Report on Dry-Contact and Noncontact Biopotential Electrodes

A Detailed Circuit-Theoretic Derivation and Technical Reflection

Rith Rajak
24116085

1 Introduction and Motivation

Biopotential measurements such as electrocardiography (ECG) and electroencephalography (EEG) are among the most widely used noninvasive physiological monitoring techniques. The amplitudes of these signals are typically very small:

- ECG: $\sim 1\,\mathrm{mV}$
- EEG: $\sim 10\,\mu\mathrm{V}$ to $100\,\mu\mathrm{V}$

Traditional clinical sensing uses wet Ag/AgCl electrodes combined with conductive gel and adhesive. Although these provide excellent signal quality and reliable contact, they are inconvenient for long-term wearable monitoring due to discomfort, irritation, dehydration of gel, and difficulty of placement for repeated use.

The paper by Chi *et al.* provides a systematic theoretical and experimental analysis of dry-contact and noncontact electrodes. A central contribution is its circuit-theoretic framework linking electrode impedance (both resistive and capacitive coupling) to noise performance and motion artifact sensitivity.

1.1 Key Questions Addressed

The paper addresses the following fundamental questions:

1. How should dry and noncontact electrodes be electrically modeled?
2. How does coupling impedance affect signal gain and noise?
3. Why is the common belief "lower electrode resistance is always better" sometimes misleading?
4. What are the practical limitations of noncontact sensing?

2 Electrical Modeling of Skin-Electrode Coupling

2.1 Why the Skin-Electrode Interface is Not a Simple Resistor

A major conceptual lesson from the paper is that the electrode-to-skin interface is not purely resistive. Instead, it is a distributed structure containing:

- ionic conduction (resistive leakage),
- dielectric layers (capacitive coupling),
- multiple skin layers (stratum corneum, epidermis, etc.),

- sweat and moisture effects.

Thus, even "noncontact" electrodes often have a small leakage conduction path, and even "dry-contact" electrodes exhibit significant capacitive coupling due to surface area contact.

2.2 Simplified Equivalent Model

The authors propose that many electrode types can be approximated by an equivalent parallel conductance-capacitance model:

$$Y_c(j\omega) = g_c + j\omega C_c \tag{1}$$

where:

- g_c is the coupling conductance ($g_c = 1/R_c$),
- C_c is the coupling capacitance,
- Y_c is the coupling admittance.

The impedance is then:

$$Z_c(j\omega) = \frac{1}{Y_c(j\omega)} = \frac{1}{g_c + j\omega C_c} \tag{2}$$

2.3 Why Admittance is Used

Admittance is the reciprocal of impedance:

$$Y = \frac{1}{Z} \tag{3}$$

Admittance is convenient because in parallel networks, admittances add:

$$Y_{\text{parallel}} = Y_1 + Y_2 + \dots \tag{4}$$

Since the electrode coupling is naturally modeled as parallel resistive and capacitive effects, Y_c is more mathematically convenient than Z_c.

3 Amplifier and Active Shield Model

3.1 Circuit Description

The paper analyzes an actively shielded amplifier topology, shown in simplified form in Fig. 3(a) of the paper. The model contains:

- source voltage v_s (biopotential at skin surface),
- coupling impedance Z_c between skin and electrode,
- amplifier input impedance Z_i to ground,
- active shield capacitance C_s between output and input node,
- amplifier voltage gain A_v,
- amplifier input voltage noise source $v_{i,n}$,
- amplifier input current noise source $i_{i,n}$.

The amplifier input admittance is similarly modeled as:

$$Y_i(j\omega) = g_i + j\omega C_i \tag{5}$$

4 Derivation of Signal Gain $G(j\omega)$

4.1 Objective

We want to derive the transfer function:

$$G(j\omega) = \frac{v_o(j\omega)}{v_s(j\omega)} \tag{6}$$

Let the amplifier input node voltage be v_i.
The amplifier output is:

$$v_o = A_v v_i \tag{7}$$

4.2 KCL at the Input Node

At node v_i, three currents are relevant:

(1) Current through coupling admittance Y_c

The current from source into the node is:

$$i_c = Y_c(v_s - v_i) \tag{8}$$

(2) Current through amplifier input admittance Y_i

The current from node to ground is:

$$i_i = Y_i v_i \tag{9}$$

(3) Current through shield capacitance C_s

The capacitor is between node v_i and output node v_o, thus:

$$i_s = j\omega C_s(v_i - v_o) \tag{10}$$

4.3 Applying KCL

By Kirchhoff's Current Law:

$$i_c = i_i + i_s \tag{11}$$

Substituting (8), (9), (10):

$$Y_c(v_s - v_i) = Y_i v_i + j\omega C_s(v_i - v_o) \tag{12}$$

Now substitute $v_o = A_v v_i$ from (7):

$$Y_c(v_s - v_i) = Y_i v_i + j\omega C_s(v_i - A_v v_i) \tag{13}$$

Factor:

$$Y_c(v_s - v_i) = Y_i v_i + j\omega C_s v_i(1 - A_v) \tag{14}$$

Expand left side:

$$Y_c v_s - Y_c v_i = Y_i v_i + j\omega C_s v_i(1 - A_v) \tag{15}$$

Bring all v_i terms to the right:

$$Y_c v_s = v_i\left(Y_c + Y_i + j\omega C_s(1 - A_v)\right) \tag{16}$$

Solve for v_i:

$$v_i = \frac{Y_c}{Y_c + Y_i + j\omega C_s(1 - A_v)} v_s \tag{17}$$

Multiply by amplifier gain A_v to obtain output:

$$v_o = A_v v_i = A_v \frac{Y_c}{Y_c + Y_i + j\omega C_s(1 - A_v)} v_s \tag{18}$$

Thus, the gain is:

$$\boxed{G(j\omega) = \frac{v_o}{v_s} = A_v \frac{Y_c(j\omega)}{Y_c(j\omega) + Y_i(j\omega) + j\omega(1 - A_v)C_s}} \tag{19}$$

This matches Eq. (2) in the paper.

4.4 Interpretation of Gain Expression

This gain expression resembles a voltage divider, except in admittance form. Important limiting cases:

- If coupling is strong ($|Y_c| \gg |Y_i|$), then $G \approx A_v$.
- If coupling is weak (small C_c or g_c), then G is significantly reduced.

Therefore, poor electrode coupling results in signal attenuation.

5 Derivation of Source Input-Referred Noise

5.1 Definition of Input-Referred Noise

The paper defines an equivalent noise source $v_{s,n}$ such that:

$$v_o = G(j\omega)\,(v_s + v_{s,n}) \tag{20}$$

The purpose is to "refer" all amplifier noise sources back to an equivalent noise at the source input.

5.2 Given Expression (Paper Eq. 3)

The paper provides:

$$\boxed{v_{s,n} = \frac{Y_c + Y_i + j\omega C_s}{Y_c} v_{i,n} + \frac{i_{i,n}}{Y_c}} \tag{21}$$

This equation shows two noise mechanisms:

1. voltage noise amplified by imperfect coupling,
2. current noise converted to voltage noise across coupling impedance.

6 Derivation of Noise Power Density $v_{s,rms}^2$

6.1 Noise Power Density Concept

Noise sources are treated as random processes. For uncorrelated sources, mean-square contributions add.

Rewrite (21) as:

$$v_{s,n} = A(\omega)v_{i,n} + B(\omega)i_{i,n} \tag{22}$$

where:

$$A(\omega) = \frac{Y_c + Y_i + j\omega C_s}{Y_c}, \qquad B(\omega) = \frac{1}{Y_c} \tag{23}$$

Assuming $v_{i,n}$ and $i_{i,n}$ are uncorrelated:

$$v_{s,rms}^2 = |A(\omega)|^2 v_{i,rms}^2 + |B(\omega)|^2 i_{i,rms}^2 \tag{24}$$

Substituting:

$$v_{s,rms}^2 = \left|\frac{Y_c + Y_i + j\omega C_s}{Y_c}\right|^2 v_{i,rms}^2 + \left|\frac{1}{Y_c}\right|^2 i_{i,rms}^2 \tag{25}$$

Thus:

$$\boxed{v_{s,rms}^2 = \frac{|Y_c + Y_i + j\omega C_s|^2}{|Y_c|^2} v_{i,rms}^2 + \frac{i_{i,rms}^2}{|Y_c|^2}} \tag{26}$$

This matches Eq. (4) in the paper.

6.2 Expanding into Conductance and Capacitance

Substitute:

$$Y_c = g_c + j\omega C_c \tag{27}$$

$$Y_i = g_i + j\omega C_i \tag{28}$$

First compute:

$$|Y_c|^2 = g_c^2 + \omega^2 C_c^2 \tag{29}$$

Next compute numerator:

$$\begin{aligned} Y_c + Y_i + j\omega C_s &= (g_c + j\omega C_c) + (g_i + j\omega C_i) + j\omega C_s && (30) \\ &= (g_c + g_i) + j\omega(C_c + C_i + C_s) && (31) \end{aligned}$$

Thus:

$$|Y_c + Y_i + j\omega C_s|^2 = (g_c + g_i)^2 + \omega^2 (C_c + C_i + C_s)^2 \tag{32}$$

Substitute into (26):

$$\boxed{v_{s,rms}^2 = \frac{(g_c + g_i)^2 + \omega^2 (C_c + C_i + C_s)^2}{g_c^2 + \omega^2 C_c^2} v_{i,rms}^2 + \frac{i_{i,rms}^2}{g_c^2 + \omega^2 C_c^2}} \tag{33}$$

This matches Eq. (5) in the paper.

7 Thermal Noise Limit (Derivation of Paper Eq. 6)

7.1 Physical Interpretation

The authors argue that in many practical dry/noncontact systems, amplifier noise is not dominant. Instead, thermal noise from the coupling conductance g_c dominates.

Thermal noise current spectral density of conductance g_c:

$$i^2_{thermal,rms} = 4kTg_c \tag{34}$$

Assume ideal amplifier:

- $g_i \approx 0$
- $v^2_{i,rms} \approx 0$
- $i^2_{i,rms} \approx 4kTg_c$

Then Eq. (33) becomes:

$$v^2_{s,rms} \approx \frac{4kTg_c}{g_c^2 + \omega^2 C_c^2} \tag{35}$$

Rewrite:

$$v^2_{s,rms} = \frac{4kTg_c}{g_c^2 + \omega^2 C_c^2} = \frac{4kT}{g_c + \frac{\omega^2 C_c^2}{g_c}} \tag{36}$$

Thus:

$$\boxed{v^2_{s,rms} \approx \frac{4kT}{g_c + \frac{\omega^2 C_c^2}{g_c}}} \tag{37}$$

This matches Eq. (6) in the paper.

8 Connecting Gain and Noise to Output RMS Voltage

8.1 Noise at Output

From the system equation:

$$v_o = G(j\omega)(v_s + v_{s,n}) \tag{38}$$

Noise component at output:

$$v_{o,n} = G(j\omega)v_{s,n} \tag{39}$$

In power spectral density form:

$$S_{v_o}(f) = |G(j2\pi f)|^2 S_{v_s}(f) \tag{40}$$

8.2 RMS Noise from PSD

The RMS noise over bandwidth $[f_1, f_2]$ is:

$$v^2_{o,rms} = \int_{f_1}^{f_2} S_{v_o}(f)\, df \tag{41}$$

Substitute:

$$\boxed{v^2_{o,rms} = \int_{f_1}^{f_2} |G(j2\pi f)|^2 S_{v_s}(f)\, df} \tag{42}$$

If gain is approximately constant over the bandwidth:

$$v_{o,rms}^2 \approx |G|^2 \int_{f_1}^{f_2} S_{v_s}(f)\, df \tag{43}$$

Thus:

$$v_{o,rms} \approx |G| \sqrt{\int_{f_1}^{f_2} S_{v_s}(f)\, df} \tag{44}$$

9 Important Observations and Insights from the Paper

9.1 Counterintuitive Noise Result

The most striking insight from Eq. (37) is that the input-referred thermal noise approaches zero in two extreme cases:

- **Very high conductance ($g_c \to \infty$):** low resistance wet-contact electrode.
- **Very low conductance ($g_c \to 0$):** nearly purely capacitive coupling.

This contradicts the common assumption that "low impedance is always best." Instead, the paper shows that the worst case occurs when g_c is intermediate.

9.2 Physical Explanation

Thermal noise requires a resistive path. If $g_c \to 0$, the resistive path disappears and thermal noise generation vanishes. However, this does not imply a perfect electrode, because weak coupling reduces signal amplitude and increases vulnerability to motion artifacts.

Thus, low thermal noise does not automatically imply good signal quality.

9.3 Frequency Dependence Intuition

Since the capacitive term grows with frequency:

$$|j\omega C_c| \propto f \tag{45}$$

noncontact electrodes improve at higher frequencies but struggle at low frequencies. This explains why:

- ECG baseline stability is difficult for noncontact sensing.
- EEG (especially below 1 Hz) is extremely challenging.

9.4 Motion Artifact Interpretation

The gain depends on coupling admittance Y_c. Motion changes Y_c (especially C_c due to distance variation). Therefore, motion introduces artifacts even if the amplifier is ideal. The paper provides balancing conditions:

$$Y_i + j\omega(1 - A_v)C_s \approx 0 \tag{46}$$

which can reduce sensitivity to coupling variation.

This reveals that motion artifacts are not only mechanical problems; they can be partially mitigated by careful circuit design.

10 Personal Understanding and Learning Reflection

10.1 What I Learned

From my perspective, this paper was valuable because it connected:

- circuit modeling (impedance/admittance),
- noise theory (thermal noise and PSD),
- biomedical instrumentation constraints (very low amplitude signals),
- real-world parasitic effects (active shield capacitance),
- practical system-level limitations (motion artifacts and settling time).

A key learning outcome was understanding why electrode performance is not solely determined by resistance but by a combination of resistive and capacitive coupling.

10.2 Intuition Developed

The main intuition I developed is that the electrode behaves like a frequency-dependent coupling network:

- At low frequency, coupling is dominated by g_c (resistive path).
- At high frequency, coupling is dominated by C_c (capacitive path).

Thus, noncontact electrodes are naturally biased toward passing higher frequency components more effectively.

10.3 Observation on "Cotton as a Bad Electrode"

One of the most interesting experimental observations in the paper is that cotton clothing introduces significant resistive coupling. This means it does not behave as a clean dielectric but as a lossy resistor-capacitor combination. As a result, cotton can add thermal noise and degrade performance.

This suggests that "noncontact" sensing through fabric is not purely capacitive, and the properties of clothing material (humidity, thickness, conductivity) may dominate system noise.

11 Novel Insight: Adaptive Impedance Estimation for Motion Artifact Reduction

One novel idea inspired by this paper is to treat motion artifacts not only as a mechanical problem but as an **electrical parameter variation problem**. The paper models the electrode-skin coupling using the admittance:

$$Y_c(j\omega) = g_c + j\omega C_c \tag{47}$$

where g_c represents the effective coupling conductance (leakage/conduction path) and C_c represents the coupling capacitance (field coupling across an insulating layer or air gap).

A major cause of motion artifacts in dry and noncontact electrodes is that both g_c and C_c are **time-varying**. In practical wearable conditions, body movement can change the electrode distance, pressure, and moisture conditions. For example:

- Small changes in electrode-skin separation strongly affect C_c.

- Sweat accumulation or drying affects g_c.
- Micro-slippage and mechanical vibration introduce rapid fluctuations in both.

Since the gain derived in the paper is:

$$G(j\omega) = A_v \frac{Y_c(j\omega)}{Y_c(j\omega) + Y_i(j\omega) + j\omega(1 - A_v)C_s} \tag{48}$$

the system gain is explicitly dependent on Y_c. Therefore, if coupling admittance varies with time, the measured signal becomes:

$$v_o(t) = G(t)\, v_s(t) \tag{49}$$

even if the physiological signal $v_s(t)$ itself is stable. This produces motion artifacts that appear as baseline drift, amplitude modulation, and transient spikes. This observation suggests that motion artifacts are fundamentally linked to variations in electrode coupling parameters.

11.1 Proposed Approach

A possible extension is to estimate the coupling admittance $Y_c(t)$ in real time and use this estimate to compensate for motion-induced gain variations. This can be done by injecting a very small calibration signal at a frequency outside the physiological band of interest. For example, a sinusoidal test tone:

$$v_{\text{test}}(t) = V_{\text{test}} \sin(2\pi f_{\text{test}} t) \tag{50}$$

can be added such that f_{test} is chosen well above the ECG/EEG frequency range (for example, $f_{\text{test}} = 500\,\text{Hz}$ or $1\,\text{kHz}$). Since ECG/EEG signals occupy mostly low frequencies (below 150 Hz), the injected tone can be filtered out after estimation.

At the test frequency, the coupling admittance becomes:

$$Y_c(f_{\text{test}}) = g_c + j(2\pi f_{\text{test}})C_c \tag{51}$$

If the amplitude and phase response of the injected signal can be measured at the amplifier input, the real and imaginary components of Y_c can be estimated:

$$g_c = \text{Re}(Y_c), \qquad C_c = \frac{\text{Im}(Y_c)}{2\pi f_{\text{test}}} \tag{52}$$

11.2 Lock-In Based Measurement Concept

A practical method to extract the response at f_{test} is a lock-in detection approach. In this method, the measured signal is multiplied by reference sinusoids at f_{test}:

$$x_I(t) = v(t)\cos(2\pi f_{\text{test}} t), \qquad x_Q(t) = v(t)\sin(2\pi f_{\text{test}} t) \tag{53}$$

and then low-pass filtered. This isolates the in-phase and quadrature components of the injected tone. The resulting components provide amplitude and phase information, enabling real-time tracking of coupling impedance.

11.3 Compensation for Motion Artifacts

Once $Y_c(t)$ is estimated, the gain $G(t)$ can be computed or approximated using the derived gain model. The physiological signal can then be reconstructed by compensating for time-varying gain:

$$\hat{v_s}(t) = \frac{v_o(t)}{G(t)} \tag{54}$$

This approach effectively treats motion artifacts as a time-varying gain distortion and attempts to remove it through adaptive calibration. Even if perfect compensation is not achievable due to nonlinearities or higher-order effects, the method can significantly reduce baseline wander and amplitude fluctuations.

11.4 Advantages and Insights

The key advantage of this approach is that it converts an unpredictable mechanical problem into an electrical estimation problem. Unlike passive solutions (tight straps, improved adhesives, or mechanical damping), this technique provides an active measurement of electrode coupling quality.

Additionally, real-time tracking of $Y_c(t)$ could enable a signal-quality index (SQI). For example, a sudden drop in estimated C_c may indicate electrode lift-off, while rapid fluctuations in g_c could indicate sweat-induced instability. Such indicators could be useful in wearable systems to detect corrupted segments and improve reliability.

11.5 Challenges

Potential limitations of this method include:

- The injected signal must be small enough to avoid saturation or interference.
- Careful frequency selection is required to avoid overlap with mains interference harmonics.
- The electrode interface may require higher-order models beyond the single-pole admittance approximation.

Despite these challenges, adaptive impedance estimation appears to be a promising direction to reduce motion artifacts and improve the practicality of dry and noncontact biopotential electrodes.

12 Conclusion

This paper provides a strong theoretical framework for analyzing dry-contact and noncontact biopotential electrodes. By modeling the coupling interface using admittance, the authors derive closed-form expressions for signal gain and input-referred noise.

The most significant insight is that minimizing electrode resistance is not always optimal; instead, electrode noise depends on the combined effect of conductance and capacitance. The paper also emphasizes that motion artifacts remain the dominant unsolved challenge for practical wearable and noncontact systems.

Overall, this paper is a strong example of how classical circuit analysis and noise theory can directly inform biomedical sensor design.

Reference

Chi, Y. M., Jung, T.-P., & Cauwenberghs, G. (2010). Dry-Contact and Noncontact Biopotential Electrodes: Methodological Review. *IEEE Reviews in Biomedical Engineering*, 3, 106–119.