PRACTICAL 4 -VARIATION OF PARAMETERS

Q1

In[43]:=
```
sol = DSolve[y''[x] + 4 * y[x] == 0, y[x], x]
```

Out[43]=
$$\{\{y[x] \to c_1 \text{Cos}[2\,x] + c_2 \text{Sin}[2\,x]\}\}$$

In[44]:=
```
y1 := Cos[2 * x];
y2 := Sin[2 * x];
f := Cos[2 * x];
w = y1 * D[y2, x] - y2 * D[y1, x];
w = Simplify[w]
```

Out[48]=
$$2$$

In[49]:=
```
yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];
yp = Simplify[yp]
```

Out[50]=
$$\frac{1}{16}(\text{Cos}[2\,x] + 4\,x\,\text{Sin}[2\,x])$$

In[51]:=
```
Out[43] + Out[50]
```

Out[51]=
$$\left\{\left\{(y[x] \to c_1 \text{Cos}[2\,x] + c_2 \text{Sin}[2\,x]) + \frac{1}{16}(\text{Cos}[2\,x] + 4\,x\,\text{Sin}[2\,x])\right\}\right\}$$

q2

In[37]:=
```
sol = DSolve[x^2 * y''[x] - x * y'[x] - 3 * y[x] == 0, y[x], x]
```

Out[37]=
$$\left\{\left\{y[x] \to \frac{c_1}{x} + x^3\, c_2\right\}\right\}$$

In[65]:=
```
y1 := 1 / x;
y2 := x^3;
f := x^2
w = y1 * D[y2, x] - y2 * D[y1, x];
w = Simplify[w]
```

Out[69]=
$$4\,x$$

In[70]:=
```
yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];
yp = Simplify[yp]
```

Out[71]=
$$\frac{x^4}{5}$$

In[72]:=
```
Out[37] + Out[71]
```

Out[72]=
$$\left\{\left\{\frac{x^4}{5} + \left(y[x] \to \frac{c_1}{x} + x^3\, c_2\right)\right\}\right\}$$

q3

In[79]:=
```
sol = DSolve[y''[x] - 4 * y'[x] + 5 * y[x] == 0, y[x], x]
```

Out[79]=
$$\{\{y[x] \to e^{2x}\, c_2\, \text{Cos}[x] + e^{2x}\, c_1\, \text{Sin}[x]\}\}$$

In[80]:=
```
y1 := Exp[2 * x] * Sin[x];
y2 := Exp[2 * x] * Cos[x];
f := Exp[2 * x] * Csc[x];
w = y1 * D[y2, x] - y2 * D[y1, x];
w = Simplify[w]
```

Out[84]=
$$-e^{4x}$$

In[85]:=
```
yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];
yp = Simplify[yp]
```

Out[86]=
$$e^{2x}\,(-x\,\text{Cos}[x] + \text{Log}[\text{Sin}[x]] \times \text{Sin}[x])$$

In[138]:=
```
Out[86] + Out[79]
```

Out[138]=
$$\{\{(y[x] \to e^{2x}\, c_2\, \text{Cos}[x] + e^{2x}\, c_1\, \text{Sin}[x]) + e^{2x}\,(-x\,\text{Cos}[x] + \text{Log}[\text{Sin}[x]] \times \text{Sin}[x])\}\}$$

q4

In[89]:=
```
sol = DSolve[y''[x] - 2 * y'[x] + y[x] == 0, y[x], x]
```

Out[89]=
$$\{\{y[x] \to e^{x}\, c_1 + e^{x}\, x\, c_2\}\}$$

In[172]:=
```
y1 := Exp[x];
y2 := x * Exp[x];
f := 6 * x ^ 2 * Exp[-1 * x];
w = y1 * D[y2, x] - y2 * D[y1, x];
w = Simplify[w]
```

Out[176]=
$$e^{2x}$$

In[177]:=
```
yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];
yp = Simplify[yp]
```

Out[178]=
$$\frac{3}{4}\, e^{-x}\left(3 + 4\,x + 2\,x^2\right)$$

In[137]:=

```
Out[109] + Out[89]
```

Out[137]=
$$\left\{\left\{\frac{3}{4}\, e^{-x}\left(3 + 4\,x + 2\,x^2\right) + (y[x] \to e^{x}\, c_1 + e^{x}\, x\, c_2)\right\}\right\}$$

q5

In[97]:=
```
sol = DSolve[y''[x] - 2 * y'[x] + y[x] == 0, y[x], x]
```

Out[97]=
$$\{\{y[x] \to e^x\, c_1 + e^x\, x\, c_2\}\}$$

In[155]:=
```
y1 := Exp[x];
y2 := Exp[x] * x;
f := 35 * x^(3 / 2) * Exp[x];
w = y1 * D[y2, x] - y2 * D[y1, x];
w = Simplify[w]
```

Out[159]=
$$e^{2\,x}$$

In[160]:=
```
yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];
yp = Simplify[yp]
```

Out[161]=
$$4\, e^x\, x^{7/2}$$

In[162]:=
```
Out[161] + Out[97]
```

Out[162]=
$$\{\{4\, e^x\, x^{7/2} + (y[x] \to e^x\, c_1 + e^x\, x\, c_2)\}\}$$

q6

In[118]:=
```
sol = DSolve[y''[x] + 2 * y'[x] + 2 * y[x] == 0, y[x], x]
```

Out[118]=
$$\{\{y[x] \to e^{-x}\, c_2\, \text{Cos}[x] + e^{-x}\, c_1\, \text{Sin}[x]\}\}$$

In[165]:=
```
y1 := Exp[-1 * x] * Sin[x];
y2 := Exp[-1 * x] * Cos[x];
f := 4 * Exp[-1 * x] * Sec^3[x]
w = y1 * D[y2, x] - y2 * D[y1, x];
w = Simplify[w]
```

Out[169]=
$$-e^{-2\,x}$$

In[170]:=
```
yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];
yp = Simplify[yp]
```

Out[171]=
$$e^{-x}\left(-4\,\text{Cos}[x]\int \text{Sec}^{3[x]}\,\text{Sin}[x]\,dx + 4\left(\int \text{Sec}^{3[x]}\,\text{Cos}[x]\,dx\right)\text{Sin}[x]\right)$$

In[126]:=
```
Out[125] + Out[118]
```

Out[126]=
$$\left\{\left\{(y[x] \to e^{-x}\, c_2\, \text{Cos}[x] + e^{-x}\, c_1\, \text{Sin}[x]) + e^{-x}\left(-4\,\text{Cos}[x]\int \text{Sec}^{3[x]}\,\text{Sin}[x]\,dx + 4\left(\int \text{Sec}^{3[x]}\,\text{Cos}[x]\,dx\right)\text{Sin}[x]\right)\right\}\right\}$$

q7

In[128]:=
```
sol = DSolve[y''[x] - y[x] == 0, y[x], x]
```

Out[128]=
$$\{\{y[x] \to e^x\, c_1 + e^{-x}\, c_2\}\}$$

In[129]:=
```
y1 := Exp[1 * x];
y2 := Exp[-1 * x];
f := 1 / Sinh[x];
w = y1 * D[y2, x] - y2 * D[y1, x];
w = Simplify[w]
```

Out[133]= -2

In[134]:=
```
yp = -y1 * Integrate[y2 * (f / w), x] + y2 * Integrate[y1 * (f / w), x];
yp = Simplify[yp]
```

Out[135]= $-e^{x}\, x + \frac{1}{2} e^{-x} \left(-1 + e^{2x}\right) \text{Log}\left[1 - e^{2x}\right]$

In[136]:=
```
Out[135] + Out[128]
```

Out[136]= $\left\{\left\{-e^{x}\, x + \frac{1}{2} e^{-x} \left(-1 + e^{2x}\right) \text{Log}\left[1 - e^{2x}\right] + (y[x] \to e^{x}\, c_1 + e^{-x}\, c_2)\right\}\right\}$