Assignment-2

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Download all python codes from here

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https://github.com/rithvikreddy6300/
Assignment-2/blob/main/Assignment-2.py
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and latex-tikz codes from

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https://github.com/rithvikreddy6300/
Assignment-2/blob/main/Assignment-2.
tex
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Gate Problem-77

If a random variable X assumes only positive integral values, with the probability

$$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, ...$$

(A) $\frac{2}{9}$ (C) 1

(B) $\frac{2}{3}$ (D) $\frac{3}{2}$

Then E(X) is ?

SOLUTION

Let Y={0,1} be a set of random variables of a Bernoulli's distribution with 0 representing a loss and 1 a win and let $Y_i \in Y$ for i=0,1,2,3...,Y_i is the outcome of i^{th} try .

For given bernouli's trail $p = \frac{2}{3}$ and $q = 1-p = \frac{1}{3}$. The given probability distribution is

$$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}$$
$$\implies P(X = x) = p(1-p)^{x-1}$$
$$\implies P(X = x) = P(Y_{i=x} = 1)P(Y_{i<x} = 0)^{x-1}$$

The expectation value of X represented by E(X) is given by

$$E(X) = \sum_{i=1}^{\infty} Pr(x = i) \times i$$

Let S=E(X),

$$\implies E(X) = S = \sum_{i=1}^{\infty} Pr(x = i) \times i \tag{1}$$
$$\implies S = \sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times i \tag{2}$$
$$\implies S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times i \tag{3}$$

Multiplying (2) with $\frac{1}{3}$ on both sides gives

$$\frac{1}{3}S = \sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i} \times i \tag{4}$$

In (3) $\sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i} \times i$ can be written as $\sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times (i-1)$

$$\implies \frac{1}{3}S = \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times (i-1) \tag{5}$$

$$\text{(3)-(5) gives :}\frac{2}{3}S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times (i-(i-1)) \tag{6}$$
$$\implies \frac{2}{3}S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \tag{7}$$
$$\implies S = 1 + \sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^{i} \tag{8}$$
$$\implies S = 1 + \frac{1/3}{1-\frac{1}{3}} \tag{9}$$
$$\implies S = \frac{3}{2} \tag{10}$$

From (10) we can say that the expectation value of X given by E(X)=S=$\frac{3}{2}$ **(Option D).**

The theoretical vs simulated probabilities are as follows,