SIGNALS & SYSTEMS

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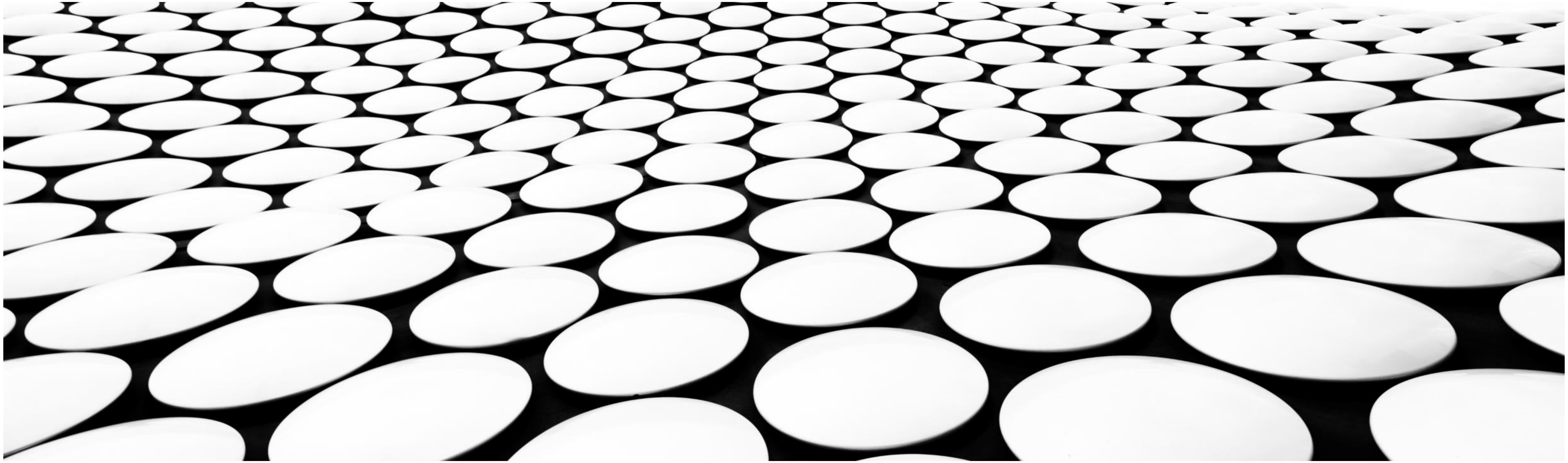


Classification of discrete time signals :-

1) Deterministic and Non-det.

2) periodicity and Non-periodic

3) Even and odd

4) Energy and power signals.

Analog domain $\rightarrow$ Integration

Discrete domain $\rightarrow$ Summation $\rightarrow$ AP | GP

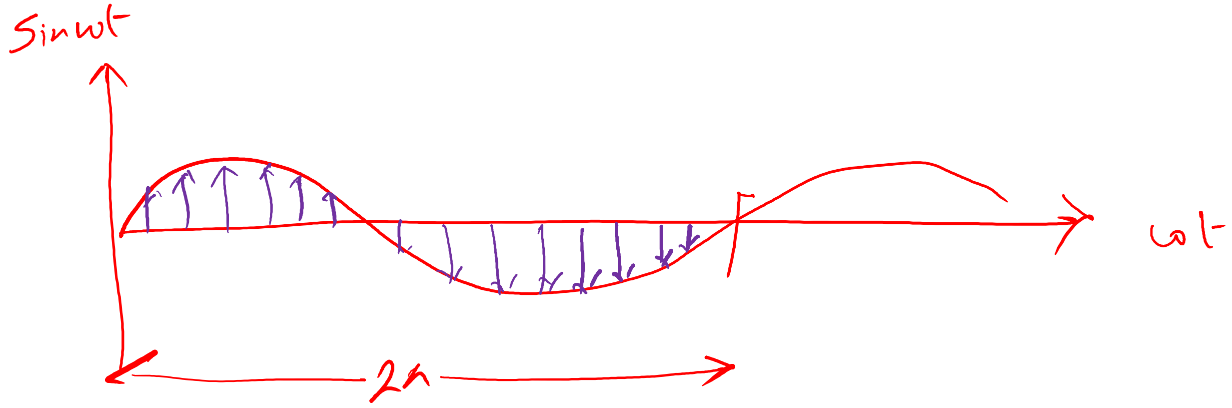
~~$x(t)$~~ is said to be periodic if $x(t+T) = x(t)$

$x(n)$

$x(n+N) = x(n)$

N → fundamental period.

$$x_a(t)\Big|_{t=nT} = x_a(nT) = x_a\left(\frac{n}{F_s}\right)$$



$M = 1$

$x(n) = \sin\left(\frac{6\pi}{7}n + 1\right)$

Since $\sin(\theta + 2\pi M) = \sin\theta$

Sol:- $x(n+N) = \sin\left(\frac{6\pi}{7}(n+N) + 1\right)$

$= \sin\left(\underbrace{\frac{6\pi}{7}n + 1}_{\theta} + \frac{6\pi N}{7}\right)$

Compare these two signals.

multiple of 2π

$\frac{6\cancel{\pi}N}{7} = 2\cancel{\pi}M$

$\Rightarrow N = \frac{2\times 7M}{\cancel{6}\,3} = \frac{7M}{3}$

$\therefore M = 3$

$N = \frac{7}{\cancel{3}} \times \cancel{3}$

$x(n+N) = \sin\left(\frac{6\pi}{7} + 1 + \frac{6\pi}{\cancel{7}} \times \cancel{7}\right)$

$= \sin\left(\frac{6\pi}{7} + 1 + 6\pi\right)$

$= \sin\left(\frac{6\pi}{7} + 1\right)$

fundamental period would be 7

$x(n) = \cos\left(\frac{n}{8} - \pi\right)$

Sol^n:-

periodic

$$\cos\left(\frac{n}{8} - \pi\right)$$

$$= \cos\left(\frac{n+N}{8} - \pi\right)$$

$$= \cos\left(\frac{n}{8} + \frac{N}{8} - \pi\right)$$

$$= \cos\left(\frac{n}{8} - \pi + \frac{N}{8}\right)$$

$\cos(\theta + 2\pi M) = \cos\theta$

$2\pi M = \frac{N}{8}$

$\Rightarrow$

16π

$$\underbrace{x(t) = x(-t) \longleftrightarrow x(n) = x(-n)}_{\text{even signal}}$$

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

$$\underbrace{x(t) = -x(-t) \longleftrightarrow x(n) = -x(-n)}_{\text{odd signal}}$$

$$x(n) = a^n$$

$$x(-n) = a^{-n}$$

even comp. $x_e(n) = \frac{1}{2}\left[a^n + a^{-n}\right]$

odd comp $x_o(n) = \frac{1}{2}\left[a^n - a^{-n}\right]$

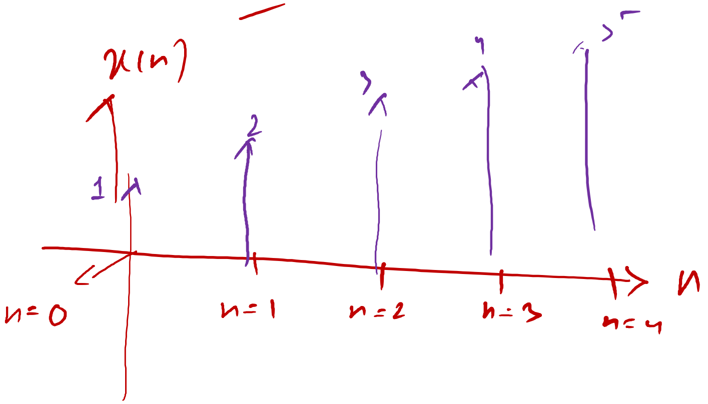
Find the odd and even part of the signal :-

$$x(n) = \{\underset{\uparrow}{4}, -4, 2, -2\}$$

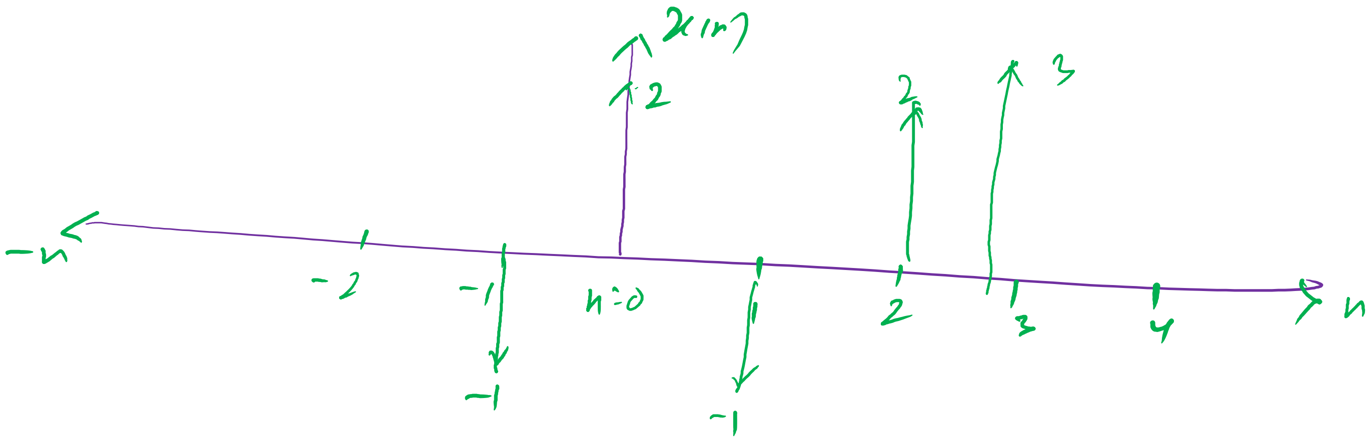
$n = 0$

$$x(n) = [\underset{\uparrow}{1}, 2, 3, 4, 5]$$

$n=0$



$$x(n) = \{\overset{n=-2}{0}, \overset{n=-1}{-1}, \underset{\underset{n=0}{\uparrow}}{2}, \overset{n=1}{-1}, \overset{2}{2}, \overset{3}{3}\}$$

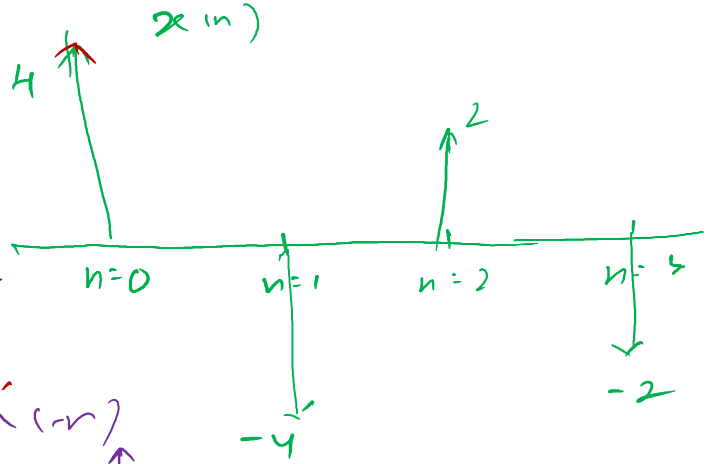


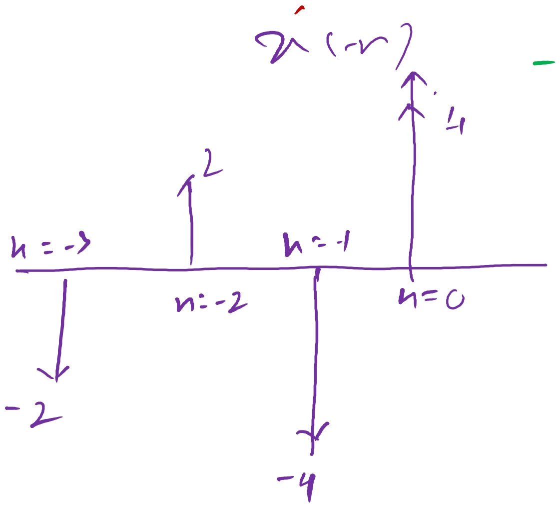
$$x(n) = \{\underset{\uparrow}{4}, -4, 2, -2\}$$

$$x(-n) = \{-2, 2, -4, \underset{\uparrow}{4}\}$$

n = 0

n ≠ 3





→ n = -3 to 0

$$(-1, 1, -2, \underset{\uparrow}{4}, -2, 1, -1)$$

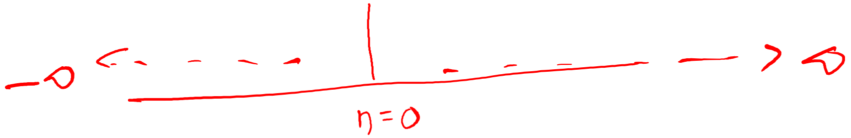
$$\frac{1}{2}[x(n) + x(-n)] = \frac{1}{2}[-2, 2, -4, \underset{\uparrow}{8}, -4, 2, -2]$$

$$(1, -1, 2, 0, -2, 1, -1)$$

$$\frac{1}{2}[x(n) - x(-n)] = \frac{1}{2}[\cancel{2}, -2, 4, \underset{\uparrow}{0}, -4, 2, -2]$$

Energy and power :-

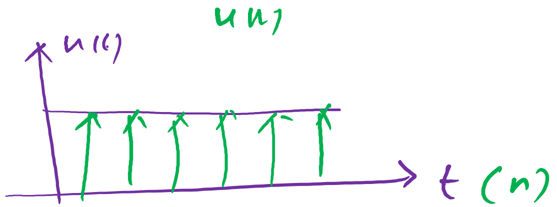
$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \sum_{n=-\infty}^{\infty} |x(n)|^2$$



$$\text{power} = \frac{1}{2T}\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$u(t) \longrightarrow u(n)$

$$x(n) = \left(\frac{1}{4}\right)^n u(n)$$



$$x(n) = \left(\frac{1}{4}\right)^n \cdot 1 \qquad \text{when } n \geq 0$$

$$= 0 \qquad , \qquad \text{otherwise.}$$

$$\boxed{x(n) = 0.25^n \;;\; n \geq 0}$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \underbrace{\sum_{n=0}^{\infty} |0.25^n|^2}_{\text{Infinite G.S}}$$

$$\sum_{n=0}^{\infty} (0.25^2)^n = \sum_{n=0}^{\infty} (0.0625)^n$$

$$= \frac{1}{1-0.0625}$$

$$= 1.0\dot{6}7 \text{ Joule.}$$

Sum of finite geometric series

$$\sum_{n=0}^{N} c^n = \frac{c^{N+1}-1}{c-1}$$

Sum^n Infinite geometric series

$$\sum_{n=0}^{\infty} c^n = \frac{1}{1-c}$$

$$\text{power} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (0.065)^n$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \frac{(0.065)^{N+1} - 1}{0.065 - 1} \rightarrow \frac{}{(2N+1)}$$

∞.

$$= \frac{1}{\infty}$$

$$= 0$$

$\therefore$ E = finite

p = 0

$\rightarrow$ Energy