SIGNALS & SYSTEMS

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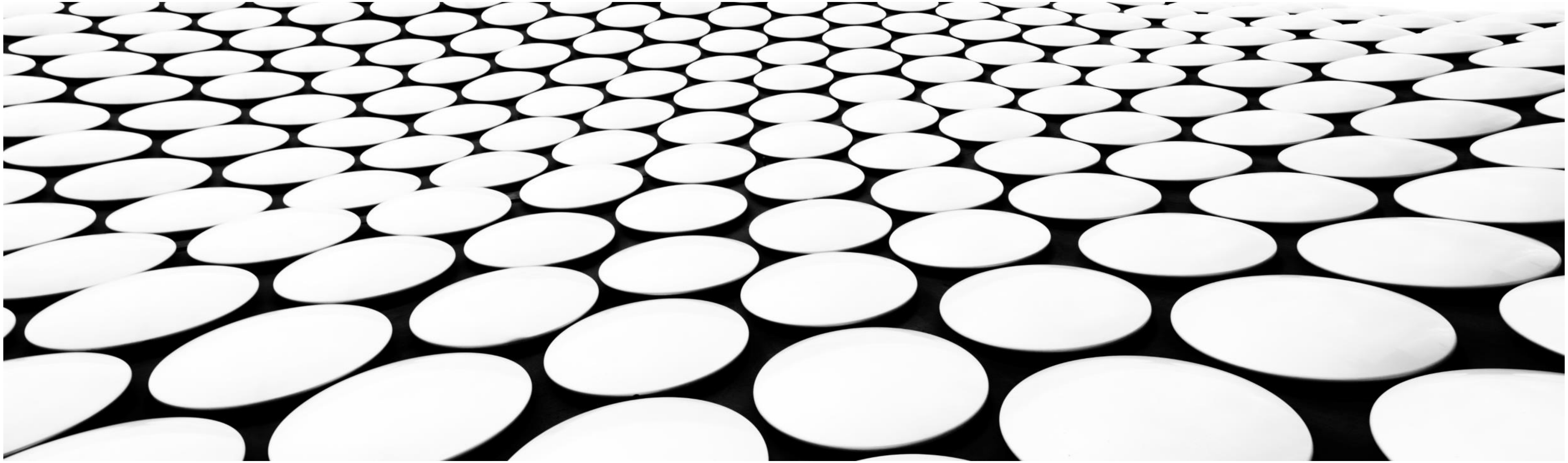


-: Complex - Exponential Fourier series: -

$x(t)$ is a given signal → Expand $x(t)$ in terms of CEFS

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \rightarrow \int x(t) \frac{1}{e^{jn\omega_0 t}} dt = C_n$$

$$C_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} dt \quad \text{—— (1)}$$

If we replace 'n' by '-n' → Reversal operation

$$C_{-n} = \frac{1}{T_0} \int_{T_0} x(t) e^{jn\omega_0 t} dt \quad \text{— (2)}$$

$n = -\infty$ to 0

↓

Negative freq.

↓

Physically can't be realized

C_n → Complex Fourier co-efficient

consider the conjugate in eqn (2)

$$C_{-n}^{*} = \frac{1}{T_0}\int_{T_0} x^{*}(t)\, e^{-jn\omega_0 t}\, dt \quad \text{--- (3)}$$

$$C_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jn\omega_0 t}\, dt \quad \text{--- (1)}$$

$$\frac{a+jb}{a-jb}$$

If C_n is complex conjugate symmetric $\Rightarrow$ $\boxed{C_n = C_{-n}^{*}}$

Now compare eq^n (1) and (3) $\Rightarrow$ $x(t) = x^{*}(t)$ $\Rightarrow$ A pure real signal.

For any real time domain signal, the exponential FS coefficient C_n will be conjugate symmetric and vice-versa is also true.

$$z = re^{i\theta} = a + ib$$

$$e^{i\theta} = \cos\theta + j\sin\theta$$

$$C_n = |C_n|\, e^{j\angle C_n} \quad \text{--- (4)}$$

Magnitude

phase

$$C_{-n} = |C_{-n}|\, e^{j\angle C_{-n}} \quad \text{--- (5)}$$

Rev. op.

Conjugate symm in eq^n (5)

$$C_{-n}^{*} = |C_{-n}^{*}|\, e^{-j\angle C_{-n}} \quad \text{--- (6)}$$

From conjugate symmetry $c_n = c_{-n}^*$

From eq^n (4) and (6) $\Rightarrow$ $|c_n| e^{i\angle c_n} = |c_{-n}|^* e^{-i\angle c_{-n}}$

Salient Features of c_n $\rightarrow$

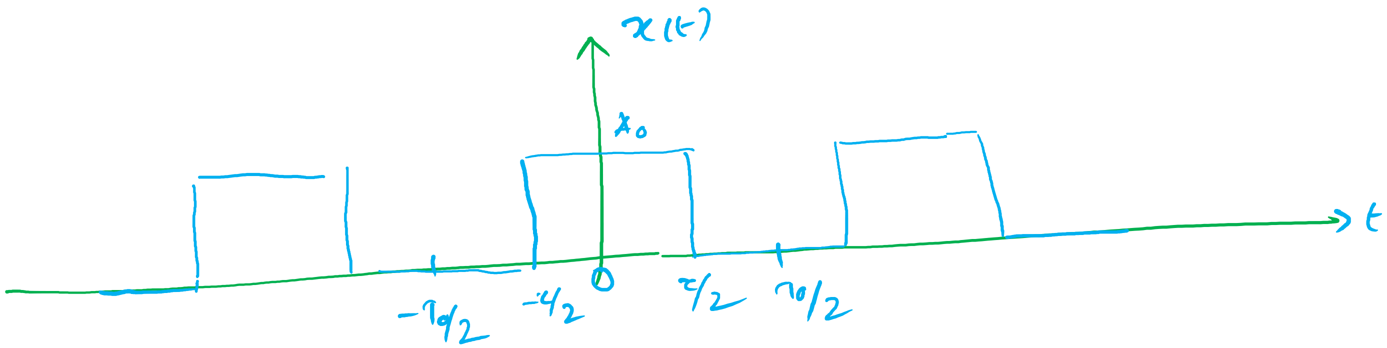
$$\boxed{\begin{array}{l} |c_n| = |c_{-n}| \rightarrow \text{even in nature (Magnitude)} \\ \angle c_n = -\angle c_{-n} \rightarrow \text{odd in nature (phase.)} \end{array}}$$

$c_n = |c_n| e^{i\angle c_n}$

$x(t) = x(-t) \rightarrow$ even

$x(t) = -x(-t) \rightarrow$ odd

Find C_n for signal:-



$$c_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} A e^{-jn\omega_0 t}\, dt \qquad = \frac{A}{T_0}\int_{-\tau/2}^{\tau/2} e^{-jn\omega_0 t}\, dt$$

$$C_n = \frac{A}{T_0} \frac{\left[e^{-jn\omega_0 t}\right]_{-\tau/2}^{\tau/2}}{-jn\omega_0}$$

$$= \frac{A}{T_0} \times -\frac{1}{jn\omega_0}\left[e^{-jn\omega_0 \tau/2} - e^{jn\omega_0 \tau/2}\right]$$

$$= \frac{A}{-jn\omega_0 T_0}\left[e^{-jn\underbrace{\omega_0\tau/2}_{\theta}} - e^{j\underbrace{n\omega_0\tau/2}_{\theta}}\right]$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i(-\theta)} = \cos(-\theta) + i\sin(-\theta)$$

$$= \cos\theta + -i\sin\theta$$

$$= \frac{A}{-jn\omega_0 T_0}\left[e^{-i\theta} - e^{j\theta}\right]$$

$$= \frac{A}{-jn\omega_0 T_0}\left[\cancel{\cos\theta} - j\sin\theta - (\cancel{\cos\theta} - j\sin\theta)\right]$$

$$= \frac{2\cancel{j}}{\cancel{j}n\omega_0 T_0}\sin\theta = \frac{2}{n\omega_0 T_0}\sin n\omega_0 \tau/2 \quad \#$$

$$\mathrm{sinc}(x) = \frac{\sin x}{x}$$

↓

sampling

$$c_n = \frac{A_0}{jn\omega_0 T_0} \times 2j\sin(n\omega_0 \tau/2)$$

$$= \frac{A_0}{\cancel{jn\omega_0} T_0} \times \cancel{2j} \left[\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}\right] \times \cancel{n\omega_0}\tau/\cancel{2}$$

→ don't cancel out.

$$\boxed{c_n = \frac{A_0 \tau}{T_0}\,\mathrm{sinc}(n\omega_0\tau/2)}$$