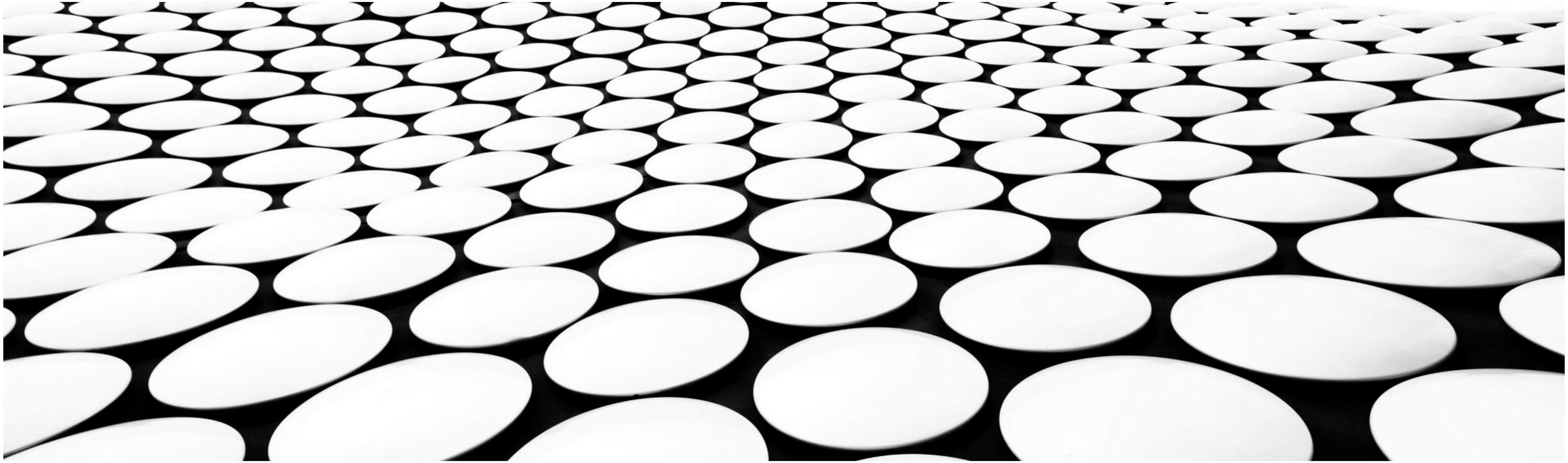
SIGNALS & SYSTEMS

MR. ANKUR JYOTI SARMAH

ASSISTANT PROF., DEPT. OF ELECTRONICS & TELECOM. ENGG.

ASSAM ENGINEERING COLLEGE



Unit step response using convolution

In general the response y(t) of a system is given by convolution of input x(t) and impulse response h(t) of the system.

$$y(t) = x(t) * h(t) = \int_{\lambda=-\infty}^{\lambda=+\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

Already discussed.

Let the input x(t) be unit step input u(t), and the corresponding response be s(t). Now the unit step response s(t) is given by,

$$\text{Unit Step Response, } s(t) = u(t) * h(t)$$

$$= h(t) * u(t) \qquad \boxed{\text{Using Commutative property}}$$

$$= \int_{\lambda=-\infty}^{\lambda=+\infty} h(\lambda)\, u(t-\lambda)\, d\lambda$$

y(t) in general

unit step response = system response in general

In the above convolution operation, $u(\lambda) = 1$ for $\lambda > 0$,

$u(-\lambda) = 1$ for $\lambda < 0$,

$u(t-\lambda) = 1$ for $\lambda < t$, and $u(t-\lambda) = 0$ for $\lambda > t$.

Therefore the unit step response s(t) is given by,

$$\text{Unit Step Response, } s(t) = \int_{\lambda=-\infty}^{\lambda=t} h(\lambda)\, d\lambda$$

Only system equation needs to be considered if i/p is an unit step signal

Perform convolution of the following causal signals.

a) $x_1(t) = 2\,u(t)$, $x_2(t) = u(t)$ b) $x_1(t) = e^{-2t}\,u(t)$, $x_2(t) = e^{-5t}\,u(t)$

c) $x_1(t) = t\,u(t)$, $x_2(t) = e^{-5t}\,u(t)$ d) $x_1(t) = \cos t\,u(t)$, $x_2(t) = t\,u(t)$

Solⁿ:- (a) First step always define the signal.

$x_1(t) = 2u(t)$

$x_2(t) = u(t)$

$u(t)$ = unit step signal and always defined for $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$

$\therefore\ x_1(t) = 2 \ ;\ t \geq 0$

and $x_2(t) = 1 \ ;\ t \geq 0$

Let us assume convoluted value would be $x_3(t)$

$\therefore\ x_3(t) = x_1(t) * x_2(t)$

$\Rightarrow x_3(t) = \int_{-\infty}^{\infty} x_1(\lambda)\, x_2(t-\lambda)\, d\lambda$

As per the formula of convolution

$$x_3(t) = \int_0^t x_1(\lambda) \cdot x_2(t-\lambda)\, d\lambda$$

→ here the change of limit from $-\infty$ to ∞ has been changed to $0 \rightarrow t$ because the given signals are causal and defined for $0 \rightarrow t$, i.e. $t \geq 0$

$$= \int_0^t (2) \cdot (1)\, d\lambda$$

$$= 2 \int_0^t d\lambda$$

$$= 2(\lambda)_0^t$$

$$= 2t$$ → Also valid for $t \geq 0$

$$\therefore x_3(t) = 2t\, u(t)$$

(b) Soln:-

$$x_1(t) = e^{-2t} u(t) = e^{-2t} \; ; \; t \geq 0$$

$$x_2(t) = e^{-5t} u(t) = e^{-5t} \; ; \; t \geq 0$$

Let $x_3(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\lambda)\, x_2(t-\lambda)\, d\lambda$

$$= \int_0^t e^{-2\lambda} \cdot e^{-5(t-\lambda)}\, d\lambda$$

Now, remember, in this problem wherever you find t that must be replaced by λ : for $x_1(t) \rightarrow t \xrightarrow[\text{by}]{\text{Replaced}} \lambda$

for $x_2(t) \xrightarrow[\text{by}]{\text{Replaced}} t-\lambda$

But in previous problem, $x_1(t)$ and $x_2(t)$ are constant.

$$x_3(t) = \int_0^t e^{-2\lambda} e^{-5t} e^{5\lambda} \, d\lambda$$

$$= \int_0^t e^{3\lambda} \cdot e^{-5t} \, d\lambda$$

constant as the integration is based on λ.

$$= e^{-5t} \int_0^t e^{3\lambda} \, d\lambda$$

$$= e^{-5t} \left[\frac{e^{3\lambda}}{3} \right]_0^t$$

$$= \frac{e^{-5t}}{3} \left[e^{3t} - 1 \right]$$

$$= \frac{1}{3} \left[e^{-2t} - e^{-5t} \right]$$

$$\therefore x_3(t) = \frac{1}{3} \left[e^{-2t} - e^{-5t} \right] u(t) \cdot \quad \text{since } t \geq 0$$