

# GASEOUS STATE

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Download all python codes from here

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https://github.com/rithvikreddy6300/  
Assignment-1/blob/main/Assignment-1.py
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and latex-tikz codes from

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https://github.com/rithvikreddy6300/  
Assignment-1/blob/main/Assignment-1.tex
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## QUESTION-4.10

Two numbers are selected at random (without replacement) from the first six positive integers. Let  $X$  denote the larger of the two numbers obtained. Find  $E(X)$ ?

## SOLUTION

Let  $X_1, X_2$  be the  $1^{st}, 2^{nd}$  numbers drawn randomly from 1 to 6 and  $X = \max(X_1, X_2)$   
let  $\max(X_1, X_2) = n$ ,  $X_i \in \{1, 2, 3, 4, 5, 6\}, i = 1, 2$  so  $X \in \{1, 2, 3, 4, 5, 6\}$ , The probability mass function is

$$p_{X_i}(n) = Pr(X_i = n) = \begin{cases} \frac{1}{6}, & \text{if } 1 \leq n \leq 6 \\ 0, & \text{otherwise} \end{cases}$$
$$p_X(n) = Pr(\max(X_1, X_2) = n)$$
$$= Pr(X_1 = n \text{ and } X_2 < n) + Pr(X_2 = n \text{ and } X_1 < n) + Pr(X_1 = X_2 = n) \quad (1)$$

Since choosing of  $X_1, X_2$  are independent events we can write

$$Pr(X_1 \text{ and } X_2) = Pr(X_1).Pr(X_2)$$

Substituting this in (1) gives us

$$p_X(n) = Pr(X_1 = n).Pr(X_2 < n) + Pr(X_2 = n).Pr(X_1 < n) + Pr(X_1 = n).Pr(X_2 = n) \quad (2)$$

$$\Rightarrow p_X(n) = Pr(X = n) = \frac{1}{6} \cdot \frac{(n-1)}{6} + \frac{1}{6} \cdot \frac{(n-1)}{6} + \frac{1}{6} \cdot \frac{1}{6}$$

$$\Rightarrow p_X(n) = Pr(X = n) = \frac{(2n-1)}{36}$$

The expectation value of  $X$  represented by  $E(X)$  is given by

$$E(x) = \sum_{X=1}^6 Pr(X = n).X$$

$$\Rightarrow E(X) = \sum_{X=1}^6 \frac{(2X-1)}{36} \cdot X \quad (3)$$

$$\Rightarrow E(X) = \sum_{X=1}^6 \frac{(2X^2 - X)}{36} \quad (4)$$

$$\Rightarrow E(X) = \frac{2}{36} \cdot \sum_{X=1}^6 X^2 - \frac{1}{36} \sum_{X=1}^6 X \quad (5)$$

$$\Rightarrow E(X) = \frac{2}{36} \cdot 91 - \frac{1}{36} \cdot 21 \quad (6)$$

$$\Rightarrow E(X) = \mathbf{4.4722} \quad (7)$$

Therefore the expectation value of  $X$ ,  $E(X) = \mathbf{4.4722}$ .