Assignment-3

S. Rithvik Reddy - cs20btech11049

Download all python codes from

https://github.com/rithvikreddy6300/Assignment-3/ tree/main/codes

and latex-tikz codes from

https://github.com/rithvikreddy6300/Assignment-3/ blob/main/Assignment-3.tex

PROBLEM-(GATE 2010 (CS) Q-27)

What is the probability that a divisor of 1099 is a multiple of 10^{96} ?

(A)
$$\frac{1}{625}$$
 (B) $\frac{4}{625}$ (C) $\frac{12}{625}$ (D) $\frac{16}{625}$

(B)
$$\frac{4}{62}$$

(C)
$$\frac{12}{62^4}$$

(D)
$$\frac{16}{625}$$

Solution

Let

$$X = \{(x, y) : 0 \le x \le 99, 0 \le y \le 99\}$$

be a set of random variables, $N = 2^x 5^y$,

$$\implies \forall (x, y) \in X$$
, N is a divisor of 10^{99} (0.0.1)

$$\implies n(X) = 100 * 100 = 10^4$$
 (0.0.2)

Let

$$Y = \{(x, y) : (x, y) \in X, x \ge 96, y \ge 96\}$$
$$n = 2^x 5^y$$

 $\implies \forall (x, y) \in Y, n \text{ is multiple of } 10^{96}, \text{ divides } 10^{99}$ (0.0.3)

$$\implies n(Y) = 4 * 4 = 16$$
 (0.0.4)

Let P denotes the probability that a divisor of 10⁹⁹ is a multiple of 10⁹⁶ then

$$P = \frac{n(Y)}{n(X)}$$

From (0.0.2) and (0.0.4) we can write

$$P = \frac{16}{10^4} = \frac{1}{625}$$

So the probability is $\frac{1}{625}$, option (A).

The comparison of probability obtained by calculation and simulation is as follows (Fig-0).

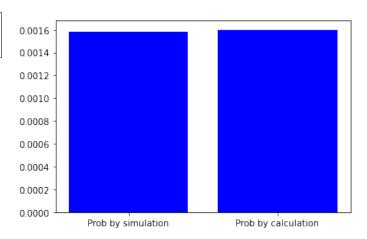


Fig. 0: a nice plot