2(a) $x = a\,\delta_k$

$a \rightarrow P(a)$ arbitary

$k \rightarrow$ uniformly distributed over $1 \dots M$

for covariance Matrix

$\mathrm{Cov}_{ij} = \mathrm{var}_i$ if $(i = j)$ - Diagonal

$= \mathrm{cor}\ x_i, x_j$ if $(i \neq j)$ Non Diagonal.

$\mathrm{cor}(x_i, x_j) = E[x_i \cdot x_j] - E[x_i]\,E[x_j]$

$\boxed{E[x] = E[a\delta_i]}$ if $(x_i \neq x_j)$

$\mathrm{cor}(x_i, x_j) = E[a\delta_i \cdot a\delta_j] - E[a\delta_i]\,E[a\delta_j]$

$(E[\delta_i] = 1/m$ uniform dist.$)$

$= E[a^2 \delta_i \delta_j] - E[a] \cdot \frac{1}{m} \cdot E[a]\ 1/m$

$= E[a^2] \cdot E[\delta_i \delta_j] - \frac{E[a]^2}{m^2}$ $(\delta_i\,\delta_j^T = 0)$ as 1 will be always be diff place

$= 0 - \frac{E[a^2]}{m^2}$

if $(x_i = x_j)$

$\mathrm{cor}(x_i, x_j) = \mathrm{var}(x_i) = E[a^2 \delta_i^2] - E[a\,\delta_i]^2$

$= E[a^2] \cdot E[\delta_i^2] - (E[a] \cdot E[\delta_i])^2$

$= \frac{E[a^2]}{m} - \frac{E[a]^2}{m^2}$ $(E[\delta_i^2] = 1/m)$

$c_{ij} = \frac{E[a^2]}{m}\,\delta_{ij} - \frac{E[a]^2}{m^2}$

where $\left(\delta_{ij} = \begin{matrix} 0 & (i \neq j) \\ 1 & (i = j) \end{matrix}\right)$

$c_{ij} = \lambda + \mu\,\delta_{ij}$

$\lambda = -E[a]^2 / m^2$

$\mu = E[a^2] / m$

2 (b)

$$C = \begin{bmatrix} \lambda+\mu & \cdots & \lambda & \lambda \\ \vdots & \lambda+\mu & \vdots & \vdots \\ \vdots & \cdots & \lambda+\mu & \vdots \\ \lambda & \cdots & \cdots & \lambda+\mu \end{bmatrix}$$

Eq^n for fending eigen value & eigen vectors

$$CX = KX$$

Taking eigen vector to be $(1 \cdots 1)$ if we can find eigen value that would mean $(1 \cdots 1]$ is valid eigen ve

$$\underset{m \times m}{\begin{bmatrix} \lambda+\mu & \cdots & \lambda & \lambda \\ \vdots & \lambda+\mu & & \vdots \\ \lambda & \cdots & & \lambda+\mu \end{bmatrix}} \begin{bmatrix} 1 \\ \vdots \\ \vdots \end{bmatrix}_{m \times 1} = K \begin{bmatrix} \\ \\ \end{bmatrix}$$

$$\underset{m \times 1}{\begin{bmatrix} \mu + \lambda m \\ \mu + \lambda m \\ \vdots \\ \mu + \lambda m \end{bmatrix}} = \begin{bmatrix} K \\ \vdots \\ K \end{bmatrix} \quad m \times 1$$

$K = \mu + m\lambda$ - eigen value

$(1, 1 \ldots 1)$ - eigen vector

~~ted~~ By observation

$$C = \mu I + \lambda e$$

$$C - \mu I = \lambda \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ & & 1 \end{bmatrix} = \begin{bmatrix} \lambda & \cdots & \lambda \\ \vdots & \lambda & \vdots \\ \lambda & \cdots & \lambda \end{bmatrix} 20$$

$$|C - \mu I| = 0$$

Rank of $|C - \mu I|$ is 1.

Hence it'll have only one eigen value & all other eigen value $= \mu$ $\therefore$ so that mean eigen vector a

2C PCA is not good. to select feature as PCA. chooses eig vector corresponding to max eigen ~~vector~~ value to proj data but in case '$m-1$' value & their eigen vector a same. so it not fesible to fend which dimension to project the data & eigen vector are same, so projection is same

$\therefore$ PCA is not good approach