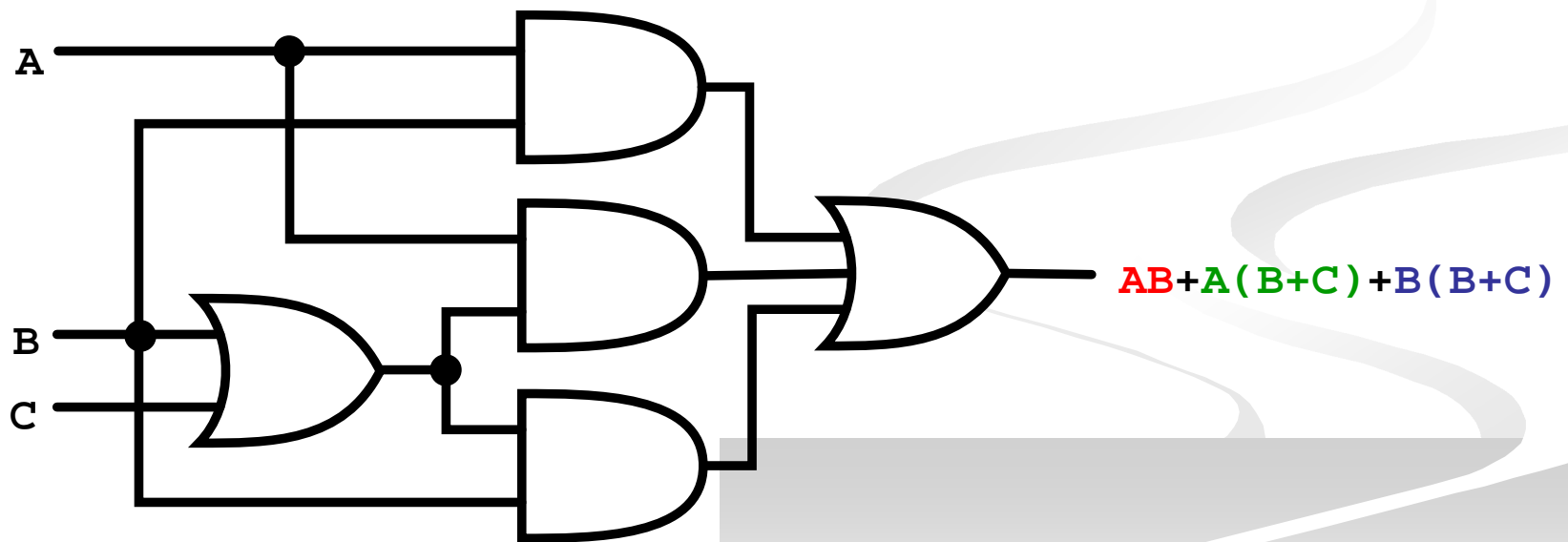


# **Lecture 7**

## **Logic Simplification**

# Simplification Using Boolean Algebra

- A simplified Boolean expression uses the fewest gates possible to implement a given expression.



# Simplification Using Boolean Algebra

■  $AB + A(B + C) + B(B + C)$

■ (distributive law)

■  $AB + AB + AC + BB + BC$

■ (rule 7;  $BB = B$ )

■  $AB + AB + AC + \mathbf{B} + BC$

■ (rule 5;  $AB + AB = AB$ )

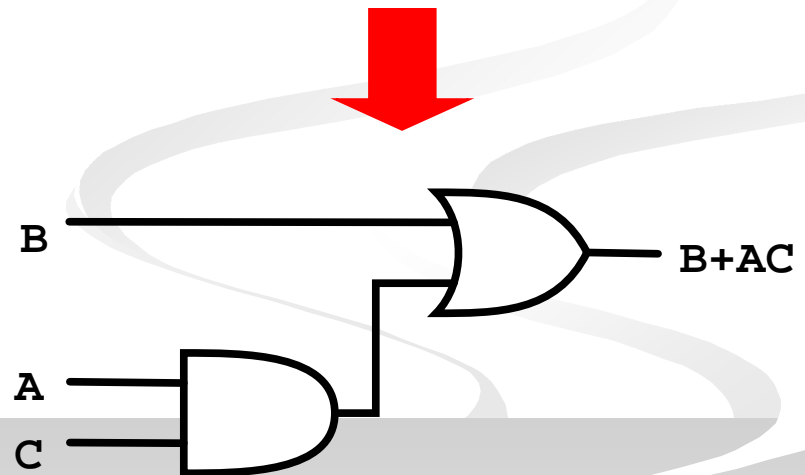
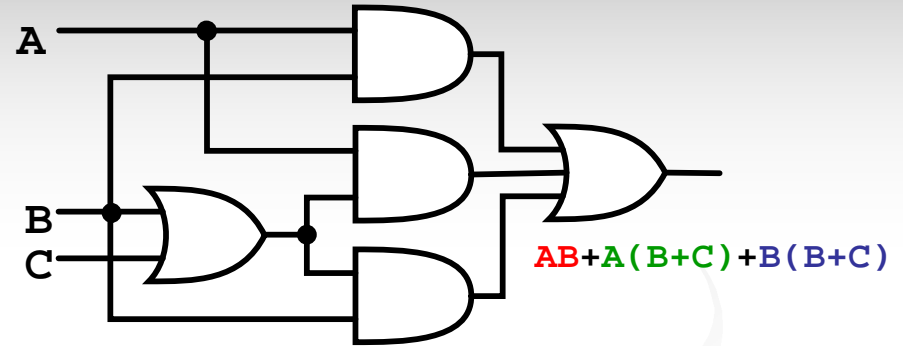
■  $\mathbf{AB} + AC + B + BC$

■ (rule 10;  $B + BC = B$ )

■  $AB + AC + \mathbf{B}$

■ (rule 10;  $AB + B = B$ )

■  $\mathbf{B} + AC$



# Simplification Using Boolean Algebra

Assignment

$$[A\bar{B}(C + BD) + \bar{A}\bar{B}]C$$

$$\bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$\overline{AB + AC} + \bar{A}\bar{B}C$$

# Standard Forms of Boolean Expressions

- All Boolean expressions, regardless of their form, can be converted into either of two standard forms:
  - The sum-of-products (SOP) form
  - The product-of-sums (POS) form
- Standardization makes the evaluation, simplification, and implementation of Boolean expressions much more systematic and easier.

# Sum-of-Products (SOP)

# The Sum-of-Products (SOP) Form

- An SOP expression → when two or more product terms are summed by Boolean addition.

- Examples:

$$AB + ABC$$

$$ABC + CDE + \overline{B}C\overline{D}$$

$$\overline{A}B + \overline{A}B\overline{C} + AC$$

- Also:

$$A + \overline{A}\overline{B}C + BCD$$

- In an SOP form, a single overbar cannot extend over more than one variable; however, more than one variable in a term can have an overbar:

- example:  $\overline{A}\overline{B}\overline{C}$  is OK!

- **But not:**  $\overline{ABC}$

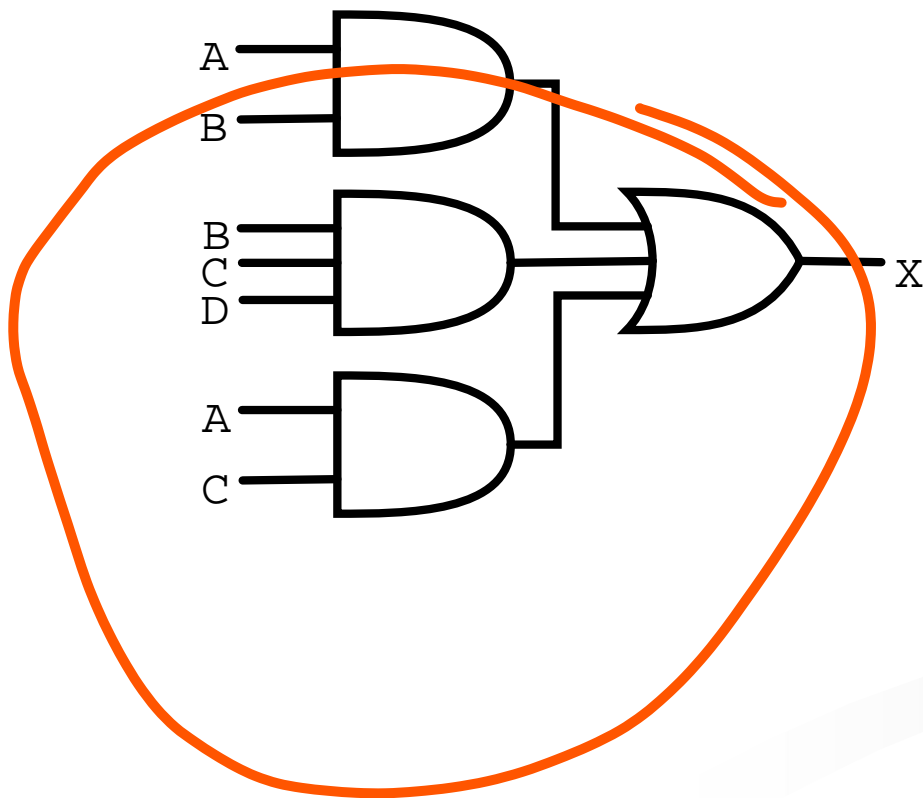
# Implementation of an SOP

$$X = AB + BCD + AC$$

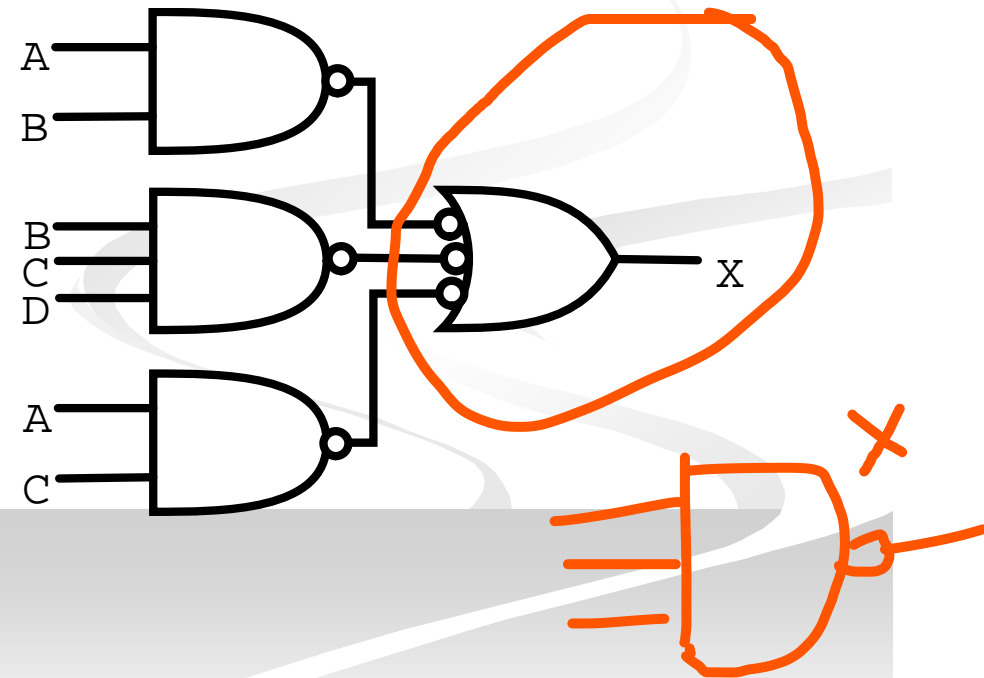
= minterm

■ AND/OR implementation

■ NAND/NAND implementation



≈





# General Expression $\rightarrow$ SOP

- Any logic expression can be changed into SOP form by applying Boolean algebra techniques.

ex:

$$A(B + CD) = AB + ACD$$

$$AB + B(\underline{CD} + \underline{EF}) = AB + BCD + BEF$$

$$(A + B)(B + C + D) = AB + AC + AD + BB + BC + BD$$

$$\overline{(\overline{A + B}) + C} = \overline{(\overline{A + B})} \cdot \overline{C} = (A + B) \cdot \overline{C} = A\overline{C} + B\overline{C}$$

# The Standard SOP Form

- A standard SOP expression is one in which *all* the variables in the domain appear in each product term in the expression.

- Example:

$$A\bar{B}CD + \bar{A}\bar{B}C\bar{D} + ABC\bar{D}$$

S S O P

- Standard SOP expressions are important in:
  - Constructing truth tables
  - The Karnaugh map simplification method

# Converting Product Terms to Standard SOP

- **Step 1:** Multiply each nonstandard product term by a term made up of the sum of a missing variable and its complement. This results in two product terms.
  - As you know, you can multiply anything by 1 without changing its value.
- **Step 2:** Repeat step 1 until all resulting product term contains all variables in the domain in either complemented or uncomplemented form. In converting a product term to standard form, the number of product terms is doubled for each missing variable.

# Converting Product Terms to Standard SOP (example)

- Convert the following Boolean expression into standard SOP form:

$$f(A, B, C, D) = \overline{A}\overline{B}C + \overline{A}\overline{B} + AB\overline{C}D$$

$$\overline{A}\overline{B}C = \overline{A}\overline{B}C(D + \overline{D}) = \overline{A}\overline{B}CD + \overline{A}\overline{B}C\overline{D}$$

$$\overline{A}\overline{B} = \overline{A}\overline{B}(C + \overline{C}) = \overline{A}\overline{B}C + \overline{A}\overline{B}\overline{C}$$

$$\overline{A}\overline{B}C(D + \overline{D}) + \overline{A}\overline{B}\overline{C}(D + \overline{D}) = \overline{A}\overline{B}CD + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}\overline{C}D + \overline{A}\overline{B}\overline{C}\overline{D}$$

$$\overline{A}\overline{B}C + \overline{A}\overline{B} + AB\overline{C}D = \overline{A}\overline{B}CD + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}\overline{C}D + \overline{A}\overline{B}\overline{C}\overline{D} + AB\overline{C}D$$

$$A + \overline{A} = 1$$

A B C D  
 0 0 0 0  
 0 0 0 1  
 ...  
 1 1 1 1

# Binary Representation of a Standard Product Term

- A standard product term is equal to 1 for only one combination of variable values.

- Example:  $A\bar{B}C\bar{D}$  is equal to 1 when  $A=1$ ,  $B=0$ ,  $C=1$ , and  $D=0$  as shown below

$$A\bar{B}C\bar{D} = 1 \cdot 0 \cdot 1 \cdot 0 = 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

- And this term is 0 for all other combinations of values for the variables.

# Product-of-Sums (POS)

# The Product-of-Sums (POS) Form

- When two or more sum terms are multiplied, the result expression is a product-of-sums (POS):

- Examples:

$$(\bar{A} + B)(A + \bar{B} + C)$$

$$(\bar{A} + \bar{B} + \bar{C})(C + \bar{D} + E)(\bar{B} + C + D)$$

$$(A + B)(A + \bar{B} + C)(\bar{A} + C)$$

- Also:

$$\bar{A}(\bar{A} + \bar{B} + C)(B + C + \bar{D})$$

- In a POS form, a single overbar cannot extend over more than one variable; however, more than one variable in a term can have an overbar:

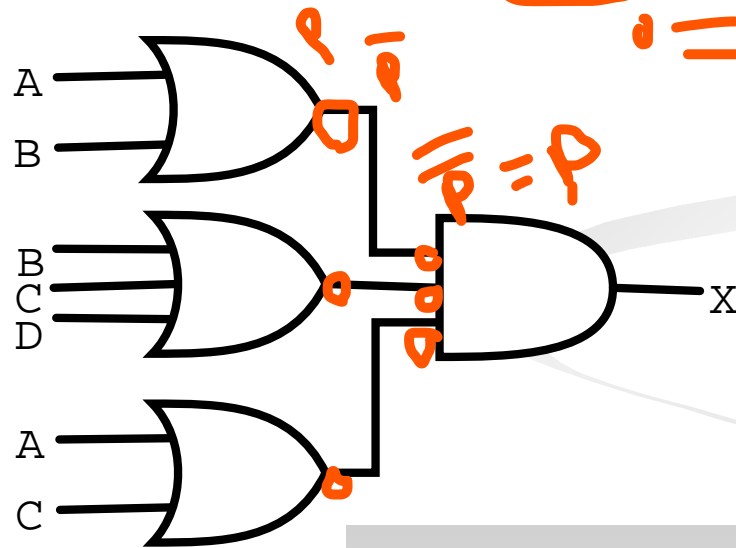
- example:  $\bar{A} + \bar{B} + \bar{C}$  is OK!

- But not:  $\overline{A + B + C}$

# Implementation of a POS

$$X = (A+B)(B+C+D)(A+C)$$

- OR/AND implementation = NOR





# The Standard POS Form

- A standard POS expression is one in which *all* the variables in the domain appear in each sum term in the expression.

- Example:  $(\bar{A} + \bar{B} + \bar{C} + \bar{D})(A + \bar{B} + C + D)(A + B + \bar{C} + D)$

- Standard POS expressions are important in:

- Constructing truth tables
- The Karnaugh map simplification method

# Converting a Sum Term to Standard POS

- **Step 1:** Add to each nonstandard product term a term made up of the product of the missing variable and its complement. This results in two sum terms.
  - As you know, you can add 0 to anything without changing its value.
- **Step 2:** Apply rule 12  $\rightarrow A + BC = (A + B)(A + C)$ .
- **Step 3:** Repeat step 1 until all resulting sum terms contain all variable in the domain in either complemented or uncomplemented form.

# Converting a Sum Term to Standard POS (example)

- Convert the following Boolean expression into standard POS form:

$$(A + \bar{B} + C)(\bar{B} + C + \bar{D})(A + \bar{B} + \bar{C} + D)$$

$$A + \bar{B} + C = A + \bar{B} + C + D\bar{D} = (A + \bar{B} + C + D)(A + \bar{B} + C + \bar{D})$$

$$\bar{B} + C + \bar{D} = \bar{B} + C + \bar{D} + A\bar{A} = (A + \bar{B} + C + \bar{D})(\bar{A} + \bar{B} + C + \bar{D})$$

$$(A + \bar{B} + C)(\bar{B} + C + \bar{D})(A + \bar{B} + \bar{C} + D) =$$

$$(A + \bar{B} + C + D)(A + \bar{B} + C + \bar{D})(A + \bar{B} + C + \bar{D})(\bar{A} + \bar{B} + C + \bar{D})(A + \bar{B} + \bar{C} + D)$$

$A \cdot A = A$



# Binary Representation of a Standard Sum Term

- A standard sum term is equal to 0 for only one combination of variable values.
  - Example:  $A + \bar{B} + C + \bar{D}$  is equal to 0 when  $A=0$ ,  $B=1$ ,  $C=0$ , and  $D=1$  as shown below

$$A + \bar{B} + C + \bar{D} = 0 + \bar{1} + 0 + \bar{1} = 0 + 0 + 0 + 0 = 0$$

- And this term is 1 for all other combinations of values for the variables.

# SOP/POS

# Converting Standard SOP to Standard POS

- The Facts:
  - The binary values of the product terms in a given standard SOP expression are not present in the equivalent standard POS expression.
  - The binary values that are not represented in the SOP expression are present in the equivalent POS expression.

# Converting Standard SOP to Standard POS

- What can you use the facts?
  - Convert from standard SOP to standard POS.
- How?
  - **Step 1:** Evaluate each product term in the SOP expression. That is, determine the binary numbers that represent the product terms.
  - **Step 2:** Determine all of the binary numbers not included in the evaluation in Step 1.
  - **Step 3:** Write the equivalent sum term for each binary number from Step 2 and express in POS form.

# Converting Standard SOP to Standard POS (example)

SOP  
SPOS

- Convert the SOP expression to an equivalent POS expression:

$$f = \overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$$

- The evaluation is as follows:

$$000 + 010 + 011 + 101 + 111$$

- There are 8 possible combinations. The SOP expression contains five of these, so the POS must contain the other 3 which are: 001, 100, and 110.

$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$

$$2^3 = 8$$

$$= 3$$

0	1
1	0
1	1



# Boolean Expressions & Truth Tables

- All standard Boolean expression can be easily converted into truth table format using binary values for each term in the expression.
- Also, standard SOP or POS expression can be determined from the truth table.

# Converting SOP Expressions to Truth Table Format

- Recall the fact:
  - An SOP expression is equal to 1 only if at least one of the product term is equal to 1.
- Constructing a truth table:
  - **Step 1:** List all possible combinations of binary values of the variables in the expression.
  - **Step 2:** Convert the SOP expression to standard form if it is not already.
  - **Step 3:** Place a 1 in the output column (X) for each binary value that makes the standard SOP expression a 1 and place 0 for all the remaining binary values.

# Converting SOP Expressions to Truth Table Format (example)

- Develop a truth table for the standard SOP expression

$$\overline{A}\overline{B}C + A\overline{B}\overline{C} + ABC$$

$A \rightarrow 1$   
 $\overline{A} \rightarrow 0$

Inputs			Output	Product Term
A	B	C	X	
0	0	0	0	
0	0	1	1	$\overline{A}\overline{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\overline{B}\overline{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	$ABC$

# Converting POS Expressions to Truth Table Format

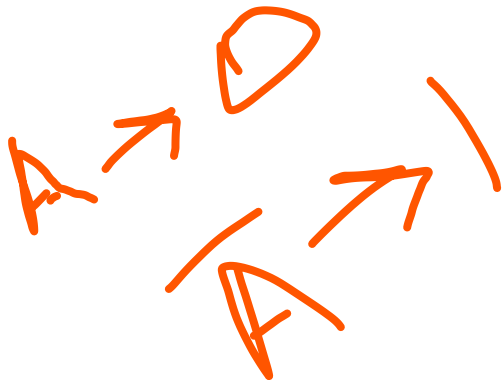
- Recall the fact:
  - A POS expression is equal to 0 only if at least one of the product term is equal to 0.
- Constructing a truth table:
  - **Step 1:** List all possible combinations of binary values of the variables in the expression.
  - **Step 2:** Convert the POS expression to standard form if it is not already.
  - **Step 3:** Place a 0 in the output column (X) for each binary value that makes the standard POS expression a 0 and place 1 for all the remaining binary values.

# Converting POS Expressions to Truth Table Format (example)

- Develop a truth table for the standard SOP expression

$$(A + B + C)(A + \bar{B} + C)(A + \bar{B} + \bar{C})$$

$$(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$$



Inputs			Output	Product Term
A	B	C	X	
0	0	0	0	$(A + B + C)$
0	0	1	1	
0	1	0	0	$(A + \bar{B} + C)$
0	1	1	0	$(A + \bar{B} + \bar{C})$
1	0	0	1	
1	0	1	0	$(\bar{A} + B + \bar{C})$
1	1	0	0	$(\bar{A} + \bar{B} + C)$
1	1	1	1	

POS  $\rightarrow$  0/1 is 0 for max

# Determining Standard Expression from a Truth Table

- To determine the standard **SOP expression** represented by a truth table.
- Instructions:
  - **Step 1:** List the binary values of the input variables for which the output is 1.
  - **Step 2:** Convert each binary value to the corresponding product term by replacing:
    - each 1 with the corresponding variable, and
    - each 0 with the corresponding variable complement.
- Example:  $1010 \rightarrow A\bar{B}C\bar{D}$

# Determining Standard Expression from a Truth Table

- To determine the standard **POS expression** represented by a truth table.
- Instructions:
  - **Step 1:** List the binary values of the input variables for which the output is 0.
  - **Step 2:** Convert each binary value to the corresponding product term by replacing:
    - each 1 with the corresponding variable complement, and
    - each 0 with the corresponding variable.
- Example:  $1001 \rightarrow \bar{A} + B + C + \bar{D}$

# Determining Standard Expression from a Truth Table (example)

I / P			O / P
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

■ There are four 1s in the output and the corresponding binary value are 011, 100, 110, and 111.

■ There are four 0s in the output and the corresponding binary value are 000, 001, 010, and 101.

$$011 \rightarrow \bar{A}BC$$

$$100 \rightarrow A\bar{B}\bar{C}$$

$$110 \rightarrow AB\bar{C}$$

$$111 \rightarrow ABC$$

$$000 \rightarrow A + B + C$$

$$001 \rightarrow A + B + \bar{C}$$

$$010 \rightarrow A + \bar{B} + C$$

$$101 \rightarrow \bar{A} + B + \bar{C}$$

$$X = \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

$$X = (A + B + C)(A + B + \bar{C})(A + \bar{B} + C)(\bar{A} + B + \bar{C})$$

$$= \Pi M(0, 1, 2, 5)$$

$$\checkmark \geq \sum m(3, 4, 6, 7)$$



# Rules of Boolean Algebra

$$1. A + 0 = A$$

$$2. A + 1 = 1$$

$$3. A \bullet 0 = 0$$

$$4. A \bullet 1 = A$$

$$5. A + A = A$$

$$6. A + \bar{A} = 1$$

$$7. A \bullet A = A$$

$$8. A \bullet \bar{A} = 0$$

$$9. \bar{\bar{A}} = A$$

$$10. A + AB = A$$

$$11. A + \bar{A}B = A + B$$

$$12. (A + B)(A + C) = A + BC$$

---

*A, B, and C can represent a single variable or a combination of variables.*

# Assignment - 7

What are the similarities and differences in SOP and POS.