SIGNALS & SYSTEMS

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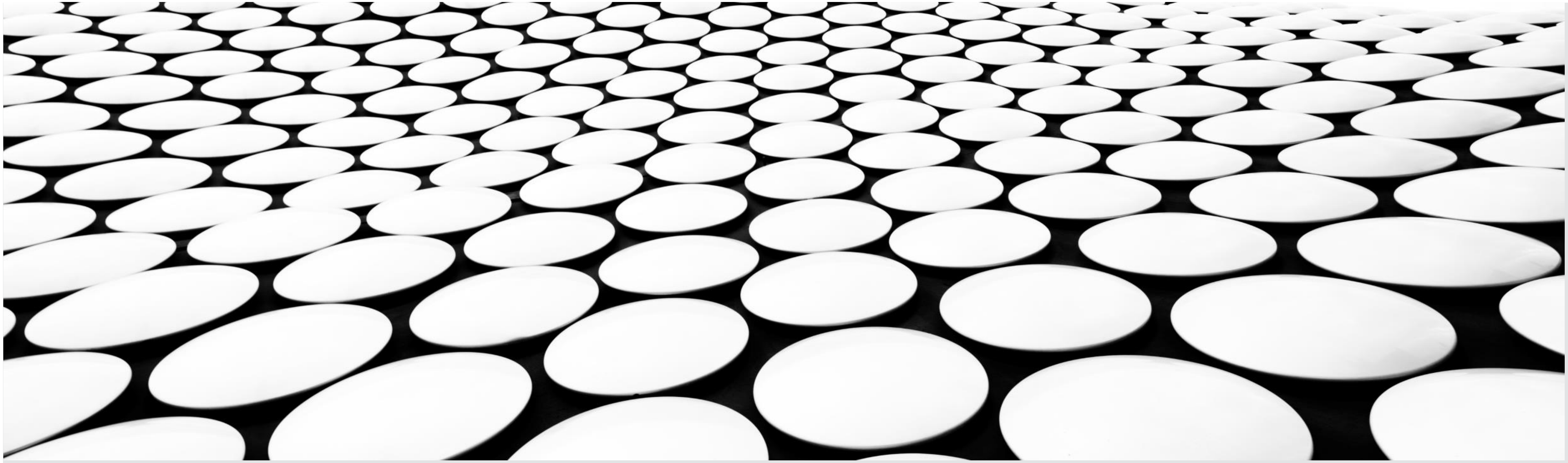
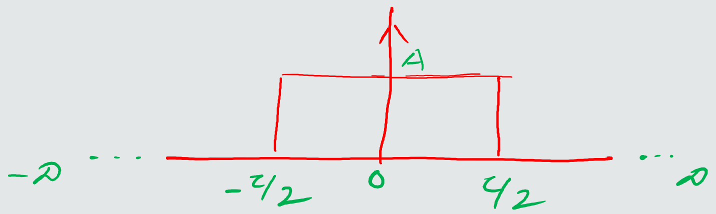


÷ Fourier Transform of Rectangular function:-

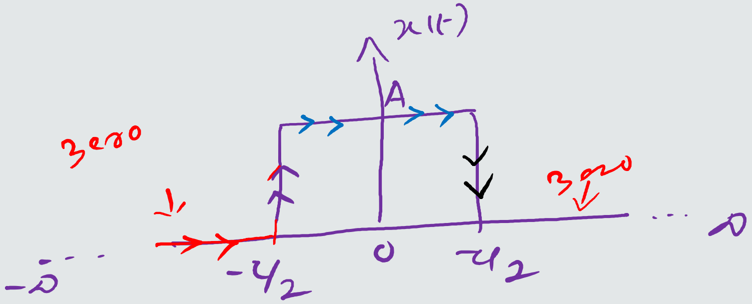
Rectangular function is given as $x(t) = A\,\mathrm{rect}(t/\tau)$

period for a pulse.

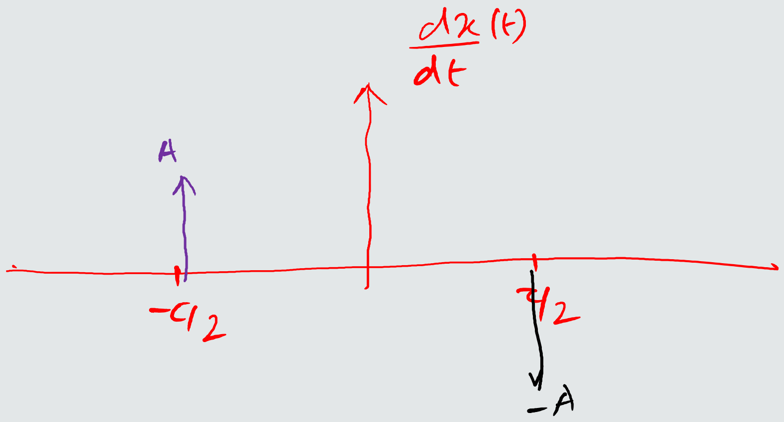


⇒ From the waveform of rectangular pulse it is clear that it is a combination of step signals. So we know that if we perform differentiation to obtain impulse then fourier transform can be easily found out.

⇒ Differentiation must perform from left to Right side i.e, from $-\infty$ to $+\infty$



Differentiation



From $-\infty$ to $-\tau/2$ $\rightarrow$ No o/p (zero)

At $t=-\tau/2$ A transition takes place.

To calculate the magnitude of transition
$(A-0)=A$.

From $t=-\tau/2$ to $t=\tau/2$, constant value so on differentiation we get zero.

At $t=\tau/2$, A downward transition takes place

$\therefore$ Magnitude $0-A=-A$

Here dc value

$\frac{A-A}{2}=0$

$$\therefore \frac{dx(t)}{dt} = A\,\delta(t+\tau/2) - A\,\delta(t-\tau/2)$$

$$\Rightarrow \frac{dx(t)}{dt} = A\left[\delta(t+\tau/2) - \delta(t-\tau/2)\right]$$

$$\Rightarrow FT\left[\frac{dx(t)}{dt}\right] = FT\left[A\,\delta(t+\tau/2) - \delta(t-\tau/2)\right]$$

$$\Rightarrow (j\omega)X(\omega) = A\left[e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right]$$

$$\Rightarrow X(\omega) = \frac{A\left(e^{j\omega\tau/2} - e^{-j\omega\tau/2}\right)}{j\omega}$$

$$= \frac{A}{\omega}\left[\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{j}\right]$$

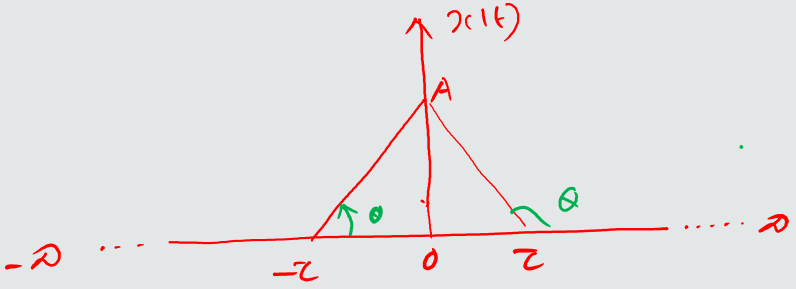
$$\boxed{X(\omega) = \frac{A}{\omega}2\sin\omega\tau/2}$$

property of time shifting

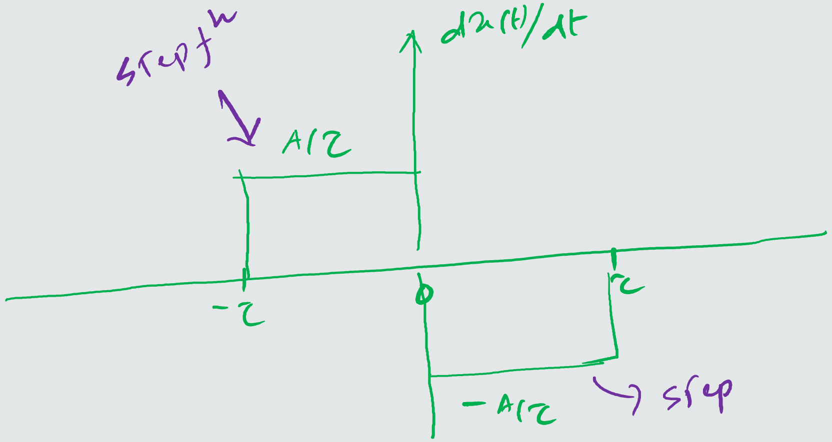
$$x(t \pm t_0) \rightleftharpoons X(\omega)e^{\pm j\omega t_0}$$

Fourier transform for a triangular pulse:-

Ramp $\xrightarrow{\text{Diff}}$ step $\xrightarrow{\text{Diff.}}$ Impulse



$\Downarrow$ Diff.



⇒ from the figure it is clear that triangular pulse function is cons^n of ramp function. so we need to perform double differentiation to get impulse function.

from $-\infty$ to $-\tau$ → No value i.e, zero.

from $-\tau$ to 0 → +ve slope since θ is acute.

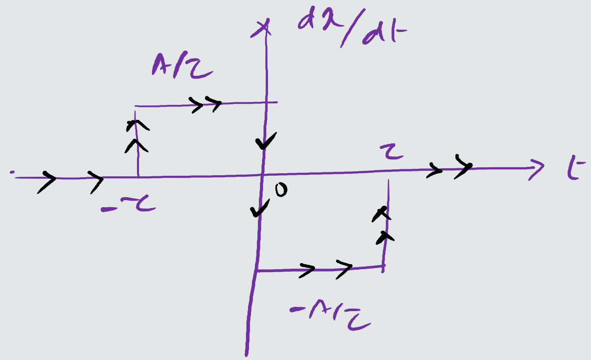
$$\Downarrow$$

$$\frac{\perp r}{\text{base}} = \frac{A-0}{0-(-\tau)} = \frac{A}{\tau}$$

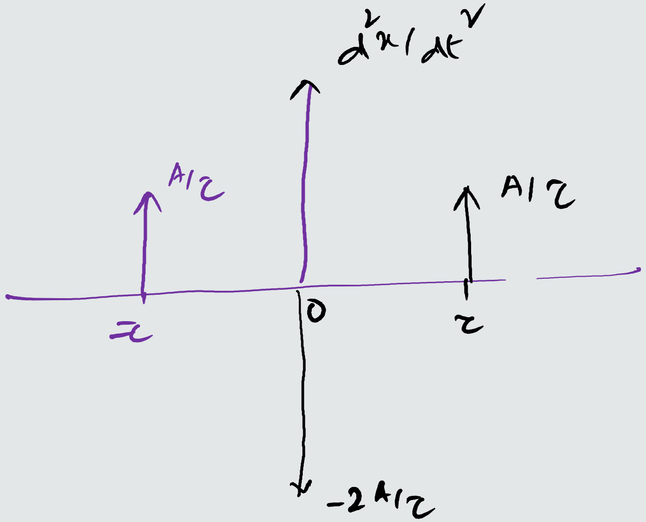
from 0 to τ → -ve slope since θ is obtuse

$$\Downarrow$$

$$\frac{\perp r}{\text{base}} = \frac{A-0}{0-\tau} = -A/\tau$$



↓ diff.



For $-\infty$ to $-\tau$ $\rightarrow$ no value i.e, zero

At $-\tau$ $\Rightarrow$ upward word switching or transition

i.e; $A/\tau - 0 = A/\tau$.

From $-\tau$ to 0 $\Rightarrow$ constant value $\Rightarrow$ so no value, i.e zero.

At 0 $\Rightarrow$ Downward switching.

i.e; $-A/\tau - A/\tau = -2A/\tau$

From 0 to τ $\Rightarrow$ constant value $\Rightarrow$ so no value; i.e zero

At τ $\Rightarrow$ upward transition

i.e. $0 - (-A/\tau) = A/\tau$

$$\therefore \quad \frac{d^2x(t)}{dt^2} = \frac{A}{\tau}\delta(t+\tau) - \frac{2A}{\tau}\delta(t) + \frac{A}{\tau}\delta(t-\tau)$$

$$\Rightarrow \frac{d^2x(t)}{dt^2} = \frac{A}{\tau}\left[\delta(t+\tau) - 2\delta(t) + \delta(t-\tau)\right]$$

Taking Fourier transform –

$$(j\omega)^2 X(\omega) = \frac{A}{\tau}\left[e^{j\omega\tau} - 2.1 + e^{-j\omega\tau}\right]$$

$$\Rightarrow -\omega^2 X(\omega) = \frac{A}{\tau}\left[e^{j\omega\tau} + e^{-j\omega\tau} - 2\right]$$

$$\Rightarrow X(\omega) = \frac{2A}{-\tau\omega^2}\left[\cos\omega\tau - 1\right]$$

$$\Rightarrow X(\omega) = \frac{2A}{\tau\omega^2}\left[1 - \cos\omega\tau\right]$$

$$\Rightarrow X(\omega) = \frac{2A}{\tau\omega^2}\, 2\sin^2 \omega\tau/2$$

$$\Rightarrow X(\omega) = \frac{4A}{\tau\omega^2}\sin^2 \omega\tau/2 \quad \#$$

Express the result in terms of sampling function.