MTH 331 – Problem 99

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Let $f : \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ given by $f(n) = (2n + 1, n - 4)$ Is f injective? Is f surjective?

Statement 1. *For f to be injective it must satifiy the following statement:* $\forall a_1, a_2 \in \mathbb{Z}$ *if* $f(a_1) = f(a_2)$ *then* $a_1 = a_2$

Proof. Let $a_1, a_2 \in$. Suppose $f(a_1) = f(a_2)$.

$$\begin{aligned}
&f(a_1) = f(a_2) \\
\Leftrightarrow &(2a_1 + 1, a_1 - 4) = (2a_2 + 1, a_2 - 4) \\
\Leftrightarrow &2a_1 + 1 = 2a_2 + 1 \wedge a_1 - 4 = a_2 - 4 \\
\Leftrightarrow &a_1 = a_2 \wedge a_1 = a_2
\end{aligned}$$

□

Statement 2. *If f is not surjective, it must satisfy the following statement:* $\exists (b, c) \in \mathbb{Z} \times \mathbb{Z} \quad \forall a \in \mathbb{Z}$ *such that* $f(a) \neq (b, c)$.

Proof.

$$\begin{aligned}
Let \quad & b = -1 \quad c = -1 \\
& b = -1 \to a = -1, \quad c = -1 \to a = 3 \\
& so\,(-1, -1) \notin f
\end{aligned}$$

□