Practical -3

3rd ORDER DIFFERENTIAL EQUATION

Q1 . y''' + 2 y' + y' = 0

In[]:=
```
sol = DSolve[y'''[x] + 2 * y'[x] + y'[x] == 0, y[x], x]
```

Out[]=
$$\left\{\left\{y[x] \to C[3] - \frac{C[2]\,\text{Cos}\left[\sqrt{3}\,x\right]}{\sqrt{3}} + \frac{C[1]\,\text{Sin}\left[\sqrt{3}\,x\right]}{\sqrt{3}}\right\}\right\}$$

In[]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
```

Out[]=
$$3 - \frac{2\,\text{Cos}\left[\sqrt{3}\,x\right]}{\sqrt{3}} + \frac{\text{Sin}\left[\sqrt{3}\,x\right]}{\sqrt{3}}$$

In[]:=
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 2, C[3] → 3}]
```

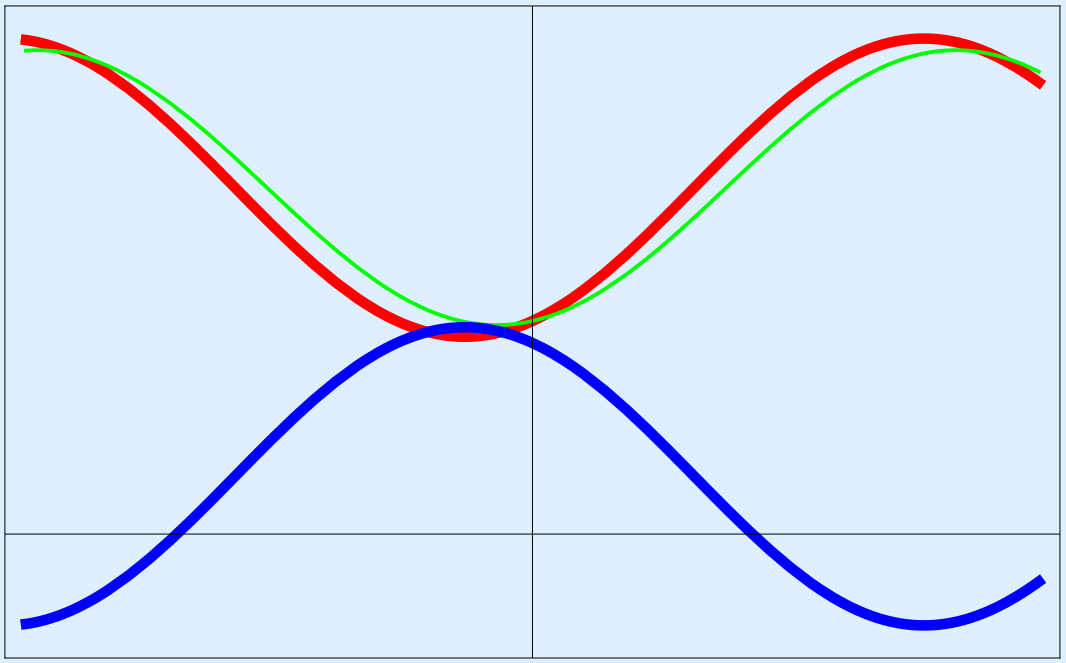
Out[]=
$$3 - \frac{2\,\text{Cos}\left[\sqrt{3}\,x\right]}{\sqrt{3}} + 0.288675\,\text{Sin}\left[\sqrt{3}\,x\right]$$

In[]:=
```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → -2, C[3] → 0.5}]
```

Out[]=
$$0.5 + \frac{2\,\text{Cos}\left[\sqrt{3}\,x\right]}{\sqrt{3}} - \frac{\text{Sin}\left[\sqrt{3}\,x\right]}{\sqrt{3}}$$

In[ ]:=
```
Plot[{sol1, sol2, sol3}, {x, -2, 2},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Blue, Thickness[0.01]}},
 PlotLegends → {sol1, sol2, sol3}, Frame → True,
 ImageSize → 550]
```

Out[ ]=



Q2.y''' + y' = Tanx

In[ ]:=
```
sol = DSolve[y'''[x] + y'[x] == Tan[x], y[x], x]
```

Out[ ]=
$$\left\{\left\{y[x] \to C[3] - C[2]\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + C[1]\,\text{Sin}[x] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right]\text{Sin}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]\text{Sin}[x]\right\}\right\}$$

In[ ]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
```

Out[ ]=
$$3 - 2\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sin}[x] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right]\text{Sin}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]\text{Sin}[x]$$

In[ ]:=
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → -2, C[3] → 1.3}]
```

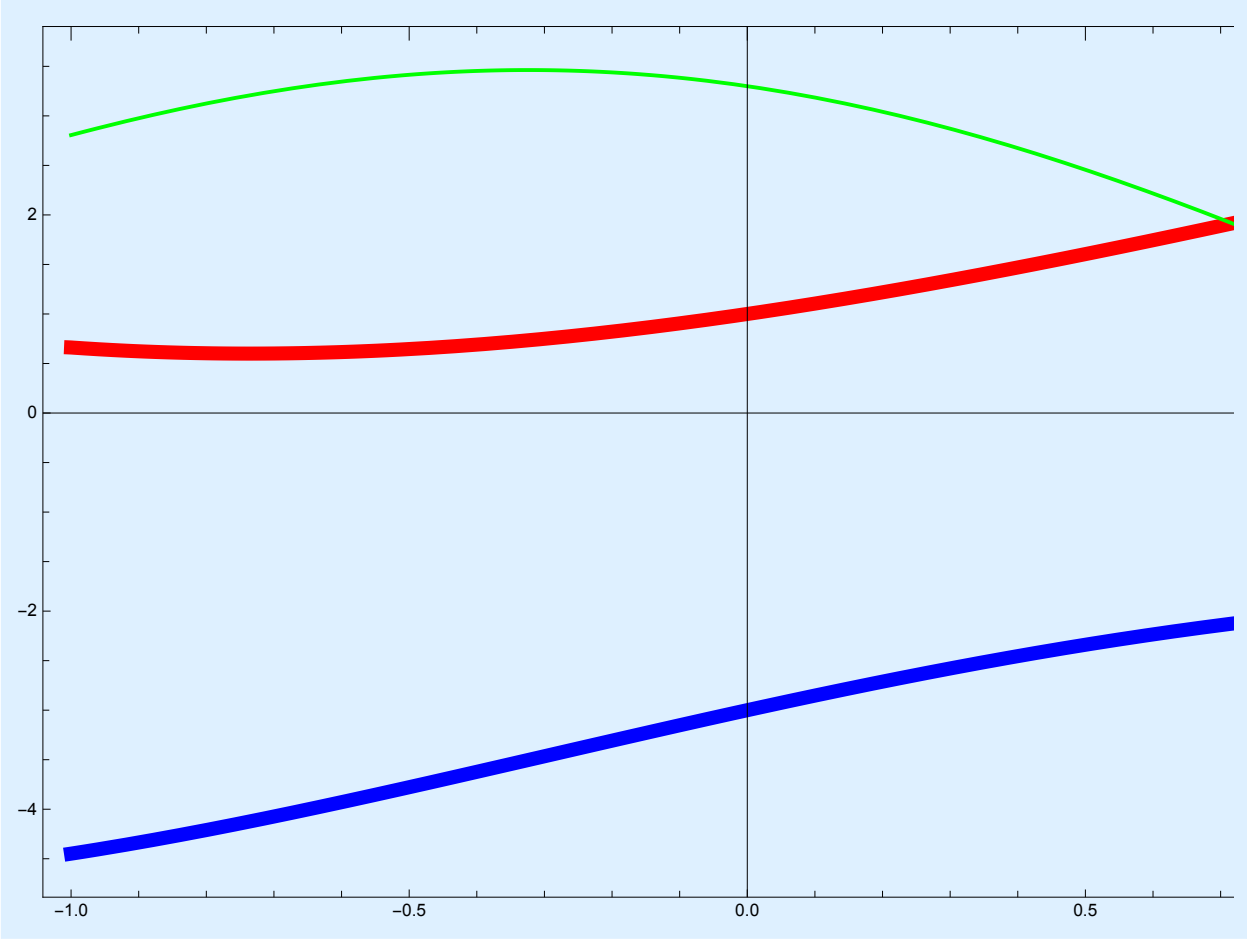
Out[ ]=
$$1.3 + 2\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Sin}[x] + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right]\text{Sin}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]\text{Sin}[x]$$

In[•]:=
```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 3/2, C[2] → 0.5, C[3] → -2.5}]
```

Out[•]=
$$-2.5 - 0.5\,\text{Cos}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \frac{3\,\text{Sin}[x]}{2} + \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right]\text{Sin}[x] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right]\text{Sin}[x]$$

In[•]:=
```
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Blue, Thickness[0.01]}},
 PlotLegends → {sol1, sol2, sol3}, Frame → True, ImageSize → 750]
```

Out[•]=



Q3. y''' + y' = 0

```
sol = DSolve[y'''[x] + y'[x] == 0, y[x], x]
```

Out[•]=
$$\{\{y[x] \to C[3] - C[2]\,\text{Cos}[x] + C[1]\,\text{Sin}[x]\}\}$$

In[•]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 2/3}]
```

Out[•]=
$$\frac{2}{3} - 2\,\text{Cos}[x] + \text{Sin}[x]$$

In[•]:=
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 0.5, C[2] → 2, C[3] → 3}]
```

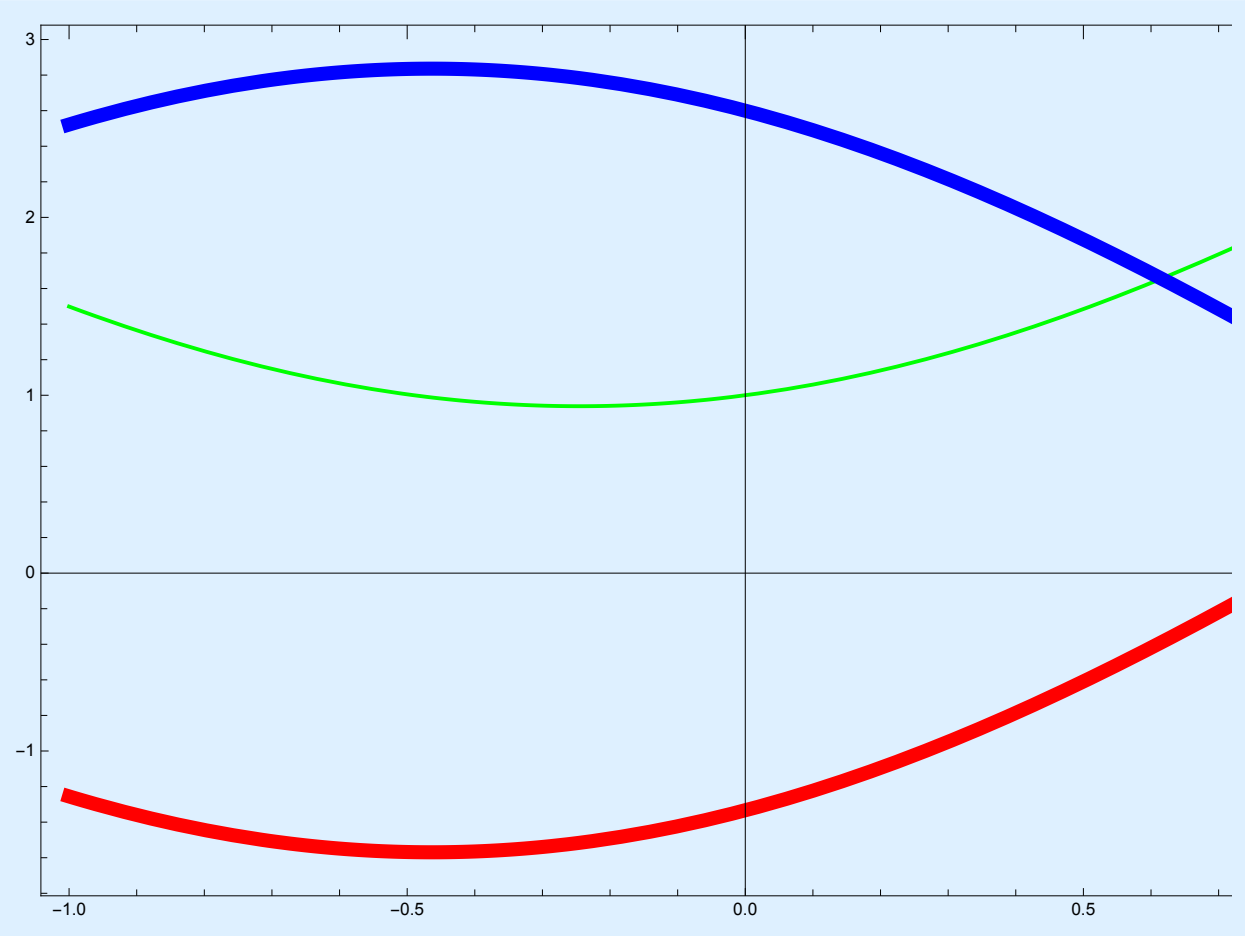
Out[•]=
$$3 - 2\,\text{Cos}[x] + 0.5\,\text{Sin}[x]$$

In[•]:=
```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → -2, C[3] → 0.6}]
```

Out[•]=
$$0.6 + 2\cos[x] - \sin[x]$$

In[•]:=
```
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Blue, Thickness[0.01]}},
 PlotLegends → {sol1, sol2, sol3}, Frame → True, ImageSize → 750]
```

Out[•]=



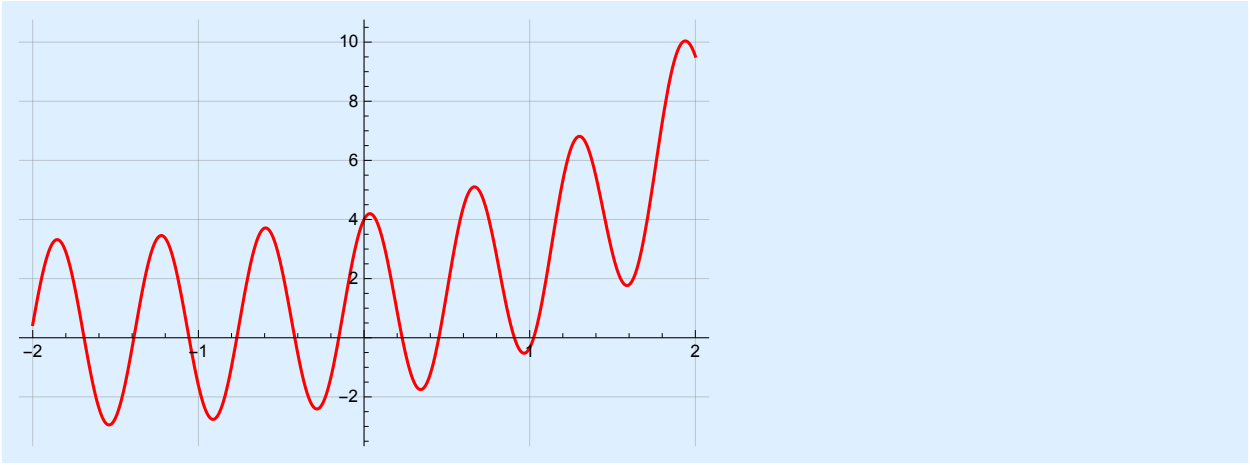
Q4. $y''' - y'' + 100\,y' - 100\,y = 0$, $y(0) = 4,\ y'(0) = 11,\ y''(0) = -299$

In[•]:=
```
sol = DSolve[{y'''[x] - y''[x] + 100 * y'[x] - 100 * y[x] == 0,
    y[0] == 4, y'[0] == 11, y''[0] == -299}, y[x], x]
```

Out[•]=
$$\{\{y[x] \to e^{x} + 3\cos[10\,x] + \sin[10\,x]\}\}$$

In[•]:=
```
Plot[y[x] /. sol, {x, -2, 2}, PlotStyle → {Red}, GridLines → Automatic]
```

Out[•]=



Q5.y''' = x

In[•]:=
```
sol = DSolve[y'''[x] == x, y[x], x]
```

Out[•]= $\left\{\left\{y[x] \to \frac{x^4}{24} + C[1] + x\, C[2] + x^2\, C[3]\right\}\right\}$

In[•]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
```

Out[•]= $1 + 2\,x + 3\,x^2 + \frac{x^4}{24}$

In[•]:=
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 3, C[3] → 2.7}]
```

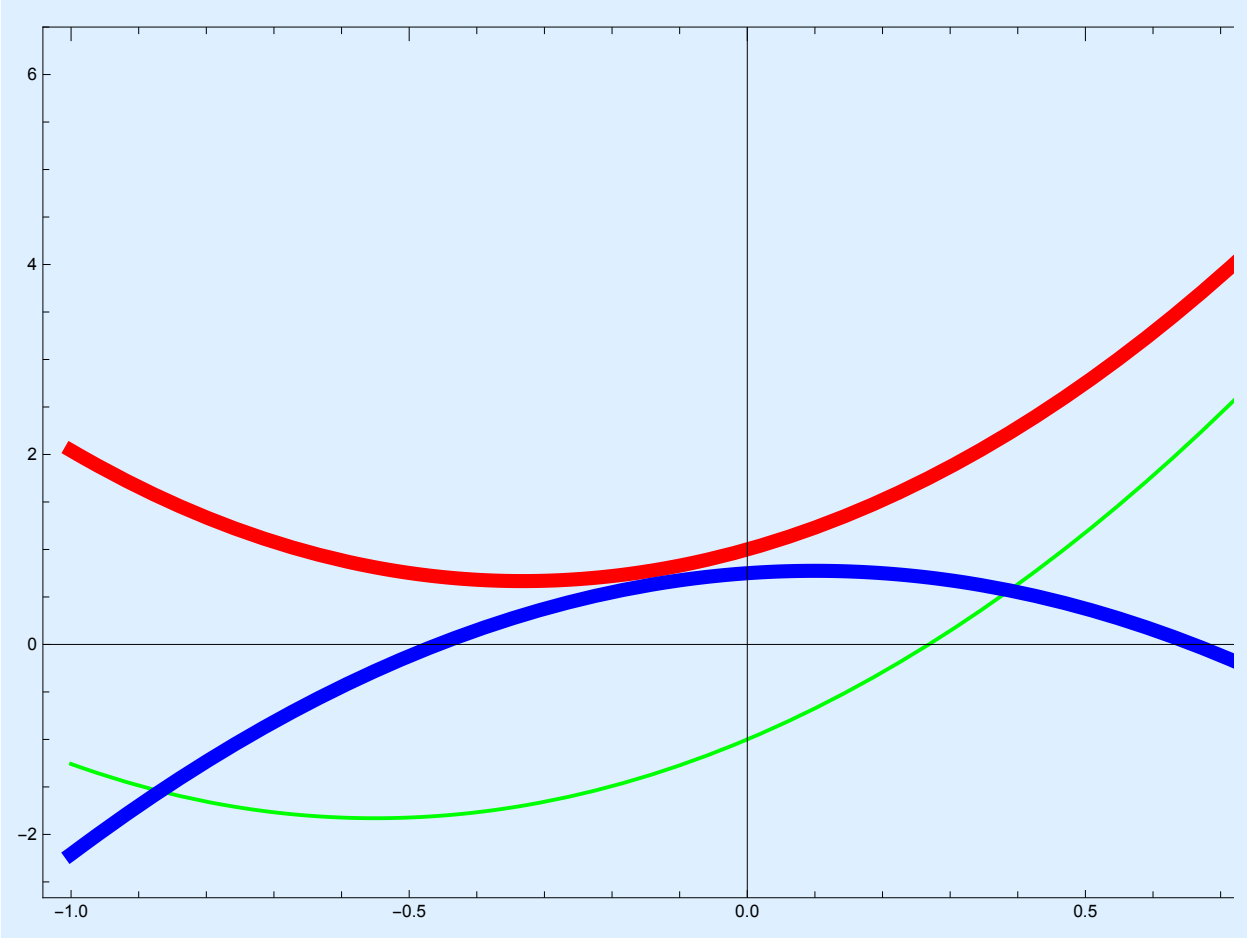
Out[•]= $-1 + 3\,x + 2.7\,x^2 + \frac{x^4}{24}$

In[•]:=
```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 3/4, C[2] → 0.5, C[3] → -2.5}]
```

Out[•]= $\frac{3}{4} + 0.5\,x - 2.5\,x^2 + \frac{x^4}{24}$

In[ ]:=
```
Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Blue, Thickness[0.01]}},
 PlotLegends → {sol1, sol2, sol3}, Frame → True, ImageSize → 750]
```

Out[ ]=



Q6.y''' + 4 y' = sec (2 x)

In[ ]:=
```
sol = DSolve[y'''[x] + 4 * y'[x] == Sec[2 x], y[x], x]
```

Out[ ]=
$$\left\{\left\{y[x] \to C[3] - C[2]\,\text{Cos}[x]^2 - \frac{1}{4}\,x\,\text{Cos}[2x] - \frac{1}{8}\,\text{Log}[\text{Cos}[x] - \text{Sin}[x]] + \frac{1}{8}\,\text{Log}[\text{Cos}[x] + \text{Sin}[x]] + \frac{1}{2}\,C[1]\,\text{Sin}[2x] + \frac{1}{8}\,\text{Log}[\text{Cos}[2x]]\,\text{Sin}[2x]\right\}\right\}$$

In[ ]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
```

Out[ ]=
$$3 - 2\,\text{Cos}[x]^2 - \frac{1}{4}\,x\,\text{Cos}[2x] - \frac{1}{8}\,\text{Log}[\text{Cos}[x] - \text{Sin}[x]] + \frac{1}{8}\,\text{Log}[\text{Cos}[x] + \text{Sin}[x]] + \frac{1}{2}\,\text{Sin}[2x] + \frac{1}{8}\,\text{Log}[\text{Cos}[2x]]\,\text{Sin}[2x]$$

```
In[•]:= sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2.5, C[2] → 1.5, C[3] → -3}]
```

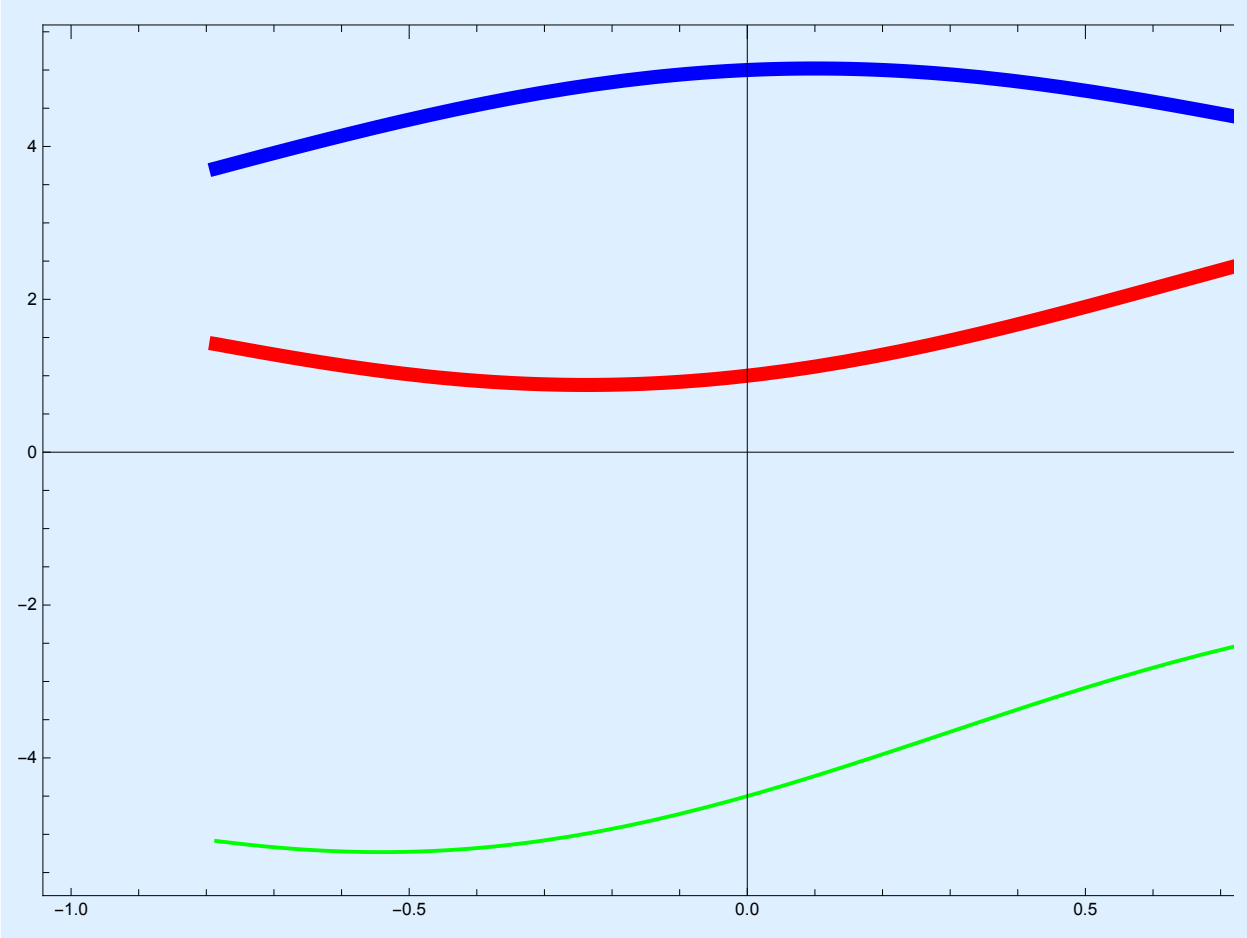
$$\text{Out}[\bullet]= -3 - 1.5\,\text{Cos}[x]^2 - \frac{1}{4}\,x\,\text{Cos}[2x] - \frac{1}{8}\,\text{Log}[\text{Cos}[x] - \text{Sin}[x]] + \frac{1}{8}\,\text{Log}[\text{Cos}[x] + \text{Sin}[x]] + 1.25\,\text{Sin}[2x] + \frac{1}{8}\,\text{Log}[\text{Cos}[2x]]\,\text{Sin}[2x]$$

```
In[•]:= sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 2/5, C[2] → -2, C[3] → 3}]
```

$$\text{Out}[\bullet]= 3 + 2\,\text{Cos}[x]^2 - \frac{1}{4}\,x\,\text{Cos}[2x] - \frac{1}{8}\,\text{Log}[\text{Cos}[x] - \text{Sin}[x]] + \frac{1}{8}\,\text{Log}[\text{Cos}[x] + \text{Sin}[x]] + \frac{1}{5}\,\text{Sin}[2x] + \frac{1}{8}\,\text{Log}[\text{Cos}[2x]]\,\text{Sin}[2x]$$

```
In[•]:= Plot[{sol1, sol2, sol3}, {x, -1, 1},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Blue, Thickness[0.01]}},
 PlotLegends → {sol1, sol2, sol3}, Frame → True, ImageSize → 750]
```

Out[•]=



Q7.y''' – 2 y' – y' + 2 y = e^4 x

```
In[•]:= sol = DSolve[y'''[x] - 3 * y'[x] + 2 * y[x] == Exp[4 * x], y[x], x]
```

$$\text{Out}[\bullet]= \left\{\left\{y[x] \to \frac{e^{4x}}{54} + e^{-2x}\,C[1] + e^{x}\,C[2] + e^{x}\,x\,C[3]\right\}\right\}$$

In[ ]:=
```
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1, C[2] → 2, C[3] → 3}]
```

Out[ ]=
$$e^{-2x} + 2e^{x} + \frac{e^{4x}}{54} + 3e^{x}x$$

In[ ]:=
```
sol2 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 5, C[2] → 2, C[3] → -1}]
```

Out[ ]=
$$5e^{-2x} + 2e^{x} + \frac{e^{4x}}{54} - e^{x}x$$

In[ ]:=
```
sol3 = Evaluate[y[x] /. sol[[1]] /. {C[1] → -1, C[2] → 2/3, C[3] → 0.5}]
```

Out[ ]=
$$-e^{-2x} + \frac{2e^{x}}{3} + \frac{e^{4x}}{54} + 0.5e^{x}x$$

In[ ]:=
```
Plot[{sol1, sol2, sol3}, {x, -2, 2},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Blue, Thickness[0.01]}},
 PlotLegends → {sol1, sol2, sol3}, Frame → True, ImageSize → 750]
```

Out[ ]=