CSE 287
Lab 2 - Surfaces

Use the Chapter 2 notes to help you complete the questions.

1. Implicit equation for 2D line
   a. Line 1
      i. Give the implicit equation for the line going between (2, 3) and (-4, 1)

         (3 – 1)x + (-4 – 2)y + 2*1 – (-4)*3 = 0
         2x - 6y = -14

      ii. Determine if the following points fall on the line: (2, 3), (-5, 1), (8, 5)

         (2,3) ➔ 2*2 - 6*3 = -14 so it's on the line
         (-5,1) ➔ 2*(-5) - 6*(1) = -16 =/= -14 so it's not on the line
         (8,5) ➔ 2*8 – 6*5 = -14 so it is on the line

   b. Line 2
      i. Give the implicit equation for the line going between (2, -3) and (2, 10)

         (-3 – 10)x + (2 – 2)y + 2*10 – 2*(-3) = 0
         -13x = -26 ➔ x = 2

      ii. Determine if the following points fall on the line: (2, 13) (4, 10)
         (2,13) is on the line since x=2, but (4,10) is not

2. Implicit equation for 2D circle
   a. Give the implicit equation for the circle centered (2, 3) having radius 5.

      (x-2)^2 + (y-3)^2 = 25

   b. Determine if the following points fall on, in, or out of the circle.
      i. (0, 5)
         (-2)^2 + (2)^2 = 8 < 25 so (0,5) is inside the circle

      ii. (-2, 8)
         (-4)^2 + (5)^2 = 41 > 25 so (-2,8) is outside the circle

      iii. (2, -10)
         (0)^2 + (-13)^2 = 169 > 25 so (2, -10) is outside the circle

3. Implicit equation for 2D ellipse

   a. Give the implicit equation for the 2D ellipse centered (2, 3) having x-radius 5 and y-radius 2.

      $(x-2)^2 / 25 + (y-3)^2 / 4 = 1$

   b. Determine if the following points fall on, in, or out of the ellipse.

      i. (2, 3)
         In the ellipse, it is the center.

      ii. (2, 8)
          $(2 - 2)^2 / 25 + (8 - 3)^2 / 4 = 25/4 > 1$ so (2,8) is outside the ellipse

      iii. (2, 1)
           $(2 - 2)^2 / 25 + (1 - 3)^2 / 4 = 4/4 = 1$ so (2,1) is on the ellipse

4. Implicit equation for 3D ellipsoid
    a. Give the implicit equation for the ellipse centered (2, 3, 4) having x-radius 5, y-radius 2, and z-radius 4.

       (x – 2)^2 / 25 + (y – 3)^2 / 4 + (z – 4)^2 / 16 = 1

    b. Determine if the following points fall on, in, or out of the ellipsoid.
       i. (3, 4, 5)
          (3 – 2)^2 / 25 + (4 – 3)^2 / 4 + (5 – 4)^2 / 16 = 1/25 + 1/4 + 1/16 < 1 so (3,4,5) is in the ellipse

       ii. (7, 3, 4)
           (7 – 2)^2 /25 + (3 – 3)^2 / 4 + (4 – 4)^2 / 16 = 25/25 = 1 so (7,3,4) is on the ellipse

5. Implicit 3D plane
    a. Plane 1 - Give the implicit equation for the plane that has a <1, -2, 3> as a normal vector and includes the point (2, 3, 4) on it surface.

       <x – 2, y - 3, z – 4> • <1, -2, 3> = 0 for point p = (x, y, z)
       (x – 2) + (-2y + 6) + (3z – 12) = 0
       x – 2y + z - 8 = 0

    b. Plane 2 - Give the implicit equation for the plane that contains the following three points, given in counterclockwise order: (1, 2, 3), (-1, 0, 4), (3, 3, 1)

       v1 = <1 – (-1), 2 – 0, 3 – 4> = <2, 2, 1>
       v2 = <3 – (-1), 3 – 0, 1 – 4> = <4, 3, -3>
       v1 x v2 = <-9, 10, -2>
       (p – (-1, 0 ,4)) • <-9, 10 , -2> = 0
       -9(x + 1) + 10(y) – 2(z – 4) = 0
       -9x + 10y – 2z - 1 = 0

    c. Confirm that all three points are on the plane defined in part b.
       (1, 2, 3)   (-1, 0, 4)   (3, 3, 1)

6. Surface properties
    a. Gradient - Determine the gradient vector for the given surface: $f(x,y,z) = x^3+y+3z^2$

       $\nabla f(x,y,z) = \langle 3x^2, 1, 6z\rangle$

    b. Determine the tangent plane to a curve at point (1, 2, 3), having a surface normal <4, 5, 6>

       4(x-1) + 5(y-2) + 6(z-2) = 0
       4x + 5y + 6z = 26

c. Approximately, determine the unit-length normal vector for a radius 1 3D sphere centered at the origin at the following points:

   i. (1, 0, 0)
   <1,0,0>

   ii. (0, -1, 0)
   <0, -1, 0>

7. In English, describe the shape of the following parametric curve: [□ □ □] = [5*cos(t) 5*sin(t) t] for t ≥ 0
   It’s a vertical, cylindrical, circular helix.