# Gauss-Jordan Matrix Elimination

-This method can be used to solve systems of **linear equations** involving two or more variables. However, the system must be changed to an augmented matrix.

-This method can also be used to find the inverse of a 2x2 matrix or larger matrices, 3x3, 4x4 etc.

**Note**: The matrix must be a square matrix in order to find its inverse.

An **Augmented Matrix** is used to solve a system of linear equations.

System of Equations $\longrightarrow$
$$\begin{aligned} a_1 x + b_1 y + c_1 z &= d_1 \\ a_2 x + b_2 y + c_2 z &= d_2 \\ a_3 x + b_3 y + c_3 z &= d_3 \end{aligned}$$

Augmented Matrix $\longrightarrow$
$$\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

-When given a system of equations, to write in augmented matrix form, the coefficients of each variable must be taken and put in a matrix.

For example, for the following system:

$$\begin{aligned} 3x + 2y - z &= 3 \\ x - y + 2z &= 4 \\ 2x + 3y - z &= 3 \end{aligned}$$

Augmented Matrix $\longrightarrow$
$$\left[\begin{array}{ccc|c} 3 & 2 & -1 & 3 \\ 1 & -1 & 2 & 4 \\ 2 & 3 & -1 & 3 \end{array}\right]$$

-There are three different operations known as **Elementary Row Operations** used when solving or reducing a matrix, using Gauss-Jordan elimination method.

1. Interchanging two rows.
2. Add one row to another row, or multiply one row first and then adding it to another.
3. Multiplying a row by any constant greater than zero.

**Identity Matrix-**is the final result obtained when a matrix is reduced. This matrix consists of ones in the diagonal starting with the first number.

-The numbers in the last column are the answers to the system of equations.

$$\left[\begin{array}{ccc||c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{array}\right] \longleftarrow \text{Identity Matrix for a 3x3}$$

$$\left[\begin{array}{cccc||c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \longleftarrow \text{Identity Matrix for a 4x4}$$

-The pattern continues for bigger matrices.

**Solving a system using Gauss-Jordan**

–The best way to go is to get the ones first in their respective column, and then using that one to get the zeros in that column.

-It is very important to understand that there is no exact procedure to follow when using the Gauss-Jordan method to solve for a system.

$$\begin{aligned} 3x + 2y - z &= 3 \\ x - y + 2z &= 4 \\ 2x + 3y - z &= 3 \end{aligned}$$

***Write as an augmented matrix.***

$$\downarrow$$

$$\left[\begin{array}{ccc||c} 3 & 2 & -1 & 3 \\ 1 & -1 & 2 & 4 \\ 2 & 3 & -1 & 3 \end{array}\right]$$

***Switch row 1 with row 2 to get a 1 in the first column***

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 3 & 2 & -1 & 3 \\ 2 & 3 & -1 & 3 \end{array}\right]$$ ***Multiply row 1 by -3 and add to row 2 to get a zero***

| | | | | |
|---|---|---|---|---|
| Row 1 multiplied by -3 ⟶ | $-3$ | $3$ | $-6$ | $-12$ |
| Row 2 ⟶ | + 3 | 2 | -1 | 3 |
| New Row 2 ⟶ | 0 | 5 | -7 | -9 |

-Put the new row 2 in the matrix, note that though row 1 was multiplied by -3, row 1 didn't change in our matrix.

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 5 & -7 & -9 \\ 2 & 3 & -1 & 3 \end{array}\right]$$

Using a similar procedure of multiplying and adding rows, obtain the following matrix

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 5 & -7 & -9 \\ 2 & 3 & -1 & 3 \end{array}\right]$$ ***Multiply row 1 by -2 and add to row3 as above.***

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 5 & -7 & -9 \\ 0 & 5 & -5 & -5 \end{array}\right]$$ ***Switch rows 2 and 3 to obtain the following***

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 5 & -5 & -5 \\ 0 & 5 & -7 & -9 \end{array}\right]$$ ***Divide the second row by 5 to obtain a 1 in the second row.***

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 5 & -7 & -9 \end{array}\right]$$ ***Add row 2 to row 1***

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 5 & -7 & -9 \end{array}\right]$$ ***Multiply and add like we did earlier*, *-5 * R2+R3***

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & -4 \end{array}\right]$$ ***Divide row 3 by -2 to obtain a 1 in the third row.***

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

-Finally, the matrix can be solved in two different ways:
**A**. Using the *1* in column 3, obtain the other zeros and the solutions.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad x = 1 \quad y = 1 \quad z = 2$$

**B**. Solve by using back substitution.

-The solution to the last row is $z = 2$, the answer can be substituted into the equation produced by the second row. $y - z = -1$ Substituting into this equation, it simplifies to:

$$y - 2 = -1$$
$$y = 1$$

-Again, substituting the answer for z into the first equation will give the answer for x.

$$x + z = 3$$
$$x + 2 = 3$$
$$x = 1$$