PRACTICAL - 1

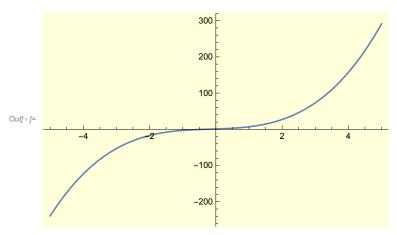
SOLUTION OF FIRST ORDER DIFFERENTIAL EQUATION

Ordinary Differential Equations(ODEs), in which there is a single independent variable and more dependent variable

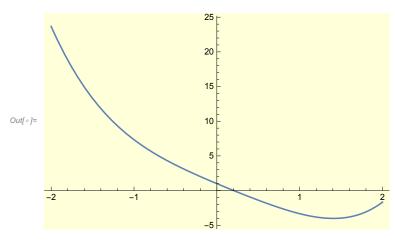
1. Solve the Differential equation $dy/dx=6(x^2)+2x+3$

$$\begin{split} & \inf_{s':=} \ sol1 = DSolve \Big[\Big\{ y' [x] == 6 * (x^2) + 2 * x + 3 \Big\}, \ y[x], \ x \Big] \\ & \text{Out}[s] = \ \Big\{ \Big\{ y[x] \to 3 \ x + x^2 + 2 \ x^3 + C[1] \Big\} \Big\} \\ & \inf_{s':=} \ A = y[x] \ /. \ sol1 \ /. \ \{ C[1] \to 1 \} \\ & \text{Out}[s] = \ \Big\{ 1 + 3 \ x + x^2 + 2 \ x^3 \Big\} \end{split}$$

In[*]:= Plot[A, {x, -5, 5}]



$$2.dy/dx=2(x^3)-x^2+x-5,y(0)=1$$



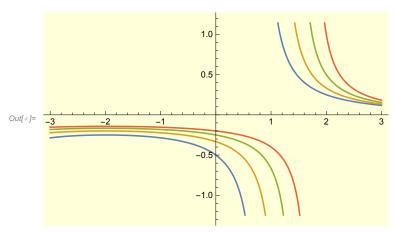
$3.dy/dx+(x+2)y^2=0$

$$ln[*]:= sol3 = DSolve[{y'[x] + (x + 2) * y[x]^2 == 0}, y[x], x]$$

$$\text{Out[s]= } \left\{ \left\{ y \left[\, x \, \right] \right. \right. \rightarrow \left. \frac{2}{4 \, x + x^2 - 2 \, C \left[\, 1 \, \right]} \right\} \right\}$$

$$ln[\circ]:= A = Table[y[x] /. sol3 /. {C[1] \rightarrow k}, {k, 2, 5}]$$

$$\textit{Out[*]=} \ \Big\{ \left\{ \frac{2}{-4+4\,x+x^2} \right\} \text{, } \Big\{ \frac{2}{-6+4\,x+x^2} \Big\} \text{, } \Big\{ \frac{2}{-8+4\,x+x^2} \Big\} \text{, } \Big\{ \frac{2}{-10+4\,x+x^2} \Big\} \Big\}$$



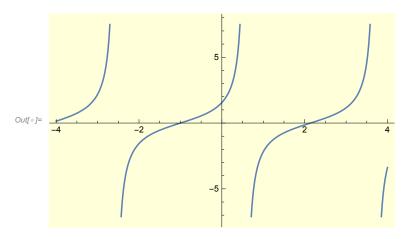
$4.dy/dx=1+y^2$

$$ln[*]:= sol4 = DSolve[y'[x] == 1 + y[x]^2, y[x], x]$$

$$\textit{Out[*]} = \; \big\{ \, \big\{ \, y \, \big[\, x \, \big] \, \rightarrow \, Tan \, \big[\, x \, + \, C \, \big[\, \boldsymbol{1} \, \big] \, \big] \, \big\} \, \big\}$$

$$ln[*]:= A = y[x] /. sol4 /. {C[1] \rightarrow 1}$$

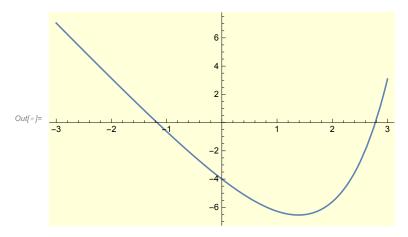
Out[
$$\bullet$$
]= { Tan [1 + x] }



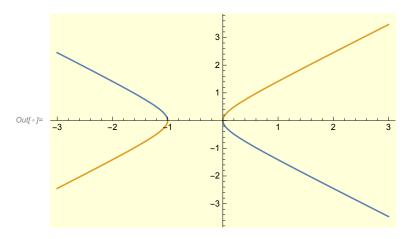
5.dy/dx=4*x+y+1

$$\begin{aligned} &\inf \{ := \ \, \text{sol5} = \text{DSolve}[\, y \, ' \, [\, x] \, == \, (4 * x) \, + y \, [\, x] \, + 1 \, , \, y \, [\, x] \, , \, x \,] \\ & \text{Out}[\, e \,] = \, \left\{ \, \left\{ \, y \, [\, x \,] \, \, \to \, -5 \, -4 \, x \, + \, \, \mathbb{e}^{\, x} \, C \, [\, 1 \,] \, \, \right\} \, \right. \\ & \inf \{ e \,] = \, \left\{ \, -5 \, + \, \, \mathbb{e}^{\, x} \, -4 \, x \, \right\} \\ & \text{Out}[\, e \,] = \, \left\{ \, -5 \, + \, \, \mathbb{e}^{\, x} \, -4 \, x \, \right\} \\ \end{aligned}$$

 $In[*]:= Plot[B, \{x, -3, 3\}]$



6. $2*x*y*dy/dx=x^2+y^2$



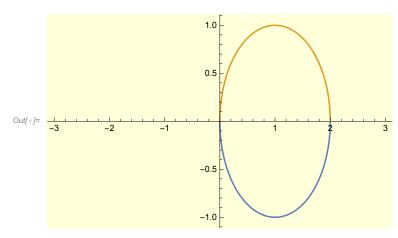
$7.dy/dx=(y^2-x^2)/2*x*y$

$$ln[*]:= sol7 = DSolve[y'[x] == (y[x]^2 - x^2) / (2 * x * y[x]), y[x], x]$$

$$\textit{Out[*]$= } \left\{ \left. \left\{ y \, [\, x \,] \right. \right. \right. \rightarrow - \sqrt{- \, x^2 \, + \, x \, C \, [\, 1\,]} \, \left. \right\}, \, \left\{ y \, [\, x \,] \right. \right. \rightarrow \sqrt{- \, x^2 \, + \, x \, C \, [\, 1\,]} \, \left. \right\} \right\}$$

$$ln[*]:= B = y[x] /. sol7 /. {C[1] \rightarrow 2}$$

Out[
$$\circ$$
]= $\left\{-\sqrt{2 x - x^2}, \sqrt{2 x - x^2}\right\}$



8.Check whether the following differential equations are exact or not.

$$a)(x+2y)dx+(2x-y)dy=0$$

$$ln[\cdot]:= P[x_{y}] := (x + 2 * y)$$

$$Q[x_{y_{1}}] := (2 * x - y)$$

$$Simplify[D[P[x, y], y] - D[Q[x, y], x]]$$

out[*]= eqn =
$$(y'[x] = -P[x, y[x]]/Q[x, y[x]])$$

 $(x) = -P[x, y[x]]/Q[x, y[x]]$

In[*]:= Expand[eqn]

Out[*]=
$$y'[x] = -\frac{x}{2x - y[x]} - \frac{2y[x]}{2x - y[x]}$$

$$lo[*]:= FullSimplify[y'[x] == -\frac{x}{2x-y[x]} - \frac{2y[x]}{2x-y[x]}]$$

Out[
$$e$$
]= $y'[x] == 2 + \frac{5x}{-2x + y[x]}$

b)
$$(x^2+2^*y^2)dx+(4^*x^*y-y^2)dy=0$$

$$\begin{array}{ll} In[*] := & P[x_{,}, y_{,}] := (x^2 + 2 * y^2) \\ & Q[x_{,}, y_{,}] := (4 * x * y - y^2) \\ & Simplify[D[P[x_{,}, y_{,}], y_{,}] - D[Q[x_{,}, y_{,}], x_{,}]] \end{array}$$

Out[•]= **0**

$$ln[x] = eqn = (y'[x] = -P[x, y[x]]/Q[x, y[x]])$$
Cutto = $y'[x] = \frac{-x^2 - 2y[x]^2}{-x^2 - 2y[x]}$

Out[
$$\sigma$$
]= $y'[x] = \frac{-x^2 - 2y[x]^2}{4xy[x] - y[x]^2}$

In[*]:= Expand[eqn]

$$Out[*]= y'[x] = -\frac{x^2}{4 x y[x] - y[x]^2} - \frac{2 y[x]^2}{4 x y[x] - y[x]^2}$$

$$c)(2*x^2+2*x*y+y^2)dx+(x^2+2*x*y)dy=0$$

$$ln[*]:= P[x_, y_] := (2 * x^2 + 2 * x * y + y^2)$$

 $Q[x_, y_] := (x^2 + 2 * x * y)$
 $Simplify[D[P[x, y], y] - D[Q[x, y], x]]$

Out[•]= 0

$$ln[*] = eqn = (y'[x] == -P[x, y[x]]/Q[x, y[x]])$$

-2x²-2xy[x]-y[x]²

$$\mbox{Out[*]= } y' \, [\, x \,] \ = \ \frac{- \, 2 \, \, x^2 - 2 \, x \, y \, [\, x \,] \, - y \, [\, x \,] \,^2}{x^2 + 2 \, x \, y \, [\, x \,]}$$

In[*]:= Expand[eqn]

$$Out[*]= y'[x] = -\frac{2x^2}{x^2 + 2xy[x]} - \frac{2xy[x]}{x^2 + 2xy[x]} - \frac{y[x]^2}{x^2 + 2xy[x]}$$