BJT Small-Signal Analysis

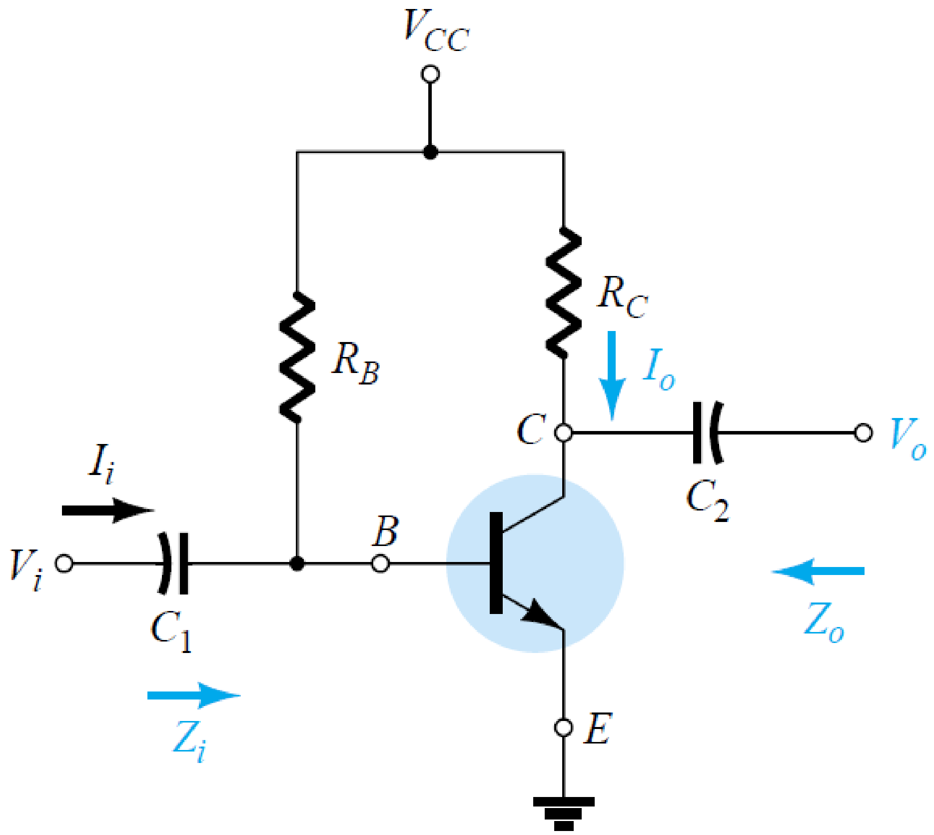
Semiconductor Devices and Circuits

(ECE 181302)

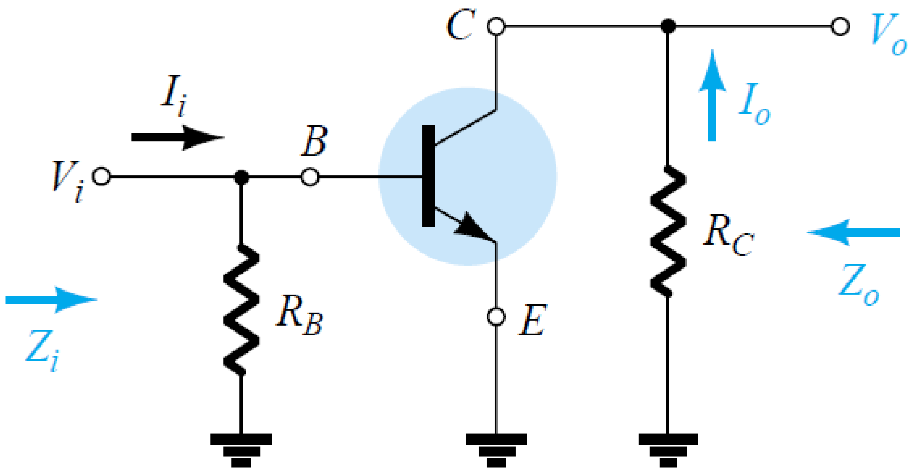
24th December 2021

BJT AC Analysis Steps

- Mark the terminals of the transistor.
- Mark the current directions and define the voltages.
- Remove the dc effects of V_{CC} by connecting it to the ground.
- Ground V_{CC}.
- Replace the dc blocking capacitors by short-circuit equivalents.
- Redraw the circuit.



V_{CC}
R_B
R_C
I_o
C
V_o
I_i
B
C_2
V_i
C_1
Z_o
Z_i
E



C
V_o
I_i
B
I_o
V_i
R_C
Z_o
R_B
Z_i
E

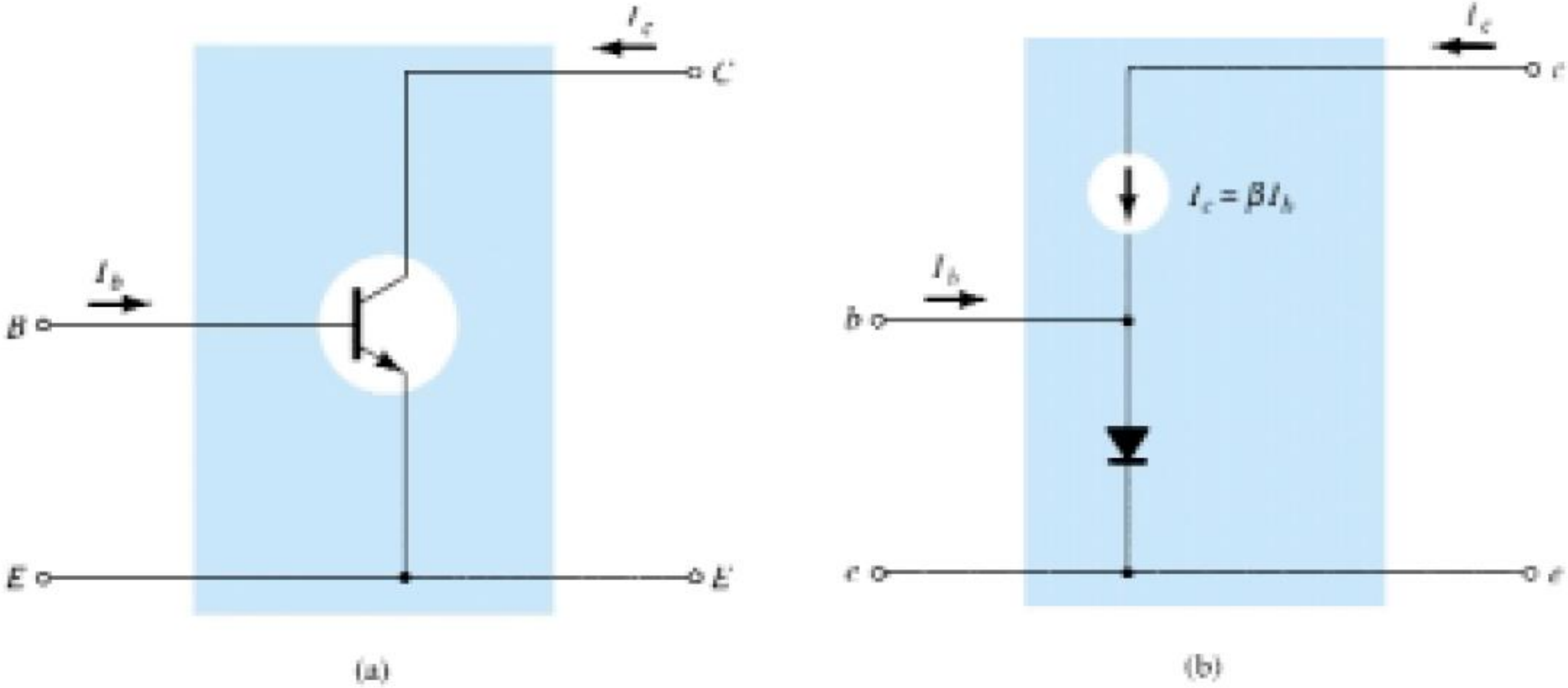
BJT r_e Model

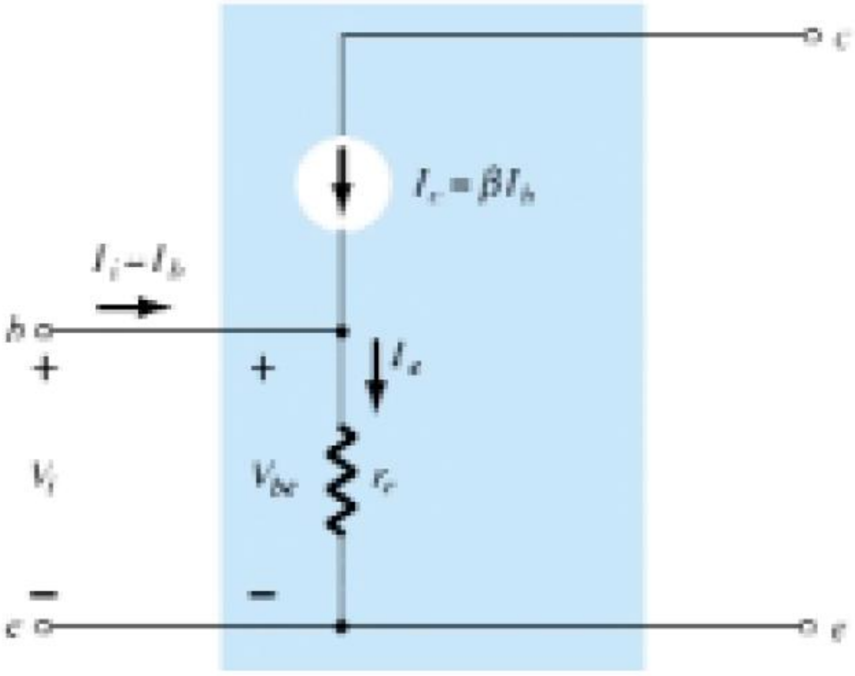
- Substitute the BJT by the r_e model into the network.
- The r_e model employs a diode and controlled current source to duplicate the behavior of a transistor in the region of interest.
- The ac resistance of a diode can be determined by the equation r_{ac} = 26 mV/I_D, where I_D is the dc current through the diode at the *Q* (quiescent) point. This same equation is used to find the ac resistance of the diode in the BJT
- Substituting the emitter current:

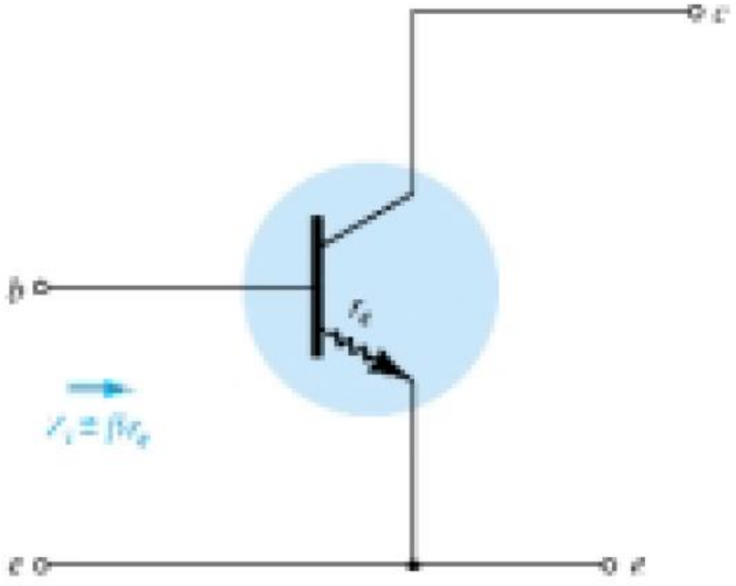
$$\boxed{r_e = \frac{26\ \text{mV}}{I_E}}$$

- The subscript *e* of r_e is chosen to emphasize that it is the dc level of emitter current that determines the ac level of the resistance of the diode.

CE *re* Model







$$\boxed{I_c = \beta I_b}$$

$$I_e = I_c + I_b = \beta I_b + I_b$$

$$\boxed{I_e = (\beta + 1)I_b}$$

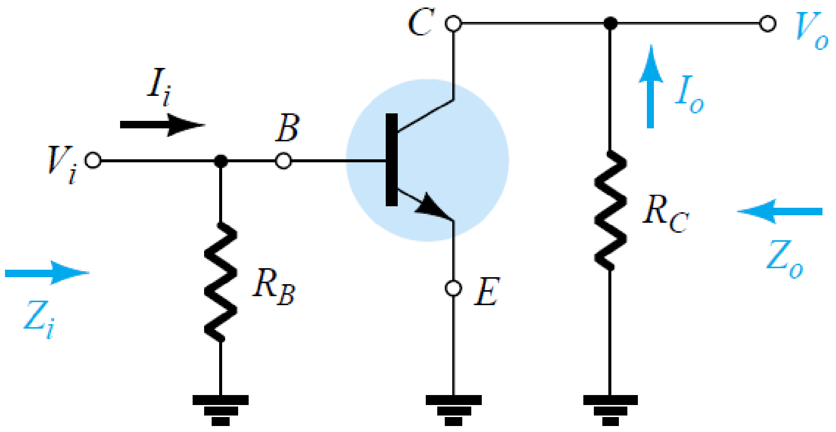
$$\boxed{I_e \cong \beta I_b}$$

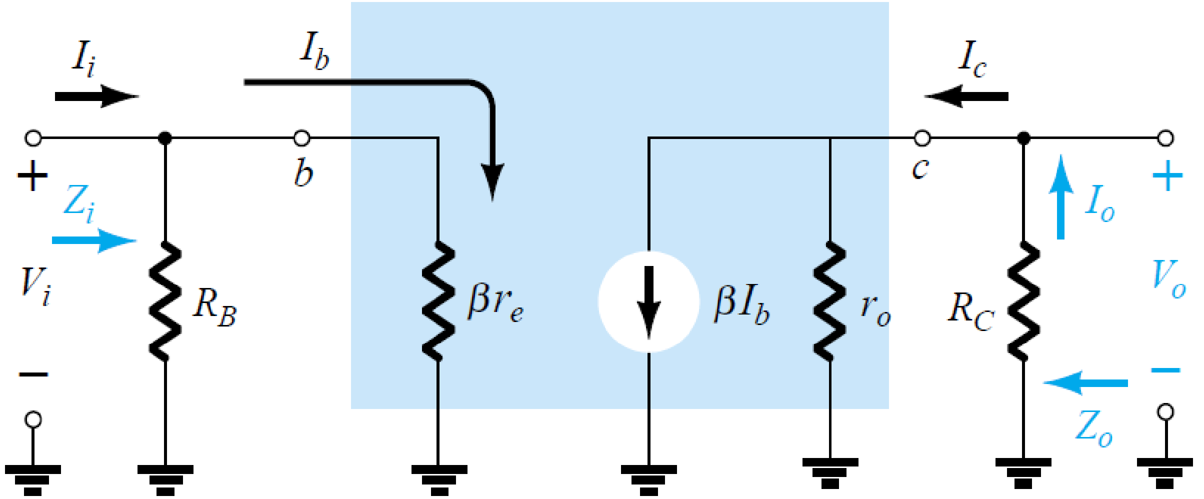
$$Z_i = \frac{V_i}{I_i} = \frac{V_{be}}{I_b}$$

$$V_i = V_{be} = I_e r_e \cong \beta I_b r_e$$

$$= \frac{V_{be}}{I_b} \cong \frac{\beta I_b r_e}{I_b}$$

$$\boxed{Z_i \cong \beta r_e}_{CE} = (160)(6.5\ \Omega) = \mathbf{1.04\ k\Omega}$$





$$\boxed{Z_i = R_B \| \beta r_e}$$ ohms Or $$\boxed{Z_i \cong \beta r_e}_{R_B \geq 10\beta r_e}$$

- *Zo*: The output impedance of any system is defined as the impedance *Zo* determined when *Vi* = 0. When *Vi* = 0, *Ii* = *Ib* = 0, resulting in an open-circuit equivalence for the current source. Therefore;

$$\boxed{Z_o = R_C \| r_o}$$ ohms $$\boxed{Z_o \cong R_C}_{r_o \geq 10R_C}$$

Example: For the network;

(a) Determine *re*.

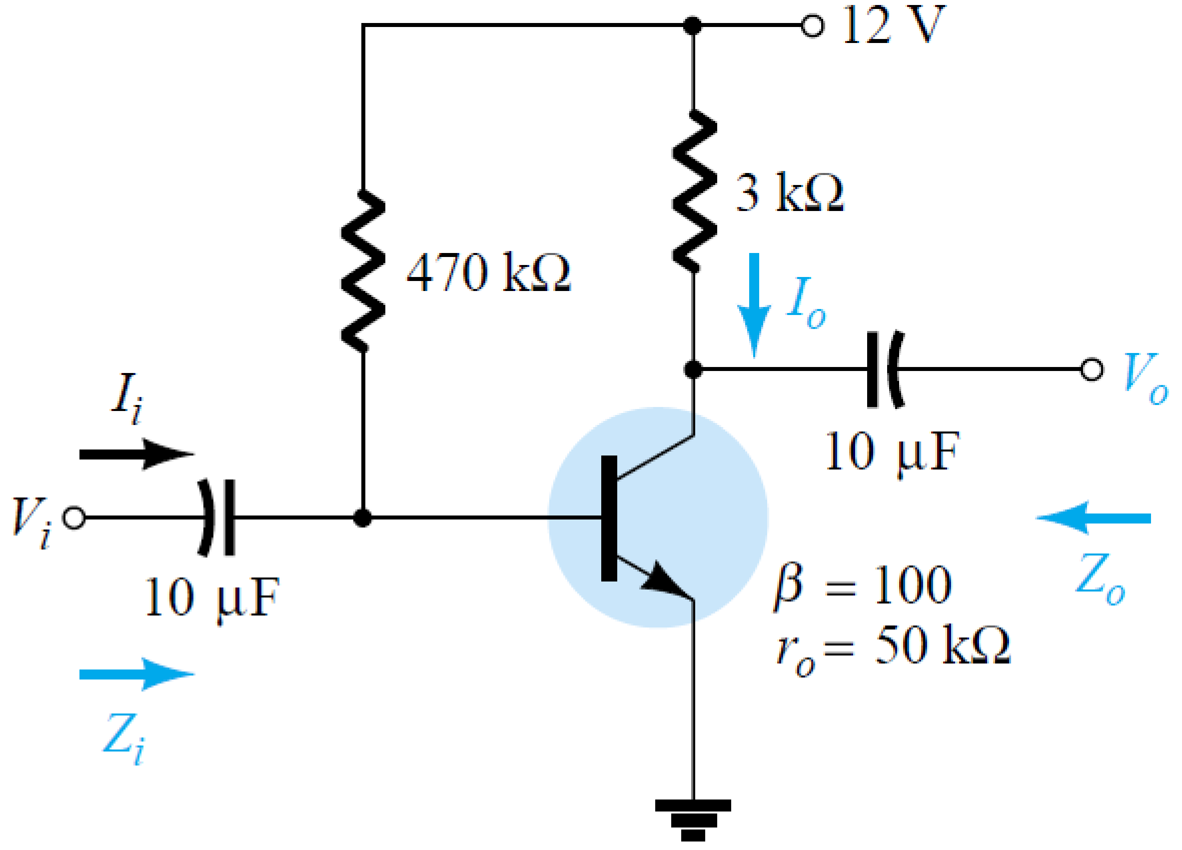
(b) Find *Zi* (with *ro* = ∞Ω).

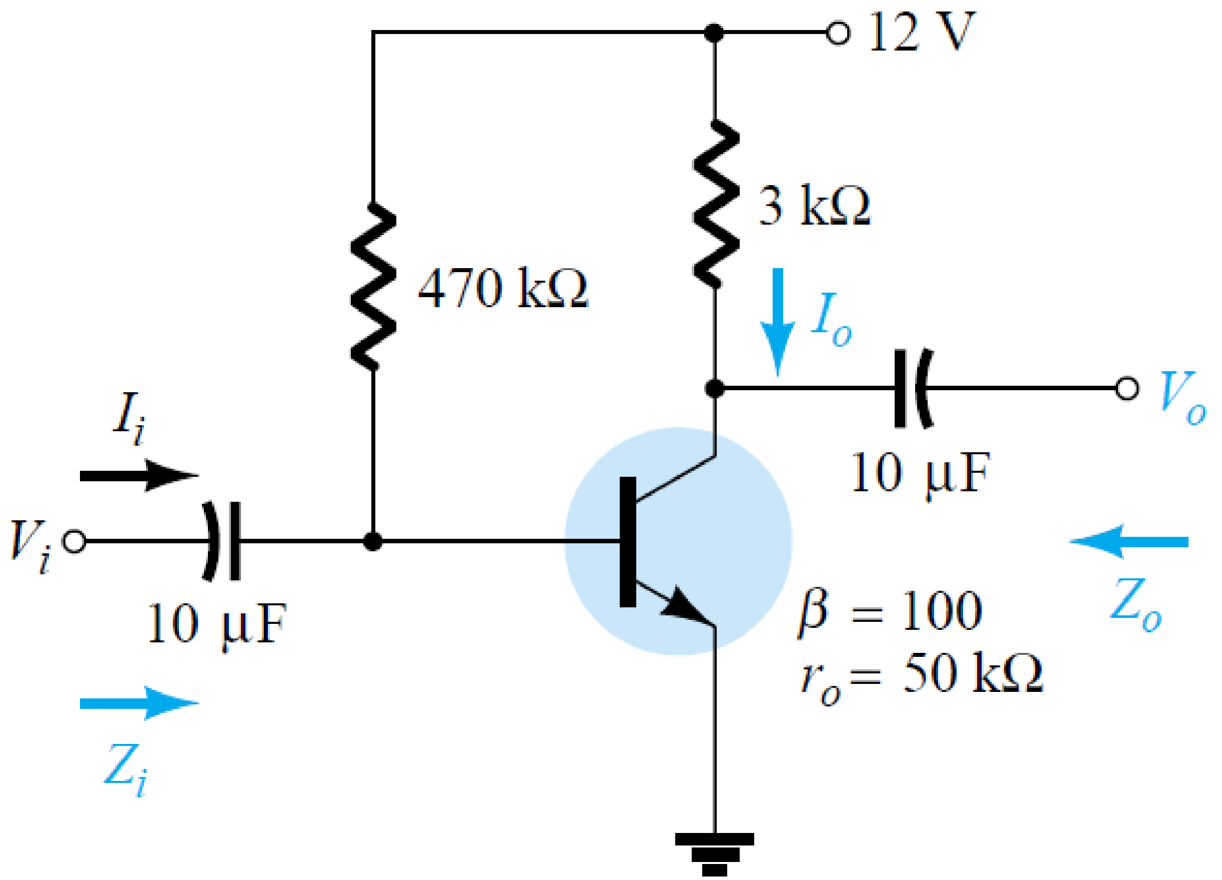
(c) Calculate *Zo* (with *ro* = ∞Ω).

(d) Determine *Av* (with *ro* = ∞Ω).

(e) Find *Ai* (with *ro* = ∞Ω).

(f) Repeat parts (c) through (e) including *ro* = 50 kΩ in all calculations and compare results.




12 V
470 kΩ
3 kΩ
I_o
V_o
I_i
V_i
10 μF
10 μF
β = 100
r_o = 50 kΩ
Z_o
Z_i

- Solution:

(a) DC analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12\text{ V} - 0.7\text{ V}}{470\text{ k}\Omega} = 24.04\ \mu\text{A}$$

$$I_E = (\beta + 1)I_B = (101)(24.04\ \mu\text{A}) = 2.428\text{ mA}$$

$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{2.428\text{ mA}} = \mathbf{10.71\ \Omega}$$

(b) $\beta r_e = (100)(10.71\ \Omega) = 1.071\text{ k}\Omega$

$Z_i = R_B \| \beta r_e = 470\text{ k}\Omega \| 1.071\text{ k}\Omega = \mathbf{1.069\ k\Omega}$

(c) $Z_o = R_C = \mathbf{3\ k\Omega}$

(d) $A_v = -\dfrac{R_C}{r_e} = -\dfrac{3\text{ k}\Omega}{10.71\ \Omega} = \mathbf{-280.11}$

(e) Since $R_B \geq 10\beta r_e (470\text{ k}\Omega > 10.71\text{ k}\Omega)$

$A_i \cong \beta = \mathbf{100}$

(f) $Z_o = r_o \| R_C = 50\ \text{k}\Omega \| 3\ \text{k}\Omega = \mathbf{2.83\ k\Omega}$ vs. $3\ \text{k}\Omega$

$$A_v = -\frac{r_o \| R_C}{r_e} = \frac{2.83\ \text{k}\Omega}{10.71\ \Omega} = \mathbf{-264.24} \text{ vs. } -280.11$$

$$A_i = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)} = \frac{(100)(470\ \text{k}\Omega)(50\ \text{k}\Omega)}{(50\ \text{k}\Omega + 3\ \text{k}\Omega)(470\ \text{k}\Omega + 1.071\ \text{k}\Omega)}$$

$$= \mathbf{94.13} \text{ vs. } 100$$

$$A_i = -A_v \frac{Z_i}{R_C} = \frac{-(-264.24)(1.069\ \text{k}\Omega)}{3\ \text{k}\Omega} = \mathbf{94.16}$$

References:

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