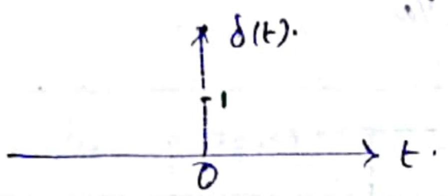
: unit ~~step~~ impulse signal :-

→ Also known as Dirac Delta function.

$$\delta(t) = \begin{cases} \infty, & t = 0 \quad \text{infinity.} \\ 0, & t \neq 0 \quad \text{zero.} \end{cases}$$



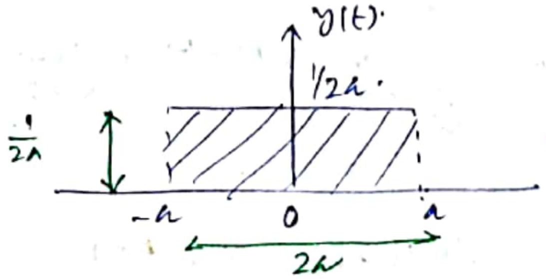
→ The area of unit impulse is always equal to 1.
ie; 'unit' terms in unit impulse signal stands for unit area.

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\, dt = 1}$$

let $y(t) = \begin{cases} \frac{1}{2a}, & -a \leq t < a \\ 0, & \text{otherwise.} \end{cases}$

waveform would be

<u>ways</u>:-



$\frac{1}{2a}$ is considered to maintain unit area

To find out the area.—

$$\int_{-\infty}^{\infty} y(t)\, dt = \int_{-a}^{a} y(t)\, dt = \cancel{\int_{-a}^{a}\left(2a \times \frac{1}{2a}\right) dt} = \cancel{\int_{-a}^{a} dt}$$

$$= 2a \times \frac{1}{2a} = 1$$

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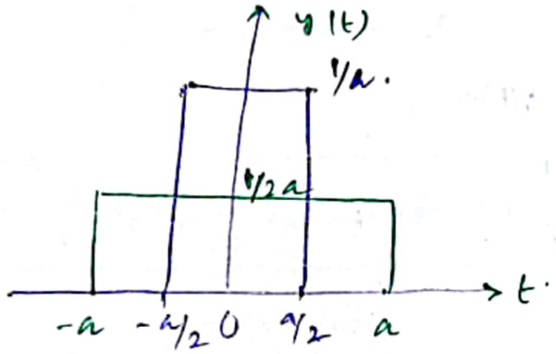
Case 2:- Let's decrease the width from 2a to a.

$-a \rightarrow -a/2$

$a \rightarrow a/2$

In this case length must be increased to maintain the unit area condition.



$Area = a \times \frac{1}{a} = 1.$

So keep on decreasing the width the height will go on increasing. i.e: $a \rightarrow 0$

$y(t) \rightarrow \infty$

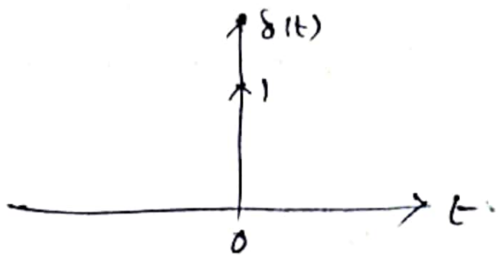
$\delta(t) = \lim_{a \to 0} y(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$

Properties of unit impulse signal:-

① $\int_{-\infty}^{\infty} \delta(t)\, dt = 1.$

$\therefore \int_{-\infty}^{\infty} \delta(t)\, dt = \int_{-\infty}^{\infty} \lim_{a \to 0} x(t)\, dt = \lim_{a \to 0} \int_{-\infty}^{\infty} x(t)\, dt$ 1

$\boxed{\text{Area of unit impulse signal} = 1.}$



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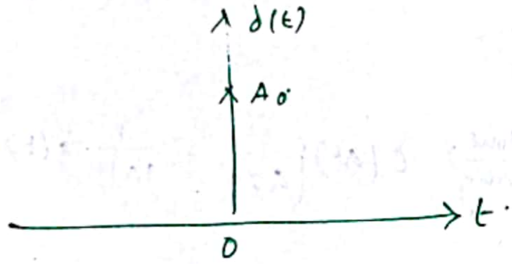
(2) Weight or strength of an impulse:-

$y(t) = A_0 \delta(t)$ → $y(t)$ weighted impulse signal since $\delta(t)$ is multiplied by A_0, a non-zero number.

$$\int_{-\infty}^{\infty} y(t)\,dt = \int_{-\infty}^{\infty} A_0 \delta(t)\,dt = A_0 \int_{-\infty}^{\infty} \delta(t)\,dt = A_0$$

$$\boxed{\text{Area of } y(t) = A_0 = \text{weight of impulse}}$$



This is the difference b/w unit impulse and normal impulse signal.

(3)

$$\int \delta(t)\,dt = u(t)$$

$$\boxed{\int A_0 \delta(t)\,dt = A_0 u(t)}$$

weight of the impulse signal = step of the step signal.

(4) $\boxed{\text{unit impulse or impulse signals are even in nature.}}$

$\therefore \delta(t) = \delta(-t)$

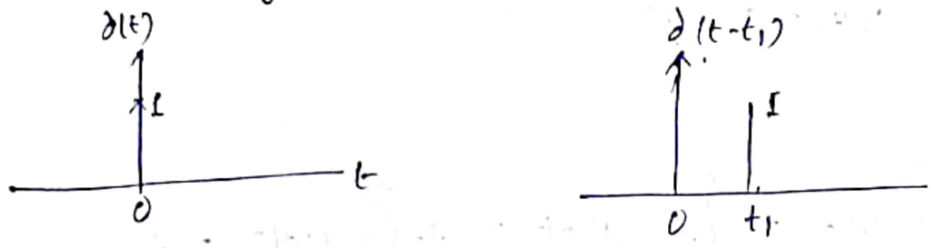
(5) When $t=0$, mag $= \infty$

Weight $\boxed{\text{Neither energy nor power}}$

Since at any instant of time if ~~power~~ magnitude is infinity then $P_\infty = \infty$.

$\therefore$ NENP

⑥ Time shifting:-



$$\boxed{\delta(t) = \delta(t-t_1)}$$

⑦ Time scaling:-

$$\delta(t) \xrightarrow[\text{scaling}]{\text{Time}} \delta(at)\Big|_{a\neq 0} = \frac{1}{|a|}\,\delta(t).$$

Ex:- (i) $\delta(-2t) = \frac{1}{|-2|}\,\delta(t) = \frac{1}{2}\,\delta(t).$

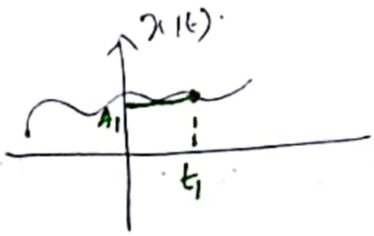
(ii) $\delta(-2t+3) = \delta[-2(t-1.5)]$

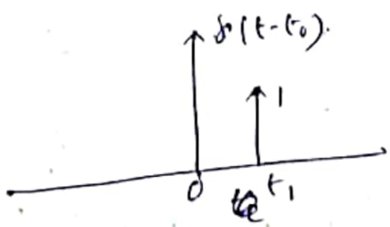
$= \frac{1}{|-2|}\,\delta(t-1.5)$

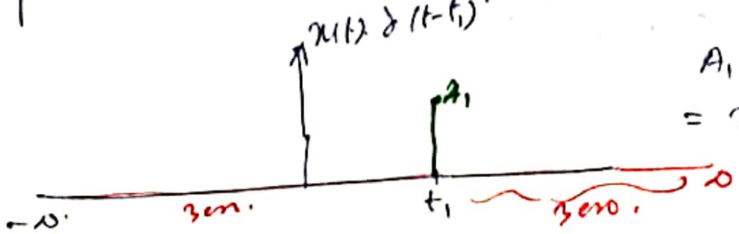
$= \frac{1}{2}\,\delta(t-1.5)$

⑧ Multiplication:-

$x(t)\cdot\delta(t-t_1) = x(t_1)\,\delta(t-t_1)$ → No change in impulse signal. but change in i/p signal.







$A_1\,\delta(t-t_1)$

$= x(t_1)\,\delta(t-t_1).$

Ex:-

i) $y(t) = 2t^2\,\delta(t-3)$.

$x(t)$

$y(t) = 2(3)^2\,\delta(t-3) = 2\times 9 = 18\,\delta(t-3)$.

ii) $y(t) = \cos 4t \cdot \delta(2t-\pi)$.

$= \cos 4t \cdot \delta[2(t-\pi/2)]$

$= \cos 4 \cdot (\pi/2)(\tfrac{1}{2})\delta(t-\tfrac{\pi}{2})$.

$y(t) = \frac{1}{2}\delta(t-\frac{\pi}{2})$.

(4) $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_1)\,dt = x(t_1) = \text{constant}$.

$\int_{-\infty}^{\infty} x(t_1)\,\delta(t-t_1)\,dt = x(t_1)\int_{-\infty}^{\infty}\delta(t-t_1)\,dt$

$= x(t_1)\int_{-\infty}^{\infty}\delta(t)\,dt$

$= x(t_1)$.

Ex: 1:

$I = \int_{-2}^{2} 2t^2\,\delta(t-4)\,dt = 0.$

$= \int_{-2/4}^{2/10} 2(4)^2\,\delta(t-4)\,dt$

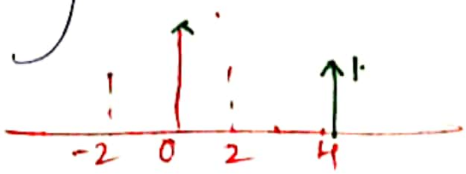
$= \int_{-12/10}^{2/10} 32\cdot\delta(t-4)\,dt$

$= 32.$

But this is not correct. Since range of the limit is included within time shifting.

here $t=4$.

limit -2 to 2.



Answer $= 0$

solve for $\int_{-10}^{10} 2t^2\,\delta(t-4)\,dt = 32$

(10) $\int_{-\infty}^{\infty} x(t) \cdot \frac{d^n}{dt^n} \delta(t-t_1)dt = (-1)^n \frac{d^n}{dt^n} x(t) \Big|_{t=t_1}$

only applicable when $x(t)|_{t=\infty} = 0$ or finite.

Ex:- $I = \int_{-\infty}^{\infty} \cos \pi t \cdot \frac{d^2}{dt^2} \delta(t-1)\, dt$.

$x(t) = \cos \pi t$ ⇒ Any $\cos \pi t$ lies in $[\omega - 1$ to $1]$.

$-1 \le \cos(\pi \infty) \le 1$.

$\therefore I = (-1)^2 \frac{d^2}{dt^2} (\cos \pi t) \Big|_{t=1}$

$= 1 \cdot \frac{d}{dt} \left[\frac{d}{dt} (\cos \pi t) \right] \Big|_{t=1}$

$= 1 \cdot \pi \frac{d}{dt} (-\sin \pi t) \Big|_{t=1}$

$= -\pi \cdot \frac{d}{dt} (\sin \pi t) \Big|_{t=1}$

$= -\pi^2 \cos \pi t \Big|_{t=1}$

$= -\pi^2 (\cos \pi)$

$= \pi^2$.

(11) $\frac{d}{dt} \delta(t)$ → Doublet function → odd signal.

Solve the following problems:-

i) $I = \int_{-5}^{4} \delta(t-5)\, dt = 0$.

ii) $I = \int_{-5}^{4} \delta(t-2)\, dt = 1$.