Assignment - 3

Time domain analysis of DT-LTI system

* Compute and plot the convolution $y(n) = x(n) * h(n)$ where $x(n) = \begin{cases} 1, & 3 \leq n \leq 8 \\ 0, & \text{otherwise} \end{cases}$ & $h(n) = \begin{cases} 1, & 4 \leq n \leq 15 \\ 0, & \text{otherwise} \end{cases}$

→ $x(n) = \{\underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1, 1, 1\}$

$h(n) = \{\underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$

using tabular method,

	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1

$y(n) = \{\underset{\uparrow}{0}, 0, 0, 0, 0, 0, 1, 2, 3, 4, 5, 6, 6, 6, 6, 6, 6, 6, 5, 4, 3, 2, 1\}$

* Compute the convolution $y(n) = x(n) * h(n)$

i) $x(n) = \delta(n) - \delta(n-2)$, $h(n) = u(n)$

$y(n) = (\delta(n) - \delta(n-2)) * u(n)$

$= \delta(n) * u(n) - \delta(n-2) * u(n)$

$= u(n) - u(n-2)$

$= \{\underset{\uparrow}{1}, 1\}$

ii) $x(n) = u(n)$, $h(n) = u(n)$

$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$

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$$y(n) = \sum_{k=-\infty}^{\infty} u(k)\, u(-k+n)$$

due to $u(k)\,u(-k+n)$ the limits will be 0 to n.

$$y(n) = \sum_{k=-\infty}^{\infty} 1 = n+1$$

* Define : Convolution sum.

Show that
i) $x(n) * \delta(n) = x(n)$
ii) $x(n) * \delta(n-n_0) = x(n-n_0)$
iii) $x(n) * u(n-n_0) = \sum_{k=-\infty}^{n-n_0} x(k)$

→ Proof of LTI system is completely characterized by unit impulse response $h(n)$.

- step-1 : $\delta(n) \xrightarrow{T} y(n) = h(n)$

step-2 : $\delta(n-k) \xrightarrow{T} y(n) = h(n-k)$

step-3 : $x(k)\,\delta(n-k) \xrightarrow{T} y(n) = x(k)\,h(n-k)$

step-4 : $\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) \xrightarrow{T} y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$

$$\therefore x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

i) $x(n) * \delta(n) = x(n)$

Since $\delta(n) = 1$ for $n=0$, the convolution of $x(n)$ with $\delta(n)$ produces the same sequence.

ii) $x(n) * \delta(n-n_0) = x(n-n_0)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

assume $x(n) = \{1, \underset{\uparrow}{1}, 1\}$ & $h(n) = \delta(n-n_0) = \{0, \underset{\uparrow}{1}\}$

$$\therefore y(n) = \{0, \underset{\uparrow}{1}, 1, 1\} = x(k-1)$$

$$\therefore y(n) = x(n) * \delta(n-n_0) = x(n-n_0)$$

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iii) $x(n) * u(n-n_0) = \sum_{k=-\infty}^{n-n_0} x(k)$

$u(n-k-n_0) = \begin{cases} 1, & n-k-n_0 > 0 \text{ or } k \leq n-n_0) \\ 0, & \text{otherwise} \end{cases}$

$\therefore x(n) * u(n-n_0) = \sum_{K=-\infty}^{n-n_0} x(k).$

* Define the condition for LTI system to be stable. Which of the following impulse response correspond to stable LTI system ? $h(n) = 3^n u(-n+10)$

→ According to condition of stability.

$$\sum_{K=-\infty}^{\infty} |h(k)| < \infty$$

$$h(k) = 3^K u(-k+10)$$

$$\sum_{K=-\infty}^{\infty} 3^K = \sum_{K=-\infty}^{-1} 3^K + \sum_{k=0}^{10} 3^K$$

$$= \sum_{K=1}^{\infty} \left(\frac{1}{3}\right)^K + \sum_{K=0}^{10} 3^K$$

$$= \frac{\left(\frac{1}{3}\right)^2}{1-\frac{1}{3}} + \frac{3^0 - 3^{11}}{1-3}$$

$$= 0.5 + 0.88753 < \infty$$

So, system is stable.

* Evalute the discrete time convolution sum given below.

$$y(n) = \beta^n u(n) * u(n-3) \quad ; |\beta| < 1$$

$$\therefore x(n) = u(n-3) \quad \& \; h(n) = \beta^n u(n)$$

$$y(k) = \sum_{K=-\infty}^{\infty} x(k) h(n-k)$$

$$= \sum_{K=-\infty}^{\infty} u(k-3) \beta^n \left(\frac{1}{\beta}\right)^k u(-k-n)$$

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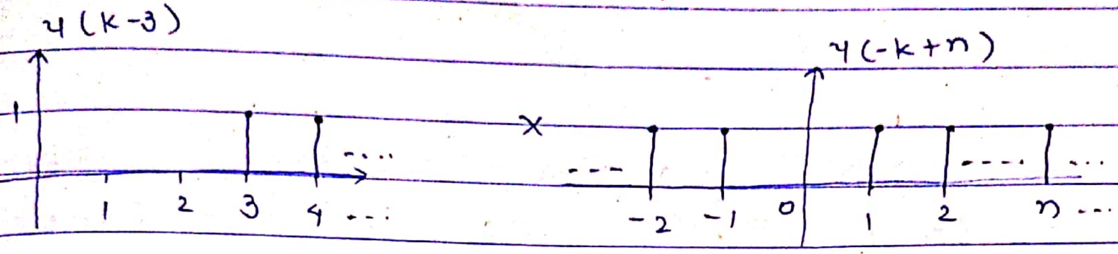
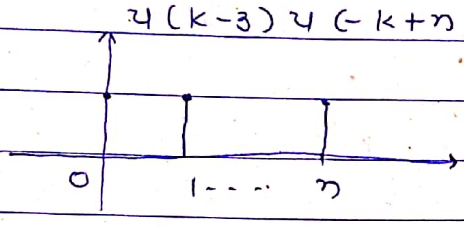
$$= \beta^n \sum_{k=-\infty}^{\infty} \left(\frac{1}{\beta}\right)^k u(k-3)\, u(-k+n)$$



$\Rightarrow$



$$\therefore\ y(k) = \beta^n \sum_{k=0}^{n} \left(\frac{1}{\beta}\right)^k$$

$$= \beta^n \left[\frac{(1/\beta)^0 - (1/\beta)^{n+1}}{1 - 1/\beta}\right]$$

$$= \beta^n \left[\frac{1 - (1/\beta)^{n+1}}{1 - 1/\beta}\right]$$

* If convolution is performed betⁿ two signals, x & h, with length N_x & N_h, then what will be the length N of resulting signal?

→ The length of resulting signal will be $N_x + N_h - 1$.

* Define invertible system.

→ A system is invertible if input of system can be recovered from the system output.

* Let $x(n) = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & \text{otherwise} \end{cases}$ & $h(n) = \begin{cases} 1, & 0 \le n \le N \\ 0, & \text{otherwise} \end{cases}$ where $n \le 9$ is an integer. Determine the value of N, given that $y(n) = x(n) * h(n)$ and $y(4) = 5$, $y(14) = 0$

→ from given information,

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$$N_x + N_h - 1 = 14$$
$$10 + (N+1) - 1 = 14$$
$$N = 4$$

* The output $y(n)$ of a discrete time LTI system is found to be $2(1/3)^n u(n)$ when the input $x(n)$ is $u(n)$. Find the impulse response $h(n)$ of the system.

→ $y(n) = 2\left(\frac{1}{3}\right)^n u(n)$ & $x(n) = u(n)$

$$Y(z) = 2 \cdot \frac{z}{z - 1/3} \quad \& \quad X(z) = \frac{z}{z-1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2z}{z - 1/3} \times \frac{z-1}{z} = \frac{2(z-1)}{z - 1/3}$$

$$H(z) = 2\frac{z}{z - \frac{1}{3}} - \frac{z}{z - 1/3}$$

$$H(n) = 2\left(\frac{1}{3}\right)^n u(n) - 2\left(\frac{1}{3}\right)^{n-1} u(n-1)$$

* Compute convolution for the following :-

$x(n) = \left(\frac{1}{5}\right)^n u(n)$, $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{5}\right)^k u(k) \left(\frac{1}{2}\right)^{n-k} u(-k+n)$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{5}\right)^k u(k) \left(\frac{1}{2}\right)^n 2^k\, u(-k+n)$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=-\infty}^{\infty} \left(\frac{2}{5}\right)^k u(k)\, u(-k+n)$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(\frac{2}{5}\right)^k$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{(2/5)^0 - (2/5)^{n+1}}{1 - 2/5} \right]$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1 - (2/5)^{n+1}}{3/5} \right]$$

* $x(n)$ & $h(n)$ are finite length discrete time signals with length of 32 & 21 respectively. If $y(n)$ is convolution of $x(n)$ & $h(n)$. Find length of $y(n)$.

→ $L = 32$, $M = 21$

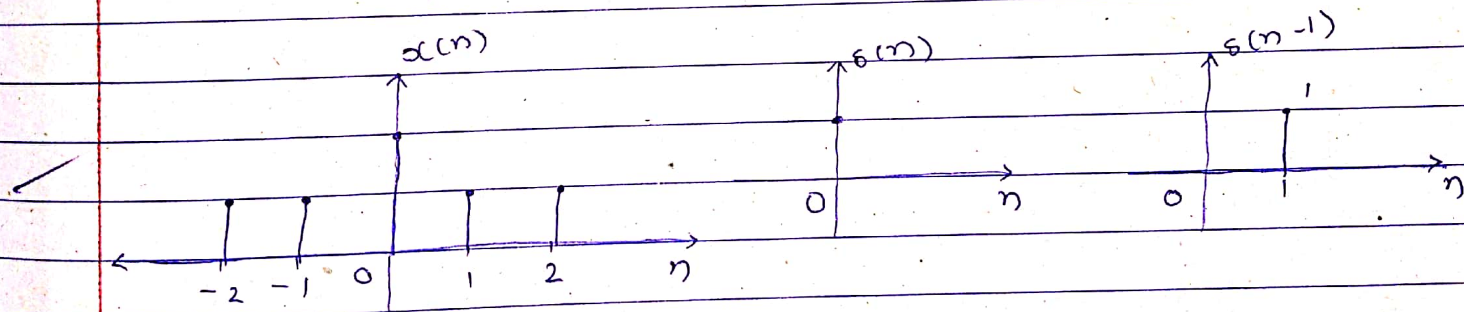
length of linear convolution is,

$N = L + M - 1 = 32 + 21 - 1 = 52$.

* Evolute the convolution $x(n) * \delta(n - n_0)$.

→ $x(n) = \{1, 1, \underset{\uparrow}{2}, 1, 1\}$, $\delta(n) = 1$, $n = 0$

$= 0$, otherwise



$\delta(n - n_0) = 1$, $n = 0$

$= 0$, $n \neq 0$

$\delta(n-1) = 1$, $n = 1$

$= 0$, $n = n_0$

$\therefore x(n)\,\delta(n-1) = x(1)$

$\therefore x(n)\,\delta(n - n_0) = x(n_0)$

* Prove that convolution is a commutative operation.

State: It states that linear convolution is commutative operation thatmeans,

$$x(n) * h(n) = h(n) * x(n)$$

proof: The linear convolution of $x(n)$ & $h(n)$ is given by,

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$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \quad \text{——} \ (1)$$

$$m = n-k \Rightarrow k = n-m$$

$$y(n) = \sum_{m=-\infty}^{\infty} x(n-m)\, h(m)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k)\, h(k) \qquad (\because m \text{ replace by } k)$$

$$y(n) = h(n) * x(n) \quad \text{——} \ (2)$$

$\therefore$ By (1) & (2)

$$x(n) * h(n) = h(n) * x(n)$$

- Graphical presentation,