Assignment-3

S. Rithvik Reddy - cs20btech11049

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PROBLEM-(GATE 2010 (CS) Q-27)

What is the probability that a divisor of 10^{99} is a multiple of 10^{96} ?

(A) $\dfrac{1}{625}$ (B) $\dfrac{4}{625}$ (C) $\dfrac{12}{625}$ (D) $\dfrac{16}{625}$

SOLUTION

Let

$$X = \{(x, y) : 0 \leq x \leq 99, 0 \leq y \leq 99\}$$

be a set of random variables, $N = 2^x 5^y$,

$$\implies \forall (x, y) \in X, \text{ N is a divisor of } 10^{99} \quad (0.0.1)$$
$$\implies n(X) = 100 \times 100 = 10^4 \quad\quad\quad (0.0.2)$$

Let

$$Y = \{(x, y) : (x, y) \in X, x \geq 96, y \geq 96\}$$
$$N_1 = 2^x 5^y$$

$$\implies \forall (x, y) \in Y, N_1 | 10^{99} \text{ and is multiple of } 10^{96} \quad (0.0.3)$$
$$\implies n(Y) = 4 \times 4 = 16 \quad\quad\quad (0.0.4)$$

Let P denotes the probability that a divisor of 10^{99} is a multiple of 10^{96} then

$$P = \frac{n(Y)}{n(X)} \quad\quad\quad (0.0.5)$$

From (0.0.2) and (0.0.4) we can write

$$P = \frac{16}{10^4} = \frac{1}{625}$$

So the probability is $\dfrac{1}{625}$, option (**A**).

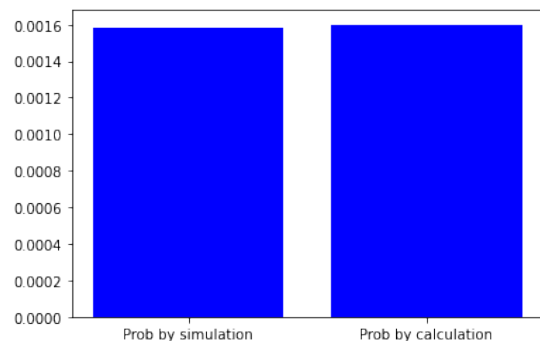The comparison of probability obtained by calculation and simulation is as follows (Fig-0).

Fig. 0: Sim Vs Cal