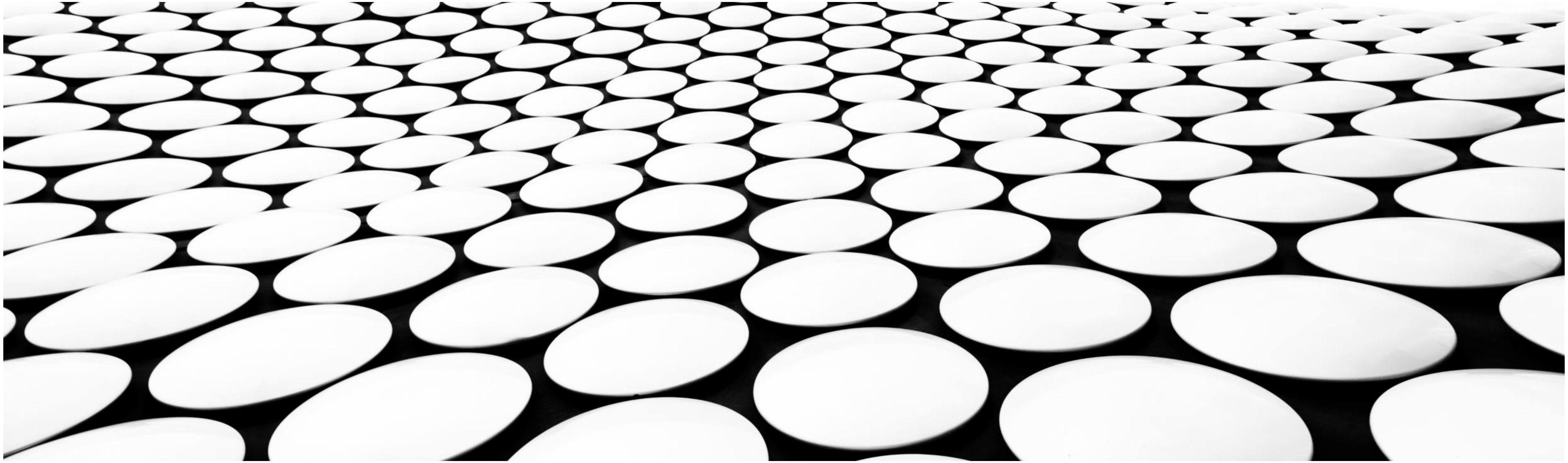
SIGNALS & SYSTEMS

MR. ANKUR JYOTI SARMAH

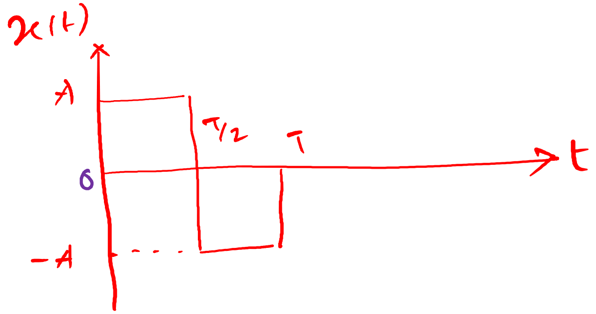
ASSISTANT PROF., DEPT. OF ELECTRONICS & TELECOM. ENGG.

ASSAM ENGINEERING COLLEGE



Determine the Laplace transform for the following:-

1)



Sol^n:- $L\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$

$$x(t) = \begin{cases} A, & \text{for } t = 0 \text{ to } T/2 \text{ or } 0 < t < T/2 \\ -A, & \text{for } T/2 < t < T \end{cases}$$

$$\therefore X(s) = \int_0^{T/2} A e^{-st}\, dt + \int_{T/2}^{T} -A e^{-st}\, dt$$

$$= A \frac{\left[e^{-st}\right]_0^{T/2}}{-s} - A \frac{\left[e^{-st}\right]_{T/2}^{T}}{-s}$$

$$= \frac{A}{-s}\left[e^{-sT/2} - e^{0}\right] + \frac{A}{s}\left[e^{-sT} - e^{-sT/2}\right]$$

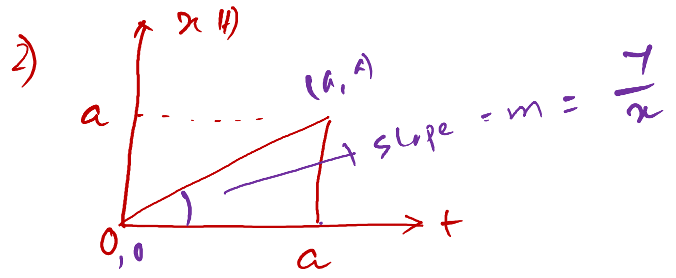
$$= \frac{-A}{s} e^{-sT/2} + \frac{A}{s} + \frac{A}{s} e^{-sT} - \frac{A}{s} e^{-sT/2}$$

$$= \frac{A}{s} + \frac{A}{s} e^{-sT} - 2\frac{A}{s} e^{-sT/2}$$

$$= \frac{A}{s}\left[1 + e^{-sT} - 2e^{-sT/2}\right] \rightarrow (a-b)^2$$

$$\boxed{X(s) = \frac{A}{s}\left[\left(1 - e^{-sT/2}\right)^2\right]}$$

2)



Sol^n:-

$$y = mx$$

$$\Rightarrow m = \frac{a}{a} = 1.$$

$$x(t) = 1.t$$

$$\Rightarrow \boxed{x(t) = t.}$$

Eq of a st. line b/w two points -

$x_1, y_1 \to 0, 0$

$x_2, y_2 \to a, a$

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\Rightarrow \frac{x(t) - 0}{0 - a} = \frac{t - 0}{0 - a}$$

$$\Rightarrow \frac{x(t)}{-a} = \frac{t}{-a}$$

$$\Rightarrow \boxed{x(t) = t}$$

$$\therefore x(t) = t \quad ; 0 < t < a$$

$$= 0 \quad ; \text{otherwise.}$$

$$X(s) = \int_0^a t \cdot e^{-st}\, dt$$

$$= t\int e^{-st}dt - \int\left[\frac{d}{dt} t \int e^{-st} dt\right] dt$$

$$= \left[t \cdot \frac{e^{-st}}{-s}\right] - \int \left[\frac{e^{-st}}{-s}\right] dt$$

$$= t\cdot\frac{e^{-st}}{-s} + \frac{1}{s}\int e^{-st}\, dt$$

$$= t\cdot\frac{e^{-st}}{-s} + \frac{1}{s}\,\frac{e^{-st}}{-s}$$

$$= \left[t\cdot\frac{e^{-st}}{-s} - \frac{e^{-st}}{s^2}\right]_0^a =$$

$$\int uv = u\int v - \int\left[du \int v\, dv\right]$$

ILATE $\rightarrow u = t$

$v = e^{-st}$

$$= a\cdot\frac{e^{-sa}}{-s} - \frac{e^{-sa}}{s^2} - \left[0 - \frac{e^{0}}{s^2}\right]$$

$$= \frac{ae^{-sa}}{-s} - \frac{e^{-sa}}{s^2} + \frac{1}{s^2}$$

$$= \frac{1}{s^2} - \frac{e^{-sa}}{s^2} - \frac{ae^{-sa}}{s}$$

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Properties:- suppose, $L\{x(t)\} = X(s)$ then $\rightarrow$

Amplitude scaling:- $L\{A\,x(t)\} = A\,X(s)$

Linearity:- If $L\{x_1(t)\} = X_1(s)$ and $L\{x_2(t)\} = X_2(s)$ then —

$$L\{a_1 x_1(t) + a_2 x_2(t)\} = a_1 X_1(s) + a_2 X_2(s)$$

Time differentiation:- $L\{\frac{d}{dt}x(t)\} = sX(s) - x(0)$

Initial cond^n

$\downarrow$

$x(t)$ at $t=0$

Time integration:- $L\{\int x(t)\,dt\} = \frac{X(s)}{s} + \frac{\left[\int x(t)\,dt\right]_{t=0}}{s}$

$$\frac{d^2x(t)}{dt^2} + 5\frac{dx(t)}{dt} + x(t) = 0$$

Taking Laplace transform

$$s^2X(s) + 5sX(s) + X(s) = 0$$

$$\Rightarrow X(s)\left[s^2 + 5s + 1\right] = 0$$

Frequency shifting :- If $L\{x(t)\} = X(s)$ then —

$$X(s) = \int x(t)\, e^{-st}\, dt$$

Proof

$$L\{e^{at}\, x(t)\} = X(s-a)$$

$$L\{e^{-at}\, x(t)\} = X(s+a)$$

Time shifting :-

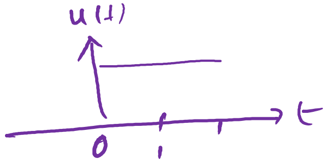
If $L\{x(t)\} = X(s)$ then

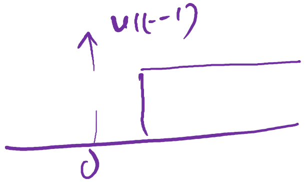
$$L\{x(t+a)\} = e^{as}\, X(s)$$

$$L\{x(t-a)\} = e^{-as}\, X(s)$$

Determine X(s)

$$x(t) = (t^2 - 2t)\, u(t-1) \quad \text{I/P}$$





$u(t-1) = 1 \; ; \; t \geq 1$

$\quad\;\; = 0 \; ; \;$ otherwise

$$X(s) = \int_1^\infty (t^2 - 2t)\, e^{-st}\, dt$$

$$= \underbrace{\int_1^\infty t^2 e^{-st}\, dt - 2\int_1^\infty t \cdot e^{-st}\, dt}_{\text{Complexity}}$$

$x(t) = (t^2 - 2t)\, \underbrace{u(t-1)}_{\text{delayed i/p}}$

$L\{x(t)\,u(t-a)\} = e^{-as} L\{x(t)\,u(t)\}$

$= t^2 u(t-1) - 2t\, u(t-1)$

$t^2 \cdot 1 = t^2 \rightarrow$ parabolic

$t \cdot 1 = t \rightarrow$ Ramp

$X(s) = L\{t^2 u(t-1)\} - 2L\{t\, u(t-1)\}$

$= e^{-s} L\{t^2 u(t)\} - 2e^{-s} L\{t\, u(t)\}$

$= e^{-s} \cdot \frac{2}{s^3} - 2e^{-s} \cdot \frac{1}{s^2} = 2e^{-s}\left[\frac{1}{s^3} - \frac{1}{s^2}\right] = 2e^{-s}\frac{(1-s)}{s^3}$ #

$x(t)$	$X(s)$
$\delta(t)$	1
$u(t)$	$1/s$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$

$\rightarrow$ Ramp: $t\,u(t) = \frac{1}{s^2}$

parabolic: $t^2 u(t) = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$

$\cos\omega_0 t\, u(t) = \frac{s}{s^2+\omega_0^2}$, $e^{-at}u(t) = \frac{1}{s+a}$, $e^{at}u(t) = \frac{1}{s-a}$

Freq. Diff :- $\mathcal{L}\{t\,x(t)\} = -\frac{d}{ds}X(s)$

Freq. Intg:- $\mathcal{L}\left\{\frac{1}{t}x(t)\right\} = \int_s^\infty X(s)\,ds$

Time scaling:- $\mathcal{L}\{x(at)\} = \frac{1}{|a|}X\left(\frac{s}{a}\right)$

Initial value theorem:- If $x(t)$ and derivatives are Laplace transformable then -

$$\lim_{t\to 0} x(t) = \lim_{s\to\infty} sX(s)$$

Final value theorem:-

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

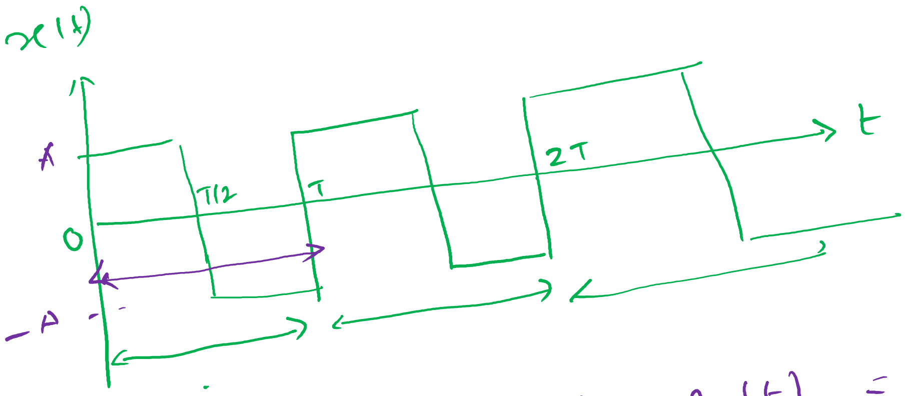
Convolution:- If $\mathcal{L}\{x_1(t)\} = X_1(s)$ and $\mathcal{L}\{x_2(t)dt\} = X_2(s)$ then -

$$\mathcal{L}\{x_1(t) * x_2(t)\} = X_1(s)\cdot X_2(s)$$

Periodicity:- If $x(t) = x(t+nT)$ and $x_1(t)$ be one period of $x(t)$

and $L\{x_1(t)\} = \int_0^T x_1(t)\, e^{-st}\, dt$ Then

$$L\{x(t+nT)\} = \frac{1}{1-e^{-sT}} \int_0^T x_1(t)\, e^{-st}\, dt$$



→ periodic signal

$$\text{Define } x_1(t) = \begin{cases} A, & \text{for } 0 < t < T/2 \\ -A, & \text{for } T/2 < t < T \end{cases}$$

$$X(s) = \frac{1}{1-e^{-sT}} \int_0^T x_1(t) e^{-st}\, dt$$

follow the soln of problem no 1

$$= \frac{1}{1-e^{-sT}} \left[\frac{A}{s} \left(1-e^{-sT/2}\right)^2 \right]$$

$$= \frac{1}{\left(1+e^{-sT/2}\right)\cancel{\left(1-e^{-sT/2}\right)}} \cdot \frac{A}{s}\left(1-e^{-sT/2}\right)$$

$$= \frac{A}{s} \cdot \frac{\left(1-e^{-sT/2}\right)}{\left(1+e^{-sT/2}\right)}$$

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