

字符串

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1. 后缀排序: DC3

DC3 后缀排序算法, 时空复杂度 $\Theta(n)$ 。字符串本体 s 数组、 sa 数组和 rk 数组都要求 3 倍空间。下标从 0 开始, 字符串长度为 n , 字符集 Σ 为 $[0, m]$ 。partial_sum 需要标准头文件 numeric。

```
1 #define CH(i, n) i < n ? s[i] : 0
2 static int ch[NMAX + 10][3], seq[NMAX + 10];
3 static int arr[NMAX + 10], tmp[NMAX + 10], cnt[NMAX + 10];
4 inline bool cmp(int i, int j) {
5     return ch[i][0] == ch[j][0] && ch[i][1] == ch[j][1] && ch[i][2]
6 } == ch[j][2];
7 }
8 inline bool sufcmp(int *s, int *rk, int n, int i, int j) {
9     if (s[i] != s[j]) return s[i] < s[j];
10    if ((i + 1) % 3 && (j + 1) % 3) return rk[i + 1] < rk[j + 1];
11    if (s[i + 1] != s[j + 1]) return s[i + 1] < s[j + 1];
12    return rk[i + 2] < rk[j + 2];
13 }
14 void radix_sort(int n, int m, int K, bool init = true) {
15     if (init) for (int i = 0; i < n; i++) arr[i] = i;
16     int *a = arr, *b = tmp;
17     for (int k = 0; k < K; k++) {
18         memset(cnt, 0, sizeof(int) * (m + 1));
19         for (int i = 0; i < n; i++) cnt[ch[a[i]][k]]++;
20         partial_sum(cnt, cnt + m + 1, cnt);
21         for (int i = n - 1; i >= 0; i--) b[--cnt[ch[a[i]][k]]] = a[i];
22     }
23 }
```

```
21     swap(a, b);
22 }
23 if (a != arr) memcpy(arr, tmp, sizeof(int) * n);
24 }
25 void suffix_sort(int *s, int n, int m, int *sa, int *rk) {
26     s[n] = 0; n++;
27     int p = 0, q = 0;
28     for (int i = 1; i < n; i += 3, p++) for (int j = 0; j < 3; j++)
29         ch[p][2 - j] = CH(i + j, n);
30     for (int i = 2; i < n; i += 3, p++) for (int j = 0; j < 3; j++)
31         ch[p][2 - j] = CH(i + j, n);
32     radix_sort(p, m, 3);
33     for (int i = 0; i < p; i++) {
34         if (!q || (q && !cmp(arr[i - 1], arr[i]))) q++;
35         s[n + arr[i]] = q;
36     }
37     if (q < p) suffix_sort(s + n, p, q, sa + n, rk + n);
38     else {
39         for (int i = 0; i < p; i++) sa[n + s[n + i] - 1] = i;
40         for (int i = 0; i < p; i++) rk[n + sa[n + i]] = i + 1;
41     }
42     m = max(m, p);
43     p = q = 0;
44     for (int i = 1; i < n; i += 3, p++) rk[i] = rk[n + p];
45     for (int i = 2; i < n; i += 3, p++) rk[i] = rk[n + p];
46     for (int i = 0; i < n; i++) if (i % 3) seq[rk[i] - 1] = i;
47     for (int i = 0; i < n; i += 3, q++) {
48         ch[i][0] = i + 1 < n ? rk[i + 1] : 0;
49         ch[i][1] = s[i];
50         arr[q] = i;
51     }
52     radix_sort(q, m, 2, false);
53     for (int i = seq[0] == n - 1, j = arr[0] == n - 1, k = 0; i <
54 p || j < q; k++) {
55         if (i == p) sa[k] = arr[j++];
56         else if (j == q) sa[k] = seq[i++];
57         else if (sufcmp(s, rk, n, seq[i], arr[j])) sa[k] = seq[i++];
58         else sa[k] = arr[j++];
59     }
60     for (int i = 0; i < n - 1; i++) rk[sa[i]] = i + 1;
61 }
```

2. AC 自动机

时间复杂度 $O(n + m + z + n|\Sigma|)$, n 是模板串总长度, m 是目标串长度, z 是总匹配次数, Σ 是字符集。如果想移掉 $n|\Sigma|$ 这一项, 需要使用哈希表。传入的字符串下标从 0 开始。

```
1 struct Node {
2     Node() : mark(false), suf(NULL), nxt(NULL) {
3         memset(ch, 0, sizeof(ch));
4     }
5     bool mark;
6     Node *suf, *nxt, *ch[SIGMA];
7 };
8 void insert(Node *x, char *s) {
9     for (int i = 0; s[i]; i++) {
10         int c = s[i] - 'a';
```

```

11     if (!x->ch[c]) x->ch[c] = new Node;
12     x = x->ch[c];
13 }
14 x->mark = true;
15 }
16 void build_automaton(Node *r) {
17     queue<Node *> q;
18     for (int c = 0; c < SIGMA; c++) {
19         if (!r->ch[c]) continue;
20         r->ch[c]->suf = r;
21         q.push(r->ch[c]);
22     }
23     while (!q.empty()) {
24         Node *x = q.front();
25         q.pop();
26         for (int c = 0; c < SIGMA; c++) {
27             Node *v = x->ch[c]; if (!v) continue;
28             Node *y = x->suf;
29             while (y != r && !y->ch[c]) y = y->suf;
30             if (y->ch[c]) y = y->ch[c];
31             v->suf = y;
32             if (y->mark) v->nxt = y;
33             else v->nxt = y->nxt;
34             q.push(v);
35 }}}}
36 void search(Node *x, char *s) {
37     for (int i = 0; s[i]; i++) {
38         int c = s[i] - 'a';
39         while (x->suf && !x->ch[c]) x = x->suf;
40         if (x->ch[c]) x = x->ch[c];
41         if (x->mark) print(i + 1, x->data);
42         for (Node *y = x->nxt; y; y = y->nxt) print(i + 1, y->data);
43     }
44 }

```

3. 后缀排序：倍增算法

倍增法后缀排序，时间复杂度为 $\Theta(n \log n)$ 。suffix_sort 是本体，结果输出到 sa 数组和 rk 数组（排名数组）。参数 s 是字符串，下标从 0 开始，n 是字符串长度，m 是字符集大小（一般为 255，字符集为 $\Sigma = \{0, 1, 2, \dots, m\}$ ，0 是保留的 \$ 字符）。算法运行完毕后 sa 数组里面存的是从 0 开始的下标，rk 数组里面存的是从 1 开始的排名值。

另外附带一个线性求 lcp 数组的代码。lcp 数组下标从 1 开始，实际上只有在 2 到 n 范围内的才是有效值。参数意义与 suffix_sort 相同。

```

1 static int sa[NMAX + 10], rk[NMAX + 10], lcp[NMAX + 10];
2 void suffix_sort(const char *s, int n, int m) {
3     static int x[NMAX + 10], y[NMAX + 10], cnt[NMAX + 10], i;
4     for (i = 0; i < n; i++) cnt[s[i]]++;
5     for (i = 1; i <= m; i++) cnt[i] += cnt[i - 1];
6     for (i = 0; i < n; i++) sa[--cnt[s[i]]] = i;
7     for (i = 1, m = 1, rk[sa[0]] = 1; i < n; i++) {
8         if (s[sa[i - 1]] != s[sa[i]]) m++;
9         rk[sa[i]] = m;
10    }
11    for (int l = 1; l < n; l <= 1) {
12        memset(cnt, 0, sizeof(int) * (m + 1));
13        for (i = 0; i < n; i++) cnt[y[i] = i + 1 < n ? rk[i + 1] :
14            0]++;

```

```

14     for (i = 1; i <= m; i++) cnt[i] += cnt[i - 1];
15     for (i = n - 1; i >= 0; i--) x[--cnt[y[i]]] = i;
16     memset(cnt, 0, sizeof(int) * (m + 1));
17     for (i = 0; i < n; i++) cnt[rk[i]]++;
18     for (i = 1; i <= m; i++) cnt[i] += cnt[i - 1];
19     for (i = n - 1; i >= 0; i--) sa[--cnt[rk[x[i]]]] = x[i];
20     for (i = 1, m = 1, x[sa[0]] = 1; i < n; i++) {
21         if (rk[sa[i - 1]] != rk[sa[i]] || y[sa[i - 1]] != y[sa[i]]) m++;
22         x[sa[i]] = m;
23     }
24     memcpy(rk, x, sizeof(int) * n);
25 }
26 void compute_lcp(const char *s, int n) {
27     int j = 0, p;
28     for (int i = 0; i < n; i++, j = max(0, j - 1)) {
29         if (rk[i] == 1) {
30             j = 0;
31             continue;
32         }
33         p = sa[rk[i] - 2];
34         while (p + j < n && i + j < n && s[p + j] == s[i + j]) j++;
35         lcp[rk[i]] = j;
36 }

```

4. 后缀排序：SA-IS

SA-IS 后缀数组排序。字符串存在 str 中，下标从 1 开始，长度为 n，并且 str[n + 1] 为哨兵字符，编号为 1。后缀数组放在 sa 中，下标从 1 开始。时空复杂度为 $\Theta(n)$ 。其中使用了 vector<bool> 来优化缓存命中率。

```

1 #define rep(i, l, r) for (register int i = (l); i <= (r); ++i)
2 #define rrep(i, r, l) for (register int i = (r); i >= (l); --i)
3 #define PUTS(x) sa[cur[tr[x]]--] = x
4 #define PUTL(x) sa[cur[tr[x]]++] = x
5 #define LMS(x) (!type[x - 1] && type[x])
6 #define RESET memset(sa + 1, 0, sizeof(int) * (n + 1));
7     memcpy(cur + 1, cnt + 1, sizeof(int) * m);
8 #define INDUCE rep(i, 1, m) cur[i] = cnt[i - 1] + 1;
9     rep(i, 1, n + 1) if (sa[i] > 1 && !type[sa[i] - 1]) PUTL(sa[i] - 1);
10    memcpy(cur + 1, cnt + 1, sizeof(int) * m);
11    rrep(i, n + 1, 1) if (sa[i] > 1 && type[sa[i] - 1]) PUTS(sa[i] - 1);
12 void sais(int n, int m, int *str, int *sa) {
13     static int id[NMAX + 10];
14     vector<bool> type(n + 2);
15     type[n + 1] = true;
16     rrep(i, n, 1) type[i] = str[i] == str[i + 1] ? type[i + 1] : s
17         tr[i] < str[i + 1];
18     int cnt[m + 1], cur[m + 1], idx = 1, y = 0, rt, lrt, *ns = str
19         + n + 2, *nsa = sa + n + 2;
20     memset(cnt, 0, sizeof(int) * (m + 1));
21     rep(i, 1, n + 1) cnt[tr[i]]++;
22     rep(i, 1, m) cnt[i] += cnt[i - 1];
23     RESET rep(i, 2, n + 1) if (LMS(i)) PUTS(i); INDUCE
24     rep(i, 2, n + 1) if (LMS(sa[i])) {
25         register int x = sa[i];

```

```

25     for (rt = x + 1; !LMS(rt); rt++) ;
26     id[x] = y && rt + y == lrt + x && !memcmp(str + x, str + y
, sizeof(int) * (rt - x + 1)) ? idx : ++idx;
27     y = x, lrt = rt;
28 }
29 int len = 0, pos[(n >> 1) + 1];
30 rep(i, 1, n) if (id[i]) {
31     ns[++len] = id[i];
32     pos[len] = i;
33 }
34 ns[len + 1] = 1, pos[len + 1] = n + 1;
35 if (len == idx - 1) rep(i, 1, len + 1) nsa[ns[i]] = i;
36 else sais(len, idx, ns, nsa);
37 RESET rrep(i, len + 1, 1) PUTS(pos[nsa[i]]); INDUCE
38 }
39 static int str[NMAX * 3 + 10], sa[NMAX * 3 + 10];

```

5. 线性筛 & 杜教筛

计算积性函数 $f(n)$ 的前缀和 $F(n) = \sum_{k=1}^n f(k)$: 先选定辅助函数 $g(n)$ 进行 Dirichlet 卷积, 得到递推公式:

$$F(n) = \frac{1}{g(1)} \left(\sum_{k=1}^n (f \times g)(k) - \sum_{k=2}^n g(k) F\left(\left\lfloor \frac{n}{k} \right\rfloor\right) \right)$$

对于 Euler 函数 $\varphi(n)$, 选定 $g(n) = 1$, 得:

$$\Phi(n) = \frac{n(n+1)}{2} - \sum_{k=2}^n \Phi\left(\left\lfloor \frac{n}{k} \right\rfloor\right)$$

对于 Mobius 函数 $\mu(n)$, 选定 $g(n) = 1$, 得:

$$M(n) = 1 - \sum_{k=2}^n M\left(\left\lfloor \frac{n}{k} \right\rfloor\right)$$

如果没有预处理, 时间复杂度为 $\Theta(n^{3/4})$, 空间复杂度为 $\Theta(\sqrt{n})$ 。如果预处理前 $\Theta(n^{2/3})$ 项前缀和, 则时空复杂度均变为 $\Theta(n^{2/3})$ 。下面的代码以 Euler 函数为例, 能够在 1s 内计算 10^{10} 内的数据。可以多次调用。

```

1 #define S 17000000 // for F(10^10)
2 static int pc, pr[S + 10];
3 static i64 phi[S + 10];
4 static unordered_map<i64, i64> dat;
5 inline void sub(i64 &a, i64 b) { a -= b; if (a < 0) a += MOD; }
6 inline i64 c2(i64 n) { n %= MOD; return n * (n + 1) % MOD * INV2 % MOD; }
7 i64 F(i64 n) { // 杜教筛
8     if (n <= S) return phi[n];
9     if (dat.count(n)) return dat[n];
10    i64 &r = dat[n] = c2(n);
11    for (i64 i = 2, l; i <= n; i = l + 1) {
12        i64 p = n / i;
13        l = n / p;
14        sub(r, (l - i + 1) * F(p) % MOD); // (l - i + 1) % MOD?
15    }
16    return r;
17 }
18 phi[1] = 1; // 线性筛
19 for (int i = 2; i <= S; i++) {
20     if (!phi[i]) {
21         pr[pc++] = i;

```

```

22         phi[i] = i - 1;
23     }
24     for (int j = 0; pr[j] * i <= S; j++) {
25         int p = pr[j];
26         if (i % p) phi[i * p] = phi[i] * (p - 1);
27         else {
28             phi[i * p] = phi[i] * p;
29             break;
30         }
31     }
32     for (int i = 2; i <= S; i++) add(phi[i], phi[i - 1]);

```

6. 类 Euclid 算法

类 Euclid 算法在模意义下计算:

$$\sum_{k=0}^n k^p \left\lfloor \frac{ak+b}{c} \right\rfloor^q$$

其中所有参数非负, 在计算过程中始终保证 $K = p + q$ 不减, $a, c \geq 1$ 且 $b \geq 0$ 。需要 Bernoulli 数 ($B_1 = +1/2$) 来计算自然数幂前缀和 $S_p(x) = \sum_{k=1}^x k^p = \sum_{k=1}^{p+1} a_k^{(p)} x^k$, 其中 $a_k^{(p)} = \frac{1}{p+1} \binom{p+1}{k} B_{p+1-k}$ 。代码中 has 为访问标记数组, 每次使用前需清空, val 为记忆化使用的数组, qpow 是快速幂, S 是自然数幂前缀和, A 记录了 $a_k^{(p)}$, C 是组合数。时空复杂度为 $O(K^3 \log \max\{a, c\})$ 。

算法主要分为三个情况, 其中 $a \geq c$ 和 $b \geq c$ 的情况比较简单。当 $a, b < c$ 时, 用 $j = \lfloor (ak+b)/c \rfloor$ 进行代换, 注意最终要转化为 $\lfloor (c(j-1) + c - b - 1)/a \rfloor < k \leq \lfloor (cj + c - b - 1)/a \rfloor$, 再进行一次分部求和即可。注意处理 $k \leq n$ 这个条件。

```

1 i64 F(i64 n, i64 a, i64 b, i64 c, int p, int q, int d = 0) {
2     if (n < 0) return 0;
3     if (has[d][p][q]) return val[d][p][q];
4     has[d][p][q] = true;
5     i64 &ret = val[d][p][q] = 0; // 后面的 d 均加 1
6     if (!q) ret = S(n, p) + (!p); // 注意 p = 0 的边界情况
7     else if (!a) ret = qpow(b / c, q) * (S(n, p) + (!p)) % MOD;
8     else if (a >= c) {
9         i64 m = a / c, r = a % c, mp = 1;
10        for (int j = 0; j <= q; j++, mp = mp * m % MOD)
11            add(ret, C[q][j] * mp % MOD * F(n, r, b, c, p + j, q - j, d) % MOD);
12    } else if (b >= c) {
13        i64 m = b / c, r = b % c, mp = 1;
14        for (int j = 0; j <= q; j++, mp = mp * m % MOD)
15            add(ret, C[q][j] * mp % MOD * F(n, a, r, c, p, q - j, d) % MOD);
16    } else {
17        i64 m = (a * n + b) / c;
18        for (int k = 0; k < q; k++) {
19            i64 s = 0;
20            for (int i = 1; i <= p + 1; i++)
21                add(s, A[p][i] * F(m - 1, c, c - b - 1, a, k, i, d) % MOD);
22            add(ret, C[q][k] * s % MOD);
23        }
24        ret = (qpow(m, q) * S(n, p) - ret) % MOD;
25    } return ret;
26 }

```

7. 带花树

```
1 class UnionFind {
2 public:
3     UnionFind() {
4         memset(set, 0, sizeof(set));
5     }
6
7     int find(int u) {
8         return _find(u);
9     }
10
11     void link(int u, int v) {
12         u = find(u);
13         v = find(v);
14
15         if (u != v)
16             set[u] = v;
17     }
18
19 private:
20     int set[NMAX + 10];
21
22     int _find(int u) {
23         return set[u] ? set[u] = _find(set[u]) : u;
24     }
25 };
26
27 class BlossomAlgorithm {
28 public:
29     BlossomAlgorithm(int _n) : n(_n) {
30         memset(match, 0, sizeof(match));
31         memset(link, 0, sizeof(link));
32         memset(type, 0, sizeof(type));
33     }
34
35     int match[NMAX + 10];
36
37     void add_edge(int u, int v) {
38         G[u].push_back(v);
39         G[v].push_back(u);
40     }
41
42     int solve() {
43         int ret = 0;
44
45         for (int i = 1; i <= n; i++) {
46             if (!match[i] && argument(i))
47                 ret++;
48         }
49
50         return ret;
51     }
52
53 private:
54     enum Type {
55         UNKNOWN, ODD, EVEN
56     };
57
```

```
58     int n;
59     vector<int> G[NMAX + 10];
60     deque<int> q;
61     UnionFind uf;
62     int link[NMAX + 10];
63     int mark[NMAX + 10];
64     Type type[NMAX + 10];
65
66     int lca(int u, int v) {
67         static int t;
68         t++;
69
70         while (u) {
71             u = uf.find(u);
72             mark[u] = t;
73             u = link[match[u]];
74         }
75
76         while (v) {
77             v = uf.find(v);
78             if (mark[v] == t)
79                 return v;
80             v = link[match[v]];
81         }
82
83         return -1;
84     }
85
86     void process(int u, int p) {
87         while (u != p) {
88             int a = match[u];
89             int b = link[a];
90
91             if (uf.find(b) != p)
92                 link[b] = a;
93
94             if (type[a] == ODD) {
95                 type[a] = EVEN;
96                 q.push_back(a);
97             }
98
99             uf.link(u, a);
100             uf.link(a, b);
101             u = b;
102         }
103     }
104
105     bool argument(int s) {
106         memset(&uf, 0, sizeof(uf));
107         memset(link, 0, sizeof(link));
108         memset(type, 0, sizeof(type));
109         q.clear();
110         q.push_back(s);
111         type[s] = EVEN;
112
113         while (!q.empty()) {
114             int u = q.front();
115             q.pop_front();
116
```

```

117     for (int v : G[u]) {
118         if (!type[v]) {
119             type[v] = ODD;
120             link[v] = u;
121
122             if (match[v]) {
123                 type[match[v]] = EVEN;
124                 q.push_back(match[v]);
125             } else {
126                 int x = v;
127                 while (link[x] != s) {
128                     int a = link[x];
129                     int b = match[a];
130                     match[x] = a;
131                     match[a] = x;
132                     x = b;
133                 }
134
135                 match[s] = x;
136                 match[x] = s;
137
138                 return true;
139             }
140         } else if (type[v] == EVEN &&
141             uf.find(u) != uf.find(v)) {
142             int p = lca(u, v);
143
144             if (uf.find(u) != p)
145                 link[u] = v;
146             if (uf.find(v) != p)
147                 link[v] = u;
148
149             process(u, p);
150             process(v, p);
151         }
152     }
153 }
154
155 return false;
156 }
157 };

```

8. Stoer Wanger

```

1 template <typename TCompare>
2 class Heap {
3 public:
4     void push(int x) {
5         s.insert(x);
6     }
7
8     void pop(int x) {
9         auto iter = s.find(x);
10
11         assert(iter != s.end());
12         s.erase(iter);
13     }
14
15     int top() {
16         return *s.begin();

```

```

17     }
18
19     size_t size() const {
20         return s.size();
21     }
22
23 private:
24     multiset<int, TCompare> s;
25 }; // class Heap
26
27 struct Edge {
28     Edge(int _u, int _v, int _w) : u(_u), v(_v), w(_w) {}
29
30     int u, v, w;
31
32     int either(int x) const {
33         return u == x ? v : u;
34     }
35 }; // struct Edge
36
37 static int n, m;
38 static vector<Edge> *G[NMAX + 10];
39 static bool marked[NMAX + 10];
40 static bool visited[NMAX + 10];
41 static int weight[NMAX + 10];
42
43 struct cmp {
44     bool operator()(const int a, const int b) const {
45         return weight[a] > weight[b] || (weight[a] == weight[b] &&
46             a < b);
47     }
48 }; // struct cmp
49
50 int find_mincut(int &s, int &t) {
51     Heap<cmp> q;
52     memset(weight, 0, sizeof(weight));
53     memset(visited, 0, sizeof(visited));
54     for (int i = 1; i <= n; i++) {
55         if (!marked[i])
56             q.push(i);
57     } // for
58
59     while (q.size() > 1) {
60         int u = q.top();
61         visited[u] = true;
62         q.pop(u);
63
64         s = u;
65         for (auto &e : G[u]) {
66             int v = e->either(u);
67
68             if (visited[v])
69                 continue;
70
71             q.pop(v);
72             weight[v] += e->w;
73             q.push(v);
74         } // foreach in G[u]
75     } // while

```

```

75
76     t = q.top();
77     return weight[t];
78 }
79
80 typedef pair<int, int> IntPair;
81
82 IntPair mincut(int cnt) {
83     if (cnt < 2)
84         return make_pair(INT_MAX, cnt);
85
86     int s, t;
87     int ans = find_mincut(s, t);
88
89     marked[t] = true;
90     for (auto &e : G[t]) {
91         if (e->u == t)
92             e->u = s;
93         else
94             e->v = s;
95
96         G[s].push_back(e);
97     } // foreach e in G[t]
98
99     IntPair ret = mincut(cnt - 1);
100     if (ans <= ret.first)
101         return make_pair(ans, cnt);
102     return ret;
103 }

```

9. 构造圆方树

G 用于存图，T 是构造的圆方树。只有一个点的点双没有添加方点。

```

1 static vector<int> G[NMAX + 10], T[NMAX + 10];
2 void bcc(int u, int f = 0) {
3     static stack<Pair> stk;
4     static bool marked[NMAX + 10];
5     static int in[NMAX + 10], low[NMAX + 10], cur;
6     in[u] = low[u] = ++cur;
7     for (int v : G[u]) {
8         if (v == f) f = 0; // 应对重边
9         else if (in[v]) low[u] = min(low[u], in[v]);
10        else {
11            stk.push(Pair(u, v)); // stk 内存储 DFS 树上的边
12            bcc(v, u);
13            low[u] = min(low[u], low[v]);
14            if (low[v] > in[u]) { // 割边 u - v
15                T[u].push_back(v);
16                T[v].push_back(u);
17                stk.pop();
18            } else if (low[v] >= in[u]) { // 可能有点双了
19                cnt++;
20                int linked = 0, p = n + cnt; // linked 点数, p 圆方
21                auto add = [p, &linked](int x) {
22                    if (!marked[x]) {
23                        marked[x] = true;
24                        T[p].push_back(x);
25                        T[x].push_back(p);

```

```

26                linked++;
27            };
28            while (!stk.empty()) {
29                Pair x = stk.top();
30                stk.pop();
31                add(x.u);
32                add(x.v);
33                if (x.u == u && x.v == v) break;
34            }
35            for (int v : T[p]) marked[v] = false;
36            if (linked == 0) cnt--; // 假点双
37        }

```

10. 最小树形图：朴素算法

给定一张 n 个点 m 条边的带权有向图，求以 r 为根的最小树形图上的边权总和，如果不存在输出 -1。时间复杂度为 $O(nm)$ 。调用 `mdst(r)` 获得答案，调用前需清空 `id` 数组。如要求不定根的最小树形图，可以额外添加一个节点，向原图中的每个点连接一条边权为 ∞ 的边。

```

1 static int n, m, G[NMAX + 10], nxt[MMAX + 10];
2 static struct Edge { int u, v, w; } E[MMAX + 10], *in[NMAX + 10];
3 static int id[NMAX + 10], mark[NMAX + 10];
4 int find(int x) { return id[x] ? id[x] = find(id[x]) : x; }
5 int dfs(int x) {
6     mark[x] = 1; int ret = 1;
7     for (int i = G[x]; i; i = nxt[i])
8         if (!mark[E[i].v]) ret += dfs(E[i].v);
9     return ret;
10 }
11 inline int detect(int x) {
12     mark[x] = x;
13     for (int y = in[x]->u; in[y]; y = in[y]->u)
14         if (mark[y]) return mark[y] == x ? y : 0;
15     else mark[y] = x;
16     return 0;
17 }
18 int mdst(int r) {
19     if (dfs(r) < n) return -1;
20     int ret = 0;
21     while (true) {
22         memset(in, 0, sizeof(in));
23         memset(mark, 0, sizeof(mark));
24         for (auto *e = E + 1; e <= E + m; e++)
25             if (e->u != e->v && e->v != r && (!in[e->v] || e->w <
26                 in[e->v]->w))
27                 in[e->v] = e;
28         int p = 0, t = 0;
29         for (int x = 1; x <= n; x++, t |= p) if (!mark[x] && in[x])
30             if (!(p = detect(x))) continue;
31             ret += in[p]->w;
32             for (int x = in[p]->u; x != p; x = in[x]->u)
33                 id[find(x)] = p, ret += in[x]->w;
34             for (auto *e = E + 1; e <= E + m; e++) {
35                 int u = find(e->u), v = find(e->v);
36                 if (u != p && v == p) e->w -= in[e->v]->w;
37                 e->u = u; e->v = v;

```

```

38     if (!t) break;
39 }
40 for (int x = 1; x <= n; x++) if (in[x]) ret += in[x]->w;
41 return ret;
42 }

```

11. 单纯型

```

1 #define EPS 1e-10
2 #define INF 1e100
3
4 class Simplex {
5 public:
6     void initialize() {
7         scanf("%d%d%d", &n, &m, &t);
8
9         memset(A, 0, sizeof(A));
10        for (int i = 1; i <= n; i++) {
11            idx[i] = i;
12            scanf("%Lf", A[0] + i);
13        }
14
15        for (int i = 1; i <= m; i++) {
16            idy[i] = n + i;
17            for (int j = 1; j <= n; j++) {
18                scanf("%Lf", A[i] + j);
19                A[i][j] *= -1;
20            }
21
22            scanf("%Lf", A[i]);
23        }
24    }
25
26    void solve() {
27        srand(time(0));
28
29        while (true) {
30            int x = 0, y = 0;
31            for (int i = 1; i <= m; i++) {
32                if (A[i][0] < -EPS && (!y || (rand() & 1)))
33                    y = i;
34            }
35
36            if (!y)
37                break;
38
39            for (int i = 1; i <= n; i++) {
40                if (A[y][i] > EPS && (!x || (rand() & 1)))
41                    x = i;
42            }
43
44            if (!x) {
45                puts("Infeasible");
46                return;
47            }
48
49            pivot(x, y);
50        }
51
52        while (true) {

```

```

53        double k = INF;
54        int x, y;
55        for (x = 1; x <= n; x++) {
56            if (A[0][x] > EPS)
57                break;
58        }
59
60        if (x > n)
61            break;
62
63        for (int i = 1; i <= m; i++) {
64            double d = A[i][x] > -EPS ? INF : -A[i][0] / A[i][
x];
65
66            if (d < k) {
67                k = d;
68                y = i;
69            }
70
71            if (k >= INF) {
72                puts("Unbounded");
73                return;
74            }
75
76            pivot(x, y);
77        }
78
79        printf("%.10Lf\n", A[0][0]);
80
81        if (t) {
82            static double ans[NMAX + 10];
83            for (int i = 1; i <= m; i++) {
84                if (idy[i] <= n)
85                    ans[idy[i]] = A[i][0];
86            }
87
88            for (int i = 1; i <= n; i++) {
89                printf("%.10Lf ", ans[i]);
90            }
91            printf("\n");
92        }
93    }
94
95 private:
96     void pivot(int x, int y) {
97         swap(idx[x], idy[y]);
98         double r = -A[y][x];
99         A[y][x] = -1;
100        for (int i = 0; i <= n; i++) {
101            A[y][i] /= r;
102        }
103
104        for (int i = 0; i <= m; i++) {
105            if (i == y)
106                continue;
107
108            r = A[i][x];
109            A[i][x] = 0;
110            for (int j = 0; j <= n; j++) {

```

```
111         A[i][j] += r * A[y][j];
112     }
113 }
114 }
115
116 int n, m, t;
117 double A[NMAX + 10][NMAX + 10];
118 int idx[NMAX + 10], idy[NMAX + 10];
119 };
```