Muon Cooling Project Updates

March 28, 2025

https://github.com/criggall/muon-cooling

Status of simplified channel

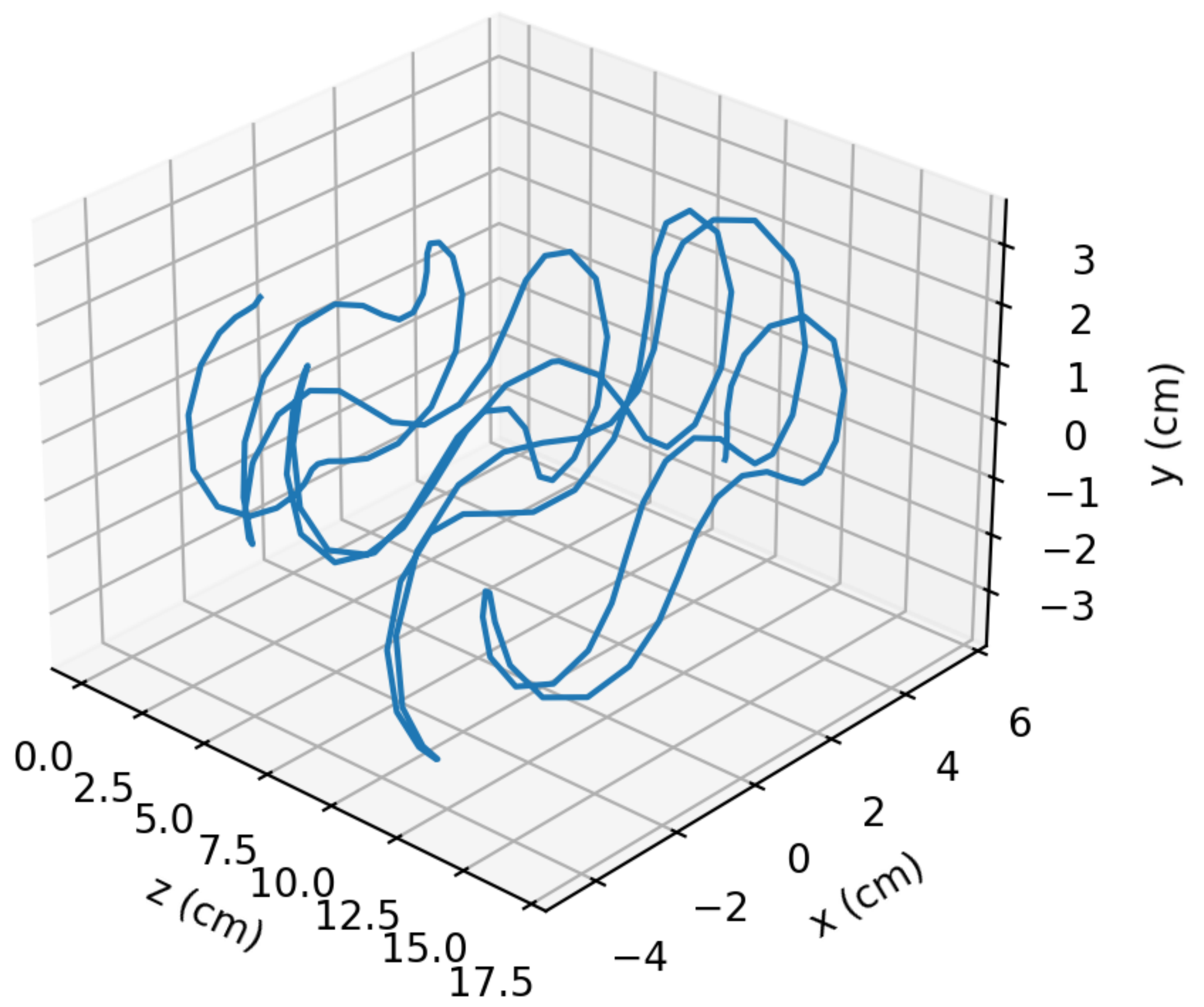
- Fixed reference particle definition to be same angle and offset as end of matching section in original HFOFO channel

- Also set constant solenoid current in simplified channel to same as at end of matching section

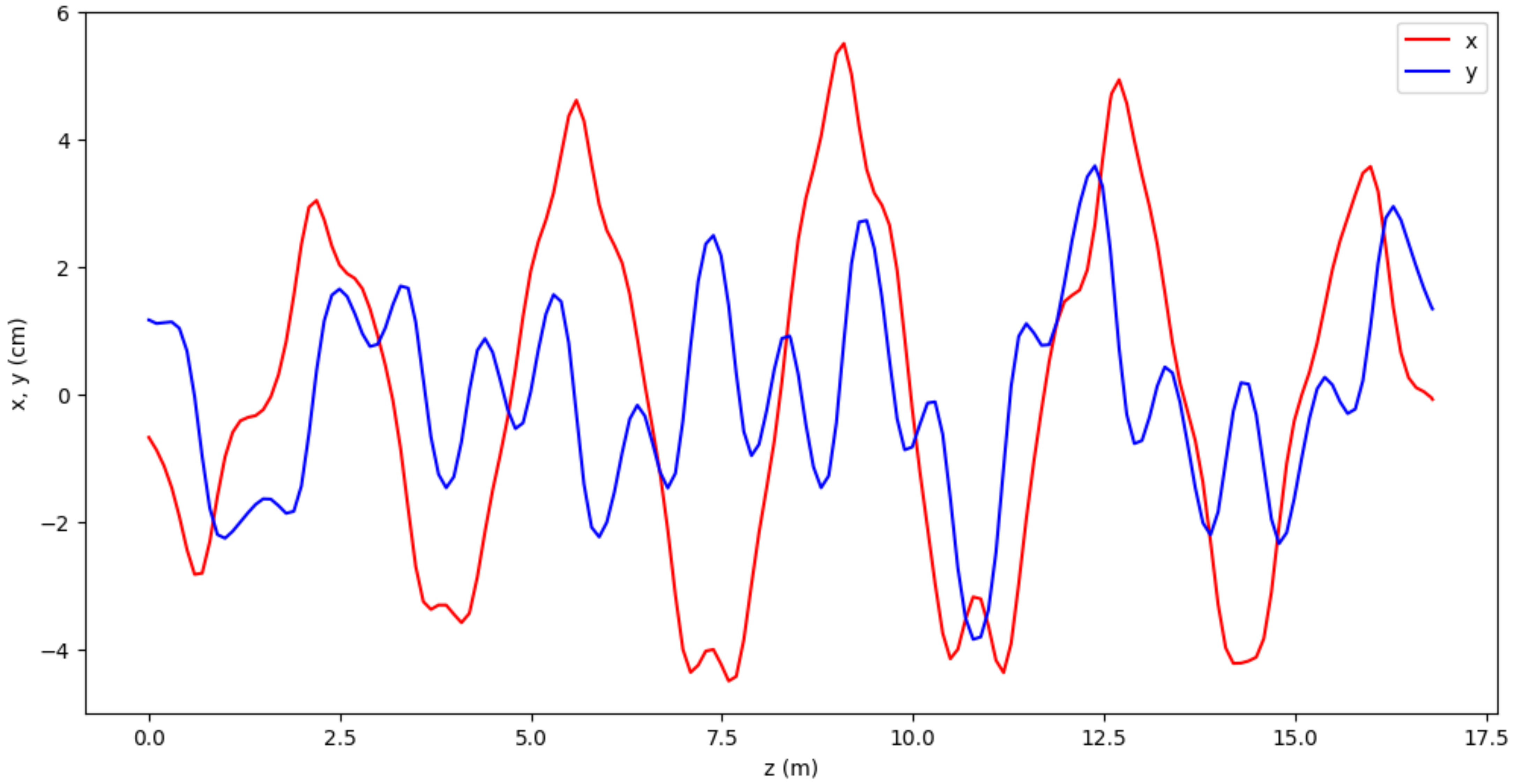
Simplified-HFOFO > simplified_hfofo_g4bl.in

```

### REFERENCE PARTICLE ###

param Xp=-3.82563/221.754
param Yp=-2.37079/221.754

reference referenceMomentum=$p particle=mu+ beamX=-6.65767 beamY=11.7673 beamZ=0.0 beamXp=$Xp beamYp=$Yp

# beamX = x leaving the matching channel
# beamY = y leaving the matching channel
# beamXp = px/pz leaving the matching channel
# beamYp = py/pz leaving the matching channel
```

Status of simplified channel



- Still not seeing a stable orbit…
    - Particle appears to be ejected prematurely
- Could this be a consequence of not properly setting the offset and angle of the reference particle due to a discrepancy in coordinate systems?

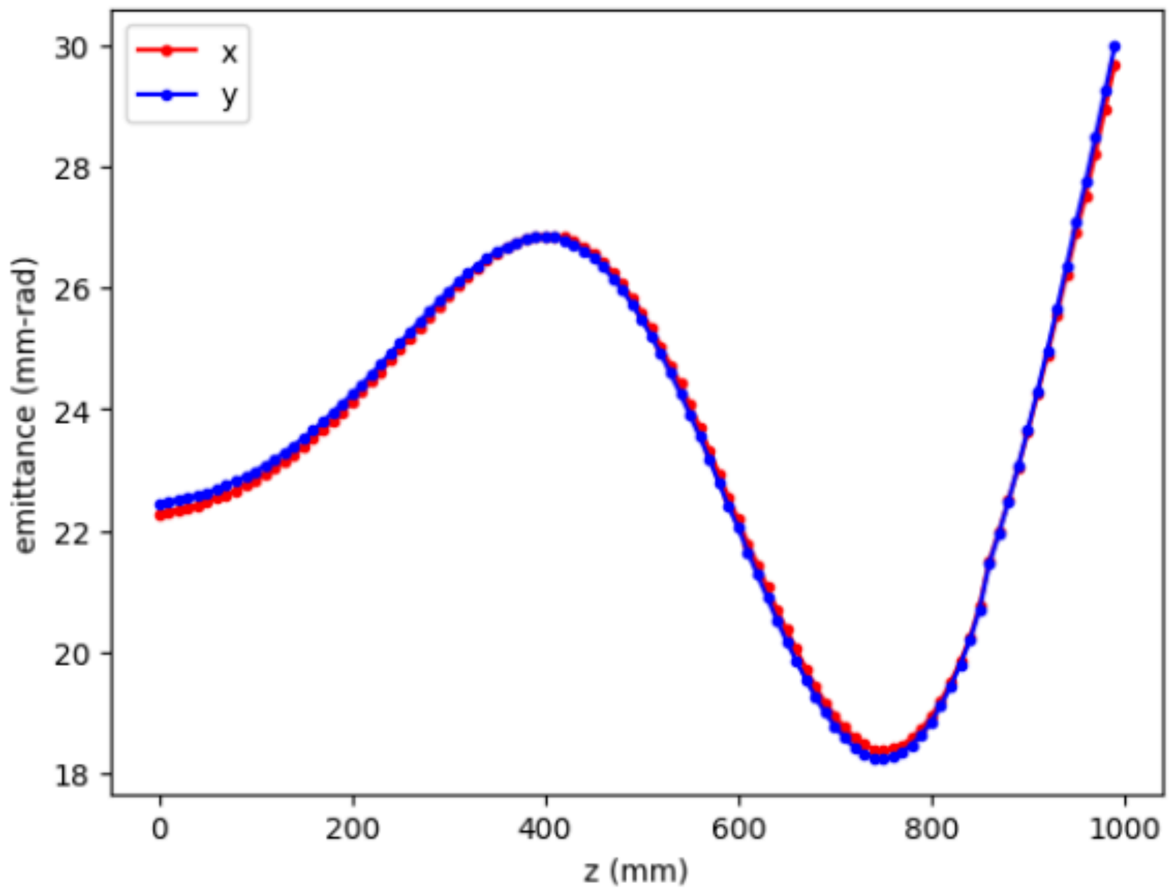


Ideas for next steps

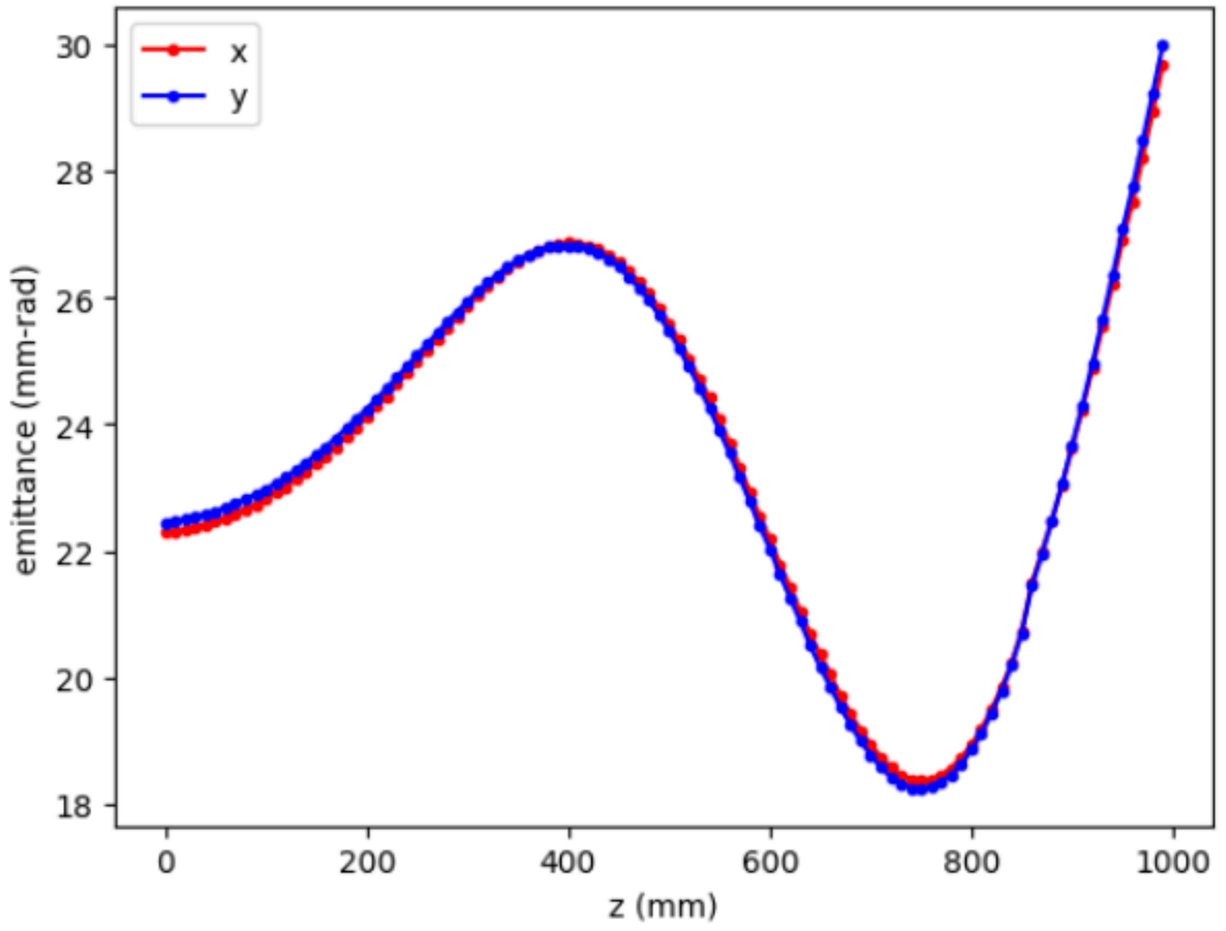
- Run particle displaced from origin without solenoid tilting
  - Slowly increase tilt while maintaining particle stability
- Place virtual detector at end of matching section and try using these x, y, px, and py values for the reference particle command
- Copy Yuri's matching section and simulate a single simplified period past it

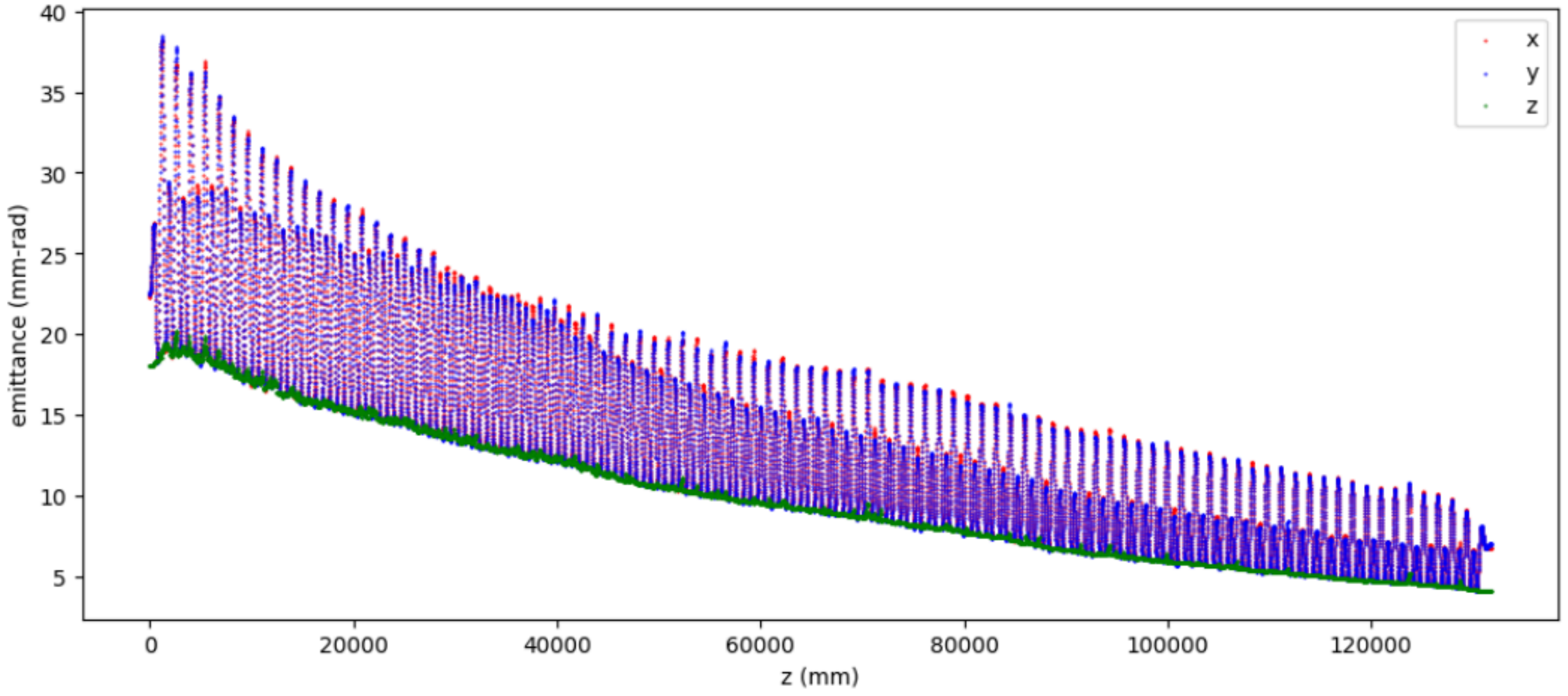
Investigating the *profile* command

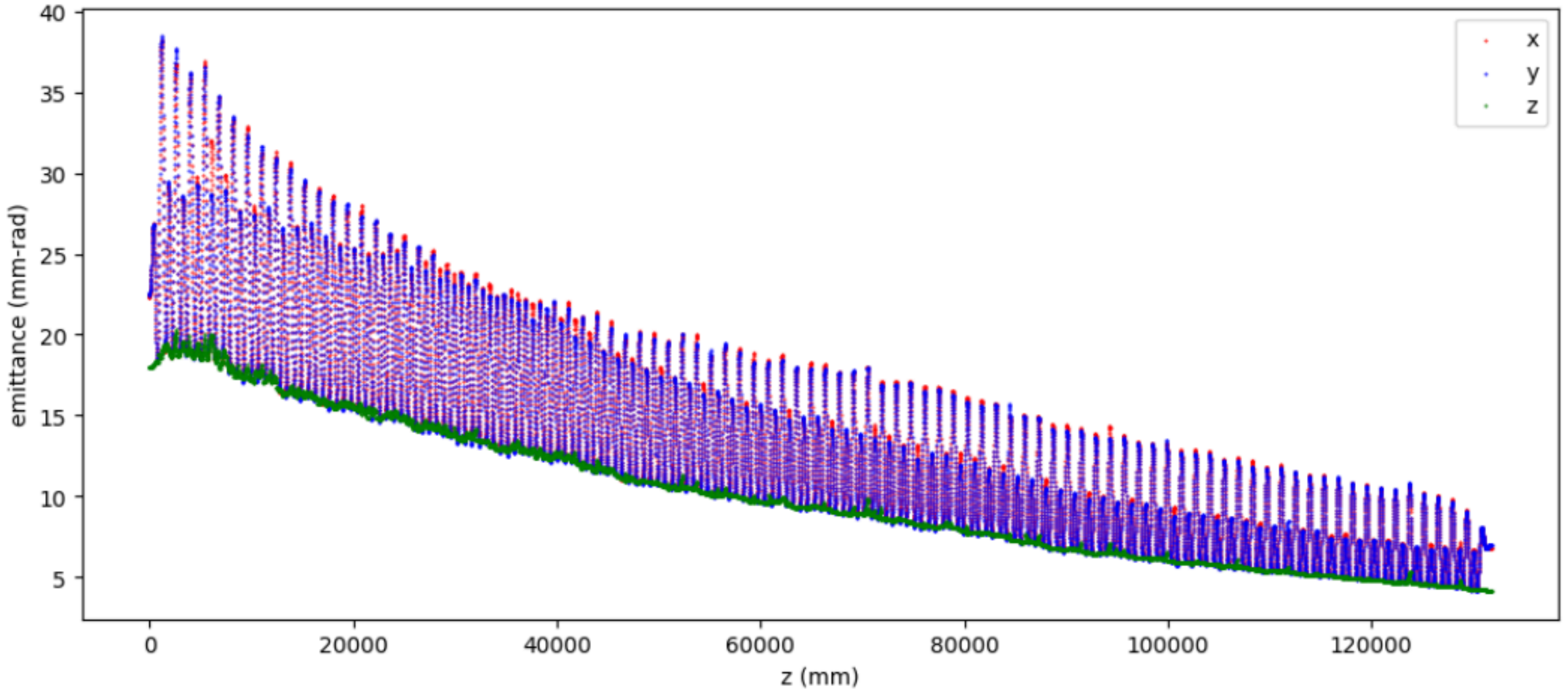
Centerline coordinates:



Reference coordinates:







Emittance Calculation

References: https://github.com/criggall/muon-cooling/issues/5

Definitions

Notation:

Underlined characters = column phase space vectors

Capital letters = matrices

Define dynamical variables as

$$\underline{z} = [x, P_x, y, P_y, s - c\beta_0 t, \delta]$$

Where s is the path length, $P_{x,y}$ are the canonical momenta, and

$$\beta_0 = \frac{p_0}{m} = \frac{p_0}{\sqrt{p_0^2 + m^2}}$$

The reference momentum can also be written as $p_0 = mc\beta_0\gamma_0$ with

$$\gamma_0 = \frac{E}{m} = \sqrt{1 + \left(p_0/m\right)^2}$$

And finally define ??? as $\delta = \dfrac{\gamma - \gamma_0}{\beta_0^2\gamma_0}$ ⟵ What is δ? And is γ found using the mechanical or canonical momentum?

Canonical Momenta

Canonical transformation:

$$P_x = \frac{p_x + \frac{e}{c}A_x}{p_0}$$

Where $A_{x,y}$ are the components of the magnetic vector potential and $p_{x,y}$ are components of the mechanical momenta

Covariance Matrix

Assumption: distribution does not contain long tails $\implies$ we can average to find elements of covariance matrix Σ

$$\Sigma_{i,j} = \frac{1}{N}\sum_{k=1}^{N} \zeta_i^{(k)}\zeta_j^{(k)}$$

Where $\underline{\zeta}^{(k)} = \underline{z}^{(k)} - \frac{1}{N}\sum_{k=1}^{N}\underline{z}^{(k)}$

In our input file, each event has the same weight; if the weights differed, we would be required to use a weighted average here.