WRITTEN QUESTION 1

Vector $V_1 = [1 \quad 1 \quad 1]^T$

vector $V_2 = [1 \quad 0 \quad 0]^T$

Point $P_1 = [3 \quad 3 \quad 3]^T$

$P_2 = [1 \quad 2 \quad 3]^T$

$P_3 = [0 \quad 0 \quad 1]^T$

$\beta^* = [\beta_1^*, \beta_2^*]^T \qquad \beta^* = \dfrac{X^T y}{X^T X}$

Vector matrix $X = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \quad \left\{ X = [V_1 \quad V_2] \right\}$

$$X^T X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1\times1+1\times1+1\times1 & 1\times1+0\times1+0\times1 \\ 1\times1+0\times1+0\times1 & 1\times1+0\times0+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

$$(X^T X)^{-1} = \frac{1}{(3\times1)-(1\times1)} \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix} \quad \left\{ \begin{array}{l} \text{if } X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ X^{-1} = \dfrac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \end{array} \right.$$

$$= \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1.5 \end{bmatrix}$$

$$(X^TX)^{-1}X^T = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1.5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5\times1 - 0.5\times1 & 0.5\times1 - 0.5\times0 & 0.5\times1 - 0.5\times0 \\ -0.5\times1 + 1.5\times1 & -0.5\times1 + 1.5\times0 & -0.5\times1 + 1.5\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0.5 & 0.5 \\ 1 & -0.5 & -0.5 \end{bmatrix}$$

Projection of point P1 on the subspace is $\hat{P}_1$

$\beta^*_{P_1}$ is $(X^TX)^{-1}X^T P1$

$$= \begin{bmatrix} 0 & 0.5 & 0.5 \\ 1 & -0.5 & -0.5 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times0 + 3\times0.5 + 3\times0.5 \\ 1\times3 - 3\times0.5 - 3\times0.5 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

Projection of point P2 on the subspace is $\hat{P}_2$

$\beta^*_{P_2}$ is $(X^TX)^{-1}X^T P2$

$$= \begin{bmatrix} 0 & 0.5 & 0.5 \\ 1 & -0.5 & -0.5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times1 + 0.5\times2 + 0.5\times3 \\ 1\times1 - 0.5\times2 - 0.5\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 2.5 \\ -1.5 \end{bmatrix}$$

Projection of point P3 on the Subspace is $\hat{P}_3$

$\beta^*_{P_3}$ is $(X^T X)^{-1} X^T P_3$

$$= \begin{bmatrix} 0 & 0.5 & 0.5 \\ 1 & -0.5 & -0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times0 + 0.5\times0 + 0.5\times1 \\ 1\times0 - 0.5\times0 - 0.5\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$$

$$\hat{P}_1 = X\beta^*_{P_1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\times3 + 1\times0 \\ 1\times3 + 0\times0 \\ 1\times3 + 0\times0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

$$\hat{P}_2 = X\beta^*_{P_2} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2.5 \\ -1.5 \end{bmatrix} = \begin{bmatrix} 2.5\times1 - 1.5\times1 \\ 1\times2.5 - 0\times1.5 \\ 1\times2.5 - 1.5\times0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2.5 \\ 2.5 \end{bmatrix}$$

$$\hat{P}_3 = X\beta^*_{P_3} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 1\times0.5 - 0.5\times1 \\ 1\times0.5 - 0\times0.5 \\ 1\times0.5 - 0\times0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$$