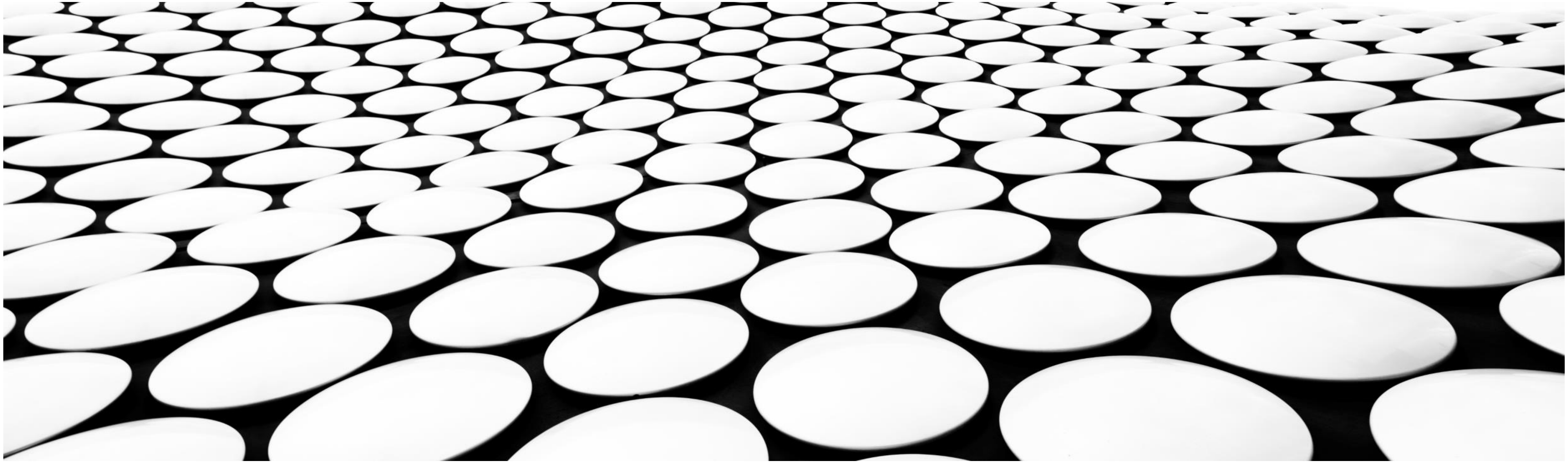
SIGNALS & SYSTEMS

MR. ANKUR JYOTI SARMAH

ASSISTANT PROF., DEPT. OF ELECTRONICS & TELECOM. ENGG.

ASSAM ENGINEERING COLLEGE



÷ Laplace Transform :- ??

Time domain $\xrightarrow{\text{Transform}}$ Complex frequency Domain or

continuous

s-domain.

Solution of differential eq^n

homogeneous sol^n

particular sol^n

Total Response = HE + PE

$$\frac{d^2 y(t)}{dt^2} + 5\,dy(t)/dt + 6\,y(t) = x(t)$$

o/p

i/p

Differential ⟶ simple algebric equation of variable 's'

Complex frequency,

Real part

$s = \sigma + j\omega \rightarrow$ Imaginary part

or

$s = \sigma + j\Omega$

$\sigma \rightarrow$ Neper frequency in neper per second

$\omega \rightarrow$ Radian (or Real) freq. in radian per second

$Ae^{st} \rightarrow$ the complex freq. is involved in the time domain

$x(t) = Ae^{st} = Ae^{(\sigma + j\omega)t} = Ae^{\sigma t} \cdot Ae^{j\omega t}$

Time domain

s-domain

$\boxed{\sigma = 0, \quad \omega = \omega_0}$

$x(t) = Ae^{st} \longrightarrow$ universal signal

$= Ae^{(\sigma + j\omega)t}$

$= Ae^{\sigma t} \cdot Ae^{j\omega_0 t}$

$= 1 . Ae^{j\omega_0 t}$

$= A[\cos\omega_0 t + j\sin\omega_0 t]$

$= A\cos\omega_0 t + jA\sin\omega_0 t$

cos

sin

$\boxed{\omega = 0}$

$x(t) = Ae^{\sigma t} \cdot \cancel{Ae^{j\omega t}}^{\,1}$

$\Rightarrow x(t) = Ae^{\sigma t}$

$\sigma \to +ve \Rightarrow x(t) \to$ Exponentially increasing

$\sigma \to -ve \Rightarrow x(t) = Ae^{-\sigma t}$

$\Rightarrow$ Exp. decaying signal

$\sigma = 0$ and $\omega = 0$

$x(t) = \cancel{Ae^{\sigma t}} \cdot \cancel{A}e^{j\omega t}$

$= A \cdot 1 \cdot 1$

$= A$

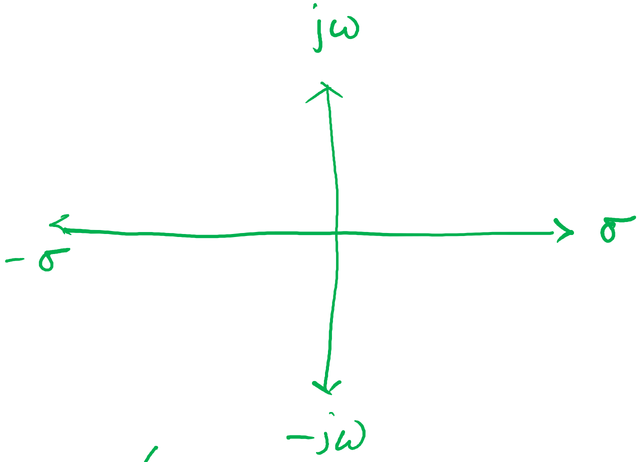
step signal

If $A = 1 \Rightarrow$ unit step signal

Ae^{st} σ, ω

s-plane

complex freq. plane



This s-plane represents complex freq. in the form poles and zeros.

$s = -a, -b$

$$X(s) = \frac{\overbrace{(s+a)(s+b)}^{\text{zeros}}}{\underbrace{(s+p)(s+q)}_{\text{poles}}}$$

poles $s = -p, -q$

Def:-

$x(t) \rightarrow$ time domain

$\downarrow$

$X(s) \rightarrow$ compl. frq or s-domain

$$X(s) = L\{x(t)\} = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} \, dt$$

Multiply $x(t)$ with e^{-st}

Integrate the multiple from $-\infty$ to ∞

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt.}$$

Inverse Laplace transform:-

$X(s) \longrightarrow x(t)$

$$x(t) = L^{-1}\{X(s)\} = \frac{1}{2\pi j} \int_{s=\sigma-j\omega}^{s=\sigma+j\omega} X(s) e^{st} \, ds.$$

Types of Laplace transform → Bilateral or two sided

→ unilateral or one-sided

$-\sigma_1 < \sigma < \sigma_2$

$S \rightarrow \underline{\underline{\sigma}} + j\omega$

+ve → RHS of s-plane

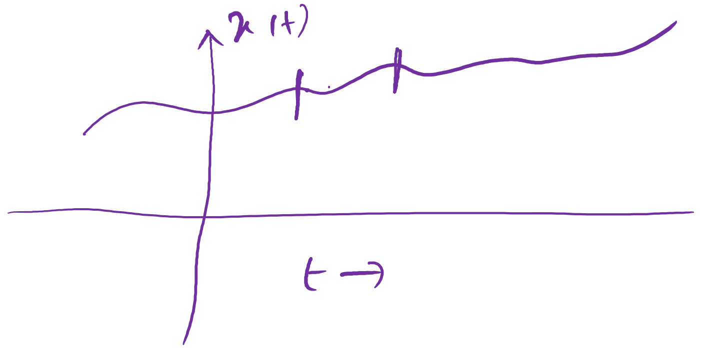
-ve → LHS of s-plane

LHS —— RHS

Bilateral | two sided.

Existance of Laplace Transform

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$ → finite value → Must converge.



$t=0 \rightarrow x(t) = \text{finite}$

$t=1 \rightarrow x(t) = \text{finite}$

$\sigma + j\omega$

$$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-(\sigma + j\omega)t}\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-\sigma t} \cdot e^{-j\omega t}\, dt.$$

approaches → zero.

Causal $\boxed{t \geq 0}$

$e^{-\sigma t}|x(t)|$ approaches to zero as t approaches to ∞

$$\boxed{\lim_{t\to\infty} e^{-\sigma t}|x(t)| = 0}$$

Ae^{st}

$$\int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt < \infty$$ → finite value

No discontinuity

Abscissa of convergence

or

Region where X(s) exists → ROC

ROC

$$x(t) = e^{-at} u(t) = e^{-at} \quad ; t \geq 0$$

Right sided signal $\rightarrow$ Causal.

$$u(t) = 1 \quad ; t \geq 0$$
$$= 0 \quad ; t < 0$$

$$L\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-(s+a)t}\, dt$$

$$= \left[\frac{e^{-(s+a)t}}{-(s+a)} \right]_{0}^{\infty}$$

$$= \frac{e^{-(\sigma + j\omega + a)\infty}}{-(s+a)} - \frac{e^{0}}{-(s+a)}$$

$$= \frac{e^{-(\sigma+a)\infty} \cdot e^{-j\omega\infty}}{-(s+a)} + \frac{1}{s+a}$$

$$X(s) = L\{x(t)\} = -\frac{e^{-(k\infty)} \cdot e^{-j\omega\infty}}{s+a} + \frac{1}{s+a}$$

$k = \sigma + a$

$\sigma > -a$

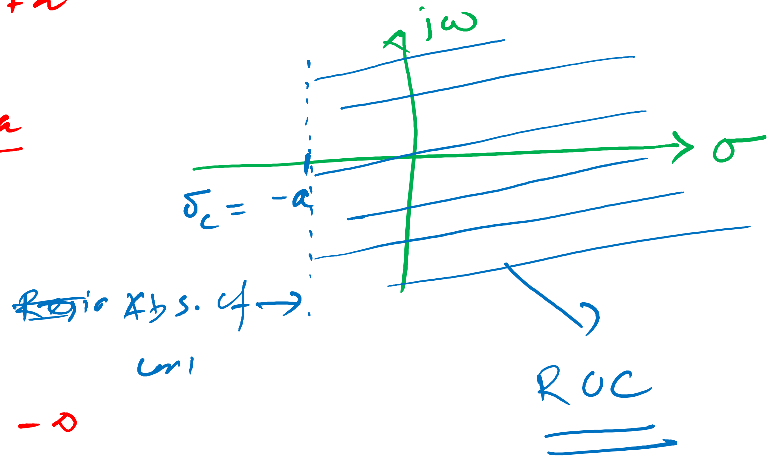
× $s = -a$ → infinity → finite

$s = \sigma + j\omega$ $\boxed{\sigma > -a}$

$\sigma < -a$

$e^{-(k\infty)}$ → positive $= e^{-\infty} = 0$

$e^{-k\infty}$ +ve → infinity



$\boxed{\sigma > -a}$ → Laplace transform exist

$$X(s) = (-)\frac{e^{-k\infty} \cdot e^{-j\omega\infty}}{s+a} + \frac{1}{s+a} = \frac{0}{s+a} + \frac{1}{s+a} = \frac{1}{s+a}$$