

# Assignment-2

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Download all python codes from here

<https://github.com/rithvikreddy6300/Assignment-2/blob/main/Assignment-2.py>

and latex-tikz codes from

<https://github.com/rithvikreddy6300/Assignment-2/blob/main/Assignment-2.tex>

## Gate Problem-77

If a random variable X assumes only positive integral values, with the probability

$$P(X = x) = \frac{2}{3} \left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, \dots$$

(A)  $\frac{2}{9}$  (C) 1

(B)  $\frac{2}{3}$  (D)  $\frac{3}{2}$

Then E(X) is ?

## SOLUTION

Let  $X = \{0, 1\}$  be a set of random variables of a Bernoulli's distribution with 0 representing a loss and 1 a win, probability of loosing =  $\frac{1}{3}$  and probability of winning is  $\frac{2}{3}$ .

The given probability distribution  $P(x=i)$  can be seen as the probability of winning a game at  $i^{th}$  try for the first time. Because to win at  $i^{th}$  try for the first time you have to loose for first  $i-1$  times whose probability is  $\left(\frac{1}{3}\right)^{i-1}$  and win at  $i^{th}$  try whose probability is  $\frac{2}{3}$

Given that  $P(x = i) = \frac{2}{3} \left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, \dots$

The expectation value of X represented by E(X) is given by

$$E(X) = \sum_{i=1}^{\infty} Pr(x = i) \times i$$

Let  $S = E(X)$ ,

$$\Rightarrow E(X) = S = \sum_{i=1}^{\infty} Pr(x = i) \times i \quad (1)$$

$$\Rightarrow S = \sum_{i=1}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \times i \quad (2)$$

$$\Rightarrow S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \times i \quad (3)$$

Multiplying (2) with  $\frac{1}{3}$  on both sides gives

$$\frac{1}{3}S = \sum_{i=1}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^i \times i \quad (4)$$

In (3)  $\sum_{i=1}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^i \times i$  can be written as  $\sum_{i=2}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \times (i-1)$

$$\Rightarrow \frac{1}{3}S = \sum_{i=2}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \times (i-1) \quad (5)$$

$$(3)-(5) \text{ gives : } \frac{2}{3}S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \times (i - (i-1)) \quad (6)$$

$$\Rightarrow \frac{2}{3}S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \quad (7)$$

$$\Rightarrow S = 1 + \sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^i \quad (8)$$

$$\Rightarrow S = 1 + \frac{1/3}{1 - \frac{1}{3}} \quad (9)$$

$$\Rightarrow S = \frac{3}{2} \quad (10)$$

From (10) we can say that the expectation value of X given by  $E(X) = S = \frac{3}{2}$  (Option D).