SIGNALS AND SYSTEMS

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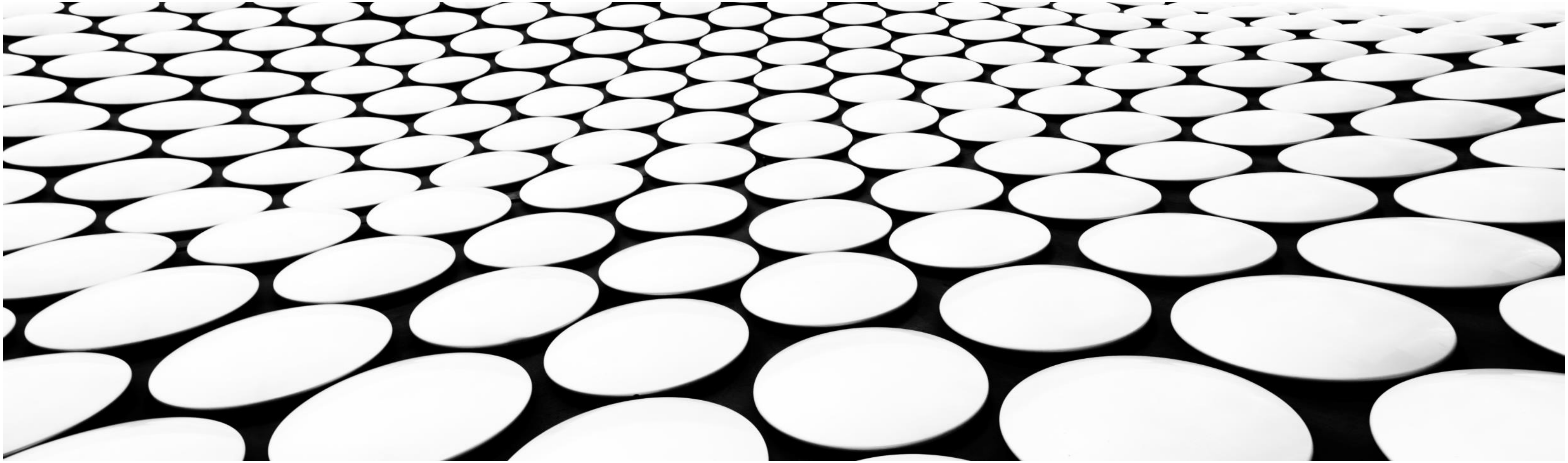


Continuous signal → Discrete Signals.

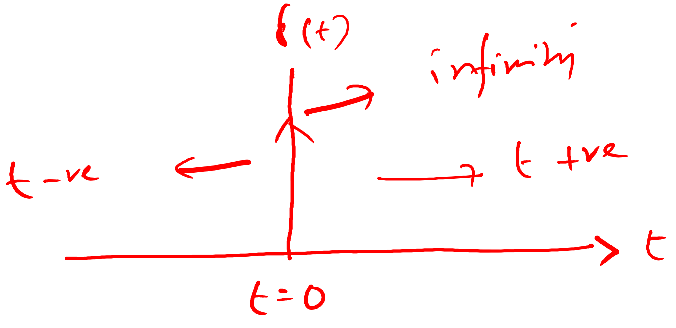
~~Impulse~~ of force = force × time, $t \to 0$

Standard Continuous Signals:-

1) Impulse Signal:-

$$\delta(t) = \infty \;;\; \text{when } t = 0$$
$$= 0 \;;\; \text{otherwise}$$

or $\boxed{\int_{-\infty}^{\infty} \delta(t)\, dt = A}$

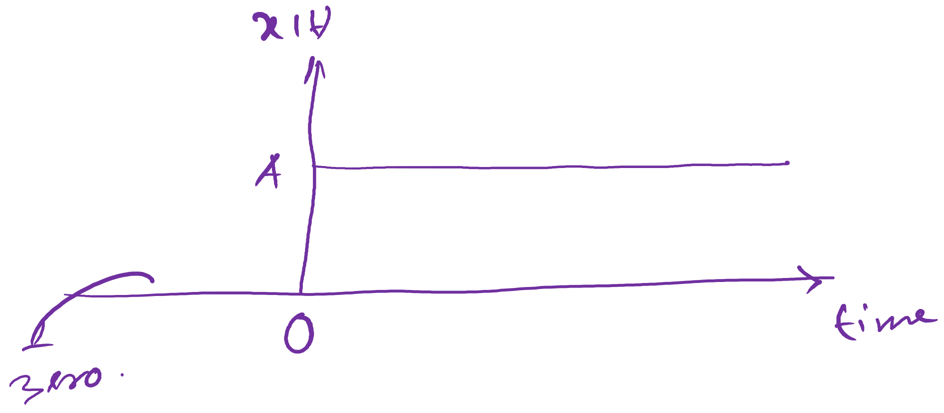


$$\left.\begin{aligned} \delta(t) &= 1 \;;\; t = 0 \\ &= 0 \;;\; \text{otherwise} \\ \int_{-\infty}^{\infty} \delta(t)\, dt &= 1 \end{aligned}\right\}$$

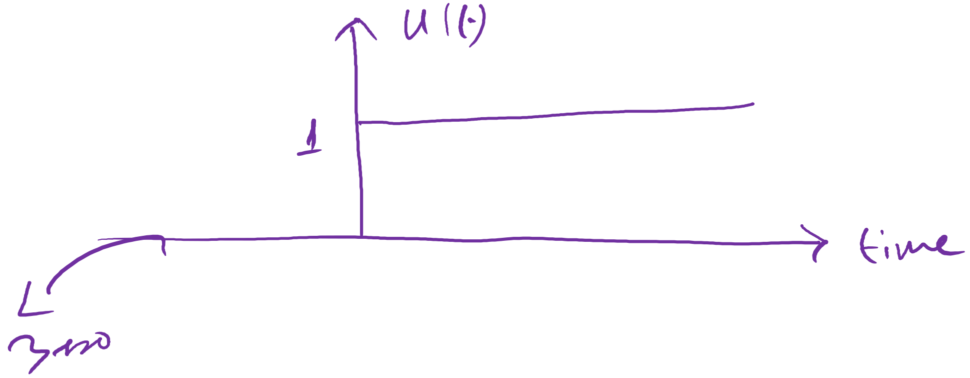
unit Impulse signal → 1

2) Step signal:-

$$x(t) = A \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$



Unit step signal $u(t) = 1 \; ; \; t \geq 0$
$$= 0 \; ; \; t < 0$$



Ramp Signal:-

$y = mx$

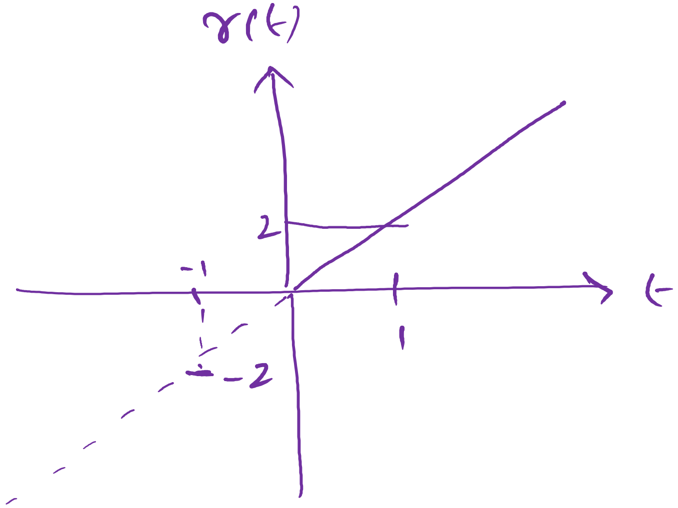
$x(t) = At \; ; \; t \geq 0$

$\quad\;\, = 0 \; ; \; t < 0$

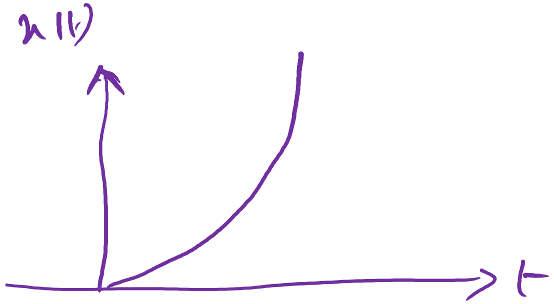
$A = 2 \quad . \; t = 1$

$x(t) = 2. \qquad t = 1$

$x(t) = -2$



Parabolic Signal :-

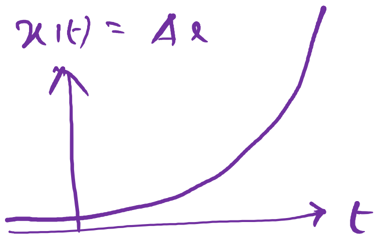


$$x(t) = A\frac{t^2}{2} \quad \text{for } t \geq 0$$

$$= 0 \; ; \; t < 0$$

Exponential Signal :-

$x(t) = Ae^{bt}$



$x(t) = Ae^{bt} \longrightarrow$ Exponentially increasing signal

$x(t) = Ae^{-bt}$

$x(t) = Ae^{-bt} \longrightarrow$ Exponentially decaying signal

Complex Exponential Signal:-

$s = vt$

$\theta = \omega t$

$$x(t) = Ae^{j\omega t}$$

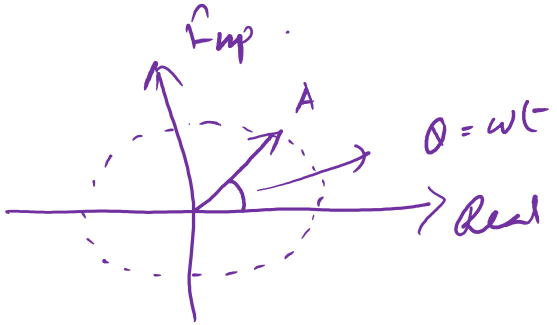
$$e^{j\theta} = \cos\theta + j\sin\theta$$

Euler's Identity

ω = Angular freq. in rad/sec.

$\omega = 2\pi f$, f = frequency (Hz) $\rightarrow$ cycles/ per.

$\Rightarrow \omega = \dfrac{2\pi}{T}$, T = Time period in second.

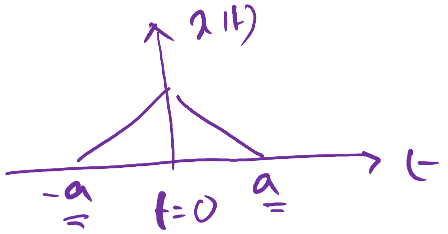


$$x(t) = Ae^{j\omega t} = A(\cos\omega t + j\sin\omega t)$$

Real part $x(t) = A\cos\omega t$

Img. part $x(t) = A\sin\omega t$

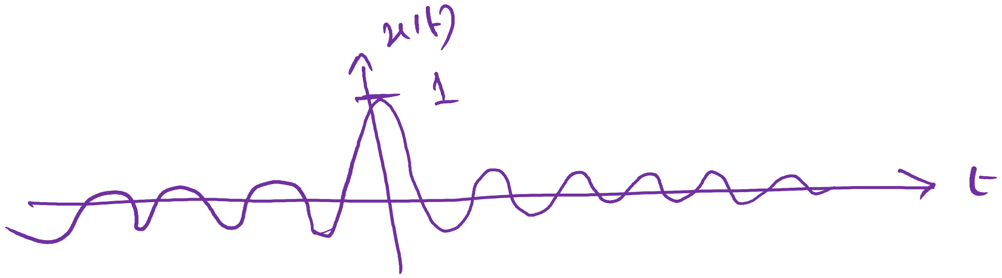
Triangular pulse :-



$$x(t) = \Delta_a(t) = 1 - \frac{|t|}{a} \quad ; \ |t| \le a$$

$$= 0 \quad ; \ |t| > a$$

Sinc function/signal



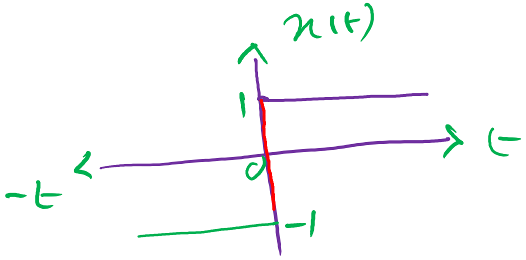
$$x(t) = \underline{\text{Sinc}}(t) = \frac{\sin t}{t} \quad ; \ -\infty < t < \infty$$

Signum signal:-

$$x(t) = \text{sgn}(t) = 1 \quad ; \ t > 0$$

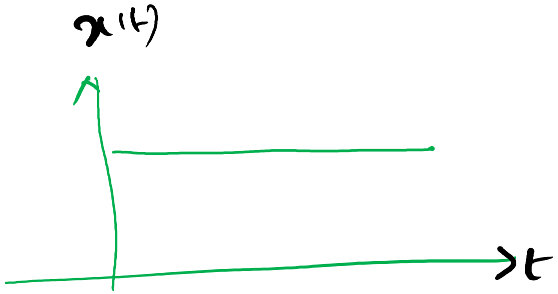
$$= 0 \quad ; \ t = 0$$

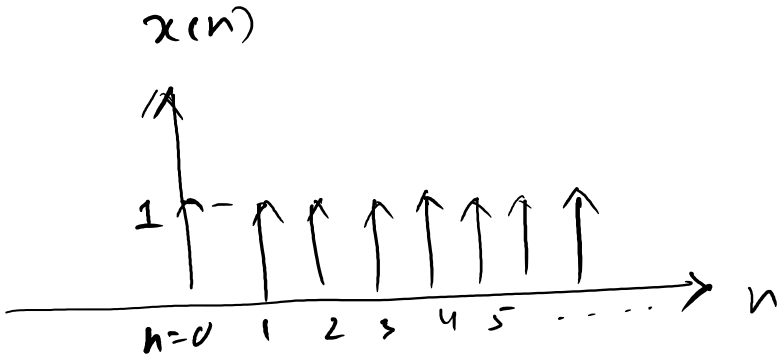
$$= -1 \quad ; \ t < 0$$



Discrete time signal:-

unit step signal





$u(t) = 1 \;;\; t \geq 0$

$\quad\;\; = 0 \;;\;$ otherwise

$$\left.\begin{aligned} u(n) &= 1 \;;\; n \geq 0 \\ &= 0 \;;\; n < 0 \end{aligned}\right\}$$ unit step signal in discrete domain.

Classification of continuous time signals:-

Depending on their characteristics

1) Deterministic and non-deterministic
2) periodic and Non-periodic
3) Symmetric and Asymmetric [even and odd]
4) Energy and power
5) Causal and Non-causal.

Deterministic :- signals that can be described / expressed using the help of a mathematical equation / Exp.

Non- Determi. :- Random nature → No definite charc. → No def. mathematical Exp.

Noise → sound coming from amplifier, Noise within the Rx.

Periodic signal: - $x(t)$

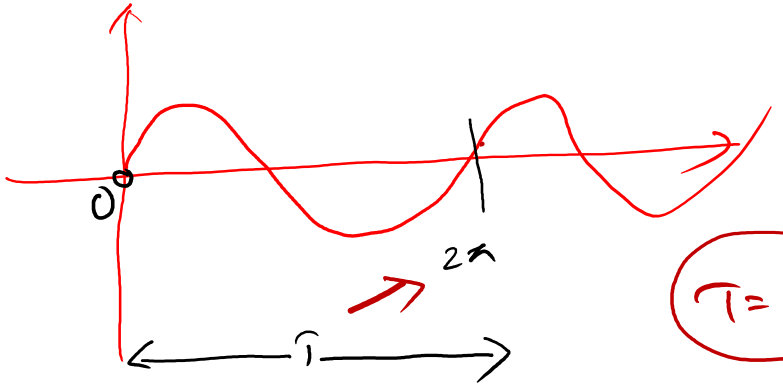
$$x(t+T) = x(t)$$

$x(t+T) \neq x(t) \rightarrow$ Aperiodic or Non-periodic

T = fundamental time period. (second)

f = fundamental freq. (Hz)

$$w_0 = \frac{2\pi}{T}$$



$T = 2\pi$

$t \rightarrow t+T$

$$x(t) = A \sin w_0 t$$

$$x(t+T) = A \sin w_0 (t+T)$$

$$= A \sin(w_0 t + w_0 T)$$

$$= A \sin(w_0 t + w_0 2\pi)$$

X

$$= A \sin\left(w_0 t + \frac{2\pi}{T} \times 2\pi\right)$$

$$= A \sin\left(w_0 t + \frac{2\pi}{\cancel{T}} \cdot \cancel{T}\right)$$

$$= A \sin(w_0 t + 2\pi)$$

$$= A \sin w_0 t$$

Complex exponential signal:-

$$x(t) = Ae^{j\omega_0 t}$$

$$x(t+T) = Ae^{j\omega_0 (t+T)}$$

$$= Ae^{j\omega_0 t} \cdot e^{j\omega_0 T}$$

$$= Ae^{j\omega_0 t} \cdot e^{j\frac{2\pi}{\cancel{T}} \cdot \cancel{T}}$$

$$= Ae^{j\omega_0 t} \cdot e^{j2\pi} \to 1$$

$$x(t+T) = Ae^{j\omega_0 t}$$

$$\therefore x(t) = x(t+T) \to$$

$$x(t) = 5e^{-3\omega t}$$

$$e^{j2\pi} = \overset{1}{\cancel{\cos 2\pi}} + j\overset{0}{\cancel{\sin 2\pi}}$$

$$=$$

$x(t) = 3\cos\left(5t + \frac{\pi}{6}\right)$

$A\sin(\omega t + \phi)$ or $A\cos(\omega t + \phi)$

All sinusoidal signals are periodic

$x(t+T) = 3\cos\left[5(t+T) + \pi/6\right] = 3\cos\left[5t + 5T + \pi/6\right] = 3\cos\left((5t + \pi/6) + 5T\right)$

$\cancel{5\pi}\ \frac{2\pi}{\cancel{5}}$

We know, $2\pi \rightarrow$ standard time period for any sinusoidal signal.

$5T = 2\pi$

$\Rightarrow T = \frac{2\pi}{5}$

$= 3\cos\left((5t + \pi/6) + 2\pi\right)$

$= 3\cos(5t + \pi/6)$

Determine the periodicity of the signal:-

1) $x(t) = 2\cos\frac{2\pi t}{3} + 3\cos\frac{2\pi t}{7}$ → $x(t+21) =$

$x(t) = x_1(t) + x_2(t)$

$x_1(t) = 2\cos\frac{2\pi t}{3}$

$\boxed{A\cos\omega_0 t}$ standard cosine f^n

$\omega_0 = \frac{2\pi}{3}$

$\omega = \frac{2\pi}{3}$

$\omega = \frac{2\pi}{T}$

$\underline{\underline{T_1 = 3}}$

$\cancel{2\pi} f = \frac{\cancel{2\pi}}{3}$

$\therefore f = \frac{1}{3}$

$T = \frac{1}{f} = 3$

$x_2(t) = 3\cos\frac{2\pi t}{7}$

$\omega = \frac{2\pi}{7}$ $\quad \omega = \frac{2\pi}{T}$

$\therefore T_2 = 7$

$\frac{T_1}{T_2} = \frac{3}{7}$

$T_0 = \frac{\text{LCM of Num } (T_1, T_2)}{\text{HCF of } \cancel{\text{Deno}} \ (T_1, T_2)} = \frac{3 \times 7}{1} = 21$

$A\cos\omega t \quad A\sin\omega t \quad 2\pi$

$x(t) = 2\cos 3t + 3\sin 7t$

$\omega_1 = 3 \qquad \omega_2 = 7$

$\Rightarrow \frac{2\pi}{T_1} = 3 \qquad \Rightarrow \frac{2\pi}{T_2} = 7$

$\Rightarrow T_1 = \frac{2\pi}{3} \qquad \Rightarrow T_2 = \frac{2\pi}{7}$

$$\frac{T_1}{T_2} = \frac{2\pi/3}{2\pi/7} = \frac{7}{3}$$

$x(t) = \boxed{5\cos 4\pi t + 3\sin 8\pi t}$

Period $= 21 \div \frac{21}{2\pi} = 2\pi$

$2\cos(3t + 2\pi) = 2\cos 3t$

$3\sin(7t + 2\pi) = 3\sin 7t$