SOLUTION OF DIFFERENTIAL EQUATION VARIATION OF PARAMETER

PRACTICE SHEET 4

Q.Solve the given non homogeneous linear ordinary differential equation by variation of parameters.

1. y'' + 4 y = cos2x

In[1]:=
```
sol1 = DSolve[y''[x] + 4 * y[x] == 0, y[x], x]
```
Out[1]=
```
{{y[x] → C[1] Cos[2 x] + C[2] Sin[2 x]}}
```

In[2]:=
```
y1 := Cos[2 x];
```

In[3]:=
```
y2 := Sin[2 x];
```

In[4]:=
```
f := Cos[2 x];
```

In[5]:=
```
w = y1 * D[y2, x] - y2 * D[y1, x];
```

In[6]:=
```
w = Simplify[w]
```
Out[6]=
```
2
```

In[7]:=
```
yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];
```

In[8]:=
```
yp = Simplify[yp]
```
Out[8]=
$$\frac{1}{16}\left(\text{Cos}[2\,x] + 4\,x\,\text{Sin}[2\,x]\right)$$

In[10]:=
```
Out[1] + Out[8]
```

In[11]:=
$$\left\{\left\{\left(y[x] \to C[1]\,\text{Cos}[2\,x] + C[2]\,\text{Sin}[2\,x]\right) + \frac{1}{16}\left(\text{Cos}[2\,x] + 4\,x\,\text{Sin}[2\,x]\right)\right\}\right\}$$

2. x^2 y'' − xy' − 3 y = x^2

In[12]:= `sol2 = DSolve[x^2 * y''[x] - x * y'[x] - 3 * y[x] == 0, y[x], x]`

Out[12]= $\left\{\left\{y[x] \to x^3\, C[1] + \frac{C[2]}{x}\right\}\right\}$

In[13]:= `y1 := Cos[2 x];`

In[14]:= `y2 := Sin[2 x];`

In[15]:= `f := x^2;`

`w = y1 * D[y2, x] - y2 * D[y1, x];`

In[16]:= `w = Simplify[w]`

Out[16]= 2

In[18]:= `yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];`

In[22]:= `yp = Simplify[yp]`

Out[22]= $\frac{1}{8}\left(-1 + 2\,x^2\right)$

In[21]:= `Out[12] + Out[19]`

Out[21]= $\left\{\left\{\frac{1}{8}\left(-1 + 2\,x^2\right) + \left(y[x] \to x^3\, C[1] + \frac{C[2]}{x}\right)\right\}\right\}$

3. y'' - 4 y + 5 y = e^(2 x) csc x

In[25]:= `sol3 = DSolve[y''[x] - 4 * y'[x] + 5 * y[x] == 0, y[x], x]`

Out[25]= $\left\{\left\{y[x] \to e^{2x}\, C[2]\, \text{Cos}[x] + e^{2x}\, C[1]\, \text{Sin}[x]\right\}\right\}$

In[26]:= `y1 := Exp[2 * x] * Cos[x];`

In[27]:= `y2 := Exp[2 * x] * Sin[x];`

In[28]:= `f := Exp[2 * x] * Csc[x];`

In[29]:= `w = y1 * D[y2, x] - y2 * D[y1, x];`

In[30]:= `w = Simplify[w]`

Out[30]= e^{4x}

In[31]:= `yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];`

In[32]:= `yp = Simplify[yp]`

Out[32]= $e^{2x}\left(-x\,\text{Cos}[x] + \text{Log}[\text{Sin}[x]]\,\text{Sin}[x]\right)$

In[33]:= `Out[25] + Out[32]`

Out[33]= $\left\{\left\{\left(y[x] \to e^{2x}\, C[2]\, \text{Cos}[x] + e^{2x}\, C[1]\, \text{Sin}[x]\right) + e^{2x}\left(-x\,\text{Cos}[x] + \text{Log}[\text{Sin}[x]]\,\text{Sin}[x]\right)\right\}\right\}$

4. y'' - 2 y' + y = 6 x^2 e^(-x)

In[34]:= `sol4 = DSolve[y''[x] - 2 * y'[x] + y[x] == 0, y[x], x]`

Out[34]= $\{\{y[x] \to e^x C[1] + e^x x C[2]\}\}$

In[35]:= `y1 := Exp[x];`

In[36]:= `y2 := x * Exp[x];`

In[37]:= `f := Exp[-1 * x] * 6 * x^2;`

In[38]:= `w = y1 * D[y2, x] - y2 * D[y1, x];`

In[39]:= `w = Simplify[w]`

Out[39]= e^{2x}

In[40]:= `yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];`

In[41]:= `yp = Simplify[yp]`

Out[41]= $\frac{3}{4} e^{-x} \left(3 + 4x + 2x^2\right)$

In[42]:= `Out[34] + Out[41]`

Out[42]= $\left\{\left\{\frac{3}{4} e^{-x} \left(3 + 4x + 2x^2\right) + \left(y[x] \to e^x C[1] + e^x x C[2]\right)\right\}\right\}$

5. y'' - 2 y' + y = 35 x^(3/2) e^x

In[43]:= `sol5 = DSolve[y''[x] - 2 * y'[x] + y[x] == 0, y[x], x]`

Out[43]= $\{\{y[x] \to e^x C[1] + e^x x C[2]\}\}$

In[44]:= `y1 := Exp[x];`

In[45]:= `y2 := x * Exp[x];`

In[46]:= `f := 35 * x^(3/2) * Exp[x];`

In[47]:= `w = y1 * D[y2, x] - y2 * D[y1, x];`

In[48]:= `w = Simplify[w]`

Out[48]= e^{2x}

In[49]:= `yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];`

In[50]:= `yp = Simplify[yp]`

Out[50]= $4 e^x x^{7/2}$

In[51]:= `Out[43] + Out[50]`

Out[51]= $\left\{\left\{4 e^x x^{7/2} + \left(y[x] \to e^x C[1] + e^x x C[2]\right)\right\}\right\}$

6. y'' + 2 y + 2 y = 4 e^(-x) sec^3 (x)

In[54]:= `sol6 = DSolve[y''[x] + 2 * y'[x] + 2 * y[x] == 0, y[x], x]`

Out[54]= $\{\{y[x] \to e^{-x} C[2] \text{Cos}[x] + e^{-x} C[1] \text{Sin}[x]\}\}$

In[55]:= `y1 := Exp[-1 * x] * Cos[x];`

In[56]:=
```
y2 := Sin[x] * Exp[-1 * x];
```

In[57]:=
```
f := 4 * (Sec[x]) ^3 * Exp[-1 * x];
```

In[58]:=
```
w = y1 * D[y2, x] - y2 * D[y1, x];
```

In[59]:=
```
w = Simplify[w]
```

Out[59]= e^{-2x}

In[60]:=
```
yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];
```

In[61]:=
```
yp = Simplify[yp]
```

Out[61]= $-2\, e^{-x} \cos[2x] \sec[x]$

In[62]:=
```
Out[54] + Out[61]
```

Out[62]=
```
{{(y[x] → e^-x C[2] Cos[x] + e^-x C[1] Sin[x]) - 2 e^-x Cos[2 x] Sec[x]}}
```

7. y'' - y = 1 / sinhx

In[63]:=
```
sol7 = DSolve[y''[x] - y[x] == 0, y[x], x]
```

Out[63]= $\{\{y[x] \to e^{x} C[1] + e^{-x} C[2]\}\}$

In[64]:=
```
y1 := Exp[x];
```

In[65]:=
```
y2 := Exp[-1 * x];
```

In[66]:=
```
f := 1 / Sinh[x];
```

In[67]:=
```
w = y1 * D[y2, x] - y2 * D[y1, x];
```

In[68]:=
```
w = Simplify[w]
```

Out[68]= -2

In[69]:=
```
yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];
```

In[70]:=
```
yp = Simplify[yp]
```

Out[70]= $\frac{1}{2} e^{x} \operatorname{Log}\left[1 - e^{-2x}\right] - \frac{1}{2} e^{-x} \operatorname{Log}\left[1 - e^{2x}\right]$

In[71]:=
```
Out[63] + Out[70]
```

Out[71]=
```
{{1/2 e^x Log[1 - e^-2 x] - 1/2 e^-x Log[1 - e^2 x] + (y[x] → e^x C[1] + e^-x C[2])}}
```