Muon Cooling Project Updates

April 18, 2025

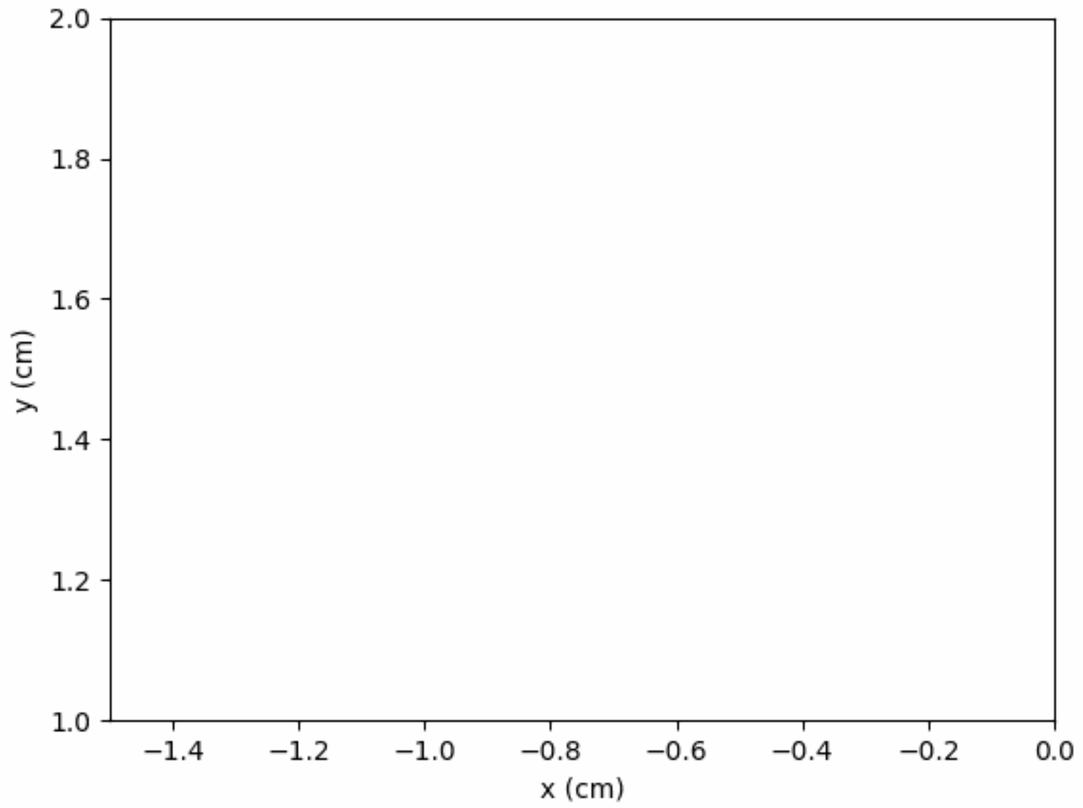
https://github.com/criggall/muon-cooling

Progress from this week

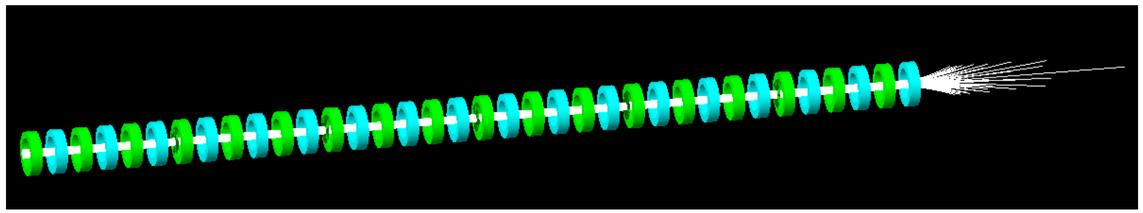
- Revisited last week's unintuitive results with solenoid tilt set to zero
- Investigated particle with small *momentum* offset from nominal reference particle
  - Performed FFT of displacement values vs. z
  - Fit results of FFT to find functional form of dispersion
- Started on investigation of particle with small *position* offset w.r.t. reference
  - Similar procedure as with dp particle
  - Objective is to find beta functions

Testing with constant solenoid polarity

Animation of reference particle trajectory in xy-plane:

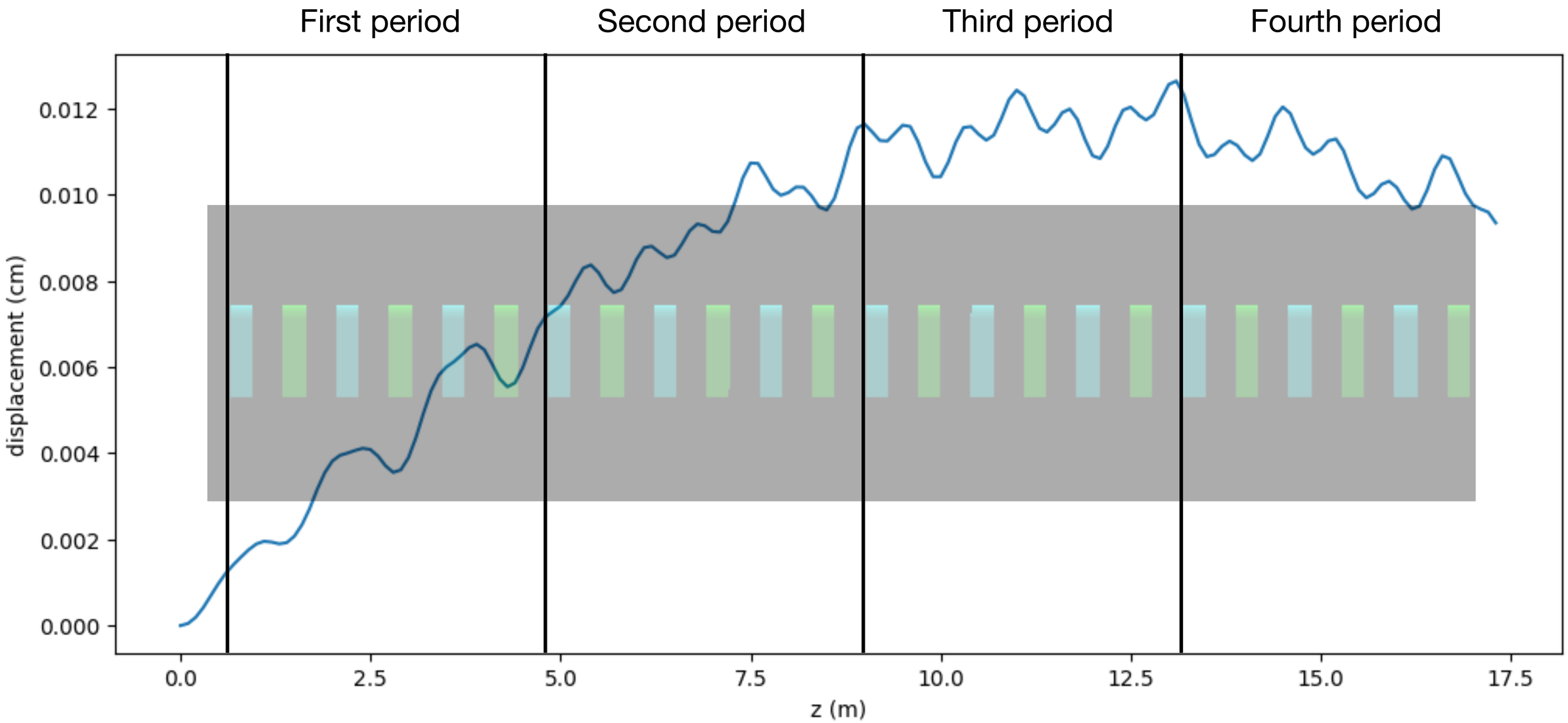


- Expect to see a circle, consistent with helical motion along z
- Suggests that the particle is not properly matched
- May explain why setting solenoid tilt to zero in alternating polarity channel produced unexpected results



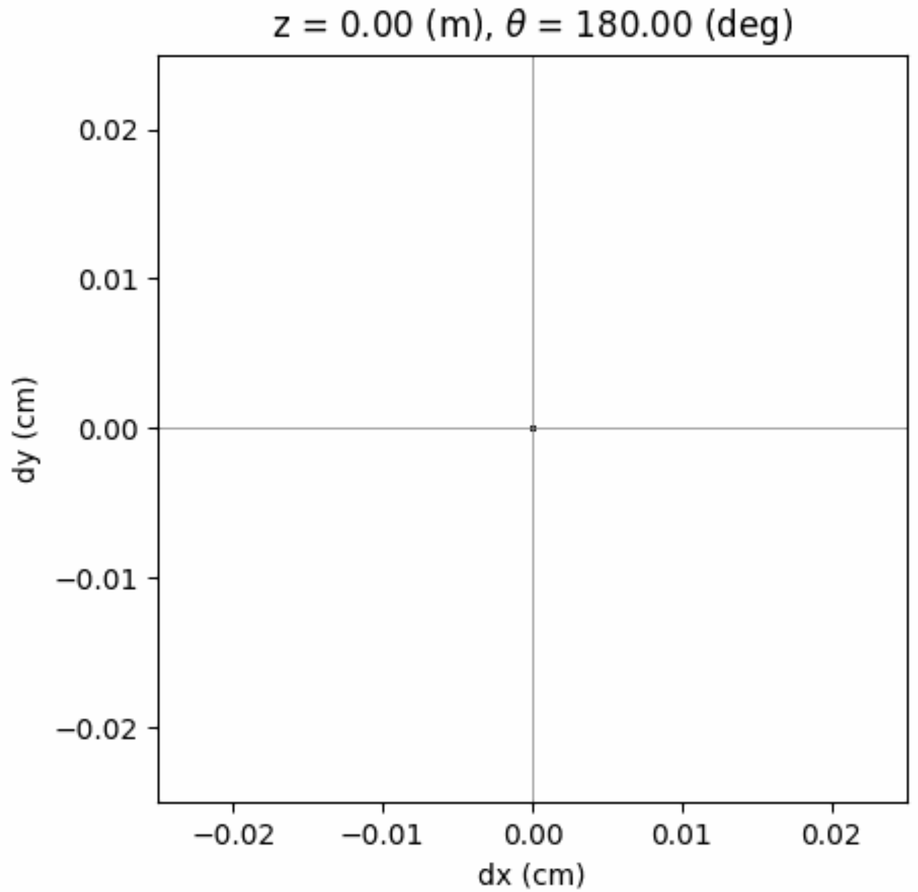
$$dp = -0.1 \text{ MeV/c}$$

Displacement from reference particle

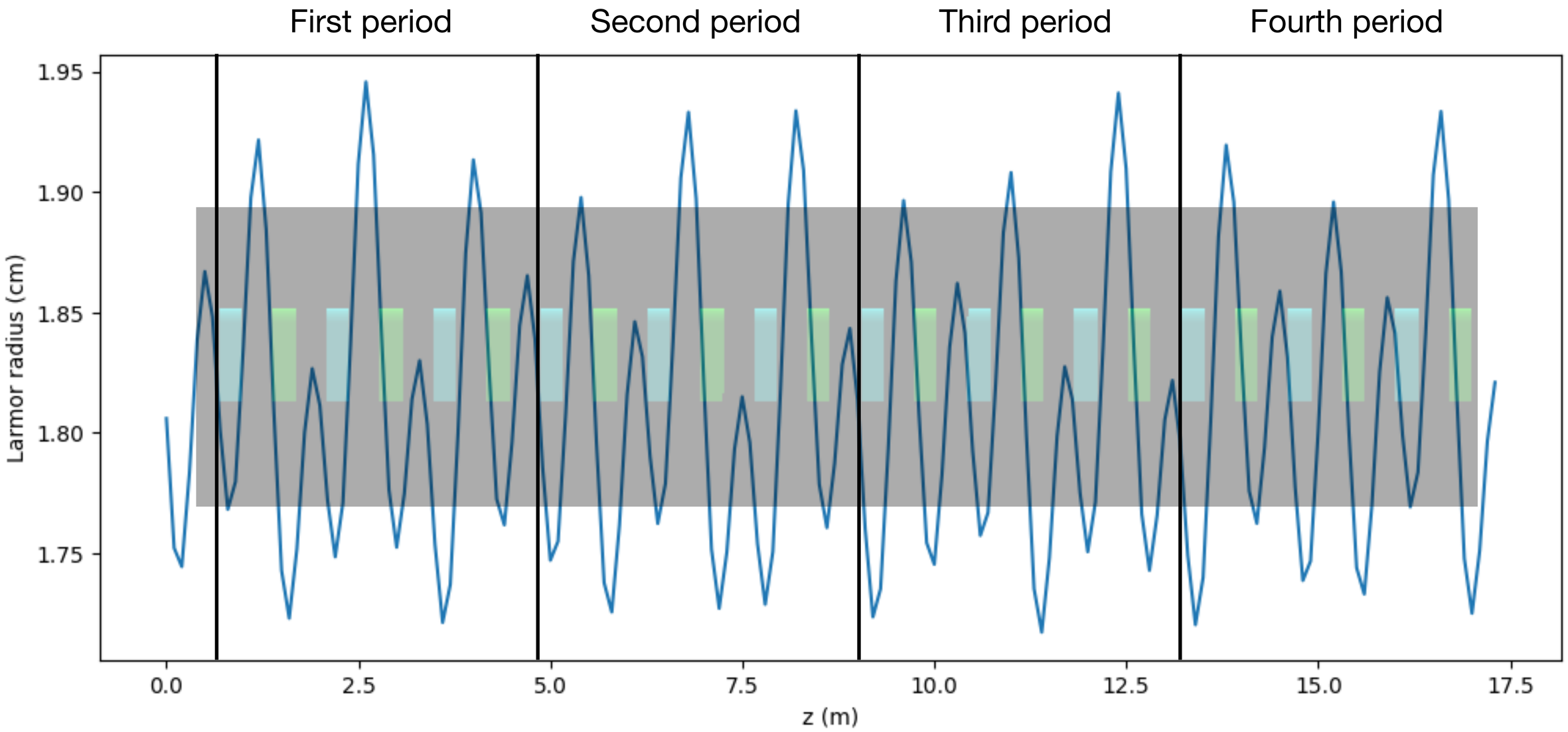


Displacement from reference particle

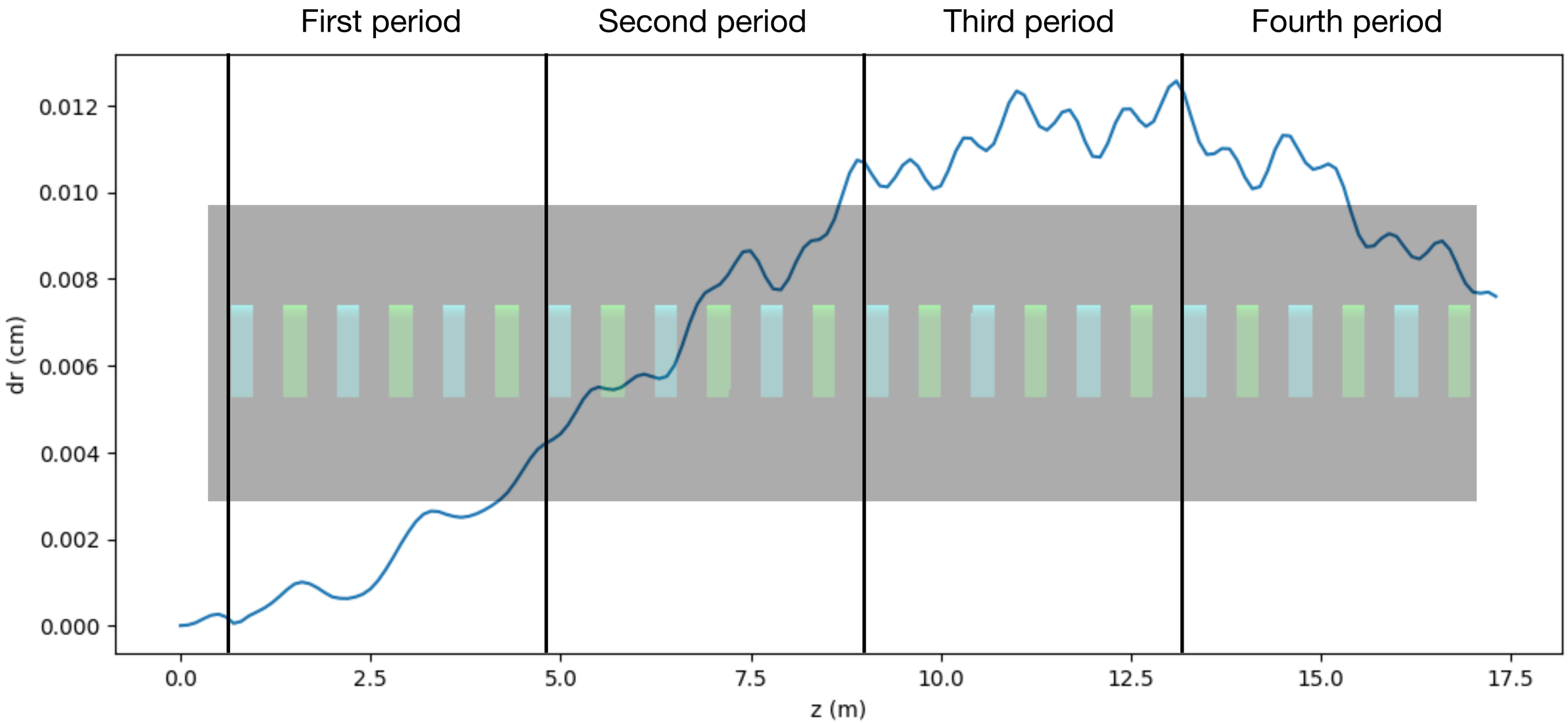
Animation of displacement vector in xy-plane:



Larmor radius



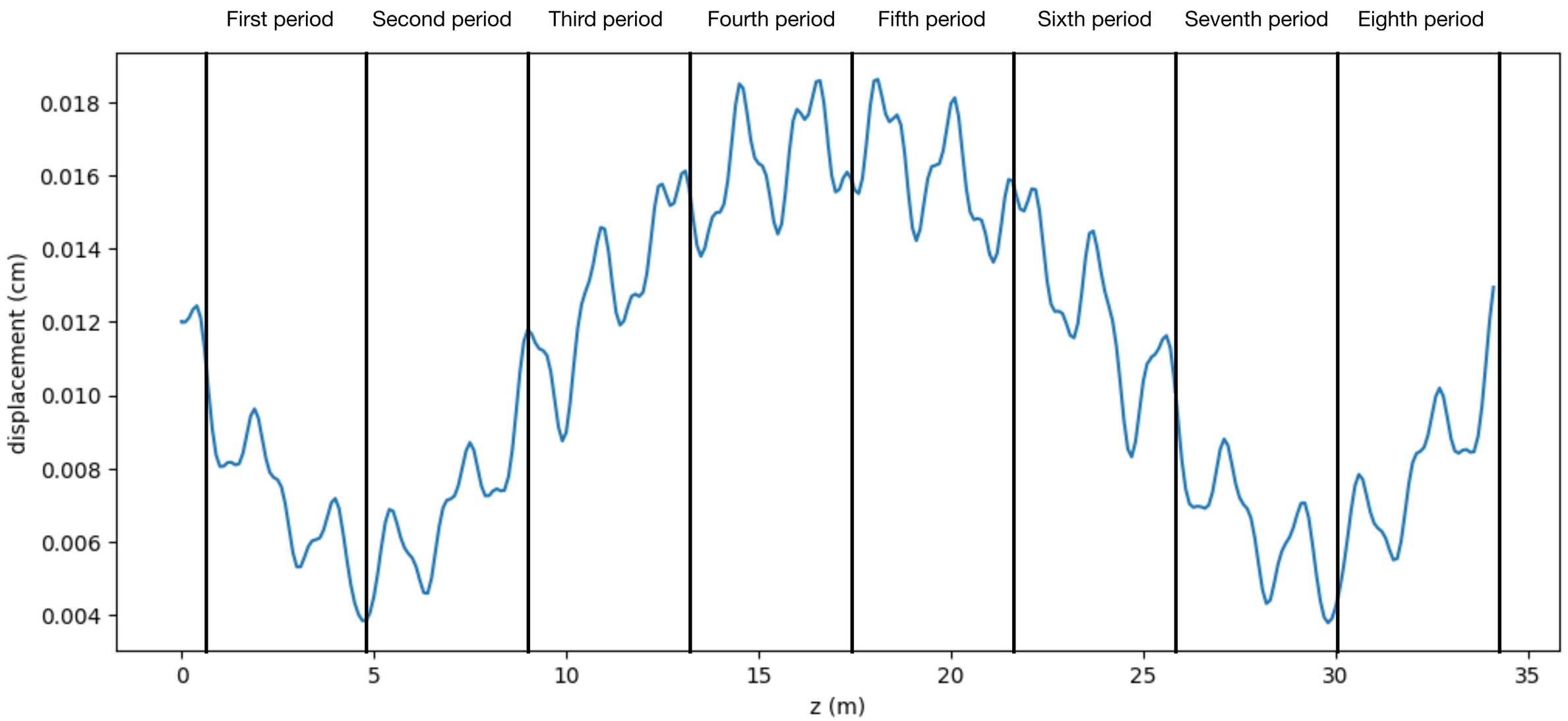
Deviation in Larmor radius w.r.t. reference particle



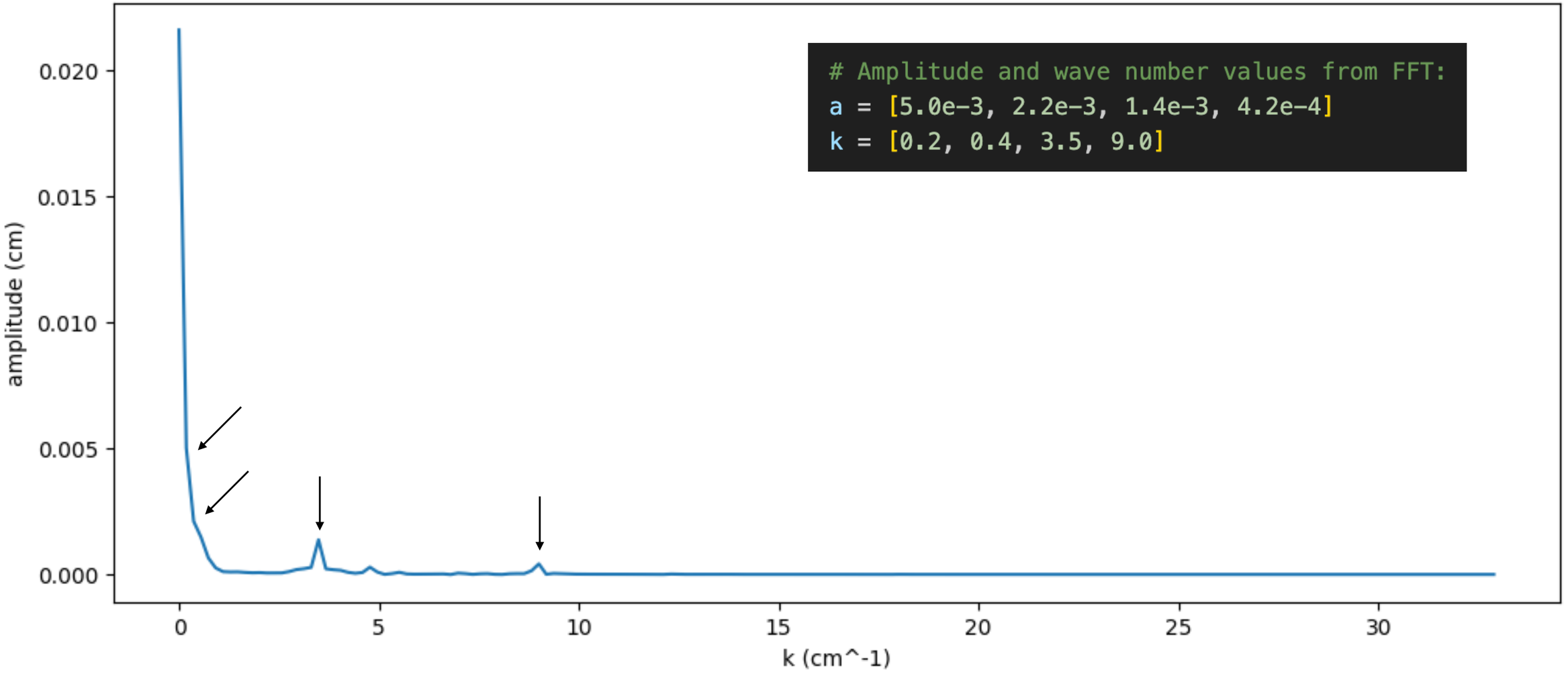
$$dp = -0.1 \text{ MeV/c}$$

$$dx = dy = -0.012/\sqrt{2} \text{ cm}$$

Displacement from reference particle (extended channel)

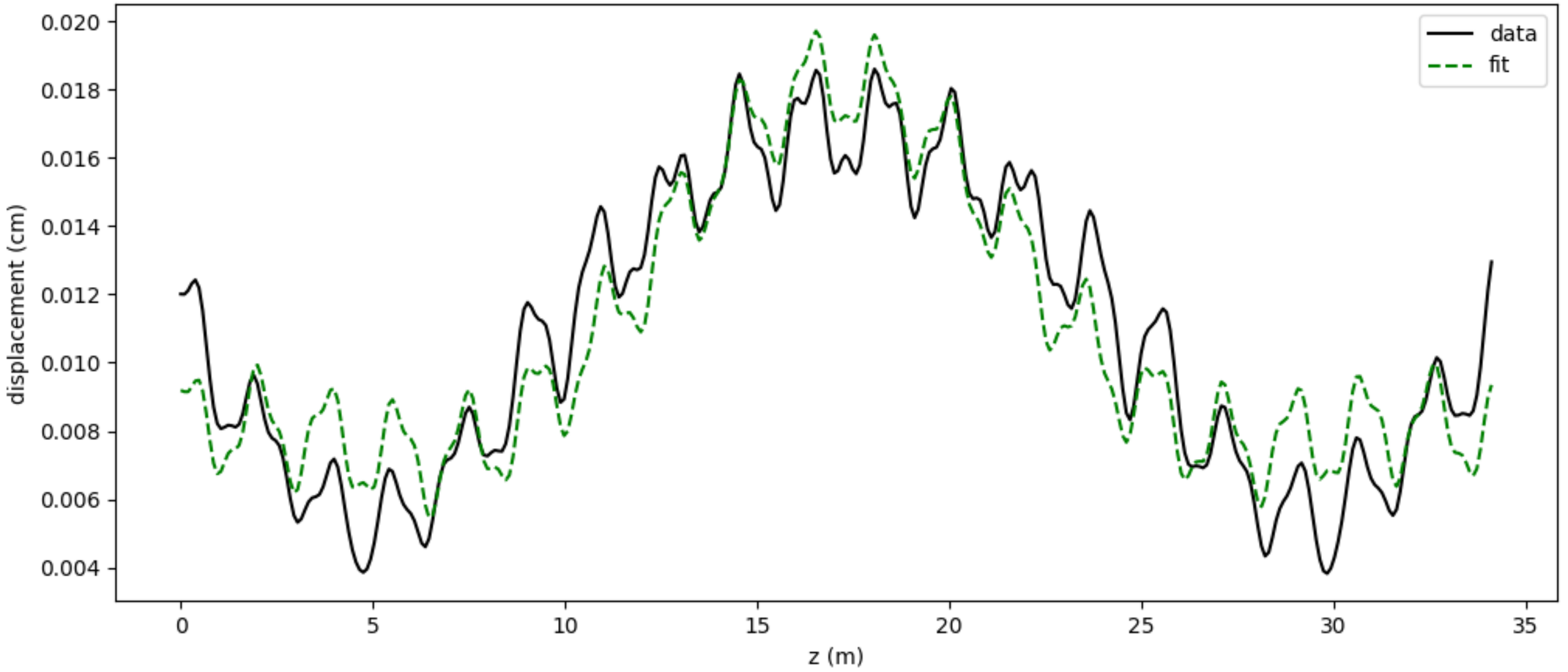


FFT of displacement




```
# Amplitude and wave number values from FFT:
a = [5.0e-3, 2.2e-3, 1.4e-3, 4.2e-4]
k = [0.2, 0.4, 3.5, 9.0]
```

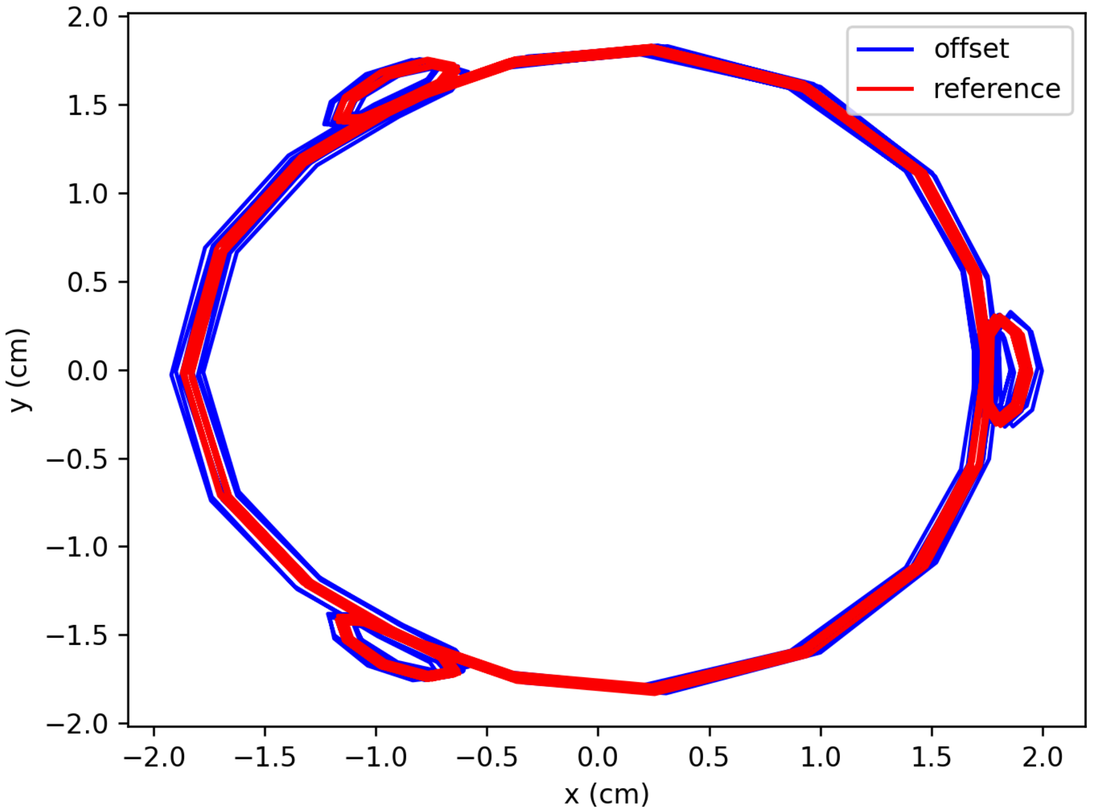
Fitting FFT result to functional form

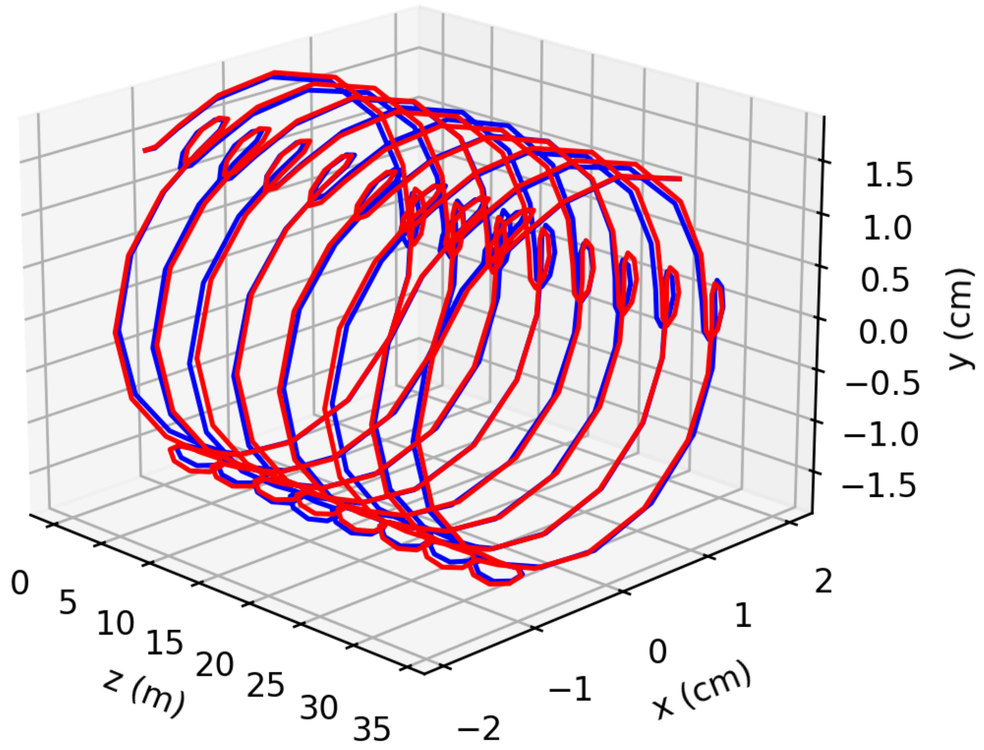


f(z) = 0.005sin(0.2z-1.88) + 0.0022sin(0.4z+1.04) + 0.0014sin(3.5z+0.75) + 0.00042sin(9.0z+2.95) + 0.01

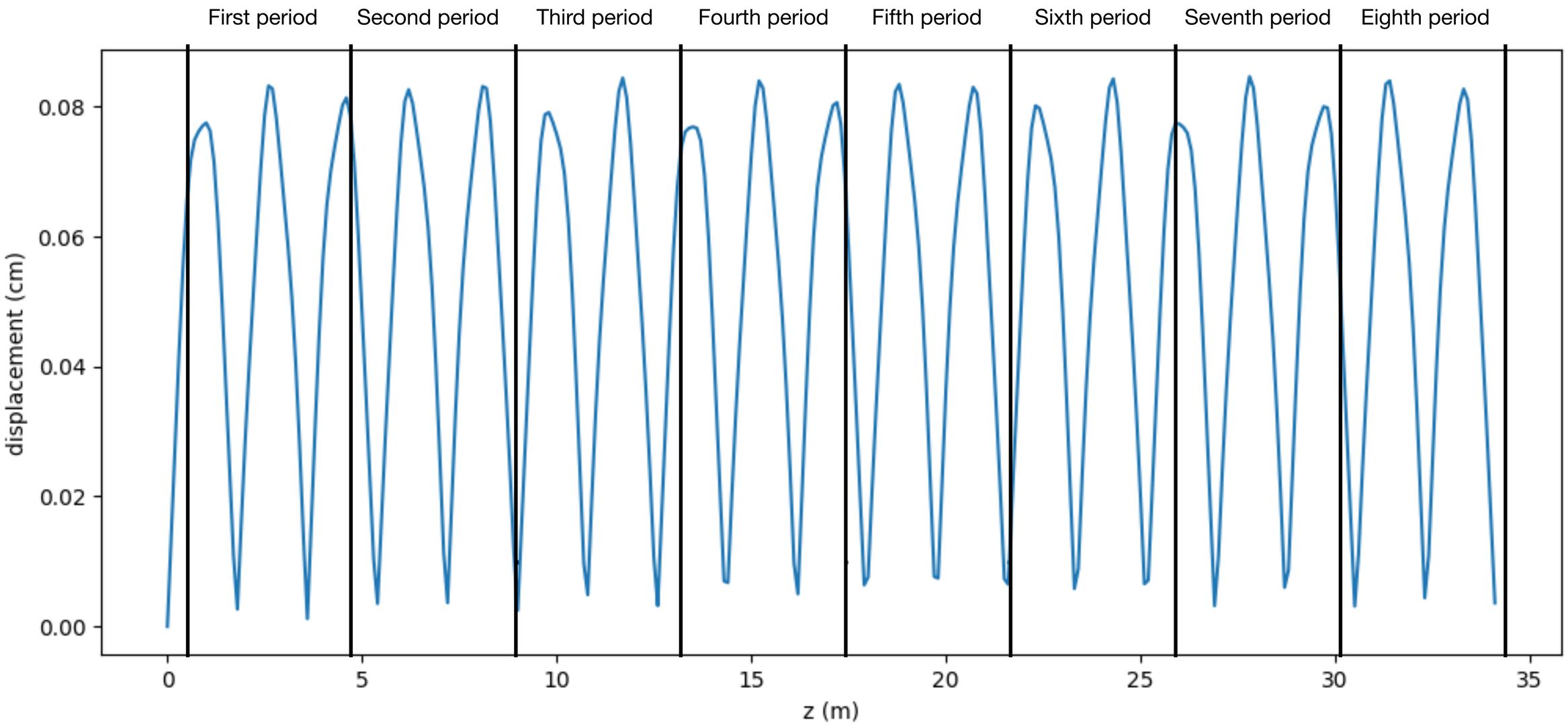
$$dxp = dyp = -0.001 \text{ radians}$$

Trajectory of dxp, dyp particle

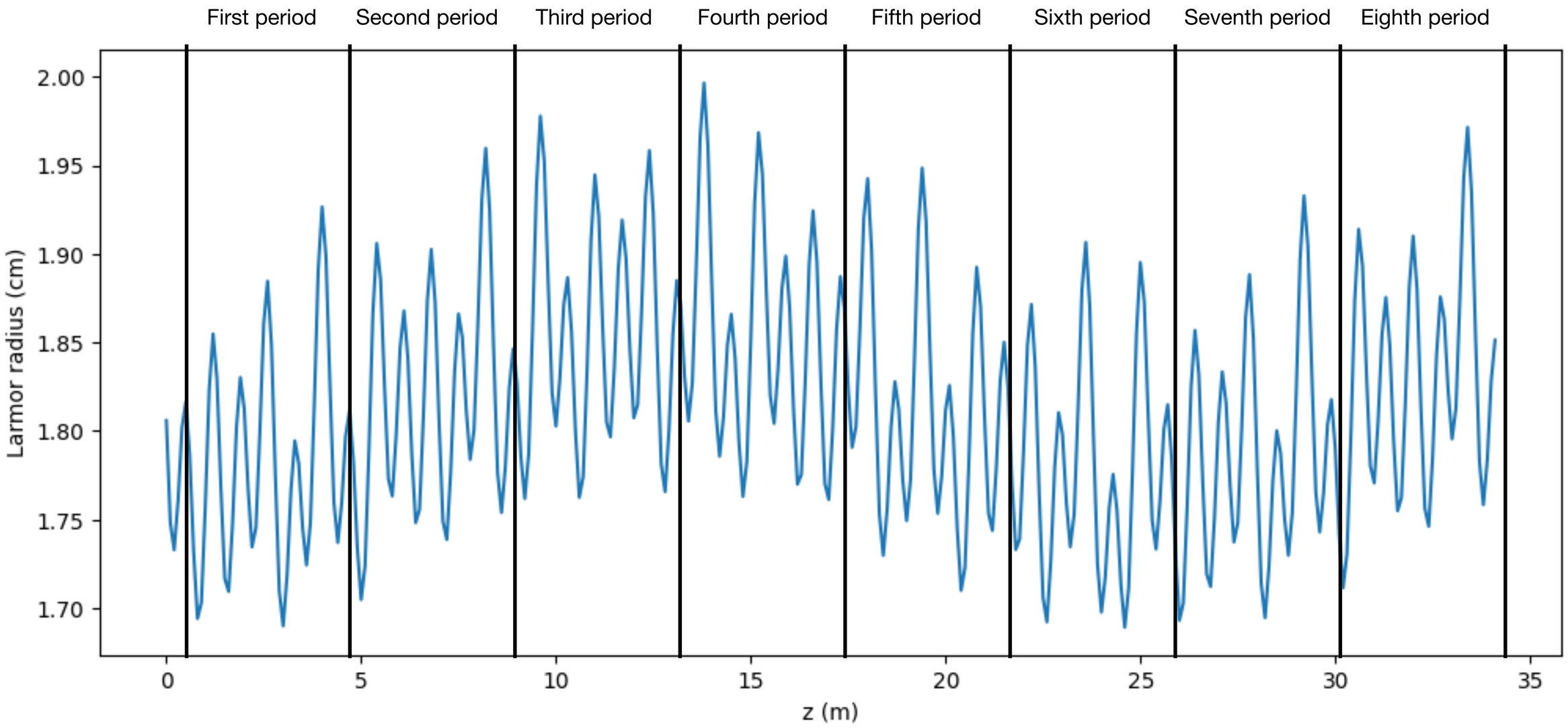




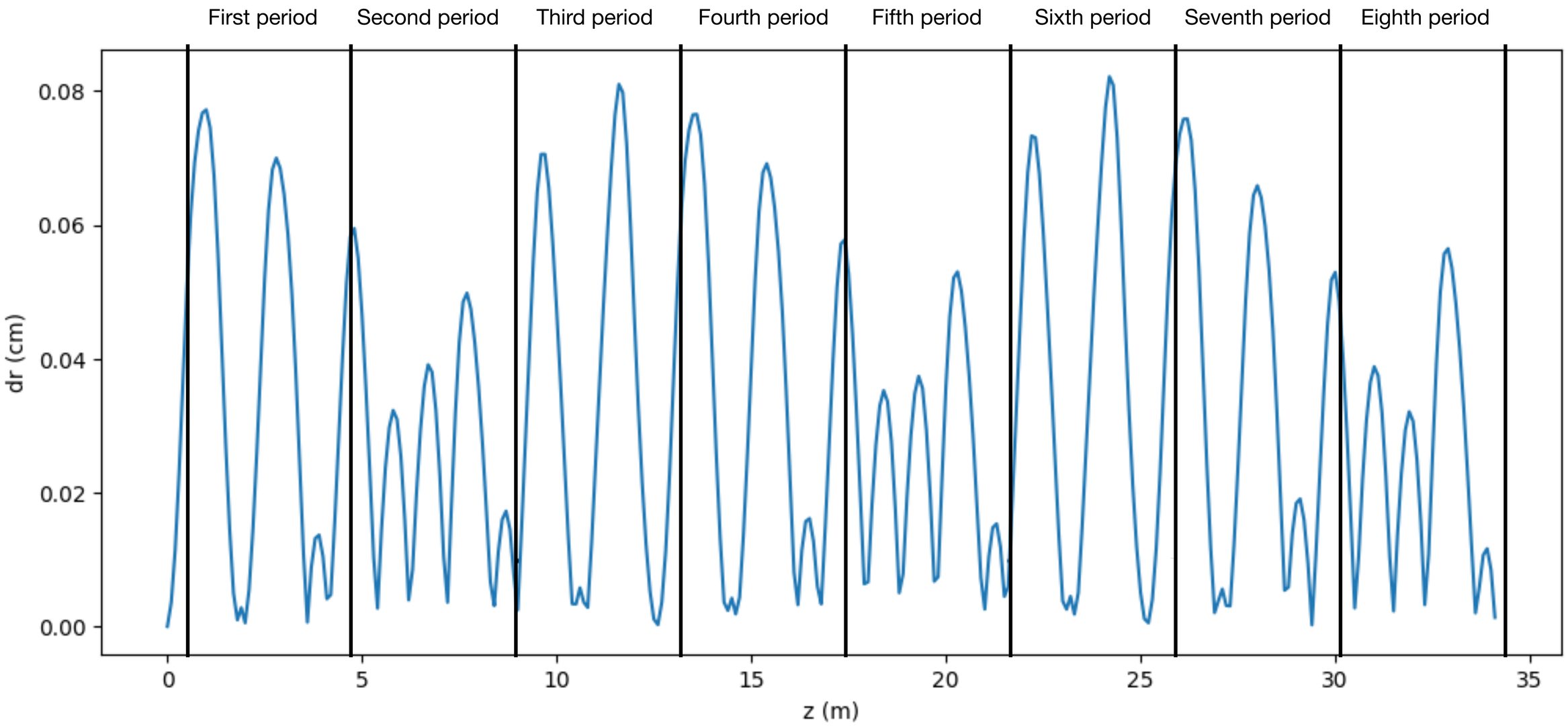
Displacement from reference particle (extended channel)



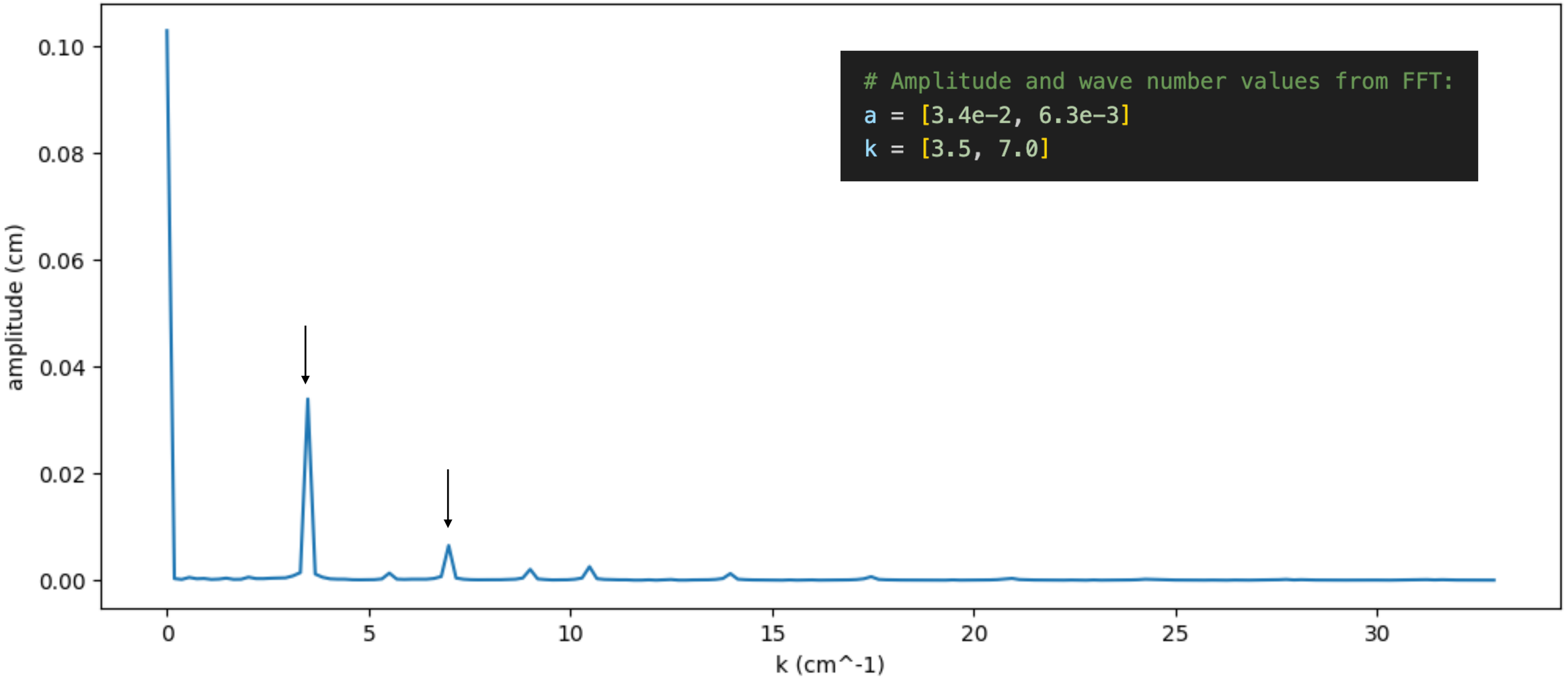
Larmor radius



Deviation in Larmor radius w.r.t. reference particle

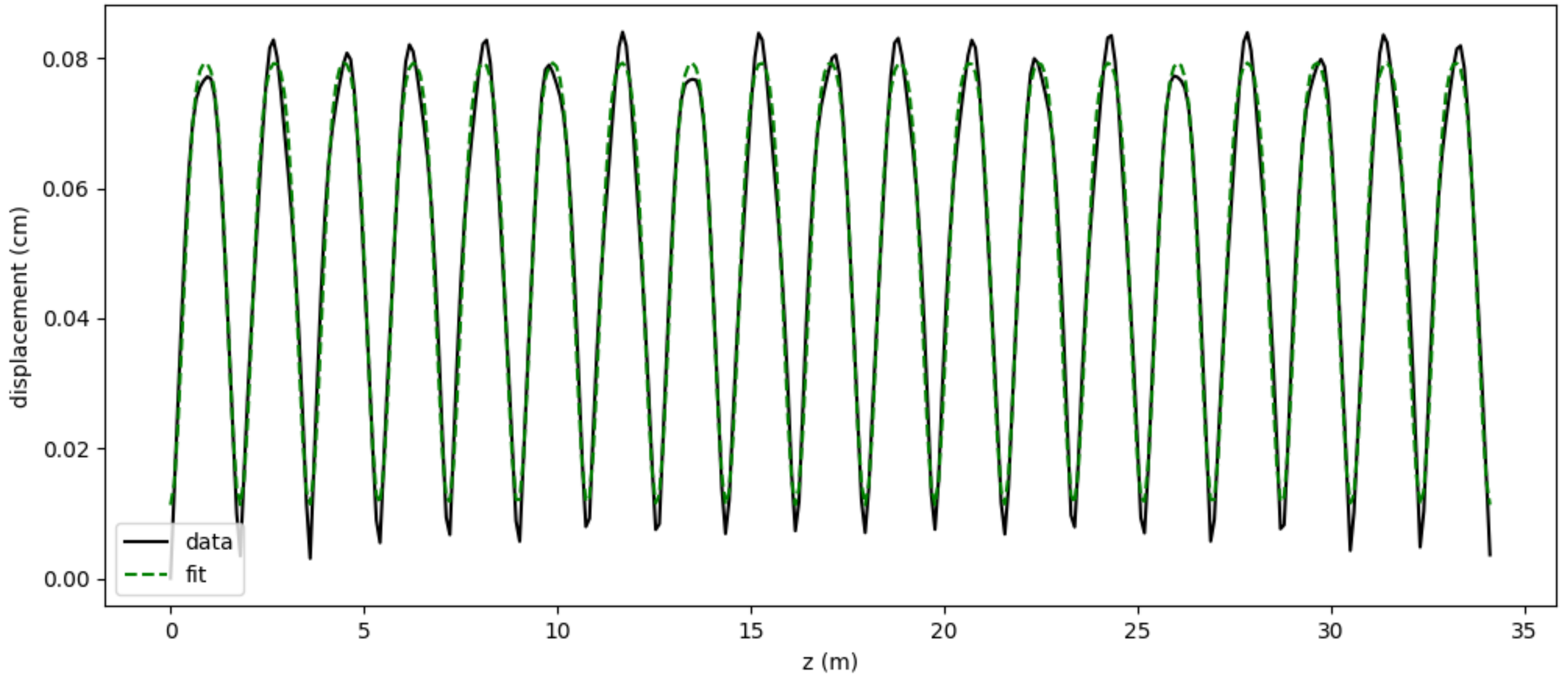


FFT of displacement




```
# Amplitude and wave number values from FFT:
a = [3.4e-2, 6.3e-3]
k = [3.5, 7.0]
```

Fitting FFT result to functional form



f(z) = 0.034sin(3.5z-1.57) + 0.0063sin(7.0z-1.56) + 0.05