q1

In[43]:=
```
sol = DSolve [y ''[x] + 4 * y[x] == 0, y[x], x]
```

Out[43]= $\{\{y[x] \to c_1 \text{Cos}[2\ x] + c_2\ \text{Sin}[2\ x]\}\}$

In[44]:=
```
y1 := Cos[2 * x];
y2 := Sin[2 * x];
f := Cos[2 * x];
w = y1 * D[y2 , x] - y2 * D[y1 , x];
w = Simplify [w]
```

Out[48]= 2

In[49]:=
```
yp = - y1 * Integrate [y2 * (f / w), x] + y2 * Integrate [y1 * (f / w), x];
yp = Simplify [yp]
```

Out[50]= $\frac{1}{16}(\text{Cos}[2\ x] + 4\ x\ \text{Sin}[2\ x])$

In[51]:=
```
Out[43] + Out[50]
```

Out[51]= $\left\{\left\{(y[x] \to c_1 \text{Cos}[2\ x] + c_2\ \text{Sin}[2\ x]) + \frac{1}{16}(\text{Cos}[2\ x] + 4\ x\ \text{Sin}[2\ x])\right\}\right\}$

q2

In[37]:=
```
sol = DSolve [x ^ 2 * y ''[x] - x * y '[x] - 3 * y[x] == 0, y[x], x]
```

Out[37]= $\left\{\left\{y[x] \to \frac{c_1}{x} + x^3\ c_2\right\}\right\}$

In[65]:=
```
y1 := 1 / x;
y2 := x ^ 3;
f := x ^ 2
w = y1 * D[y2 , x] - y2 * D[y1 , x];
w = Simplify [w]
```

Out[69]= 4 x

In[70]:=
```
yp = - y1 * Integrate [y2 * (f / w), x] + y2 * Integrate [y1 * (f / w), x];
yp = Simplify [yp]
```

Out[71]= $\frac{x^4}{5}$

In[72]:=
```
Out[37] + Out[71]
```

Out[72]=
$$\left\{\left\{\frac{x^4}{5} + \left(y[x] \to \frac{c_1}{x} + x^3\, c_2\right)\right\}\right\}$$

q3

In[79]:=
```
sol = DSolve [y''[x] - 4 * y'[x] + 5 * y[x] == 0, y[x], x]
```

Out[79]=
$$\{\{y[x] \to e^{2x}\, c_2 \operatorname{Cos}[x] + e^{2x}\, c_1 \operatorname{Sin}[x]\}\}$$

In[80]:=
```
y1 := Exp[2 * x] * Sin[x];
y2 := Exp[2 * x] * Cos[x];
f := Exp[2 * x] * Csc[x];
w = y1 * D[y2 , x] - y2 * D[y1 , x];
w = Simplify [w]
```

Out[84]=
$$-e^{4x}$$

In[85]:=
```
yp = -y1 * Integrate [y2 * (f / w), x] + y2 * Integrate [y1 * (f / w), x];
yp = Simplify [yp]
```

Out[86]=
$$e^{2x}\,(-x \operatorname{Cos}[x] + \operatorname{Log}[\operatorname{Sin}[x]] \times \operatorname{Sin}[x])$$

In[138]:=
```
Out[86] + Out[79]
```

Out[138]=
$$\{\{(y[x] \to e^{2x}\, c_2 \operatorname{Cos}[x] + e^{2x}\, c_1 \operatorname{Sin}[x]) + e^{2x}\,(-x \operatorname{Cos}[x] + \operatorname{Log}[\operatorname{Sin}[x]] \times \operatorname{Sin}[x])\}\}$$

q4

In[89]:=
```
sol = DSolve [y''[x] - 2 * y'[x] + y[x] == 0, y[x], x]
```

Out[89]=
$$\{\{y[x] \to e^{x}\, c_1 + e^{x}\, x\, c_2\}\}$$

In[172]:=
```
y1 := Exp[x];
y2 := x * Exp[x];
f := 6 * x ^ 2 * Exp[-1 * x];
w = y1 * D[y2 , x] - y2 * D[y1 , x];
w = Simplify [w]
```

Out[176]=
$$e^{2x}$$

In[177]:=
```
yp = -y1 * Integrate [y2 * (f / w), x] + y2 * Integrate [y1 * (f / w), x];
yp = Simplify [yp]
```

Out[178]=
$$\frac{3}{4}\, e^{-x}\left(3 + 4x + 2x^2\right)$$

In[137]:=

```
Out[109] + Out[89]
```

Out[137]=
$$\left\{\left\{\frac{3}{4}\, e^{-x}\left(3 + 4x + 2x^2\right) + (y[x] \to e^{x}\, c_1 + e^{x}\, x\, c_2)\right\}\right\}$$

q5

In[97]:=
```
sol = DSolve [y ''[x] – 2 * y '[x] + y[x] == 0, y[x], x]
```

Out[97]= $\{\{y[x] \to e^x\, c_1 + e^x\, x\, c_2\}\}$

In[155]:=
```
y1 := Exp[x];
y2 := Exp[x] * x;
f := 35 * x ^ (3 / 2) * Exp[x];
w = y1 * D[y2 , x] – y2 * D[y1 , x];
w = Simplify [w]
```

Out[159]= $e^{2\,x}$

In[160]:=
```
yp = – y1 * Integrate [y2 * (f / w), x] + y2 * Integrate [y1 * (f / w), x];
yp = Simplify [yp]
```

Out[161]= $4\, e^x\, x^{7/2}$

In[162]:=
```
Out[161] + Out[97]
```

Out[162]= $\{\{4\, e^x\, x^{7/2} + (y[x] \to e^x\, c_1 + e^x\, x\, c_2)\}\}$

q6

In[118]:=
```
sol = DSolve [y ''[x] + 2 * y '[x] + 2 * y[x] == 0, y[x], x]
```

Out[118]= $\{\{y[x] \to e^{-x}\, c_2\, \text{Cos}[x] + e^{-x}\, c_1\, \text{Sin}[x]\}\}$

In[165]:=
```
y1 := Exp[– 1 * x] * Sin[x];
y2 := Exp[– 1 * x] * Cos[x];
f := 4 * Exp[– 1 * x] * Sec ^ 3[x]
w = y1 * D[y2 , x] – y2 * D[y1 , x];
w = Simplify [w]
```

Out[169]= $-e^{-2\,x}$

In[170]:=
```
yp = – y1 * Integrate [y2 * (f / w), x] + y2 * Integrate [y1 * (f / w), x];
yp = Simplify [yp]
```

Out[171]= $e^{-x}\left(-4\,\text{Cos}[x]\int \text{Sec}^{3[x]}\,\text{Sin}[x]\,dl\,x + 4\left(\int \text{Sec}^{3[x]}\,\text{Cos}[x]\,dl\,x\right)\text{Sin}[x]\right)$

In[126]:=
```
Out[125] + Out[118]
```

Out[126]= $\left\{\left\{(y[x] \to e^{-x}\, c_2\, \text{Cos}[x] + e^{-x}\, c_1\, \text{Sin}[x]) + e^{-x}\left(-4\,\text{Cos}[x]\int \text{Sec}^{3[x]}\,\text{Sin}[x]\,dl\,x + 4\left(\int \text{Sec}^{3[x]}\,\text{Cos}[x]\,dl\,x\right)\text{Sin}[x]\right)\right\}\right\}$

q7

In[128]:=
```
sol = DSolve [y ''[x] – y[x] == 0, y[x], x]
```

Out[128]= $\{\{y[x] \to e^x\, c_1 + e^{-x}\, c_2\}\}$

In[129]:=
```
y1 := Exp[1 * x];
y2 := Exp[-1 * x];
f := 1 / Sinh[x];
w = y1 * D[y2, x] - y2 * D[y1, x];
w = Simplify [w]
```

Out[133]= -2

In[134]:=
```
yp = -y1 * Integrate [y2 * (f / w), x] + y2 * Integrate [y1 * (f / w), x];
yp = Simplify [yp]
```

Out[135]= $-e^{x}\, x + \frac{1}{2}\, e^{-x} \left(-1 + e^{2\, x}\right) \text{Log}\left[1 - e^{2\, x}\right]$

In[136]:=
```
Out[135] + Out[128]
```

Out[136]= $\left\{\left\{-e^{x}\, x + \frac{1}{2}\, e^{-x} \left(-1 + e^{2\, x}\right) \text{Log}\left[1 - e^{2\, x}\right] + (\text{y[x]} \to e^{x}\, \text{C}_1 + e^{-x}\, \text{C}_2)\right\}\right\}$