

Repeated Rank

If any two or more individuals have same rank or the same value then the formula for rank correlation fails.

In this case, each individual is given an average ~~rank~~ rank. This common average rank is the average of the ranks which these individuals ~~are~~ would have assumed if they were slightly different from each other.

If two individuals are ranked equal at the 6th place, ~~they~~ (in descending order), they would have assumed the 6th and 7th ranks if they were ranked slightly different. Their common rank is $\frac{6+7}{2} = 6.5$.

Adjustment: we have to add $\frac{1}{12} m(m-1)$ to $\sum d^2$ where m stands for the number of times an item is repeated.

The formula for correlation coefficient for repeated rank is,

$$r = 1 - \frac{6 \left[\sum d^2 + \frac{1}{12} m_1(m_1-1) + \frac{1}{12} m_2(m_2-1) + \dots \right]}{n(n-1)}$$

Ex:→ A sample of 12 fathers and their eldest sons gave the following data about their heights in inches.

Father: 65 63 67 64 68 62 70 66 68 67

Son: 68 66 68 65 69 66 68 65 71 67

Father: 69 71

Son: 68 70

Solⁿ Let us consider Father's height as X and Son's height as Y in inches.

X	65	63	67	64	68	62	70	66	68	67	69	71	Total
Y	68	66	68	65	69	66	68	65	71	67	68	70	
Rank in X (X _i)	9	11	6.5	10	4.5	12	2	8	4.5	6.5	3	1	
Rank in Y (Y _i)	5.5	9.5	5.5	11.5	3	9.5	5.5	11.5	1	8	5.5	2	
d _i	3.5	1.5	1	-1.5	1.5	2.5	-3.5	-3.5	3.5	-1.5	-2.5	-1	= 0
d ²	12.25	2.25	1	2.25	2.25	6.25	12.25	12.25	12.25	2.25	6.25	1	72.5

In X, 68 repeat 2 times and would have been ranked 4 and 5 if one 68 is slightly different. So, we will give average rank $\frac{4+5}{2} = 4.5$ to these two positions. Similarly

67 is repeated 2 times.

In Y, 65 is repeated 4 times and would have been ranked 4, 5, 6, 7 to them if they are slightly different from 65. So, we give average rank to them as $= \frac{4+5+6+7}{4} = 5.5$.

lly we give rank to others.

$$\text{Here, } m_1 = 2, m_2 = 2, \\ m_3 = 4, m_4 = 2, m_5 = 2.$$

∴ Correlation coefficient,

$$r = 1 - \frac{6 \sum f^2 \cdot x}{12 \times (144 - 1)}$$

$$r = 1 - \frac{6 [72 \cdot 5 + \frac{1}{12} \times 2(4-1) + \frac{1}{12} \times 2(4-1) + \frac{1}{12} \times 4(16-1) + \frac{1}{12} \times 2(4-1) + \frac{1}{12} \times 2(4-1)]}{12 \times (144 - 1)}$$

$$= 1 - \frac{72 \cdot 5 + 7}{2 \times 286}$$

$$= 723$$