Assignment-2

S. RITHVIK REDDY- cs20btech11049

Download all python codes from here

```
NA
```

and latex-tikz codes from

```
https://github.com/rithvikreddy6300/
Assignment-2/blob/main/Assignment-2.
tex
```

Gate Problem-77

If a random variable X assumes only positive integral values, with the probability

$$P(X=x)=\frac{2}{3}\left(\frac{1}{3}\right)^{x-1},x=1,2,3,...$$

(A) $\frac{2}{9}$ (C) 1

(B) $\frac{2}{3}$ (D) $\frac{3}{2}$

Then E(X) is ?

SOLUTION

Given that $P(X=x)=\frac{2}{3}\left(\frac{1}{3}\right)^{x-1},x=1,2,3,...$

The expectation value of X represented by E(X) is given by

$$E(X)=\sum_{i=1}^{\infty}Pr(x=i)\times i$$

Let S=E(X),

$$\implies E(X)=S=\sum_{i=1}^{\infty}Pr(x=i)\times i \quad (1)$$

$$\implies S=\sum_{i=1}^{\infty}\frac{2}{3}\left(\frac{1}{3}\right)^{i-1}\times i \quad (2)$$

$$\implies S=\frac{2}{3}+\sum_{i=2}^{\infty}\frac{2}{3}\left(\frac{1}{3}\right)^{i-1}\times i \quad (3)$$

Multiplying (2) with $\frac{1}{3}$ on both sides gives

$$\frac{1}{3}S=\sum_{i=1}^{\infty}\frac{2}{3}\left(\frac{1}{3}\right)^{i}\times i \quad (4)$$

In (3) $\sum_{i=1}^{\infty}\frac{2}{3}\left(\frac{1}{3}\right)^{i}\times i$ can be written as $\sum_{i=2}^{\infty}\frac{2}{3}\left(\frac{1}{3}\right)^{i-1}\times(i-1)$

$$\implies \frac{1}{3}S=\sum_{i=2}^{\infty}\frac{2}{3}\left(\frac{1}{3}\right)^{i-1}\times(i-1) \quad (5)$$

$$\text{(3)-(5) gives :}\frac{2}{3}S=\frac{2}{3}+\sum_{i=2}^{\infty}\frac{2}{3}\left(\frac{1}{3}\right)^{i-1}\times(i-(i-1)) \quad (6)$$

$$\implies \frac{2}{3}S=\frac{2}{3}+\sum_{i=2}^{\infty}\frac{2}{3}\left(\frac{1}{3}\right)^{i-1} \quad (7)$$

$$\implies S=1+\sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^{i} \quad (8)$$

$$\implies S=1+\frac{1/3}{1-\frac{1}{3}} \quad (9)$$

$$\implies S=\frac{3}{2} \quad (10)$$

From (10) we can say that the expectation value of X given by E(X)=S=$\frac{3}{2}$ **(Option D).**

Comment : There is no simulation or graph possible for this question as the probability P(X) is fixed and the sample space is a countable infinite set.