## Assignment-3

## S. Rithvik Reddy - cs20btech11049

Download all python codes from

https://github.com/rithvikreddy6300/Assignment-3/ tree/main/codes

and latex-tikz codes from

https://github.com/rithvikreddy6300/Assignment-3/ blob/main/Assignment-3.tex

PROBLEM-(GATE 2010 (CS) Q-27)

What is the probability that a divisor of  $10^{99}$  is a multiple of  $10^{96}$ ?

(A) 
$$\frac{1}{625}$$
 (B)  $\frac{4}{625}$  (C)  $\frac{12}{625}$  (D)  $\frac{16}{625}$ 

(B) 
$$\frac{4}{62^4}$$

(C) 
$$\frac{12}{625}$$

(D) 
$$\frac{16}{625}$$

Solution

Let

$$X = \{(x, y) : 0 \le x \le 99, 0 \le y \le 99\}$$

be a set of random variables,  $N = 2^x 5^y$ ,

$$\implies \forall (x, y) \in X$$
, N is a divisor of  $10^{99}$  (0.0.1)

$$\implies n(X) = 100 \times 100 = 10^4$$
 (0.0.2)

Let

$$Y = \{(x, y) : (x, y) \in X, x \ge 96, y \ge 96\}$$
$$N_1 = 2^x 5^y$$

⇒ 
$$\forall (x, y) \in Y, N_1 | 10^{99}$$
 and is multiple of  $10^{96}$  (0.0.3)

$$\implies n(Y) = 4 \times 4 = 16 \tag{0.0.4}$$

Let P denotes the probability that a divisor of 10<sup>99</sup> is a multiple of 10<sup>96</sup> then

$$P = \frac{n(Y)}{n(X)}$$

From (0.0.2) and (0.0.4) we can write

$$P = \frac{16}{10^4} = \frac{1}{625}$$

So the probability is  $\frac{1}{625}$ , option (A).

The comparison of probability obtained by calculation and simulation is as follows (Fig-0).

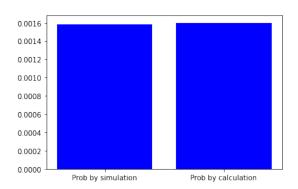


Fig. 0: Sim Vs Cal