Communication Systems (EE321A), Spring 2019
Indian Institute of Technology Kanpur
Computer Assignment 1

Student Name: Ritesh Kumar
Roll Number: 160575
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QUESTION 1

Problems:

1. Using the union bound, derive the average probability of symbol error for the 32-PSK constellation having unity radius.
   Solution:
   The minimum distance (d_{min}) for M-ary PSK is given by

$$d_{min} = 2R\sin\left(\frac{\pi}{M}\right)$$

   Assuming that the all symbols are equally likely, we get the average probability of symbol error using union bound as

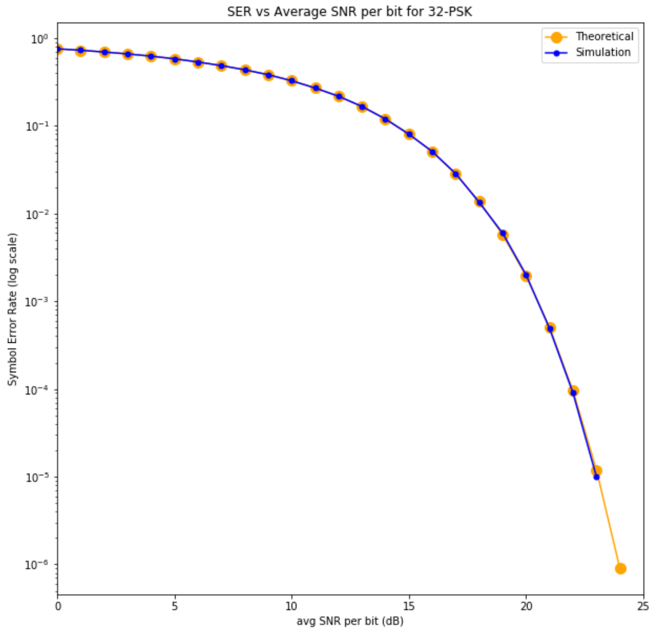
$$P(e) \leq \sum_{i=1}^{M} \frac{G_i}{2M} \operatorname{erfc}\left(\sqrt{\frac{\|d\|^2}{8\sigma_w^2}}\right)$$

   where G_i is number of nearest neighbours for symbol i.
   In 32-PSK, all symbols have two nearest neighbours (i.e. $G_i = 32$). So the error rate is

$$\begin{aligned} P(e) &\leq \sum_{i=1}^{M} \frac{2}{2M} \operatorname{erfc}\left(\sqrt{\frac{\|d\|^2}{8\sigma_w^2}}\right) \\ &= \frac{32}{32} \operatorname{erfc}\left(\frac{2R\sin\left(\frac{\pi}{M}\right)}{\sqrt{8\sigma_w^2}}\right) \\ &= \operatorname{erfc}\left(\frac{2R\sin\left(\frac{\pi}{M}\right)}{\sqrt{8\sigma_w^2}}\right) \end{aligned}$$

   Note that here $R = 1$ and $M = 32$.

2. Plot the theoretical symbol-error-rate vs the average SNR per bit and compare with the computer simulations in the same plot. Take the average SNR per bit in the range 0 to 24 dB (both inclusive), in steps of 1 dB.
   Solution:



Plot for theoritical and simulation of Symbol Error Rate vs Average SNR per bit for 32-PSK

3. Clearly define the average SNR per bit.
   Solution:
   We know that

$$\text{Average SNR, SNR}_{avg} = \frac{P_{avg}}{2\sigma_w^2}$$

therefore

$$\text{Average SNR per bit, SNR}_{avg,b} = \frac{P_{avg}}{2\kappa\sigma_w^2}$$

Where κ is number of bits required to represent the symbol. Therefore, $\kappa = \log(32) = 5$
Now, average power

$$P_{avg} = \frac{1}{M}\sum_{m=1}^{M} R^2$$

For 32-PSK $P_{avg} = 1$ W. So

$$\text{SNR}_{avg,b} = \frac{1}{10\sigma_w^2}$$

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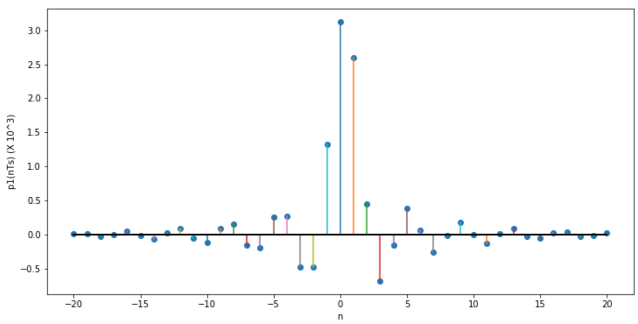
QUESTION 2

Let $p(t)$ denote the time response corresponding to the root-raised cosine frequency spectrum, with $B = 5$ MHz and $\rho = 0.11$.

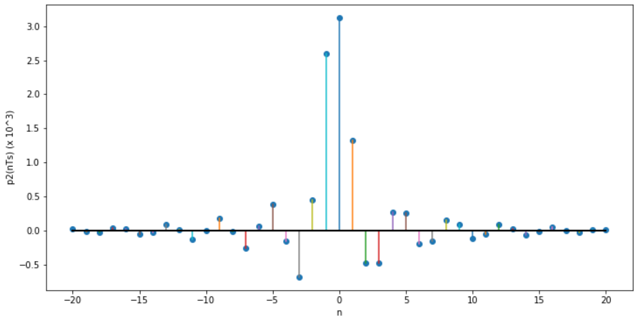
Problems

1. What is the minimum sampling frequency, which is also an integer multiple of the symbol-rate? Denote this sampling frequency as $F_s = \frac{1}{T_s}$.
   Solution:
   We know that $B = 5$ MHz, $\rho = 0.11$ and so $F_1 = B(1-\rho) = 4.45$ MHz. The raised cosine spectrum will be in range $[-2B + F_1, 2B - F_1]$.
   Now, we know that the minimum sampling frequency must be at least twice the bandwidth of a spectrum.So, $F_s \geq 2(2B - F_1) = 11.1$ MHz. Given F_s is an integer multiple of F, and $F = 2B = 10$ MHz, minimum F_s must be 20 MHz.

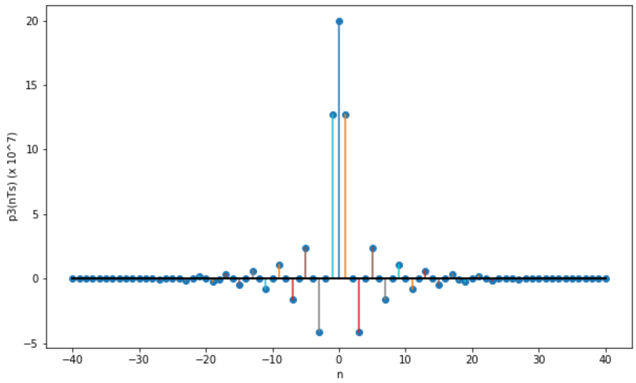
2. Obtain $p_1(nT_s) = p(nT_s - \alpha)$, for $\alpha = 0.3T_s$ and $-20 \leq n \leq 20$, for integer n. Obtain $p_2(nT_s)$ that is matched to $p_1(nT_s)$. Obtain $p_3(nT_s) = p_1(nT_s) \star p_2(nT_s)$. Plot $p_1(nT_s), p_2(nT_s)$ and $p_3(nT_s)$ in three separate figures.
   Solution:
   Note that $p_2(nT_s)$ is the reverse of $p_1(nT_s)$. The figures are below. Yes, it verifies the figure E.4 given in the reference textbook.



Plot for $p_1(nT_s)$ for $\alpha = 0.3T_s$



Plot for $p_2(nT_s)$ for $\alpha = 0.3T_s$

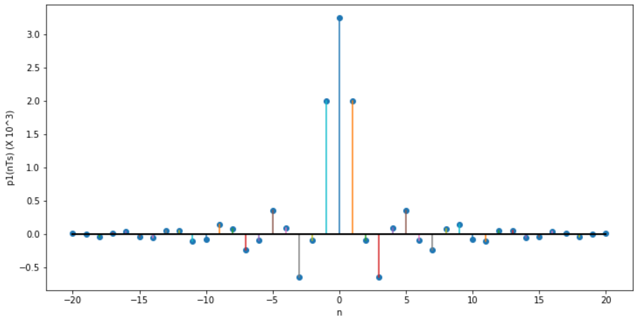


Plot for $p_3(nT_s) = p_1(nT_s) \;\star\; p_2(nT_s)$ for $\alpha = 0.3T_s$

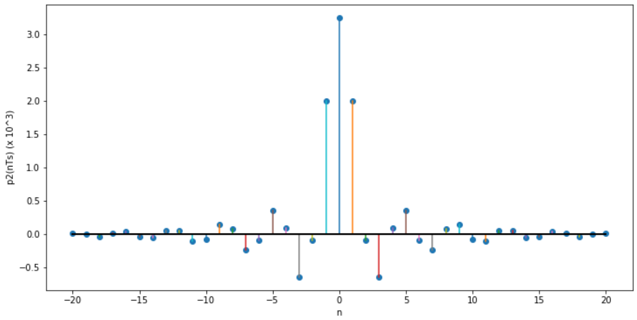
3. Repeat above steps for $\alpha = 0$.
   Solution:
   To obtain for $n = 0$, we have to use L'Hospital Rule. The value is

$$p(0) = \frac{8B\rho + 2\pi B(1-\rho)}{\pi\sqrt{2B}}$$

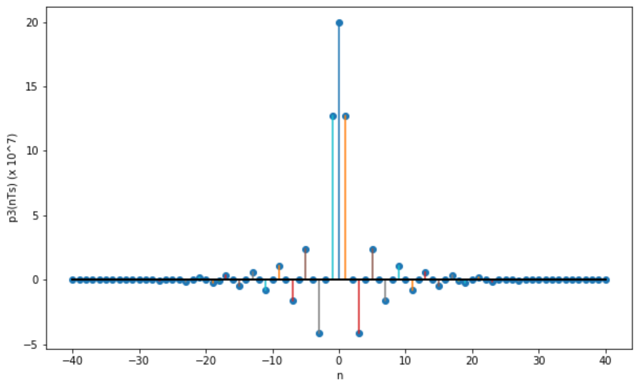
   Plot is below. Yes, it verifies figure E.3 given in reference textbook.



Plot for $p_1(nT_s)$ for $\alpha = 0$



Plot for $p_2(nT_s)$ for $\alpha = 0$



Plot for $p_3(nT_s) = p_1(nT_s) \star p_2(nT_s)$ for $\alpha = 0$