# SIGNALS & SYSTEMS

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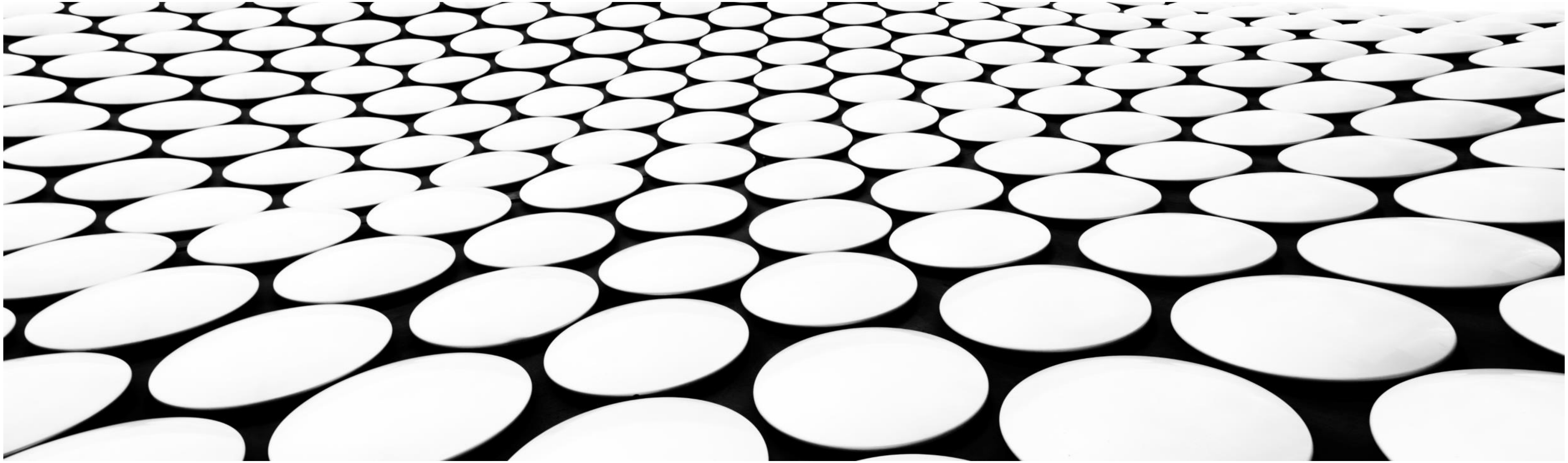

Fourier Transform:-

For non-periodic signal

Absolutely integrable

→ Energy signal ✓

× → Impulse rela ted signal → NENP

× → power signal

Laplace transform

Energy, power signal,

NENP signals

limited case

$$\int_{-\infty}^{\infty} |x(t)|\,dt < \infty$$

$x(t) \xrightarrow{FT} X(j\omega)$ or $X(\omega)$

$$X(j\omega) \text{ or } X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$L\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$s = \sigma + j\omega$

$\sigma = 0 \quad s = j\omega$

Conditions for existence of FT:-

1) signal should have finite no. maxima and minima over any finite interval.

2) signal should have finite no discontinuities over any finite interval.

3) $\int_{-\infty}^{\infty} |x(t)| < \infty$ × may not be satisfied

✓ The conditions are sufficient but not necessary.

Fourier Transform → Freq. domain → ($\omega$)

## Properties of FT:-

1) Linearity:-

$\alpha x_1(t) \longrightarrow X_1(\omega)$

$\beta x_2(t) \longrightarrow X_2(\omega)$

$$\boxed{\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha X_1(\omega) + \beta X_2(\omega)}$$

2) Conjugation:-

$x^*(t) \longrightarrow X^*(-j\omega)$.

3) Time reversal:-

$x(-t) \longrightarrow X(-j\omega)$

4) Time shifting:-

$x(t) \xrightarrow{FT} X(\omega)$

$x(t+t_0) \longrightarrow X(\omega)\, e^{j\omega t_0}$

$x(t-t_0) \longrightarrow X(\omega)\, e^{-j\omega t_0}$

5) Time scaling:-   $x(at) \rightarrow \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$   where $a \neq 0$

6) Freq. shifting:-   $e^{j\omega_0 t} \cdot x(t) \rightarrow X[j(\omega - \omega_0)]$

$x(t) \rightarrow X(j\omega)$   $e^{-j\omega_0 t} \cdot x(t) \rightarrow X[j(\omega + \omega_0)]$

7) Convolution in time domain:-   $x_1(t) \rightarrow X_1(\omega)$

$x_2(t) \rightarrow X_2(\omega)$

$$x_1(t) * x_2(t) \rightarrow X_1(\omega) \cdot X_2(\omega)$$

8) Multiplication in time domain:-   $x_1(t) \cdot x_2(t) \rightarrow \frac{1}{2\pi}\{X_1(\omega) * X_2(\omega)\}$

9) Differentiation in time :-

$$x(t) \longrightarrow X(j\omega)$$

$$\frac{d^n x(t)}{dt^n} \longrightarrow (j\omega)^n X(j\omega)$$

$$L\left[\frac{dx(t)}{dt}\right] \rightarrow sX(s)$$

10) Integration in time :-

$$\int_{-\infty}^{t} x(t)\,dt \longrightarrow \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$$

$$L\left\{\int x(t)\,dt\right\} \rightarrow \frac{X(s)}{s}$$

11) Differentiation in freq :-

$$t^n x(t) \xrightarrow{FT} (j)^n \frac{d^n X(j\omega)}{d\omega^n}$$

## Parseval's Energy Theorem:-

If $x(t) \rightarrow X(j\omega)$ then

$$E(x(t)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega$$

## DC signal:-

$x(t) = A_0$

condition for absolutely integrable $\rightarrow$ ??

$$\int_{-\infty}^{\infty} |x(t)| \, dt$$

$$= \int_{-\infty}^{\infty} A_0 \, dt$$

$$= A\int_{-\infty}^{\infty} dt = A[\infty - (-\infty)] = \infty.$$

$$x(t) \xrightarrow{FT} X(\omega)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} A_0\, e^{-j\omega t}\, dt$$

$$= A_0 \int_{-\infty}^{\infty} e^{-j\omega t}\, dt$$

$$= A_0 \left[ \frac{e^{-j\omega t}}{-j\omega} \right]_{-\infty}^{\infty}$$

$$= \frac{A_0}{-j\omega} \left[ e^{-j\omega \cdot \infty} - e^{-j\omega(-\infty)} \right]$$

$$= \frac{A_0}{\omega} \left[ \frac{e^{-j\infty} - e^{j\infty}}{-j} \right]$$

$$= \frac{A_0}{\omega} \left[ \frac{e^{j\infty} - e^{-j\infty}}{j} \right] \rightarrow \text{Euler's identity}$$

$$= \frac{A_0}{\omega}\, 2\sin(\infty) \rightarrow \underline{\text{Undefined}}$$

Let us consider fourier transform pair —

$$x(t) \xrightarrow{FT} \underline{X(j\omega)} = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) \xrightarrow{IFT} x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$x(t_1)\cdot\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

The given signal is $x(t) = A_0$

$$\downarrow FT$$

$$\boxed{X(j\omega) = A_0\,\delta(\omega)}$$

$$\therefore x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_0\,\delta(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{A_0}{2\pi}\int_{-\infty}^{\infty} \delta(\omega)\, e^{j\omega t}\, d\omega \qquad = \frac{A_0}{2\pi}\int_{-\infty}^{\infty} \delta(\omega - 0)\, e^{j\omega t}\, d\omega$$

$$e^{j\omega t} \rightarrow e^{j0t} \rightarrow e^{0} = 1$$

$$= \frac{A_0}{2\pi}\int_{-\infty}^{\infty} \cancel{\delta(\omega)}^{\,1}\, d\omega$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \quad \rightarrow \quad \int_{-\infty}^{\infty} \delta(\omega)\,d\omega = 1$$

$$A_0 \qquad x(t) \xrightarrow{FT} X(j\omega) = \underline{A_0\,\delta(\omega)}$$

$$\therefore \; x(t) = \frac{A_0}{2\pi}\cdot 1 \qquad \therefore \; x(t) = \frac{A_0}{2\pi}$$

$$\therefore \; x(t) = \frac{A_0}{2\pi}$$

$A_0 = 5$ unit.

FT of $A_0 = 5$ will be –

$$2\pi \times (5)\,\delta(\omega)$$

$$= \underline{10\pi\,\delta(\omega)}$$

<u>using linearity property :-</u>

$$\frac{A_0}{2\pi} \rightleftharpoons A_0\,\delta(\omega)$$

$$\Rightarrow \frac{A_0}{\cancel{2\pi}} \times \cancel{2\pi} \rightleftharpoons 2\pi A_0\,\delta(\omega)$$

$$\Rightarrow \boxed{A_0 \rightleftharpoons 2\pi A_0\,\delta(\omega)}$$

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