

Assignment-3

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Download all python codes from

<https://github.com/rithvikreddy6300/Assignment-4/tree/main/codes>

and latex-tikz codes from

<https://github.com/rithvikreddy6300/Assignment-4/blob/main/Assignment-4.tex>

PROBLEM-(GATE 2011 (CS) Q-3)

If two fair coins are flipped and atleast one of the outcomes is known to be a head, what is the probability that both outcomes are heads ?

- (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

SOLUTION

Let $X = \{0, 1\}$ be the set of random variables where 0 represents **Tail**, 1 represent **Head**, x_1, x_2 represent the outcomes of coins 1 and 2.

$$\Pr(X = n) = \begin{cases} \frac{1}{2} & \text{if } n = 0 \\ \frac{1}{2} & \text{if } n = 1 \end{cases} \quad (0.0.1)$$

Let P denote the probability that both outcomes are heads given atleast one is a head,

$$\Rightarrow P = \Pr(x_1 x_2 = 1 | x_1 \text{ or } x_2 = 1) \quad (0.0.2)$$

$$\Rightarrow P = \frac{\Pr((x_1 x_2 = 1) \cap (x_1 \text{ or } x_2 = 1))}{\Pr(x_1 \text{ or } x_2 = 1)} \quad (0.0.3)$$

$$\Rightarrow P = \frac{\Pr(x_1 x_2 = 1)}{\Pr(x_1 \text{ or } x_2 = 1)} \quad (0.0.4)$$

$$\Rightarrow P = \frac{\Pr(x_1 x_2 = 1)}{\Pr(x_1 = 1) + \Pr(x_2 = 1) - \Pr(x_1 x_2 = 1)} \quad (0.0.5)$$

Since x_1 and x_2 are independent events we can write

$$\Pr(x_1 x_2 = 1) = \Pr(x_1 = 1) \Pr(x_2 = 1) \quad (0.0.6)$$

$$\Rightarrow P = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} + \frac{1}{2} - \frac{1}{2} \times \frac{1}{2}} \quad (0.0.7)$$

$$\Rightarrow P = \frac{1}{3} \quad (0.0.8)$$

Therefore the probability of both outcomes are heads given atleast one outcome is head is $P = \frac{1}{3}$ option (A).

The probability obtained by simulation versus the calculated prob is as shown in Fig(0).

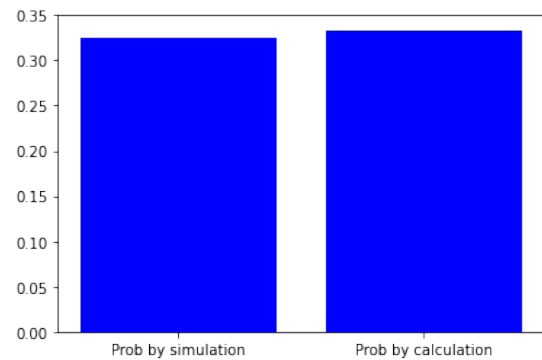


Fig. 0: Sim Vs Cal