

# Assignment-3

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Download all python codes from

<https://github.com/rithvikreddy6300/Assignment-3/tree/main/codes>

and latex-tikz codes from

<https://github.com/rithvikreddy6300/Assignment-3/blob/main/Assignment-3.tex>

PROBLEM-(GATE 2010 (CS) Q-27)

What is the probability that a divisor of  $10^{99}$  is a multiple of  $10^{96}$  ?

- (A)  $\frac{1}{625}$       (B)  $\frac{4}{625}$       (C)  $\frac{12}{625}$       (D)  $\frac{16}{625}$

SOLUTION

Let  $X = \{(x, y) : 0 \leq x \leq 99, 0 \leq y \leq 99\}$  be a set of random variables. Let  $N = 2^x 5^y$ , then  $\forall (x, y) \in X$ ,  $N$  is a divisor of  $10^{99}$ .

$$\Rightarrow n(X) = 100 * 100 = 10^4 \quad (0.0.1)$$

Let  $Y = \{(x, y) : (x, y) \in X, x \geq 96, y \geq 96\}$  Let  $n = 2^x 5^y$  then  $\forall (x, y) \in Y$ ,  $n$  is a divisor of  $10^{99}$  and multiple of  $10^{96}$

$$\Rightarrow n(Y) = 4 * 4 = 16 \quad (0.0.2)$$

Let  $P$  denotes the probability that a divisor of  $10^{99}$  is a multiple of  $10^{96}$  then

$$P = \frac{n(Y)}{n(X)}$$

From (0.0.1) and (0.0.2) we can write

$$P = \frac{16}{10^4} = \frac{1}{625}$$

So the probability is  $\frac{1}{625}$ , option (A).

The comparison of probability obtained by calculation and simulation is as follows (Fig-0).

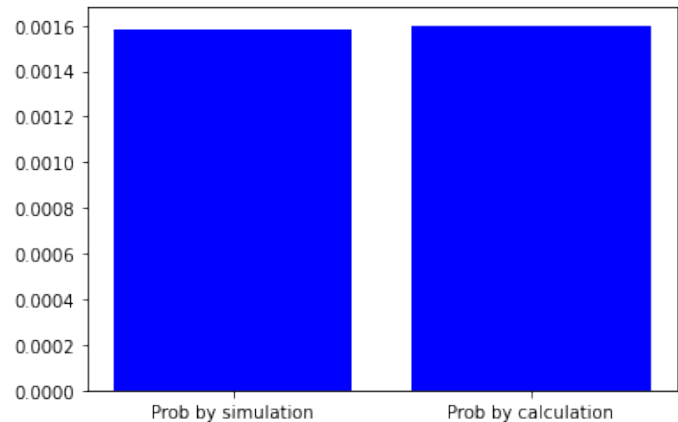


Fig. 0: a nice plot