MTH 331 – Statement 30

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September 3, 2015

Statement 30. *Let a, b, and c be integers. If a divides bc, and $gcd(a, b) = 1$ then a divides c.*

Proof. Suppose that a divides bc and that $gcd(a, b) = 1$, there exists an integer k such that $bc = ak$ and according to theorem 18 there exist integers m and n such that $gcd(a, b) = am + bn$.

$$\begin{aligned} am + bn &= 1 \\ ak(am + bn) &= ak \\ (ak)am + akbn &= ak \\ bcam + akbn &= bc \\ cam + akn &= c \\ a(cm + kn) &= c \end{aligned}$$

$cm + kn$ is an integer, thus a divides c. □