Assignment-2

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Download all python codes from here

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https://github.com/rithvikreddy6300/
Assignment-2/tree/main/code
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and latex-tikz codes from

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https://github.com/rithvikreddy6300/
Assignment-2/blob/main/Assignment-2.
tex
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Gate Problem-77

If a random variable X assumes only positive integral values, with the probability

$$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, ...$$

(A) $\frac{2}{9}$ (C) 1

(B) $\frac{2}{3}$ (D) $\frac{3}{2}$

Then E(X) is ?

SOLUTION

Let Y={0,1} be a set of random variables of a Bernoulli's distribution with 0 representing a loss and 1 a win and let $Y_i \in Y$ for i=1,2,3...,Y_i is the outcome of i^{th} try of choosing 0 or 1 from Y.
So the Random variable X is generated by assigning value of i to X where $Y_i = 1$ for the first time.

$$X = \{x : Y_{i=x} = 1, Y_{i<x} = 0\}$$
$$\implies X = \{Y_1 = 0, Y_2 = 0, Y_3 = 0, ..., Y_x = 1\}$$

For given bernouli's trail $p = \frac{2}{3}$ and $q = 1 - p = \frac{1}{3}$.
The given probability distribution is

$$P(X = x) = P(Y_{i=x} = 1)P(Y_{i<x} = 0)$$
$$\implies P(X = x) = p(1 - p)^{x-1}$$
$$\implies P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}$$

The expectation value of X represented by E(X) is given by

$$E(X) = \sum_{i=1}^{\infty} Pr(x = i) \times i$$

Let S=E(X),

$$\implies E(X) = S = \sum_{i=1}^{\infty} Pr(x = i) \times i \tag{1}$$
$$\implies S = \sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times i \tag{2}$$
$$\implies S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times i \tag{3}$$

Multiplying (2) with $\frac{1}{3}$ on both sides gives

$$\frac{1}{3}S = \sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i} \times i \tag{4}$$

In (3) $\sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i} \times i$ can be written as $\sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times (i-1)$

$$\implies \frac{1}{3}S = \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times (i-1) \tag{5}$$

(3)-(5) gives :
$$\frac{2}{3}S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \times (i - (i-1)) \tag{6}$$
$$\implies \frac{2}{3}S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \tag{7}$$
$$\implies S = 1 + \sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^{i} \tag{8}$$
$$\implies S = 1 + \frac{1/3}{1 - \frac{1}{3}} \tag{9}$$
$$\implies S = \frac{3}{2} \tag{10}$$

The Variance Var(X) is given by $\sum x^2P(x) - E(X)$ for the given distribution,

$$Var(X) = \sum_{i=1}^{\infty} i^2 P(x=i) - E(X) \tag{11}$$

$$\text{let } S = \sum_{i=1}^{\infty} i^2 P(x=i) = \sum_{i=1}^{\infty} i^2 \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \tag{12}$$

$$S/3 = \sum_{i=1}^{\infty} i^2 \frac{2}{3}\left(\frac{1}{3}\right)^{i} = \sum_{i=0}^{\infty} i^2 \frac{2}{3}\left(\frac{1}{3}\right)^{i} \tag{13}$$

$$= \sum_{i=1}^{\infty} (i-1)^2 \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \tag{14}$$

(12)-(14) gives us

$$\frac{2S}{3} = \sum_{i=1}^{\infty} (i^2 - (i-1)^2) \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \tag{15}$$

$$S = \sum_{i=1}^{\infty} (2i-1)\left(\frac{1}{3}\right)^{i-1} \tag{16}$$

$$\implies S = 3\sum_{i=1}^{\infty} \frac{2}{3}\left(\frac{1}{3}\right)^{i-1} i - \sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^{i-1} \tag{17}$$

$$\implies S = 3E(X) - \frac{1}{1-1/3} \tag{18}$$

$$\implies S = \frac{9}{2} - \frac{3}{2} = 3 \tag{19}$$

From (19) and (11) we can write

$$Var(X) = 3 - \frac{3}{2} = \frac{3}{2}$$

From (10) we can say that the expectation value of X given by E(X)=S=$\frac{3}{2}$ and $Var(X) = \frac{3}{2}$ **(Option D).**

The theoretical vs simulated probabilities are as follows,