

Assignment-2

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Download all python codes from here

NA

and latex-tikz codes from

<https://github.com/rithvikreddy6300/Assignment-2/blob/main/Assignment-2.tex>

Gate Problem-77

If a random variable X assumes only positive integral values, with the probability

$$P(X = x) = \frac{2}{3} \left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, \dots$$

(A) $\frac{2}{9}$ (C) 1

(B) $\frac{2}{3}$ (D) $\frac{3}{2}$

SOLUTION

Given that $P(X = x) = \frac{2}{3} \left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, \dots$

The expectation value of X represented by E(X) is given by

$$E(X) = \sum_{i=1}^{\infty} Pr(x = i) \times i$$

Let $S = E(X)$,

$$\Rightarrow E(X) = S = \sum_{i=1}^{\infty} Pr(x = i) \times i \quad (1)$$

$$\Rightarrow S = \sum_{i=1}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \times i \quad (2)$$

$$\Rightarrow S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \times i \quad (3)$$

Multiplying (3) with $\frac{1}{3}$ on both sides gives

$$\frac{1}{3}S = \sum_{i=1}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^i \times i \quad (4)$$

In (3) $\sum_{i=1}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^i \times i$ can be written as

$$\sum_{i=2}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \times (i-1)$$

$$\Rightarrow \frac{1}{3}S = \sum_{i=2}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \times (i-1) \quad (5)$$

$$(3)-(5) \text{ gives : } \frac{2}{3}S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \times (i - (i-1)) \quad (6)$$

$$\Rightarrow \frac{2}{3}S = \frac{2}{3} + \sum_{i=2}^{\infty} \frac{2}{3} \left(\frac{1}{3}\right)^{i-1} \quad (7)$$

$$\Rightarrow S = 1 + \sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^i \quad (8)$$

$$\Rightarrow S = 1 + \frac{1/3}{1 - \frac{1}{3}} \quad (9)$$

$$\Rightarrow S = \frac{3}{2} \quad (10)$$

From (10) we can say that the expectation value of X given by $E(X) = S = \frac{3}{2}$ (**Option D**).

Comment : There is no simulation or graph possible for this question as the probability P(X) is fixed and the sample space is a countable infinite set.