Assignment-2

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Download all python codes from here

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https://github.com/rithvikreddy6300/
Assignment-2/blob/main/Assignment-2.py
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and latex-tikz codes from

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https://github.com/rithvikreddy6300/
Assignment-2/blob/main/Assignment-2.
tex
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Gate Problem-77

If a random variable X assumes only positive integral values, with the probability

$$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, ...$$

(A) $\frac{2}{9}$ (C) 1

(B) $\frac{2}{3}$ (D) $\frac{3}{2}$

Then E(X) is ?

SOLUTION

Let X={0,1} be a set of random variables of a Bernoulli's distribution with 0 representing a loss and 1 a win, probability of loosing $=\frac{1}{3}$ and probability of winning is $\frac{2}{3}$.

The given probability distribution P(x=i) can be seen as the probability of winning a game at i^{th} try for the first time. Because to win at i^{th} try for the first time you have to loose for first i-1 times whose probability is $\left(\frac{1}{3}\right)^{i-1}$ and win at i^{th} try whose probability is 2/3

Given that $P(x = i) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, ...$

The expectation value of X represented by E(X) is given by

$$E(X) = \sum_{i=1}^{\infty} Pr(x = i) \times i$$

Let S=E(X),

$$\implies E(X) = S = \sum_{i=1}^{\infty} Pr(x = i) \times i \quad (1)$$

$$\implies S = \sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times i \quad (2)$$

$$\implies S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times i \quad (3)$$

Multiplying (2) with $\frac{1}{3}$ on both sides gives

$$\frac{1}{3}S = \sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i} \times i \quad (4)$$

In (3) $\sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i} \times i$ can be written as $\sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times (i-1)$

$$\implies \frac{1}{3}S = \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times (i-1) \quad (5)$$

$$\text{(3)-(5) gives :}\frac{2}{3}S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times (i-(i-1)) \quad (6)$$

$$\implies \frac{2}{3}S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \quad (7)$$

$$\implies S = 1 + \sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^{i} \quad (8)$$

$$\implies S = 1 + \frac{1/3}{1-\frac{1}{3}} \quad (9)$$

$$\implies S = \frac{3}{2} \quad (10)$$

From (10) we can say that the expectation value of X given by E(X)=S=$\frac{3}{2}$ **(Option D).**

In order to simulate the above question in python I used bernouli.rvs function to generate an array of size i with a probability of an element being 1 is 2/3, I did the same for say 1000 times and each time I checked for an array whose first (i-1) terms are 0 and i^{th} term is 1 as only this array will contribute towards P(i) dividing the number of such arrays with 1000 gives me P(i). I repeated the process for i=0 to 1000. Then found the P(i) for i=0 to 1000 and stored them in an array b to latter use them to calculate the Expectation value.
The theoretical vs simulated probabilities are as follows,