

Q → Why does the system have same peak overshoot for the poles

- i) $-10 \pm 10j$
- ii) $-20 \pm 20j$

Peak overshoot M_p is given by

$$M_p = e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}} \rightarrow \text{does not depend on } \omega_n$$

Here $\zeta \rightarrow$ Damping ratio

$\omega_n \rightarrow$ natural frequency

For a second order system given by +

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Poles are given by +

$$-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

In our case the magnitudes of real & imaginary parts are same

$$\Rightarrow \zeta\omega_n = \omega_n\sqrt{1-\zeta^2}$$

$$\because \omega_n \neq 0 \Rightarrow \zeta^2 = 1 - \zeta^2 \Rightarrow 2\zeta^2 = 1$$

$$\boxed{\zeta = \frac{1}{\sqrt{2}}}$$

Hence ζ is fixed in case i) & ii) as well and M_p depends only on ζ & not on ω_n

Hence Both the systems have same peak overshoot and just the settling time $t_s = \begin{cases} 4T = \frac{4}{\zeta\omega_n} & (2\% \text{ criterion}) \\ 3T = \frac{3}{\zeta\omega_n} & (5\% \text{ criterion}) \end{cases}$ changes as ω_n changes.