NUMERICAL PROBLEMS

Formulas at glance

Energy of a photon $E = h\vartheta$ -> $\mathrm{E} = \frac{\mathrm{hc}}{\lambda}$

Momentum $\boldsymbol{P} = \sqrt{\boldsymbol{2mE}}$

Kinetic energy $\mathrm{E} = \frac{\mathrm{P}^2}{2\mathrm{m}} = \frac{\mathrm{h}^2}{2\mathrm{m}\lambda^2}$

De – Broglie wavelength of photon/electron $\boldsymbol{\lambda} = \frac{\mathbf{h}}{\boldsymbol{p}} = \frac{\mathbf{h}}{\boldsymbol{mv}}$

De – Broglie wavelength of electron/any particle $\boldsymbol{\lambda} = \frac{\mathbf{h}}{\sqrt{\boldsymbol{2mE}}} = \frac{\mathbf{h}}{\sqrt{\boldsymbol{2meV}}}$

De – Broglie wavelength of accelarated electron $\boldsymbol{\lambda} = \frac{\mathbf{1.226}}{\sqrt{\boldsymbol{V}}}\ \boldsymbol{nm}$

$$p = mv_{particle}$$

$$v_{group} = v_{particle}$$

$$v_{group}\ X\ v_{phase} = c^2$$

$$\Delta x \,.\, \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \geq \frac{h}{4\pi\ X\ \Delta p}$$

$$\Delta x \geq \frac{h}{4\pi\ X\ \Delta\,(mv)} = \frac{h}{4\pi\ Xm\ \Delta v}$$

1] Calculate the De – Broglie wavelength associated with an electron having a kinetic energy of 100 eV

$$\boldsymbol{\lambda} = \frac{\mathbf{h}}{\sqrt{2mE}}$$

Data Given

$E = 100$

$E = 100\ X\ 1.602\ X\ 10^{-19}$ J

$\lambda = ?$

$$\lambda = \frac{6.63 \text{ X } 10^{-34}}{\sqrt{2\text{X } 9.1\ X 10^{-31} X\ 100\ X\ 1.602\ X\ 10^{-19}}}$$

$$\lambda = 1.227 \text{ X } 10^{-10} m$$

$$\boxed{\boldsymbol{\lambda} = \mathbf{1.227}\ \text{Å}}$$

2] The velocity of an electron of a hydrogen atom in the ground state is $2.19\ X\ 10^6\ m/s$. Calculate the wavelength of the De – Broglie waves associated with its motion

Data Given

$v = 2.19\ X\ 10^6\ m/s$ $\lambda =$?

$$\boldsymbol{\lambda} = \frac{\mathbf{h}}{\boldsymbol{mv}}$$

$$\lambda = \frac{6.63 \text{ X } 10^{-34}}{9.1\ X 10^{-31} X\ 2.19\ X\ 10^{6}}$$

$$\boxed{\boldsymbol{\lambda} = \mathbf{3.326}\ \text{Å}}$$

3] Calculate the De – Broglie wavelength of an electron accelerated under a potential difference of 100 V.

Data Given

$potential\ V = 100\ volts$

$\lambda = ?$

$$\boldsymbol{\lambda} = \frac{\mathbf{1.226}}{\sqrt{\boldsymbol{V}}}\ \boldsymbol{nm}$$

$$\lambda = \frac{1.226\ 10^{-9}}{\sqrt{100}}$$

$$\boxed{\boldsymbol{\lambda} = \mathbf{1.226}\ \text{Å}}$$

4] Calculate the de Broglie's wavelength associated with 400g cricket ball with a speed of90Km/hr.

$m = 400\ g = 0.4\ kg$

$v = \frac{90}{hr}\ km = \frac{90\ X\ 10^3}{60\ X\ 60} = 25\ m/s$

$\lambda\ = ?$

$$\lambda\ =\ \frac{h}{mv}$$

$$\lambda\ =\ \frac{6.63\ X\ 10^{-34}}{0.4\ X\ 25}$$

$$\boxed{\lambda = 6.63\ X\ 10^{-35}\ m}$$

5] Compare the de Broglie's wavelength associated with (a) 10g bullet travelling at 500m/s. (b)An electron with kinetic energy 100Mev.

$v_B = 500\ m/s$

$m_B = 10\ g = 0.01\ kg$

K.E of electron -> $100 MeV$ $E = 100\ X\ 10^6 X\ 1.602\ X\ 10^{-19}$

$\frac{\lambda_B}{\lambda_e} = ?$

$$\lambda_B\ =\ \frac{h}{mv}$$

$$\lambda_B\ =\ \frac{6.63\ X\ 10^{-34}}{0.01\ X\ 500}$$

$$\boldsymbol{\lambda_B\ =\ 1.326\ X\ 10^{-34}\ m}$$

$$\lambda_e\ =\ \frac{h}{\sqrt{2mE}}$$

$$\lambda\ =\ \frac{6.63\ X\ 10^{-34}}{\sqrt{2X\ 9.1\ X 10^{-31} X\ 100\ X\ 1.602\ X\ 10^{-19}}}$$

$$\boldsymbol{\lambda\ =\ 1.227\ X\ 10^{-10} m}$$

$$\boldsymbol{\frac{\lambda_B}{\lambda_e} = \frac{1.326\ X\ 10^{-34}}{1.227\ X\ 10^{-10}}}$$

$$\boxed{\boldsymbol{\frac{\lambda_B}{\lambda_e} = 1.080\ X\ 10^{-24}}}$$

6] Calculate the de Broglie's wavelength of proton whose kinetic energy is equal to the rest massenergy of electron and mass of proton is equal to 1836 times the mass of electron.

$m_P = 1836 X 9.1 X\ 10^{-31}$

$m_P = 1.670\ X\ 10^{-27}$

K.E of proton = rest mass energy -> K.E of proton = $m_0 c^2$ $E = 9.1 X\ 10^{-31} X\ (3\ X 10^8\ )^2$ -> $E = 8.19 X\ 10^{-14}$

$\lambda_p = ?$

$$\lambda_p = \frac{\mathbf{h}}{\sqrt{2mE}}$$

$$\lambda_p = \frac{6.63\ \mathrm{X}\ 10^{-34}}{\sqrt{2\ X\ 1.670\ X\ 10^{-27} X\ 8.19 X\ 10^{-14}}}$$

$$\boxed{\boldsymbol{\lambda = 4.00\ \mathrm{X}\ 10^{-14} m}}$$

7] Calculate the De – Broglie wavelength and momentum of an electron, if its kinetic energy is 1.5 keV.

Data Given

$E = 1.5\ keV$

$E = 1.5\ X 10^3\ X 1.602\ X\ 10^{-19}$

$\lambda = ?$

$P = ?$

$$\boldsymbol{P = \sqrt{2mE}}$$

$$P = \sqrt{2\mathrm{X}\ 9.1\ X 10^{-31} X\ 1.5\ X 10^3\ X 1.602\ X\ 10^{-19}}$$

$$\boldsymbol{P = 2.09}\ X\ 10^{-19}\ \boldsymbol{kgm/s}$$

$$\boldsymbol{\lambda = \frac{h}{P}} \rightarrow \lambda = \frac{6.63\ \mathrm{X}\ 10^{-34}}{\mathbf{2.09}\ X\ 10^{-19}}$$

$$\boxed{\boldsymbol{\lambda = 0.317}\ \text{Å}}$$

8] Calculate the De – Broglie wavelength of neutron moving with one tenth part of the velocity of light. Given the mass of neutron = $1.674\ X 10^{-27}\ kg$

$m = 1.674\ X 10^{-27}$

Velocity of neutron

$$\boldsymbol{v = \frac{1}{10}\ c}$$

$$\boldsymbol{v} = \frac{3\ X 10^8}{10}$$

$$\boldsymbol{v} = 3\ X 10^7\ m/s$$

$\lambda = ?$

$$\boldsymbol{\lambda = \frac{h}{mv}}$$

$$\lambda = \frac{6.63\ \mathrm{X}\ 10^{-34}}{1.674\ X 10^{-27} X\ 3\ X\ 10^7}$$

$$\boxed{\boldsymbol{\lambda = 1.320\ \mathrm{X}}\ 10^{-14}\mathbf{m}}$$

9] For a particle moving in a free space, prove that $\lambda\sqrt{\vartheta} = constant$ where λ id the de Broglie'swavelength and ϑ is the frequency associated with the quantum energy carried by the electron.

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{h}{m\,(\vartheta\lambda)} \qquad \text{(because} = \vartheta\lambda\text{)}$$

$$\lambda^2\vartheta = \frac{h}{m} = \lambda\sqrt{\vartheta} = \sqrt{\frac{h}{m}} \qquad (\sqrt{\frac{h}{m}} = constant)$$

$$\boxed{\boldsymbol{\lambda\sqrt{\vartheta} = Constant}}$$

10] Find the de Broglie's wavelength of an electron accelerated through a potential difference of 182V and object of mass 1Kg moving with a speed of 1m/s. compare the result and comment.

$v_{object} = 1\, m/s$

$m_{object} = 1\, kg$

$V = 182\, V$

$\frac{\lambda_e}{\lambda_O} = ?$

$$\boldsymbol{\lambda_e = \frac{1.226}{\sqrt{V}}\, nm}$$

$$\lambda_e = \frac{1.226 \text{ X } 10^{-9}}{\sqrt{182}} \rightarrow \lambda_e = 0.91\text{Å}$$

$$\lambda_O = \frac{h}{mv} = \frac{6.63 \text{ X } 10^{-34}}{1X\,1} \rightarrow \lambda_O = 6.63 \text{ X } 10^{-34}$$

$$\frac{\lambda_e}{\lambda_O} = \frac{0.91\, X\, 10^{-10}}{6.63 \text{ X } 10^{-34}}$$

$$\boxed{\boldsymbol{\frac{\lambda_e}{\lambda_O} = 1.372\, X\, 10^{23}}}$$

11] Estimate the potential difference through which a proton is needed to be accelerated so that its De – Broglie wavelength becomes equal to . 1Å. Given its mass is = $1.674\, X10^{-27}\, kg$

Data Given

$m = 1.674\, X10^{-27}$ $\lambda =$ 1Å

$\lambda = 1\, X10^{-10}\, m$

$Potential = V = ?$

$$\boldsymbol{\lambda = \frac{h}{\sqrt{2meV}}} \;\text{->}\; \lambda^2 = \frac{h^2}{2meV} \;\text{->}\; V = \frac{h^2}{2me\lambda^2}$$

$$V = \frac{\left(6.63 \text{ X } 10^{-34}\right)^2}{2 \text{ X } 1.674\, X10^{-27} X 1.602\, X\, 10^{-19}\, 1\, X(10^{-10})^2}$$

$$\boxed{\boldsymbol{V = 0.0819\, volts}}$$

12] Evaluate De – Broglie wavelength of Helium nucleus that is accelerated through 500 V. Given its mass of proton is = $1.67\ X10^{-27}\ kg$

Data Given

$m = 1.67\ X10^{-27}$

$V = 500\ volts$

$\lambda = ?$

As the helium nucleus has 2 protons and 2 neutrons

Mass of helium nucleus = $4\ m = 4\ X\ 1.67\ X\ 10^{-27}$

Since there are 2 protons, the charge on the nucleus = $2\ X\ 1.602\ X\ 10^{-19}$

$$\boldsymbol{\lambda = \frac{h}{\sqrt{2meV}}}$$

$$\lambda = \frac{6.63\ \text{X}\ 10^{-34}}{\sqrt{2\text{X}\ (4\ X\ 1.67\ X\ 10^{-27})(2\ X\ 1.602\ X\ 10^{-19})\ \text{X}500}}$$

$$\boxed{\boldsymbol{\lambda = 4.531\ \text{X}\ 10^{-13}\ m}}$$

13] A particle of mass $0.5\ MeV/c^2$ has kinetic energy 100 eV. Find its De – Broglie wavelength, where ‘c ‘is the velocity of light

Data given

$E = 100\ eV$

$E = 100\ X\ 1.602\ X\ 10^{-19}\ J$

Mass of the particle

$m = 0.5\ MeV/c^2$

$$m = \frac{0.5\ X\ 10^6 X\ 1.602\ X\ 10^{-19}}{(3\ X\ 10^8)^2}$$

$m = 8.9\ X\ 10^{-31}\ kg$

$\lambda = ?$

$$\boldsymbol{\lambda = \frac{h}{\sqrt{2mE}}}$$

$$\lambda = \frac{6.63\ \text{X}\ 10^{-34}}{\sqrt{2\text{X}\ (8.9\ X\ 10^{-31})(100\ X\ 1.602\ X\ 10^{-19})}}$$

$$\lambda = 1.241\ \text{X}\ 10^{-10}\ m$$

$$\boxed{\boldsymbol{\lambda = 1.241}\ \text{Å}}$$

14] Compare the momentum, total energy and the kinetic energy of an electron with a De-Broglie wavelength of 1Å with that of a photon with same wavelength.

Given:

$\lambda_1 = \lambda_2 = 10^{-10}\text{m}$

$\frac{P_e}{P_p} = ?$

$\frac{E_e}{E_p} = ?$

$\frac{E_{k_e}}{E_{k_p}} = ?$

$$P_e = \frac{h}{\lambda_e} = \frac{6.63 \text{ X } 10^{-34}}{10^{-10}} = 6.63 \text{ X } 10^{-24}$$

$$P_p = \frac{h}{\lambda_p} = \frac{6.63 \text{ X } 10^{-34}}{10^{-10}} = 6.63 \text{ X } 10^{-24}$$

$$\boxed{\frac{\boldsymbol{P_e}}{\boldsymbol{P_p}} = \mathbf{1}}$$

Total energy of electron = kinetic energy + rest mass energy

$$E_e = \frac{P^2}{2m} + m_0c^2$$

$$E_e = \frac{(6.63 \text{ X } 10^{-24})^2}{2 \text{ X } 9.1 \text{ X } 10^{-31}} + 9.1 \text{ X } 10^{-31}(3 \text{ X } 10^8)^2$$

$$E_e = 2.413 \text{ X } 10^{-17} + 8.19 \text{ X } 10^{-14} \text{ -> } \mathbf{E_e = 8.192 \text{ X } 10^{-14} \text{ J}}$$

Total energy of photon is given by

$$E_p = h\vartheta = \frac{hc}{\lambda} = \frac{6.63 \text{ X } 10^{-34} \text{ X } 3 \text{ X } 10^8}{10^{-10}} \text{ -> } \mathbf{E_p = 1.989 \text{ X } 10^{-15} \text{ J}}$$

$$\frac{E_e}{E_p} = \frac{8.192 \text{ X } 10^{-14}}{1.989 \text{ X } 10^{-15}} \text{ -> } \boxed{\frac{\mathbf{E_e}}{\mathbf{E_p}} = \mathbf{41.18}}$$

Kinetic energy of electron is given by $E_{k_e} = \frac{P^2}{2m}$

$$E_{k_e} = \frac{(6.63 \text{ X } 10^{-24})^2}{2 \text{ X } 9.1 \text{ X } 10^{-31}} \rightarrow E_{k_e} = 2.413 \text{ X } 10^{-17} \text{ J}$$

$$\frac{E_{k_e}}{E_{k_p}} = \frac{= 2.413 \text{ X } 10^{-17}}{1.989 \text{ X } 10^{-15}}$$

$$\boxed{\frac{\mathbf{E_{k_e}}}{\mathbf{E}} = \mathbf{0.012}}$$

15] Compare the energy of a photon with that of a neutron when both are associated with a wavelength of 1Å . Given that the mass of neutron is $1.678\ X\ 10^{-27} kg$

Given:

$$\lambda_1 = \lambda_2 = 10^{-10}\mathrm{m}$$

$$m_n = 1.678\ X\ 10^{-27} kg$$

$$\frac{\mathrm{E_p}}{\mathrm{E_n}} = ?$$

$$\mathrm{E_p} = \mathrm{h}\vartheta = \frac{\mathrm{hc}}{\lambda} = \frac{6.63\ \mathrm{X}\ 10^{-34}\ \mathrm{X}\ 3\ \mathrm{X}\ 10^{8}}{10^{-10}}$$

$$\mathbf{E_p = 1.989\ X\ 10^{-15}\ J}$$

$$\mathrm{E_n} = \frac{\mathrm{P}^2}{2\mathrm{m}} = \frac{\mathrm{h}^2}{2\mathrm{m}\lambda^2}$$

$$\mathrm{E_n} = \frac{(6.63\ \mathrm{X}\ 10^{-34})^2}{2\ \mathrm{X}\ 1.678\ X\ 10^{-27}\ \mathrm{X}\ (10^{-10})^2}$$

$$\mathbf{E_n = 1.3098\ X\ 10^{-20}\ J}$$

$$\frac{\mathrm{E_p}}{\mathrm{E_n}} = \frac{1.989\ \mathrm{X}\ 10^{-15}}{1.3098\ \mathrm{X}\ 10^{-20}}$$

$$\boxed{\frac{\mathbf{E_p}}{\mathbf{E_n}} = \mathbf{1.53\ X\ 10^{5}}}$$

16] If an electron has a De – Broglie wavelength of 2nm, find its kinetic energy and group velocity,

Given:

$$\lambda = 2nm$$

$$\lambda = 2\ X\ 10^{-9}\ m$$

$$\mathbf{E} = ?$$

$$v_{group=\,?}$$

$$\mathrm{E} = \frac{\mathrm{P}^2}{2\mathrm{m}} = \frac{\mathrm{h}^2}{2\mathrm{m}\lambda^2}$$

$$\mathrm{E} = \frac{(6.63\ \mathrm{X}\ 10^{-34})^2}{2\ \mathrm{X}\ 9.1\ X\ 10^{-31}\ \mathrm{X}\ (2\ \mathrm{X}\ 10^{-9})^2}$$

$$\mathbf{E = 6.038\ X\ 10^{-20}\ J}$$

$$E = \frac{1}{2} m v_{particle}^2 \rightarrow v_g = \sqrt{\frac{2E}{m}} \rightarrow v_{particle} = \sqrt{\frac{2\ X\ 6.038\ \mathrm{X}\ 10^{-20}}{9.1\ X\ 10^{-31}}}$$

$$v_{particle} = 3.642\ \mathrm{X}\ 10^{5}\ m/s$$

As $v_{group=} v_{particle}$

$$\boxed{\boldsymbol{v_{group} = 3.642\ X\ 10^{5}\ m/s}}$$

NUMERICALS ON PHASE VELOCITY AND GROUP VELOCITY

17] A fast moving neutron is found to have an associated De – Broglie wavelength of $2\ X\ 10^{-10} m$. Find its kinetic energy and the group velocity of the De – Broglie waves (Given $m_n = 1.675\ \mathrm{X}\ 10^{-27}$ kg).

Given:

$\lambda = 2nm$

$\lambda = 2\ X\ 10^{-9}\ m$

$m_n = 1.675\ \mathrm{X}\ 10^{-27}$ kg

$\mathbf{E} = ?$

$v_{group} = ?$

$P = \frac{h}{\lambda} = \frac{6.63\ \mathrm{X}\ 10^{-34}}{2\ X\ 10^{-9}}$ -> $\boldsymbol{P = 3.315\ X\ 10^{-24}\ kgm/s}$

$\mathrm{E} = \frac{\mathrm{P}^2}{2\mathrm{m}} = \frac{(\mathbf{3.315\ X\ 10^{-24}})^2}{2\ \mathrm{X}\ 1.675\ \mathrm{X}\ 10^{-27}}$ -> $\boxed{\boldsymbol{E = 3.280\ X\ 10^{-21}}\ \mathbf{J}}$

w.k.t $p = mv_{particle} \rightarrow v_{particle} = \frac{p}{m} \rightarrow v_{particle} = \frac{\mathbf{3.315\ X\ 10^{-24}}}{1.675\ \mathrm{X}\ 10^{-27}}$

$v_{particle} = 1.979\ \mathrm{X}\ 10^3\ m/s$

As $v_{group} = v_{particle}$ $\boxed{\boldsymbol{v_{group} = 1.979\ \mathrm{X}\ 10^3\ m/s}}$

18] A particle of mass $0.5\ MeV/c^2$ has kinetic energy 80 eV. Find its De – Broglie wavelength, Phase velocity and group velocity, where ‘c ‘is the velocity of light

Given:

$E = 80\ eV$

$E = 80\ X\ 1.602\ X\ 10^{-19}\ J$

Mass of the particle

$m = 0.5\ MeV/c^2$

$$m = \frac{0.5\ X\ 10^6 X\ 1.602\ X\ 10^{-19}}{(3\ X\ 10^8)^2}$$

$m = 8.9\ X\ 10^{-31}\ kg$

$\lambda = ?$

$v_{group} = ?$

$v_{phase} = ?$

$$\boldsymbol{\lambda = \frac{h}{\sqrt{2mE}}}$$

$$\lambda = \frac{6.63\ \mathrm{X}\ 10^{-34}}{\sqrt{2\mathrm{X}\ (8.9\ X\ 10^{-31})(80\ X\ 1.602\ X\ 10^{-19})}}$$

$$\boxed{\boldsymbol{\lambda = 1.388\ \mathrm{X}\ 10^{-10}\ m}}$$

$\lambda = \frac{h}{P} = \frac{h}{mv_{particle}}$

As $v_{group} = v_{particle}$, $v_{group} = \frac{h}{m\lambda}$

$$v_{group} = \frac{6.63\ \mathrm{X}\ 10^{-34}}{8.9\ X\ 10^{-31}\ X\ 1.388\ \mathrm{X}\ 10^{-10}}$$

$$\boxed{\boldsymbol{v_{group} = 5.367\ \mathrm{X}\ 10^6\ m/s}}$$

W.K.T $v_{group}\ X\ v_{phase} = c^2$ -> $v_{phase} = \frac{c^2}{v_{group}} = \frac{(3\ X\ 10^8)^2}{5.367\ \mathrm{X}\ 10^6}$

$$\boxed{\boldsymbol{v_{phase} = 1.676\ \mathrm{X}\ 10^{10}\ m/s}}$$

19] If the group velocity of a particle is $3\ X\ 10^6\ m/s$. What would be its phase velocity

$$v_{phase} = \frac{c^2}{v_{group}} = \frac{(3\ X\ 10^8)^2}{3\ \mathrm{X}\ 10^6}$$

$$\boxed{\boldsymbol{v_{phase} = 3\ \mathrm{X}\ 10^{10}\ m/s}}$$

NUMERICALS ON HEISENBERG UNCERTAINTY PRINCIPLE

20] The position and momentum of an electron with energy 0.5 keV are determined. What is the minimum percentage of uncertainty in its momentum, if the uncertainty in the measurement of its position is 0.5 Å

$$E = 0.5\ keV$$

$$E = 0.5\ X\ 10^3\ X 1.602\ X\ 10^{-19}$$

$$\Delta x = 0.5\ \text{Å}$$

$$\Delta x = 0.5\ X\ 10^{-10} m$$

Percentage of uncertainty in momentum = ?

$$p = \sqrt{2mE}$$

$$p = \sqrt{2\ X\ 9.1\ X\ 10^{-31} X\ 0.5\ X\ 10^3\ X 1.602\ X\ 10^{-19}}$$

$$\boldsymbol{p = 1.207\ X\ 10^{-23}\ kgm/s}$$

$$\Delta x\ .\ \Delta p \geq \frac{h}{4\pi}$$

$$\Delta p \geq \frac{6.63\ X\ 10^{-34}}{4\ X\ 3.14\ X\ 0.5\ X\ 10^{-10}}$$

$$\boldsymbol{\Delta p \geq 1.055\ X\ 10^{-24}}$$

% uncertainty in momentum = $\frac{\Delta p}{P}\ X\ 100\ \%$

% uncertainty in momentum = $\frac{1.055\ X\ 10^{-24}}{1.207\ X\ 10^{-23}}\ X\ 100\ \%$

% uncertainty in momentum = 8.74 %

21] In a measurement of position and momentum, the inherent uncertainty involved in the determination of position of electron is $\frac{6.63\ X\ 10^{-10}}{4\pi}\ m$, then what would be the uncertainty in the determination of its momentum.

$$\Delta x = \frac{6.63\ X\ 10^{-10}}{4\pi}$$

$$\Delta p = ?$$

$$\Delta x\ .\ \Delta p \geq \frac{h}{4\pi} \rightarrow \Delta p \geq \frac{h}{4\pi\ X\ \Delta x}$$

$$\Delta p \geq \frac{6.63\ X\ 10^{-34}}{4\ X\ \pi\ X\ \frac{6.63\ X\ 10^{-10}}{4\pi}}$$

$$\boxed{\Delta p \geq \boldsymbol{10^{-24}\ kg\ m/s}}$$

22] In a measurement of position and momentum that involved an uncertainty of 0.003%, the speed of an electron was found to be 800 m/s. Calculate the corresponding uncertainty that arises in the determination of posi

$$v = 800\, m/s$$

Uncertainty in velocity = 0.003 %

$$\Delta v = 800\ X\ \frac{0.003}{100}$$

$$\Delta v = 0.024\, m/s$$

$$\Delta x\, .\, \Delta p\ \geq \frac{h}{4\pi}$$

$$\Delta x\ \geq \frac{h}{4\pi\ X\ \Delta p}$$

$$\Delta x\ \geq \frac{h}{4\pi\ X\ \Delta\,(mv)} = \frac{h}{4\pi\ Xm\ \Delta v}$$

$$\Delta x\ \geq \frac{6.63\ X\ 10^{-34}}{4\ X\ \pi\ X\ 9.1\ X\ 10^{-31}\ X\ 0.024}$$

$$\boxed{\boldsymbol{\Delta x\ \geq\ 2.416\ X\ 10^{-3}\ m}}$$

23] In a measurement of position and velocity of an electron moving with a speed of $6\ X\ 10^5\ m/s$. Calculate the highest accuracy with which its position could be determined. If the inherent error in the measurement of its velocity is 0.01 % for the speed.

$$v = 6\ X\ 10^5\ m/s$$

Uncertainty in velocity = 0.01%

$$\Delta v = 6\ X\ 10^5\ \ X\ \frac{0.01}{100}$$

$$\Delta v = 60\ m/s$$

$$\Delta x\, .\, \Delta p\ \geq \frac{h}{4\pi}$$

$$\Delta x\ \geq \frac{h}{4\pi\ X\ \Delta p}$$

$$\Delta x\ \geq \frac{h}{4\pi\ X\ \Delta\,(mv)} = \frac{h}{4\pi\ Xm\ \Delta v}$$

$$\boxed{\boldsymbol{\Delta x\ \geq 9.667\ X\ 10^{-7}\ m}}$$

24] A spectral line of wavelength 5461Å has width of 10^{-4}Å. Evaluate the minimum time spent by the electrons in the upper energy state.

$\lambda = 5461$ Å

Width of spectral line or uncertainty in wavelength is $\Delta\lambda = 10^{-4}$ Å

$$E = h\vartheta \;=\; \frac{hc}{\lambda}$$

(Diffrentiating w.r .t λ), $\Delta E = -\,\frac{hc}{\lambda^2}\,.\,\Delta\lambda$

W.K.T $\Delta E\,.\,\Delta t \;\geq\; \frac{h}{4\pi}$ -> $\Delta t \;\geq\; \frac{h}{4\pi\; X\; \Delta E}$

$$\Delta t \;\geq\; \frac{h}{4\pi\; X\, \frac{hc}{\lambda^2}\; .\Delta\lambda}$$ (neglecting the negative sign)

$$\Delta t \;\geq\; \frac{\lambda^2}{4\pi\; X\;\; \Delta\lambda\; X\; c}$$

$$\Delta t \;\geq\; \frac{(5461\; X\; 10^{-10})^2}{4\; X\; 3.14\; X\;\; 5461\; X\; 10^{-14} X\; 3\; X\; 10^8}$$

$$\boxed{\Delta t \;\geq 7.9\;\; X\;\; 10^{-9}\; seconds}$$

25] The inherent uncertainty in the measurement of time spent by Iridium – 191 nuclei in the exited state is found to be $1.4\; X 10^{-10} s$. Estimate the uncertainty that results in its energy in the exited state.

$\Delta t = \; 1.4\; X\;\; 10^{-10}\; seconds$
$\Delta E = ?$

W.K.T $\Delta E\,.\,\Delta t \;\geq\; \frac{h}{4\pi}$ -> $\Delta E \;\geq\; \frac{h}{4\pi\; X\; \Delta t}$

$$\Delta E \;\geq\; \frac{6.63\; X\; 10^{-10}}{4\; X\; 3.14\; X\;\; 5461\; X\; 1.4\; X\;\; 10^{-10}}$$

$$\Delta E \;\geq\; \frac{3.770\; X\; 10^{-25}}{1.602\; X\;\; 10^{-19}}\;\; \mathrm{J}$$

$$\boxed{\Delta E \;\geq 2.353\; X\; 10^{-6}\; eV}$$

26] Write down the uncertainty relation connecting position and momentum variables. Show that this relation can be written as $\Delta x \,.\, \Delta\lambda \ \geq \ \lambda^2/4\pi$

(Diffrentiating w.r .t λ),

W.K.T $\Delta x \,.\, \Delta p \ \geq \frac{h}{4\pi}$ -> $\Delta x \ \geq \dfrac{h}{4\pi \ X \ \Delta p} \rightarrow \Delta x \ \geq \dfrac{h}{4\pi \ X \ \Delta\frac{h}{\lambda}}$

$\Delta x \ \geq \dfrac{h}{4\pi h \ (\frac{1}{\lambda^2})}$ (neglecting the negative sign)

$$\boldsymbol{\Delta x \,.\, \Delta\lambda \ \geq \ \lambda^2/4\pi}$$

27] Show that $\Delta t \,.\, \Delta\lambda \ \geq \dfrac{2}{4\pi c}$ for a photon

W . K . T $E = h\vartheta \ = \dfrac{hc}{\lambda}$

(Diffrentiating w.r .t λ), $\Delta E = - \dfrac{hc}{\lambda^2} \,.\, \Delta\lambda$

W.K.T $\Delta E \,.\, \Delta t \ \geq \frac{h}{4\pi}$ -> $\Delta t \ \geq \frac{h}{4\pi \ X \ \Delta E}$

$\Delta t \ \geq \dfrac{h}{4\pi \ X \frac{hc}{\lambda^2} \ .\Delta\lambda}$ (neglecting the negative sign)

$$\boxed{\Delta t \,.\, \Delta\lambda \ \geq \dfrac{\lambda^2}{4\pi c}}$$

28] An electron is confined to a box of length 10^{-9}m, calculate the minimum uncertainty in its velocity.

$$\Delta x \,.\, \Delta p \ \geq \frac{h}{4\pi}$$

$$\Delta x \ \geq \frac{h}{4\pi \ X \ \Delta p}$$

$$\Delta x \ \geq \frac{h}{4\pi \ X \ \Delta \ (mv)} = \frac{h}{4\pi \ Xm \ \Delta v} \rightarrow \Delta v = \frac{h}{4\pi \ Xm \ \Delta x}$$

$$\Delta v \ \geq \frac{6.63 \ X \ 10^{-34}}{4 \ X \ \pi \ X \ 9.1 \ X \ 10^{-31} \ X \ 10^{-9}} \rightarrow \boldsymbol{\Delta v \ \geq \ 57.9 \ m/s}$$

29] An excited atom has an average time of 10^{-8}s. During this period, it emits a photon and returnsto the ground state. What is the minimum uncertainty in the frequency of photon.

$\Delta t = 10^{-8}\, seconds$ W.K.T $\Delta E\, .\Delta t \geq \frac{h}{4\pi}$ -> $\Delta\,(h\vartheta).\Delta t \geq \frac{h}{4\pi} \rightarrow \Delta\vartheta.\Delta t \geq \frac{h}{4\pi h}$

$\Delta\boldsymbol{\vartheta}$ = ?

$$\Delta\vartheta \geq \frac{1}{4\pi\, \Delta t}$$

$$\Delta\vartheta \geq \frac{1}{4\pi\, \Delta t}$$

$$\Delta\vartheta \geq \frac{1}{4\, X\, 3.14\, X\, 10^{-8}}$$

$$\boxed{\Delta\boldsymbol{\vartheta} \geq \mathbf{8\, X\, 10^{6}}\ \boldsymbol{Hz}}$$