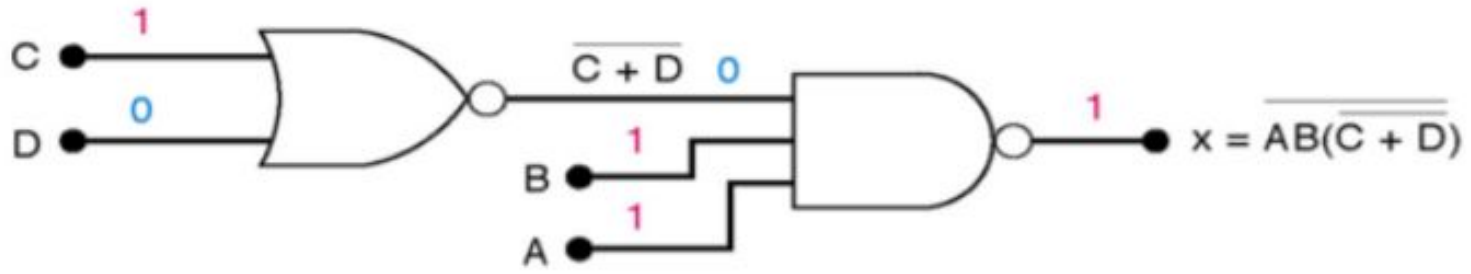
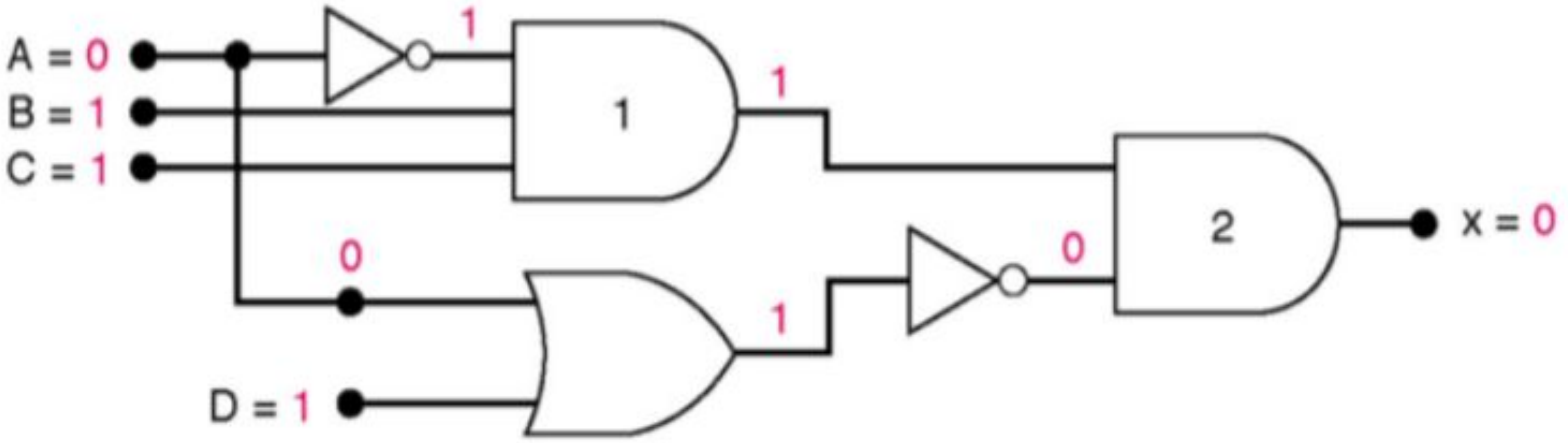
1. Boolean algebra
2. De Morgan’s law
3. Positive and negative logic

Determining output value from a diagram

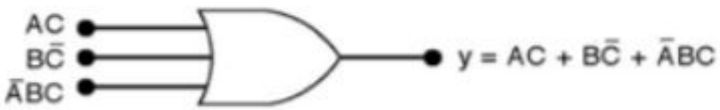


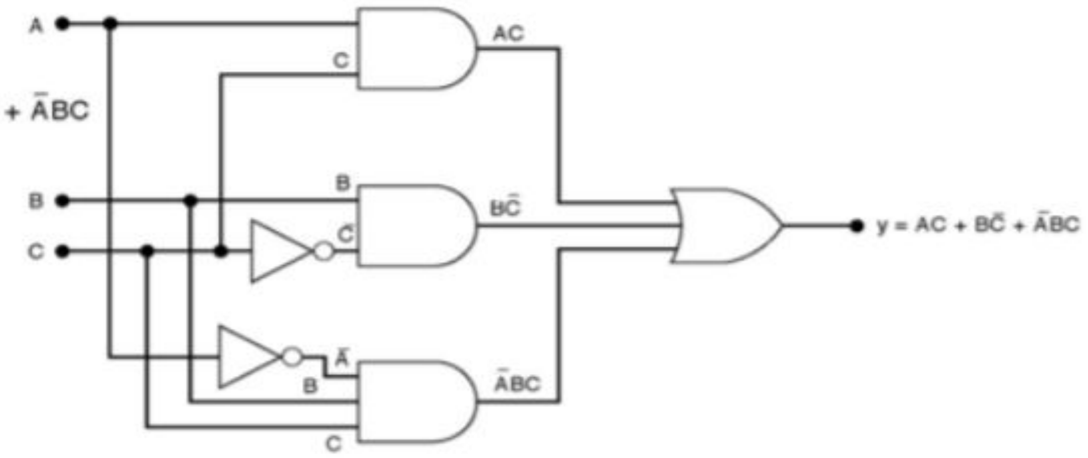


Reference: www.ee.ic.ac.uk/pcheung/teaching/DE1_EE/

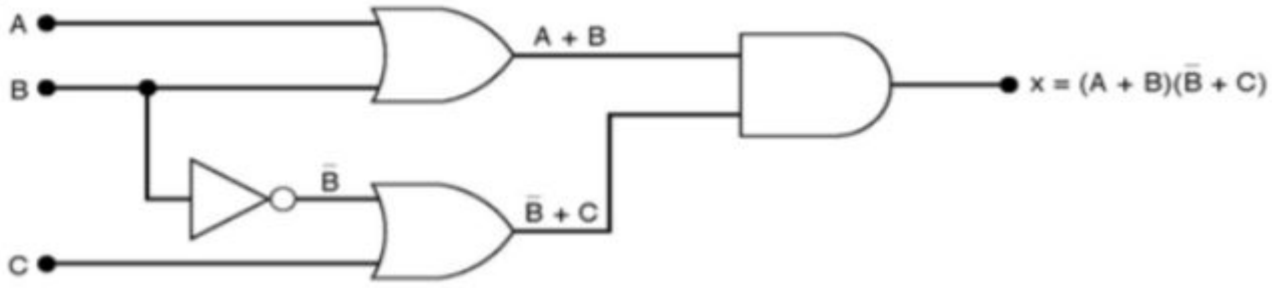
Implementing Circuits From Boolean Expressions

- When the operation of a circuit is defined by a Boolean expression, we can *draw a logic-circuit* diagram directly from that expression.





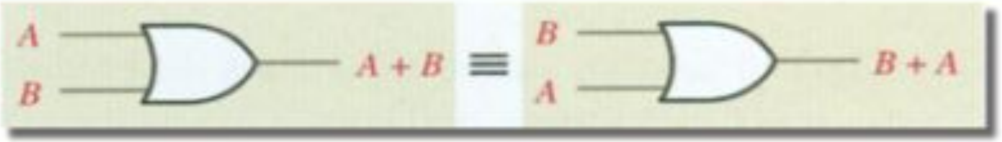
$$x = (A + B)(\overline{B} + C)$$



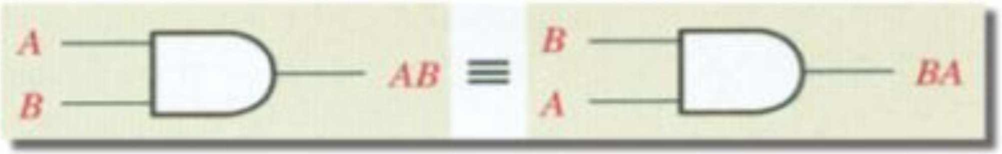
Laws of Boolean Algebra

- Commutative Laws

  $A + B = B + A$



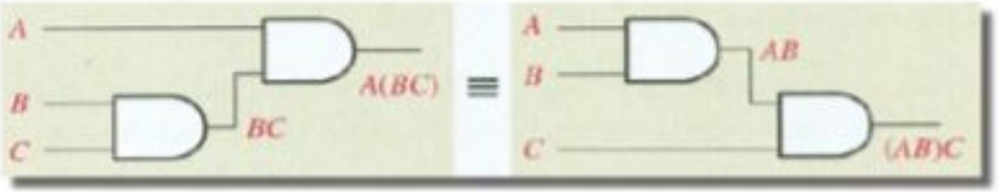
  $A \cdot B = B \cdot A$



- Associative Laws

  $A + (B + C) = (A + B) + C$

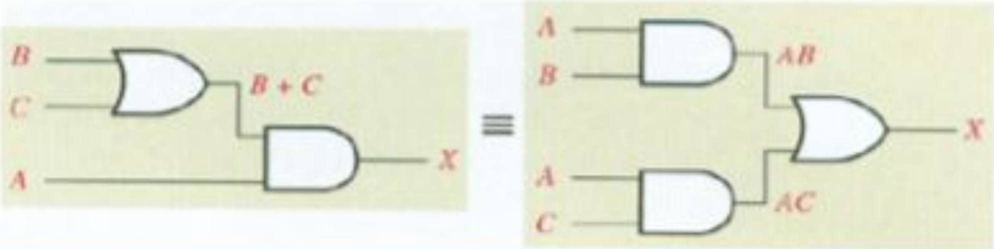
  $A \cdot (B \cdot C) = (A \cdot B) \cdot C$



- Distributive Law

  $A \cdot (B + C) = A \cdot B + A \cdot C$

  $A\,(B + C) = A\,B + A\,C$

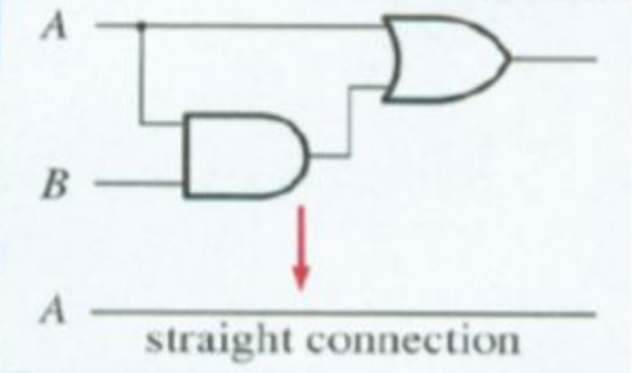


Rules of Boolean Algebra

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

- Rules 1 to 9 are obvious.
- Rule 10: A + AB = A

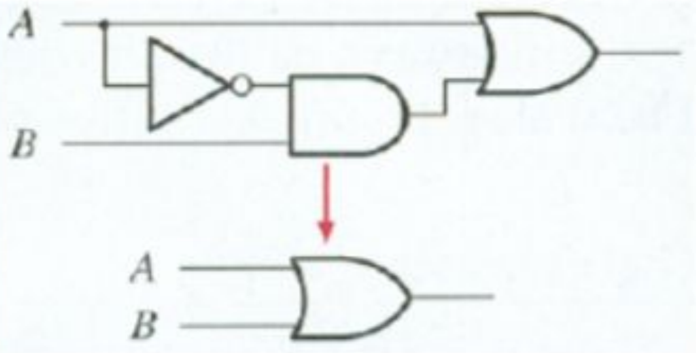
A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1



Rules 11 and 12 of Boolean Algebra

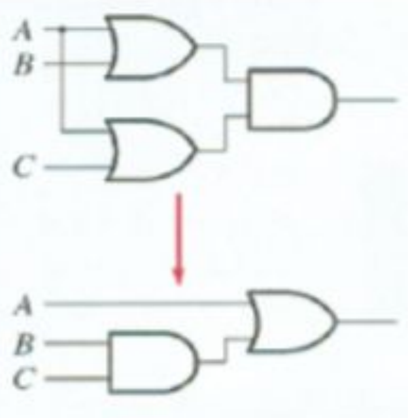
- Rule 11: $A + \overline{A}B = A + B$

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1



- Rule 12: $(A + B)(A + C) = A + BC$

A	B	C	$A + B$	$A + C$	$(A + B)(A + C)$	BC	$A + BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1



Using Boolean Algebra to simplify expressions

$$y = A\overline{B}D + A\overline{B}\,\overline{D} \longrightarrow y = A\overline{B}$$

$$z = \left(\overline{A} + B\right)\left(A + B\right) \longrightarrow z = B$$

$$x = ACD + \overline{A}BCD \longrightarrow x = ACD + BCD$$

DeMorgan's Theorems

- Theorem 1 $$\overline{(x+y)} = \overline{x} \cdot \overline{y}$$
- Theorem 2 $$\overline{(x \cdot y)} = \overline{x} + \overline{y}$$

Remember:

"Break the bar, change the operator"

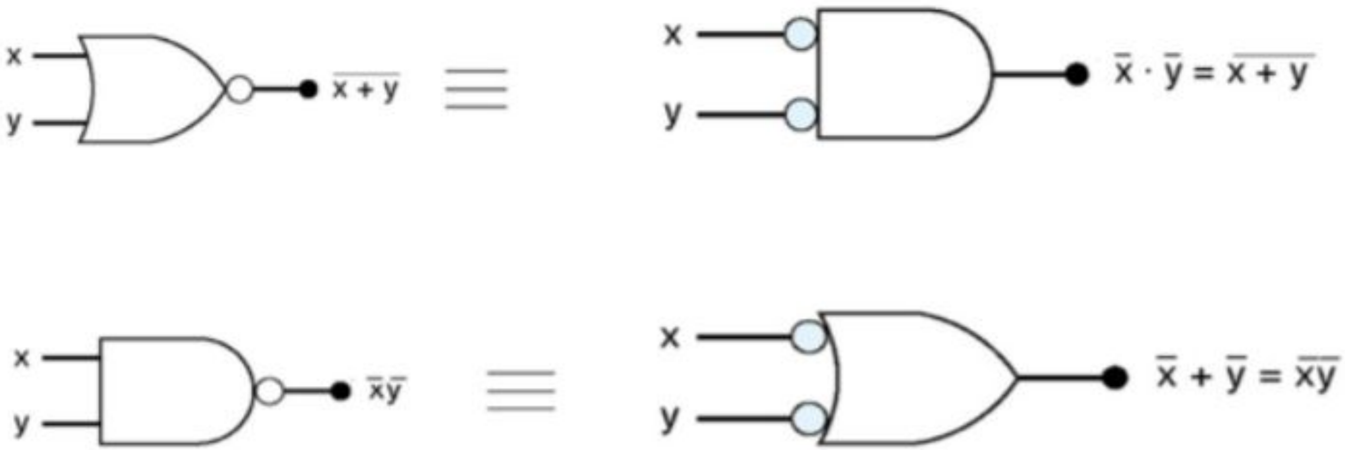
- DeMorgan's theorem is very useful in digital circuit design
- It allows ANDs to be exchanged with ORs by using invertors
- DeMorgan's Theorem can be extended to any number of variables.

$$F = \overline{X.Y} + \overline{P.Q} \longleftarrow \text{2 NAND plus 1 OR}$$

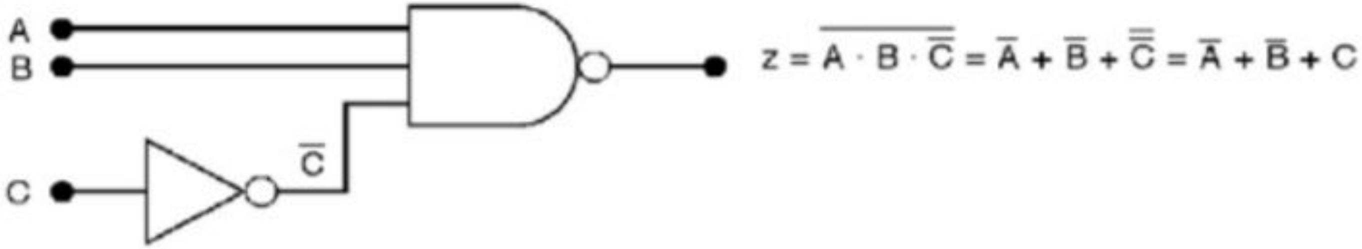
$$= \overline{X} + \overline{Y} + \overline{P} + \overline{Q} \longleftarrow \text{1 OR with some input invertors}$$

$$z = \overline{(\overline{A} + C)} \cdot \overline{(B + \overline{D})} \longrightarrow z = A\overline{C} + \overline{B}D$$

Implications of DeMorgan's Theorems(I)



- Determine the output expression for the circuit below and simplify it using DeMorgan's Theorem



Verify De Morgan’s law using Truth Table:

A	B	$\overline{A}$	$\overline{B}$	A+B	$A \cdot B$	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1	0	0	1	1	1	1
0	1	1	1	1	0	0	0	1	1
1	0	0	0	1	0	0	0	1	1
1	1	0	0	1	1	0	0	0	0

Positive and negative logic