BJT Hybrid Equivalent Model

Semiconductor Devices and Circuits

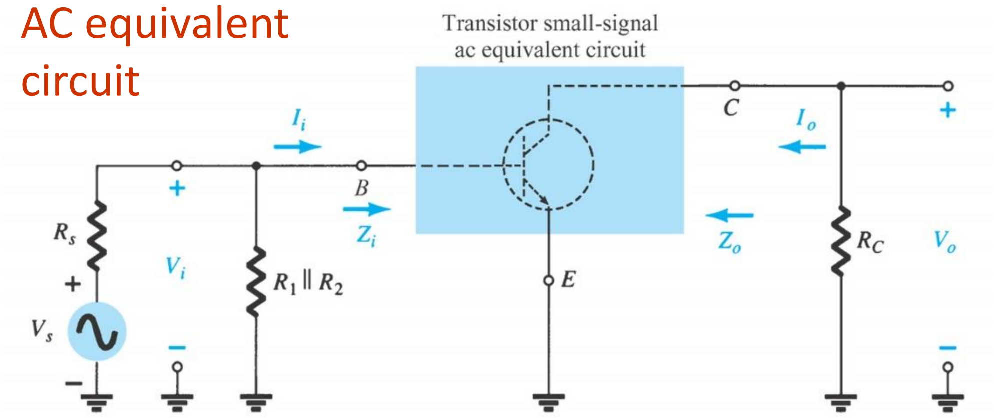
(ECE 181302)

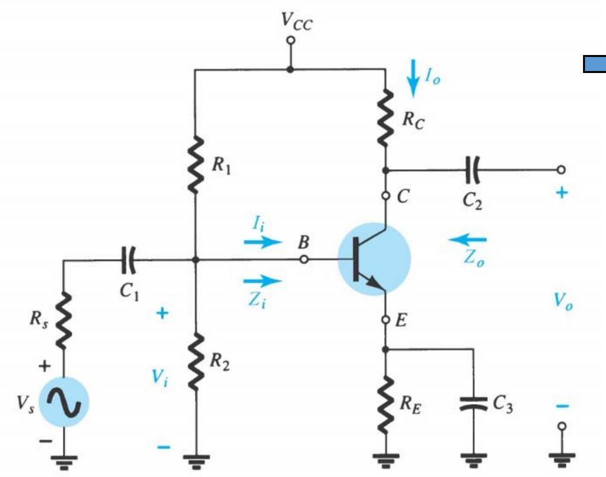
31st December 2021

BJT Transistor Modeling

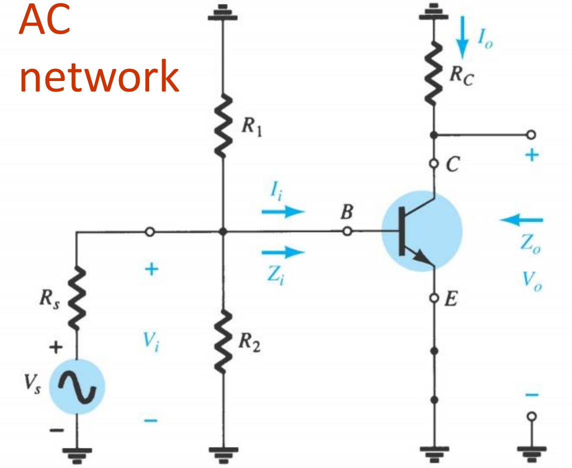
- An equivalent circuit that represents the AC characteristics of the transistor.
- Uses circuit elements that approximate the behavior of the transistor.
- There are three models commonly used in small signal AC analysis of a transistor:
  - r_e model
  - Hybrid equivalent model
  - Hybrid ∏ model

AC equivalent circuit





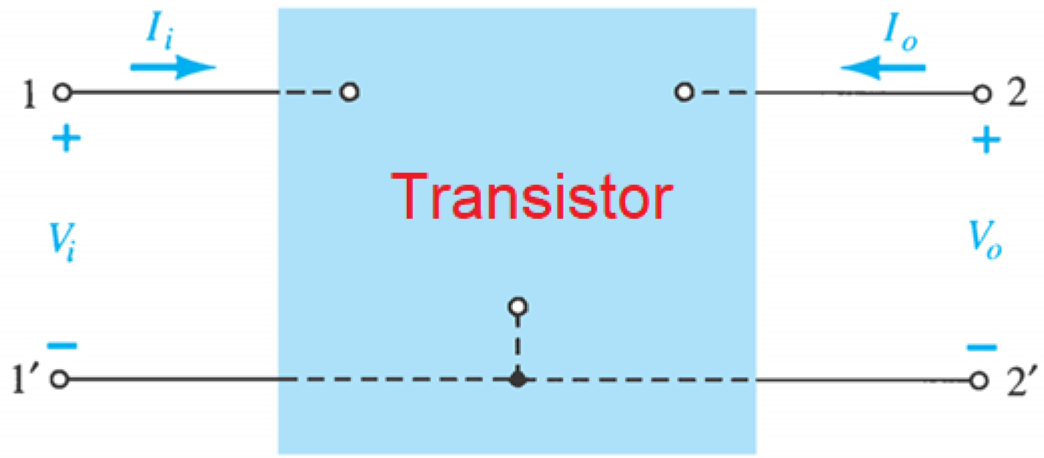
AC network

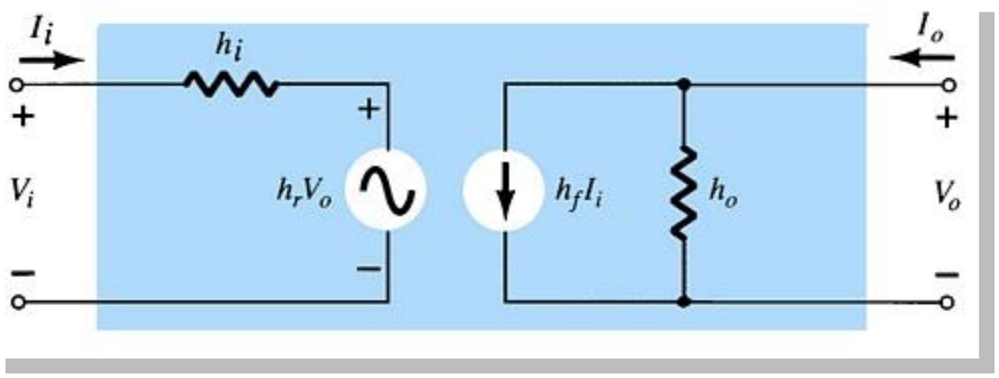


Sketch an AC network:

1. Remove DC supplies (replaced by short)
2. The coupling capacitor and bypass capacitor can be replaced by a short

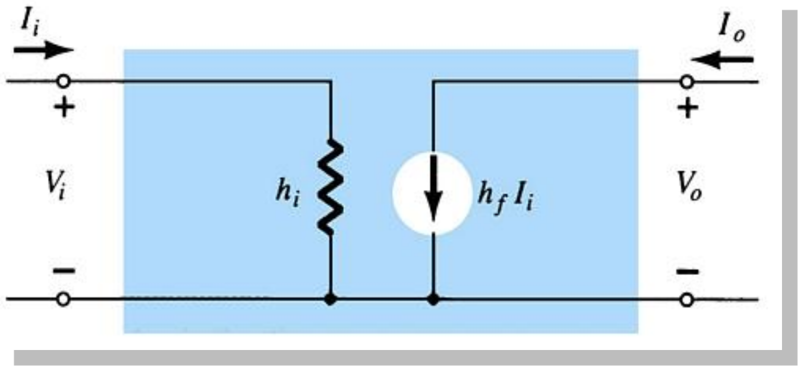
The Hybrid Equivalent Model





- h_i = input resistance
- h_r = reverse transfer voltage ratio $(V_i/V_o) \cong 0$
- h_f = forward transfer current ratio (I_o/I_i)
- h_o = output conductance $\cong \infty$

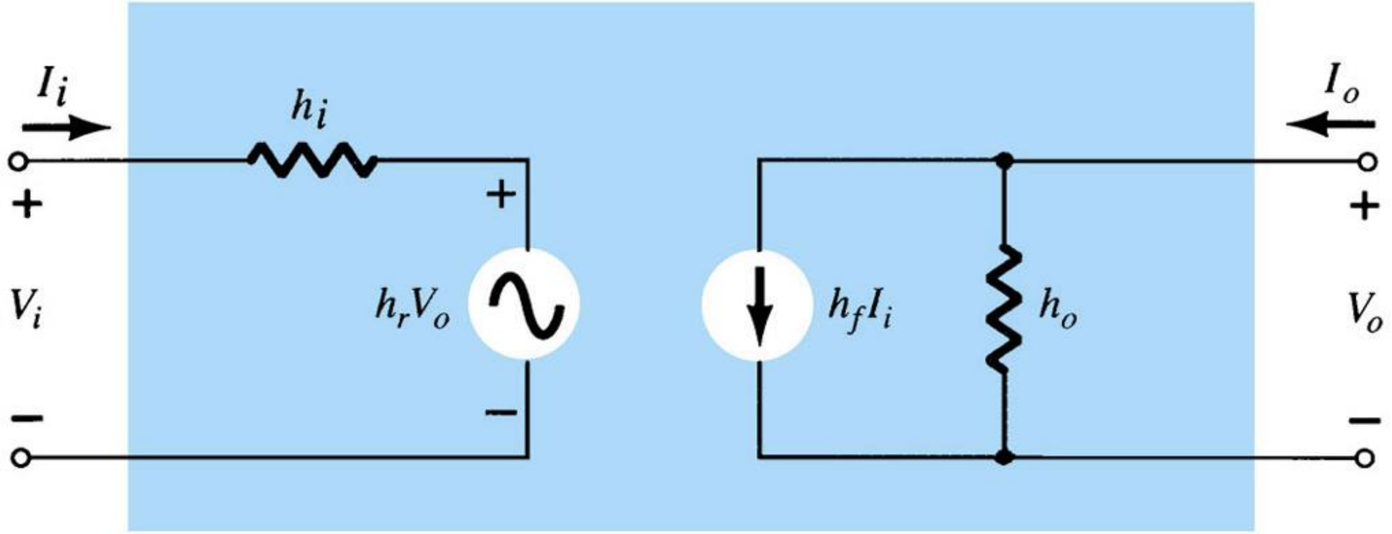
Simplified General H-Parameter Model: Approximate hybrid equivalent model



- h_i = input resistance
- h_r = reverse transfer voltage ratio $(V_i/V_o) \cong 0$
- h_f = forward transfer current ratio (I_o/I_i)
- h_o = output conductance $\cong \infty$

Hybrid Equivalent Model

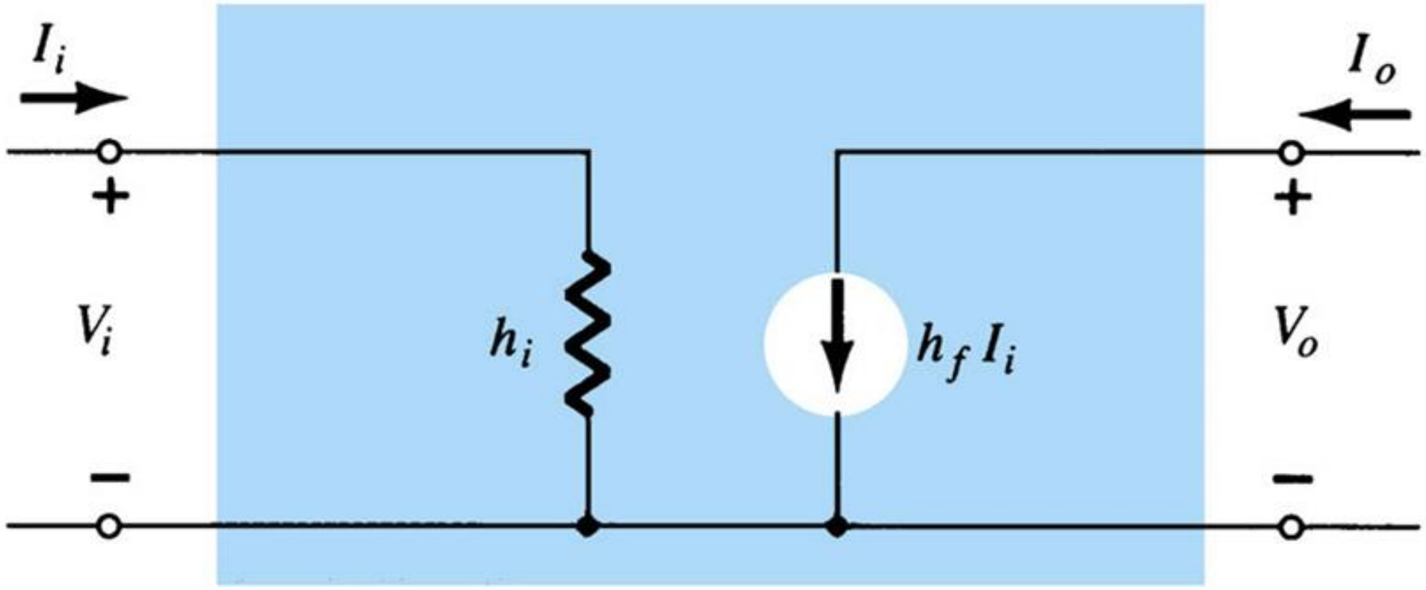
- General h-Parameters: hie, hre, hfe, hoe are developed and used to model the transistor.
- The h-Parameters can be found in a specification sheet for a transistor.



hi = input resistance
hr = reverse transfer voltage ratio (Vi/Vo)
hf = forward transfer current ratio (Io/Ii)
ho = output conductance

Simplified General h-parameter Model

- The general h-parameter model can be simplified based on approximations:
- hr $\cong$ 0 therefore hrVo = 0 and ho $\cong \infty$



Two Port Networks

- The standard configuration of a two port:

Network Equations:

Impedance Z parameters

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$V_2 = b_{11}V_1 - b_{12}I_1$$
$$I_2 = b_{21}V_1 - b_{22}I_1$$

Admittance Y parameters

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

Hybrid H parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

Transmission A, B, C, D parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

- From the four variables V_1, V_2, i_1 and i_2, two can be selected as independent variables and the remaining two can be expressed in terms of these independent variables.
- This leads to various two part parameters out of which the following three are more important.

Hybrid Parameters or h-parameters

- If the input current i_1 and output Voltage V_2 are takes as independent variables,
- Then input voltage V_1 and output current i_2 can be written as:

$$V_1 = h_{11}\, i_1 + h_{12}\, V_2$$
$$i_2 = h_{21}\, i_1 + h_{22}\, V_2$$

- The four hybrid parameters : h_{11}, h_{12}, h_{21} and h_{22} are defined as follows.

  - $h_{11} = [V_1 / i_1]$ with $V_2 = 0$, :I/P Resistance with output port short circuited.
  - $h_{12} = [V_1 / V_2]$ with $i_1 = 0$, : reverse voltage transfer ratio with i/p port open circuited.
  - $h_{21} = [i_2 / i_1]$ with $V_2 = 0$, : Forward current gain with output part short circuited.
  - $h_{22} = [i_2 / V_2]$ with $i_1 = 0$, : Output admittance with input part open circuited.

$h_{11} = [V_1 / i_1]$ with $V_2 = 0$

= Input Impedance with output part short circuited.

$h_{22} = [i_2 / V_2]$ with $i_1 = 0$

= Output admittance with input part open circuited.

$h_{12} = [V_1 / V_2]$ with $i_1 = 0$

= Reverse voltage transfer ratio with input part open circuited.

$h_{21} = [i_2 / i_1]$ with $V_2 = 0$

= Forward current gain with output part short circuited.

Dimensions of the h-parameters

h_{11} - Ω

h_{12} – dimension less

h_{21} – dimension less

h_{22} – mhos

- As the dimensions are not alike, (i.e) they are hybrid in nature, these parameters are called as hybrid parameters.

Equations for the Hybrid Parameters

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2 = 0} \qquad h_{12} = \frac{V_1}{V_2}\bigg|_{I_1 = 0}$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2 = 0} \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1 = 0}$$

$V_1 = h_{11}\, i_1 + h_{12}\, V_2$

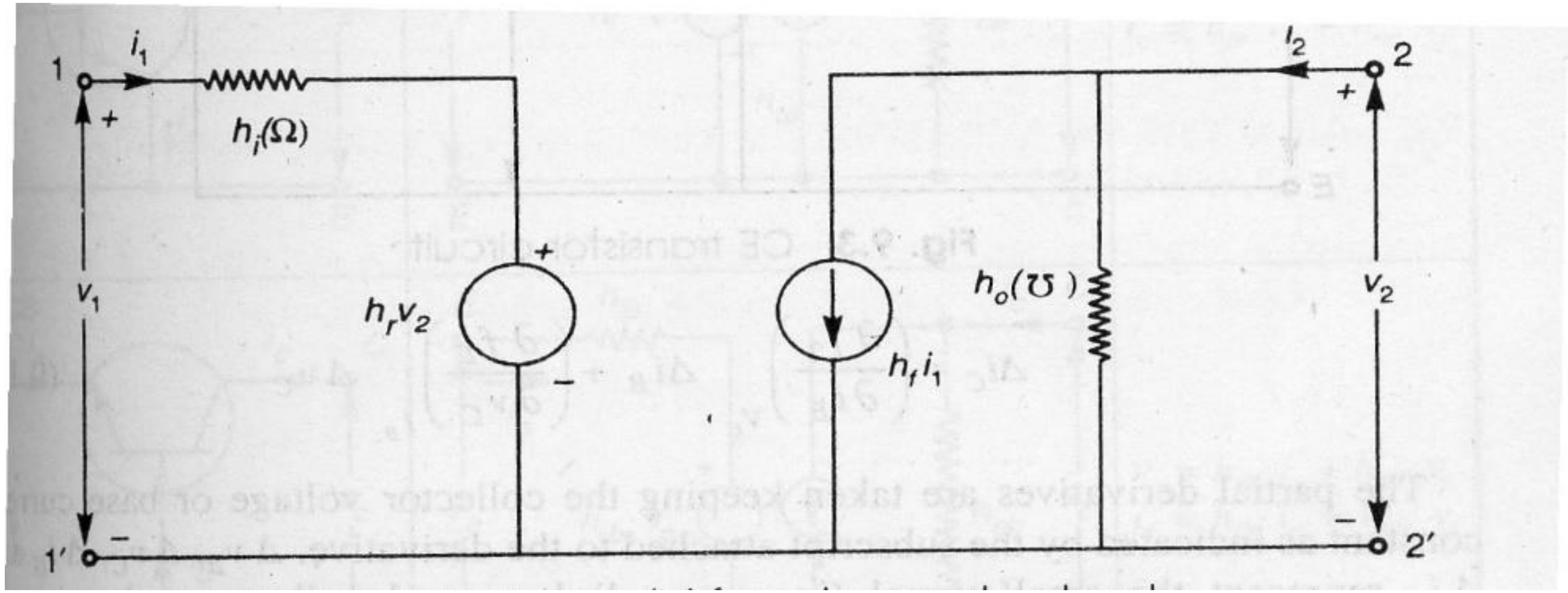
$\downarrow$

$V_1 = h_i\, i_1 + h_r\, V_2$

$I_2 = h_{21}\, i_1 + h_{22}\, V_2$

$\downarrow$

$I_2 = h_f\, i_1 + h_0\, V_2$

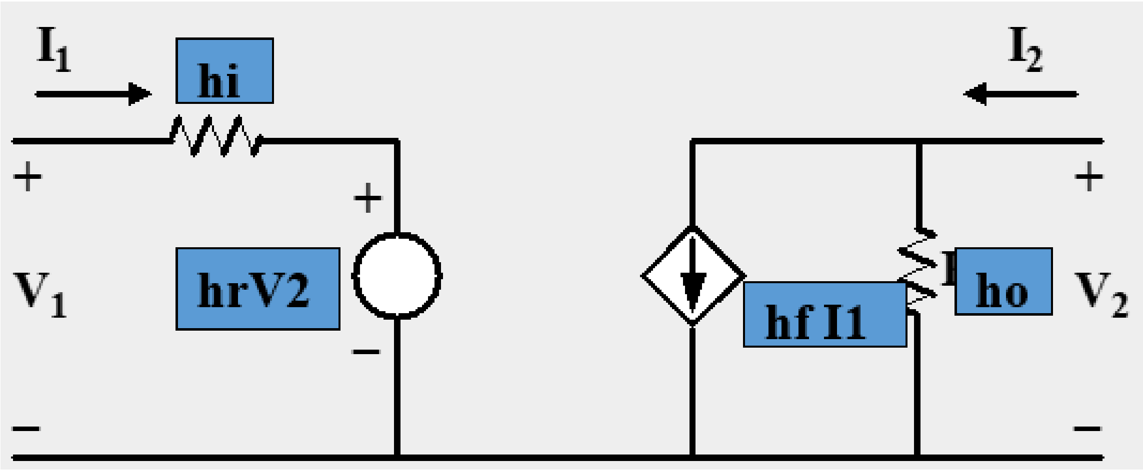


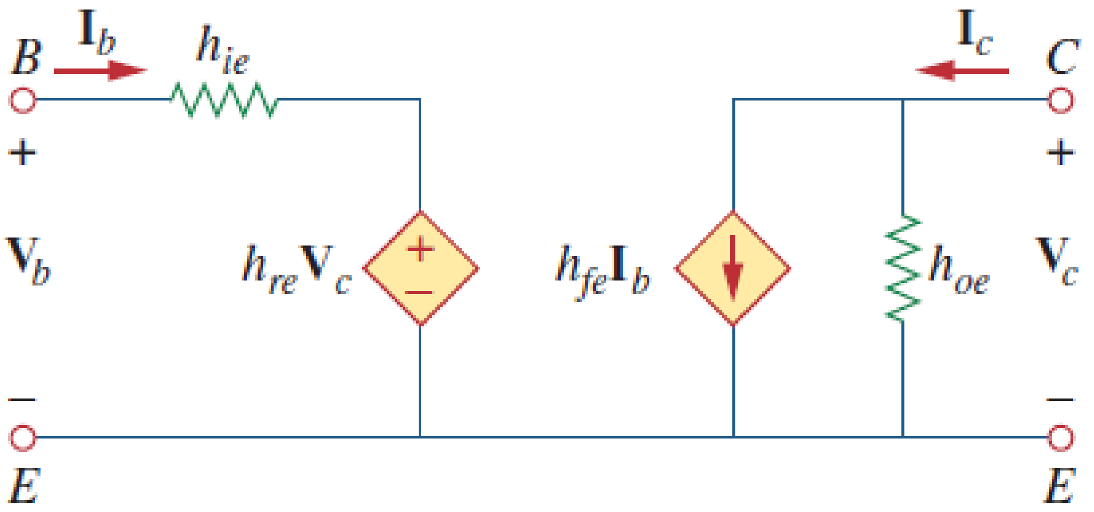
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_i & h_r \\ h_f & h_o \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_i = \frac{V_1}{I_1} \Bigg|_{V_2 = 0} \qquad h_r = \frac{V_1}{V_2} \Bigg|_{I_1 = 0}$$

$$h_f = \frac{I_2}{I_1} \Bigg|_{V_2 = 0} \qquad h_o = \frac{I_2}{V_2} \Bigg|_{I_1 = 0}$$

Transistor Model





- We can write the following equations:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_i & h_r \\ h_f & h_o \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

h-parameters Advantages

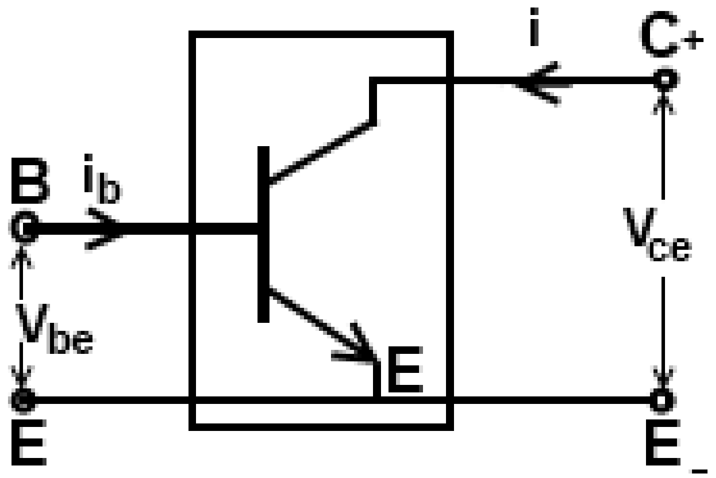
- h-parameters are real numbers up to radio frequencies.
- They are easy to measure.
- They can be determined from the transistor static characteristics curves.
- They are convenient to use in circuit analysis and design.
- Easily convert able from one configuration to other.
- Readily supplied by manufactories.

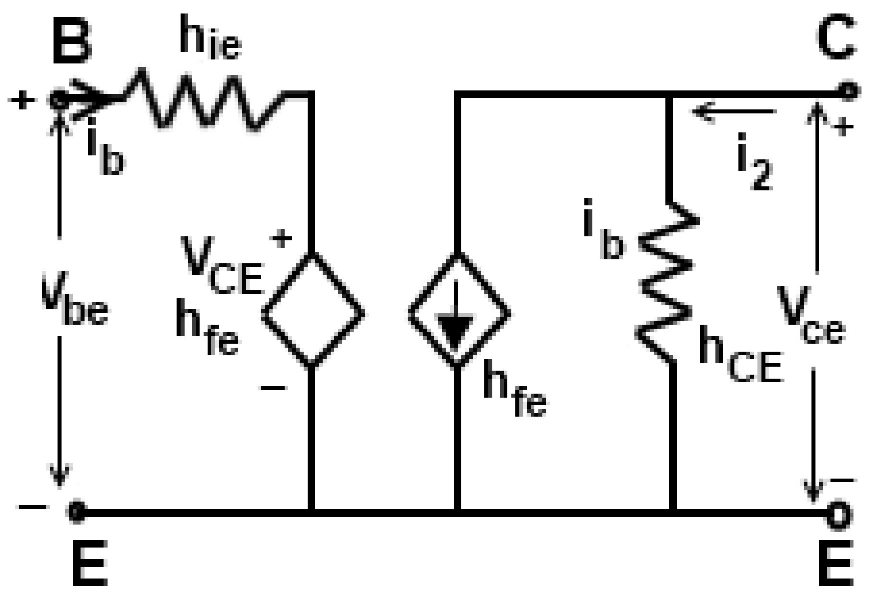
Hybrid Model CE Configuration

- In the common emitter transistor configuration, the input signal is applied between the base and emitter terminals of the transistor and output appears between the collector and emitter terminals.
- The input voltage (V_{be}) and the output current (i_c) are given by the following equations:

$V_{be} = h_{ie}.i_b + h_{re}.V_c$

$i_e \quad = h_{fe}.i_b + h_{oe}.V_c$

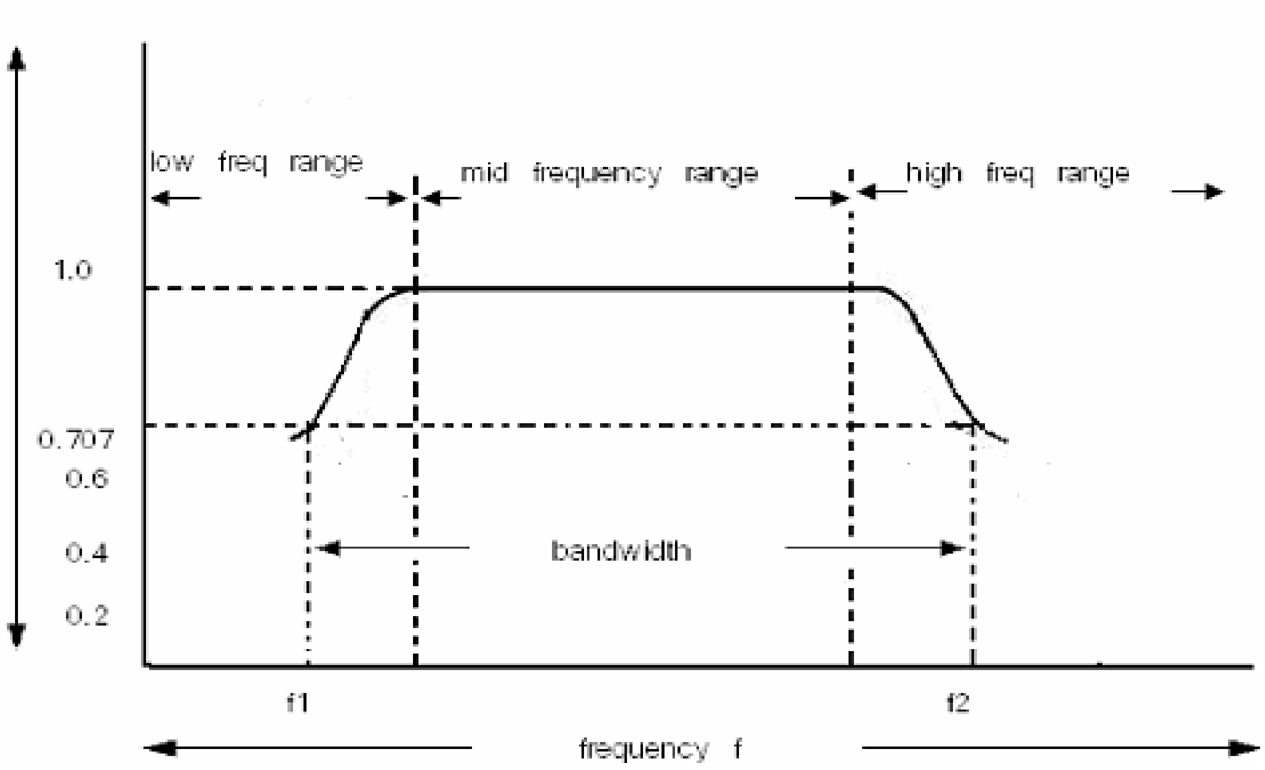




Hybrid h-Parameter Model for an Amplifier

- The equivalent circuit of a transistor can be drawn using simple approximation by retaining its essential features.
- The equivalent circuit aids in analyzing transistor circuits easily and rapidly.
- A transistor can be treated as a two part network.
- The terminal behavior of any two part network can be specified by the terminal voltages V_1 & V_2 at parts 1 & 2 respectively and current i_1 and i_2, entering parts 1 & 2, respectively, as shown in figure.

RC Coupled Amplifier



- Response of an amplifier depends on frequency considerations.
- There are 3 regions of frequency : low , mid and high
- The difference between high and low frequency is the bandwidth

Transistor Amplifier Analysis using Complete h-Parameter Model

- Small signal response is analyzed using the h-parameter model
- In the h-parameter model consider the load Resistance R_L and input signal V_s.
- The expressions for Current gain, Voltage gain ,input and output impedance are:

1. Current Gain:

$A_i = -h_f/(1+h_o R_L)$

- Where A_i is the current amplification or current gain.

- The overall current gain taking source resistance is given by:

  $A_{is}=A_i * (R_s/Z_i + R_s)$

- Where,
  - Z_i input impedance
  - R_s source resistance

2) Input Impedance(Zi)

   $Z_i= h_i+h_rA_iR_L$

3) Voltage Gain(A_v):

   $A_v=(A_i * R_L)/ Z_i$

- Voltage gain taking source resistance is given by

  $A_{vs}=(A_v * Z_i)/(Z_i+R_s)$

4) Output Admittance(Y_o)

   $Y_o=h_o-h_f * h_r/(h_i+R_s)$

Analysis of Transistor Amplifier using Simplified h-Parameter Model

- Common Emitter Configuration
- Fixed Bias configuration:

  Input Impedance $Z_i = R_B \,||\, h_{ie}$

  Output Impedance $Z_o=R_C \,||\, (1/h_{oe})$

  Voltage gain $A_v=-h_{fe} * (R_C \,||\, (1/h_{oe}) /h_{ie}$

  Current Gain $A_i=h_{fe} * R_B/(R_B + h_{ie})$

- Voltage Divider Configuration:

  Input impedance $Z_i=(R_{B1} \,||\, R_{B2})||\, h_{ie}$

  Output Impedance $Z_o=R_C \,||(1/h_{oe})$

  Voltage gain $Av=-h_{fe} * [R_C \,||\, (1/h_{oe})]/h_{ie}$

  Current gain $A_i=h_{fe} * (R_{B1}||R_{B2})/(R_{B1}||\, R_{B2}) + h_{ie}$

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