WRITTEN QUESTION 2

Given probability of heads is $\frac{2}{3}$

$P(H) = \frac{2}{3} \Rightarrow$ Probability of tails $P(T) = \frac{1}{3}$

We need the probability of heads less than or equal to 50

$$P(H \leq 50) = P(H=50) + P(H=49) + P(H=48) + P(H=47) + \cdots + P(H=0)$$

$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

Here $p = \frac{2}{3}$

$q = \frac{1}{3}$

$n = 100$

$$P(H=50) = {}^{100}C_{50}\left(\frac{2}{3}\right)^{50}\left(\frac{1}{3}\right)^{50}$$

$$P(H=49) = {}^{100}C_{49}\left(\frac{2}{3}\right)^{49}\left(\frac{1}{3}\right)^{51}$$

$$\vdots$$

$$P(H=0) = {}^{100}C_{0}\left(\frac{2}{3}\right)^{0}\left(\frac{1}{3}\right)^{100}$$

${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$

$$\therefore P(H \leq 50) = {}^{100}C_{50}\left(\frac{2}{3}\right)^{50}\left(\frac{1}{3}\right)^{50} + {}^{100}C_{49}\left(\frac{2}{3}\right)^{49}\left(\frac{1}{3}\right)^{51} + \cdots + {}^{100}C_{0}\left(\frac{2}{3}\right)^{0}\left(\frac{1}{3}\right)^{100}$$

This computation can be performed in python as shown in the jupyter notebook below.

The computed probability for 50 or fewer heads if $P(H) = \frac{2}{3}$ is 0.00032845 (or) 0.032845 %