Assignment 1

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QUESTION

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X)?

SOLUTION

Let X_1, X_2 be the $1^{st}, 2^{nd}$ numbers drawn randomly from 1 to 6 and X = max (X_1, X_2)
let max (X_1, X_2)=n let $X_i \in \{1,2,3,4,5,6\}, i = 1,2$ so $X \in \{1,2,3,4,5,6\}$, The probability mass function is

$$p_{X_i}(n) = Pr(X_i = n) = \begin{cases} \frac{1}{6}, if 1 \leq n \leq 6 \\ 0, otherwise \end{cases}$$

$$p_X(n) = Pr(max(X_1, X_2) = n)$$

$$= Pr(X_1 = n \text{ and } X_2 < n) + Pr(X_2 = n \text{ and } X_1 < n) + Pr(X_1 = X_2 = n) \tag{0.0.1}$$

Since choosing of X_1, X_2 are independent events we can write

$$Pr(X_1 \text{ and } X_2) = Pr(X_1).Pr(X_2)$$

Substituting this in (0.0.1) gives us

$$p_X(n) = Pr(X_1 = n).Pr(X_2 < n) + Pr(X_2 = n).Pr(X_1 < n) + Pr(X_1 = n).Pr(X_2 = n)$$

$$\implies p_X(n) = Pr(X = n) = \frac{1}{6}.\frac{(n-1)}{6} + \frac{1}{6}.\frac{(n-1)}{6} + \frac{1}{6}.\frac{1}{6}$$

$$\implies p_X(n) = Pr(X = n) = \frac{(2n-1)}{36}$$

The expectation value of X represented by E(X) is given by

$$E(x) = \sum_{X=1}^{6} Pr(X = n).X$$

$$\implies E(X) = \sum_{X=1}^{6} \frac{(2X-1)}{36}.X$$

$$\implies E(X) = \sum_{X=1}^{6} \frac{(2X^2 - X)}{36}$$

$$\implies E(X) = \frac{2}{36}.\sum_{X=1}^{6} X^2 - \frac{1}{36}\sum_{X=1}^{6} X$$

$$\implies E(X) = \frac{2}{36}.91 - \frac{1}{36}.21$$

$$\implies E(X) = 4.4722$$