SIGNALS & SYSTEMS

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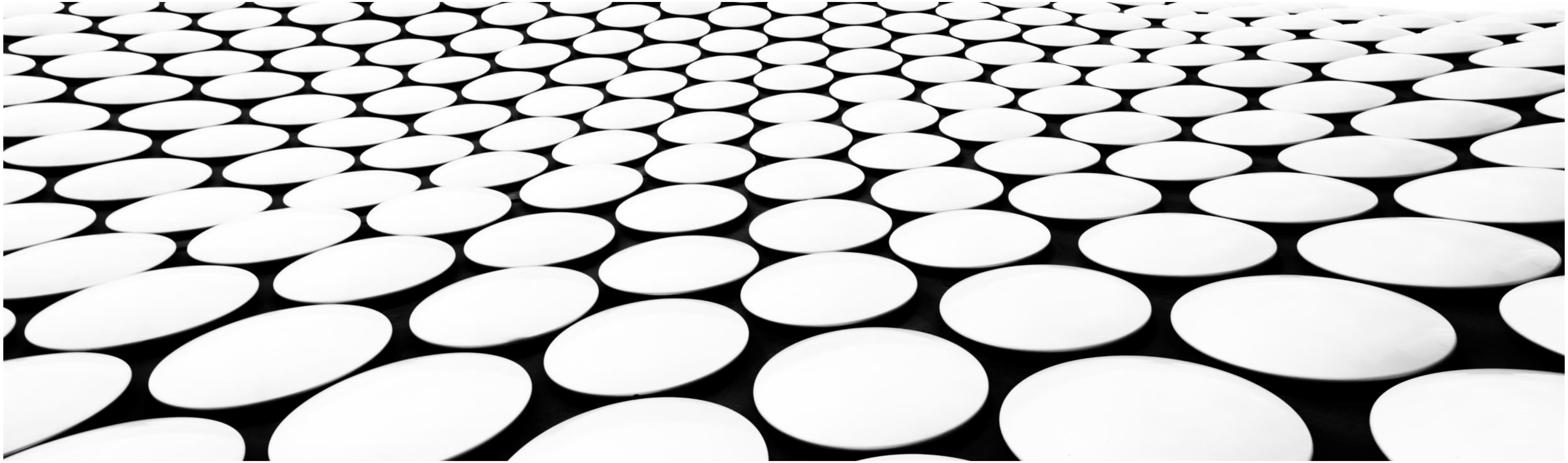


Here $X(s)$ is the Laplace transform of a signal $x(t)$.
and $X(s)$ is expressed in terms of $X(s) = \frac{P(s)}{Q(s)}$, then $X(s)$ is called as Rational function.

$$X(s) = \frac{P(s)}{Q(s)}$$

$$= \frac{b_0 s^M + b_1 s^{M-1} + b_2 s^{M-2} + \ldots + b_{M-1} s + b_M}{a_0 s^N + a_1 s^{N-1} + a_2 s^{N-2} + \ldots + a_{N-1} s + a_N}$$

where, $P(s)$ = Numerator polynomial of $X(s)$

$Q(s)$ = Denominator polynomial of $X(s)$

here, $b_0, b_1 \ldots b_M$ are co-efficients of Numerator polynomial

$a_0, a_1 \ldots a_N$ are " " Denominator polynomial

$$X(S) = \frac{b_0\left(s^M + \frac{b_1}{b_0} s^{M-1} + \frac{b_2}{b_0} s^{M-2} + \ldots\ldots + \frac{b_{M-1}}{b_0} s + \frac{b_M}{b_0}\right)}{a_0\left(s^N + \frac{a_1}{a_0} s^{N-1} + \frac{a_2}{a_0} s^{N-2} + \ldots\ldots + \frac{a_{N-1}}{a_0} s + \frac{a_N}{a_0}\right)}$$

$$= G\,\frac{(s - z_1)(s - z_2)\ldots\ldots(s - z_M)}{(s - p_1)(s - p_2)\ldots\ldots(s - p_N)}$$

where, $G = \frac{b_0}{a_0}$ = Scaling factor

$z_1, z_2, \ldots\ldots, z_M$ = Roots of numerator polynomial, P(s)

$p_1, p_2, \ldots\ldots, p_N$ = Roots of denominator polynomial, Q(s)

The zeros and poles are two critical frequencies at which rational function's takes two extreme values, zero or infinity.

zeros at which X(s) becomes zero.

poles at which X(s) becomes infinity.

Representation of Poles and Zeros in s-Plane

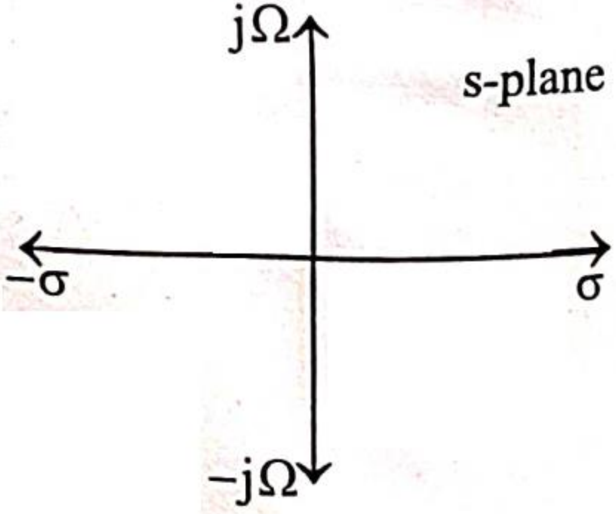
We know that, Complex frequency, $s = \sigma + j\Omega$

where, σ = Real part of s, Ω = Imaginary part of s

Hence the s-plane is a complex plane, with σ on real axis and Ω on imaginary axis as shown in fig 3.5. In the s-plane, the zeros are marked by small circle " o " and the poles are marked by letter "X".

zero

Pole



For example consider the rational function of s shown below.

$$X(s) = \frac{(s+2)(s+5)}{s(s^2+6s+13)}$$

$$s = \frac{-6 \pm \sqrt{36 - 4 \times 13}}{2} = \frac{-6 \pm j4}{2} = -3 + j2, \quad -3 - j2$$

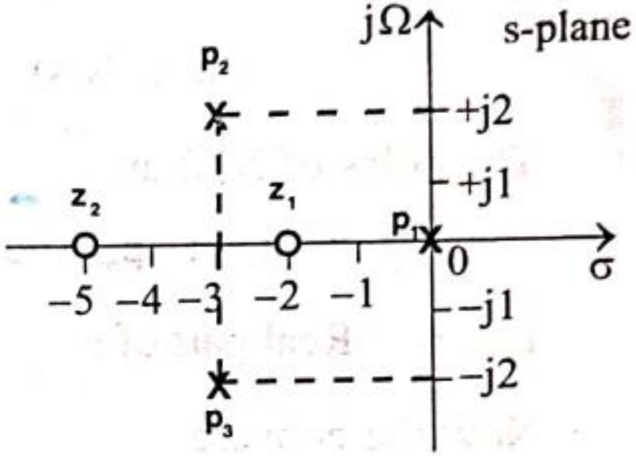
$$\therefore s^2 + 6s + 13 = (s + 3 - j2)(s + 3 + j2)$$

$$\therefore X(s) = \frac{(s+2)(s+5)}{s(s^2 + 6s + 13)} = \frac{(s+2)(s+5)}{s(s+3-j2)(s+3+j2)}$$

The zeros of the above function are,

$z_1 = -2$

$z_2 = -5$



The poles of the above function are,

$p_1 = 0$

$p_2 = -3 + j2$

$p_3 = -3 - j2$