Operations on Signals

$x(n) \rightarrow x(n-k)$ ←

$x(n) \rightarrow x(n+k)$ →

$x(-n) \rightarrow x(-n+k)$ ⟷

$x(-n) \rightarrow x(-n-k)$

Time Shifting

$x(n) \longrightarrow x(n-t)$ [delay]

$x(n) \longrightarrow x(n+t)$ [Advance]

$x(n) = \{1, 2, \underset{\uparrow}{1}, 3, 4\} \longrightarrow x(n-2) = \{\underset{\uparrow}{1}, 2, 1, 3, 4\}$

$x(n) = \{1, 2, \underset{\uparrow}{1}, 3, 4\} \longrightarrow x(n+2) = \{1, 2, 1, 3, \underset{\uparrow}{4}\}$

Folding

$x(n) \longrightarrow x(-n)$

$x(n) = \{1, 2, \underset{\uparrow}{1}, 3, 4\} \longrightarrow x(-n) = \{4, 3, \underset{\uparrow}{1}, 2, 1\}$

Scaling

- Amplitude scaling

$x'(n) = K\,x(n)$

→ $x(n) = \{1, 2, \underset{\uparrow}{1}, 3, 4\}$

→ $y(n) = 2\,x(n)$

$= \{2, 4, \underset{\uparrow}{2}, 6, 8\}$

Arithmatic Operation

- Addition
- Subtraction
- Multiplication
- division.

$$x(n) = \{\overset{-1}{1}, \underset{\uparrow}{\overset{0}{2}}, \overset{1}{3}, \overset{2}{1}\}$$

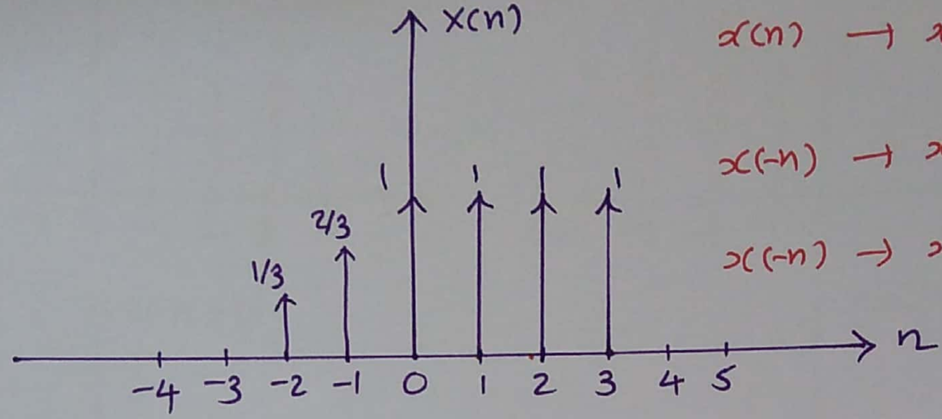
$$y(n) = \{\overset{-2}{1}, \overset{-1}{1}, \underset{\uparrow}{\overset{0}{2}}, \overset{1}{2}\}$$

→ $z(n) = x(n) + y(n)$,

$= \{1, 2, \underset{\uparrow}{4}, 5, 1\}$.

Examples on Signal Operations

x(n) is given by



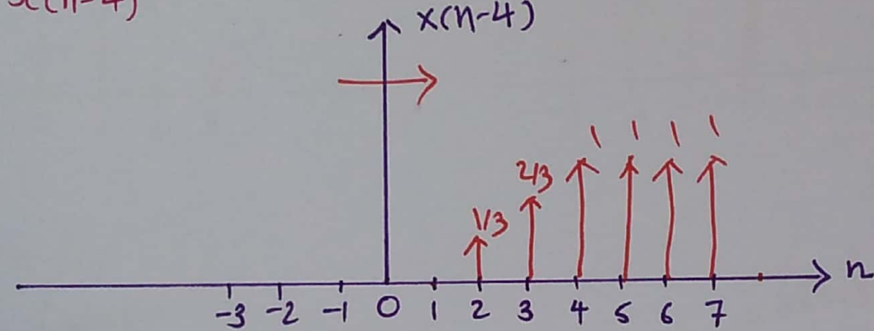
$x(n) \rightarrow x(n-4)$ →

$x(n) \rightarrow x(n+4)$ ←

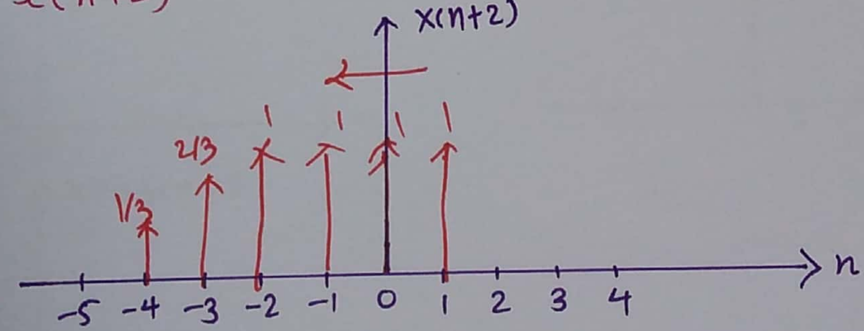
$x(-n) \rightarrow x(-n-4)$ ←

$x(-n) \rightarrow x(-n+4)$ →

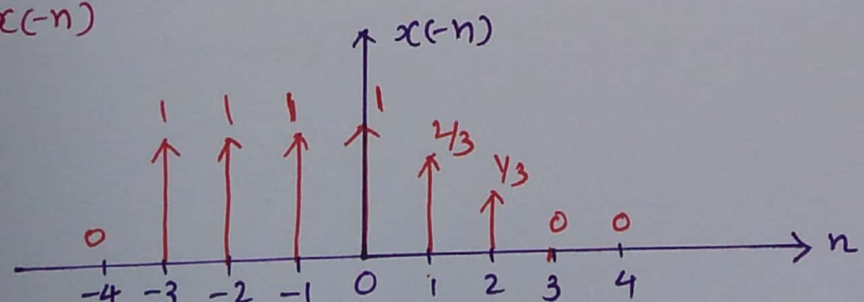
Find ① $x(n-4)$



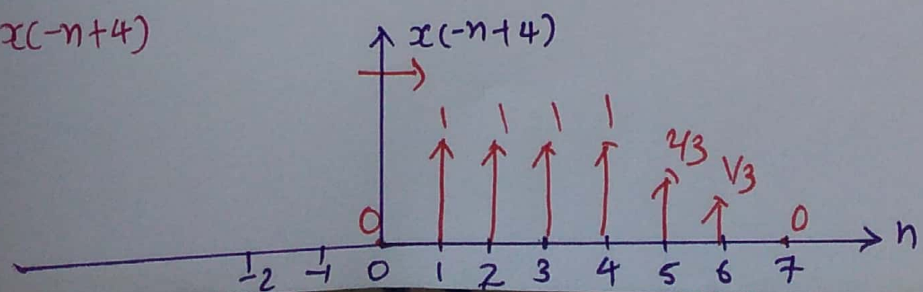
② $x(n+2)$



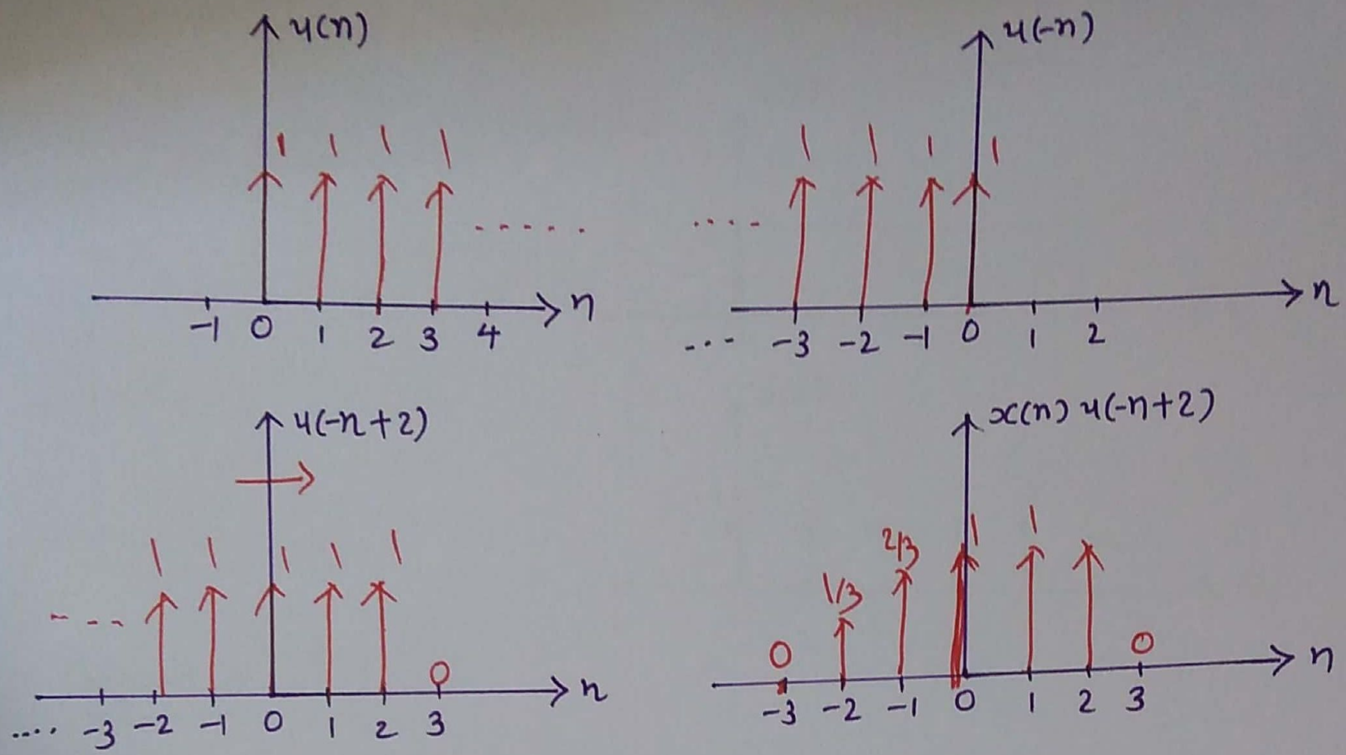
③ $x(-n)$



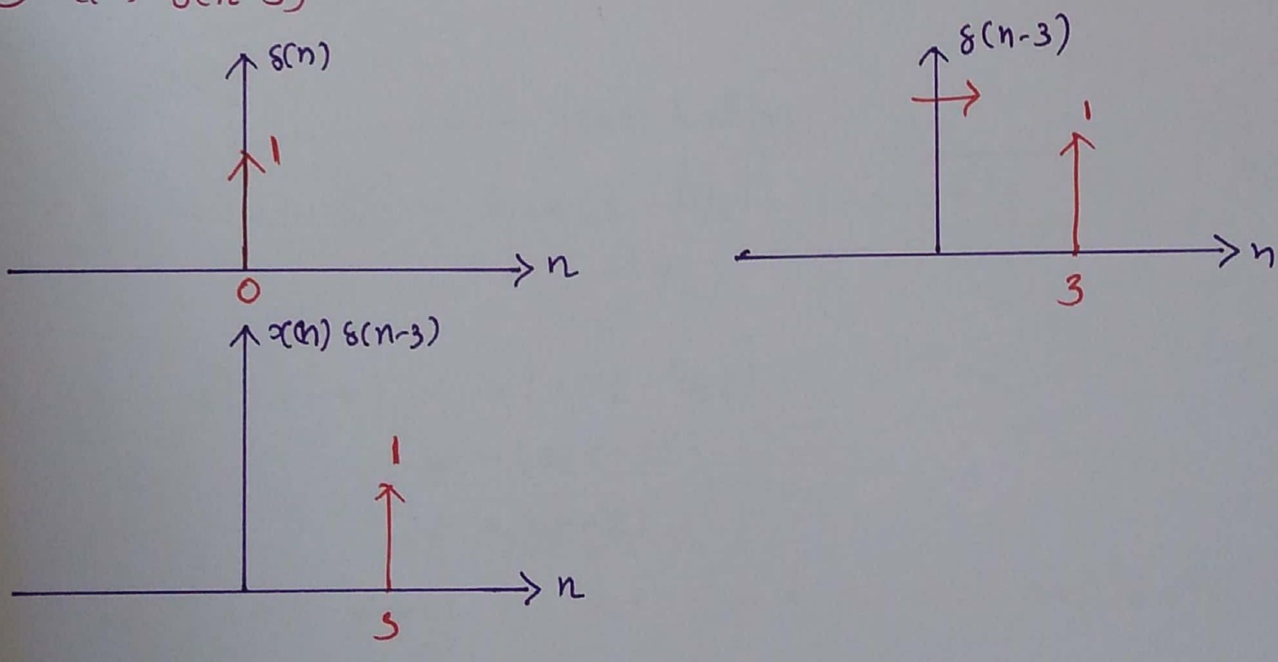
④ $x(-n+4)$



(5) $x(n)\,u(-n+2)$

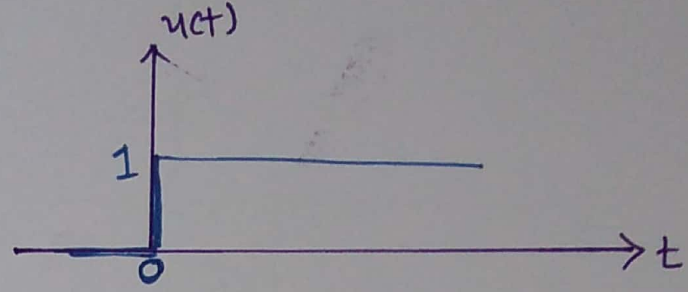


(6) $x(n)\,\delta(n-3)$

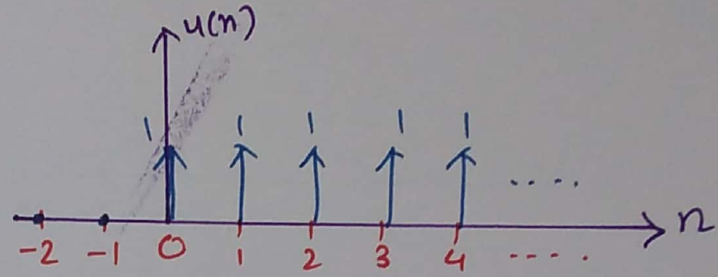


Unit step Signal

- It is denoted by $u(t)$ in Continuous time & $u(n)$ in descrete time.



- $u(t) = 1 \quad t \geq 0$
  $\quad\;\; = 0 \quad t < 0$



- $u(n) = 1 \quad n \geq 0$
  $\quad\;\; = 0 \quad n < 0$

Properties

1) $(u(t))^n = u(t)$
2) $(u(t-t_0))^n = u(t-t_0)$
3) $u(at) = u(t)$

time Scaling.

e.g. $u(at - t_0) = u(a(t - t_0/a))$ $= \boxed{u(t - at_0)}$ ×

$= u(t - t_0/a)$.

e.g. $u(2t-4) = u(2(t-4/2))$

$= u(2(t-2))$

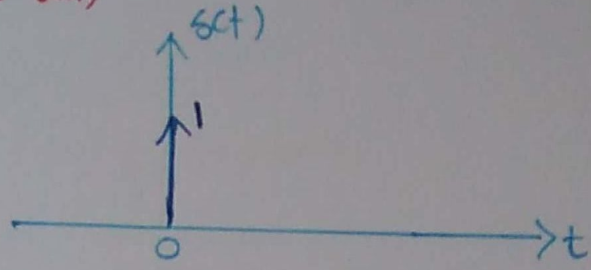
$= u(t-2)$

- Unit step Signal is best signal to test any System.

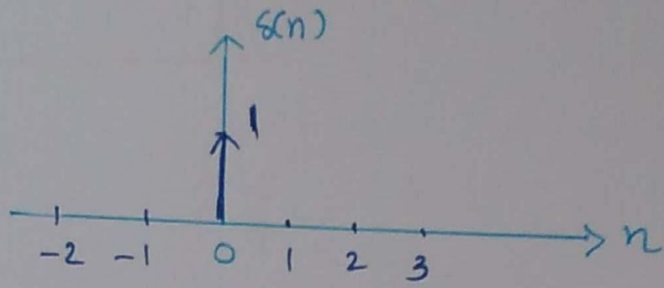
Unit Impulse function

→ It is denoted by $\delta(t)$ or $\delta(n)$

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$



$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

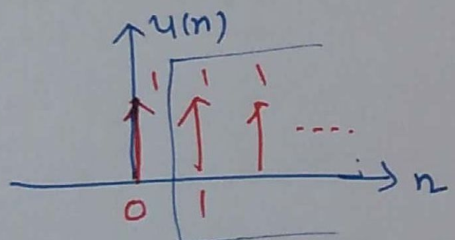


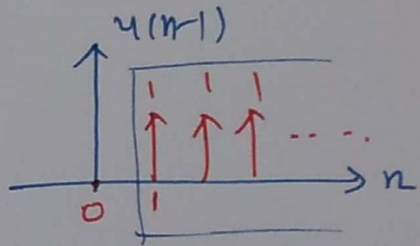
Properties

① $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

② $\delta(n-k) = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$

③ $\delta(n) = u(n) - u(n-1)$





④ $f(t)\,\delta(t) = f(0)$

⑤ $f(t)\,\delta(t-t_0) = f(t_0)$

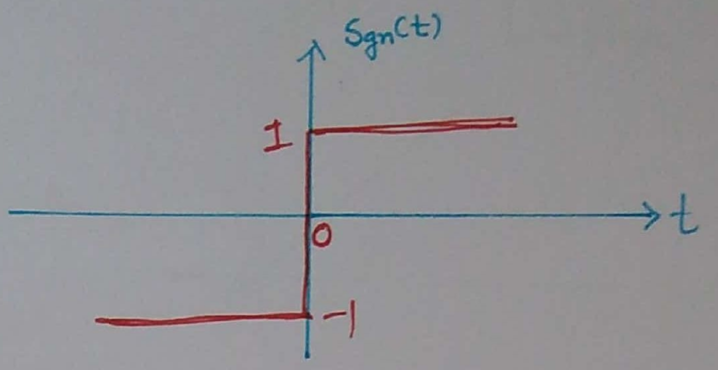
⑥ $\delta(kt) = \frac{1}{|k|}\,\delta(t)$

⑦ $\delta(-t) = \delta(t)$ [Unit Impulse function is even function].

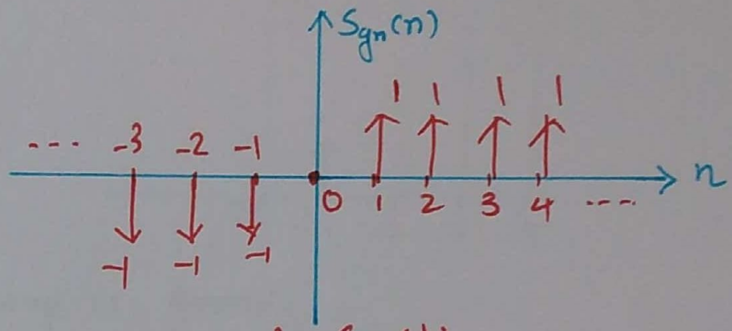
Signum function

- It is denoted by $Sgn(t)$ and $Sgn(n)$.

$$Sgn(t) = \begin{cases} 1 & t>0 \\ 0 & t=0 \\ -1 & t<0 \end{cases}$$

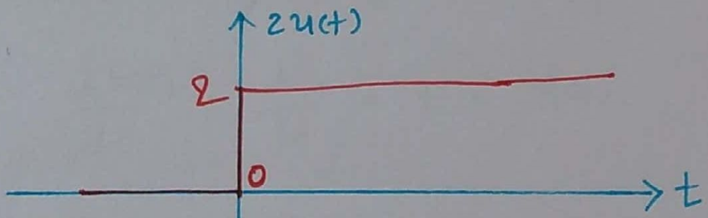


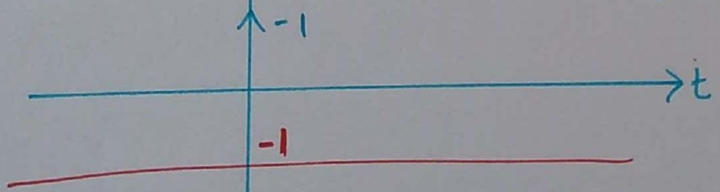
$$Sgn(n) = \begin{cases} 1 & n>0 \\ 0 & n=0 \\ -1 & n<0 \end{cases}$$

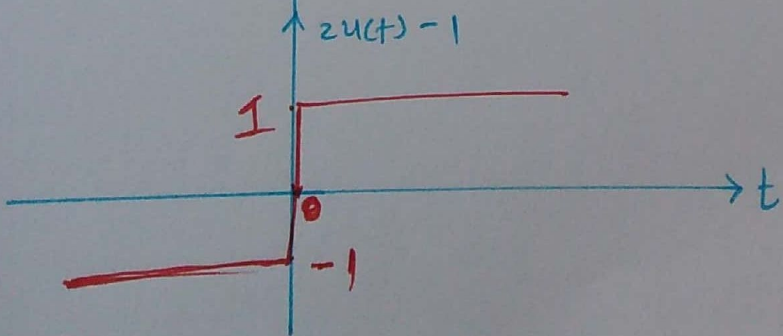


→ Relationship between $u(t)$ & $Sgn(t)$.

$$\underline{Sgn(t) = 2u(t) - 1}$$







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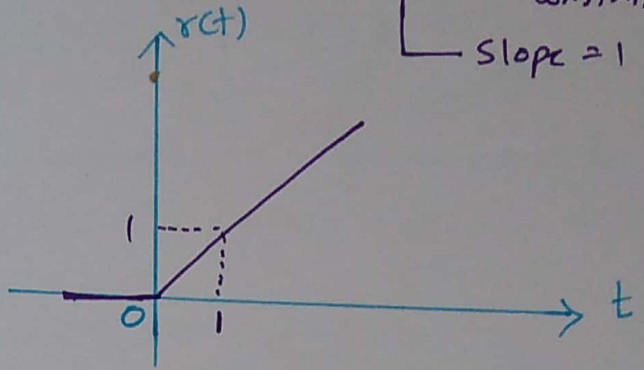
Unit Ramp Signal

- It is denoted by $r(t)$ and $r(n)$.

$y = mx + c$

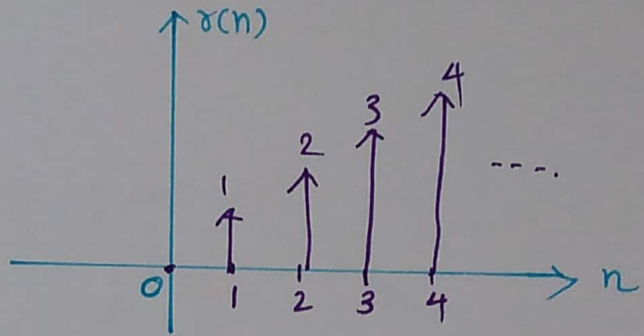
Constant $= 0$

Slope $= 1$



- $r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$

- $r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$



- Integration of step is ramp.

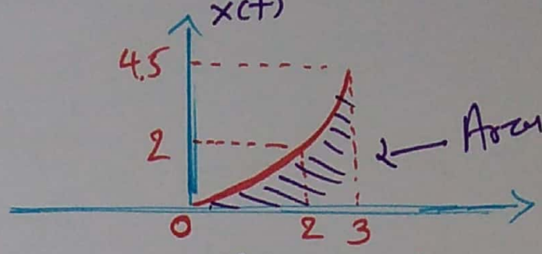
$$\int_{-\infty}^{\infty} u(t)\, dt = \int_{0}^{t} 1\, dt = t = r(t)$$

- differentiation of ramp is step.

$$\frac{d\,r(t)}{dt} = \frac{dt}{dt} = 1 = u(t).$$

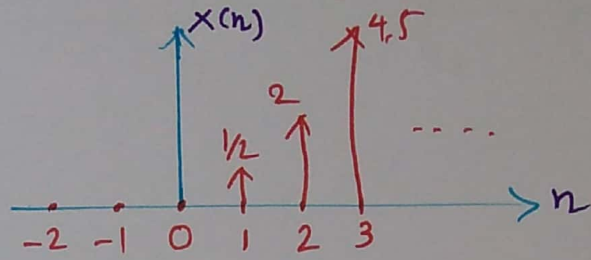
Unit Parabolic Signal

$$x(t) = \begin{cases} t^2/2 , & t \geq 0 \\ 0 , & t < 0 \end{cases}$$



Area covered by Unit Parabolic Signal is always unity.

$$x(n) = \begin{cases} n^2/2 , & n \geq 0 \\ 0 , & n < 0 \end{cases}$$



→ **Integration**

$$\int u(t)\, dt = r(t) \longrightarrow \int r(t)\, dt$$

$$= \int t\, dt = t^2/2$$

→ **Differentiation**

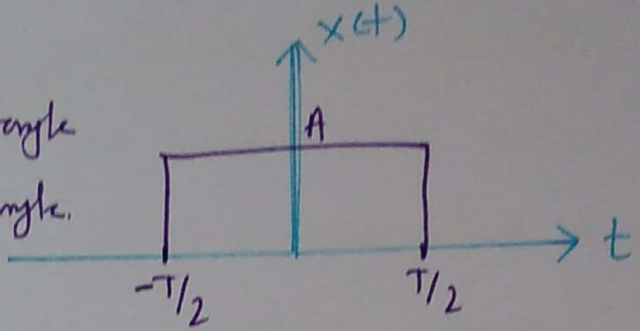
$$\frac{d(t^2/2)}{dt} = \frac{1}{2}\frac{dt^2}{dt} = \frac{1}{2}(2t) = t = r(t).$$

Rectangular Pulse

- $x(t) = A \; rect\left(\frac{t}{T}\right)$

where A = Amplitude of rectangle

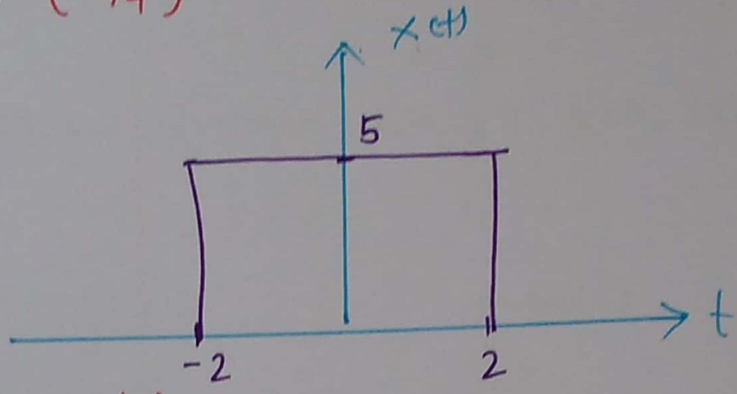
T = Periode of rectangle.



- e.g. $x(t) = 5 \; rect\left(t/4\right)$

$A = 5$

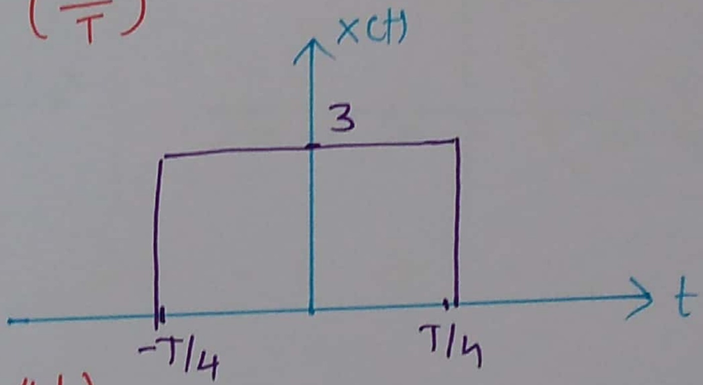
$T = 4$



- e.g. $x(t) = 3 \; rect\left(\frac{2t}{T}\right)$

$A = 3$

$T' = T/2$

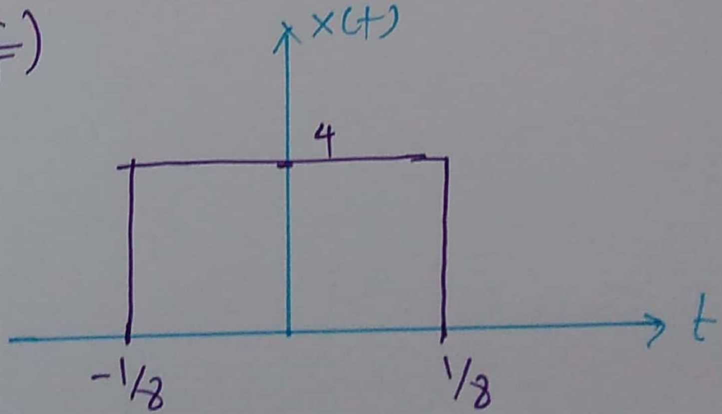


- e.g. $x(t) = 4 \; rect(4t)$

$= A \; rect\left(\frac{t}{T}\right)$

$A = 4$

$T = 1/4$



Sinc & Sampling functions

Sinc function

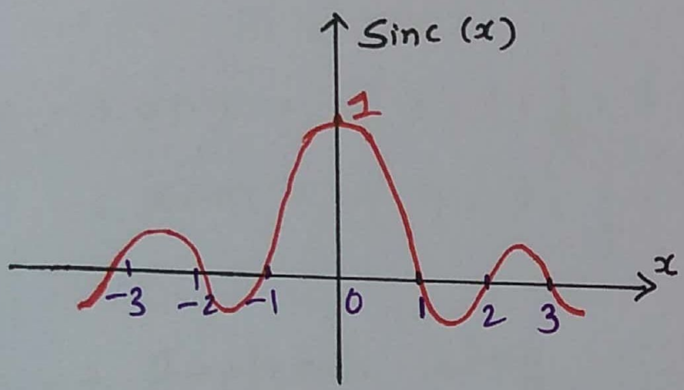
→ Denoted with sinc (x)

→ It is Normalized function

→ $\text{sinc}(x) = \frac{\sin \pi x}{\pi x}$

$= 0$

where $x = \pm 1, \pm 2, \pm 3, \ldots$



Sampling function

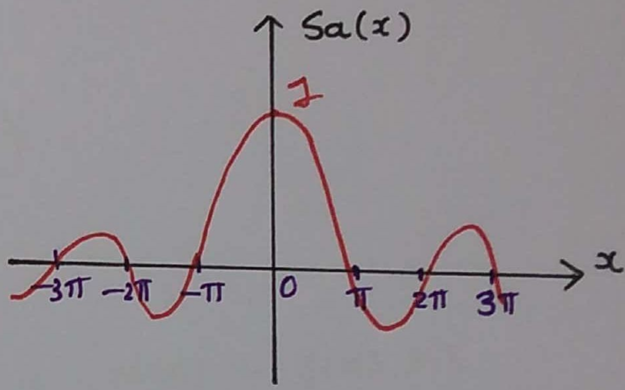
→ Denoted with Sa (x)

→ It is unnormalized function.

→ $Sa(x) = \frac{\sin x}{x}$

$= 0$

where $x = \pm \pi, \pm 2\pi, \pm 3\pi, \ldots$



$\text{sinc}(x) = Sa(x) = 1$ at $x = 0$

Operations on Signal

- Time shifting

→ $x(n) \xrightarrow[\text{delay}]{} x(n-k)$

→ $x(n) \xrightarrow[\text{Advance}]{} x(n+k)$

→ $x(n) = \{1, 2, \underset{\uparrow}{1}, 3, 4\}$

→ $x(n-2) = \{1, \underset{\uparrow}{2}, 1, 3, 4\}$

→ $x(n+2) = \{\overset{\uparrow}{\ }1, 2, 1, 3, \underset{\uparrow}{4}\}$

$x(n) \longrightarrow x(n-k)$ ⟹

$x(n) \longrightarrow x(n+k)$ ⟸

$x(-n) \longrightarrow x(-n-k)$ ⟸

$x(-n) \longrightarrow x(-n+k)$ ⟹

- Folding

→ $x(n) \longrightarrow x(-n)$

→ $x(n) = \{1, 2, \underset{\uparrow}{1}, 3, 4\}$

→ $x(-n) = \{4, 3, \underset{\uparrow}{1}, 2, 1\}$

- Scaling

→ Amplitude scaling.

$x(n) = y(n) \times K$

→ $y(n) = \{1, 2, \underset{\uparrow}{1}, 3, 4\}$

$K = 2$

$x(n) = y(n) \times K$

$= \{2, 4, \underset{\uparrow}{2}, 6, 8\}$

- Arithmatic Operation.

- Addition
- Subtraction
- Multiplication
- Devision.

→ $z(n) = x(n) + y(n)$

$x(n) = \{1, \underset{\uparrow}{2}, 1, 3\}$

$y(n) = \{2, 3, 1, \underset{\uparrow}{1}\}$

→ $z(n) = \{2, 3, 2, \underset{\uparrow}{3}, 1, 3\}$

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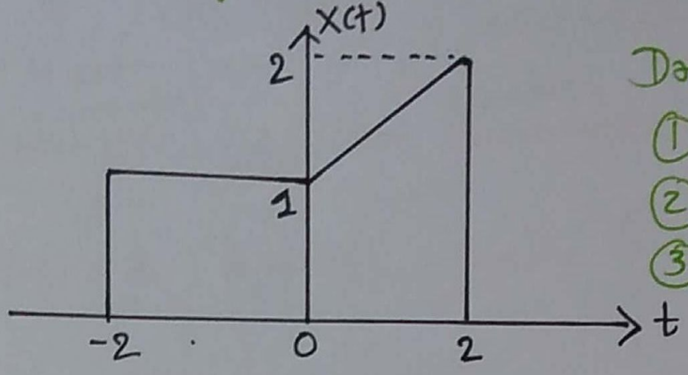
Operations on Signal [Scaling Operation]

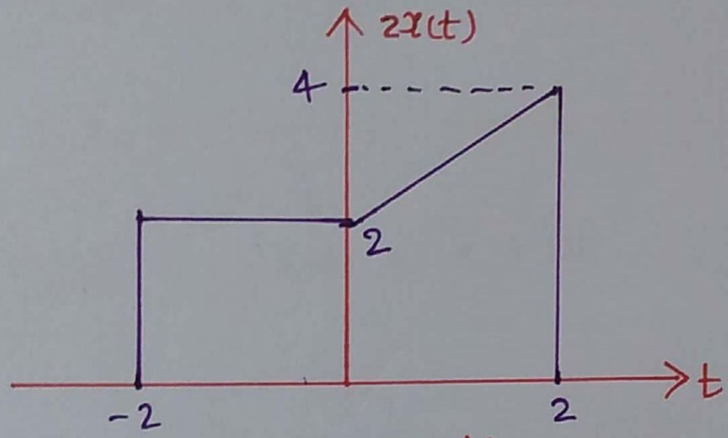
- Example on Amplitude and time Scaling

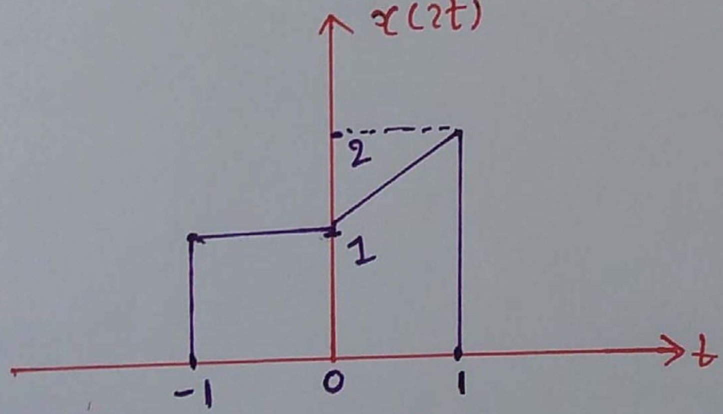
* Perform following Operations on Signal $x(t)$.

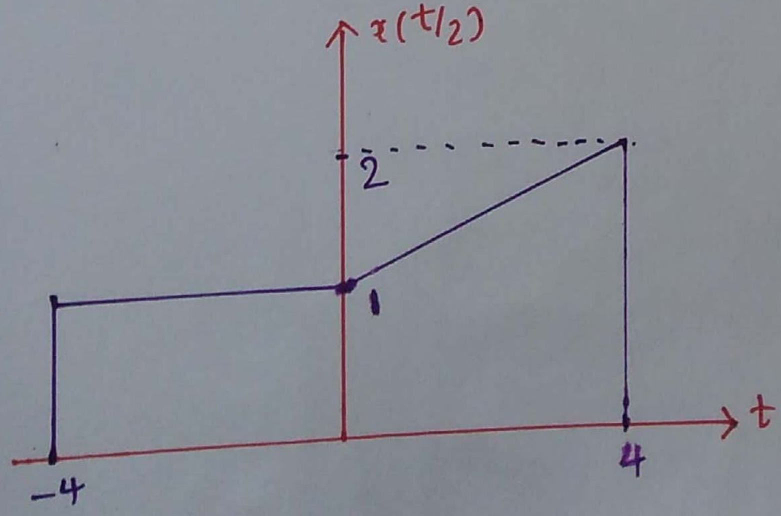


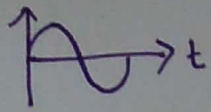
Draw

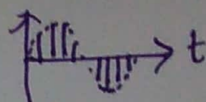
1. $2x(t)$ [Amplitude]
2. $x(2t)$ } [time]
3. $x(t/2)$ }







→ Classification of signals 



1. Continous time and Discrete time signals

Symetric Asymetric

2. Even & Odd signals

$x(t) = x(-t)$ [Even]

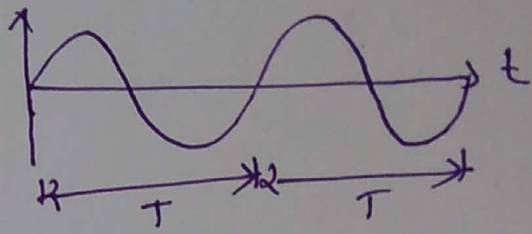
$x(t) = -x(-t)$ [odd]

Define it with functions

(Random)

3. Deterministic & Non deterministic signals

4. Periodic & Aperiodic signals



5. Energy & Power signals

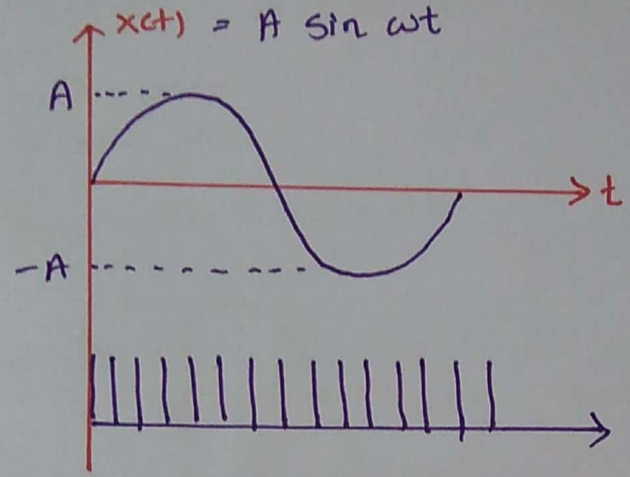
6. Real & Imaginary signals.

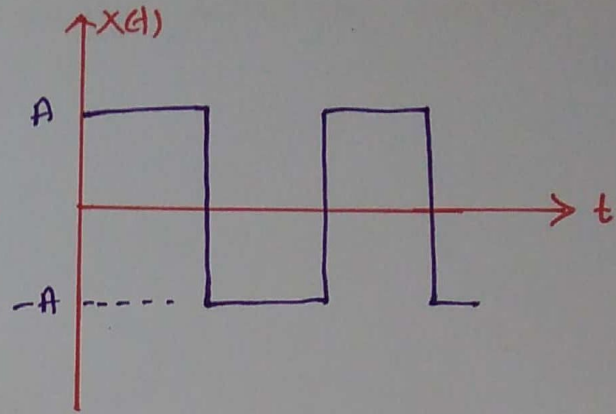
$x(t) = a + jb$

Continous time & discrete time signals

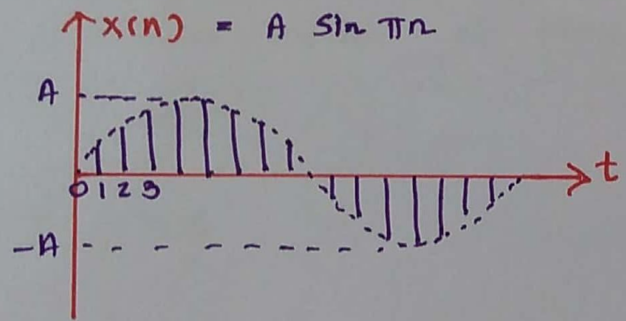
- A signal which is defined for all values of time t is called continous time signal.

- e.g.





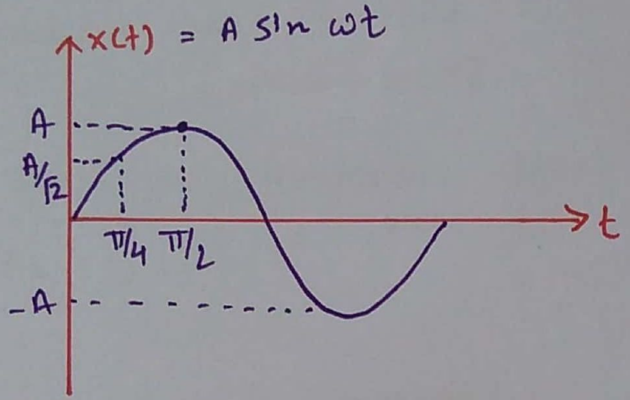
- A signal which is defined only at discrete time interval of time t is called discrete signal.



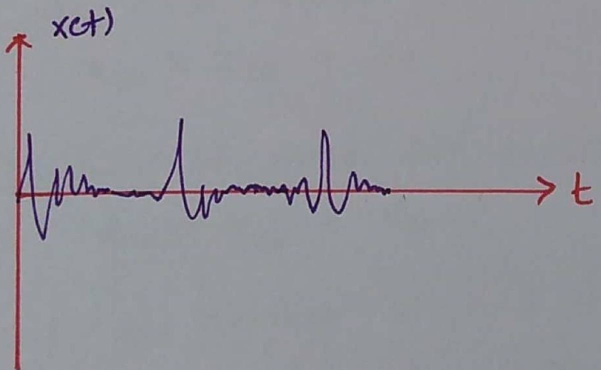
- For discrete time signals, time is discrete & amplitude is continous.
- For digital signals, both time & Amplitude is discrete.

Deterministic & Non deterministic (Random) Signals

- A signal is said to be deterministic, If there is no uncertainity with respect to its value at any instant of time.
- Deterministic signal can be easily define by function.
- e.g. - Sine wave
  - Cosine wave



- Random signal is one where there is uncertainity at any particular instant of time.
- e.g. voice signal.



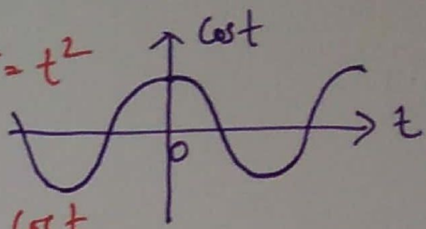
Even & Odd Signals

- A signal is said to even when it satisfies the cond.^{n}

$$x(t) = x(-t)$$

$x(t) = t^2$

$x(-t) = (-t)^2 = t^2$



- Even signal is symetric signal.
- e.g. $\underline{\cos} \, t, \; t^2, \; t^4, \; t^{2n}$

$x(t) = \cos t$

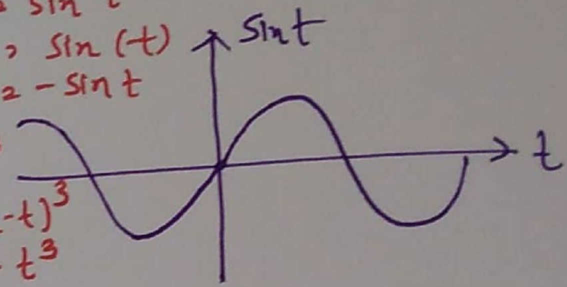
$x(-t) = \cos(-t) = \cos t$

- A signal is said to odd when it satisfies the cond.^{n}

$$x(t) = -x(-t)$$

$x(t) = \sin t$

$x(-t) = \sin(-t) = -\sin t$



- Odd signal is Asymetric signal.
- e.g. $\underline{\sin} \, t, \; t, \; t^3, \; t^{2n+1}$

$x(t) = t^3$

$x(-t) = (-t)^3 = -t^3$

Important Notes for examples.

- Sum of two or more even function is even function.

$$x_{1e} + x_{2e} = x_e$$

- Product of two or more even function is even function.

$$x_{1e} \times x_{2e} = x_e$$

- Product of even number of odd function is even function.

$$x_{1o} \times x_{2o} = x_e$$

- Sum of two or more odd functions is odd function.

$$x_{1o} + x_{2o} = x_o$$

- Product of odd number of odd function results odd function.

$$x_{1o} \times x_{2o} \times x_{3o} = x_o$$

Even & Odd Components of Signal with example

$\rightarrow \; x(t) = \underbrace{x_e(t)}_{\text{even}} + \underbrace{x_o(t)}_{\text{odd}} \quad \text{—} ①$

$\rightarrow \; x(-t) = x_e(t) - x_o(t) \quad \text{—} ②$

$\rightarrow$ Add ① & ②.

$\Rightarrow x(t) + x(-t) = 2x_e(t)$

$\Rightarrow \boxed{x_e(t) = \dfrac{x(t) + x(-t)}{2}} \quad \text{—} (A)$

$\rightarrow$ Sub ① & ②

$\Rightarrow x(t) - x(-t) = 2x_o(t)$

$\Rightarrow \boxed{x_o(t) = \dfrac{x(t) - x(-t)}{2}} \quad \text{—} (B)$

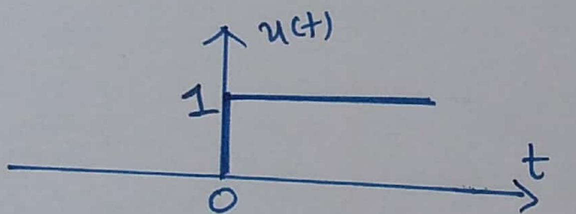
Find even & odd component of unit step signal.

$\rightarrow u(t) = 1 \quad t \geq 0$

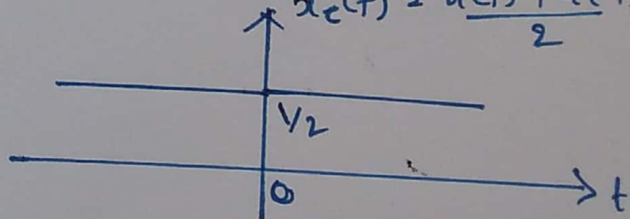
$\quad\quad\; = 0 \quad t < 0$

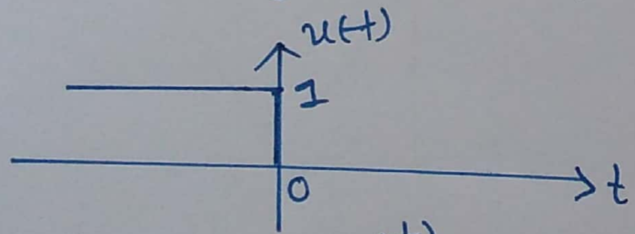
$x_e(t) = \dfrac{x(t) + x(-t)}{2}$

$x_o(t) = \dfrac{x(t) - x(-t)}{2}$

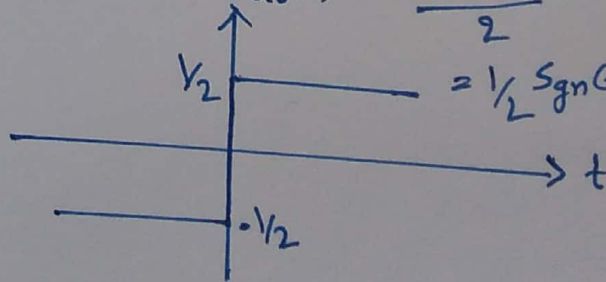


$x_e(t) = \dfrac{u(t) + u(-t)}{2} = \dfrac{1}{2}$

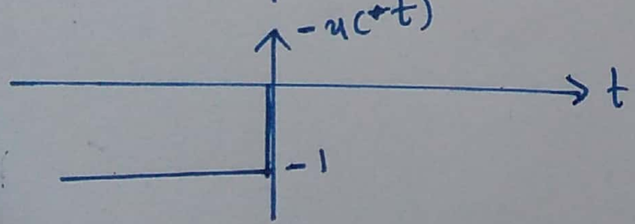




$x_o(t) = \dfrac{u(t) - u(-t)}{2}$

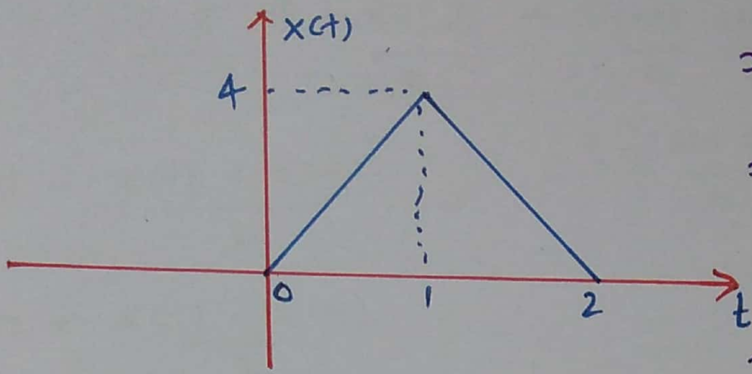


$= \frac{1}{2} \operatorname{Sgn}(t)$



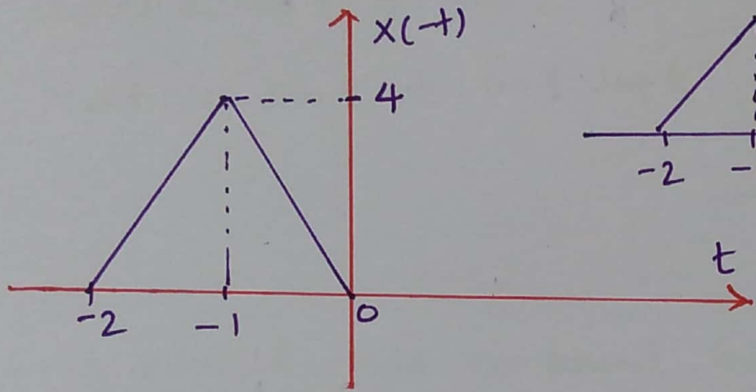
Example of even & odd component of signal based on graph

- Find even and odd component of given signal. $x(t)$.

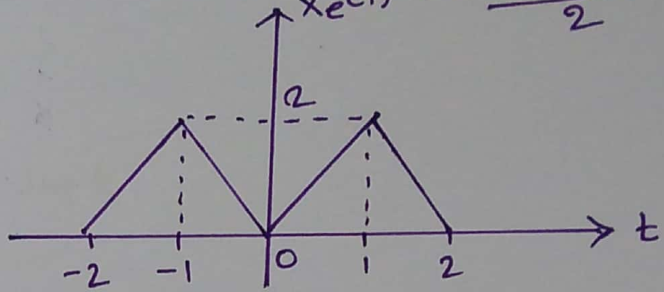


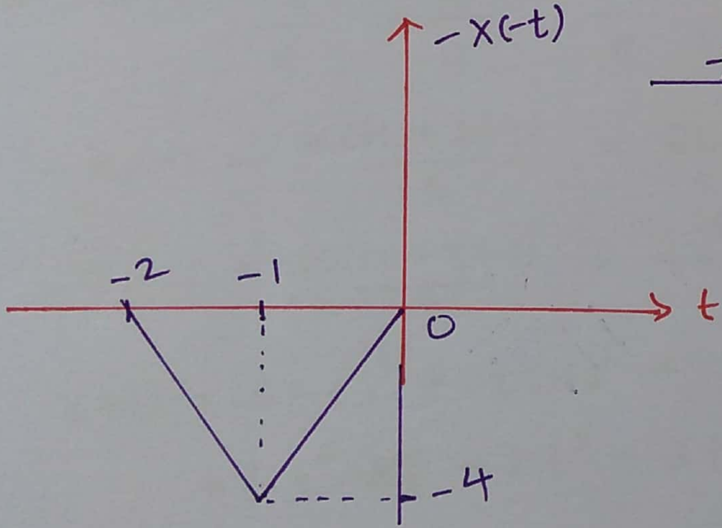
$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

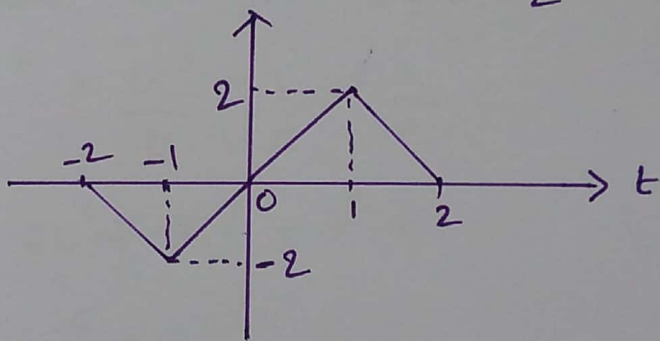


$$x_e(t) = \frac{x(t) + x(-t)}{2}$$





$$x_o(t) = \frac{x(t) - x(-t)}{2}$$



Examples of even & odd component of signal based on function

- Find even & odd component of given signal x(t).

$$x(t) = \underset{e}{\underline{\cos t}} + \underset{o}{\underline{\sin t}} + \underset{e}{\underline{\cos t}}\ \underset{o}{\underline{\sin t}} + \underline{\sin^2 t}$$

$$= \frac{1}{2}\underset{o}{\underline{\sin 2t}} \qquad = \underset{e}{\underline{\frac{1-\cos 2t}{2}}}$$

- $x_e(t) = \dfrac{x(t) + x(-t)}{2}$

- $x_o(t) = \dfrac{x(t) - x(-t)}{2}$

- $x_e(t) = \cos t + \sin^2 t$

- $x_o(t) = \sin t + \cos t \sin t$

- Find even & odd component of given signal x(t)

$$x(t) = \underset{e}{\underline{5t^6}} + \underset{o}{\underline{2t^5}} + \underset{e}{\underline{3t^4}} + \underset{o}{\underline{2t^3}} + \underset{e}{\underline{t^2}} + \underset{e}{\underline{1}}$$

- $x_e(t) = \dfrac{x(t) + x(-t)}{2} = 5t^6 + 3t^4 + t^2 + 1$

- $x_o(t) = \dfrac{x(t) - x(-t)}{2} = 2t^5 + 2t^3$

$$x(-t) = 5(-t)^6 + 2(-t)^5 + 3(-t)^4 + 2(-t)^3 + (-t)^2 + 1$$

$$= 5t^6 - 2t^5 + 3t^4 - 2t^3 + t^2 + 1$$

- $x_e(t) = \dfrac{x(t) + x(-t)}{2} = \dfrac{5t^6 + \cancel{2t^5} + 3t^4 + \cancel{2t^3} + t^2 + 1 + 5t^6 - \cancel{2t^5} + 3t^4 - \cancel{2t^3} + t^2 + 1}{2}$

$$\boxed{x_e(t) = 5t^6 + 3t^4 + t^2 + 1}$$

- $x_o(t) = \dfrac{x(t) - x(-t)}{2} = \dfrac{\cancel{5t^6} + 2t^5 + \cancel{3t^4} + 2t^3 + \cancel{t^2} + \cancel{1} - \cancel{5t^6} + 2t^5 - \cancel{3t^4} + 2t^3 - \cancel{t^2} - \cancel{1}}{2}$

$$= 2t^5 + 2t^3$$

Cheeking of Periodicity

- Method 1 :- Ratio

- Consider two signals $x_1(t)$ & $x_2(t)$ with with periods T_1 & T_2 respectively. when They are summed, The resultant Signal is said to be periodic when $\frac{T_1}{T_2}$ = rational number.

- Method 2 :- GCD

- If GCD of freq is possible. Then said to be periodic otherwise aperiodic.

- e.g. ① $x(t) = \cos 5t + \sin \pi t$

  - $\omega_1 = 5$ $\omega_2 = \pi$
  - $T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{5}$ $T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{\pi} = 2$
  - $\frac{T_1}{T_2} = \frac{2\pi/5}{2} = \frac{\pi}{5}$ = It is not a rational number.
  - This is aperiodic signal.
  - GCD is not possible so given signal is aperiodic

② $x(t) = \cos \pi t + \sin 4\pi t$

$\Rightarrow \omega_1 = \pi$ $\omega_2 = 4\pi$

$\Rightarrow T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\pi} = 2$ $T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{4\pi} = 1/2$

$\Rightarrow \frac{T_1}{T_2} = \frac{2}{1/2} = 4$ = It is rational number.

$\Rightarrow$ So this is periodic signal.

$\Rightarrow$ Here GCD of two signal is possible. so given signal is periodic signal.

③ $x(t) = \cos \pi t + \cos 3\pi t + \sin 5\pi t$

$\Rightarrow \omega_1 = \pi$ $\omega_2 = 3\pi$ $\omega_3 = 5\pi$

$\Rightarrow T_1 = \frac{2\pi}{\omega_1} = 2$ $T_2 = \frac{2\pi}{\omega_2} = \frac{3}{2}$ $T_3 = \frac{2\pi}{\omega_3} = \frac{2}{5}$

$\Rightarrow T_1 : T_2 : T_3 = 2 : \frac{3}{2} : \frac{2}{5} = \boxed{20 : 15 : 4}$

$\rightarrow$ So given signal is periodic Signal.

Energy & Power Signal.

Energy Signal

- Energy is finite

$$0 < E < \infty$$

- For finite duration

$$E = \int_{-T}^{T} x^2(t)\, dt$$

- For Infinite duration.

$$E = \lim_{T \to \infty} \int_{-T}^{T} x^2(t)\, dt$$

- For energy signal

$$P = 0$$

- Almost all non-periodic signals are energy signal

- Energy signals exist over a short period of time. They are time limited.

Power Signal

- Power is finite.

$$0 < P < \infty$$

- For finite duration

$$P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

- For Infinite duration

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

- For power signal

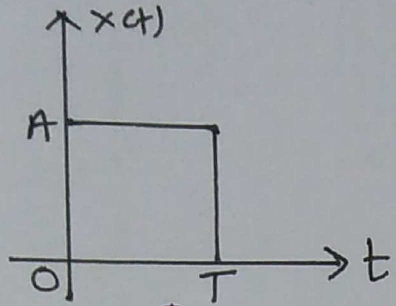
$$E = \infty$$

- Almost all the periodic signals are power signals.

- Power signals exist over an infinite time. They are not time limited.

Energy and Power of Rectangular Pulse and Square wave

- Find Energy and Power of given Rectangular Pulse



$\rightarrow x(t) = A \quad 0 < t < T$

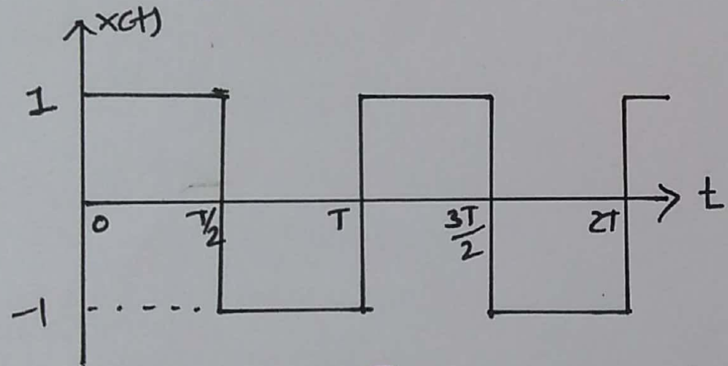
$= 0 \quad$ else.

$$\rightarrow E = \int_{-T}^{T} x^2(t)\, dt = \int_0^T A^2\, dt = A^2 \int_0^T 1\, dt = A^2 [t]_0^T$$

$$= A^2 [T - 0] = \boxed{A^2 T}$$

$\rightarrow$ Here signal is energy signal with energy A^2T so Power P of this signal is 0.

- Find Energy and Power of given Square wave.



$\rightarrow$ Periodic signal

$x(t) = 1 \quad 0 < t < T/2$

$= -1 \quad T/2 < t < T$

$$\rightarrow P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt = \lim_{T \to \infty} \frac{1}{2T} \int_0^T x^2(t)\, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[\int_0^{T/2} x^2(t)\, dt + \int_{T/2}^{T} x^2(t)\, dt \right]$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[\int_0^{T/2} 1\, dt + \int_{T/2}^{T} 1\, dt \right]$$

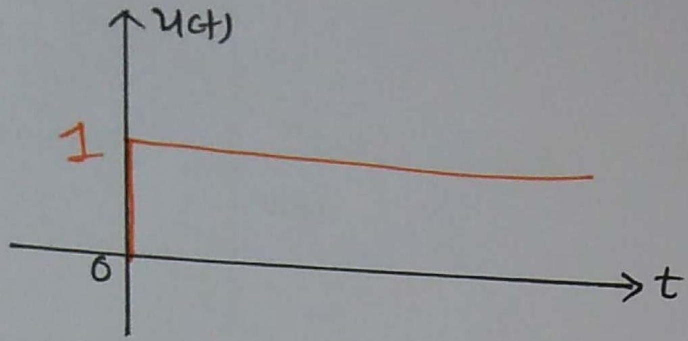
$\rightarrow$ Energy of this signal will be infinite.

$$= \lim_{T \to \infty} \frac{1}{2T} \left[[t]_0^{T/2} + [t]_{T/2}^{T} \right]$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[T/2 + T/2 \right] = \lim_{T \to \infty} \frac{T}{2T} = \frac{1}{2}$$

Energy and Power of Unit Step and exponential function

- Find Energy and Power of Unit step function



$u(t) = 1 \quad t \geq 0$

$= 0 \quad$ else.

$\rightarrow E = \lim_{T \to \infty} \int_{-T}^{T} x^2(t)\, dt$

$= \lim_{T \to \infty} \int_{0}^{T} 1\, dt$

$= \lim_{T \to \infty} [t]_0^T$

$= \lim_{T \to \infty} T = \boxed{\infty}$

$\rightarrow P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt$

$= \lim_{T \to \infty} \frac{1}{2T} \int_{0}^{T} 1\, dt$

$= \lim_{T \to \infty} \frac{1}{2\cancel{T}} [\cancel{T}]$

$= \boxed{1/2 \text{ W}}$

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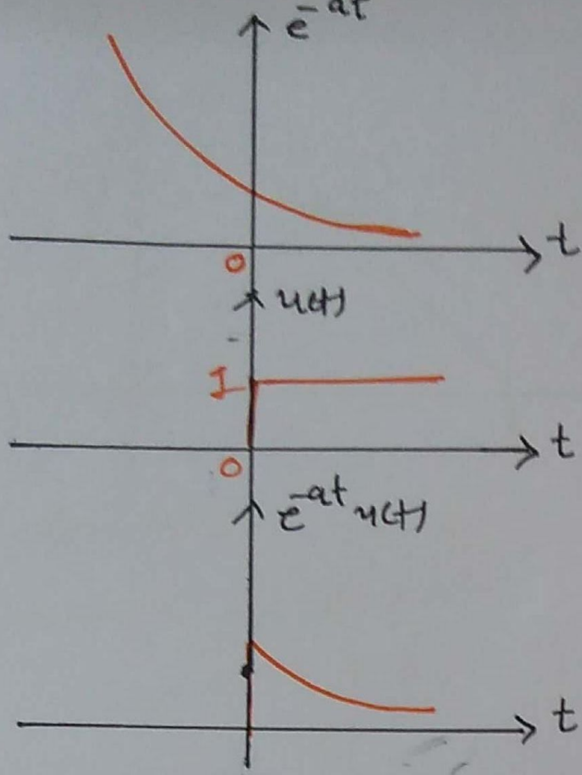
- Find Energy and Power of exponential signal $e^{-at}u(t)$.



$x(t) = e^{-at} \quad t \geq 0$

$= 0 \quad$ else.

$$\rightarrow E = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)\, dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} (e^{-at})^2\, dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} e^{-2at}\, dt$$

$$= \lim_{T\to\infty} \left[\frac{e^{-2at}}{-2a} \right]_0^T$$

$$= \lim_{T\to\infty} \left[-\frac{e^{-2aT}}{2a} + \frac{e^{-0}}{2a} \right]$$

$$= \lim_{T\to\infty} \left[\frac{1}{2a} - \frac{e^{-2aT}}{2a} \right]$$

$$= \frac{1}{2a} - 0 = \boxed{1/2a}$$

→ As this energy signal with energy $1/2a$, it's power will be 0.

- If the energy of the signal $x(t) = e^{-5t}u(t)$ is $1/10$. Then the energy of the time scaled version of signal $x(2t)$ is $\underline{1/20}$

→ Energy of $x(t) = E$

→ Energy of $x(at) = E/a$

→ Energy of $x(2t) = E/2 = \frac{1}{10} \times \frac{1}{2} = \boxed{\frac{1}{20}}$

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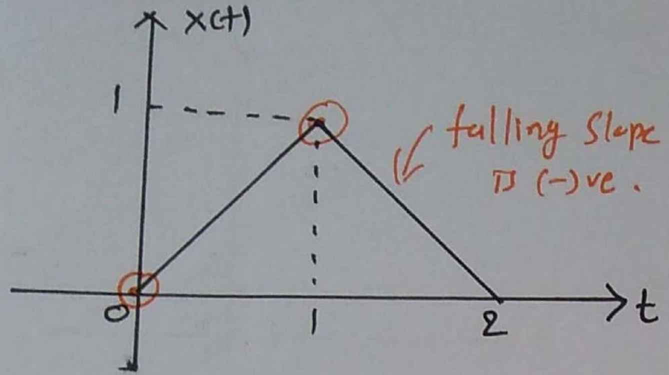
Energy & Power of triangular function

- Find total Energy and Power of given triangular function



→ For time 0 to 1

$x(t) = mt + c$

$= (1)t + c$

$= t + c$

→ $(t, x(t)) = (0,0)$

$\Rightarrow 0 = 0 + c$

$\Rightarrow c = 0$

→ $x(t) = t$

→ For time 1 to 2

$x(t) = mt + c$

$= (-1)t + c$

$= -t + c$

→ Put $(t, x(t)) = (1,1)$

$\Rightarrow 1 = -1 + c$

$\Rightarrow c = 2$

→ $x(t) = -t + 2$

→ function :

$x(t) = t \quad 0 < t < 1$

$= -t + 2 \quad 1 < t < 2$

→ $$E = \int_{-T}^{T} x^2(t)\, dt = \int_{0}^{2} x^2(t)\, dt = \int_{0}^{1} x^2(t)\, dt + \int_{1}^{2} x^2(t)\, dt$$

$$= \int_{0}^{1} (t)^2 dt + \int_{1}^{2} (-t+2)^2\, dt$$

$$= \int_{0}^{1} t^2\, dt + \int_{1}^{2} (t^2 - 4t + 4)\, dt$$

$$= \left[\frac{t^3}{3}\right]_0^1 + \left[\frac{t^3}{3} - 4\left(\frac{t^2}{2}\right) + 4(t)\right]_1^2$$

$$= \left[\frac{1}{3} - 0\right] + \left[\frac{8}{3} - \cancel{8} + \cancel{8} - \frac{1}{3} + 2 - 4\right]$$

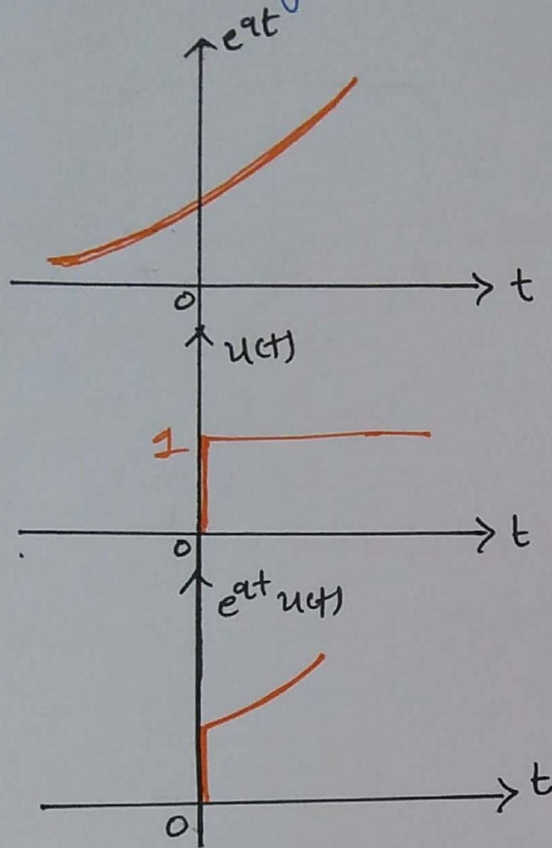
$$= \cancel{\frac{1}{3}} + \frac{8}{3} - \cancel{\frac{1}{3}} - 2$$

$$= \frac{8}{3} - 2 = \frac{8-6}{3} = \boxed{\frac{2}{3}}$$

→ as given signal is energy signal, power of this signal should ~~be~~ 0.

Energy and Power of exponential signal with positive coefficient

- Find Energy and Power of exponential signal with +ve coefficient. signal is $e^{at}u(t)$.



$$x(t) = e^{at} \quad t \geq 0$$
$$= 0 \quad \text{else.}$$

$$\rightarrow E = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)\, dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} (e^{at})^2\, dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} e^{2at}\, dt$$

$$= \lim_{T\to\infty} \left[\frac{e^{2at}}{2a} \right]_0^T$$

$$= \lim_{T\to\infty} \left[\frac{e^{2aT}}{2a} - \frac{e^{0}}{2a} \right]$$

$$= \lim_{T\to\infty} \left[\frac{e^{2aT}}{2a} - \frac{1}{2a} \right]$$

$$= \boxed{\infty}$$

$$\rightarrow P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} (e^{at})^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} e^{2at}\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{2at}}{2a} \right]_0^T$$

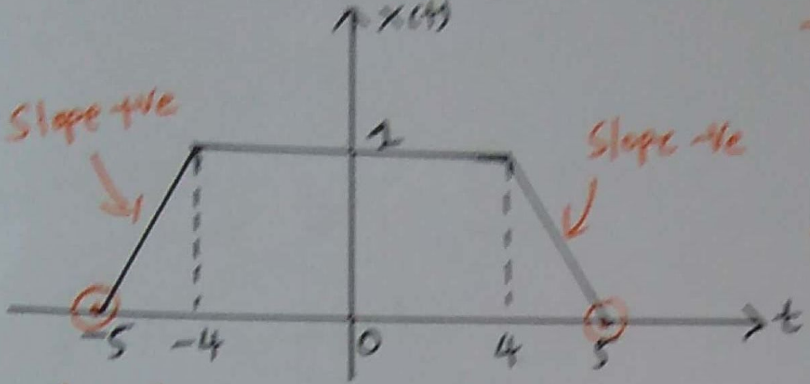
$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{2aT}}{2a} - \frac{1}{2a} \right]$$

$$= \boxed{0}$$

$\rightarrow$ this signal is not energy as well as power signal.

Example on Energy and Power of given Signal

- Find energy and power of given Signal.



→ For time -5 to -4

$x(t) = mt + c$

$= t + c$

→ Put $(t, x(t)) = (-5, 0)$

$\Rightarrow 0 = -5 + c$

$\Rightarrow c = 5$

→ $\boxed{x(t) = t + 5}$

→ For time -4 to 4

$\boxed{x(t) = 1}$

→ for time 4 to 5

$x(t) = mt + c$

$= -t + c$

→ Put $(t, x(t)) = (5, 0)$

$\Rightarrow 0 = -5 + c$

$\Rightarrow c = 5$

→ $\boxed{x(t) = -t + 5}$

$$x(t) = \begin{cases} t+5 & -5 < t < -4 \\ 1 & -4 < t < 4 \\ -t+5 & 4 < t < 5 \end{cases}$$

$$\rightarrow E = \int_{-T}^{T} x^2(t)\, dt = \int_{-5}^{5} x^2(t)\, dt$$

$$= \int_{-5}^{-4} (t+5)^2\, dt + \int_{-4}^{4} (1)^2\, dt + \int_{4}^{5} (-t+5)^2\, dt$$

$$= \int_{-5}^{-4} (t^2 + 10t + 25)\, dt + \int_{-4}^{4} 1\, dt + \int_{4}^{5} (t^2 - 10t + 25)\, dt$$

$$= \left[\frac{t^3}{3} + 5t^2 + 25t\right]_{-5}^{-4} + \left[t\right]_{-4}^{4} + \left[\frac{t^3}{3} - 5t^2 + 25t\right]_{4}^{5}$$

$$= \boxed{8.66\ J}$$

Energy & Power of discrete Signal

→ If discrete signal is given by $x(n)$

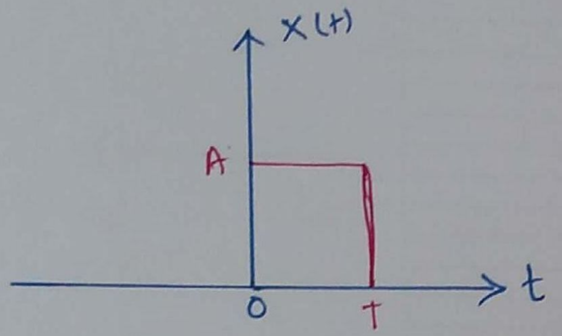
$$E = \lim_{N\to\infty} \sum_{n=-\infty}^{\infty} |x(n)|^2$$

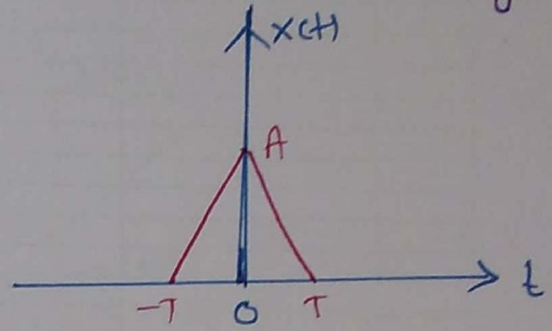
$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} |x(n)|^2$$

→ $E = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)\,dt$

→ $P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\,dt$

→ All finite duration signals of finite amplitude are energy signal.





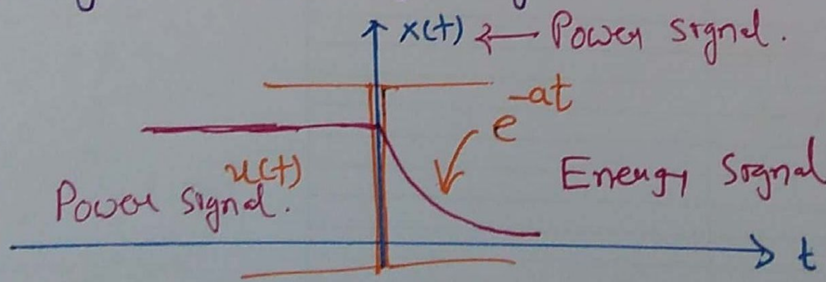
→ All periodic signals are power signals but all power signals are not periodic signals.

$$P_{avg} = \left(\frac{V_m}{\sqrt{2}}\right)^2 = \frac{V_m^2}{2} = V_{rms}^2 \quad \left(V_{rms} = \frac{V_m}{\sqrt{2}}\right)$$

Ex. $x(t) = 5 \sin \omega t$

→ $V_m = 5$ → $P = \frac{V_m^2}{2} = \frac{25}{2} = 12.5$

→ Power Signal + Energy Signal = Power Signal.



→ Continous impulse is neither energy nor power signal.

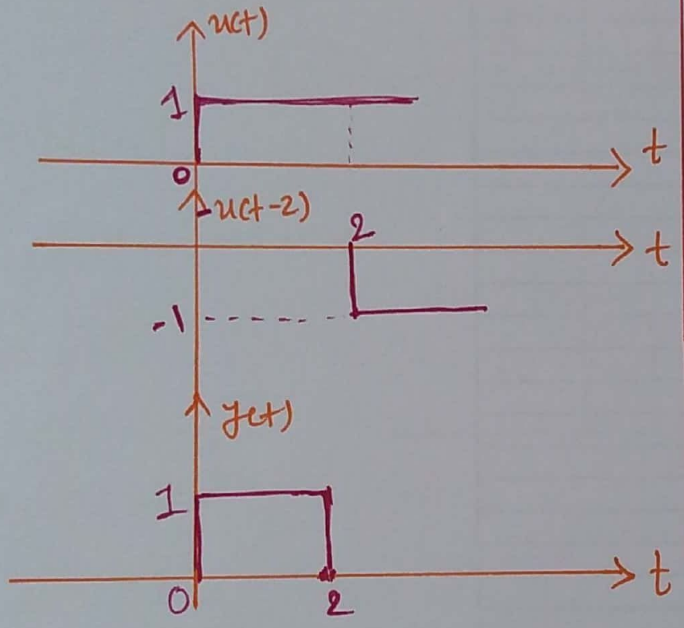
* Example of energy and Power Signal.

* If $x(t) = r(t) - r(t-2)$, then calculate energy and Power of a signal $y(t) = \frac{d\,x(t)}{dt}$.

$\rightarrow$ $y(t) = \frac{d\,x(t)}{dt}$

$= \frac{d(r(t) - r(t-2))}{dt}$

$= u(t) - u(t-2)$



$\rightarrow$ $y(t) = 1 \quad 0 \le t \le 2$

$= 0 \quad \text{else.}$

$\rightarrow$ $E = \int_{-T}^{T} y^2(t)\, dt$

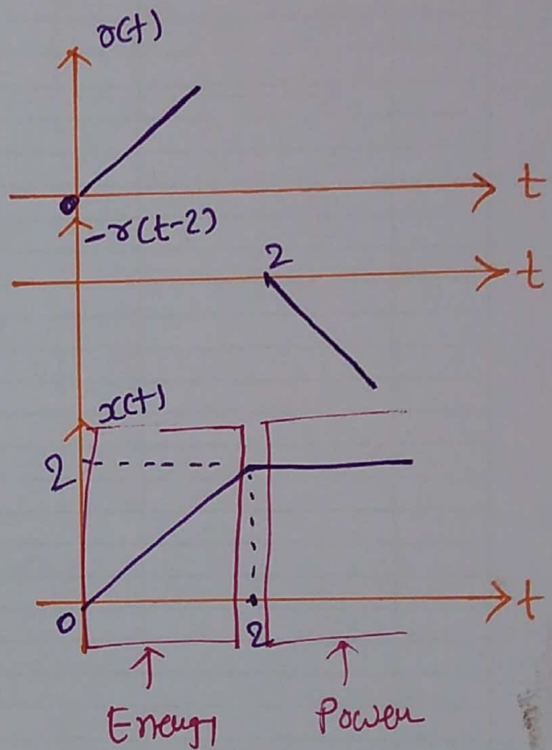
$= \int_{0}^{2} (1)^2\, dt$

$= [t]_0^2$

$\boxed{E = 2\,J}$

$\rightarrow$ $\boxed{P = 0\,W}$

$\rightarrow$ $x(t) = r(t) - r(t-2)$



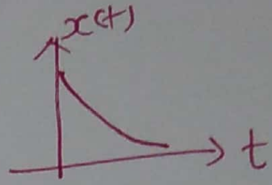
$\rightarrow$ $x(t)$ is Energy + Power signal.

$\rightarrow$ Resultant signal is Power signal.

Estimation of Energy & Power for discrete Signal.

Check signal is energy or Power for different cond.^{n} of variable a.

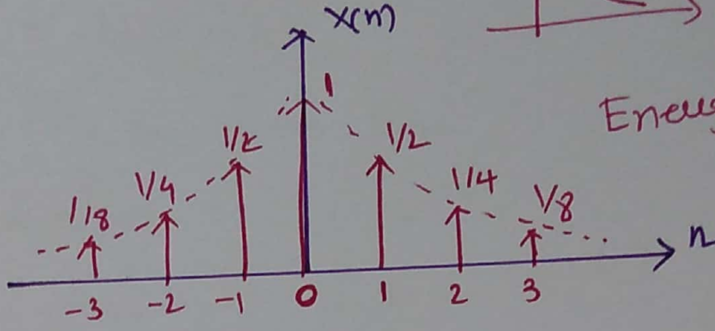
$$x(n) = a^n u(n)$$



* $|a| < 1$

$|a| = 1/2$

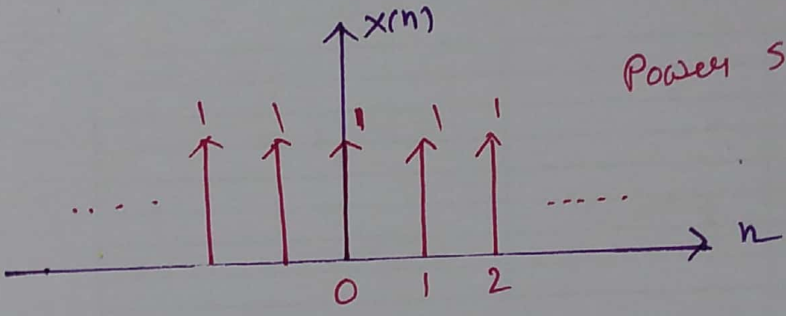
$x(n) = (1/2)^n u(n)$



Energy Signal.

* $|a| = 1$

$x(n) = (1)^n u(n)$

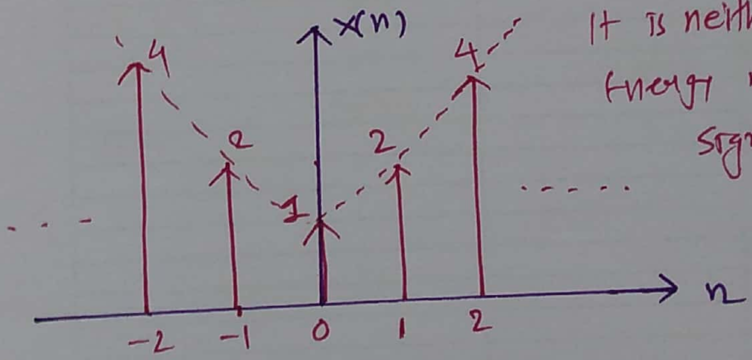


Power signal.

* $|a| > 1$

$|a| = 2$

$x(n) = 2^n u(n)$

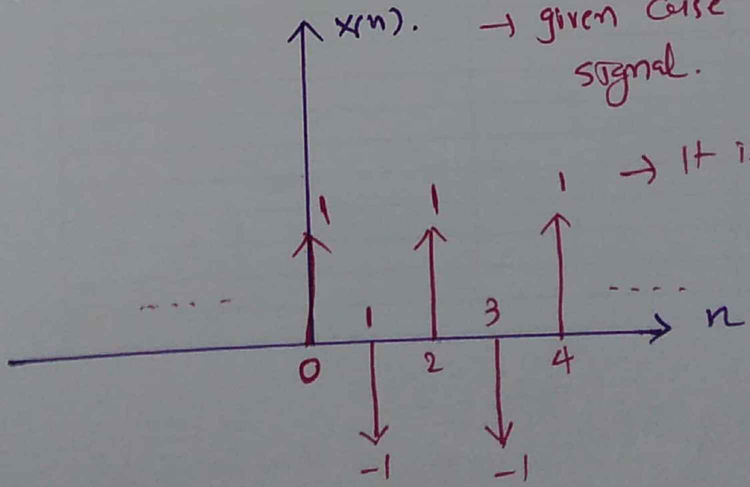


It is neither Energy nor power signal.

* $a =$ – Negative Value

→ $a = -1$

$x(n) = (-1)^n u(n)$



→ given case is periodic signal.

→ It is power signal.

Real and Imaginary Signals.

- For a real signal, Imaginary part is zero.
- For an Imaginary signal, Real part is zero.
- Real signal, Satisfies given Cond^n

$$x(t) = x^*(t)$$

- Imaginary signal, Satisfies given Cond^n

$$x(t) = -x^*(t)$$

*→ $x(t) = at \longrightarrow x^*(t) = at$ [Real signal]

*→ $x(t) = \hat{i}\,at \longrightarrow x^*(t) = -\hat{i}\,at$ [Imag. Signal]

*→ $x(t) = e^{j\omega t} = \underset{\text{Real}}{\underline{\cos \omega t}} + j\,\underset{\text{Imag}}{\underline{\sin \omega t}}$ = complex signal.

*→ $x(t) = e^{-j\omega t} = \underset{\text{Real}}{\underline{\cos \omega t}} - j\,\underset{\text{Imag}}{\underline{\sin \omega t}}$ = Complex Signal

Classification of Systems

1.) linear and Non linear System
   - If System follows super position theorem. then System will be linear.

2) Time Variant & Time Invariant System
   - If System's char. does not change w.r.t time, then System will be time Invariant.

3) Linear time variant (LTV) & linear time Invariant (LTI)

4) Static & Dynamic System
   - Static system is memoryless system.

5) Causal & Noncausal System
   (Present & Past Input) (future Input).

6) Invertible & Noninvertible system

7) Stable & Unstable system

Linear & Non Linear System

- A System is said to be linear if it satisfies the Super position principle.
- Consider a system with Inputs $x_1(t)$, $x_2(t)$ and their Output are $y_1(t)$ & $y_2(t)$. then
- For linearity

$$T[a_1 x_1(t) + a_2 x_2(t)] = a_1 T[x_1(t)] + a_2 T[x_2(t)]$$

example $\quad y(t) = x^2(t)$

- $T[x_1(t)] = x_1^2(t)$
- $T[x_2(t)] = x_2^2(t)$
- $\text{RHS} = a_1 T[x_1(t)] + a_2 T[x_2(t)]$
  $= a_1 x_1^2(t) + a_2 x_2^2(t)$
- $\text{LHS} = T[a_1 x_1(t) + a_2 x_2(t)]$.
  $= [a_1 (x_1(t)) + a_2 x_2(t)]^2$
  $= a_1^2 x_1^2(t) + a_2^2 x_2^2(t) + 2a_1 a_2 x_1(t) x_2(t)$.
  $\neq \text{RHS}$

→ this is non linear System.

example $\quad y(t) = x(t)$.

- $T[x_1(t)] = x_1(t)$
- $T[x_2(t)] = x_2(t)$.
- $\text{R.H.S.} = \cancel{a_1 T} \; a_1 T[x_1(t)] + a_2 T[x_2(t)]$
  $= a_1 x_1(t) + a_2 x_2(t)$
- $\text{LHS} = T[a_1 x_1(t) + a_2 x_2(t)]$
  $= a_1 x_1(t) + a_2 x_2(t)$
  $= \text{R.H.S}$

→ This is linear System.

Time Variant & Time Invariant System

- A System is said to be time variant if its Input - Output characteristics changes with time, Otherwise said to be time invariant.

- The Condition for time Invariance is

$$y(n,k) = y(n-k)$$

where $y(n,k) = T[x(n-k)]$

- example $y(n) = x(n) + x(n-3)$

  - $y(n,k) = T[x(n-k)]$
    $= x(n-k) + x(n-k-3)$

  - $y(n-k) = x(n-k) + x(n-k-3)$
    $= y(n,k)$

- It is time Invariant System.

- example $y(n) = n\,x(n) + x(n-2)$

  - $y(n,k) = T[x(n-k)]$
    $= n\,x(n-k) + x(n-k-2)$

  - $y(n-k) = (n-k)\,x(n-k) + x(n-k-2)$
    $\neq y(n,k)$

- It is time variant system.

- example $y(n) = x^2(n) + 2n$

  - $y(n,k) = T[x(n-k)]$
    $= x^2(n-k) + 2n$

  - $y(n-k) = x^2(n-k) + 2(n-k)$
    $\neq y(n,k)$.

- It is time variant system.

... Variant (LTV) & Linear time Invariant (LTI)

- A System is said to be LTV when it satisfies both linearity & time variance.
- A System is said to be LTI when it satisfies both linearity & time invariance.

example $y(n) = n\,x(n)$

- $T[x_1(n)] = n\,x_1(n)$
- $T[x_2(n)] = n\,x_2(n)$
- $R.H.S = a_1 T[x_1(n)] + a_2 T[x_2(n)]$

$= a_1 n\,x_1(n) + a_2 n\,x_2(n)$

- $L.H.S = T[a_1 x_1(n) + a_2 x_2(n)]$

$= n[a_1 x_1(n) + a_2 x_2(n)]$

$= a_1 n x_1(n) + a_2 n x_2(n)$

$= R.H.S$

- It is linear system

- $y(n,k) = T[x(n-k)]$

$= n\,x(n-k)$

- $y(n-k) = (n-k)\,x(n-k)$
- there

$y(n,k) \neq y(n-k)$

- It is time variant system.

→ given system is linear time variant (LTV)

example $y(n) = n\,x^2(n)$

- $T[x_1(n)] = n\,x_1^2(n)$
- $T[x_2(n)] = n\,x_2^2(n)$
- $R.H.S = a_1 T[x_1(n)] + a_2 T[x_2(n)]$

$= a_1 n\,x_1^2(n) + a_2 n\,x_2^2(n)$

- $L.H.S = T[a_1 x_1(t) + a_2 x_2(t)]$

$= n[a_1 x_1(t) + a_2 x_2(t)]^2$

$= a_1^2 n x_1^2(t) + a_2^2 n x_2^2(t) + 2a_1a_2 x_1(t)\,x_2(t).$

$\neq RHS.$

- It is non linear system.

- $y(n,k) = T[x(n-k)]$

$= n\,x^2(n-k)$

- $y(n-k) = (n-k)\,x^2(n-k)$
- $y(n,k) \neq y(n-k)$
- It is time variant system.

→ Given system is neither LTV nor LTI system.

Static & Dynamic System

- Static system is memory less system [- Present Input]
- Dynamic system is memory system

① $y(n) = x(n)$

- If $n=0$

$y(0) = x(0)$

- o/p depends on present Input, so given system is static.

② $y(n) = x^2(n) + x(n-2)$

- If $n=0$

$y(0) = x^2(0) + x(-2)$

- o/p depends on present & Past Input, so given system is dynamic.

③ $y(n) = x(n) + x(n+3)$

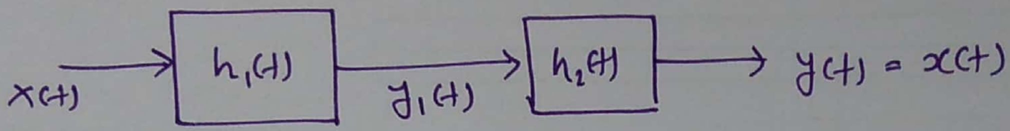
- If $n=0$

$y(0) = x(0) + x(3)$

- o/p depends on present & future Input, so given system is dynamic.

Invertible & Non - Invertible System

- A system is said to be If the Input of system appears at the Output



- Response of $h_1(t)$

$$y_1(t) = x(t) * h_1(t)$$

- In freq. domain

$$y_1(f) = x(f)\, h_1(f)$$

→ Response of $h_2(t)$

$$y(t) = y_1(t) * h_2(t)$$
$$= x(t) * h_1(t) * h_2(t)$$

→ In freq. domain

$$y(f) = x(f)\, h_1(f)\, h_2(f).$$

→ If $\boxed{h_1(f)\, h_2(f) = 1}$

$$\boxed{y(f) = x(f)}$$

→ by Inverse X'form

$$\boxed{y(t) = x(t).}$$

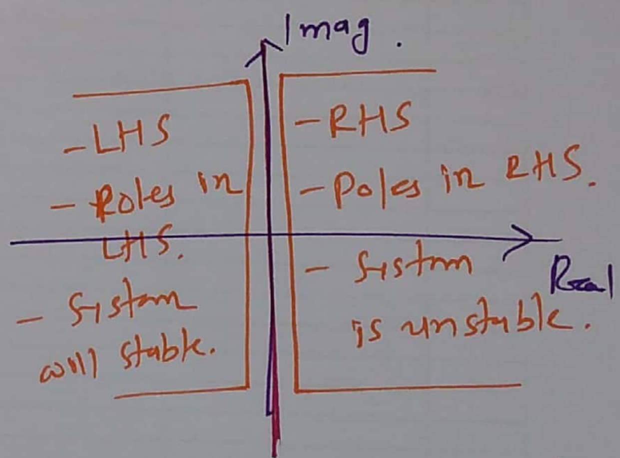
Stable & Unstable System

- A System is said to stable System when it produces Bounded Output for a bounded Input.

- Ex $y(n) = x^2(n)$
  - $\boxed{x(n) = u(n)}$
  - $\Rightarrow y(n) = u^2(n)$
  - $\Rightarrow \boxed{y(n) = u(n)}$
- Hence Bounded o/p is appearing.
- So system is stable.

- Ex $y(t) = 2x^2(t) + x(t)$
  - $\boxed{x(t) = u(t)}$
  - $\Rightarrow y(t) = 2u^2(t) + u(t)$
  - $= 2u(t) + u(t)$
  - $= 3u(t)$.
- Hence BIBO is appearing
- So System is stable.

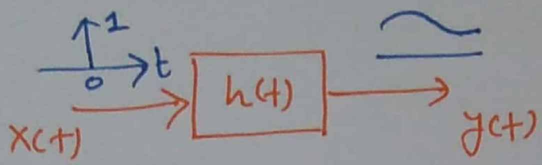
- Ex $y(t) = \int x(t)\, dt$
  - $\boxed{x(t) = u(t)}$
  - $\Rightarrow y(t) = \int u(t)\, dt$
  - $= r(t)$.
- o/p is unbounded
- So System is unstable.



- Poles on Imag axis
- Critically stable.

Impulse Response of a System

- The response of the system for an impulse input is called impulse response of the system.
- Impulse response denoted as $h(t)$ for continous domain and $h(n)$ in discrete domain.



- In time domain

$$y(t) = x(t) * h(t)$$

- In freq. domain

$$\Rightarrow Y(\omega) = X(\omega)\, h(\omega) \quad \Big| \quad Y(s) = X(s)\, h(s) \quad \Big| \quad Y(z) = X(z)\, h(z)$$

$$\Rightarrow \boxed{h(\omega) = \frac{Y(\omega)}{X(\omega)}} \longleftarrow \text{Transfer function}$$

- Inverse fourier X'form

$$\Rightarrow F^{-1}[h(\omega)] = h(t)$$

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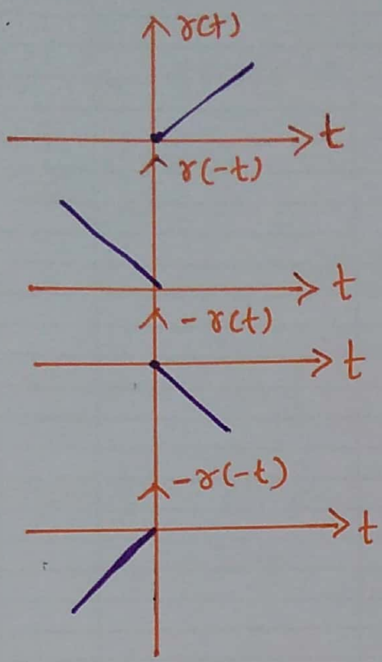
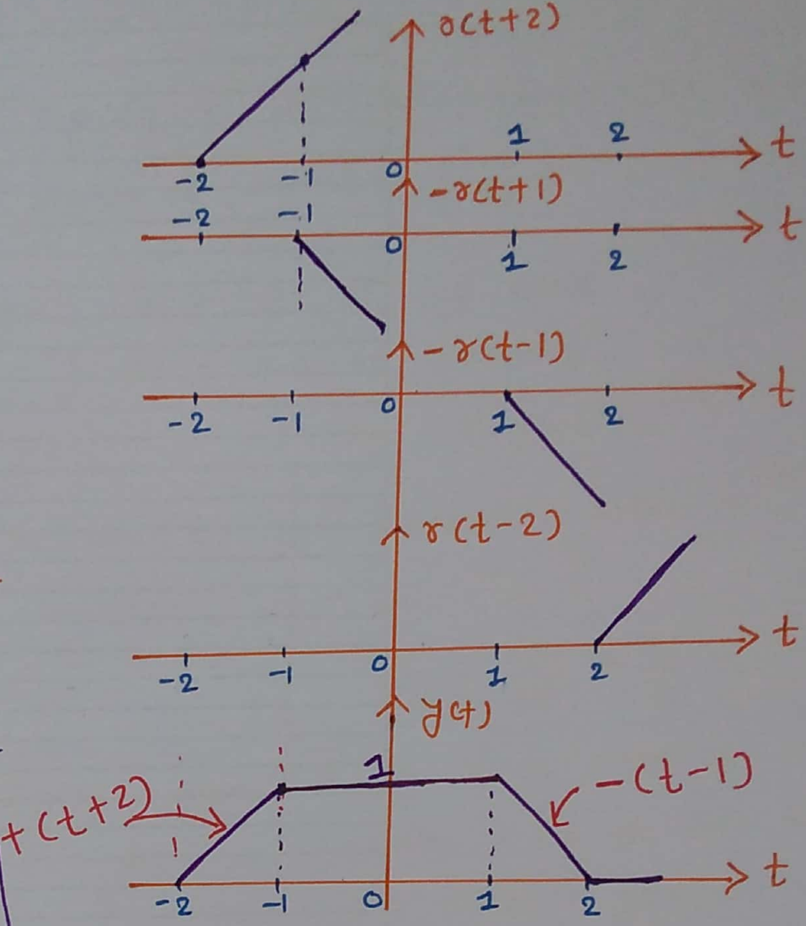
Ramp signal example based on graph & function

- Draw a signal and represent it in form of function

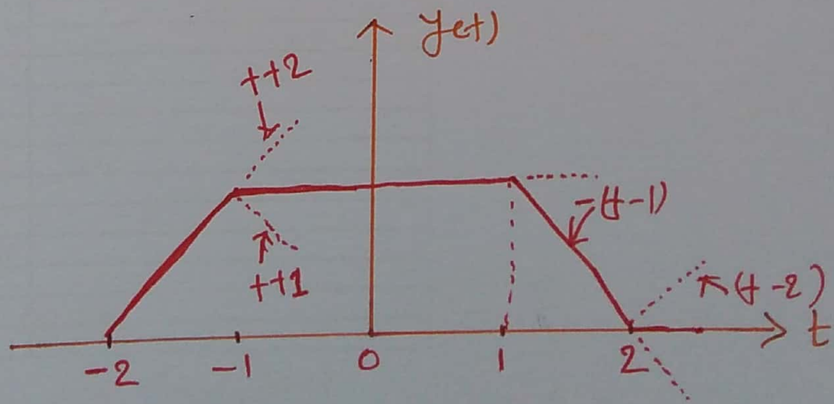
$$y(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$$





$$\boxed{\begin{aligned} y(t) &= t+2 && -2 < t < -1 \\ &= 1 && -1 < t < 1 \\ &= -t+1 && 1 < t < 2 \end{aligned}}$$

$$y(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$$



Checking of Periodicity of discrete Signal

- A discrete signal $x(n)$ is said to be periodic when

$$x(n) = x(n+N)$$

- If a discrete signal is periodic then the ratio $\frac{\omega_0}{2\pi}$ must be rational number $\left[\frac{m}{N}\right]$.

where, m = no of full cycle
N = no of samples.

Ex $x(n) = \cos\left(\frac{n}{4}+\pi\right)$

- Compare it with $\cos(\omega n + \phi)$
- $\omega = \frac{1}{4}$, $\phi = \pi$
- $\frac{\omega}{2\pi} = \frac{1/4}{2\pi} = \frac{1}{8\pi}$
- $\omega/2\pi$ is not a rational number so given signal is aperiodic.

Ex $x(n) = e^{\frac{j\pi n}{4}}$

- Compare it with $e^{j\omega n}$
- $\omega = \pi/4$
- $\frac{\omega}{2\pi} = \frac{\pi/4}{2\pi} = \frac{1}{8} = \frac{m}{N}$
- $\frac{\omega}{2\pi}$ is a rational number with $N=8$ number samples.
- So signal is periodic with 8 numbers of sample.

Ex $x(n) = \sin\left(\frac{\pi n}{2}\right) + \cos\left(\frac{\pi n}{4}\right)$

- $\omega_1 = \pi/2$
- $\frac{\omega_1}{2\pi} = \frac{\pi/2}{2\pi} = \frac{1}{4} = \frac{m}{N_1}$
- $N_1 = 4$

- $\omega_2 = \pi/4$
- $\frac{\omega_2}{2\pi} = \frac{\pi/4}{2\pi} = \frac{1}{8} = \frac{m}{N_2}$
- $N_2 = 8$

- Resultant Periodicity

$$N = \text{LCM}(N_1, N_2)$$
$$= \text{LCM}(4, 8)$$
$$= 8$$

4	8
×2 →	2
×2 →	2
	2

- Resultant Signal $x(n)$ is periodic with $N = 8$ samples.

Impulse function Examples based on Integration

① $\int_{-3}^{\infty} (t+1)\, \delta(t)\, dt$

- $\delta(t)$ means at $t=0$, there is impulse
- $t=0$ is available in limit range from -3 to ∞

$= (t+1)\big|_{t=0} = 0+1 = \boxed{1}$

② $\int_{-3}^{-5} (t^2+1)\, \delta(t)\, dt$

- $\delta(t)$ means at $t=0$, there is impulse.
- $t=0$ is not available in limit range from -3 to -5

$= 0$

③ $\int_{-\infty}^{\infty} e^{(2-t)}\, \delta(t-2)\, dt$

- $\delta(t-2)$ means at $t=2$, there is impulse.
- $t=2$ is available in limit range from $-\infty$ to ∞

$= e^{(2-t)}\big|_{t=2} = e^{2-2} = e^{0} = \boxed{1}$

④ $\int_{-\infty}^{\infty} \sin(3(t-1))\, \delta(3t+4)\, dt$

$= \int_{-\infty}^{\infty} \sin(3(t-1))\, \delta(3(t+4/3))\, dt$

$\left[\delta(at) = \frac{1}{a}\delta(t)\right]$

$= \int_{-\infty}^{\infty} \sin(3(t-1)) \left[\frac{1}{3}\delta(t+4/3)\right] dt$

$= \int_{-\infty}^{\infty} \frac{1}{3} \sin(3(t-1))\, \delta(t+4/3)\, dt$

- $\delta(t+4/3)$ means at $t=-4/3$, there is impulse.
- $t=-4/3$ is available in limit range from $-\infty$ to ∞

$= \frac{1}{3}\sin(3(t-1))\big|_{t=-4/3}$

$= \frac{1}{3}\sin(3(-4/3-1)) = \frac{1}{3}\sin\left(\cancel{3}\left(-\frac{7}{\cancel{3}}\right)\right) = \boxed{\frac{-1}{3}\sin 7}$

Examples of Stability Check.

- Process – For stability check, System should give BIBO.
  - As a test Input, we use unit step $u(t)$ signal.

① $y(t) = x^2(t)$

- If $x(t) = u(t)$
- $y(t) = (u(t))^2$
  $= u(t)$
- Here we have BIBO
- given System is Stable.

② $y(t) = x(t) \cos \omega t$

- If $x(t) = u(t)$
- $y(t) = u(t) \cos \omega t$
  $= \cos \omega t \mid t \in (0, \infty)$
- $\cos \omega t$ varries in between -1 to 1
- hence we have BIBO, so System is stable.

③ $y(t) = x(t-2)$

- If $x(t) = u(t)$
- $y(t) = u(t-2)$
- Here we have BIBO
- So given System is stable.

④ $y(t) = \int x(t)\,dt$

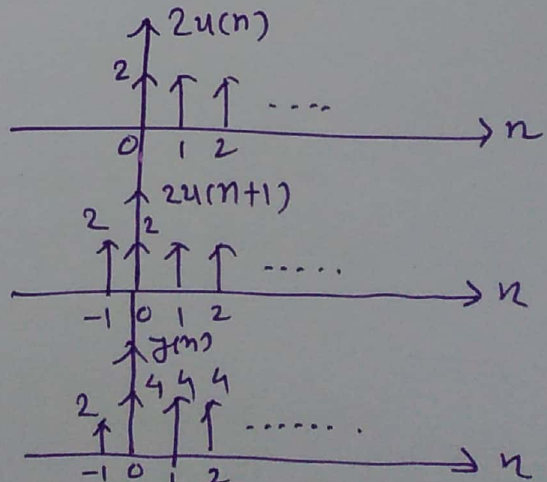
- If, $x(t) = u(t)$
- $y(t) = \int u(t)\,dt$
  $= r(t)$.
- Ramp signal is not bounded as It's Amplitude is ∞ at $t = \infty$.
- So given System is unstable.

⑤ $y(t) = \dfrac{d\,x(t)}{dt}$

- If $x(t) = u(t)$
- $y(t) = \dfrac{d\,u(t)}{dt}$
  $= \delta(t)$
- Impulse Signal $\delta(t)$ is not bounded Signal.
- So given System is unstable.

⑥ $y(n) = 2x(n) + 2x(n+1)$

- If $x(n) = u(n)$
- $y(n) = 2u(n) + 2u(n+1)$



- Here BIBO, So System is Stable.

Checking Causality

- For Causality - System response should be depending on present or past Inputs
  - If system response depends upon future Input then system is non Causal.

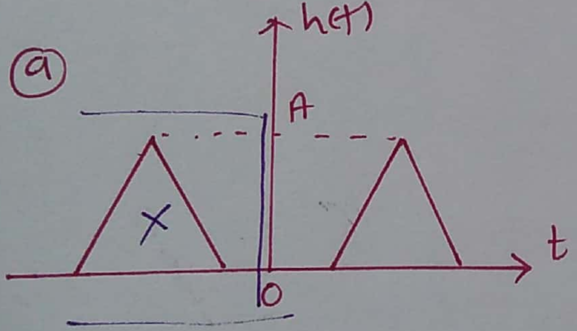
An excitation is applied to a system at time $t = T$. and its response is zero for $-\infty < t < T$, such a system is

→ Input is applied at $\boxed{t = T}$

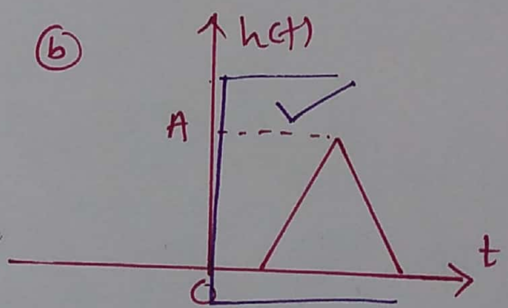
(a) Non causal
(b) Stable
(c) Causal system ✓
(d) Unstable system

which of the given system is causal system

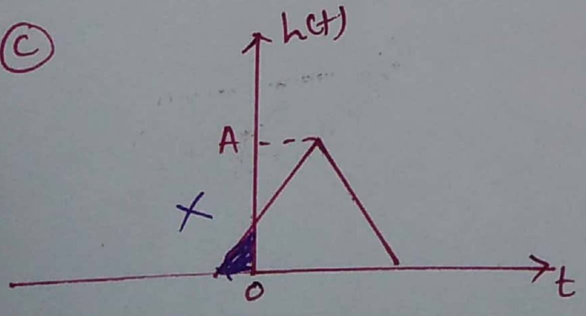
(a)



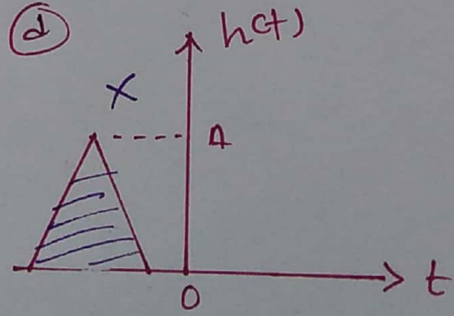
(b)



(c)



(d)



Cheeck causality for given ~~system~~. System.

$$h(n) = \underbrace{u(n+3)}_{\text{future}} + \underbrace{u(n-2)}_{\text{Past}} + \underbrace{u(n)}_{\text{Present}}$$

- this is non Causal system.