Fitting of a straight line:

Let (x_i, y_i), $i = 1, 2 \ldots n$ be n sets of observations of related data and $y = ax + b$ → ①

be the st. line to be fitted.

∴ The error at $x = x_i$ is,

$$e_i = y_i - f(x_i)$$

$$= y_i - (ax_i + b), \quad i = 1, 2 \ldots n.$$

$$\therefore E = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} [y_i - (ax_i + b)]^2$$

∴ By the principle of least squares, E must be minimum.

For extreme values, we know that

$$\frac{\partial E}{\partial a} = 0 \rightarrow ② \text{ and } \frac{\partial E}{\partial b} = 0 \rightarrow ③$$

$$\therefore ② \Rightarrow \sum_{i=1}^{n} 2(y_i - ax_i - b) \cdot (-x_i) = 0$$

$$\Rightarrow \sum_{i=1}^{n} (y_i x_i - a x_i^2 - b x_i) = 0 \rightarrow ④$$

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(3) $\Rightarrow \sum_{i=1}^{n} 2(y_i - ax_i - b)(-1) = 0$

$$\Rightarrow \sum_{i=1}^{n} (y_i - ax_i - b) = 0 \quad \rightarrow (5)$$

$\therefore$ (4) $\Rightarrow \sum_{i=1}^{n} y_i x_i = a \sum_{i=1}^{n} x_i^2 + b \sum_{i=1}^{n} x_i \rightarrow (6)$

(5) $\Rightarrow \sum_{i=1}^{n} y_i = a \sum_{i=1}^{n} x_i + nb \ldots \rightarrow (7)$.

Since, x_i, y_i are known, eq^n (6) and (7) give two eq^{n}s in a and b. Solving eq^n (6) and (7) we can get the required st. line to fit the given data.

<u>Note:</u> The eq^n (6) and (7) are called normal equation ~~for~~ to fit a given data by straight line.

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