Date: Day 1

$x(n) = x(-n) \Rightarrow$ even signal

$x(n) = -x(-n) \Rightarrow$ odd signal

Cont. time signal

$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^T |x(t)|^2 dt$$

$$\text{power} = \frac{1}{T}\int_0^T |x(t)|^2 dt = \underbrace{\frac{1}{2N+1} \sum_{n=0}^{N} |x(n)|^2}_{\text{discrete time signal}}$$

Let $x(t) = \underbrace{2\sin(\omega_n t)}_{T_1} + \underbrace{3\cos(5\omega_n t)}_{T_2}$

For the signal to be periodic, $\frac{T_1}{T_2}$ should be rational

and time period of the composite signal will be $\frac{\text{LCM of } (T_1 \,\&\, T_2)}{\text{HCF of } (T_1 \,\&\, T_2)}$

Sin $\omega_n t$ is periodic with 2π i.e. ω_n

$\therefore$ period of $\sin \omega_n t$ is $\frac{\omega_n}{\omega_n} = 1$

Cos $5\omega_n t$ is periodic with 2π i.e. ω_n

$\therefore$ perid of $\cos 5\omega_n t$ is $\frac{\omega_n}{5\omega_n} = \frac{1}{5}$

$\therefore$ Lcm of T_1 & T_2 = 8, HCF = 1

$\therefore$ Time period of the composite signal is $\frac{8}{1} = 8$

4.5 box @

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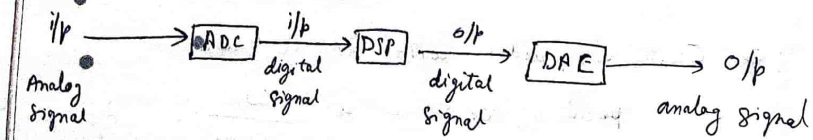
Date : ....

Q What is DSP?

ans Numerical processing of signal on a digital platform such as computer, mobile etc, is called digital signal processing

Q What are the disadvantages of ~~DSP?~~ Analog Signal Processing?

ans
(i) Limited Accuracy
(ii) Limited dynamic range
(iii) Limited repeatability due to temperature or ageing
(iv) Limited processing speed
(v) Lack of flexibility
(vi) Difficulty in implementing useful operation such as non-linear operations ~~in time~~ and time varying operations.
(vii) High cost.



Block diagram of DSP

Discrete

Convolution

Date : ....

Z- Transform :- Basic Tool

Convol^n of a signal $x(n)$ & $h(n)$ is given as

$$y(n) = x(n) * h(n)$$

$$= \sum_{K=-\infty}^{\infty} x(n)\, h(n-K)$$

K = arbitrary const

Step 1
n is replaced by K

$$\underbrace{y(n)}_{\text{Fixed}} = x(K) * \underbrace{h(K)}_{h(-K)}$$

Step 2 Time reversal of $h(K)$ to $h(-K)$

Step 3 Sliding over $x(K)$

$h(n-K)$
↓ multiplication
$x(K)\, h(n-K) \longrightarrow$ Addition

Linear & Circular ?

LC is used for aperiodic signal
CC " " " periodic "

Q Find the Conv. of $x(n)$

$$x(n) = \{\overset{n=0}{\underset{\uparrow}{1}}, \overset{1}{2}, \overset{2}{3}, \overset{3}{4}\}$$

$$h(n) = \{\overset{n=0}{\underset{\uparrow}{4}}, \overset{1}{4}, \overset{2}{3}, \overset{3}{2}\}$$

↑ = initialiser

No of samples N_1 for $x(n)$ $\quad n_1$
" " " N_2 " $h(n)$ $\quad n_2$ } Initial index Value

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☆ The length of the convoluted output is : (for $x(n)$ & $h(n)$)

$$\boxed{N_1 + N_2 - 1}$$

☆ The initial starting sequence of the convoluted output is

$$\boxed{n_1 + n_2}$$

A/Q

$n_1 = 0, \quad n_2 = 0$

$\therefore \ n_1 + n_2 = 0$

☆ The final sequence is given by:

$$\underset{0}{n_1} + \underset{0}{n_2} + \underset{4}{N_1} + \underset{4}{N_2} - 2$$

$= 6$

$$y(n) = \{ \underset{n=0}{4} \quad \underset{1}{12} \quad \underset{2}{23} \quad \underset{3}{36} \quad \underset{4}{18} \quad \underset{5}{8} \quad \underset{6}{} \}$$

Shortcut Method. multiply row & column.

$x(n)$ \ $h(n)$	4	4	3	2
1	4	4	3	2
2	8	8	6	4
	12	12	9	6
4	16	16	12	8

add them groupwise and write in the $y(n)$

Convolution is also called Convoluted Sum

Date : ....

Role of initializer

$$x(n) = \{ \underset{-1}{1} \quad \underset{n=0}{\underset{\uparrow}{2}} \quad \underset{1}{3} \quad \underset{2}{4} \}$$

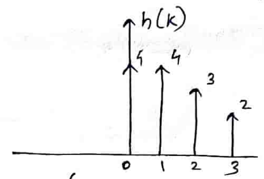
$$h(n) = \{ \underset{-2}{4} \quad \underset{-1}{4} \quad \underset{n=0}{\underset{\uparrow}{3}} \quad \underset{1}{2} \}$$

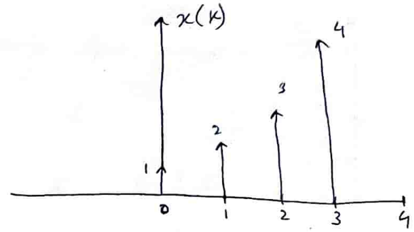
$\therefore \ n_1 + n_2 + N_1 + N_2 - 2 = 3$

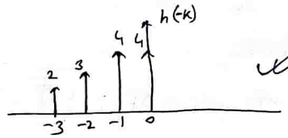
$$y(n) = \{ \underset{n=0}{4} \quad \underset{1}{12} \quad \underset{2}{23} \quad \underset{\underset{\uparrow}{3}}{36} \quad \underset{4}{29} \quad \underset{5}{18} \quad \underset{6}{8} \}$$

☆ Graphical Method

Dont use matrix method







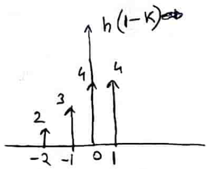
Shifting is done on this!!

$n - k = 0 \implies -k = -n$

No ustadi

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

when $n = 0$ $\quad y(0) = \sum_{k=-\infty}^{\infty} x(k) h(0-k) = 4$



$n = 1 \quad y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k) = 4 + 8 = 12$

when $n = 1$, the time shifting we perform on $h(-k)$

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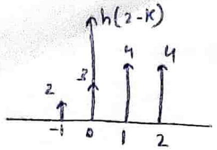
For $n=2$



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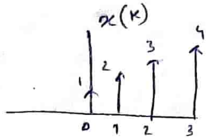


$= 3 + 8 + 12 = 23$

From Monday — Personality Development & Aptitude

Volunteer to browse through aptitude QP

10Q – 30min, 20min, 10min

Trick from monday –

IIT Madras – Personality Grooming Course

Next 15min CV writing — we should sit for achievement

Room Safa
Glass Safa
Go ask bearer

Date : ....

Find a Psycologist : – Psycometric Test – Ankur & Mukut* (Hold on) End of march

Read sampling from Simon Hykin and for interview – Auriphinilis. ✗ Concept & Question.

in Sampling & Aliasing - read mod ~~log~~ function.

DSP → 1 → Mitra

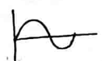
2 → Opena Sepher

3 → Proakis — 7300

Sharma / Nagur – Indian for problem solving.

Signal Causality

non causal
–ve have values



causal
only +ve values

System Causality
we know it
past present etc

We do not use non causal system as ~~i/p~~ o/p depend on future i/p.

Clear out stability — we get a measurable o/p.

Parameter of system in time domain are the constants.

like in convolution, $y[n] = \sum_{n=0}^{N-1} h[n]\, x[n-m]$

It gives the system ch in time domain.

Imp of TF = freq^n domain

Fourier Series = ?

Only applicable to Periodic signal — $E = \frac{Q}{t} P \times t$.

Fourier Transform :– applicable to aperiodic signal.

Laplace deals with Continuous time ~~&~~ signal

Z - Trans . ,, ,, discrete ,, ,,

System Stability cannot be analysed with impulse f^n

For eg:- $h[n] = [\underset{\uparrow}{-1}\ 2\ 1]$

BIBO stability & Parseval Theorem.

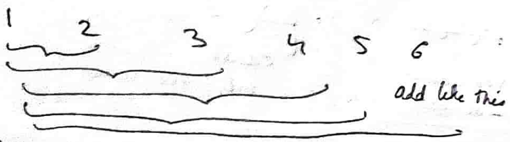
Add the elements in the [ ] and take | |

It should be $< \infty$

Constraint :- Input should be bounded.

Recursive & Non recursive system

Say ~~Accumulator~~ to add sum of n numbers



The Accumulator keeps the result of previous two ~~result~~ operation Then we add the next

System eqn $y[5] = y[4] + x[5]$

Generalised exp: $y[n] = y[n-1] + x[n]$ — recursive

Non recursive method (not optimised)

$$y[n] = \sum_{m=0}^{n-1} x[m]$$

Finite Impulse Response (FIR) → recursive & IR → non recursive

Do not consider the previous inputs

Date: 13/02/23

Properties of Linear Convolution

i) Commutative

ii) Associative

iii) Distributive

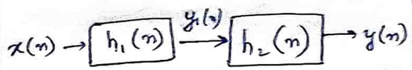
i) Com $x_1(n) * x_2(n) = x_2(n) * x_1(n)$

ii) Asso $[x_1(n) * x_2(n)] * x_3(n) = x_1(n) * [x_2(n) * x_3(n)]$

iii) Distri:- $x_1(n) * [x_2(n) + x_3(n)] = x_1(n) * x_2(n) + x_1(n) * x_3(n)$

Interconnection

Cascade



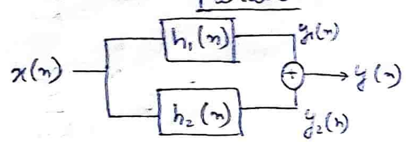
⇓ $y_1(n) = x(n) * h(n)$

$y(n) = [x(n) * h(n)] * h_2(n)$

$= x(n) * \underbrace{[h_1(n) * h_2(n)]}_{h(n)}$

($h(n)$ is the impulse response)

Parallel



$y(n) = y_1(n) + y_2(n)$

$= x(n) * h(n) + x(n) * h_2(n)$

$= x(n) * \underbrace{[h_1(n) + h_2(n)]}_{h(n)}$

Q) Determine the impulse response for cascade of two LTI systems

$h_1(n) = \left(\frac{1}{2}\right)^n u(n) \quad h_2(n) = \left(\frac{1}{4}\right)^n u(n) \quad \} \ n \geq 0$

Ans ① Determination of given signal

Both the given signal $h_1(n)$ & $h_2(n)$ exist for $n \geq 0$

Find $h(n)$ (Impulse response)

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Date : ....

$h(n) = h_1(n) * h_2(n)$ [Cascade]

$= \sum_{k=0}^{n} h_1(k)\, h_2(n-k)$ (Conv^n sum formula)

Replace 'n' in both the equations by K & (n-K) respectively.

$\therefore \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} \left(\frac{1}{4}\right)^{(n-k)}$

$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} \underbrace{\left(\frac{1}{4}\right)^{n}}_{\text{constant}} \left(\frac{1}{4}\right)^{-k}$

Finite Geometric Series

$\sum_{n=0}^{N} C^n = \frac{C^{N+1}-1}{C-1}$

$= \left(\frac{1}{4}\right)^{n} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} \left(\frac{1}{4}\right)^{-k}$

$= \left(\frac{1}{4}\right)^{n} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{k} 4^{k}$

$= \left(\frac{1}{4}\right)^{n} \sum_{k=0}^{n} 2^{k}$

Using finite geometric series sum formula

$= \left(\frac{1}{4}\right)^{n} \frac{2^{n+1}-1}{2-1}$

$= \left(\frac{1}{4}\right)^{n} \left(2^{n+1}-1\right) u(n)$

Date : ....

Circular Convolution :-

The CC of two periodic discrete time sequences $x_1(n)$ & $x_2(n)$ with periodicity of N samples is defined as

$x_3(n) = \sum_{m=0}^{N-1} x_1(m) \cdot x_2((n-m))_N$

or $\sum_{m=0}^{N-1} x_2(m) \cdot x_1((n-m))_N$

To perform circular Conv, any of the two sequences $x_1(n)$ or $x_2(n)$ needs to be periodic. Shifting is done in periodic signal only.

(out of $x_1(n)$ or $x_2(n)$ any of the two is periodic)

The CC of finite duration sequences can be performed only if both the sequences consists of same no of samples.

If the sequences have diff. no. of samples, then convert the smaller size seq^n to the length of the longer size seq^n by appending zeros (padding)

20- Pass

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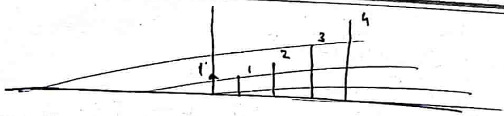
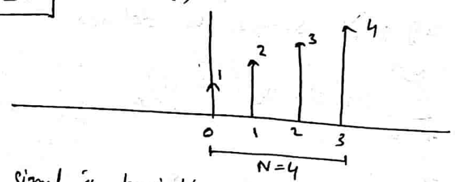


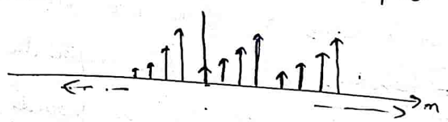
20/Feb/22

$x(n)$



If this signal is periodic, how will we ~~do~~ check?

After $n=4$, the same signal will repeat (also before 0)



☆ Periodic signals are of infinite length. ($-\infty$ to $+\infty$)

Q Let $x_1(m) = \{2, 1, 2, 1\}$

$x_2(m) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Perform circular convolution.

ans Method 1 :- Matrix Method

Fix $x_1(m)$ and rotate x_2 (shifting)

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 1 & 3 \\ 2 & 4 \end{bmatrix} \quad \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix} \quad \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix}$$

1+

N.B:- To check whether you have done it correctly or not, see the element of 1st row 1st colm should be equal to the element of last row last column

Date :

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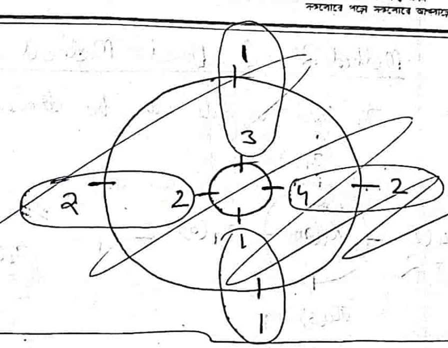
Put $n=3$

$3\times1 + 4\times2 + 1\times1 + 2\times2$

$= 14$



ans:- Method 1 :- Matrix Method (Don't do in exam)

Make two matrices of $x_1(n)$ and $x_2(n)$ such that one is fixed and the other is kept shifting. Let us keep $x_1(n)$ fixed.

If $x_1(n) = \{x_1(0), x_1(1), x_1(2), x_1(3)\}$ — Fix

and $x_2(n) = \{x_2(0), x_2(1), x_2(2), x_2(3)\}$ — Rotating.

Then, the matrices are

$$\underbrace{\begin{bmatrix} x_2(0) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(0) \end{bmatrix}}_{4\times4} \times \underbrace{\begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix}}_{4\times1}$$

Matrix multiply and we will get the result.

A/Q $x_1(0) = 2$, $x_1(1) = 1$, $x_1(2) = 2$, $x_1(3) = 1$

$x_2(0) = 1$, $x_2(1) = 2$, $x_2(2) = 3$, $x_2(3) = 4$

∴ Convoluted result by matrix multiplication method is given by:

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

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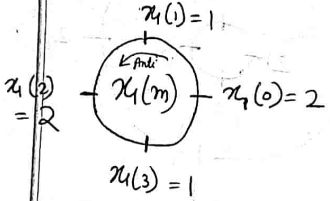
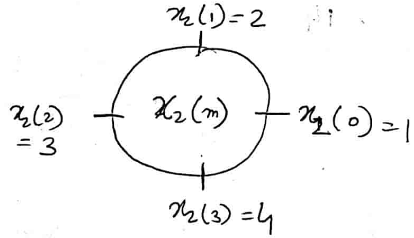
Method 2:- Graphical Method:- (Do in exam)

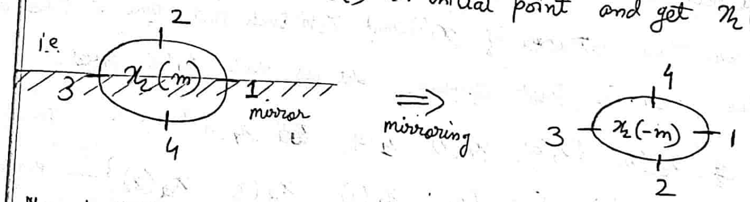
The two signals can be drawn as follows'





Rotate $x_2(m)$ about $x_2(0)$ i.e. initial point and get $x_2(-m)$

i.e



Now, to determine circular convoluted o/p, $x_3(m)$, we will use

$$x_3(m) = \sum_{n=0}^{N-1} x_1(m)\, x_2(n-m)\Big|_N$$

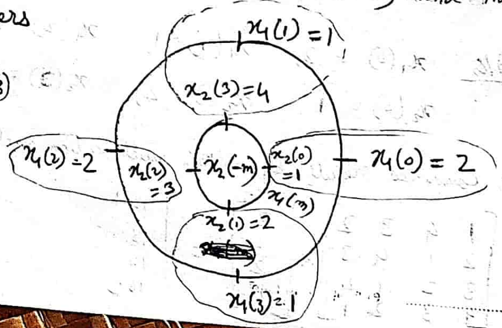
Here $N=4$, $\therefore N-1=3$. $\therefore$ we will go from 0 to 3

i.e. rotate 3 times.

For $n=0$, Superimpose $x_1(m)$ over $x_2(-m)$ and multiply the adjecent numbers

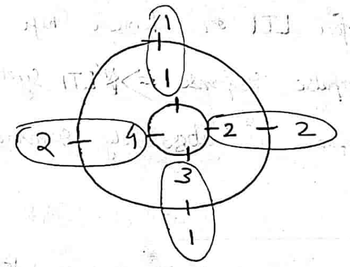
$(1\times4)+(2\times1)+(1\times2)+(2\times3)$

$= 14$



For $n=1$, x_2 will take a form of $x_2(n-m+1)$

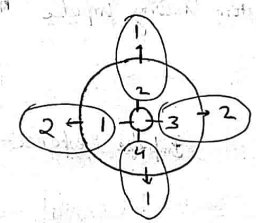
So, rotate the 2nd sequence i.e. x_2 in anticlockwise directi



$= (1\times1)+(2\times2)+(3\times1)+(2\times4)$

$= 16$

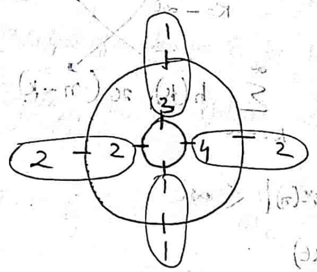
For $n=2$



$= (1\times2)+(2\times3)+(1\times4)+(1\times2)$

$= 14$

For $n=3$



$= (1\times3)+(2\times4)+(1\times1)+(2\times2)$

$= 16$

outer circle is fixed. Inner circle rotates anticlockwise

Date : ....

Stability of a System

A discrete time system is stable is bounded input produces bounded output. "Bounded" means: ~~bounded~~ BIBO

Stability criteria for LTI or Linear Shift invarient (LSI) in terms of unit impulse response $\Rightarrow$ ~~A~~ LTI system is stable if if its impulse response is absolutely summable — Derive (3m)

Proof :-

Let us consider an LTI system having impulse ~~ht~~ response $h(n)$

$$Y(s) = H(s)\,X(s)$$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} \qquad \text{Impulse response} = ILT \text{ of } H(s) = h(t)$$

Let $x(n)$ be the i/p applied to the system.

As per convolution, $$y(n) = \sum_{K=-\infty}^{\infty} x(K)\,h(n-K) \quad \text{——} (1)$$

we can interchange $$\Rightarrow y(n) = \sum_{K=-\infty}^{\infty} h(K)\,x(n-K) \quad \text{——} (2)$$

$\because x(n)$ is bounded, then $|x(n)| < \infty$

$\therefore |x(n)| \leq M_x$ (ut)

$\Rightarrow |x(n)| \leq M_x < \infty$

Date : ....

$$(2) \Rightarrow |y(n)| = \left|\sum_{K=-\infty}^{\infty} h(K)\,x(n-K)\right| \quad \text{——} (3)$$

$$\therefore \left|\sum_{K=-\infty}^{\infty} h(K)\,x(n-K)\right| \leq \sum_{K=-\infty}^{\infty} |h(K)|\,|x(n-K)| \quad \text{——} (4)$$

Always absolute value of sum of terms is less than or equal to the sum of absolute value of terms.

From eq (3) & (4), we can write

$$|y(n)| \leq \sum_{n=-\infty}^{\infty} |h(K)|\,|x(n-K)| \quad \text{——} (5)$$

Since $|x(n)| \leq M_x$ (M_x = constant)

$\Rightarrow |x(n-K)| \leq M_x$

$$(5) \Rightarrow |y(n)| \leq \sum_{K=-\infty}^{\infty} |h(n)|\,M_x$$

$$\Rightarrow |y(n)| \leq M_x \sum_{K=-\infty}^{\infty} |h(K)|$$

Since, the output of a system to be bounded, $\therefore |y(n)| < \infty$.

This implies, $M_x \sum_{K=-\infty}^{\infty} |h(K)| < \infty$

Replacing 'K' by 'n', $M_x \sum_{K=-\infty}^{\infty} |h(n)| < \infty$

or $\int_{-\infty}^{\infty} h(t)\,dt < \infty$ for time domain

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Say $y(t) = \sin \omega t \rightarrow$ Its bounded ~~as~~ from -1 to 1

Tan is not

Date: Causality of LTI System

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A discrete time system is said to be Causal.

if output of the system depends only on present and the past value of the input and not on the future

Causal — Present + Past

Non causal — Present + Past + Future

Anti causal — Future.

☆ LTI System is causal if its impulse response $= 0$ for $n < 0$

$$\therefore \boxed{h(n) = 0 \ ; \ n < 0}$$

A/ Linear convolution. Let, $x(n) = i/p$, $y(n) = o/p$, $h(n) =$ imp resp.

$$y(n) = \sum_{n=-\infty}^{\infty} x(k)\, h(n-k)$$

$$\Rightarrow y(n) = \sum_{n=-\infty}^{\infty} h(k)\, x(n-k)$$

$$\Rightarrow y(n) = \sum_{K=0}^{\infty} h(k)\, x(n-k) + \sum_{K=-\infty}^{-1} h(k)\, x(n-k)$$

~~[h(0) x(0-k) + h(1) x(1-k) + h(2) x(2-k) ...]~~
~~+ [h(-1) x(~~

$$[h(0)\, x(n) + h(1)\, x(1-k) + h(2)\, x(2-k) \ldots$$
$$+ h(-1)\, x(n+1) + \cancel{h(-2)\, x(-2-k)}\ h(-2)\, x(n+2) \ldots]$$

(Put $K = 0, -1$)

Date:

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If we consider $n=0$ (present input) then $x(n-1)$, $x(n-2)$.... are delayed i/p

If we consider $n=-1$ (past i/p), then $x(n+1)$, $x(n+2)$ .... are advanced i/p

But $x(n+1)$, $x(n+2)$ can't be used (as they are future i/p)

$$\therefore y(n) = \underbrace{\sum_{K=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{K=-\infty}^{\infty} h(k)\, x(n-k)}$$

$$\Rightarrow \sum_{K=0}^{\infty} x(k)\, h(n-k) = \sum_{K=0}^{\infty} h(k)\, x(n-k)$$

We are not allowed to consider $n < 0$ as we are considering only ~~future~~ present i/p $\rightarrow$ Causality criterion

$\boxed{h(n) = 0 \text{ if } n < 0}$ will be required in FIR & IIR

Teacher's Signature ....

Finite Impulse Response (FIR)

Date: 23/02/22

We know that, $y(n) = \sum_{K=-M}^{M} h(n)\,x(n-K) \longrightarrow$ Non Causal

$$\therefore y(n) = \sum_{K=0}^{M} h(K)\,x(n-K) \longrightarrow \text{causal}$$

expanding $\Rightarrow y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \dots + h(M)x(n-M)$ (M is a finite number)

—ve value of K will give us the future i/p

M is a memory element. For 8 bit K range from 0 to 7

It has a finite storage capacity.

Used in FIR filters.

FIR has finite memory requirement

IIR ~~Infinite Impulse Response~~

$$y(n) = \sum_{K=-\infty}^{\infty} h(k)\,x(n-k) \longrightarrow \text{Non causal}$$

$$y(n) = \sum_{K=0}^{\infty} h(k)\,x(n-k) \text{ —— causal}$$

$$\Rightarrow y(n) = \underbrace{h(0)\,x(n)}_{\text{Present i/p}} + h(1)\underbrace{x(n-1)}_{\text{past i/p}} + \dots + h(\infty)\,x(n-\infty)$$

IIR has infinite memory requirement. (drawback)

can't be realised in practise.

Both take in account – Causality

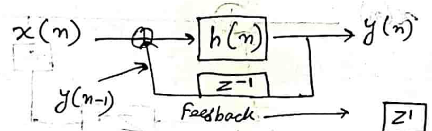
Date:

Depending upon the output of the system, the discrete time domain system can be classified as non Recursive & Recursive system

Non Recursive — Without feedback

Recursive — With feedback

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$



Z^{1} or Z^{-1}

Advance Delay

$$\therefore \underbrace{y(n)}_{\text{present o/p}} = \underbrace{x(n)}_{\text{i/p}} + \underbrace{y(n-1)}_{\text{previous value of o/p}}$$

Non recursive

This System does not require any ~~past~~ past o/p sample to calculate the present o/p

Let us consider a causal LTI system as:-

$$y(n) = \sum_{K=0}^{M} x(K)\,h(n-K) \quad \times$$

$$\text{or } y(n) = \sum_{K=0}^{M} h(K)\,x(n-K) \quad \checkmark$$

In DSP
Input is varied
System is fixed

Property of convolution
→ we can interchange x and h

We need $x(n)$ for causality.

$$= h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \dots + h(M)x(n-M)$$

All non recursive systems are FIR

Teacher's Signature

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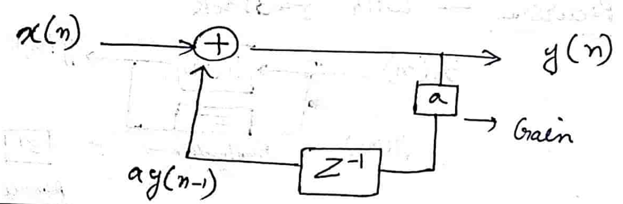
Recursive System

A discrete time system in which the o/p $y(n)$ depends on present, past ~~as well as~~ i/p as well as previous outputs.



$$\boxed{y(n) = x(n) + ay(n-1)}$$

$n=0 \Rightarrow \underbrace{y(0)}_{\text{present o/p}} = \underbrace{x(0)}_{\text{present i/p}} \cancel{+ a(y)} \underbrace{ay(-1)}_{\text{past o/p}}$

Z-Transform

Date : ....

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$x(n) \rightarrow$ Discrete time signal

$\downarrow$ Z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

Formula :-

① $\delta(n) \xrightarrow{ZT} 1$

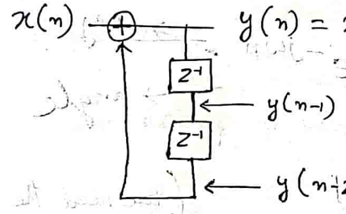
② $\delta(n-1) \xrightarrow{ZT} z^{-1}$

③ $\delta(n-2) \xrightarrow{ZT} z^{-2}$

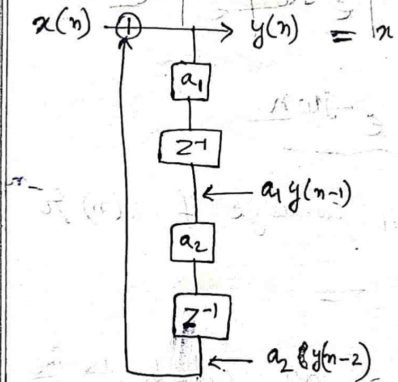
④ $\delta(n+1) \xrightarrow{ZT} z^{1}$

⑤ $u(n) \xrightarrow{ZT} \frac{z}{z-1} = \frac{1}{1-\frac{1}{z}}$

⑥ $a^n u(n) \xrightarrow{ZT} \frac{z}{z-a} = \frac{1}{1-\frac{a}{z}}$



$y(n) = x(n) + y(n-1) + y(n-2)$



$y(n) = x(n) + a_1 y(n-1) + a_2 y(n-2)$

This is a difference equation.

$\frac{d^2x(t)}{dt^2} + 2\frac{dx(t)}{dt} + 3$ (for eg)

is a differential equation.

Teacher's Signatura ....

$$X[Z] = \sum_{n=-\infty}^{\infty} x(n)\, Z^{-n}$$

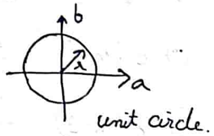
Date : $n=-\infty$ to -1 → Left signal signal ; $n = 0$ to ∞ → Right sided signal

Region of Convergence

We Know, $X[Z] = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$

$$= \sum_{n=-\infty}^{\infty} x(n)\,(re^{j\omega})^{-n}$$

where, $Z = re^{j\omega} = a + ib$



$$\boxed{\begin{array}{l} \text{ROC for} \\ \text{Z transform} \rightarrow \text{circle} \\ \text{Laplace} \;\;\text{"} \rightarrow \text{plane} \end{array}}$$

$$\sum_{n=-\infty}^{\infty} \{x(n)\, r^{-n}\}\, e^{-j\omega n} \;\; \begin{array}{l} \nearrow \text{mag} \\ \rightarrow \text{angle} \end{array}$$

when $r = 1$, then $|z| = 1$

$$|z| = r|e^{j\omega}| = |e^{j\omega}| = 1$$ (we need the magnitude of Z transform to be 1)

Hence $\underbrace{\sum_{n=-\infty} \{x(n)\, r^{-n}\}\, e^{-j\omega n}}$

This particular term will converge if $x(n)\, r^{-n}$ is absolutely summable.

$\therefore \sum_{n=-\infty} x(n)\, r^{-n} < \infty$ which is the condition of existance of "Z-transform"

Infinite Series formula

$$\sum_{n=0}^{\infty} c^n = \frac{1}{1-c}$$

Date :

Q Determine the Z-transform and ROC for the following signal

$x(n) = a^n$ for $n \geq 0$

$= 0$ for $n < 0$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} = \sum_{n=0}^{\infty} x(n)\, z^{-n} = \sum_{n=0}^{\infty} a^n z^{-n}$$

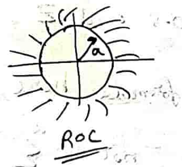
$$= \underbrace{\sum_{n=0}^{\infty} (az^{-1})^n} = \frac{1}{1 - az^{-1}}$$

Convert to infinite series expression "c^n"

$$= \frac{1}{1 - \frac{a}{z}} = \frac{z}{z-a}$$ Ans

To calculate the ROC, always look for $\frac{a}{z}$ =

For ROC to be valid, $\frac{1}{1-\frac{a}{z}} \neq \infty$



$\therefore z \neq 0$ or $\frac{z}{z-a} \neq \infty$

or $\left|\frac{a}{z}\right| < 1 \Rightarrow |z| > a$

We can't take "less than" condition too as $n \geq 0$, $0; n < 0$

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Q $x(n) = 0.8^n \, u(-n-1)$. Determine the Z transform & ROC

It is a left sided signal

$-n-1 = 0$

$\Rightarrow n = -1$

∴ The range is $-\infty$ to -1

∴ $X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n} = \sum_{n=-\infty}^{-1} x(n) z^{-n}$

$= \sum_{n=-\infty}^{-1} 0.8^n Z^{-n}$

$= \sum_{n=1}^{\infty} 0.8^{-n} Z^{n}$!!

To interchange the limit of summation, we need to change the signs of the powers of 0.8 and Z

Imp: We can't apply infinite sum series formula here, as the sum range is $\sum_{n=1}^{\infty}$ and not $\sum_{n=0}^{\infty}$

∴ Change!

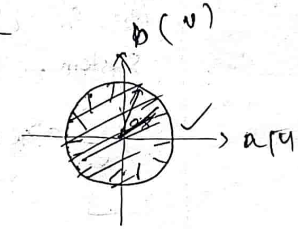
$\sum_{n=0}^{\infty} 0.8^{-n} Z^{n} - 0.8^{0} Z^{-0}$

$= \sum_{n=0}^{\infty} 0.8^{-n} Z^{n} - 1$

$= \sum (0.8^{-1} Z)^{n} - 1$

$= \frac{1}{1-0.8^{-1}Z} - 1$

$\frac{1}{1-\frac{Z}{0.8}} = \frac{0.8}{0.8-Z} \longrightarrow$ Ans



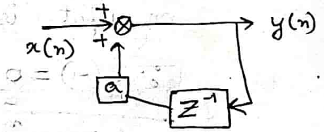
$0.8 - Z = 0 \Rightarrow Z = 0.8$

$|Z| < 0.8$

23/Feb LTI system characterised by constant coefficient difference Eq^n

Let us ~~cond~~ consider a recursive system

$y(n) = x(n) + a y(n-1)$ —①



for $n=0 \Rightarrow y(n) = x(0) + a y(-1)$ —②

for $n=1 \Rightarrow y(1) = x(1) + a y(0)$

$= x(1) + a[x(0) + a y(-1)]$

$= x(1) + a x(0) + a^2 y(-1)$

∴ $y(1) = a^2 y(-1) + a x(0) + x(1)$ —③

for $n=2 \Rightarrow y(2) = a^3 y(-1) + a^2 x(0) + a x(1) + x(2)$ —④

In general,

$y(n) = a^{n+1} y(-1) + a^n x(0) + a^{n-1} x(1) + \dots + a x(n-1) + x(n)$

$$\boxed{\Rightarrow y(n) = a^{n+1} y(-1) + \sum_{k=0}^{n} a^k x(n-k)}$$

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Date: ....

This o/p $y(n)$ is defined for $n \geq 0$ i.e. for causal system. $y(-1)$ is the initial condition of the system. (feedback)

$\therefore \sum_{k=0}^{n} a^k x(n-k)$ is the response of the system to the input $x(n)$

Zero state response or Forced response

If $y(-1)=0$ denoted by $y_{zs}(n)$

$\therefore$ The system is forced to the input i.e. completely depends on input is called forced response.

$\therefore y(-1)=0$ it is also called as Relaxed System.

$$\boxed{y(n) = \sum_{k=0}^{m} a^k x(n-k)}$$

Zero input response or Natural Response

Input is not applied to the system & initial condition is not equals to zero. Thus, this kind of o/p without applying the i/p $x(n)$ is called ZIR or NR

Since, $\underbrace{|y(-n)|}_{\text{generalised initial condition}} \neq 0$ is called as non relaxed initial condition.

$$\boxed{\therefore y(n) = y_{zi}(n) = a^{n+1} y(-1)}$$

$\therefore$ overall response of the system is

$$y(n) = y_{zi}(n) + y_{zs}(n)$$

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$$\boxed{\Rightarrow y(n) = -\underbrace{\sum_{k=1}^{N} a_k y(n-k)}_{\text{initial response}} + \underbrace{\sum_{k=0}^{M} b_k x(n-k)}_{\text{zero state response}}}$$ Memorise

Linear Constant coefficient difference eq^n

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

where a_k & b_k are coefficients & N is the order of the system (Numbers ~~Laplace~~ of past ~~first~~ o/p, present i/p and M is the no. of past i/p where $N<M$ $M \leq N$

(*) The constant or coefficients are inserted for o/p signals because o/p signals are feedback from o/p to i/p. The constant are inserted for i/p signals because i/p signals are fed forward from i/p to o/p

The general diff eq^n governing first order (difference) ~~first or~~ discrete time LTI system is ~~y(n)~~

$$y(n) = a_1 y(n-1) + b_0 x(n) + b_1 x(n-1).$$ sol^n of this expression can be obtained by homogeneous complementary or by implementing particular sol^n

Q) Construct the block diagram & SFG of the discrete time system whose i/p & o/p relationships where i/p & o/p are described by the difference eq^n.

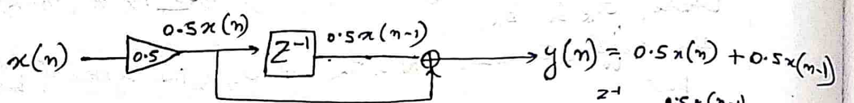
Teacher's Signature ....

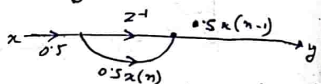
Date : ....

materials required :-

i/p, o/p, adder, multiplier, Delay element, Advanced elmt

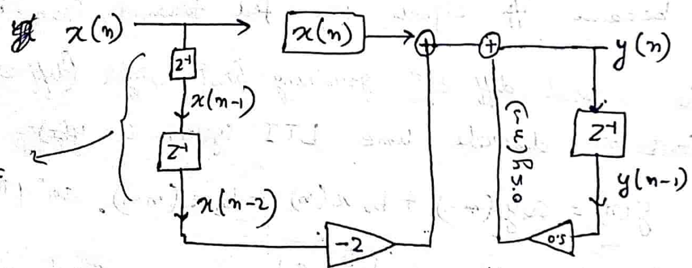
ⓐ $y(n) = 0.5x(n) + 0.5x\underbrace{(n-1)}_{\text{delay}}$ $\boxed{x(n+1) = \text{Advanced}}$



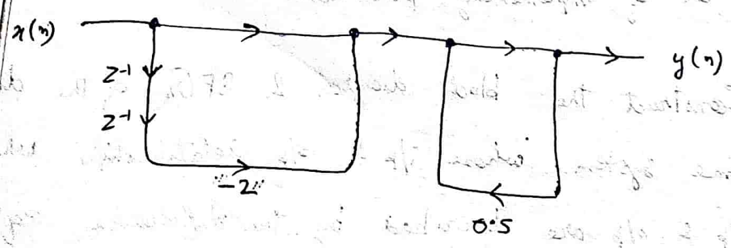


Always introduce a delay ~~block~~ element when there is a term $(n-k)$
and " " advance element " " " " $(n+k)$

ⓑ $y(n) = 0.5y(n-1) + x(n) - 2x(n-2)$



Don't put $\boxed{Z^{-2}}$



Date : ....

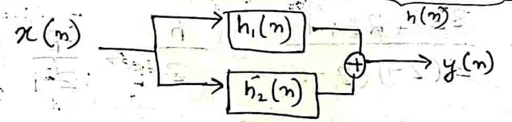
Interconnection of discrete time system

① Cascaded Connection

ⓐ $x(n) \longrightarrow \boxed{h_1(n)} \xrightarrow{y_1(n)} \boxed{h_2(n)} \longrightarrow y(n)$

$$y(n) = x(n) * \underbrace{[h_1(n) * h_2(n)]}_{h(n)}$$

ⓑ



$$y(n) = x(n) * \underbrace{[h_1(n) + h_2(n)]}_{h(n)}$$

To find the system response, Find h(n).

Then $h(n) = \dfrac{y(n)}{x(n)}$

$$H(z) = \frac{Y(z)}{X(z)}$$

Q Determine inverse Z transform for $X(z) = \dfrac{3 + 2z^{-1} + z^{-2}}{1 - 3z^{-1} + 2z^{-2}}$

(Find $\xrightarrow[\text{ZT}]{\text{Inv}}$ x(n))

-2 is the highest power here.

$$X(z) = \frac{\cancel{z^{-2}}\{3z^2 + 2z + 1\}}{\cancel{z^{-2}}\{z^2 - 3z + 2\}}$$

$$= \frac{3z^2 + 2z + 1}{z^2 - 3z + 2}$$

$$x(z) = \frac{3z^2 + 2z + 1}{(z-1)(z-2)}$$

$$\frac{x(z)}{z} = \frac{3z^2 + 2z + 1}{z(z-1)(z-2)}$$

$$\therefore \frac{x(z)}{z} = \frac{3z^2+2z+1}{z(z-1)(z-2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2}$$

$$3z^2+2z+1 = A(z-1)(z-2) + B(z)(z-2) + C(z)(z-1)$$

when $z=0$

$$1 = A(-1)(-2)$$

$$A = \frac{1}{2}$$

when $z=1$

$$B(1)(1-2) = 3\times1^2 + 2\times1 + 1$$

$$\Rightarrow -B = 6$$

$$\Rightarrow B = -6$$

when $z=2$

$$C(2)(2-1) = 3\times2^2 + 2\times2 + 1$$

$$\Rightarrow 2C = 17$$

$$\Rightarrow C = \frac{17}{2} = 8.5$$

$$\frac{X(z)}{z} = \frac{1/2}{z} + \frac{-6}{z-1} + \frac{8.5}{z-2}$$

$$\Rightarrow X(z) = \frac{1}{2} - 6\frac{z}{z-1} + 8.5\frac{z}{z-2}$$

Properties

$$Z\{\delta(n)\} = 1$$

$$Z\{u(n)\} = \frac{z}{z-1}$$

$$Z\{a^n u(n)\} = \frac{z}{z-a}$$

Taking inverse z transform

$$x(n) = \frac{1}{2}\delta(n) - 6u(n) + 8.5[2^n u(n)]$$

LTI is always causal

$\therefore$ Take $n \geq 0$

If $n=0$, $x(n) = \frac{1}{2} - 6 + 8.5 = 3$

$2^0 = 1$, $u(n) = 1$, $\delta(n) = 1$

If $n=1$

$$x(n) = 0 - 6 + 8.5\times2 = 11$$

$\delta(n) = 0$, $u(n) = 1$, $2^1 = 1$

If $n=2$

$$x(n) = 0 - 6 + 8.5\times4 = 28$$

$$\therefore x(n) = \{\underset{\uparrow}{3}, 11, 28, \ldots\}$$

Memorise Z transform formulae

Date : ....

Transfer f^n of LTI discrete time system

Imp

Let $x(n)$ and $y(n)$ be the input and o/p of discrete time LTI system. We know $y(n) = -\sum_{m=1}^{N} a_m\, y(n-m) + \sum_{m=0}^{M} b_m\, x(n-m)$ — ①

Considering z transform for eq ① with zero initial condition i.e.

$y(n) = 0$ for $n < 0$ and

$x(n) = 0$ for $n < 0$

Z transform

$$\therefore Z\{y(n)\} = Z\left\{-\sum_{m=1}^{N} a_m y(n-m) + \sum_{m=0}^{M} b_m x(n-m)\right\}$$

$$\Rightarrow Y(z) = Z\left\{-\sum_{m=1}^{N} a_m y(n-m)\right\} + Z\left\{\sum_{m=0}^{M} b_m x(n-m)\right\}$$

$$\Rightarrow Y(z) = -\sum_{m=1}^{N} a_m Z\{y(n-m)\} + \sum_{m=0}^{M} b_m Z\{x(n-m)\}$$

$$\Rightarrow Y(z) = -\sum_{m=1}^{N} a_m z^{-m} Y(z) + \sum_{m=0}^{M} b_m z^{-m} X(z)$$

$$\Rightarrow Y(z)\left[1 + \sum_{m=1}^{N} a_m z^{-m}\right] = \sum_{m=0}^{M} b_m z^{-m} X(z)$$

$$\boxed{\frac{Y(z)}{X(z)} = \frac{\sum_{m=0}^{M} b_m z^{-m}}{1 + \sum_{m=1}^{N} a_m z^{-m}}}$$

Looks like CS but in discrete domain.

Limit { – Feedback starts from 0
i/p starts from 1

Date : ....

On expanding

$$\frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots b_m z^{-m}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots a_N z^{-N}}$$

$$\therefore \frac{Y(z)}{X(z)} = G \frac{(z-z_1)(z-z_2)\dots(z-z_m)}{(z-p_1)(z-p_2)\dots(z-p_N)}$$

zeroes

Poles.

Impulse Response and Transfer Function

$x(n)$, $y(n)$, $h(n)$, → we need these.

We know: $y(n) = x(n) * h(n)$

$y(n) = \sum_{m=-\infty}^{\infty} x(m)\, h(n-m)$

$\Rightarrow Z\{y(n)\} = Z\left\{\sum_{m=-\infty}^{\infty} x(m)\, h(n-m)\right\}$

$\Rightarrow Y(z) = X(z)\, H(z)$

$H(z) = \frac{Y(z)}{X(z)} = TF$

If $Z\{x(m)\} = X(z)$ and $Z(h(n)) = H(z)$ then, $Z\{x(n) * h(n)\} = X(z)\, H(z)$

Convolution property

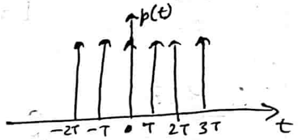
$\frac{o/p}{i/p}$ = profit/loss

Date: 6/03/22

Relationship betn Laplace Transform and Z-transform.

Consider a periodic impulse ~~string~~ train P_t with time period T as shown in fig below

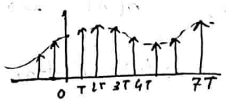
$$P(t) = \sum_{n=-\infty}^{\infty} \delta(t-n\delta)$$



When a continuous time signal $x(t)$ is multiplied by the impulse train P_t the product signal will have impulses

As per the statement, we are multiply  $\times \; p(t)$

The result will be



(Sampling $t \to nT$)

Here $x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$ ——— ①

Taking Laplace Transform of eq ①

$$L\{x_p(t)\} = X_p(s) = L\left\{\sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)\right\}$$

$$= \sum_{n=\infty}^{\infty} x(nT)\; L\{\delta(t-n)\}$$

$$= \sum_{n=-\infty}^{\infty} x(nT)\, e^{-nT_s} \cdot 1$$

1 → Lap transform of $\delta(t)$

$$\boxed{X_p(s) = \sum_{n=-\infty}^{\infty} x(nT)\,(e^{sT})^{-n}}$$

If $L\{x(t)\} = X(s)$, then time shifting prop $L\{x(t-a)\}=$

Z transform was

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

Date: ....

★★ Considering $e^{st} = z$ → Lawaris Statement

$$\boxed{X_p(s) = \sum_{n=-\infty}^{\infty} x(nT)\, z^{-n} = X(z)}$$

of a discrete time signal $x(nt)$ ~~$x(nt)$~~ is a sampled version of $x(t)$

The Z Transform of discrete time can be obtained from Laplace Transform of sampled version of $x(t)$ by choosing $e^{st} = z$ known as impulse invarient transformation.

30 marks

The relationship betn freqn of continuous time & discrete time signal

$\Omega \to$ Ohmega
ω = chota omega

Let The freqn of continuous time signal in rad/signal $\to \Omega$

and The " " discrete " " " $\to \omega$

Let $z = re^{j\omega}$ be a point on the z-plane and $s = \sigma + j\Omega$

Comparing, $z = e^{st}$

$$\Rightarrow re^{j\omega} = e^{sT}$$

$$\Rightarrow re^{j\omega} = e^{(\sigma + j\Omega)T}$$

$$\Rightarrow re^{j\omega} = e^{\sigma T} \cdot e^{j\Omega T}$$

$$\boxed{\omega = \Omega T}$$ → Important during filter design

Freqn in DTS is somewhat discretised too.

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Q. Determine the impulse response of the system described by 2nd order diff eqⁿ: $y(n)-4y(n-1)+4y(n-2)=x(n-1)$

3-4 marks

ans: Steps: Determine TF = $H(z)=\dfrac{Y(z)}{X(z)}$

Take IZT $h(n)$

Taking ZT of eqⁿ

$$Y(z)-4z^{-1}Y(z)+4z^{-2}Y(z)=z^{-1}X(z)$$

$$Y(z)\{1-4z^{-1}+4z^{-2}\}=z^{-1}X(z)$$

$$\frac{Y(z)}{X(z)}=\frac{z^{-1}}{1-4z^{-1}+4z^{-2}}$$

$$=\frac{}{z^{2}\{\frac{1}{z^{-2}}}$$

$$=\frac{z}{(z-2)^2}$$

Property :- $\left[Z\{na^n u(n)\}=\dfrac{az}{(z-a)^2}\right]$

$$=\frac{1}{2}\cdot\frac{2z}{(z-2)^2}$$

$\left\{na^n u(n)=Z^{-1}\left\{\dfrac{az}{(z-a)^2}\right.\right.$

Taking IZT of $H(z)$, $h(n)=$ $a=2$

$=\frac{1}{2}n2^n u(n)$ or $n2^{n-1}u(n)$ Ans.

Q. Determine the response of DT LTI system governed by eq:

8-10 marks

$$y(n)-2y(n-1)-3y(n-2)=x(n)+4x(n-1) \quad \text{——— (1)}$$

for the input $x(n)=2^n u(n)$ and with initial condition $y(-2)=0$ & $y(-1)=5$

// whenever there is past value, there is initial condition. Modify the formula.

ans:

~~Z transform of this eqⁿ (1)~~

Formula

$$Z\{x(n-m)\}=z^{-m}X(z)+\underbrace{\sum_{i=1}^{m}x(-i)z^{-(m-i)}}_{\text{initial value term}}$$

Sort out the initial value term at first:

Compare $x(n-m)$ with $y(n-2)$ first $\therefore m=2$.

$$\therefore \sum_{i=1}^{2}y(-i)z^{-(2-i)}$$

$$=y(-1)z^{-(2-1)}+y(-2)z^{-(2-2)}$$

$$=y(-1)z^{-1}+y(-2)$$

Now find $z^{-2}Y(z)+y(-1)z^{-1}+y(-2)$

Z transform of eq (1) is

~~$Y(z)-2z^{-1}Y(z)-3z^{-2}$~~

$$Y(z)-2[z^{-1}Y(z)+\overset{5}{y(-1)}]-3[z^{-2}Y(z)+z^{-1}\overset{5}{y(-1)}+\overset{0}{y(-2)}]$$
$$=X(z)+4\{z^{-1}X(z)+\underset{0}{x(-1)}\}$$

because $x(n)=2^n u(n)$ and $u(n)=0$ for $n<0$.

$$Y(z)-2[z^{-1}Y(z)+5]-3[z^{-2}Y(z)+5z^{-1}+0]$$
$$=X(z)+4z^{-1}X(z)$$

$$Y(z) - 2z^{-1}Y(z) - 10 - 3z^{-2}Y(z) - 15z^{-1} = X(z) + 4z^{-1}X(z)$$

$$\Rightarrow Y(z)\{1 - 2z^{-1} - 3z^{-2}\} - 15z^{-1} - 10 = X(z)\{1 + 4z^{-1}\}$$

$$\Rightarrow Y(z)\left\{1 - \frac{2}{z} - \frac{3}{z^2}\right\} - \left\{\frac{15}{z} + 10\right\} = X(z)\left[1 + \frac{4}{z}\right]$$

$$\Rightarrow Y(z)\left[\frac{z^2 - 2z - 3}{z^2}\right] - \left(\frac{15 + 10z}{z}\right) = X(z)\left(\frac{z+4}{z}\right) \quad \text{——— (i)}$$

// Taking ZT of input signal, $x(n) = 2^n u(n)$

$$\xrightarrow{ZT} X(z) = \frac{z}{z-2}$$

$$\text{(ii)} \Rightarrow Y(z)\left[\frac{z^2 - 2z - 3}{z^2}\right] - \left\{\frac{15 - 10z}{z}\right\} = \frac{z}{z-2}\left[\frac{z+4}{z}\right]$$

$$\cancel{Y(z)} \quad Y(z)\left\{\frac{(z-3)(z+1)}{z^2}\right\} = \frac{11z^2 - z - 30}{z(z-2)}$$

$$Y(z) = \frac{\cancel{z^2}(11z^2 - z - 30)}{\cancel{z}(z-2)(z-3)(z+1)}$$

$$\frac{Y(z)}{z} = \frac{11z^2 - z - 30}{(z-2)(z-3)(z+1)} = \frac{A}{z-2} + \frac{B}{z-3} + \frac{C}{z+1}$$

Using partial fraction $A = -4$, $B = \frac{33}{2}$, $C = \frac{-3}{2}$

$$\therefore \frac{Y(z)}{z} = \frac{-4}{z-2} + \frac{33}{2}\frac{1}{z-3} - \frac{3}{2}\frac{1}{z+1}$$

$$\Rightarrow Y(z) = -4\left(\frac{z}{z-2}\right) + \frac{33}{2}\left(\frac{z}{z-3}\right) - \frac{3}{2}\left(\frac{z}{z+1}\right)$$

By taking IZT, $y(n) = -4 \times 2^n u(n) + \frac{33}{2} \times 3^n u(n) - \frac{3}{2} \times (-1)^n u(n)$

$$\boxed{Z\{a^n u(n)\} = \frac{z}{z-a}}$$
Formula

Date: CAT I Last Q

Q A linear shift invariant system is described by the difference eqn

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + x(n-1)$$

with $y(-1)=0$, & $y(-2) = -1$

Find a) The natural response of the system

b) The forced " " " for step input

c) " freq " " " "

Solⁿ a) To calculate the natural response, ~~and~~ make $x(n)=0$. This is due to the initial ~~and~~ condition only i.e. $x(n) = x(n-1) = 0$

$\therefore y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 0$

Taking Z transform

~~$Y(z) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 0$~~

$$Y(z) - \frac{3}{4}\left[z^{-1}Y(z) + y(-1)\right] + \frac{1}{8}\left[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)\right] = 0$$

$\Rightarrow Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] - \frac{1}{8} = 0$

$\Rightarrow Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = \frac{1}{8}$

$\Rightarrow Y(z) = \dfrac{1/8}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$

$\Rightarrow \quad = \dfrac{1/8}{z^{-2}\left(z^2 - \frac{3}{4}z + \frac{1}{8}\right)}$

$= \dfrac{1/8\, z^2}{\frac{8z^2 - 6z + 1}{8}}$

$= \dfrac{z^2}{8z^2 - 6z + 1} = \dfrac{z^2}{8z^2 - 4z - 2z + 1} = \dfrac{z^2}{4z(2z-1) - 1(2z-1)}$

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$\therefore Y(z) = \dfrac{z^2}{(2z-1)(4z-1)}$

$\Rightarrow \dfrac{Y(z)}{z} = \dfrac{z}{(2z-1)(4z-1)} = \dfrac{z}{8(z-\frac{1}{2})(z-\frac{1}{4})}$

$\Rightarrow \dfrac{Y(z)}{z} = \dfrac{1}{8}\dfrac{z}{(z-\frac{1}{2})(z-\frac{1}{4})}$

using partial fraction

$$\frac{z/8}{(z-\frac{1}{2})(z-\frac{1}{4})} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{4}}$$

$\Rightarrow \dfrac{z}{8} = A(z-\frac{1}{4}) + B(z-\frac{1}{2})$

Put $z = 1/4$, $B = 1/8$

Put $z = 1/8$ $A = 1/4$

$\therefore \dfrac{Y(z)}{z} = \dfrac{1/4}{z-1/2} - \dfrac{1/8}{z-1/4}$

$Y(z) = 1/4\left\{\dfrac{z}{z-1/2}\right\} - \dfrac{1}{8}\left\{\dfrac{z}{z-1/4}\right\}$

Taking IZT

$y(n) = \frac{1}{4}\left(\frac{1}{2}\right)^n u(n) - \frac{1}{8}\left(\frac{1}{4}\right)^n u(n)$

ans

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(b) To determine the ~~fixed~~ forced response, we need to consider $x(n)$: Here $x(n) = u(n)$ as per equation. Hence we are going to neglect all the initial conditions

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = u(n) + u(n-1)$$

Taking Z-Transfer for both sides

$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = U(z) + z^{-1}U(z)$$

Here, $Z[u(n)] = U(z) = \frac{z}{z-1}$

$Z[u(n-1)] = z^{-1}U(z) = \frac{1}{z}\frac{z}{z-1} = \frac{1}{z-1}$

$\therefore U(z) + z^{-1}U(z) = \frac{z}{z-1} + \frac{1}{z-1} = \frac{z+1}{z-1}$

$$\therefore Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = \frac{z+1}{z-1}$$

$$\Rightarrow Y(z)\left\{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right\} = \frac{z+1}{z-1}$$

$$\Rightarrow Y(z) = \frac{z+1}{(z-1)\left(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right)}$$

$$= \frac{z^2(z+1)}{(z-1)\left(z^2 - \frac{3}{4}z + \frac{1}{8}\right)}$$

$$= \frac{z^2(z+1)}{(z-1)\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$$

Refer to sol^n

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$$\therefore \frac{Y(z)}{z} = \frac{z(z+1)}{(z-1)\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} = \frac{A}{z-1} + \frac{B}{z-\frac{1}{2}} + \frac{C}{z-\frac{1}{4}}$$

$$\therefore \frac{z(z+1)}{(z-1)\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} = A\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right) + B(z-1)\left(z-\frac{1}{4}\right) + C(z-1)\left(z-\frac{1}{2}\right)$$

Sol^n The above eq^n using partial fraction

$$\frac{Y(z)}{z} = \frac{16/3}{z-1} - \frac{6}{z-\frac{1}{2}} + \frac{5/3}{z-\frac{1}{4}}$$

$$Y(z) = \frac{16}{3}\frac{z}{z-1} - 6\frac{z}{z-1/2} - 5/3\frac{z}{z-1/4}$$

Taking IZT

$$y(n) = \frac{16}{3}u(n) - 6\left(\frac{1}{2}\right)^n u(n) - \frac{5}{3}\left(\frac{1}{4}\right)^n u(n) \quad \text{ans}$$

(c) The $freq^n$ response of the system $H(z)$ can be calculated by putting $z = e^{j\omega}$ in $H(z)$

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + x(n-1)$$

Taking Z-Transform for the above eq^n —

$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}Y(z)z^{-2} = X(z) + z^{-1}X(z)$$

Here neglect all initial conditions

$$\Rightarrow Y(z)\left\{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right\} = X(z)\left\{1 + z^{-1}\right\}$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

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$$= \frac{z^{-1}(z+1)}{z^{-2}\left(z^2 - \frac{3}{4}z + \frac{1}{8}\right)}$$

$$= \frac{z(z+1)}{z^2 - \frac{3}{4}z + \frac{1}{8}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z(z+1)}{z^2 - \frac{3}{4}z + \frac{1}{8}}$$

$$H(\omega) = \frac{e^{j\omega}(e^{j\omega}+1)}{(e^{j\omega})^2 - \frac{3}{4}e^{j\omega} + \frac{1}{8}}$$

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Realisation of Digital System

Date: 20/3/23

We Know,

$$y(n) = -\sum_{m=1}^{N} a_m y(n-m) + \sum_{m=0}^{M} b_m x(n-m) \quad \text{—— (1)}$$

Z-Transf

$$\frac{Y(z)}{X(z)} = H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 - a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}} \quad \text{—— (ii)}$$

→ Discrete time domain IIR System response

We can use eq (1) - in various sets of different equation

M = no. of zeros
N = no of poles

to reduce the computation complexity, memory requirement and finite word length effect in computation.

(i) Direct form I
(ii) " " II
(iii) Cascade form
(iv) Parallel "

(1) Direct form (1)

We know that $H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 - a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$

$$y(n) = -\sum_{m=1}^{N} a_m y(n-m) + \sum_{m=0}^{M} b_m x(n-m) \quad \text{(1)}$$

expanding

$$\Rightarrow y(n) = -a_1 y(n-1) - a_2 y(n-2) - \dots - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \dots + b_M x(n-M)$$

On taking Z transform

$$Y(z) = a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) \dots - a_N z^{-N} Y(z) + b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + \dots + b_M z^{-M} X(z)) \quad \text{(3)}$$

$w(z)$

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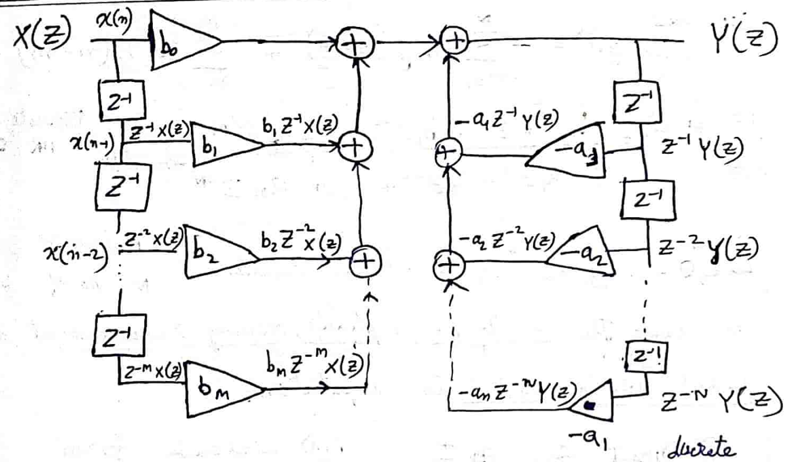
Date :



The direct form I provides a direct relation bet^n time domain and z domain.

The direct form I structure uses separate delays (z^{-1}) for input and output samples. Hence for realising DF I structure, more memory requirement is there.

mcq To realisation of an N^{th} order discrete time system with M no of zeros and N numbers of poles we need N+M+1 multipliers, N+M numbers of adders and N+M number of delays are required. when the number of delays in the structure is equals to order of the system, then the structure is called Canonic structure. Hence direct form I is non canonical structure.

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Date : 21 march

Q Obtain the DF II ~~fo~~ for the DLTI system.

y(n)

We know $y(n) = -\sum_{m=1}^{N} a_m y(n-m) + \sum_{m=0}^{M} b_m x(n-m)$

on expanding $y(n) = -a_1 y(n-1) - a_2 y(n-2) \dots - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \dots + b_m x(n-m)$

On taking Z Transform

$$Y(z) = -a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) \dots - a_N z^{-N} Y(z) + b_0 X(z) + b_1 z^{-1} X(z) + \dots + b_m z^{-m} X(z)$$

$$\boxed{\frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_m z^{-m}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_N z^{-N}}}$$

Let $\frac{Y(z)}{X(z)} = \frac{W(z)}{X(z)} \times \frac{Y(z)}{W(z)}$

where, $\frac{W(z)}{X(z)} = \frac{1}{1 - a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$ —— (1)

$\frac{Y(z)}{W(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_m z^{-1}$ —— (2)

eq (1) $\Rightarrow W(z) + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N} = X(z)$

$\Rightarrow W(z) = X(z) - a_1 z^{-1} W(z) - a_2 z^{-2} W(z) \dots - a_N z^{-N} W(z)$ —— (3)

(2) $\Rightarrow y(z) = b_0 W(z) + b_1 W(z) z^{-1} + \dots + b_m z^{-m} W(z)$ —— (4)

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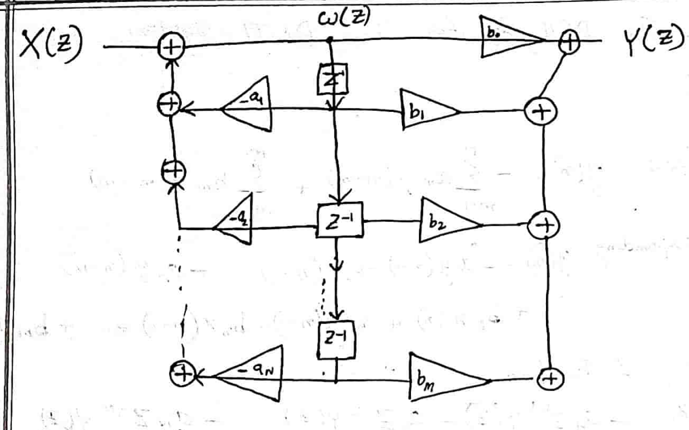
Date :



Q Obtain direct form I and DF II for the given discrete LTI system.

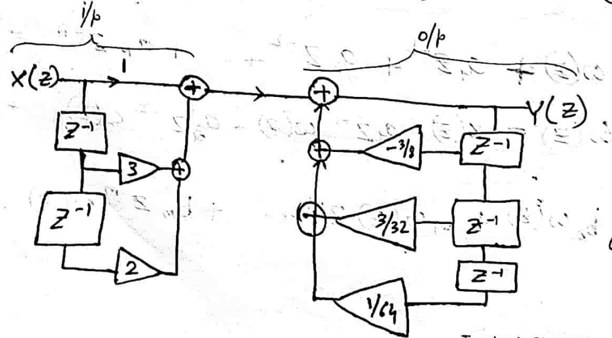
$$y(n) = -\frac{3}{8}\, y(n-1) + \frac{3}{32}\, y(n-2) + \frac{1}{64}\, y(n-3)$$
$$+\, x(n) + 3x(n-1) + 2x(n-2)$$

ans: Sol^n for

DFI

Taking ZT of the eq^n:

$$Y(z) = \frac{-3}{8} z^{-1} Y(z) + \frac{3}{32} z^{-2} Y(z) + \frac{1}{64} z^{-3} Y(z)$$
$$+\, X(z) + 3z^{-1} X(z) + 2z^{-2} X(z)$$



delay = 5

Sum = 4

Gain = 5

SS → Anand Kumar Ramesh

DSP → Prokais

EMW → Shevgaonkar

Date : .... OFC → John Senior

Sol^n for DF II

$$Y(z) + \frac{3}{8} z^{-1} Y(z) - \frac{3}{32} z^{-2} Y(z) - \frac{1}{64} z^{-3} Y(z)$$
$$= X(z) + 3z^{-1} X(z) + 2z^{-2} X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1 + 3z^{-1} + 2z^{-2}}{1 + \frac{3}{8} z^{-1} - \frac{3}{32} z^{-2} - \frac{1}{64} z^{-3}}$$

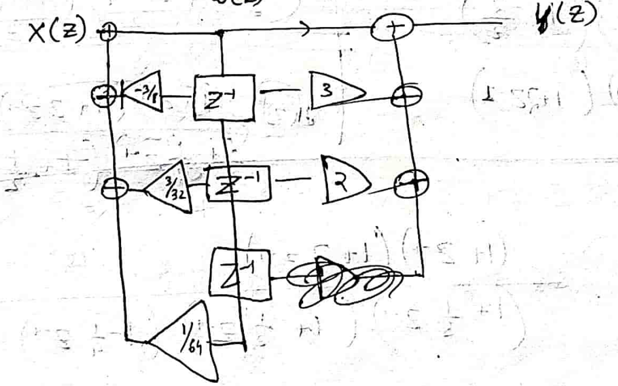
$$\frac{\omega(z)}{X(z)} \times \frac{Y(z)}{\omega(z)} = \frac{1}{1 + \frac{3}{8} z^{-1} - \frac{3}{32} z^{-2} - \frac{1}{64} z^{-3}} \times \frac{(1 + 3z^{-1} + 2z^{-2})}{1}$$

$$\Rightarrow \omega(z) + \frac{3}{8} z^{-1} \omega(z) - \frac{3}{32} z^{-2} \omega(z) - \frac{1}{64} z^{-3} \omega(z) = X(z)$$

$$\Rightarrow \omega(z) = X(z) - \frac{3}{8} z^{-2} \omega(z) + \frac{3}{32} z^{-2} \omega(z) + \frac{1}{64} z^{-2} \omega(z) \quad \text{— (i)}$$

$$Y(z) = \omega(z) + 3z^{-1} + 2z^{-2} \quad \text{— (ii)}$$

using (i) & (ii), draw the block diagram.



Cascaded Format :—

Date: 29/03/23

$$H(z) = \frac{Y(z)}{X(z)} = \underbrace{H_1(z).H_2(z).....H_n(z)}$$

Individual blocks can be realized with the help of direct form I. and/or direct form II.

Taking the eq from last class

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+3z^{-1}+2z^{-2}}{1+\frac{3}{8}z^{-1}-\frac{3}{32}z^{-2}-\frac{1}{64}z^{-3}} \quad (CF)$$

In CF, the numerator & denominator of polynomial should be expressed in factored form.

NUMERATOR

$\Rightarrow 1+3z^{-1}+2z^{-2}$

$\Rightarrow z^{-2}(z^{2}+3z+2)$

$\Rightarrow z^{-2}(z+2)(z+1)$

$\frac{(z+1)}{z}\left(\frac{z+2}{z}\right)$

$=\left(1+\frac{1}{z}\right)\left(1+\frac{2}{z}\right)$

$=(1+z^{-1})(1+2z^{-1})$

DENOMINATOR

$z^{-3}\left(z^{3}+\frac{3}{8}z^{2}-\frac{3}{32}z-\frac{1}{64}\right)$

$z^{-3}(z+0.5)(z-0.25)(z+1/8)$

$(z^{-3})\left(z+\frac{1}{2}\right)\left(z-\frac{1}{4}\right)\left(z+\frac{1}{8}\right)$

$\left(\frac{z+1/2}{z}\right)\left(\frac{z-\frac{1}{4}}{z}\right)\left(\frac{z+\frac{1}{8}}{z}\right)$

$\cancel{H(z)\frac{(1+2z^{-1})(1+2z^{-1})}{\left(1+\frac{1}{8}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}\right)}}$

$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{\left(1+\frac{1}{8}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$\therefore H(z) = \frac{Y(z)}{X(z)} =$ "

$$\left(\frac{1+z^{-1}}{1+\frac{1}{8}z^{-1}}\right)\left(\frac{1+2z^{-1}}{1+\frac{1}{2}z^{-1}}\right)\left(\frac{1}{1+\frac{1}{4}z^{-1}}\right)$$

$= H_1(z) \,.\, H_2(z) \,.\, H_3(z)$

$$H_1(z) = \frac{Y_1(z)}{X_1(z)} = \frac{W_1(z)}{X(z)} \times \frac{Y_1(z)}{W_1(z)}$$

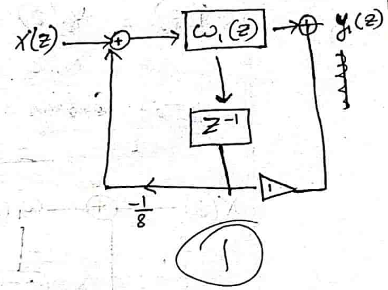
Now, $\frac{W_1(z)}{X(z)} = \frac{1}{1+\frac{1}{8}z^{-1}}$

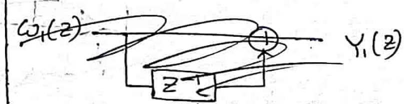
$\Rightarrow W_1(z) + \frac{1}{8}z^{-1}W_1(z) = X(z)$

$\Rightarrow W_1(z) = X(z) - \frac{1}{8}z^{-1}W_1(z)$

& $\frac{Y_1(z)}{W_1(z)} = 1+z^{-1}$

$\Rightarrow Y_1(z) = W_1(z) + z^{-1}W_1(z)$





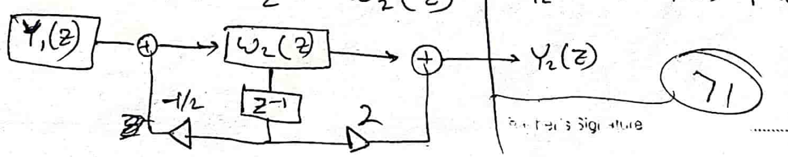
$$H_2(z) = \frac{Y_2(z)}{Y_1(z)} = \frac{W_2(z)}{Y_1(z)} \times \frac{Y_2(z)}{W_2(z)} = \frac{1+2z^{-1}}{1+\frac{1}{2}z^{-1}}$$

$\frac{W_2(z)}{Y_1(z)} = \frac{1}{1+\frac{1}{2}z^{-1}}$ & $\frac{Y_2(z)}{W_2(z)} = 1+2z^{-1}$

$\Rightarrow W_2(z) + \frac{1}{2}z^{-1}W_2(z) = Y_1(z)$

$\Rightarrow W_2(z) = Y_1(z) - \frac{1}{2}z^{-1}W_2(z)$ & $Y_2 = W_2 + 2z^{-1}W($

$\Rightarrow$



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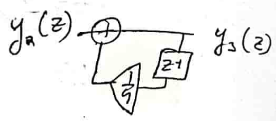
Date : ....

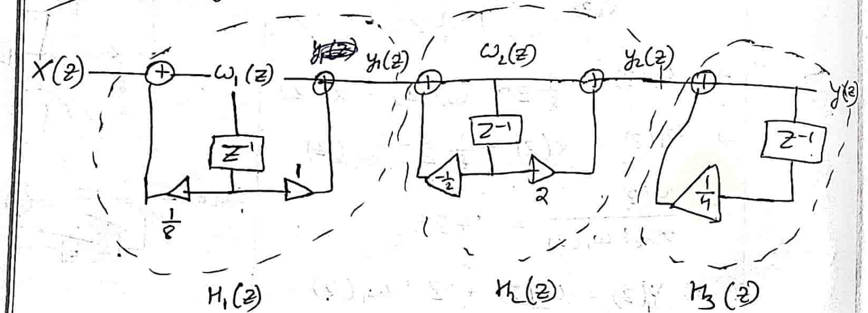
Now, $H_3(z) = \frac{Y_3(z)}{Y_2(z)} = \frac{1}{1-\frac{1}{4}z^{-1}}$



$Y_3(z) - \frac{1}{4}z^{-1}Y_3(z) = Y_2(z)$

$Y_3(z) = Y_2(z) + \frac{1}{4}z^{-1}Y_3(z)$ (iii)

~~$y_3(z)$~~ Combining (i), (ii) & (iii)



FFT of the signal. (spectrum analysis)

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Date : 27 march

1 $H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{8}z^{-1})(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-2})}$

Use parallel form to realize the system.

Solⁿ → * Using partial fraction method.

* Parallel form structure $H(z) = \frac{Y(z)}{X(z)} = H_1(z) + H_2(z) + \dots + H_m(z)$

$H(z) = \overbrace{\left(\frac{A}{1+\frac{1}{8}z^{-1}}\right)}^{H_1(z)} + \overbrace{\left(\frac{B}{1+\frac{1}{2}z^{-1}}\right)}^{H_2(z)} + \overbrace{\left(\frac{C}{1-\frac{1}{4}z^{-1}}\right)}^{H_3(z)}$

Solⁿ for A

$\frac{(1+z^{-1})(1+2z^{-1})}{\cancel{(1+\frac{1}{8}z^{-1})}(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} \times \cancel{\left(1+\frac{1}{8}z^{-1}\right)}\Big|_{z^{-1}=-8}$

$= \frac{(1-8)(1-16)}{(1+4)(1+2)} = \frac{-7\times(-\cancel{15})}{-\cancel{3}\times 3} = \frac{35}{-3}$

Solⁿ for B

$\frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{8}z^{-1})\cancel{(1+\frac{1}{2}z^{-1})}(1-\frac{1}{4}z^{-1})} \times \cancel{\left(1+\frac{1}{2}z^{-1}\right)}\Big|_{z^{-1}=-2}$

$= \frac{(1-2)(1-4)}{(1-\frac{1}{4})(1+\frac{1}{2})} = \frac{-1\times -3}{(-0.75)(1.5)} = \frac{3}{-0.5625} = -2.667$

Solⁿ C

$\frac{(1+z^{-1})(1+2z^{-1}) \times \cancel{(1-\frac{1}{4}z^{-1})}}{(1+\frac{1}{8}z^{-1})(1+\frac{1}{2}z^{-1})\cancel{(1-\frac{1}{4}z^{-1})}}\Big|_{z^{-1}=+4}$

$= \frac{(1+4)(1+8)}{(1+\frac{1}{2})(1+2)} = 10$

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Date : ....

$A = \frac{-35}{3}$ $\quad B = \frac{8}{3}$ $\quad C = 10$

$$H_1(z) = \frac{-35/3}{1+\frac{1}{8}z^{-1}}$$

$$\Rightarrow H_1(z) = \frac{Y_1(z)}{X_1(z)} = \frac{-35/3}{1+\frac{1}{8}z^{-1}}$$

$$y_1(z) = \frac{-35}{3}X_1(z) - \frac{1}{8}z^{-1}Y_1(z)$$

$$H_2(z) = \frac{Y_2(z)}{X_2(z)} = \frac{8/3}{1+\frac{1}{2}z^{-1}}$$

$$y_2(z) = \frac{8}{3}X_2(z) - \frac{1}{2}z^{-1}X_2(z)$$

$$H_3(z) = \frac{Y_3(z)}{X_3(z)} = \frac{10}{1-\frac{1}{4}z^{-1}}$$

$$y_3(z) = 10X_3(z) + \frac{1}{4}z^{-1}Y_3(z)$$

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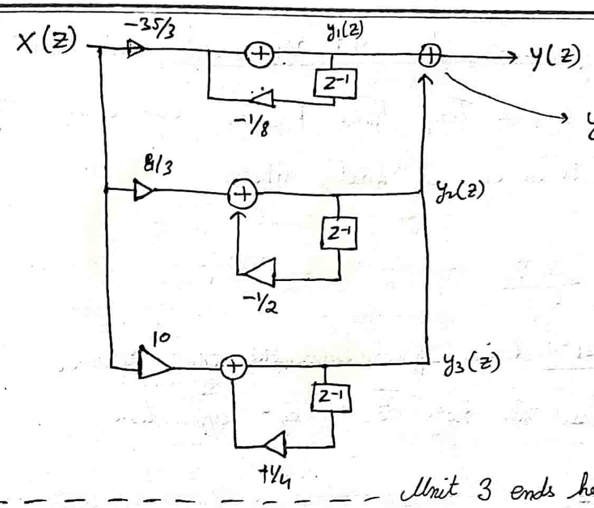
BVEC → Borak Valley

Date : ....



$y(z) = y_1(z)+y_2(z)+y_3(z)$

Unit 3 ends here

: Design & Analysis of IIR Digital Filters

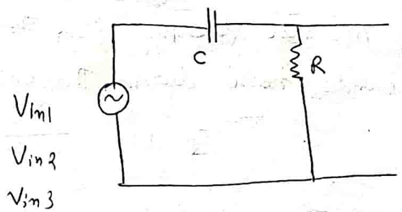
28/3/23 **Filtering Concepts** :- Analog filters are designed using analog components like resistors, caps and inductors. On the other hand DF are ~~implet~~ implemented using difference eqn.

Advantages :-

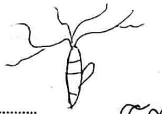
(1) Many i/p signals can be filtered out by one DF without replacing the hardware

AF = SISO

DF = MIMO



(ii) DF have characteristics like linear phase response. Such Ch. is not possible to be obtained in case of AF.



$R \propto \frac{1}{\tau}$ relaxation time.

TAT

Date : ....

(iii) Performance does not vary with environmental parameters. However, e.g. like: Temp, Humi, Pressure can change the value of components in case of analog filters.

(iv) Portable

(v) Flexible

(vi) Using VLSI Technique, power consumption can be reduced

It can be used for very low freq^n application

Disadvantage

1) Speed limitation (~~under~~ DAC & ADC are used. So the speed of the filters depends upon the conversion time of ADC & sampling time of DAC. and also the BW of DF are much lower than the AF.)

(ii) Finite Word length effect

> Accuracy of DF depends on the wordlength used to encode them in binary form. Wordlength should be long enough to obtain the required accuracy.

> It is also affected by ADC noise resulting from the quantisation of cont. signal & roundoff noise during the compution of wordlength.

(iii) Long design & development time : Initial design & dev time for ~~AF~~ DF hardware is more than AF

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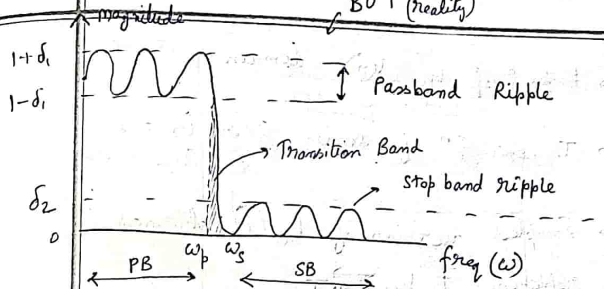
DCT → FFT Trans

Date : 4/4/23

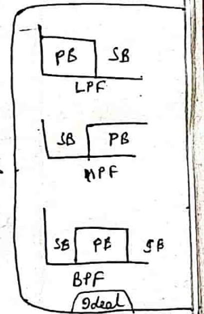
Ideally

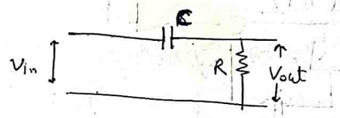
BUT (reality)



1) LPF
2) HPF
3) BPF
4) BRF
5) All pass filter

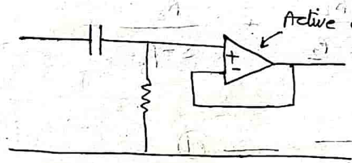
Magnitude characteristics of physically realizable filter.





Analog low pass filter uses only passive elements

In digital domain, we need active components



(some sort of digital filter)

Q Design of IIR filter using impulse invarient transformation or impulse invarient method.

δ_1 = passband ripple

δ_2 = stop " "

ω_p = passband edge freq

ω_s = stop " " "

derive the TF of IIR filter using Imp. Invarient Method

(TF will not be given)

If given, its a numerical.

ans: Obsessive Compulsive Impulse Invarient method.

In this method, the impulse response of both the analog & digital filter are designed to be same.

Learn & Compare formulas

Date: ....

Steps()

Here, $h_a(t)$ → Imp resp in time domain.

$H_a(s)$ → Transfer f^n of analog filter in s domain (Laplace Transform)

$h_a(nT)$ → Sampled version of $h_a(t)$ obtained by replacing t by nT

$H(z)$ = Z-transform of $h(nT)$.

This is the digital response of the filter.

Let the TF of the analog filter be

$$H_a(s) = \frac{A_1}{s-p_1} + \frac{A_2}{s-p_2} + \frac{A_3}{s-p_3} + \dots$$

$$\therefore H_a(s) = \sum_{i=1}^{N} \frac{A_i}{s-p_i} \quad \text{—— (1)}$$

Where, $A_1, A_2, \dots A_N$ → Coefficient of partial frac.

$p_1, p_2 \dots p_N$ = Location of the poles

Taking ILT of (1).

$$h_a(t) = \sum_{i=1}^{N} A_i e^{p_i t} \quad \text{—— (2)}$$

Now, unit imp response for discrete structure is obtained by sampling $h_a(t)$. This means $h(n)$ can be obtained from $h_a(t)$ by ~~rep~~ replacing t by nT in eq (2)

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Fourier → Time to freq^n

DTFT → DFT → FFT

~~DTFT~~ → DFT → DFT is sampled version of DTFT

(fast fourier Trans)

Date: .... Same as $t = nT$, $\omega = \Omega T$ → samp. of freq^n.

fiddle factor

radix method

$$\therefore h(n) = \sum_{i=1}^{N} A_i e^{p_i(nT)} \quad \text{—— (3)} \quad (T = \text{Sampling time})$$

Now, taking Z Transform of exp (3)

i.e. $\sum_{n=0}^{\infty} h(n) z^{-n}$

$$= \sum_{n=0}^{\infty} \left[\sum_{i=1}^{N} A_i e^{p_i(nT)} \right] e^{-n}$$

$$= \sum_{i=1}^{N} A_i \sum_{n=0}^{\infty} e^{p_i nT} z^{-n}$$

$$\Rightarrow H(z) = \sum_{i=1}^{N} A_i \sum_{n=0}^{\infty} \left(e^{p_i T} z^{-1}\right)^n$$

A - analog

Using standardised geometrical sum formula $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$

$$\boxed{H(z) = \sum_{i=1}^{N} A_i \frac{1}{1 - e^{p_i T} z^{-1}}}$$ Req TF of digital filter

Mapping of Poles

TF of analog filter is given by $\frac{1}{s-p_i}$ and the TF of digital filter is given by $\frac{1}{1-e^{p_i T} z^{-1}}$

The mapping of poles from analog domain to g digital domain is obtained by putting $z = e^{p_i T}$

We know, $\omega = \Omega T$

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Drawback of Impulse Invarience Method

i) Analog freq Ω ranges from $\frac{-\pi}{T}$ to $\frac{\pi}{T}$.
while digital freq ω " " $-\pi$ to π
i.e. mapping of $\frac{-\pi}{T}$ to $\frac{\pi}{T} \rightarrow -\pi$ to π is not ~~one~~ one-to-one mapping but many to one mapping.

ii) Analog factors are not band limited. So, There ~~were~~ will be aliasing due to the ~~freq~~ sampling process. Because of This aliasing, The freq resp of resulting digital filter will not be identical to the original freq resp of analog filter.

iii) The change in The value of sampling time 'T' has no effect on The amount of aliasing.

Q:- Determine $H(z)$ using ~~freq~~ IIM at 5Hz sampling freq from $H_a(s)$ given below

$$H_a(s) = \frac{2}{(s+1)(s+2)}$$

Ans:-

$$H_a(s) = \frac{2}{(s+1)(s+2)}$$

$$\Rightarrow H_a(s) = \frac{2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \left.\frac{2}{(s+1)(s+2)} \times (s+1)\right|_{s=-2}$$

$\therefore 2 = A(s+2) + B(s+1)$

$s=-1$ ~~$2 = B(-1+1) = 3A$~~ $2 = A(-1+2) \Rightarrow A = 2$

$s=-2$ $2 = B(-2+1) \Rightarrow B = -2$

$$\therefore H(s) = \frac{2}{s+1} - \frac{2}{s+2}$$

$$h(t) = \text{ILT of } H(s) = 2e^{-t}u(t) - 2e^{-2t}u(t)$$

$$= 2[e^{-t} - e^{-2t}]u(t)$$

We know, $\frac{1}{s+p_i} \xrightarrow{\text{ILT}} \frac{1}{1-e^{p_iT}z^{-1}}$

Here, $f = 5\text{Hz}$

$\therefore T = \frac{1}{f_s} = \frac{1}{5} = 0.2$ sec

We have poles at $p_1 = -1$ & $p_2 = -2$

$$\frac{1}{s+1} \rightarrow \frac{1}{1-e^{-1\times 0.2}z^{-1}} = \frac{1}{1-e^{-0.2}z^{-1}}$$

$$\& \quad \frac{1}{s+2} \rightarrow \frac{1}{1-e^{-2\times 0.2}z^{-1}} = \frac{1}{1-e^{-0.4}z^{-1}}$$

$\therefore$ TF of Digital filter

$$= H(z) = \sum_{i=1} \frac{A_i}{1-e^{p_iT}z^{-1}}$$

$$= \frac{1}{1-e^{-0.2}z^{-1}} + \frac{1}{1-e^{-0.4}z^{-1}} = \frac{0.29z}{z^2 - 1.49z + 0.54}$$

To find impulse invarient method

Date: 25/ April

① Deduce digital Transfer function

for this we need analog TF

analog TF $\xrightarrow{ILT}$ Sampling $\xrightarrow{ZT}$ Digital TF

Problem associated with impulse invariance method

1) Mapping

TF of analog filter $= \frac{1}{S - p_i}$

TF of digital filter $= \frac{1}{1 - e^{p_i T} z^{-1}}$

$= \frac{z}{z - e^{p_i T}}$

The poles of digital filters are located at $Z = e^{p_i T}$. ∴ The poles of analog filter at $S = p_i$ are transformed into the digital form at $Z = e^{p_i T}$.

∴ $\boxed{Z = e^{sT}}$

$re^{j\omega} = e^{(\sigma + j\Omega)T}$

=> ~~$\Omega = \omega T$~~ ~~analog freq~~ ~~digital freq~~

$re^{j\omega} = e^{\sigma T} \cdot e^{j\Omega T}$

$r = e^{\sigma T}$ & $\omega = \Omega T$ (Comparing)

Case 1

Consider $r = e^{\sigma T}$

① When $\sigma < 1$

∴ $r = e^{-ve\ something}$

∴ e^{-ve} will be $< 0\ -1$

∴ $r < 1$

$0 < r < 1$

IRNSS

navic

Date: .... navigation with indian Constellation.

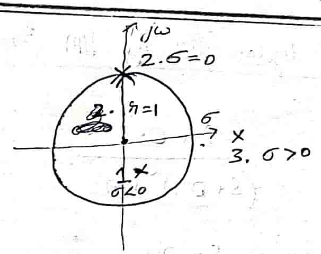
(ii) $\sigma = 0$

$r = 1$

(iii) $\sigma > 0$

$r = e^{+ve}$

∴ $r > 1$



This is the mapping of pole from analog to digital or S to Z plane precisely, it is many to one mapping.

This is the disadvantage of IIM.

Drawbacks of IIM

① Analog frequency 'Ω' ranges from 0 to 2π while the digital freq 'ω' varies from −π to π. This is not 1-1 mapping. This is many−1 mapping

(ii) Analog filters are not band limited, so there will be aliasing effect due to sampling ~~process~~ process.

Because of this aliasing the freq^n response of resulting digital filter will not be identical to the original freq^n response of AF.

(iii) The change in the value of sampling time T has no effect on the aliasing.

Date : ....

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Imp

Q design a digital filter by IIM for $H_a(s) = \frac{s+0.1}{(s+0.1)^2+9}$

$$H_a(s) = \frac{s+0.1}{(s+0.1)^2+9}$$

$$= \frac{s+0.1}{s+0.1^2 - (j3)^2}$$

$$= \frac{s+0.1}{(s+0.1+j3)(s+0.1-j3)}$$

$$\frac{s+0.1}{(s+0.1+j3)(s+0.1-j3)} = \frac{A}{(s+0.1+j3)} + \frac{B}{(s+0.1-j3)}$$

$$\Rightarrow s+0.1 = (s+0.1-j3)A + (s+0.1+3j)B$$

$$A = \frac{1}{2} \qquad B = \frac{1}{2}$$

$$\frac{1}{2}\left(\frac{1}{\underbrace{s+0.1}_{a}+j3}\right) + \frac{1}{2}\left(\frac{1}{\underbrace{s+0.1}_{a}-j3}\right) = \frac{s+0.1}{(s+0.1)^2+9}$$

$e^{-at}u(t) \to \frac{1}{s+a}$

Takin ILT

$$h_a(t) = \frac{1}{2}e^{-(0.1+j3)t}u(t) + \frac{1}{2}e^{-(0.1-j3)t}u(t)$$

$=$

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Sampling Replacing t by nT_s

$$h_a(t) = \frac{1}{2}\left[e^{-(0.1+3j)nT_s} + e^{-(0.1-3j)nT_s}\right]u(t)$$

Now

$$ZT\{h_a(nT_s)\} = \sum_{n=0}^{\infty} h_a(nT_s)z^{-n}$$

↳ for $u(t)$

$$= \frac{1}{2}\sum_{n=0}^{\infty} e^{(-0.1+3j)nT_s} z^{-n} + \frac{1}{2}\sum_{n=0}^{\infty} e^{-(0.1-3j)nT_s} z^{-n}$$

Solve this as assignment

ans:- $$H(z) = \frac{1-(e^{0.1T_s}\cos 3T)z^{-1}}{1-(2e^{-0.1T_s}\cos 3T_s)z^{-1}+e^{-0.2T_s}z^{-2}}$$

Standard exp

① $$\frac{1}{s-p_i} \to \frac{1}{1-e^{p_iT}z^{-1}}$$

② $$\frac{s+a}{(s+a)^2+b^2} \to \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

③ $$\frac{b}{(s+a)^2+b^2} \to \frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

Date: ....

Design of IIR filters by Bilinear Transformation

One to one mapping — no aliasing

> Give rise to non linearity.

Numerial Integration Method

① Trapezoidal Rule ⟶ we will use this method

(ii) Simpson's $\frac{1}{3}$ rule — odd only (when n = odd or even)

(iii) Simpson's $\frac{3}{8}$ rule — even only

① Trapezoidal rule

$$\int_a^b f(x)\,dx$$

$$= h\left(\frac{y_0 + y_n}{2},\ y_1 + y_2 + \cdots + y_{n-1}\right)$$

In this rule, divide the integration limit (a, b) into n equal parts with a step size 'h'

Let $a = x_0$

$x_1 = x_0 + h$

$x_2 = x_1 + h$

$\vdots$

$x_n = x_{n-1} + h$

Date: ....

Let us consider a system as shown in the fig below.

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$$

$H(s) = 1/s$

Considering the interval in between $nT_s - T_s$ (lower limit) to nT_s (upper limit), T_s → step size.

$\therefore y = \int x(t)\,dt$ ~~and~~

By implementing Trapezoidal rule

$$\frac{Y(s)}{X(s)} = \frac{1}{s}$$

$$X(s) = s\,Y(s) \quad \text{—①}$$

$$\left[y(t)\right]_{nT_s - T_s}^{nT_s} = \int_{nT_s - T_s}^{nT_s} x(t)\,dt$$

$$= T_s\left[\frac{x(n-1)T_s + x(nT_s)}{2}\right]$$

Now, we are going to make ~~at Ts~~ $f(T_s) = 1$ not the coefficients!

$$\Rightarrow y(n\cancel{T_s}) - y(n\cancel{T_s} - \cancel{T_s}) = T_s\left[\frac{x(n-1)T_s + x(nT_s)}{2}\right]$$

$$\Rightarrow y(n) - y(n-1) = T_s\left[\frac{x(n-1) + x(n)}{2}\right]$$

Now taking ZT

$$Y(z) - z^{-1}Y(z) = T_s\left[\frac{z^{-1}X(z) + X(z)}{2}\right]$$

$$\Rightarrow Y(z) - z^{-1}Y(z) = T_s\left[\frac{z^{-1}X(z) + X(z)}{2}\right]$$

$$\Rightarrow Y(z)\left[1 - z^{-1}\right] = \frac{T_s}{2}\left[X(z)\left[z^{-1} + 1\right]\right]$$

$$\boxed{X(z) = \frac{2}{T_s}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)Y(z)} \quad \text{—②}$$

ω = digital freq

Ω - analog ..

Date : ....

From eq ① & ②

$$\boxed{s = \frac{2}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} \quad \text{or} \quad \boxed{s = \frac{2}{T_s}\left(\frac{z-1}{z+1}\right)}$$

Let $s = \sigma + j\Omega$ → analog freq

$z = re^{j\omega}$

$$\sigma + j\Omega = \frac{2}{T_s}\left(\frac{z-1}{z+1}\right) = \frac{2}{T_s}\left(\frac{re^{j\omega}-1}{re^{j\omega}+1}\right)$$

Rationalizing

$$= \frac{2\,(re^{j\omega}-1)(re^{j\omega}+1)}{T_s\,(re^{j\omega}+1)(re^{j\omega}+1)}$$

$$= \frac{2}{T_s} \times \frac{re^{2j\omega}-1}{re^{2j\omega}+2re^{j\omega}+1}$$

$$\sigma+j\Omega = \frac{2}{T_s}\left[\frac{r^2+re^{j\omega}-re^{j\omega}-1}{r^2+re^{j\omega}+r^{-j\omega}+1}\right]$$

$$= \frac{2}{T_s}\left[\frac{(r^2-1)+r(e^{j\omega}-e^{-j\omega})}{r^2+1+r(e^{j\omega}+e^{j\omega})}\right]$$

$$= \frac{2}{T_s}\left[\frac{r^2-1+r2j\sin\omega}{r^2+1+r2\cos\omega}\right]$$

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Date : ....

Separating real and img part

Real part $\frac{2}{T_s}\,\frac{r^2-1}{r^2+1+2r\cos\omega}$ —(iii)

Img part $\frac{2}{T_s}\,\frac{2rj\sin\omega}{r^2+1+r2\cos\omega}$ —(iv)

we obtain σ = ↑ and jΩ = ↗

eq (iv) ⇒

$$\Omega = \frac{2}{T_s}\,\frac{2r\sin\omega}{r^2+1+2r\cos\omega}$$

If r=1, $\Omega = \frac{2}{T_s}\left[\frac{2\sin\omega}{2+2\cos\omega}\right]$

$$= \frac{2}{T_s}\,\frac{\sin\omega}{1+\cos\omega}$$

$$= \frac{2}{T_s}\,\frac{2\sin\omega/2\,\cos\omega/2}{1+(2\cos^2\frac{\omega}{2}-1)}$$

$$\boxed{\Omega = \frac{2}{T_s}\tan\frac{\omega}{2}} \text{ —(v)}$$

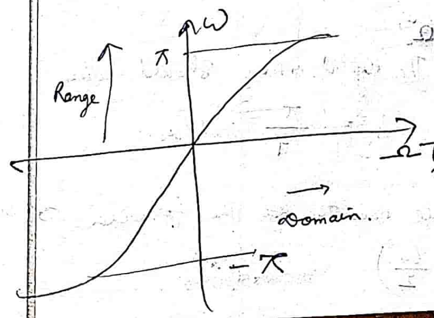
r is taken 1 ∵ we are taking the unit circle

Put ω = ΩTs in numericals.

Now considering exp (v) $\Omega = \frac{2}{T_s}\tan\frac{\omega}{2}$

$$\Rightarrow \frac{\Omega T_s}{2} = \tan\frac{\omega}{2}$$

$$\Rightarrow \cancel{\frac{\omega}{2} = \tan\frac{\omega}{2}}\ \tan^{-1}\left(\frac{\Omega T_s}{2}\right) = \omega/2$$



$\Omega T_s \Rightarrow \omega = 2\tan^{-1}\left(\frac{\Omega T_s}{2}\right)$

Problem:- Frequency Wrapping because of non linear mapping

Basic design method — Imp. / Bilinear } Used to design IIR filters

Butterworth, Elliptical, Sebasice — IIR

Date: 04/May/2023

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Q Apply Bilinear Transformation to obtain digital filter response for the following TF $H(s) = \frac{2}{(s+1)(s+3)}$ with $T = 0.1s$

ans:- Replacing S by $\frac{2}{T_s}\left(\frac{z-1}{z+1}\right)$

A/Q $s = \frac{2}{0.1}\ \frac{z-1}{z+1} = \frac{2}{\left[\frac{2}{0.1}\left(\frac{z-1}{z+1}\right)\right]\left[\frac{2}{0.1}\left(\frac{z-1}{z+1}\right)\right]}$

(given)

$\therefore$

$$= \frac{2}{\left[\frac{2}{0.1}\left(\frac{z-1}{z+1}\right)+1\right]\left[\frac{2}{0.1}\left(\frac{z-1}{z+1}\right)+3\right]}$$

$$= \frac{2(z+1)^2}{(21z-19)(23z-17)}$$ Do upto this much for 4 marks

$$= \frac{0.0041(1+z^{-1})^2}{1-1.644z^{-1}+0.668z^{-2}}$$ If you can solve do this. for 8 marks

Butterworth filter design will be attatched to this question. (10-12 marks)

Q Convert the analog filter ~~to~~ with system f^n ~~H(s)~~

$H(s) = \frac{s+0.1}{(s+0.1)^2+9}$ $\rightarrow (\Omega_c^2)$ into a ~~dgt~~ digital IIR filter using bilinear transformation. The digital filter should have a ~~and~~ resonant freq $\omega_r = \frac{\pi}{4}$ rad/sec

[Here T_s is not given, we have to use the formula to map

$\Omega = \frac{2}{T_s}\tan\left(\frac{\omega}{2}\right)$

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Date: ....

$$\frac{7.243(z-1)+0.1(z+1)}{z+1} = \frac{7.243z-7.243+0.1z+0.1}{(z+1)}$$

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$*\ \Omega_c = \frac{2}{T_s}\tan\left(\frac{\omega_r}{2}\right)\ *$

A/Q $9^2 = \Omega_c^2 \Rightarrow \omega_c = \pm 3$ Take $+3$ as freq can't be $-ve$

$\therefore\ 3 = \frac{2}{T_s}\tan\left(\frac{\pi/4}{2}\right)$

$T_s = \frac{2}{3}\tan\left(\frac{\pi}{8}\right) = 0.27614$ rad/sec

$S = \frac{2}{0.276}\left(\frac{z-1}{z+1}\right)$

$= 7.243\left(\frac{z-1}{z+1}\right)$

$$H(z) = \frac{7.243\left(\frac{z-1}{z+1}\right)+0.1}{\left[7.243\left(\frac{z-1}{z+1}\right)+0.1\right]^2+9}$$

$$= \frac{(7.246z-7.246+0.1z-0.1)/(z+1)}{\left(\frac{7.246z-7.246+0.1z-0.1}{z+1}\right)^2+9}$$

$$= \frac{(7.346z-7.146)(z+1)}{(7.346-7.146)^2+9(z+1)^2}$$

$$= \frac{7.346z^2+0.2z-7.146}{(53.96z^2+51.06-104.98z)+(9z^2+18z+9)}$$

$$= \frac{7.346z^2+0.2z-7.146}{62.96z^2-86.98z+60.06}$$

$$= \frac{\cancel{z^2}(7.346+0.2z^{-1}-7.146z^{-2})}{\cancel{z^2}(62.96-86.98z+60.06z^{-2})}$$

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$=$

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$$\frac{\cancel{7.346}\ 7.346\left(1+0.027z^{-1}-0.972z^{-2}\right)}{7.346\left(8.57-11.84z^{-1}+8.175z^{-2}\right)}$$

$$\boxed{H(z)=\frac{1+0.027z^{-1}-0.972z^{-2}}{8.57-11.84z^{-1}+8.175z^{-2}}}$$

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$$\Omega_c=\frac{2}{T_s}\tan\left(\frac{\omega_r}{2}\right)$$

A/o $9^2=\Omega_c^2 \Rightarrow \omega_c=\pm 3$ Take $+3$ as freq cant be $-$ve

$$\therefore 3=\frac{2}{T_s}\tan\left(\frac{\pi/4}{2}\right)$$

$$T_s=\frac{2}{3}\tan\left(\frac{\pi}{8}\right)=0.27614 \rightarrow \text{rad/sec.}$$

$$\therefore \boxed{H(z)=\frac{1+0.027z^{-1}-0.973z^{-2}}{8.572-11.84z^{-1}+8.177z^{-2}}}$$

$$s=\frac{2}{0.2761}\left(\frac{z-1}{z+1}\right)$$

$$\frac{Az^2\pm Bz+C}{Pz^2\pm qz+r}$$

$$\therefore H(z)=\frac{\left[\frac{2}{0.2761}\left(\frac{z-1}{z+1}\right)+0.1\right]}{\left(\frac{2}{0.2761}\left(\frac{z-1}{z+1}\right)+0.1\right)^2+9}$$

$$=\frac{\frac{7.2437z-7.2437}{z+1}+0.1}{\left(\frac{7.2437z-7.2437}{z+1}+0.1\right)^2+9}$$

$$=\frac{\frac{7.2437z-7.2437+0.1z+0.1}{z+1}}{\left(\frac{7.2437z-7.2437+0.1z+0.1}{z+1}\right)^2+9}$$

$$=\frac{7.3437z-7.3437/z+1}{\frac{(7.3437z-7.3437)^2}{(z+1)^2}+9}$$

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$$= \frac{(7.3437z - 7.3437)/\cancel{(z+1)}}{\dfrac{(7.3437z - 7.3437)^2 + 9(z+1)^2}{(z+1)^{\cancel{2}}}}$$

$$= \frac{(7.3437z - 7.3437)(z+1)}{53.875z^2 - 107.854z + 53.876 + 9z^2 + 18z + 9}$$

$$= \frac{7.3437z^2 - 7.3437z + 7.3437z - 7.3437}{62.875z^2 - 89.84z + 62.875}$$

Filter Specs (in Hz)

F_s — sampling freq

f_c — passband cutoff

f_s — stopband "

Δ_p — attenuation in dB at f_c

Δ_s = " " " " f_s

ω_p = digital passband freq

ω_s = " stop " " in rad/s

IIM

If sampling freq is given,

$$\omega = 2\pi f$$

$$\omega_p = 2\pi\left(\frac{f_p}{F_s}\right)$$

$$\omega_s = 2\pi\left(\frac{f_s}{F_s}\right)$$

otherwise keep $F_s = 1$

Find: Order of N, Cutoff freq

Q. Design a 2nd order discrete time butterworth filter with cutoff freq of 1KHz & sampling freq 10^4 sam/sec by implementing Bilinear Transformation

an. In this particular prob, spec of digital filter is not ~~se~~ given directly.

∴ obtain a digital filter & convert the same into Analog filter

Given, $N = 2$

Analog ~~Cut~~ cutoff freq $F_c = 1KHz = 1000 Hz$

Sampling freq $F_s = 10^4$ samples/sec.

First we have to convert continuous freq into discrete freq.

$$f_c = \frac{F_c}{F_s} = \frac{10^3}{10^4} = 10^{-1} \text{ cycles/sec}$$

Now angular freq

$$\omega_c = 2\pi f_c = 0.2\pi \text{ rad/sec}$$

For ~~bise~~ BLT method $\Omega_c = \frac{2}{T} \tan\left(\frac{\omega_c}{2}\right)$

In this problem, sampling time is directly not given

$$\therefore T_s = \frac{1}{F_s} = \frac{1}{10^4} = 10^{-4} \text{ sec}$$

$$\therefore \Omega_c = \frac{2}{10^{-4}} \tan\frac{0.2\pi}{2} = 6.49 \times 10^3 \text{ rad/s}$$

Cal^n of poles

$$P_i = \pm \Omega_c e^{j(N+2i+1)\frac{\pi}{2N}} \quad i = 0, 1, 2, \ldots N-1$$

For $i = 0$

$$P_0 = \pm 6.5 \times 10^3 e^{j(2+2\times 0+1)\frac{\pi}{2\times 2}} = 6.5 \times 10^3 e^{(j3\pi/4)}$$

$$= \pm 6.5 \times 10^3 \left[\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right]$$

$$P_{0+} = -4.596 \times 10^3 + j4.596 \times 10^3$$

$$P_{0-} = 4.596 \times 10^3 - j4.596 \times 10^3$$

Skipped 1,2,3,4

Date : ....

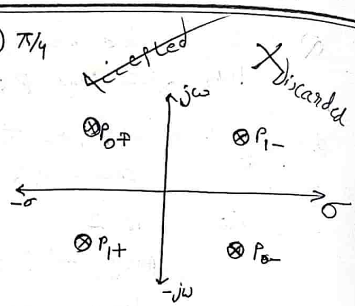
Illy For i=1

$$P_1 = \pm 6.5\times10^3\, e^{j(2+2\times1+1)\pi/4}$$

$$= \pm 6.5\times10^3\, e^{j\frac{5\pi}{4}}$$

$P_{1+} = -4595.05 - j4595.05$

$P_{1-} = -4595.05 + j4595.05$



Analog filter is stable if all the poles lie on the LHS of S plane.
∴ To determine the analog TF. Consider the poles that lie on LHS and discard the poles ~~be~~ lying on the RHS To determine $H_a(s)$

Step 10
To eliminate poles in RHS

Now ~~$H_a(s)$~~ determine the analog transfer function.

Take i=1

$$H_a(s) = \frac{\Omega_c^N}{(s-s_1)(s-s_1^*)}$$

$s_1 = -4595.05 + j4595.05$

$s_1^* = -4595.05 - j4595.05$ Complex conjugate

$$\therefore H_a(s) = \frac{\Omega_c^N}{[s-(-4595.05+j4595.05)][s-(-4595.05 \mp j4595.05)]}$$

$$= \frac{(6.4\times10^3)^2}{s^2 + 9190.1 s + 42.22\times10^3}$$

By replacing s by $\frac{2}{T_s}\left(\frac{z-1}{z+1}\right)$

Altered Step 11 from Table (expanded form)

$$H_a(s) = \frac{6.4\times10^6}{\left[\frac{2}{T_s}\left(\frac{z-1}{z+1}\right)\right]^2 + 9190.1\,\frac{2}{T_s}\,\frac{z-1}{z+1} + 42.22\times10^3}$$

The End

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Date : ....

Ex 1 H/W Ans

Numerical design example

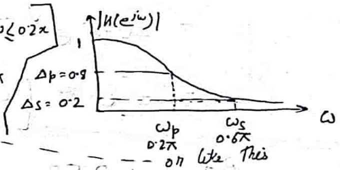
Sampling freq $F_s = 20\text{KHz}$, passband freq $f_p = 2\text{KHz}$, Stopband freq $f_s = 3\text{KHz}$ with $\Delta_p \geq -1\text{dB}$ and $\Delta_{stop} \leq -15\text{dB}$.

Using BLT design a Butterworth filter which specifies the following conditions

$0.8 \leq |H(e^{j\omega})| \leq 1$ for $0\leq\omega\leq0.2\pi$

$|H(e^{j\omega})| \leq 0.2$ for $0.6\pi\leq\omega\leq\pi$

Q can be given like this - or like this



Given

$\Delta_p = 0.8$, $\Delta_s = 0.2$, $\omega_p = 0.2\pi$ $\omega_s = 0.6\pi$

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} = 2\tan\frac{0.2\pi}{2} = 0.65$$ Take T = 1

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} = 2\tan\frac{0.6\pi}{2} = 2.752$$

$$N = \left[\frac{1}{2}\;\frac{\log(\alpha_p-1)-\log(\alpha_s-1)}{\log(\Omega_p)-\log(\Omega_s)}\right] = \frac{1}{2}\left[\frac{\log[0.8-1]-\log[0.2-1]}{\log 0.65 - \log 2.75}\right]$$

Don't use this

$$N = \frac{1}{2}\;\frac{\log\left[\frac{1}{\Delta_s^2}-1\right]/\left[\frac{1}{\Delta_p^2}-1\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} \simeq 1.3$$ Use this

Use both formula instead of pdf

$\therefore N = \text{ceil } 1.3 = 2$

$$\text{Cutoff freq}^n\ \Omega_c = \frac{\Omega_p}{\left(\frac{1}{\Delta_p^2}-1\right)^{1/2N}} = \frac{0.65}{\left(\frac{1}{(0.8)^2}-1\right)^{\frac{1}{2\times2}}} = 0.75$$

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Data : ....

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Determination of Poles

$$P_i = \pm \Omega_c \, e^{\frac{j(N+2i+1)\pi}{2N}}$$

$N-1 = 2-1 = 1$

$i = 0 \to 1$

$$P_0 = \pm 0.75 \, e^{j\frac{(2+2\times0+1)\pi}{2\times2}}$$

$$= \pm 0.75 \, e^{j\frac{3\pi}{4}}$$

$$= \pm 0.75 \left[\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right]$$

$P_{0+} = -0.53 + j0.53$

$P_{0-} = 0.53 - j0.53$

$$P_1 = \pm \Omega_c \, e^{\frac{j(N+2\times1+1)\pi}{2N}}$$

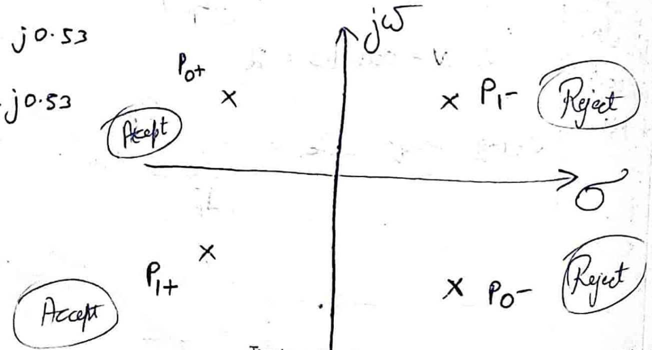
$$= \pm 0.75 \, e^{j\frac{(2+2\times1+1)\pi}{2\times2}}$$

$$= \pm 0.75 \, e^{j\frac{5\pi}{4}}$$

$$= \pm 0.75 \left[\cos\frac{5\pi}{4} + j\sin\frac{5\pi}{4}\right]$$

$P_{1+} = -0.53 - j0.53$

$P_{1-} = 0.53 + j0.53$



Date : ....

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$$H_a(s) = \frac{\Omega_c^N}{(s-s_1)(s-s_1)^*}$$

$$= \frac{0.75^2}{[s-(-0.53+j0.53)][s-(-0.53-j0.53)]}$$

$$= \frac{0.5625}{[s+0.53-j0.53][s+0.53+j0.53]}$$

$$= \frac{0.5625}{\underbrace{s^2+0.53s}+\cancel{j0.53s}+\underbrace{0.53s}+0.2809+\cancel{0.2809j}-\cancel{js0.53}-\cancel{0.2809j}+0.2809}$$

$$= \frac{0.5625}{s^2+1.065s+0.56}$$

$\therefore H(z)$ -

Put $s = \frac{2}{T}\frac{z-1}{z+1}$

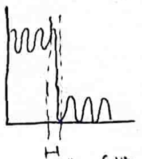
in $H_a(s)$ to get $H(z)$

$$H(z) = \frac{0.56\,(z+1)^2}{6.68z^2 - 6.88z + 2.44}$$

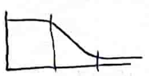
Chebyshev Filter Approximation

Date: 16/05/23

i) Type-I Chebyshev Filter → All pole filter
→ In passband equiripple behaviour
→ Monotonic characteristics in stop band or equiripple

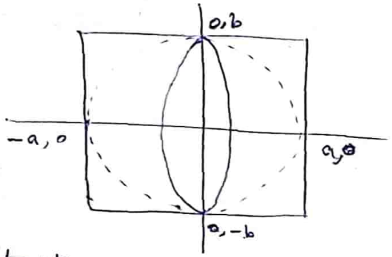


Transition filter of Cheby



⊛ Poles of butterworth filter lie on a circle where as the ~~poles~~ poles of chebyshev " " " an ~~ellipse~~ eclipse

Transition from ~~SB~~ PB to SB is better in Chebyshev than butter



~~Design a low pass filter with a cutoff freq 1rad/sec~~

Design a low pass 1rad/sec bandwidth $\to \Omega_p$ chebyshev filter with following characteristics

(i) Acceptable passband ripple of 2dB. $\to A_p$
(ii) Cutoff freq of 1rad/sec $\to \Omega_c$
(iii) Stopband attenuation of −20 dB or beyond 1.3 rad/sec.

passband equiripple

stopband equiripple

C Type I

We use type I if not given in Q

monotonic passband

C-type 2

Sol: ∴ Cutoff freq is given as 1rad/sec which is also normalized lowpass chebyshev filter characteristics.

Take 1rad/sec By default

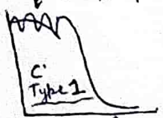
Golden rule ☆☆

Stopband attenuation value needs to be fixed as $|H(j\Omega)|$ to calⁿ the order of filter

$$|H(j\omega)| = -20\,dB \text{ in this problem}$$

Stopband equiripple

equiripple pB



C Type 1

monotonic stopband

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Transfer fⁿ for Chebyshev filter

$$|H(\Omega)|^2 = \frac{1}{1+\varepsilon_p^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)}$$

where $C_N\left(\frac{\Omega}{\Omega_p}\right) \to$ Chebyshev polynomial of order N

$\varepsilon_p \to$ Ripple parameter in PB

$\Omega_p \to$ passband freq

$$C_{N+1}(x) = 2x C_N(x) - C_{N-1}(x)$$

with $C_0(x) = 1$ and $C_1(x) = x$

if N=1

$$C_2(x) = 2x C_1(x) - C_0(x)$$
$$\Rightarrow C_2(x) = 2x\cdot x - 1$$
$$\boxed{\Rightarrow C_2(x) = 2x^2 - 1}$$

If N = 2

$$C_3(x) = 2x C_2(x) - C_1(x)$$
$$= 2x(2x^2-1) - x$$
$$= 4x^3 - 2x - x$$
$$\boxed{C_3(x) = 4x^3 - 3x}$$

???

$$\therefore |H(\Omega)|^2 = \frac{1}{1+\varepsilon_p^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)}$$

$$\varepsilon_p = \left(10^{0.1 A_p (\text{in dB})} - 1\right)^2$$

if A_p is not in dB then $\varepsilon_p = \left[\frac{1}{A_p^2} - 1\right]^{1/2}$

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Date :

Cutoff freq if $\Omega_c = 1 \rightarrow$ Normalized filter

then $|H(j\Omega)| = \frac{1}{\sqrt{1-\varepsilon^2}}$

$$\boxed{\text{order of filter } \cancel{H(j\Omega)}\ -A_s(dB) = -20\log_{10}\varepsilon - 6(N-1)\ 20N\log(\Omega_s)}$$

$$\varepsilon = \left[10^{0.1A_p} - 1\right]^{1/2} = \left(10^{0.1\times 2} - 1\right]^{1/2} = 0.765$$

we have been given

$\Omega_c = 1$ rad/sec

$\Omega_p = 1$ rad/sec

$\Omega_s = 1.3$ rad/sec

Finding N

$\cancel{-20dB = -20\log_{10} \times 0.765 - 6[N-1]\ 20N\log(1.3)}$

$\cancel{\Rightarrow N = 3.421 \approx 4}$ | $\cancel{1.87 \approx 2}$

$$-20 = -20\log_{10}(0.765) - 6(N-1) - 20N\log_{10}(1.3)$$

$$6(N-1) + 20N\log_{10}(1.3) = 20 - 20\log_{10}(0.765)$$

$$6N - 6 + 2.278N = 22.3267$$

$$8.278N = 28.3267$$

$$N = \frac{28.3267}{8.378} = 3.421 \approx 4$$

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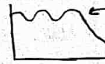
Date : 18/05/23

~~Here we will see Type I Chebyshev Filter~~

$$\boxed{S_p = r\cos\theta_i + jR\sin\theta_i}$$ Determine the poles with

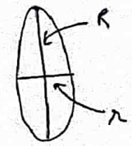


pole positions are given by this formula

r is the minor axis and R is the major axis.

i range from 0 to 3 i.e. (N-1) i.e. 0, 1, 2, 3

where $\boxed{\theta_i = \frac{\pi}{2} + \frac{(2i+1)\pi}{2N}}$



$\boxed{r = \Omega_p \frac{\beta^2-1}{2\beta}}$ $\boxed{R = \Omega_p \frac{\beta^2+1}{2\beta}}$

$$\boxed{\beta = \left[\frac{\sqrt{1+\varepsilon^2}+1}{\varepsilon}\right]^{1/N}}$$

$$\beta = \left[\frac{\sqrt{1+0.765^2}+1}{0.765}\right]^{1/4} = 1.31089$$

$$\therefore r = 1 \times \frac{1.3108^2 - 1}{2\times 1.31} = 0.274$$

$$R = 1 \times \frac{1.3108^2 + 1}{2\times 1.31} = 1.0366$$

$$\theta_i = \frac{\pi}{2} + \frac{(2i+1)\pi}{2\times 4}$$

$\theta_0 = \frac{\pi}{2} + \frac{(2\times 0+1)\pi}{8} = 1.96$ ($5\pi/8$)

$\theta_1 = \frac{\pi}{2} + \frac{(2\times 1+1)\pi}{8} = 2.74$ ($7\pi/8$)

$\theta_2 = \frac{\pi}{2} + \frac{(2\times 2+1)\pi}{8} = 3.53$ ($9\pi/8$)

$\theta_3 = \frac{\pi}{2} + \frac{(2\times 3+1)\pi}{8} = 4.32$ ($11\pi/8$)

Keep it in $x\frac{\pi}{y}$ form

$$S_p = r\cos\theta_i + jR\sin\theta_i$$

$S_0 = 0.274\cos\theta_0 + j1.0366\sin\theta_0 = -0.1 + j0.93$

$S_1 = -0.25 + 0.39$

$S_2 = -0.253 - 0.398$

$S_3 = -0.1047 - 0.93$

cc

cc

It should be complex conjugates

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analog TF.

Date : ....

Now, $H_a(s) = \dfrac{i}{(s-s_0)(s-s_1).(s-s_2)(s-s_3)}$ (Eq)

$$= \frac{i}{(s+0.1-j0.96)(s+0.25-j0.4)(s+0.25+j0.4)(s+0.1+j0.96)}$$

$$= \frac{1}{s^4 + 0.7s^3 + 1.2s^2 + 0.51s + 0.207}$$

Denominator should be decreasing order of s i.e. $As^4 + Bs^3 + Cs^2 + Ds + Es^0$

Here $N=4$ which is even,

$$\boxed{i = \frac{b_0}{\sqrt{1+\varepsilon^2}}}$$

If it is odd $\boxed{i = b_0}$

$\therefore b_0 =$ const term in deno $= 0.207$

$$i = \frac{0.207}{\sqrt{1+0.765^2}} = 0.16$$

After finding $H_a(s)$. put $s = \frac{2}{T}\left[\frac{z-1}{z+1}\right]$ and get $H(z)$ i.e. digital filter response

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