15/11/2021

Q. Determine the initial value $x(0)$ and find value $x(\infty)$ for the following $X(s) = \dfrac{s+1}{s^2+2s+2}$

Sol: $$x(0) = \lim_{s\to\infty} s\,X(s) = \lim_{s\to\infty} s \cdot \frac{s+1}{s^2+2s+2}$$ (Initial value them)

$$= \lim_{s\to\infty} \cancel{s} \cdot \frac{\cancel{s}\left(1+\frac{1}{s}\right)}{\cancel{s^2}\left(1+\frac{2}{s}+\frac{2}{s^2}\right)}$$

$$= \lim_{s\to\infty} \frac{1+1/s}{1+2/s+2/s^2}$$

$$= \frac{1+0}{1+0+0}$$

$$\Rightarrow x(0) = 1$$

(final value theorem)

And, $$x(\infty) = \lim_{s\to 0} s\,x(s)$$

$$= \lim_{s\to 0} s \cdot \frac{(s+1)}{s^2+2s+2}$$

$$= 0 \cdot \frac{0+1}{0+0+2}$$

$$= 0$$

$$x(s) = \frac{s+5}{s^2(s+9)}$$

$$= \lim_{s\to\infty} s \cdot \frac{s+5}{s^2(s+9)} = \lim_{s\to\infty} \frac{s^2(\cancel{0}\ 1+5/s)}{s^2(s+9)} = \lim_{s\to\infty} \frac{1+5/s}{s+9} = 0$$

$$\lim_{s\to 0} s \cdot \frac{s+5}{s^2(s+9)} = \lim_{s\to 0} \frac{s \cdot s+5}{s^2(s+9)} = \infty$$

Note : Convolution → (i) If $X_1(s) = L\{x_1(t)\}$ & $X_2(s) = L\{x_2(t)\}$, then

$$L\{x_1(t) * x_2(t)\} = X_1(s) \cdot X_2(s)$$

(ii) If $X_1(s) = L\{x_1(t)\}$ then

$$L\{x(t+a)\} = e^{+as} \cdot X(s)$$

Q: $x_1(t) = u(t+5)$ → Delayed unit step sig.

$x_2(t) = \delta(t-7)$ → Delayed impulse sig.

Sol^n: $X_1(s) = L\{x_1(t)\}$

$= L\{u(t+5)\}$

$= e^{5s} \cdot \frac{1}{s}$

$= \frac{e^{5s}}{s}$ $[\because L\{u(t)\} = 1/s]$

$X_2(s) = L\{x_1(t)\}$

$= L\{\delta(t-7)\}$

$= e^{-7s} \cdot 1$ $[\because L\{\delta(t)\} = 1]$

$\Rightarrow X_2(s) = e^{-7s}$

According to the convolution property of Laplace ~~the~~ transformation

$L\{x_1(t) * x_2(t)\} = X_1(s) \cdot X_2(s)$

$= \frac{e^{5s}}{s} \cdot e^{-7s}$

$= \frac{e^{-2s}}{s}$

$\Rightarrow x_1(t) * x_2(t) = L^{-1}\{X_1(s) \cdot X_2(s)\}$

$= L^{-1}\{e^{-2s}/s\}$

$= L^{-1}\{\frac{1}{s}\}_{t=t-2}$

$\Rightarrow x_1(t) * x_2(t) = u(t-2)$ //

Q. Perform the convolution of $x_1(t)$ & $x_2(t)$ using convolution theorem of Laplace transform & sketch the resultant waveform.

$$x_1(t) = u(t) - u(t-1)$$
$$x_2(t) = u(t) - u(t-2)$$

Sol^n:
$$X_1(s) = L\{u(t) - u(t-1)\}$$
$$= L\{u(t)\} - L\{u(t-1)\}$$
$$= \frac{1}{s} - \frac{e^{-s}}{s}$$

$$X_2(s) = L\{u(t) - u(t-2)\}$$
$$= L\{u(t)\} - L\{u(t-2)\}$$
$$= \frac{1}{s} - \frac{e^{-2s}}{s}$$

~~$x_1(t) + x_2(t) =$~~

Acc. to conv. th^m of Laplace transform,

$$L\{x_1(t) * x_2(t)\} = X_1(s) \cdot X_2(s)$$
$$= \left(\frac{1}{s} - \frac{e^{-s}}{s}\right)\left(\frac{1}{s} - \frac{e^{-2s}}{s}\right)$$
$$= \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-3s}}{s^2}$$

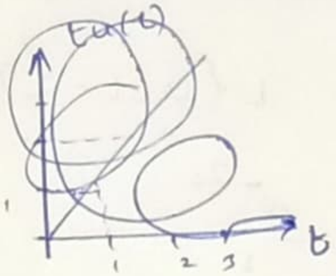
By taking Inverse Laplace transform,

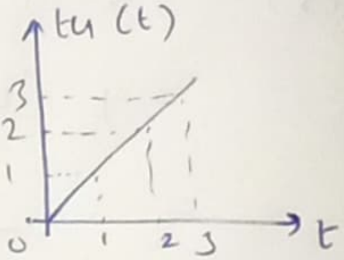
$$\Rightarrow x_1(t) * x_2(t) = L^{-1}\left\{\frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-3s}}{s^2}\right\}$$
$$= tu(t) - (t-2)u(t-2) - (t-1)u(t-1) + (t-3)u(t-3)$$

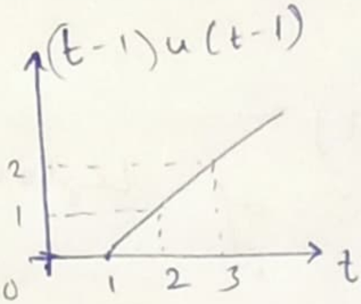
[Formula next page]

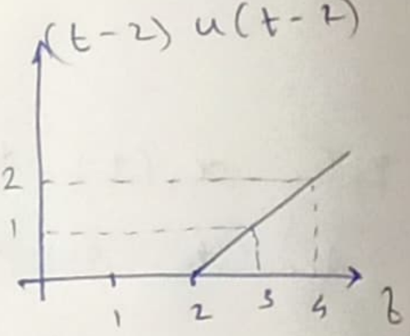
$$x_1(t) * x_2(t) = \underset{t\geq 0}{tu(t)} - \underset{t \geq 1}{(t-1)u(t-1)} - \underset{t\geq 2}{(t-2)u(t-2)} + \underset{t\geq 3}{(t-3)u(t-3)}$$

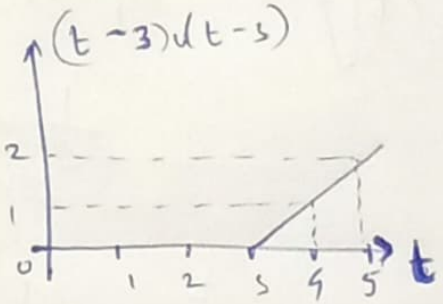
[Graph next page]











From, $t = 0$ to 1, $y(t) = t\,u(t) = t$.

From $t = 1$ to 2, $u(t) = 1$

$$u(t-1) = 1$$

$$u(t-2) = 0$$

$$u(t-3) = 0$$

$$\therefore x_1(t) * x_2(t) = t \cdot 1 - (t-1) \cdot 1$$

$$= t - t + 1$$

$$= 1$$

From, 2 to 3, $u(t)=1,\ u(t-1)=1,\ u(t-2)=1,\ u(t-3)=0$

$$x_1(t) * x_2(t) = t.1 - (t-1).1 - (t-2).1 + 0$$
$$= t - t + 1 - t + 2$$
$$= 3 - t$$

From $t > 3$, $u(t) = u(t-1) = u(t-2) = u(t-3) = 1$

$$\Rightarrow x_1(t) * x_2(t) = t - t + 1 - t + 2 - t + 3$$
$$= 0$$

$\therefore$