SIGNALS & SYSTEMS

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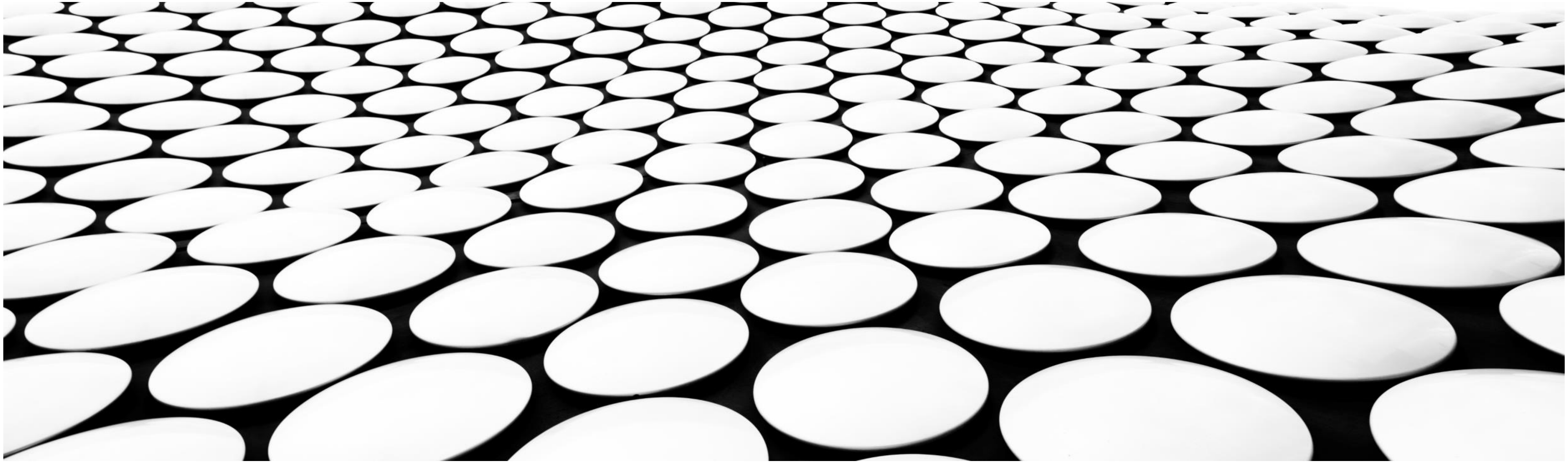


Find the Fourier transform of $x(t) = \cos\omega_0 t$.

Soln:- To determine the Fourier transform of $x(t) = \cos\omega_0 t$

First use Euler's identity - i.e. $\cos\omega_0 t = \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$$\Rightarrow \cos\omega_0 t = \frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$$

$$\Rightarrow FT[\cos\omega_0 t] = \frac{1}{2}\left[FT\left[e^{j\omega_0 t}\right] + FT\left[e^{-j\omega_0 t}\right]\right] \quad \text{--- (1)}$$

Here $e^{j\omega_0 t}$ is complex exponential function. Let us first calculate the FT of it.

As $e^{j\omega_0 t}$ is not absolutely integrable function we can't use direct formula to calculate the FT of it.

Let us first find → $x'(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X'(\omega)\, e^{j\omega_0 t}\, d\omega$ and to determine this let's consider

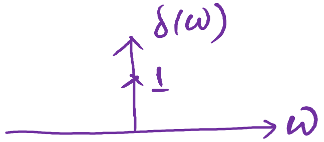
$$\boxed{X'(\omega) = \delta(\omega-\omega_0)} \quad \text{— (2)}$$

$$\Rightarrow x'(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\, e^{j\omega_0 t}\, d\omega$$

$$\Rightarrow x'(t) = \frac{e^{j\omega_0 t}}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\, d\omega$$

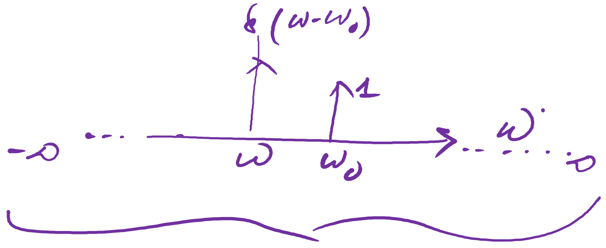
$$\Rightarrow x'(t) = \frac{e^{j\omega_0 t}}{2\pi}\cdot 1$$

$$\Rightarrow \boxed{x'(t) = \frac{e^{j\omega_0 t}}{2\pi}} \quad \text{— (3)}$$



∴ $\underline{\delta(\omega-\omega_0)}$

frequency shifting



We are integrated this signal over $-\infty$ to ∞ therefore over all result of integration will be 1.

From all these discussions we obtained that –

$$x'(t) \xrightarrow{FT} \delta(\omega-\omega_0)$$ [we have assumed from eqn (2)]

From eqn (3)

$$\Rightarrow \frac{e^{j\omega_0 t}}{2\pi} \longrightarrow \delta(\omega-\omega_0)$$

$$\Rightarrow \cancel{2\pi} \cdot \frac{e^{j\omega_0 t}}{\cancel{2\pi}} \longrightarrow 2\pi\,\delta(\omega-\omega_0)$$ [Multiplying 2π on both the sides as we need to obtain $e^{j\omega_0 t}$] → homogeneity property.

$$\Rightarrow \boxed{e^{j\omega_0 t} \longrightarrow 2\pi\,\delta(\omega-\omega_0)} \quad \text{— (4)}$$

Similarly for $$\boxed{e^{-j\omega_0 t} \longrightarrow 2\pi\,\delta(\omega+\omega_0)} \quad \text{— (5)}$$

$\therefore$ (i) $\Rightarrow$ $FT[\cos\omega_0 t] = \frac{1}{2}\left[2\pi\delta(\omega-\omega_0) + 2\pi\delta(\omega+\omega_0)\right]$

$$= \frac{1}{\cancel{2}} \times \cancel{2}\pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

$$\boxed{FT[\cos\omega_0 t] = \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]}$$

Similarly, we can prove that $FT[\sin\omega_0 t] = j\pi\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$ #

$\frac{1+j\omega}{\cdot}$