08/Nov/2021

Q Determine The laplace Transform of The following Signal and Sketch The ROC

$$x(t) = e^{-4|t|}$$

$$\therefore x(t) = \begin{cases} e^{-4t} \; ; \; t \geq 0 & |t| = t \\ e^{4t} \; ; \; t < 0, & |t| = -t \end{cases}$$

In time domain	In S-plane
we can't analyse The signal/system (It's in diff eq)	Freq domain $s = \sigma + j\omega$ (It's in algebric eq)

$$x(t) \longrightarrow X(s)$$

$$X(s) = L\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

Then ROC will converge.

Step1:- Analyse The signal. If not defined properly, we hv to define it.

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt \longrightarrow \text{def of LT}$$

$$= \int_{-\infty}^{0} e^{4t} \cdot e^{-st}\, dt + \int_{0}^{\infty} e^{-4t}\, e^{-st}\, dt$$

Write all terms in $(s+a)$ form or $(s-b)$ form

$$= \int_{-\infty}^{0} e^{-(s-4)t}\, dt + \int_{0}^{\infty} e^{-(s+4)t}\, dt$$

$$= \frac{\left[e^{-(s-4)t}\right]_{-\infty}^{0}}{-(s-4)} + \frac{\left[e^{-(s+4)t}\right]_{0}^{\infty}}{-(s+4)}$$

$$= \frac{1}{-(s-4)}\left[\underbrace{e^{-(s-4)\times 0}}_{1} - \underbrace{e^{-(s-4)\times(-\infty)}}_{\infty} + \underbrace{e^{-(s-4)\infty}}_{0} - \underbrace{e^{-(s-4)\times 0}}_{1}\right]$$

$$= \frac{1}{s+4} - \frac{1}{s-4}$$

$$= \frac{s-4-s-4}{(s+4)(s-4)}$$

$$= \frac{-8}{s^2-16}$$

$$\therefore X(s) = \frac{-8}{s^2-16} = \frac{-8}{(s+4)(s-4)}$$

How to find ROC quickly? We will find out so

<u>Property of ROC</u>

$$\text{Transfer } f^n = \frac{\text{LT of O/p } [y(s)] \rightarrow \text{zeros}}{\text{LT of i/p } [x(s)] \rightarrow \text{poles}}$$

$$TF = \frac{(s-a)(s-b) \rightarrow \text{zero}}{(s+p)(s+q) \rightarrow \text{pole}}$$

Zeroes → Values of S for which TF = 0

Poles → Values of S for which TF = ∞

at S = a & S = b Zeros exist

Prop 1 ROC does not include any pole.

Date: LSS   RSS

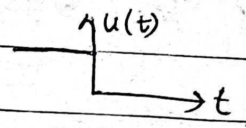
Prop 2

Left sided & Right sided Signal

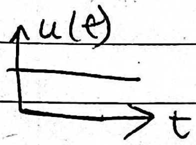
→ $t < 0 \rightarrow u(-t)$

→ $t \geq 0$, $u(t)$ } finit valeees

eg $e^{4t}\, u(-t)$

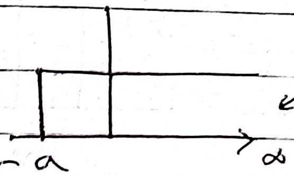


eg - $e^{4t}\, u(t)$



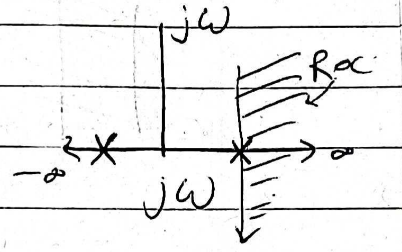
For RSS, ROC is right side to The right most pole
For LSS, ROC is left side " " left " "

For eg:-

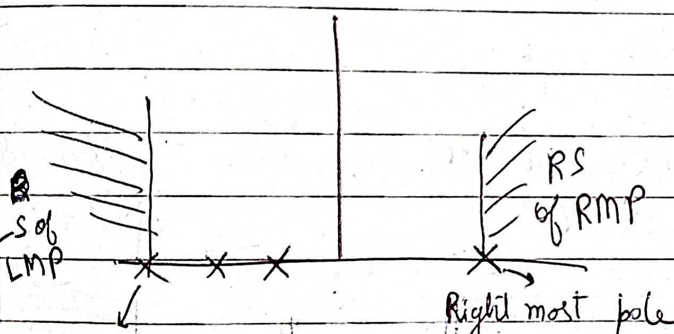


Consider This signal

⇒



Pole = X
Zero = O



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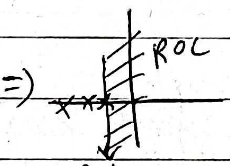
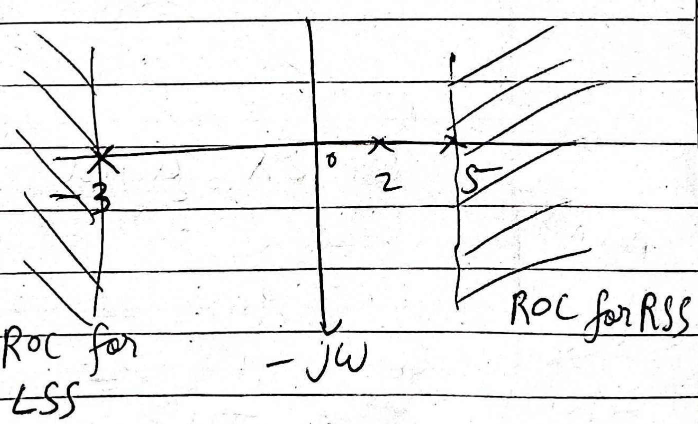


For eg $X(s) = \dfrac{4}{(s-2)(s+3)(s-5)}$

$s + j\omega$

$s = 2$ $s = -3$ $s = 5$



If $X(s)$ is RSS
Then ROC will be at extreme Right

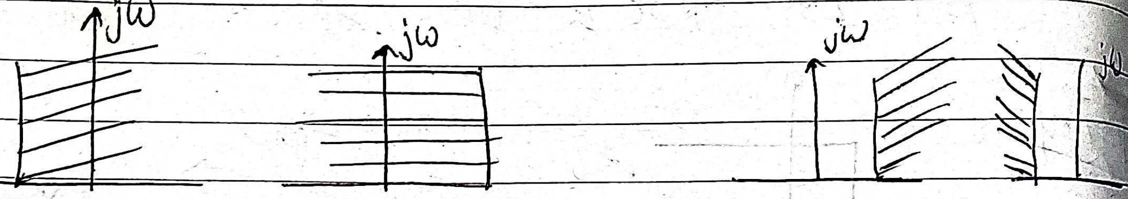
Ily for LSS extreme left

Prop 3 — For the absolutely integrable system / Signal ROC should include img axis.

This gives stability of the system.

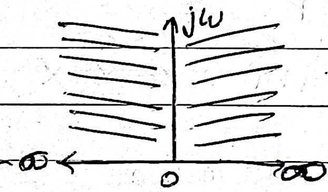
To become a stable system, the ROC of the system must include img axis.

eg



Stable   unstable.

Prop 4 — For finite duration signal, ROC is the entire s-plane excluding $s=0$



Short cut method.

$s = \underbrace{\sigma}_{\text{real}} + \underbrace{j\omega}_{\text{img}}$

$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$

step 1 — compare σ with real part of the coefficient of t in powers ∴ a is −a

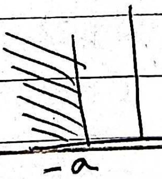
Ex. Signal :- $\underbrace{x(t) = e^{-at}\, u(-t)}$

Step 2 : (It is LSS)

Determine LSS or RSS

$\sigma < -a$

∴

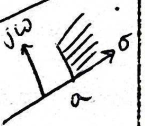


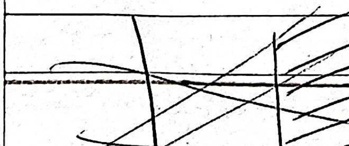
Our ROC will be like this

For RSS

$e^{at}\, u(t)$ → RSS

$\sigma > a$





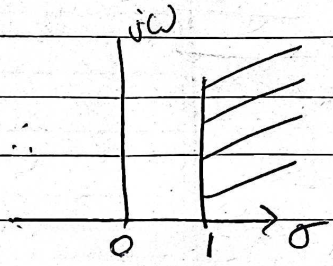
eg

$$x(t) = e^{(1+3j)t}\, u(t)$$

Step 1 Check for real part from power of e ~~& compare~~ ~~with~~ It is 1

$$\therefore \sigma > 1$$

Also it is an RSS



Our ~~Previous~~ Main Problem

$$x(t) = e^{-4(t)}$$

$$\Rightarrow x(t) = \underline{e^{-4t}} + \underline{e^{4t}}$$

$t \geq 0$ → RSS $t < 0$ → LSS

$$\underbrace{\sigma_1 > -4 \qquad \sigma_2 < 4}_{\text{Intersection}}$$