Practical 7

Plotting the characteristics of the first order PDE

(y^2up/x)+ xup = y^2

In[∘]:=
```
a[u_, x_, y_] := y[x]^2 * u[x, y] / x
```

In[∘]:=
```
b[u_, x_, y_] := u[x, y] x
```

In[∘]:=
```
sol = DSolve[y'[x] == b[u, x, y] / a[u, x, y], y[x], x]
```

Out[∘]= $\left\{\left\{y[x] \to \left(x^3 + 3\,C[1]\right)^{1/3}\right\}, \left\{y[x] \to -(-1)^{1/3} \left(x^3 + 3\,C[1]\right)^{1/3}\right\}, \left\{y[x] \to (-1)^{2/3} \left(x^3 + 3\,C[1]\right)^{1/3}\right\}\right\}$

In[∘]:=
```
tab1 = Table[y[x] /. sol[[1, 1]] /. C[1] → i, {i, -2, 2}]
```

Out[∘]= $\left\{\left(-6 + x^3\right)^{1/3}, \left(-3 + x^3\right)^{1/3}, \left(x^3\right)^{1/3}, \left(3 + x^3\right)^{1/3}, \left(6 + x^3\right)^{1/3}\right\}$

In[∘]:=
```
tab2 = Table[y[x] /. sol[[2, 1]] /. C[1] → i, {i, -2, 2}]
```

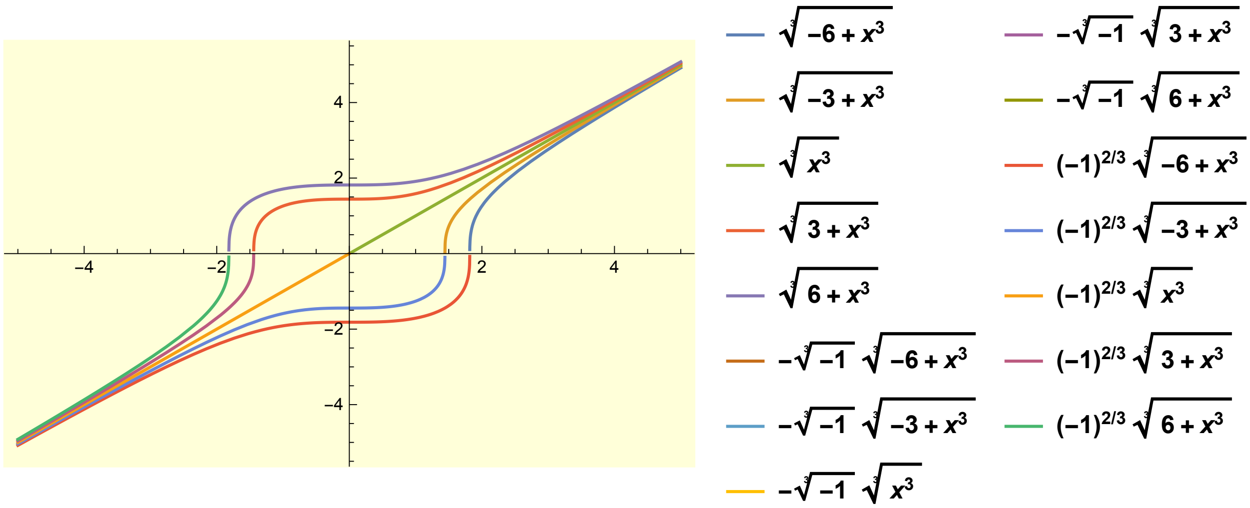
Out[∘]= $\left\{-(-1)^{1/3} \left(-6 + x^3\right)^{1/3}, -(-1)^{1/3} \left(-3 + x^3\right)^{1/3}, -(-1)^{1/3} \left(x^3\right)^{1/3}, -(-1)^{1/3} \left(3 + x^3\right)^{1/3}, -(-1)^{1/3} \left(6 + x^3\right)^{1/3}\right\}$

In[∘]:=
```
tab3 = Table[y[x] /. sol[[3, 1]] /. C[1] → i, {i, -2, 2}]
```

Out[∘]= $\left\{(-1)^{2/3} \left(-6 + x^3\right)^{1/3}, (-1)^{2/3} \left(-3 + x^3\right)^{1/3}, (-1)^{2/3} \left(x^3\right)^{1/3}, (-1)^{2/3} \left(3 + x^3\right)^{1/3}, (-1)^{2/3} \left(6 + x^3\right)^{1/3}\right\}$

In[∘]:=
```
Plot[{Evaluate[tab1], Evaluate[tab2], Evaluate[tab3]},
 {x, -5, 5}, PlotLegends → "Expressions"]
```

Out[ ]=



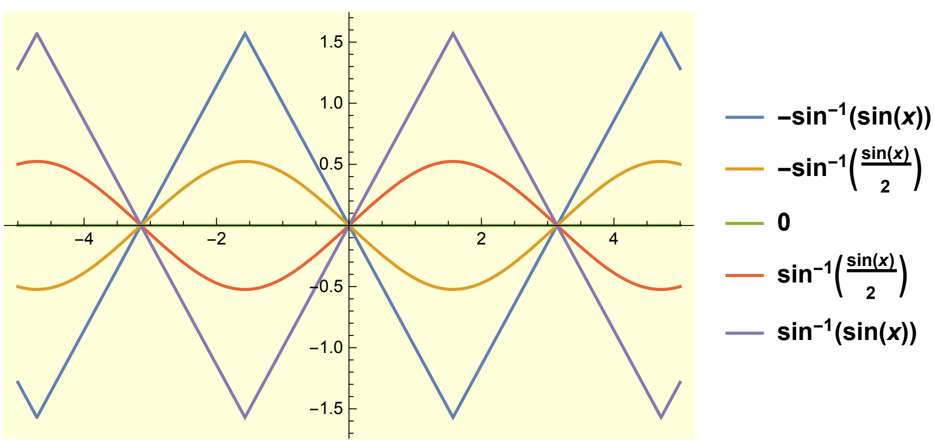
ptanx+qtany=tanz

```
In[ ]:= a[u_, x_, y_] := Tan[x]
In[ ]:= b[u_, x_, y_] := Tan[y[x]]
In[ ]:= sol = DSolve[y'[x] == b[u, x, y] / a[u, x, y], y[x], x]
```

Out[ ]= $\{\{y[x] \to \text{ArcSin}[\frac{1}{2} C[1] \text{Sin}[x]]\}\}$

```
tab1 = Table[y[x] /. sol[[1, 1]] /. C[1] → i, {i, -2, 2}]
In[ ]:= Plot[{Evaluate[tab1]}, {x, -5, 5}, PlotLegends → "Expressions"]
```

Out[ ]=

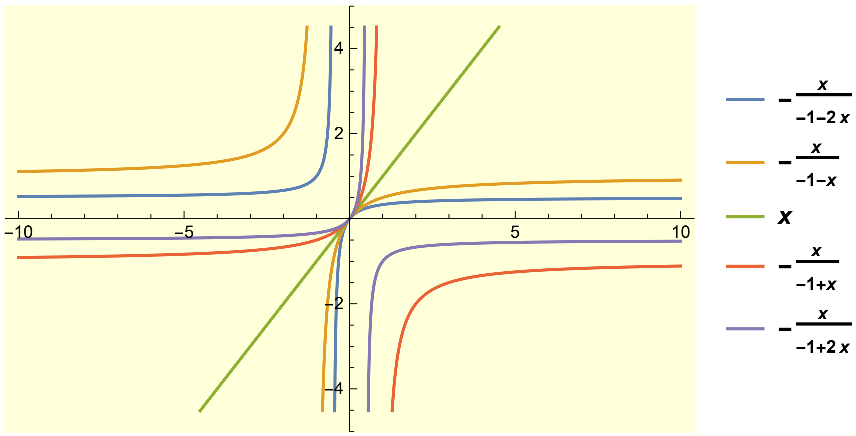


x(y^2-z^2)q-y(z^2+x^2)q=z(x^2+y^2)

```
In[ ]:= a[u_, x_, y_] := x^2
In[ ]:= b[u_, x_, y_] := y[x]^2
```

In[∘]:=
```
sol = DSolve[y'[x] == b[u, x, y] / a[u, x, y], y[x], x]
```
Out[∘]= $\left\{\left\{y[x] \to -\frac{x}{-1 + x\, C[1]}\right\}\right\}$

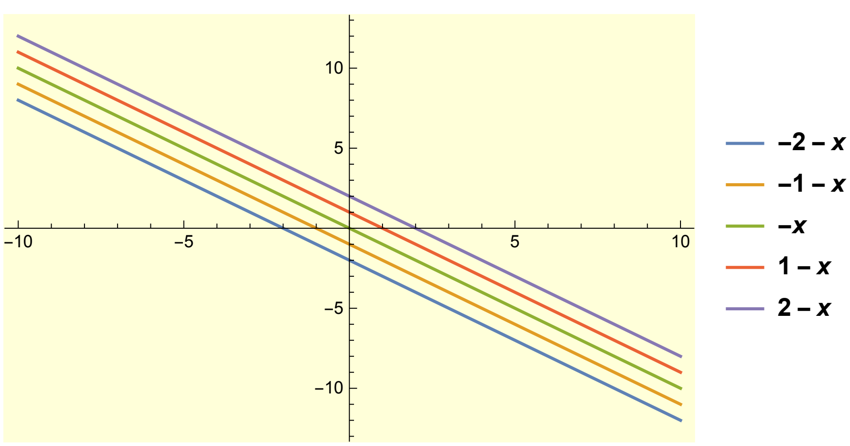
In[∘]:=
```
tab1 = Table[y[x] /. sol[[1, 1]] /. C[1] → i, {i, -2, 2}]
Plot[{Evaluate[tab1]}, {x, -10, 10}, PlotLegends → "Expressions"]
```
Out[∘]= $\left\{-\frac{x}{-1 - 2x}, -\frac{x}{-1 - x}, x, -\frac{x}{-1 + x}, -\frac{x}{-1 + 2x}\right\}$

Out[∘]=


u_x - u_y = 1

In[∘]:=
```
a[u_, x_, y_] := 1
```
In[∘]:=
```
b[u_, x_, y_] := -1
```
In[∘]:=
```
sol = DSolve[y'[x] == b[u, x, y] / a[u, x, y], y[x], x]
```
Out[∘]= $\{\{y[x] \to -x + C[1]\}\}$

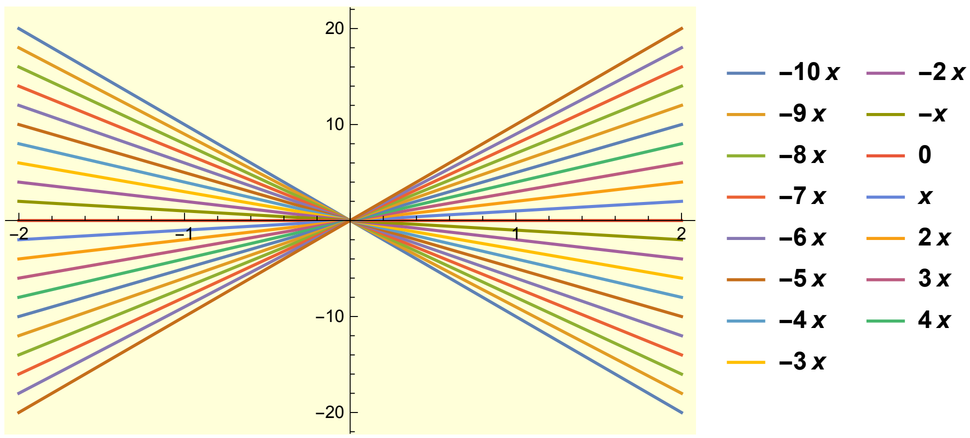
In[∘]:=
```
tab1 = Table[y[x] /. sol[[1, 1]] /. C[1] → i, {i, -2, 2}]
Plot[{Evaluate[tab1]}, {x, -10, 10}, PlotLegends → "Expressions"]
```
Out[∘]= $\{-2 - x, -1 - x, -x, 1 - x, 2 - x\}$

Out[∘]=


xu_x+ yu_y = u

```
In[◦]:= a[x_, y_, u_] := x

In[◦]:= b[x_, y_, u_] := y[x]

In[◦]:= sol = DSolve[y'[x] == b[x, y, u] / a[x, y, u], y[x], x]

Out[◦]= {{y[x] → x C[1]}}

In[◦]:= tab = Table[y[x] /. sol /. C[1] → i, {i, -10, 10}] // Flatten

Out[◦]= {-10 x, -9 x, -8 x, -7 x, -6 x, -5 x, -4 x, -3 x,
  -2 x, -x, 0, x, 2 x, 3 x, 4 x, 5 x, 6 x, 7 x, 8 x, 9 x, 10 x}

In[◦]:= Plot[Evaluate[tab], {x, -2, 2}, PlotLegends → "Expressions"]
```

Out[◦]=




```
In[◦]:= Clear[y, x, u, a, b, sol]
```

$u\,(x+y)\,u_x + u\,(x-y)\,u_y = \text{x^2 + y^2}$

```
In[◦]:= a[u_, x_, y_] := u[x, y] (x + y[x])

In[◦]:= b[u_, x_, y_] := u[x, y] (x - y[x])

In[◦]:= sol = DSolve[y'[x] == b[u, x, y] / a[u, x, y], y[x], x]
```

Out[◦]= $\left\{\left\{y[x] \to -x - \sqrt{e^{2\,C[1]} + 2\,x^2}\right\}, \left\{y[x] \to -x + \sqrt{e^{2\,C[1]} + 2\,x^2}\right\}\right\}$

```
In[◦]:= tab1 = Table[y[x] /. sol[[1, 1]] /. C[1] → i, {i, -2, 2}]
```

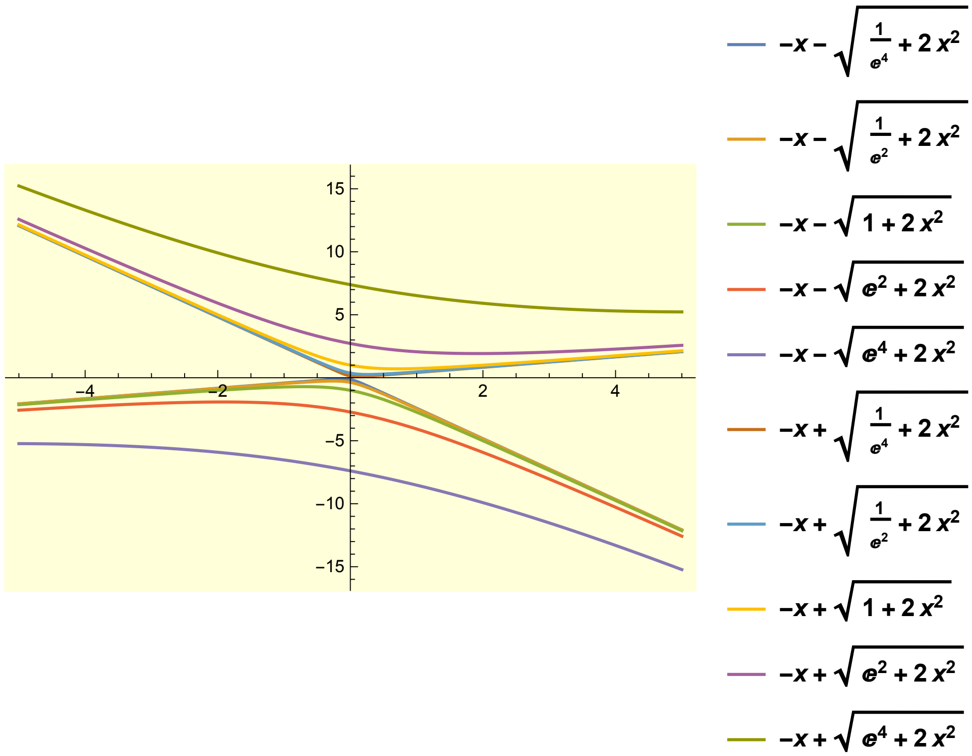
Out[◦]= $\left\{-x - \sqrt{\frac{1}{e^4} + 2\,x^2},\ -x - \sqrt{\frac{1}{e^2} + 2\,x^2},\ -x - \sqrt{1 + 2\,x^2},\ -x - \sqrt{e^2 + 2\,x^2},\ -x - \sqrt{e^4 + 2\,x^2}\right\}$

```
In[◦]:= tab2 = Table[y[x] /. sol[[2, 1]] /. C[1] → i, {i, -2, 2}]
```

Out[◦]= $\left\{-x + \sqrt{\frac{1}{e^4} + 2\,x^2},\ -x + \sqrt{\frac{1}{e^2} + 2\,x^2},\ -x + \sqrt{1 + 2\,x^2},\ -x + \sqrt{e^2 + 2\,x^2},\ -x + \sqrt{e^4 + 2\,x^2}\right\}$

```
In[◦]:= Plot[{Evaluate[tab1], Evaluate[tab2]}, {x, -5, 5}, PlotLegends → "Expressions"]
```

Out[ ]=


15
10
5
−4
−2
2
4
−5
−10
−15
$-x-\sqrt{\frac{1}{e^4}+2x^2}$
$-x-\sqrt{\frac{1}{e^2}+2x^2}$
$-x-\sqrt{1+2x^2}$
$-x-\sqrt{e^2+2x^2}$
$-x-\sqrt{e^4+2x^2}$
$-x+\sqrt{\frac{1}{e^4}+2x^2}$
$-x+\sqrt{\frac{1}{e^2}+2x^2}$
$-x+\sqrt{1+2x^2}$
$-x+\sqrt{e^2+2x^2}$
$-x+\sqrt{e^4+2x^2}$