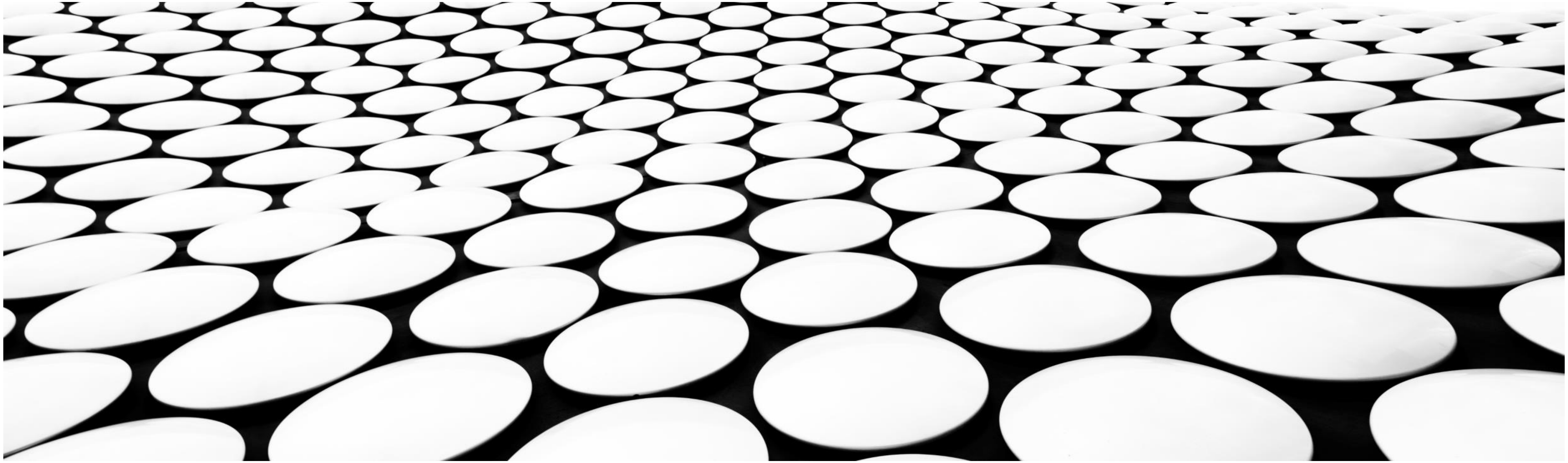
SIGNAL & SYSTEMS

MR. ANKUR JYOTI SARMAH

ASSISTANT PROF., DEPT. OF ELECTRONICS & TELECOM. ENGG.

ASSAM ENGINEERING COLLEGE



Stable and unstable system :-

BIBO → Bounded i/p → finite value

Bounded o/p → finite value

h(t) → system Response

$$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty \longrightarrow \text{stable system.}$$

$< \infty$ → less than infinity

$\boxed{\text{LTI system}}$ → Linear Time invariant system

Causal

$y(t) = \cos(x(t))$

Sinusoidal i/p → cos(i/p)

-1 to 1

finite

BIBO stable

$y(t) = x(-t-2)$

folding shifting

i/p for this case assume bounded

A bounded i/p will always remain bounded after folding and shifting operation.

O/p bounded

System is stable.

$$y(t) = t \cdot x(t) \begin{cases} \rightarrow \text{stable} \\ \rightarrow \text{unstable} \end{cases}$$

$$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t)$$

impulse response of system $(h(t))$

$t \to \infty \qquad y(t) = \infty \longrightarrow$ unstable

$t \to 0 \qquad y(t) = 0 \longrightarrow$ stable

$$\underline{y(t) = x(t)}$$

Test the stability of the LTI system, whose impulse response is given as

$$h(t) = e^{-5|t|}$$

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} \left| e^{-5|t|} \right| dt = \int_{-\infty}^{\infty} e^{-5|t|}\, dt$$

$$= \int_{-\infty}^{0} e^{-s(-t)}\,dt + \int_{0}^{\infty} e^{-s(t)}\,dt$$

$$= \int_{-\infty}^{0} e^{st}\,dt + \int_{0}^{\infty} e^{-st}\,dt$$

$$= \left(\frac{e^{st}}{s}\right)_{-\infty}^{0} + \left(\frac{e^{-st}}{-s}\right)_{0}^{\infty}$$

$$= \frac{1}{s}\cancel{\frac{e^{s.0}}{s}} - \cancel{\frac{e^{-s\infty}}{s}} + \cancel{\frac{e^{-s\infty}}{(-s)}} - \frac{e^{\overset{1}{-s.0}}}{(-s)}$$

$$0 \qquad 0$$

$$= \frac{2}{s}$$

$-t \to -\infty < t < 0$

$t \to 0 < t < \infty$

$$2\int_{0}^{\infty} e^{st}\,dt = \left(\frac{e^{st}}{s}\right)_{0}^{\infty}$$

$$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$h(t) = e^{4t} u(t)$

$h(t) = e^{4t}$ $\quad u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

Only $t \geq 0$.

$$\int_0^{\infty} e^{4t}\, dt = \frac{1}{4}\left[e^{4t}\right]_0^{\infty} = \frac{1}{4}\left[e^{\infty} - e^{0}\right] = \frac{1}{4}\left[\infty - 1\right] = \infty$$

Unstable stable.

$h(t) = e^{-4t} u(t)$

$$\int_0^\infty e^{-4t}\,dt = \frac{1}{-4}\left[e^{-4t}\right]_0^\infty = \frac{1}{-4}\left[\cancel{e^{-\infty}}^{\,0} - e^{0}\right]$$

$$= \frac{1}{-4}\left[-1\right]$$

$$= \frac{1}{4}.$$

Stable system.

$$h(t) = t\cos u(t).$$

$$\int u v = u\int v - \int\left[du \int v\right]$$

ILATE

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{0}^{\infty} \underset{u}{t}\,\underset{v}{\cos t}\,dt =$$

$$= \left[t\int \cos t\,dt - \int\left[\cancel{\frac{d}{dt}t}^{\,1}\int \cos t\,dt\right]dt\right]_0^{\infty}$$

$$= \left[t\sin t\right]_0^{\infty} - \left[\int \sin t\,dt\right]_0^{\infty}$$

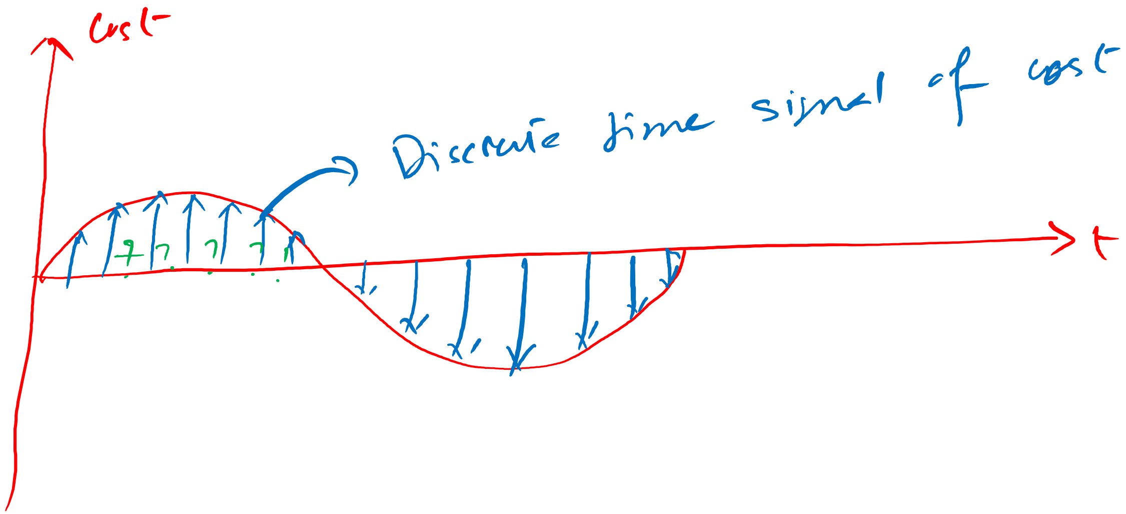
$$\left[t\sin t + \cos t\right]_0^{\infty}$$

$$= \left[t\sin t\right]_0^{\infty} - \left[-\cos t\right]_0^{\infty}$$

$$= \left[\underbrace{t\sin t}_{\infty}\right]_0^{\infty} + \left[\cos t\right]_0^{\infty} = \infty$$

Analog domain → Real life incidents.

Continuous time domain ⟶ Discrete time domain system.



what is sampling ?

$\Rightarrow$ process to convert continuous time signal into discrete time signal.

$x_a(t)$ = Analog / continuous time signal

$x(n)$ = Discrete time signal $\rightarrow$ obtained by sampling $x_a(t)$

$x(n) = x_a(t)\Big|_{t=nT} = x_a(nT) = x_a\left(\frac{n}{F_s}\right)$

$T \rightarrow$ sampling period

$F_s = \frac{1}{T} \Rightarrow$ sampling freq.

Range of $n \Rightarrow -\infty < n < \infty$

$\therefore x(n) = x_a\left(\frac{n}{F_s}\right)$

$x_a(t) = A\cos(\Omega_0 t + \theta) = A\cos(2\pi F_0 t + \theta)$

$\Omega_0 \rightarrow$ Analog freq $\rightarrow$ rad/s

$\Omega_0 = 2\pi F_0$

$\Rightarrow F_0 = \frac{\Omega_0}{2\pi} = \text{Hz}$

Let $x_a(t)$ be sampled at intervals T seconds to get $x(n)$

$T = \frac{1}{F_s}$

$\omega = 2\pi f$

$x(n) = x_a(t)\big|_{t=nT} = A\cos(\Omega_0 t + \theta)\big|_{t=nT}$

$= A\cos(\Omega_0 nT + \theta)$

$= A\cos(2\pi F_0 n \cdot \frac{1}{F_s} + \theta)$

$\frac{F_0}{F_s} = f_0$

$x(t) = A\cos(\Omega_0 t + \theta)$

$x(n) = A\cos(\omega_0 n + \theta)$

$= A\cos\left(\frac{2\pi F_0}{F_s} \cdot n + \theta\right)$

$= A\cos\left(2\pi \frac{F_0}{F_s} \cdot n + \theta\right) = A\cos(2\pi f_0 n + \theta) = A\cos(\omega_0 n + \theta)$