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⇒ Poisson's and Laplace's Equation:-

$\nabla \cdot D = \rho_v$

WKT

$E = -\nabla V$

$\nabla(\varepsilon_0 E) = \rho_v$

$\nabla(\varepsilon_0(-\nabla V)) = \rho_v$

$-\nabla^2 V = \frac{\rho_v}{\varepsilon_0}$

$\nabla^2 V = -\frac{\rho_v}{\varepsilon_0}$ — ①

eqn ① is poisson's eqn

For in freespace, $\rho_v = 0$

$\therefore \nabla^2 V = 0$ — ②

Eqn ② is called as Laplace's Equation

Q11 Determine whether or not the following potential fields satisfies Laplace eqn.

a) $V = x^2 - y^2 + z^2$. b) $V = r\cos\phi + z$ c) $V = r\cos\theta + \phi$

Sol:

$\nabla^2 V$ in 3-coordenates

In cartesian

$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$ (cartesian coordinates) in cylindrical c.s

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$$ (cylindrical c.s)

in spherical

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial r}{\partial \theta}\right) +$$

$$\frac{1}{r^2 \sin^2\theta}\left(\frac{\partial^2 V}{\partial \phi^2}\right)$$ (spherical)

a) $V = x^2 - y^2 + z^2$

$\nabla^2 V = 0$

$$\nabla^2 V = \frac{\partial^2 (x^2-y^2+z^2)}{\partial x^2} + \frac{\partial^2 (x^2-y^2+z^2)}{\partial y^2} + \frac{\partial^2 (x^2-y^2+z^2)}{\partial z^2}$$

$= 2 - 2 + 2$

$= \underline{2}$ $\quad \therefore \nabla^2 V \neq 0$

Hence the potential field doesn't satisfy the laplace eqn

b) $V = r\cos\phi + z$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}[r\cos\phi] + \frac{r}{r^2}[-\cos\phi] + 0$$

$$= \frac{\cos\phi}{r} - \frac{\cos\phi}{r}$$ $\quad \therefore \nabla^2 V = 0$

$= \underline{0}$

Hence the given potential field satisfy Laplace eqn

c) $V = r\cos\theta + \phi$

$1-\cos^2\theta$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cos\theta\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin^2\theta\, v\right] + \frac{1}{r^2\sin^2\theta}\left[0\right]$$

$$\nabla^2 V = \frac{2r\cos\theta}{r^2} + \frac{2\cos\theta\left[-\sin\theta\right]}{r\sin\theta}$$

$$= \frac{2\cos\theta}{r} - \frac{\sin 2\theta}{r\sin\theta}$$

$$= \frac{2\cos\theta}{r} - \frac{2\cos\theta}{r} = 0$$

∴ Hence the given potential field satisfy eqn

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⇒ **Uniqueness Theorem**

Under the given boundary conditions laplace eqn has one and only one soln. The same holds good for poissons eqn

Consider laplace eqn

$$\nabla^2 V = 0 \quad -(1)$$

Let there be 2 solns V_1, V_2 which obey the above eqn and both satisfy boundary conditions for the given case

$$\nabla^2 V_1 = 0 \quad -(2) \qquad \nabla^2 V_2 = 0 \quad -(3)$$

Subtraction eqn 3 from (2) $\quad$ (2) − (3)

$$\nabla^2 (V_1 - V_2) = 0$$

$$\nabla^2 f = 0$$

where $f = V_1 - V_2$ is a scalar function

Let the values of 2 solns at the boundary be V_{1b} & V_{2b}

∴ $V_{1b} = V_{2b}$ (since both satisfy boundary conditions)

$$V_{1b} = V_{2b} = V_b$$

$V_b \rightarrow$ potential at boundary

$$V_{1b} - V_{2b} = 0 = f_b = f$$

from vector calculus we have

$$\nabla(f.\vec{A}) = f.(\nabla\vec{A}) + \vec{A}.\nabla f$$

$\therefore$ Let us chose $\vec{A} = \nabla f$

is can be any vector

$$\therefore \nabla(f \cdot \nabla f) = f \cdot (\nabla \cdot \nabla f) + \nabla f \cdot \nabla f$$

Integrate above eqn throughout the volume

$$\int \nabla(f \cdot \nabla f) \cdot dv = \int f \cdot [\nabla \cdot \nabla f] dv + \int (\nabla f)^2 dv \quad - (4)$$

From divergence Theorem

$$\int_V \nabla \vec{A}\, dv = \oint \vec{A} \cdot \vec{ds}$$

$$\int \nabla \cdot \nabla f\, dv = \oint \nabla f \cdot ds$$

Multiply bothsides by f

$$\int \nabla [f \cdot \nabla f]\, dv = \oint f \cdot \nabla f \cdot ds \quad - (5)$$

The RHS of above eqn is an integral over through out surface that surrounds the volume. Obviously this surface involves the boundaries at which we know

$f = f_b = 0$

is Since $\nabla^2 f = 0$ eq.n (4)

$$\int (\nabla f)^2 dV = 0$$

$$\int [\nabla(V_1 - V_2)]^2 dv = 0$$

An integral over certain volume can be zero under 2 condition

(1) If the integral has +ve values at some points & -ve at some other points of the volume. So that the sum could be zero.

(2) If the integrant is zero at every point in volume

Since eqn (6) integrant consists of a square, it can't be -ve at any point which rules out the 1st possibility

$$\therefore [\nabla(V_1 - V_2)]^2 = 0$$

$$\nabla(V_1 - V_2) = 0$$

It is possible only if $(V_1 - V_2)$ is constant

The statement $V_1 - V_2 =$ constant throughout the volume implies that the value it has at the boundary inside volume should be equal.

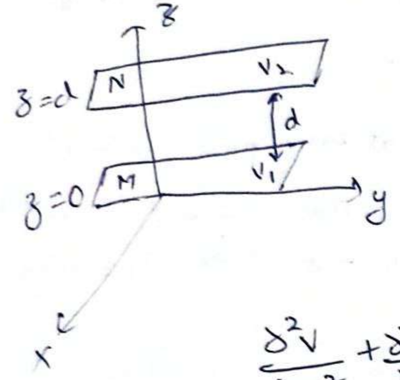
$$\therefore V_1 - V_2 = V_{1b} - V_{2b} = 0$$

$$V_1 = V_2$$

Thus 2 soln are identical and hence the Uniqueness theorem.

Application of laplace eqn:-

1) Two uniformly changed llel planes of infinite extent (case of llel plate capacitor)



Applying laplace eqn to the region b/w two planes

$$\nabla^2 V = 0$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad -①$$

$$\therefore \frac{\partial V}{\partial y} = \frac{\partial V}{\partial x} = 0$$

$$\therefore \frac{\partial^2 V}{\partial z^2} = 0 \quad -②$$

soln of above eqn

$$V = Az + B \quad -③$$

where A & B are constants apply boundary conditions

At $z=0$ $V=V_1$

$$V_1 = A(0) + B$$

$$B = V_1$$

put in eqn ③ $\quad V = Az + V_1 \quad -④$

At $z=d$, $V=V_2$

Substitute in eqn ④

$$V_2 = Ad + V_1$$

$$\frac{V_2 - V_1}{d} = A$$

$$\boxed{V = \left(\frac{V_2 - V_1}{d}\right) z + V_1}$$

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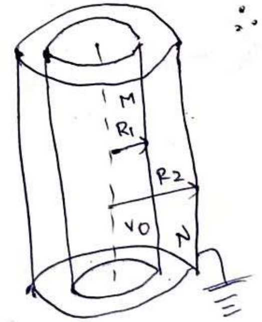
$\therefore$ WKT

$$E = -\nabla V$$

$$= -\left[\cancel{\frac{\partial V}{\partial x}}^{0} \hat{a}_x + \cancel{\frac{\partial V}{\partial y}}^{0} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z\right]$$

$$\vec{E} = -\left[\frac{V_2 - V_1}{d}\right] \hat{a}_z$$

② Two concentric cylinders of infinite length. (coaxial cable)



$$\therefore \nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial V}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} +$$

$$\frac{\partial^2 V}{\partial z^2} = 0$$

Since the field exists only along radial direction

$$\frac{\partial V}{\partial \phi} = \frac{\partial V}{\partial z} = 0$$

$$\therefore \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial V}{\partial r}\right]=0$$

$$\partial\left[r\frac{\partial V}{\partial r}\right]=0$$

$$r\frac{\partial V}{\partial r}=A$$

$$\frac{\partial V}{\partial r}=\frac{A}{r}$$

$$V=A\ln r+B \quad -(1)$$

Apply boundary conditions

at $r = R_1$

$$V=V_0$$

$$V_0 = A\ln R_1 + B \quad -(2)$$

at $r=R_2$, $V=0$ (as it is grounded)

$$0=A\ln R_2+B$$

$$B=-A\ln R_2 \quad -(3)$$

substitute (3) in (2)

$$V_0=A\ln R_1 - A\ln R_2$$

$$V_0 = A\ln\left[\frac{R_1}{R_2}\right]$$

$$A=\frac{V_0}{\ln(R_1/R_2)} \quad -(4)$$

(3) in (1)

$$V = A\ln r - A\ln R_2$$

$$= A\ln\left(\frac{r}{R_2}\right)$$

$$V = V_0\frac{\ln(r/R_2)}{\ln(R_1/R_2)}$$

WKT $E=-\nabla V$

$$E=-\frac{d}{dr}\left[V_0\frac{\ln[R_2/r]}{\ln[R_2/R_1]}\right]$$

$$E=\frac{-V_0}{\ln[R_2/R_1]}\frac{d}{dr}\left[\ln R_2-\ln r\right]$$

$$E=\frac{-V_0}{\ln\left[\frac{R_2}{R_1}\right]}\left[0-\frac{1}{r}\right]=\frac{V_0}{r\ln\left[\frac{R_2}{R_1}\right]}$$

$$\therefore \vec{E}=\frac{V_0}{r\ln\left(\frac{R_2}{R_1}\right)}\hat{a}_r \quad -(5)$$

Also we have

$$\vec{E}=\frac{\rho_L}{2\pi\varepsilon r}\hat{a}_r \quad -(6)$$

Compare (5) & (6)

$$\frac{V_0}{r\ln[R_2/R_1]}=\frac{\rho_L}{2\pi\varepsilon r}$$

$$\frac{V_0}{\ln\left[\frac{R_2}{R_1}\right]} = \frac{Q}{L\, 2\pi\varepsilon}$$

$$C = \frac{Q}{V_0} = \frac{2\pi\varepsilon L}{\ln\left[R_2/R_1\right]}$$

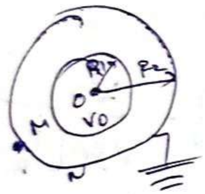
$Q = CV$

$C = Q/V$

capacitance per unit length

$$\frac{C}{L} = \frac{2\pi\varepsilon}{\ln\left[R_2/R_1\right]}$$

(3) Two concentric spheres



$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta \frac{\partial V}{\partial \theta}\right]$$

$$+ \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

$\frac{\partial V}{\partial \theta} = \frac{\partial V}{\partial \phi} = 0$, potential remain constant along θ & ϕ

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right]\right] = 0$$

$$r^2\frac{\partial V}{\partial r} = A$$

$$\therefore \frac{\partial V}{\partial r} = \frac{A}{r^2}$$

$$V = \frac{-A}{r} + B \quad - (1)$$

Apply boundary conditions

at $r = R_2$, $V = 0$

$$0 = \frac{-A}{R_2} + B$$

$$B = A/R_2 \quad - (2)$$

at $r = R_1$, $V = V_0$

$$\therefore V_0 = \frac{-A}{R_1} + B$$

$$V_0 = \frac{-A}{R_1} + \frac{A}{R_2}$$

$$V_0 = A\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$$

$$\therefore A = \frac{V_0}{\left[\frac{1}{R_2} - \frac{1}{R_1}\right]} \quad - (3)$$

substitute (2) in (1)

$$V = -\frac{A}{r} + \frac{A}{R_2}$$

$$V = A\left[\frac{-1}{r} + \frac{1}{R_2}\right]$$

$$V = \frac{V_0}{\frac{1}{R_2} - \frac{1}{R_1}}\left[\frac{1}{R_2} - \frac{1}{r}\right]$$

$$V = \frac{V_0\left[\frac{1}{r} - \frac{1}{R_2}\right]}{\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

$$E = -\nabla V$$

$$= -\frac{\partial}{\partial r}\left[\frac{V_0\left[\frac{1}{r} - \frac{1}{R_2}\right]}{\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}\right]$$

$$= \frac{-V_0}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}\left[-\frac{1}{r^2}\right]$$

$$\boxed{\vec{E} = \frac{1}{r^2}\frac{V_0}{\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}\hat{a}_r} \rightarrow (4)$$

Also we have

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r \quad - (5)$$

$$\frac{Q}{4\pi\varepsilon_0 r^2} = \frac{1}{r^2}\frac{V_0}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$$\therefore C = \frac{Q}{V_0} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

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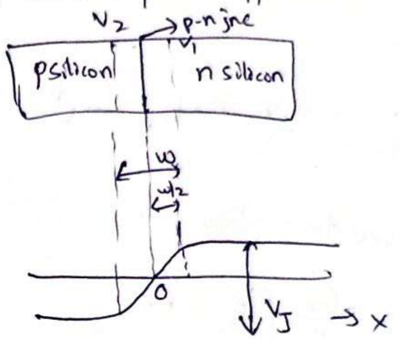
Application of Poisson's Eqn

(i) Solution of poisson's eqn as applicable to p-n jnc.



→ Exactly at junction, i.e at $x=0$, $V=0$ — (1)

→ At $x = w/2$, $\frac{dV}{dx} = 0$ — (2)

→ $\frac{dV}{dy} = \frac{dV}{dz} = 0$ all over the material → (3)

From poisson's eqn

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

$$\frac{\partial^2 V}{dx^2} + \cancelto{0}{\frac{\partial^2 V}{dy^2}} + \cancelto{0}{\frac{\partial^2 V}{dz^2}} = -\frac{\rho_v}{\varepsilon}$$

from condition (3)

$$\frac{\partial^2 V}{dx^2} = \frac{-\rho_v}{\varepsilon} \quad \text{(partial diff)}$$

Since we are differentiating only x

∴ It is complete differential

$$\frac{d^2V}{dx^2} = \frac{-\rho_v}{\varepsilon}$$

$$\frac{d}{dx}\left(\frac{dV}{dx}\right) = \frac{-\rho_v}{\varepsilon}$$

$$d\left(\frac{dV}{dx}\right) = \frac{-\rho_v}{\varepsilon}\,dx$$

Integrate

$$\frac{dV}{dx} = \frac{-\rho_v}{\varepsilon}x + k_1 \quad -① \qquad k_1 \rightarrow \text{integration constant}$$

From condition ②

$$0 = \frac{-\rho_v}{\varepsilon}\left(\frac{W}{2}\right) + k_1$$

$$k_1 = \frac{\rho_v}{\varepsilon}\frac{w}{2} \quad -②$$

substitute k_1 in eqn ①

$$\frac{dV}{dx} = \frac{-\rho_v}{\varepsilon}x + \frac{\rho_v}{\varepsilon}\left(\frac{w}{2}\right)$$

$$dV = \cancel{8} - \frac{\rho_v}{\varepsilon}x\,dx + \frac{\rho_v(\omega)}{2\varepsilon}dx$$

Integrate

$$V = \frac{-\rho_v}{\varepsilon}\frac{x^2}{2} + \frac{\rho_v(\omega)}{2\varepsilon}x + k_2 \quad -③$$

condition ① $\quad V=0 \quad x=0$

$$0 = k_2$$

substitute in ③

$$V = \frac{\rho_v \omega x}{2\varepsilon} - \frac{\rho_v x^2}{2\varepsilon}$$

$$\boxed{\therefore V = \frac{\rho_v x}{2\varepsilon}(\omega - x)}$$

Let the potential at $x = \frac{\omega}{2}$ be V_1

$$V_1 = \frac{\rho_v \omega}{2\varepsilon}\left(\frac{\omega}{2}\right) - \frac{\rho_v}{2\varepsilon}\left(\frac{\omega}{2}\right)^2$$

$$V_1 = \frac{\rho_v w^2}{4\varepsilon} - \frac{\rho_v w^2}{8\varepsilon}$$

$$\boxed{V_1 = \frac{\rho_v \cdot w^2}{8\varepsilon}}$$

Let the potential at $x = \frac{-\omega}{2}$ be V_2

$$V_2 = \frac{\rho_v \omega}{2\varepsilon}\left(\frac{-\omega}{2}\right) - \frac{\rho_v}{2\varepsilon}\left(\frac{\omega}{2}\right)^2$$

$$V_2 = \frac{-\rho_v w^2}{4\varepsilon} - \frac{\rho_v w^2}{8\varepsilon}$$

$$\boxed{V_2 = \frac{-3\rho_v w^2}{8\varepsilon}}$$

$\therefore$ The junction potential V_J

$$V_J = V_1 - V_2$$

$$= \frac{\rho_v w^2}{8\varepsilon} + \frac{3\rho_v w^2}{8\varepsilon}$$

$$= \frac{1}{2}\frac{\rho_v w^2}{5}$$

$$\boxed{V_J = \frac{\rho_v w^2}{2\varepsilon}}$$

(2) electric field across the junction

$$E = -\nabla V$$

$$= -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z \right]$$

from condition (3)

$$E = -\frac{\partial V}{\partial x}\hat{a}_x$$

$$V = \frac{\rho_v w x}{2\varepsilon} - \frac{\rho_v x^2}{2\varepsilon}$$

$$E = -\left[\frac{d}{dx}\left\{ \frac{\rho_v w x}{2\varepsilon} - \frac{\rho_v x^2}{2\varepsilon} \right\}\right]\hat{a}_x$$

$$E = -\left[\frac{\rho_v w}{2\varepsilon} - \frac{2\rho_v x}{2\varepsilon} \right]\hat{a}_x$$

$$\boxed{E = \frac{\rho_v}{2\varepsilon}\left[x - \frac{w}{2} \right]\hat{a}_x}$$

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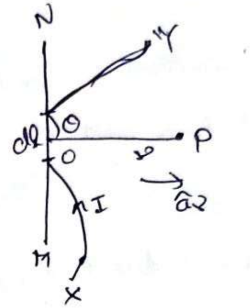
Module-3

Biot - Savart's Law



statement: The magnitude of the magnetic field intensity $\vec{dH}$ at P due to current in the differential element is directly proportional to the product of the current I, the magnitude of the length of the differential element $d\ell$, and the sine of the angle b/w the tangent drawn to the element and the line joining the point P and the element and it is inversely proportional to the square of the distance b/w the point and the element.

$$\therefore dH \propto \frac{I d\ell \sin\theta}{r^2}$$

$$dH = \frac{K I d\ell \sin\theta}{r^2}$$

where K is the proportionality constant it is given by

$$K = \frac{1}{4\pi}$$

$$dH = \frac{I d\ell \sin\theta}{4\pi r^2}$$

The direction of $\vec{dH}$ at P as per the law is $\perp$ on to the plane containing the tangent drawn to the element & the line joining the point and the element.

It is expressed in vector notation as

$$d\vec{H} = \frac{I\vec{dl} \times \hat{a}_r}{4\pi r^2}$$

where $\vec{dl}$ is directed along the direction of current and $\hat{a}_r$ is the unit vector directed from point 'r' towards 'P'.

Q) Find the magnetic field at $A(2,3,-2)$ due to a current element $I\vec{dl} = \pi(0.5\hat{a}_x - 0.6\hat{a}_y + 0.8\hat{a}_z)\,\mu Am$ situated at point $B(3,-2,4)$.

Sol:- $A(2,3,-2)$ $B(3,-2,4)$

$I\vec{dl} = \pi(0.5\hat{a}_x - 0.6\hat{a}_y + 0.8\hat{a}_z)\,\mu Am$

$$dl \quad B(3,-2,4) \longrightarrow A(2,3,-2) \qquad \hat{a}_r = \frac{\vec{r}}{|\vec{r}|}$$

$\vec{r} = -\hat{a}_x + 5\hat{a}_y - 6\hat{a}_z$ $\qquad |\vec{r}| = 7.87 = \sqrt{62}$

$$\hat{a}_r = \frac{-\hat{a}_x + 5\hat{a}_y - 6\hat{a}_z}{\sqrt{62}}$$

$\hat{a}_r = -0.127\hat{a}_x + 0.63\hat{a}_y - 0.76\hat{a}_z$

$$I\vec{dl} \times \hat{a}_r = \pi \times 10^{-6}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0.5 & -0.6 & 0.8 \\ -0.127 & 0.63 & -0.76 \end{vmatrix}$$

$= \pi \times 10^{-6}\{-0.048\hat{a}_x + 0.278\hat{a}_y + 0.23\hat{a}_z\}$

$$d\vec{H} = \frac{\pi \times 10^{-6}[-0.048\hat{a}_x + 0.278\hat{a}_y + 0.238\hat{a}_z]}{4\pi(62)}$$

$d\vec{H} = 10^{-6}[\,.$

$|d\vec{H}| = 0.004\,\mu Am$

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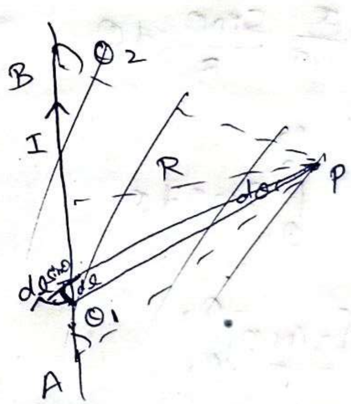
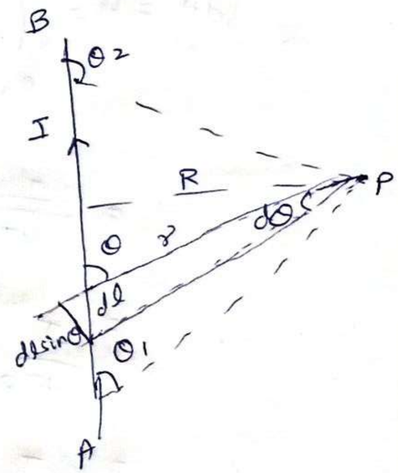
Applications of Biot Savart's Law:-

i) Magnetic field intensity at a point due to a current in a straight conductor of finite length:-





From Biot Savart's law

$$d\vec{H} = \frac{I\vec{dl} \times \hat{a}_r}{4\pi r^2}$$

Its magnitude

$$dH = \frac{I\,dl\sin\theta}{4\pi r^2}$$

From the figure

$$dl \sin\theta = r\,d\theta$$

$$\therefore dH = \frac{I\,r\,d\theta}{4\pi r^2} = \frac{I}{4\pi r}\,d\theta$$

from fig

$$\sin\theta = \frac{R}{r}$$

$$\therefore \frac{1}{r} = \frac{\sin\theta}{R}$$

$$\therefore dH = \frac{I}{4\pi}\frac{\sin\theta}{R}\,d\theta$$

$\therefore$ Total field at Point 'p' due to entire ~~conductor~~ conductor

$$\int dH = H = \int_{\theta_1}^{\theta_2} \frac{I}{4\pi}\frac{\sin\theta}{R}\,d\theta$$

$$= \frac{I}{4\pi R}\int_{\theta_1}^{\theta_2} \sin\theta\,d\theta$$

$$= \frac{I}{4\pi R}\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= \frac{I}{4\pi R}\left[\cos\theta_1 - \cos\theta_2\right)$$

2) Magnetic field intensity at a point due to a current in an infinitely long length along straight conductor:-

Same as prev diagram

for infinite length $\theta_1 = 0$ & $\theta_2 = \pi$

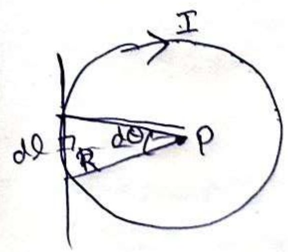
$$H = \frac{I}{4\pi R}\left[\cos\theta_1 - \cos\theta_2\right)$$

$$= \frac{I}{4\pi R}\left[\cos 0 - \cos\pi\right]$$

$$= \frac{I}{4\pi R}(2)$$

$$H = \frac{I}{2\pi R}$$

3) Magnetic field intensity at the centre of circular current loop :-



From Biot-Savart's law

$$d\vec{H} = \frac{I\,d\vec{l} \times \hat{a}_r}{4\pi r^2}$$

$$dH = \frac{I\,dl \sin\theta}{4\pi R^2}$$

from the fig $\theta = 90°$ & $dl = R\,d\theta$

$$\therefore dH = \frac{I \times R\,d\theta \sin 90°}{4\pi R^2}$$

$$dH = \frac{I\,d\theta}{4\pi R}$$

$$H = \frac{I}{4\pi R}\int d\theta$$

$$H = \frac{I}{4\pi R}(2\pi)$$

$$H = \frac{I}{2R}$$

$$\boxed{\vec{H} = \frac{I}{2R}\hat{a}_\phi}$$

⇒ **Ampere's Circuital Laws**

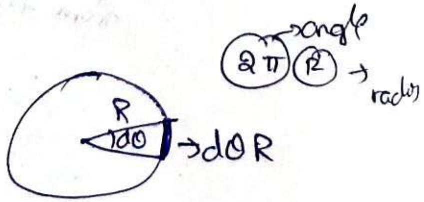
Statement:- Ampere's law states that line integral of $\vec{H}$ about any closed path is equal to the current enclosed by the path. Its representation as integration

$$\oint \vec{H}\cdot\vec{dl} = I$$

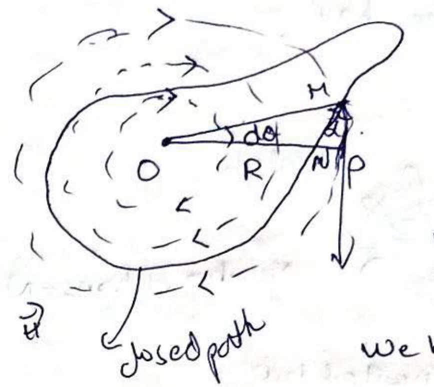
or

$$\oint \frac{\vec{B}}{\mu}\vec{dl} = I$$

$$\oint \vec{B}\cdot\vec{dl} = \mu I$$



$MN = dl$



from fig

$MP = dl\cos\alpha$

&

$Rd\theta = MP$

where 'R' is the distance of element from 'O',

We know that magnetic flux density due to a current carrying straight conductor

$$B = \frac{\mu I}{2\pi R}$$

∴ The flux through the current element $= \vec{B}\cdot\vec{dl}$

The flux through the entire closed path $= \oint\vec{B}\cdot\vec{dl}$

$$\therefore \oint\vec{B}\cdot\vec{dl} = \oint B\,dl\cos\alpha$$

$$\oint\vec{B}\cdot\vec{dl} = \oint BR\,d\theta$$

$$= \oint \frac{\mu I}{2\pi R}R\,d\theta$$

$$= \frac{\mu I}{2\pi}\int d\theta$$

$$\oint\vec{B}\cdot\vec{dl} = \frac{\mu I}{2\pi}(2\pi)$$

$$\boxed{\oint\vec{B}\cdot\vec{dl} = \mu I}$$

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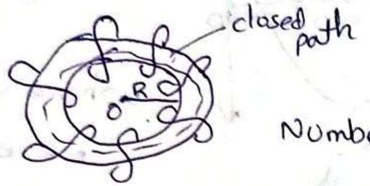
Application of Ampere's Law:-

① Toroidal coil:-



Number of turns = N

From Ampere's Circuital law

$\oint \vec{H}\cdot\vec{dl} = N \cdot I$

$H \oint \vec{dl} = NI$

Since H remains constant

$H(2\pi R) = NI$ — closed path is a circle of radius R

$$\boxed{H = \frac{NI}{2\pi R}}$$

② Straight solid cylindrical conductor:-

closed path inside the conductor

From Ampere's Circuital Law

$\oint \vec{H}\cdot\vec{dl} = I$

→ The current density through the cross section $= \frac{I}{\pi R^2} = \frac{\text{Current}}{\text{area}}$

③ Coaxial cube (refer notes)

→ Area of cross section enclosed by the closed path $= \pi r^2$

→ Current enclosed by the closed path

$= \frac{I}{\pi R^2} \times \pi r^2$

$= \frac{I r^2}{R^2}$

From Ampere's Law

$\oint \vec{H}\cdot\vec{dl} = I = \frac{I r^2}{R^2}$

$H \oint \vec{dl} = \frac{I r^2}{R^2}$

$H(2\pi r) = \frac{I r^2}{R^2}$

$H = \frac{I r}{2\pi R^2}$

② If consider H at the surface i.e $r = R$

$H = \frac{IR}{2\pi R^2}$

$$\boxed{H = \frac{I}{2\pi R}}$$

③ Consider H outside the surface i.e $r > R$

From Ampere's Law

$\oint \vec{H}\cdot\vec{dl} = I$

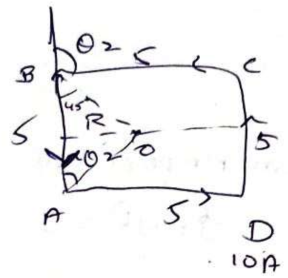
$H \oint \vec{dl} = I$

$H(2\pi r) = I$

$$\boxed{H = \frac{I}{2\pi r}}$$

1. Find the magnetic field intensity at the centre of the square of sides equal to 5 m and carrying a current of 10A

Sol: $I = 10A$



$R = 2.5$

$\tan\theta_1 = \frac{2.5}{2.5} = 1$

$\theta = 45°$

$\theta_2 = 180° - 45 = 135°$ $\boxed{\theta_2 = 135°}$

$\vec{H} = \frac{I}{4\pi R}[\cos\theta_1 - \cos\theta_2]$

$\vec{H} = \frac{10}{4\pi(2.5)}(\cos 45° - \cos 135°)$

$\vec{H} = \frac{10}{4\pi(2.5)}[1.714]$

$\boxed{\vec{H} = 0.45 \text{ A/m}}$ considering one side

(For 4 sides): The field strength at center 'o' due to all 4 sides of square

$\vec{H} = 4(0.45)$

$\boxed{\vec{H} = 1.8 \text{ A/m}}$

2. An air cored toroid having a nonsectional area of $6cm^2$ and mean radius 15cm is bound uniformly with 500 turns carrying current of 4A. Determine the magnetic flux density and field intensity of the toroid.

Sol: $H = \frac{NI}{2\pi R}$

Given

$N = 500$, R = Mean Radius = 0.15 m

$I = 4$

$H = \frac{(500)(4)}{2\pi \times (0.15)}$

$H = 2121.79$ A/m

$B = \mu H$

$B = \mu_0 \mu_r H$

$B = 4\pi \times 10^{-7} \times 1 \times 2121.79$

$B = 2.6 \times 10^{-3}$

$B = 2.6$ m Tesla or A/m²

⇒ Magnetic forces:

A positive charge 'q' moving with a velocity 'v' in a uniform magnetic field of flux density $\vec{B}$ experiences force $\vec{F_1}$ given by

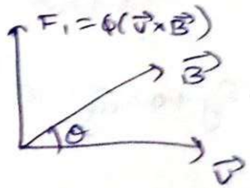
$\vec{F_1} = Q(\vec{v} \times \vec{B}) \rightarrow (1)$

It's magnitude is given by

$|\vec{F_1}| = QvB\sin\theta$ (angle b/w v & B)

velocity = v

B - flux density



$\theta \rightarrow$ is angle b/w velocity vector 'v' and flux density B

The direction of force is ⊥er to plane containing V & B

If the charge is subjected to only the influence of an electric field of strength F, then the force F_2 is

$$\boxed{F_2 = Q_2\vec{E}} \quad (2)$$

If it is subjected to the combined influence of a magnetic field of flux density $\vec{B}$ and electric field of strength $\vec{E}$ then the resulted force vector $\vec{F}$ will be the sum of 2 forces $\vec{F_1}$ & $\vec{F_2}$

$$F = Q(\vec{E} + \vec{V} \times \vec{B})$$

↳ Lorentz force eqn

(1) A point charge $q = 18nC$ has a velocity of $V = 5\times10^6$ m/s in direction $\vec{a} = 0.6\hat{a}_x + 0.75\hat{a}_y + 0.3\hat{a}_z$. Calculate the magnitude of the force exerted on charge by the field $\vec{B} = -3\hat{a}_x + 4\hat{a}_y + 6\hat{a}_z$ mT

Sol:

$\vec{B} = -3\hat{a}_x + 4\hat{a}_y + 6\hat{a}_z$

$V = 5\times10^6$ m/s

$Q = 18nC$

$\vec{a} = 0.6\hat{a}_x + 0.75\hat{a}_y + 0.3\hat{a}_z$

$\vec{F} = Q(\vec{V}\times\vec{B})$

$$\vec{V}\times\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0.6 & 0.75 & 0.3 \\ -3 & 4 & 6 \end{vmatrix} \times 5\times10^6$$

$$\vec{V}\times\vec{B} = (3.3\hat{a}_x - 4.5\hat{a}_y + 4.65\hat{a}_z) \times 5\times10^6$$

$$\vec{F} = 18\times10^{-9} \times 5\times10^6 \left[3.3\hat{a}_x - 4.5\hat{a}_y + 4.65\hat{a}_z\right]$$

$$\vec{F} = 90\times10^{-3}\left[3.3\hat{a}_x - 4.5\hat{a}_y + 4.65\hat{a}_z\right]$$

$$|\vec{F}| = \sqrt{(0.297)^2 + (0.405)^2 + (0.4185)^2} \times10^{-3}$$

$$|\vec{F}| = 0.65374\times10^{-3}$$

$$|\vec{F}| = 653.7\ \mu N$$

(2) $Q = 18nC$, $V = 5\times10^6$ m/s

$\hat{a} = 0.6\hat{a}_x + 0.75\hat{a}_y + 0.3\hat{a}_z$

$\vec{B} = -3\hat{a}_x + 4\hat{a}_y + 4\hat{a}_z$ mT

Sol

$$\vec{V}\times\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0.6 & 0.75 & 0.3 \\ -3 & 4 & 4 \end{vmatrix} \; 5\times10^6 \times10^{-3}$$

$$= (1.8\hat{a}_x - 3.3\hat{a}_y + 4.65\hat{a}_y)5\times10^3$$

$$\vec{F} = 90\times10^{-6}\left\{1.8\hat{a}_x - 3.3\hat{a}_y + 4.65\hat{a}_z\right\}$$

$$\vec{F} = \left\{162\hat{a}_x - 297\hat{a}_y + 418.5\hat{a}_z\right\}\times10^{-6}$$

$$|\vec{F}| = \sqrt{(16)^2 + (297)^2 + (418.5)^2} \times10^{-6}$$

$$F = 538.14\times10^{-6}\ N$$

$$\boxed{F = 538.14\ \mu N}$$