SIGNALS & SYSTEMS

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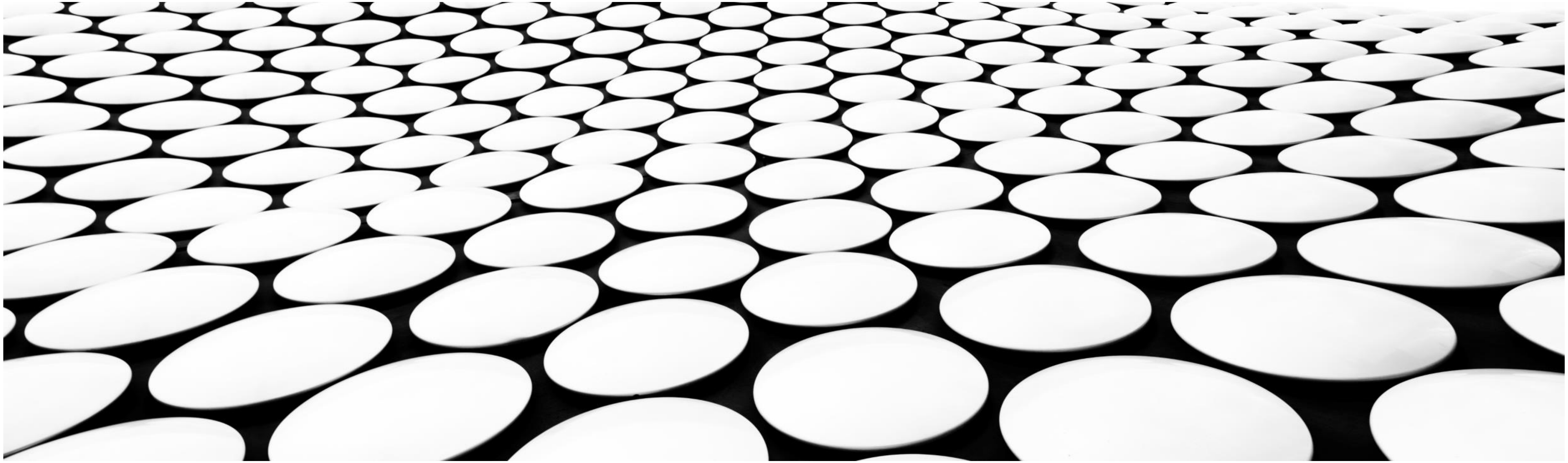


Region of Convergence: The laplace transform of a system is given by $\int_{-\infty}^{+\infty} x(t)e^{-st}dt$. The values of s for which the integral $\int_{-\infty}^{+\infty} x(t)e^{-st}dt$ converges is called ***region of convergence (ROC).***

ROC of three types.

1) Right sided (causal) signal.
2) Left sided(anti- causal) signal.
3) Two sided signal

Can be termed as unilateral

Bilateral

Roc for right sided (causal) signal:-

Let us consider a signal $x(t) = e^{-at} u(t)$

$u(t)$ ↳ defined for $t \geq 0 \rightarrow$ causal

$\therefore u(t) = 1; \ t \geq 0$

$= 0; \ \text{otherwise}$

$$\Rightarrow x(t) = e^{-at} \cdot 1 \quad ; t \geq 0$$

$$\Rightarrow x(t) = e^{-at}$$

Now using Laplace transform formula →

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

Here change of limit from $-\infty \rightarrow +\infty$ to 0 to ∞ because $x(t)$ is defined for $t \geq 0 \rightarrow \infty$

$$= \int_{0}^{\infty} e^{-at} e^{-st}\, dt$$

$$X(s) = \int_0^{\infty} e^{-(s+a)t}\, dt$$

$$= \left[\frac{e^{-(s+a)t}}{-(s+a)} \right]_0^{\infty}$$

$$= \frac{e^{-(s+a)\infty}}{-(s+a)} - \frac{\overset{\to 1}{\cancel{e^{-(s+a)0}}}}{-(s+a)}$$

$$\therefore X(s) = \frac{e^{-(s+a)\infty}}{-(s+a)} + \frac{1}{s+a}$$

Now, consider $s = \sigma + j\omega$ $\Rightarrow X(s) = \frac{e^{-(\sigma+j\omega+a)\infty}}{-(s+a)} + \frac{1}{s+a}$

only for exponential Term
not for denominator

$$\therefore X(s) = \frac{e^{-[(\sigma+a)+j\omega]\infty}}{-(s+a)} + \frac{1}{s+a}$$

Rearranging real and imaginary parts.

$$\Rightarrow X(s) = \frac{e^{-(\sigma+a)\infty} \cdot e^{-j\omega\infty}}{-(s+a)} + \frac{1}{s+a}$$

Consider $\sigma + a = K$.

$$\Rightarrow X(s) = \frac{e^{-K\infty} \cdot e^{-j\omega}}{-(s+a)} + \frac{1}{s+a}$$

So, $\sigma + a = K$ can be written as

$$K = \sigma - (-a)$$

Now if $\sigma > -a \Rightarrow K = \sigma - (-a) \Rightarrow K$ becomes +ve

$\Rightarrow e^{-K\infty}$ becomes $e^{-\infty}$

$$\Downarrow$$

$$e^{-\infty} = 0$$

Meaning of convergence of an integral means the value of the integral must be finite; i.e;

$$\int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt < \infty$$

$$\Downarrow$$

So it is quite evident that if $s = -a \Rightarrow X(s)$ becomes infinite.

on the other hand; $\sigma < -a \Rightarrow K = \sigma - (-a) \Rightarrow K$ becomes negative

$e^{-K\sigma}$ = becomes positive

$\Downarrow$

$e^{K\sigma} \Rightarrow$ Infinity.

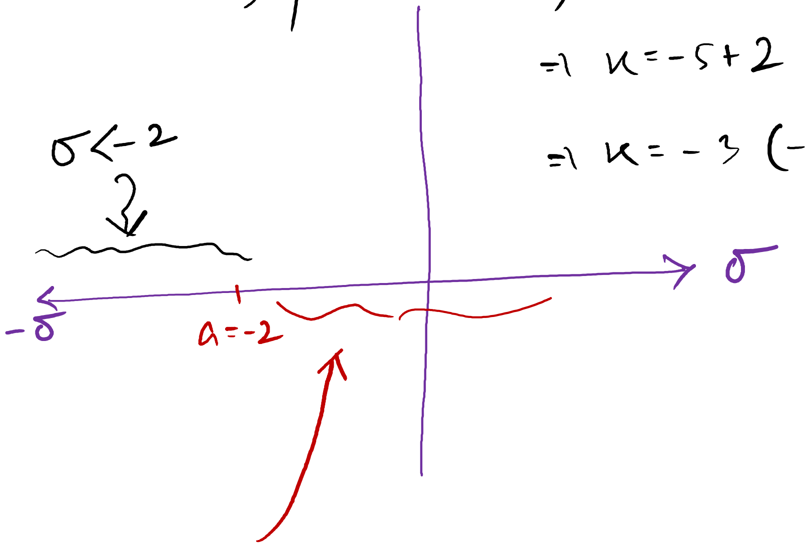
when $\sigma < -2 \Rightarrow K = \sigma - (-a)$

say $\sigma = -5$

$\Rightarrow K = -5 - (-2)$

$\Rightarrow K = -5 + 2$

$\Rightarrow K = -3$ (-ve).



when $\sigma > -2$

$K = \sigma - (-a)$.

say $\sigma = 5$

$\therefore \underbrace{K = 5 - (-2) = 7}_{\text{positive}}$.

From the above discussion it is clear that $X(s)$ converges when $\sigma > -a$ and fails to converge when $\sigma < -a$.

Considering $\sigma < -a$. We can say that $\sigma = -a$

Abscissa of convergence.

$$\therefore X(s) = \frac{\overbrace{e^{-\infty}}^{0 \text{ for } \sigma<-a} \cdot e^{-j\omega\infty}}{-(s+a)} + \frac{1}{s+a}$$

$$\Rightarrow \boxed{X(s) = \frac{1}{s+a}}$$

For causal signal ROC includes all points on s-plane to the right of abscissa of convergence.

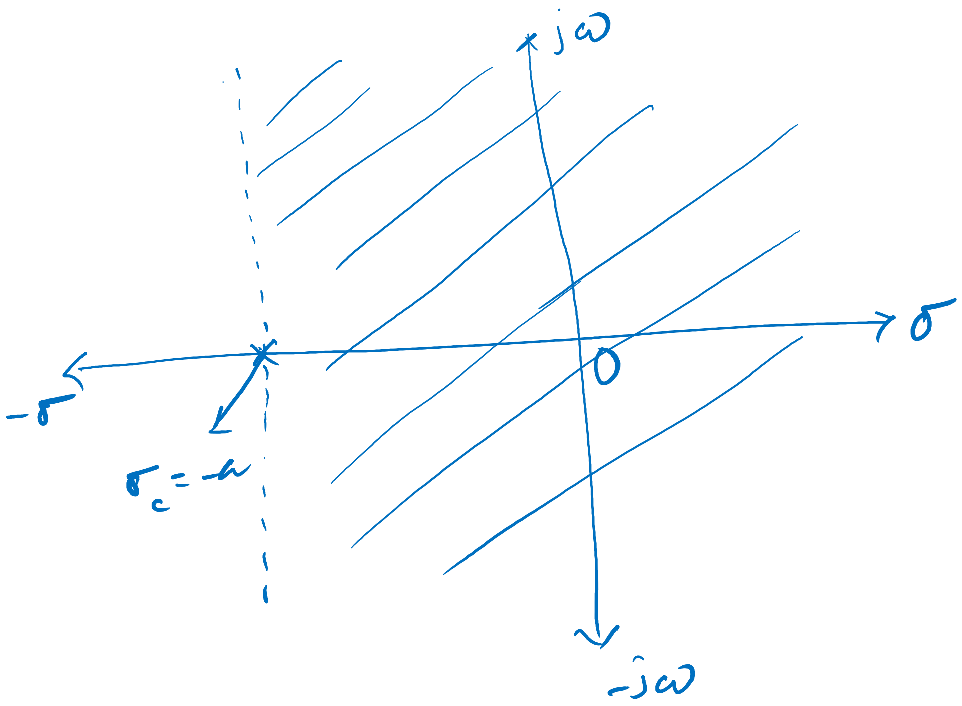


Fig: ROC of $e^{-at}u(t)$