PRACTICAL 5

SOLUTION OF SYSTEM OF ODE

Solve the system of ODE

y1'=-3y1+y2

y2'=y1-3y2

Clear[sol, eq1, eq2, y, x, t]

In[ ]:=
```
eq1 = y'[t] == -3 y[t] + x[t]
eq2 = x'[t] == y[t] - 3 x[t]
```

Out[ ]= $y'[t] == x[t] - 3\,y[t]$

Out[ ]= $x'[t] == -3\,x[t] + y[t]$

In[ ]:=
```
sol = DSolve[{eq1, eq2}, {y[t], x[t]}, t]
```

Out[ ]= $\left\{\left\{x[t] \to \frac{1}{2} e^{-4t}\left(1+e^{2t}\right) C[1] + \frac{1}{2} e^{-4t}\left(-1+e^{2t}\right) C[2],\ y[t] \to \frac{1}{2} e^{-4t}\left(-1+e^{2t}\right) C[1] + \frac{1}{2} e^{-4t}\left(1+e^{2t}\right) C[2]\right\}\right\}$

In[ ]:=
```
sol[[1, 1]]
sol[[1, 2]]
```

Out[ ]= $x[t] \to \frac{1}{2} e^{-4t}\left(1+e^{2t}\right) C[1] + \frac{1}{2} e^{-4t}\left(-1+e^{2t}\right) C[2]$

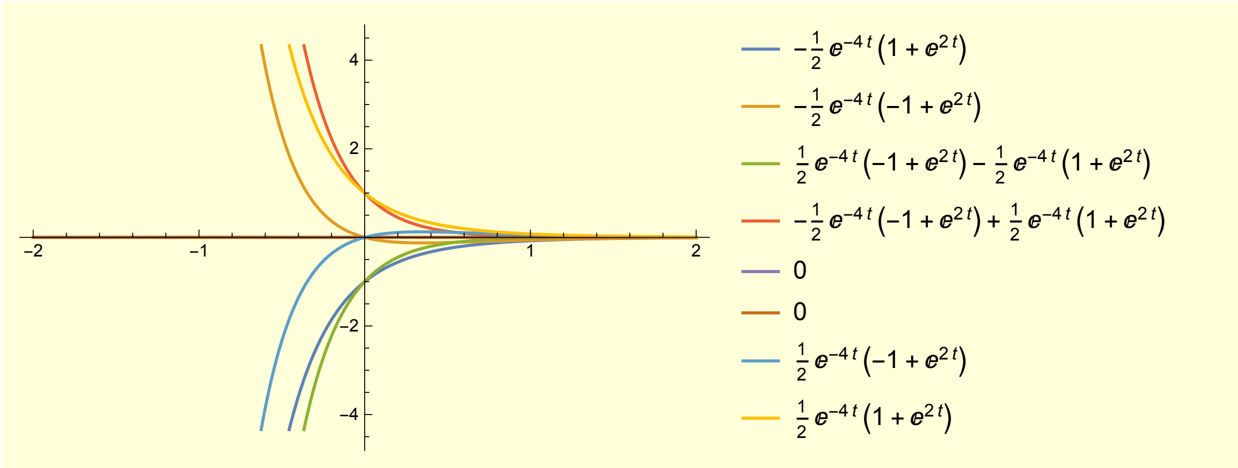
Out[ ]= $y[t] \to \frac{1}{2} e^{-4t}\left(-1+e^{2t}\right) C[1] + \frac{1}{2} e^{-4t}\left(1+e^{2t}\right) C[2]$

In[ ]:=
```
tab = Table[{x[t] /. sol[[1, 1]], y[t] /. sol[[1, 2]]} /. {C[1] → i, C[2] → j},
   {i, -1, 0}, {j, 0, 1}] // Flatten
```

Out[ ]= $\left\{-\frac{1}{2} e^{-4t}\left(1+e^{2t}\right),\ -\frac{1}{2} e^{-4t}\left(-1+e^{2t}\right),\ \frac{1}{2} e^{-4t}\left(-1+e^{2t}\right) - \frac{1}{2} e^{-4t}\left(1+e^{2t}\right),\ -\frac{1}{2} e^{-4t}\left(-1+e^{2t}\right) + \frac{1}{2} e^{-4t}\left(1+e^{2t}\right),\ 0,\ 0,\ \frac{1}{2} e^{-4t}\left(-1+e^{2t}\right),\ \frac{1}{2} e^{-4t}\left(1+e^{2t}\right)\right\}$

In[∘]:=
```
Plot[Evaluate[tab], {t, -2, 2}, PlotLegends → "Expressions"]
```

Out[∘]=



y1’=-5y1+2y2
y2’=2y1-2y2,y1(0)=1,y2(0)=-2

Clear[x, y, t, eq1, eq2, sol]

In[∘]:=
```
eq1 = x'[t] == -5 x[t] + 2 y[t]
eq2 = y'[t] == 2 x[t] - 2 y[t]
```

Out[∘]= $x'[t] == -5\,x[t] + 2\,y[t]$

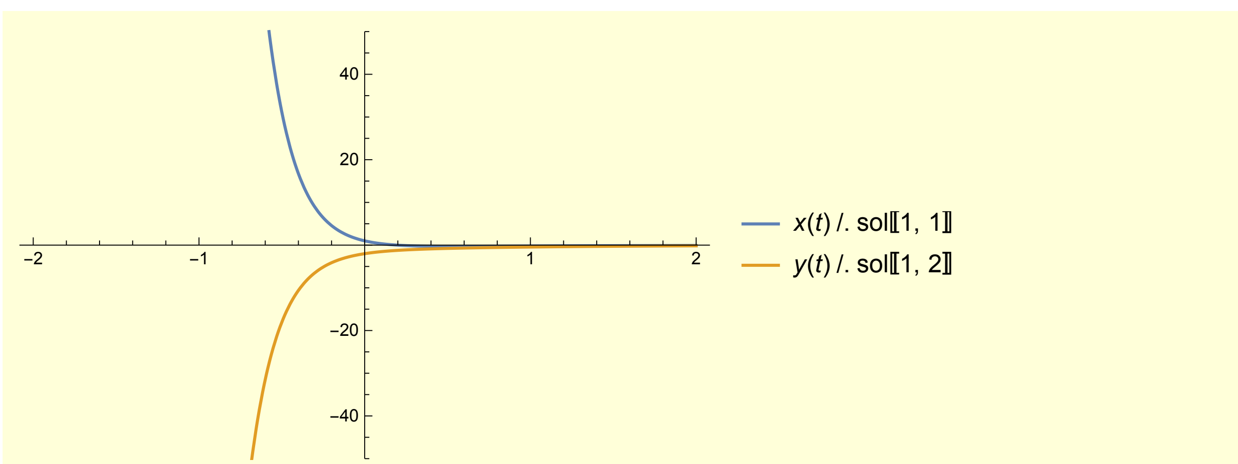
Out[∘]= $y'[t] == 2\,x[t] - 2\,y[t]$

In[∘]:=
```
sol = DSolve[{eq1, eq2, x[0] == 1, y[0] == -2}, {x[t], y[t]}, t]
```

Out[∘]= $\left\{\left\{x[t] \to -\frac{1}{5} e^{-6t}\left(-8+3e^{5t}\right),\ y[t] \to -\frac{2}{5} e^{-6t}\left(2+3e^{5t}\right)\right\}\right\}$

In[∘]:=
```
Plot[{x[t] /. sol[[1, 1]], y[t] /. sol[[1, 2]]},
 {t, -2, 2}, PlotRange → {-50, 50}, PlotLegends -> "Expressions"]
```

Out[∘]=



*Alternative Method

y1’=-3y1+y2
y2’=y1-3y2

In[∘]:=
```
matrixa = {{-3, 1}, {1, -3}}
polya = CharacteristicPolynomial[matrixa, lambda]
```
Out[∘]= {{-3, 1}, {1, -3}}

Out[∘]= $8 + 6\,\text{lambda} + \text{lambda}^2$

In[∘]:=
```
eigs = Solve[polya == 0, lambda] (*Distinct and real*)
```
Out[∘]= {{lambda → -4}, {lambda → -2}}

In[∘]:=
```
system1 = (matrixa - (lambda /. eign[[1]]) IdentityMatrix[2]).{y1, y2}
```
Out[∘]= {y1 + y2, y1 + y2}

In[∘]:=
```
system2 = (matrixa - (lambda /. eign[[2]]) IdentityMatrix[2]).{x1, x2}
```
Out[∘]= {-x1 + x2, x1 - x2}

In[∘]:=
```
Solve[system1 == 0, y2]
```
Out[∘]= {{y2 → -y1}}

In[∘]:=
```
Solve[system2 == 0, x2]
```
Out[∘]= {{x2 → x1}}

In[∘]:=
```
X1 = {1, -1};
X2 = {1, 1};
```

In[∘]:=
```
sol = c1 X1 E^(eigs[[1, 1]] x) + c2 X2 E^(eigs[[1, 2]] x) (*general sol*)
```
Out[∘]= $\left\{c1\, e^{-4x} + c2\, e^{-2x},\ -c1\, e^{-4x} + c2\, e^{-2x}\right\}$

Alternative(using Eigensystem)

In[∘]:=
```
matrix = {{-3, 1}, {1, -3}}
```
Out[∘]= {{-3, 1}, {1, -3}}

In[∘]:=
```
matrix // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -3 & 1 \\ 1 & -3 \end{pmatrix}$$

```
In[•]:= eigs = Eigensystem[matrix]
        (*-4 is an eigenvalueof the matrix extracted with eigs from eigs[[1,1]] &
         {-1,1} is corresponding eigenvector extracted with eigs from eign[[2,1]],
        similarly for eigenvalue -2*)(*Distinct and real *)

Out[•]= {{-4, -2}, {{-1, 1}, {1, 1}}}

In[•]:= eigs[[1, 1]]

Out[•]= -4

In[•]:= eigs[[1, 2]]

Out[•]= -2

In[•]:= (matrix - eigs[[1, 1]] IdentityMatrix[2]).eigs[[2, 1]] // Simplify

Out[•]= {0, 0}

In[•]:= (matrix - eigs[[1, 2]] IdentityMatrix[2]).eigs[[2, 2]] // Simplify

Out[•]= {0, 0}

In[•]:= sol = c1 eigs[[2, 1]] E^(eigs[[1]] x) + c2 eigs[[2, 2]] E^(eigs[[1, 2]] x) (*general sol*)
```

Out[•]= $\left\{-c1\, e^{-4x} + c2\, e^{-2x},\ c1\, e^{-2x} + c2\, e^{-2x}\right\}$

y1'=5y1+3y2

y2'=4y1+y2

```
In[•]:= Clear[sol, eq1, eq2, y, x, t]
        eq1 = x'[t] == 5 x[t] + 3 y[t]
        eq2 = y'[t] == 4 x[t] + y[t]

Out[•]= x'[t] == 5 x[t] + 3 y[t]

Out[•]= y'[t] == 4 x[t] + y[t]

In[•]:= sol = DSolve[{eq1, eq2}, {x[t], y[t]}, t]
```

Out[•]= $\left\{\left\{x[t] \to \frac{1}{4} e^{-t}\left(1+3 e^{8t}\right) C[1] + \frac{3}{8} e^{-t}\left(-1+e^{8t}\right) C[2],\ y[t] \to \frac{1}{2} e^{-t}\left(-1+e^{8t}\right) C[1] + \frac{1}{4} e^{-t}\left(3+e^{8t}\right) C[2]\right\}\right\}$

```
In[•]:= sol[[1, 1]]
```

Out[•]= $x[t] \to \frac{1}{4} e^{-t}\left(1+3 e^{8t}\right) C[1] + \frac{3}{8} e^{-t}\left(-1+e^{8t}\right) C[2]$

In[•]:=
```
sol[[1, 2]]
```

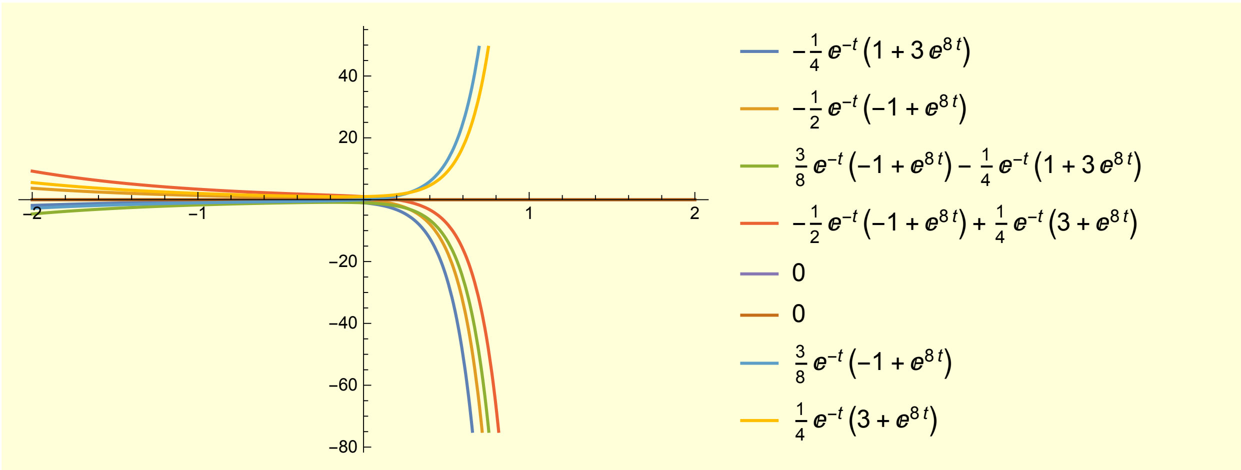
Out[•]= $y[t] \to \frac{1}{2} e^{-t} \left(-1 + e^{8t}\right) C[1] + \frac{1}{4} e^{-t} \left(3 + e^{8t}\right) C[2]$

In[•]:=
```
tab = Table[{x[t] /. sol[[1, 1]], y[t] /. sol[[1, 2]]} /. {C[1] → i, C[2] → j},
   {i, -1, 0}, {j, 0, 1}] // Flatten
```

Out[•]= $\left\{-\frac{1}{4} e^{-t} \left(1 + 3 e^{8t}\right), -\frac{1}{2} e^{-t} \left(-1 + e^{8t}\right), \frac{3}{8} e^{-t} \left(-1 + e^{8t}\right) - \frac{1}{4} e^{-t} \left(1 + 3 e^{8t}\right), -\frac{1}{2} e^{-t} \left(-1 + e^{8t}\right) + \frac{1}{4} e^{-t} \left(3 + e^{8t}\right), 0, 0, \frac{3}{8} e^{-t} \left(-1 + e^{8t}\right), \frac{1}{4} e^{-t} \left(3 + e^{8t}\right)\right\}$

In[•]:=
```
Plot[Evaluate[tab], {t, -2, 2}, PlotLegends → "Expressions"]
```

Out[•]=



y1’=3y1+5y2

y2’=-2y1+5y2 , x(0)=5,y(0)=-1

In[•]:=
```
Clear[x, y, t, eq1, eq2, sol]
```

In[•]:=
```
eq1 = x'[t] == 3 x[t] + 5 y[t]
eq2 = y'[t] == -2 x[t] + 5 y[t]
```

Out[•]= $x'[t] == 3\,x[t] + 5\,y[t]$

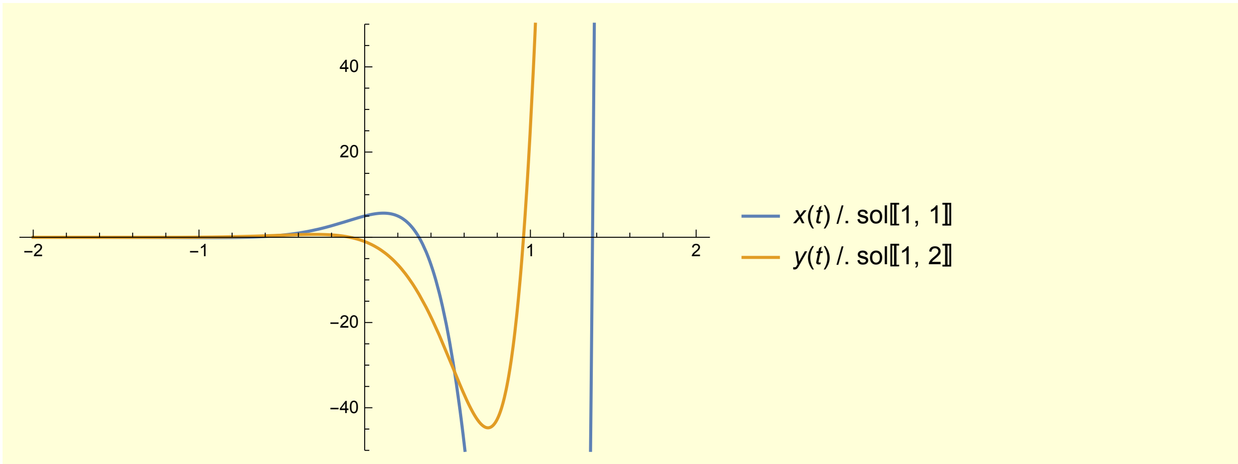
Out[•]= $y'[t] == -2\,x[t] + 5\,y[t]$

In[•]:=
```
sol = DSolve[{eq1, eq2, x[0] == 5, y[0] == -1}, {x[t], y[t]}, t]
```

Out[•]= $\left\{\left\{x[t] \to \frac{5}{3} e^{4t} \left(3\,\text{Cos}[3t] - 2\,\text{Sin}[3t]\right), y[t] \to -\frac{1}{3} e^{4t} \left(3\,\text{Cos}[3t] + 11\,\text{Sin}[3t]\right)\right\}\right\}$

In[∘]:=
```
Plot[{x[t] /. sol[[1, 1]], y[t] /. sol[[1, 2]]},
 {t, -2, 2}, PlotRange → {-50, 50}, PlotLegends → "Expressions"]
```

Out[∘]=



y1'=7y1+4y2

y2'=-1y1+3y2

In[∘]:=
```
Clear[sol, eq1, eq2, y, x, t]
```

In[∘]:=
```
eq1 = x'[t] == 7 x[t] + 4 y[t]
eq2 = y'[t] == -x[t] + 3 y[t]
```

Out[∘]= $x'[t] == 7\,x[t] + 4\,y[t]$

Out[∘]= $y'[t] == -x[t] + 3\,y[t]$

In[∘]:=
```
sol = DSolve[{eq1, eq2}, {x[t], y[t]}, t]
```

Out[∘]= $\{\{x[t] \to e^{5t}(1+2t)\,C[1] + 4\,e^{5t}\,t\,C[2],\ y[t] \to -e^{5t}\,t\,C[1] - e^{5t}(-1+2t)\,C[2]\}\}$

In[∘]:=
```
sol[[1, 1]]
```

Out[∘]= $x[t] \to e^{5t}(1+2t)\,C[1] + 4\,e^{5t}\,t\,C[2]$

In[∘]:=
```
sol[[1, 2]]
```

Out[∘]= $y[t] \to -e^{5t}\,t\,C[1] - e^{5t}(-1+2t)\,C[2]$

In[∘]:=
```
tab2 = Table[{x[t] /. sol[[1, 1]], y[t] /. sol[[1, 2]] /. C[1] → i, C[2] → j},
   {i, -1, 0}, {j, 0, 1}] // Flatten
```

Out[∘]= $\{e^{5t}(1+2t)\,C[1] + 4\,e^{5t}\,t\,C[2],\ e^{5t}\,t - e^{5t}(-1+2t)\,C[2],$
$C[2] \to 0,\ e^{5t}(1+2t)\,C[1] + 4\,e^{5t}\,t\,C[2],\ e^{5t}\,t - e^{5t}(-1+2t)\,C[2],$
$C[2] \to 1,\ e^{5t}(1+2t)\,C[1] + 4\,e^{5t}\,t\,C[2],\ -e^{5t}(-1+2t)\,C[2],$
$C[2] \to 0,\ e^{5t}(1+2t)\,C[1] + 4\,e^{5t}\,t\,C[2],\ -e^{5t}(-1+2t)\,C[2],\ C[2] \to 1\}$

In[ ]:=
```
Plot[Evaluate[tab2], {t, -2, 2}, PlotLegends → "Expressions"]
```

Out[ ]=