Assignment - 1

Date

Introduction to signals & system

1] For each of the following system. 1) $y(t) = x(t-2) + x(2-t)$. 2) $y(n) = nx(n)$. Determine which of properties memoryless, time invariant, linear, causal holds and justify your answer.

→ 1) $y(t) = x(t-2) + x(2-t)$

not memoryless

$T[x_1(t)] = x_1(t-2) + x_1(2-t)$, $T[x_2(t)] = x_2(t-2) + x_2(2-t)$

$\therefore a_1 T[x_1(t)] + a_2 T[x_2(t)]$

$= a_1 x_1(t-2) + a_1 x_1(2-t) + a_2 x_2(t-2) + a_2 x_2(2-t)$

$T[a_1 x_1(t) + a_2 x_2(t)] = a_1 x_1(t-2) + a_1 x_1(t-2) + a_2 x_2(2-t$

$\therefore$ This system is linear.

$y(t)$ is depends on past so this system is causal.

$\therefore$ This system is dynamic.

2) $y(n) = nx(n)$

memoryless

$y(n-k) = (n-k)x(n-k)$

$y(n,k) = nx(n-k)$

$\therefore y(n-k) \neq y(n,k)$

$\therefore$ This system is time varient.

$T[x_1(n)] = nx_1(n)$, $T[x_2(n)] = nx_2(n)$

$\therefore a_1 T[x_1(n)] + a_2 T[x_2(n)] = n a_1 x_1(n) + n a_2 x_2(n)$

$T[a_1 x_1(n) + a_2 x_2(n)] = n a_1 x_1(n) + n a_2 x_2(n)$

$\therefore$ This system is linear.

$\therefore y(n)$ is depends on present so this system is causal and static.

2] Determine whether or not each of the following signals is periodic. If the signal is periodic determine its fundamental period.

→ 1) $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2$

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$$= \left[1 + \cos\left(4t - \frac{2\pi}{3}\right)\right] / 2$$

periodic signal

period $= \frac{2\pi}{4} = \frac{\pi}{2}$

2) $x(n) = \cos\left(\frac{n^2\pi}{8}\right)$

periodic signal

period $= 8$

3] Define : Signal. Find the fundamental periods of the following periodic signals.

→ Signal is one or more variable changes with variable time of with respect to variable.

- Those variables may be any information in terms of voltage, current, amplitude etc.

1) $x(t) = \cos(3\pi t) + 2\sin(4\pi t)$

period $= 2$, $\frac{T_1}{T_2} = \frac{2}{13} \times \frac{4}{2} = \frac{4}{13}$

2) $x(n) = e^{j7.351n\pi}$

period = non periodic

$\frac{2\pi kn}{N} = 7.351\pi n$ ∴ $\frac{k}{N} = \frac{7.351}{2}$

4] Define : System. Determine whether the system $y(t) = tx(t)$ is 1) Memoryless 2) Linear 3) time invarient 4) Casual 5] BIBO Stable. Justify your ans.

→ A system is any process that produce an output signal in response to an input signal.

$y(t) = tx(t)$

1) Memory less

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2) $T[x_1(t)] = t x_1(t)$, $T[x_2(t)] = t x_2(t)$

$a_1 T[x_1(t)] + a_2 T[x_2(t)] = t a_1 x_1(t) + t a_2 x_2(t)$

$T[a_1 x_1(t)] + a_2 x_2(t)] = t a_1 x_1(t) + t a_2 x_2(t)$

$\therefore$ linear system

3) $y(t-k) = (t-k)\, x(t-k)$

$y(t,k) = t\, x(t-k)$

$y(t-k) \neq y(t,k)$

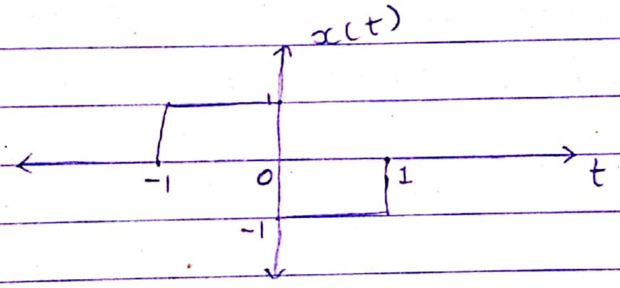
$\therefore$ Time varient system

4) System is depends on present so it is casual.

5) System depends on present so static dyn system

5] Sketch the waveform of the following signal:

$x(t) = u(t+1) - 2u(t) + u(t-1)$



6] Evaluate the following integrals:

1) $\int_1^2 (3t^2+1)\,\delta(t)\,dt = \left[(3t^2+1)u(t)\right]_1^2 - \int_1^2 6t\, u(t)\,dt$

$= (13-4)u(t) - 6\left[t\, r(t)\right]_1^2 + 6\left[r(t)\right]_1^2$

$= 9 - 6(4-1) + 6(2-1)$

$= -3$

2) $\int_0^\infty t^2 \delta(t-6)\,dt = \left[t^2 u(t-6)\right]_0^\infty - \int_0^\infty 2t\, u(t-6)\,dt$

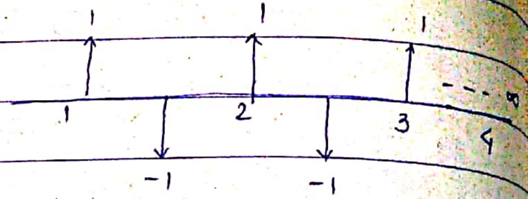
$= \infty - \left[2t\, r(t-6)\right]_0^\infty + \left[2 r(t-6)\right]_0^\infty$

$= \infty$

7] Determine whether following signal is periodic or not

→ If it is periodic, find fundamental period.

$x(n) = (-1)^n$.



It is period with period $t_0 = 2$ samples.

8] Integration of unit impulse function over $(-\infty, \infty)$ yields unit step signal and differentiating a unit ramp function yields unit step signal.

9] Find the even and odd components of following signal:

$x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$

→ $x(-t) = 1 - t + 3t^2 - 5t^3 + 9t^4$

$\text{even} = \dfrac{x(t) + x(-t)}{2} = \dfrac{2 + 6t^2 + 18t^4}{2} = 1 + 3t^2 + 9t^4$

$\text{odd} = \dfrac{x(t) - x(-t)}{2} = \dfrac{2t + 10t^3}{2} = t + 5t^3$

10] Categerize the following signals as an energy or power signal and find energy or power of the signals.

01] $x(t) = 5\cos(\pi t) + \sin(5\pi t)$, $-\infty < t < \infty$

$$E_{total} = \lim_{T \to \infty} \int_{-T}^{T} x^2(t)\,dt$$

$$= \int_{-\infty}^{\infty} 25\cos^2(\pi t) + 10\cos(\pi t)\sin(5\pi t) + \sin^2(5\pi t)\,dt$$

but here $\int_{-\infty}^{\infty} \cos^2(\pi t)\,dt = \infty$

$\therefore E = \infty$

$$P_{total} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

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$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 25\cos^2(\pi t) + 10\cos(\pi t)\sin(5\pi t) + \sin^2(5\pi t)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{25}{2}(1+\cos(2\pi t)) + 10\cos(\pi t)\sin(5\pi t) + \frac{1-\cos(10\pi t)}{2}\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left\{ \frac{25}{2}\left[t + \frac{\sin(2\pi t)}{2\pi}\right]_{-T/2}^{T/2} + 0 + \frac{1}{2}\left[t - \frac{\sin(10\pi t)}{10\pi}\right]_{-T/2}^{T/2} \right.$$

$$= \lim_{T\to\infty} \frac{25}{2} + \frac{1}{T}\left[\frac{\sin(2\pi t)}{2\pi}\right]_{-T/2}^{T/2} + \frac{1}{2} - \frac{1}{T}\left[\frac{\sin(10\pi t)}{10\pi}\right]_{-T/2}^{T/2}$$

$$= \frac{25}{2} + \frac{1}{2} = 13$$

$\therefore$ It is power signal.

Q2) $x(n) = \begin{cases} \cos(n\pi) & ; \pi > 0 \\ 0 & ; \text{otherwise} \end{cases}$

$$E_{total} = \lim_{n\to\infty} \int_{-n}^{n} x^2(n)\, dn$$

$$= \lim_{n\to\infty} \int_{-n}^{n} \cos^2(n\pi)\, dn$$

$$= \lim_{n\to\infty} \int_{-n}^{n} \left(\frac{1+\cos(2n\pi)}{2}\right) dn$$

$$= \frac{1}{2} \lim_{n\to\infty} \left[n + \frac{\sin(2n\pi)}{2\pi}\right]_{-n}^{n}$$

$$= \frac{1}{2}\left[n + \frac{\sin(2n\pi)}{2\pi}\right]_{\pi}^{\infty}$$

$$= \frac{\pi}{2}$$

$\therefore$ This is energy signal

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11] Periodic signals are
$x(t+T) = x(t)$

12] Even signal satisfies
$x(-t) = x(t)$

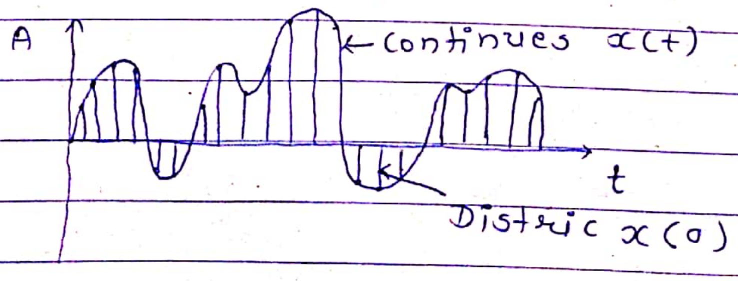
13] Which system is non casual system
$y(t) = x(t+1)$

14] Which signal is non casual
$x(t) = 0 \quad , t > 0$

15] In memory less system
zero input response is zero.

16] Signal $x(t)$ is odd signal if
$x(t) = -x(-t)$

17] Explain classification of signal.
1) Continues and district signal :-
→ Continues signal is vary continuesly with respect to time. District signal having amplitude & fixc intervals.



2) Periodic and Aperiodic signal :-
→ A signal repeated after finite time duration T is periodic signal. If signal is periodic signal interval with T then $x(t) = x(t+T)$.

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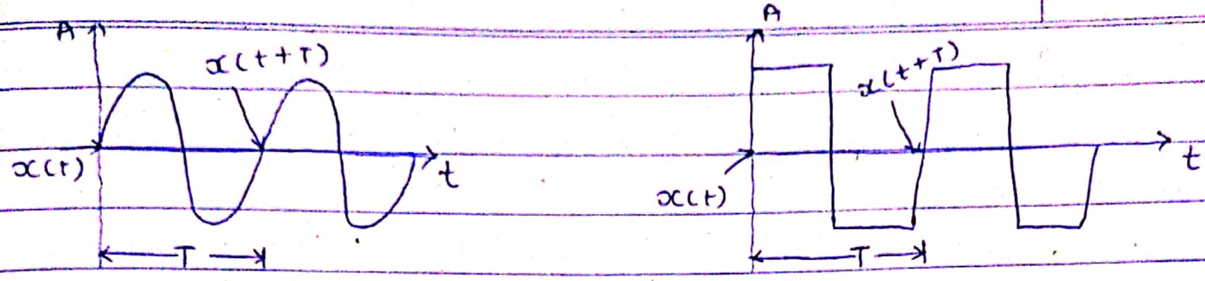
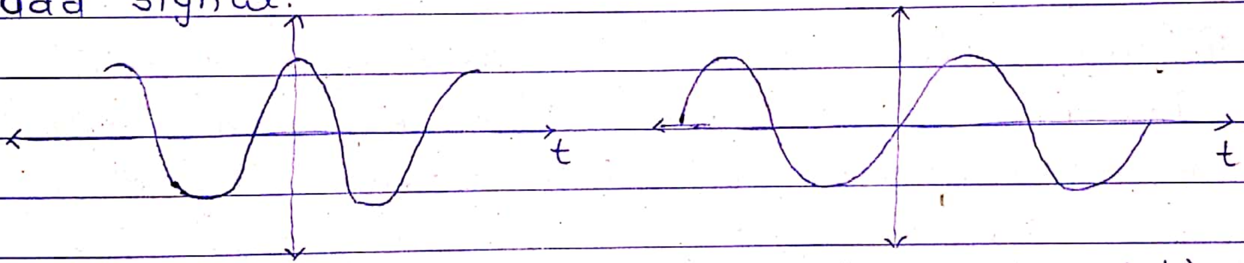


3) Deterministic and nondeterministic signal :-

→ Determinestic signal can be expressed in terms of mathematical function. Nondeterminestic signal is randomly nature.

4) Even and odd signals :-

→ If signal is symmetric with respect to t = 0 time interval is called even signal. If signal is asymetric with respect to t = 0 time interval is called odd signal.



Even , $x(t) = x(-t)$ odd : $x(t) = -x(-t)$

5) Energy or power signal :-

→ Signal can be energy/power signal if it has finite energy/power. If signal is energy signal then power should be zero. If signal is power signal then energy should be infinite.

$$E = \int_{-T}^{T} x^2(t)\,dt \Rightarrow E_{total} = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)\,dt$$

$$P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt \Rightarrow P_{total} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$$

6) Real and imagenary signal :-

→ Real signal does not have any imagenary ~~power~~ ~~should~~. component. Imagenary signal has imagenary component.

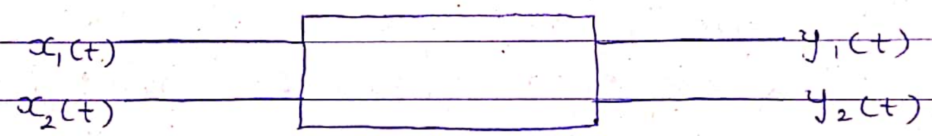
18] Explain classification of system.

1) Linear and non linear system :-

→ If system follows linearly then it should follow super position theorem.

- If we give $x_1(t)$ & $x_2(t)$ inputs to the system and corresponding outputs are $y_1(t)$ & $y_2(t)$.

$$T[a_1 x_1(t) + a_2 x_2(t)] = a_1 T[x_1(t)] + a_2 T[x_2(t)]$$



2) Time varient and time invarient system:

→ If response of system changes with respect to time then system is time varient system.

$$y(n,k) = y(n-k)$$

3) Linear time varient and linear time invarient system:

4) Static and dynamic system :-

→ If system response is only depending on present input then it is said to be static system.

- If system response based on present, past and future inputs then it is said to be dynamic system.

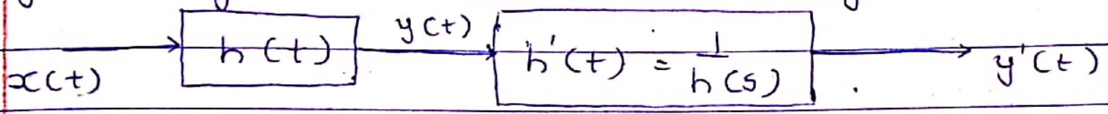
5) Causal and non causal system :-

→ If system response depends on present and past inputs then system is said to be causal system.

- If system response depends on future input then system is said to be non causal system.

6) Invertable and non invertable system :-

→ If we can extract input from the response then given system is invertable system.



$y(t) = x(t) * h(t)$

In freq. domain $y(s) = x(s)h(s)$

$y'(t) = y(t) * h'(t)$

In freq. domain $y'(s) = x(s)h(s)\left(\frac{1}{h(s)}\right)$

$y'(s) = x(s)$

7) Stable and unstable system :-

→ If system produces bounded output with given bounded input which system said to be stable system.

19] A system has the input - output relation given by $y(n) = T[x(n)] = nx(n)$ determine whether the system is, memoryless, causal, linear, time invarient stable.

→ $y(n) = nx(n)$

∴ System is memoryless and causal system.

$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 x_1(n) \cdot n + a_2 n x_2(n)$

$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 n x_1(n) + a_2 n \cdot x_2(n)$

∴ This system is linear system.

$y(n, k) = nx(n-k)$

$y(n-k) = (n-k)x(n-k)$

$y(n, k) \neq y(n-k)$

∴ This is time varient system.

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20] Consider a system S with input $x(n)$ and o/p $y(n)$ related by $y(n) = x(n)(g(n) + g(n-1))$. i) if $g(n) = 1$ for all n, show that S is time invarient ii) if $g(n) = n$, show that S is not time invarient iii) if $g(n) = 1 + (-1)^n$ show that S is time invarient.

$$y(n) = x(n)(g(n) + g(n-1))$$

i) If $g(n) = 1$

If wt change input by k positions. output will also change by same amount. so it is time invariant.

ii) If $g(n) = n$

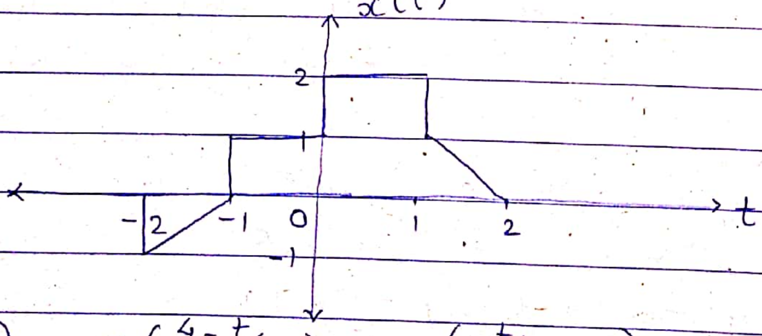
$$y(n) = x(n) + (n-1) \times x(n)$$

If this case n is outside the term $x(n)$. So, it is not time invarient.

iii) If $g(n) = 1 + (-1)^n$

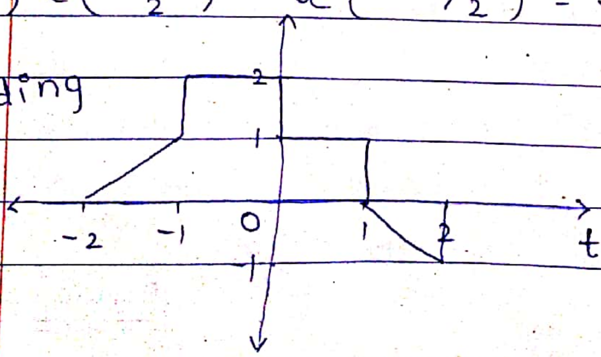
If we change $x(n)$ by some amount then $y(n)$ will also be changed by same amount. so system is time invarient.

21] A continuous time signal $x(t)$ is shown in fig. sketch and label carefully each of the following signals

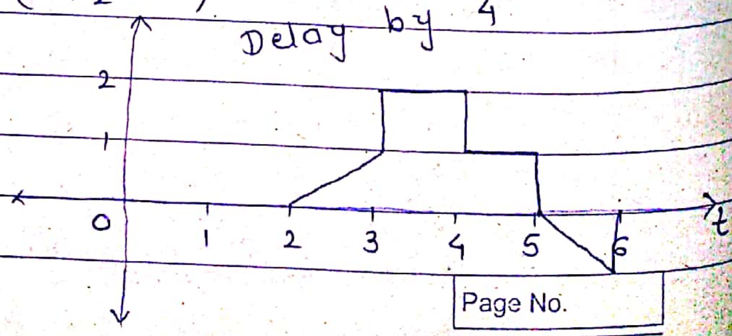


i) $x\left(\frac{8-t}{2}\right) = x\left(4 - t/2\right) = x\left(-t/2 + 4\right)$

folding



Delay by 4



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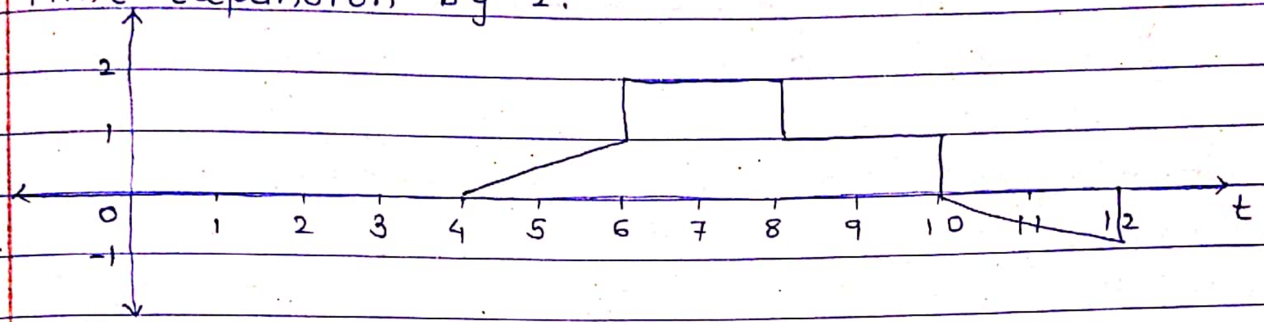
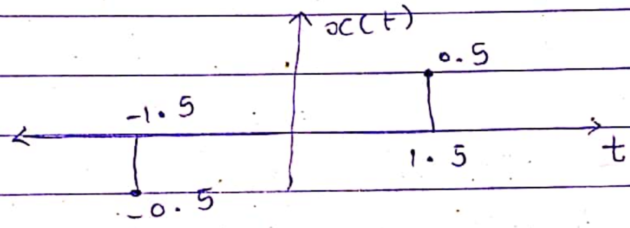
Time expansion by 2.



ii) $x(t)\left[\delta\left(t+\frac{3}{2}\right)-\delta\left(t-\frac{3}{2}\right)\right]$

The signal $\delta\left(t+3/2\right)$ will exist at $t=-1.5$ at $\delta(t-3/2)$ will exist at $t=1.5$. Thus $x(t)\left[\delta\left(t+\frac{3}{2}\right)-\delta\left(t-\frac{3}{2}\right)\right]$ is shown.



22] Determine whether each of them is: 1. Memoryless 2. stable 3. causal and 4. linear

→ 1) $y(n) = 2x(2^n)$

1. It does not contain past sample. So it is memoryless.
2. For bounded input bounded output is obtained so it is stable.
3. It does not contain any future input, so it is causal.
4. It is exponential. So it is non linear.

2) $y(t) = x(t/2)$

1. Due to past sample it is not memoryless.
2. Bounded input produces bounded output so it is stable.
3. It contains past sample, so it is causal.
4. It is time expansion operation. So it is linear.

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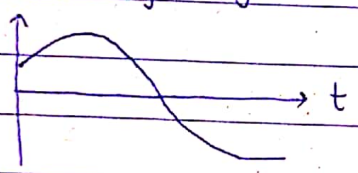
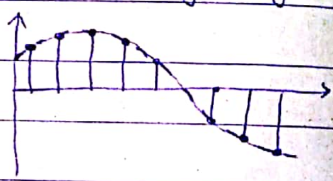
23] Rondom signal can be modeled by
→ stistical equation

24] Explain sampling and quantization:
→ Sampling process :-
- In the pulse modulation & digital modulation systems. the signal to be transmitted must be in the discrate time form.

Continous time analog signal → Sampler → Discrete time analog signal





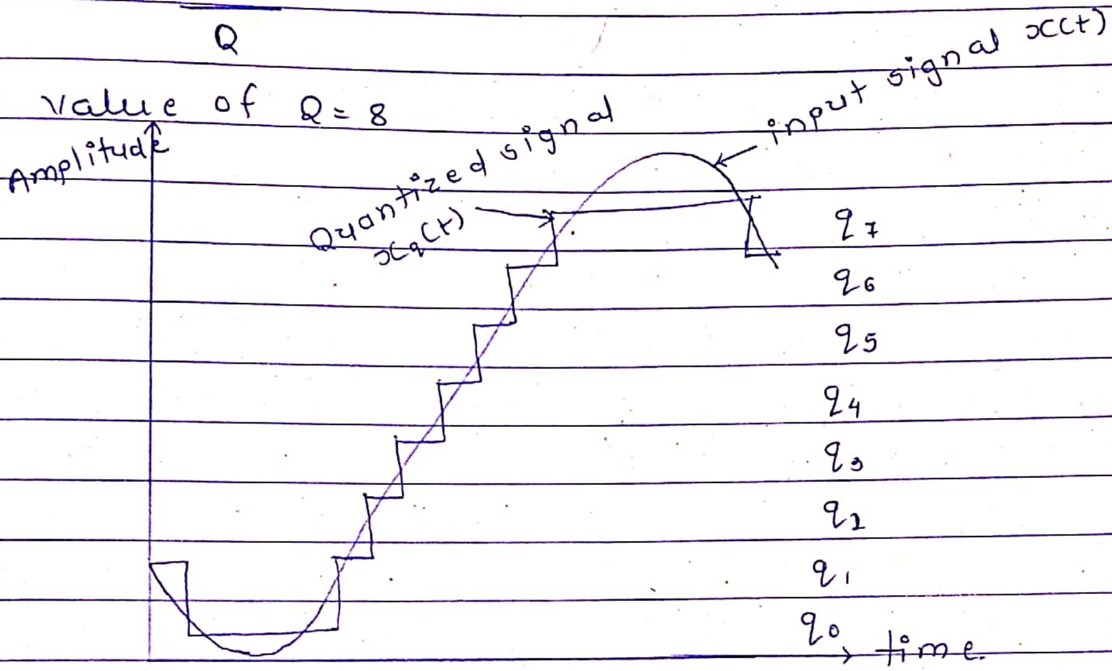
- using the sampling process convert a continuous time signal into a discrete time signal. for the sampling process to be of practical utility it is necessary to choose the sampling rate property.
1. Sampled ~~rate~~ property. signal should represent the original signal faithfully.
2. We should be able to reconstruct the original signal from its sampled version.
- Thus sampling is the process of converting a continuous analog signal is to a discrete analog signal & the sampled signal is the discrete time representation of the original analog signal.

→ Quantization process :-
- Quantization is a process of approximation or rounding off. The sampled signal in PCM transmitted is applied to the quntizer signal block. Quantizer converts the sampled signal into an approximate quantized signal which consists of only a finite number of

predecided voltage level. Those standard levels are known as the quantization levels.

$$S = \frac{V_K - V_L}{Q}$$

- The value of $Q = 8$



25] Determine whether or not each of the following signals is periodic. If the signal is periodic. Find the fundamental the period.

→ 1) $x(t) = \left[\sin\left(2t - \frac{\pi}{3}\right)^2\right]$

$= \frac{1}{2}\left[1 - \cos\left(4t - \frac{2\pi}{3}\right)\right] \qquad \left(\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$

$= \frac{1}{2} - \frac{1}{2}\cos\left(4t - \frac{2\pi}{3}\right)$

Here, $2\pi/3$ is phase shift

$\cos 4t = \cos 2\pi f_0 t$

$2\pi f_0 t = 4t$

$f_0 = 4/2\pi$ signal is non periodic.

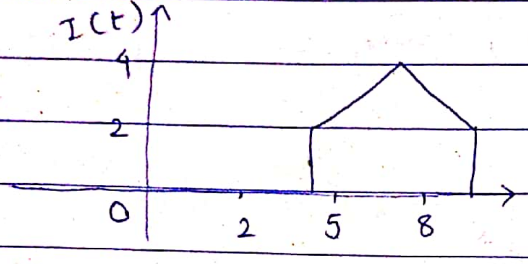
2) $x(n) = \cos\left(n\pi/8\right)$

$x(n) = A\cos(2\pi f_0 n)$

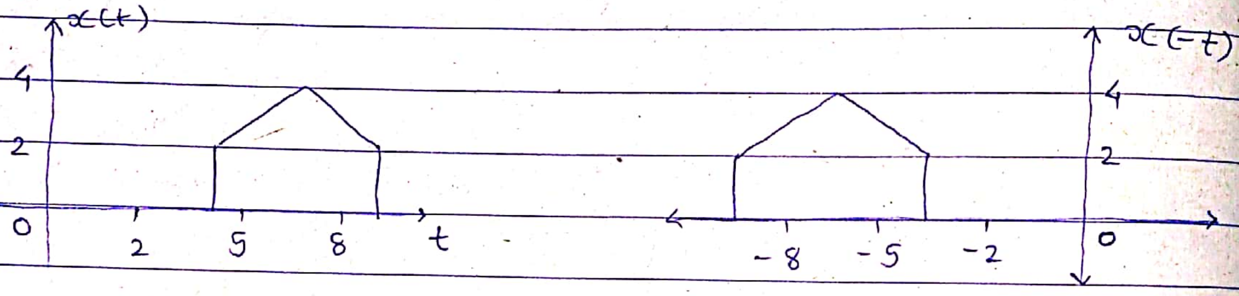
$2\pi f_0 n = \frac{n\pi}{8}$ signal is periodic with

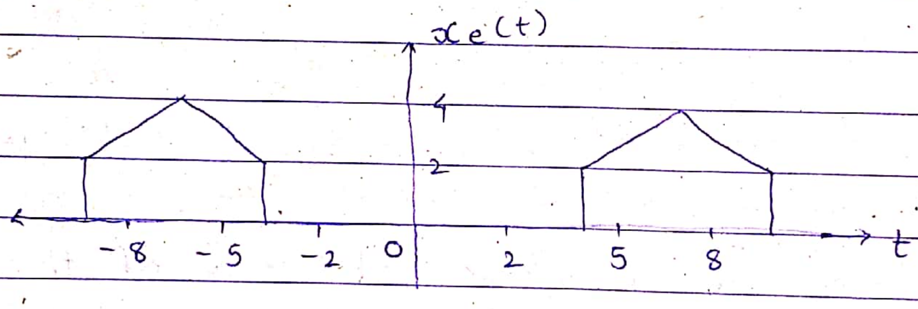
$f_0 = \frac{1}{16} = K/N$ Period $N = 16$

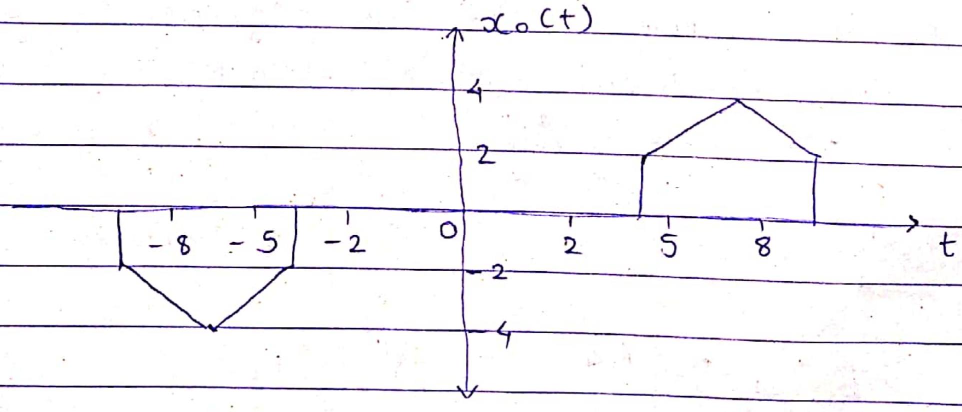
26] Find out the even & odd part of the following signal



$x_e(t) = \frac{1}{2}(x(t) + x(-t))$, $x_o(t) = \frac{1}{2}(x(t) - x(-t))$







27] Differentiate betn continuous and discrete time signals.

- Continous time signal :- A signal of continuous amplitude & time is known as a continuous time signal or an analog signal. This signal will have some finite value at every instant at time.
- Discrete time signal :- If the signal has finite values only at discrete instants of time, then it

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is known as a discrete time signals. The discrete time signals have values only at certain instants of time.

28] State the condition for a discrete time sinusoidal signal to be Periodic.

→ For the discrete time signal, the condition of periodicity is, $x(n) = x(n+N)$.
The smallest value of N for which the condition of periodicity exists is called as fundamental period.

29] Define : Aliazing

→ If the signal $x(t)$ is not strically bandlimited and / or if the sampling freq. f_s is less than 2W. then an error called aliazing or foldover error is observed. The adjucent spectrum overlap if $f_s < 2W$.

30] $x(n)\,\delta(n-k) = x(n-k)$.

31] Justify : static system are causal but all causal system are not static.

→ The output of static system dependent on present i/p only and o/p of causal system are causal but all causal system are not static.

32] Determine whether signal $x(n) = e^{j4n}$ is energy signal or power signal.

→ $x(n) = e^{j4n} = \cos 4\Omega + j \sin 4\Omega$

It is periodic signal, so it is a power signal.

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |e^{j4n}|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

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$$= \lim_{N \to \infty} \frac{1}{2N+1} \times 2N+1$$

$$= 1 \ W$$

33] Continous time signals $x_1(t)$ & $x_2(t)$ are periodic with T_1 & T_2 respectively. Signal $x_3(t) = x_1(t) + x_2(t)$. Find condition for $x_3(t)$ to be periodic.

→ $x_3(t) = x_1(t) + x_2(t)$

$x_1(t)$ is periodic,

$x_1(t) = x_1(t + mT_1)$

$x_2(t)$ is periodic,

$x_2(t) = x_2(t + mT_2)$

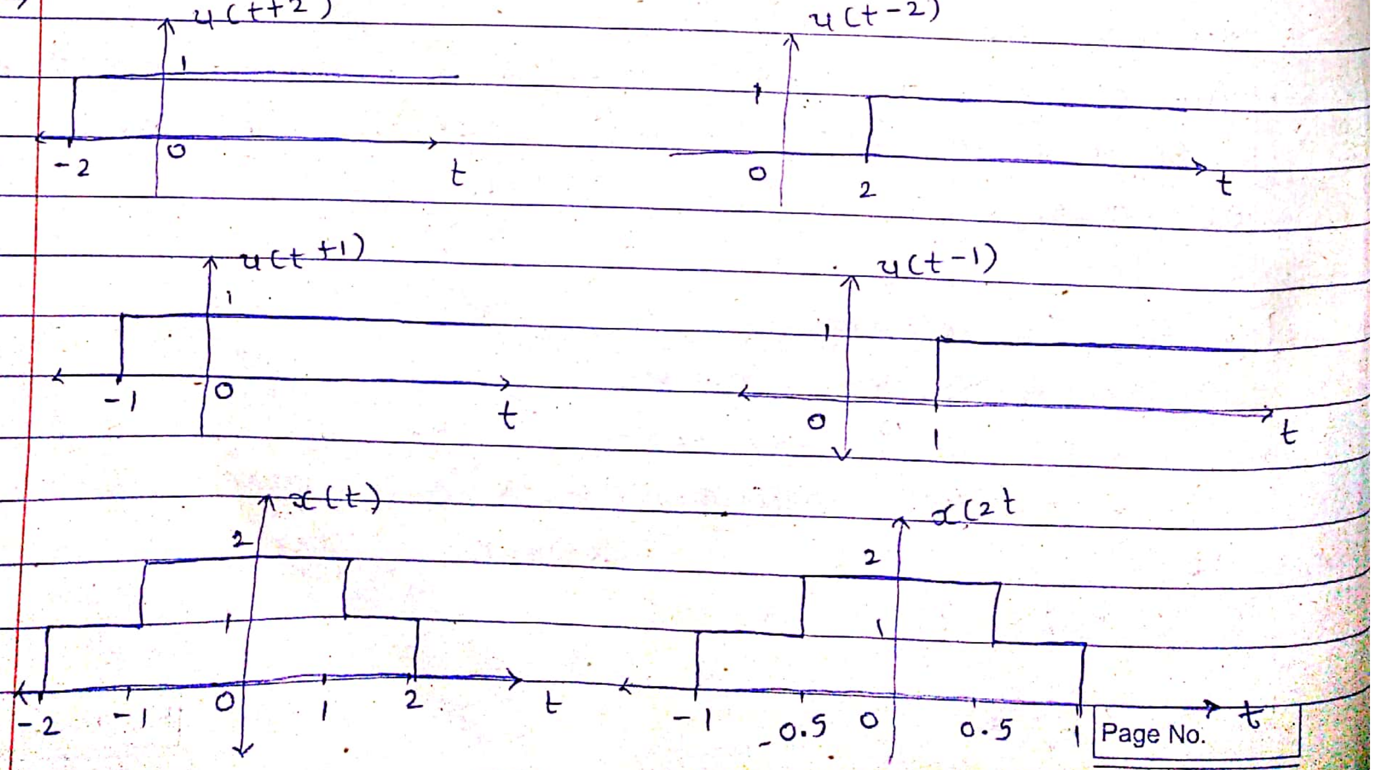
$x_3(t)$ is periodic if $mT_1 = nT_2 = T_0$

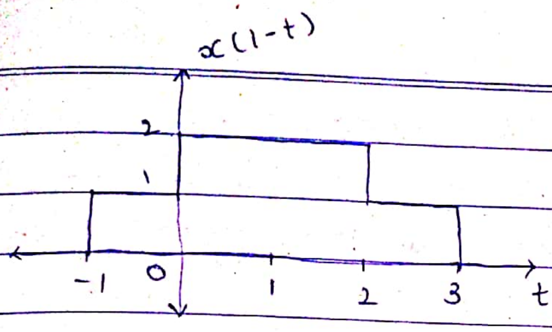
$$\therefore \frac{T_1}{T_2} = \frac{n}{m}$$

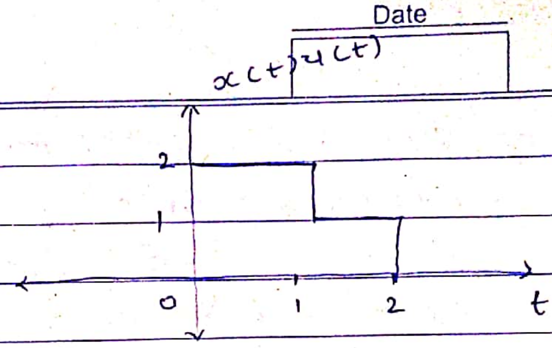
The period of $x_3(t)$ will be least common multiplexer of T_1 & T_2.

34] Sketch signal $x(t) = u(t+2) - u(t-2) + u(t+1) - u(t-1)$. also sketch i) $x(2t)$ ii) $x(1-t)$ iii) $x(t)\,u(t)$.

→



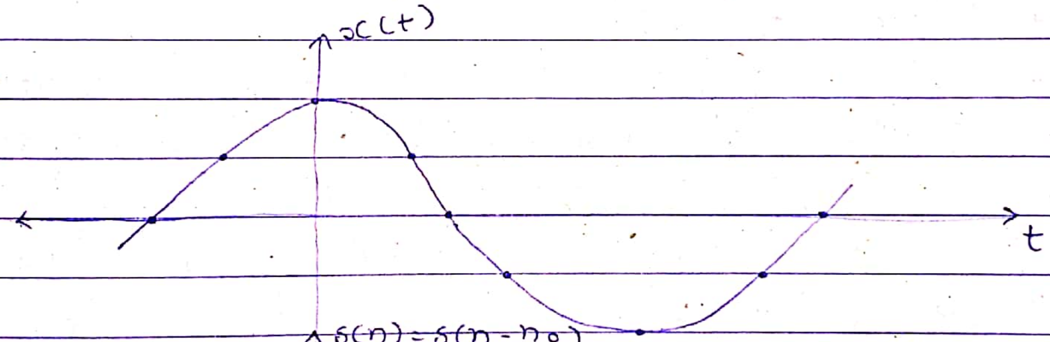


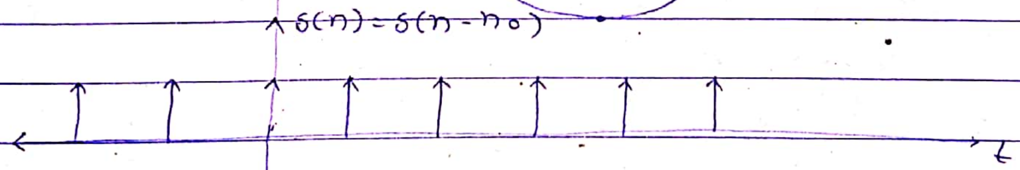


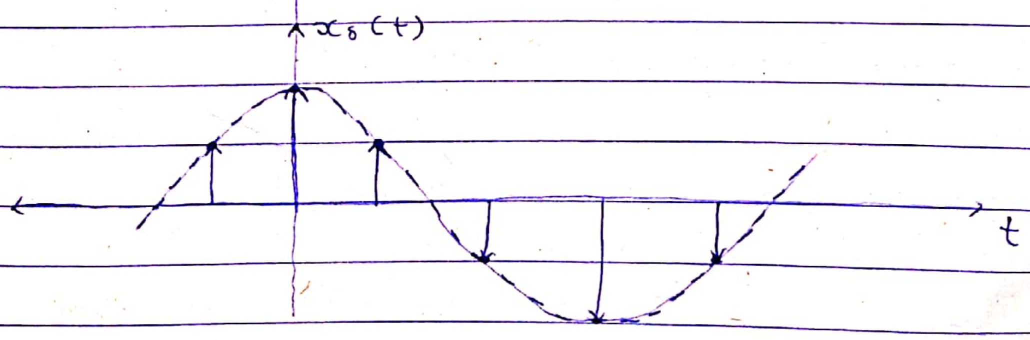
36] State and prove Sampling Theorem.

→ If a finite energy signal $x(t)$ contains nofrequncy higher then W Hz then it is completely determine by specifying its values at the instants of time which are spaced $(\frac{1}{2}W)$ seconds apart.

→ If a finite energy signal $x(t)$ contains no freq. components higher than W Hz then it may be compelety recovered from its samples which are spaced $(\frac{1}{2}W)$ seconds aparts.







$T_s = \frac{1}{f_s}$ = Sampling period

$f_s = \frac{1}{T_s}$ = Sampling rate.

Date

37] Determine if given system is linear, time invariant, causal, stable & static, Justify your answer.
$y(n) = nx(-n)$. Here $y(n)$ represents system output & $x(n)$ represents system input.

- If $x(n)=0$ then $y(n)=0$, so system is initially relaxed.

$$x(n) = x_1(n) + x_2(n)$$

$$x_1(n) \xrightarrow{T} nx_1(-n) \quad , \quad x_2(n) \xrightarrow{T} nx_2(-n)$$

$$y'(n) = nx_1(-n) + nx_2(-n) = n[x_1(-n) + x_2(-n)]$$

$$[x_1(n) + x_2(n)] \xrightarrow{T} y''(n) = n[x_1(-n) + x_2(-n)]$$

$\therefore y'(n) = y''(n)$. It is linear system.

$$y(n,k) = nx(-n+k)$$

$$y(n-k) = (n-k)x(-n+k)$$

$\therefore y(n,k) \neq y(n-k)$. It is time varient system

It is non causal system.

for bounded i/p: o/p is bounded. It is stable.

output contains folded i/p. It is dynamic system.

38] Identify if the system $y(t) = Ax(t) + B$ is linear, Justify your answer.

→ $x(t) = 0 \Rightarrow y(t) = B$

$$x_1(t) \xrightarrow{T} Ax_1(t) + B$$

$$x_2(t) \xrightarrow{T} Ax_2(t) + B$$

$$y'(t) = Ax_1(t) + B + Ax_2(t) + B$$

$$[x_1(t) + x_2(t)] \xrightarrow{T} y''(t) = Ax_1(t) + B + Ax_2(t) + B$$

$$y'(t) = y''(t)$$

It is linear system.

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