interior sheet is used for guiding the light back to the exterior layer. Fortunately, the Kepler profile (5) does not lead to total reflection if $r_0 \geq 2|w_2 - w_1|$. In this case, the invisible area is largest for

$$r_0 = 2|w_2 - w_1| \qquad (7)$$

Figure 3 illustrates the light propagation in a dielectric invisibility device based on the simple map (3) and the Kepler profile (5) with $r_0 = 8a$. Here n ranges from 0 to about 36, but this example is probably not the optimal choice. One can choose from infinitely many conformal maps $w(z)$ that possess the required properties for achieving invisibility: $w(z) \sim z$ for $z \to \infty$ and two branch points w_1 and w_2. The invisible region may be deformed to any simply connected domain by a conformal map that is the numerical solution of a Riemann-Hilbert problem (*16*). We can also relax the tacit assumption that w_1 connects the exterior to only one interior sheet, but to m sheets where light rays return after m cycles. If we construct $w(z)$ as $af(z/a)$ with some analytic function $f(z)$ of the required properties and a constant length scale a, the refractive-index profile $|dw/dz|$ is identical for all scales a. Finding the most practical design is an engineering problem that depends on practical demands. This problem may also inspire further mathematical research on conformal maps in order to find the optimal design and to extend our approach to three dimensions.

Finally, we ask why our scheme does not violate the mathematical theorem (*3*) that perfect invisibility is unattainable. The answer is that waves are not only refracted at the boundary between the exterior and the interior layer, but also are reflected, and that the device causes a time delay. However, the reflection can be substantially reduced by making the transition between the layers gradual over a length scale much larger than the wavelength $2\pi/k$ or by using anti-reflection coatings. In this way, the imperfections of invisibility can be made as small as the accuracy limit of geometrical optics (*1*), i.e., exponentially small. One can never completely hide from waves, but can from rays.

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Controlling Electromagnetic Fields

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Using the freedom of design that metamaterials provide, we show how electromagnetic fields can be redirected at will and propose a design strategy. The conserved fields—electric displacement field D, magnetic induction field B, and Poynting vector B—are all displaced in a consistent manner. A simple illustration is given of the cloaking of a proscribed volume of space to exclude completely all electromagnetic fields. Our work has relevance to exotic lens design and to the cloaking of objects from electromagnetic fields.

To exploit electromagnetism, we use materials to control and direct the fields: a glass lens in a camera to produce an image, a metal cage to screen sensitive equipment, "blackbodies" of various forms to prevent unwanted reflections. With homogeneous materials, optical design is largely a matter of choosing the interface between two materials. For example, the lens of a camera is optimized by altering its shape so as to minimize geometrical aberrations. Electromagnetically inhomogeneous materials offer a different approach to control light; the introduction of specific gradients in the refractive index of a material can be used to form lenses and other optical elements, although the types and ranges of such gradients tend to be limited.

A new class of electromagnetic materials (*1*, *2*) is currently under study: metamaterials, which owe their properties to subwavelength details of structure rather than to their chemical composition, can be designed to have properties difficult or impossible to find in nature. We show how the design flexibility of metamaterials can be used to achieve new electromagnetic devices and how metamaterials enable a new paradigm for the design of electromagnetic structures at all frequencies from optical down to DC.

Progress in the design of metamaterials has been impressive. A negative index of refraction (*3*) is an example of a material property that does not exist in nature but has been enabled by using metamaterial concepts. As a result, negative refraction has been much studied in recent years (*4*), and realizations have been reported at both GHz and optical frequencies (*5*–*8*). Novel magnetic properties have also been reported over a wide spectrum of frequencies. Further information on the design and construction of metamaterials may be found in (*9*–*13*). In fact, it is now conceivable that a material can be constructed whose permittivity and permeability values may be designed to vary independently and arbitrarily throughout a material, taking positive or negative values as desired.

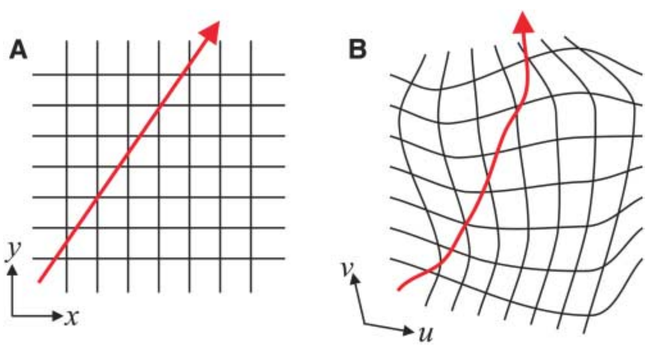

Fig. 1. (**A**) A field line in free space with the background Cartesian coordinate grid shown. (**B**) The distorted field line with the background coordinates distorted in the same fashion. The field in question may be the electric displacement or magnetic induction fields **D** or **B**, or the Poynting vector **S**, which is equivalent to a ray of light.

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