

Assignment-4

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Download all python codes from

<https://github.com/rithvikreddy6300/Assignment-4/tree/main/codes>

and latex-tikz codes from

<https://github.com/rithvikreddy6300/Assignment-4/blob/main/Assignment-4.tex>

PROBLEM-(GATE 2011 (CS) Q-3)

- 1) If two fair coins are flipped and atleast one of the outcomes is known to be a head, what is the probability that both outcomes are heads ?

- a) $\frac{1}{3}$
- b) $\frac{1}{4}$
- c) $\frac{1}{2}$
- d) $\frac{2}{3}$

SOLUTION

Let $X = \{0, 1\}$ be the set of random variables where 0 represents **Tail**, 1 represent **Head**, x_1, x_2 represent the outcomes of coins 1 and 2. Let A,B be the events that $x_1 = 1, x_2 = 1$ respectively.

$$\Pr(X = n) = \begin{cases} \frac{1}{2} & \text{if } n = 0 \\ \frac{1}{2} & \text{if } n = 1 \end{cases} \quad (0.0.1)$$

Let P denote the probability that both outcomes are heads given atleast one is a head,

$$\Rightarrow P = \Pr(AB|A + B) \quad (0.0.2)$$

$$\Rightarrow P = \frac{\Pr(AB(A + B))}{\Pr(A + B)} \quad (0.0.3)$$

$$\text{But, } AB(A + B) = ABA + ABB = AB + AB = AB \quad (0.0.4)$$

$$\Rightarrow P = \frac{\Pr(AB)}{\Pr(A + B)} \quad (0.0.5)$$

$$\Rightarrow P = \frac{\Pr(AB)}{\Pr(A) + \Pr(B) - \Pr(AB)} \quad (0.0.6)$$

Since A and B are independent events we can write

$$\Pr(AB) = \Pr(A) \Pr(B) \quad (0.0.7)$$

$$\Rightarrow P = \frac{\Pr(A) \Pr(B)}{\Pr(A) + \Pr(B) - \Pr(A) \Pr(B)} \quad (0.0.8)$$

$$\Rightarrow P = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} + \frac{1}{2} - \frac{1}{2} \times \frac{1}{2}} \quad (0.0.9)$$

$$\Rightarrow P = \frac{1}{3} \quad (0.0.10)$$

Therefore the probability of both outcomes are heads given atleast one outcome is head is $P = \frac{1}{3}$ option (A).

The probability obtained by simulation versus the calculated prob is as shown in Fig(1).

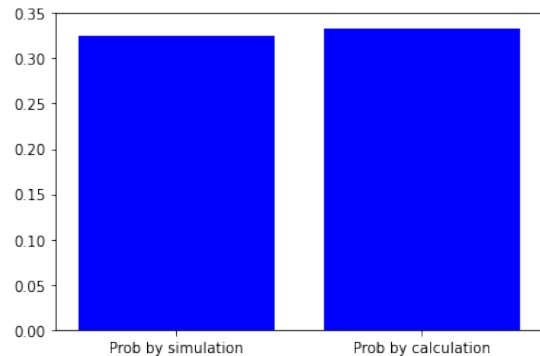


Fig. 1: Sim Vs Cal