SIGNALS & SYSTEMS

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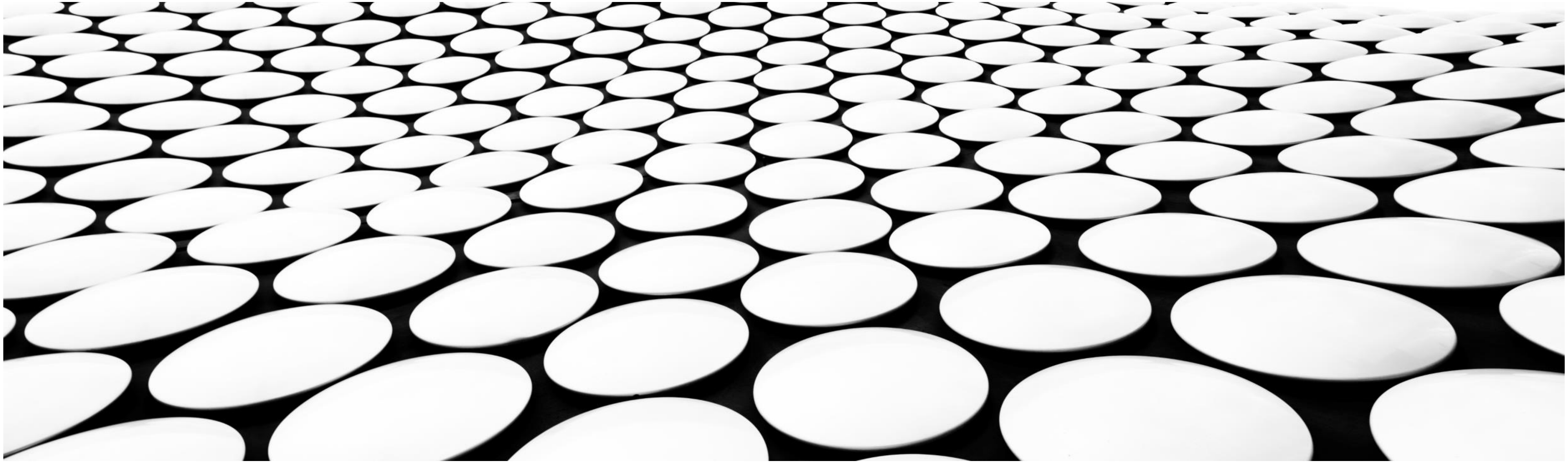


1) Fourier transform for impulse signal

unit imp. signal.

$$x(t) = \delta(t) = \begin{cases} 1, & t=0 \\ 0, & \text{otherwise} \end{cases}$$

$$x(t_1)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

$$\int_{-\infty}^{\infty} |x(t)|\,dt = \text{unit area} = 1$$

$$x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t-0)\, e^{-j\omega t}\, dt$$

$$x(\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{0}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t)\, dt$$

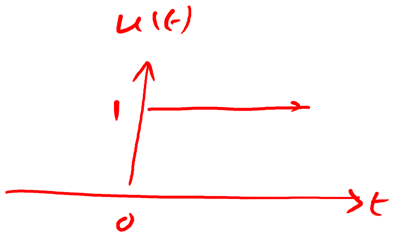
$$= 1.$$

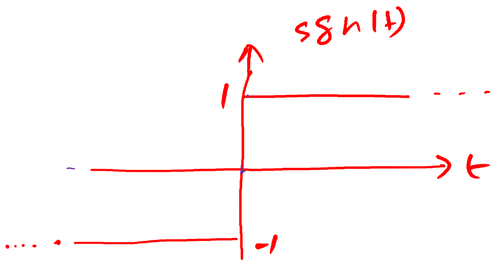
$\therefore$ FT of unit impulse fn $\rightarrow 1$

$$\boxed{x(\omega) = 1}$$

2) unit step $\longrightarrow$ signum f^n

Fourier transform for signum f^n





$$x(t) = sgn(t) = \begin{cases} 1, & t>0 \\ 0, & t=0 \\ -1, & t<0 \end{cases}$$

$$\int_{-\infty}^{\infty} |u(t)|\,dt \to \infty$$

↓

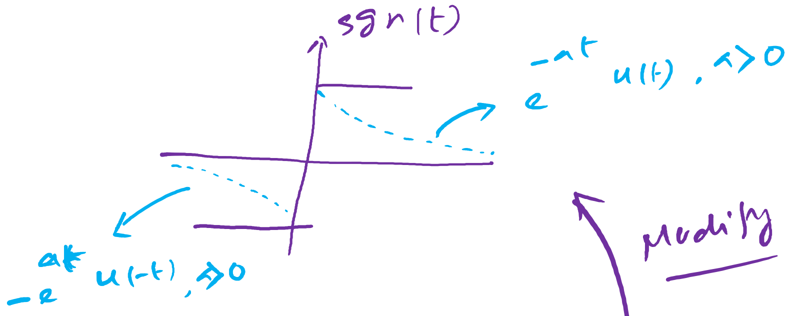
Modify

↓

converse

→ Also not absolutely integrable

$$\int_{-\infty}^{\infty} |sgn(t)|\,dt = \infty$$



Modify

$$u(t) = \lim_{a\to 0} e^{-at} u(t)$$

$$u(-t) = \lim_{a\to 0} e^{at} u(-t)$$

$e^{j\omega t}$ Freq. shifting

if $x(t) \rightarrow X(\omega)$

Similarly we can find the FT of

$$e^{at} u(-t) = \frac{1}{a-j\omega}$$

FT of Exponential signal

$$x(t) = e^{-at} u(t)$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot 1 \cdot e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$$

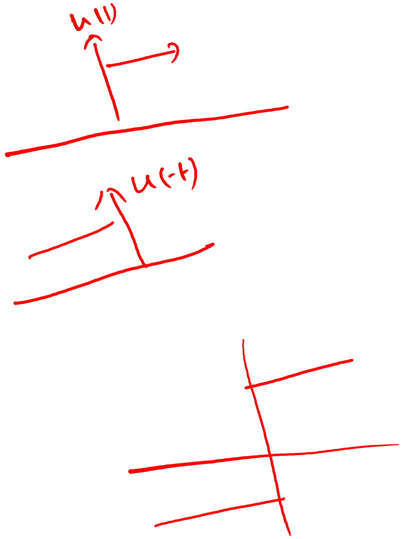
$$= \frac{1}{-(a+j\omega)} \left[\cancel{e^{-(a+j\omega)\infty}}^{\,0} - \cancel{e^{-(a+j\omega)\cdot 0}}^{\,1} \right]$$

$$\therefore X(\omega) = \frac{1}{a+j\omega}, \text{ when } a>0$$

H/W. $x(t) = e^{-a|t|} u(t)$; $a>0$

Acc. to the property of signum f^n —

$$sgn(t) = u(t) - u(-t) = \lim_{a\to 0} e^{-at}u(t) - \lim_{a\to 0} e^{at}u(-t).$$



Let us consider, the FT of sgn(t) is $X(\omega)$

$$sgn(t) = \lim_{a\to 0}\left[e^{-at}u(t) - e^{at}u(-t)\right]$$

$$FT\left[sgn(t)\right] = X(\omega) = \lim_{a\to 0}\left[\frac{1}{a+j\omega} - \frac{1}{a-j\omega}\right]$$

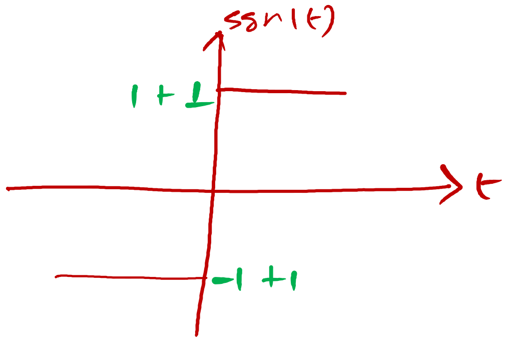
$$= \lim_{a\to 0}\left[\frac{\cancel{a}-j\omega-\cancel{a}-j\omega}{a^2+\omega^2}\right]$$

$$= \lim_{a\to 0}\frac{-2j\omega}{a^2+\omega^2}$$

$$= \frac{-2j\cancel{\omega}}{\omega^{\cancel{2}}} = \frac{-2j}{\omega}$$

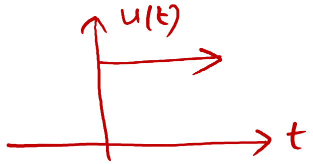
$$\therefore \boxed{X(\omega) = \frac{-2j}{\omega} = \frac{2}{j\omega}}$$

Fourier Transform of signum f^n is $\frac{2}{j\omega}$

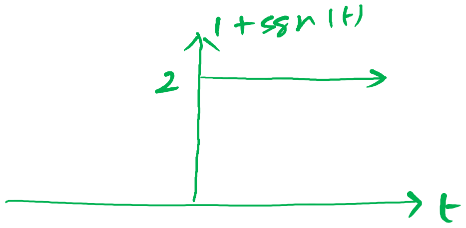


$\Rightarrow$ Modify sgn(t) to u(t)

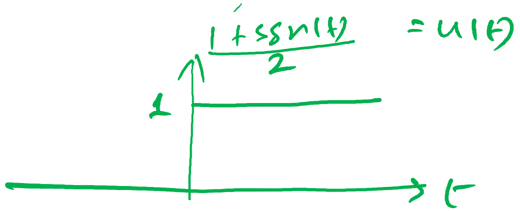
Amplitude scaling (add 1)



$2\pi A_0 \delta(\omega)$



Divide 1+sgn(t) by 2



$$u(t) = \frac{1+sgn(t)}{2}$$

$$\Rightarrow u(t) = \frac{1}{2} + \frac{sgn(t)}{2}$$

$$\Rightarrow FT[u(t)] = FT\left[\frac{1}{2} + \frac{sgn(t)}{2}\right]$$

$$= FT\left[\frac{1}{2}\right] + FT\left[\frac{sgn(t)}{2}\right]$$

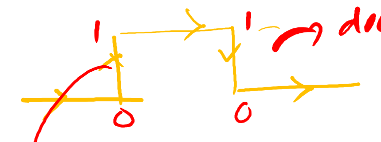
$$= \cancel{2}\pi \times \frac{1}{\cancel{2}}\delta(\omega) + \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{j\omega}$$

$$\boxed{FT[u(t)] = \pi\delta(\omega) + \frac{1}{j\omega}}$$

Ramp $\xrightarrow{\text{diff}}$ step $\xrightarrow{\text{diff}}$ Impulse

Ramp $\xleftarrow[\text{int}]{}$ step $\xleftarrow[\text{int}]{}$ Imp.

Diff $\rightarrow$ $\dfrac{dx(t)}{dt}$ $\Rightarrow$ slope $\rightarrow$ Transition.

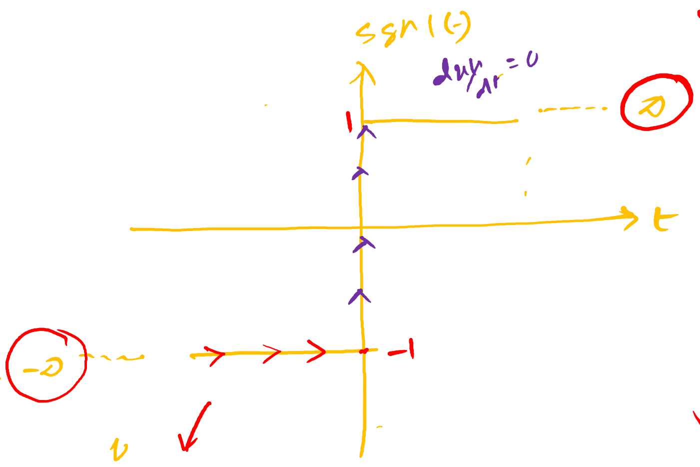


downward switching

upward switching

$\downarrow$

discontinuity

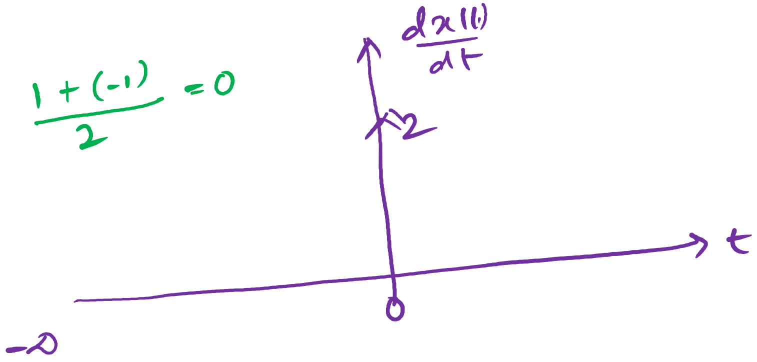


differentiate fn's entire curve

$1-(-1) = \underline{\underline{2}}$

$\dfrac{dx(t)}{dt} = 0$

DC value | Av. value $= \dfrac{1+(-1)}{2} = 0$



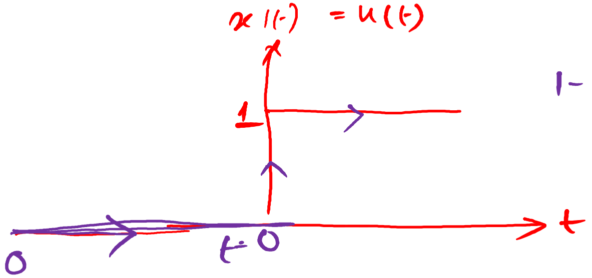
To have a impulse waveform

$\dfrac{dx(t)}{dt} = 2\delta(t)$ $\rightarrow$ FT

$\Rightarrow \dfrac{dx(t)}{dt} = 2\delta(t-0)$ + DC value

$(j\omega)\,X(\omega) = 2.1 + 0$

$\Rightarrow X(\omega) = \dfrac{2}{j\omega}$



$1 - 0 = 1$

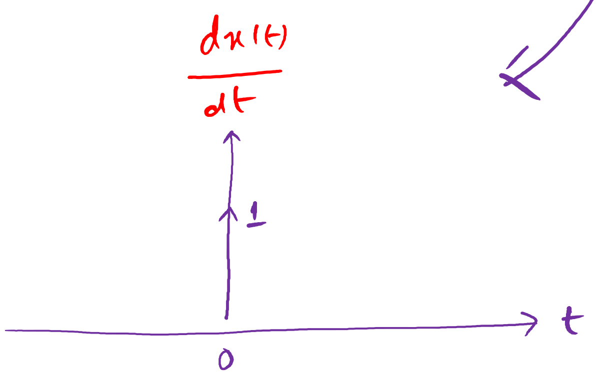
After differentiating unit step signal $\longrightarrow$ unit impulse signal

$$\frac{dx(t)}{dt} = 1.\delta(t) + \text{DC value} \qquad x(t) \xrightarrow{FT} X(\omega)$$

After taking FT

$$(j\omega) X(\omega) = 1 + \cancel{2}\pi \times \left(\frac{1}{\cancel{2}}\right) \delta(\omega)$$

$$\Rightarrow \quad X(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$$



$$\text{DC value} = \frac{1+0}{2} = \frac{1}{2}$$