TESSELLATE PRESENTS

STEMS

Scholastic Test of Excellence in Mathematical Sciences

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Computer Science Category B

Exam Date	**:**	**12th January 2019**
Exam Timing	**:**	**4pm - 7pm**

Rules and Regulations

Marking Scheme

1. The question paper is divided in two parts -
   Objective (8 questions) + Subjective (5 questions).
2. Each objective question is worth 10 marks each.
   - If you mark exactly the set of correct answers as correct, you get 10 marks
   - Otherwise, you
     - gain 2 points for every answer you marked and is actually correct
     - and loose 1 point for every answer you marked but is actually incorrect.
3. For Objective questions having choice, each problem has at least one correct answer: *however, there might be many.* Choose ALL that apply, unless indicated otherwise.
4. Each subjective problem is worth 20 marks each.
5. **You are not expected to solve all problems in the subjective section.** It is advisable to read the problems and decide on the ones which you'd be willing to solve. Marks will be awarded for partially correct solutions.
6. The subjective problems will be graded only if you score above a certain cut-off (to be decided later) in the objective section of the paper.
7. **For the final score, your total score (subjective + objective) will be taken into consideration.**

Solution guidelines

1. You are **NOT** required to show your work for the objective part of the paper.
   Only tick your option choices in a tabular format drawn on a blank sheet.
   A sample is shown below.

Q. No.	(a)	(b)	(c)	(d)
1		✓		
2			✓	
3		✓		
4	✓			
5			✓	✓

Fig. - Sample objective answer submission

2. Provide a complete solution/proof for subjective questions. The solutions must be correct, original, detailed and clearly understandable for full credit.
   Partial credit might be awarded to incomplete proofs, based on the progress made towards solving the problem.

3. Draw clear, well-labeled diagrams wherever necessary.

Miscellaneous

1. During the examination, you are allowed to use the internet and books as resources. But answers should be your own and should reflect your independent thinking process.

2. Do **NOT** post the questions on any forums or discussion groups. It will result in immediate disqualification of involved candidates when caught.

3. Answers should be written clearly, in a legible way. Formal proofs are required wherever asked for. Unclear reasoning might not be awarded points, draw clear diagrams wherever necessary.

4. Sharing/discussion aimed towards solving or distribution of problems appearing in the contest while the contest is live in any kind of online platform/forum shall be considered as a failure in complying with the regulations.

5. Any form of plagiarism or failure to comply with aforementioned regulations may lead to disqualification.

6. If you had sent an email regarding alternate time slot, please refer to the alternate time slot schedule on our webpage (`tessellate.cmi.ac.in/stems`)

Contact details - ONLY for subject related queries

- Please do not call these people for technical problems or submission inquiries. Only if you find an ambiguity in a question and need clarification, use these contacts.
- As our phone numbers will be busy, **we prefer WhatsApp & email queries** instead. Only call us if absolutely necessary.
- Try to solve all your submission related doubts from information in the next page ONLY. We have included all details in the next section.

Agnishom Chattopadhyay - 9903675222	Ashwin Bhaskar - 7738543688
Query email - stems.2019.cs@gmail.com	

- **Do NOT call any number to ask if your submission has reached us. If you send your submission to the right email address with mentioned details, we will receive it. We will contact all participants who fail to submit, so please be patient.**

How to submit your answers

1. Write the following details **as per your registration** on the first page of your submission file/photographs -

Name	:	Your full name
School/College name	:	
Class/Year of Study	:	Class 8, Class 11, Undergrad 1st year etc.
Registered Email address	:	
Mode of S.T.E.M.S. Registration	:	Online (TheCollegeFever) / Through School / Other (mention details)

2. Write all your answers on sheets of paper, following all the solution guidelines.
   Write the page number in the top right corner of every sheet.

3. Scan your answers or take clear photographs of your response sheets.
   Compile them into a single PDF file or send all pictures in the right sequence.

   If you have a limited file size issue when sending your submission, make a new Google Drive folder with title '**Your Name - CS B submission**'.

4. Send the submission file or Google Drive link from your registered email -

Submission email address	:	**stems.2019.cs.B@gmail.com**
Subject of email	:	Computer Science Category B Submission - STEMS
Submission deadline	:	12th January 2019 - 19:00

Good luck, happy problem-solving!

Objective Questions

Problem 1. *Eku usually keeps books on his bookshelf ordered in ascending order. But today they have been jumbled up. To reorder them, he can perform the following operation: A book could be removed from its current position and then placed elsewhere. For example, if the sequence of the books were 4,3,1,2,5 then one could remove the book with label 3, and place it in between the books 1 and 2 so that the sequence reads 4,1,3,2,5.*

What is the least number of such operations he could perform such that he can re-arrange the books in sequence if they are currently in this order?

$$13, 15, 20, 25, 23, 1, 10, 26, 6, 14, 21, 24, 12, 16, 5, 9, 22, 24, 2, 19, 3, 7, 17, 8, 4, 11, 18$$

(a) *7*

(b) *17*

(c) *5*

(d) *19*

Problem 2. *Which of the following are **not** regular?*

(a) $\{a^n \cdot w \cdot b^n \mid w \in \{a^* b^*\}\}$

(b) *Context free languages over the alphabet* $\{a\}$

(c) $\{a^k \cdot u \cdot a^k \mid k \in \mathbb{N}, u \in \{a, b\}^*\}$

(d) *Any language* L *such that* L^* *is regular*

Problem 3. *If* $\chi(G)$ *denotes the chromatic number of a graph,* $\alpha(G)$ *the size of the largest independent set,* $\tau(G)$ *denote the size of the smallest vertex cover and* $\omega(G)$ *the size of the largest clique that is a sub-graph of* G.

*Which of the following statements are **false**?*

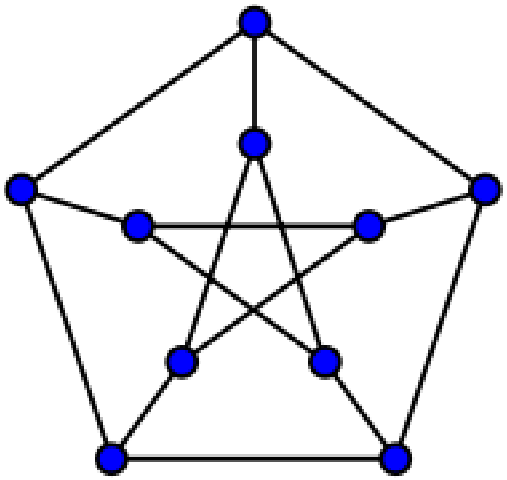
(a) $\alpha(G) + \tau(G) = |V|$

(b) $\alpha(G).\chi(G) \geq |V|$

(c) $\omega(G) \leq \chi(G)$

(d) *None of the above*

Problem 4. *Which of the following are true about the following graph?*



(a) *It is a planar graph*

(b) *It is not a planar graph*

(c) *It has K_5 as a minor*

(d) *It has $K_{3,3}$ as a minor*

Problem 5. *Which of the following are true?*

(a) $BPP \subseteq \Sigma_p^5$

(b) $EXP \subseteq NSPACE(n^{100})$

(c) $\Sigma_p^{10} \subseteq P^{PP}$

(d) $P = NP \implies EXP = NEXP$

Problem 6. *Assume that we are given an alphabet Σ. If $w = a_1\ a_2 \cdots a_n \in \Sigma^*$, then define $w^r := a_n\ a_{n-1} \cdots a_1$, (i.e) w^r is the reverse of the word w.*

Which of the following are true?

(a) *If $P \neq NP$, then the set of all planar graphs which can be 3 colored can be recognized in deterministic polynomial time.*

(b) *If $P \neq NP$, then the set of all planar graphs which can be 4 colored can be recognized in deterministic polynomial time*

(c) *The language $\{ww^r : w \in \Sigma^*\}$ cannot be recognized by a DFA*

(d) *The language $\{ww^r w : w \in \Sigma^*\}$ can be recognized by a non-deterministic pushdown automata*

Problem 7. *Consider the following problem.*

- ***Input:*** *A graph G with labels on edges from a finite alphabet Σ, distinguished vertices s and t on G. A regular expression e.*
- ***Question:*** *Does there exist a path from s to t such that the word formed by the labels satisfy e?*

Which of the following are true about this problem? (You may assume $P \neq NP$)

(a) *This problem can be solved in polynomial time*

(b) *This problem is NP-complete*

(c) *This problem is PSPACE-complete*

(d) *This problem is decidable*

Problem 8. *Here are two problems,* ***A*** *and* ***B****.*

- ***A:*** *Does a graph G have a cycle of length at most k for a given G, k?*
- ***B:*** *Given two graphs G and H. Does graph G have a subgraph, that is isomorphic to the graph H?*

Which of the following are true? (You may assume $P \neq NP$)

(a) *A is in P*

(b) *B is in P*

(c) *A is NP-complete*

(d) *B is NP-complete*

Subjective Questions

Problem 9. *A sequence of positive integers $a_1, a_2, \dots a_n$ is called a degree sequence if there is a graph G with n vertices $v_1, v_2 \dots v_n$ such that the degree of v_i is a_i for $1 \leq i \leq n$.*

Prove that if a sequence S is the degree sequence of a graph G, then there exists a ***connected graph*** *H such that S is also the degree sequence of H.*

Problem 10. *Let $\{0,1\}^n$ be the set of all n-size bitvectors. Let S be a subset of $\{0,1\}^n$. Now, suppose ψ is a formula in n variables $x_1, x_2 \cdots x_n$ built using $\wedge, \vee$ and $\neg$. With the usual semantics, ψ defines a subset of $\{0,1\}^n$, namely the set of assignments which make ψ evaluate to 1.*

S is said to be shattered by formulae if given any $T \subseteq S$, it is possible to ensure that there is a formula φ that is evaluated to true on points in T and to false on points in $S \setminus T$.

1. *What is the cardinality of the maximal set that can be shattered? (**8 points**)*
2. *Suppose the formulae that we use are restricted to only using $\wedge$ and $\vee$, and $\neg$ is not allowed. In this case, exhibit a set of cardinality $\binom{n}{\lfloor \frac{n}{2} \rfloor}$ which can be shattered. (**12 points**)*

Problem 11. *Suppose $A = (Q, \Sigma, \delta, q_0, F)$ is a DFA with Q as the set of states, Σ as the alphabet, $\delta : Q \times \Sigma \to Q$ is the transition function, $q_0 \in Q$ is the initial state and $F \subseteq Q$ is the set of accepting states. Define $\hat{\delta} : Q \times \Sigma^* \to Q$ inductively as follows: $\hat{\delta}(q, \epsilon) = q$ and $\hat{\delta}(q, aw) = \hat{\delta}(\delta(q,a), w)$ where $a \in \Sigma, w \in \Sigma^*$.*

A state $q \in Q$ is called a rendez-vous state if $\exists\ w \in \Sigma^*$ s.t. $\forall q_1, q_2 \in Q$, $\hat{\delta}(q_1, w) = \hat{\delta}(q_2, w) = q$. Give a polynomial time algorithm which when given a DFA A and a state q as input, will decide if q is a rendez-vous state or not.

Problem 12. *Given a graph G the score of a vertex v is the maximum number of disjoint paths from the vertex v to some other vertex u. The score of the graph G is the minimum score over all the vertices. Give an algorithm to calculate the score that runs in polynomial time in the number of vertices.*

You may consider using Menger's Theorem and a suitable max-flow algorithm.

Problem 13. *Consider a binary tree T with sufficiently many nodes. Let's say that sufficiently many of the nodes in T have been colored red.*

Call a subtree of T good if it has at least 10 red nodes.

1. *Does there always exist a good subtree with less than 20 red nodes? (**8 points**)*
2. *Design an algorithm that can efficiently find such a subtree (or correctly claim that it does not exist). Give a reasonable analysis of the efficiency of your algorithm. (**12 points**)*