

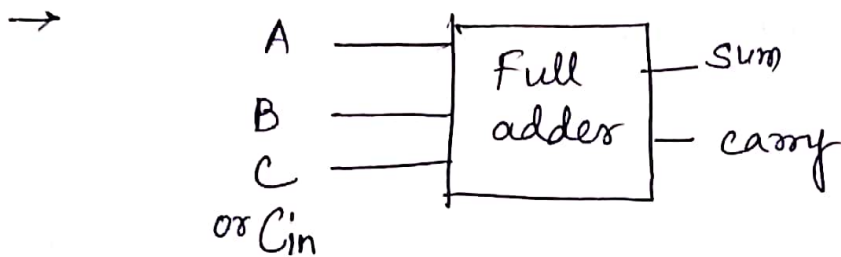
Full adder

→ 3 i/p to add → no. of o/p = ?

→ Similarly 4 i/p to add

$\begin{array}{c} \text{---} \\ | \\ \text{---} \\ 100 \\ \text{---} \end{array}$ 3 o/p bits.

$\begin{array}{c} 0 \\ 0 \\ 0 \\ \text{---} \\ 0 \\ \text{min}^m \end{array}$ $\begin{array}{c} 1 \\ | \\ \text{---} \\ 11 \\ \text{max}^m \end{array}$



Truth table

A	B	C	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$\text{Sum} = \sum m(1, 2, 4, 7)$$

$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC \quad (\text{SOP form})$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

$$= (A \oplus B) \oplus C = A \oplus B \oplus C$$

→ So, sum of any i/p bits follow the XOR pattern

Half adder sum, $\text{sum} = A \oplus B$

full adder " , $\text{sum} = A \oplus B \oplus C$

Similarly for a 8 bit adder, $\text{sum} = A \oplus B \oplus C \oplus \dots \oplus 1$

$$\text{Carry} = \sum m(3, 5, 6, 7)$$

$$= \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

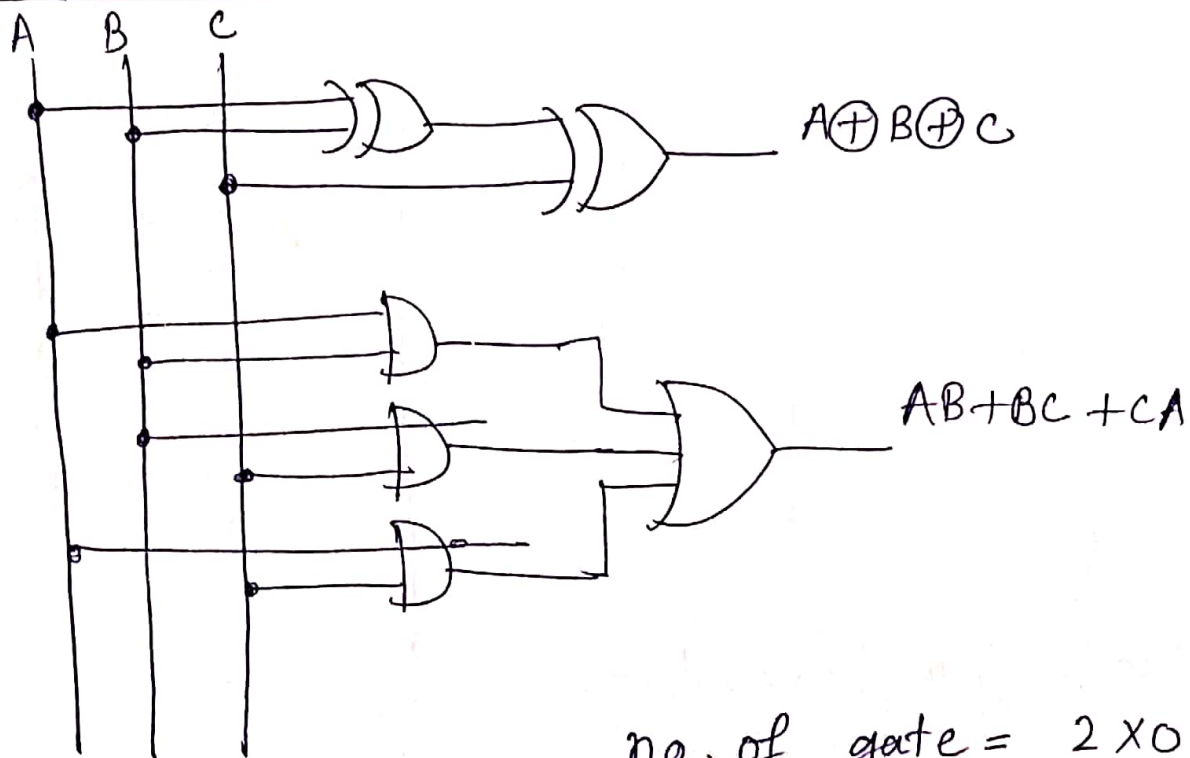
$$= BC(A + \bar{A}) + A\bar{B}C + AB\bar{C}$$

$$= BC + A(B \oplus C) = AB + C(A \oplus B) + CA + B(A \oplus C)$$

← map
minimization

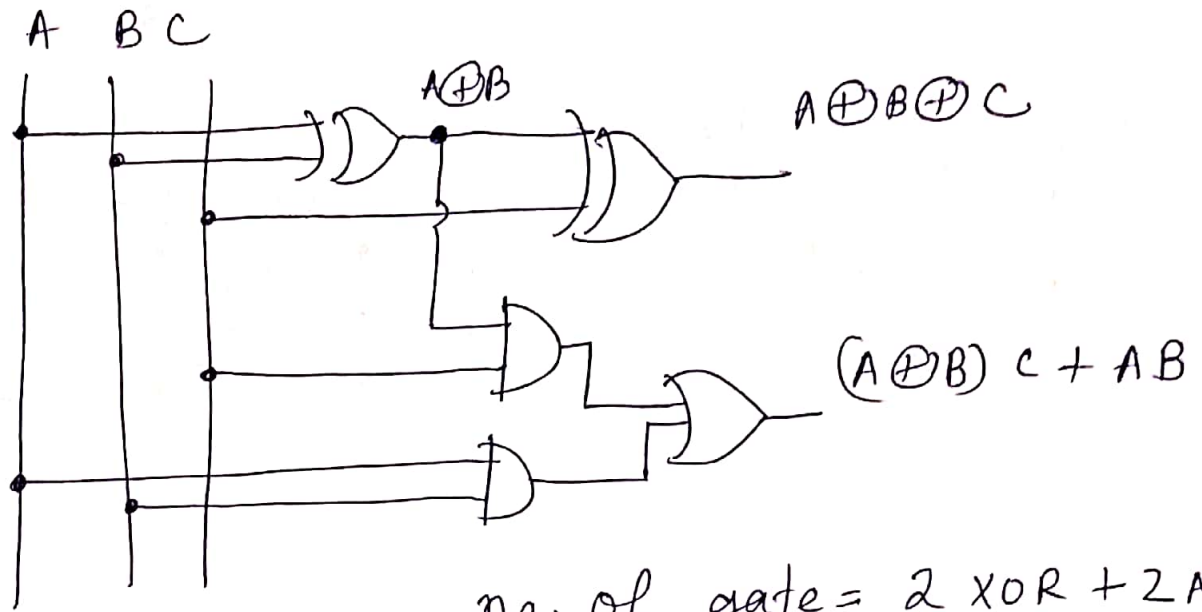
$$= AB + BC + CA \quad \rightarrow \textcircled{1}$$

Logic circuit:



$$\text{no. of gate} = 2 \text{ XOR} + 3 \text{ AND} + 1 \text{ OR}$$

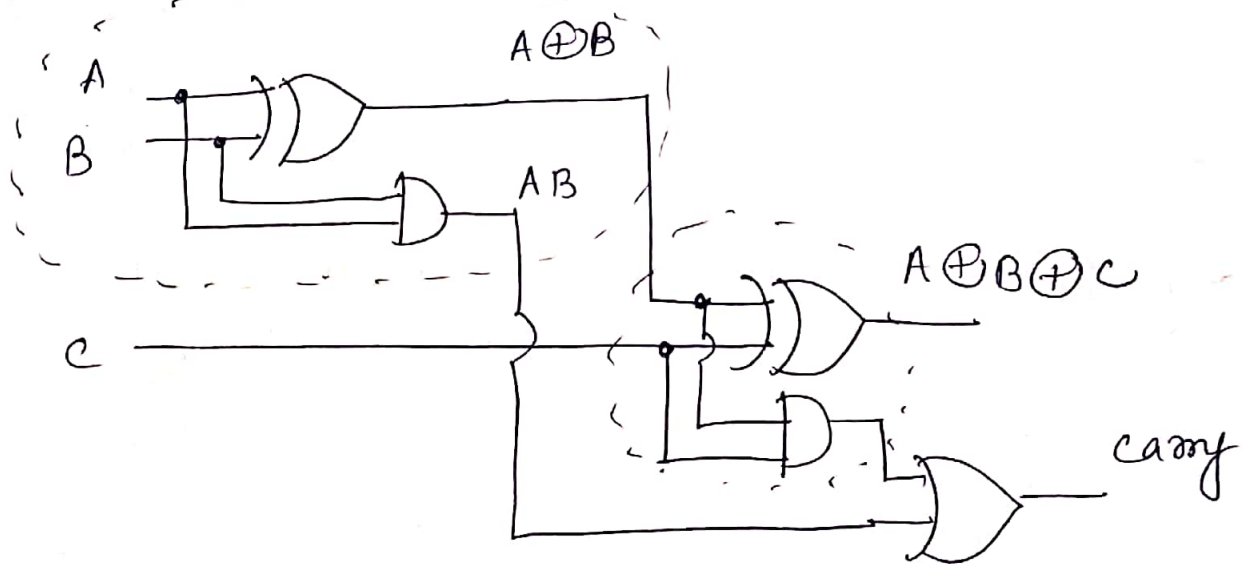
or, logic circuit:



no. of gate = 2 XOR + 2 AND + 1 OR

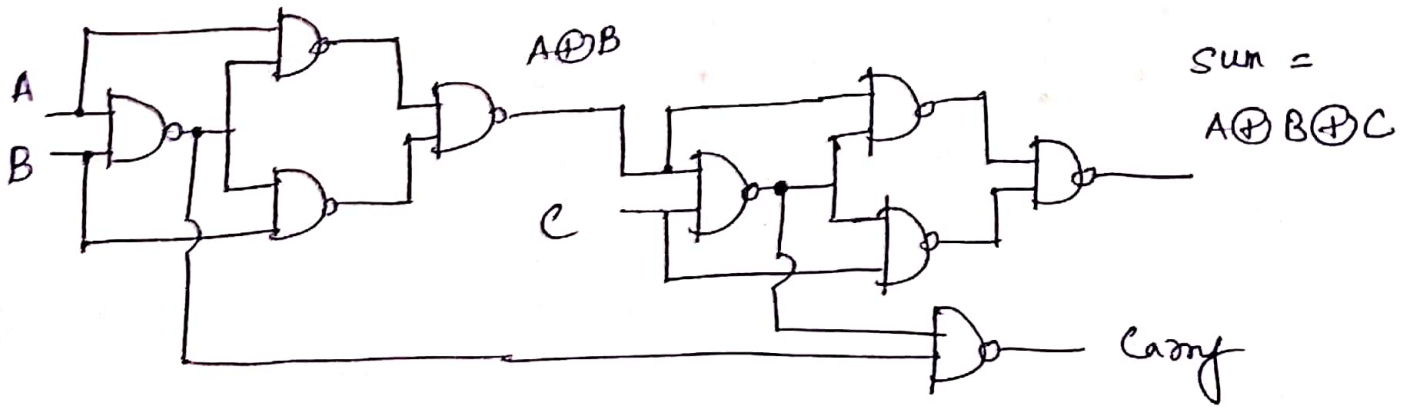
or, two other ways a/c eqⁿ ①

Same diagram,



So, FA = 2HA + 1 OR gate

Full adder using NAND :

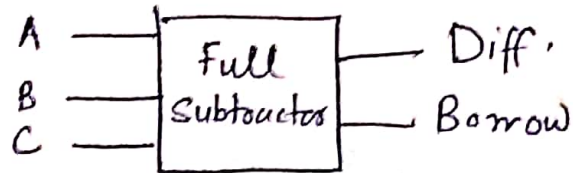


$$\begin{aligned} \text{Carry} &= AB + (A \oplus B)C = \overline{\overline{AB + (A \oplus B)C}} \\ &= \overline{\overline{AB} \cdot \overline{(A \oplus B)C}} \end{aligned}$$

total NAND gate required to design Full adder = 9
 " " " " " " Sum = 8
 " " " " " " Carry = 6

try full adder using NOR: . do it yourself.

Full subtractor:



$(A-B-C)$

Truth table:

A	B	C	Diff	Borrow	Diff,
0	0	0		0	0
0	0	1		1	1
0	1	0		1	1
0	1	1		1	0
1	0	0		0	1
1	0	1		0	0
1	1	0		0	0
1	1	1		1	1

$$\text{Diff.} = \sum m(1, 2, 4, 7) = A \oplus B \oplus C \quad [\text{same as sum of full adder}]$$

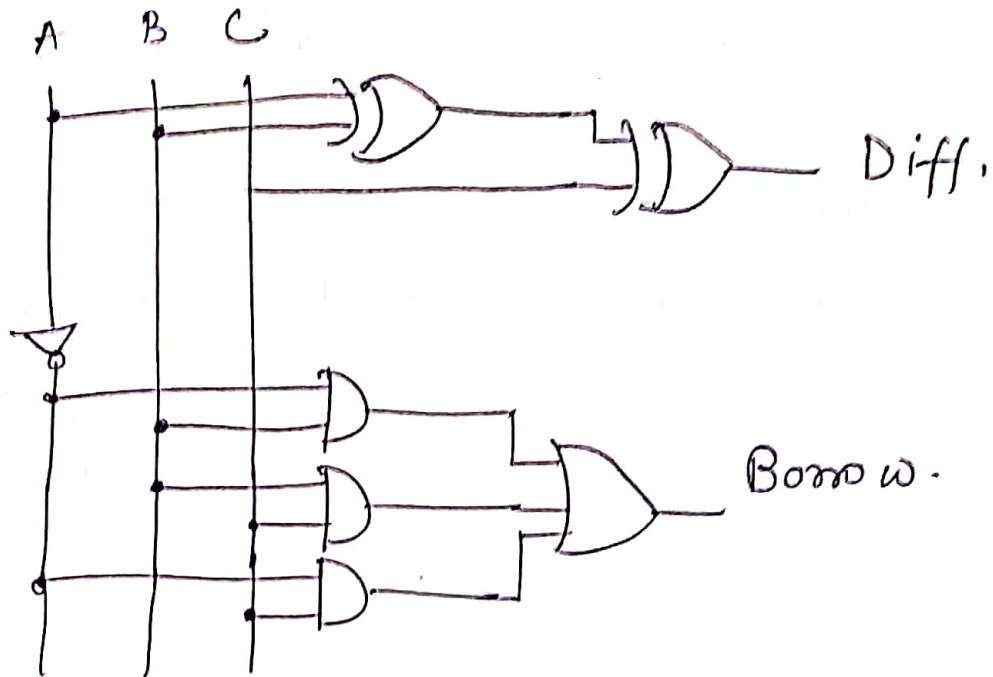
$$\text{Borrow} = \sum m(1, 2, 3, 7)$$

$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$$

K-map minimization

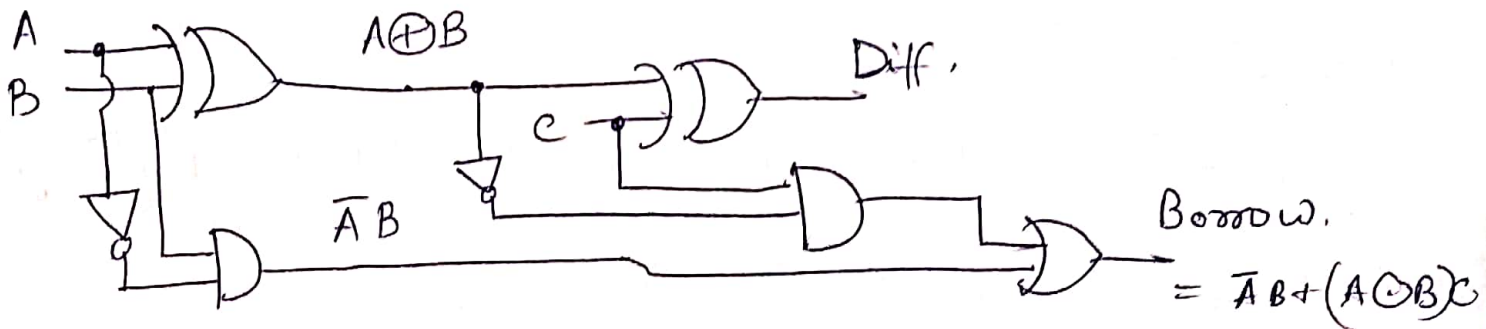
$$\begin{aligned} &= \bar{A}C(B + \bar{B}) + \bar{A}B(C + \bar{C}) + BC(A + \bar{A}) \\ &\downarrow \\ &= \bar{A}C + \bar{A}B + BC \end{aligned}$$

Logical circuit:



Borrow can also be written as $= \bar{A}B + (A \oplus B) \cdot C$

Full subtractor using half subtractor:



try Full subtractor using NAND only & NOR only:

* Calculate no. of gates required.

Full adder as a counter:

Truth table:

A	B	C	Cy	Sum		
0	0	0	0	0	→	0
0	0	1	0	1	→	1
0	1	0	0	1	→	1
0	1	1	1	0	→	2
1	0	0	0	1	→	1
1	0	1	1	0	→	2
1	1	0	1	0	→	2
1	1	1	1	1	→	3

it counts no. of
i/p which are '1'