Problems:

Ex:- Ten students got the following percentage of marks in two subjects as:

Roll No	1	2	3	4	5	6	7	8	9	10
Sub-1	78	36	98	25	75	82	90	62	65	39
Sub-2	84	51	91	60	68	62	86	58	53	47

Calculate the coefficient of correlation.

~~Ex:-) Ten students~~

Sol^{n}:-> Let us consider the two subjects by x and y respectively.

Since, the values of x and y are large, so we change the scale and origin (to reduce calculation) by choosing,

$$u = x - 65, \quad v = y - 66$$

Now, we make the following table

$$u = \frac{x-a}{h}, \quad v = \frac{y-b}{k}$$

Here, $a = 65$, $b = 66$, $k = h = 1$

x	y	$u = x - 65$	$v = y - 66$	u^2	v^2	uv
78	84	13	18	169	324	234
36	51	-29	-15	841	225	435
98	91	33	25	1089	625	825
25	60	-40	-6	1600	36	240
75	68	10	2	100	4	20
82	62	17	-4	289	16	-68
90	86	25	20	625	400	500
62	58	-3	-8	9	64	24
65	53	0	-13	0	169	0
39	47	-26	-19	676	361	494
Total		0	0	5398	2224	2704

Now, $\bar{u} = \frac{1}{n}\sum' u = 0$, $\bar{v} = \frac{1}{n}\sum' v = 0$

$$\therefore r_{xy} = r_{uv} = \frac{\frac{1}{n}\sum' u_i v_i - \bar{u}\,\bar{v}}{\sqrt{\left(\frac{1}{n}\sum' u_i^2 - \bar{u}^2\right)\left(\frac{1}{n}\sum' v_i^2 - \bar{v}^2\right)}}$$

[Just write the formula for r_{xy} and replace ~~xxx~~ x and y by u and v respectively]

$$= \frac{\frac{1}{10} \times 2704 - 0}{\sqrt{\left(\frac{1}{10} \times 5398 - 0^2\right)\left(\frac{1}{10} \times 2224 - 0^2\right)}}$$

$$= \frac{270.4}{\sqrt{539.8 \times 222.4}}$$

$$= .78$$

Here, $r_{xy} > 0$, so the correlation is positive.

Ex:- A computer while calculating correlation co-efficient between two variables X and Y from 25 pairs of observations obtained the following results:

$n = 25$, $\sum X = 125$, $\sum X^2 = 650$,

$\sum Y = 100$, $\sum Y^2 = 460$, $\sum XY = 508$

It was however later discovered at the time of checking that he had copied down two pairs as

X	Y
6	14
8	6

while the correct values were,

X	Y
8	12
6	8

Obtain the correct value of correlation co-efficient.

Soln Corrected $\sum X = 125 - 6 - 8 + 8 + 6 = 125$.

Corrected $\sum Y = 100 - 14 - 6 + 12 + 8 = 100$

" $\sum X^2 = 650 - 6^2 - 8^2 + 8^2 + 6^2 = 650$

" $\sum Y^2 = 460 - 14^2 - 6^2 + 12^2 + 8^2 = 436$

" $\sum XY = 508 - 6 \times 14 - 8 \times 6 + 8 \times 12 + 6 \times 8 = 520$

$$\therefore \bar{x} = \frac{1}{n}\sum x = \frac{1}{25} \times 125 = 5 \,,$$

$$\bar{y} = \frac{1}{n}\sum y = \frac{1}{25} \times 100 = 4 \quad \text{and}$$

$\sum fx$

$$\therefore \text{Correlated, } r_{xy} = \frac{\frac{1}{n}\sum xy - \bar{x}\,\bar{y}}{\sqrt{\left(\frac{1}{n}\sum x^2 - \bar{x}^2\right)\left(\frac{1}{n}\sum y^2 - \bar{y}^2\right)}}$$

$$= \frac{\frac{1}{25} \times 520 - 5 \times 4}{\sqrt{\left(\frac{1}{25} \times 650 - 25\right)\left(\frac{1}{25} \times 436 - 16\right.}}$$

$$= \frac{20.8 - 20}{\sqrt{1 \times 1.44}}$$

$$= 0.66$$

Ex:->

Ex:- If $z = ax + by$ and r is the correlation co-efficient betⁿ x and y, show that,

$$\sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2abr\sigma_x\sigma_y$$

Sol^n $z_i = ax_i + by_i$. (given)

$$\Rightarrow \bar{z} = a\bar{x} + b\bar{y}$$

$$\therefore (z - \bar{z}) = a(x - \bar{x}) + b(y - \bar{y})$$

Now, $\sigma_z^2 = \frac{1}{n}\sum (z - \bar{z})^2$

$$= \frac{1}{n}\sum [a(x-\bar{x}) + b(y-\bar{y})]^2$$

$$= \frac{1}{n}\sum [a^2(x-\bar{x})^2 + b^2(y-\bar{y})^2 + 2ab(x-\bar{x})(y-\bar{y})]$$

$$= a^2\frac{1}{n}\sum (x-\bar{x})^2 + b^2\frac{1}{n}\sum (y-\bar{y})^2 + 2ab\frac{1}{n}\sum (x-\bar{x})(y-\bar{y})$$

$$= a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\sigma_x\sigma_y \frac{\frac{1}{n}\sum (x-\bar{x})(y-\bar{y})}{\sigma_x\sigma_y}$$

$$= a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\,r\,\sigma_x\sigma_y$$

where, $r = \dfrac{\frac{1}{n}\sum (x-\bar{x})(y-\bar{y})}{\sigma_x\sigma_y}$.