BJT Small-Signal Analysis

Semiconductor Devices and Circuits

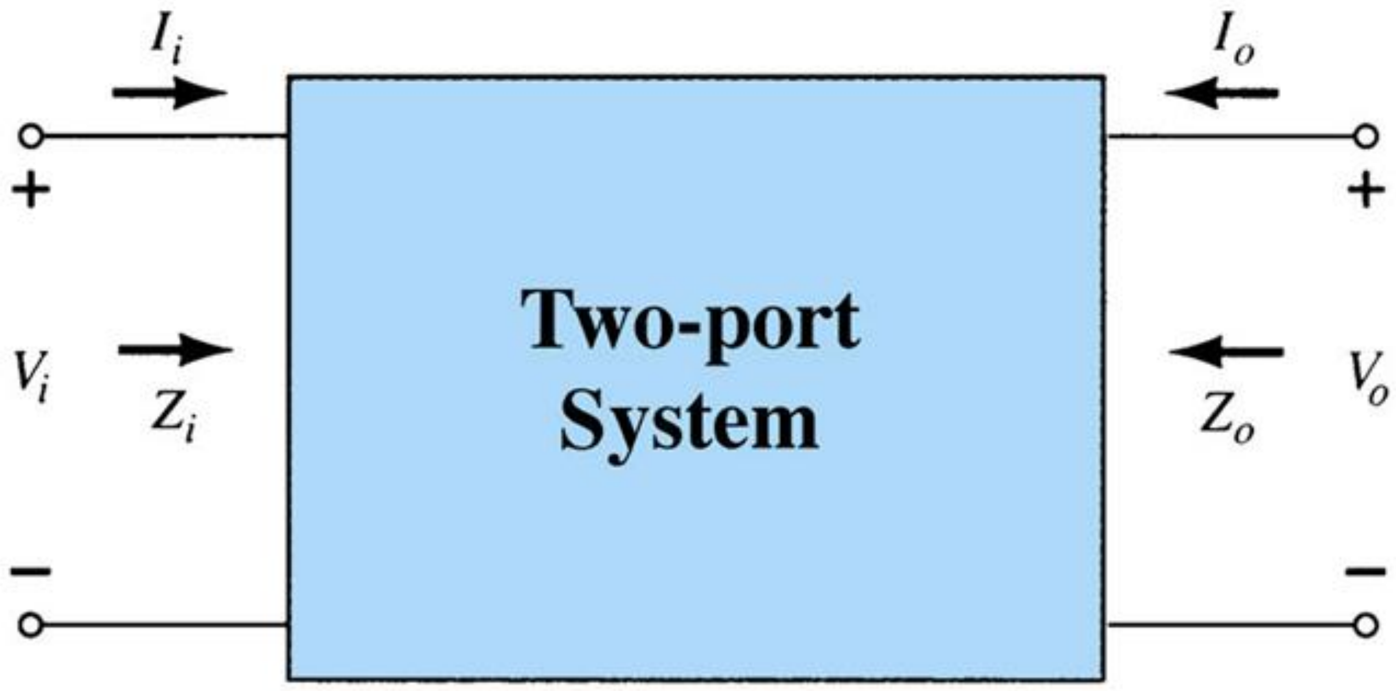
(ECE 181302)

28th December 2021

- BJT small-signal model is the equivalent circuit that represents the AC characteristics of the transistor.
- It uses circuit elements that approximate the behavior of the transistor.
- 2 models are commonly used in small signal AC analysis of a transistor:
    - r_e model
    - hybrid equivalent model

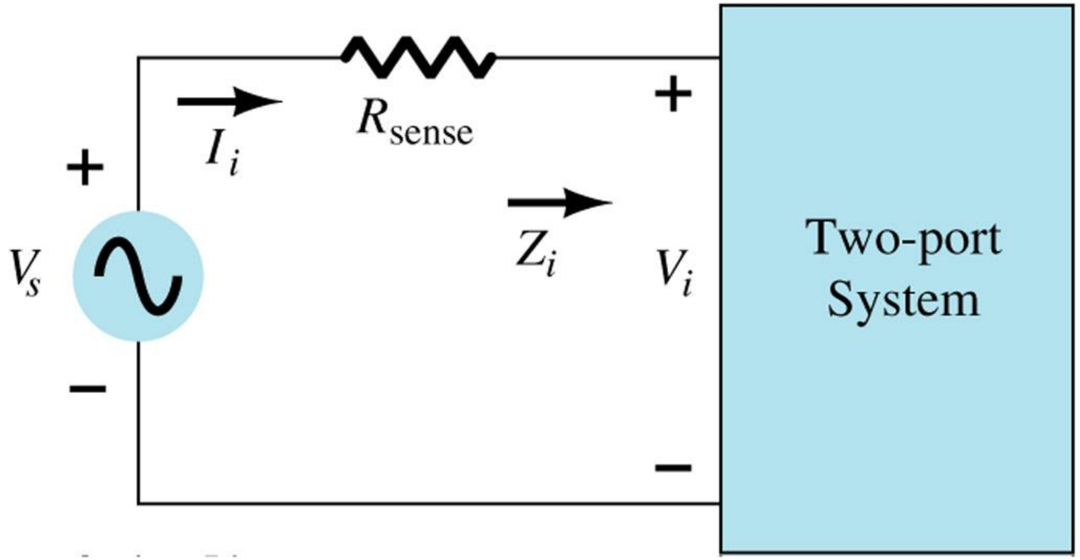
Important Parameters

- Zi, Zo, Av, Ai are important parameters for the analysis of the AC characteristics of a transistor circuit.



Input Impedance, Zi

- To determine Ii: insert a “sensing resistor”



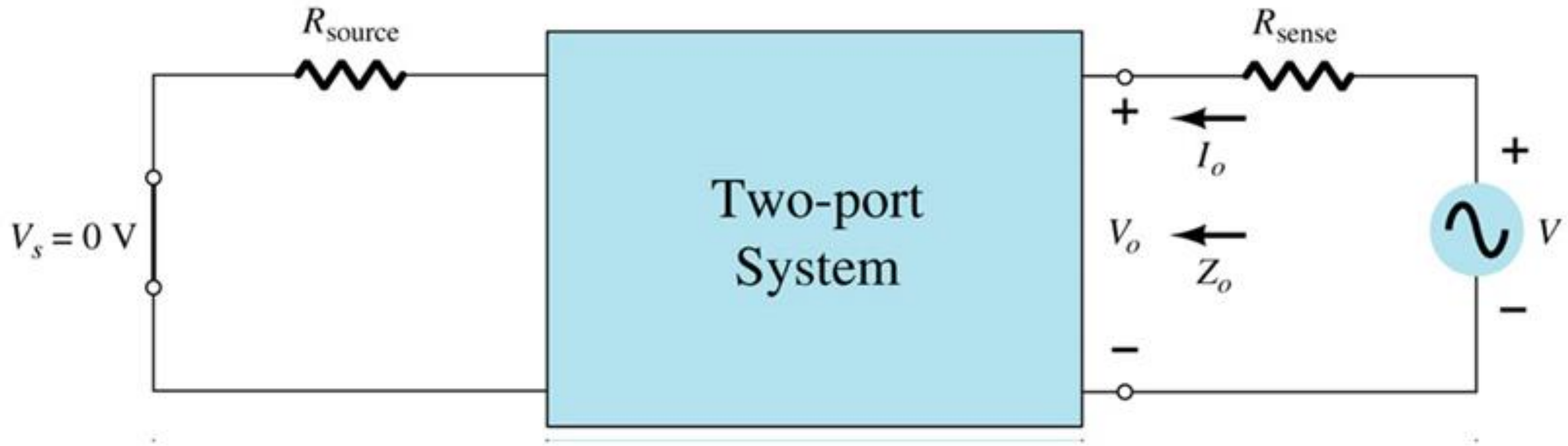
then calculate Ii:

$$Ii = \frac{Vs - Vi}{Rsense}$$

$$Zi = \frac{Vi}{Ii}$$

Output Impedance, Zo

- To determine Io: insert a “sensing resistor”



then calculate Io:

$$Io = \frac{V - Vo}{Rsense}$$

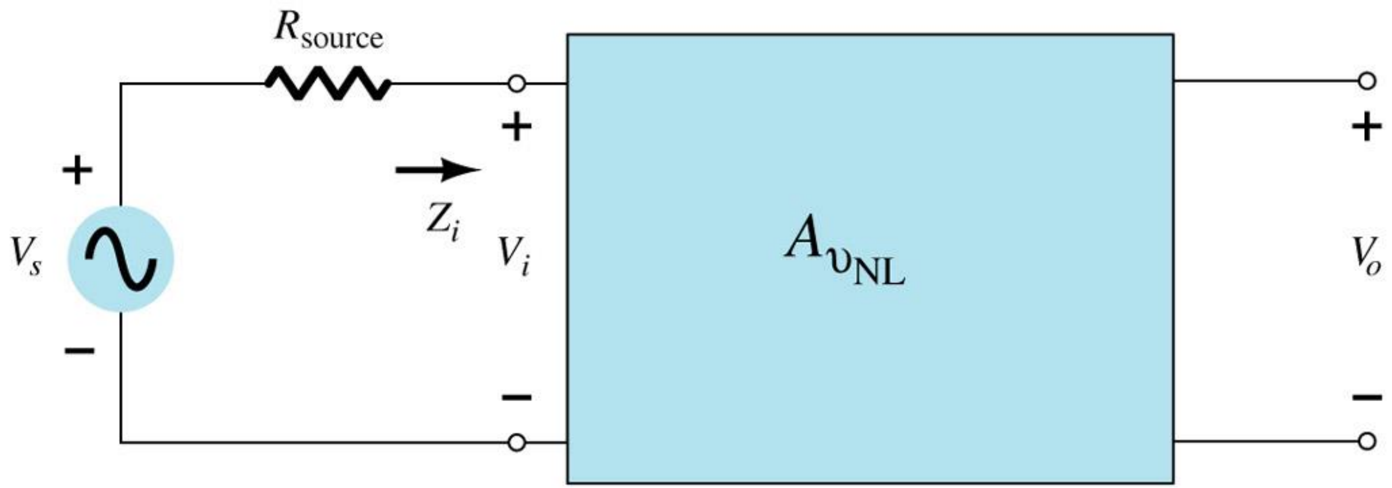
$$Zo = \frac{Vo}{Io}$$

Voltage Gain, Av

- For an amplifier with no load:

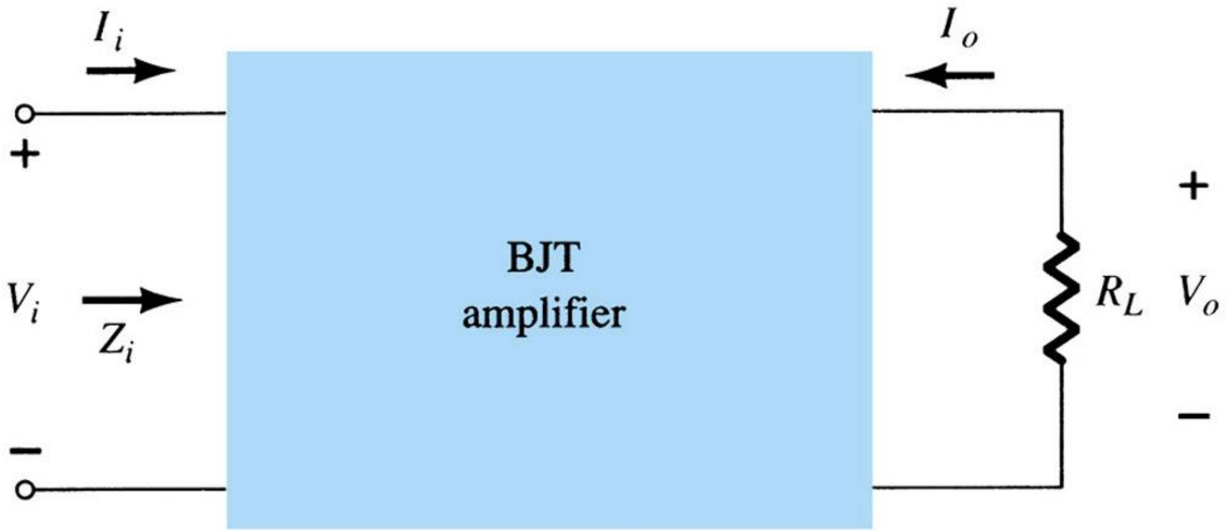
$$Av = \frac{Vo}{Vi}$$

$$A_{VNL} = \frac{Vo}{Vi} \Big/ R_L = \infty\Omega (\text{opencircuit})$$



- ***Note:*** the no-load voltage gain (A_{VNL}) is always greater
  - than the loaded voltage gain (A_V).

Current Gain, Ai



- The current gain (Ai) also be calculated using the voltage gain (Av):

$$Ai = -Av \frac{Zi}{R_L}$$

$$Ai = \frac{Io}{Ii}$$

Phase Relationship

- The phase relationship between input and output depends on the amplifier configuration circuit.

  - Common – Emitter ~ 180 degrees

  - Common - Base ~ 0 degrees

  - Common – Collector ~ 0 degrees

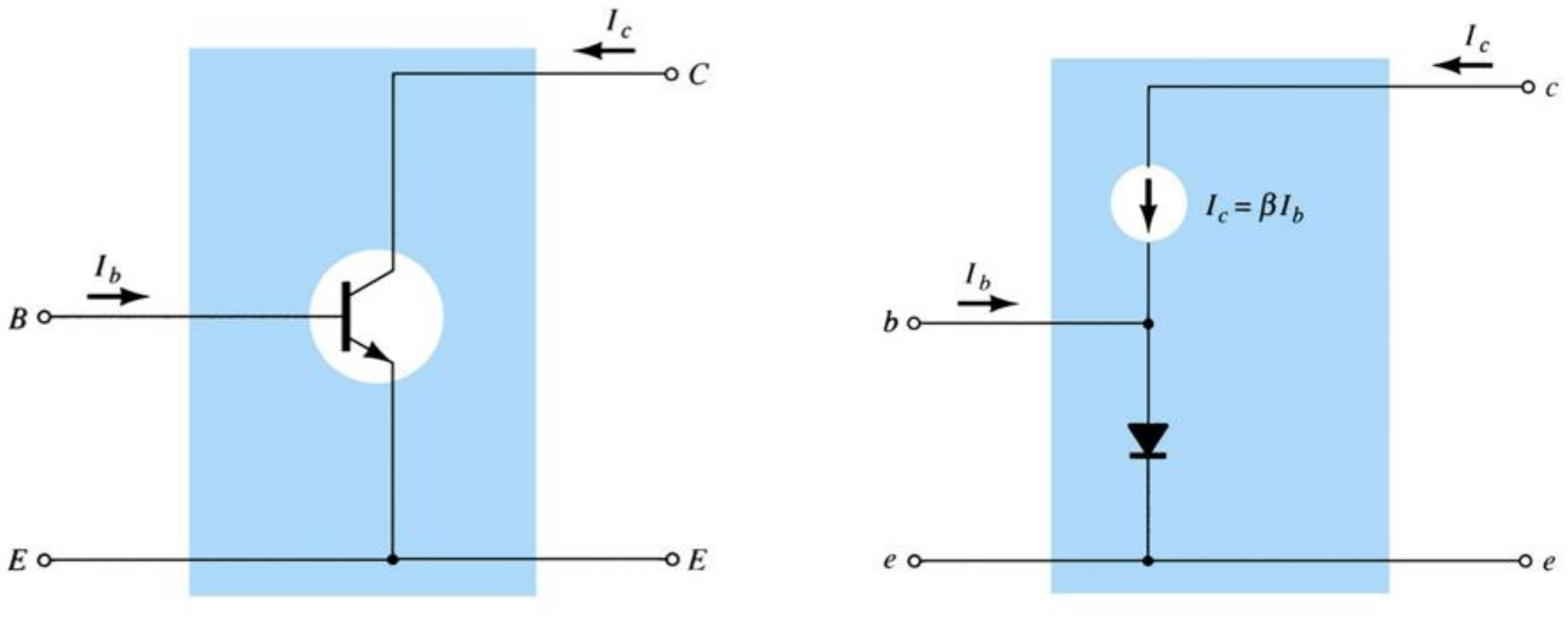
BJT *re* Model

- BJTs are basically current controlled devices, therefore the re model uses a diode and a current source to duplicate the behavior of the transistor.
- The r_e model employs a diode and controlled current source to duplicate the behavior of a transistor in the region of interest.
- The ac resistance of a diode can be determined by the equation r_{ac} = 26 mV/I_D, where I_D is the dc current through the diode at the *Q* (quiescent) point. This same equation is used to find the ac resistance of the diode in the BJT.
- Substituting the emitter current:

$$\boxed{r_e = \frac{26\ \text{mV}}{I_E}}$$

- The subscript *e* of r_e is chosen to emphasize that it is the dc level of emitter current that determines the ac level of the resistance of the diode.
- One disadvantage to this model is its sensitivity to the DC level.

CE *re* Model

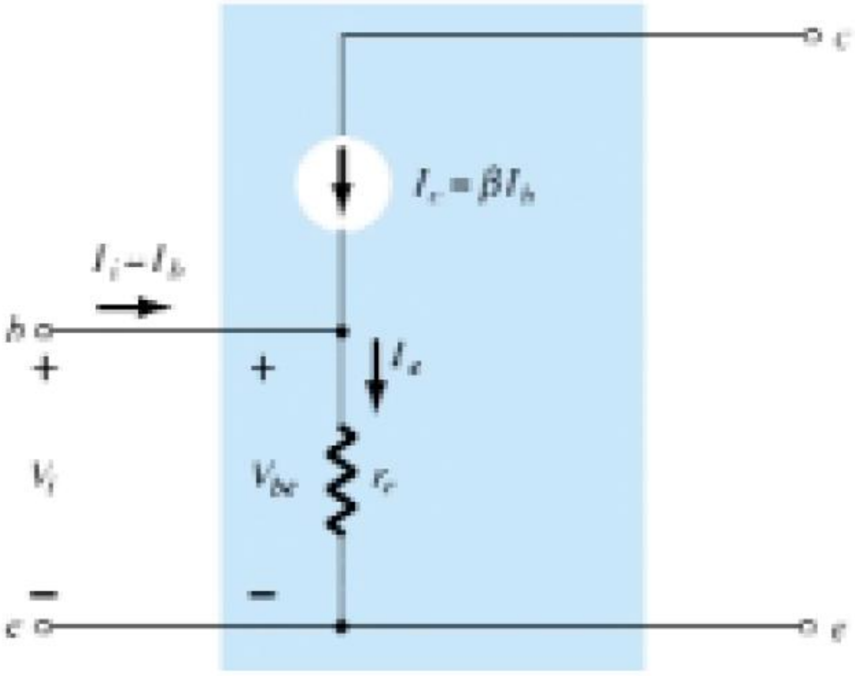


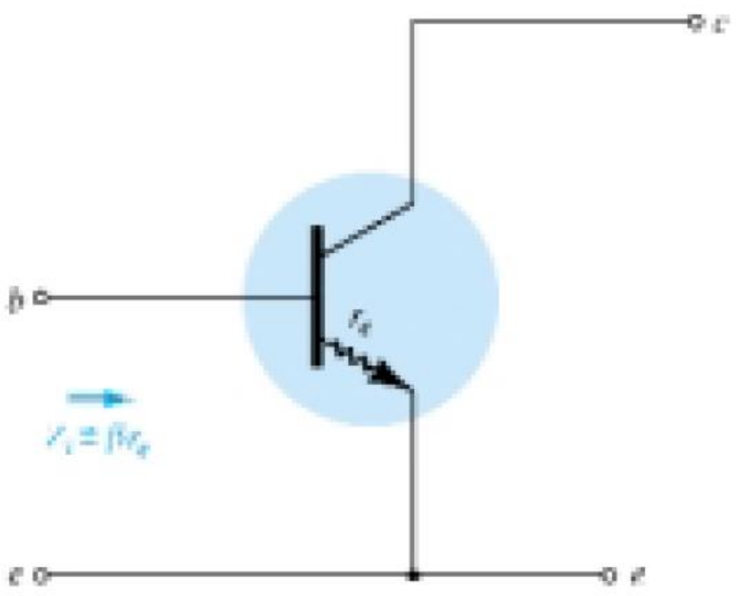
The base current is the input and the collector is the output.
This model indicates:

$$Ic = \mathrm{B}Ib$$

$$Ie = (\mathrm{B}+1)Ib$$

$$Ie \cong \mathrm{B}Ib$$





$$\boxed{I_c = \beta I_b}$$

$$I_e = I_c + I_b = \beta I_b + I_b$$

$$\boxed{I_e = (\beta + 1) I_b}$$

$$\boxed{I_e \cong \beta I_b}$$

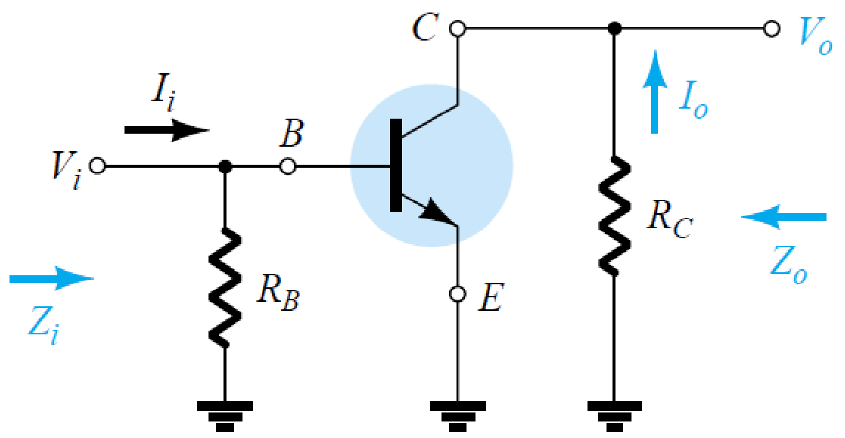
Impedance in Common-Emitter Configuration

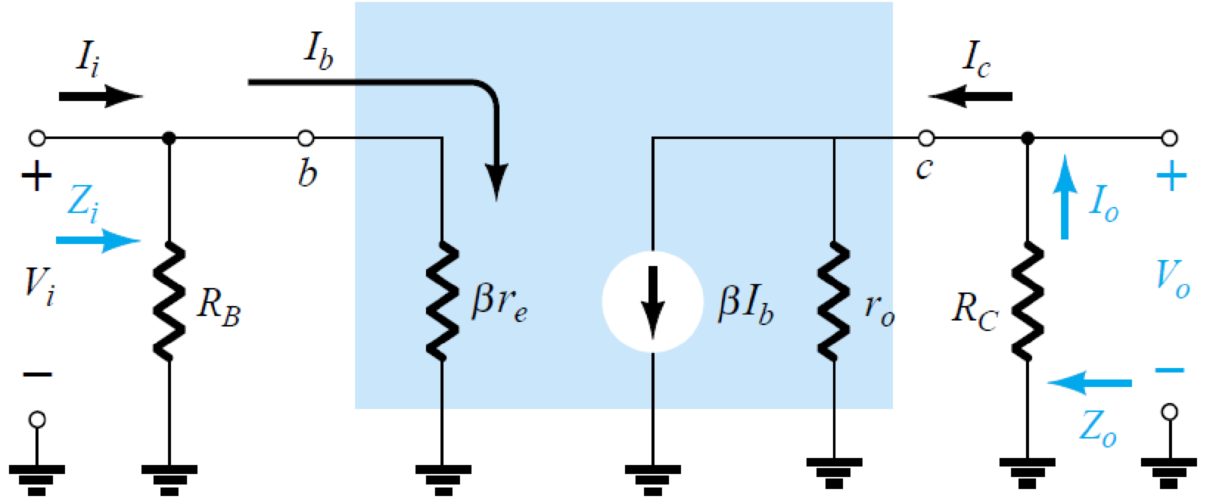
$$Z_i = \frac{V_i}{I_i} = \frac{V_{be}}{I_b}$$

$$V_i = V_{be} = I_e r_e \cong \beta I_b r_e$$

$$= \frac{V_{be}}{I_b} \cong \frac{\beta I_b r_e}{I_b}$$

$$\boxed{Z_i \cong \beta r_e}_{CE} = (160)(6.5\ \Omega) = \mathbf{1.04\ k\Omega}$$





$\boxed{Z_i = R_B \| \beta r_e}$ ohms Or $\boxed{Z_i \cong \beta r_e}_{R_B \geq 10\beta r_e}$

- *Zo*: The output impedance of any system is defined as the impedance *Zo* determined when *Vi* = 0. When *Vi* = 0, *Ii* = *Ib* = 0, resulting in an open-circuit equivalence for the current source. Therefore;

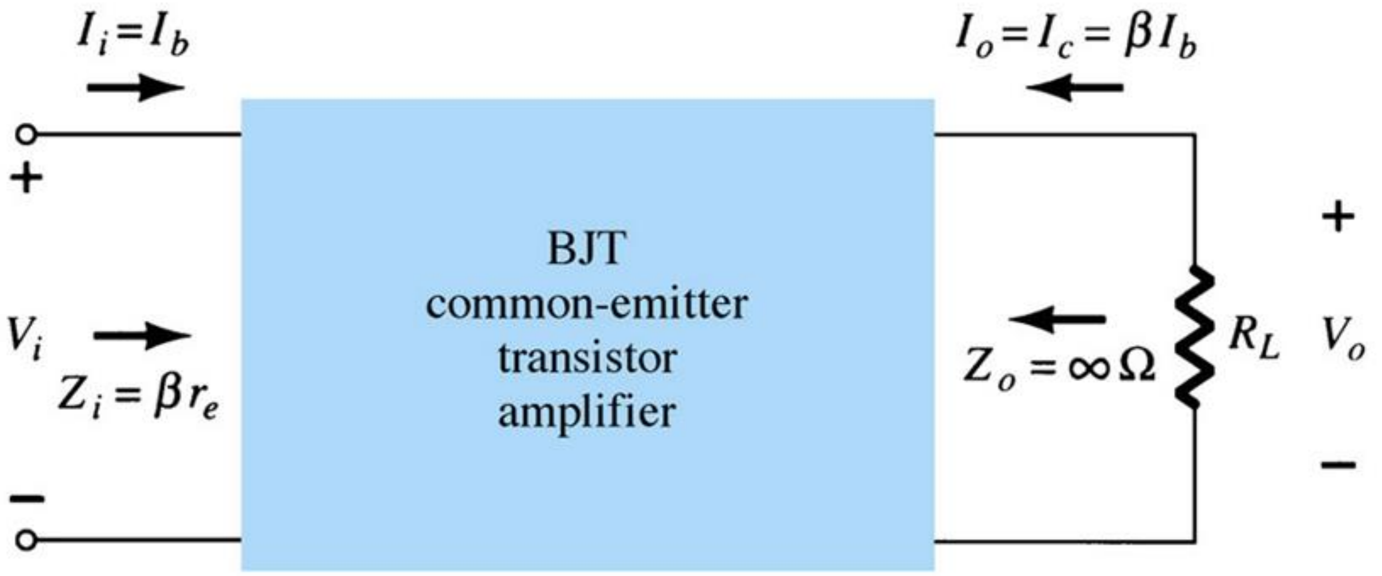
$$Zo = ro$$

$$Zo \cong \infty\Omega$$

$\boxed{Z_o = R_C \| r_o}$ ohms

$\boxed{Z_o \cong R_C}_{r_o \geq 10R_C}$

Gain calculations for the Common-Emitter using the re model



Voltage Gain (Av): $$Av = -\frac{R_L}{r_e}$$

Current Gain (Ai): $$Ai = B \Big/ ro = \infty\Omega$$

Example: For the network;

(a) Determine *re*.

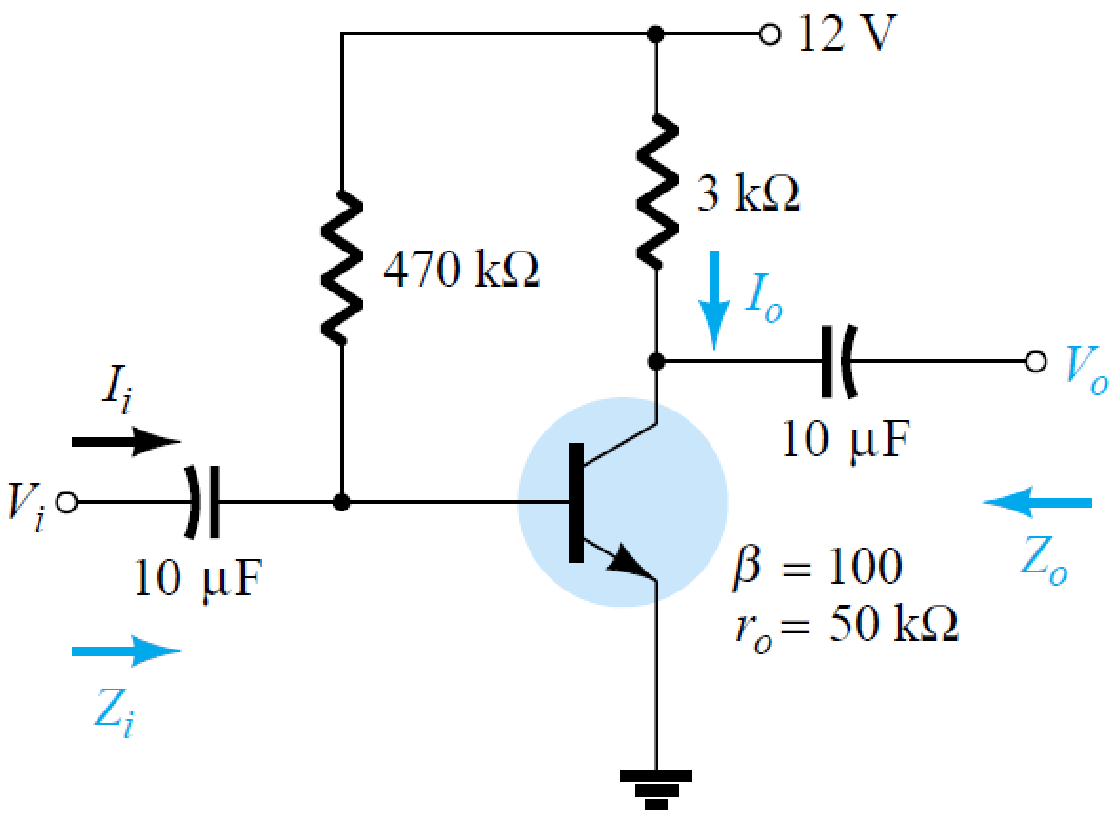
(b) Find *Zi* (with *ro* = ∞Ω).

(c) Calculate *Zo* (with *ro* = ∞Ω).

(d) Determine *Av* (with *ro* = ∞Ω).

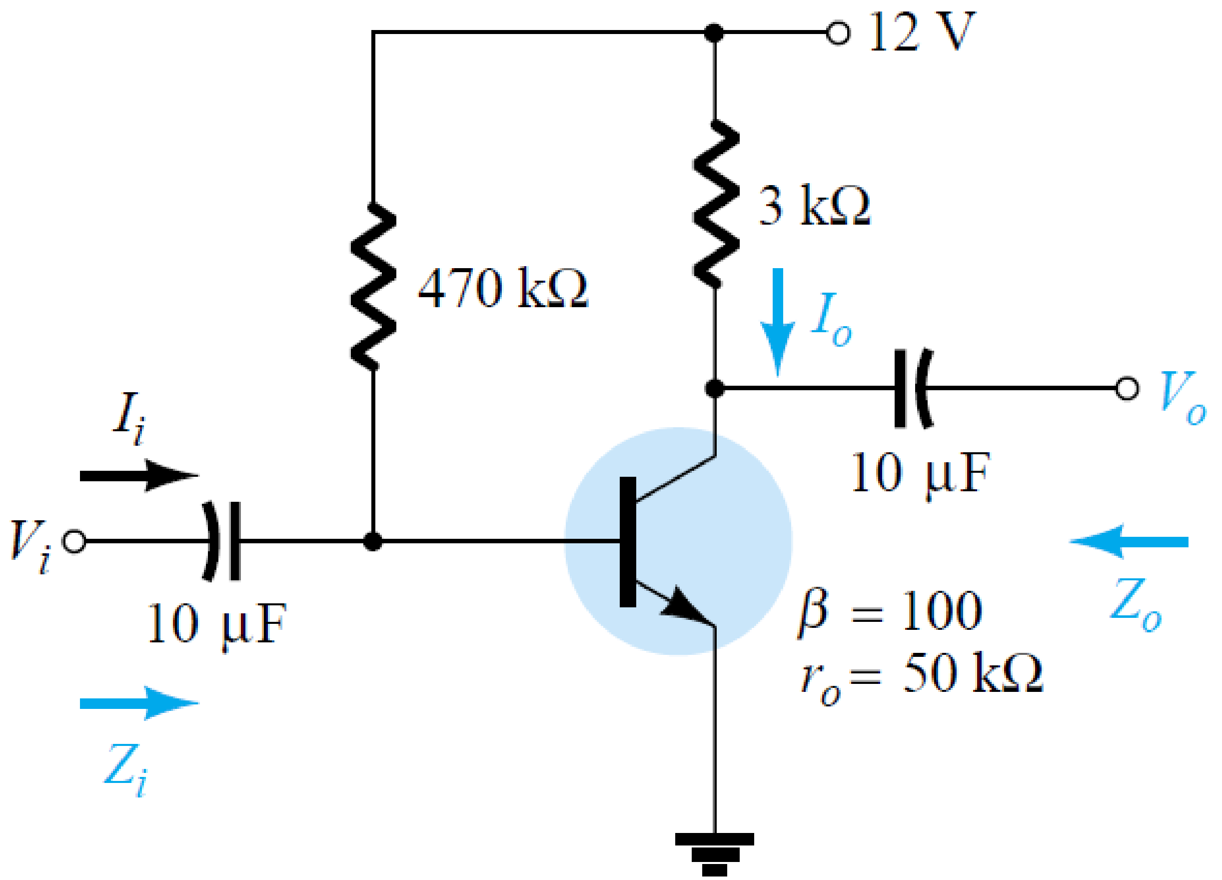
(e) Find *Ai* (with *ro* = ∞Ω).

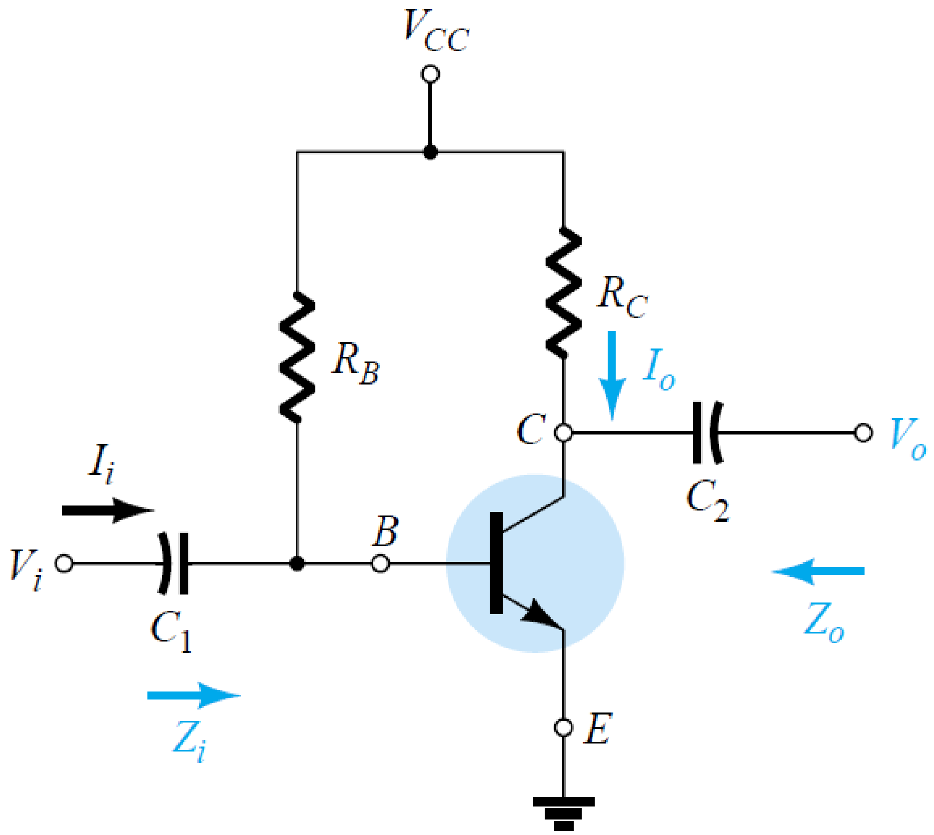
(f) Repeat parts (c) through (e) including *ro* = 50 kΩ in all calculations and compare results.



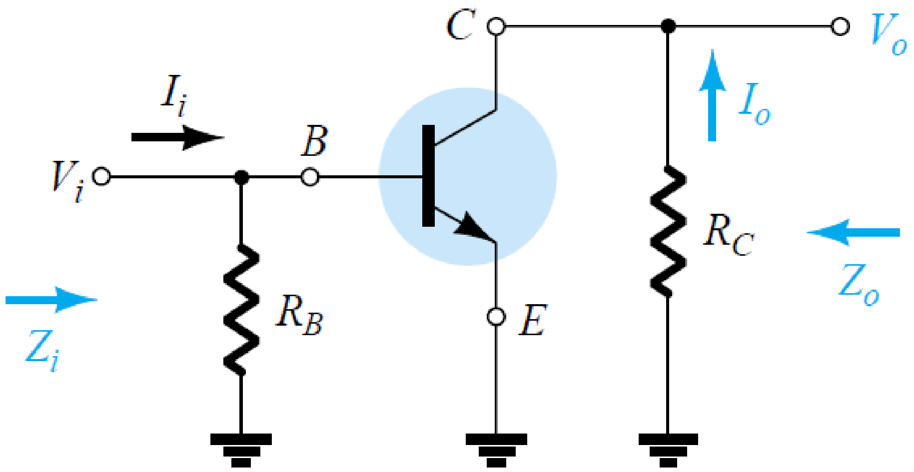
BJT AC Analysis Steps

- Mark the terminals of the transistor.
- Mark the current directions and define the voltages.
- Remove the dc effects of V_{CC} by connecting it to the ground.
- Ground V_{CC}.
- Replace the dc blocking capacitors by short-circuit equivalents.
- Redraw the circuit.
- Substitute the BJT by the r_e model into the network.


12 V
3 kΩ
470 kΩ
I_o
V_o
I_i
10 μF
V_i
10 μF
$\beta = 100$
$r_o = 50$ kΩ
Z_o
Z_i



V_{CC}
R_B
R_C
I_o
C
V_o
C_2
I_i
V_i
C_1
B
E
Z_o
Z_i



C
V_o
I_i
B
I_o
V_i
R_C
R_B
E
Z_o
Z_i

- Solution:

(a) DC analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12\text{ V} - 0.7\text{ V}}{470\text{ k}\Omega} = 24.04\ \mu\text{A}$$

$$I_E = (\beta + 1)I_B = (101)(24.04\ \mu\text{A}) = 2.428\text{ mA}$$

$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{2.428\text{ mA}} = \mathbf{10.71\ \Omega}$$

(b) $\beta r_e = (100)(10.71\ \Omega) = 1.071\text{ k}\Omega$

$$Z_i = R_B \| \beta r_e = 470\text{ k}\Omega \| 1.071\text{ k}\Omega = \mathbf{1.069\text{ k}\Omega}$$

(c) $Z_o = R_C = \mathbf{3\text{ k}\Omega}$

(d) $A_v = -\dfrac{R_C}{r_e} = -\dfrac{3\text{ k}\Omega}{10.71\ \Omega} = \mathbf{-280.11}$

(e) Since $R_B \geq 10\beta r_e (470\text{ k}\Omega > 10.71\text{ k}\Omega)$

$$A_i \cong \beta = \mathbf{100}$$

(f) $Z_o = r_o \| R_C = 50\text{ k}\Omega \| 3\text{ k}\Omega = \mathbf{2.83\ k\Omega}$ vs. $3\text{ k}\Omega$

$$A_v = -\frac{r_o \| R_C}{r_e} = \frac{2.83\text{ k}\Omega}{10.71\ \Omega} = \mathbf{-264.24}\text{ vs. } -280.11$$

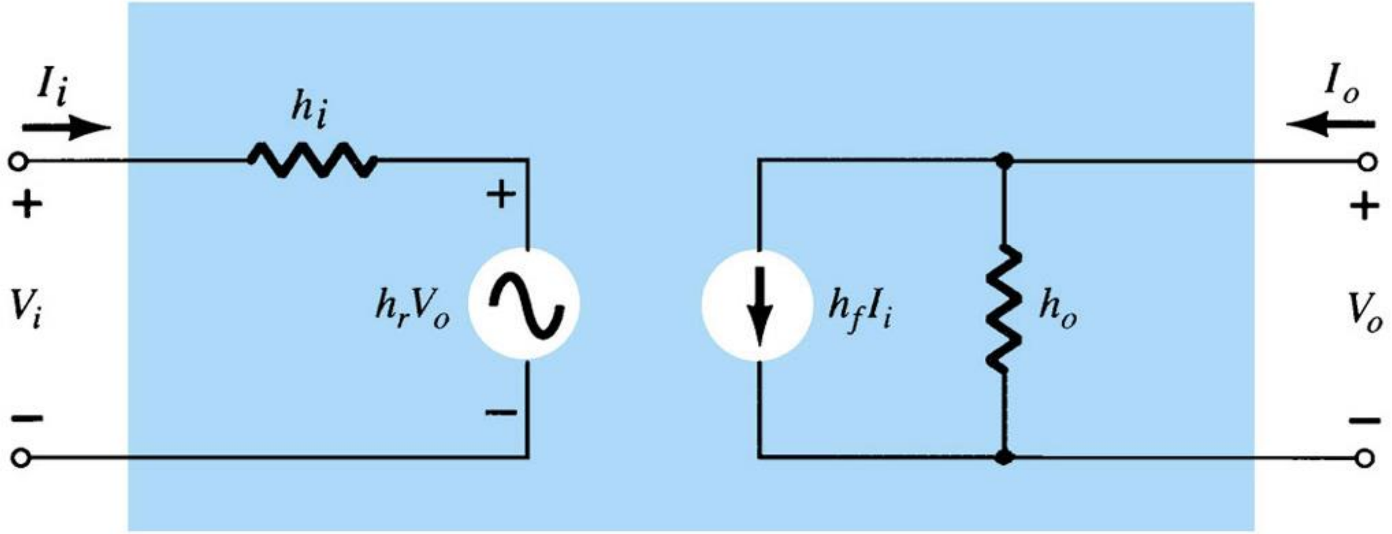
$$A_i = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)} = \frac{(100)(470\text{ k}\Omega)(50\text{ k}\Omega)}{(50\text{ k}\Omega + 3\text{ k}\Omega)(470\text{ k}\Omega + 1.071\text{ k}\Omega)}$$

$$= \mathbf{94.13}\text{ vs. } 100$$

$$A_i = -A_v\frac{Z_i}{R_C} = \frac{-(-264.24)(1.069\text{ k}\Omega)}{3\text{ k}\Omega} = \mathbf{94.16}$$

Hybrid Equivalent Model

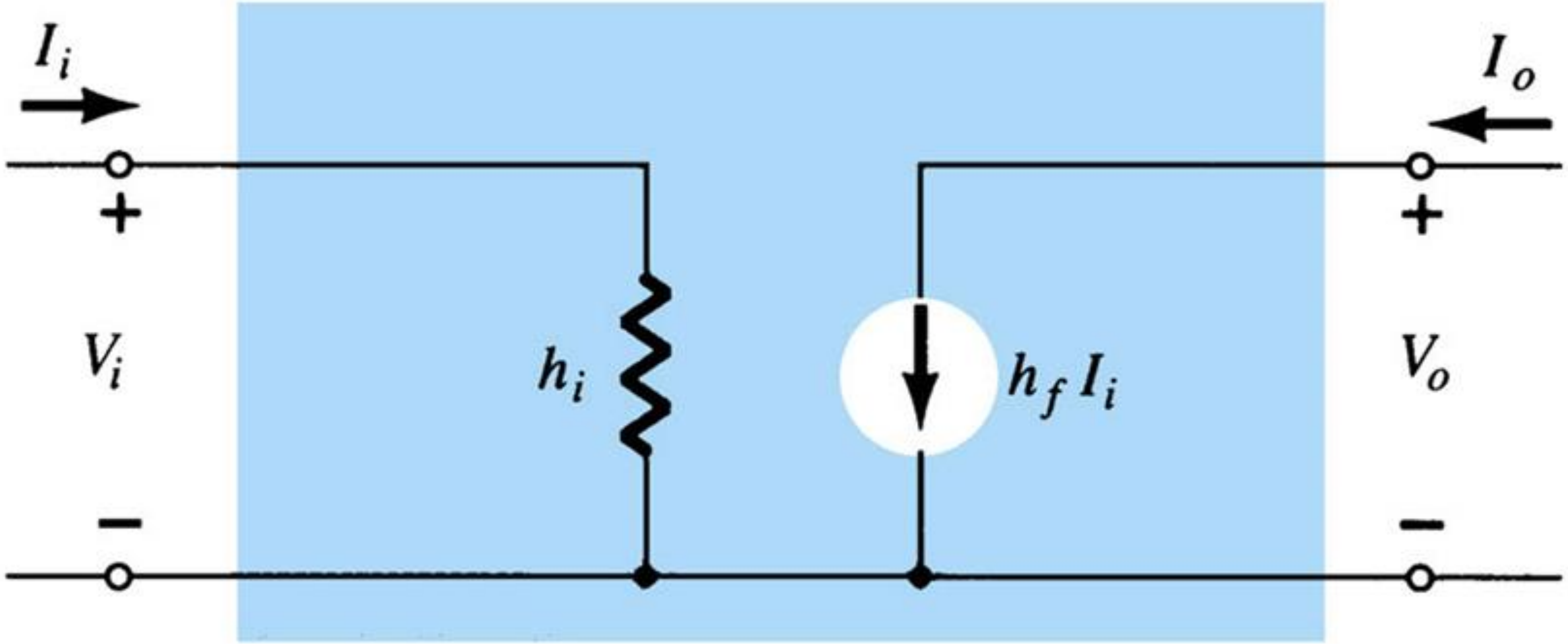
- General h-Parameters: hie, hre, hfe, hoe are developed and used to model the transistor.
- The h-Parameters can be found in a specification sheet for a transistor.



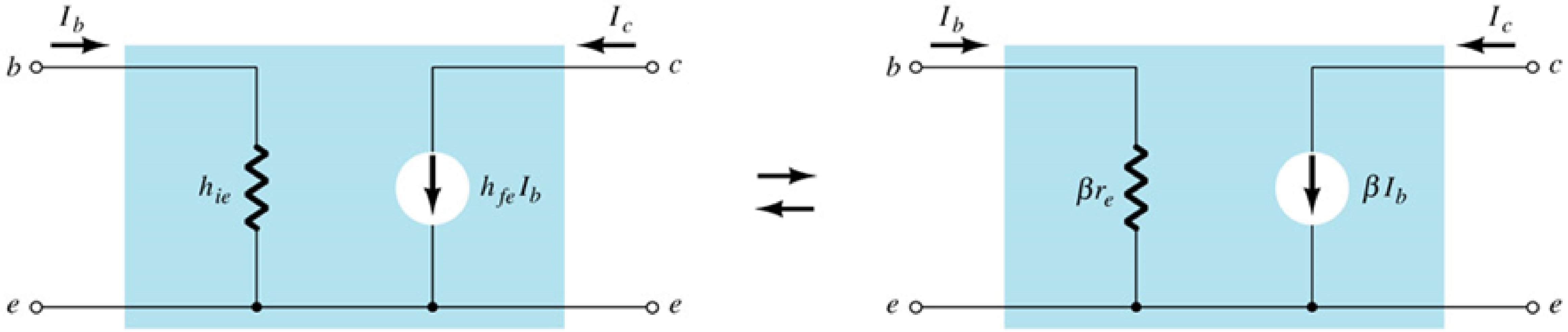
hi = input resistance
hr = reverse transfer voltage ratio (Vi/Vo)
hf = forward transfer current ratio (Io/Ii)
ho = output conductance

Simplified General h-Parameter Model

- The general h-parameter model can be simplified based on approximations:
- hr $\cong 0$ therefore hrVo = 0 and ho $\cong \infty$



Common-Emitter re vs. h-Parameter Model



hie = βre
hfe = β
hoe = 1/ro

Common-Emitter h-Parameters:

$$h_{ie} = \mathrm{B} r_e$$

$$h_{fe} = \mathrm{B}_{ac}$$

References:

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- http://ecee.colorado.edu/~bart/book/book/contents.htm
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