Ex:- By the method of least squares, find the straight line that best fits the following data.

x :	1	2	3	4	5
y :	14	27	40	55	68

Sol^n Let the st. line to fit the given data is, $y = ax + b$

Now, the normal eqn for st. lines are

$$\sum x_i y_i = a \sum x_i^2 + b \sum x_i \longrightarrow (I)$$

$$\sum y_i = a \sum x_i + nb \longrightarrow (II)$$

Now, we make the following table.

x_i	y_i	x_i^2	$x_i y_i$
1	14	1	14
2	27	4	54
3	40	9	120
4	55	16	220
5	68	25	340
$\sum x_i = 15$	$\sum y_i = 204$	$\sum x_i^2 = 55$	$\sum x_i y_i = 748$

Putting the values of $\sum x_i y_i$, $\sum x_i^2$, $\sum y_i$, $\sum x_i$ in (I) & (IV) we get,

$$748 = 55a + 15b \quad \rightarrow (III)$$

$$204 = 15a + 5b \quad \rightarrow (IV)$$

These are two linear eqns in two variable a and b.

Solving (III) & (IV) (upto you, how you solve it)

we get, $a = 13.6$, $b = 0$.

Now, the required straight line to fit given data is,

$$y = 13.6x$$

Ex:- Fit an equation of the form $y = a e^{bx}$ for the following data.

x :	1	2	3	4
y :	1·65	2·7	4·5	7·35

Sol^n The given curve is,

$$y = a e^{bx}.$$

$$\Rightarrow \log_e y = \log_e a + bx \quad [\text{taking log base } e]$$

$$\Rightarrow Y = A + bx. \rightarrow (1)$$

Now, the normal eq^n for (1) are,

$$\sum x_i Y_i = A \sum x_i + b \sum x_i^2 \rightarrow (II)$$

$$\sum Y_i = 4A + b \sum x_i \longrightarrow (III)$$

Now we make the following table.

x	y	$Y = \log y$	$x_i Y_i$	x_i^2
1	1·65	·50	·50	1
2	2·7	·99	1·98	4
3	4·5	1·50	4·5	9
4	7·35	1·99	7·96	16

$\sum x_i = 10$ $\quad \sum Y_i = 4·98$ $\quad \sum x_i Y_i = 14·94$ $\quad \sum x_i^2 = 30$

Putting in eqⁿ (II) & (III) we get,

$$14·94 = 30b + 10A \longrightarrow (IV)$$

$$4·98 = 10b + 4A \longrightarrow (V).$$

Solving (IV) & (V) we get,

$$A = 0, \quad b = ·498.$$

$$\Rightarrow \log_e a = 0$$

$$\Rightarrow a = 1.$$

∴ The required curve is, $y = e^{·498x}$