

2

$$y_i = \sum_{j=1}^n \omega_j h_j(x_i) + \epsilon_i, \quad h - \text{basis } f^0$$

$$\omega = (H^T H)^{-1} H^T y \quad (H - n \times m) \quad (H_{ij} = h_j(x_i))$$

$$\begin{aligned} \hat{y} &= h(x)^T \omega = h(x)^T (H^T H)^{-1} H^T y \\ &= H (H^T H)^{-1} h(x))^T y \end{aligned}$$

$$l(x) = H (H^T H)^{-1} h(x)$$

$$\hat{y} = l(x)^T y \rightarrow \text{Linear Smoother}$$

2 a) $k(x_i, x) = \exp \frac{\|x_i - x\|_2^2}{\sigma}$

$$\hat{y} = \frac{\sum_{i=1}^n \omega_i y_i}{\sum_{i=1}^n \omega_i}$$

$$l_i(x) = \frac{\omega_i}{\sum_{i=1}^n \omega_i}$$

$$\hat{y} = ?$$

$$\left(\omega_i = \exp \left(- \frac{D(x_i, x)^2}{k^2 \omega} \right) \right)$$

we expanded this
eqn is form of
 $l(x)^T y$

$$\boxed{\hat{y} = l(x)^T y}$$

So the above
Linear Regression is
Kernel smoother.

Ans 2(b)

sum of residual ~~error~~ square $\|HW - Y\|_2$

There is no way we can convert $\sum \text{abs}(\text{error})$ that will be minimize

Optimal w has same # of +ve & -ve error $(x_i \rightarrow \text{input})$

$\rightarrow x_i = 1$ (or any constant)
for different y
 $w = \text{median}(y)$
 w is not linear in any of (y_1, \dots, y_n)
(Median \propto Rank of ~~matrix~~ matrix)

2C

$$\hat{y} = \frac{1}{|B_k|} \sum_{x_i \in B_k} y_i \quad \text{for } x \in B_k$$

$$I_{ij}(x) = \frac{\mathbb{I}(x_j \in B_i)}{|B_i|} \quad \mathbb{I} \rightarrow \text{indicator Random Variable}$$

Since \mathbb{I} is indicator Random Variable
we can say regression is linear sm