Practical 6

Solution of Cauchy problem for first order partial differential order

Q1.SOLVE the PDE $xu_x + yu_y = u + 1$

In[ ]:=
```
Eqn1 := x * D[u[x, y], x] + y * D[u[x, y], y] == u[x, y] + 1;
```

In[ ]:=
```
Sol = DSolve[Eqn1, u[x, y], {x, y}]
```

Out[ ]= $\{\{u[x, y] \to -1 + x\, C[1][\frac{y}{x}]\}\}$

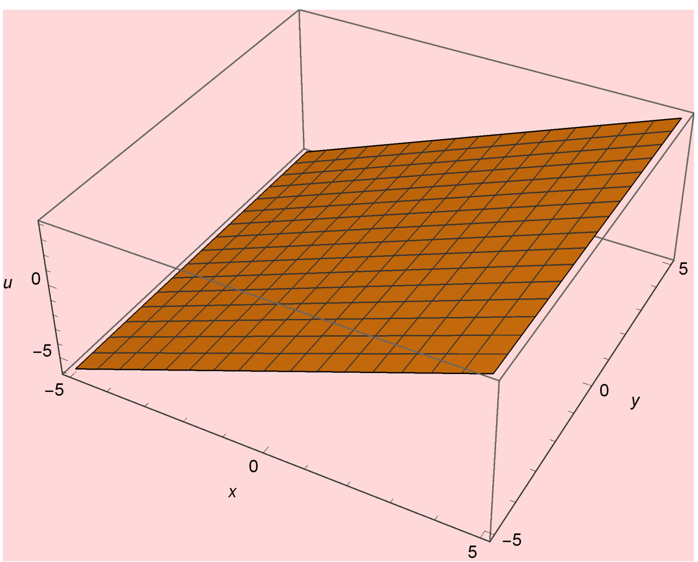
$\{\{u[x, y] \to -1 + x\, C[1][\frac{y}{x}]\}\}$ (*general solution*)

In[ ]:=
```
Par = u[x, y] /. Sol /. C[1][y / x] → 1
```

{-1 + x} (*particular solution*)

In[ ]:=
```
Plot3D[Par, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}]
```

Out[ ]=

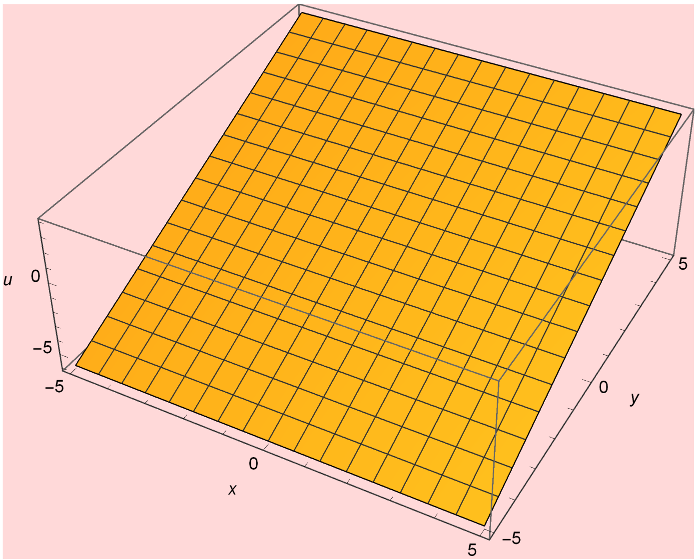


In[ ]:=
```
Par1 = u[x, y] /. Sol /. C[1][y / x] → y / x
```

Out[ ]= {-1 + y}

In[∘]:= `Plot3D[Parl, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}]`

Out[∘]=



Question 2 : Solve the pde $3\,u_x + 2\,u_y = 0$

In[∘]:= `Eqn2 := 3 D[u[x, y], x] + 2 * D[u[x, y], y] == 0;`

In[∘]:= `Sol = DSolve[Eqn2, u[x, y], {x, y}]`

Out[∘]= $\left\{\left\{u[x, y] \to C[1]\left[\frac{1}{3}\left(-2x + 3y\right)\right]\right\}\right\}$

In[∘]:= $\left\{\left\{u[x, y] \to C[1]\left[\frac{1}{3}\left(-2x + 3y\right)\right]\right\}\right\}$ `(*General solution*)`

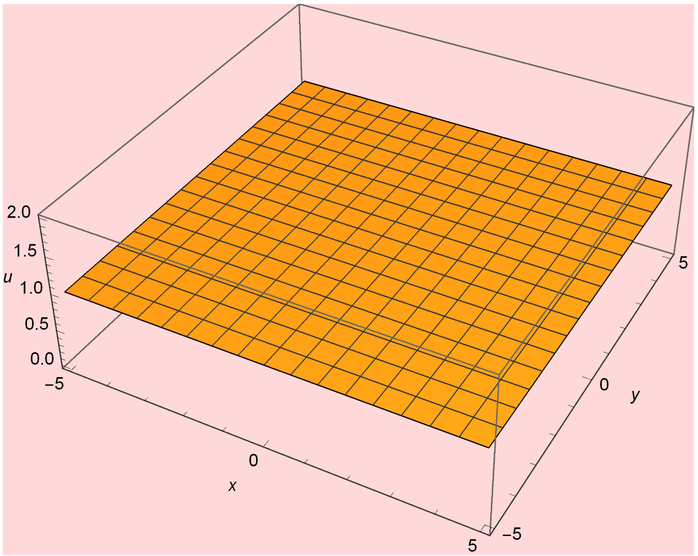
`Par1 = u[x, y] /. Sol /. C[1][1/3 * (-2 * x + 3 * y)] → 1`

Out[∘]= $\left\{\left\{u[x, y] \to C[1]\left[\frac{1}{3}\left(-2x + 3y\right)\right]\right\}\right\}$

`{1}(*Particular solution*)`

In[∘]:= `Plot3D[Par1, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}]`

Out[ ]=



Q3.Solve the PDE $3\,u_x + 2\,u_y = 0$ given $u[x, 0] = \text{Sin}[x]$

In[ ]:=
```
Eqn3 := 3 * D[u[x, y], x] + 2 * D[u[x, y], y] == 0;
```

In[ ]:=
```
Sol3 = DSolve[{Eqn3, u[x, 0] == Sin[x]}, u[x, y], {x, y}]
```

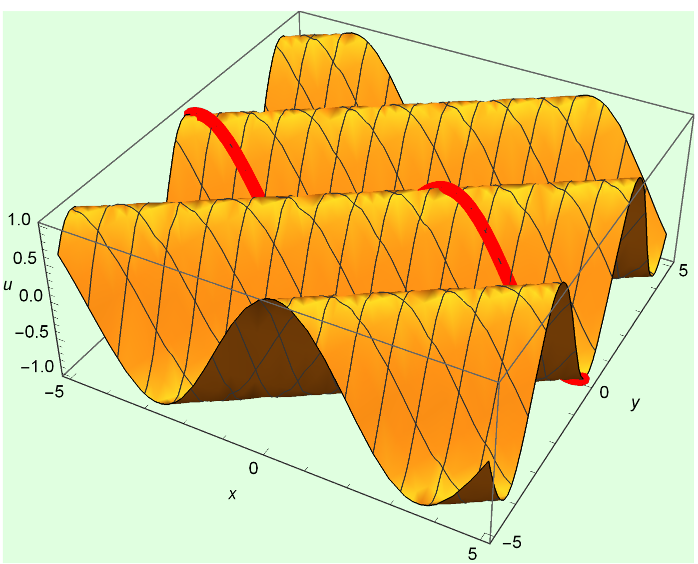
Out[ ]= $\left\{\left\{u[x, y] \to \text{Sin}\left[\frac{1}{2}\,(2\,x - 3\,y)\right]\right\}\right\}$

In[ ]:=
```
p1 = Plot3D[u[x, y] /. Sol3, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}];
```

In[ ]:=
```
p3 = ParametricPlot3D[{x, 0, Sin[x]}, {x, -5, 5}, PlotStyle → {Red, Thickness[0.02]}];
```

In[ ]:=
```
Show[p1, p3]
```

Out[ ]=



Observation-above graph shows the particular solution of the given PDE

```
Soln1 = DSolve[3 * y'[x] - 2 == 0, y[x], x] (*Characteristic Equation*)
```

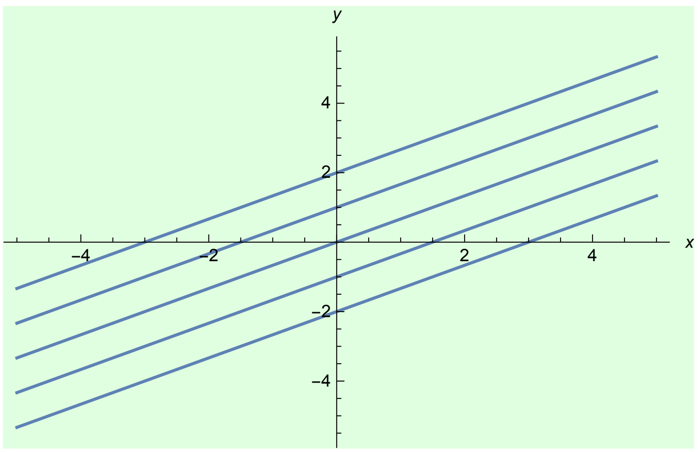
Out[∘]= $\left\{\left\{y[x] \to \frac{2x}{3} + C[1]\right\}\right\}$

```
In[∘]:= Par = y[x] /. Soln1 /. C[1] → {-2, -1, 0, 1, 2}
```

Out[∘]= $\left\{\left\{-2 + \frac{2x}{3}, -1 + \frac{2x}{3}, \frac{2x}{3}, 1 + \frac{2x}{3}, 2 + \frac{2x}{3}\right\}\right\}$

```
In[∘]:= Plot[Par, {x, -5, 5}, AxesLabel → {x, y}]
```

Out[∘]=



Observation-characteristics are family of Parallel Straight lines

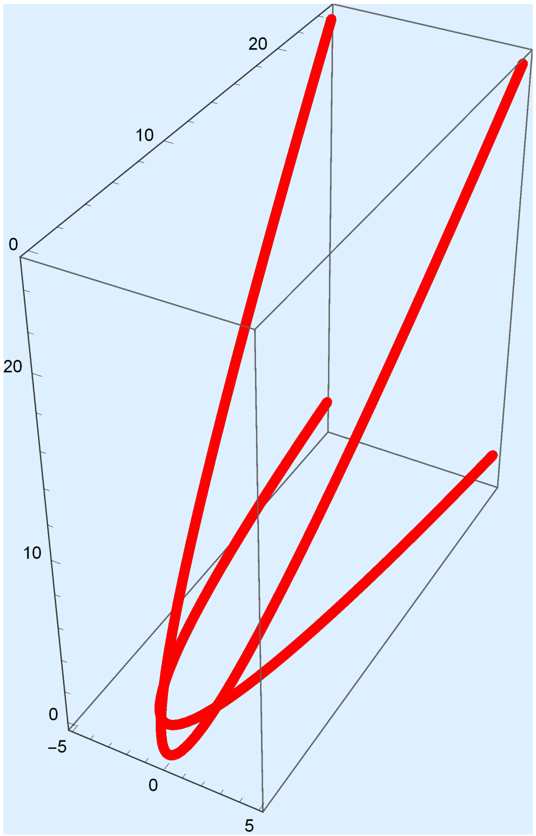
Q4.Solve the PDE $xu_x + yu_y$==u+1 with initial conditions u (x,x^2)=x^2

```
In[∘]:= Eqn4 := x * D[u[x, y], x] + y * D[u[x, y], y] == u[x, y] + 1;

In[∘]:= Sol4 = DSolve[{Eqn4, u[x, x^2] == x^2}, u[x, y], {x, y}]
```

Out[∘]= $\left\{\left\{u[x, y] \to \frac{x^2 - y + y^2}{y}\right\}\right\}$

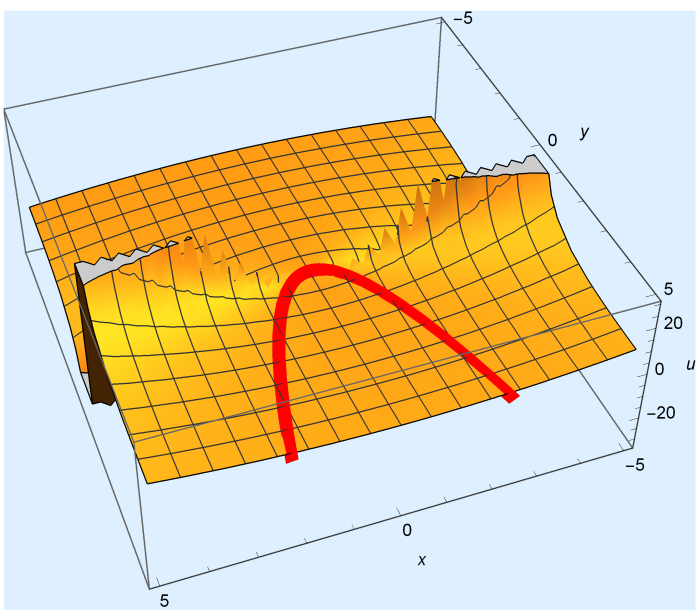
```
In[∘]:= p1 = Plot3D[u[x, y] /. Sol4, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}];

In[∘]:= p2 = ParametricPlot3D[{x, x^2, x^2}, {x, -5, 5}, PlotStyle → {Red, Thickness[0.02]}];

In[∘]:= p3 = ParametricPlot3D[{x, x^2, 2}, {x, -5, 5}, PlotStyle → {Red, Thickness[0.02]}];

In[∘]:= Show[p2, p3]
```

Out[•]=



In[•]:= **Show[p1, p2]**

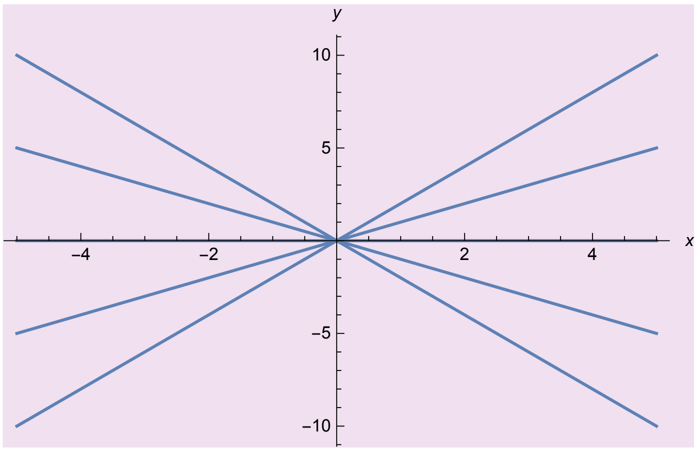
Out[•]=



In[•]:= **Soln2 = DSolve[x * y'[x] - y[x] == 0, y[x], x] (*Characteristic Equation*)**

Out[•]= {{y[x] → x C[1]}}

In[∘]:=
```
Par1 = y[x] /. Soln2 /. C[1] → {-2, -1, 0, 1, 2}
```
Out[∘]=
```
{{-2 x, -x, 0, x, 2 x}}
```
In[∘]:=
```
Plot[Par1, {x, -5, 5}, AxesLabel → {x, y}]
```
Out[∘]=



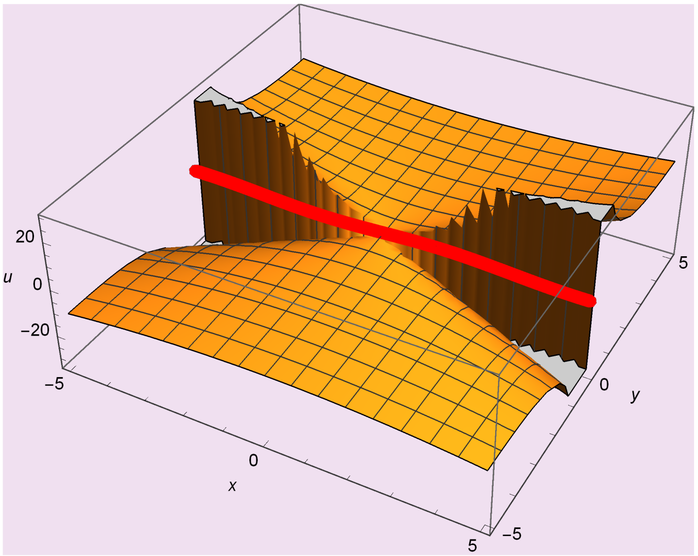
Observation-Characteristics are straight lines passing through the origin

Q5. Solve the PDE 3 Subscript[*u*, *x*] + 2 u_y==0, with initial condition u (x,0)=sin[x]

In[∘]:=
```
Eqn5 := 3 * D[u[x, y], x] + 2 * D[u[x, y], y] == 0;
```
In[∘]:=
```
Sol5 = DSolve[{Eqn5, u[x, 0] == Sin[x]}, u[x, y], {x, y}]
```
Out[∘]= $\{\{u[x, y] \to \mathrm{Sin}[\frac{1}{2}(2x - 3y)]\}\}$

$\{\{u[x, y] \to \mathrm{Sin}[\frac{1}{2}(2x - 3y)]\}\}$ (*Particular solution*)

In[∘]:=
```
p1 = Plot3D[u, [x, y] /. Sol5, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}];
```
In[∘]:=
```
p2 = ParametricPlot3D[{x, 0, Sin[x]}, {x, -5, 5}, PlotStyle → {Red, Thickness[0.02]}];
```
In[∘]:=
```
Show[p1, p2]
```

Out[ ]= 

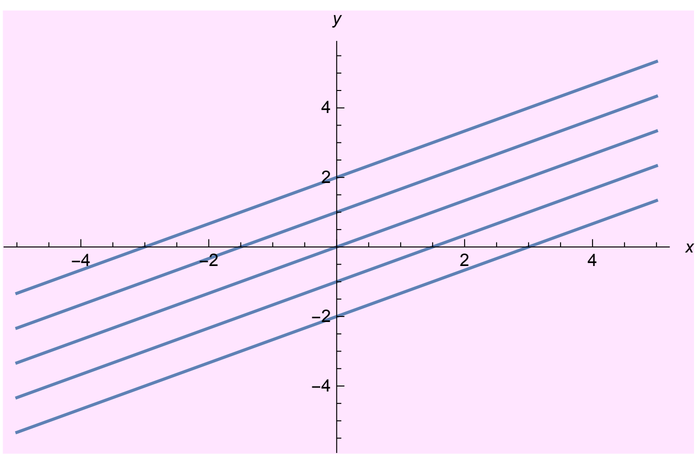
In[ ]:=
```
Soln5 = DSolve[3 * y'[x] - 2 == 0, y[x], x] (*Characteristic Equation*)
```

Out[ ]= $\left\{\left\{y[x] \to \frac{2x}{3} + C[1]\right\}\right\}$

In[ ]:=
```
Part1 = y[x] /. Soln5 /. C[1] → {-2, -1, 0, 1, 2}
```

Out[ ]= $\left\{\left\{-2 + \frac{2x}{3}, -1 + \frac{2x}{3}, \frac{2x}{3}, 1 + \frac{2x}{3}, 2 + \frac{2x}{3}\right\}\right\}$

In[ ]:=
```
Plot[Part1, {x, -5, 5}, AxesLabel → {x, y}]
```

Out[ ]= 

Observation-Characteristics are family of Parallel Straight lines

Q6:Solve the PDE $yu_x + xu_y = 0$ with given initial condition u[0,y]=e^-y^2

In[ ]:=
```
Eqn6 := y * D[u[x, y], x] + x * D[u[x, y], y] == 0;
```

In[ ]:=
```
Sol6 = DSolve[{Eqn6, u[0, y] == Exp[-y^2]}, u[x, y], {x, y}]
```

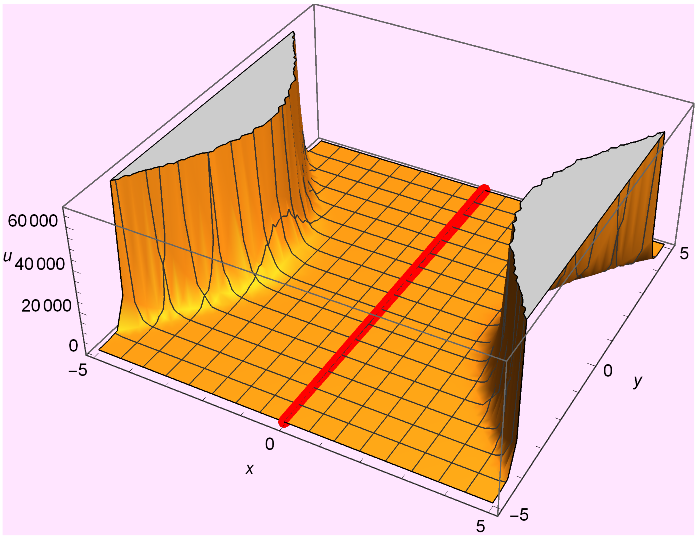
Out[ ]= $\left\{\left\{u[x, y] \to e^{x^2-y^2}\right\}\right\}$

In[ ]:=
```
p1 = Plot3D[u[x, y] /. Sol6, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}];
```

In[∘]:=
```
p2 = ParametricPlot3D[{0, y, Exp[-y^2]}, {y, -5, 5}, PlotStyle → {Red, Thickness[0.02]}];
```

In[∘]:=
```
Show[p1, p2]
```

Out[∘]=



In[∘]:=
```
Soln6 = DSolve[y[x] * y'[x] - x == 0, y[x], x] (*Characteristic Equation*)
```

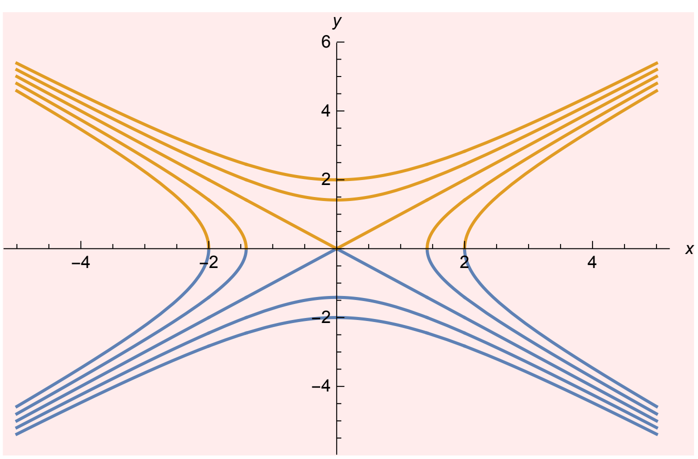
Out[∘]= $\{\{y[x] \to -\sqrt{x^2 + 2\,C[1]}\}, \{y[x] \to \sqrt{x^2 + 2\,C[1]}\}\}$

In[∘]:=
```
Par1 = y[x] /. Soln6 /. C[1] → {-2, -1, 0, 1, 2}
```

Out[∘]= $\{\{-\sqrt{-4 + x^2}, -\sqrt{-2 + x^2}, -\sqrt{x^2}, -\sqrt{2 + x^2}, -\sqrt{4 + x^2}\},$
$\{\sqrt{-4 + x^2}, \sqrt{-2 + x^2}, \sqrt{x^2}, \sqrt{2 + x^2}, \sqrt{4 + x^2}\}\}$

In[∘]:=
```
Plot[Par1, {x, -5, 5}, AxesLabel → {x, y}]
```

Out[∘]=



Observation - Characteristics are Hyperbolic curves wit two straight lines passing through origin and asymptotic to hyperbolic curves.

Q7:Solve the PDE xu_x+ yu_y = xy, with given initial condition u[x,x^2]=2

In[∘]:=
```
Eqn7 := x * D[u[x, y], x] + y * D[u[x, y], y] == x * y;
```

In[ ]:= Sol7 = DSolve[{Eqn7, u[x, x^2] == 2}, u[x, y], {x, y}]

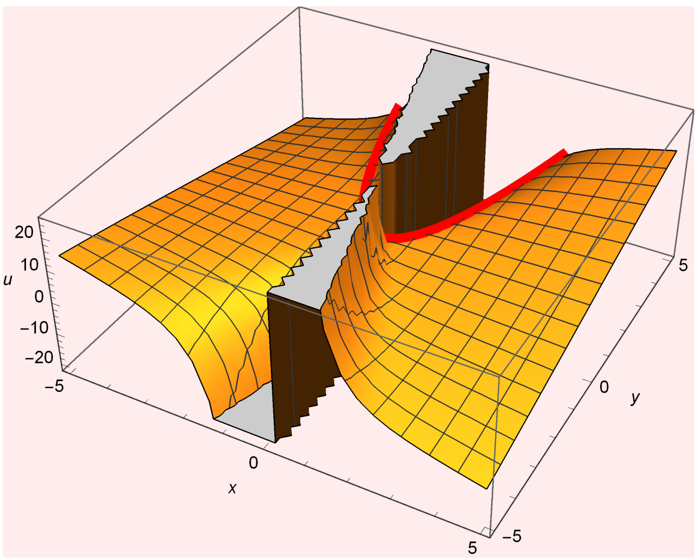
Out[ ]= $\left\{\left\{u[x, y] \to \frac{4 x^3 + x^4 y - y^3}{2 x^3}\right\}\right\}$

$\left\{\left\{u[x, y] \to \frac{4 x^3 + x^4 y - y^3}{2 x^3}\right\}\right\}$ (*Particular solution*)

In[ ]:= p1 = Plot3D[u[x, y] /. Sol7, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}];

In[ ]:= p2 = ParametricPlot3D[{x, x^2, 2}, {x, -5, 5}, PlotStyle → {Red, Thickness[0.02]}];

In[ ]:= Show[p1, p2]

Out[ ]=


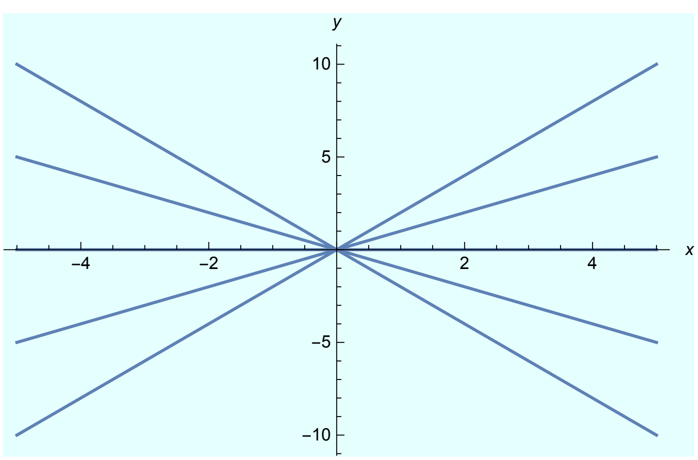
In[ ]:= Soln7 = DSolve[x * y'[x] - y[x] == 0, y[x], x] (*Characteristic Equation*)

Out[ ]= {{y[x] → x C[1]}}

In[ ]:= Par1 = y[x] /. Soln7 /. C[1] → {-2, -1, 0, 1, 2}

Out[ ]= {{-2 x, -x, 0, x, 2 x}}

In[ ]:= Plot[Par1, {x, -5, 5}, AxesLabel → {x, y}]

Out[ ]=


Observation - Characteristics are Straight lines passing through the origin

Q8:Solve the PDE Subscript[*u*, *x*] + xu_y==0, with given initial condition u[0,y]=Sin[y]

In[◦]:=
```
Eqn8 := D[u[x, y], x] + x * D[u[x, y], y] == 0;
```

In[◦]:=
```
Sol8 = DSolve[{Eqn8, u[0, y] == Sin[y]}, u[x, y], {x, y}]
```

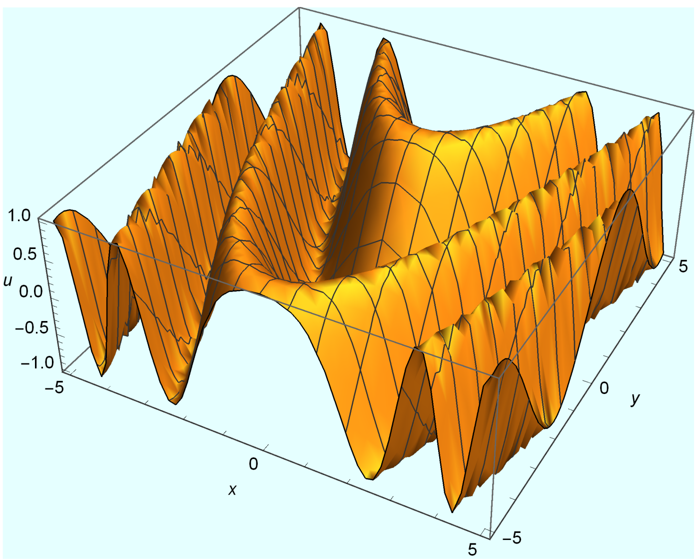
Out[◦]= $\{\{u[x, y] \to \text{Sin}[\frac{1}{2}(-x^2 + 2y)]\}\}$

In[◦]:=
```
p1 = Plot3D[u[x, y] /. Sol8, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}];
```

In[◦]:=
```
p2 = ParametricPlot3D[{0, y, Sin[y]}, {Y, -5, 5}, PlotStyle → {Red , Thickness[0.02]}];
```

In[◦]:=
```
Show[p1, p2]
```

Out[◦]=



In[◦]:=
```
Soln8 = DSolve[y'[x] == x, y[x], x] (*Characteristic Euation*)
```

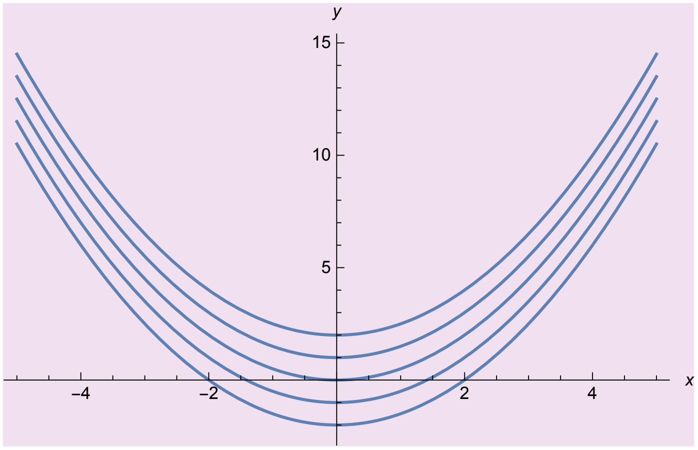
Out[◦]= $\{\{y[x] \to \frac{x^2}{2} + C[1]\}\}$

In[◦]:=
```
Par1 = y[x] /. Soln8 /. C[1] → {-2, -1, 0, 1, 2}
```

Out[◦]= $\{\{-2 + \frac{x^2}{2}, -1 + \frac{x^2}{2}, \frac{x^2}{2}, 1 + \frac{x^2}{2}, 2 + \frac{x^2}{2}\}\}$

In[◦]:=
```
Plot[Par1, {x, -5, 5}, AxesLabel → {x, y}]
```

Out[∘]=



Observation - Characteristics are family of Parabolic Curves

Q9: Solve the PDE u_x+ xu_y = (y-((x^2)/2))^2, with the given initial condition u[0,y] = e^y

In[∘]:=
```
Eqn9 := 1 * D[u[x, y], x] + x * D[u[x, y], y] == (y - ((x^2) / 2)) ^2;
```

In[∘]:=
```
Sol9 = DSolve[{Eqn9, u[0, y] == Exp[y]}, u[x, y], {x, y}]
```

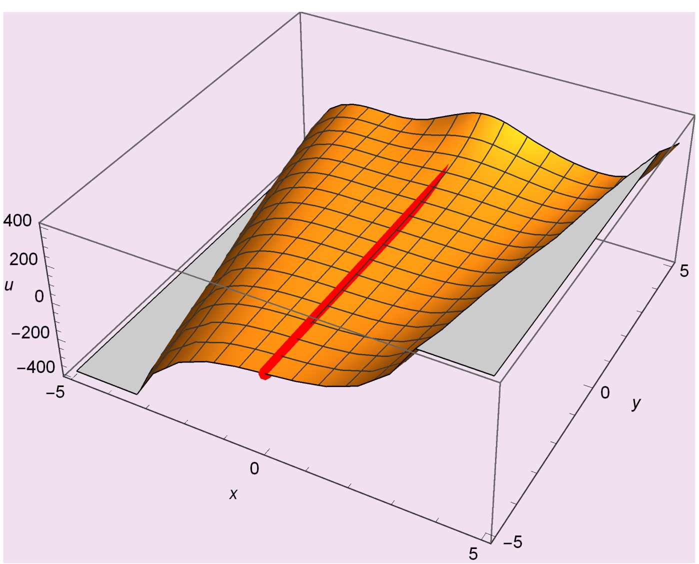
$$\left\{\left\{u[x, y] \to \frac{1}{4} e^{-\frac{x^2}{2}} \left(4 e^y + e^{\frac{x^2}{2}} x^5 - 4 e^{\frac{x^2}{2}} x^3 y + 4 e^{\frac{x^2}{2}} x y^2\right)\right\}\right\}$$ (*Particular solution*)

In[∘]:=
```
p1 = Plot3D[u[x, y] /. Sol9, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}];
```

In[∘]:=
```
p2 = ParametricPlot3D[{0, y, Sin[y]}, {y, -5, 5}, PlotStyle → {Red, Thickness[0.02]}];
```

In[∘]:=
```
Show[p1, p2]
```

Out[∘]=



In[∘]:=
```
Soln9 = DSolve[y'[x] == x, y[x], x] (*Characteristic Equation*)
```

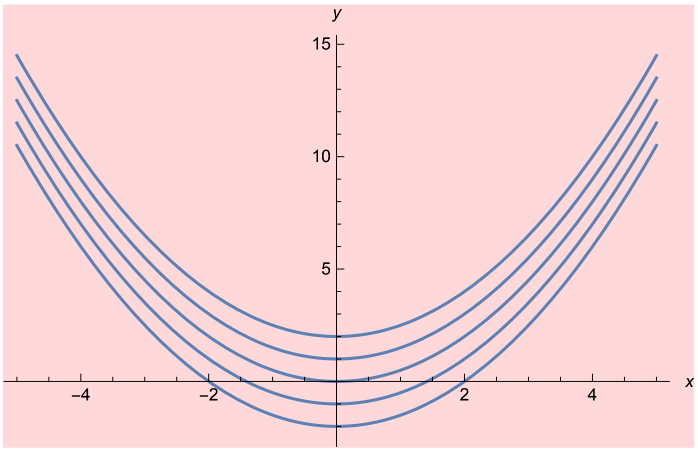
Out[∘]= $\left\{\left\{y[x] \to \frac{x^2}{2} + C[1]\right\}\right\}$

In[∘]:=
```
Par1 = y[x] /. Soln9 /. C[1] → {-2, -1, 0, 1, 2}
```

Out[∘]= $\left\{\left\{-2+\frac{x^2}{2}, -1+\frac{x^2}{2}, \frac{x^2}{2}, 1+\frac{x^2}{2}, 2+\frac{x^2}{2}\right\}\right\}$

In[∘]:=
```
Plot[Par1, {x, -5, 5}, AxesLabel → {x, y}]
```

Out[∘]=



Observation - Characteristics are family of Parabolic Curves

Q10.Solve the PDE u_x + xu_y =y, with the initial condition u[1,y]=2y

In[∘]:=
```
Eqn10 := 1 * D[u[x, y], x] + x * D[u[x, y], y] == y;
```

In[∘]:=
```
Sol10 = DSolve[{Eqn10, u[1, y] == 2 * y}, u[x, y], {x, y}]
```

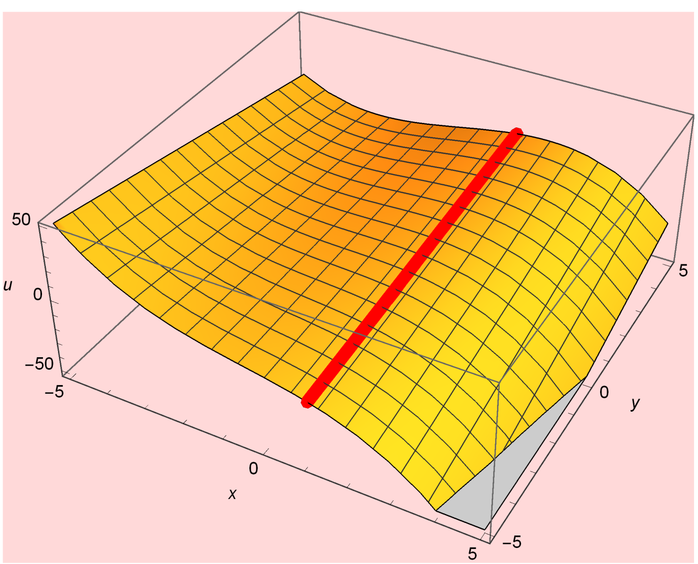
$\left\{\left\{u[x, y] \to \frac{1}{6}\left(5 - 3x^2 - 2x^3 + 6y + 6xy\right)\right\}\right\}$ (*Particular Solution*)

In[∘]:=
```
p1 = Plot3D[u[x, y] /. Sol10, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}];
```

In[∘]:=
```
p2 = ParametricPlot3D[{1, y, 2 * y}, {y, -5, 5}, PlotStyle → {Red , Thickness[0.02]}];
```

In[∘]:=
```
Show[p1, p2]
```

Out[∘]=



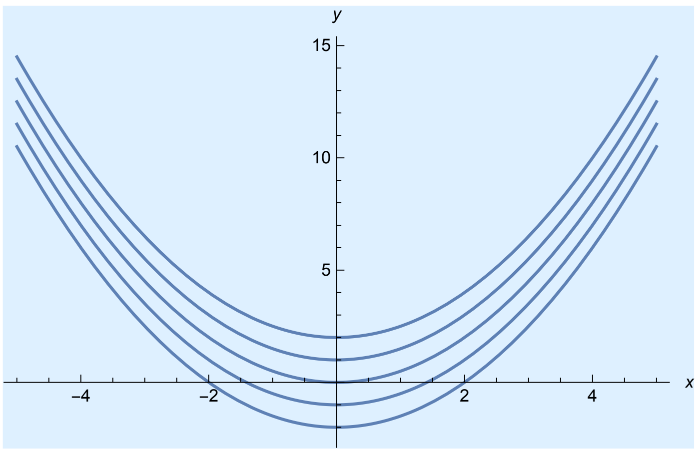
In[•]:=
```
Soln10 = DSolve[y'[x] == x, y[x], x] (*Characteristics equation*)
```

Out[•]= $\left\{\left\{y[x] \to \frac{x^2}{2} + C[1]\right\}\right\}$

In[•]:=
```
Par1 = y[x] /. Soln10 /. C[1] → {-2, -1, 0, 1, 2}
```

Out[•]= $\left\{\left\{-2 + \frac{x^2}{2}, -1 + \frac{x^2}{2}, \frac{x^2}{2}, 1 + \frac{x^2}{2}, 2 + \frac{x^2}{2}\right\}\right\}$

In[•]:=
```
Plot[Par1, {x, -5, 5}, AxesLabel → {x, y}]
```

Out[•]=


Observation - Characteristics are family of Parabolic curves

Q10. Solve the PDE $u_x + xu_y = y$, with given initial condition u[0,y]=y^2

In[•]:=
```
Eqn11 := 1 * D[u[x, y], x] + x * D[u[x, y], y] == y;
```

In[•]:=
```
Sol11 = DSolve[{Eqn11, u[0, y] == y^2}, u[x, y], {x, y}]
```

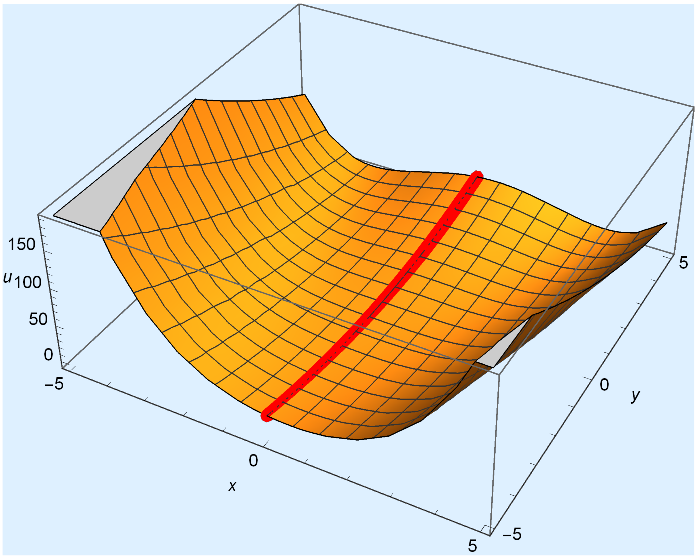
Out[•]= $\left\{\left\{u[x, y] \to \frac{1}{12}\left(-4 x^3 + 3 x^4 + 12 x y - 12 x^2 y + 12 y^2\right)\right\}\right\}$

In[•]:=
```
p1 = Plot3D[u[x, y] /. Sol11, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}];
```

In[•]:=
```
p2 = ParametricPlot3D[{0, y, y^2}, {y, -5, 5}, PlotStyle → {Red, Thickness[0.02]}];
```

In[•]:=
```
Show[p1, p2]
```

Out[•]=



In[•]:= `Soln11 = DSolve[y'[x] == x, y[x], x] (*Characteristic Equation*)`

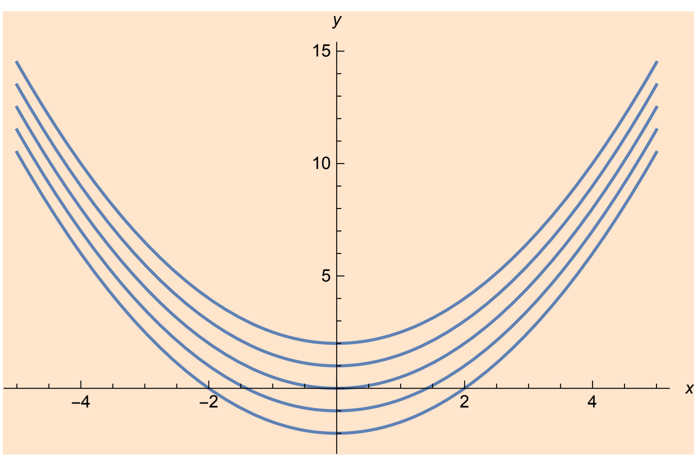
Out[•]= $\left\{\left\{y[x] \to \frac{x^2}{2} + C[1]\right\}\right\}$

In[•]:= `Par1 = y[x] /. Soln11 /. C[1] → {-2, -1, 0, 1, 2}`

Out[•]= $\left\{\left\{-2 + \frac{x^2}{2}, -1 + \frac{x^2}{2}, \frac{x^2}{2}, 1 + \frac{x^2}{2}, 2 + \frac{x^2}{2}\right\}\right\}$

In[•]:= `Plot[Par1, {x, -5, 5}, AxesLabel → {x, y}]`

Out[•]=



Observation - Characteristics are family of Parabolic curves

Q12. Solve the PDE Subscript[xu, *x*] + (*x* + *y*) u_y=u+1 , with given initial condition [x,0]=x^2

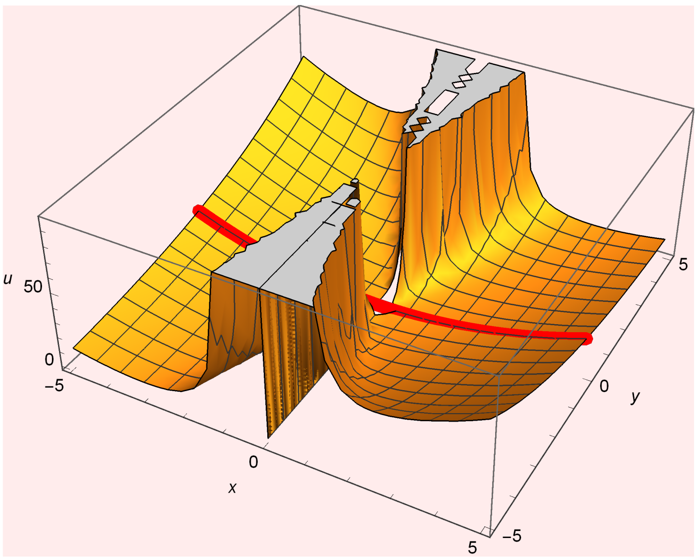
In[•]:= `Eqn12 := x * D[u[x, y], x] + (x + y) * D[u[x, y], y] == u[x, y] + 1;`

In[ ]:=
```
Sol12 = DSolve[{Eqn12, u[x, 0] == x^2}, u[x, y], {x, y}]
```

Out[ ]= $\left\{\left\{u[x, y] \to e^{-\frac{y}{x}} \left(-e^{\frac{y}{x}} + e^{\frac{2y}{x}} + x^2\right)\right\}\right\}$

```
p1 = Plot3D[u[x, y] /. Sol12, {x, -5, 5}, {y, -5, 5}, AxesLabel → {x, y, u}];
p2 = ParametricPlot3D[{x, 0, x^2}, {x, -5, 5}, PlotStyle → {Red, Thickness[0.02]}];
```

In[ ]:=
```
Show[p1, p2]
```

Out[ ]=


In[ ]:=
```
Soln12 = DSolve[y'[x] == 1 + (y[x] / x), y[x], x] (*Characteristics Equation*)
```

Out[ ]= {{y[x] → x C[1] + x Log[x]}}

In[ ]:=
```
Par1 = y[x] /. Soln12 /. C[1] → {-2, -1, 0, 1, 2}
```

Out[ ]= {{-2 x + x Log[x], -x + x Log[x], x Log[x], x + x Log[x], 2 x + x Log[x]}}

In[ ]:=
```
Plot[Par1, {x, -5, 5}, AxesLabel → {x, y}]
```

Out[ ]=