Sol 1 (a)

$K(x,z) = k_1(x,z) + k_2(x,z)$ is a valid kernel f^n

$C = C_{ij} = k_1(x_i x_j)$

$D = d_{ij} = k_2(x_i, x_j)$ $\quad U^T E U \geq 0$

$E = e_{ij} = C_{ij} + d_{ij}$

for validity of kernel f^n K it must satisfy PSD condition $\quad U^T E U \geq 0 \quad \forall U \in R^n$

$e_{ij} = k_1(x_i, x_j) + k_2(x_i x_j)$

$= U^T K_1 U + U^T K_2 U$

Since K_1 & K_2 are valid kernel f^n

$U^T K_1 U \geq 0$ & $U^T K_2 U \geq 0$

hence

$\boxed{U^T K_1 U + U^T K_2 U \geq 0}$ valid kernel

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1 (b) $C = C_{ij} = k_1(x_i, x_j)$

$D = (d_{ij}) = d_{ij} = k_2(x_i, x_j)$ $\quad U^T E U \geq 0$

$E = (e_{ij}) = e_{ij} = c_{ij} \cdot d_{ij}$ $\quad C = A^T A \quad A = (a_1 \cdots a_n)$

$D = B^T B \quad B = (b_1 \cdots b_n)$

& $U^T E U = \sum_{ij} U_i U_j c_{ij} d_{ij}$

$d_{ij} = b_i^T b_j = \sum_l b_{il} b_{jl}$

$= \sum U_{ij} \left(\sum_k a_{ik} a_{jk}\right)\left(\sum_l b_{il} b_{jl}\right)$ $\quad c_{ij} = a_i^T a_j$

$\quad \quad = \sum_k a_{ik} a_{jk}$

$= \sum_{kl} \sum_{ij} U_i U_j a_{ik} a_{jk} b_{il} b_{jl}$

$$= \sum_{kl} \left(\sum_i v_i a_{ik} b_{il} \right)^2 \geq 0$$

is a valid kernel

Ans 1 (c) $k(x, z) = h(k_1(x, z))$

h is polynomial f^n with positive coeff.

$$h(k_1(x, z)) = a_0 [k_1(x, z)]^0 + a [k_1(x, z)]^1 + a_2 [k_1(x, z)]^2 \ldots$$

$a_0 \cdot a_1 \ldots a_n \geq 0$ (# coefficient

after adding +ve constant it still hold validity

$$= \underbrace{a_0 + a_1 [k_1(x, z)]}_{\text{linear}} + \underbrace{a_2 [k(x, z) \times k_1(x, z)]}_{\text{product}} \ldots$$

→ from 1a, 1(b) we have seen that linear combining & product of two or more valid kernel holds true

So this holds true

Sol 1 (d) $k(x, z) = \exp(k_1(x, z))$

taylor series $e^x \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots\right)$

$$e^{k_1(x, z)} = \underbrace{1 + k(x, z)}_{\text{linear combi}} + \underbrace{\frac{[k_1(x, z)]^2}{2!}}_{\text{product} \cdot \text{by itself}} \ldots$$

we ~~saw~~ have seen in 1(a) 1(b) ~~two~~ combination of valid kernel this is same as them.

so $\exp(k_1(x, z))$ is also valid kernel

1(e)

$$k(x,z) = \exp\left(\frac{-\|x-z\|^2}{\sigma^2}\right)$$

$$= \exp\left(\frac{-\|x\|^2 - \|z\|^2 + 2x^T z}{\sigma^2}\right)$$

$$= \exp\left(\frac{-\|x\|^2}{\sigma^2}\right) \cdot \exp\left(\frac{-\|z\|^2}{\sigma^2}\right) \cdot \exp\left(\frac{2x^T z}{\sigma}\right)$$

$$= g(x) \cdot g(z) \exp\left(k_3(x,z)\right)$$

$$g(x) = \exp\left(\frac{-\|x\|^2}{\sigma^2}\right)$$

$$g(z) = \exp\left(\frac{-\|z\|^2}{\sigma^2}\right)$$

} gaussian kernel (valid kernel)

$$k_3(x,z) = \frac{2x^T z}{\sigma^2}$$

$g(x) \cdot g(z)$ — valid gaussian kernel

$\cdot \exp(k_3(x,z))$ — valid kernel (1(d))

=) Since product of 2 valid kernel is also valid ~~so~~ (1(b)) so these are also valid Kernel.