SIGNALS & SYSTEMS

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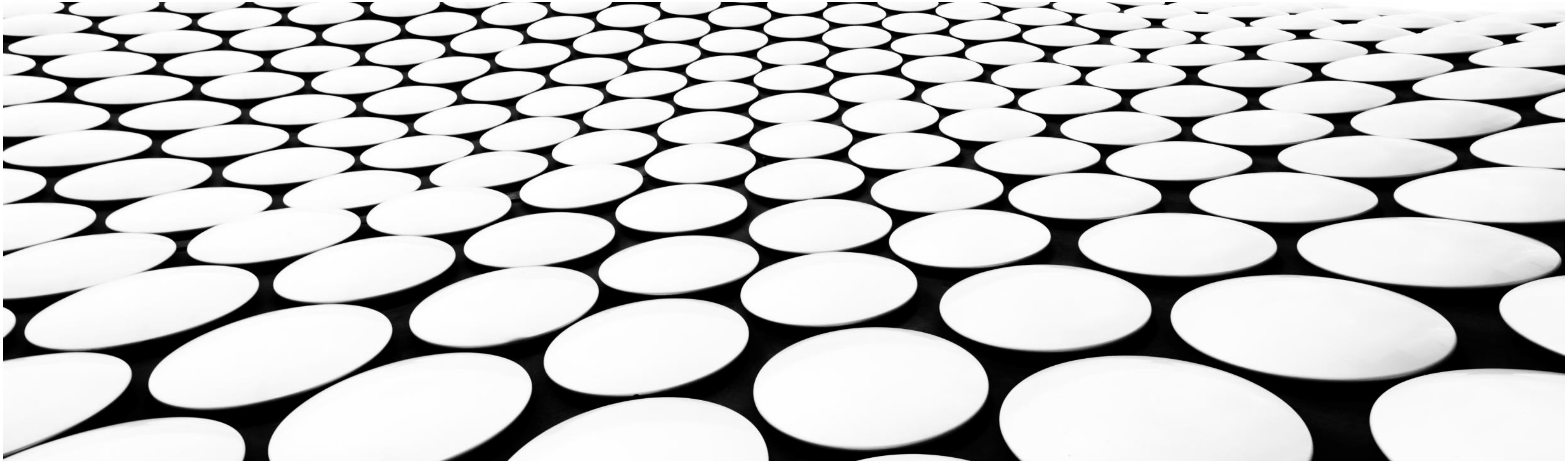

Q. Find c_n for the signal given below:-

$$x(t) = 3 + 2\sin\omega_0 t + \cos\omega_0 t + \cos(2\omega_0 t + \pi/4)$$

Sol^n:- complex-exponential Fourier Expansion of a signal $x(t)$ is given as -

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$\Rightarrow x(t) = -\infty \; c_{-3} e^{-j3\omega_0 t} \cdots + c_{-2} e^{-j2\omega_0 t} + c_{-1} e^{-j1\omega_0 t} + \cancel{c_0 e^{j0\omega_0 t}} \; c_0 e^0 + c_1 e^{j1\omega_0 t} + c_2 e^{j2\omega_0 t} + c_3 e^{j3\omega_0 t} \cdots + \infty$$

— (P) → standard expanded version of $x(t)$

Given signal $x(t) = 3 + 2\sin\omega_0 t + \cos\omega_0 t + \cos(2\omega_0 t + \pi/4)$

From Euler's formula → $\cos\alpha = \dfrac{e^{i\alpha} + e^{-i\alpha}}{2}$, $\sin\alpha = \dfrac{e^{i\alpha} - e^{-i\alpha}}{2j}$

$$x(t) = 3 + \cancel{2} \times \frac{1}{\cancel{2}j}\left[e^{j\omega_0 t} - e^{-j\omega_0 t}\right] + \frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right] + \frac{1}{2}\left[e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}\right]$$

$$= 3 + \frac{1}{j}e^{j\omega_0 t} - \frac{1}{j}e^{-j\omega_0 t} + \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t} + \frac{1}{2}e^{j2\omega_0 t} \cdot e^{j\pi/4} + \frac{1}{2}e^{-j2\omega_0 t} \cdot e^{-j\pi/4}$$

$$a^m \cdot a^n = a^{m+n}$$

$$e^{j\pi/4} = \cos\pi/4 + j\sin\pi/4 = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} = \frac{1+j}{\sqrt{2}}$$

$$e^{-j\pi/4} = \frac{1-j}{\sqrt{2}}$$

$$= 3 + \left(\frac{1}{j} + \frac{1}{2}\right)e^{j\omega_0 t} + \left(-\frac{1}{j} + \frac{1}{2}\right)e^{-j\omega_0 t} + \frac{1}{2}e^{j2\omega_0 t} \cdot \frac{(1+j)}{\sqrt{2}} + \frac{1}{2}e^{-j2\omega_0 t} \cdot \frac{(1-j)}{\sqrt{2}}$$

$$x(t) = 3e^0 + \left(\frac{1}{2} - j\right)e^{j\omega_0 t} + \left(\frac{1}{2} + j\right)e^{-j\omega_0 t} + \frac{1+j}{2\sqrt{2}}e^{j2\omega_0 t} + \frac{1-j}{2\sqrt{2}}e^{-j2\omega_0 t} \quad \text{—} \quad (Q)$$

$$\frac{1}{j} = \frac{1}{j} \times \frac{j}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$$

Comparing eqn (P) and (Q) $\Rightarrow$ $c_0 = 3$, $c_1 = \frac{1}{2} - j$, $c_2 = \frac{1+j}{2\sqrt{2}}$

$c_{-1} = \frac{1}{2} + j$, $c_{-2} = \frac{1-j}{2\sqrt{2}}$ #

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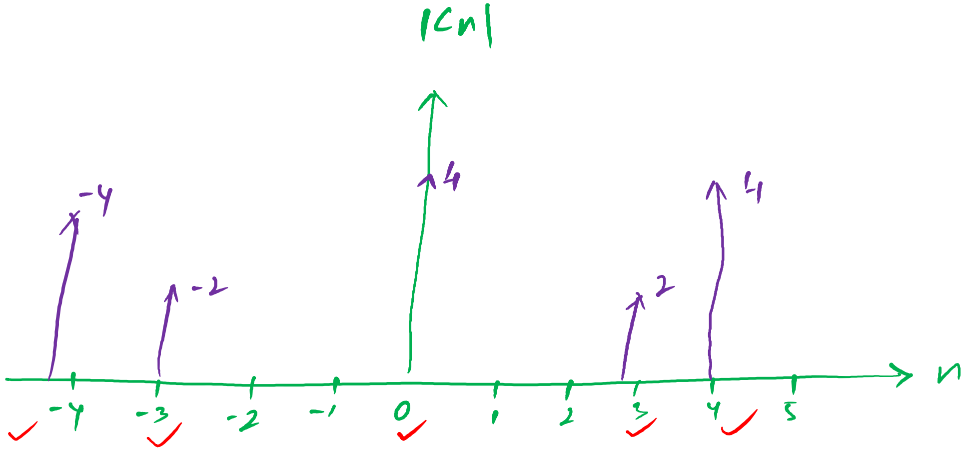
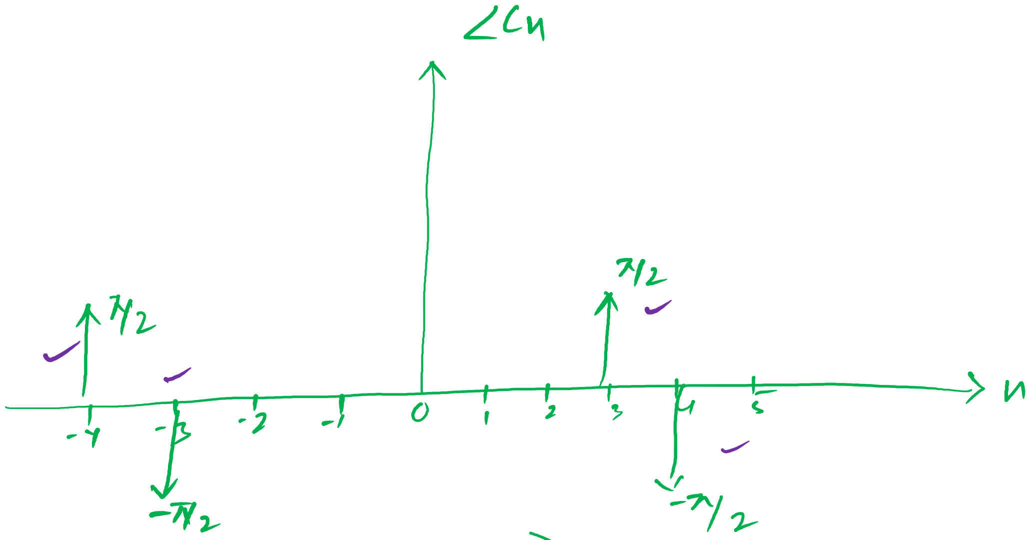
Q. Find $x(t)$ from the given figures:-



→ magnitude plot of cn



→ phase plot of Cn

Sol^n:- C_n → complex exponential fourier coefficient

$$\boxed{c_n = |c_n| e^{j\angle c_n}} \Rightarrow c_n = |c_n| [\cos\angle c_n + j \sin\angle c_n]$$

$$c_0 = 4$$

$$c_3 = 2e^{j\pi/2}$$

$$c_4 = 4e^{-j\pi/2}$$

$$c_{-3} = 2e^{-j\pi/2}$$

$$c_{-4} = 4e^{j\pi/2}$$

$$c_n = |c_n| e^{j\angle c_n}$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$= c_0 e^{0} + c_3 e^{j3\omega_0 t} + c_4 e^{j4\omega_0 t} + c_{-3} e^{-j3\omega_0 t} + c_{-4} e^{-j4\omega_0 t}$$

$$= 4 + 2e^{j\pi/2} \cdot e^{j3\omega_0 t} + 4e^{-j\pi/2} e^{j4\omega_0 t} + 2e^{-j\pi/2} \cdot e^{-j3\omega_0 t} + 4e^{j\pi/2} e^{-j4\omega_0 t}$$

$$= 4 + 2e^{j(\pi/2 + 3\omega_0 t)} + 4e^{j(4\omega_0 t - \pi/2)} + 2e^{-j(3\omega_0 t + \pi/2)} + 4e^{-j(4\omega_0 t - \pi/2)}$$

$$= 4 + 2\left[e^{j(3\omega_0 t + \pi/2)} + e^{-j(3\omega_0 t + \pi/2)}\right] + 4\left[e^{j(4\omega_0 t - \pi/2)} + e^{-j(4\omega_0 t - \pi/2)}\right]$$

Euler's Identity, $\dfrac{e^{i\theta}+e^{-i\theta}}{2} = \cos\theta$, $\dfrac{e^{i\theta}-e^{i\theta}}{2j} = \sin\theta$

$\Rightarrow e^{i\theta}+e^{-i\theta} = 2\cos\theta$

$\therefore x(t) = 4 + 2\times 2\cos(3\omega_0 t + \pi/2) + 4\times 2\cos(4\omega_0 t - \pi/2)$

$\Rightarrow x(t) = 4 + 4\cos(3\omega_0 t + \pi/2) + 8\cos(4\omega_0 t - \pi/2)$.

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Q. The signal $x(t)$ has the period equal to 1 and the following fourier coefficients given as

$$C_n = \begin{cases} (-1/3)^n & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$$

Determine the signal $x(t)$.

Soln:-

$T_0 = 1 \rightarrow$ fundamental period

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{1} \Rightarrow \boxed{\omega_0 = 2\pi}$$

$$\Rightarrow x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn2\pi t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn2\pi t}$$

$$= \cancelto{0}{\sum_{n=-\infty}^{0} \underbrace{c_n}_{\text{zero}} e^{jn2\pi t}} + \sum_{n=0}^{\infty} c_n e^{jn2\pi t}$$

$$= \sum_{n=0}^{\infty} c_n e^{jn2\pi t}$$

$$= \sum_{n=0}^{\infty} \left(-\frac{1}{3}\right)^n e^{jn2\pi t}$$

$$= \sum_{n=0}^{\infty} \left(-\frac{1}{3} e^{j2\pi t}\right)^n$$

$$x(t) = 1 + \left(-\frac{1}{3}\right)e^{j2\pi t} + \left(-\frac{1}{3}e^{j2\pi t}\right)^2 + \left(-\frac{1}{3}e^{j2\pi t}\right)^3 + \cdots\cdots + \infty$$

This $x(t)$ is a geometric progression. with first term $a = 1$

and with common ratio, $r = -\frac{1}{3}e^{j2\pi t}$

$$S_n = \frac{a(1-r^n)}{1-r} \qquad \boxed{-1 < r < 1} \qquad (\text{fraction})^{\infty} = 0$$

$$\therefore S_\infty = \frac{a}{1-r}$$

$$x(t) = \frac{1}{1-\left(-\frac{1}{3}e^{j2\pi t}\right)} = \frac{1}{1+\frac{1}{3}e^{j2\pi t}}.$$

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