Karl Pearson's β and γ co-efficients.

Karl Pearson defined following 4 co-efficients based upon the first four moments about mean.

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}, \quad \gamma_1 = \sqrt{\beta_1}$$

&

$$\beta_2 = \frac{\mu_4}{\mu_2^2}, \quad \cancel{\gamma_2 = \beta_2} \quad \gamma_2 = \beta_2 - 3.$$

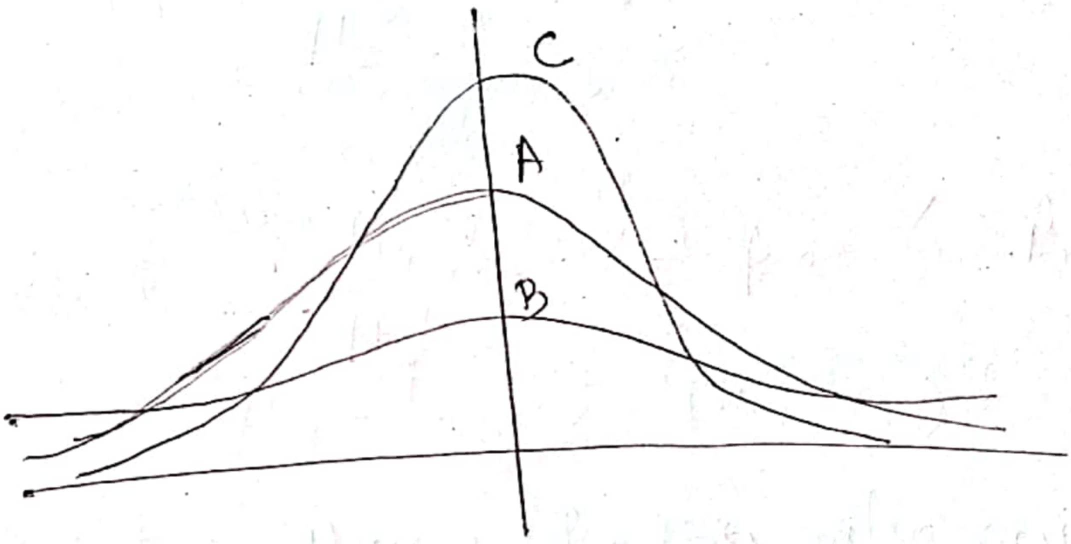
Note: Here, β_1 ~~gi~~ also gives ~~means~~ measures of ~~seq~~ skewness.

If $\beta_1 = 0$, then the variates are symmetrically distributed.

Scanned with CamScanner

Kurtosis:

The relative flatness of the top of a curve is called Kurtosis and it is measured by β_2.



Normal or Mesokurtic curve:

Curve which are neither flat nor sharply peaked are called normal curve "mesokurtic curves". (As is the figure curve A). For such curve, $\beta = 3$ and hence $\gamma_2 = 0$.

Scanned with CamScanner

Platykurtic curve: Curves which are flatter than normal curve (as in the figure curve B) are called Platykurtic curve. For such curves, $\beta_2 < 3$ and so $\gamma_2 < 0$

Leptokurtic curve:

Curves which are more sharply peaked than the normal curve (as the curve C in the figure) are called Leptokurtic curve. For such curve, $\beta > 3$ and $\gamma_2 > 0$.

Ex:- Calculate β_1 and β_2 for the following distribution and obtain the nature of kurtosis.

x :	0	1	2	3	4	5	6	7	8
f :	1	8	28	56	70	56	28	8	1

Soln. → Let us calculate the first four moments about $x = 4$. (i.e. moment about mean).

$$\mu_r = \frac{1}{N} \sum f(x - \bar{x})^r$$

$$= \frac{1}{N} \sum f(x-4)^r.$$

Let, $d = x - 4$.

$$\therefore \mu_r = \frac{1}{N} \sum f d^r.$$

Scanned with CamScanner

Now, we make the following table.

x	f	$d = x-4$	fd	fd^2	fd^3	~~$d f^4$~~ fd^4
0	1	-4	-4	16	-64	256
1	8	-3	-24	72	-216	648
2	28	-2	-56	112	-224	448
3	56	-1	-56	56	-56	56
4	70	0	0	0	0	0
5	56	1	56	56	56	56
6	28	2	56	112	224	448
7	8	3	24	72	216	648
8	1	4	4	16	64	256
	$N=256$		$\sum fd = 0$	$\sum fd^2 = 512$	$\sum fd^3 = 0$	$\sum fd^4 = 2816$

$\therefore \quad \mu_1 = \frac{1}{N}\sum fd = 0$ (always)

$\mu_2 = \frac{1}{N}\sum fd^2 = \frac{512}{256} = 2$ [σ^2 i.e. the variance]

$$\mu_3 = \frac{1}{N}\sum fd^3$$

$$= \frac{1}{256} \times 0 = 0 .$$

$$\mu_4 = \frac{1}{N}\sum fd^4 = \frac{1}{256} \times 2816$$

$$= 11 .$$

$\therefore \beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0$, i.e. the curve is symmetrical.

and $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{11}{4}$

$$= 2 \cdot 75 < 3$$

and $\gamma_2 = \beta_2 - 3 = -\cdot 25 < 0$

Hence, the curve is ~~Platykurtic~~.

Platykurtic. (more flat).

Scanned with CamScanner