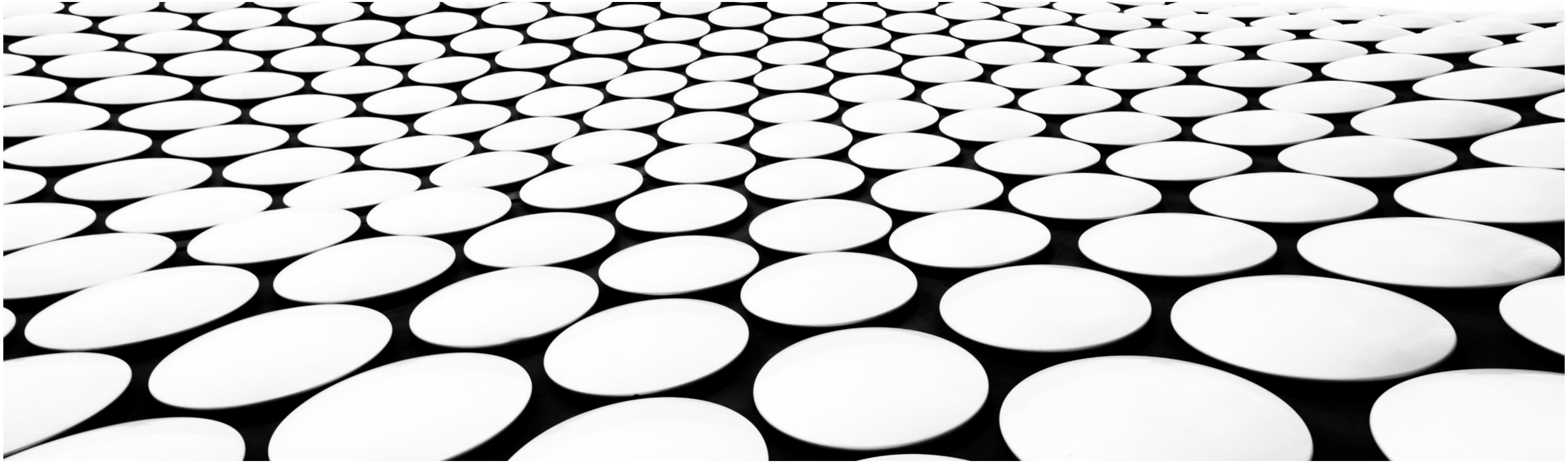
SIGNALS & SYSTEMS

MR. ANKUR JYOTI SARMAH

ASSISTANT PROF., DEPT. OF ELECTRONICS & TELECOM. ENGG.

ASSAM ENGINEERING COLLEGE



Fourier series Representation of any periodic signals : $x(t)$

Expansion of $x(t)$ in terms of various harmonics.

$$x(t) = \text{DC Term} + \sum \text{cosine harmonics} + \sum \text{sine harmonics}$$

$$\Rightarrow x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

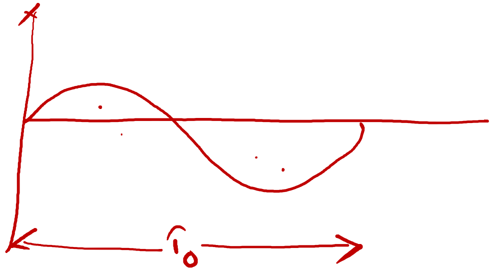
Fourier coeff.

a_0 = dc term or av. value term = Area of a signal over the fundamental time period

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$

$$a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega_0 t\,dt$$

$$b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega_0 t\,dt$$



$$\frac{\int \sin\omega_0 t\,dt}{T/2} = \frac{2}{T_0}\int \sin\omega_0 t\,dt$$

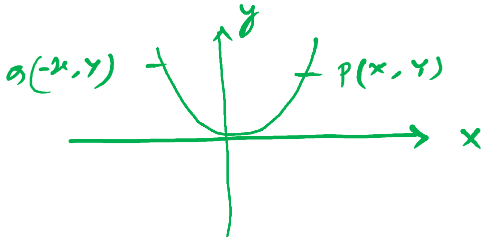
$$x(t) = a_0 + \underbrace{\sum_{n=1}^{\infty} a_n \cos n\omega_0 t}_{\text{even}} + \underbrace{\sum_{n=1}^{\infty} b_n \sin n\omega_0 t}_{\text{odd}}$$

$x(t) \rightarrow$ symmetrical about x-axis $\Rightarrow a_0 = 0$

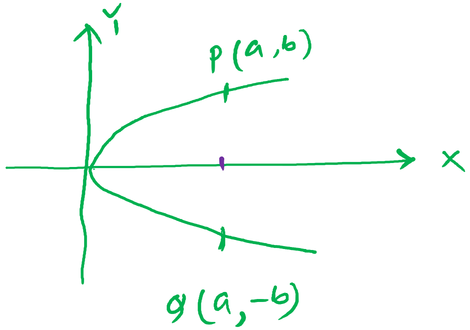
$\rightarrow$ even $\Rightarrow b_n = 0$

$\rightarrow$ odd $\Rightarrow a_n = 0$

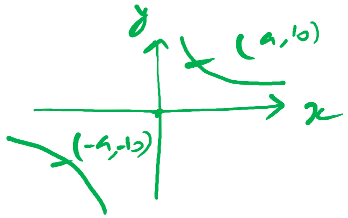
Symmetrical about Y-axis



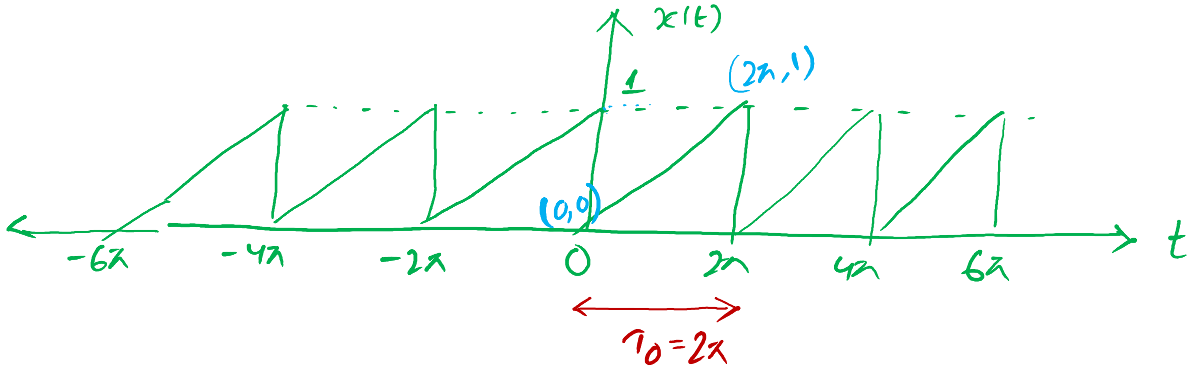
Symmetrical about x-axis:-



Symmetrical about origin



Q. Expand the following signal with the help of fourier series expansion —



Here, fundamental period $= T_0$

$$\therefore \frac{2\pi}{\omega_0} = 2\pi$$

$$\Rightarrow \omega_0 = 1 \text{ rad/sec.}$$

Solⁿ:-

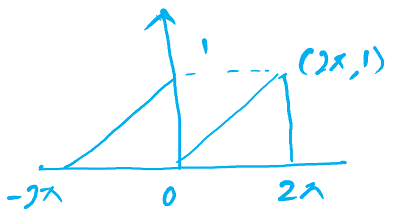
$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$y = mx + c$$

$$\Rightarrow y = mx$$

$$\Rightarrow x(t) = \frac{1-0}{2\pi - 0} t$$

$$\Rightarrow x(t) = \frac{t}{2\pi}$$, define the signal for fundamental time period.



$(2\pi, -1)$

Not symmetrical about x-axis $\Rightarrow a_0 \neq 0$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$

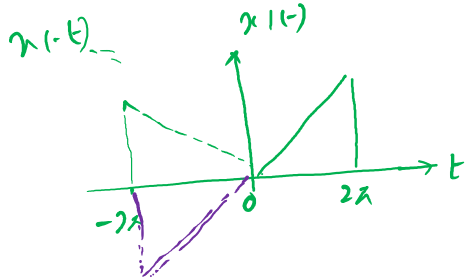
$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{t}{2\pi}\,dt$$

$$= \frac{1}{4\pi^2}\int_0^{2\pi} t\,dt$$

$$= \frac{1}{4\pi^2}\left[\frac{t^2}{2}\right]_0^{2\pi}$$

$$= \cancel{\frac{1}{4\pi^2}} \times \frac{1}{2} \times \cancel{4\pi^2} \qquad \therefore \boxed{a_0 = \frac{1}{2}}$$

$\underline{x(t) = x(-t)}$ $\Rightarrow$ even



$x(t) = -x(-t)$

The given is actually neither odd nor even

$a_n \neq 0$ and

$b_n \neq 0$

$$a_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cos n\omega_0 t \, dt$$

$$= \frac{\cancel{2}}{\cancel{2}\pi}\int_0^{2\pi} \frac{t}{2\pi}\cos n\cdot 1\cdot t \, dt$$

$$= \frac{1}{2\pi^2}\int_0^{2\pi} t\cdot\cos nt \, dt$$

$$= \frac{1}{2\pi^2}\left[2\pi\cdot\cancel{\frac{\overset{0}{\sin n2\pi}}{n}} + \frac{\overset{1}{\cancel{\cos n2\pi}}}{n^2} - \cancel{0\cdot\frac{\overset{0}{\sin 0}}{n}} - \frac{\overset{1}{\cancel{\cos 0}}}{n^2}\right]$$

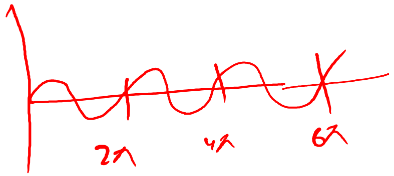
$$a_n = \frac{1}{2\pi^2}\left[\frac{1}{n^2} - \frac{1}{n^2}\right] = 0.$$

I → Inv. Trig.
L → log.
A → Arithmetic
T → Trigo
E → Exp.

$$\int uv = u\int v - \int\left[du\int v\, dv\right]$$

$u \to t$

$v \to \cos nt$

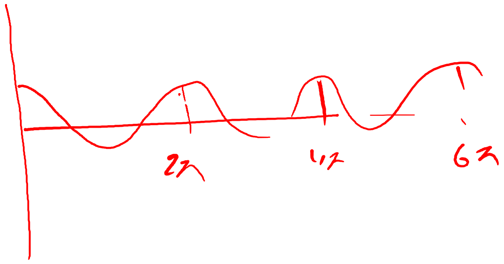


$$\int t\cos nt \, dt$$

$$= t\int \cos nt \, dt - \int\left[\frac{\overset{1}{\cancel{dt}}}{\cancel{dt}}\int \cos nt\, dt\right] dt$$

$$= t\,\frac{\sin nt}{n} - \int \frac{\sin nt}{n}\, dt$$

$$= \boxed{t\cdot\frac{\sin nt}{n} + \frac{\cos nt}{n^2}}$$



$$b_n = \frac{2}{T_0}\int_0^{T_0} x(t)\sin n\omega_0 t\,dt$$

$$= \frac{\cancel{2}}{2\pi}\int_0^{2\pi} \frac{t}{2\pi}\sin nt\,dt$$

$$= \frac{1}{2\pi^2}\int_0^{2\pi} t\sin nt\,dt$$

$$b_n = -\frac{1}{n\pi}.$$

$$\int t\sin nt\,dt$$

$$= t\,\frac{-\cos nt}{n} - \int\left(\int \sin nt\,dt\right)dt$$

$$= \frac{-t\cos nt}{n} + \int\frac{\cos nt\,dt}{n}$$

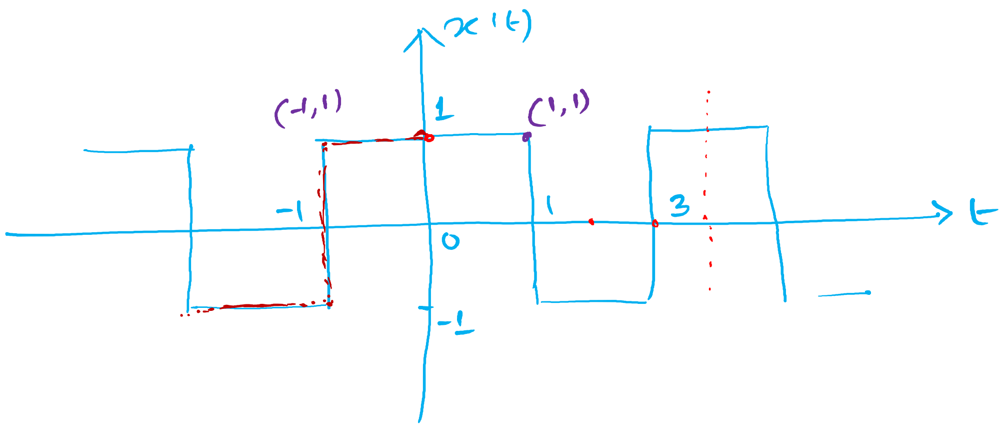
$$= \left[\frac{-t\cos nt}{n} + \frac{\sin nt}{n^2}\right]_0^{2\pi}$$

$$\therefore \text{F.S of } x(t) = \frac{1}{2} + \sum_{n=1}^{\infty} a_n\cos n\omega_0 t + \sum_{n=1}^{\infty} -\frac{1}{n\pi}\sin nt$$

$$= \frac{1}{2} - \sum_{n=1}^{\infty}\frac{1}{n\pi}\sin nt$$

$$x(t) = \frac{1}{2} - \frac{1}{\pi}\sin t - \frac{1}{2\pi}\sin 2t - \frac{1}{3\pi}\sin 3t \cdots\cdots$$

2.



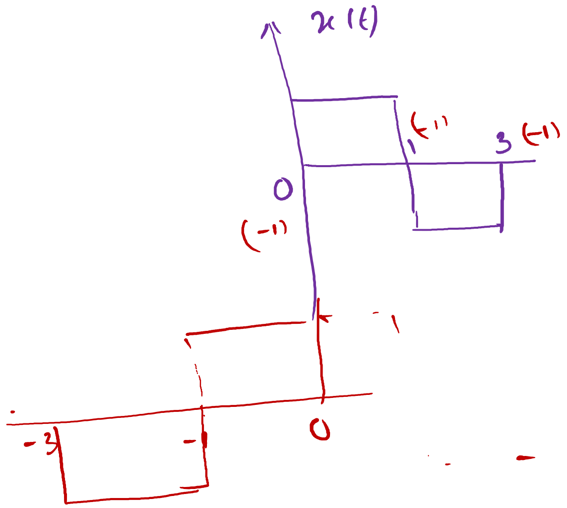
Sol^n:-

-1 to $3 \Rightarrow T_0 = 4$ sec.

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ rad/sec}$$

$$x(t) = 1, \text{ for } -1 < t < 1$$

$$= -1 \text{ for } 1 < t < 3$$



$b_n = 0$

$a_n \neq 0$

$$a_0 = \frac{1}{T_0}\int x(t)\,dt$$

$$= \frac{1}{T_0}\int_{-1}^{1} 1\,dt + \frac{1}{T_0}\int_{1}^{3} (-1)\,dt$$

$$= \frac{1}{T_0}\left[t\right]_{-1}^{1} - \frac{1}{T_0}\left[t\right]_{1}^{3}$$

$$= \frac{1}{T_0}\left[1+1\right] - \frac{1}{T_0}\left[3-1\right]$$

$$= \frac{1}{T_0}\left[2-2\right] = 0$$

$$a_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cos n\omega_0 t \, dt$$

$$= \frac{2}{4}\int_{-1}^{3} x(t)\cos\left(n\frac{\pi}{2}t\right)dt$$

$$= \frac{1}{2}\int_{-1}^{3} x(t)\cos n\frac{\pi}{2}t \, dt$$

$$a_n = \int_{-1}^{1} \cos\left(n\frac{\pi}{2}t\right)dt - \int_{1}^{3}\cos\left(n\frac{\pi}{2}t\right)dt$$

$$= \left[\frac{\sin n\frac{\pi}{2}t}{n\frac{\pi}{2}}\right]_{-1}^{1} - \left[\frac{\sin n\frac{\pi}{2}t}{n\frac{\pi}{2}}\right]_{1}^{3}$$

$$= \frac{2}{n\pi}\left[\sin n\frac{\pi}{2}t\right]_{-1}^{1} - \frac{2}{n\pi}\left[\sin n\frac{\pi}{2}t\right]_{1}^{3}$$

$$= \frac{2}{n\pi}\left[\sin n\frac{\pi}{2} - \sin\left(n\frac{\pi}{2}(-1)\right)\right] - \frac{2}{n\pi}\left[\sin n\frac{\pi}{2}\cdot 3 - \sin n\frac{\pi}{2}\right]$$

$$= \frac{2}{n\pi}\left[\sin n\frac{\pi}{2} + \sin n\frac{\pi}{2}\right] - \frac{2}{n\pi}\left[\sin 3n\frac{\pi}{2} - \sin n\frac{\pi}{2}\right]$$

$$= \frac{2}{n\pi}\left[\sin n\frac{\pi}{2} + \sin n\frac{\pi}{2} - \sin 3n\frac{\pi}{2} + \sin n\frac{\pi}{2}\right]$$

$$a_n = \frac{2}{n\pi}\left[\sin n\frac{\pi}{2} + \cancel{\sin\frac{n\pi}{2}} - \cancel{\sin\frac{3n\pi}{2}} + \sin\frac{n\pi}{2}\right]$$

$\sin n\pi/2$

$$\Rightarrow a_n = \frac{2}{n\pi}\left[2\sin n\pi/2\right]$$

$$= \frac{\cancel{2}}{\cancel{2}\times\pi}\left[2\sin(2\pi/2)\right]$$

$$= \frac{2}{\cancel{4}}\frac{1}{\pi}\left[2\sin\pi\right]$$

$$= 0$$

$a_n = 0$, when n = even

when n = even

$$\sin\frac{3n\pi}{2} = \sin\left(n\pi + \frac{n\pi}{2}\right)$$

$$\Rightarrow \sin\frac{3n\pi}{2} = \sin\frac{n\pi}{2}$$

when n = odd