Assignment-1

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Download all python codes from here

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https://github.com/rithvikreddy6300/
Assignment-1/blob/main/Assignment-1.py
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and latex-tikz codes from

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https://github.com/rithvikreddy6300/
Assignment-1/blob/main/Assignment-1.
tex
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QUESTION-4.10

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X)?

SOLUTION

The question can be seen as choosing a number first from 1 to 6 numbers and then choosing one more from the remaining 5 numbers, Let X_1 be the 1^{st} numbers drawn randomly from 1 to 6 and X_2be the 2^{nd} number drawn from remaining and $X = \max\ (X_1, X_2)$

$$Pr(X_1 = n_1) = \begin{cases} \frac{1}{6}, & \text{if } 1 \leq n_1 \leq 6 \\ 0, & \text{otherwise} \end{cases}$$

$$Pr(X_2 = n_2) = \begin{cases} \frac{1}{5}, & \text{if } 1 \leq n_2 \leq 6 \text{ and } n_2 \neq n_1 \\ 0, & \text{otherwise} \end{cases}$$

let $\max\ (X_1, X_2) = i$ and $Pr(X = i)$ denotes the probability that $X = \max\ (X_1, X_2) = i$

$$\begin{aligned} Pr(X = i) = Pr(X_1 = i \text{ and } X_2 < i) \\ + Pr(X_2 = i \text{ and } X_1 < i) \end{aligned} \tag{1}$$

since choosing of X_1, X_2 are independent events, so we can write

$$Pr(X_1 \text{ and } X_2) = Pr(X_1)Pr(X_2)$$

Substituting this in (1) gives us

$$\begin{aligned} Pr(X = i) = Pr(X_1 = i)Pr(X_2 < i) + \\ Pr(X_2 = i)Pr(X_1 < i) \end{aligned} \tag{2}$$

$$\implies Pr(X = i) = \frac{1}{6} \times \frac{(i-1)}{5} + \frac{(i-1)}{6} \times \frac{1}{5} \tag{3}$$

$$\implies Pr(X = n) = \frac{(i-1)}{15} \tag{4}$$

The expectation value of X represented by E(X) is given by

$$E(X) = \sum_{i=1}^{6} Pr(X = i) \times i$$

$$\implies E(X) = \sum_{i=1}^{6} \frac{(i-1)}{15} \times i \tag{5}$$

$$\implies E(X) = \sum_{i=1}^{6} \frac{(i^2 - i)}{15} \tag{6}$$

$$\implies E(X) = \frac{1}{15} \sum_{i=1}^{6} i^2 - \frac{1}{15} \sum_{i=1}^{6} i \tag{7}$$

$$\implies E(X) = \frac{1}{15} \times 91 - \frac{1}{15} \times 21 \tag{8}$$

$$\implies E(X) = \mathbf{4.6667} \tag{9}$$

Therefore the expectation value of X,
E(X)= **4.6667**.