 \times \frac{10^{-3} \times 10\times10^{-9}}{(\sqrt{14})^3} (-3, 1, 2)$$

$$= 1.72\times10^{-3} (-3, 1, 2) \text{ N}$$

$$= (-5.16, 1.72, 3.44) \text{ mN}$$

$$\rightarrow \vec{F_2} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_2^2} \left(\frac{\vec{r_2}}{r_2}\right)$$

$$\rightarrow \vec{r_2} = (-1, -1, 4) - (0, 3, 1)$$

$$= (-1, -4, 3)$$

$$\rightarrow r_2 = \sqrt{1+16+9} = \sqrt{26}$$

$$\rightarrow \vec{F_2} = 9\times10^9 \times \frac{2\times10^{-3} \times 10\times10^{-9}}{(\sqrt{26})^3} (-1, -4, 3)$$

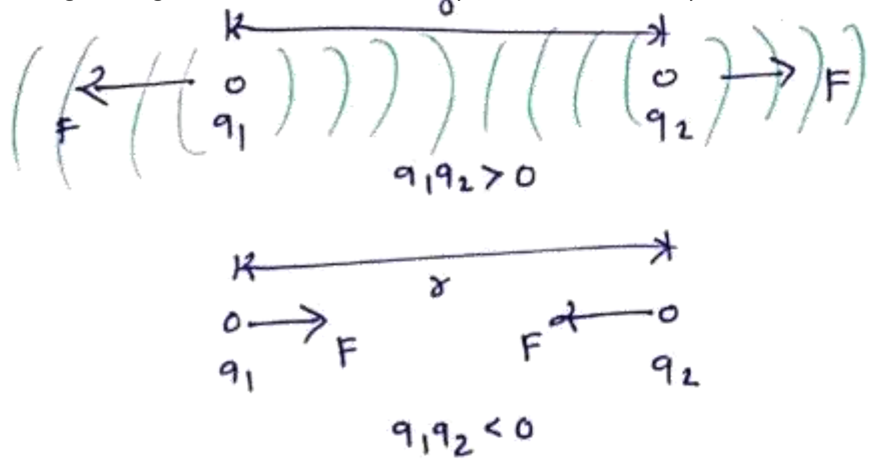
$$= 1.35\times10^{-3} (-1, -4, 3) \text{ N} = (-1.35, -5.4, 4.05) \text{ mN}$$

$$\rightarrow \vec{F} = \vec{F_1} + \vec{F_2}$$

$$= (-5.16, 1.72, 3.44) + (-1.35, -5.4, 4.05)$$

$$= (-6.51, -3.68, 7.49) \text{ mN}$$

Electric field by point charge.

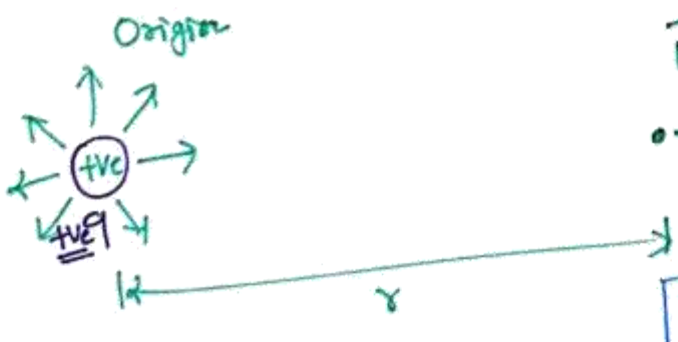


$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

- Electric field by charge Q is amount of force on 1C charge.

$$E = \frac{F}{q} = \frac{F}{1C} = \frac{1}{q}\left[\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\right] = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

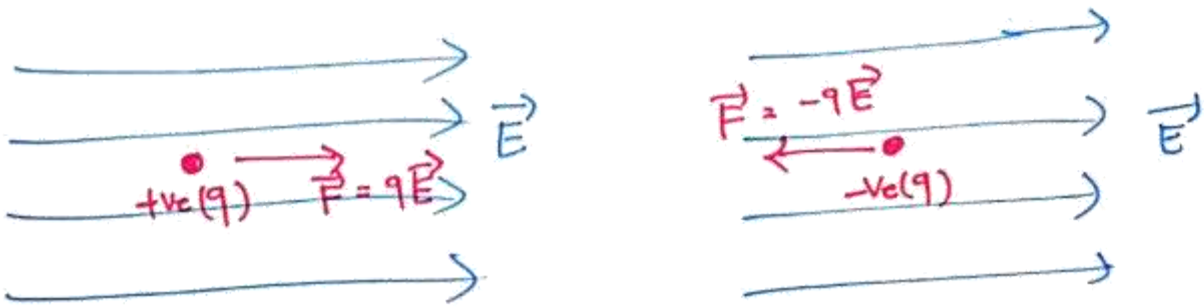
- For +ve charge electric field will be in outward from charge and for -ve charge electric field will be in inward direction to the charge.



$$\vec{E} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}\,\hat{r}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}}$$



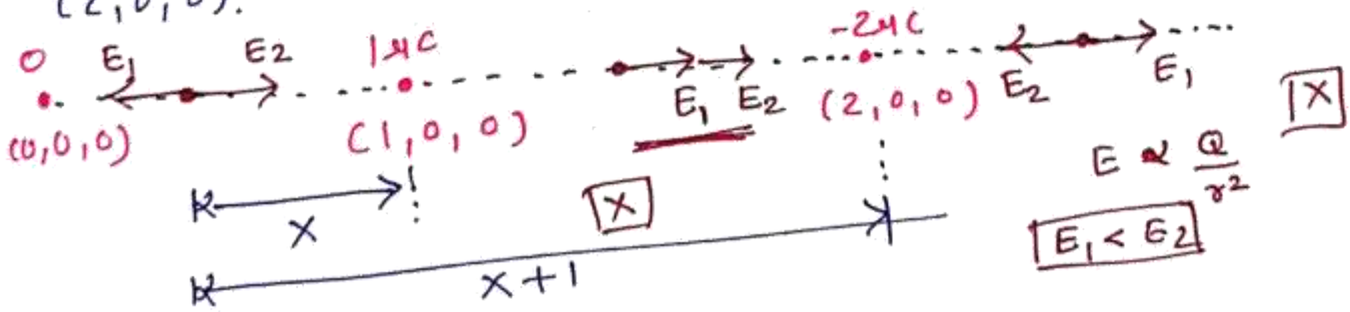
Examples on Electric field due to point charge.

Find the electric field $\vec{E}$ at $(0,3,4)$ m due to a point charge $Q = 0.5\,\mu C$ placed at the origin.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(\frac{\vec{r}}{r}\right)$$

$$\vec{r} = (0,3,4) - (0,0,0)$$
$$= (0,3,4)\ m$$
$$r = \sqrt{0^2+3^2+4^2} = 5\ m$$
$$\vec{E} = 9\times10^9 \times \frac{0.5\times10^{-6}}{5^3}\times(0,3,4)$$
$$= 36(0,3,4)\ V/m \text{ or } N/C$$
$$= (0, 108, 144)\ V/m.$$

Find the position at which electric field is zero. If $1\mu C$ is located at $(1,0,0)$ and $-2\mu C$ is located at $(2,0,0)$.



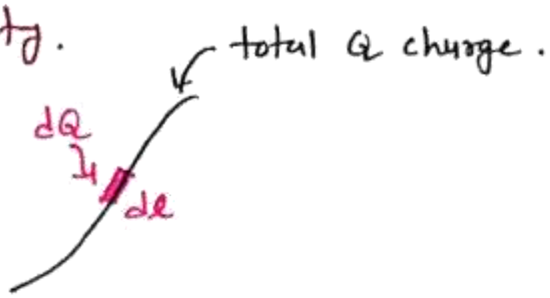
$$\Rightarrow E_1 = E_2$$
$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_2^2}$$
$$\Rightarrow \frac{1\times10^{-6}}{x^2} = \frac{2\times10^{-6}}{(x+1)^2}$$
$$\Rightarrow (x+1)^2 = 2x^2$$
$$\Rightarrow x^2 - 2x - 1 = 0$$

$$x = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{2\pm\sqrt{4+4}}{2}$$
$$= \frac{2\pm\sqrt{8}}{2}$$
$$= \boxed{2.41} \text{ or } \boxed{-0.41}$$

$$(x',0,0) = ((1-2.41), 0, 0)$$
$$= (-1.41, 0, 0)$$

line charge density, Surface charge density & Volume charge density.

line charge density.

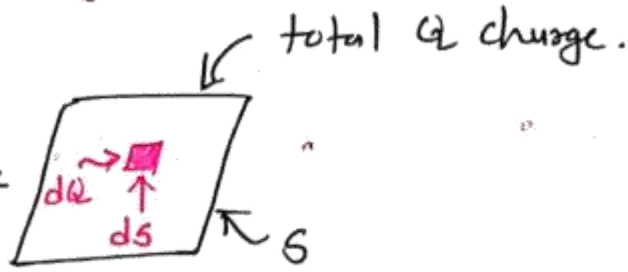


$\rightarrow \rho_L = \dfrac{\text{total charge}}{\text{length of line.}}$

$\rightarrow \rho_L = \dfrac{dQ}{dl} \Rightarrow dQ = \rho_L \, dl$

$\Rightarrow \int dQ = Q = \int \rho_L \, dl$

Surface charge density

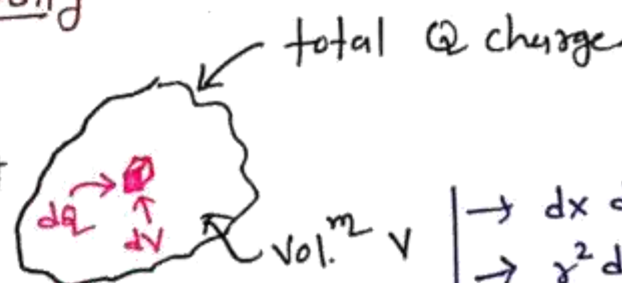


$\rightarrow \rho_S = \dfrac{\text{total Charge}}{\text{Area of Surface}}$

$\rightarrow \rho_S = \dfrac{dQ}{dS} \Rightarrow dQ = \rho_S \, dS$

$\Rightarrow \int dQ = Q = \int \rho_S \, dS$

Volume charge density



$\rightarrow \rho_V = \dfrac{\text{total charge}}{\text{total Vol}^m \text{ of object}}$

$\rightarrow \rho_V = \dfrac{dQ}{dV} \Rightarrow dQ = \rho_V \, dV$

$\Rightarrow \int dQ = Q = \int \rho_V \, dV$

$\rightarrow dx\, dy\, dz$

$\rightarrow r^2 dr\, d\theta\, d\phi$

$\rightarrow r\, dr\, d\phi\, dz.$

Examples based on Volume charge density.

The volume charge density $\rho_v = \rho_0 e^{-|x|-|y|-|z|}$ exits over all free space. Calculate the total charge present.

$$\rho_v = \rho_0 e^{-|x|-|y|-|z|}$$

$$\rightarrow Q = \int \rho_v \, dV$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho_0 e^{-|x|-|y|-|z|} \, dx\, dy\, dz$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho_0 e^{-|x|} e^{-|y|} e^{-|z|} \, dx\, dy\, dz.$$

$$= \rho_0 \int_{0}^{\infty} 2 e^{-x} dx \int_0^{\infty} 2 e^{-y} dy \int_0^{\infty} 2 e^{-z} dz$$

$$= 8\rho_0 \left[\frac{e^{-x}}{-1}\right]_0^{\infty} \left[\frac{e^{-y}}{-1}\right]_0^{\infty} \left[\frac{e^{-z}}{-1}\right]_0^{\infty}$$

$$= 8\rho_0 [0+1][0+1][0+1]$$

$$\boxed{Q = 8\rho_0}$$

The spherical volume charge is given by

$$\rho_v = \rho_{v0}\left(1 - \frac{r^2}{a^2}\right), \quad r < a$$

$$= 0, \quad r > a$$

Calculate the total charge Q.

$$\rightarrow Q = \int \rho_v \, dV = \int_V \rho_v (r^2 \sin\theta)\, dr\, d\theta\, d\phi.$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{0}^{a} \rho_{v0}\left(1 - \frac{r^2}{a^2}\right) r^2 \sin\theta \, dr\, d\theta\, d\phi.$$

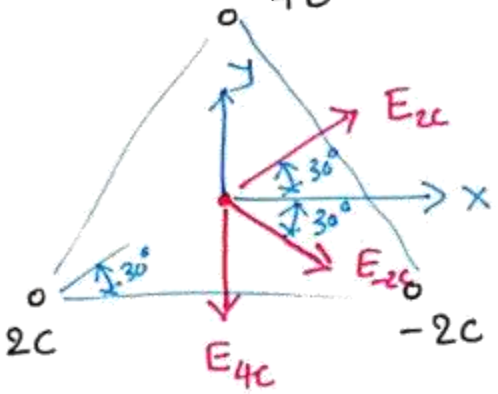
$$= \rho_{v0} [\phi]_0^{2\pi} [-\cos\theta]_0^{\pi} \left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right]_0^a$$

$$= \rho_{v0} [2\pi][2]\left[\frac{a^3}{3} - \frac{a^3}{5}\right]$$

$$= 4\pi \rho_{v0} \left[\frac{2}{15} a^3\right]$$

$$\boxed{Q = \frac{8\pi}{15} \rho_{v0} a^3}$$

Examples of Electric field for charges located on triangle.



– For equilateral triangle of side a = 1 m, find electric field at center.

$$\rightarrow \vec{E}_{4c} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}(-\hat{j}) = -9\times10^9 \times \frac{4}{(1/\sqrt{3})^2}\hat{j} = -108\times10^9\hat{j}$$

$$\rightarrow \vec{E}_{2c} = E_{2c}\cos 30\,\hat{i} + E_{2c}\sin 30\,\hat{j}$$

$$= \frac{1}{4\pi\varepsilon_0}\times\frac{2}{(1/\sqrt{3})^2}\cos 30\,\hat{i} + \frac{1}{4\pi\varepsilon_0}\times\frac{2}{(1/\sqrt{3})^2}\sin 30\,\hat{j}$$

$$= 9\times10^9\times3\times\cancel{2}\times\frac{\sqrt{3}}{\cancel{2}}\hat{i} + 9\times10^9\times2\times3\times\left(\frac{1}{2}\right)\hat{j}$$

$$= 27\sqrt{3}\times10^9\hat{i} + 27\times10^9\hat{j}$$

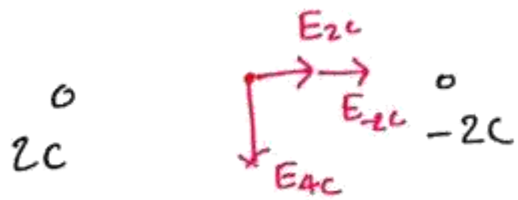
$$\rightarrow \vec{E}_{-2c} = E_{2c}\cos 30\,\hat{i} + E_{-2c}\sin 30\,(-\hat{j})$$

$$= \frac{1}{4\pi\varepsilon_0}\times\frac{2}{(1/\sqrt{3})^2}\cos 30\,\hat{i} - \frac{1}{4\pi\varepsilon_0}\times\frac{2}{(1/\sqrt{3})^2}\sin 30\,\hat{j}$$

$$= 9\times10^9\times3\times\cancel{2}\times\frac{\sqrt{3}}{\cancel{2}}\hat{i} - 9\times10^9\times3\times\cancel{2}\times\frac{1}{\cancel{2}}\hat{j}$$

$$= 27\sqrt{3}\times10^9\hat{i} - 27\times10^9\hat{j}$$

$$\rightarrow \vec{E}_{total} = \vec{E}_{4c} + \vec{E}_{2c} + \vec{E}_{-2c} = \boxed{54\sqrt{3}\times10^9\hat{i} - 108\times10^9\hat{j}}\ N/C$$



$$\rightarrow \vec{E}_{2c} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{i} = 9\times10^9\times\frac{2}{(1/2)^2}\hat{i} = 72\times10^9\hat{i}$$

$$\rightarrow \vec{E}_{-2c} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{i} = 9\times10^9\times\frac{2}{(1/2)^2}\hat{i} = 72\times10^9\hat{i}$$

$$\rightarrow \vec{E}_{4c} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{j} = -9\times10^9\times\frac{4}{(\sqrt{3}/2)^2}\hat{j}$$

$$= -48\times10^9\hat{j}$$

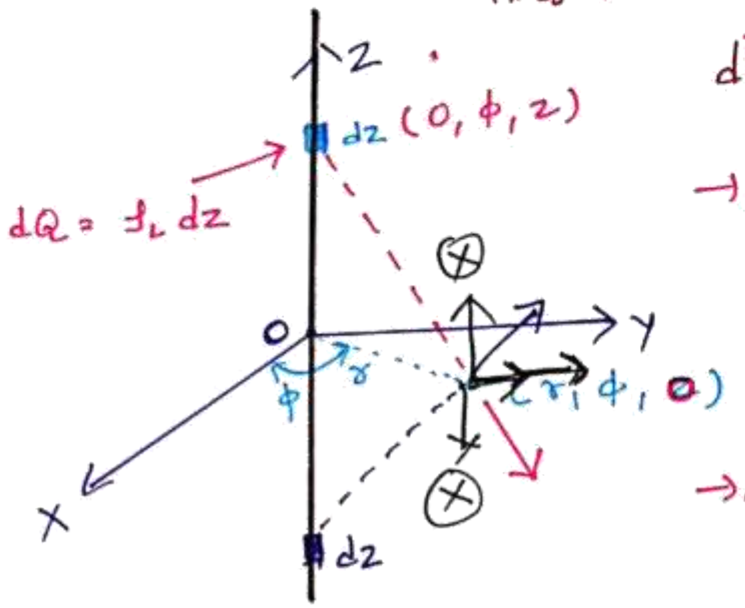
$$\rightarrow \vec{E}_{total} = \vec{E}_{2c} + \vec{E}_{-2c} + \vec{E}_{4c}$$

$$= \boxed{144\times10^9\hat{i} - 48\times10^9\hat{j}\ N/C}$$

Electric field due to line charge density.

→ We already know electric field due to point charge.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$



$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{R^2}\hat{R}' = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{R^3}\vec{R}$$

$$\rightarrow \vec{R}' = (r, \phi, 0) - (0, \phi, z)$$

$$= (r, 0, -z)$$

$$\rightarrow R = \sqrt{r^2 + z^2}$$

$$\rightarrow d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{(r^2+z^2)^{3/2}}(r, 0, -z)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\rho_L\, dz}{(r^2+z^2)^{3/2}}(\underline{r}, 0, \underline{-z})$$

→ So here z component will get cancelled and r component will be added.

$$\rightarrow d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_L\, dz}{(r^2+z^2)^{3/2}}(r, 0, 0)$$

$$\rightarrow \vec{E} = \int_{-\infty}^{\infty}\frac{1}{4\pi\varepsilon_0}\frac{\rho_L\, dz}{(r^2+z^2)^{3/2}}(r, 0, 0)$$

$z = r\tan\theta \Rightarrow d\theta = r\sec^2\theta\, d\theta$.

$-\infty \rightarrow -\pi/2$

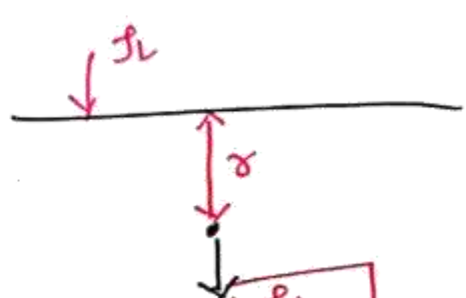
$\infty \rightarrow \pi/2$

$$\rightarrow \vec{E} = \int_{-\pi/2}^{\pi/2}\frac{1}{4\pi\varepsilon_0}\frac{\rho_L\, r^2\sec^2\theta\, d\theta}{r^3\sec^3\theta}\hat{a}_r$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta\,\hat{a}_r$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r}\left[\sin\theta\right]_{-\pi/2}^{\pi/2}\hat{a}_r$$

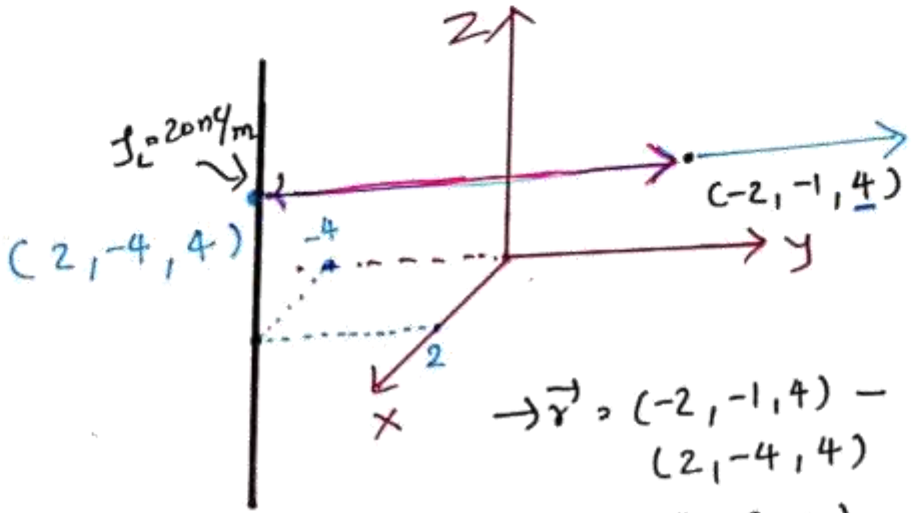
$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{a}_r}$$



$$\boxed{\frac{\rho_L}{2\pi\varepsilon_0 r}}$$

Examples on Electric field due to line charge density.

- Uniform line charge distribution $\rho_L = 20\ nC/m$ is there on line $x = 2m$, $y = -4m$. Determine $\vec{E}$ at $(-2,-1,4)\ m$.



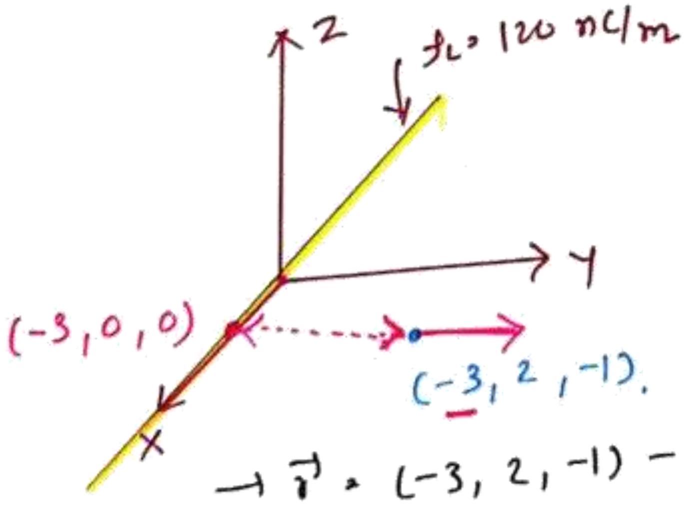
$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{r} = \frac{\rho_L}{2\pi\varepsilon_0 r}\left(\frac{\vec{r}}{r}\right) = \frac{\rho_L}{2\pi\varepsilon_0 r^2}\vec{r}$$

$\rightarrow \vec{r} = (-2,-1,4) - (2,-4,4)$
$= (-4,3,0)$

$\rightarrow r = \sqrt{4^2+3^2} = 5$

$$\rightarrow \vec{E} = \frac{20\times10^{-9} \times 18\times10^{9}}{25}(-4,3,0)$$
$$= 14.4(-4,3,0) = (-57.6, 43.2, 0)\ N/C.$$

- Uniform line charge of $120\ nC/m$ lie along the entire x-axis. find Electric field at $P(-3,2,-1)m$.



$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{r} = \frac{\rho_L}{2\pi\varepsilon_0 r}\left(\frac{\vec{r}}{r}\right) = \frac{\rho_L}{2\pi\varepsilon_0 r^2}\vec{r}$$

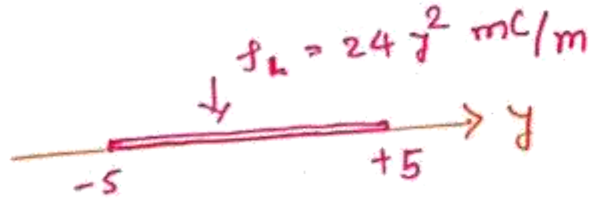
$\rightarrow \vec{r} = (-3,2,-1) - (-3,0,0) = (0,2,-1)$

$\rightarrow r = \sqrt{0^2+2^2+1} = \sqrt{5}$

$$\rightarrow \vec{E} = \frac{18\times10^{9} \times 120\times10^{-9}}{5}(0,2,-1)$$
$$= 432(0,2,-1) = (0, 864, -432)\ V/m.$$

Examples on line charge, Surface charge & Vol.^{m} charge

If the line charge density $\rho_L = 24y^2 \frac{mC}{m}$. Find the total charge on Y axis from -5 to $+5$ m.



$$Q = \int \rho_L \, dl$$

$$= \int_{-5}^{+5} 24y^2 \, dy \times 10^{-3}$$

$$= 24 \times 10^{-3} \left[\frac{y^3}{3} \right]_{-5}^{+5}$$

$$= 8 \times 10^{-3} [125 + 125]$$

$$= 2000 \times 10^{-3} = 2\,C.$$

Find the total charge contained in Cylindrical Vol.^{m} defined by $\rho \le 2$, $0 \le z \le 3$, $\rho_v = 20\rho z \frac{mC}{m^3}$.

$$Q = \int \rho_v \, dv$$

For Cylinder.

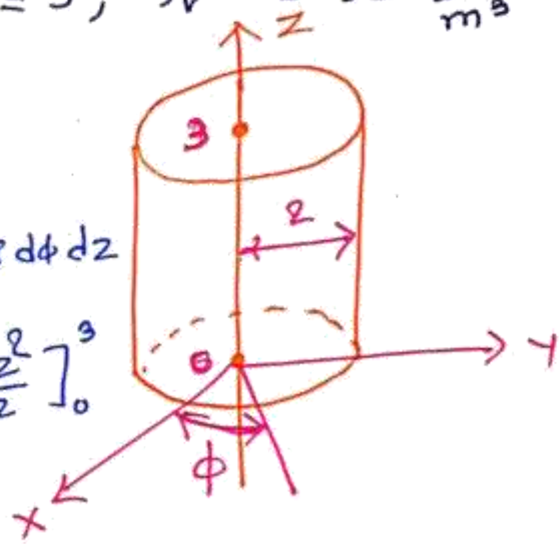
$$= \iiint \rho_v \, \boxed{\rho} \, d\rho \, d\phi \, dz$$

$$= \int_0^3 \int_0^{2\pi} \int_0^2 20\rho z \times 10^{-3} \times \rho \, d\rho \, d\phi \, dz$$

$$= 20 \times 10^{-3} \left[\frac{\rho^3}{3} \right]_0^2 [\phi]_0^{2\pi} \left[\frac{z^2}{2} \right]_0^3$$

$$= 20 \times 10^{-3} \times \frac{8}{3} \times 2\pi \times \frac{9}{2}$$

$$= 1.5072\ C$$



A Squre plate residing in a xy plane is situated in the space defined by $-3 \le x \le 3$ and $-3 \le y \le 3$. find the total charge on the plate If Surface charge density is given by $\rho_s = 4y^2 \,\mu C/m^2$.

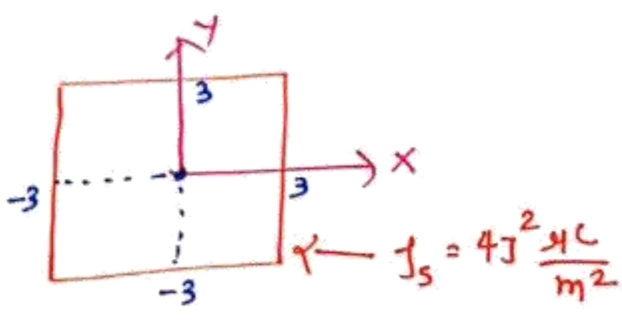
$$Q = \int \rho_s \, ds$$

$$= \int_{-3}^{3} \int_{-3}^{3} 4y^2 \, dx \, dy \times 10^{-6}$$

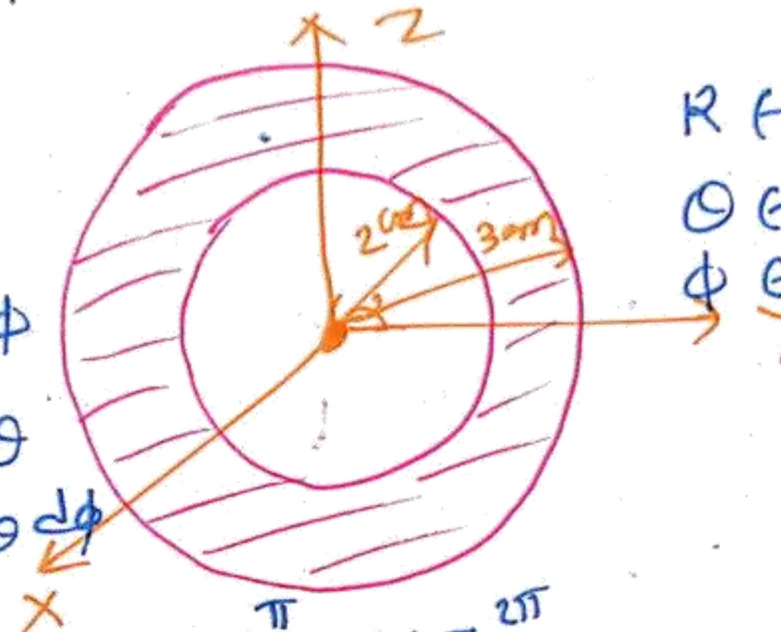
$$= 4 [x]_{-3}^{3} \left[\frac{y^3}{3} \right]_{-3}^{3} \times 10^{-6}$$

$$= 4 \times 10^{-6} \times 6 \times \frac{1}{\cancel{3}} \times [9 + 9]$$

$$= 0.432\ mC.$$



A thick spherical shell centered at the origin extends bet.^{n} $R = 2$ cm and $R = 3$ cm. If the vol.^{m} charge density is $\rho_v = 3R \times 10^{-4}$ C/m^3. Find the total charge contained in the shell.



$R \in (2\times10^{-2}, 3\times10^{-2})$ m

$\theta \in (0, \pi)$.

$\phi \in (0, 2\pi)$.

$$Q = \int \rho_v \, dV$$

$$= \int \rho_v R^2 \sin\theta \, dR \, d\theta \, d\phi$$

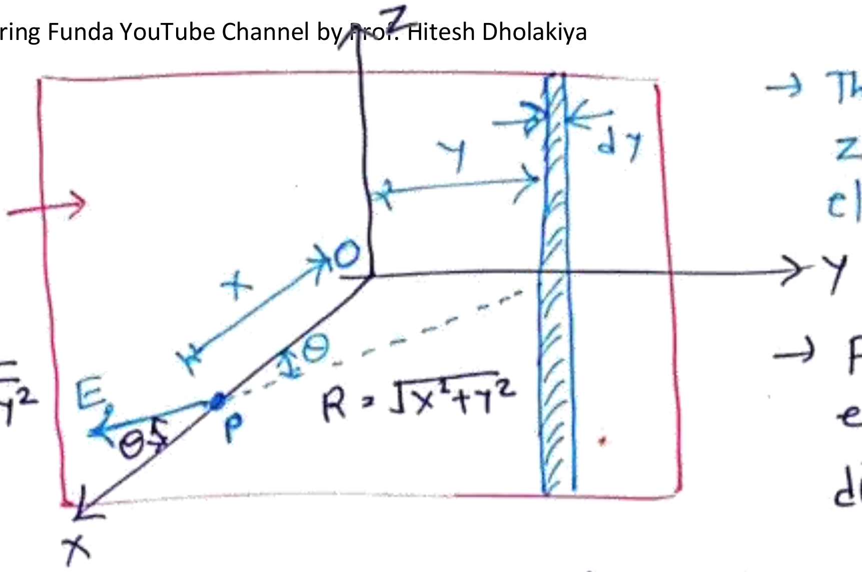
$$= \int_0^{2\pi}\int_0^{\pi}\int_{2\times10^{-2}}^{3\times10^{-2}} 3R\times10^{-4} \times R^2 \sin\theta \, dR \, d\theta \, d\phi$$

$$= 3\times10^{-4}\left[\frac{R^4}{4}\right]_{2\times10^{-2}}^{3\times10^{-2}} [-\cos\theta]_0^{\pi} [\phi]_0^{2\pi}$$

$$= 3\times10^{-4} \times \frac{1}{4}[81\times10^{-8} - 16\times10^{-8}][2][2\pi]$$

$$= 612.3 \times 10^{-12} \text{ C} = 0.612 \text{ nC}.$$

Electric field due to Surface Charge density.



$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$$

→ There is no y & z component of electric field.

→ For line charge electric field

$$d\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 R}.$$

→ Relation bet.ⁿ line charge & surface charge.

$$\rho_L = dy\,\rho_s$$

→ So,

$$d\vec{E} = \frac{dy\,\rho_s}{2\pi\varepsilon_0\sqrt{x^2+y^2}}\cos\theta = \frac{dy\,\rho_s}{2\pi\varepsilon_0\sqrt{x^2+y^2}} \times \frac{x}{\sqrt{x^2+y^2}}$$

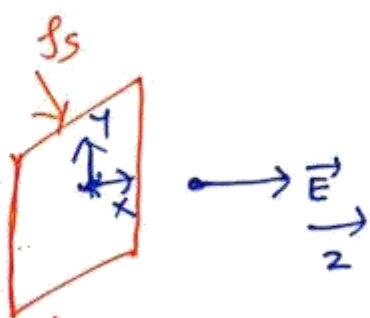
$$d\vec{E} = \frac{\rho_s\, x\, dy}{2\pi\varepsilon_0(x^2+y^2)}$$

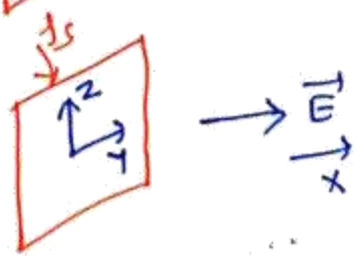
→ $$E = \int d\vec{E} = \int_{-\infty}^{\infty} \frac{\rho_s\, x\, dy}{2\pi\varepsilon_0(x^2+y^2)}$$

$$= \frac{\rho_s}{2\pi\varepsilon_0}\int_{-\infty}^{\infty} \frac{x\,dy}{x^2+y^2}$$

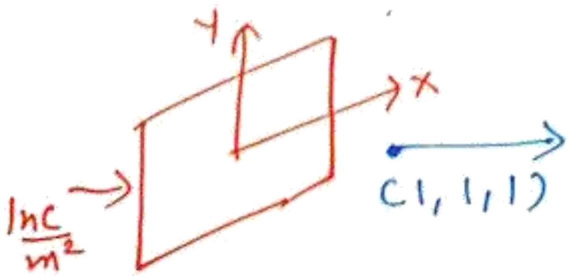
$$= \frac{\rho_s}{2\pi\varepsilon_0}\left[\tan^{-1}\left(\frac{y}{x}\right)\right]_{-\infty}^{\infty}$$

$$\boxed{E = \frac{\rho_s}{2\varepsilon_0}}$$





- IF $1\,nC/m^2$ Charge Sheet is there on XY plane the E field at (1, 1, 1) m.



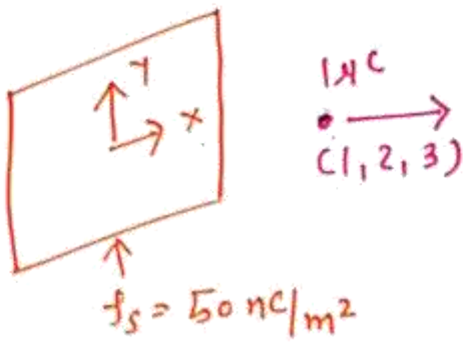
$$E = \frac{\rho_s}{2\varepsilon_0} = \frac{1\times10^{-9}}{2\times 8.854\times10^{-12}}$$

$$= 56.47\ V/m\ (\hat{k})$$

Examples on Electric field due to Surface charge density.

If Surface charge of $50\ nC/m^2$ is lie on xy plane. then find force on $1\mu C$ charge located at $(1,2,3)$ m.

$\rightarrow \vec{F} = q\vec{E}$



$\rightarrow$ Electric field at $(1,2,3)$

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{k}$$

$$= \frac{50\times10^{-9}}{2\times8.854\times10^{-12}}\hat{k}$$

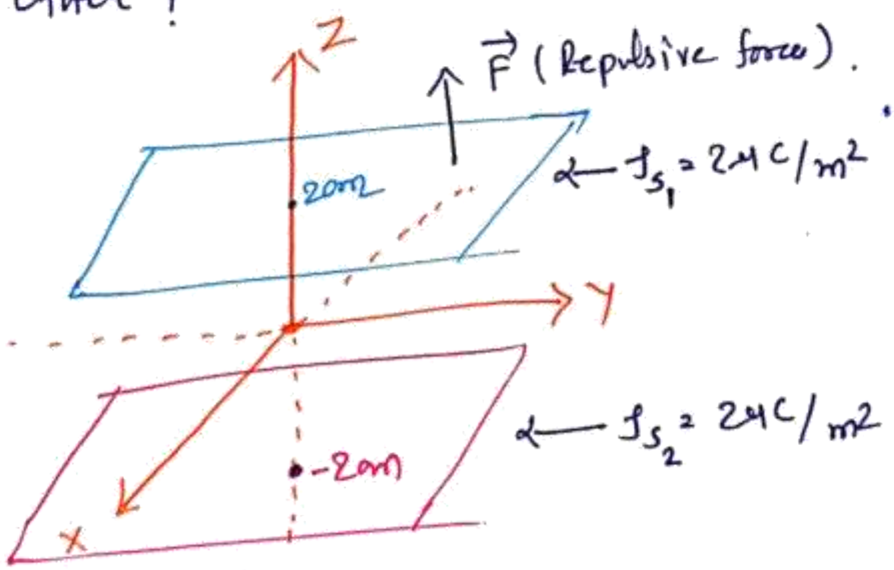
$$= 2.82\times10^{3}\ \frac{V}{m}\ \hat{k}$$

$$\rightarrow \vec{F} = q\vec{E}$$

$$= 1\times10^{-6}\times2.82\times10^{3}$$

$$= 2.82\times10^{-3}\ N\ \hat{k}$$

* If $\rho_s = 2\mu C/m^2$, existing in free space at $z = \pm 2$ cm what force per unit area does each sheet exert on the other?



$\rightarrow$ There will be repulsive force on each other.

$$\rightarrow \vec{F} = q\vec{E}$$

$$\rightarrow \frac{\vec{F}}{A} = \frac{q}{A}\vec{E}$$

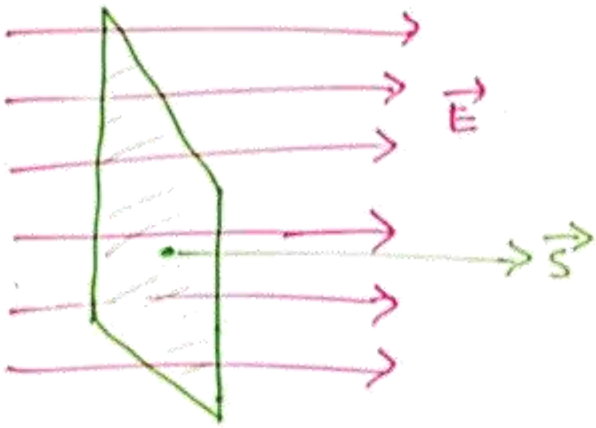
$$= \rho_{s1}\vec{E_2}$$

$$= \rho_{s1}\left(\frac{\rho_{s2}}{2\varepsilon_0}\right)$$

$$= \frac{\rho_{s1}\rho_{s2}}{2\varepsilon_0} = \frac{2^2\times10^{-12}}{2\times8.854\times10^{-12}} = 0.225\ \frac{N}{m^2}\ (\hat{k})$$

Electric flux

- Electric field lines passing through any surface is Electric flux.



Electric flux

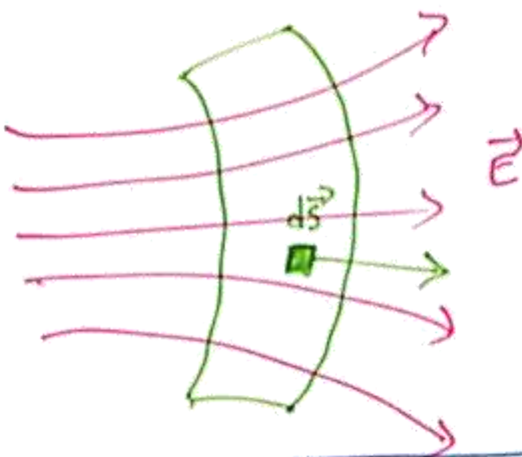
$\psi = \vec{E}.\vec{S}$

$= ES\cos\theta$

- $\theta = 0$

$\psi = E.S$

- Unit of Electric flux <u>wb</u> or <u>Vm</u>.



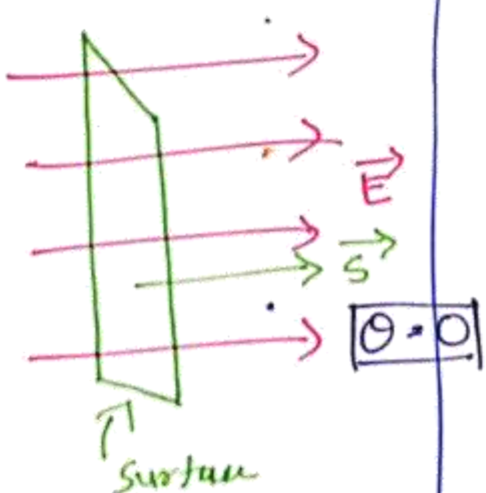
→ for small surface $d\vec{s}$ flux will be

$d\psi = \vec{E}.d\vec{s}$

→ For total electric flux

$\psi = \int d\psi = \int \vec{E}.d\vec{s}$

Case-I $\theta = 0$

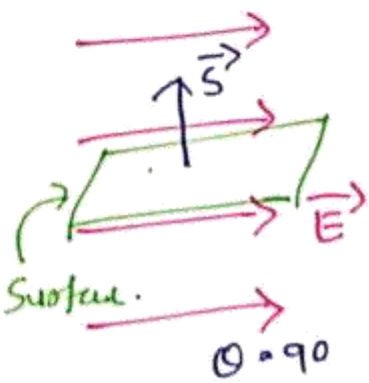


$\psi = \vec{E}.\vec{S}$

$= ES\cos\theta$

$= ES$

Case-II $\theta = 90$

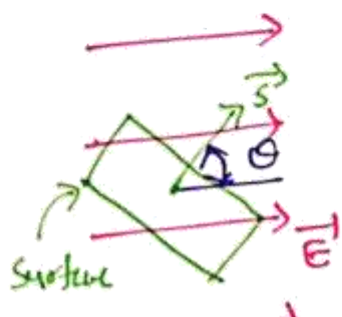


$\psi = \vec{E}.\vec{S}$

$= ES\cos\theta$

$= 0$

Case-III

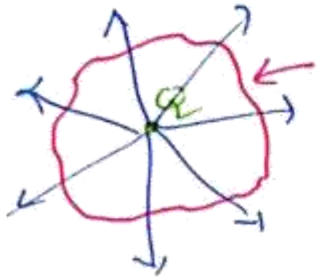


$\psi = \vec{E}.\vec{S}$

$= ES\cos\theta.$

Gauss's Law for Electric field.

- It states that total Electric flux passing through any closed surface is equal to enclosed charge by that surface devided by ε_0.

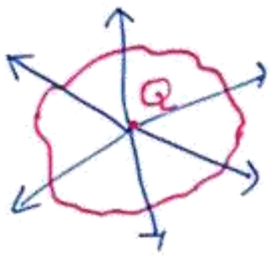


- Electric flux.

$$\psi = \oint \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$$

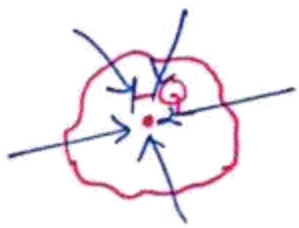
$$\psi = \oint \vec{D}\cdot d\vec{S} = Q_{enc}.$$

- Case-I



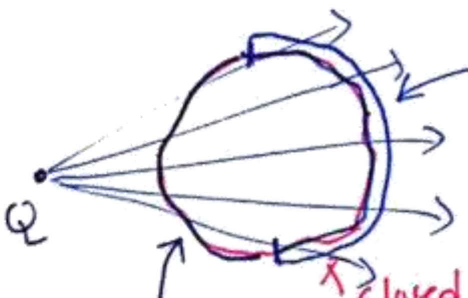
$$\psi = \oint \vec{D}\cdot d\vec{S} = Q_{enc}.$$

- Case-II



$$\psi = \oint \vec{D}\cdot d\vec{S} = -Q$$

- If flux is leaving enclosed surface then Electric flux is positive and if Electric flux is entering inside enclosed surface then Electric flux is positive.

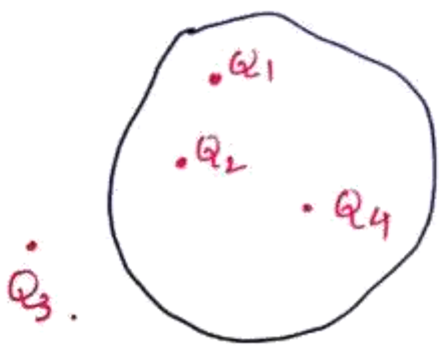


flux is leaving surface.
- So flux is (+ve).

- So total flux will be zero.

- flux is entering inside surface.
- So ψ is (-ve)

-



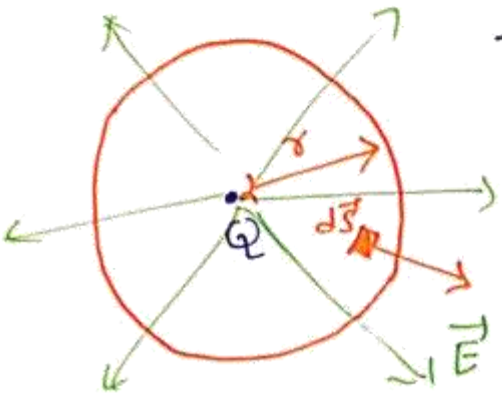
- $\psi = \oint \vec{D}\cdot d\vec{S} = (Q_1 + Q_2 + Q_4).$

- $\psi = \oint \vec{E}\cdot d\vec{S} = \dfrac{(Q_1 + Q_2 + Q_4)}{\varepsilon_0}.$

Applications of Gauss's law for electric field.

① Electric field due to point charge.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$



→ As per gauss law

$$\oint \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0}.$$

→ $\vec{E} \parallel d\vec{s}$, $\theta = 0$.

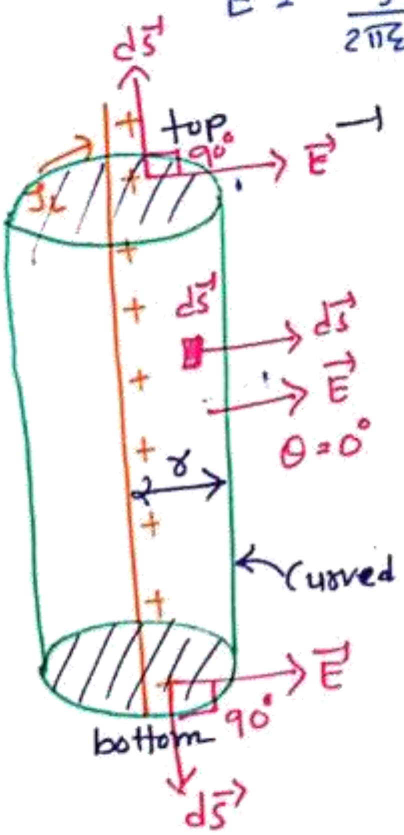
$$\Rightarrow \oint E\,ds = \frac{Q}{\varepsilon_0}.$$

$$\Rightarrow E(4\pi R^2) = \frac{Q}{\varepsilon_0}.$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\hat{R}$$

② Electric field due to line charge ρ_L.

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{r}$$



→ As per gauss's law.

$$\Rightarrow \int \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0}.$$

$$\Rightarrow \int_{top} \vec{E}\cdot d\vec{s} + \int_{Bottom} \vec{E}\cdot d\vec{s} + \int_{Curved} \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0}$$

top: $= \vec{E}\cdot d\vec{s} = E\,ds\cos 90 = 0$

Bottom: $= \vec{E}\cdot d\vec{s} = E\,ds\cos 90 = 0$

Curved: $= \vec{E}\cdot d\vec{s} = E\,ds\cos 0 = E\,ds.$

$$\Rightarrow \int E\,ds = \frac{Q}{\varepsilon_0}.$$

$$\Rightarrow E(2\pi r l) = \frac{Q}{\varepsilon_0}.$$

$$\Rightarrow E = \frac{Q}{2\pi r l\varepsilon_0}$$

$$\Rightarrow \boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{r}}$$

→ line charge

$$\rho_L = \frac{Q}{l}$$

Maxwell's 1st eq.n with Integral form & differential form.

→ As per gauss's law.

$$\oint \vec{D} \cdot d\vec{s} = Q. \quad \text{—} (1)$$

→ For vol.m charge distribution ρ_v

$$Q = \int \rho_v \, dV. \quad \text{—} (2).$$

→ from (1) & (2).

$$\oint \vec{D} \cdot d\vec{s} = \int \rho_v \, dV \quad \text{—} (A).$$

→ eq.n (A) is Integral form of Maxwell's 1st eq.n

→ As per divergence theorem

$$\oint \vec{A} \cdot d\vec{s} = \int \vec{\nabla} \cdot \vec{A} \, dV \quad \text{—} (3).$$

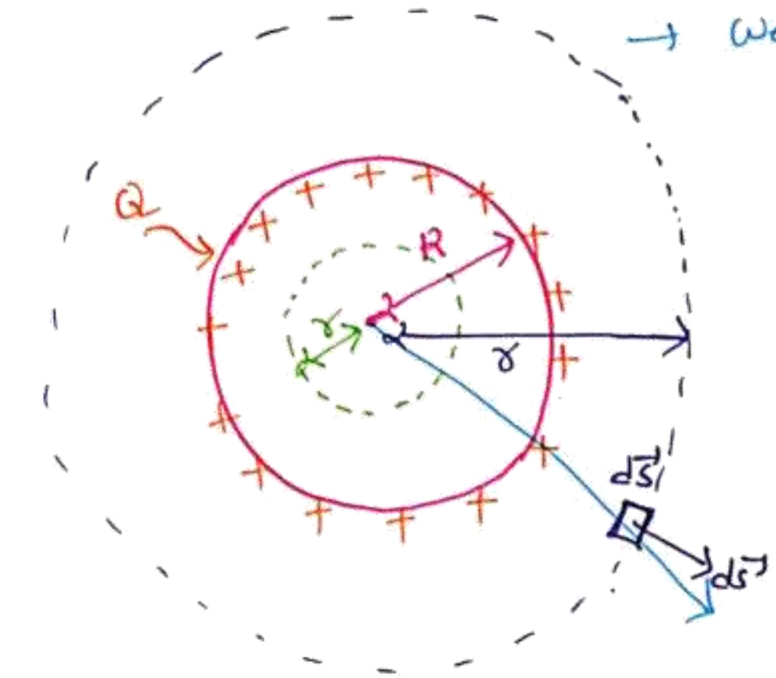
→ from eq.n (3), apply in eq.n (A).

$$\Rightarrow \int \vec{\nabla} \cdot \vec{D} \, dV = \int \rho_v \, dV$$

$$\Rightarrow \vec{\nabla} \cdot \vec{D} = \rho_v \quad \text{—} (B)$$

→ eq.n (B) is differential form of Maxwell's 1st eq.n or Point form of Maxwell's 1st eq.n

Electric field Intensity due to Conducting Sphere.



→ We will Calculated electric field at

$r > R$

$r = R$

$r < R$

→ Case I, $r > R$.

→ As per gauss's law.

$$\psi = \oint \vec{E}\, d\vec{S} = \frac{Q}{\varepsilon_0}.$$

$\vec{E} \parallel d\vec{S}$, $\theta = 0$.

$$\Rightarrow E(4\pi r^2) = \frac{Q}{\varepsilon_0}.$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

→ $r = R$, Case-II

→ As per gauss's law

$$\Rightarrow \oint \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}.$$

$\vec{E} \parallel d\vec{S}$, $\theta = 0$

$$\Rightarrow E(4\pi R^2) = Q/\varepsilon_0$$

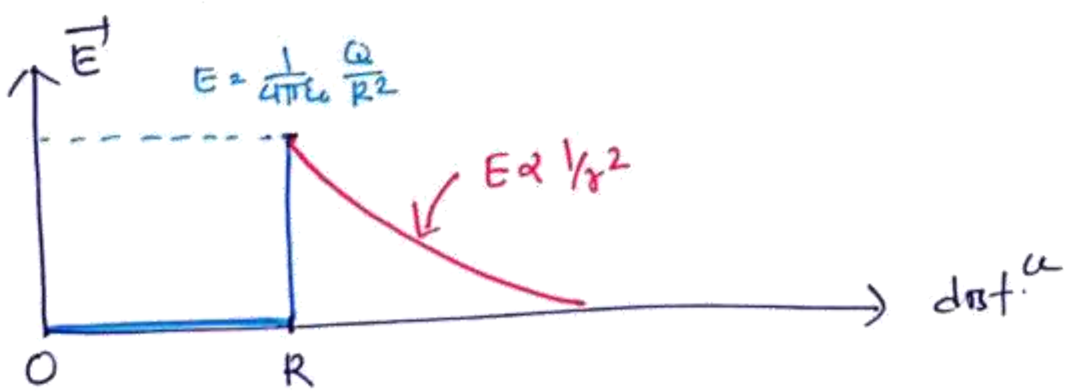
$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

→ Case-III, $r < R$

→ As per gauss's law

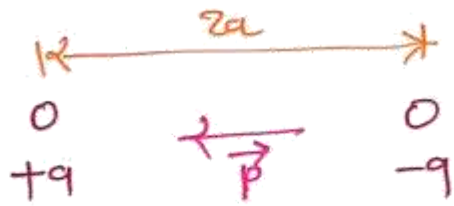
$$\Rightarrow \oint \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0} = 0.$$

$$\Rightarrow \boxed{\vec{E} = 0}$$



* Electric dipole & Dipole moment

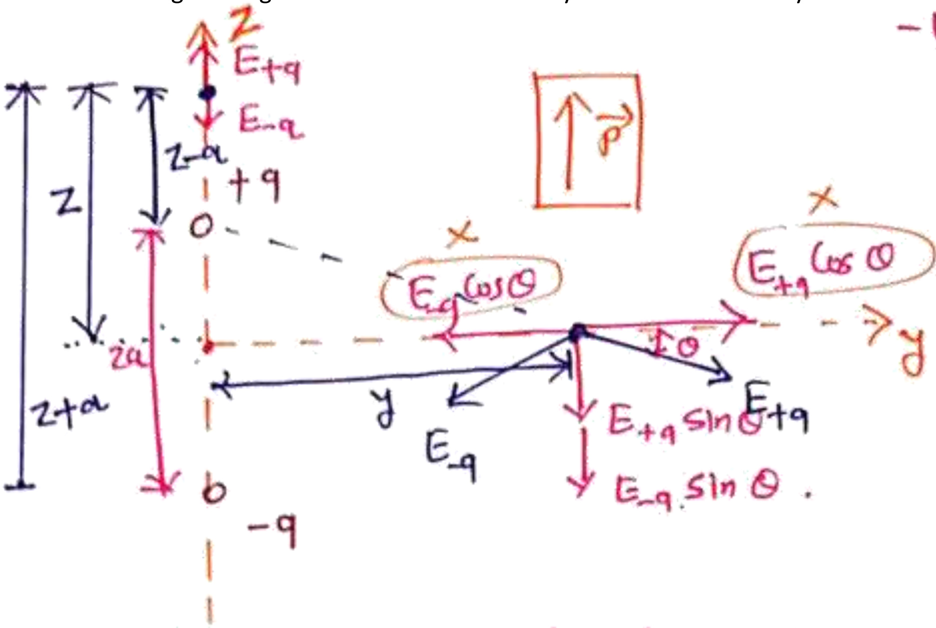
- Electric dipole is a pair of charges with same magnitude with opposite polarity



- Dipole moment

$$\vec{p} = (2\vec{a})q$$

Electric field due to dipole.



- Electric field on equator of dipole.

$$\vec{E} = -(E_{+q}\sin\theta + E_{-q}\sin\theta)\,\hat{p}$$

$$= -2E\sin\theta\,\hat{p}$$

$$= -2\left(\frac{kq}{(a^2+y^2)}\right)\frac{a}{\sqrt{a^2+y^2}}\,\hat{p}$$

$$= -\frac{k(2aq)\hat{p}}{(a^2+y^2)^{3/2}}$$

- If $a >> y$, $\vec{p} = (2aq)\hat{p}$

$$\boxed{\vec{E} = -\frac{k\vec{p}}{y^3}}$$

- Electric field on dipole axis.

$$E = E_{+q} + E_{-q}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(z-a)^2} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(z+a)^2}$$

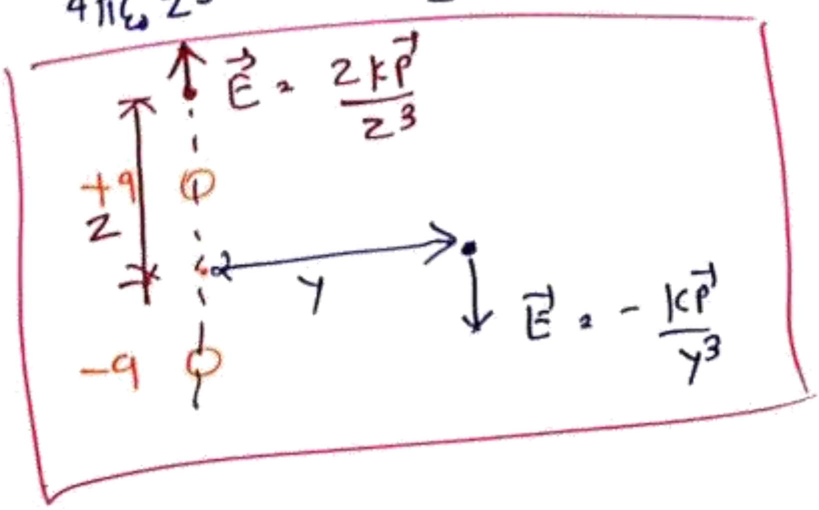
$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(z-a)^2} - \frac{1}{(z+a)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(z+a)^2-(z-a)^2}{(z^2-a^2)^2}\right]$$

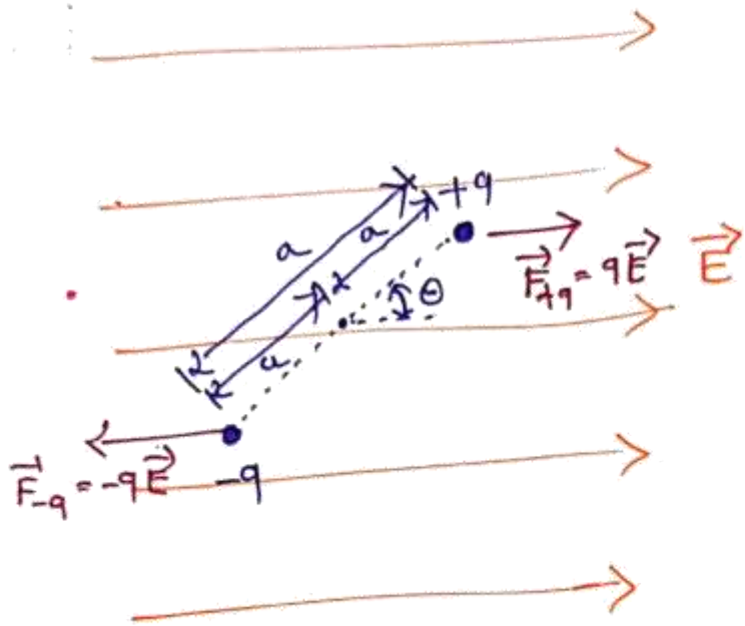
$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{4za}{(z^2-a^2)^2}\right]$$

- If $z >> a$, $p = (2a)q$

$$\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 z^3} = \frac{2k\vec{p}}{z^3}$$



Force & Torque on dipole due to uniform Electric field



→ So total force on dipole due to uniform electric field

$$\vec{F} = \vec{F}_{+q} + \vec{F}_{-q}$$
$$= q\vec{E} - q\vec{E}$$
$$= 0$$

→ Consider clockwise torque is +ve.

- torque on +q charge

$$\tau_{+q} = \vec{a} \times \vec{F}_{+q}$$
$$= aF_{+q}\sin\theta$$

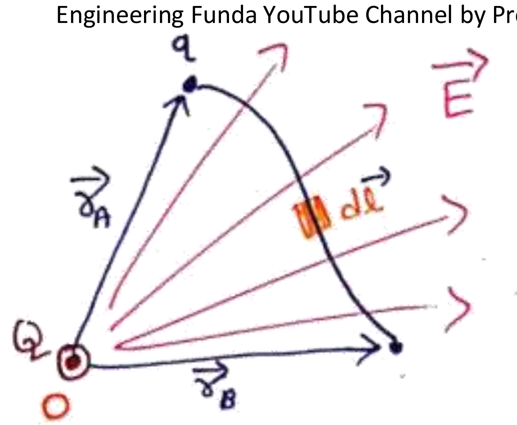
- torque on -q charge.

$$\tau_{-q} = \vec{a} \times \vec{F}_{-q}$$
$$= aF_{-q}\sin\theta$$

- So total torque on dipole

$$\tau = \tau_{+q} + \tau_{-q}$$
$$= aF_q\sin\theta + aF_{+q}\sin\theta$$
$$= aqE\sin\theta + aqE\sin\theta$$
$$= (2aq)E\sin\theta \quad (\because \text{dipole moment } p = 2aq).$$
$$= pE\sin\theta$$

Potential



- Force on point charge q due to Electric field is

$$\vec{F} = q\vec{E}$$

- for dl movement work done

$$\Rightarrow dw = \vec{F}.d\vec{l}$$
$$\Rightarrow dw = q\vec{E}.d\vec{l}$$

- For total work done

$$w = \int dw$$
$$= \int q\vec{E}.dl$$
$$\boxed{w = \int_A^B q\vec{E}.d\vec{l}}$$

- Potential difference

- It is work to move charge per unit charge (Colomb).

$$\Rightarrow V_{AB} = \frac{w}{q} = -\int_A^B \vec{E}.dl$$

$$\Rightarrow \boxed{V_{AB} = -\int_A^B \vec{E}.dl}$$

[$\because$ -ve sign is due to work done by external force.].

- At origin Q charge is placed. so, Electric field due to +Q charge.

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}}$$

$$\Rightarrow V_{AB} = -\int_{r_A}^{r_B} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr = \frac{-Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_A}^{r_B} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$\Rightarrow V_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_B} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_A} = V_B - V_A$$

- If we bring the charge from infinity to given position r, then work done per unit charge is potential at point.

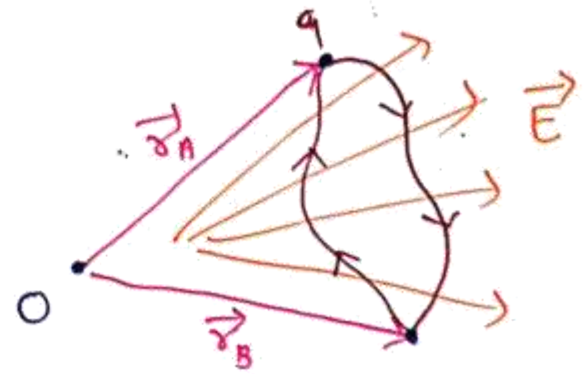
$$\rightarrow V_A = -\int_{\infty}^{r_A} \vec{E}.dl$$
$$= -\int_{\infty}^{r_A} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dr = \frac{-Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r_A} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_A}}$$

1. V_{AB} = -Ve (work done happend by $\vec{E}$ field)
   = +ve (work done happend by External force).
2. Potential difference is independent on path.
3. Unit Volt or J/C
4. $V_{AB} = V_B - V_A = -\int_A^B \vec{E}.d\vec{l}$

Maxwell's 2^{nd} eq.^{n} with Integral & differencial form.

- we have calculated potential difference by

$$V_{AB} = -\int_A^B \vec{E}.d\vec{l}$$



- Potential difference for closed path

$$\Delta V = -\oint \vec{E}.dl$$

$$= -\int_A^B \vec{E}\,d\vec{l} + \left(-\int_B^A \vec{E}.d\vec{l}\right)$$

$$= V_{AB} + V_{BA}$$

$$= (V_B - V_A) + (V_A - V_B)$$

$$= 0$$

$$\Rightarrow \boxed{\oint \vec{E}.d\vec{l} = 0} \quad \text{—— (A)}$$

→ It is maxwell's 2^{nd} eq.^{n} with Integral form.

→ It explain's work done to move charge in closed path under external field is zero.

→ As per Stock's theorm

$$\oint \vec{A}.d\vec{l} = \int \vec{\nabla}\times\vec{A}.ds \quad \text{—— (1)}$$

→ So apply (1) in eq.^{n} (A).

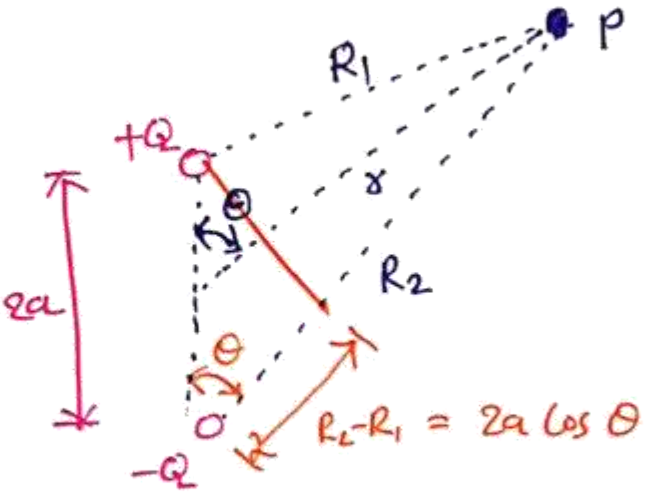
$$\Rightarrow \int(\vec{\nabla}\times\vec{E}).d\vec{s} = 0$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = 0} \quad \text{—— (B)}$$

→ It is maxwell's 2^{nd} eq.^{n} with differencial form or point form.

→ Maxwell's 2^{nd} eq.^{n} shows conservative nature of E field.

Potential due to Dipole.



→ total Potential at point P

$$V = V_{+q} + V_{-q}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_1} + \frac{1}{4\pi\varepsilon_0}\frac{-Q}{R_2}$$

$$= KQ\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= kQ\left[\frac{R_2 - R_1}{R_1 R_2}\right]$$

→ For $r >> a$, $R_1 R_2 = r^2$, $R_2 - R_1 = 2a\cos\theta$

$$V = KQ\left[\frac{2a\cos\theta}{r^2}\right]$$

$$= \frac{K(2aQ)}{r^2}\cos\theta$$

→ Dipole moment $P = (2a)Q$

$$\boxed{V = \frac{kP}{r^2}\cos\theta}$$

Examples on Electric potential.

Three point charges $-4\mu C$, $5\mu C$ and $3\mu C$ are located at $(2,-1,3)$, $(0,4,-2)$ and $(0,0,0)$ respectively. Find the potential at $(-1,5,2)$.

$q_1 = -4\mu C$, $\vec{r}_1 = (2,-1,3)\,m$

$q_2 = 5\mu C$, $\vec{r}_2 = (0,4,-2)\,m$

$q_3 = 3\mu C$, $\vec{r}_3 = (0,0,0)\,m$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = K\frac{q}{r}$$

$\vec{r}_1' = (-1,5,2) - (2,-1,3) = (-3,6,-1)$, $r_1' = \sqrt{9+36+1} = \sqrt{46}$

$\vec{r}_2' = (-1,5,2) - (0,4,-2) = (-1,1,4)$, $r_2' = \sqrt{1^2+1+16} = \sqrt{18}$

$\vec{r}_3' = (-1,5,2) - (0,0,0) = (-1,5,2)$, $r_3' = \sqrt{1+25+4} = \sqrt{30}$.

$\rightarrow$ $V = V_1 + V_2 + V_3$

$= K\frac{q_1}{r_1'} + K\frac{q_2}{r_2'} + K\frac{q_3}{r_3'}$

$= K\left[\frac{q_1}{r_1'} + \frac{q_2}{r_2'} + \frac{q_3}{r_3'}\right]$

$= 9\times10^9\left[\frac{-4\times10^{-6}}{\sqrt{46}} + \frac{5\times10^{-6}}{\sqrt{18}} + \frac{3\times10^{-6}}{\sqrt{30}}\right]$

$\boxed{V = 10.22\ KV}$

- In Free Space, $V = x^2y(z+3)$ V. Find
  ① $\vec{E}$ at $(3,4,-6)$
  ② the charge within the cube $0 < xyz < 1$

$\rightarrow$ $\vec{E} = -\nabla V \big|_{(3,4,-6)}$

$= -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right]$

$= -\left[2xy(z+3)\,\hat{a}_x + x^2(z+3)\,\hat{a}_y + x^2y\,\hat{a}_z\right]$

$= -\left[2\times3\times4(-3)\,\hat{a}_x + 3^2(-3)\,\hat{a}_y + 3^2\times4\,\hat{a}_z\right]$

$= -\left[-72\,\hat{a}_x + (-27)\,\hat{a}_y + 36\,\hat{a}_z\right]$

$= 72\,\hat{a}_x + 27\,\hat{a}_y - 36\,\hat{a}_z$ (V/m)

$\rightarrow$ $|\vec{E}| = \sqrt{72^2 + 27^2 + (-36)^2}$

$= 84.9$ V/m

$\rightarrow$ $\nabla\cdot\vec{D} = \rho_v$

$\rightarrow$ $Q = \int \rho_v\, dV$

$\rightarrow$ $\vec{D} = \varepsilon_0\vec{E}$

$\vec{D} = -\varepsilon_0\left[2xy(z+3)\hat{a}_x + x^2(z+3)\hat{a}_y + x^2y\,\hat{a}_z\right]$.

$\rightarrow$ $\rho_v = \nabla\cdot\vec{D}$

$= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$.

$\boxed{\rho_v = -\varepsilon_0 2y(z+3)}$

$\rho_v = -\varepsilon_0 2y(z+3)$

$Q = \iiint \rho_v\, dV$

$= \int_0^1\int_0^1\int_0^1 -\varepsilon_0 2y(z+3)\, dx\,dy\,dz$

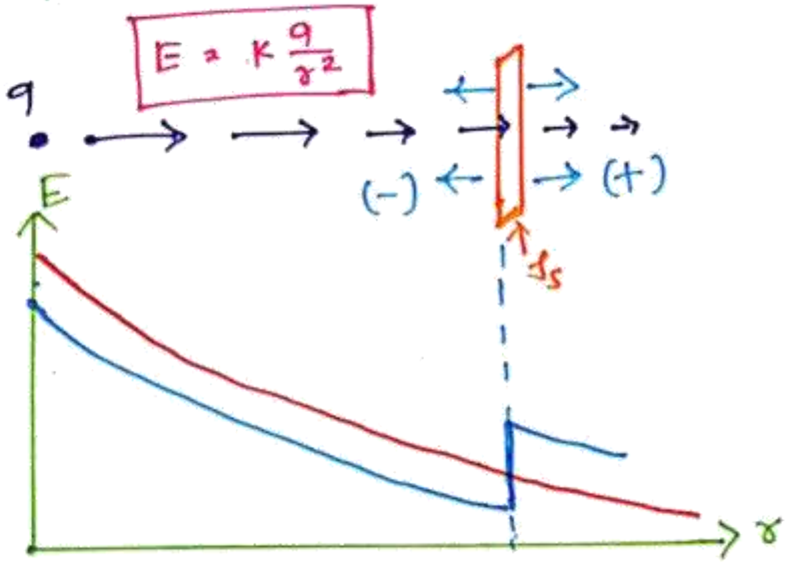
$= -\varepsilon_0[x]_0^1[y^2]_0^1\left[z^2/2 + 3z\right]_0^1$

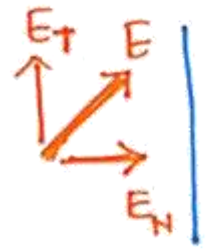
$= -\varepsilon_0\times1\times1\times7/2$

$= -30.9$ pC

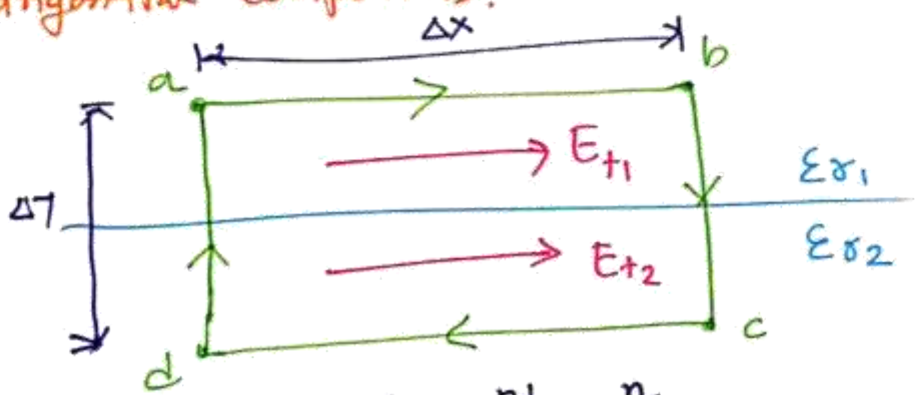
Boundary Conditions for Electric field.

$$\boxed{E = K\frac{q}{r^2}}$$





Tangential components.



→ As per Maxwell's 2^{nd} eq^n

$$\Rightarrow \oint \vec{E}\cdot d\vec{l} = 0$$

$$\Rightarrow \int_a^b E\,dl + \int_b^c E\,dl + \int_c^d E\,dl + \int_d^a E\,dl = 0$$

$$\Rightarrow E_{t_1}\Delta x + 0 - E_{t_2}\Delta x + 0 = 0$$

$$\Rightarrow \boxed{E_{t_1} = E_{t_2}}$$

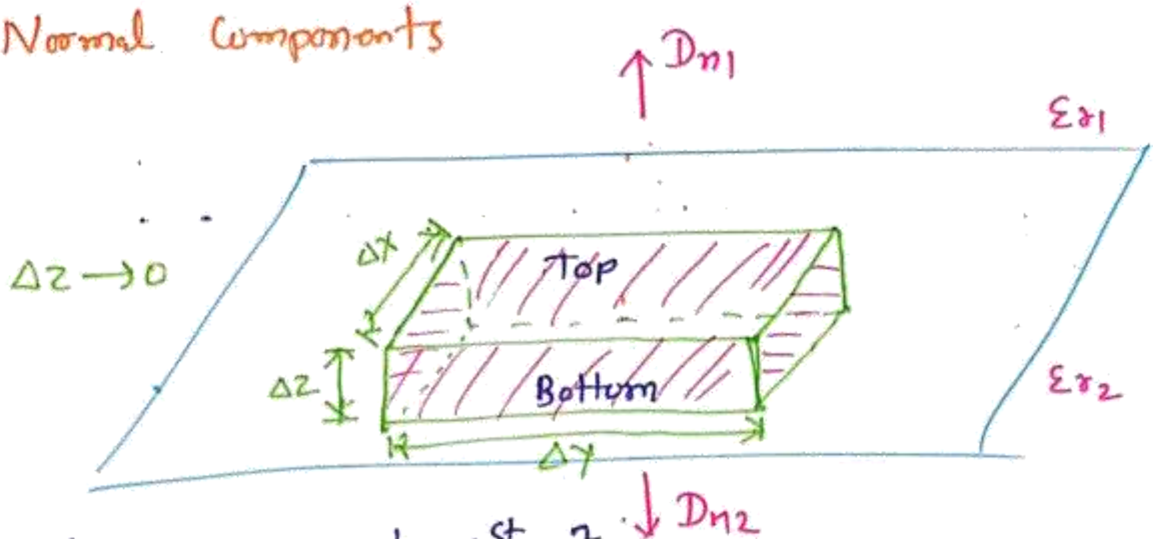
→ Electric field is continuous for tangential component.

→ $\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$

$$\Rightarrow \frac{D_{t1}}{\varepsilon_{r1}} = \frac{D_{t2}}{\varepsilon_{r2}}$$

$$\Rightarrow \boxed{\frac{D_{t1}}{D_{t2}} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$

Normal Components



→ As per maxwell's 1st eqn

$$\Rightarrow \oint \vec{D} \cdot d\vec{s} = Q$$

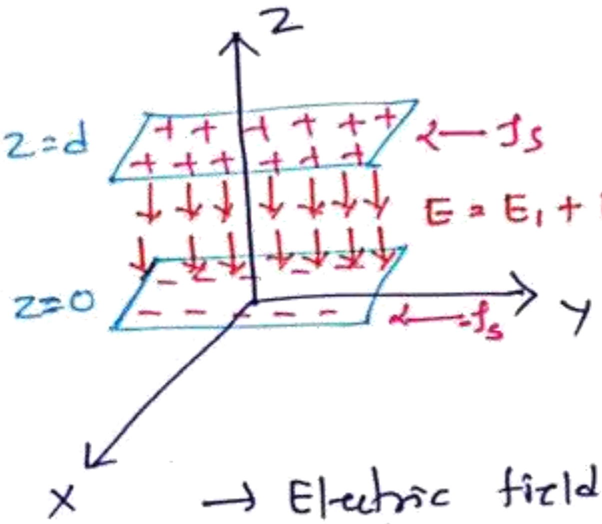
$$\Rightarrow \int_{top} D \cdot ds + \int_{Bottom} D \cdot ds + \int_{left} D \cdot ds + \int_{right} D \cdot ds + \int_{front} D \cdot ds + \int_{back} D \cdot ds = Q$$

$$\Rightarrow D_{n1}(\Delta y \Delta x) - D_{n2}(\Delta y \Delta x) + 0 + 0 + 0 + 0 = Q$$

$$\Rightarrow D_{n1} - D_{n2} = Q / (\Delta y \Delta x)$$

$$\Rightarrow \boxed{D_{n1} - D_{n2} = \rho_s}$$

Parallel Plate Capacitor



→ $C = \dfrac{Q}{V}$ — (A)

→ Charge Q

$Q = \rho_s A$ — (1)

→ Voltage V

$V = -\int \vec{E} \cdot d\vec{l}$.

→ Electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\rho_s}{2\varepsilon_0}(-\hat{a}_z) + \frac{\rho_s}{2\varepsilon_0}(-\hat{a}_z)$$

$$= -\frac{\rho_s}{\varepsilon_0}\hat{a}_z$$

→ Voltage V

→ $V = -\int_0^d \left(-\frac{\rho_s}{\varepsilon_0}\right) dz$

$$= \frac{\rho_s}{\varepsilon_0}[z]_0^d$$

$$= \frac{\rho_s d}{\varepsilon_0} \quad \text{— (2)}$$

→ Put (1) & (2) in eq.^{n} (A)

$$C = \frac{Q}{V} = \frac{\rho_s A}{\rho_s d/\varepsilon_0} = \boxed{\varepsilon_0 \varepsilon_r \frac{A}{d}}$$

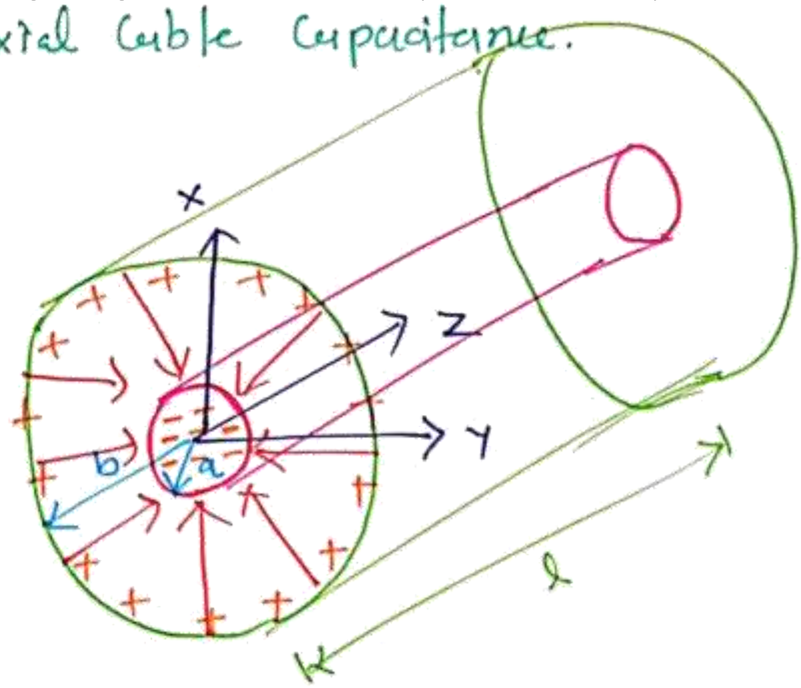
→ Energy Stored In Capacitor.

$$E = \frac{1}{2} C V^2 \quad (J)$$

→ Energy density in Capacitor.

$$\omega = \frac{1}{2}\varepsilon_0 |E|^2 \quad (J/m^3)$$

Co-axial Cable Capacitance.



→ Capacitance

$$C = \frac{Q}{V} \quad \text{— (A)}$$

→ Charge Q

$$Q = \rho_L L \quad \text{— (1)}$$

→ Voltage V

$$V = -\int \vec{E} \cdot d\vec{l}$$

→ Electric field due to line charge ρ_L

$$\vec{E} = \frac{-\rho_L}{2\pi\varepsilon_0 r}\hat{r}$$

→ So potential V

$$\Rightarrow V = -\int_a^b \left(\frac{-\rho_L}{2\pi\varepsilon_0 r}\right) dr$$

$$= \frac{\rho_L}{2\pi\varepsilon_0}\left[\ln r\right]_a^b$$

$$= \frac{\rho_L}{2\pi\varepsilon_0}\left[\ln(b/a)\right] \quad \text{— (2)}$$

→ Put (1) & (2) in (A)

$$\Rightarrow C = \frac{Q}{V} = \frac{\rho_L L}{\frac{\rho_L}{2\pi\varepsilon_0}\ln(b/a)}$$

$$\Rightarrow \boxed{C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}}$$

If Inner radius of Co-axial Cable is 1mm & Outer radius is 3 mm for Co-axial Cable with dielectric material of $\varepsilon_r = 4.5$ then calculate Capacitance per unit length.

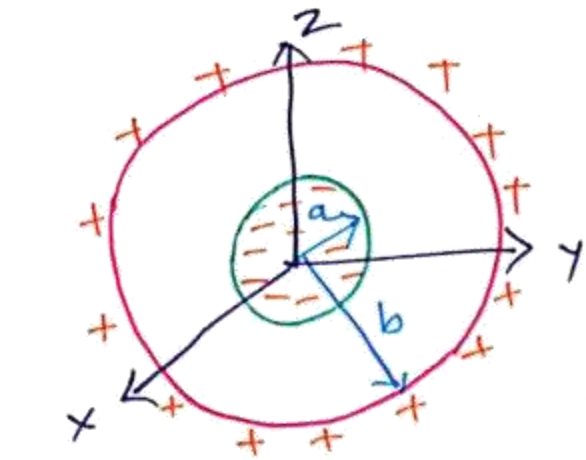
- $a = 1\text{ mm}$
  $b = 3\text{ mm}$
  $l = 1\text{ m}$
  $\varepsilon_r = 4.5$

- $$C = \frac{2\pi\varepsilon_0\varepsilon_r l}{\ln(b/a)} = \frac{2 \times \pi \times 8.854 \times 10^{-12} \times 4.5 \times 1}{\ln(3)}$$

$$= 0.27 \times 10^{-9}\text{ F}$$

$$= 0.27\text{ nF}$$

Spherical Capacitor.



→ Capacitance $C = \frac{Q}{V}$

→ Voltage V

$$V = -\int \vec{E}\cdot d\vec{l}$$

$$= +\int_a^b \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,dr$$

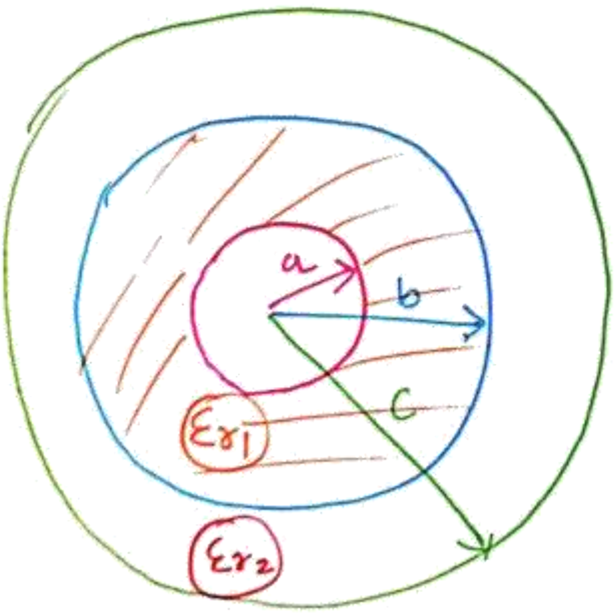
$$= \frac{+Q}{4\pi\varepsilon_0}\int_a^b \frac{1}{r^2}\,dr$$

$$= +\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_a^b$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]$$

→ Capacitance

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a}-\frac{1}{b}\right]} = \boxed{\frac{4\pi\varepsilon_0\varepsilon_r}{\left[\frac{1}{a}-\frac{1}{b}\right]}}$$



→ Resultant capacitance is there as per series connection

→ For a to b

$$C_1 = \frac{4\pi\varepsilon_0\varepsilon_{r1}}{\left[\frac{1}{a}-\frac{1}{b}\right]}$$

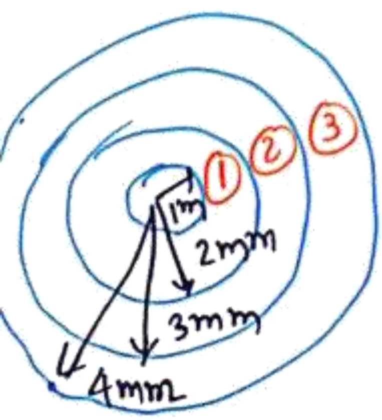
→ For b to c

$$C_2 = \frac{4\pi\varepsilon_0\varepsilon_{r2}}{\left[\frac{1}{b}-\frac{1}{c}\right]}.$$

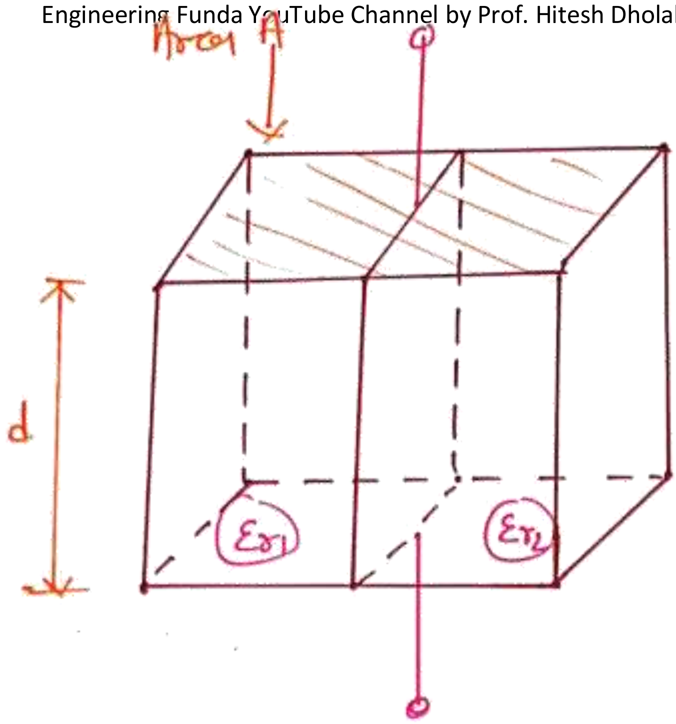
→ For series capacitance

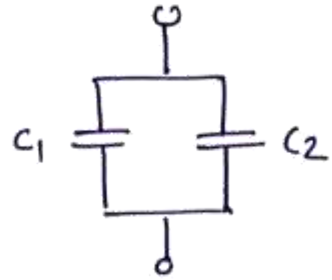
$$\Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \boxed{\frac{1}{C} = \frac{\left[\frac{1}{a}-\frac{1}{b}\right]}{4\pi\varepsilon_0\varepsilon_{r1}} + \frac{\left[\frac{1}{b}-\frac{1}{c}\right]}{4\pi\varepsilon_0\varepsilon_{r2}}}$$



Examples based on Capacitance.



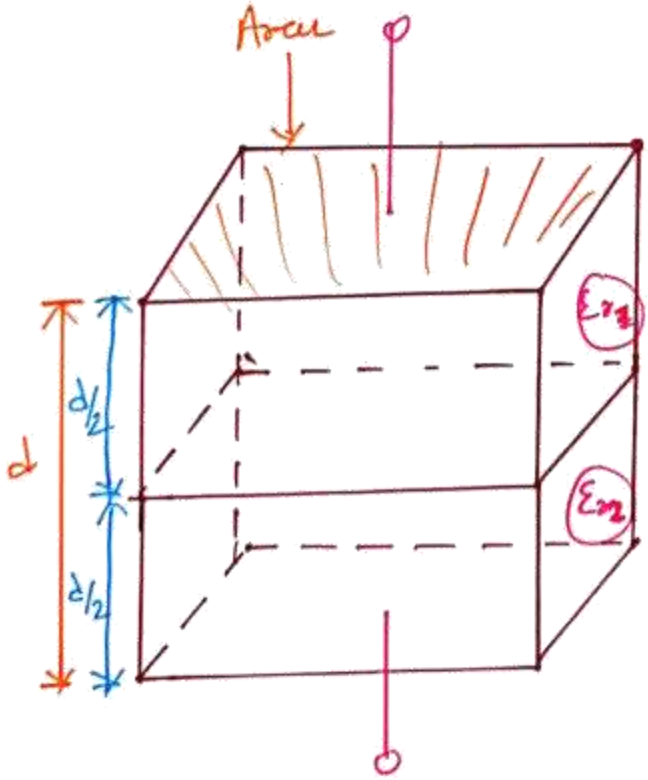


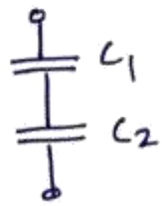
$\rightarrow C_1 = \varepsilon_0 \varepsilon_{r1} \dfrac{(A/2)}{d} = \varepsilon_0 \varepsilon_{r1} \left(\dfrac{A}{2d}\right)$

$\rightarrow C_2 = \varepsilon_0 \varepsilon_{r2} \dfrac{(A/2)}{d} = \varepsilon_0 \varepsilon_{r2} \left(\dfrac{A}{2d}\right)$

$\rightarrow C = C_1 + C_2$

$$\boxed{C = \varepsilon_0 \frac{A}{2d} (\varepsilon_{r1} + \varepsilon_{r2})}$$





$\rightarrow C_1 = \varepsilon_0 \varepsilon_{r1} \left(\dfrac{A}{(d/2)}\right) = \varepsilon_0 \varepsilon_{r1} \left(\dfrac{2A}{d}\right)$

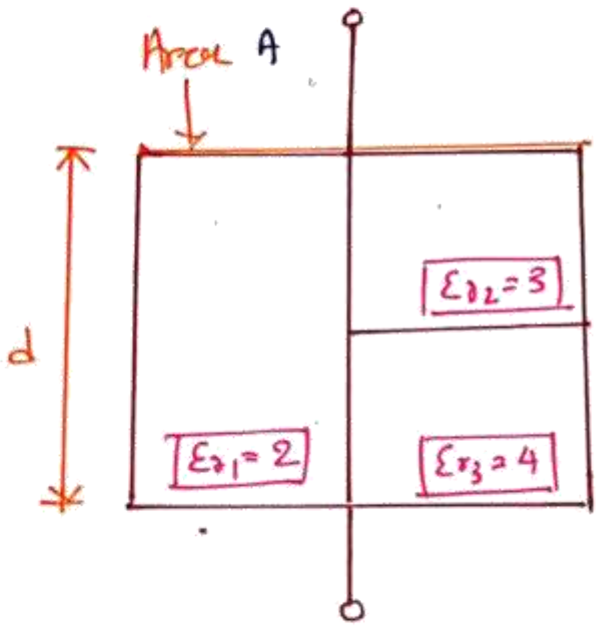
$\rightarrow C_2 = \varepsilon_0 \varepsilon_{r2} \left(\dfrac{A}{(d/2)}\right) = \varepsilon_0 \varepsilon_{r2} \left(\dfrac{2A}{d}\right)$

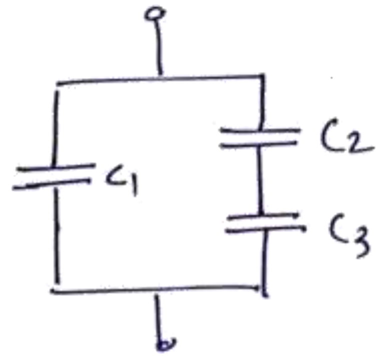
$\rightarrow C = \dfrac{C_1 C_2}{C_1 + C_2}$

$$= \frac{\varepsilon_0 \varepsilon_{r1} \left(\frac{2A}{d}\right) \varepsilon_0 \varepsilon_{r2} \left(\frac{2A}{d}\right)}{\varepsilon_0 \varepsilon_{r1} \left(\frac{2A}{d}\right) + \varepsilon_0 \varepsilon_{r2} \left(\frac{2A}{d}\right)}$$

$$= \varepsilon_0 \left(\frac{2A}{d}\right) \left[\frac{\varepsilon_{r1} \varepsilon_{r2}}{\varepsilon_{r1} + \varepsilon_{r2}}\right]$$

Find Resultant dielectric Constant





$$\rightarrow C = C_1 + \frac{C_2 C_3}{C_2 + C_3}.$$

$$\rightarrow C_1 = \varepsilon_0 \varepsilon_{r_1} \left(\frac{A/2}{d}\right) \quad \rightarrow C_2 = \varepsilon_0 \varepsilon_{r_2} \left(\frac{A/2}{d/2}\right) \quad \rightarrow C_3 = \varepsilon_0 \varepsilon_{r_3} \frac{(A/2)}{(d/2)}$$

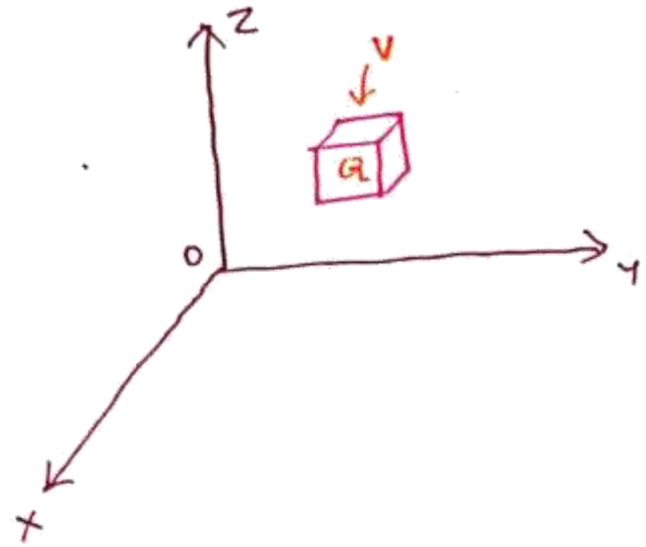
$$\Rightarrow C = \varepsilon_0 \varepsilon_{r_1} \left(\frac{A}{2d}\right) + \frac{\varepsilon_0 \varepsilon_{r_2} \left(\frac{A}{d}\right) \varepsilon_0 \varepsilon_{r_3} \left(A/d\right)}{\varepsilon_0 \varepsilon_{r_2} \left(\frac{A}{d}\right) + \varepsilon_0 \varepsilon_{r_3} \left(A/d\right)}$$

$$\Rightarrow C = \varepsilon_0 \varepsilon_{r_1} \left(\frac{A}{2d}\right) + \varepsilon_0 \left(\frac{A}{d}\right) \left[\frac{\varepsilon_{r_2} * \varepsilon_{r_3}}{\varepsilon_{r_2} + \varepsilon_{r_3}}\right]$$

$$\Rightarrow C = \varepsilon_0 \frac{A}{d} \left[\frac{\varepsilon_{r_1}}{2} + \frac{\varepsilon_{r_2} * \varepsilon_{r_3}}{\varepsilon_{r_2} + \varepsilon_{r_3}}\right] = C = \varepsilon_0 \boxed{\varepsilon_r} \frac{A}{d}$$

$$\rightarrow \varepsilon_r = \frac{\varepsilon_{r_1}}{2} + \frac{\varepsilon_{r_2} * \varepsilon_{r_3}}{\varepsilon_{r_2} + \varepsilon_{r_3}} = \frac{2}{2} + \frac{3 * 4}{3 + 4} = 1 + \frac{12}{7} = \boxed{2.71}$$

* Continuity of Current



→ Current in closed surface

$$I = -\frac{dQ}{dt} \quad \text{—} ①$$

[∵ (-)ve sign is due to decrease of charge w.r.t time t.]

→ Current in closed surface

$$I = \oint J \cdot ds \quad \text{—} ②$$

→ from ① & ②

$$\Rightarrow \oint J\, ds = -\frac{dQ}{dt} \quad \text{—} (A)$$

→ As per gauss divergence theorem

$$\oint_s J\, ds = \int \vec{\nabla} \cdot \vec{J}\, dv \quad \text{—} ③$$

→ As per vol.^{m} charge density.

$$Q = \int_v \rho_v\, dv \quad \text{—} ④$$

→ Apply ③ & ④ in eq.^{n} (A)

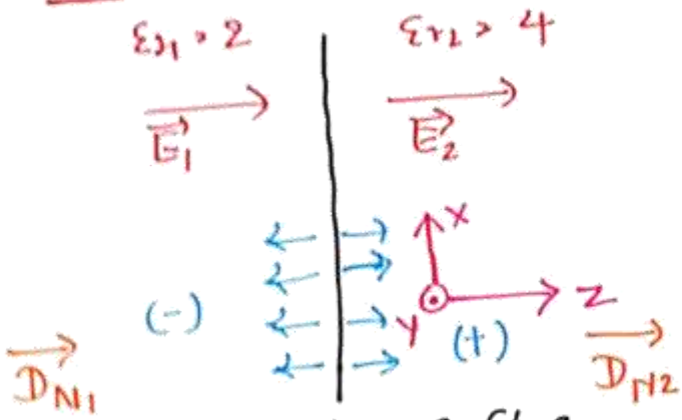
$$\Rightarrow \int \vec{\nabla} \cdot \vec{J}\, dv = -\frac{d}{dt} \int \rho_v\, dv$$

$$\Rightarrow \int \vec{\nabla} \cdot \vec{J}\, dv = -\int \frac{d\rho_v}{dt}\, dv$$

$$\Rightarrow \boxed{\vec{\nabla} \cdot \vec{J} = -\frac{d\rho_v}{dt}}$$

Example on boundary condition

→ $\vec{E}_1 = 2\hat{i}_x + 4\hat{i}_y + 6\hat{i}_z$ V/m



Find $\vec{E}_2$.

$\rho_s = 4\varepsilon_0 \ C/m^2$

① For tangential component

$$E_{t1} = E_{t2}$$

② For Normal component

$$D_{N1} - D_{N2} = \rho_s$$

$$D_{N2} - D_{N1} = \rho_s$$

→ $\vec{E}_1 = 2\hat{i}_x + 4\hat{i}_y + 6\hat{i}_z$ (V/m)

→ $\vec{E}_2 = 2\hat{i}_x + 4\hat{i}_y + E_{N2}\hat{i}_z$

→ $D_{N2} - D_{N1} = \rho_s \Rightarrow D_{N2} = D_{N1} + \rho_s$

$$= \varepsilon_0 \varepsilon_{r1} E_{N1} + \rho_s$$

$$= \varepsilon_0 (2) 6 + 4\varepsilon_0$$

$$= 16\varepsilon_0.$$

→ $E_{N2} = \dfrac{D_{N2}}{\varepsilon_0 \varepsilon_{r2}} = \dfrac{16\varepsilon_0}{\varepsilon_0 (4)} = 4$

→ $\boxed{\vec{E}_2 = 2\hat{i}_x + 4\hat{i}_y + 4\hat{i}_z \ (V/m)}$

Poission & Laplace eq.^{n}

→ Electric field based on potential

$$\vec{E} = -\vec{\nabla} V \quad \text{— (1)}$$

→ As per gauss law of Electric field

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_v}{\varepsilon_0} \quad \text{— (2)}$$

→ Apply eq.^{n} (1) in eq^n (2).

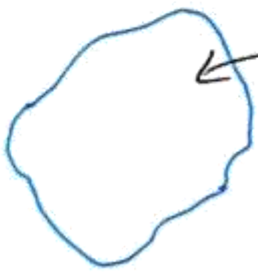
$$\Rightarrow \vec{\nabla} \cdot (-\vec{\nabla} V) = \frac{\rho_v}{\varepsilon_0}$$

$$\Rightarrow \vec{\nabla} \cdot (\vec{\nabla} V) = -\frac{\rho_v}{\varepsilon_0}$$

$$\Rightarrow \boxed{\boxed{\nabla^2} V = -\frac{\rho_v}{\varepsilon_0}} \quad \text{— (A)}$$

(∇^2 is referred as Laplacian operator).

→ eq.^{n} (A) is referred as poission eq.^{n}

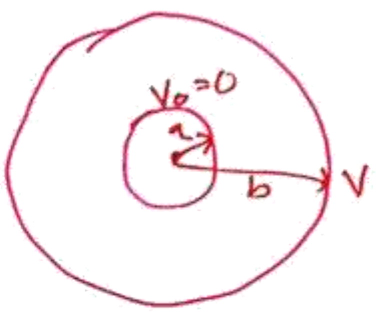


← If enclosed surface does not have any vol.^{m} charge ρ_v

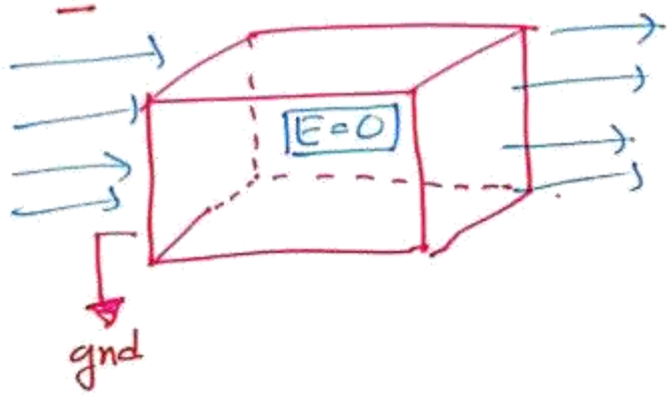
$\Rightarrow \rho_v = 0$

$$\Rightarrow \boxed{\nabla^2 V = 0} \quad \text{— (B)}$$

→ eq.^{n} (B) is referred as Laplace eq.^{n}



→ V = ? → $\vec{E} = -\vec{\nabla} V$ → $C = \frac{Q}{V}$



⁂ Uniqueness theorm

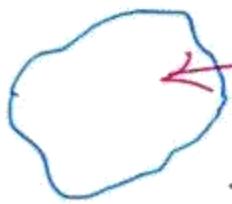
- It explains unique sol^n to the Laplacian eq^n

$$\boxed{\nabla^2 V = 0}$$

→ Sol^n of V is unique.

→ As poission $eq.^n$

$$\boxed{\nabla^2 V = -\frac{\rho_V}{\varepsilon_0}}$$



← there is no $v.l^m$ charge inside

$$\boxed{\rho_V = 0}$$

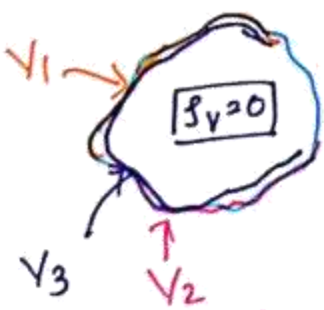
$$\boxed{\nabla^2 V = 0}$$

→ If there are two $sol.^n$, V_1 & V_2

$$\boxed{\nabla^2 V_1 = 0} \;,\; \& \;\boxed{\nabla^2 V_2 = 0}$$

→ If $V_3 = V_1 - V_2$, for same boundary.

Q1 $\boxed{\nabla^2 V_3 = 0}$



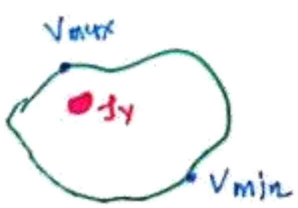
→ For metallic boundry

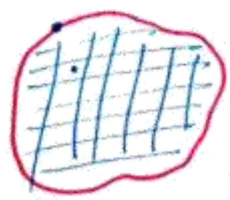
$V_3 = 0 \Rightarrow V_1 - V_2 = 0$

$\Rightarrow \boxed{V_1 = V_2}$

Q2

→ $\boxed{\nabla^2 V = 0}$

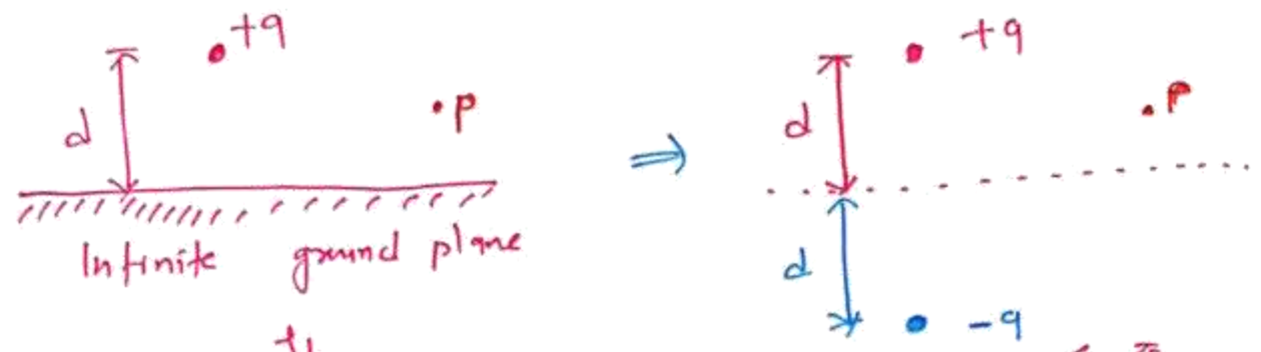


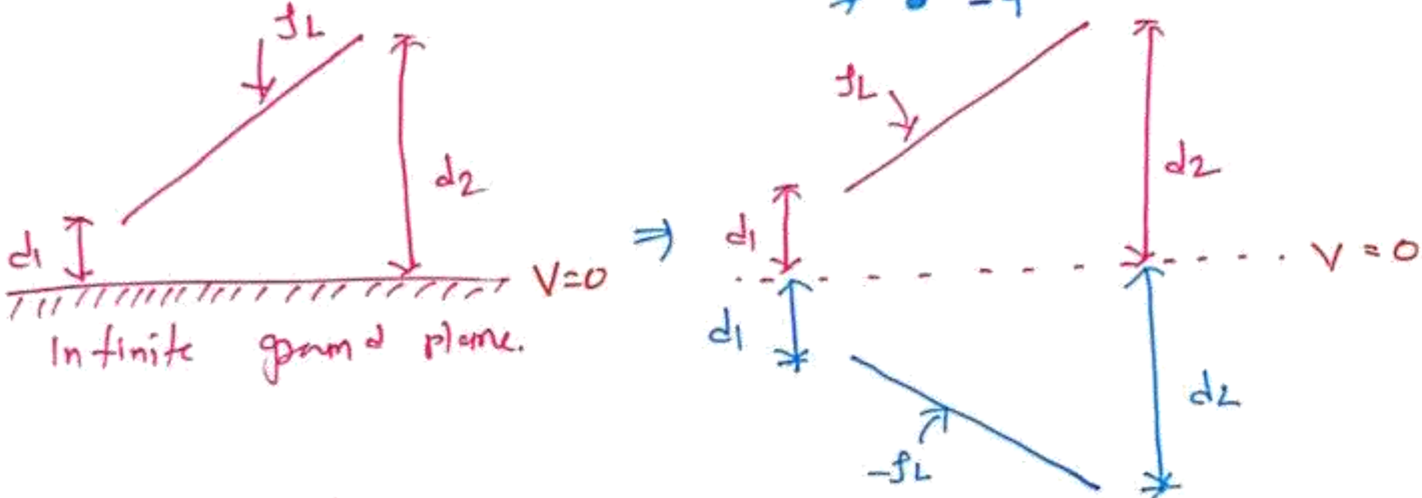


- In free Space, there is no fix rettoun.
- There is max & min. for enclosed Surface.

Image Theory of Charges.

- It states that if any charge placed above infinite grounded perfect conducting plane then it is replaced by charge with it's image and with opposite polarity.

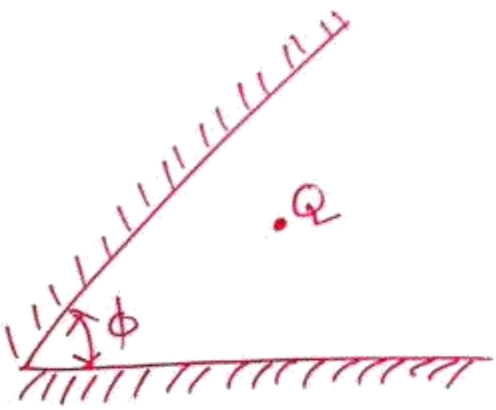




Important points.

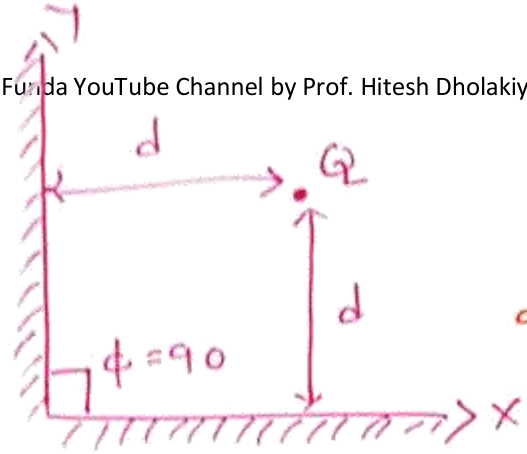
1. Image charges must be there inside the conducting region.

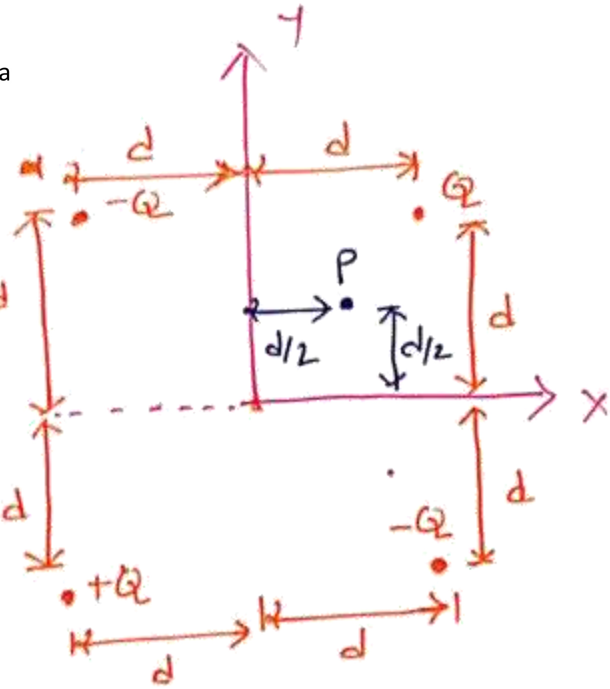
2. Image charge must be located in such a way that on perfect conducting plane has $V=0$ on it's surface.



- Number of Image charges

$$\boxed{N = \frac{360}{\phi} - 1}$$





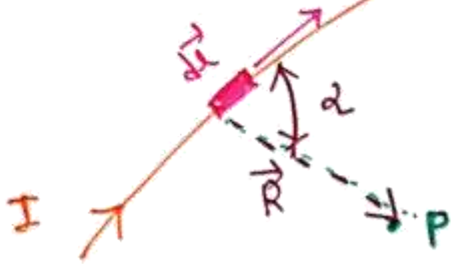
Number of Image charges

$$N = \frac{360}{90} - 1$$

$$\boxed{N = 3}$$

Biot Savart's Law

→ It states that the differential magnetic field Intensity $d\vec{H}$ produced at any point P by the differential current element Idl is proportional to product Idl and sine of angle betn element and the line joining P to the element and inversely proportinal to the square of the distance betn P & element.



$$\Rightarrow dH \propto \frac{Idl \sin\alpha}{R^2}$$

$$\Rightarrow dH = \frac{1}{4\pi} \frac{Idl \sin\alpha}{R^2}$$

$$\Rightarrow d\vec{H} = \frac{1}{4\pi} \frac{I d\vec{l} \times \hat{R}}{R^2}$$

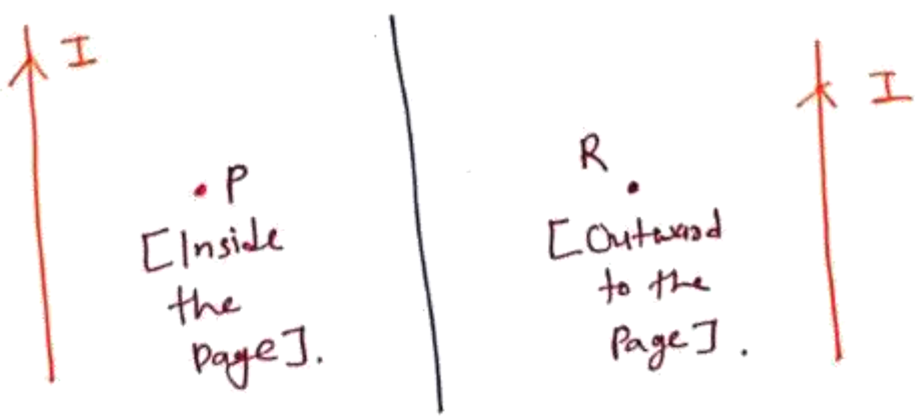
$$\Rightarrow \boxed{d\vec{H} = \frac{1}{4\pi} \frac{I d\vec{l} \times \vec{R}}{R^3}}$$

→ Total magnetic field Intensity

$$\vec{H} = \int d\vec{H}$$

$$\boxed{\vec{H} = \int \frac{1}{4\pi} \frac{I d\vec{l} \times \vec{R}}{R^3}}$$

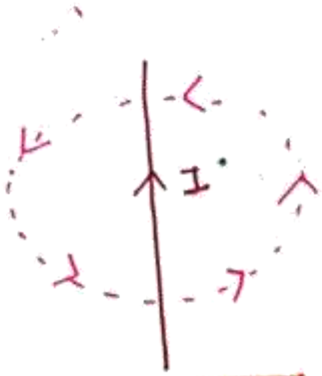
→ Unit of magnetic field Intensity A/m.



Ampere's Law

→ line Integral of magnetic field Intensity around Closed loop is equal to total current enclosed by that Closed loop.

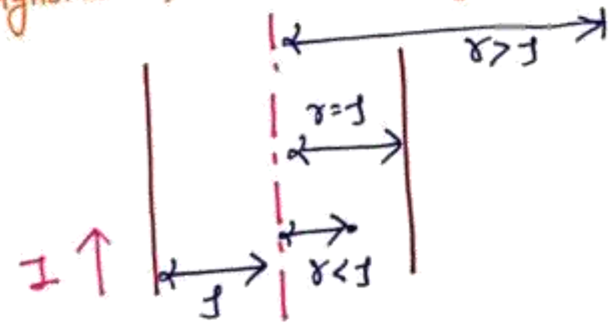
$$\boxed{\oint \vec{H}\cdot d\vec{l} = I}$$



$$\boxed{\oint \vec{H}\cdot d\vec{l} = I}$$

$$\oint \vec{H}\cdot d\vec{l} = (I_1 + I_3 + I_4 - I_2)$$

→ Magnetic field Intensity for straight wire.



Case-I Outside the wire $\boxed{r > a}$

$$\Rightarrow \int \vec{H}\, d\vec{l} = I$$

$$\Rightarrow H(2\pi r) = I$$

$$\Rightarrow \vec{H} = \frac{I}{2\pi r}\,\hat{a}_\phi$$

→ Case-II On the wire $\boxed{r = a}$

$$\Rightarrow \int \vec{H}\cdot d\vec{l} = I$$

$$\Rightarrow H(2\pi a) = I$$

$$\Rightarrow \vec{H} = \frac{I}{2\pi a}\,\hat{a}_\phi$$

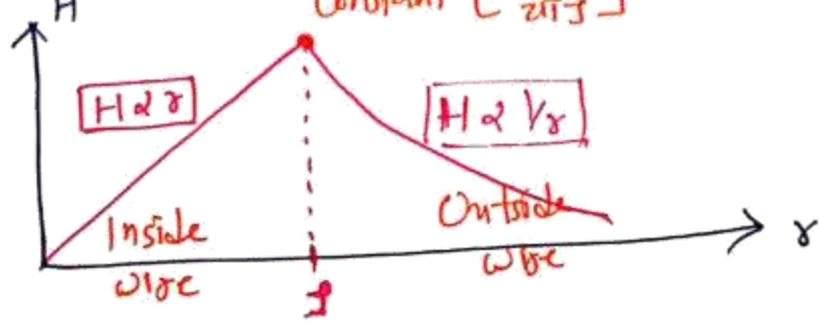
→ Case-III Inside the wire $\boxed{r < a}$

$$\Rightarrow \int \vec{H}\cdot d\vec{l} = I'$$

$$\Rightarrow H(2\pi r) = I\,\frac{(\pi r^2)}{(\pi a^2)}$$

$$\Rightarrow H = r\left[\frac{I}{2\pi a^2}\right]$$

Constant $\left[\frac{I}{2\pi a}\right]$



Maxwell's 3^{rd} eq.^{n} with Integral & differential Form

→ As per Ampere Circuit law.

$$\Rightarrow \oint \vec{H} \cdot \vec{dl} = I \qquad [\, I = I_c + I_d \,]$$

Differential Current ↓, ↑ Conduction current

[For time varying field].

$$\Rightarrow \oint \vec{H} \cdot \vec{dl} = I_c + I_d \quad \text{— (1)}$$

→ As per current density J.

$$\Rightarrow I = \int \vec{J} \cdot \vec{dS}$$

$$\Rightarrow I = \int (\vec{J} + \vec{J}_D) \cdot \vec{dS} \quad \text{— (2).}$$

→ As per eq.^{n} (1) & (2).

$$\Rightarrow \boxed{\oint \vec{H}\, \vec{dl} = \int \vec{J} \cdot \vec{dS}} \quad \text{— (A)}$$

$$\boxed{\oint \vec{H}\, \vec{dl} = \int (\vec{J} + \vec{J}_d) \cdot \vec{dS}}$$

↑ For time varying field.

→ eq.^{n} (A) is maxwell's 3^{rd} eq.^{n} with Integral form.

→ As per stock's theorem

$$\oint \vec{H} \cdot \vec{dl} = \int (\vec{\nabla} \times \vec{H}) \cdot \vec{dS}$$

$$\rightarrow \Rightarrow \int (\vec{\nabla} \times \vec{H}) \cdot \vec{dS} = \int \vec{J} \cdot \vec{dS}$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{H} = \vec{J}} \quad \text{— (B)}$$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}_D}$$

↑ For time varying field.

→ eq.^{n} (B) is maxwell's 3^{rd} eq.^{n} with differential form or point form.

Magnetic field

→ Relation in betn magnetic field and magnetic field Intensity

$\Rightarrow \vec{B} = \mu \vec{H}$

$\Rightarrow \boxed{\vec{B} = \mu_0 \mu_r \vec{H}}$

→ By direction magnetic field and magnetic field Intensity has same direction.

$\mu_0 = 4\pi \times 10^{-7}$ SI [obsault Perеability].

μ_r = Realative Permeability

→ Unit of Magnetic field Tesla or $\frac{N}{Am}$ or $\frac{Wb}{m^2}$ or Gauss.

$\Rightarrow \vec{F_m} = q(\vec{v} \times \vec{B})$

$\Rightarrow N = (A\ sec)\left(\frac{m}{sec}\right)(T)$

$\Rightarrow \boxed{T = \frac{N}{Am}}$

$\Rightarrow \phi = \int \vec{B}\, d\vec{s}$

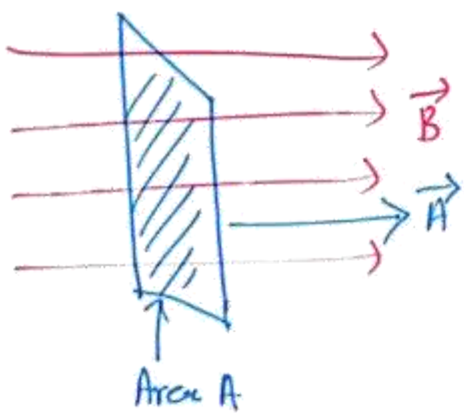
$\Rightarrow Wb = (T)(m^2)$

$\Rightarrow \boxed{T = \frac{Wb}{m^2}}$

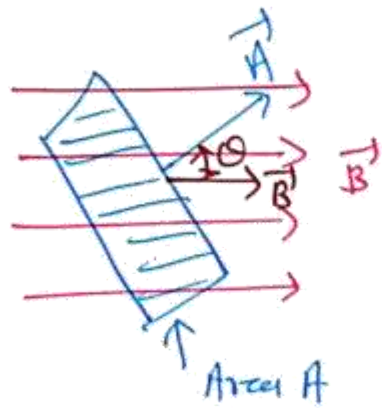
$\boxed{1T = 10000\ G}$

Magnetic flux

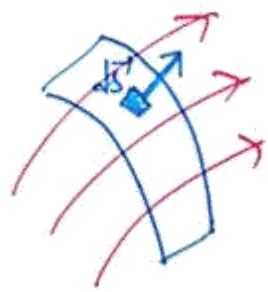
→ magnetic flux $\phi_m = \int \vec{B} . d\vec{s}$



$\phi_m = BA$



$\phi_m = BA \cos\theta$
$= \vec{B} . \vec{A}$



$\boxed{\phi_m = \int \vec{B} . d\vec{s}}$

→ Unit of magnetic flux $\boxed{Wb}$

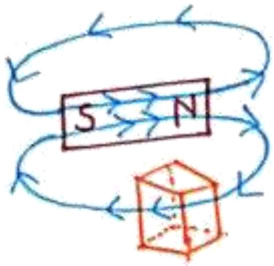
$\Rightarrow Wb = T(m^2) = \left(\frac{N}{Am}\right) m^2 = \boxed{\frac{Nm}{A}}$

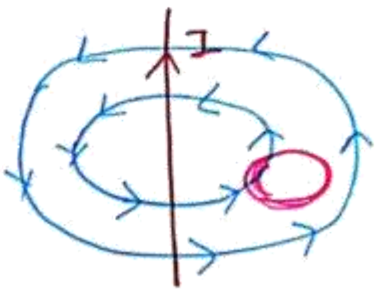
Gauss's law for magnetic field.

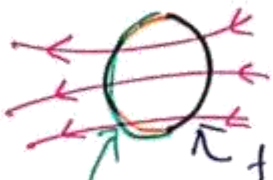
→ For Closed Surface, Magnetic flux is zero.

$$\oint \vec{B} \cdot \vec{ds} = 0$$

→ Magnetic field always stays in closed loop







$$\text{total flux} = \phi_E + \phi_L$$
$$= 0$$

Maxwell's forth eq.^{n} with Integral and differencial Form.

- As per gauss's law for magnetic field.

$$\Rightarrow \oint \vec{B} \cdot \vec{ds} = 0 \quad \text{--- (A)}$$

- eq.^{n} (A) is considered as Integral form of maxwell's forth eq.^{n}

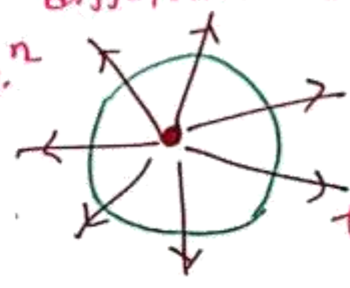
→ There is no existence of magnetic mono pole.

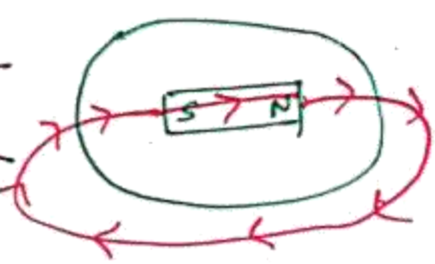
- As per divergence theorm

$$\Rightarrow \oint \vec{B} \cdot \vec{ds} = \int \vec{\nabla} \cdot \vec{B} \, dV = 0$$

$$\Rightarrow \vec{\nabla} \cdot \vec{B} = 0 \quad \text{--- (B).}$$

- eq.^{n} (B) considered as differencial form of maxwell's forth eq.^{n}





Faraday's Law.

→ Static magnetic field produces no flow of current.

→ Time varying magnetic field produces flow of current in circuit.

→ As per Faraday's law, it states "Rate of change of magnetic flux will generate induced emf."

$$V_{emf} = -\frac{d\psi_m}{dt}$$

-Ve sign is due to emf (Induced) current direction and time varying mag. direction is opposite.

$$\boxed{V_{emf} = -\frac{d\left(\int \vec{B}\cdot d\vec{s}\right)}{dt}}$$

→ $V_{emf} = +\int \vec{E}\cdot d\vec{l}$

+Ve sign shows work done on charges.

→ $\Rightarrow \boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{d\left(\int \vec{B}\cdot d\vec{s}\right)}{dt}}$ — (A)

→ As per Stock's theorm

$$\Rightarrow \oint \vec{E}\, d\vec{l} = \int \nabla\times\vec{E}\, ds$$

→ so from above eq^n

$$\Rightarrow \int \vec{\nabla}\times\vec{E}\, ds = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ — (B)

→ eq^n (A) & (B) with maxwell's 2nd eq.^{n} for time varying field

Maxwell's eq.^{n}

Gauss Electric field

$\vec{\nabla}\cdot\vec{D} = \rho_v$ $\oint \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0}$

[existence of electric flux only if charge is enclosed by that closed surface.]

Potential.

$\oint \vec{E}\cdot d\vec{l} = 0$ $\left[\oint \vec{E}\cdot d\vec{l} = -\frac{\partial\phi}{\partial t}\right]$ $\vec{\nabla}\times\vec{E} = 0$. $\left[\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}\right]$

[for closed path, Potential difference (at point) is zero]

[Change in Mag. flux will Induce potential].

Ampere's ckt law.

→ $\oint \vec{H}\cdot d\vec{l} = \int \vec{J}\cdot d\vec{s}$ $\vec{\nabla}\times\vec{H} = \vec{J}$

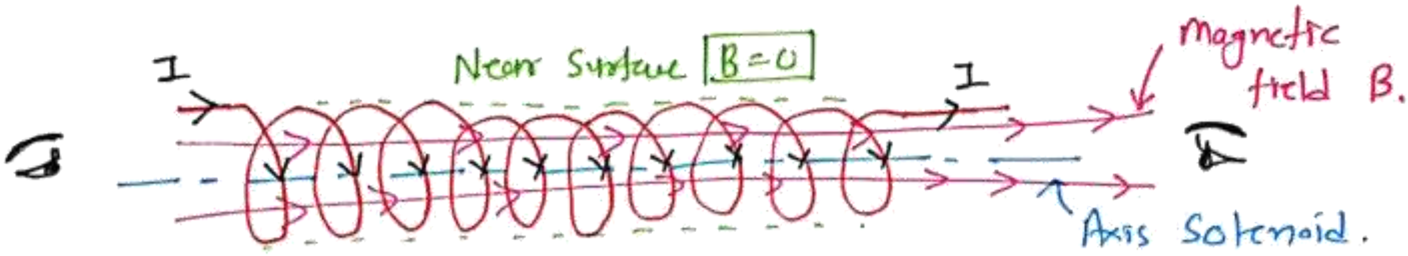
→ $\oint \vec{H}\cdot d\vec{l} = \int(\vec{J} + \vec{J}_D)\, d\vec{s}$ $\vec{\nabla}\times\vec{H} = \vec{J} + \vec{J}_D$

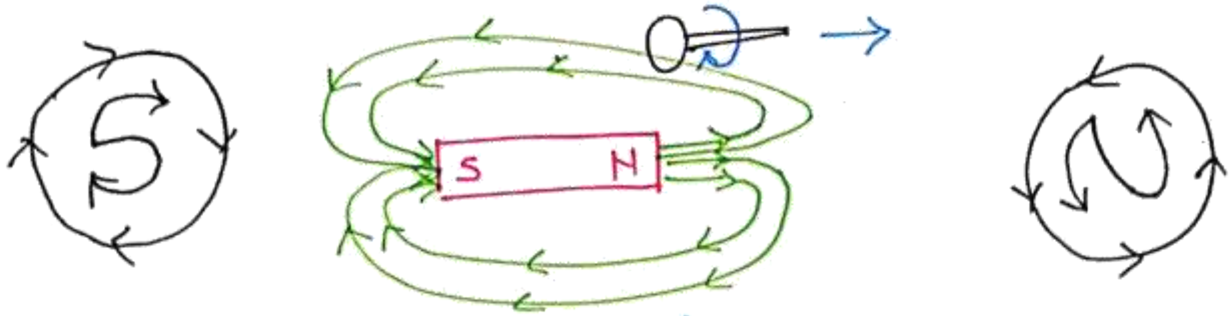
Gauss's for mag field.

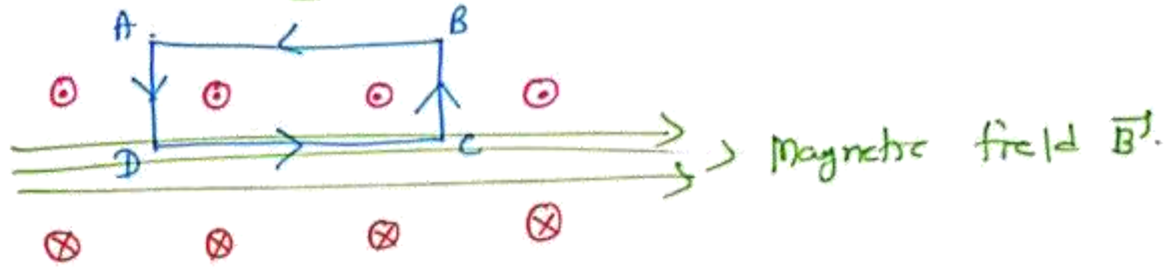
$\oint \vec{B}\cdot d\vec{s} = 0$ $\vec{\nabla}\cdot\vec{B} = 0$.

[Isolated magnetic charge does not exist].

Solenoid







→ As per Ampere's Law.

$$\boxed{\vec{H} = \frac{\vec{B}}{\mu}}$$

$$\Rightarrow \oint \vec{H}.\vec{dl} = I_{enc}.$$

$$\Rightarrow \oint \vec{B}.\vec{dl} = \mu I_{enc}$$

$$\Rightarrow \underbrace{\int_D^C \vec{B}.\vec{dl}}_{\theta=0} + \underbrace{\underbrace{\int_C^B \vec{B}.\vec{dl}}_{\theta=90}}_{0} + \underbrace{\underbrace{\int_B^A \vec{B}.\vec{dl}}_{\theta=180}}_{0} + \underbrace{\underbrace{\int_A^D \vec{B}.\vec{dl}}_{\theta=90}}_{0} = \mu I_{enc}$$

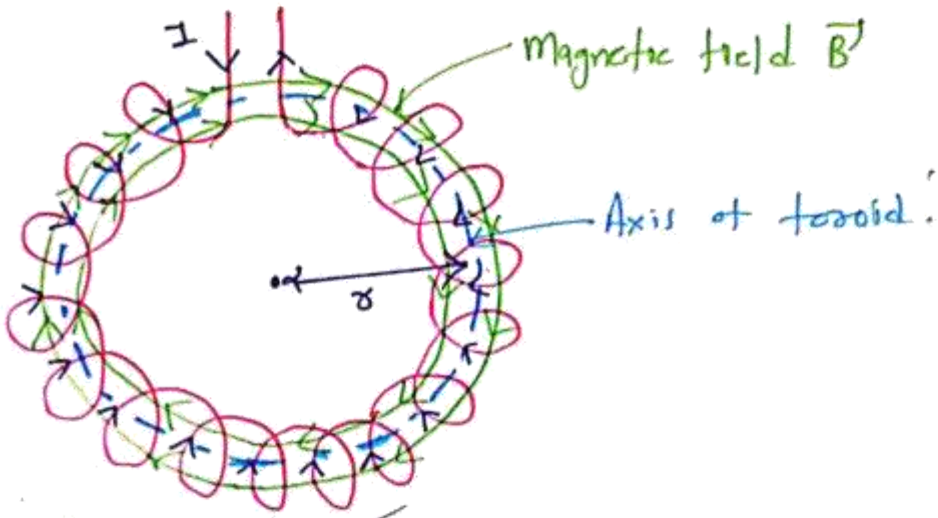
$$\Rightarrow B(L) = \mu I_{enc}.$$

$$\Rightarrow B = \frac{\mu I_{enc}}{L}$$

→ for N numbers of turns

$$\Rightarrow B = \mu \frac{N}{L} I_{enc} = \boxed{\mu n I_{enc}} \quad \left[n = \frac{N}{L} \right]$$

Toroid



→ As per Ampere's law.

$\Rightarrow \int \vec{H} \cdot d\vec{l} = I_{enc}$. $\quad \left[\vec{H} = \frac{\vec{B}}{\mu} \right]$.

$\Rightarrow \int \underbrace{\vec{B} \cdot d\vec{l}}_{\theta = 0} = \mu I_{enc}$.

$\Rightarrow B \int d\vec{l} = \mu I_{enc}$

$\Rightarrow B(2\pi r) = \mu I_{enc}$

$\Rightarrow B = \frac{\mu I_{enc}}{2\pi r}$

→ For N number of turns

$\Rightarrow B = \mu \frac{N}{2\pi r} I_{enc}$.

$\Rightarrow \boxed{B = \mu n I_{enc}}$

$\left[n = \frac{N}{2\pi r} \right]$

→ For toroid, radius is 0.2 m and number of turns is 50. Find magnetic field if we give 1 A current to toroid.

→ $r = 0.2\,m$
$N = 50.$
$I = 1\,A$
$B = ?$

→ $n = \frac{N}{2\pi r} = \frac{50}{2\pi \times 0.2} = 39.8$

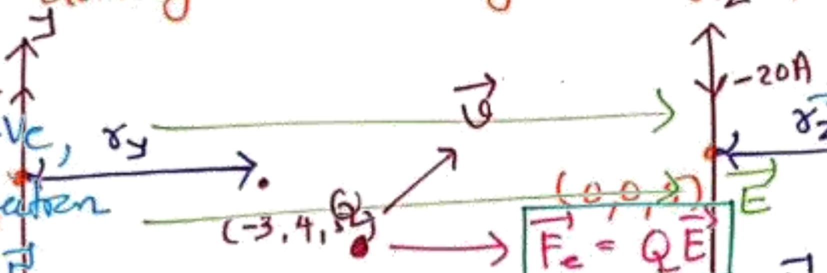
→ $B = \mu n I$
$= \mu_0 n I$
$= 4\pi \times 10^{-7} \times 39.8 \times 1$
$= 5 \times 10^{-5}\ T$

Lorentz Force / Magnetic field Intensity due to Current carrying wire

- Lorentz has calculated Force on moving charge q due to electric and magnetic field.

* Two long parallel wires carrying current of 10 A and −20 A, respectively. Find $\vec{H}$ at $(-3, 4, 5)$ m.

- Force acting on moving charge q due to E-field.



If Q is +Ve, then direction of $\vec{E}$ & $\vec{F}_e$ will be same.

$$\boxed{\vec{F}_e = Q\vec{E}}$$

If Q is −Ve, then direction of $\vec{E}$ & $\vec{F}_e$ Opposite.

$$\vec{H}_1 = \frac{I}{2\pi r_y}\hat{a}_{\phi y} \qquad \vec{H}_2 = \frac{I}{2\pi r_2}\hat{a}_{\phi z}$$

$\rightarrow \vec{r}_y = (-3, 4, 5) - (0, 4, 0) = (-3, 0, 5)$ m

- Force acting on moving charge due to B-field

$\rightarrow \vec{r}_2 = (-3, 4, 5) - (0, 0, 5) = (-3, 4, 0)$ m

$\rightarrow r_y = \sqrt{3^2 + 0^2 + 5^2} = \sqrt{34}$

$\rightarrow r_2 = \sqrt{3^2 + 4^2} = 5$

$$\boxed{\vec{F}_B = Q(\vec{u} \times \vec{B})}$$

$\rightarrow \hat{a}_{\phi y} = \hat{a}_y \times \hat{r}_y$

$= \hat{a}_y \times \frac{(-3\hat{a}_x + 5\hat{a}_z)}{\sqrt{34}}$

$= 3\hat{a}_z + 5\hat{a}_x / \sqrt{34}$

$\rightarrow \hat{a}_{\phi z} = \hat{a}_z \times \frac{(-3\hat{a}_x + 4\hat{a}_y)}{5}$

$= (-3\hat{a}_y - 4\hat{a}_x)/5$

$\rightarrow \vec{H}_1 = \frac{10}{2\pi \times \sqrt{34}} \frac{(3\hat{a}_z + 5\hat{a}_x)}{\sqrt{34}}$

$\rightarrow \vec{H}_2 = \frac{-20}{2\pi \times 5} \times \frac{(-3\hat{a}_y - 4\hat{a}_x)}{5}$

- Resultant Force due to E-field and H-field

$\vec{F} = \vec{F}_E + \vec{F}_B$

$= 0.14\hat{a}_z + 0.234\hat{a}_x$

$= 0.3821\hat{a}_y + 0.5095\hat{a}_x$

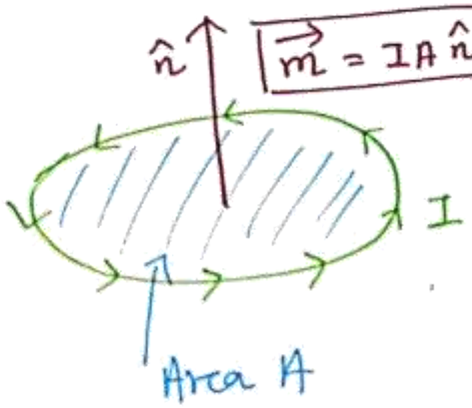
$$\boxed{\vec{F} = Q[\vec{E} + \vec{u} \times \vec{B}]}$$

$\rightarrow \vec{H} = \vec{H}_1 + \vec{H}_2$

$= 0.7435\hat{a}_x + 0.3821\hat{a}_y + 0.14\hat{a}_z$ (A/m)

Magnetic Dipole Moment, Magnetization, Magnetic Susceptibility and Permeability.

- Magnetic Dipole Moment



$$\boxed{\vec{m} = IA\hat{n}}$$

→ Unit of magnetic dipole moment is Am^2.

→ It direction is perpendicular to the surface.

- Magnetization

→ magnetization is depending on material.

→ Magnetization is magnetic dipole moment per unit vol.^{m}

→ Unit $= \left[\frac{Am^2}{M^3}\right] = A/m$

→ $\vec{m} = X_m \vec{H}$ (where X_m = Magnetic susceptibility).

→ $\vec{B} = \mu_0(\vec{H} + \vec{m})$

$= \mu_0(\vec{H} + X_m\vec{H})$

$= \mu_0(1+X_m)\vec{H}$

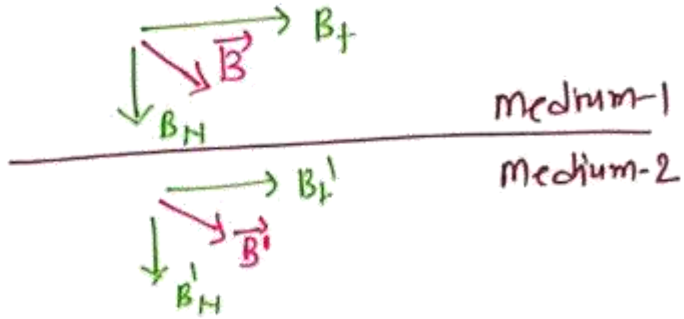
$$\boxed{\vec{B} = \mu_0\mu_r\vec{H}}$$

→ $\boxed{\mu_r = 1 + X_m}$

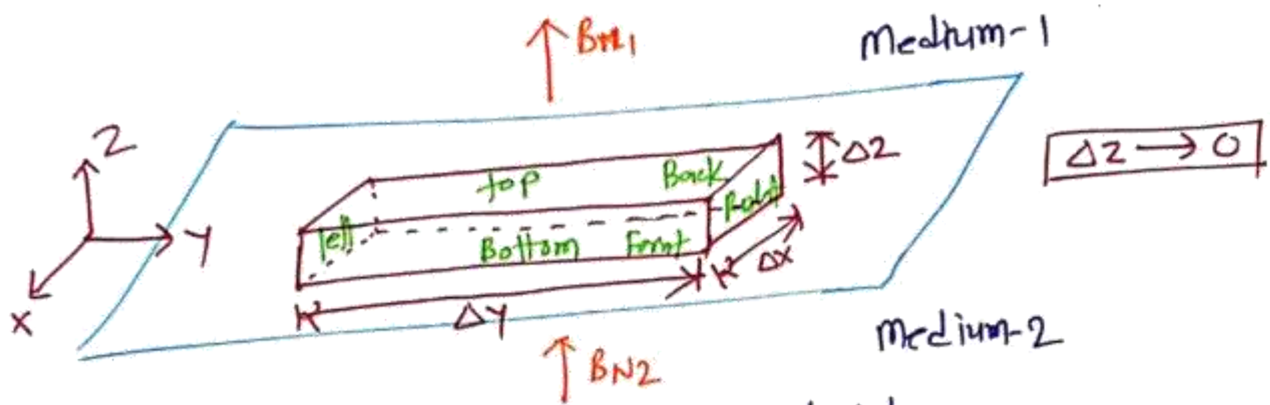
→ Relative permeability μ_r

Magnetic boundary Condition.

→ When magnetic field enters from one medium to other medium, there may be discontinuity in magnetic field, which can be understand by magnetic boundry conditions.



– Boundry conditions for normal component of mag. field.



→ As per gauss's law for magnetic field

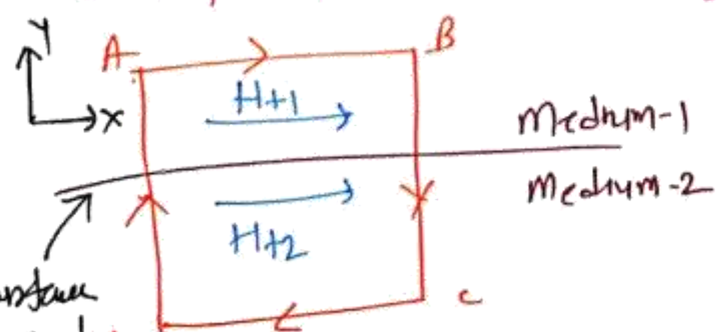
$$\Rightarrow \oint \vec{B}\cdot d\vec{s} = 0$$

$$\Rightarrow \int_{top} \vec{B}\cdot d\vec{s} + \int_{Bottom} \vec{B}\cdot d\vec{s} + \underbrace{\int_{Left} \vec{B}\cdot d\vec{s} + \int_{Right} \vec{B}\cdot d\vec{s} + \int_{front} \vec{B}\cdot d\vec{s} + \int_{Back} \vec{B}\cdot d\vec{s}}_{0} = 0$$

$$\Rightarrow B_{N1}(\Delta X \Delta Y) - B_{N2}(\Delta X \Delta Y) = 0$$

$$\Rightarrow \boxed{B_{N1} = B_{N2}}$$

– Boundry Conditions for tangential Component



Surface current per unit length is K

→ As per Ampere's law.

$$\Rightarrow \int \vec{H}\cdot d\vec{l} = I_{enc} = K\Delta X$$

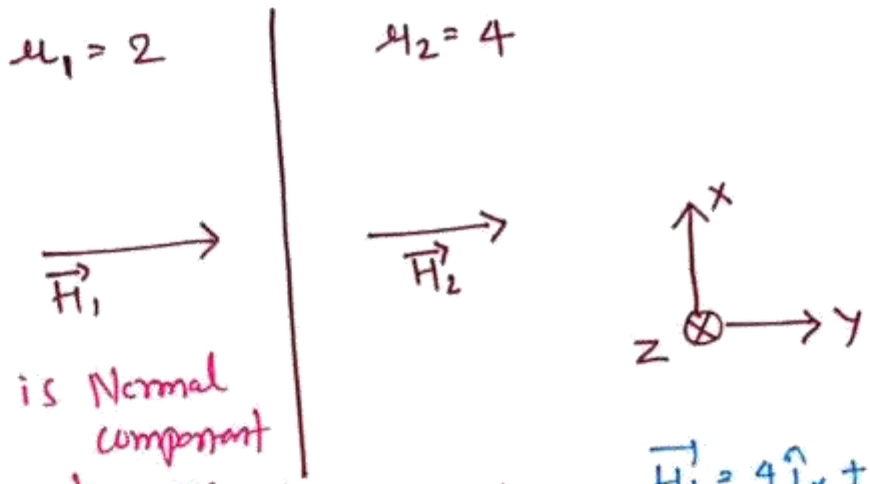
$$\Rightarrow \int_A^B H\,dl + \underbrace{\int_B^C H\,dl}_{0} + \int_C^D H\,dl + \underbrace{\int_D^A H\,dl}_{0} = K\Delta X$$

$$\Rightarrow H_{t1}\Delta X - H_{t2}\Delta X = K\Delta X$$

$$\Rightarrow \boxed{H_{t1} - H_{t2} = K}$$

Example on Magnetic Boundary Conditions.

→ $\vec{H_1} = 4\hat{i}_x + 6\hat{i}_y + 2\hat{i}_z$, then Find $\vec{H_2} = ?$



→ y component is Normal component

→ x & z components are tangential components $\vec{K} = 2\hat{i}_x + 4\hat{i}_z$

$\vec{H_1} = 4\hat{i}_x + \underline{6\hat{i}_y} + 2\hat{i}_z$

→ For Normal components, $B_{H_1} = B_{H_2}$

$\Rightarrow \mu_0\mu_1 H_{H_1} = \mu_0\mu_2 H_{H_2}$

$\Rightarrow 2 \times 6 = 4 H_{H_2} \Rightarrow \boxed{H_{H_2} = 3\hat{i}_y}$

→ For Tangential Components, $H_{t_1} - H_{t_2} = K$

– For x component $\perp^{r}$ to H_{t_1} with K Direction of propagation

$H_{t_2} = H_{t_1} - K \underline{\downarrow} \;\; \underline{\downarrow}$

$= 4\hat{i}_x + 4\hat{i}_z \times \hat{i}_y$

$= 4\hat{i}_x + 4(-\hat{i}_x)$

$= 0\hat{i}_x$

– For z Component $\perp^{r}$ to H_{t_1} with K Direction of propagation

$H_{t_2} = H_{t_1} - K \underline{\downarrow} \;\; \underline{\downarrow}$

$= 2\hat{i}_z + 2\hat{i}_x \times \hat{i}_y$

$= 2\hat{i}_z + 2\hat{i}_z$

$= 4\hat{i}_z$

→ $\vec{H_2} = 0 \times \hat{i}_x + 3\hat{i}_y + 4\hat{i}_z = 3\hat{i}_y + 4\hat{i}_z$

EM wave in free Space and in Medium.

- For EM wave, electric field [free Space]

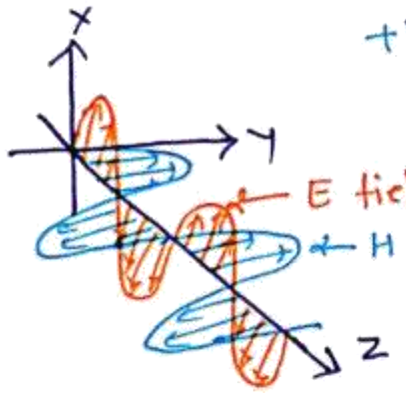
$$\vec{E} = E_0 \cos(\omega t - \beta z)\ \hat{a}_x$$

E_0 ← Amplitude; ω ← freq; β ← phase Constant; z ← Direction of propagation; $\hat{a}_x$ ← Direction of Amplitude

- For EM wave, magnetic field in free Space.

$$\vec{H} = H_0 \cos(\omega t - \beta z)\ \hat{a}_y$$

−ve Sign, propagation in +ve z direction
+ve Sign, propagation in −ve z direction.

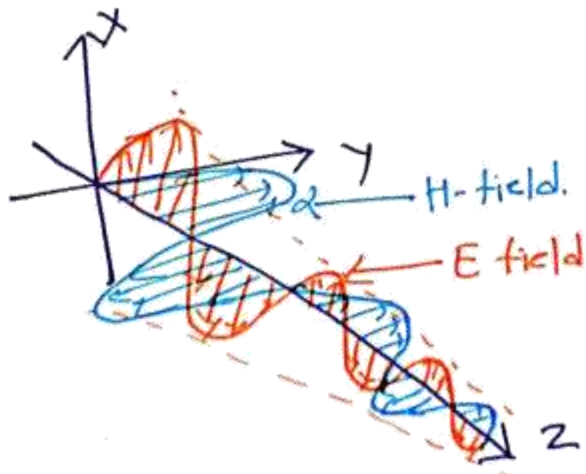


→ For EM wave, E-field in medium.

$$\vec{E} = E_0 \boxed{e^{-\alpha z}} \cos(\omega t - \beta z)\ \hat{a}_x$$

→ For EM wave, H-field In medium

$$\vec{H} = H_0 \boxed{e^{-\alpha z}} \cos(\omega t - \beta z)\ \hat{a}_y$$



Wave equation in EM wave.

→ As per Faraday's law

$$\Rightarrow \nabla \times E = -\frac{dB}{dt}$$

→ take curl on Both side

$$\Rightarrow \nabla \times \nabla \times E = -\frac{d(\nabla \times B)}{dt}$$

$$\Rightarrow \nabla(\nabla . E) - \nabla^2 E = -\frac{d(\nabla \times B)}{dt}$$

→ As per gauss law of electric field

$$\Rightarrow \nabla . D = \rho_v$$

$$\Rightarrow \nabla . (\varepsilon_0 E) = \rho_v$$

$$\Rightarrow \nabla . E = \frac{\rho_v}{\varepsilon_0}$$

→ For Free space $\rho_v = 0$.

$$\Rightarrow \boxed{\nabla . E = 0}$$

→ from above eq^n

$$\Rightarrow +\nabla^2 E = +\frac{d(\nabla \times B)}{dt} = \mu_0 \frac{d(\nabla \times H)}{dt}$$

→ As per Ampere circuit law for time varying field

$$\Rightarrow \nabla \times H = J + \frac{\partial D}{\partial t}$$

→ For Free Space $J = 0$

$$\Rightarrow \nabla \times H = \frac{\partial D}{\partial t} = \varepsilon_0 \frac{\partial E}{\partial t}$$

→ From above eq^n

$$\Rightarrow \nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

→ For Free Space, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$$\Rightarrow \nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$$

$$\Rightarrow \boxed{c^2 \nabla^2 E = \frac{\partial^2 E}{\partial t^2}} \quad \text{— (A)}$$

$$\boxed{\nabla^2 E = \gamma^2 E} \quad \text{— (B)}$$

→ Here eq^n (A) & (B) are wave eq^n

plane wave (EM) in Medium

→ As per wave eq.ⁿ

$$\nabla^2 E = \gamma^2 E$$

→ γ is propagation Constant

$$\gamma = \alpha + j\beta$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

→ Sol.ⁿ of above eq.ⁿ

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} - 1\right]^{1/2}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} + 1\right]^{1/2}$$

Lossless Medium

- there is no loss of signal during propagation.
- Charge carrier absorbs energy of EM wave.
- There is no free charge carrier in lossless medium.

$[\sigma = 0]$

→ $\boxed{\alpha = 0}$

→ $\boxed{\beta = \omega\sqrt{\mu\varepsilon}}$

Lossy Medium

- there is a loss of signal during propagation $\boxed{\frac{\sigma}{\omega\varepsilon} >> 1}$

→ $\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\sqrt{\frac{\sigma}{\omega\varepsilon}} = \sqrt{\frac{\omega\mu\sigma}{2}}$

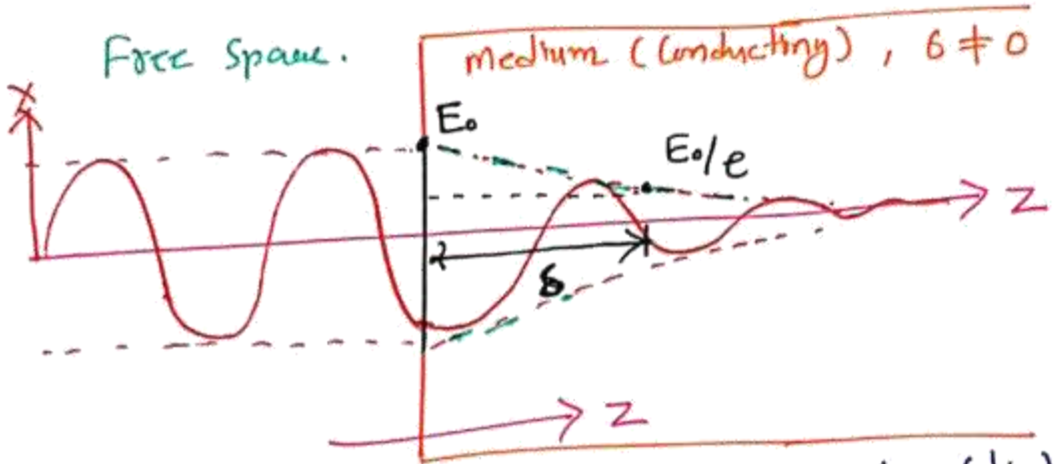
→ $\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\sqrt{\frac{\sigma}{\omega\varepsilon}} = \sqrt{\frac{\omega\mu\sigma}{2}}$

Skin Effect

→ When signal passing through any conducting medium, It has loss as it passes through medium.

$$E = \underbrace{E_0 e^{-\alpha z}}_{\text{Amplitude}} \cos(\omega t - \beta z)\ \hat{a}_x$$

z is direction of propagation.



→ When amplitude of signal decreases to $(1/e)^{th}$ of it's initial amplitude at certain depth δ. that depth is reffered as skin depth.

$$\Rightarrow \frac{E_0}{e} = E_0 e^{-\alpha\delta}$$

$$\Rightarrow e = e^{\alpha\delta}$$

$$\Rightarrow \boxed{\delta = \frac{1}{\alpha}}$$

→ α for loss medium

$$\alpha = \sqrt{\frac{\mu\sigma\omega}{2}}$$

$$\Rightarrow \boxed{\delta = \sqrt{\frac{2}{\mu\sigma\omega}}}$$

Intrinsic Impedance

→ It is a Impedance of medium, in which wave is travelling.

→ So based on medium we can define, Intrinsic Impedance

$$* \quad \eta = \frac{E}{H}$$

$$* \quad \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

→ Loss less medium.

→ Loss less medium has no loss of signal.

→ So free charge carrier in loss less medium will be zero.

$$\boxed{\sigma = 0}$$

$$\rightarrow \eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$$

→ For free space, $\mu_r = 1$, $\varepsilon_r = 1$

$$\rightarrow \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}} = \boxed{377} \text{ or } \boxed{120\pi}\ \Omega$$

→ For lossy medium

→ For lossy medium, It has loss of signal due to Free charge Carriers.

$$\boxed{\sigma >> 1}$$

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma}}}\ \Omega$$

Loss tangent

→ It is based on medium.

→ It is angle of loss for medium.

→ As per maxwell's eq.^{n}

$$\Rightarrow \nabla \times H = J_c + J_d$$

$$\Rightarrow \nabla \times H = J_c + \frac{\partial D}{\partial t}$$

$[D = \varepsilon_0 E]$

$$\Rightarrow \boxed{\nabla \times H = \underbrace{J_c}_{\text{lossy term}} + \underbrace{\varepsilon_0 \frac{\partial E}{\partial t}}_{\text{loss less term}}}$$

→ loss tangent

$$\tan \theta_e = \frac{J_c}{J_D} = \frac{\sigma E}{\varepsilon \omega E} = \frac{\sigma}{\varepsilon \omega}$$

→ θ_n is Intrinsic Impedance angle

$$\boxed{2\theta_n = \theta_e}$$

$$\Rightarrow \boxed{\tan 2\theta_n = \frac{\sigma}{\varepsilon \omega}}$$

Poynting Theorm

→ Poynting Theorm is used to find power of EM wave.

→ Poynting theorm explains power density of EM wave.

→ Power density

$$\Rightarrow \vec{P} = \vec{E} \times \vec{H}$$

→ $\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z + \theta_1)\, \hat{a}_x$

$\vec{H} = H_0 e^{-\alpha z} \cos(\omega t - \beta z + \theta_2)\, \hat{a}_y$

$$\Rightarrow \vec{P} = E_0 e^{-\alpha z} \cos(\omega t - \beta z + \theta_1)\, \hat{a}_x \times H_0 e^{-\alpha z} \cos(\omega t - \beta z + \theta_2)\, \hat{a}_y$$

$$\Rightarrow \vec{P} = E_0 H_0 e^{-2\alpha z} \cos(\omega t - \beta z + \theta_1) \cos(\omega t - \beta z + \theta_2)\, \hat{a}_z$$

→ Here, $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$$\Rightarrow \vec{P} = \frac{E_0 H_0 e^{-2\alpha z}}{2} [\cos(2\omega t - 2\beta z + \theta_1 + \theta_2) + \cos(\theta_1 - \theta_2)]\, \hat{a}_z$$

→ for avg. power density

$$\Rightarrow P_{avg} = \frac{1}{T} \int_0^T P\, dt$$

$$\Rightarrow P_{avg} = \frac{1}{T} \int_0^T \frac{E_0 H_0 e^{-2\alpha z}}{2} [\underbrace{\cos(2\omega t - 2\beta z + \theta_1 + \theta_2)}_{0} + \cos(\theta_1 - \theta_2)]\, dt$$

$$= \frac{1}{\cancel{T}} \left[\frac{E_0 H_0 e^{-2\alpha z}}{2} \right] [0 + \cos(\theta_1 - \theta_2) \times [\cancel{T}]]$$

$$= \frac{E_0 H_0 e^{-2\alpha z}}{2} \cos(\theta_1 - \theta_2)$$

→ $\eta = \frac{E \angle \theta_1}{H \angle \theta_2} = \frac{E_0}{H_0} \angle \theta_1 - \theta_2 = \frac{E_0}{H_0} \angle \theta_n$

$$\Rightarrow P_{avg} = \frac{E_0 H_0}{2} e^{-2\alpha z} \cos \theta_n$$

→ $\eta = \frac{E_0}{H_0} \Rightarrow H_0 = \frac{E_0}{\eta}$

$$\Rightarrow P_{avg} = \frac{E_0^2}{2\eta} e^{-2\alpha z} \cos \theta_n$$

→ For Lossless medium

$\alpha = 0,\ \theta_n = 0$

→ $\boxed{P_{avg} = \frac{E_0^2}{2\eta}}$

→ $P_{avg} = \left[\frac{V}{m}\right]\left[\frac{A}{m}\right] = \left[\frac{W}{m^2}\right]$

→ Avg. Power

$P = P_{avg}(S)$

Example based on EM wave and Poynting theorm

The time domain expression for the magnetic field of plane wave travelling in a non-magnetic medium is given by

$$H(x,t) = 0.5 \cos(6\pi \times 10^8 t - 10x)\, \hat{a}_z \ (A/m).$$

(a) Find the direction of wave propagation
(b) The dielectric constant & Intrinsic Impedance
(c) The time domain expression of E field
(d) ~~Time~~ Average power density.

→ $\mu_r = 1$

$H(x,t) = 0.5 \cos(6\pi \times 10^8 t - 10x)\, \hat{a}_z \ (A/m).$

$H(x,t) = H_0 e^{-\alpha x} \cos(\omega t - \beta x)\, \hat{a}_z \ (A/m).$

Direction of propagation is in +ve x direction.

$\omega = 6\pi \times 10^8$

$\beta = 10$

$H_0 = 0.5$

→ For loss less medium

$\Rightarrow \beta = \omega\sqrt{\mu\varepsilon}$

$\Rightarrow \beta = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$

$\Rightarrow 10 = 6\pi \times 10^8 \sqrt{\mu_0\varepsilon_0}\ \sqrt{\varepsilon_r}$

$\Rightarrow \sqrt{\varepsilon_r} = \dfrac{10 \times c}{6\pi \times 10^8} = \dfrac{10 \times 3 \times 10^8}{6\pi \times 10^8} = 1.59$

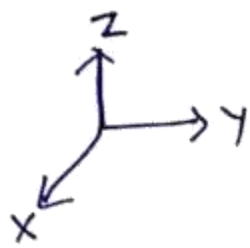
$\Rightarrow \boxed{\varepsilon_r = 2.54}$

→ For loss less medium

$\eta = \sqrt{\dfrac{\mu}{\varepsilon}} = \sqrt{\dfrac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$

$= \sqrt{\dfrac{4\pi \times 10^{-7} \times 1}{8.854 \times 10^{-12} \times 2.54}}$

$\boxed{\eta = 236\ \Omega}$



→ $E = E_0 \underbrace{e^{-\alpha x}}_{=1} \cos(\omega t - \beta x)\, \hat{a}_y \ \left(\frac{V}{m}\right)$

→ $\eta = \dfrac{E_0}{H_0} \Rightarrow E_0 = \eta H_0 = 236 \times 0.5 = 118$

→ $\boxed{E = 118 \cos(6\pi \times 10^8 t - 10x)\, \hat{a}_y \ (V/m).}$

→ $P = \dfrac{E_0^2}{2\eta} = \dfrac{118^2}{2 \times 236} = \boxed{29.5\ W/m^2}$