(iii) ~~E&H~~ $\mathcal{E}$ & H are perpendicular to each other & to the dir^n of propagation. Hence, they are called transverse EM Wave.

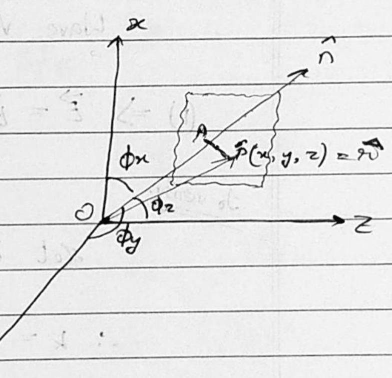
(iv) A uniform plane wave is one in which $\mathcal{E}$ & H lie in a plane and ~~the~~ have the same value everywhere in that plane.

(v) For a uniform plane wave travelling in z-direction, the space variations of $\mathcal{E}$ & H are zero over $Z = K$ (constant) ~~plane~~.

Date:- 17/03

Plane Wave in Arbitrary Dir^n :-

$$\hat{n} = \cos\phi_x \hat{x} + \cos\phi_y \hat{y} + \cos\phi_z \hat{z}$$



$$\vec{OP} = x\hat{x} + y\hat{y} + z\hat{z}$$

Eq^n of constant phase gain is given by:-

$$\vec{r}\cdot\hat{n} = \vec{OA} = \text{const.}$$

$$\Rightarrow (x\hat{x} + y\hat{y} + z\hat{z})\cdot(\cos\phi_x\hat{x} + \cos\phi_y\hat{y} + \cos\phi_z\hat{z}) = \text{const.}$$

$$\Rightarrow x\cos\phi_x + y\cos\phi_y + z\cos\phi_z = \text{Const.}$$

Let β be the phase constant,

$\therefore$ Phase at pt. A $\longrightarrow \beta\cdot\vec{OA}$

$$= \beta(\hat{n}\cdot\vec{r})$$

Elc. field, at that point,

$$\vec{E} = \vec{E_0} = e^{-j\beta(\hat{n}\cdot\vec{r})} \longrightarrow (1)$$

So a transverse EMW travelling in some arbitrary dir^n with unit vector $\hat{n}$ is given by the eq^n (1).

$$\vec{E_0} = \vec{E}_{0x}\hat{x} + \vec{E}_{0y}\hat{y} + \vec{E}_{0z}\hat{z}$$

Now,

$$E_0 \cdot \hat{n} = 0 \quad [\because \vec{E_0} \perp \hat{n} \; ; \; \hat{n} \rightarrow \text{dir}^n \text{ of propagation}]$$

Wave Vector/Wave Number :-

Wave vector, $k = \beta \cdot \hat{n}$

$$(1) \Rightarrow \vec{E} = \vec{E_0}\, e^{-j\vec{k}\cdot\vec{r}} \longrightarrow (2) \; \{\text{Generalised Eq}^n\}$$

To verify:

Let $\hat{n}$ is in z-direction, i.e. $\hat{n} \equiv \hat{z}$

$$\therefore k = \beta\cdot\hat{n} = \beta\cdot\hat{z}$$

$$(2) \Rightarrow \vec{E} = E_0\, e^{-j\beta(\hat{z}\cdot\vec{r})}$$

$$\Rightarrow \boxed{\vec{E} = \vec{E_0}\, e^{-j\beta \vec{z}}}$$; This verifies the relation.

For H,

From Maxwell's Eq^{n}s for non-conducting and source free medium $\rightarrow$

$$\nabla \times E = -j\omega\mu H$$

$$\Rightarrow H = \frac{-1}{j\omega\mu}(\nabla \times E)$$

$$= -\frac{1}{j\omega\mu}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \longrightarrow (1)$$

Again,

$$\vec{E} = \vec{E_0} e^{-j\vec{k}\cdot\vec{r}}$$

$$= (E_{ox}\hat{x} + E_{oy}\hat{y} + E_{oz}\hat{z}) e^{-j\vec{k}\cdot\vec{r}}$$

$$\text{or, } \left(\frac{\partial \vec{E}}{\partial x}\right) = -jk_x \vec{E}$$

$$\frac{\partial}{\partial x} = -jk_x$$

$$\frac{\partial}{\partial y} = -jk_y$$

$$\frac{\partial}{\partial z} = -jk_z$$

$$(1) \Rightarrow H = -\frac{1}{j\omega\mu}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -jk_x & -jk_y & -jk_z \\ E_x & E_y & E_z \end{vmatrix}$$

$$\Rightarrow H = -\frac{1}{j\omega\mu}(-j)\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ k_x & k_y & k_z \\ E_x & E_y & E_z \end{vmatrix}$$

$$\Rightarrow H = \frac{1}{\omega\mu}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ k_x & k_y & k_z \\ E_x & E_y & E_z \end{vmatrix} = \frac{1}{\omega\mu}(k \times E)$$

$$\Rightarrow H = \frac{1}{\omega\mu}(k \times \vec{E}) \longrightarrow (2)$$

Phase Velocity in any dir^n (z-direction):

We have,

$$\vec{E} = \vec{E}_0 \, e^{-j\vec{k}\cdot\vec{r}}$$

$$= \vec{E}_0 \, e^{-j(\beta\cos\phi_x \hat{x} + \beta\cos\phi_y \hat{y} + \beta\cos\phi_z \hat{z})}$$

$$= \vec{E}_0 \, e^{-j\beta(\cos\phi_x \hat{x} + \cos\phi_y \hat{y})} \, e^{-j\beta\cos\phi_z \hat{z}}$$

$$= \vec{P}_0 \cdot e^{-j\beta\cos\phi_z \hat{z}} \quad ; \text{ where } \vec{P}_0 = \vec{E}_0 e^{-j\beta(\cos\phi_x \hat{x} + \cos\phi_y \hat{y})}$$

∴ The wave phase constant in z-dir^n is given by :-

$$\beta_z = \beta\cos\phi_z$$

& Phase velocity, $V_{P_z} = \dfrac{\omega}{\beta_z} = \dfrac{\omega}{\beta\cos\phi_z}$

$$= \frac{V_0}{\cos\phi_z}, \quad V_0 = \frac{\omega}{\beta}$$

where V_0 is the velocity of wave in dir^n perpendicular to constant phase front

IIly, $V_{P_x} = \dfrac{V_0}{\cos\phi_x}$ & $V_{P_y} = \dfrac{V_0}{\cos\phi_y}$

Here, $0 \leq |\cos\phi_x| \leq 1 \Rightarrow \underbrace{\infty \leq V_{P_x} \leq V_0}$

But this is not possible ∵ speed cannot exceed 'c' classmate

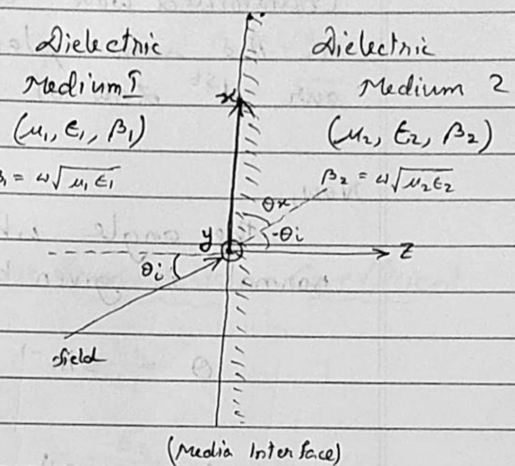
Date:- 24/03

Plane Wave at Dielectric Interface:

$\vec{F} = \vec{F}_0 e^{-jkr}$

$\vec{F} = \vec{F}_0 e^{-j\beta_1(\cos\theta_x x + \cos\theta_y y + \cos\theta_z z)}$ → (1)

where, $\theta_x = \frac{\pi}{2} - \theta_i$
$\theta_y = \pi/2$
$\theta_z = \theta_i$



(1) ⟹ $\theta_x = \frac{\pi}{2} - \theta_i \Rightarrow \cos\theta_x = \cos(\frac{\pi}{2} - \theta_i) = \sin\theta_i$

$\theta_y = \frac{\pi}{2} \Rightarrow \cos\theta_y = \cos\frac{\pi}{2} = 0$

&, $\theta_z = \theta_i \Rightarrow \cos\theta_z = \cos\theta_i$

$\therefore$ (1) $\Rightarrow \vec{F} = \vec{F}_0 e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$ → (2), field eqn

To see the amplitude variation of the field in x-y plane, we can write

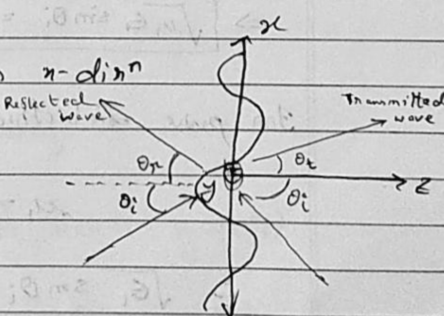
$Re\{\vec{F}_i\} = \vec{F}_0 \cos(x\beta_1 \sin\theta_i)$ → This is the reqd. amplitude

| $z = 0$ in x-y plane
& $e^{j\theta} = \cos\theta + j\sin\theta$

The spatial phase gradient in x-dirn is:

Phase Gradient $= -\beta_1 \sin\theta_i$

Phase Constant $= -\beta_1 x \sin\theta_i$



Variation of amp. in x-y plane

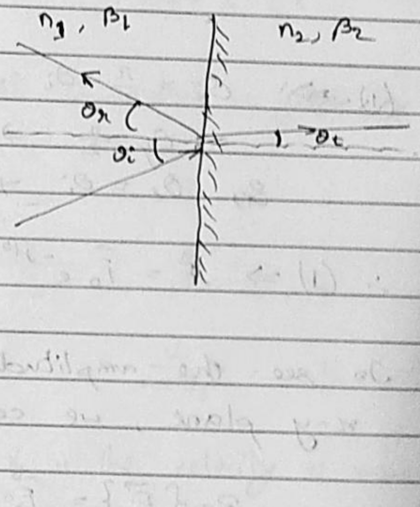
Hence, we can say that the incident wave, the transmitted wave and the reflected wave lie on the same plane of incidence which shows our 1st Law of Reflection.

Now,

∵ The angle which the wave makes with its normal is given by :-

$$\theta = \sin^{-1}\left\{\frac{-\text{ phase gradient}}{\text{phase constant}}\right\}$$

$$\Rightarrow \theta_r = \sin^{-1}\left(\frac{-(-\beta_1 \sin\theta_i)}{\beta_1}\right)$$



$$\rightarrow \boxed{\theta_r = \theta_i}$$

$$\Rightarrow \theta_t = \sin^{-1}\left(\frac{-(-\beta_1 \sin\theta_i)}{\beta_2}\right)$$

$$= \sin^{-1}\left(\frac{\beta_1 \sin\theta_i}{\beta_2}\right)$$

$$\therefore \Rightarrow \beta_2 \sin\theta_t = \beta_1 \sin\theta_i$$

$$\Rightarrow \cancel{\omega}\sqrt{\mu_2\epsilon_2}\sin\theta_t = \cancel{\omega}\sqrt{\mu_1\epsilon_1}\sin\theta_i$$

$$\Rightarrow \boxed{\sqrt{\mu_1\epsilon_1}\sin\theta_i = \sqrt{\mu_2\epsilon_2}\sin\theta_t} \longrightarrow \text{Snell's Law}$$

For pure dielectric Medium, we have

$$\mu_1 = \mu_2 = \mu$$

$$\therefore \sqrt{\epsilon_1}\sin\theta_i = \sqrt{\epsilon_2}\sin\theta_t \quad \text{[From Snell's Law]}$$

$$\Rightarrow \sqrt{\epsilon_0\epsilon_{r_1}}\sin\theta_i = \sqrt{\epsilon_0\epsilon_{r_2}}\sin\theta_t$$

classmate

Date

Page

$$\Rightarrow \sqrt{\epsilon_{r_1}} \sin\theta_i = \sqrt{\epsilon_{r_2}} \sin\theta_t$$

or $\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$; $n_1 = \sqrt{\epsilon_{r_1}}$

$n_2 = \sqrt{\epsilon_{r_2}}$

27-03-23

Reflection Co-efficient & Transmission Co-efficient:

→ Reflection Co-efficient, $= \frac{E_r}{E_i}$

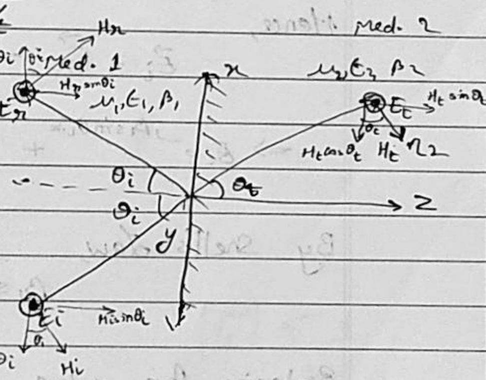
→ Transmission Co-efficient, $\tau = \frac{E_t}{E_i}$

→ Intrinsic impedance $= \eta = \sqrt{\mu/\epsilon}$

$$\eta_1 = \sqrt{\mu_1/\epsilon_1}$$

$$\eta_2 = \sqrt{\mu_2/\epsilon_2}$$

$$\frac{E_t}{H_t} = \eta_2$$



→ The arbitrary direction of E & H can be decomposed into a perpendicular & parallel to the plane of incidence.

For perpendicular polarisation:

$$\vec{F_i} = \vec{F_{oi}} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

For incident wave, $\vec{E_i} = E_i e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$

For reflected wave, $\vec{E_r} = E_r e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$

For transmitted wave, $\vec{E_t} = E_t e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$

$$\vec{E_i} = E_i e^{-j\beta_1 x \sin\theta_i} \longrightarrow (1)$$

$$\vec{E_r} = E_r \cdot e^{-j\beta_1 x \sin\theta_i} \longrightarrow (2)$$

$$\vec{E_t} = E_t e^{-j\beta_2 \sin\theta_t} \longrightarrow (3)$$

$\because z=0$ at boundary

We know,

The Tangential component of E should be continuous across the dielectric - dielectric boundary. At interface, i.e. $z=0$ plane, the elc fields are given by the above eq^{n}s (1), (2), (3).

Hence,

$$\vec{E_i} + \vec{E_r} = \vec{E_t}$$

$$\Rightarrow E_i e^{-j\beta_1 \sin\theta_i \cdot x} + E_r e^{-j\beta_1 \sin\theta_i x} = E_t e^{-j\beta_2 \sin\theta_t \cdot x} \rightarrow (4)$$

By Snell's Law,

$$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

Replacing this value in the above eq^n (4), we get

$$\therefore (4) \Rightarrow E_i + E_r = E_t \longrightarrow (5)$$

From Poynting vector,

Dir^n of energy flow of EM Wave $= \vec{E} \times \vec{H}$ (Right hand thumb rule)

$\because$ There are no surface currents, so tangential component of H is also continuous

$$H_i \cos\theta_i - H_r \cos\theta_i = H_t \cos\theta_t \quad (6)$$

$$\Rightarrow \frac{E_i}{\eta_1}\cos\theta_i - \frac{E_r}{\eta_1}\cos\theta_i = \frac{E_t}{\eta_2}\cos\theta_t \quad (6) \qquad \because \eta = \frac{E}{H}$$

Reflection coefficient,

$$\Gamma = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \qquad \Gamma \rightarrow \text{Gamma}$$

& the transmission co-efficient is given by :-

$$\tau = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_i \cos\theta_t}$$

Now,

$$(5) \Rightarrow E_i + E_r = E_t$$

$$\Rightarrow 1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$\Rightarrow 1 + \Gamma_\perp = \tau_\perp \qquad |\Gamma_\perp| \leq 1$$

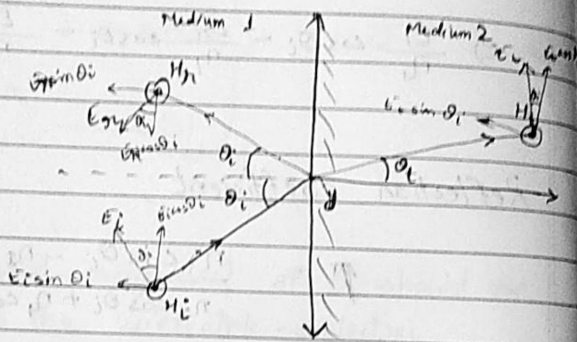
$$|\tau_\perp| \leq 1$$

Here,

$E_t > E_i$ but it cannot violate law of conservation of energy, hence $H_t < H_i$ - to compensate for the excess E.

Date:- 29/3/23

For parallel polarisation :



$$H_i + H_r = H_t$$

$$\Rightarrow \frac{E_i}{\eta_1} + \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2} \longrightarrow (1)$$

$$\Rightarrow \frac{1}{\eta_1} + \frac{1}{\eta_1}\left(\frac{E_r}{E_i}\right) = \frac{1}{\eta_2}\left(\frac{E_t}{E_i}\right)$$

$$\Rightarrow 1 + \Gamma_{\parallel} = \frac{\eta_1}{\eta_2}\tau_{\parallel}$$

Also,

$$E_i \cos\theta_i - E_r \cos\theta_i = E_t \cos\theta_t \longrightarrow (2)$$

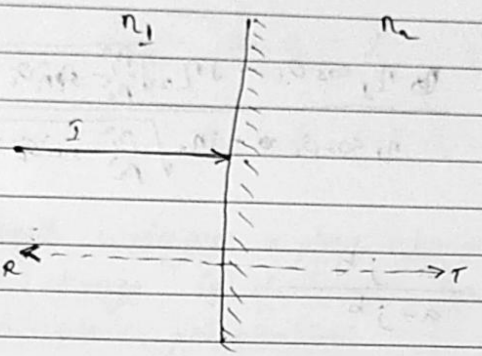
Then,

$$\Gamma_{\parallel} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$$\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

For

For normal incidence :



$$\theta_i = \theta_r = \theta_t = 0^\circ$$

$$\Gamma_1 = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$T_1 = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Total Internal Reflection (TIR) :

From Snell's Law,

$$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

$$\Rightarrow \sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_i$$

Note :- $\left(\frac{\beta_1}{\beta_2}\sin\theta_i\right)$ can be greater than 1 i.e $\left(\frac{\beta_1}{\beta_2}\sin\theta_i\right) > 1$

But $\sin\theta_t$ cannot be greater than 1 for any value

$\therefore$ In case of TIR, transmission is 0.

Also, $\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1 - \frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i}$

$$= j\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}$$

$$\Gamma_{\parallel} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

$$= \frac{\eta_1 \cos\theta_i - j\eta_2\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}{\eta_1 \cos\theta_i - j\eta_2\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}$$

$$= \frac{a - jb}{a + jb}$$

$$\therefore |\Gamma_{\parallel}| = 1 \quad \& \quad \angle = \left(-2\tan^{-1} b/a\right)$$

Date: 1/4/23

Total Internal Reflection:

$$\frac{\beta_1}{\beta_2}\sin\theta_i > 1$$

$$\Rightarrow \cancel{\cos\sqrt{\mu_1\varepsilon_1}} \quad \frac{\cancel{\omega}\sqrt{\mu_1\varepsilon_1}}{\cancel{\omega}\sqrt{\mu_2\varepsilon_2}} \times \sin\theta_i > 1$$

$$\Rightarrow \frac{\sqrt{\mu_1\varepsilon_1}}{\sqrt{\mu_2\varepsilon_2}}\sin\theta_i > 1$$

(i) For non-magnetic:

$$\mu_1 = \mu_2 \Rightarrow \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}\sin\theta_i > 1$$

$$\Rightarrow \frac{n_1}{n_2}\sin\theta_i > 1$$

$$\Rightarrow \frac{n_1}{n_2} > 1$$

$$\therefore n_1 > n_2$$

For critical angle :-

$$\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin\theta_c = 1$$

$$\Rightarrow \frac{n_1}{n_2}\sin\theta_c = 1 \Rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

(2) ~~Wavelength~~ undergoes a phase change at TIR & the phase change is different for parallel and perpendicular polarisation:

$$\phi_{\parallel} = -2\tan^{-1}\frac{n_2\sqrt{\frac{\beta_2^2}{\beta^2}\sin^2\theta_i - 1}}{n_1\cos\theta_i}$$

$$\phi_{\perp} = -2\tan^{-1}\frac{n_1\sqrt{\frac{\beta_2^2}{\beta^2}\sin^2\theta_i - 1}}{n_2\cos\theta_i}$$

(3) Fields in medium 2:

$$E_T = E_t e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$= E_t e^{-j\beta_2\left(x\sin\theta_t \pm jz\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}\right)}$$

$$= E_t e^{-j\beta_2 x\sin\theta_t \pm z\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}$$

$$\Rightarrow E_T = \underbrace{E_t e^{\pm z\sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}}_{\text{Magnitude}}\underbrace{e^{-j\beta_2 x\sin\theta_t}}_{\text{Phase}}$$

At $\theta_i = \theta_c$,

$$E_T = E_t\,(\text{phase term})$$

(4) Fields in medium 1:

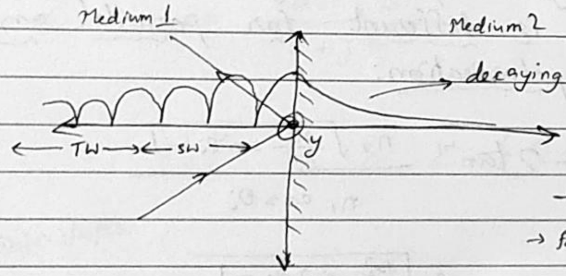
$$\vec{E_i} = E_i e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{E_r} = E_r e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} = E_i e^{j\phi} e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)}$$

$$\therefore \vec{E}_{mod\,1} = \vec{E_i} + \vec{E_r}$$

$$= \underbrace{E_i e^{-j\beta_1 x \sin\theta_i}}_{\text{Travelling wave in x-direction}} \underbrace{\left(e^{-j\beta_1 z\cos\theta_i} + e^{j\phi} e^{j\beta_1 z \cos\theta_i}\right)}_{\text{standing wave in z-dir}^n}$$

Total Field:



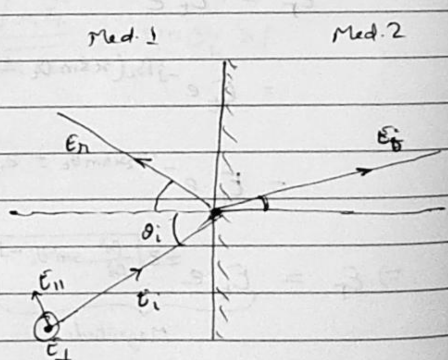
→ decaying field is also called evenescent field

→ close to boundary = standing wave

→ far away from boundary = travelling wave

Date:- 05/04/23

Polarisation at Media Interface:



$$E_i = E_{||} + E_\perp e^{j\phi}$$

$$E_r = E_{r_{||}} + E_{r_\perp}$$

$$= \Gamma_{||} E_{i_{||}} + \Gamma_\perp E_{i_\perp} e^{j\phi}$$

$$\Rightarrow E_t = E_{t_{||}} + E_\perp$$

$$= \tau_{||} E_{i_{||}} + \tau_\perp E_{i_\perp} e^{j\phi}$$

- Γ is real for normal reflection
- Γ is complex for TIR reflection.

Linear Polarisation :

$\phi = 0$

(a) Normal reflection $\longrightarrow$ $\Gamma'_\perp$, $\Gamma'_\parallel$ are real

$\Gamma'_\perp \neq \Gamma'_\parallel$ (Linear polarization)

(b) TIR $\longrightarrow$ $\Gamma'_\perp$, $\Gamma'_\parallel$ are complex

$|\Gamma'_\perp| = |\Gamma'_\parallel| = 1$ (Elliptical polarization)

For parallel polarization :

$$\Gamma'_\parallel = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$\therefore \Gamma'_\parallel = 0$ when $\eta_1 \cos\theta_i = \eta_2 \cos\theta_t$

Similarly, for $\Gamma'_\perp = 0$ when

$\eta_2 \cos\theta_i = \eta_1 \cos\theta_t$ } For perpendicular polarisation

$$\tan\theta_i = \frac{\beta_2}{\eta_1}\left\{\frac{\eta_2 - \eta_1}{\beta_2 - \beta_1}\right\}$$

$$\Rightarrow \tan\theta_{B_\perp} = \frac{\beta_2}{\eta_1}\left\{\frac{\eta_2 - \eta_1}{\beta_2 - \beta_1}\right\}^{-1/2} \longrightarrow (1) \; ; \; \theta_{B_\perp} \rightarrow \text{Brewster angle}$$

$$\rightarrow \tan\theta_{B_\parallel} = \cancel{\frac{\beta_2}{\beta_1}} \; \frac{\beta_2}{\eta_2}\left\{\frac{\eta_1 - \eta_2}{\beta_2 - \beta_1}\right\}^{1/2} \longrightarrow (2)$$

If $\Gamma'_\parallel = 0$, then the angle is the Brewster's Angle for parallel polarisation

$\eta_1 = \frac{1}{\sqrt{\epsilon_1}}$

If the $\Gamma_{\perp} = 0$, then the angle is the Brewster's Angle for perpendicular polarisation.

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \quad ; \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

$$\beta_1 = \omega\sqrt{\mu_1\varepsilon_1} \quad ; \quad \beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$\theta_{B\perp} = \tan^{-1}\left\{\sqrt{\frac{\mu_1}{\mu_2}}\left[\frac{\mu_2\varepsilon_1 - \mu_1\varepsilon_2}{\mu_2\varepsilon_2 - \mu_1\varepsilon_1}\right]\right\}^{1/2}$$

$$\& \ \theta_{B\parallel} = \tan^{-1}\left\{\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\left[\frac{\mu_1\varepsilon_2 - \mu_2\varepsilon_1}{\mu_2\varepsilon_2 - \mu_1\varepsilon_1}\right]\right\}^{1/2}$$

Under the condⁿ, $\mu_1 = \mu_2$

$\theta_{B\perp}$ is imaginary angle

$$\& \ \theta_{B\parallel} = \tan^{-1}\left[\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right]$$

$$= \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

$\because$ Refractive index, $n = \sqrt{\varepsilon}$