MTH 331 – Problem 31

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(i) If a vector x_0 is a linear combination of $\{x_1, ..., x_n\}$, does it follow that the set $\{x_0, x_1, ..., x_n\}$ is dependant?

(ii) If, conversely, a finite set $\{x_0, x_1, ..., x_n\}$ of vectors is dependant, does it follow that at least one of them is a linear combination of the others?

Proof. *(i)* Suppose x_0 is a linear combination of $\{x_1, ..., x_n\}$. Let $\alpha_1, \alpha_2, ..., \alpha_n$ be scalars.
$x_0 = \alpha_1 x_1 + \alpha_2 x_2 ... + \alpha_n x_n \rightarrow (\alpha_1 x_1 + \alpha_2 x_2 ... + \alpha_n x_n) + (-1)x_0 = 0$ so $\{x_0, x_1, ..., x_n\}$ is dependant.

(ii) Suppose $\{x_0, x_1, ..., x_n\}$ is dependant. Let $\alpha_0, \alpha_1, ..., \alpha_n$ be scalars where $\alpha_i \neq 0$.
$x_i = (\frac{-1}{\alpha_i})(\alpha_0 x_0 + \alpha_1 x_1 + ... + \alpha_{i-1} x_{i-1} + \alpha_{i+1} x_{i+1} + ... + \alpha_n x_n)$

□