Small-signal Diode Model

Semiconductor Devices and Circuits

(ECE 181302)

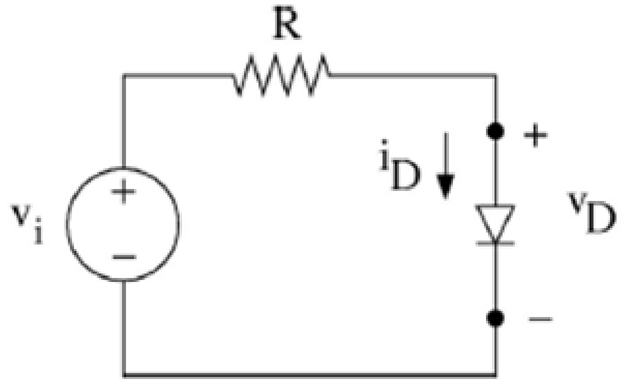
8th October 2021

Diode Circuit Equations are Nonlinear

KCL : current i_D in all elements

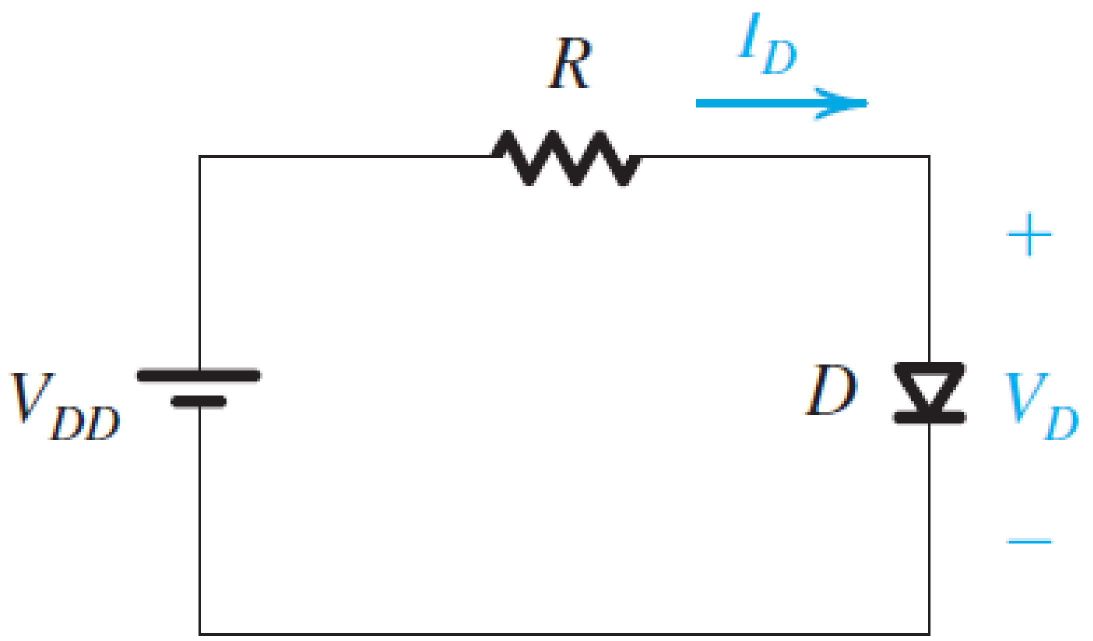
KVL : $v_i = Ri_D + v_D$

$$i_D = I_S\left(e^{v_D/nV_T} - 1\right)$$



- Two equation in two-unknowns to solve for i_D and v_D
- Non-linear equation: cannot be solved analytically
- **Solution methods:**
  - Numerical (PSpice)
  - Graphical (load-line)
  - Approximation to get linear equations (diode piece-linear model)

Diode Circuit Analysis

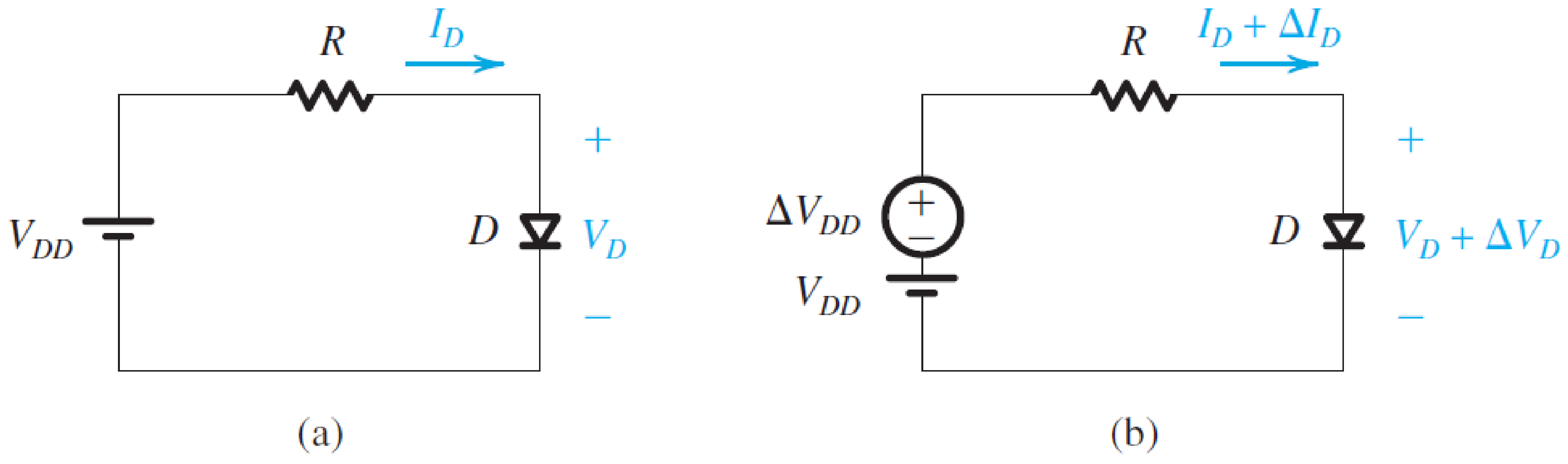


- I_D and V_D can be solved using:
  - ✓Exponential model
  - ✓Ideal diode model
  - ✓Constant voltage-drop model
  - ✓Piecewise linear model

Modeling the Diode Forward Characteristic

- Analysis of circuits employing forward conducting diodes.
- Simplified diode models are better suited for use in circuit analysis and design of diode circuits:
  - ✓ Exponential model
  - ✓ Ideal diode model
  - ✓ Constant voltage-drop model
  - ✓ Piecewise linear model
  - ✓ Small-signal (linearization) model

Small-signal Diode Model

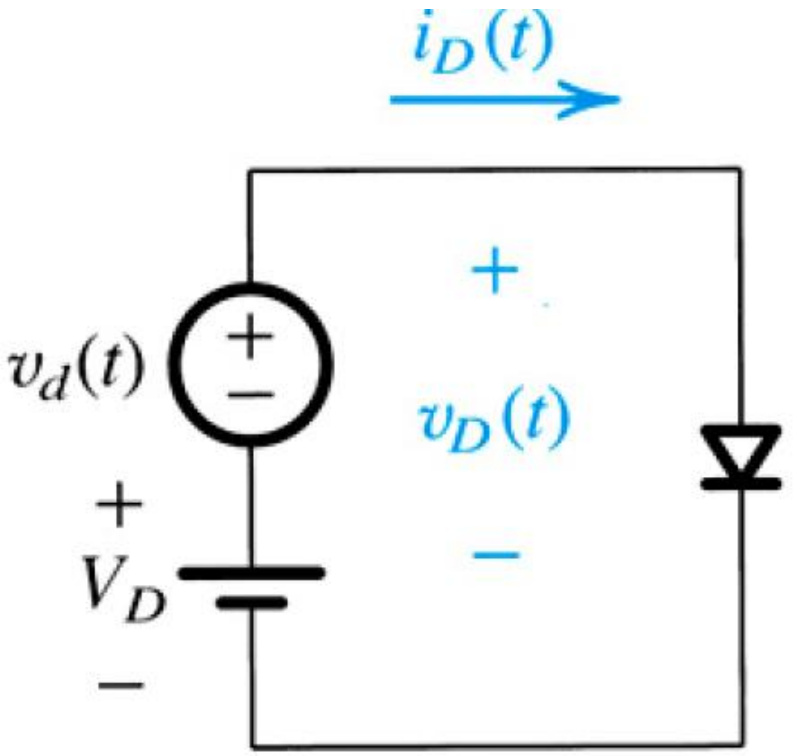


(a) A simple diode circuit; **(b)** the situation when V_{DD} changes by ΔV_{DD}.

- Voltage V_{DD} undergoes a small change ΔV_{DD}.
  - ΔV_{DD}. Is time-varying.
- Current I_D changes by an increment ΔI_D.
- Diode voltage V_D changes by an increment ΔV_D.

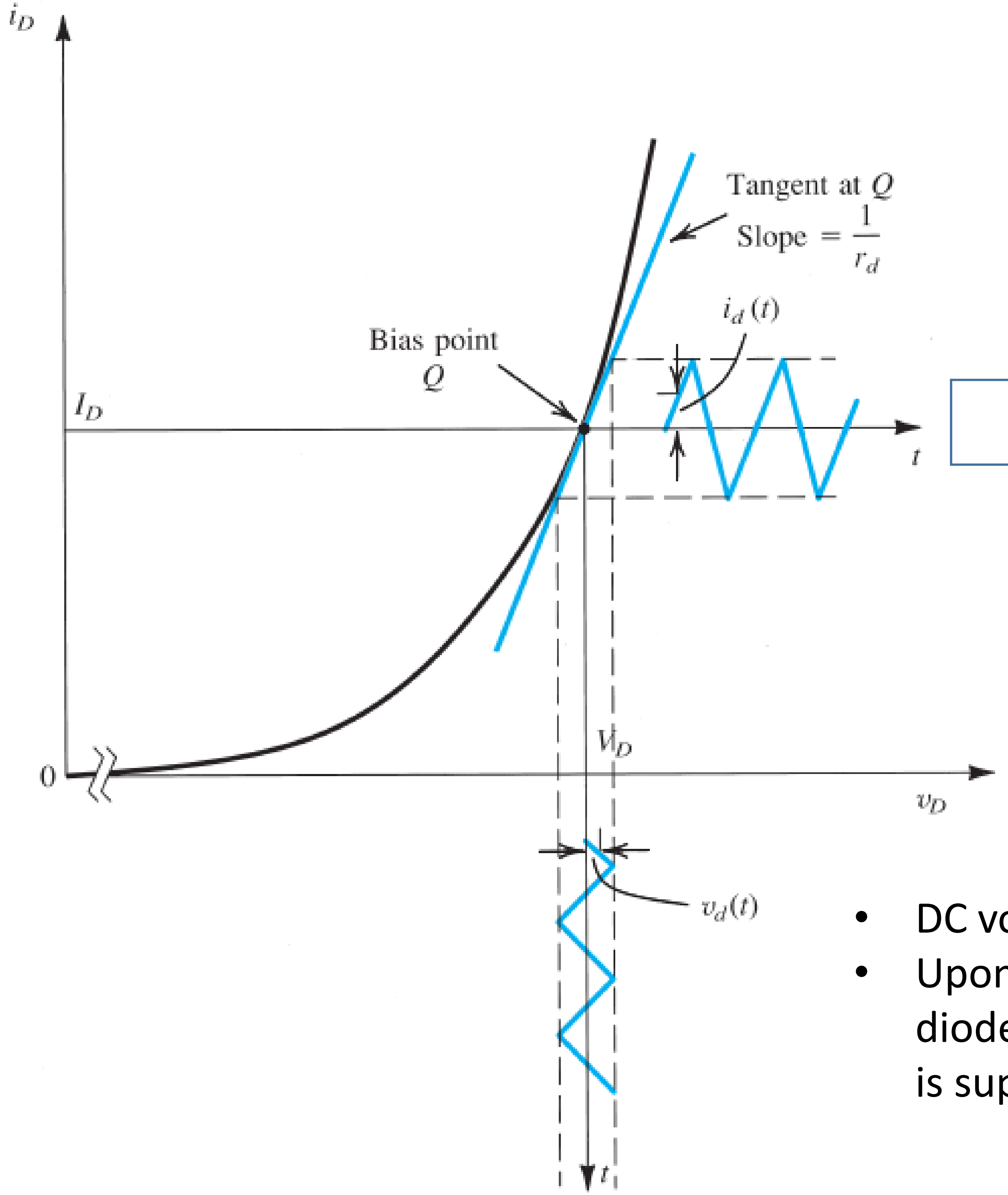
- Voltage across the diode is the sum of the dc voltage V_D and the time-varying signal $v_d(t)$
- Total instantaneous voltage across the diode is: $v_D(t) = V_D + v_d(t)$
- Get the total instantaneous diode current will be by substituting v_D in the diode current equation $i_D(t) = I_S e^{v_D/V_T}$
- Therefore:

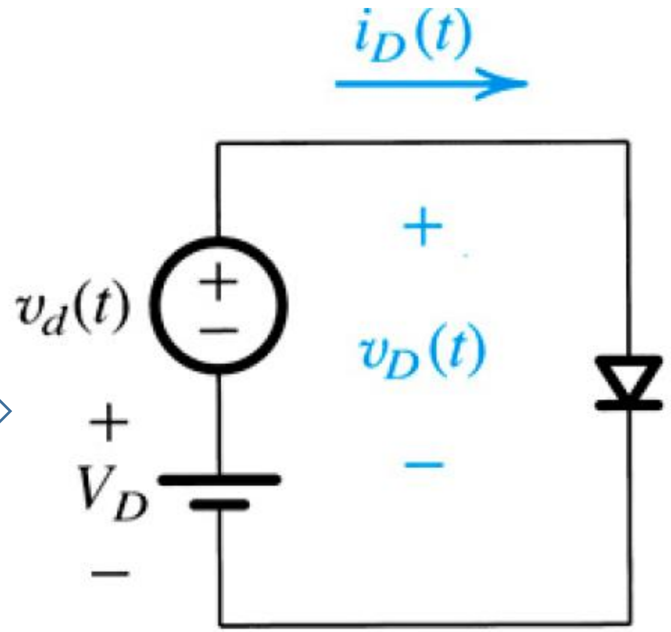
$$i_D(t) = I_S e^{(V_D + v_d)/V_T}$$
$$= I_S e^{V_D/V_T} e^{v_d/V_T}$$



Conceptual circuit of small-signal diode model

- DC only – upper-case w/ upper-case subscript
- Time-varying only – lower-case w/ lower-case subscript
- Total instantaneous – lower-case w/ upper-case subscript DC + time-varying





- DC voltage (V_D) is applied to the diode
- Upon V_D, at the operating point (V_D, I_D) on the diode *I-V* plot, arbitrary time-varying signal $v_d(t)$ is super-imposed

- In the absence of the signal $v_d(t)$ he diode voltage is equal to V_D, and the diode current is I_D :

$$I_D = I_S e^{V_D/V_T}$$

- Therefore total instantaneous diode current is: $i_D(t) = I_D e^{v_d/V_T}$

- Apply the power series expansion to the diode current:

$$i_D(t) = I_D \overbrace{\underbrace{\left(1 + \frac{v_d}{V_T} + \overbrace{\left[\left(\frac{v_d}{V_T}\right)^2 \frac{1}{2!}\right] + \left[\left(\frac{v_d}{V_T}\right)^3 \frac{1}{3!}\right]}^{\text{because } v_d/V_T \ll 1\text{, these terms are assumed to be negligible}} + \ldots\right)}_{\text{power series expansion of } e^{v_d/V_T}}}^{\text{apply power series expansion}}$$

- Because $\frac{v_d}{V_T} \ll 1$ higher terms are neglected and the series is truncated after the first two terms to obtain the approximate expression:

$$i_D(t) \simeq I_D\left(1 + \frac{v_d}{V_T}\right)$$

- This is the **small-signal approximation** and it is valid for signals whose amplitudes are smaller than about 5 mV (recall that V_T = 25 mV)

- Total instantaneous diode curren $i_D(t) = I_D + \frac{I_D}{V_T} v_d$

- We have a signal current component directly proportional to the signal voltage v_d superimposed on the dc current I_D, i.e.

$$i_D = I_D + i_d$$

where $i_d = \frac{I_D}{V_T} v_d$

- The quantity relating the signal current i_d to the signal voltage v_d has the dimensions of conductance, mhos $(\mho)$, and is called the **diode small-signal conductance**.
- The inverse of this parameter is the **diode small-signal resistance**, or **incremental resistance**, r_d ,

$$r_d = \frac{V_T}{I_D}$$

Small-signal Resistance r_d

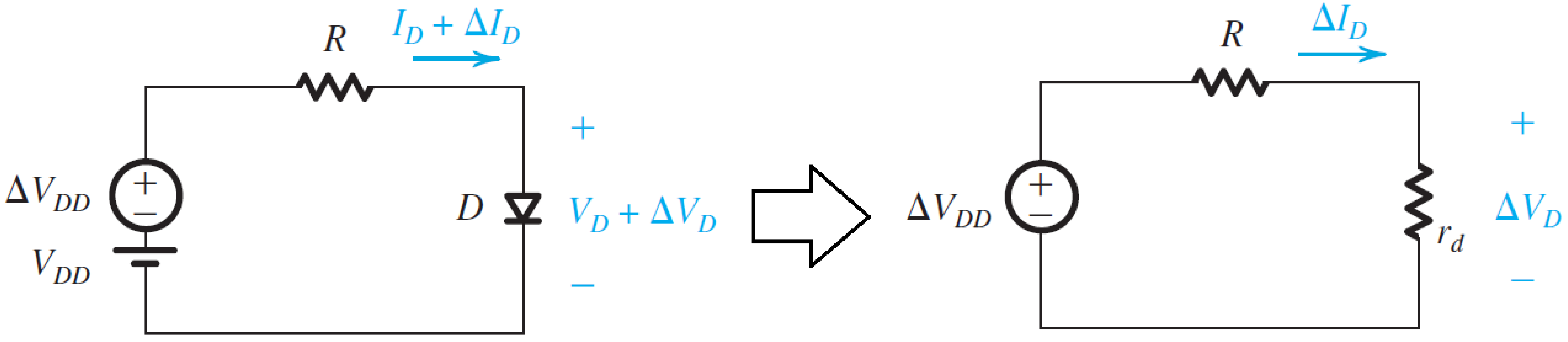
- Assuming that the signal amplitude is sufficiently small such that the excursion along the *I-V* curve is limited to a short almost-linear segment, then

$$r_d = 1 \Big/ \left[\frac{\partial i_D}{\partial v_D}\right]_{i_D = I_D}$$

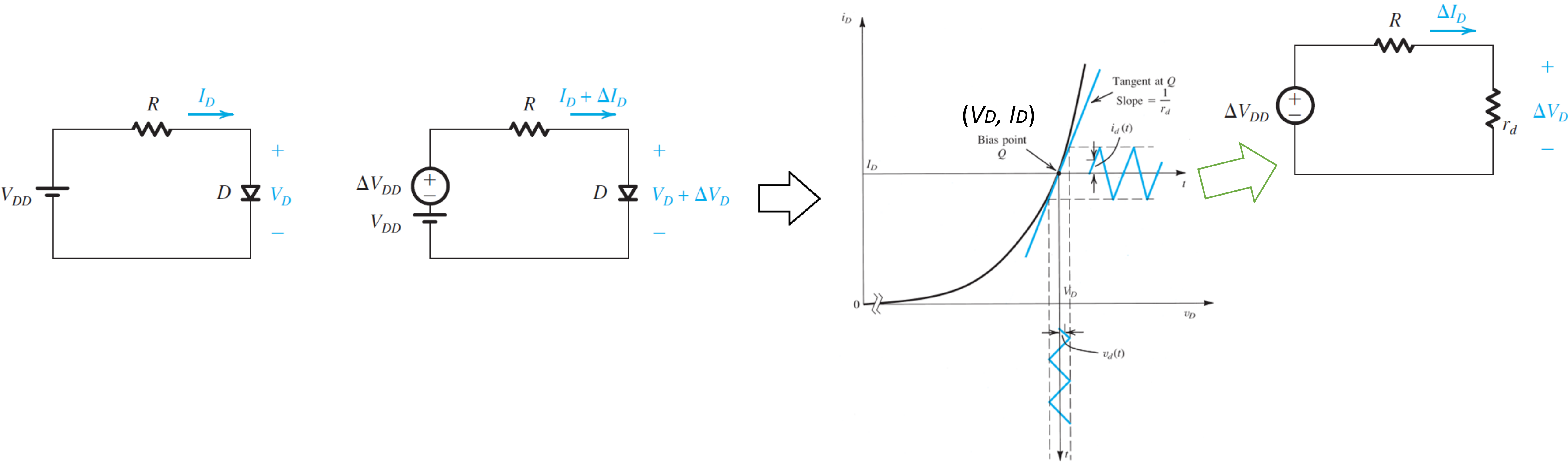
- This method may be used to approximate any function $y = \mathbf{f}(x)$ around an operating point (x_0, y_0).

$$y(t) = y_0 + \left(\left.\frac{\partial y}{\partial x}\right|_{y=Y} \right)^{-1} \overbrace{\left(x(t) - x_0\right)}^{\Delta x}$$

Small-signal Equivalent Circuit



- The small-signal equivalent circuit determines the incremental quantities I_D and V_D for the circuit by replacing the diode with its small-signal resistance r_d results in a linear circuit.



- Diode circuit operates at a dc biased point on the forward *I-V* characteristic and a small ac signal superimposed on the dc.
- The dc operating point (V_D, I_D) is determined by other model (e.g. constant voltage drop model or CVDM).
- Diode small-signal is modeled as variable resistor (= inverse of the slope of the tangent to exponential *I-V* characteristic at the bias point (V_D, I_D).
- Value of variable resistor is defined via linearization of exponential model around bias point defined by CVDM.

- The total instantaneous circuit is divided into steady-state and time varying components, which may be analyzed separately and solved via algebra:
    - ➢In steady-state, diode represented as CVDM (Constant Voltage Drop Model).
    - ➢In time-varying, diode represented as resistor.

- Neither of these circuits employ the exponential model – simplifying the “solving” process.

- Difficulty of diode circuit analysis due to nonlinear nature is avoided.

References:

- Microelectronic Circuits, 7th edition by Adel S. Sedra Kenneth C. Smith.
- G. Streetman, and S. K. Banerjee, “Solid State Electronic Devices,” 7th edition, Pearson,2014.
- D. Neamen, D. Biswas, "Semiconductor Physics and Devices," McGraw-Hill Education.
- Electronic Devices and Circuit Theory 11th Edition by Boylestad, Robert . L, Louis Nashelskyl.
- http://ecee.colorado.edu/~bart/book/book/contents.htm
- http://www.ecse.rpi.edu/~schubert/Course-ECSE-2210-Microelectronics-Technology-2010/