10/09/2021

Graph Theory

Graph of a N/W

- Collection of **nodes** and **branches.**
- Represent N/W elements (R, L, C) by **lines**.
- **Voltage** and **current** source by their internal impedance.

S.C   O.C

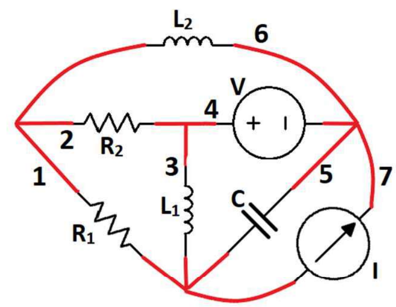


Fig: Arbitrary N/W

- **Node** → Small Letters (a,b,c,….)
- **Elements** → Numbers (1,2,…..)
- **No.** of **branch** in the **graph** ≤ **No.** of **branch** in the **N/W**

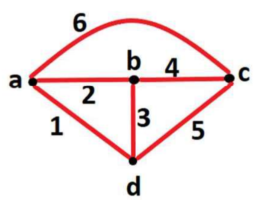


Fig: Undirected Graph

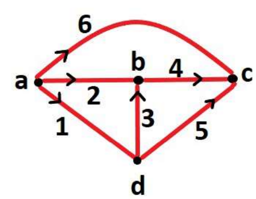


Fig: Directed Graph

Sum of Degree of nodes = no. of branches

Terminology used in N/W graph

- **Undirected Graph**
- **Directed/Oriented Graph**
- **Complete Graph** → Graph in which **each node pair** should have **a line segment**.

  No. of trees = n^{n-2} = No. of Co-tree   Since Complementary tree exist for each tree.

  Complementary

  No. of branches = ${}^{n}C_2 = \frac{n(n-1)}{2}$ = no. of node pair voltage

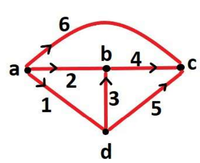


Fig: Complete Graph

$4^{4-2} = 4^2 = 16$

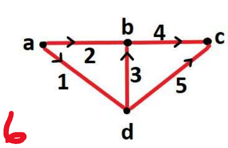


Fig: Incomplete Graph_1

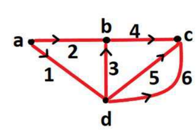
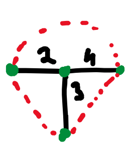
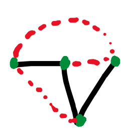


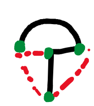
Fig: Incomplete Graph_2

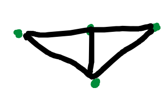
Twigs + Links = B



n = 4
T = (n-1)
= 3







- **Incomplete Graph**

  no. of node pair voltage = $\frac{n(n-1)}{2}$

- **Connected Graph** → There is a path from any point (Node) to any other point (Node) in the graph.

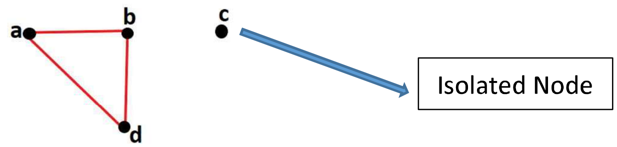


Fig: Unconnected Graph

- **Subgraph** → Graph in which **no. of branches atleast less by one** of original graph.
- **Spanning Subgraph** → Spanning subgraph **consists of all the nodes** of original graph.
- **Branch, B → line segments** → represents **one N/W element** or a **combination of elements** between two points
- **Node, n → End point** of a line segment.
- **Degree of a Node** → No. of branches incident to it.
- **Tree → Connected subgraph** which consists of
  - → **All Nodes** of the graph **without** forming **any close loop**.
  - → **(n-1) branches**
  - → **No. of branches in the graph = Twigs + Links**

B = T + L

=> L = B – (n-1)

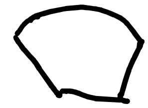
→ **Rank of graph/tree = (n-1)**

- **Twig** → Tree branch
- **Tree Link or Chord or Link** → **Co-tree branch** are referred as links
- **Loop** → Closed contour
- **Planar Graph** → Graph which can be drawn on a plane surface **without a crossover**
- **Non-Planar Graph** → Graph which **cannot be drawn on plane surface** without a crossover.

14/09/2021

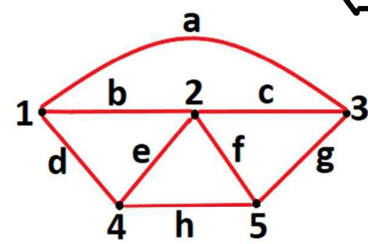
GATE EC 1999

Q1 Identify which of the following is **NOT** a tree of the graph shown in fig.



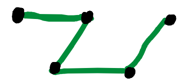
=

a) begh b) defg c) adhg d) aegh



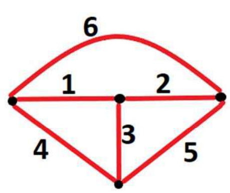
$n = 5$

$T = (n-1) = 4$

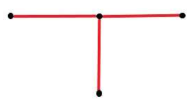


ESE EC 2004

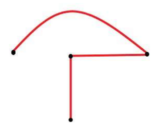
Q2 Consider the following graph. Which one of the following is **not** a tree of the below graph?



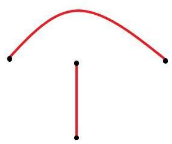
a)



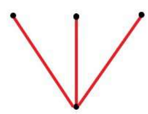
b)



c)



d)

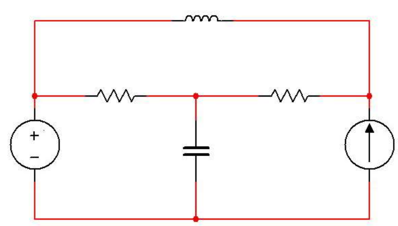


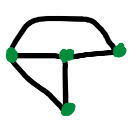
$T = (n-1)$
$= 4-1 = 3$

GATE EE 2008

Q3 The **no. of chords** in the graph of the given circuit will be

a) 2 b) 3 c) 4 d) 4





$n = 4$

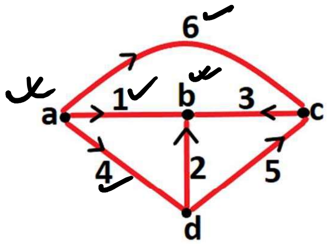
$B = 5$

$l = B - (n-1)$

$= 5 - (4-1) = 5 - 3 = 2$

Incidence Matrix $[A]_{nxB}$

Any oriented graph can be described completely in a **compact matrix form**.



Rules

Outgoing Branches → +1

Incoming Branches → -1

Not associated → 0

$$[A]_{nXB} = [A]_{4X6} = \begin{array}{c c} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \end{bmatrix}_{4x6} \end{array}$$

Information

- **Orientation** (incoming/outgoing from a node) of **each branch**.
- **Nodes** at which this **branch is incident**.

Properties

- Algebric **sum** of all **column entries** will be **zero**.
- If **n = B** ie. Square matrix **|A| = 0** ie. Singular matrix.

<u>21/09/2021</u>

<u>Reduced Incidence Matrix $[R]_{(n-1)xB}$</u>

When **one row is deleted** from the complete incidence matrix, the remaining matrix is then termed as reduced incidence matrix.

$$[R]_{(n-1)XB} = [A]_{3X6} = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix}_{3x6} \end{array}$$

<u>Adv</u>

- ➔ Reduce space
- ➔ Less complex
- ➔ Cost of designing R.I.M. will be less **without any loss of info**.

<u>For any graph i.e. complete or incomplete</u>

No. of trees = det $[R.R^T]$

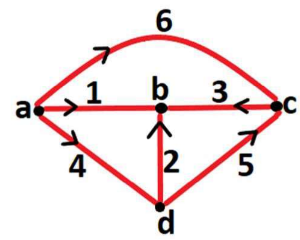
ESE EE 2015

Q4 Reduced incidence matrix of a graph is given by

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & -1 & -1 & 0 \end{bmatrix}$$

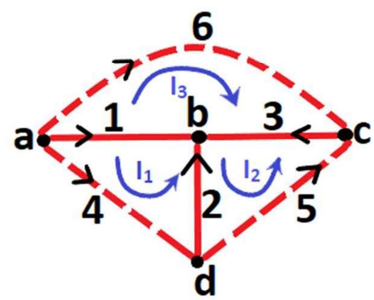
The no. of possible trees are a) 8 b) 9 c) 10 d) 11

Tie-set matrix/Fundamental loop matrix $[T]_{LXB}$



- **Link dependent** matrix.
- **Direction** of fundamental loop current **depends on direction of link**.
- One fundamental loop has **only one link**.

Rules



- **Follow direction** of link current → +1
- **Oppose direction** of link current → -1
- **Not associated** with the loop → 0

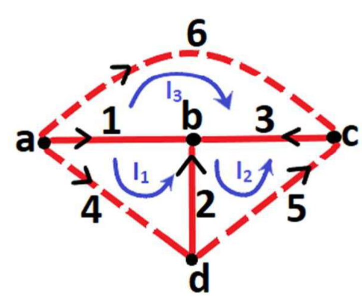
$$[T]_{3X6} = \begin{array}{c c} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} I_1 \\ I_2 \\ I_3 \end{array} & \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}_{3x6} \\ & \begin{array}{cccccc} J_1 & J_2 & J_3 & J_4 & J_5 & J_6 \end{array} \end{array}$$

Branch Currents

Loop Currents

- In tie set matrix we determine **branch current(J) in terms of loop currents(I)**.

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$



In matrix form, $\boxed{J_b = T^T I_L}$ where J_b is a **column vector of branch currents**

I_L is a **column vector of loop currents**

$J_1 = -I_1 - I_3$ $\quad$ $J_4 = I_1$

$J_2 = I_1 - I_2$ $\quad$ $J_5 = I_2$

$J_3 = I_2 + I_3$ $\quad$ $J_6 = I_3$

- **Rank [Tie-set] = L = B – (n-1)**
- No. of fundamental ***loop currents*** = L = B – (n-1)

- The tie set matrix **satisfies** voltage equilibrium i.e. **KVL equation.**
- **No. of KVL eq^n = No. of links = No. of fundamental loop current = B – (n – 1)**

$$\begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = 0$$

In matrix form, $\boxed{\mathbf{T\,V_b = 0}}$ where V_b is a **column vector of branch voltages**.

KVL equations

$-V_1 + V_2 + V_4 = 0$

$-V_2 + V_3 + V_5 = 0$

$-V_1 + V_3 + V_6 = 0$

- Since each tree has one tie set matrix,

  For a **complete graph**,

  No. of tie set matrix = n^{n-2} = no. of trees

$$[T]_{3X6} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}_{3x6} \end{array}$$

By viewing **identity matrix we can find links and twigs** of a given graph.

GATE EC 1998 $n = 7$, $5 = l = B-(n-1)$
$5 = B-(7-1) \Rightarrow B = 5+6$
$= 11$

Q A network has 7 nodes and 5 independent loops. The no. of branches in the N/W is

a) 13 b) 12 c) 11 d) 10

ESE EC 2009 $B = 12$ $n = 5$ $l = B-(n-1)$

$= 12-(5-1)$

Q In a N/W with twelve circuit elements and five nodes, what is the minimum no. of mesh equations?

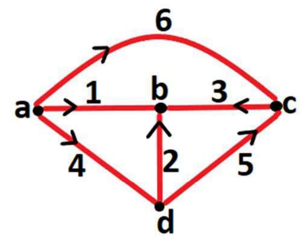
$= 12-4$

a) 24 b) 12 c) 10 d) 8

$= 8$

22/09/2021

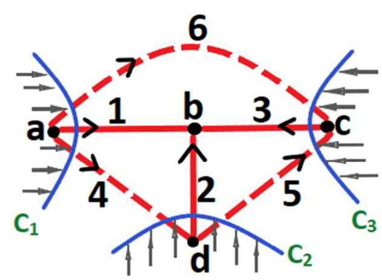
Fundamental Cut set matrix $[C]_{TXB}$



- **Twig dependent** matrix.
- In cut-set matrix, cut set **depends** on the **orientation of twig**.
- One cut set can have **only one twig**.

Rules

- **Same orientation** as twig → +1
- **Opposite** orientation as twig → -1
- **Not associated** → 0



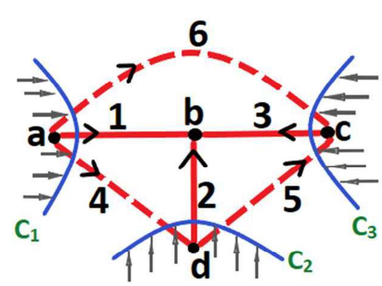
$$[C]_{3x6} = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix}_{3x6} \\ \begin{array}{cccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{array} \end{array} \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ \\ \end{array}$$

Branch Voltage

Induced cut-set voltage

- In case of cut-set matrix, we determine **branch voltage(V)** in terms of **induced cut-set voltage (twig voltage, e)**.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$



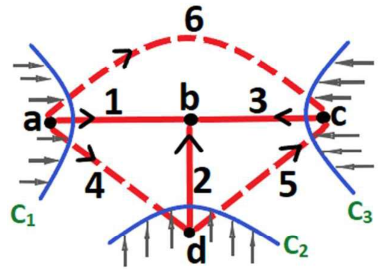
In matrix form, $\boxed{V_b = C^T V_t}$ where V_b is a **column vector of branch Voltage**

V_t is a **column vector of twig voltage**

$V_1 = e_1$ $\qquad$ $V_4 = e_1 - e_2$

$V_2 = e_2$ $\qquad$ $V_5 = e_2 - e_3$

$V_3 = e_3$ $\qquad$ $V_6 = e_1 - e_3$



- **No. of cut-set** = T = n-1
- **Rank** [cut-set] = n-1
- For a **complete graph**, No. of cut-set matrix = $\mathbf{n^{n-2}}$
- Cut set matrix satisfies the **KCL equation** i.e. current equilibrium equation.
- **No. of KCL equation = T = n-1**

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix} = 0$$

J → **Branch Currents**

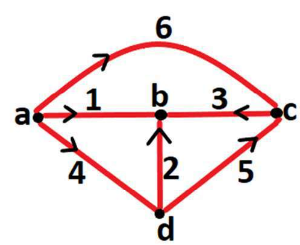
In matrix form, $\boxed{\mathbf{C\ J_b = 0}}$ where J_b is a **column vector of branch currents**.

<u>KCL equations</u>

$J_1 + J_4 + J_6 = 0$

$J_2 - J_4 + J_5 = 0$

$J_3 - J_5 - J_6 = 0$



- $$[C]_{3X6} = \begin{array}{c c} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix}_{3x6} \end{array} \quad \begin{array}{c} C_1 \\ C_2 \\ C_3 \end{array}$$

By viewing **identity matrix we can find links and twigs** of a given graph.

24/09/2021

GATE EE 2016

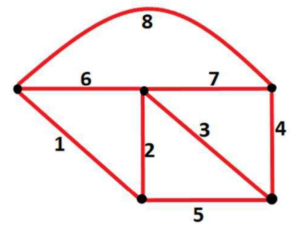
Q The graph associated with an electrical N/W has 7 branches and 5 nodes. The number of independent KCL equations and the number of independent KVL equations, respectively are

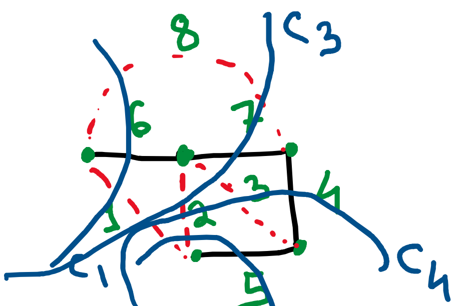
a) 2 & 5 b) 5 & 2 c) 3 & 4 d) 4 & 3

ESE EC 1997

Q In the graph shown in the fig., one possible tree is formed by the branches 4,5,6,7. Then one possible fundamental cut-set is

a) 1,2,3,8 b) 1,2,5,6 c) 1,5,6,8 d) 1,2,3,7,8 ✓





$n = 5$

$B = 8$

$T = n - 1 = 4$

no. of cut set = T = 4

$C_1 \rightarrow (1, 6, 8)$

$C_2 \rightarrow (1, 2, 5)$

$C_3 \rightarrow (1, 2, 3, 7, 8)$

$C_4 \rightarrow (1, 2, 3, 4)$

ESE EC 1997 $B = 6$ $T = 3$

Q The graph of a N/W has six branches with three tree branches. The minimum number of equations required for the solution of the N/W is

a) 2 b) 3 ✓ c) 4 d) 5

KVL → 3

KCL → 3

min (KVL, KCL)

min (l, T) $T = n-1$

$l = B - (n-1)$

$= 6 - (3)$

$= 3$

Duality

Two circuits that are described by ***equations of the same form***, but in which the ***variables are interchanged***, are said to be dual to each other.

Two circuits are said to be duals of one another if they are described by the ***same characterizing equations*** with **dual quantities interchanged**.

The *mesh (or nodal) equations* of the original circuit are **similar** to the *nodal (or mesh) equations* of the dual circuit.

- ❖ Note that **power has no dual**. Since Power is ***not linear***, duality does not apply.

  Exception
- ❖ Even when the principle of linearity applies, a circuit element or variable may not have a dual. For example, **Mutual inductance has no dual**.

Dual Pairs	
Resistance R	Conductance G
Inductance L	Capacitance C
Voltage v	Current i
Voltage Source	Current Source
Node	Mesh
Series Path	Parallel Path
Open Circuit	Short Circuit
KVL	KCL
Thevenin	Norton

Graphical technique to construct a dual circuit of a given ***planar circuit***.

1) Place a **node** at the **center of each mesh** of the given circuit. Place the **reference node** (the ground) of the dual circuit **outside** the given circuit.
2) **Draw lines** between the nodes such that each line crosses an element. **Replace** that element by its **dual**.
3) To determine the polarity of voltage sources and direction of current sources, follow this rule: A voltage **source** that produces a positive (**clockwise**) mesh current has as its dual a current source whose reference **direction** is from the **ground to the nonreference node**.

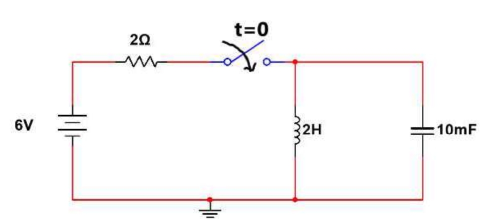


Fig1: Example 1 for duality

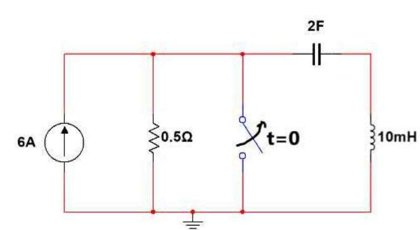
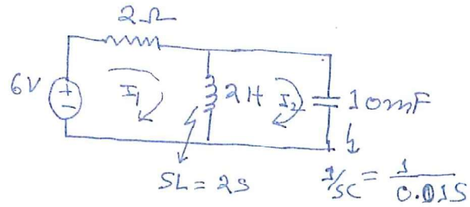


Fig2: Dual of fig1



$$6 - 2I_1 - 2s(I_1 - I_2) = 0$$

$$\Rightarrow 2(1+s)I_1 - 2sI_2 = 6$$

$$\Rightarrow (1+s)I_1 - sI_2 = 3 \quad ---(i)$$

$$\frac{1}{0.01s} I_2 + 2s(I_2 - I_1) = 0$$

$$\Rightarrow -2sI_1 + \left(\frac{1}{0.01s} + 2s\right) I_2 = 0$$

$$\Rightarrow -0.02s^2 I_1 + (1 + 0.02s^2) I_2 = 0 \quad ---(ii)$$

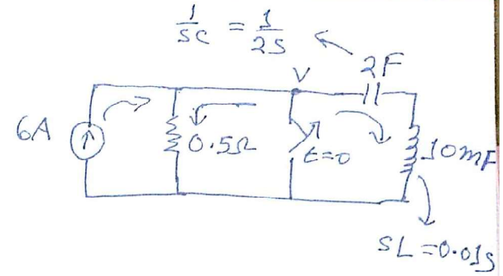
From (i), $\Rightarrow (1+s)I_1 - s\left(\frac{0.02s^2}{1+0.02s^2} I_1\right) = 3$

$$\Rightarrow (1+s)(1+0.02s^2)I_1 - 0.02s^3 I_1 = 3 + 0.06s^2$$

$$\Rightarrow (1 + 0.02s^2 + s + 0.02s^3)I_1 - 0.02s^3 I_1 = 3 + 0.06s^2$$

$$\Rightarrow I_1 + 0.02s^2 I_1 + sI_1 + \cancel{0.02s^3 I_1} - \cancel{0.02s^3 I_1} = 3 + 0.06s^2$$

$$\Rightarrow \boxed{(1+s)I_1 + 0.02 I_1 s^2 - 0.06s^2 = 3}$$



$$\frac{V}{0.5} + \frac{V}{\frac{1}{2s} + 0.01s} = 6$$

$$\Rightarrow 2V + \frac{2sV}{1 + 0.02s^2} = 6$$

$$\Rightarrow 2V + 0.04Vs^2 + 2sV = 6 + 0.12s^2$$

$$\Rightarrow \boxed{(1+s)V + 0.02Vs^2 - 0.06s^2 = 3}$$

01/10/2021

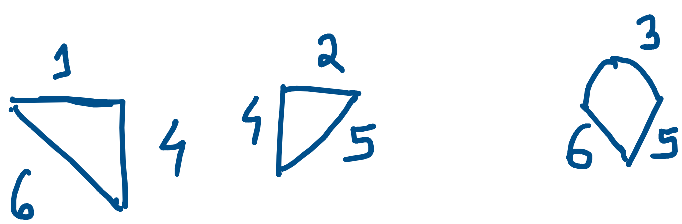
Q Branch current and loop-current relationships are expressed in matrix form as

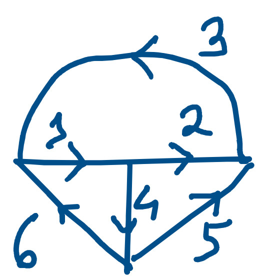
$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ J_5 \\ J_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Draw the oriented graph.

link Twig's

$$[T]_{l \times B} = \begin{array}{c} \\ l_1 \\ l_2 \\ l_3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix} \end{array}$$





Q The reduced incidence matrix of an oriented graph is

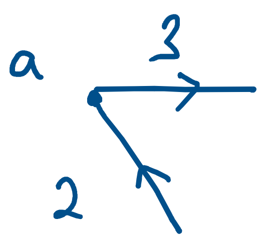
$$A = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

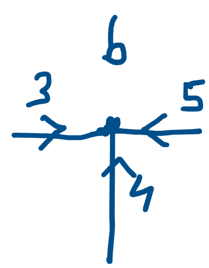
i) Draw the graph.

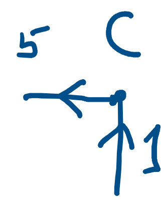
ii) How many trees are possible for this graph?

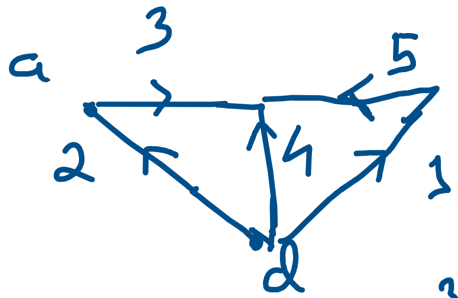
iii) Write the tie-set and f-cut-set matrices.

$$[A_c]_{n\times B} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

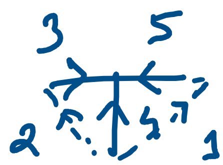




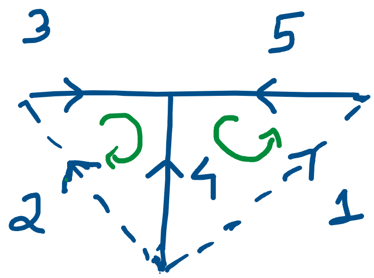




The no. of possible tree's $= \underline{\underline{8}}$

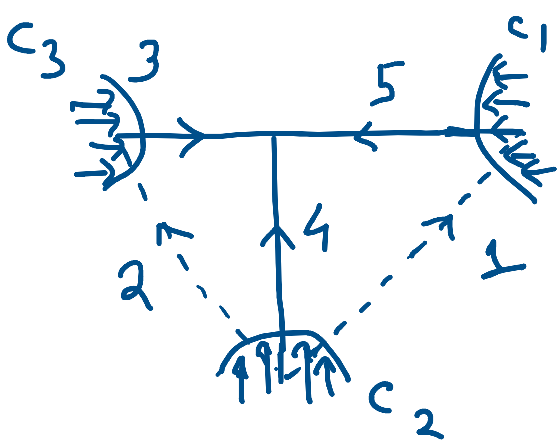


04/10/2021



Tie set matrix

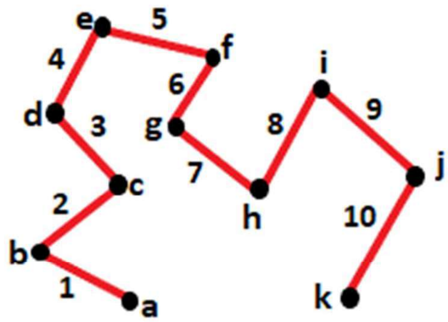
$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ [T] = & \begin{bmatrix} 0 & 1 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$



f - cut set matrix

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ [C] = & \begin{bmatrix} -1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Forest - A forest is the **disjoint**(disconnected) **union of trees**.



By deleting

Some edges

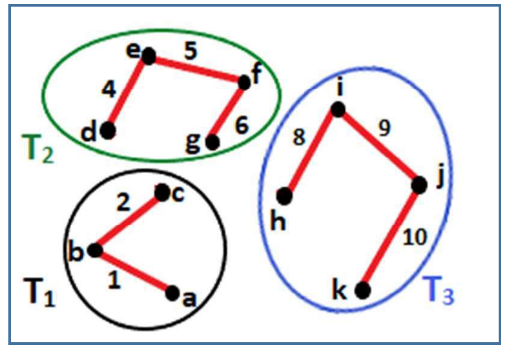


Fig: Tree

Fig: Forest

Nodes, n = 11

Branches, B = (n-1) = 10

{1,2}, {4,5,6}, {8,9,10}

T_1 T_2 T_3

- Every tree is a forest but every forest is not a tree.
- If there are **'n' nodes** and **'K' components in a forest**, then **no. of edges = (n-K)**

<u>Proof</u>

Let $T_1, T_2, \ldots\ldots\ldots\ldots T_k$ be K trees with $n_1, n_2, \ldots\ldots\ldots n_k$ nodes respectively.

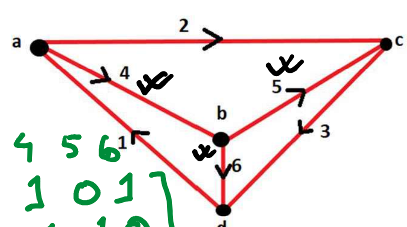
Then, $n_1 + n_2 + \ldots\ldots\ldots n_k = n$

& $(n_1 - 1) + (n_2 - 1) + \ldots\ldots\ldots\ldots + (n_k - 1) =$ No. of edges in the forest

$\Rightarrow (n_1 + n_2 + \ldots\ldots\ldots n_k) - k =$ No. of edges in the forest

$\Rightarrow$ $\boxed{\textbf{No. of edges in the forest} = (n - K)}$

Q For the graph shown, write the i) incidence matrix ii) tie-set matrix iii) f-cut-set matrix



Tie-set Matrix

$$[T]_{3\times6} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix} \end{array}$$

Incidence Matrix

$$[I]_{4\times6} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

f-cut set Matrix

$$[C]_{3\times6} = \begin{array}{c} \\ c_4 \\ c_5 \\ c_6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

05/10/2021

May be **voltage driven** or **current driven**

Two Port Network

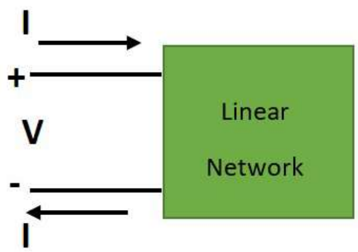


Fig: One Port N/W

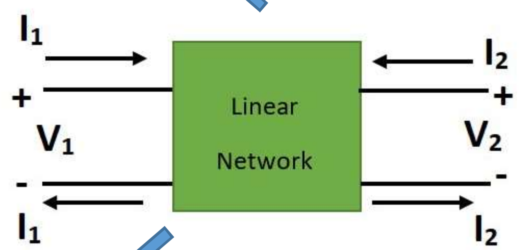


Fig : Two Port N/W

Two port ckt. contains **No Independent Sources**, although can contain **dependent sources**

- Port
- Examples of one port N/W
- Examples of two port N/W
- Application of two port N/W
- Terminal quantities of two port N/W
- Parameters (6 Sets)
- Relation between these parameters
- Series, parallel and cascade connections of two port N/W

Impedance Parameter

Uses/ Application

- Synthesis of **filters**
- Design and analysis of **impedance-matching** networks and **power distribution N/Ws**

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

Z terms are called the **impedance Parameters** or simply **Z Parameters** and have a units of **Ohms**.

In Matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$Z_{11} \rightarrow$ Open Circuit i/p impedance or i/p **driving point** impedance with the o/p port O.C.

$Z_{12} \rightarrow$ Open Circuit **transfer** impedance from port 1 to 2

$Z_{21} \rightarrow$ Open Circuit **transfer** impedance from port 2 to 1

$Z_{22} \rightarrow$ Open Circuit output impedance or o/p **driving point** impedance with the i/p port O.C.

<u>**08/10/2021**</u>

<u>**Symmetry Condition**</u>

$Z_{11} = Z_{22}$

N/W has mirrorlike symmetry about some center line i.e. **a line** can be found that divides the N/W into **two similar halves**.

<u>**Reciprocal Condition**</u>

$Z_{12} = Z_{21}$

If the points of excitation and response are interchanged, the transfer impedances remain the same.

- Any **two port** that is made entirely of **resistors, capacitors and inductors** must be **reciprocal. (**when two port N/W is **linear** and has **no dependent sources)**

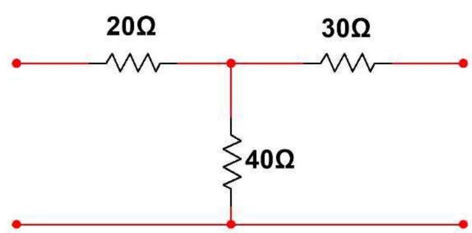
Linearity = Homogeneity (scaling) + Additivity

Q1 Determine the Z parameter for the circuit.

<u>Sol^n</u>:

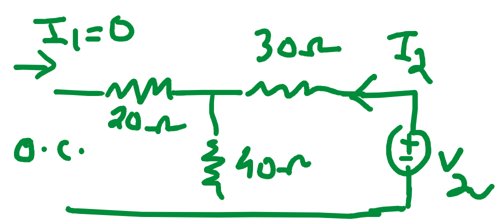
$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$



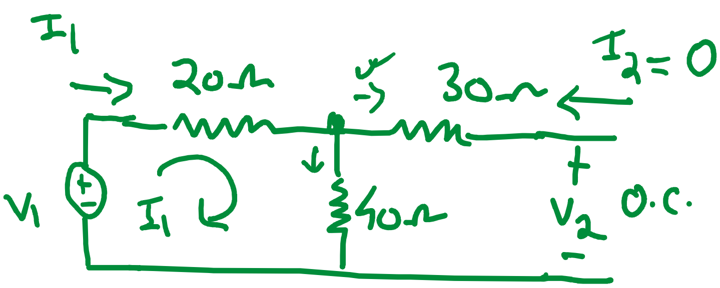
✓ $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$, $Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$

✓ $Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$, $Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$



$Z_{12} = 40\,\Omega$

$Z_{22} = 70\,\Omega$



$V_2 = 40 I_1$

$\frac{V_2}{I_1} = Z_{21} = 40\,\Omega$

$V_1 = (20+40) I_1 = 60 I_1$

$\Rightarrow \frac{V_1}{I_1} = Z_{11} = 60\,\Omega$

$[Z] = \begin{bmatrix} 60\Omega & 40\Omega \\ 40\Omega & 70\Omega \end{bmatrix}$

Sr. No.	Parameter	Equations in Matrix form	Symmetry Condition	Reciprocity Condition
1	**Z**- Parameter or O.C. Impedance Parameters	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
2	**Y**-Parameter or S.C. Admittance Parameters	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$	$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
3	Transmission Parameter or ABCD Parameter (**T**)	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$	A = D	AD – BC = 1 Or $\Delta T = 1$
4	Inverse Transmission Parameter or $A'B'C'D'$ Parameter (**T′**)	$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$	$A' = D'$	$A'D' - B'C' = 1$ Or $\Delta T' = 1$
5	Hybrid Parameter or H-Parameter (**h**)	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$	$h_{11}h_{22} - h_{12}h_{21} = 1$ Or $\Delta h = 1$	$h_{12} = - h_{21}$
6	Inverse Hybrid Parameter or G-Parameter (**g**)	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$	$g_{11}g_{22} - g_{12}g_{21} = 1$ Or $\Delta g = 1$	$g_{12} = - g_{21}$

<u>23/10/2021 (2hr Class)</u>

<u>Hybrid Parameter or H-Parameter (h)</u>

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

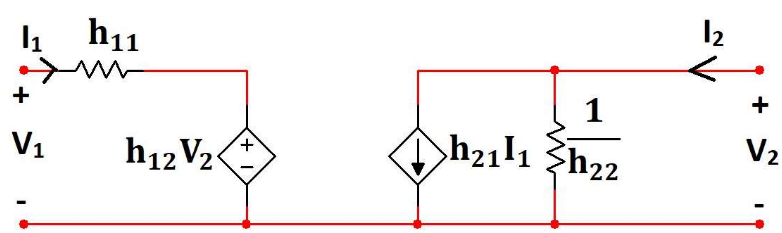
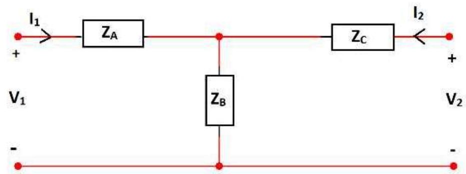


Fig: h-parameter equivalent network of a two-port N/W

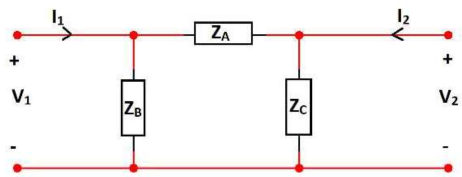
Remember the results



$V_1 = I_1 Z_A + (I_1 + I_2) Z_B = (Z_A + Z_B) I_1 + Z_B I_2$

$V_2 = I_2 Z_C + (I_1 + I_2) Z_B = Z_B I_1 + (Z_B + Z_C) I_2$

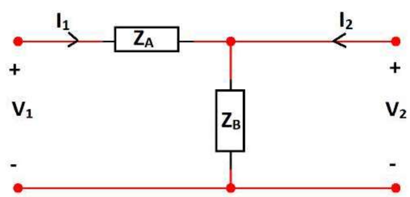
$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z_A + Z_B} & \mathbf{Z_B} \\ \mathbf{Z_B} & \mathbf{Z_B + Z_C} \end{bmatrix}$$



$I_1 = \frac{V_1}{Z_B} + \frac{V_1 - V_2}{Z_A} = \left(\frac{1}{Z_A} + \frac{1}{Z_B}\right) V_1 - \frac{1}{Z_A} V_2$

$I_2 = \frac{V_2}{Z_C} + \frac{V_2 - V_1}{Z_A} = -\frac{1}{Z_A} V_1 + \left(\frac{1}{Z_A} + \frac{1}{Z_C}\right) V_1$

$$\mathbf{Y} = \begin{bmatrix} \frac{1}{Z_A} + \frac{1}{Z_B} & -\frac{1}{Z_A} \\ -\frac{1}{Z_A} & \frac{1}{Z_A} + \frac{1}{Z_C} \end{bmatrix}$$



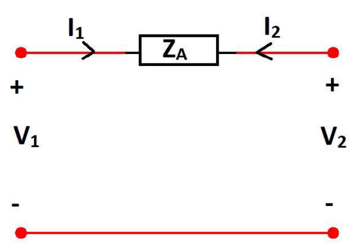
$V_1 = I_1 Z_A + (I_1 + I_2) ZB = (Z_A + Z_B) I_1 + Z_B I_2$ (i)

$V_2 = (I_1 + I_2) Z_B \Rightarrow I_1 = \frac{1}{Z_B} V_2 - I_2$

(i) $\Rightarrow V_1 = (Z_A + Z_B)\left(\frac{V_2}{Z_B} - I_2\right) + Z_B I_2$

$= \left(1 + \frac{Z_A}{Z_B}\right) V_2 - Z_A I_2$

$$\mathbf{T} = \begin{bmatrix} \mathbf{1} + \frac{Z_A}{Z_B} & \mathbf{Z_A} \\ \frac{1}{Z_B} & \mathbf{1} \end{bmatrix}$$

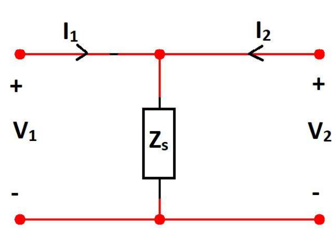


$I_1 = \frac{V_1 - V_2}{Z_A} = \frac{1}{Z_A} V_1 + \left(-\frac{1}{Z_A}\right) V_2$

$I_2 = \frac{V_2 - V_1}{Z_A} = \left(-\frac{1}{Z_A}\right) V_1 + \frac{1}{Z_A} V_2$

$$\mathbf{Y} = \begin{bmatrix} \frac{1}{Z_A} & -\frac{1}{Z_A} \\ -\frac{1}{Z_A} & \frac{1}{Z_A} \end{bmatrix}$$

[Z] don't exist since |Y| = 0



$V_1 = (I_1 + I_2) Z_S = Z_S I_1 + Z_S I_2$

$V_2 = (I_1 + I_2) Z_S = Z_S I_1 + Z_S I_2$

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z_S} & \mathbf{Z_S} \\ \mathbf{Z_S} & \mathbf{Z_S} \end{bmatrix}$$

[Y] don't exist since |Z| = 0

Interrelationship between various parameters

Y Parameters in terms of Z parameter

Z Parameter

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

Y Parameter

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \Rightarrow I_1 = \frac{V_1 - Z_{12}I_2}{Z_{11}} = \frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}}\left(\frac{V_2 - Z_{21}I_1}{Z_{22}}\right)$$

$$= \frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}Z_{22}}V_2 + \frac{Z_{12}Z_{21}}{Z_{11}Z_{22}}I_1$$

$$\Rightarrow I_1\left(1 - \frac{Z_{12}Z_{21}}{Z_{11}Z_{22}}\right) = \frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}Z_{22}}V_2$$

$$\Rightarrow I_1\frac{\Delta Z}{Z_{11}Z_{22}} = \frac{V_1}{Z_{11}} - \frac{Z_{12}}{Z_{11}Z_{22}}V_2$$

$$\Rightarrow \boxed{I_1 = \underbrace{\left(\frac{Z_{22}}{\Delta Z}\right)}_{Y_{11}}V_1 + \underbrace{\left(-\frac{Z_{12}}{\Delta Z}\right)}_{Y_{12}}V_2}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \Rightarrow I_2 = \frac{V_2}{Z_{22}} - \frac{Z_{21}}{Z_{22}}\left[\left(\frac{Z_{22}}{\Delta Z}\right)V_1 + \left(-\frac{Z_{12}}{\Delta Z}\right)V_2\right]$$

$$= \left(-\frac{Z_{21}}{\Delta Z}\right)V_1 + \left[\frac{1}{Z_{22}} + \frac{Z_{12}Z_{21}}{\Delta Z \times Z_{22}}\right]V_2$$

$$= \left(-\frac{Z_{21}}{\Delta Z}\right)V_1 + \left(\frac{Z_{11}Z_{22} - Z_{12}Z_{21} + Z_{12}Z_{21}}{\Delta Z \times Z_{22}}\right)V_2$$

$$\Rightarrow \boxed{I_2 = \underbrace{\left(-\frac{Z_{21}}{\Delta Z}\right)}_{Y_{21}}V_1 + \underbrace{\left(\frac{Z_{11}}{\Delta Z}\right)}_{Y_{22}}V_2}$$

h Parameters in terms of Z parameter

Z Parameter

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

h Parameter

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

$$h = \begin{bmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{bmatrix}$$

Interconnection of N/W's

When two port N/Ws are connected in

- **Series,** the **Z parameter** for the **overall network** is given by **sum** of the **Z parameters** for the **individual networks**.

$$[Z] = [Z_A] + [Z_B]$$

- **Parallel**, the **Y parameter** for the **overall network** is given by **sum** of the **Y parameters** for the **individual networks**.

$$[Y] = [Y_A] + [Y_B]$$

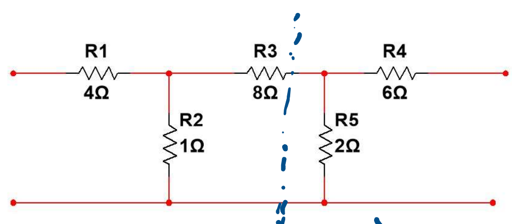
- **Cascade**, the **T parameter** for the **overall network** is given by **product** of the **T parameters** for the **individual networks**.

$$[T] = [T_A] \times [T_B]$$

Q1 Find the transmission parameter for the circuit.

$$[T] = \begin{bmatrix} 27 & 206\Omega \\ 5.5\,S & 42 \end{bmatrix}$$

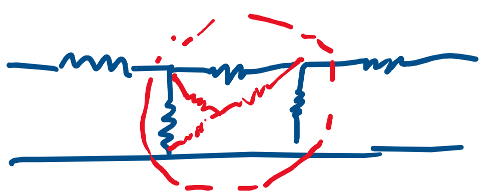
① $Z_1 \downarrow T_1$



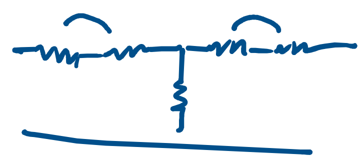
$\rightarrow Z_2 \rightarrow T_2$

$[T] = [T_1] \times [T_2]$

②

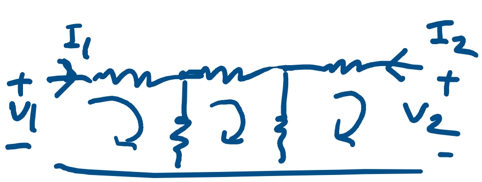


$\Rightarrow$

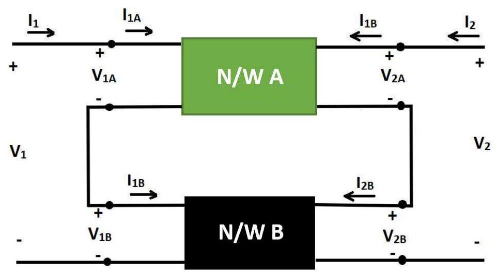


$\rightarrow Z \rightarrow T$

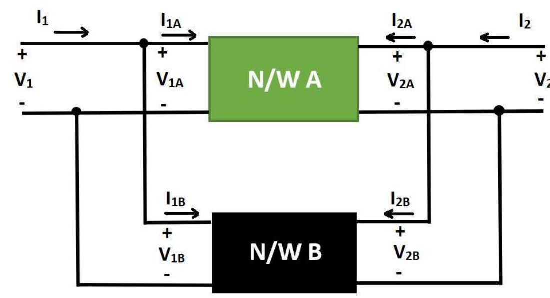
③



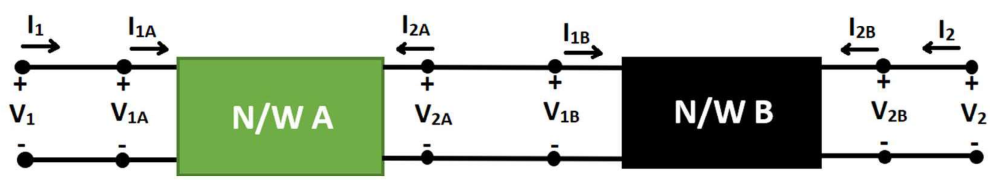
Series Connection



Parallel Connection



Cascade Connection

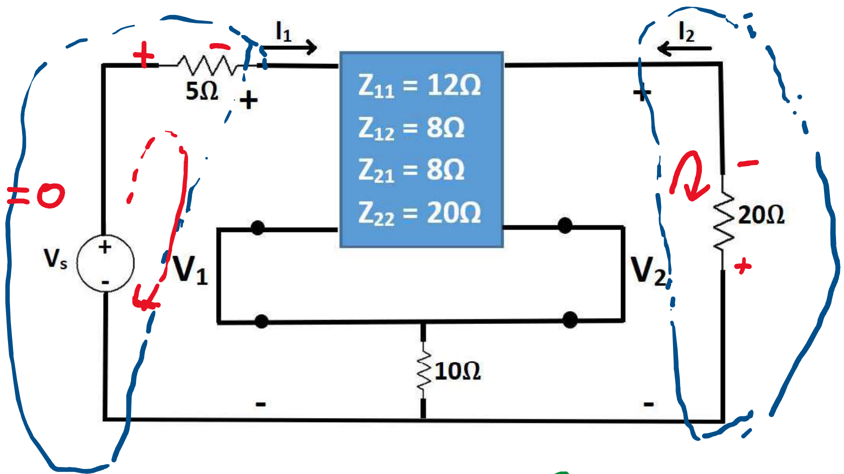


02/11/2021

Q2 Evaluate V_2/V_s in the circuit.

$V_s - I_1 5 - V_1 = 0$

$V_s = 5I_1 + V_1$



$V_2 + I_2 20 = 0$

$V_2 = -20 I_2$

Eqv $[Z] = \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$

$V_1 = 22 I_1 + 18 I_2$

$V_2 = 18 I_1 + 30 I_2$

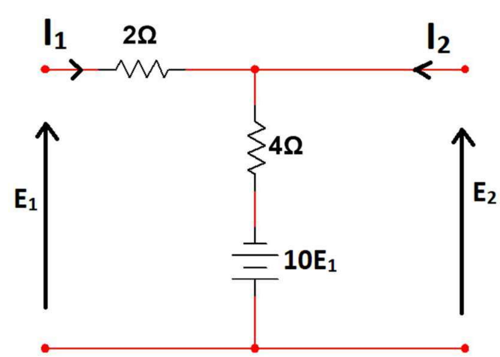
$V_2 = -20 I_2$

$V_S = 5I_1 + V_1$

$\frac{V_2}{V_s} = 0.3509$

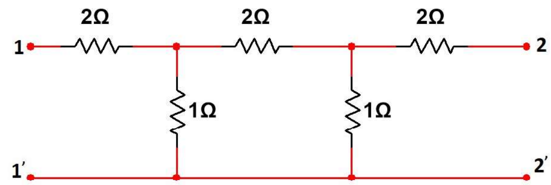
GATE 2001

Q3 The Z parameters Z_{11} and Z_{21} for the 2-port network in the figure are



GATE 2003

Q4 The impedance parameters Z_{11} and Z_{21} of the 2-port network in the figure are



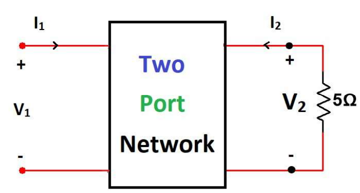
$$[Z] = \begin{bmatrix} 11/4\,\Omega & 1/4\,\Omega \\ 1/4\,\Omega & 15/4\,\Omega \end{bmatrix}$$

Q5 The following equation gives the voltage and current at the input port of a two-port network shown in fig shown.

$$V_1 = 5V_2 - 3I_2$$

$$I_1 = 6V_2 - 2I_2$$

A load resistance of 5Ω is connected across the output port. Calculate the input impedance.



$$Z_i = 7/8\,\Omega$$

09/11/2021

Network Function

Mathematically, a **network function** is defined as the **ratio** of laplace transform of **output** (response) of the network to the laplace transform of **input** (excitation), **with all initial conditions as zero**.

Network function is broadly classified as **driving point** and **transfer function**.

In **time-domain analysis**, the general **solution** of a **differential equation** is given as

$$\boxed{i(t) = k_n e^{S_n t}}$$

where S_n is a complex number, which is **root of the characteristic equation** and is expressed as

$$\boxed{S_n = \sigma_n + j\omega_n}$$

Complex frequency

Neper frequency

Governs the **exponential decay or rise**

Radian frequency

Governs the **Sinusoidal Oscillations**

Driving Point Function

If **excitation** and **response** are measured at the **same ports**, the network function is known as the **driving point function**.

i) Driving point **Impedance Function**: $Z(s) = \frac{V(S)}{I(S)}$

ii) Driving point **Admittance Function**: $Y(s) = \frac{I(S)}{V(S)}$

Transfer Point Function

Network should have **at least two ports**.

i) **Voltage Transfer Function**: $G_{12}(S) = \frac{V_2(S)}{V_1(S)}$

$G_{21}(S) = \frac{V_1(S)}{V_2(S)}$

ii) **Current Transfer Function**: $\alpha_{12}(S) = \frac{I_2(S)}{I_1(S)}$

$\alpha_{21}(S) = \frac{I_1(S)}{I_2(S)}$

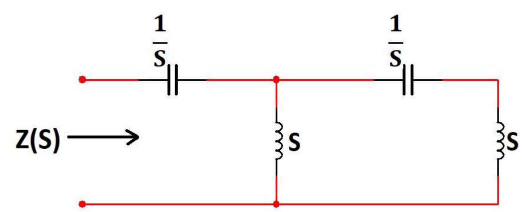
iii) **Transfer Impedance Function**: $Z_{12}(S) = \frac{V_2(S)}{I_1(S)}$

$$Z_{21}(S) = \frac{V_1(S)}{I_2(S)}$$

iv) **Transfer Admittance Function**: $Y_{12}(S) = \frac{I_2(S)}{V_1(S)}$

$$Y_{21}(S) = \frac{I_1(S)}{V_2(S)}$$

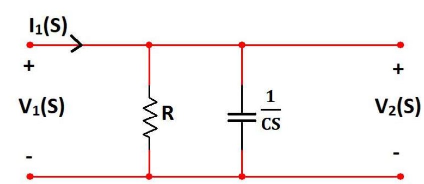
Q1 Determine the driving point impedance of network shown in fig below.



<u>**Sol**</u>

<u>Sol^n</u> $Z(S) = \frac{S^4+3S^2+1}{2S^3+S}$

Q2 Find the transfer impedance function $Z_{12}(S)$ for the network shown below.



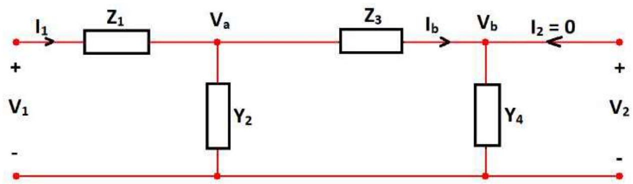
<u>Sol^n</u>

$$Z_{12}(S) = \frac{V_2(S)}{I_1(S)} = \frac{1}{C\left(S+\frac{1}{RC}\right)}$$

Analysis of ladder N/W

- A N/W composed of a **sequence of H, L, T or pi networks** connected in tandem; chiefly used as an **electric filter**. Also known as series-shunt N/W.
- A N/W consisting of circuit elements (R, L, C) connected in **series and in parallel fashion**.
- A ladder N/W may be described as a **cascade connection of a series of symmetrical T-section or pi-section** four-terminal N/Ws.

Analysis is done by writing the set of equations by **starting at Port 2 of the ladder and working towards Port 1**.



$V_b = V_2$

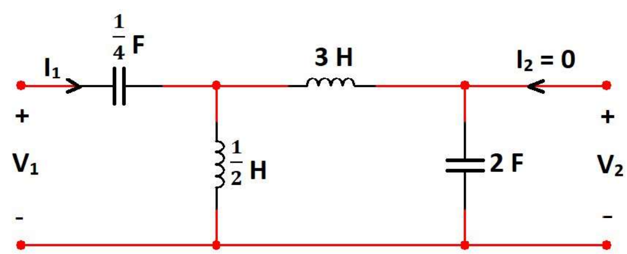
$I_b = Y_4 V_2$

$V_a = Z_3 I_b + V_2 = (Z_3 Y_4 + 1)\ V_2$

$I_1 = Y_2 V_a + I_b = [Y_2(Z_3 Y_4\ +\ 1) +\ Y_4]\ V_2$

$V_1 = Z_1 I_1 + V_a = [Z_1\{Y_2(Z_3 Y_4\ +\ 1) +\ Y_4\} +\ (Z_3 Y_4\ +\ 1)]\ V_2$

Q1 Find the network functions $\frac{V_1}{I_1}$, $\frac{V_2}{V_1}$ and $\frac{V_2}{I_1}$ for the network shown below.



Sol^n

$\frac{V_1}{I_1} = \frac{6S^4 + 57S^2 + 8}{14S^3 + 2S}$

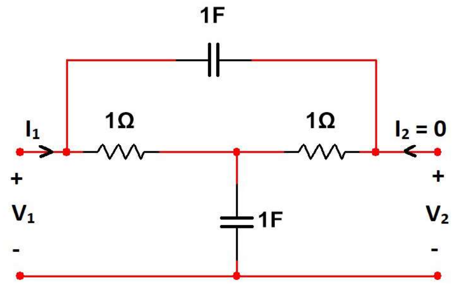
$\frac{V_2}{V_1} = \frac{S^2}{6S^4 + 57\ {}^2 + 8}$

$\frac{V_2}{I_1} = \frac{S}{14S^2 + 2}$

Analysis of non-ladder networks

For such type of networks, it is necessary to express the network functions as a quotient of determinants, formulated on **KVL and KCL** basis.

Q2 For the network shown, determine $Z_{11}(s)$, $Z_{12}(s)$ and $G_{12}(s)$



<u>Soln</u>

$$Z_{11}(s) = \frac{V_1}{I_1} = \frac{S^2+3\ \ +1}{S(2S+1)}$$

$$Z_{12}(s) = \frac{V_2}{I_1} = \frac{S^2+2\ \ +1}{S(2S+1)}$$

$$G_{12}(s) = \frac{V_2}{V_1} = \frac{S^2+2S+1}{S^2+3S+1}$$

<u>Poles and Zeros of N/W functions</u>

The **network function F(s)** can be written as a ratio of two polynomials

$$F(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1}s^{n-1} + \ldots\ldots + a_1 s + a_0}{b_m s^m + b_{m-1}s^{m-1} + \ldots\ldots + b_1 s + b_0} = H\frac{(s-z_1)(s-z_2)\ldots\ldots\ldots(s-z_n)}{(s-p_1)(s-p_2)\ldots\ldots\ldots(s-p_m)}$$

where $a_0, a_1, \ldots\ldots, a_n$ are **coefficients** of the polynomial **N(s)**

$b_0, b_1, \ldots\ldots, b_m$ are **coefficients** of the polynomial **D(s)**

These are real and positive for N/W's of passive elements

$z_1, z_2, \ldots\ldots, z_n$ are **roots** of **N(s) = 0**

$p_1, p_2, \ldots\ldots, p_m$ are **roots** of **D(s) = 0**

Are complex frequencies.
$z_1, z_2, \ldots\ldots, z_n$ → Zeros of N/W function
$p_1, p_2, \ldots\ldots, p_m$ → Poles of N/W function

$H = \frac{a_n}{b_m}$ is a constant called **scale factor.**

- If the poles or zeros are **not repeated**, then the function is said to be having **simple poles or simple zeros**.
- If the poles or zeros are **repeated**, then the function is said to be having **multiple poles or multiple zeros**.
- In addition to **finite** poles and zeros of a network function, if poles and zeros located at zero and infinity are considered, then for a **network function** the

  Total no. of zeros = Total no. of poles
  - When n>m, then (n-m) poles are at s = ∞
  - For m>n, (m-n) zeros are at s = ∞
- The graphical symbol for a **pole is X** and for a **zero is O**.

16/11/2021

Necessary conditions for Driving-Point functions

The necessary conditions for a N/W function to be a driving-point function with common factors in numerator polynomial N(s) and denominator polynomial D(s) cancelled are as follows:

1. The coefficients in the polynomials N(s) and D(s) must be real and positive.
2. The poles and zeros, if complex or imaginary, must occur in conjugate pairs.
3. The real part of all poles and zeros must be negative or zero, i.e., the poles and zeros must lie in left half of S-plane.
4. If the real part of pole or zero is zero, then that pole or zero must be simple.
5. The polynomials N(s) and D(s) may not have missing terms between those of highest and lowest degree, unless all even or all odd terms are missing.
6. The degree of N(s) and D(s) may differ by either zero or one only. This condition prevents multiple poles and zeros at $S = \infty$.
7. The terms of lowest degree in N(s) and D(s) may differ in degree by one at most. This condition prevents multiple poles and zeros at S=0.

Q Test whether the following represent **driving-point immittance functions**.

a) $\frac{5S^4+3S^2-2 \quad +1}{S^3+6S+2}$ b) $\frac{S^3+S^2+5S+2}{S^4+6S^3+9S^2}$ c) $\frac{S^2+3S+2}{S^2+6S+2}$

a) 1,5

b) 7

c) Satisfies all

Necessary conditions for Transfer functions

The necessary conditions for a N/W function to be a transfer function with common factors in numerator polynomial N(s) and denominator polynomial D(s) cancelled are as follows:

1. The coefficients in the polynomials N(s) and D(s) must be real and those for D(s) must be positive.
2. The poles and zeros, if complex or imaginary, must occur in conjugate pairs.
3. The real part of poles must be negative or zero. If the real part is zero, then that pole must be simple.
4. The polynomial D(s) may not have any missing terms between that of highest and lowest degree, unless all even or all odd terms are missing.
5. The polynomial N(s) may have missing terms between the terms of lowest and highest degree, and some of the coefficients may be negative.
6. The degree of N(s) may be as small as zero, independent of the degree of D(s).
7. For voltage and current transfer functions, the maximum degree of N(s) is the degree of D(s).
8. For transfer impedance and admittance functions, the maximum degree of N(s) is the degree of D(s) plus one.

Q Test whether the following represent **transfer functions**

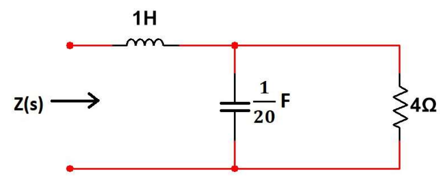
a) $G_{21} = \frac{3S + 2}{5S^3+4S^2+1}$ b) $\alpha_{12} = \frac{2S^2 + 5S + 1}{S + 7}$ c) $Z_{21} = \frac{1}{S^3 + 2}$

a) 4

b) 7

c) Satisfies all

Q Determine Z(s) in the network shown. Find poles and zeros of Z(s) and plot them on s-plane.



Sol^n Z(s) = $\frac{(s+2.5+j3.71)(s+2.5-j3.71)}{(s+5)}$

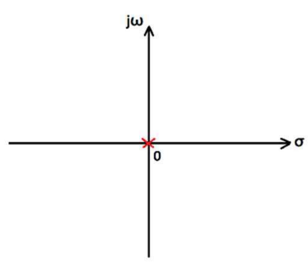
Time domain behaviour from the pole-zero plot

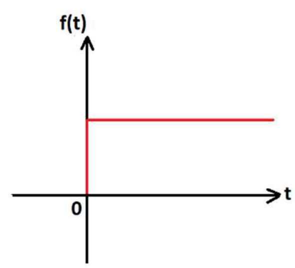
Consider a network function

F(s) = H$\frac{(s-z_1)(s-z_2)\ldots\ldots\ldots(s-z_n)}{(s-p_1)(s-p_2)\ldots\ldots\ldots(s-p_m)}$

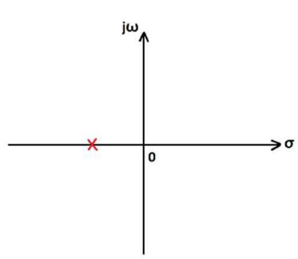
- The **poles** of this function determine the **time-domain behaviour of f(t)**
- The function f(t) can be determined from the knowledge of the **poles**, the **zeros** and the **scale factor H**.

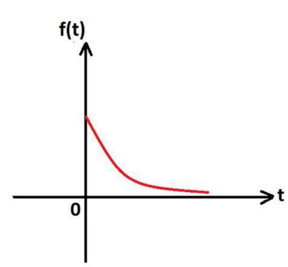
I) When pole is at origin, i.e., at s = 0, the function f(t) represents steady-state response of the circuit i.e., **dc value.**



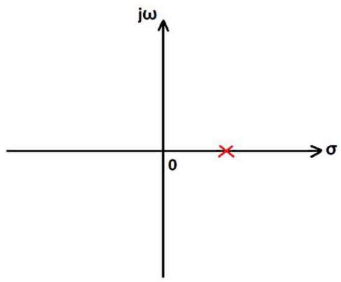


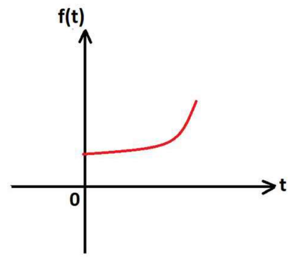
II) When **pole** lies in the **left half of the s-plane**, the response decreases exponentially



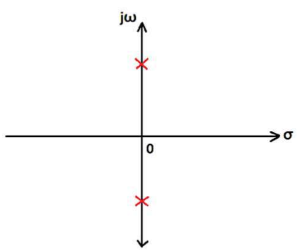


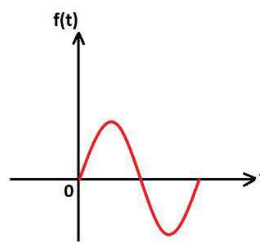
III) When pole lies in the right half of the s-plane, the response increases exponentially. A **pole** in the **right half plane** gives rise to **unbounded response** and **unstable system**.



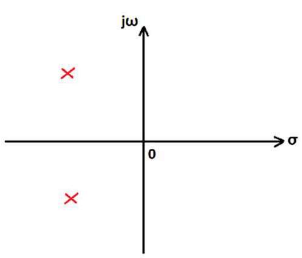


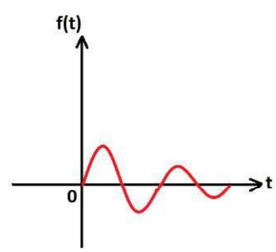
IV) For s = 0 ± jω$_n$, the response becomes f(t) = k$_n$ $e^{\pm j\omega_n t}$ = k$_n$ (cos $\omega_n t$ ± j sin $\omega_n t$). The variation of exponential function $e^{\pm j\omega_n t}$ with time is sinusoidal and hence, constitutes the case of **sinusoidal steady state**.

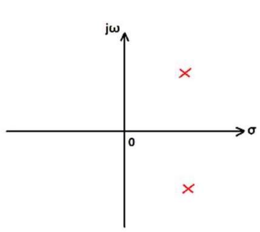


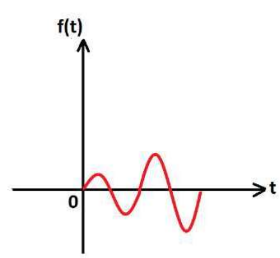


V) For s = σ$_n$ + jω$_n$, the response becomes f(t) = k$_n$ $e^{s_n t}$ = k$_n$ $e^{(\sigma_n + j\omega_n)t}$ = k$_n$ $e^{\sigma_n t} e^{j\omega_n t}$. The response $e^{\sigma_n t}$ is an exponentially increasing or decreasing function. The response $e^{j\omega_n t}$ is a sinusoidal function. Hence, the response of the product of these responses will be **over damped sinusoids** or **under damped sinusoids**.

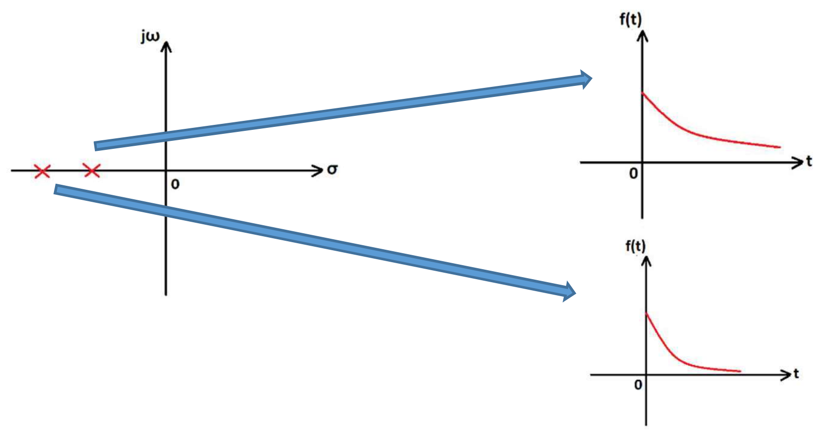








VI) The real part of s of the pole is the displacement of the pole from the imaginary axis. Since σ is also the damping factor, a greater value of σ (i.e., a greater displacement of the pole from the imaginary axis) means that response decays more rapidly with time.



Stability of the Network

Stability of the network is directly related to the **location of poles** in the s-plane

i) When all the **poles** lie in the **left half** of the s-plane, the N/W is said to be **stable**.
ii) When the **poles** lie in the **right half** of the s-plane, the N/W is said to be **unstable**.
iii) When the **poles** lie on the **jω axis**, the N/W is said to be **marginally stable**.
iv) When there are **multiple poles on the jω axis**, the network is said to be **unstable**.
v) When the **poles move away from jω axis** towards the **left half** of the s-plane, the relative **stability** of the network **improves**.

Routh-Hurwitz criterion

Nyquist criterion

Bode Plot

Root locus plot

23/11/2021

Introduction to transients

In **N/W** we have **3 elements** → **Resistor** → **Static element** → **no memory** capability

→**Inductor**
→**Capacitor** } **Dynamic element**/memory element i.e. it **can store energy**. **Present value** may depend on **past value**

Transient analysis or Switching analysis

Transient analysis doesn't exist/valid for resistive N/W, **it needs at least one dynamic element** L or C, since concept of transient is based on charging and discharging of inductor and capacitor.

Switching → to **charge** or **discharge** an inductor/capacitor. Can handle both past and present state with the help of switch.

Before and after switching we actually discuss charging and discharging of inductor and capacitor.

Standard discharging equation

$Y(t) = Y_0\, e^{-t/\tau}$, where **τ** is **time constant**

$Y(t=0) = Y_0\, e^{-0} = Y_0$

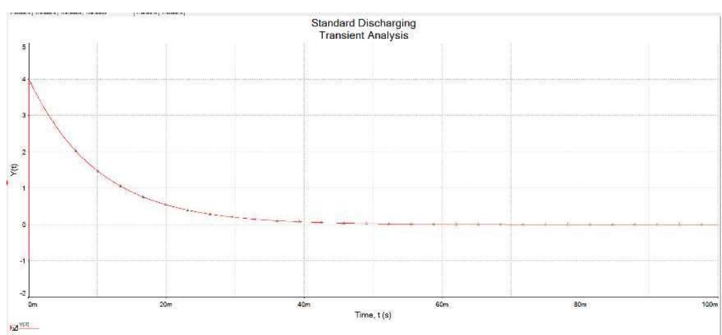
$Y(t=\tau) = Y_0\, e^{-1} = 0.368\ Y_0$

$Y(t=2\tau) = Y_0\, e^{-2} = 0.135\ Y_0$

$Y(t=3\tau) = Y_0\, e^{-3} = 0.049\ Y_0$

$Y(t=4\tau) = Y_0\, e^{-4} = 0.018\ Y_0$

$Y(t=5\tau) = Y_0\, e^{-5} = 0.0067\ Y_0$



………

$Y(t=\infty) = Y_0\, e^{-\infty} = 0$

Time constant→ the time in which response will be **36.8%** (only **valid for standard discharging equation**) of its maximum value or **initial value**.

T_{settle} → Settle down i.e. time at which the **discharge** that is taking place will **settle down to zero**.

Ideal settle time will be **infinite**.

CS → 4τ (<2% error)

N/W → 5τ

Normal discharging equation

Y(t) = K + $Y_0\, e^{-t/\tau}$, where **τ** is **time constant**

K is a **constant**

Y(t=0) = K + $Y_0\, e^{-0}$ = K + Y_0

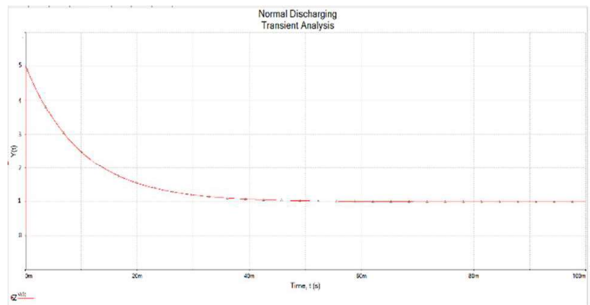
Y(t=τ) = K + $Y_0\, e^{-1}$ = K + 0.368 Y_0

Y(t=2τ) =k + $Y_0\, e^{-2}$ = K + 0.135 Y_0

Y(t=3τ) = K + $Y_0\, e^{-3}$ = K + 0.049 Y_0

Y(t=4τ) = K + $Y_0\, e^{-4}$ = K + 0.018 Y_0

Y(t=5τ) =K + $Y_0\, e^{-5}$ = K + 0.0067 Y_0



………

Y(t=∞) = K + $Y_0\, e^{-\infty}$ = K

This equation is also a discharging equation. The value at infinite time will be K. It will not go to zero.

Therefore, for **Standard discharging** equation → the graph will go to **Zero**

Normal discharging equation → the graph will go to a **finite value**.

Standard charging equation

Y(t) = $Y_0(\,1 - e^{-t/\tau}\,)$, where **τ** is **time constant**

Y(t=0) = $Y_0\,(1 - e^{-0})$ = (1 – 1) Y_0 =0

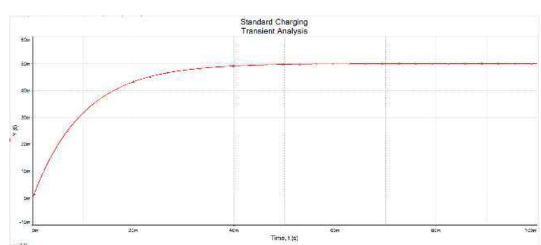
Y(t=τ) = $Y_0\,(1 - e^{-1})$ = (1 - 0.368) Y_0 = 0.632 Y_0

Y(t=2τ) = $Y_0\,(1 - e^{-2})$ = (1 - 0.135) Y_0 = 0.865 Y_0

Y(t=3τ) = $Y_0\, 1 - e^{-3})$ = (1 - 0.049) Y_0 = 0.951 Y_0

Y(t=4τ) = $Y_0\, 1 - e^{-4})$ = (1 - 0.018) Y_0 = 0.98 Y_0

Y(t=5τ) = $Y_0\,(1 - e^{-5})$ = (1 - 0.0067) Y_0 = 0.993 Y_0



………

Y(t=∞) = $Y_0\,(1 - e^{-\infty})$ = (1 – 0) Y_0 = Y_0

Standard charging equation always starts from zero.

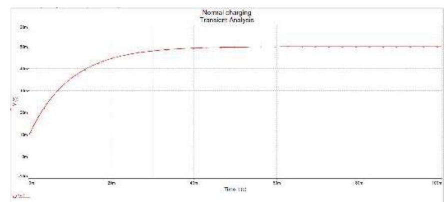
Time constant → The time at which the **response** will be **63.2 % of its final value**.

Normal charging equation

$Y(t) = K + Y_0(\, 1 - e^{-t/\tau}\,)$, where **τ** is **time constant**

Y(t=0) = K + $Y_0\,(1 - e^{-0})$ = K + (1 – 1) Y_0 = K

Y(t=τ) = K + $Y_0\,(1 - e^{-1})$ = K + (1 - 0.368) Y_0 = K + 0.632 Y_0



Y(t=2τ) = K + $Y_0\,(1 - e^{-2})$ = K + (1 - 0.135) Y_0 = K + 0.865 Y_0

Y(t=3τ) = K + $Y_0\, 1 - e^{-3})$ = K + (1 - 0.049) Y_0 = K + 0.951 Y_0

Y(t=4τ) = K + $Y_0\, 1 - e^{-4})$ = K + (1 - 0.018) Y_0 = K + 0.98 Y_0

Y(t=5τ) = K + $Y_0\,(1 - e^{-5})$ = K + (1 - 0.0067) Y_0 = K + 0.993 Y_0

………

Y(t=∞) = K + $Y_0\,(1 - e^{-\infty})$ = K + (1 – 0) Y_0 = K + Y_0

Both standard charging equation and normal charging equation are increasing function/ charging equation but in case of **standard charging equation starts from zero**.

These **equations** are taken for **1st order N/W** and a **single time constant circuit**.

Whether it is a **RL, RC N/W** (1st order N/W), **current/voltage waveform** follows **one of the below figure's** only.

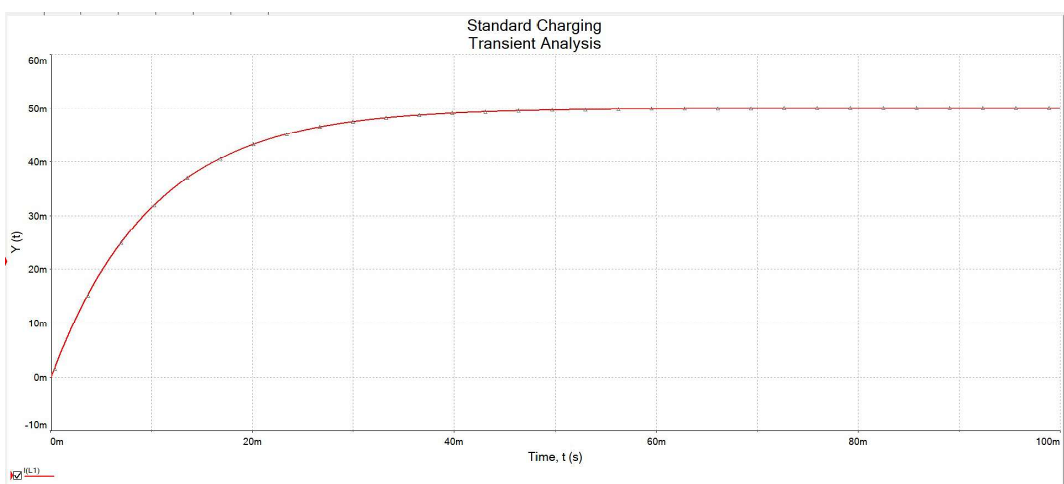


Fig : Standard Charging

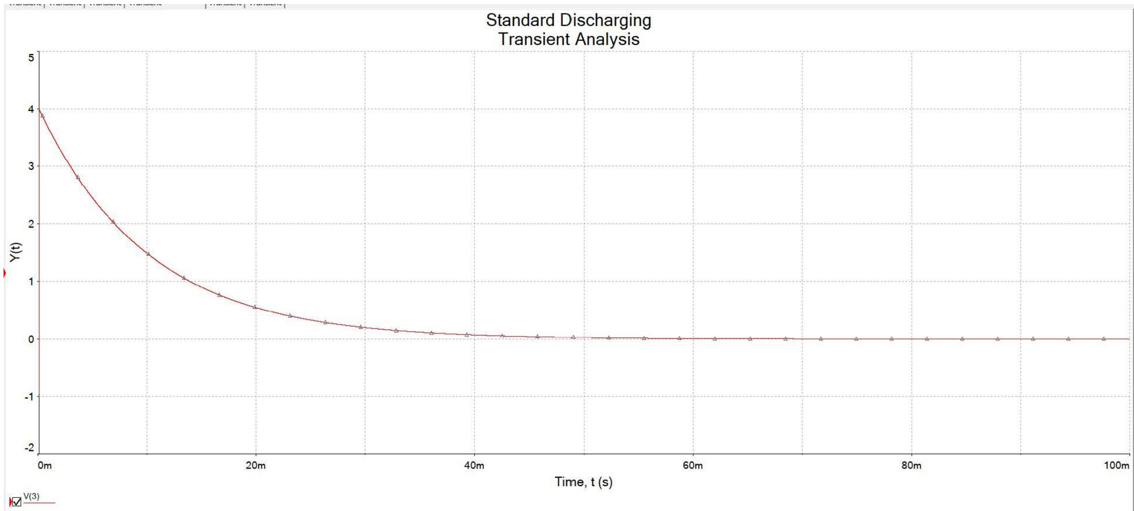


Fig : Standard Discharging

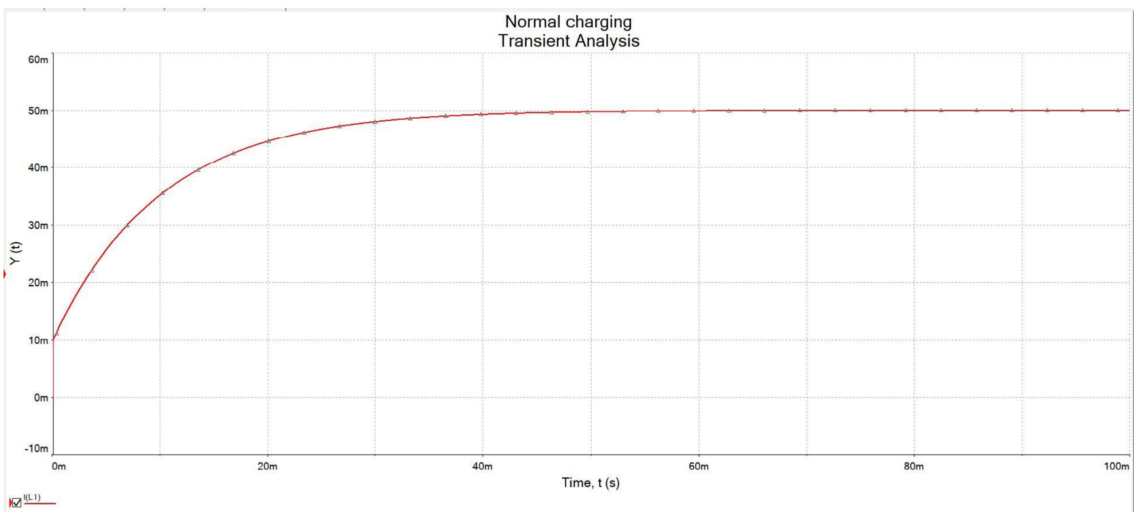


Fig : Normal Charging

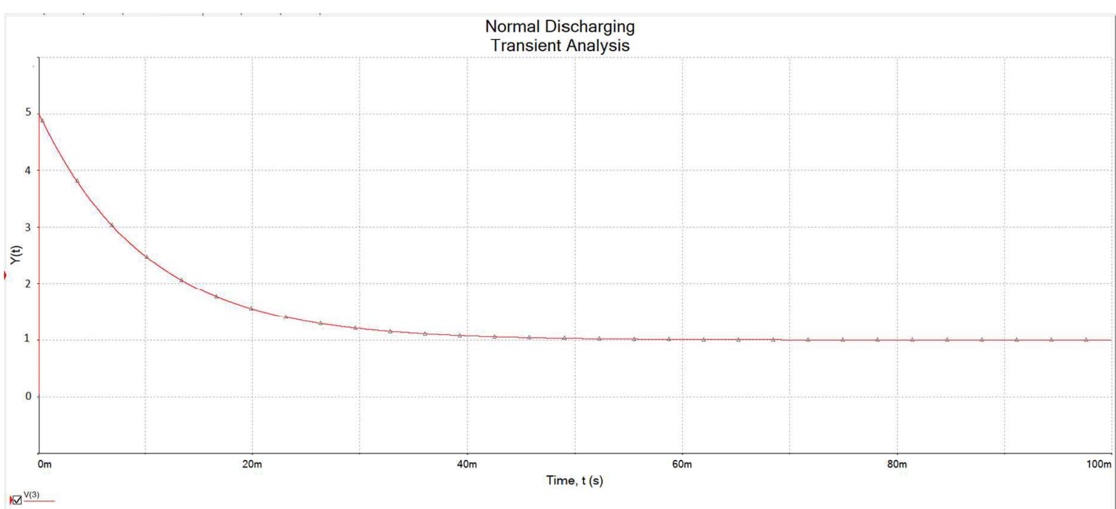
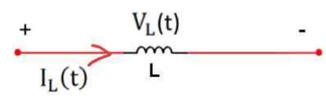


Fig : Normal Discharging

<u>26/11/2021</u>

Inductor, L

According to the **ohm's law**,



Voltage across the **inductor,** $V_L(t) = L\frac{dI_L(t)}{dt}$

$$I_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(t)\,dt = \frac{1}{L}\int_{-\infty}^{0} V_L(t)\,dt + \frac{1}{L}\int_{0}^{t} V_L(t)\,dt$$

Therefore, $\boxed{I_L(t) = I_L(0^-) + \frac{1}{L}\int_{0}^{t} V_L(t)\,dt}$

Discharge of Inductor

Inductor **store** the **energy** in the form of **current.**

Energy stored across inductor, $E_L(t) = \frac{1}{2} L\, I_L^{\,2}(t)$ J

During discharge → i) $E_L(t) \downarrow$ $E_L(\infty)$ = min/0

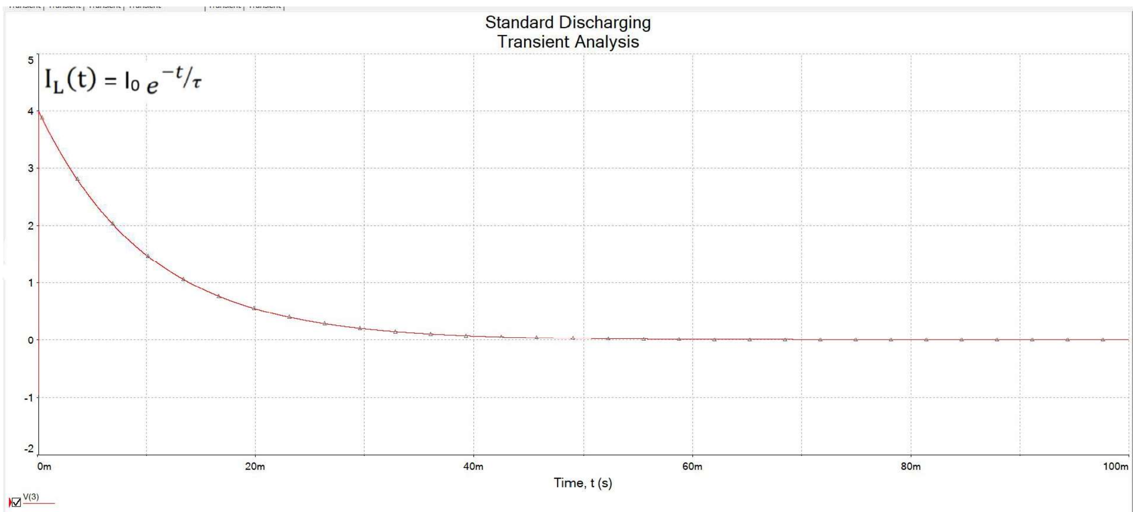
ii) $I_L(t) \downarrow$ $I_L(\infty)$ = min/0

iii) $\frac{dI_L(t)}{dt}$ = -ve (since derivative of decreasing function is -ve)

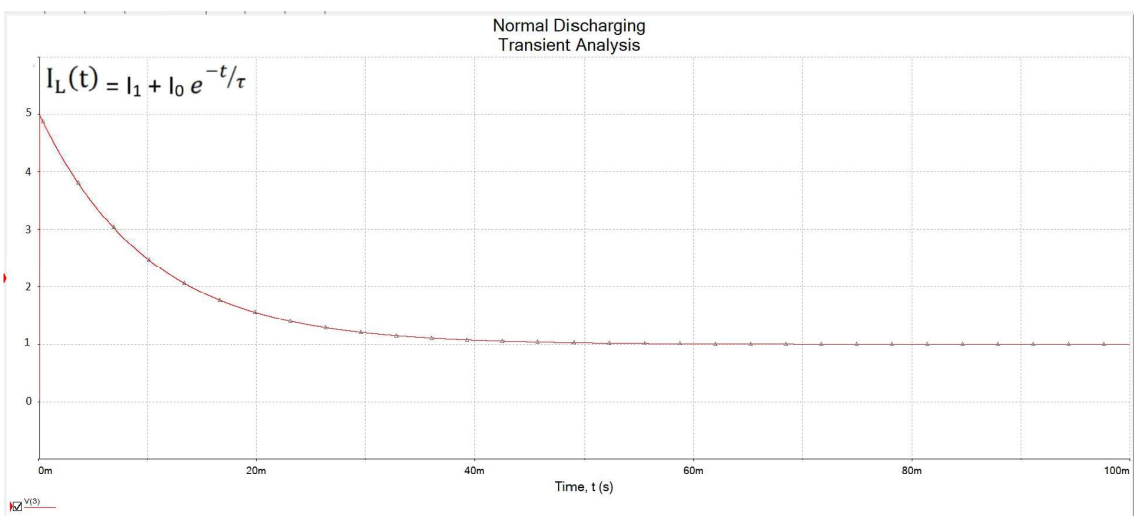
iv) $V_L(t) = L\frac{dI_L(t)}{dt}$ = -ve

v) $V_L(\infty) = L\frac{dI_L(\infty)}{dt} = L\frac{dI_L(\text{min})}{dt}$ = 0 Volt that means <u>**S.C.**</u> at <u>**S.S.**</u>

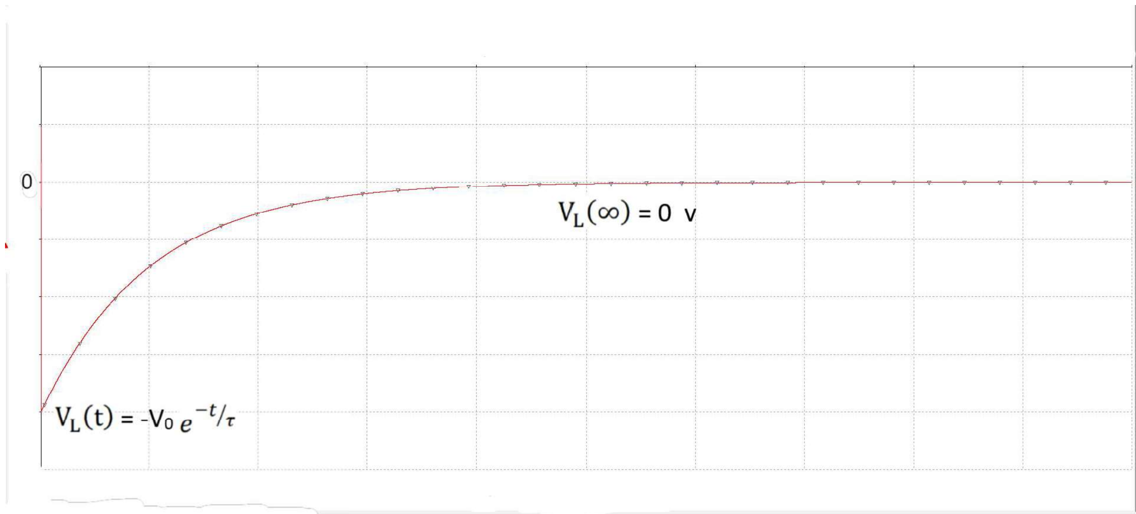
1st order, RL N/W, inductor current, $I_L(t)$ during discharging



Standard Discharging
Transient Analysis
$I_L(t) = I_0\, e^{-t/\tau}$
5
4
3
2
1
0
-1
-2
0m
20m
40m
60m
80m
100m
Time, t (s)
V(3)



Normal Discharging
Transient Analysis
$I_L(t) = I_1 + I_0\, e^{-t/\tau}$
5
4
3
2
1
0
0m
20m
40m
60m
80m
100m
Time, t (s)
V(3)



- ✓ inductor **volt -ve** during discharge and **0** at **S.S.** (For both standard & normal discharge case)
- ✓ inductor current decrease during discharge.

<u>Charging of Inductor</u>

$E_L(t) = \frac{1}{2} L\, I_L^{\,2}(t)$ J

i) $E_L(t) \uparrow \quad E_L(\infty)$ = max

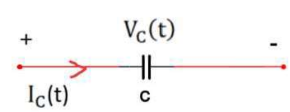
ii) $I_L(t) \uparrow \quad I_L(\infty)$ = max

iii) $\frac{dI_L(t)}{dt}$ = +ve (since derivative of increasing function is +ve)

iv) $V_L(t) = L\frac{dI_L(t)}{dt}$ = +ve

v) $V_L(\infty) = L\frac{dI_L(\infty)}{dt} = L\frac{dI_L(\max)}{dt}$ = 0 Volt   that means <u>**S.C.**</u> at <u>**S.S.**</u>

<u>Capacitor, C</u>



According to the **ohm's law**,

Current through the **capacitor**, $I_C(t) = C\frac{dV_C(t)}{dt}$

$$V_C(t) = \frac{1}{C}\int_{-\infty}^{t} I_C(t)\, dt = \frac{1}{C}\int_{-\infty}^{0} I_C(t)\, dt + \frac{1}{C}\int_{0}^{t} I_C(t)\, dt$$

Therefore, $\boxed{V_C(t) = V_C(0^-) + \frac{1}{C}\int_0^t I_C(t)\, dt}$

Discharge of capacitor

Capacitor store the **energy** in the form of **voltage**.

Energy stored across capacitor, $E_C(t) = \frac{1}{2} C V_C^2(t)$ J

During discharge → i) $E_C(t) \downarrow \quad E_C(\infty) = \text{min}/0$

ii) $V_C(t) \downarrow \quad V_C(\infty) = \text{min}/0$

iii) $\frac{dV_C(t)}{dt} = -ve$ (since derivative of decreasing function is -ve)

iv) $I_C(t) = C\frac{dV_C(t)}{dt} = -ve$

v) $I_C(\infty) = C\frac{dV_C(\infty)}{dt} = C\frac{d(\text{min})}{dt} = 0$ A that means **<u>O.C.</u>** at **<u>S.S.</u>**

1st order, RC N/W, inductor current, $V_C(t)$ during discharging

Charging of capacitor

$E_C(t) = \frac{1}{2} C V_C^2(t)$ J

i) $E_C(t) \uparrow \quad E_C(\infty) = \text{max}$

ii) $V_C(t) \uparrow \quad V_C(\infty) = \text{max}$

iii) $\frac{dV_C(t)}{dt} = +ve$ (since derivative of increasing function is +ve)

iv) $I_C(t) = C\frac{dV_C(t)}{dt} = +ve$

v) $I_C(\infty) = C\frac{dV_C(\infty)}{dt} = L\frac{d(\text{max})}{dt} = 0$ A that means **<u>O.C.</u>** in **<u>S.S.</u>**

For **inductor** and **capacitor**, **current** and **voltage expressions** are just **opposite** of each other because **energy storage capability** is based on **different parameter**. **Capacitor** stores energy in the form of **voltage**, **inductor** stores the energy in the form of **current**. So, their **property is interchanged** in terms of current and voltage.

30/11/2021

Concept of 0^- - 0 – 0^+ in transient Analysis

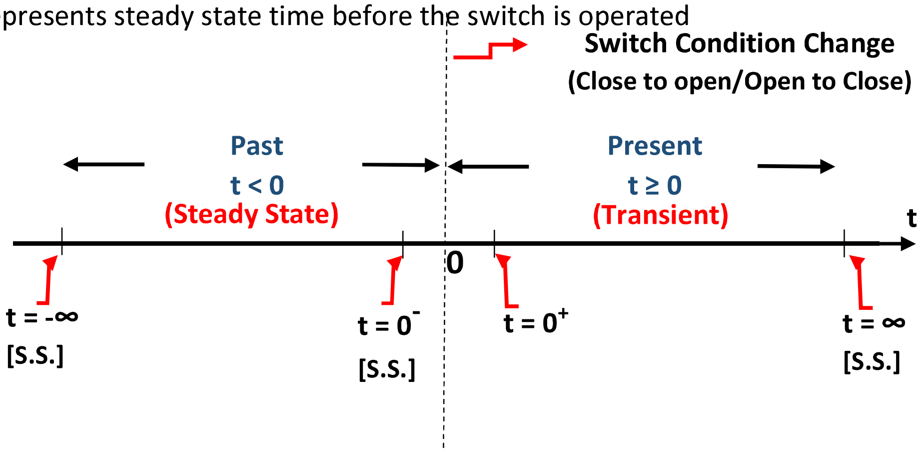
t = 0^+ represents moment of time after the switch is operated

t = 0^- represents moment of time before the switch is operated

t = 0 represents transition time at which the switch is operated

t = ∞ represents steady state time after the switch is operated

t = -∞ represents steady state time before the switch is operated



- In the full analysis of transient, steady state (**S.S.**) comes **two times**.
  - Past ie. **t < 0**
  - In Present **t = ∞**
- At **S.S.** the **capacitor** acts as **O.C** and **inductor** acts as **S.C.**

Case 1: **No initial charge** across capacitor

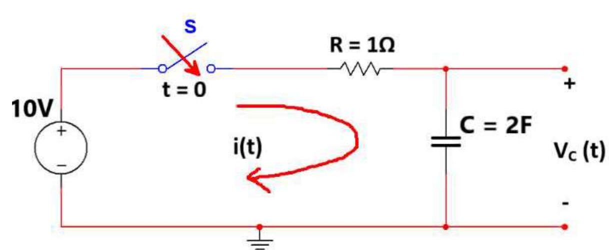


Fig: RC circuit

Transient equation

$$\boxed{X(t) = X(\infty) + [X(0^+) - X(\infty)]\, e^{-t/\tau} \quad, t \geq 0}$$

- Applies only to **step responses,** i.e. when the input excitation is constant.
- Valid for **finite time constant circuit**. So, for $\tau = 0$ or ∞, this equation is not valid.
- Valid for **1st order N/W**
- **Can't** be used for Case 3, where **capacitor doesn't follows its property**.

(i)

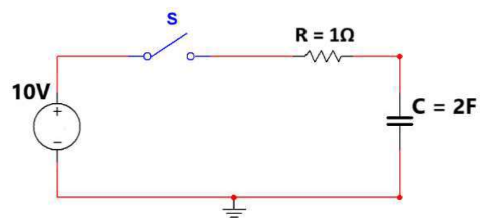


Fig: t < 0

$V_C(0^-) = 0$ Volt

i.e. uncharged capacitor

By **capacitor Property**,

$\boxed{V_C(0^-) = V_C(0^+)}$

Since it **opposes the change of voltage**.

(ii)

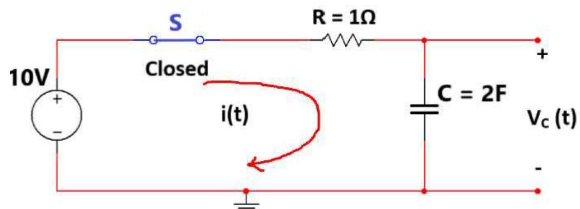


Fig: t ≥ 0 (transient)

Time constant

$\tau = RC$ sec if single R and single C

$= R_{th}C$ sec if combination of R and single C

$= RC_{eq}$ sec if single R and capacitor's in || or series that can form eqv. but not 2nd order.

$= R_{th}C_{eq}$ sec if combination of R and capacitor's in || or series that can form eqv. but not 2nd order.

Note: Time constant is always **calculated** during transient i.e. **for t ≥ 0**.

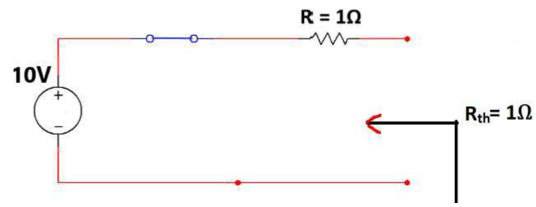


Fig: Calculation of R_{th}

$\tau = RC = 1 \times 2 = 2$ Sec, $t_{settle} = 5\,\tau = 5 \times 2 = 10$ sec

(iii)

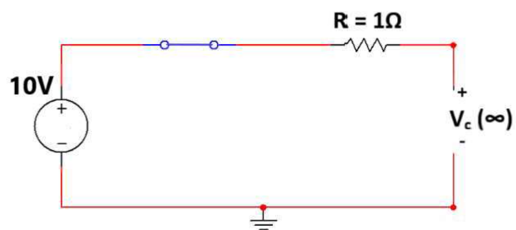


Fig: t = ∞

Therefore, $V_C(t) = V_C(\infty) + [\,V_C(0^+) - V_C(\infty)]\, e^{-t/\tau}$, $t \geq 0$

$$=> \boxed{V_C(t) = 10 + [0 - 10]\, e^{-t/2} = 10\,(1 - e^{-t/2})}$$

$$i(t) = i_C(t) = C\frac{dV_C(t)}{dt} = 2\,[\,-10\left(\frac{-1}{2}\right) e^{-t/2}\,]$$

$$=> \boxed{i_C(t) = 10\, e^{-t/2}}$$

- **At t = 0**, capacitor current is a discontinuous function. i.e. $\mathbf{i_C(0^-) \neq i_C(0^+)}$
- **At t = 0**, capacitor voltage is a continuous function. i.e. $\mathbf{V_C(0^-) = V_C(0^+)}$

__Case 2:__ **Initial charge** stored across the capacitor is **given**.

Say, initial charge in the capacitor $q(0^-) = 10$ C and at t = 0, switch S is closed. Which means charged capacitor is available. So we need not have to analyze the past or t < 0.

Now, $q = CV \Rightarrow V = \frac{q}{C}$

Therefore, $V_C(0^-) = \frac{q(0^-)}{C} = \frac{10}{2} = 5$ Volts

$V_C(0^-) = V_C(0^+) = 5$ Volts

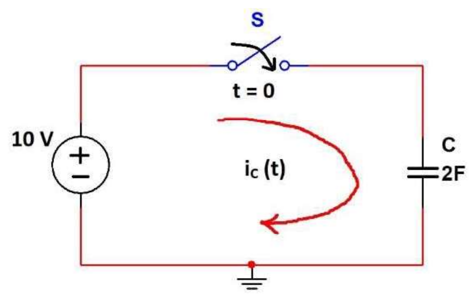
$V_C(t) = V_C(\infty) + [V_C(0^+) - V_C(\infty)]\, e^{-t/\tau} = 10 + (5 - 10)\, e^{-t/2} = 10 - 5e^{-t/2} = 5 + 5(1 - e^{-t/2})$

=> $\boxed{V_C(t) = 5 + 5(1 - e^{-t/2})}$

$i_C(t) = C\frac{dV_C(t)}{dt} = 2\,(-5)\left(\frac{-1}{2}\right) e^{-t/2} = 5\, e^{-t/2}$

=> $\boxed{i_C(t) = 5\, e^{-t/2}}$

Case 3: Without resistor R



$V_C(0^-) = 0$ Volt

$V_C(0^+) \neq 0$ Volt

$V_C(0^+) = 10$ Volt (Since in | |, voltage remains same)

When ideal voltage source is directly connected across capacitor, then capacitor doesn't follow it's own property. That means,

$$V_C(0^+) \neq V_C(0^-)$$

Therefore, $V_C(t) = 10\, U(t)$

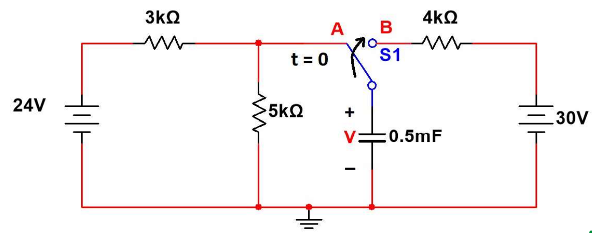
$i_C(t) = C\frac{dV_C(t)}{dt} = 2 \times 10 \times \delta(t) = 20\, \delta(t)$

Therefore, the capacitor current will be a impulse signal and capacitor voltage will be a step signal.

$\tau = 0$ Sec, tsettle = 0 Sec that means it settles abruptly. It can happen in case of step signal.

03/12/2021

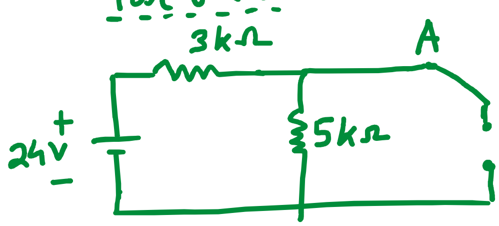
Q1 The switch in the fig has been in position A for a long time. At t=0, the switch moves to B. Determine v(t) for t > 0 and calculate its value for t = 1 and 4s.



Sol^n:

$$V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)]\,e^{-t/\tau} ; \quad t \geq 0$$

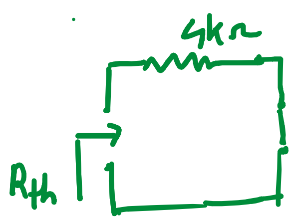
For t < 0



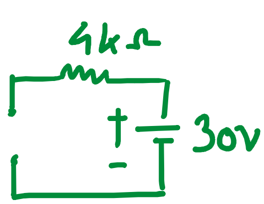
$$V_c(0^-) = \frac{24}{8} \times 5 = 15V = V_c(0^+)$$

For t ≥ 0

$$\tau = R_{th}C = 4 \times 0.5 = 2 \text{ sec}$$



For t = ∞



$V_c(\infty)$

$= 30V$

$$V_c(t) = 30 + [15 - 30]\,e^{-t/2} , \quad t \geq 0$$

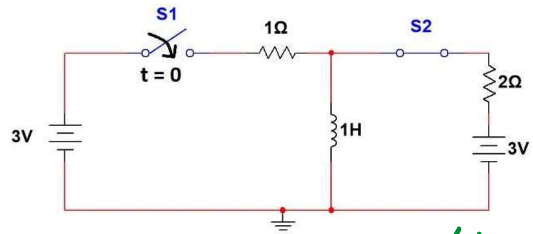
$$= 30 - 15e^{-t/2} , \quad t \geq 0$$

$$V_c(1) = 30 - 15e^{-1/2} = 20.9V$$

$$V_c(4) = 30 - 15e^{-4/2} = 27.9V$$

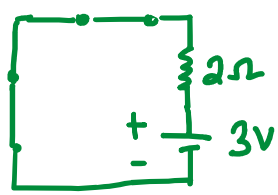
GATE 2016 EE

Q2 In the circuit shown, switch S_2 has been closed for a long time. At time t = 0 switch S_1 is closed. At t = 0^+, the rate of change of current through the inductor, in amperes per second, is?



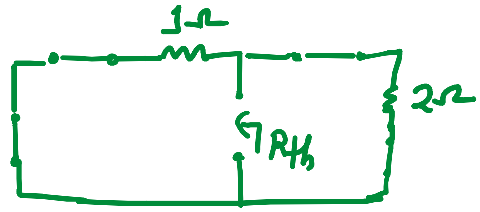
Soln 2, $I_L(t) = I_L(\infty) + [I_L(0^+) - I_L(\infty)]\, e^{-t/\tau}, \quad t \geq 0$

For t < 0



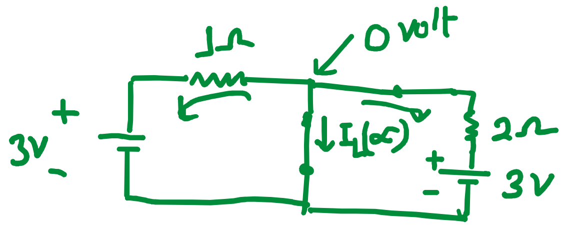
$I_L(0^-) = \frac{3}{2}$ A $= I_L(0^+)$

For t ≥ 0



$\tau = \frac{L}{R_{th}} = \frac{1}{1\|2} = \frac{1}{\frac{1\times 2}{1+2}} = \frac{3}{2}$ sec

For t = ∞



$\frac{0-3}{1} + \frac{0-3}{2} + I_L(\infty) = 0$

$\Rightarrow -3 - \frac{3}{2} + I_L(\infty) = 0$

$\Rightarrow -3 - 1.5 + I_L(\infty) = 0$

$\Rightarrow I_L(\infty) = 4.5$ A

$I_L(t) = 4.5 + [1.5 - 4.5]e^{-2t/3}$

$\frac{d I_L(t)}{dt} = -\cancel{3} \times \frac{-2}{\cancel{3}} e^{-2t/3} = 2e^{-2t/3}$

$\therefore \frac{d}{dt} I_L(0^+) = 2e^{-2(0)/3} = 2$ A/sec

Alternative way

$$V_L(t) = L \frac{d}{dt} I_L(t)$$

$$V_L(0^+) = L \frac{d}{dt} I_L(0^+)$$

$$\frac{d}{dt} I_L(0^+) = \frac{V_L(0^+)}{L}$$

$$= V_L(0^+)$$

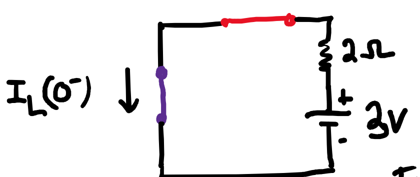


Fig t<0

$$I_L(0^-) = \frac{3}{2} = 1.5A = I_L(0^+)$$

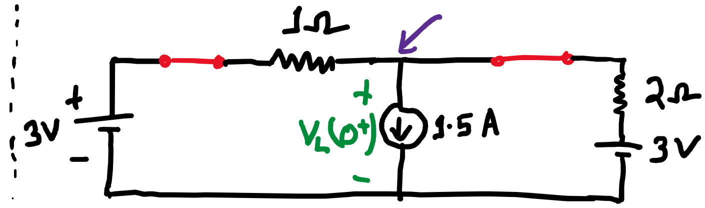


Fig: $t = 0^+$

$$\frac{V_L(0^+) - 3}{1} + \frac{V_L(0^+) - 3}{2} + 1.5 = 0$$

$$\Rightarrow \quad 3V_L(0^+) - 6 - 3 + 3 = 0$$

$$\Rightarrow \quad V_L(0^+) = \frac{6}{3} = 2$$

$$\therefore \frac{d}{dt} I_L(0^+) = V_L(0^+) = 2 \text{ A/sec}$$

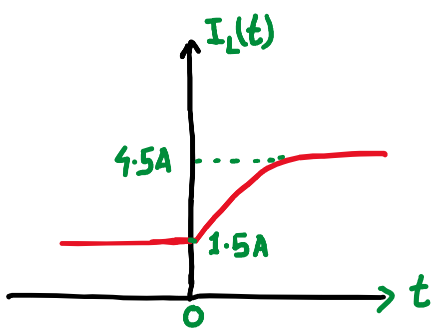


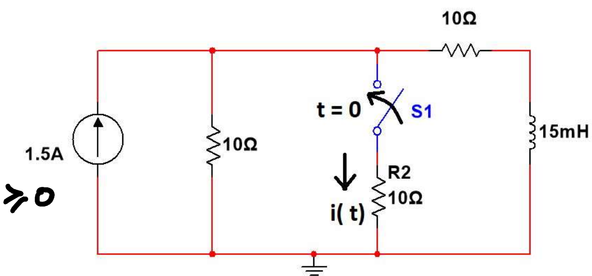
Fig:- $I_L(t)$

$$E_L(0^-) = \frac{1}{2} \times 1 \times (1.5)^2 = 1.125 \text{ J}$$

$$E_L(\infty) = \frac{1}{2} \times 1 \times (4.5)^2 = 10.125 \text{ J}$$

GATE EC, 2010

Q3 In the circuit shown, the switch S is open for a long time and is closed at t = 0. The current i(t) for t ≥ 0^+ is

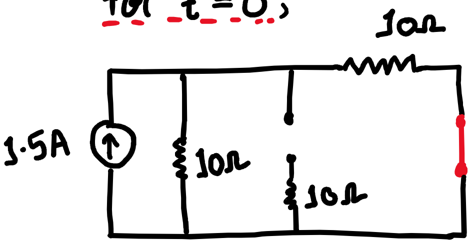


Sol^n For 1st order RL N/W,

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}, \; t \geq 0$$

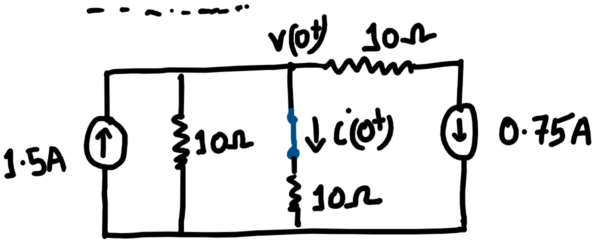
Here, $i(0^-) \neq i(0^+)$

For $t = 0^-$,



$$i_L(0^-) = \frac{1.5}{2} = 0.75A = i_L(0^+)$$

At $t = 0^+$



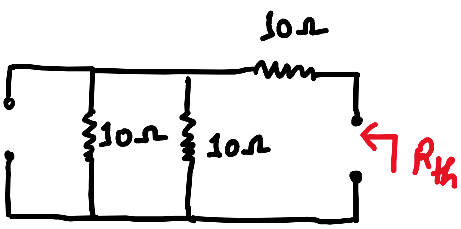
$$\frac{V(0^+)}{10} + \frac{V(0^+)}{10} + 0.75 - 1.5 = 0$$

$$2V(0^+) = 0.75 \times 10 = 7.5$$

$$\Rightarrow V(0^+) = 3.75V$$

$$\therefore i(0^+) = \frac{V(0^+)}{10} = \frac{3.75}{10} = 0.375A$$

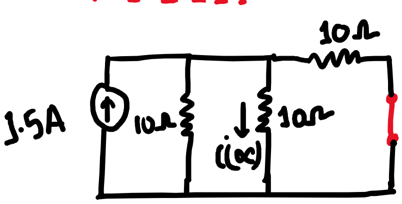
At $t \geq 0$



$$R_{th} = (10 \| 10) + 10 = 5 + 10 = 15\Omega$$

$$\therefore \tau = \frac{L}{R_{th}} = \frac{15 \text{ mH}}{15\,\Omega} = 1 \text{ msec} = 10^{-3} \text{sec}$$

At $t = \infty$



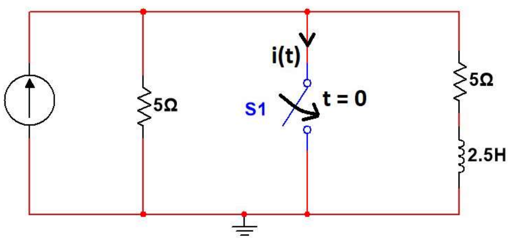
$$i(\infty) = \frac{1.5}{3} = 0.5A$$

$$\therefore i(t) = 0.5 + (0.375 - 0.5)e^{-t/10^{-3}}$$

$$= 0.5 - 0.125e^{-1000t}$$

GATE 2017

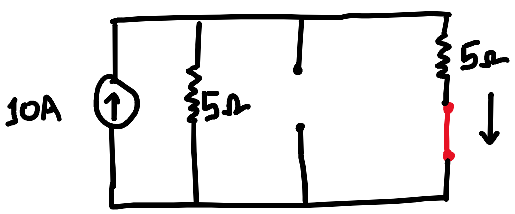
Q4 The switch in the circuit, shown in the figure, was open for a long time and is closed at t = 0. The current i(t) (in ampere) at t = 0.5 seconds is _______



Solⁿ $i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right] e^{-t/\tau}, \; t \geq 0$

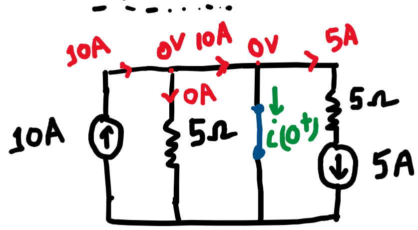
Here, $i(0^-) \neq i(0^+)$

At $t < 0$



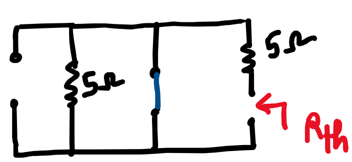
$\downarrow i_L(0^-) = \frac{10}{2} = 5\,A = i_L(0^+)$

At $t = 0^+$



$i(0^+) = 10 - 5 = 5\,A$

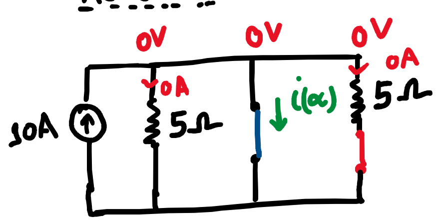
At $t \geq 0$



$R_{th} = 5\Omega$

$\therefore \; \tau = \frac{L}{R_{th}} = \frac{2.5}{5} = \frac{1}{2}$ sec

At $t = \infty$



$i(\infty) = 10A$

$\therefore \; i(t) = 10 + (5 - 10)\, e^{-t/\frac{1}{2}}$

$= 10 - 5\, e^{-2t}$

$i(t)\big|_{t = 0.5\,sec} = 10 - 5\, e^{-2 \times 0.5} = 8.16A$

13/12/2021

Properties of the Laplace Transform

TABLE 15.1

Properties of the Laplace transform.

Property	$f(t)$	$F(s)$
Linearity	$a_1f_1(t) + a_2f_2(t)$	$a_1F_1(s) + a_2F_2(s)$
Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Frequency shift	$e^{-at}f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \cdots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(x)dx$	$\frac{1}{s}F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds}F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1-e^{-sT}}$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

TABLE 15.2

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t + \theta)$	$\frac{s\sin\theta + \omega\cos\theta}{s^2+\omega^2}$
$\cos(\omega t + \theta)$	$\frac{s\cos\theta - \omega\sin\theta}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

*Defined for $t \geq 0$; $f(t) = 0$, for $t < 0$.

Laplace Transform to analyze circuits

Resistor

$v(t) = Ri(t)$

Taking L.T. both sides, we get

$\boxed{V(s) = RI(s)}$

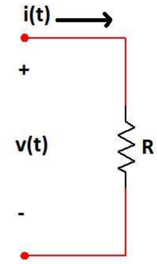


Fig: Time domain Representation

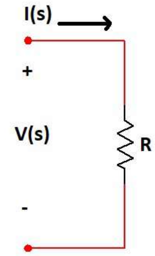


Fig: S-domain Representation

Inductor

$v(t) = L\frac{di(t)}{dt}$

Taking L.T. of both sides, we get

$\boxed{V(s) = L[sI(s) - i(0^-)] = sLI(s) - L\,i(0^-)}$

Or

$\boxed{I(s) = \frac{V(s)}{sL} + \frac{i(0^-)}{s}}$

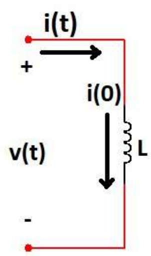


Fig : Time domain representation of inductor

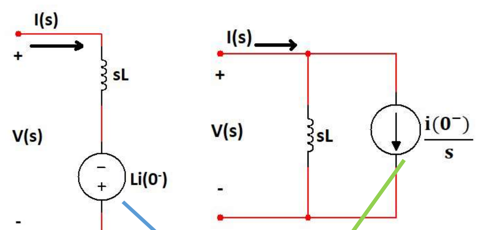


Fig: s-domain equivalents

Initial condition is modeled as a **voltage/current source**

Capacitor

$i(t) = C\frac{dv(t)}{dt}$

Taking L.T. of both sides, we get

$\boxed{I(s) = C[sV(s) - v(0^-)] = sCV(s) - Cv(0^-)}$

Or

$\boxed{V(s) = \frac{I(s)}{sC} + \frac{v(0^-)}{s}}$

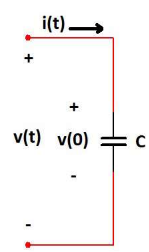


Fig : Time domain representation of capacitor

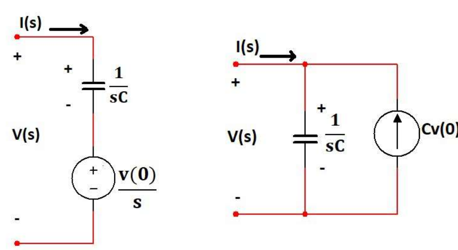
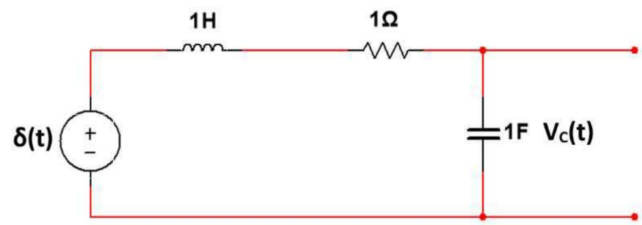


Fig: s-domain equivalents

GATE 2008

Q The following series RLC circuit with zero initial conditions is excited by a unit impulse function δ(t).



i) For t>0, the o/p voltage Vc(t) is

a) $\frac{2}{\sqrt{3}}(e^{-\frac{1}{2}t} - e^{-\frac{\sqrt{3}}{2}t})$ b) $\frac{2}{\sqrt{3}} t e^{-\frac{1}{2}t}$ c) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \cos(\frac{\sqrt{3}}{2}t)$ d) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin(\frac{\sqrt{3}}{2}t)$

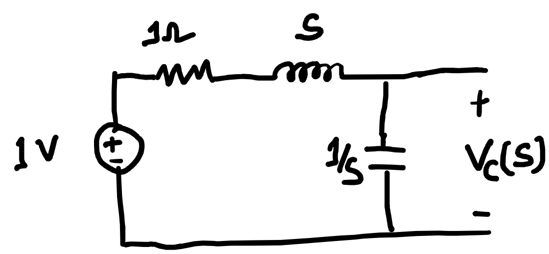
ii) For t>0 voltage across the resistor is

a) $\frac{1}{\sqrt{3}}(e^{-\frac{\sqrt{3}}{2}t} - e^{-\frac{1}{2}t})$ b) $e^{-\frac{1}{2}t}[\cos(\frac{\sqrt{3}}{2}t) - \frac{1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}t)]$ c) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin(\frac{\sqrt{3}}{2}t)$ d) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \cos(\frac{\sqrt{3}}{2}t)$

Sol^n zero initial condition

$V_c(0^-) = 0V$

$I_L(0^-) = 0A$



i) $V_c(t)$

$$V_c(s) = \frac{1}{1+s+1/s} \times \frac{1}{s} = \frac{1}{s^2+s+1} = \frac{1}{s^2+2\times\frac{1}{2}s+\frac{1}{4}-\frac{1}{4}+1}$$

$$= \frac{1}{(s+\frac{1}{2})^2+\frac{3}{4}} = \frac{1}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

$$\Rightarrow V_c(s) = \frac{\sqrt{3}/2}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} \times \frac{2}{\sqrt{3}}$$

$$L[\sin\omega t] = \frac{\omega}{s^2+\omega^2}$$

$$L[e^{-at}\sin\omega t] = \frac{\omega}{(s+a)^2+\omega^2}$$

$$\therefore V_c(t) = \frac{2}{\sqrt{3}} e^{-t/2} \sin\frac{\sqrt{3}}{2}t$$

ii) $\underline{V_R(t)}$

$$V_R(t) = i(t) \times R = 1 \times i(t) = i(t)$$

& $\because$ it is a series ckt

$$\therefore \quad i_R(t) = i_L(t) = i_C(t) = i(t)$$

$$i(t) = C\,\frac{dV_C(t)}{dt} = \frac{d}{dt}\left(\frac{2}{\sqrt{3}}\,e^{-t/2}\sin\frac{\sqrt{3}}{2}t\right)$$

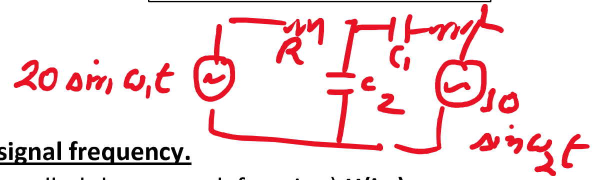
$$= \frac{2}{\sqrt{3}}\left[e^{-t/2} \times \frac{\sqrt{3}}{2}\cos\frac{\sqrt{3}}{2}t + \sin\frac{\sqrt{3}}{2}t \times \left(-\frac{1}{2}\right) \times e^{-t/2}\right]$$

$$= e^{-t/2}\left[\cos\frac{\sqrt{3}}{2}t - \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right] = V_R(t)$$

14/12/2021

Frequency Response

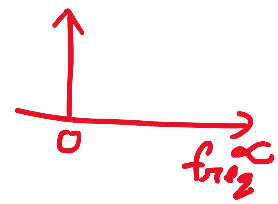
Sinusoidal Steady State Analysis→ Calculated i) Voltages
ii) Currents } with **Constant frequency Source**

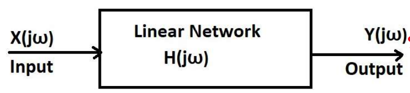


Frequency response of a circuit

- is the **variation** in its behavior, **with change in signal frequency.**
- is the plot of the circuit's **transfer function** (also called the network function) **H(jω) =** H(ω)∠ϕ versus ω, with ω varying from **ω = 0 to ω = ∞.**

D.C





$= H(j\omega)\,X(j\omega)$

Fig: Block diagram representation of a linear Network

$10\log_{10}\frac{P_o}{P_i}$

$20\log_{10}\frac{V_o}{V_i}$, $20\log_{10}\frac{I_o}{I_i}$

Four possible **transfer functions**→ H(jω) = Voltage gain = $\frac{V_o(j\omega)}{V_i(j\omega)}$

H(jω) = Current gain = $\frac{I_o(j\omega)}{I_i(j\omega)}$

H(jω) = Transfer impedance = $\frac{V_o(j\omega)}{I_i(j\omega)}$

H(jω) = Transfer admittance = $\frac{I_o(j\omega)}{V_i(j\omega)}$

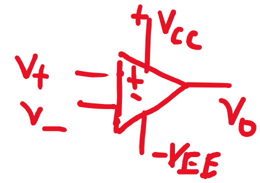
Applications

- Electric filters→ **Blocks**/eliminates/attenuate signals with **unwanted frequencies**

  → **Pass** signals of the **desired frequencies**.

Filters → Used in radio, TV, telephone systems **to separate one broadcast frequency from another.**

Filters (Freq. selective device)



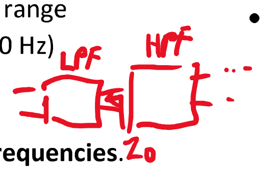
Passive filter

★ Consist of

- Only **Passive elements** (R,L,C)

★ **Limitations**

- The **maximum gain** of a passive filter is **unity.**
- May require bulky and expensive inductors
- **Performs poorly** at frequencies **below the audio frequency** range (300 Hz < f <3,000 Hz)



★ Useful at **high frequencies.**

Active filter

★ Consist of

- **Active elements** (such as transistors and op-amps)

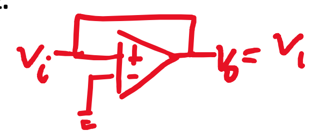
  \+

- **Passive elements** (R,L,C)

★ **Adv over Passive filter**

- Can provide **amplifier gain** in addition to providing **same freq response** as RLC filters.
- Often **smaller and less expensive**, because they do not require inductors. This makes **feasible the IC realizations** of filters.
- **Can be combined with buffer amplifiers** (volt. Follower) to **isolate each stage** of the filter from source and load impedance effects.

★ The **practical limit** of most active filters is **about 100 kHz.**



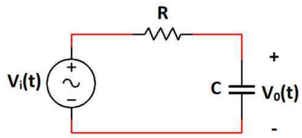
Steps to obtain frequency response

$R \to R$, $C \to \frac{1}{j\omega C}$, $L \Rightarrow j\omega L$

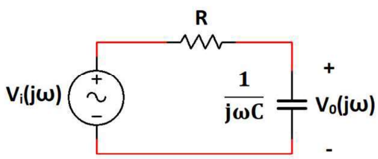
- ➢ Obtain the **frequency domain** equivalent of the **ckt**.
- ➢ Use any ckt technique(s) to obtain the appropriate quantity.
- ➢ Plot → **Magnitude of T.F. vs frequency**

  → **Phase of T.F. vs frequency**

Low Pass Filter

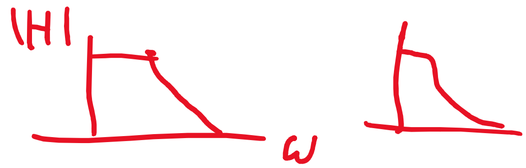


Step 1



$= \dfrac{V_i(j\omega)}{R+\frac{1}{j\omega C}} \times \dfrac{1}{j\omega C}$

Fig: Frequency domain ckt.



$$H(j\omega) = \frac{\left(1+\frac{j\omega}{Z_1}\right)}{\left(1+\frac{j\omega}{P_1}\right)}$$

Step 2

By voltage division, the **transfer function** is given by

$$H(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{1+j\omega RC} = \frac{1}{1+\frac{j\omega}{1/RC}}$$

$1+j\omega RC = 0$

$\Rightarrow j\omega = -\frac{1}{RC}$

Step 3

Magnitude and phase of H(jω) can be expressed as

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}} \quad , \quad \phi = -\tan^{-1}\frac{\omega}{\omega_0} \quad , \text{ where } \omega_0 = \frac{1}{RC}$$

$a+jb \quad \emptyset = \tan^{-1}\frac{b}{a} = 0$

$\tan^{-1}\omega RC$

$\dfrac{\angle 0}{\angle \tan^{-1}\omega RC}$

$= \angle(0 - \tan^{-1}\omega RC)$

$= \angle -\tan^{-1}\frac{\omega}{1/RC}$

At ω = 0, | H(jω)|= 1

ω = ∞, | H(jω)|= 0

Cut-off freq. (in context of filters) or Half-power freq. or corner freq. (Bode Plots) or roll off freq.

- The cut off freq. is the freq. at which the **T.F.** drops in **magnitude** to **70.71% (or 1/$\sqrt{2}$ times) of its maximum value.**
- Freq at which the **power dissipated** in a ckt is **half of its maximum value.**

Cut off freq, ω_c is obtained by setting the mag. of H(jω) equal to $\frac{1}{\sqrt{2}}$.

$$|H(j\omega_c)| = \frac{1}{\sqrt{1+\omega^2R^2C^2}} = \frac{1}{\sqrt{2}}$$

or $\boxed{\omega_c = \frac{1}{RC}}$

16/12/2021

$X_L = j\omega L$

$X_C = \frac{1}{j\omega C}$

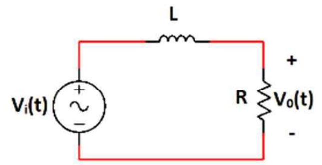
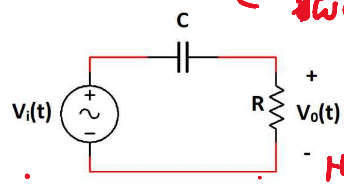


Fig: LPF

		gain
$\omega \to 0$	S.C.	1
$\omega \to \infty$	O.C.	0



H.P.F

Fig:

	gain
O.C.	0
S.C.	1

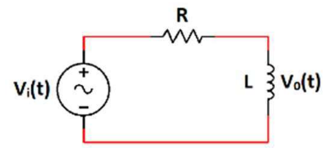


Fig:

Resonance

LC Series → S.C.

LC || → O.C.

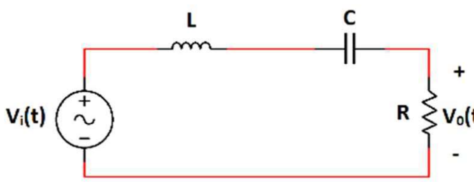
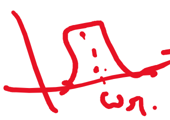


Fig:



BPF

	gain
$\omega \to 0$	0
$\omega \to \infty$	0
$\omega \to \omega_r$	1

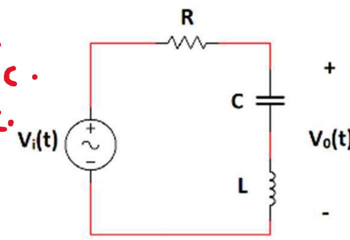


Fig: BRF

	gain
$\omega \to 0$	1
$\omega \to \infty$	1
$\omega \to \omega_r$	0

Metric (SI) Prefixes

Purpose	Prefix Name	Prefix Symbol	Value	
Larger quantities or whole units	tera	T	10^{12}	Trillion
	giga	G	10^{9}	Billion
	mega	M	10^{6}	Million
	kilo	k	10^{3}	Thousand
	hecto	h	10^{2}	Hundred
	deka	da	10^{1}	Ten
Smaller quantities or sub units	deci	d	10^{-1}	Tenth
	centi	c	10^{-2}	Hundredth
	milli	m	10^{-3}	Thousandth
	micro	µ	10^{-6}	Millionth
	nano	n	10^{-9}	Billionth
	pico	p	10^{-12}	Trillionth
	femto	f	10^{-15}	Quadrillionth

1 m = 10dm

Properties of logarithms

1. $\log P_1P_2 = \log P_1 + \log P_2$
2. $\log \frac{P_1}{P_2} = \log P_1 - \log P_2$
3. $\log P^n = n \log P$
4. $\log 1 = 0$

Bel (Logarithmic unit)

In communication systems, **gain** is measured in ***bels***.

Gain, G = No. of **bels** = $\log_{10} \frac{P_2}{P_1}$

Gain in **dB**, $G_{dB} = 10 \log_{10} \frac{P_2}{P_1}$

$G_{dB} = 20 \log_{10} \frac{V_2}{V_1}$

$G_{dB} = 20 \log_{10} \frac{I_2}{I_1}$

when $R_1 = R_2$

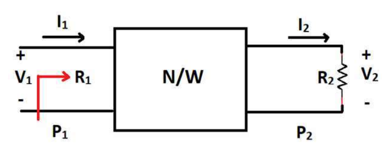


Fig: Four terminal N/W

$$G_{dB} = 10\log\frac{I_2^2 R_2}{I_1^2 R_1} = 10\log\left(\frac{I_2}{I_1}\right)^2\left(\frac{R_2}{R_1}\right)$$

$$= 10\log\left(\frac{I_2}{I_1}\right)^2 + 10\log\frac{R_2}{R_1}$$

- **Decibel (dB)** value is a **logarithmic measurement** of the **ratio of** one variable to another of the **same type**.

17/12/2021

to view large **range of frequency** with other advantages

Bode plots → are the **industry-standard way of presenting frequency response**.

- Magnitude Vs frequency
  (decibel, dB) (logarithm)
- Phase Vs frequency
  (degrees) (logarithm)

Semilogarithmic plots of the **T.F.** known as ***Bode Plots***.

- **Straight line plots** that **approximate the actual plot** to a reasonable degree of accuracy.
- A **decade** is an interval between two frequencies **with a ratio of 10**.
- The special case of **dc (ω = 0) does not appear** on Bode plots because log 0 = -∞, implying that zero frequency is **infinitely far to the left of the origin** of Bode plots.

Standard form of representing T.F.

$$H(\omega) = \frac{K(j\omega)^{\pm 1}\left(1+{}^{j\omega}/_{z_1}\right)\left[1+j2\zeta_1{}^{\omega}/_{\omega_k}+({}^{j\omega}/_{\omega_k})^2\right]\ldots\ldots}{\left(1+{}^{j\omega}/_{P_1}\right)\left[1+j2\ {}_2{}^{\omega}/_{\omega_n}+({}^{j\omega}/_{\omega_n})^2\right]\ldots\ldots}$$

Different factors that appear in the above T.F. are

1. A **gain** K
2. A **pole** $(j\omega)^{-1}$ or **zero** (jω) at the **origin**
3. A **simple pole** $\left(1+{}^{j\omega}/_{P_1}\right)$ or **zero** $\left(1+{}^{j\omega}/_{z_1}\right)$
4. A **quadratic pole** $1/\left[1+j2\zeta_2{}^{\omega}/_{\omega_n}+({}^{j\omega}/_{\omega_n})^2\right]$ or **zero** $\left[1+j2\zeta_1{}^{\omega}/_{\omega_k}+({}^{j\omega}/_{\omega_k})^2\right]$ where ζ is the damping factor.

➔ In constructing a Bode plot, **each factor is plotted separately** and then **added them graphically (possible because of log involved).**

In Bode plot,

Magnitude or gain, H_{dB} = 20 $\log_{10}$ |H|

Phase,

(a + jb)

ϕ = $\tan^{-1}$(b/a)

Table: Specific gain and their decibel values

Magnitude H	20 log |H| (dB)
0.001	-60
0.01	-40
✓ 0.1	✓ -20
0.5	-6
$1/\sqrt{2}$	-3
1	0
$\sqrt{2}$	3
2	6
✓ 10	✓ 20
20	26
100	40
1000	60

$$H(\omega) = \frac{K(j\omega)^{\pm 1}\left(1+{}^{j\omega}/_{z_1}\right)\left[1+j2\ {}_{1}{}^{\omega}/_{\omega_k}+({}^{j\omega}/_{\omega_k})^2\right]\ldots\ldots}{\left(1+{}^{j\omega}/_{P_1}\right)\left[1+j2\zeta_2{}^{\omega}/_{\omega_n}+({}^{j\omega}/_{\omega_n})^2\right]\ldots\ldots}$$

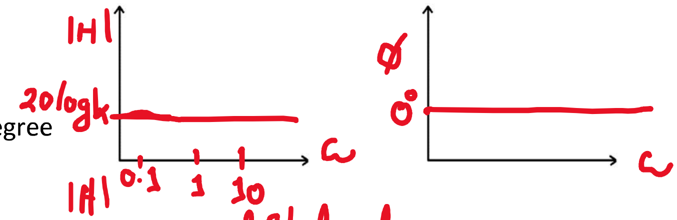
Plot for constant term

Gain, K

Mag, H_{dB} = 20 log K = constant

Phase, $\phi = \tan^{-1}(0/K) = \tan^{-1} 0 = 0$ degree

If K is -ve, Phase = $\tan^{-1}(-0) = \pm 180^0$

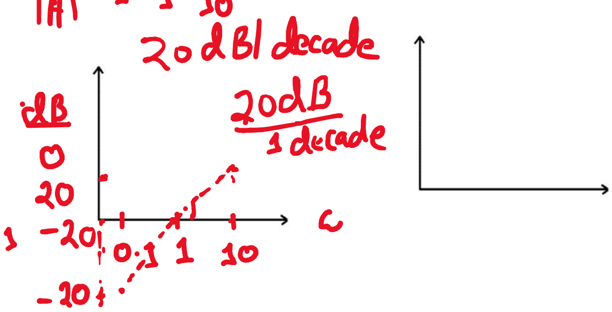


Plot for pole/zero at the origin

- **Zero at origin, (jω)**

Mag, H_{dB} = 20 log ω

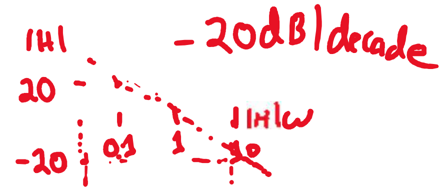
Phase, $\phi = \tan^{-1}(\omega/0) = \tan^{-1}(\infty) = 90^0$



- **Pole at origin, $\frac{1}{j\omega}$**

Mag, H_{dB} = 20 log (1/ω) = -20 log ω

Phase, $\phi = -\tan^{-1}(\omega/0) = -\tan^{-1}(\infty) = -90^0$



In general **$(j\omega)^N$,** where N is an integer, the **magnitude plot** will have a **slope of 20N dB/decade**, while the **phase** is **90N degrees**.

Plot for simple Pole/zero

- **Simple zero, $(1 + \frac{j\omega}{Z_1})$**

Mag, H_{dB} = 20 log $|1 + \frac{j\omega}{Z_1}|$

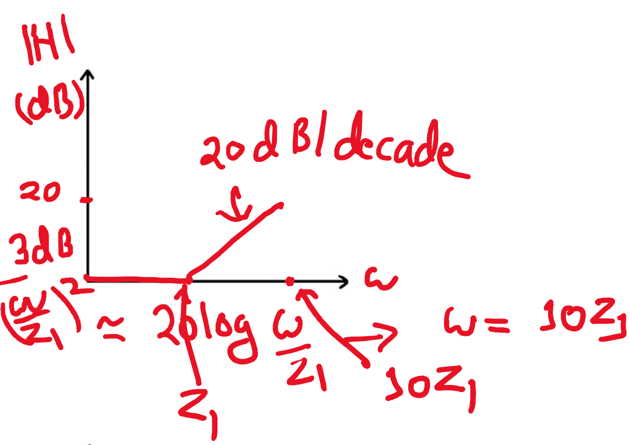
HdB

At ω = 0 — 0

ω = Z_1 — $20\log\sqrt{2} \simeq 3$dB

ω → ∞ — $20\log\sqrt{1+(\frac{\omega}{Z_1})^2} \simeq 20\log\frac{\omega}{Z_1}$

Phase, $\phi = \tan^{-1}(\omega/Z_1)$



- **Simple pole, $\frac{1}{\left(1+\frac{j\omega}{P_1}\right)}$**

Mag, H_{dB} = - 20 log $|1 + \frac{j\omega}{P_1}|$

Phase, $\phi = -\tan^{-1}(\omega/P_1)$