

Date-26/10/21

Ex. Find first four moments about mean from the following data.

$x:$ 10 15 20 25 30

$f:$ 2 5 4 9 5

$$\bar{x} = \frac{\sum fx}{N}$$

Soln: $\bar{x} = 22$

$$= \frac{550}{25} = 22$$

| x | f | $x - \bar{x}$ | $f(x - \bar{x})$ | $f(x - \bar{x})^2$ | $f(x - \bar{x})^3$ | $f(x - \bar{x})^4$ |
|-----|-----|---------------|------------------|--------------------|--------------------|--------------------|
| 10 | 2 | -12 | -24 | 288 | -3456 | 41472 |
| 15 | 5 | -7 | -35 | 245 | -1715 | 12005 |
| 20 | 4 | -2 | -8 | 16 | -32 | 64 |
| 25 | 9 | 3 | 27 | 81 | 243 | 729 |
| 30 | 5 | 8 | 40 | 320 | 2560 | 20480 |

$$\sum f(x - \bar{x})^2 = 950, \quad \sum f(x - \bar{x})^3 = -2400, \quad \sum f(x - \bar{x})^4 = 74750$$

Now,

$$\mu_1 = 0$$

$$\mu_2 = \frac{950}{25}$$

$$= 38$$

$$\mu_3 = \frac{-2400}{25}$$

$$= -96$$

$$\mu_4 = \frac{74750}{25}$$

$$= 2990$$

Ex. The first four moments about any point 28.5 of a distribution are 0.294, 7.144, 42.409, 454.98.

Calculate the mean and first four moments about mean.

Solⁿ: Given, $\mu_1' = 0.294$, $\mu_2' = 7.144$, $\mu_3' = 42.409$, $\mu_4' = 454.98$.

Now,

$$\mu_1' = 0.294$$

$$\Rightarrow \frac{1}{N} \sum f(x - 28.5) = 0.294$$

$$\Rightarrow \frac{1}{N} (\sum fx - 28.5 \sum f) = 0.294$$

$$\Rightarrow \frac{1}{N} \sum fx - \frac{28.5}{N} \times N = 0.294$$

$$\Rightarrow \bar{x} = 28.794$$

Now,

Obviously, $\mu_1 = 0$

$$\begin{aligned} \mu_2 &= \mu_2' - \mu_1'^2 = 7.144 - (0.294)^2 \\ &= 7.058. \end{aligned}$$

$$\begin{aligned}
 \mu_3 &= \mu'_3 - 3\mu'_1\mu'_2 + 2(\mu'_1)^3 \\
 &= 42.409 - 3 \times 0.294 \times \overset{7.144}{\cancel{7.058}} + 2 \times (0.294)^3 \\
 &= 42.409 - \overset{6.301}{\cancel{6.225}} + 0.051 \\
 &= \cancel{36.235} \quad 36.159
 \end{aligned}$$

$$\begin{aligned}
 \mu_4 &= \mu'_4 - 4\mu'_1\mu'_3 + 6\mu'_2(\mu'_1)^2 + 4(\mu'_1)^4 \\
 &= 454.98 - 4 \times 0.294 \times 42.409 + 6 \times 7.144 \times (0.294)^2 \\
 &\quad + 4 \times (0.294)^4 \\
 &= 454.98 - 49.87 + 3.70 + 0.03 \\
 &= 408.84
 \end{aligned}$$