Assignment - 2

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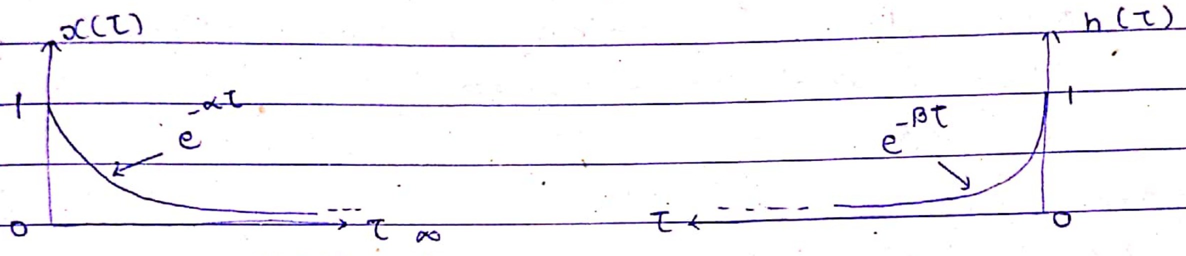
Time domain analysis of CT - LTI system

1] Using the convolution integral to find the response $y(t)$ of the LTI system with impulse response $h(t) = e^{-\beta t} u(t)$ to the input $x(t) = e^{-x(t)} u(t)$ for $\alpha = \beta$ and $\alpha \neq \beta$

$\rightarrow$ $x(t) = e^{-\alpha t} u(t) \Rightarrow x(\tau) = e^{-\alpha \tau} u(t)$

$h(t) = e^{-\beta t} u(t) \Rightarrow h(\tau) = e^{-\beta \tau} u(t)$



$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

Case I : When $t < 0$.

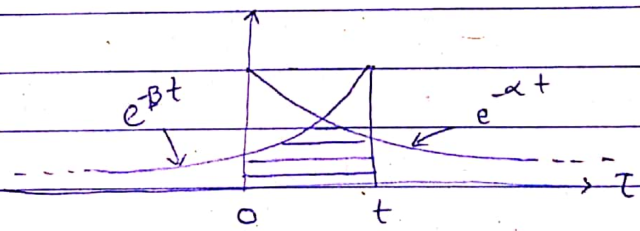
There will not be any overlapping.

$\therefore y(t) = 0$

Case - 2 : When $t > 0$

Here overlapping from 0 to t.

$$y(t) = \int_0^t e^{-\alpha \tau} e^{-\beta \tau} d\tau$$



$$y(t) = \int_0^t e^{-\tau(\alpha+\beta)} d\tau$$

$$= \left[\frac{e^{-\tau(\alpha+\beta)}}{-(\alpha+\beta)} \right]_0^t$$

$$= \frac{-1}{\alpha+\beta} \left(e^{-t(\alpha+\beta)} - e^0 \right)$$

$$= \frac{1 - e^{-t(\alpha+\beta)}}{\alpha+\beta} \quad ; \text{for } \alpha \neq \beta$$

If $\alpha = \beta$ then,

$$y(t) = \frac{1}{2}\left(1 - e^{-2t}\right) \quad ; \text{for } \alpha = \beta$$

2] Find the convolution of two signal $x(t)$ & $y(t)$

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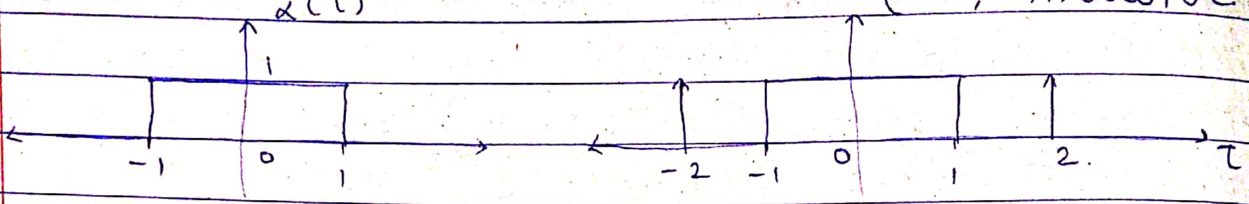
$x(t) = \begin{cases} 1 & , |t| \leq 1 \\ 0 & , \text{otherwise} \end{cases}$, $y(t) = \begin{cases} 1 & , |t| \leq 1 \\ \delta(t+1) + \delta(t+2) & \\ 0 & , \text{otherwise} \end{cases}$



according to diffination of convolution.

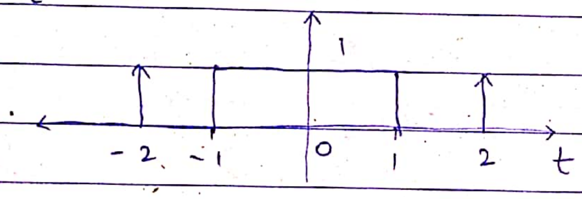
$$\delta(t) = \int_{-\infty}^{\infty} y(z)\, x(-z+t)\, dz$$

Case-1

$t < -2$ these will not be any overlaping $\delta(t) = 0$

Case-2

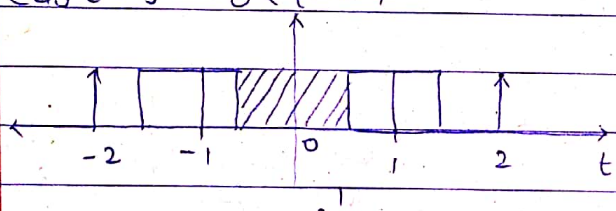
$t < -1$



Here overlaping is all only are position.

It is at $t = -2$, $y(t) = 1$

Case-3 $0 < t < 1$



Here overlaping is from -2 to 1.

$$y(t) = \int_{-2+t}^{1} 1 \cdot 1 \, dz = (z)^2 + t$$

$$1 - (-1+t) = 2 - t$$

Case-4

$t > 2$ simillar to condition 2 overlaping will at $t = 2$

Thus $y(t) = 1$

3) If $\alpha(s)$ is impulse transform o/p signal $x(t)$ & impulse transform of $x(t-T)$.

given $x(t) \xrightarrow{L.T} X(s)$

using time shifting property.

$\alpha(t-T) \longrightarrow e^{-st} x(s)$

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5] If to response of LTI continuous time system to unit step signal $\frac{1}{2} - \frac{1}{2} e^{-2t}$ then find impulse response of the system.

$$s(t) = \frac{1}{2} - \frac{1}{2} e^{-2t} \quad \& \quad x(t) = u(t)$$

$$s(t) = \int_0^t h(\tau)\, d\tau$$

$$h(t) = \frac{d}{dt} s(t) = \frac{d}{dt}\left(\frac{1}{2} - \frac{1}{2} e^{-2t}\right)$$

$$= 0 - \frac{1}{2}(-2)e^{-2t}$$

$$= e^{-2t}$$

7] Find the o/p of an LTI system with impulse response $h(t) = \delta(t-3)$ for the i/p $x(t) = \cos 4t + \sin 7t$.

$$h(t) = \delta(t-3), \quad x(t) = \cos 4t + \sin 7t$$

Now, o/p $y(t) = x(t) * h(t)$

$$= (\cos 4t + \sin 7t) * \delta(t-3)$$

$$= \cos 4t * \delta(t-3) + \sin 7t * \delta(t-3)$$

$$= \cos 4(t-3) + \sin 7(t-3)$$

4] Define laplace transform. Prove linearity property for laplace transform. State how ROC of laplace transform is usefull in defining stability of system.

→ Defination of laplace transform :-

- Consider $x(t)$ is a function of t for all $t > 0$. Then the laplace transform of $x(t)$ is denoted by $L\{x(t)\}$ and it is defined as, $L\{x(t)\} = \int_0^\infty x(t) e^{-st} dt = x(s)$.

→ Stability of system :-

- LTI system can be completly characterised by its impulse response $h(t)$. The input & output of

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system are related through convolution integral

$$x(t) \longrightarrow \boxed{\begin{array}{c} CT-LTI \\ \text{System } h(t) \end{array}} \longrightarrow y(t)$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

$$L\{y(t)\} = L\{x(t) * h(t)\}$$

$$Y(s) = X(s)\, H(s)$$

$$H(s) = \frac{Y(s)}{X(s)}$$

→ Linearity property :-

state :- Consider two time domain signals $x_1(t)$ & $x_2(t)$

If $x_1(t) \xleftrightarrow{LT} X_1(s)$ & $x_2(t) \xleftrightarrow{LT} X_2(s)$

Then $[a_1 x_1(t) + a_2 x_2(t)] \xleftrightarrow{LT} a_1 x_1(s) + a_2 x_2(s)$

proof :- $X(s) = L\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$

$$L\{a_1 x_1(t) + a_2 x_2(t)\} = \int_{-\infty}^{\infty} (a_1 x_1(t) + a_2 x_2(t)) e^{-st}\, dt$$

$$= a_1 \int_{-\infty}^{\infty} x_1(t)\, e^{-st}\, dt + a_2 \int_{-\infty}^{\infty} x_2(t)\, e^{-st}\, dt$$

$$= a_1 X_1(s) + a_2 X_2(s)$$

→ ROC :

- The resultant ROC is an intersection of ROC of $X_1(s)$ & $X_2(s)$.

6] Find inverse laplace transform of $(1 - e^{-2s}/s)$

$$x(s) = \frac{1}{s}(1 - e^{-2s})$$

$$x(s) = \frac{1}{s} - \frac{e^{-2s}}{s}$$

using shifting property,

$$x(t) = u(t) - u(t-2)$$

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12] Laplace transform of $u(t)$ is : $1/s$

13] Inverse laplace transform of $\frac{1}{(s+a)^2}$ is : $te^{-at}u(t)$

14] Inverse laplace transform of $\frac{10}{s^2+2s+5}$ is : $5e^{-t}\sin 2t$

10] Evaluate the step response for the LTI system response by following impulse response : $h(t) = e^{-|t|}$

→ It can be expresed as,

$$h(t) = e^{5t}u(-t) + e^{-5t}u(t)$$

step response to terms of impulse response is,

$$s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

$$s(t) = \int_{-\infty}^{t} e^{5\tau}\,d\tau + \int_{0}^{t} e^{-5\tau}\,d\tau$$

$$= \left[\frac{e^{5\tau}}{5}\right]_{-\infty}^{t} + \left[\frac{e^{-5t}}{-5}\right]_{0}^{t}$$

$$= \frac{1}{5}e^{5t} - \frac{1}{5}\left(e^{-5t} - 1\right)$$

$$= \frac{1}{5}\left(e^{5t} - e^{-5t} + 1\right)$$

11] Determine the output of the system described by the following differential equation with input & initial condition as specified.

$$\frac{d}{dt}y(t) + 10y(t) = 2x(t), \quad y(0) = 1, \quad x(t) = u(t)$$

taking laplace transform of both side,

$$sy(s) - y(0) + 10y(s) = 2\cdot\frac{1}{s}$$

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$$sY(s) - 1 + 10Y(s) = \frac{2}{s}$$

$$Y(s)(s+10) = \frac{s+2}{s}$$

$$Y(s) = \frac{s+2}{s(s+10)} = \frac{A_1}{s} + \frac{A_2}{s+10}$$

$$A_1 = \left.\frac{s+2}{s+10}\right|_{s=0} = \frac{2}{10} = 0.2$$

$$A_2 = \left.\frac{s+2}{s}\right|_{s=-10} = \frac{-8}{-10} = +0.8$$

$$\therefore Y(s) = \frac{0.2}{s} + \frac{0.8}{s+10}$$

taking inverse laplace,

$$y(t) = 0.2\,u(t) + 0.8\,e^{-10t}\,u(t).$$

16] Derive the convolution integral of CTS.

→ Convolution is a mathematical operation which is used as a tool by communication enginners for system analysis, probability theory & transform calculations. convolution can be performed in time as well as freq. domain.

→ The convolution of two fu^n $x(t)$ & $y(t)$ is defined as,

$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

- The integration is always performed with respect to a dummy variable such as τ & t treated as a constant so far as the integration is concerned.
- The process of convolution involves following operations of $y(\tau)$ while the signal $x(\tau)$ remain unchanged.

1) Folding or time reversal to obtain $y(-\tau)$.

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2) Time shifting the folded signal $y(-\tau)$ to obtain $y(t-\tau)$.

3) Mutiplication of $x(\tau)$ & $y(t-\tau)$.

4) Integration of the product term $x(\tau)y(t-\tau)$.

15] Consider the system described by $y'(t) + 2y(t) = x(t) + x'(t)$ find the impulse response $h(t)$ of the system.

→ $$\frac{dy(t)}{dt} + 2y(t) = x(t) + \frac{dx(t)}{dt}$$

$$sy(s) + 2y(s) = x(s) + sx(s)$$

$$y(s)(s+2) = x(s)(s+1)$$

$$H(s) = \frac{y(s)}{x(s)} = \frac{s+1}{s+2}$$

$$\frac{H(s)}{s} = \frac{s+1}{s(s+2)} = \frac{A_1}{s} + \frac{A_2}{s+2}$$

$$A_1 = \left.\frac{s+1}{s+2}\right|_{s=0} = \frac{1}{2}$$

$$A_2 = \left.\frac{s+1}{s}\right|_{s=-2} = \frac{1}{2}$$

$$\frac{H(s)}{s} = \frac{1}{2} \times \frac{1}{s} + \frac{1/2}{s+2}$$

$$H(s) = \frac{1}{2} + \frac{1}{2} \times \frac{s}{s+2}$$

$$h(t) = \frac{1}{2}\delta(t) + \frac{1}{2} t e^{-2t} u(t)$$

17] Define convolution & explain initial value & final value theorem.

→ Convolution :-

- It is a mathematical operation & the convolution bet^n $x(n)$ & $h(n)$ is denoted by $y(n) = x(n) + h(n)$.

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It is given by,

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

→ Initial value theorem :-

- It $x(n)$ is a causal sequnce then its initial value is given by, $x(0) = \lim_{z\to\infty} x(z)$

→ Final value theorem :-

- If $x(n) \xleftrightarrow{z} x(z)$ then $x(\infty) = \lim_{z\to 1} [(z-1)x(z)]$

19] Define :- Impulse response.

- If applied input to the system is unit impulse then output of system is called as impulse response.

8] Define the condition for LTI system to be stable, Which of the following impulse response correspond to stable LTI system. $h_1(t) = e^{-(1-2j)t} u(t)$.

→ Stability criteria for LTI system :-

State :- LTI system is stable if its impulse response is absolutely summable.

proof :-

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \quad \text{--- (1)}$$

or

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) \quad \text{--- (2)}$$

$$|x(n)| \leq Mx \quad \text{--- (3)}$$

$$-|x(n)| \leq Mx < \infty \quad \text{--- (4)}$$

$$\therefore |y(n)| = \left|\sum_{k=-\infty}^{\infty} h(k)x(n-k)\right| \quad \text{--- (5)}$$

Thus $\left|\sum_{k=-\infty}^{\infty} h(k)x(n-k)\right| \leq \sum_{k=-\infty}^{\infty} |h(k)||x(n-k)|$ --- (6)

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$$|y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)|\,|x(n-k)| \quad \text{--- (7)} \qquad (\because (5))$$

$$|x(n)| \leq M_x$$

$$\therefore |x(n-k)| \leq M_x$$

eq.ⁿ (7) $\Rightarrow |y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)|\, M_x$

$$\therefore |y(n)| \leq M_x \sum_{k=-\infty}^{\infty} |h(k)| \quad \text{--- (8)}$$

$$\therefore \sum_{k=-\infty}^{\infty} |h(k)| < \infty \qquad = S$$

→ $h_1(t) = e^{-(1-2j)t}\, u(t)$

According to condition of stability,

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

Here, $h(t) = e^{-t}\, e^{2jt}\, u(t)$

$$\int_{0}^{\infty} e^{-t}\, dt = \left[\frac{e^{-t}}{-1}\right]_0^{\infty}$$

$$= -1\,[e^{-\infty} - e^{0}] \qquad = 1 < \infty$$

So, System is stable.

8] The following is the impulse response of LTI system Determine whether the system is causal and / or stable. Justify your answers.

$$h(t) = e^{2t}\, u(-1-t)$$

$$= e^{2t}\, u(-t-1)$$

$u(-t-1)$ is existing from $-\infty$ to -1.

So, it is not causal.

According to condition of stability.

$$S = \int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

$$= \int_{-\infty}^{-1} e^{2t}\, dt = \left[\frac{e^{2t}}{2}\right]_{-\infty}^{-1} = \frac{1}{2}\,[e^{-2} - e^{-\infty}]$$

$$= -0.432 < \infty$$

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