SIGNALS & SYSTEMS

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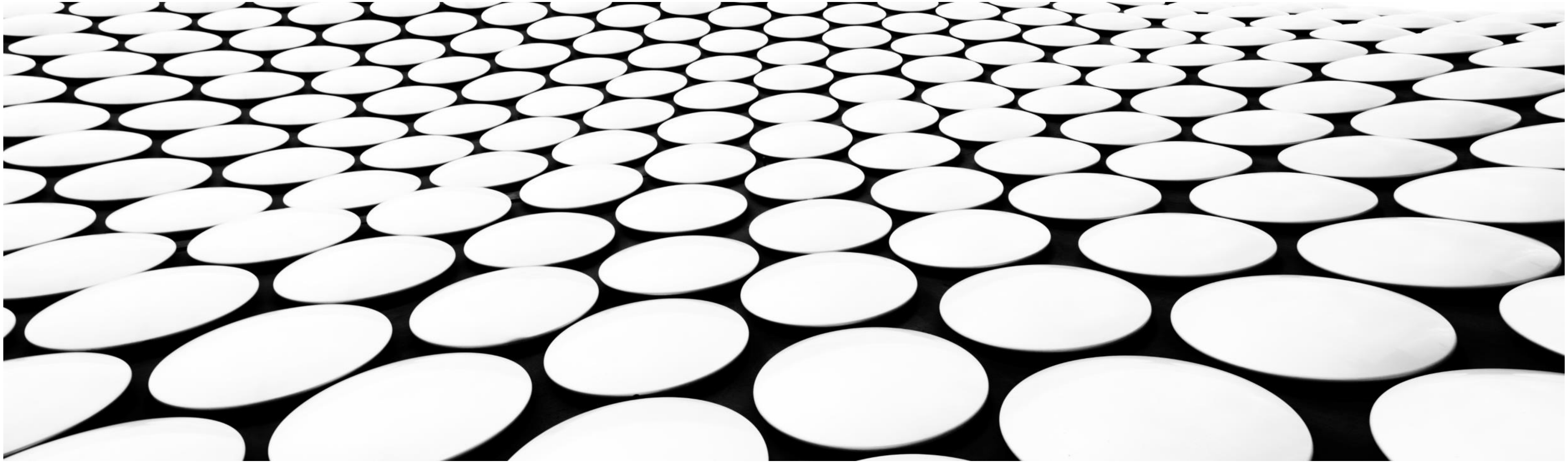


-: $x(t)$ – C_n pairs: -

$x(t)$	C_n
Real (R)	Conjugate symmetric (CS)
CS	R
Img	Con: Anti symmetric (CAS)
CAS	Img
~~R~~ + E	R + E
I + E	I + E
R + O	I + O
I + O	R + O

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$$

Is very helpful to solve MCQ

$$x(t) = \sum_{n=\infty}^{\infty} \underbrace{j \cos n\pi}\, e^{jn\pi t}$$

↳ C_n is Img and even

$n = 1, -1$

$$\therefore x(t) = \cdots + \underbrace{j\cos\pi e^{j\pi t}}_{n=1} + \underbrace{j\cos(-\pi) e^{-j\pi t}}_{n=-1} + \cdots$$

$$\cdots (-) j e^{j\pi t} - j e^{-j\pi t} + \cdots$$

$$\underbrace{-j\left[e^{j\pi t} - e^{-j\pi t}\right]}$$

$$-2j\cos \pi t \rightarrow$$

I + E

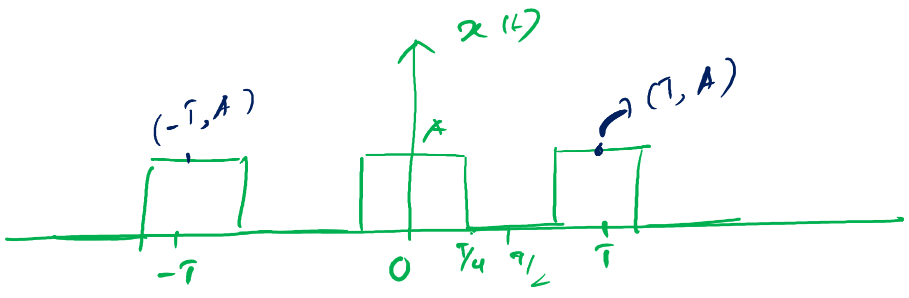
$$\frac{e^{i\alpha} + e^{-i\alpha}}{2i}$$

$\therefore C_n \rightarrow I + E$

$\rightarrow x(t) \rightarrow I + E$

Square, Triangular, saw-tooth → periodic.

Q Determine the FS coefficients for given periodic signal $x(t)$ is —



(a) $\frac{A}{j\pi k}\sin(\pi/2\, k)$ ×

(b) $\frac{A}{j\pi k}\cos(\pi/2\, k)$ ×

(c) $\frac{A}{\pi k}\sin(\pi/2\, k)$

(d) $\frac{A}{\pi k}\cos(\pi/2\, k)$

Sol^n: $\underbrace{x(-t) \rightarrow x(t)}_{\text{equal}} \Big\} \Rightarrow$ even and Real.

$\therefore c_n \rightarrow R + E$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega t}$$

$$c_n = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jn\omega t}\, dt$$

(d) $C_k = \frac{A}{\pi k}\cos(\pi/2\, k)$

$C_{-k} = \frac{A}{\pi(-k)}\cos\left[\pi/2\,(-k)\right]$

$= \frac{A}{-\pi k}\cos(\pi/2\, k)$

$C_k \neq C_{-k}$

$c_k = \frac{A}{\pi k}\sin(\pi/2\, k)$

$C_{-k} = \frac{A}{\pi(-k)}\sin(\pi/2\,(-k))$

$= \frac{A}{-\pi k}\cdot -\sin(\pi/2\, k)$

$= \frac{A}{\pi k}\sin(\pi/2\, k)$

$C_k = C_{-k}$

Symmetricities of FS:-

Even symmetry:- $x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$

$b_n = 0 \rightarrow$ only cos term, a_0

Odd symmetry:- $\left.\begin{array}{l} a_0 = 0 \\ a_n = 0 \end{array}\right\} b_n \neq 0 \rightarrow$ Sine term.

Half wave symmetry:- → In each cycle/period consists of two equal and opposite half cycle.

→ $\underbrace{\text{has alternative +ve and -ve}}_{\text{Alternate symmetry}}$ half cycles.

$\boxed{\begin{array}{l} \text{Av. value } a_0 = 0 \\ \qquad\qquad c_0 = 0 \end{array}}$

$\boxed{x(t) = -x\left(t + \frac{T_0}{2}\right)} \rightarrow$ HFS

$\boxed{\text{HWS contains only odd harmonics}}$

Odd hws:-

$b_n \neq 0$

$\left.\begin{array}{l} a_n = 0 \\ a_0 = 0 \end{array}\right\}$ symmetric about time axis (x-axis)

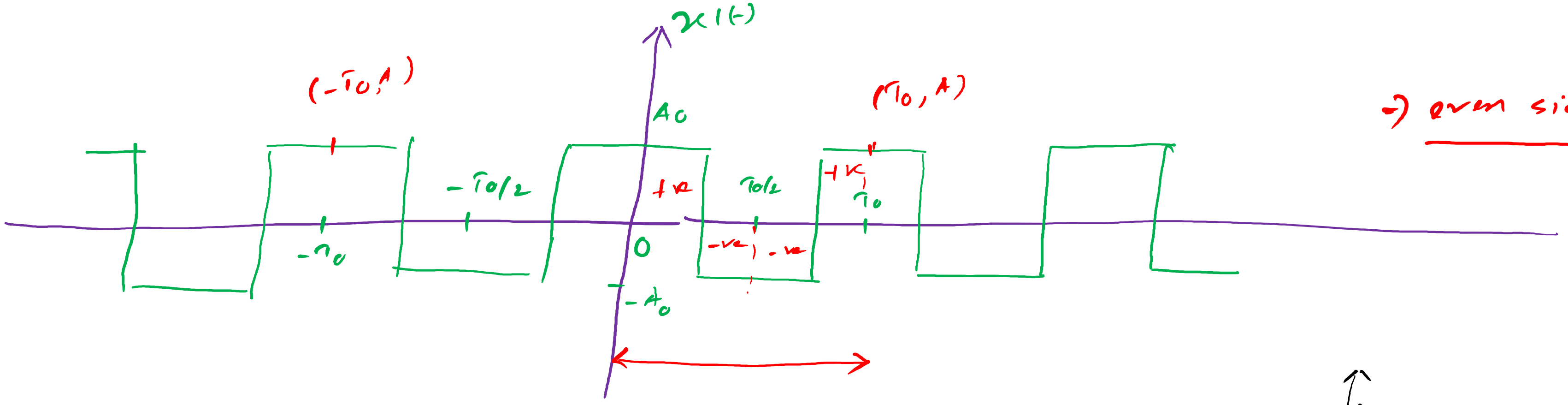
only odd harmonics. n = 1, 3, 5, 7 ...... (sine terms are available)

Even hws:-

$\left.\begin{array}{l} a_n = 0 \\ b_n \neq 0 \\ a_0 \neq 0 \end{array}\right\}$ even symm.

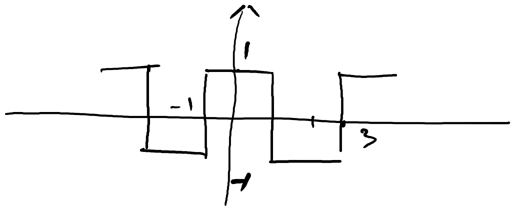
odd harmonics → of cosine Terms.

TFS $\boxed{a_0 \to c_0}$



→ even signal

$\boxed{\text{even + HWS}}$



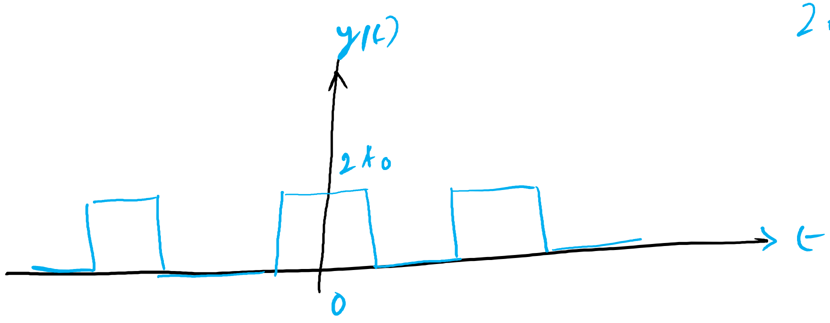
$$x(t) = \cancel{a_0}^{\,0} + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \cancel{\sum_{n=1}^{\infty} b_n \sin n\omega_0 t}^{\,0}$$

$$= \frac{4}{\pi}\left[\cos \frac{\pi}{2} t - \frac{1}{3}\cos \frac{3\pi}{2} t + \frac{1}{5}\cos \frac{5\pi}{2} t \ldots\right]$$

Only odd harmonics of cosine must present.

$$x(t) = a_1 \cos \omega_0 t + a_3 \cos 3\omega_0 t + a_5 \cos 5\omega_0 t + \ldots\ldots$$

Hidden symmetry:-



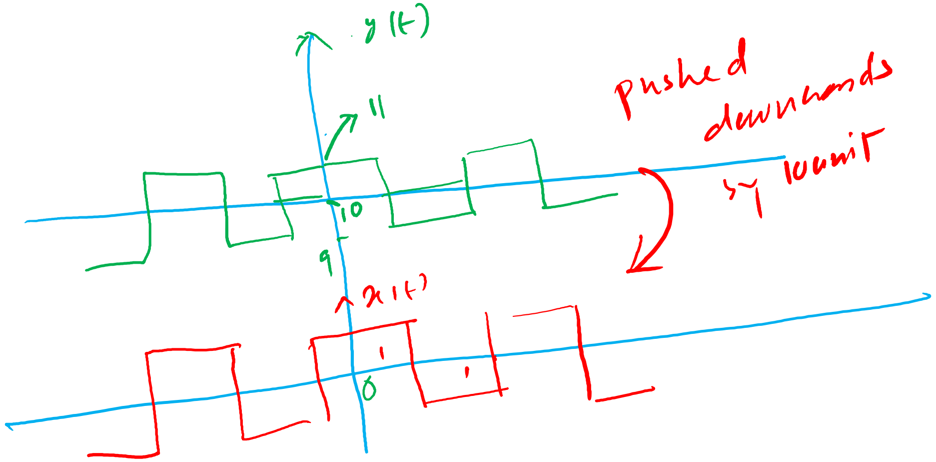
$2A_0 = A_0 + A_0$

$2x(t)$

$y(t) = 1 + x(t)$

$y(t) = A_0 + x(t)$

$a_0 + a_1 \cos \omega_0 t + a_3 \cos 3\omega_0 t \; \ldots\ldots$



pushed downwards by 10 unit

$y(t) = 10 + x(t)$

$\Rightarrow x(t) = y(t) - 10$

Relationships b/w Fourier Coeffi

TFS :- $x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$

CEFS :- $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \rightarrow \sum_{n=-\infty}^{-1} (\ \ ) + c_0 + \sum_{1}^{\infty} (\ \ )$

$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2j}$

$\frac{1}{j} = -j$

$x(t) = a_0 + \sum_{n=1}^{\infty} \frac{a_n}{2}\left(e^{jn\omega_0 t} + e^{-jn\omega_0 t}\right) + \sum_{n=1}^{\infty} \frac{-jb_n}{2}\left(e^{jn\omega_0 t} - e^{-jn\omega_0 t}\right)$

$= a_0 + \sum_{n=1}^{\infty}\left[\frac{a_n}{2}\left(e^{jn\omega_0 t} + e^{-jn\omega_0 t}\right) - \frac{jb_n}{2}\left(e^{jn\omega_0 t} - e^{-jn\omega_0 t}\right)\right]$

$= a_0 + \sum_{n=1}^{\infty}\left[\left(\frac{a_n - jb_n}{2}\right)e^{jn\omega_0 t} + \left(\frac{a_n + jb_n}{2}\right)e^{-jn\omega_0 t}\right]$

Assume $m = -n$

$= a_0 + \sum_{n=1}^{\infty} c_n e^{jn\omega_0 t} + \sum_{n=1}^{\infty} c_{-n} e^{-jn\omega_0 t}$

$$x(t) = a_0 + \sum_{n=1}^{\infty} c_n e^{jn\omega_0 t} + \sum_{n=-\infty}^{-1} c_n e^{jn\omega_0 t}$$

$\boxed{c_0 \rightarrow a_0}$

$c_n = \frac{1}{2}(a_n - jb_n)$

Valid for all types of signals.

Now, c_n is CEFC, $c_n \rightarrow R$, $c_n \rightarrow Img$

$$c_n = Re(c_n) + j\,Img(c_n)$$

$$\Rightarrow \frac{1}{2}[a_n] - \frac{1}{2}j(b_n) = Re(c_n) + j\,Img(c_n)$$

$Re(c_n) = \frac{1}{2}a_n \Rightarrow a_n = 2Re(c_n)$

$Img(c_n) = -\frac{1}{2}b_n \Rightarrow b_n = -2\,Img(c_n)$

Ex:- Determine the Fourier Series Representation of Half-wave Rectifier O/P.