SIGNALS & SYSTEMS

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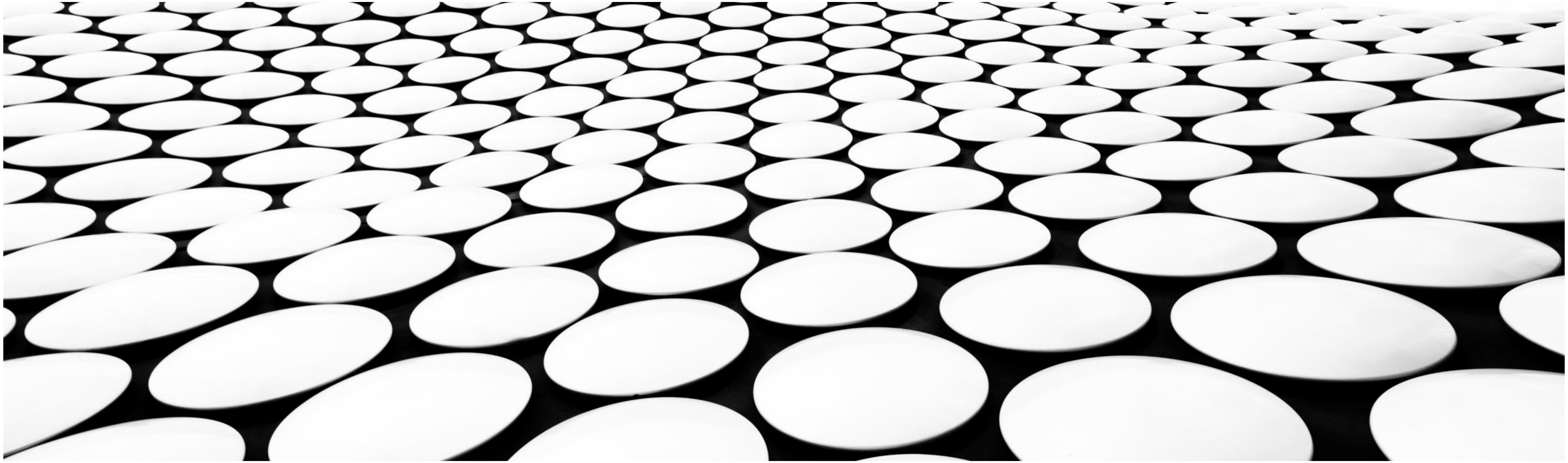
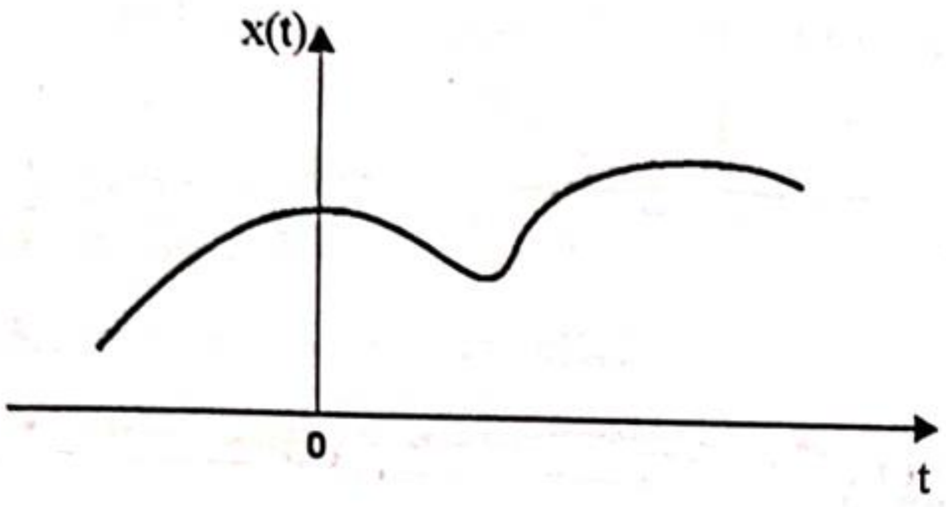
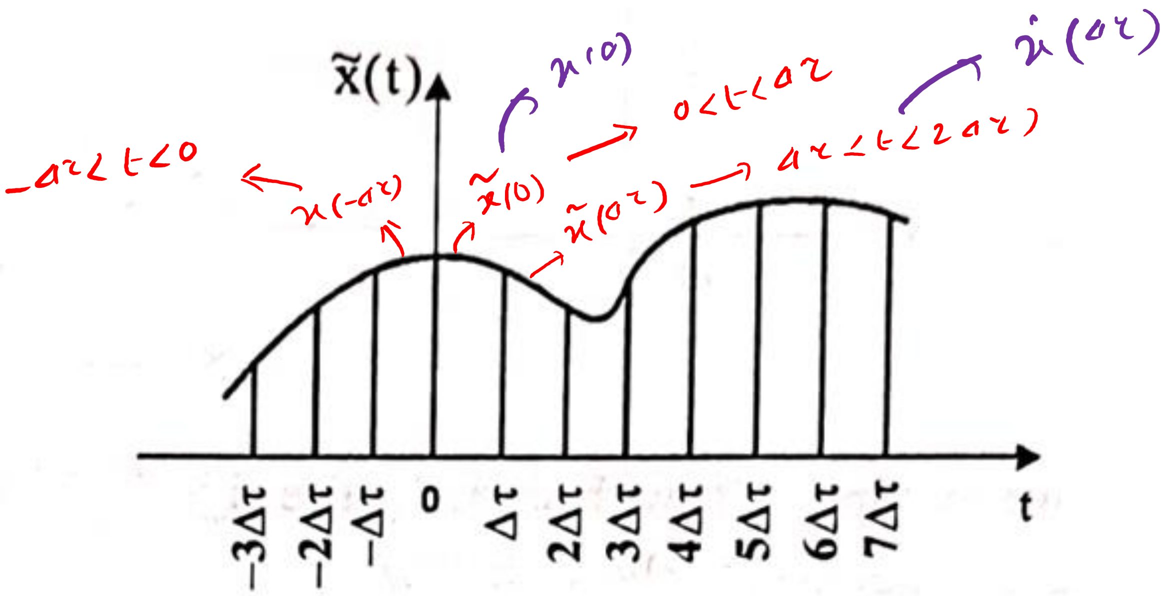


Representation of continuous time signal as an integral of impulse





Δτ

Let us divide x(t) as narrow pulses of width Δτ as shown in fig

Now the signal x(t) can be expressed as,

$$x(t) = \lim_{\Delta\tau \to 0} \tilde{x}(t) \quad \longrightarrow \quad \text{Eqn 1}$$

$$\vdots$$

$$\left.\begin{aligned}
x(-2\Delta\tau) &= \tilde{x}(t) \;; \quad \text{for} \quad -2\Delta\tau < t < -\Delta\tau \\
x(-\Delta\tau) &= \tilde{x}(t) \;; \quad \text{for} \quad -\Delta\tau < t < 0 \\
x(0) &= \tilde{x}(t) \;; \quad \text{for} \quad 0 < t < \Delta\tau \\
x(\Delta\tau) &= \tilde{x}(t) \;; \quad \text{for} \quad \Delta\tau < t < 2\Delta\tau \\
x(2\Delta\tau) &= \tilde{x}(t) \;; \quad \text{for} \quad 2\Delta\tau < t < 3\Delta\tau
\end{aligned}\right\}$$

$\tilde{x}(t) = x(-2\Delta\tau)$,

$\tilde{x}(t) = x(-\Delta\tau)$

$$\vdots$$

$$\therefore\; x(t) = \underset{\Delta\tau \to 0}{\mathrm{Lt}} \; \tilde{x}(t)$$

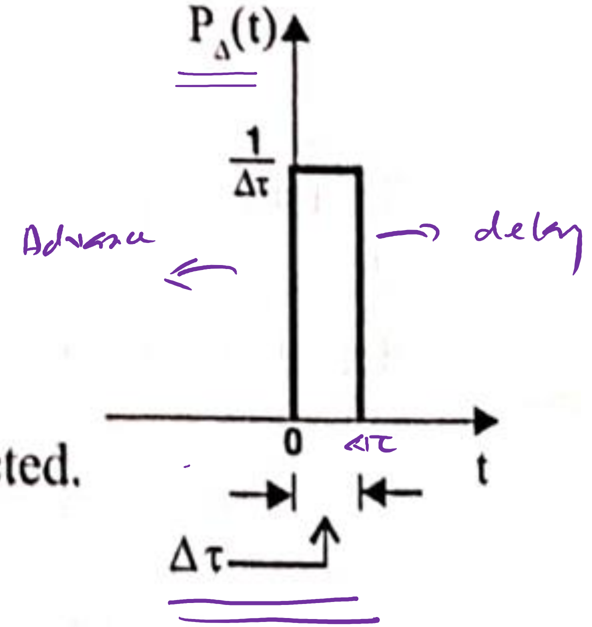
$$= \underset{\Delta\tau \to 0}{\mathrm{Lt}} \; [\ldots\ldots x(-2\Delta\tau) + x(-\Delta\tau) + x(0) + x(\Delta\tau) + x(2\Delta\tau) + \ldots\ldots]$$

Eqn 2

Consider the pulse signal of width $\Delta\tau$ and height $1/\Delta\tau$ as shown in fig. This pulse signal can be expressed as,

$$
\begin{aligned}
P_\Delta(t) &= \frac{1}{\Delta\tau} \quad ; \quad 0 \le t \le \Delta\tau \\
&= 0 \quad ; \quad \text{otherwise}
\end{aligned}
$$

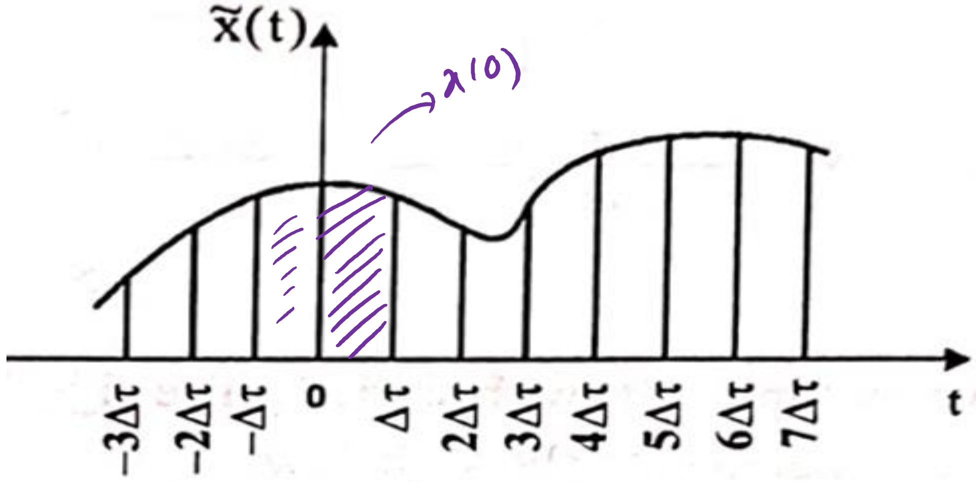
$\frac{1}{\Delta\tau} \times \Delta\tau = 1$



Now, $P_\Delta(t) \times \Delta\tau$ = A pulse of unit amplitude.

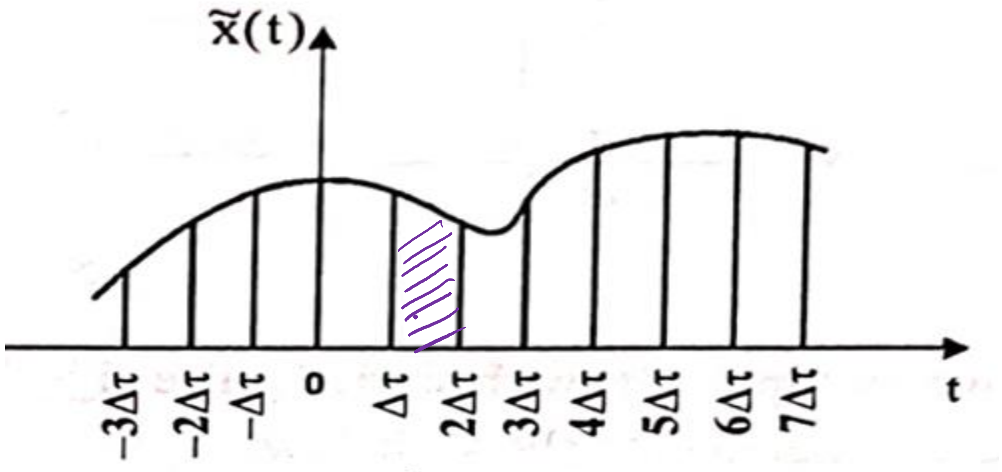
$\therefore$ On multiplying $P_\Delta(t) \times \Delta\tau$ with the signal $\tilde{x}(t)$, the signal $x(0)$ is selected.

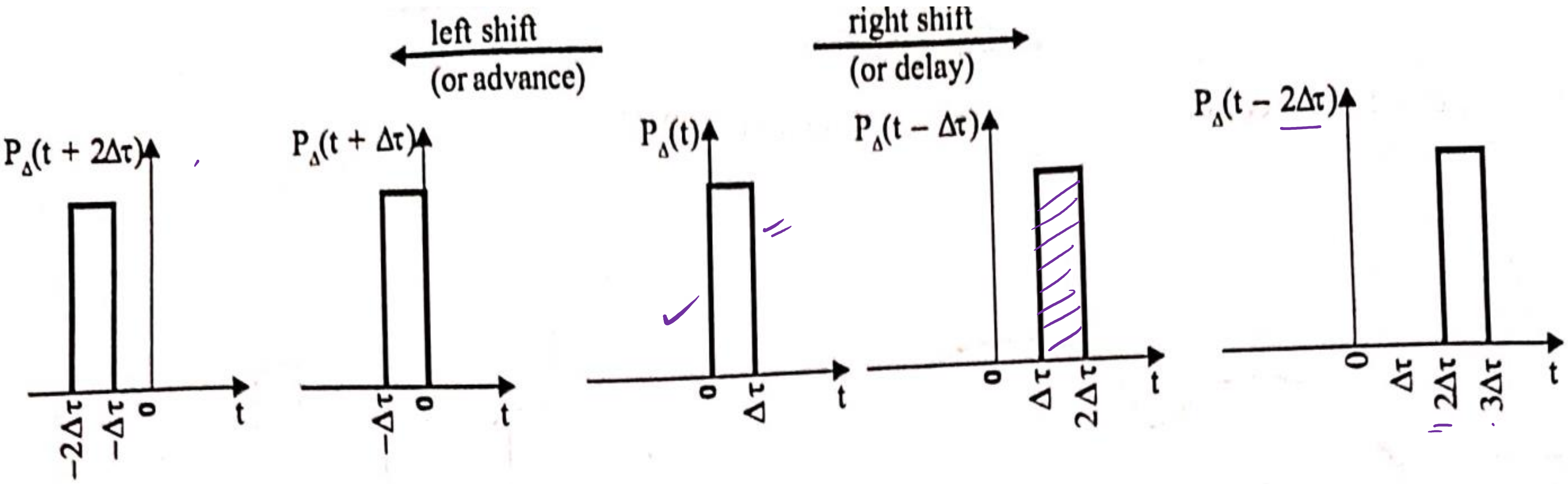
$$\therefore \; x(0) = \tilde{x}(t)\, P_\Delta(t)\, \Delta\tau$$



$p_\Delta(t) \times \Delta\tau \times \tilde{x}(t) = x(0)$

$x(\Delta\tau) = p_\Delta(t-\Delta\tau) \times \Delta\tau \times \tilde{x}(t)$





If we multiply $\tilde{x}(t)$ with shifted pulse signals shown in fig , then each product will select one pulse of the signal $\tilde{x}(t)$ as shown below.

$$\begin{aligned}
&\vdots \\
x(-2\Delta\tau) &= \tilde{x}(t)\ P_\Delta(t+2\Delta\tau)\ \Delta\tau \\
x(-\Delta\tau) &= \tilde{x}(t)\ P_\Delta(t+\Delta\tau)\ \Delta\tau \\
x(0) &= \tilde{x}(t)\ P_\Delta(t)\ \Delta\tau \\
x(\Delta\tau) &= \tilde{x}(t)\ P_\Delta(t-\Delta\tau)\ \Delta\tau \\
x(2\Delta\tau) &= \tilde{x}(t)\ P_\Delta(t-2\Delta\tau)\ \Delta\tau \\
&\vdots
\end{aligned}$$

In the above equation $\tilde{x}(t)$ can be replaced by respective selected pulses itself as shown below,

$$\vdots$$

$$\therefore\ x(-2\Delta\tau) = \tilde{x}(-2\Delta\tau)\ P_{\Delta}(t+2\Delta\tau)\ \Delta\tau$$

$$x(-\Delta\tau) = \tilde{x}(-\Delta\tau)\ P_{\Delta}(t+\Delta\tau)\ \Delta\tau$$

$$x(0) = \tilde{x}(0)\ P_{\Delta}(t)\ \Delta\tau$$

$$x(\Delta\tau) = \tilde{x}(\Delta\tau)\ P_{\Delta}(t-\Delta\tau)\ \Delta\tau$$

$$x(2\Delta\tau) = \tilde{x}(2\Delta\tau)\ P_{\Delta}(t-2\Delta\tau)\ \Delta\tau$$

$$\vdots$$

On substituting the above equations in equation 2 we get,

$$\tilde{x}(t) = \underset{\Delta\tau \to 0}{\mathrm{Lt}} \left[\begin{array}{l} \ldots\ldots \tilde{x}(-2\Delta\tau)\ P_\Delta(t+2\Delta\tau)\,\Delta\tau + \tilde{x}(-\Delta\tau)\ P_\Delta(t+\Delta\tau)\,\Delta\tau + \tilde{x}(0)\ P_\Delta(t)\,\Delta\tau \\ \quad + \tilde{x}(\Delta\tau)\ P_\Delta(t-\Delta\tau)\,\Delta\tau + \tilde{x}(2\Delta\tau)\ P_\Delta(t-2\Delta\tau)\,\Delta\tau + \ldots\ldots\ldots\ldots\ldots \end{array} \right]$$

$$= \underset{\Delta\tau \to 0}{\mathrm{Lt}} \sum_{n=-\infty}^{+\infty} \tilde{x}(n\Delta\tau)\ P_\Delta(t-n\Delta\tau)\,\Delta\tau$$

On applying limit $\Delta\tau \to 0$ the signal $\tilde{x}(n\Delta\tau)$ becomes continuous, the signal $P_\Delta(t-n\Delta\tau)$ becomes an impulse and so the summation becomes integration.

Hence the above equation can be expressed as,

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

$$x(t) = \lim_{\Delta\tau \to 0} \tilde{x}(t)$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$\boxed{x(t) = \int_{-\infty}^{+\infty} x(\tau)\,\delta(t-\tau)\,d\tau}$$