



give me Vibrating Cantilever Beam example of state space control model which is "controllable" and "observable" with state space equations and matrix values a,b,c,d



Certainly! A vibrating cantilever beam is a classic example in control theory and structural dynamics. In this example, I'll provide a state-space representation of a vibrating cantilever beam that is controllable and observable. The state-space equations will describe the motion of the beam.



The equations for a vibrating cantilever beam can be represented as follows:

1. State Variables:

- x_1 : Displacement of the beam at the free end (vertical deflection)
- x_2 : Velocity of the beam at the free end

2. System Dynamics (Continuous-time):

- $\dot{x}_1 = x_2$
- $\dot{x}_2 = -\left(\frac{k}{m}\right) x_1 - \left(\frac{c}{m}\right) x_2 + \left(\frac{1}{I}\right) u$

Where:

- k is the stiffness of the beam.
- m is the mass of the beam.
- c is the damping coefficient.
- I is the moment of inertia of the beam.
- u is the control input (force applied at the free end).

Now, we can put this system into state-space form ($\dot{x} = Ax + Bu$, $y = Cx + Du$):

State vector $x = [x_1, x_2]$


Input vector $u = [u]$

Output vector $y = [x_1]$

The state-space representation is as follows:

A matrix:


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$$A = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}$$

B matrix:


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$$B = \begin{bmatrix} 0 & 1/m \end{bmatrix}$$

C matrix:


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$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

D matrix:

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$$D = \begin{bmatrix} 0 \end{bmatrix}$$

Now, to check for controllability and observability, we need to evaluate the controllability and observability matrices:

Controllability Matrix, Q_c :


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$$Q_c = \begin{bmatrix} B & A*B \end{bmatrix}$$

Observability Matrix, Q_o :

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Qo = [C  
      C*A]
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You can use the controllability matrix Q_c to check if the system is controllable. If the rank of Q_c is equal to the number of states (2 in this case), then the system is controllable. Similarly, use the observability matrix Q_o to check if the system is observable. If the rank of Q_o is equal to the number of states, the system is observable.

In this example, assuming appropriate values for k , m , and c , you should be able to determine if the system is controllable and observable using the matrices A , B , C , and D as described.



give Controllability Matrix



The controllability matrix, often denoted as Q_c , for a state-space representation of a system is constructed by stacking the system's A and B matrices in a specific way. For a system with n state variables (i.e., n dimensions), the Q_c matrix is of size $n \times (n*m)$, where m is the number of control inputs.



For your vibrating cantilever beam system with the A and B matrices as previously defined, the controllability matrix Q_c is constructed as follows:

$$Q_c = [B \ A*B]$$

In your case, B is a 2×1 matrix, and $A*B$ is also a 2×1 matrix. So, Q_c will be a 2×2 matrix.

$$Q_c = \begin{bmatrix} 0 & 1 \\ 1/m & -k/m \end{bmatrix}$$

This is the controllability matrix for your vibrating cantilever beam system. To check for controllability, you can calculate the rank of this matrix. If the rank is equal to the number of state variables (2 in this case), then the system is controllable.



give Observability Matrix



The observability matrix, often denoted as Q_o , for a state-space representation of a system is constructed by stacking the system's C and A matrices in a specific way. For a system with n state variables (i.e., n dimensions), the Q_o matrix is of size $(n \times p) \times n$, where p is the number of output variables (sensors).



For your vibrating cantilever beam system with the A and C matrices as previously defined, the observability matrix Q_o is constructed as follows:

$$Q_o = \begin{bmatrix} C \end{bmatrix}$$

$$\begin{bmatrix} C \cdot A \end{bmatrix}$$

In your case, C is a 1×2 matrix, and $C \cdot A$ is also a 1×2 matrix. So, Q_o will be a 2×2 matrix.

$$Q_o = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -k/m & -c/m \end{bmatrix}$$

This is the observability matrix for your vibrating cantilever beam system. To check for observability, you can calculate the rank of this matrix. If the rank is equal to the number of state variables (2 in this case), then the system is observable.