

Assignment-3

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Download all python codes from

<https://github.com/rithvikreddy6300/Assignment-3/tree/main/code>

and latex-tikz codes from

<https://github.com/rithvikreddy6300/Assignment-3/blob/main/Assignment-3.tex>

PROBLEM-(GATE 2010 (CS) Q-27)

What is the probability that a divisor of 10^{99} is a multiple of 10^{96} ?

- (A) $\frac{1}{625}$ (B) $\frac{4}{625}$ (C) $\frac{12}{625}$ (D) $\frac{16}{625}$

SOLUTION

Let $X = \{(x, y) : 0 \leq x \leq 99, 0 \leq y \leq 99\}$ be a set of random variables. Let $N = 2^x 5^y$, then $\forall (x, y) \in X$, N is a divisor of 10^{99} .

$$\Rightarrow n(X) = 100 * 100 = 10^4 \quad (0.0.1)$$

Let $Y = \{(x, y) : (x, y) \in X, x \geq 96, y \geq 96\}$ Let $n = 2^x 5^y$ then $\forall (x, y) \in Y$, n is a divisor of 10^{99} and multiple of 10^{96}

$$\Rightarrow n(Y) = 4 * 4 = 16 \quad (0.0.2)$$

Let P denotes the probability that a divisor of 10^{99} is a multiple of 10^{96} then

$$P = \frac{n(Y)}{n(X)}$$

From (0.0.1) and (0.0.2) we can write

$$P = \frac{16}{10^4} = \frac{1}{625}$$

So the probability is $\frac{1}{625}$, option (A).

The comparison of probability obtained by calculation and simulation is as follows (Fig-0).

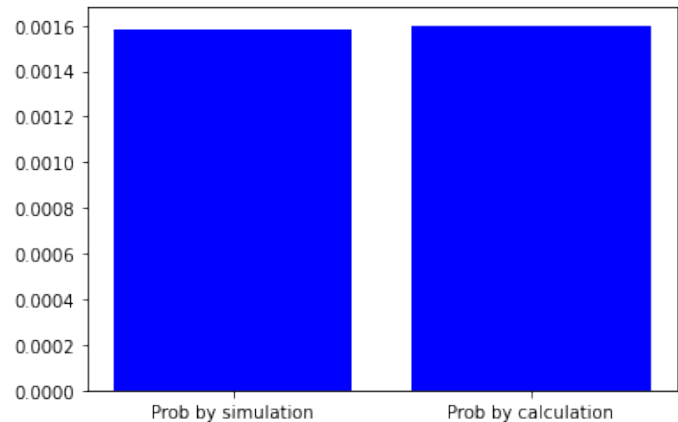


Fig. 0: a nice plot