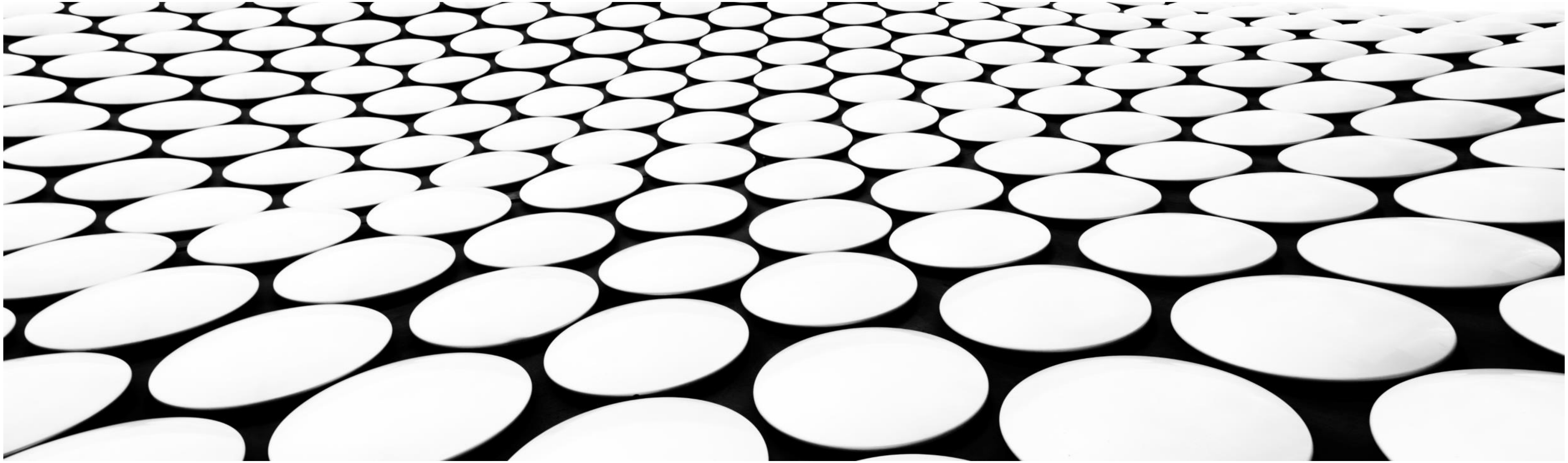
SIGNALS & SYSTEMS

MR. ANKUR JYOTI SARMAH

ASSISTANT PROF., DEPT. OF ELECTRONICS & TELECOM. ENGG.

ASSAM ENGINEERING COLLEGE



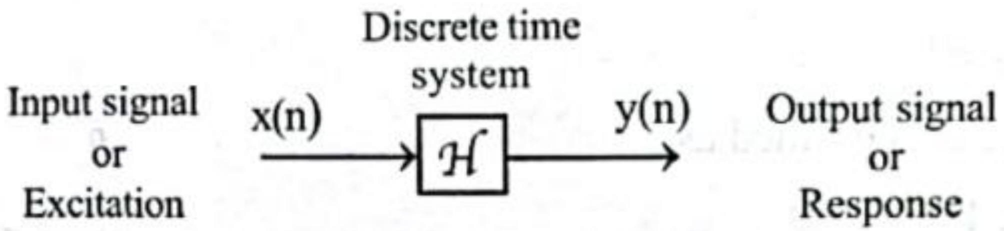
Discrete Time System:

A ***discrete time system*** is a device or algorithm that operates on a discrete time signal, called the input or excitation, according to some well defined rule, to produce another discrete time signal called the output or the response of the system. We can say that the input signal x(n) is transformed by the system into a signal y(n), and the transformation can be expressed mathematically as shown in equation

The diagrammatic representation of discrete time system is shown in fig

Response, $y(n) = \mathcal{H}\{x(n)\}$

where, $\mathcal{H}$ denotes the transformation (also called an operator).



Classification of discrete time systems:

1. Static and dynamic systems
2. Time invariant and time variant systems
3. Linear and nonlinear systems
4. Causal and noncausal systems
5. Stable and unstable systems
6. FIR and IIR systems
7. Recursive and nonrecursive systems

Static and dynamic systems:

A discrete time system is called ***static*** or ***memoryless*** if its output at any instant n depends at mos on the input sample at the same time but not on the past or future samples of the input. In any other case the system is said to be ***dynamic*** or to have memory.

Example :

$$\left.\begin{aligned} y(n) &= a\,x(n)\\ y(n) &= n\,x(n) + 6\,x^3(n)\end{aligned}\right\}\text{Static systems}$$

$$\left.\begin{aligned}&\left.\begin{aligned} y(n) &= x(n) + 3\,x(n-1)\\ y(n) &= \sum_{m=0}^{N} x(n-m)\end{aligned}\right\}\text{Finite memory is required}\\ &\left. y(n) = \sum_{m=0}^{\infty} x(n-m)\right\}\text{Infinite memory is required}\end{aligned}\right\}\text{Dynamic systems}$$

Time Variant and Time Invariant Systems:

A system is said to be ***time invariant*** if its input-output characteristics do not change with time.

Definition : A relaxed system $\mathcal{H}$ is ***time invariant*** or ***shift invariant*** if and only if

$$x(n) \xrightarrow{\mathcal{H}} y(n) \text{ implies that, } x(n-m) \xrightarrow{\mathcal{H}} y(n-m)$$

for every input signal x(n) and every time shift m.

i.e., in time invariant systems, if $\mathbf{y(n) = \mathcal{H}\{x(n)\}}$ then $\mathbf{y(n-m) = \mathcal{H}\{x(n-m)\}}$.

Alternative Definition for Time Invariance

A system $\mathcal{H}$ is ***time invariant*** if the response to a shifted (or delayed) version of the input is identical to a shifted (or delayed) version of the response based on the unshifted (or undelayed) input.

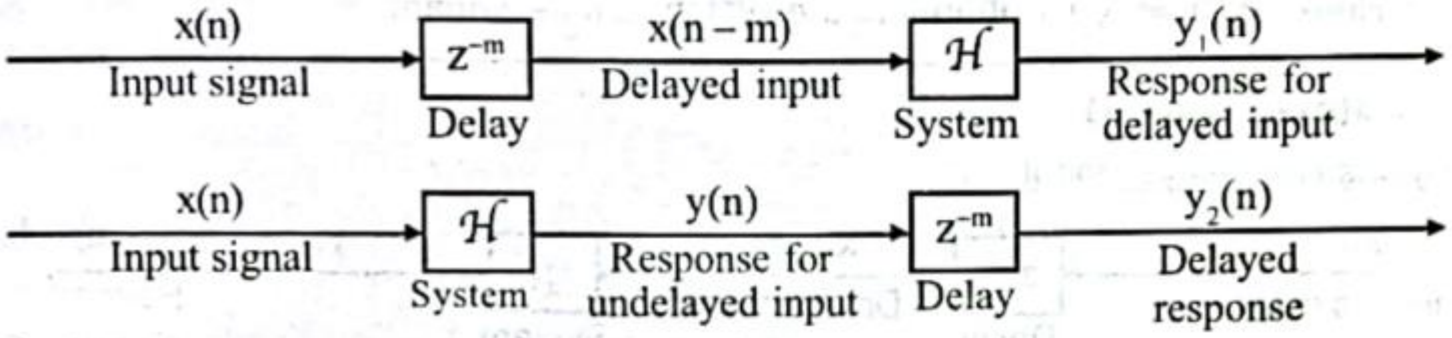
i.e., In a time invariant system, $\mathcal{H}\{\mathbf{x(n-m)}\} = \mathbf{z^{-m}}\,\mathcal{H}\{\mathbf{x(n)}\}$; for all values of m .

The operator z^{-m} represents a signal delay of m samples.

Time Variant and Time Invariant Systems:

Procedure to test time variance of a system:

1. Delay the input signal by m units of time and determine the response of the system for this delayed input signal. Let this response be $y_1(n)$.
2. Delay the response of the system for undelayed input by m units of time. Let this delayed response be $y_2(n)$.
3. Check whether $y_1(n) = y_2(n)$. If they are equal then the system is time invariant. Otherwise the system is time variant



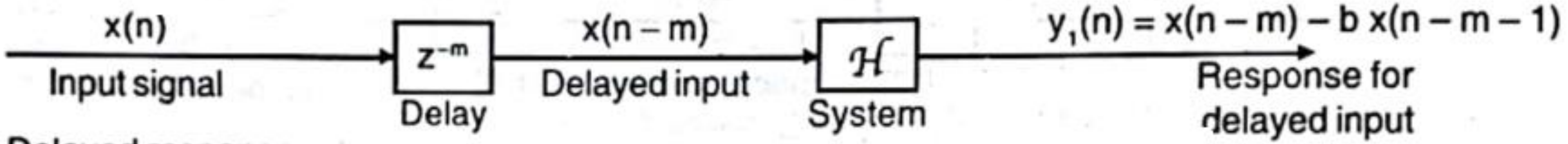
If, $y_1(n) = y_2(n)$, then the system is time invariant

Diagrammatic explanation of time invariance.

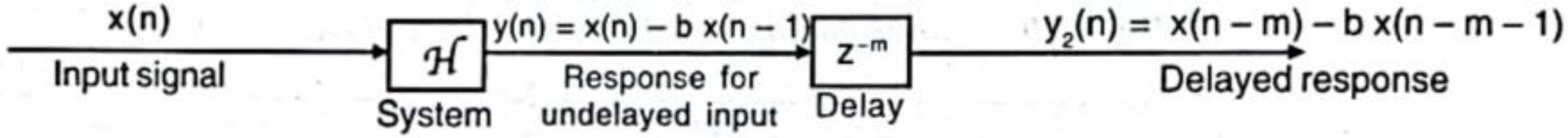
Determine the variance of the following systems:

$$y(n) = x(n) - b\,x(n-1)$$

Test 1 : Response for delayed input



Test 2 : Delayed response

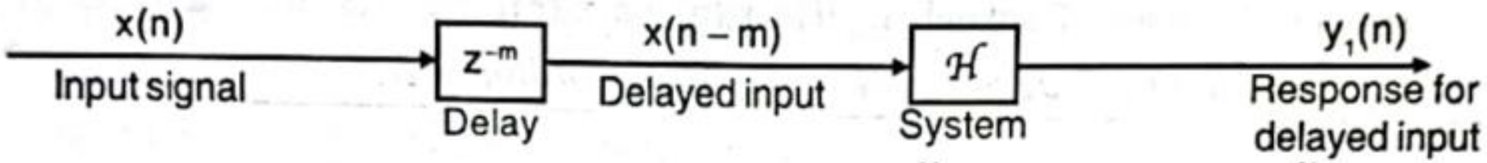


Conclusion : Here, $y_1(t) = y_2(t)$, therefore the system is time invariant.

Determine the variance of the following systems:

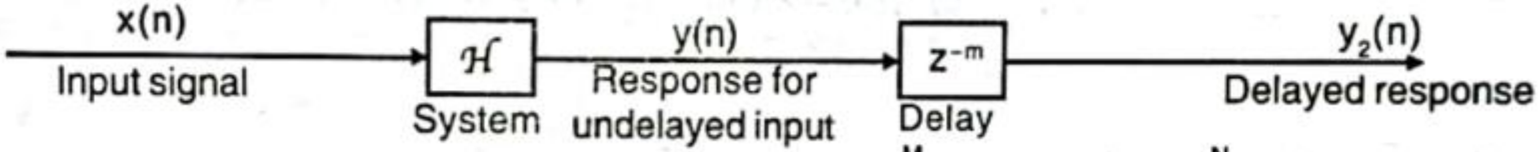
$$y(n) = \sum_{k=0}^{M} b_k\, x(n-k) - \sum_{k=1}^{N} a_k\, y(n-k)$$

Test 1 : Response for delayed input



Response for delayed input, $y_1(n) = \mathcal{H}\{x(n-m)\} = \sum_{k=0}^{M} b_k\, x(n-m-k) - \sum_{k=1}^{N} a_k\, y(n-m-k)$

Test 2 : Delayed response



Response for undelayed input $= \mathcal{H}\{x(n)\} = y(n) = \sum_{k=0}^{M} b_k\, x(n-k) - \sum_{k=1}^{N} a_k\, y(n-k)$

Delayed response, $y_2(n) = z^{-m}\, \mathcal{H}\{x(n)\}$

$$= z^{-m}\left[\sum_{k=0}^{M} b_k\, x(n-k) - \sum_{k=1}^{N} a_k\, y(n-k)\right]$$

$$= \sum_{k=0}^{M} b_k\, x(n-m-k) - \sum_{k=1}^{N} a_k\, y(n-m-k)$$

Conclusion : Here, $y_1(n) = y_2(n)$, therefore the system is time invariant.

Linear & Non- linear Systems:

A ***linear system*** is one that satisfies the superposition principle. The ***principle of superposition*** requires that the response of the system to a weighted sum of the signals is equal to the corresponding weighted sum of the responses of the system to each of the individual input signals.

Definition : A relaxed system $\mathcal{H}$ is ***linear*** if

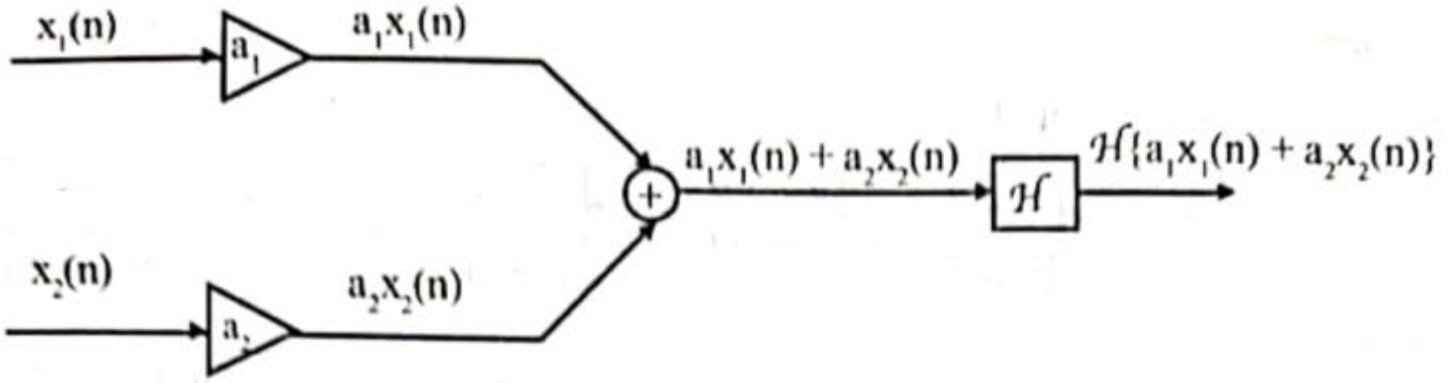
$$\mathcal{H}\{a_1 x_1(n) + a_2 x_2(n)\} = a_1 \mathcal{H}\{x_1(n)\} + a_2 \mathcal{H}\{x_2(n)\}$$

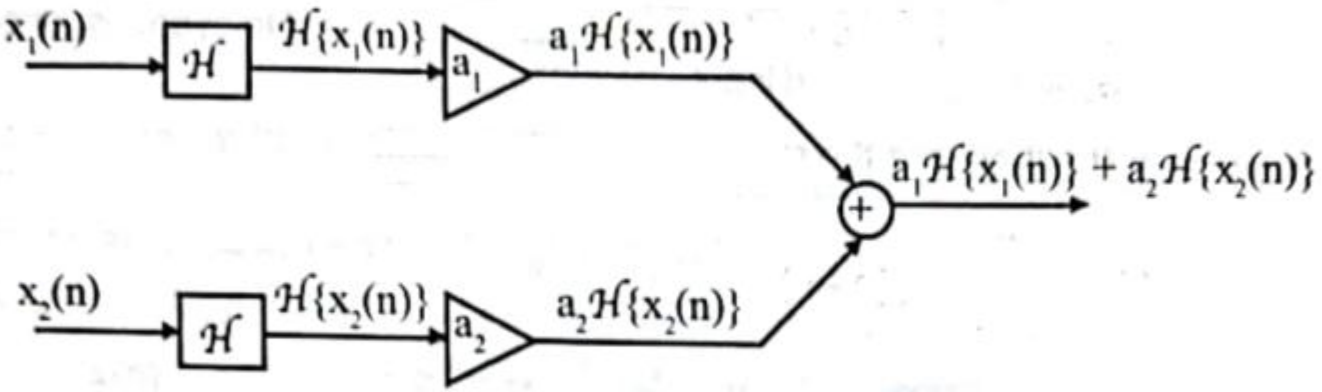
for any arbitrary input sequences $x_1(n)$ and $x_2(n)$ and for any arbitrary constants a_1 and a_2.

If a relaxed system does not satisfy the superposition principle as given by the above definition, the system is ***nonlinear***.

Procedure to test time variance of a system:

1. Let $x_1(n)$ and $x_2(n)$ be two inputs to system $\mathcal{H}$, and $y_1(n)$ and $y_2(n)$ be corresponding responses.
2. Consider a signal, $x_3(n) = a_1\, x_1(n) + a_2\, x_2(n)$ which is a weighed sum of $x_1(n)$ and $x_2(n)$.
3. Let $y_3(n)$ be the response for $x_3(n)$.
4. , Check whether $y_3(n) = a_1\, y_1(n) + a_2\, y_2(n)$. If they are equal then the system is linear, otherwise it is nonlinear.





The system, $\mathcal{H}$ is linear if and only if, $\mathcal{H}\{a_1\, x_1(n) + a_2\, x_2(n)\} = a_1\, \mathcal{H}\{x_1(n)\} + a_2\, \mathcal{H}\{x_2(n)\}$

Determine the linearity of the system

$$y(n) = a^{x(n)}$$

Let $\mathcal{H}$ be the system represented by the equation, $y(n) = a^{x(n)}$ and the system $\mathcal{H}$ operates on $x(n)$ to produce, $y(n) = \mathcal{H}\{x(n)\} = a^{x(n)}$.

Consider two signals $x_1(n)$ and $x_2(n)$.

Let $y_1(n)$ and $y_2(n)$ be the response of the system $\mathcal{H}$ for inputs $x_1(n)$ and $x_2(n)$ respectively.

$$\therefore\ y_1(n) = \mathcal{H}\{x_1(n)\} = a^{x_1(n)}$$

$$y_2(n) = \mathcal{H}\{x_2(n)\} = a^{x_2(n)}$$

$$\therefore\ a_1 y_1(n) + a_2 y_2(n) = a_1 a^{x_1(n)} + a_2 a^{x_2(n)} \qquad .....(1)$$

Consider a linear combination of inputs, $a_1x_1(n) + a_2x_2(n)$. Let the response of the system for this linear combination of inputs be $y_3(n)$.

$$\therefore\ y_3(n) = \mathcal{H}\{a_1 x_1(n) + a_2 x_2(n)\} = a^{[a_1 x_1(n) + a_2 x_2(n)]}$$

$$= a^{a_1 x_1(n)}\ a^{a_2 x_2(n)} \qquad .....(2)$$

The condition to be satisfied for linearity is, $y_3(n) = a_1 y_1(n) + a_2 y_2(n)$.

From equations (1) and (2) we can say that, $y_3(n) \neq a_1 y_1(n) + a_2 y_2(n)$. Hence the system is nonlinear.