Unit-1:

a. COULOMB'S LAW AND ELECTRIC FIELD INTENSITY

Experimental law of Coulomb, Electric field intensity, Field due to continuous volume charge distribution, Field of a line charge. 3hours

INTRODUCTION:

Electrostatics is a very important step in the study of engineering electromagnetic. Electrostatics is sciences related to the electric charges which are static i.e are at rest. An electric charge has its effect in a region or space around it. This region is called an **electric field** of that charge. Such an electric field produced due to stationary electric charge does not vary with time. It is time invariant and called **static electric field**. The study of such time invariant electric fields in a space or vacuum, produced by various types of static charge distributions is called **electrostatics**.

APPLICATIONS OF ELECTROSTATICS:

- A very common example of such a field is used in cathode ray tube for focusing and deflecting a beam.
- Most computer peripheral devices like keyboards, touch pads, liquid crystal displays etc. work on the principle of electrostatics.
- A variety of machines such as X-ray machine and medical instruments used for electrocardiograms, scanning etc. use the principle of electrostatics.
- Many industrial processes like spray painting, electro deposition etc.
- Agricultural activities like sorting seeds, spraying to plants etc.
- Components such as resistors, capacitors etc.
- Devices such as BJTs, FETs function based on electrostatics.

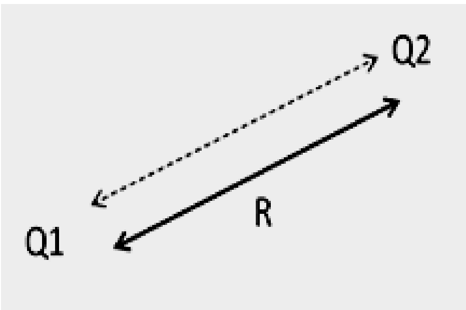
COULOMB'S LAW:

The study of electrostatics starts with the study of the results of the experiments performed by an engineer from the French army engineers, **Colonel Charles Coulomb**. The experiments are related to the force exerted between the two point charges, which are placed near each others. The force exerted due to the electric fields produced by the point charges.

A **point charge** means that electric charge which is spreaded on a surface or space whose geometrical dimensions are very small compared to the other dimensions, in which the effect of its electric field is to be studied. Thus a point charge has a location but not the dimensions.

Statement: Coulomb's law states that "The force of attraction or repulsion between two point charges Q_1 and Q_2 is,

a. Along the line joining them
b. Directly proportional to the product of charges Q1 and Q2
c. Inversely proportional to the square of distance between the two point charges
d. Depends on the medium in which they are located.



$$F \propto Q1\, Q2 \quad \left[\text{where Q1 and Q2 be two point charges in coulomb's}\right]$$

$$F \propto \frac{1}{R^2} \quad \left[\text{where } R \text{ be the distance between the 2 point charges}\right]$$

$$F = k\frac{Q1Q2}{R^2} \quad [where\ k\ is\ proportionality\ constant]$$

The force is of attraction type if charges Q1 and Q2 are of opposite polarity and repulsive type if charges are of same polarity.

If the SI units are used, Q is measured in coulombs (C), R is measured in meters (m) and force should be in Newton's (N). This will be achieved if the constant of proportionality k is written as

$$k = \frac{1}{4\pi\varepsilon} \quad \varepsilon - absolute\ permittivity\ of\ the\ medium$$

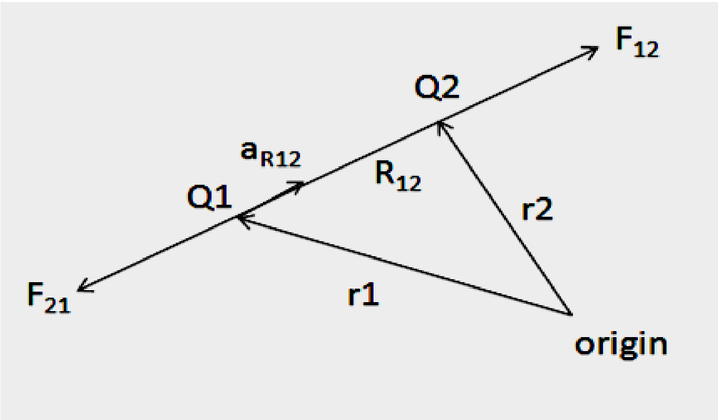
$$F = \frac{Q1Q2}{4\pi\varepsilon R^2} \qquad \varepsilon = \varepsilon_0\varepsilon_r$$

$$where\ \varepsilon_0 - permittivity\ of\ free\ space = 8.854 \times 10^{-12}\ \text{F/m}$$

$$\varepsilon_r - relative\ permittivity = 1\ for\ air\ or\ vacuum$$

$$F = \frac{Q1Q2}{4\pi\varepsilon_0 R^2} N = 9 \times 10^9 \frac{Q1Q2}{R^2} N \qquad \text{---------------} (1)$$

Vector form of Coulomb's law:



In order to write the vector form of equation (1), we need the additional fact that the force acts along the line joining the two charges and is repulsive if the charges are alike in sign and attractive if they are of opposite sign. Let the vector r1 locate Q1 while r2 locate Q2. Then the vector R_{12} =r2-r1 represents the directed line segment from Q1 to Q2 as in fig. The vector F_{12} is the force on Q2 and is shown for the case where Q1 and Q2 have the same sign. The vector form of coulomb's law is

$$F_{12} = \frac{Q1Q2}{4\pi\varepsilon_0 |R_{12}|^2} a_{R_{12}}$$

Where a_{R12}=a unit vector is the direction of R12, as

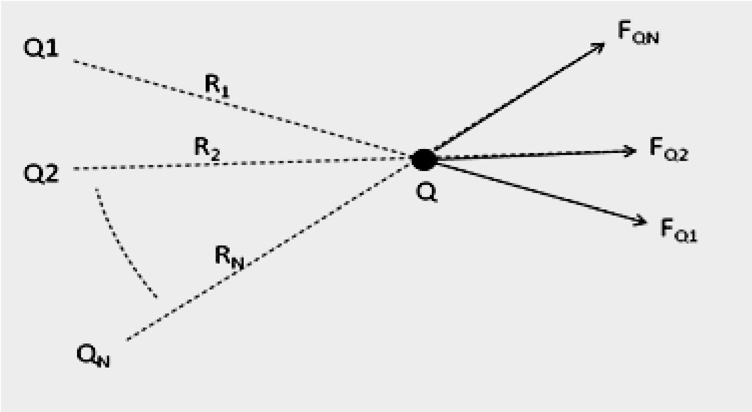
$$a_{R_{12}} = \frac{\overrightarrow{R_{12}}}{|\overrightarrow{R_{12}}|} = \frac{r2 - r1}{|r2 - r1|}$$

$$F_{12} = \frac{Q1Q2}{4\pi\varepsilon_0 |R_{12}|^3} \overrightarrow{R_{12}}$$

The force F_{21} on Q1 due to Q2 is given by

$F_{21} = -F_{12}$ bcz $a_{R12} = -a_{R21}$

Force due to several charges:



The superposition of forces due to Q1, Q2, ______________ Q_N on a charge Q is

$$F = F_1\widehat{a_{R1}} + F_2\widehat{a_{R2}} + ----------- + F_N\widehat{a_{R_N}}$$

$$F = \frac{QQ1}{4\pi\varepsilon_0|R_1|^3}\overrightarrow{R_1} + \frac{QQ2}{4\pi\varepsilon_0|R_2|^3}\overrightarrow{R_2} + -------- + \frac{QQ_N}{4\pi\varepsilon_0|R_N|^3}\overrightarrow{R_N}$$

$$F = \frac{Q}{4\pi\varepsilon_0}\left[\frac{Q1}{|R_1|^3}\overrightarrow{R_1} + \frac{Q2}{|R_2|^3}\overrightarrow{R_2} + ----------- + \frac{Q_N}{|R_N|^3}\overrightarrow{R_N}\right]$$

$$F = \frac{Q}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{Q_i}{|R_i|^3}\overrightarrow{R_i}$$

If $Q_1 = Q_2 =$ ------------ $= Q_N = Q_1$ then

$$F = \frac{QQ1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{\overrightarrow{R_i}}{|R_i|^3}$$

ELECTRIC FIELD INTENSITY:

Definition: Electric field Intensity, at any point in an electric field is the force experienced by a unit positive charge placed at that point. Its unit is Newtons/coulomb and represented by E.

If we now consider one charge fixed in position, say Q1 and move a second charge slowly around, we note that there exists everywhere a force on this second charge, in other words, this second charge is displaying the existence of a force ***field***. Call this second charge a test charge Q_t. The force on it is given by coulomb's law,

$$F_t = \frac{Q_1Q_t}{4\pi\varepsilon_0|R_{1t}|^2}\widehat{a_{1t}}$$

Writing this force as a force per unit charge gives

$$\frac{F_t}{Q_t} = \frac{Q_1}{4\pi\varepsilon_0|R_{1t}|^2}\widehat{a_{1t}}$$

The quantity on the right side of the above equation is a function only of Q1 and the directed line segment from Q1 to the position of the test charge. This describes a vector field and is called the **electric field intensity.**

We define the electric field intensity as the vector force on a unit positive test charge. We would not measure it experimentally by finding the force on a 1-C test charge, however for this would probably cause such a force on Q1 as to change the position of that charge.

Electric field intensity must be measured by the unit N/C. But, electric potential has units J/C i.e. (Nm/C). Hence E is also measured in units V/m (volts/meter). Using a capital letter E for electric field intensity we have finally

$$F = \frac{F_t}{Q_t} \quad ------(1)$$

$$E = \frac{Q1}{4\pi\varepsilon_0 |R_{1t}|^2}\widehat{a_{1t}} \quad -------(2)$$

Equation (1) is the defining expression for electric field intensity and equation (2) is the expression for the electric field intensity due to a single point charge Q1 in a vacuum. Let us dispense with most of the subscripts in equation (2),

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}\widehat{a_R} \quad ---------(3)$$

Let us arbitrarily locate Q1 at the centre of a spherical coordinate system. The unit vector a_R then becomes the radial unit vector a_r, and R is r. Hence,

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}\widehat{a_r} \quad ----------(4)$$

Note: Here unit positive charge doesn't mean that a test charge which is small enough so as not to disturb original charge configuration

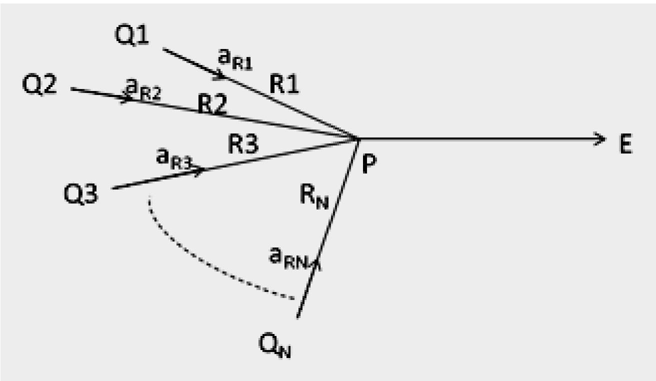
$$\vec{E} = \lim_{\Delta Q \to 0} \frac{\Delta F}{\Delta Q}$$

$$E = \frac{F}{Q} \qquad or \qquad F = EQ$$

<u>Electric field Intensity at a point due to 'n' number of charges:</u>

If we have more than one point charge producing the field, then the principle of superposition is valid for electric forces, the electric field intensity can be found by employing equation (4) computing electric field due to each charge and finally adding the individual contributions. Assume that we have n point charges Q1, Q2, ________, Qn. The electric field intensity at a point that is at distances r1, r2, _________, r_n from the charges is then given by

$$E = \sum_{i=1}^{n} \frac{1}{4\pi\varepsilon} \frac{Q_i}{{r_i}^2}\widehat{a_{r_i}} \; V/m$$



The electric field intensity at a point 'P' due to n-number of charges Q1, Q2, Q3, _______, Q_n is given by the vector sum of electric field Intensities due to individual charges i.e.,

$$E = E_1 + E_2 + E_3 + ---------- + E_n$$

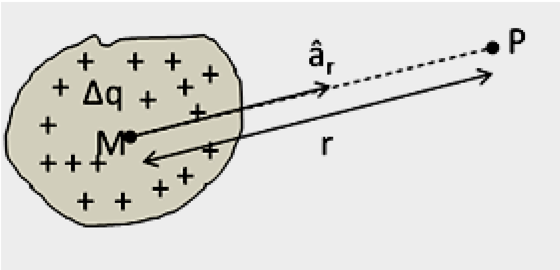
$$E = \frac{Q_1}{4\pi\varepsilon_0|R_1|^2}R_1 + \frac{Q_2}{4\pi\varepsilon_0|R_2|^2}R_2 + \frac{Q_3}{4\pi\varepsilon_0|R_3|^2}R_3 + ------ + \frac{Q_n}{4\pi\varepsilon_0|R_n|^2}R_n \ V/m$$

$$E = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{Q_i}{|R_i|^2}R_i \quad V/m$$

Field due to continuous distribution of charges:

a. Electric field Intensity at a point due to infinite number of point charges:

When the number of charges is infinite in a space of finite dimensions the distance of separation between the individual charges tend to become zero. Then it amounts to continuous distribution of charges.



Let us consider such a distribution of charges where in the number of charges is infinite. Let P be the point at which the electric field intensity is to be determined. Consider an elementary space which comprises of a very small charge Δq of the charge distribution at M from where the distance of P is r. Compared to r, the dimensions of the elementary space is negligible because of which Δq can be approximated to be a point charge.

Electric field intensity at P due to the charge Δq is

$$\overrightarrow{\Delta E} = \frac{\Delta q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Where $\hat{r}$ is the unit vector directed from M to P. Since we are treating Δq to be equivalent to a point charge, we must consider the limit in which spatial dimensions over which the elementary charge is spread as tending to zero. In the same limit $\Delta q \rightarrow dq$ at which time we can say that the field intensity $\overrightarrow{dq}$ at P due dq as,

$$\overrightarrow{dq} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

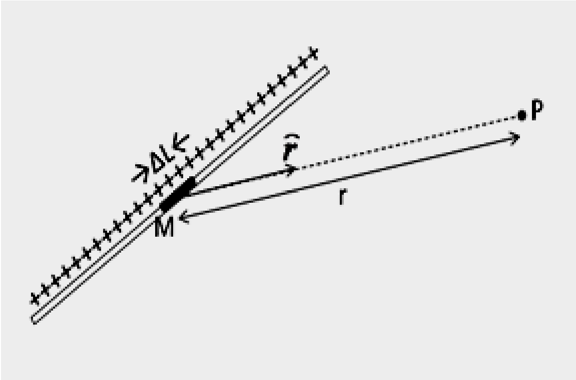
The electric field intensity at P due to the entire charge distribution i.e. due to all the infinite number of charges is

$$\vec{E} = \int \overrightarrow{dE} = \int \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

Note: It is because that the charge distribution is continuous, $\vec{E}$ can be obtained by integrating $\overrightarrow{dE}$.

b. Electric field intensity at a point due to line charge distribution:

If the charge distribution is such that the charges are aligned continuously along a line, then it is referred to as line charge.



For a line charge, a line charge density ρ_L could be defined as the charge/unit length.

E.g.: a charged wire of uniform cross section serves as a line charge.

If Δq is the charge over a length ΔL, then ρ_L can be defined in the limits.

$$\rho_L = \lim_{\Delta L \to 0}\left[\frac{\Delta q}{\Delta L}\right] \qquad or \qquad \rho_L = \frac{dq}{dL} \quad -----------(1)$$

In the limits when ΔL→0, it becomes a point where the charge present is dq. In other words, dq is a point charge. Thus the electric field intensity at P due to entire line charge can be written as,

$$\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\hat{r}$$

From equation (1),

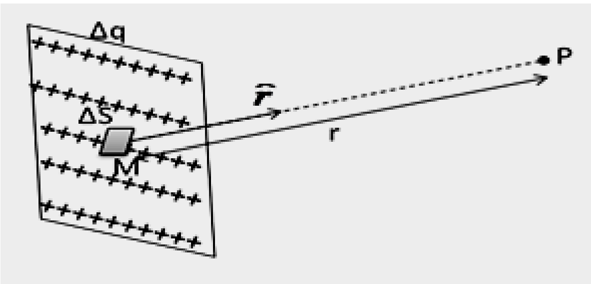
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_L . dL}{r^2}\hat{r}$$

If the charges are distributed uniformly on the length then, ρ_L becomes a constant.

$$\therefore \qquad \vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int \frac{dL}{r^2}\hat{r} \quad -----------(2)$$

c. <u>Electric field intensity at a point due to surface charge distribution:</u>

If the charge distribution is such that the charges are continuously distributed on a two dimensional surface, then it is referred to as surface charge distribution, or as sheet of charge.



For a surface charge distribution, surface charge density ρ_S could be defined as the charge/unit area. The case of charged parallel plate capacitor falls under this category.

If Δq is charge over a small area ΔS, then

$$\rho_S = \lim_{\Delta S \to 0}\left[\frac{\Delta q}{\Delta S}\right] = \frac{dq}{dS}$$

In the limit ΔS→0, dq represents a point charge, due to which the field at P is,

$$\overrightarrow{dE} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

Since the charge distribution is continuous, the field at P, due to the entire charged sheet is,

$$\vec{E} = \int \overrightarrow{dE} = \int \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{1}{4\mu\varepsilon_0}\int \frac{dq}{r^2}\hat{r}$$

$$dq = \rho_S dS$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\rho_S dS}{r^2}\hat{r}$$

If the charges are distributed uniformly over the surface, then ρ_S remains constant over the surface.

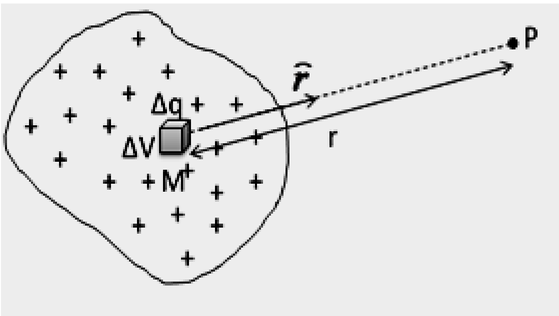
$$\vec{E} = \frac{\rho_S}{4\pi\varepsilon_0}\int_S \frac{\rho_S}{r^2}\hat{r}$$

If the dimensions of the sheet of charge is large compared to the distance at which its electrical effects are considered, and then it is referred to as **infinite sheet of charge**.

d. <u>Electric field intensity at a point due to continuous volume charge distribution:</u>

If the charge distribution is such that the charges are distributed continuously in a volume, then it is referred to as a volume charge distribution.

For a volume charge distribution, the volume charge density ρ_V can be defined as the charge/unit volume. The case of space charge region such as that is a vacuum tube falls under this category.



If Δq is the charge in a small volume ΔV then,

$$\rho_V = \left[\frac{\Delta q}{\Delta V}\right]_{\Delta V \to 0} = \left[\frac{dq}{dV}\right]$$

The field at P due to the point charge dq is

$$\overrightarrow{dE} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

Since the charge distribution is continuous, the field at P due to the entire volume charge distribution is,

$$\vec{E} = \int \overrightarrow{dE} = \int_v \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\int_v \frac{dq}{r^2}\hat{r}$$

$$dq = \rho_V dV$$

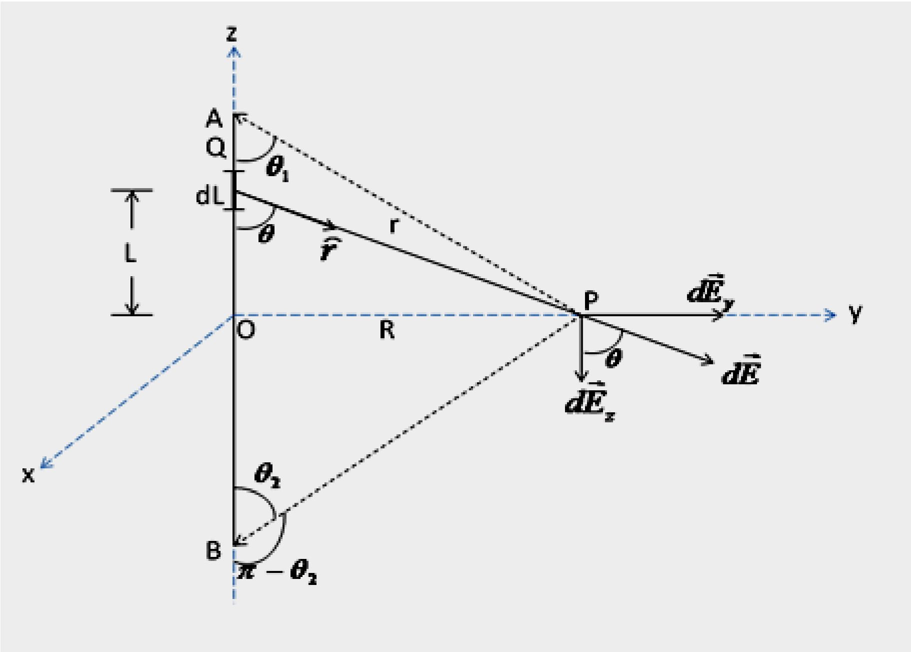
$$\vec{E} = \frac{\rho_V}{4\pi\varepsilon_0}\int_v \frac{dV}{r^2}\hat{r}$$

Electric field due to a line charge (uniformly charged wire):

Let AB be a straight line charge along the z-axis. Let its linear charge density be ρ_L.

∴ If dq is the charge over a length dL, then $\rho_L = \frac{dq}{dL}, \quad or \quad dq = \rho_L dL \; - - - (1)$

Let P be the point at which the electric field due to the line charge is required to be determined. Let PO be drawn perpendicular to the line charge and let PO=R. Now, the line AB can be considered to be made of a number of infinitesimally small elementary incremental lengths, each of lengths carry a charge dq which act independently as point charges. Let us consider one such incremental length dL at Q, which is at a distance r from P and at a distance L above O.



The electric field $\vec{dE}$ at P due to the incremental length at Q is given by,

$$\vec{dE} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

Where $\hat{r}$ is the unit vector along QP.

$\vec{dE}$ can be resolved into two rectangular components $\vec{dE_y}$ and $\vec{dE_z}$ along and perpendicular to the y-direction, as shown in fig.

$$\vec{dE} = \vec{dE_y} + \vec{dE_z}$$

$$Here \quad \vec{dE_y} = dE_y\hat{a_y} \quad and \quad \vec{dE_z} = dE_z(-\hat{a_z})$$

Where dE_y and dE_z are the magnitudes of $\vec{dE_y}$ and $\vec{dE_z}$ and $\hat{a_y}$ and $\hat{a_z}$ are the unit vectors along y and z directions.

From fig. we have,

$$dE_y = dE \; \sin\theta$$

Let us evaluate dE_y (i.e. the magnitude of $\overrightarrow{dE_y}$)

$$dE_y = dE \ \sin\theta$$

$$dE_y = \frac{dq}{4\pi\varepsilon_0 r^2}\sin\theta$$

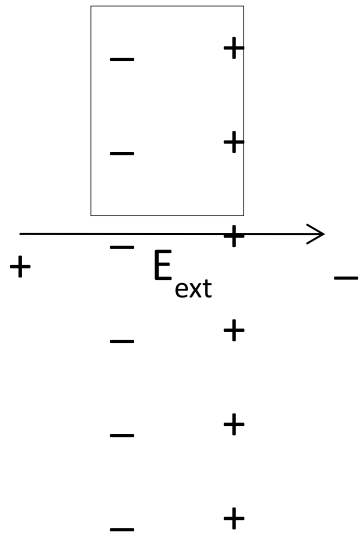
But $dq = \rho_L dL$ from eqn(1)

$$dE_y = \frac{\rho_L dL}{4\pi\varepsilon_0 r^2}\sin\theta$$

And

$$E_y = \int_A^B dE_y = \int_A^B \frac{\rho_L dL}{4\pi\varepsilon_0 r^2}\sin\theta$$

Now, $L = R\cot\theta$ or $dL = -R cosec^2\theta \ d\theta$ & $r = R \ cosec\theta$. Let the line joining the ends A and B of the line charge to P make angles θ_1 and θ_2 with the line charge as shown in below fig.



$$E_y = \int_{\theta_1}^{\pi-\theta_2} \frac{\rho_L(-R cosec^2\theta \ d\theta)}{4\pi\varepsilon_0(R^2 cosec^2\theta)}\sin\theta$$

$$E_y = \frac{-\rho_L}{4\pi\varepsilon_0 R}\int_{\theta_1}^{\pi-\theta_2}\sin\theta \ d\theta = \frac{-\rho_L}{4\pi\varepsilon_0 R}[-\cos\theta_2 - \cos\theta_1] = \frac{\rho_L}{4\pi\varepsilon_0 R}[\cos\theta_1 + \cos\theta_2]$$

$$\vec{E_y} = \frac{\rho_L}{4\pi\varepsilon_0 R}[\cos\theta_1 + \cos\theta_2]\widehat{a_y}$$

Note that R is the perpendicular distance of P from the line charge. Now, let us consider the evaluation of dE_z.

Again we have,

$$dE_z = dE\cos\theta = \frac{dq}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$E_z = \int_A^B dE_z = \int_A^B \frac{\rho_L dL}{4\pi\varepsilon_0 r^2}\cos\theta$$

Following the same steps as in the earlier case, we have

$$E_z = \frac{-\rho_L}{4\pi\varepsilon_0 R}\int_{\theta_1}^{\pi-\theta_2}\cos\theta\, d\theta = \frac{\rho_L}{4\pi\varepsilon_0 R}(\sin\theta_2 - \sin\theta_1)$$

Using $\overrightarrow{dE_z} = dE_z(-\widehat{a_z})$ we have,

$$\overrightarrow{E_z} = \frac{-\rho_L}{4\pi\varepsilon_0 R}(\sin\theta_2 - \sin\theta_1)(-\widehat{a_z}) = \frac{\rho_L}{4\pi\varepsilon_0}(\sin\theta_2 - \sin\theta_1)\widehat{a_z}$$

But the total field $\vec{E}$ at P is

$$\vec{E} = \overrightarrow{E_y} + \overrightarrow{E_z} = \frac{\rho_L}{4\pi\varepsilon_0 R}\left[(\cos\theta_2 + \cos\theta_1)\widehat{a_y} + (\sin\theta_2 - \sin\theta_1)\widehat{a_z}\right] ----(2)$$

Electric field due to line charge of infinite length:

In this case, the line charge extends from $L = -\infty \; to \; +\infty$.

But, when A is at $+\infty, \theta_1 = 0$

Similarly, B at $-\infty, \theta_2 = 0$

For line charge of infinite length, eqn (2) becomes,

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0 R}\left[(\cos 0 + \cos 0)\widehat{a_y} + (\sin 0 - \sin 0)\widehat{a_z}\right] = \frac{2\rho_L}{4\pi\varepsilon_0 R}\widehat{a_L}$$

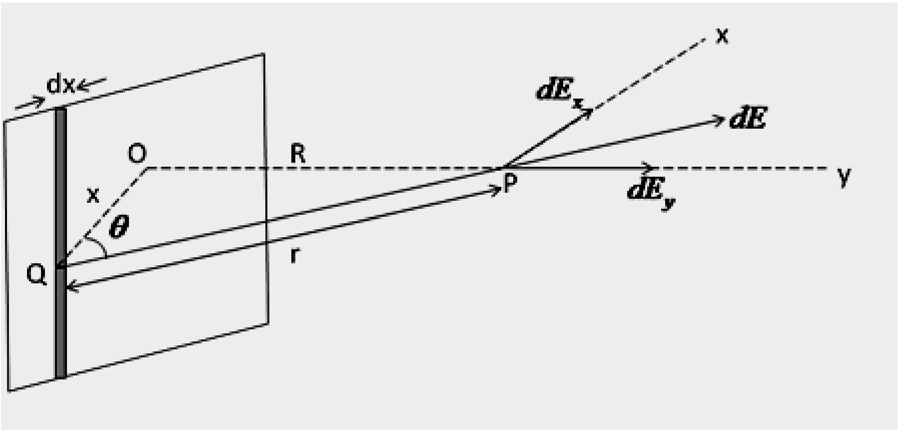
The field due to infinite line charge is given by,

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 R}\widehat{a_L}$$

Electric field due to sheet of charge:

Consider a infinite sheet of charge with uniform surface charge density ρ_S. Let the sheet be aligned in the x-z plane. The sheet of charge may be assumed to be made up of a very large number of line charges of infinite length each of thickness dx and aligned parallel to z-axis. If ρ_L is the linear charge density for these line charges, then it is related to ρ_S through the relation,

$$\rho_L = \rho_S dx$$



Let P be the point at which the electric field due to the sheet of charge is required to be determined. Let PO be drawn perpendicular to the plane of the sheet charge, and let PO=R, which is along the y-axis. Let OQ be the perpendicular from O to the elementary line charge under consideration and let OQ=x and PQ=r.

Now, the electric field dE at P due to the elementary line charge at Q is given by

$$\overrightarrow{dE} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{r}$$

Where $\hat{r}$ is the unit vector along QP and r is the perpendicular distance of P from the elementary line charge $\overrightarrow{dE}$ can be resolved into two rectangular components dE_y and dE_x, along and perpendicular to the y-direction as shown in fig.

Let us evaluate dE_y,

$$now \quad dE_y = dE \ \sin\theta$$

Taking the magnitude of dE, the above equation becomes,

$$dE_y = \frac{\rho_L}{2\pi\varepsilon_0 r}\sin\theta = \frac{\rho_S dx}{2\pi\varepsilon_0 r}\sin\theta$$

The total field at P is due to the sum of the fields due to all the line charge elements of which the sheet of charge is made.

$$E = \int dE = \int dE_y + \int dE_x$$

Now, taking O as the origin of the coordinate system, the dE_x component provided by the line charge elements at Q cancels with the dE_x component due to a corresponding line charge element located at equal distance to the other side of O. since this effect is true for every line charge element in the sheet of charge,

$$\int dE_x = 0$$

$$E = \int dE = \int_{x=-\infty}^{x=+\infty} \frac{\rho_S dx}{2\pi\varepsilon_0 r}\sin\theta$$

Now, $x = R\ cot\theta, \ \ and \quad r = R\ cosec\theta$

Also for $x = -\infty \quad \theta = \pi \quad and \quad for\ \ x = +\infty \quad \theta = 0$

$$E = \frac{\rho_S}{2\pi\varepsilon_0}\int_{\pi}^{0}\frac{R(-cosec^2\theta\ d\theta}{R\ cosec\theta} sin\theta = \frac{\rho_S}{2\pi\varepsilon_0}\int_{0}^{\pi} d\theta = \frac{\rho_S}{2\pi\varepsilon_0}[\theta]_0^{\pi}$$

$$E = \frac{\rho_S}{2\varepsilon_0}$$

If $\widehat{a_S}$ is the unit vector aligned normal to the sheet, and directed outwards, the electric field due to the infinite sheet of charge at any point is given by,

$$E = \frac{\rho_S}{2\varepsilon_0}\hat{a}_S$$

In the above equation it can be observed that the strength of the electric field at any point due to an infinite sheet of charge is independent of the distance from the sheet.

Electric field at a point on the axis of charged circular ring:

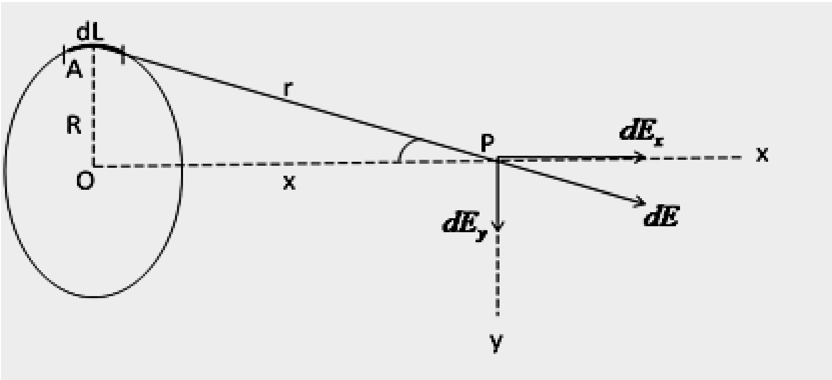
Consider a circular loop of radius R with a total charge q uniformly distributed along its circumference.

Consider a point P on its axis at a distance x from its centre. Consider a differential element of length dL at A, which is at the top of the ring. Let the charge on this element (which can be considered as a point charge) be dq.

Also, ρ_L is the linear charge density, then, $\rho_L = \frac{dq}{dL}$

$$\therefore \qquad dq = \rho_L dL \qquad ------(1)$$

The electric field intensity at P due to the element at A is dE. Since the element can be considered as a point charge, by applying Coulomb's law we have,



$$\overrightarrow{dE} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r} \qquad ---------(2)$$

where $\hat{r}$ is the unit vector along AP.

$\overrightarrow{dE}$ can be resolved into two rectangular components dE_x and dE_y as shown.

Now, $$dE_x = dE\ cos\theta$$

Taking the magnitude of dE from equation (2), the above equation becomes,

$$dE_x = \frac{dq}{4\pi\varepsilon_0 r^2}cos\theta$$

But, $cos\theta = \frac{x}{r}$ also substituting for dq from equation (1), we have,

$$dE_x = \rho_L dL\frac{x}{4\pi\varepsilon_0 r^3} = \frac{\rho_L x}{4\pi\varepsilon_0 r^3}dL$$

The component dE_y is directed downwards.

If we consider an element of the ring at a point diametrically opposite to A, then its dE_y component points upwards, and hence cancels with that due to the element at A. but the dE_x components due to both the elements add up. This is true for all the elements of the ring.

$$\int dE_y = 0$$

The total field at P is sum of the fields due to all the elements of the ring.

$$E = \int dE = \int dE_x + \int dE_y = \int dE_x$$

$$E = \int dE_x = \frac{\rho_L x}{4\pi\varepsilon_0 r^3}\int_{L=0}^{2\pi R} dL$$

$$= \frac{\rho_L x}{4\pi\varepsilon_0 r^3}(2\pi R)$$

But, $r = (R^2 + x^2)^{1/2}$

$$E = \frac{2\pi R \rho_L x}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}} \quad -------(3)$$

If $\hat{a}_x$ is the unit vector along the axis, then the field at P is given by,

$$\vec{E} = \left(\frac{\rho_L R x}{2\varepsilon_0 (R^2 + x^2)^{3/2}}\right)\hat{a}_x$$

In the limiting case, when the point P is at the centre of the ring, x=0. Then $\vec{E}$=0. On the other hand, if P is at a very large distance on the axis from the centre, then x>>R, and this ignoring R^2 in equation (3) $\vec{E} = \frac{q}{4\pi\varepsilon_0 x^2}\hat{a}_x$, [since q is the total charge on the ring, $\rho_L = \frac{q}{2\pi R} \quad or \quad \rho_L(2\pi R) = q$] i.e., it approximates to the field due to a point charge q.