SIGNALS & SYSTEMS

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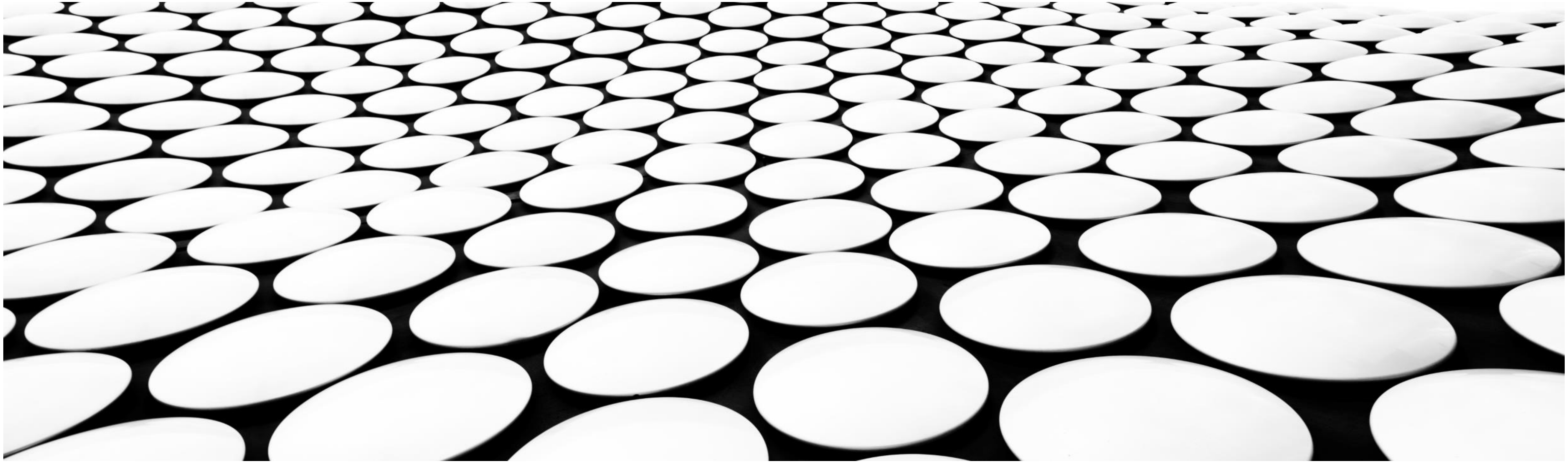


:- Relationship between C_n and Fourier transform :-

→ We all know that C_n is complex exponential fourier coefficient.

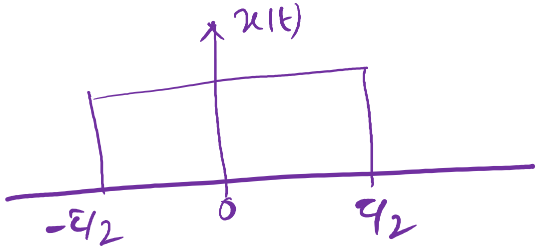
C_n is given as $\quad C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)\, e^{-jn\omega_0 t}\, dt \quad$ —(1)

→ We also know that, fourier transform for a signal $x(t)$ is $X(\omega)$

i.e., $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega_0 t}\, dt.$

Say, in our discussion,

$x(t)$ is a rectangular pulse shown in the figure



Let us consider $\omega = n\omega_0$

Then (2) $\Rightarrow$ $$X(n\omega) = \int_{-\tau/2}^{\tau/2} x(t)\, e^{-jn\omega_0 t}\, dt \quad \text{—— (3)}$$

Similarly we can modify eq^n (1) as $$C_n = \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} x(t)\, e^{-jn\omega_0 t}\, dt \quad \text{—— (4)}$$

$$\Rightarrow C_n = \frac{1}{T_0} X(n\omega)$$

$$\therefore \boxed{C_n = \frac{X(n\omega)}{T_0}}$$

Fourier transform can also be determined for periodic signal.

x(t) can be expressed as $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$ in terms of fourier series.

To establish this fact, we us first recall the fourier transform of a DC signal.

i.e., $A_0 \xrightarrow{FT} 2\pi A_0 \delta(\omega)$

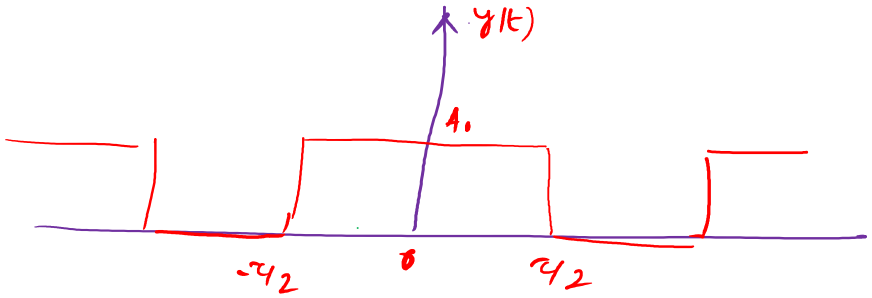
If $A = 1 \Rightarrow 1 \xrightarrow{FT} 2\pi(1)\delta(\omega) = 2\pi\delta(\omega)$

$\Rightarrow c_n \times 1 \xrightarrow{FT} 2\pi c_n \delta(\omega)$ [as per homogeneity]

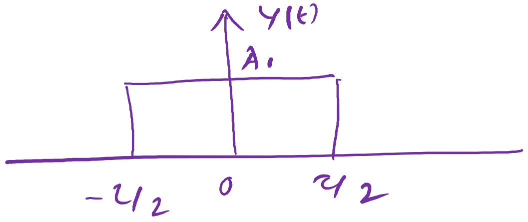
$\Rightarrow c_n e^{jn\omega_0 t} \xrightarrow{FT} 2\pi c_n \delta(\omega-\omega_0)$ [as per time shifting property]

$$\Rightarrow \boxed{\sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \xrightarrow{FT} \sum_{n=-\infty}^{\infty} 2\pi c_n \delta(\omega-\omega_0)}$$

Q. Find the fourier transform for rectangular pulse shown in fig –



Sol:- To determine the FT consider



This out of the entire figure

We can consider this as it is aperiodic signal.

As we know that for periodic signal fourier expansion exists, therefore

$$X(\omega) = \sum_{n=-\infty}^{\infty} 2\pi c_n \, \delta(\omega - \omega_0) \quad \text{--- (1)}$$

To determine c_n :

We know that fourier transform of rectangular pulse

$$X(\omega) = A_0 \tau \, Sa(\omega\tau/2)$$

and $c_n = \dfrac{X(n\omega_0)}{T_0}$

so replace $\omega \to n\omega_0$

$$\therefore X(n\omega) = A_0 \tau \, Sa\left(\frac{n\omega_0\tau}{2}\right)$$

and so $c_n = \dfrac{A_0 \tau \, Sa(n\omega_0\tau/2)}{T_0}$

$$\therefore (1) \Rightarrow X(\omega) = \sum_{n=-\infty}^{\infty} 2\pi \frac{A_0\tau}{T_0} Sa\left(\frac{n\omega_0\tau}{2}\right) \delta(\omega - n\omega_0)$$

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