Diode Circuit Analysis

Semiconductor Devices and Circuits

(ECE 181302)

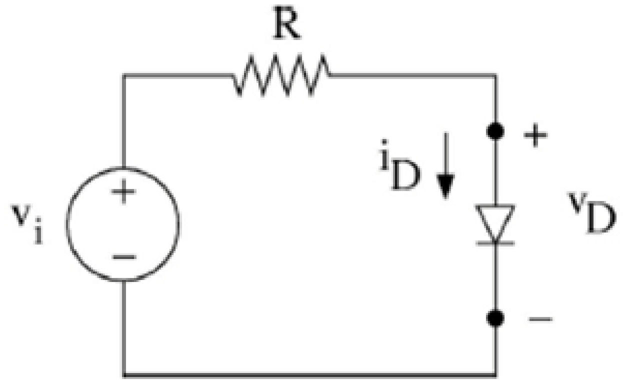
5th October 2021

Diode Circuit Equations are Nonlinear

KCL : current i_D in all elements

KVL : $v_i = Ri_D + v_D$

$$i_D = I_S\left(e^{v_D/nV_T} - 1\right)$$



- Two equation in two-unknowns to solve for i_D and v_D
- Non-linear equation: cannot be solved analytically
- **Solution methods:**
  - Numerical (PSpice)
  - Graphical (load-line)
  - Approximation to get linear equations (diode piece-linear model)

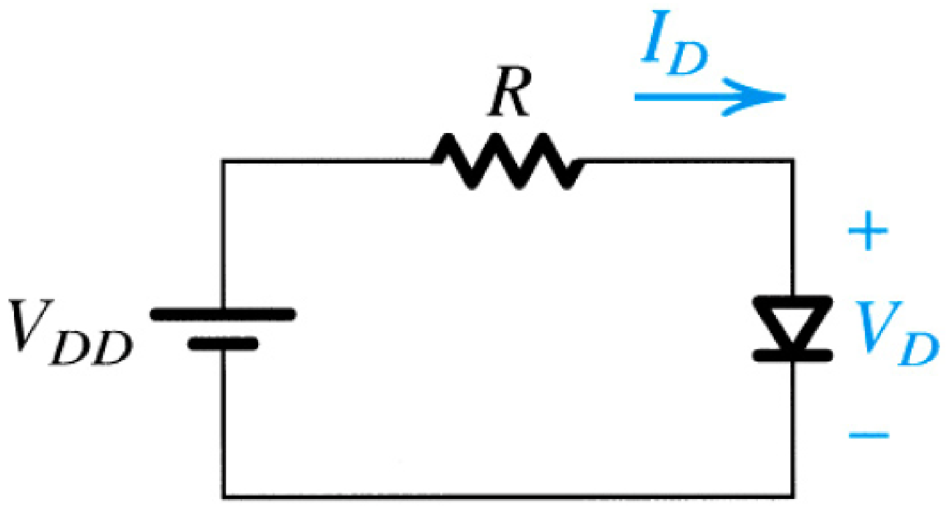
Modeling the Diode Forward Characteristic

- To aid in analysis, define a robust set of diode models.
- Simplified diode models are better suited for use in circuit analysis and design of diode circuits.
- Exponential diode model based on Shockley's Equation:

$$i_D = I_S\left[\exp\left(\frac{qv_D}{nkT}\right)-1\right] = I_S\left[\exp\left(\frac{v_D}{nV_T}\right)-1\right]$$

  - Most accurate.
  - Most difficult to employ in circuit analysis due to nonlinear nature.

Exponential Diode Model

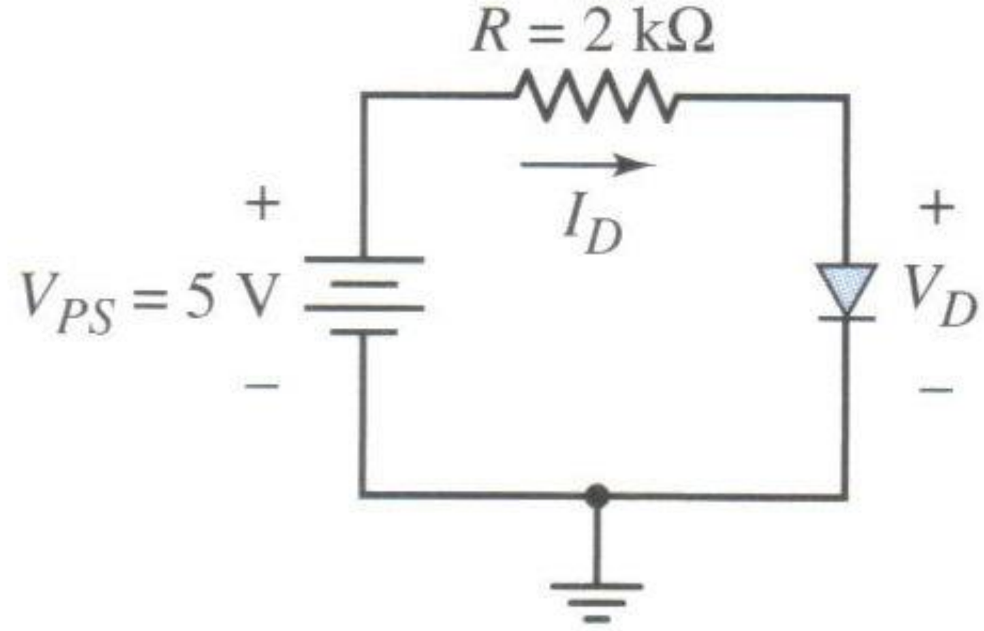


$$I_D = I_S e^{V_D / V_T}$$ from diode current equation.

Also $I_D = \dfrac{V_{DD} - V_D}{R}$ from network equation or KVL.

Example:

Question**:** How does one solve for I_D in the circuit?



- Consider $I_S = 10^{-13}$ A.

- So:

$$V_{PS} = I_D R + V_D$$

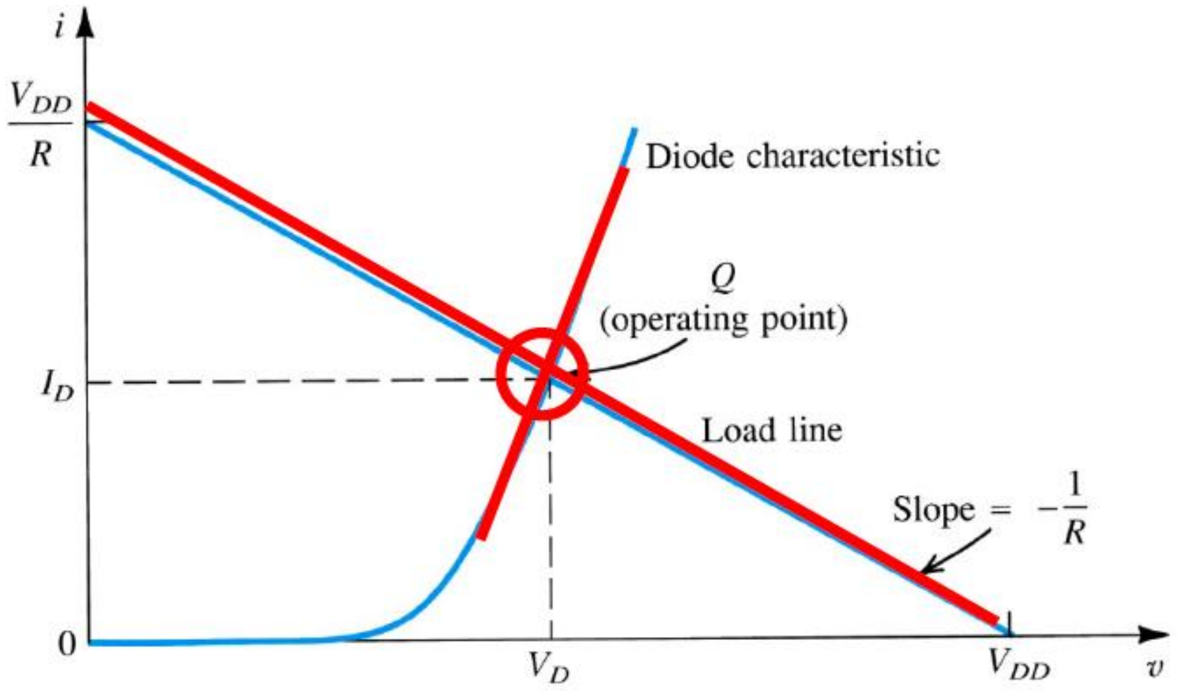
$$5 = (2 \text{ x } 10^3)\,(10^{-13})\,[\, e^{\,(V_D / 0.026)} - 1 \,] + V_D$$

Answer: Two methods exist…

- Graphical method
- Iterative method

Graphical Analysis using Exponential Model

- Step 1: Plot the diode current equation and KVL on single graph
- Step 2: Find intersection of the network equation (KVL) $I_D = \frac{V_{DD} - V_D}{R}$ and diode current $I_D = I_S e^{V_D / V_T}$

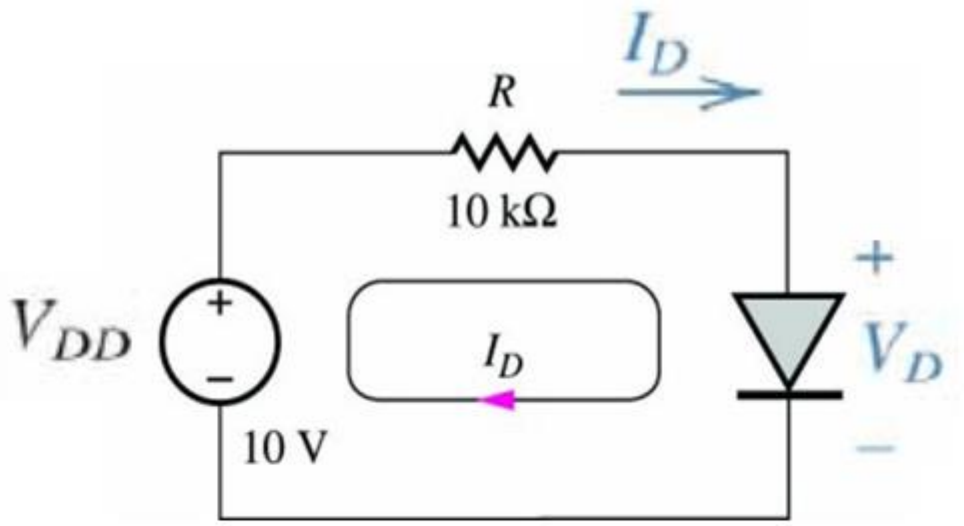


- Operating point (Q): intersection of load line and diode characteristic

Load Line Analysis

- Load line: network equation of the diode circuit (e.g. KVL).
- The load of a circuit determines the point or the region of operation of a diode (or device).
- The point of operation (called Q-point) is the point of intersection of the device characteristic (i-v curve plotted by Shockley's Equation) and the network equation (KVL).
- Method: A line is drawn on the characteristic of the device. The intersection point gives the point of operation (called Q-point).

Load Line

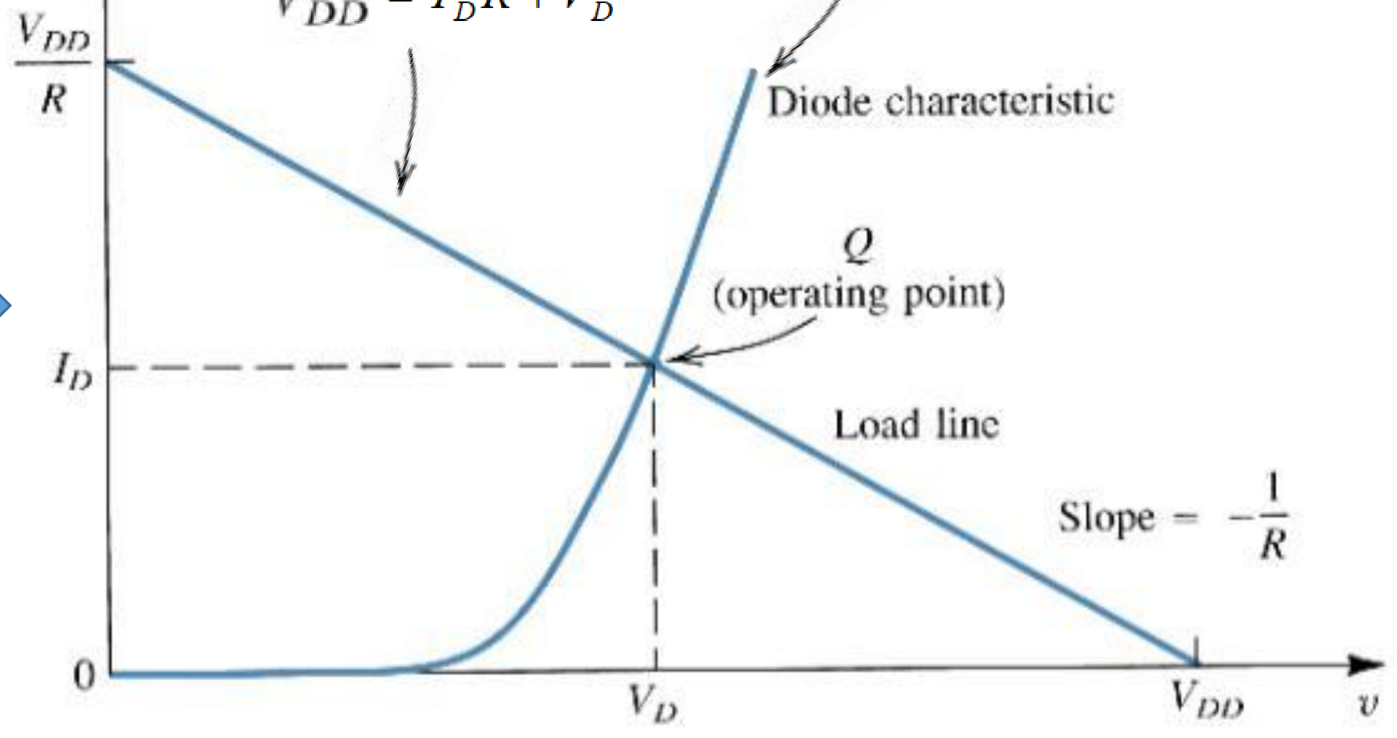


(Shockley's Equation)

$$i_D = I_S\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right]$$

(Network equation: KVL)

$$V_{DD} = I_D R + V_D$$



Graphical Analysis Pro’s and Con’s

- Pro’s
  - Intuitive because of visual nature
- Con’s
  - Poor Precision
  - Not Practical for Complex Analyses
    - Multiple lines required

Iterative Analysis using Exponential Model

- Step 1: Start with initial guess of V_D
    - $V_D^{(0)}$
- Step 2: Use nodal / mesh analysis to solve I_D
- Step 3: Use exponential model to update V_D
    - $V_D^{(1)} = \mathbf{f}(V_D^{(0)})$
- Step 4: Repeat these steps until $V_D^{(k+1)} = V_D^{(k)}$

➢ Upon convergence, the new and old values of V_D will match

Iterative Analysis Pro’s and Con’s

- Pro’s
  - High precision
- Con’s
  - Not intuitive
  - Not Practical for Complex Analyses
    - 10+ iterations maybe required

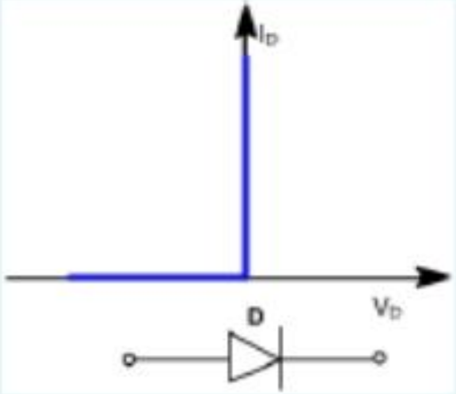
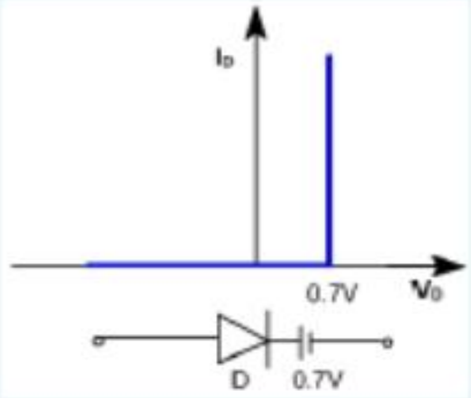
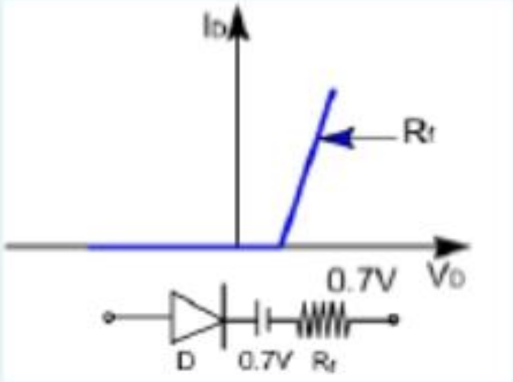
The Need for Rapid Analysis

- Analyze the diode-based circuit more efficiently
  - Rapid circuit analysis with a simpler model
  - Further refine and “fine-tune” the design in almost final design
  - Perform with the aid of a computer circuit analysis program (SPICE)

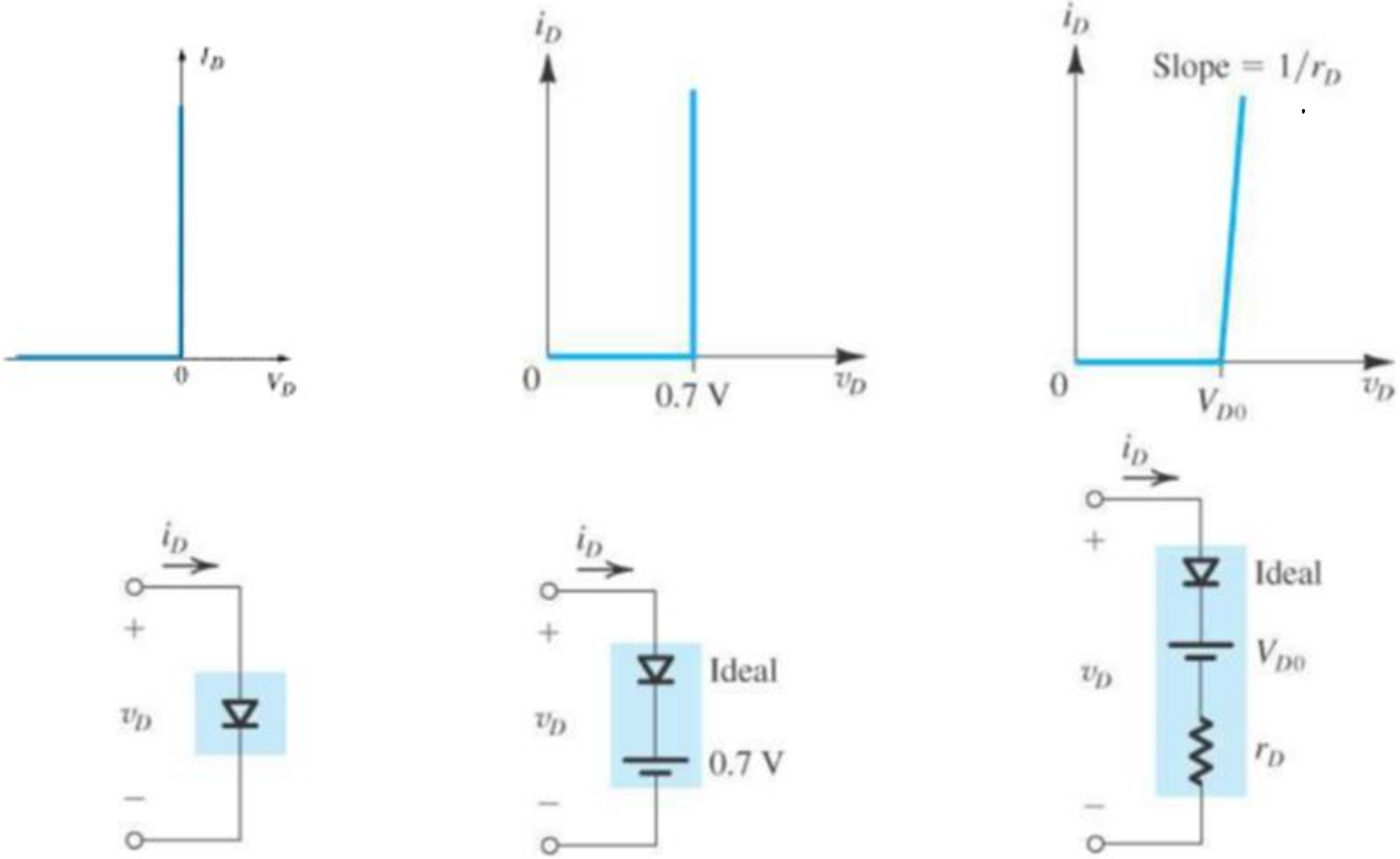
- To aid in analysis, define a robust set of diode models.
- Simplified diode models are better suited for use in circuit analysis and design of diode circuits:
  - ✓ Ideal diode model
  - ✓ Constant voltage-drop model
  - ✓ Piecewise linear model

Diode Equivalent Circuits

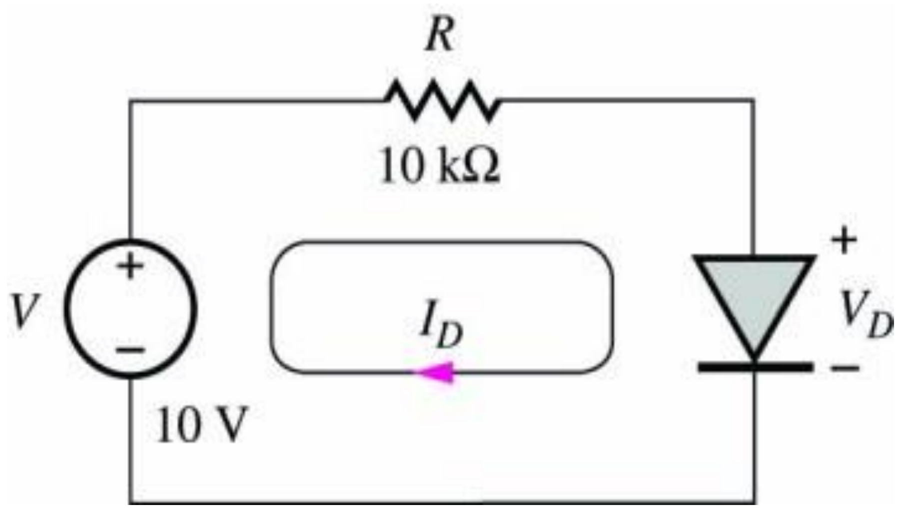
- To aid in analysis, define a robust set of diode models.
- Simplified diode models are better suited for use in circuit analysis and design of diode circuits:
  - ✓ Ideal diode model
  - ✓Constant voltage-drop model
  - ✓ Piecewise linear model

Ideal diode	2^{nd} approximation: const. voltage drop model	3rd approximation: Piecewise linear model
		
When diode is forward biased, resistance offered is zero, When it is reverse biased resistance offered is infinity. It acts as a perfect switch	When forward voltage is more than 0.7 V, for Si diode then it conducts and offers zero resistance. The drop across the diode is 0.7V. When reverse biased it offers infinite resistance.	When forward voltage is more than 0.7 V, for Si diode then it conducts and offers resistance. When reverse biased it offers very high resistance but not infinity.

Forward Bias I-V Characteristics



Diode Circuit Analysis: Basics



Objective of diode circuit analysis is to find **quiescent/ operating point** for diode, consisting of dc current and voltage that define diode's *i-v* characteristic.

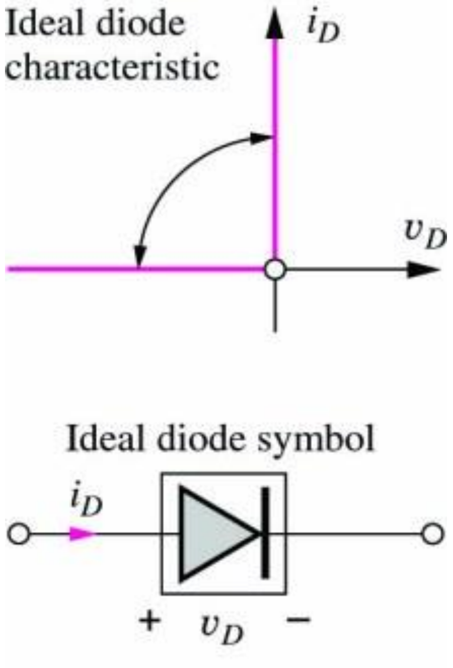
Loop equation for given circuit is:

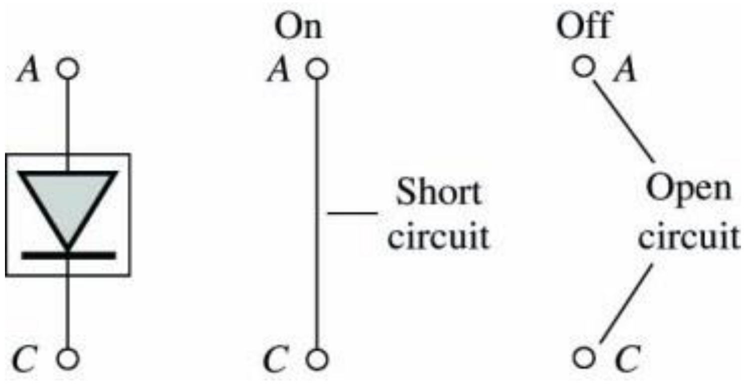
$$V = I_D R + V_D$$

This is also called the **load line** for the diode. Solution to this equation can be found by:

1. Simplified analysis with ideal diode model.
2. Simplified analysis using constant voltage drop model.
3. Graphical analysis using load-line method.
4. Analysis with diode's mathematical model.

Analysis Using Ideal Diode Model





If diode is forward-biased, voltage across diode is zero. If diode is reverse-biased, current through diode is zero.

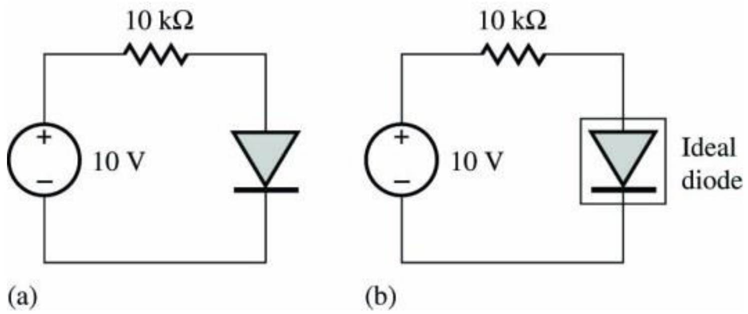
$v_D = 0$ for $i_D > 0$ and $i_D = 0$ for $v_D < 0$

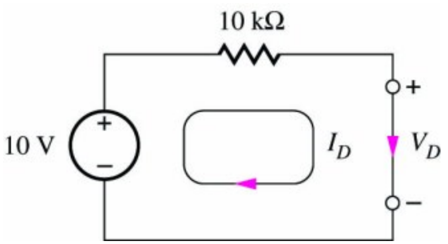
Thus diode is assumed to be either on or off.

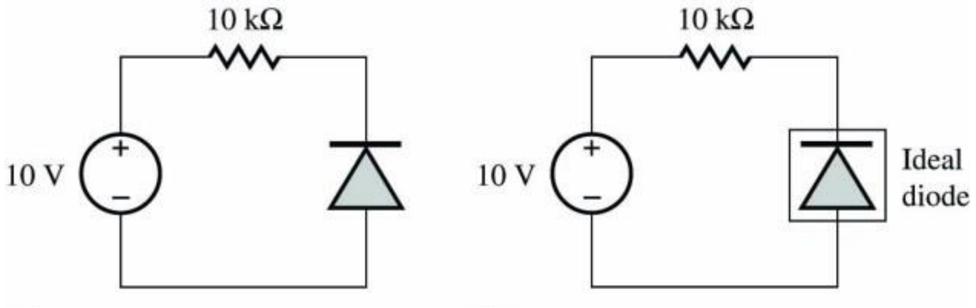
<u>Analysis is conducted in following steps:</u>

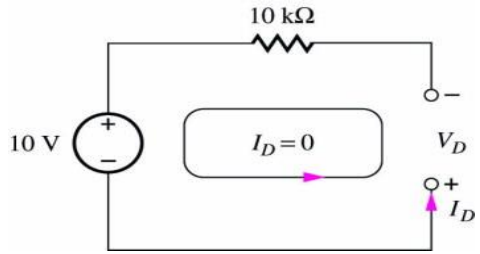
1. Select diode model.
2. Identify anode and cathode of diode and label v_D and i_D.
3. Guess diode's region of operation from circuit.
4. Analyze circuit using diode model appropriate for assumed operation region.
5. Check results to check consistency with assumptions.

Example:









Since source is forcing positive current through diode assume diode is on.

$$I_D = \frac{(10-0)\text{V}}{10\text{k}\Omega} = 1\text{m}A$$

$$\because I_D \geq 0$$

Q-point is(1 mA, 0V)

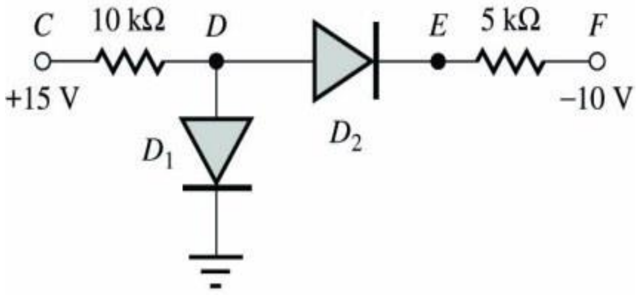
Since source is forcing current backward through diode assume diode is off. Hence I_D =0 . Loop equation is:

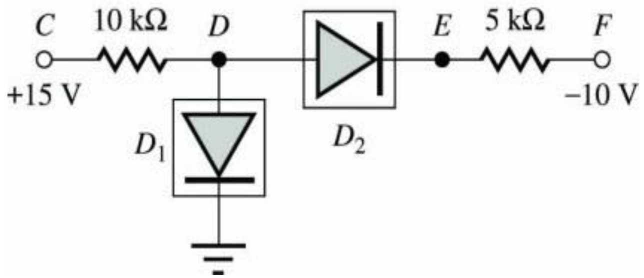
$$10 + V_D + 10^4 I_D = 0$$

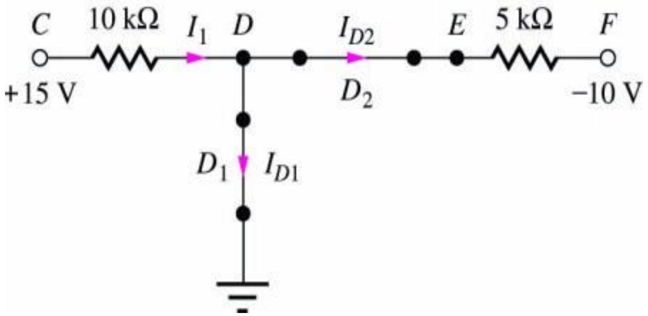
$$\therefore V_D = -10\text{V}$$

Q-point is (0, -10 V)

Two-Diode Circuit Analysis







Analysis: Ideal diode model is chosen. Since 15V source is forcing positive current through D_1 and D_2 and -10V source is forcing positive current through D_2, assume both diodes are on.

Since voltage at node D is zero due to short circuit of ideal diode D_1,

$$I_1 = \frac{(15-0)\text{V}}{10\text{k}\Omega} = 1.50\text{mA} \qquad I_{D2} = \frac{0-(-10)\text{V}}{5\text{k}\Omega} = 2.00\text{mA}$$

$$I_1 = I_{D1} + I_{D2} \qquad \therefore I_{D1} = 1.5 - 2 = -0.50\text{mA}$$

Q-points are (-0.5 mA, 0 V) and (2.0 mA, 0 V)

But, I_{D1} <0 is not allowed by diode, so ?.

References:

- Microelectronic Circuits, 7th edition by Adel S. Sedra Kenneth C. Smith.
- G. Streetman, and S. K. Banerjee, “Solid State Electronic Devices,” 7th edition, Pearson,2014.
- D. Neamen, D. Biswas, "Semiconductor Physics and Devices," McGraw-Hill Education.
- Electronic Devices and Circuit Theory 11th Edition by Boylestad, Robert . L, Louis Nashelskyl.
- http://ecee.colorado.edu/~bart/book/book/contents.htm
- http://www.ecse.rpi.edu/~schubert/Course-ECSE-2210-Microelectronics-Technology-2010/