3.1 INTRODUCTION

In Chapter 2, we stated the laws of demand and supply and studied the effects of shifts in the demand and supply curves on the equilibrium prices and quantities. We also studied the effects of a per unit tax, import tariff, export subsidy, and so on. In all these cases we made statements concerning the direction of change in price and quantity. In this chapter we examine the magnitudes of these changes. For example, the law of demand states that if the price rises by 10 percent, then quantity demanded will decline. But will it decline by 10 percent? By more? By less? Just how responsive is quantity demanded to a change in price?

The measures of responsiveness we will examine are *elasticities*. The ratio of percentage change in the quantity demanded to percentage change in price is called the *price elasticity of demand*. (This ratio is, however, generally expressed as an absolute value.) Similarly, the ratio of percentage change in quantity supplied to percentage change in price is called the *price elasticity of supply*. Thus what we are asking is whether the elasticity of demand is equal to 1, less than 1, or greater than 1.

We will consider elasticity of demand from the seller's point of view. Several revenue concepts will be introduced, and their relationship to demand elasticities will be explored. What, for example, happens to total expenditure on a product as the price of that product rises or falls? The answer depends on the price elasticity of demand.

We will also quantify many of the applications of demand and supply analysis which we looked at in Chapter 2. For instance, in our discussion of the per unit tax, we said that a tax T results in an increase in the price paid by buyers and a decrease in the price the seller gets. In other words, the tax is "shared" by buyers and sellers. An important question is: Whose share of the burden of the tax is bigger? We will show that the answer depends on the price elasticities of demand and supply.

Finally, we will review some problems with the estimation of demand and supply functions from actual data on prices and quantities. And we will look at a simple dynamic model of price determination.

Chapter 3 is an important chapter. It is one of the "cornerstones" of this text. Although some of it should be review, students should make certain that they fully understand these concepts.

3.2 PRICE ELASTICITY OF DEMAND AND SUPPLY

We define the price elasticity of demand as the absolute value of the ratio of percentage change in the quantity demanded to the percentage change in price, ceteris paribus. Note that this is the same as the ratio of *relative* changes or *proportionate* changes. It is customary to use the Greek symbol η (eta) for price elasticity of demand. Thus, price elasticity can be written as

$$\eta = \left|\frac{\Delta Q/Q}{\Delta P/P}\right| = \left|\frac{\Delta Q}{\Delta P}\cdot\frac{P}{Q}\right|$$

The vertical lines denote that we take the absolute value of the ratio, and ΔQ and ΔP denote the changes in quantity and price.[1]

The price elasticity of supply can be similarly defined as the ratio of percentage change in quantity supplied to the percentage change in price, ceteris paribus. Using ϵ (epsilon) to denote the price elasticity of supply, we can write

$$\epsilon = \frac{\Delta Q/Q}{\Delta P/P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

Clearly the definition of ϵ is very similar to that of η. The only difference is that Q now denotes the quantity supplied instead of the quantity demanded. Also, the absolute value signs have been dropped, because the law of supply tells us that ϵ is already positive (why?) and, hence, the absolute value signs are no longer needed.

Now we are ready to consider why we work with proportionate or percentage changes rather than simple magnitudes of changes. Why don't we just calculate $|\Delta Q/\Delta P|$ and use this to measure responsiveness of quantity demanded (or supplied) to a change in price? Graphically this number is the inverse of the slope of the demand curve (or supply curve) expressed in absolute terms.

One reason is because $|\Delta Q/\Delta P|$ depends on the units of measurement for both P and Q and, hence, is somewhat difficult to interpret. Suppose, for instance, that we are told that $|\Delta Q/\Delta P|$ for corn is 7.3. How responsive is the quantity demanded of corn to a change in its price? What if P is measured in cents and Q is in millions of bushels? A 1-cent increase in the price of corn would cause a 7.3 million bushel decrease in the quantity demanded of corn. But suppose instead that P is measured in dollars and Q is in bushels. The picture changes dramatically.

Elasticities, however, are pure numbers and are thus much less cumbersome to interpret. If the price elasticity of demand for corn is 4.1, a 1 percent increase in the price of corn leads to a 4.1 percent reduction in quantity demanded, holding other things constant. The units of measurement for P and Q are immaterial.

EXAMPLE 3.1 Price Elasticity of Demand for Whole Life Insurance

In the insurance literature, life insurance has often been characterized as a "sold good, not a bought good" in the sense that the initiative in a life insurance transaction in the ordinary (individual) life insurance market comes typically from the seller (insurance agent who works on commissions) rather than the buyer. Since the complexity of a whole life insurance contract is not easily amenable to price comparisons, it is inferred that consumers are insensitive to variations in the price of life insurance.

Some other indirect evidence has also been suggested to argue that quantity demanded of life insurance is not especially sensitive to price. Most of the studies conducted into the competitiveness of policy pricing among insurers have found wide ranges in the prices of ostensibly similar policies. It has also been observed that few consumers make the comparisons that characterize consumer shopping behavior for other goods and services of similar price. Some have even argued that the mere existence of whole life contracts is evidence of consumer irrationality (the

[1]In terms of derivatives we have $\eta = |dQ/dP \cdot P/Q|$ or $|(d \log Q)/(d \log P)|$.

argument being that a rational person could be economically ahead by purchasing term insurance and investing the difference).

Babble discusses all these arguments and demonstrates why they are not valid.[2] He constructs a real price index for whole life insurance sold in the United States from 1953 to 1979. He finds a price elasticity of 0.71 to 0.92 for nonparticipating and 0.32 to 0.42 for participating individuals. He also finds an income elasticity of 0.62 to 0.98. Thus, new purchases of whole life insurance are responsive to changes in the price index, contrary to what has been accepted in the insurance literature but consistent with economic theory. This, he argues, does not however ensure that the insurance industry manifests a high degree of competition.

3.3 ARC AND POINT ELASTICITIES

We defined the elasticity of demand as

$$\eta = \left| \frac{\Delta Q/Q}{\Delta P/P} \right|$$

The quantities ΔQ and ΔP, which are the changes in Q and P, respectively, are easy to define. But the question arises as to what value of Q and P we use. Are we supposed to take beginning values or final values or some average? For example, suppose the price per unit of a watch went up from \$10 to \$11, and the number of watches demanded went down from 100 to 95. Clearly, $\Delta P = \$1$ and $\Delta Q = -5$. If we take the initial values, then $(\Delta Q)/Q = -5/100 = -1/20$ and $(\Delta P)/P = 1/10$ and $\eta = 0.5$. It seems reasonable to consider the starting values for both P and Q. However, economists more frequently calculate the *arc elasticity*, which uses the average of the initial and final values. Thus, if P_1 and Q_1 are the initial price and quantity respectively, and P_2 and Q_2 are the final price and quantity respectively, then $\Delta Q = Q_2 - Q_1$ and $\Delta P = P_2 - P_1$. For the divisors we use the average quantity and average price, which are $(Q_1 + Q_2)/2$ and $(P_1 + P_2)/2$.

Thus, the arc elasticity is (cancelling the factor 2 in both the denominator and numerator)

$$\eta = \left| \frac{\Delta Q/\Delta P}{(Q_1 + Q_2)/(P_1 + P_2)} \right| = \left| \frac{\Delta Q}{\Delta P} \cdot \frac{P_1 + P_2}{Q_1 + Q_2} \right|$$

And in the example from the preceding paragraph

$$\eta = \left| \frac{-5}{1} \cdot \frac{21}{195} \right| = 0.54$$

Figure 3.1 shows the arc elasticity. As the point B gets closer to the point

[2]David Babble, "Price Elasticity of Demand for Whole Life Insurance," *Journal of Finance*, March 1985, pp. 225–239. The paper contains numerous references to early studies on this topic.

$$\text{Arc elasticity} = \frac{|(Q_2 - Q_1)/P_2 - P_1)|}{|(Q_1 + Q_2)/P_1 + P_2)|}$$

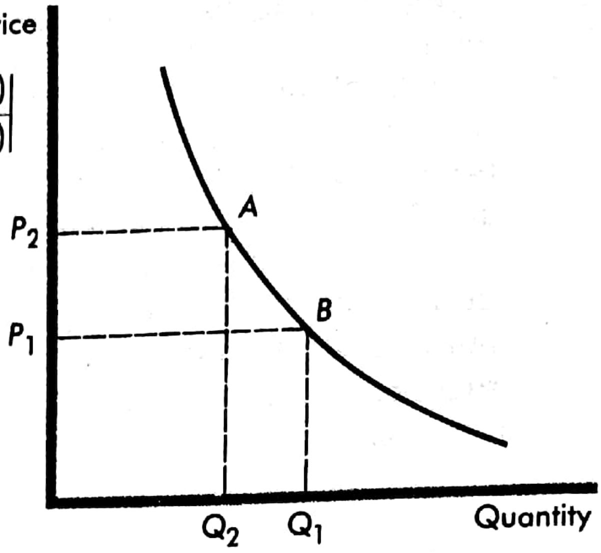


FIGURE 3.1 Arc elasticity.

A, so that the changes in price and quantity get smaller, we get the *point elasticity* at A.[3]

EXAMPLE 3.2 Elasticity of Demand for Gambling

Estimates of elasticity of demand for legalized gambling are important for use by governments in devising the tax rates that maximize the revenue to the government. Suits estimates these elasticities and finds them considerably higher than unity (in absolute value).[4] The estimates he obtains are in the range of 1.6 to 2.7. This high elasticity of demand, he argues, places severe limits on gambling as a source of tax revenue. An important source of this high price elasticity is the availability of illegal establishments.

Gambling is a recreational activity. For most gamblers the purpose of gambling is not to get rich but to have fun and to experience excitement. Gamblers are perfectly aware that they will lose on the average, but they view this expectation of loss as the price paid to engage in the game.

What is the price and what is the quantity in the case of gambling? Gambling establishments pay out in winnings only a fraction of the total dollars bet. The fraction of the total that the gambling establishment withholds is called the *take-out rate.* This is the price for any game. In the case of casinos the take-out rate is highest for slot machines (usually around 0.90) and lowest for games such as roulette (which is around 0.06). The rate is usually 0.15 to 0.20 at race tracks and 0.50 or more for lotteries. Different games constitute different products and are not even close substitutes in the minds of the players. For any particular game the take-out rate is the price P for playing that game. The quantity Q is the total value of bets placed and is called the *handle.* It is measured in dollars, but it is not revenue. The revenue for the gambling establishment is PQ, that is, take-out rate multiplied by the total value bet.

To derive an elasticity of demand, Suits looked at the experience of Nevada betting parlors when at the end of 1974 the federal excise tax on bookmaking was

[3] As ΔP and $\Delta Q \to 0$ the arc elasticity becomes $|(dQ)/(dP)| \cdot P/Q$ or $(d \log Q)/(d \log P)$.

[4] D. B. Suits, "The Elasticity of Demand for Gambling," *The Quarterly Journal of Economics*, February 1979, pp. 155–162.

reduced from 10 percent to 2 percent of handle. The per-quarter betting rates preceding the reduction of the tax rate and following it were as follows:

Activity	Year	Federal Tax (%)	Take-Out Rate (P) Operator (%)	Total (%)	Handle (Q) ($ million)
Off-track Horse Betting	1974	9.1	14.5	23.6	5.1
	1975	2.0	14.0	16.0	9.6
Sports Betting Parlors	1974	10.0	8.0	18.0	1.4
	1975	2.0	8.0	10.0	5.0

The arc elasticities are

$$\text{Off-track horse betting} = \left| \frac{4.5}{7.6} \cdot \frac{39.6}{14.7} \right| = 1.6$$

$$\text{Sports betting parlors} = \left| \frac{3.6}{8.0} \cdot \frac{28.0}{6.4} \right| = 2.0$$

Consider the sports betting parlors. The government revenues have declined from \$140,000 to \$100,000, that is, by \$40,000. The gambling establishments' revenues have increased from \$112,000 to \$400,000 or by \$288,000 or seven times the loss in government revenue. The gambling and racing establishments and the gamblers and racing fans all constitute a strong lobby against increased taxes, since the lobby has much to gain and the state has relatively little to lose from low tax rates. More empirical evidence and more detailed calculations can be found in the paper by Suits.

3.4 GEOMETRIC REPRESENTATION OF POINT ELASTICITY

We all know that the slope of a straight line is constant and that the slope between any two points is given by the change in y divided by the change in x. Thus, with a linear demand curve $\Delta Q/\Delta P$ is constant. However, since Q/P is not constant, the demand elasticity will be different at different points on the demand curve. There is, however, a simple rule for finding the demand elasticity at any point. This rule, for a linear demand function, is

$$\text{Point elasticity of demand} = \frac{\text{Distance of the point from the } Q \text{ axis}}{\text{Distance of the point from the } P \text{ axis}}$$

with both distances *measured along a demand curve.* Figure 3.2 illustrates this.

This result can be proven as follows: $\Delta Q/\Delta P = EC/AE$, and at the point C we have $Q/P = OF/OE$. Hence, $\eta = (EC/AE)/(OF/OE) = OE/AE$ since $EC = OF$. Since CF and OA are parallel lines, $\eta = OE/AE = CB/CA$.

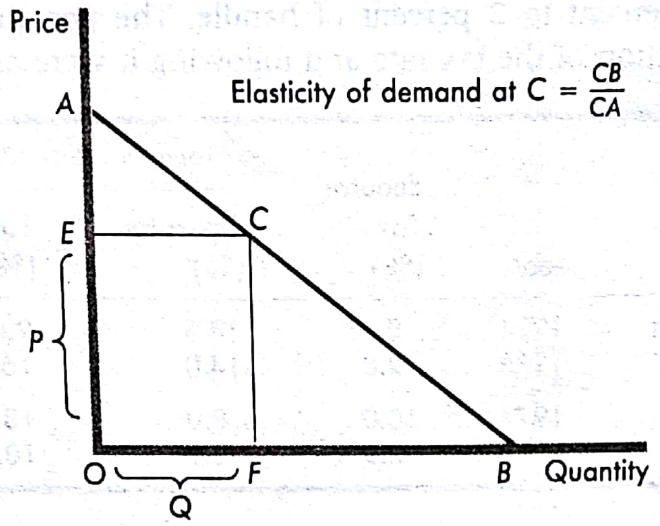


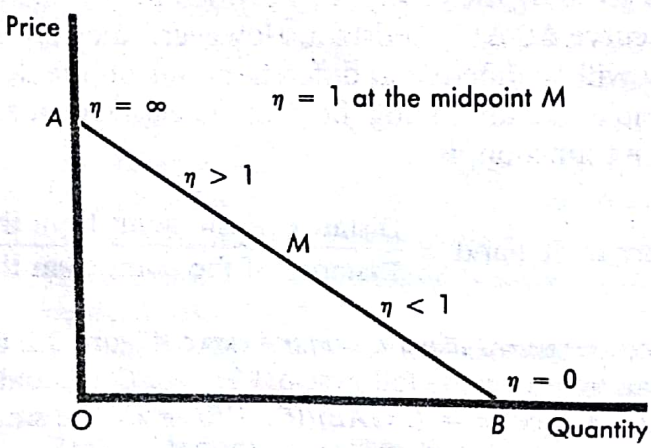
FIGURE 3.2 Geometric representation of elasticity for a linear demand curve.

As a consequence, we get the result that $\eta = 1$ at the midpoint M of AB, $\eta > 1$ for points on the demand curve between A and M and $\eta < 1$ for points on the demand curve between M and B (Why?). This is shown in Figure 3.3.

For a nonlinear demand curve the same rule for calculating elasticity applies except that we have to consider the tangent to the demand curve in place of the linear demand curve we considered earlier. (Note that the slope at a point on a curve is the slope of the tangent to that curve at that point.) This is shown in Figure 3.4. All we have to do is draw a tangent to the demand curve at the point we are considering and then use the rule for the elasticity at a point on a linear demand curve given earlier.

Let's now turn our attention to the elasticity of supply. With a linear demand curve, as we have shown in Figure 3.3, $\eta = 1$ at the midpoint of the demand curve, $\eta < 1$ below the midpoint, and $\eta > 1$ above the midpoint. With a linear supply curve, the elasticity can equal, be less than, or be greater than 1 at *all* points on the supply curve. In Figure 3.5 we show supply curves with elasticity of less than, greater than, and equal to 1 at all points.

FIGURE 3.3 Elasticity of demand at the different points on a linear demand curve.



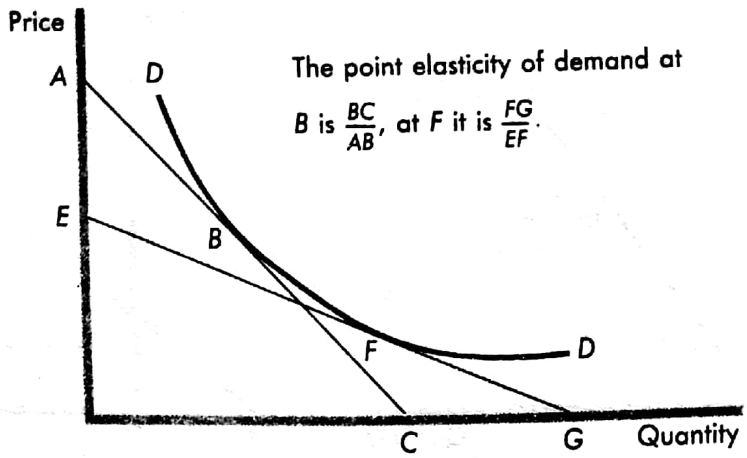


FIGURE 3.4 Price elasticity of a curvilinear demand curve.

The rule for calculating the elasticity of supply at a point can be derived the same way we did for elasticity of demand. Consider, for instance, Figure 3.5(*b*).

$$\text{Elasticity of supply } \epsilon = \frac{\Delta Q/Q}{\Delta P/P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

But
$$\frac{\Delta Q}{\Delta P} = \frac{1}{\text{slope}} = \frac{AD}{CD}$$

and
$$\frac{P}{Q} = \frac{CD}{OD}$$

Hence,
$$\epsilon = \frac{AD}{CD} \cdot \frac{CD}{OD} = \frac{AD}{OD}$$

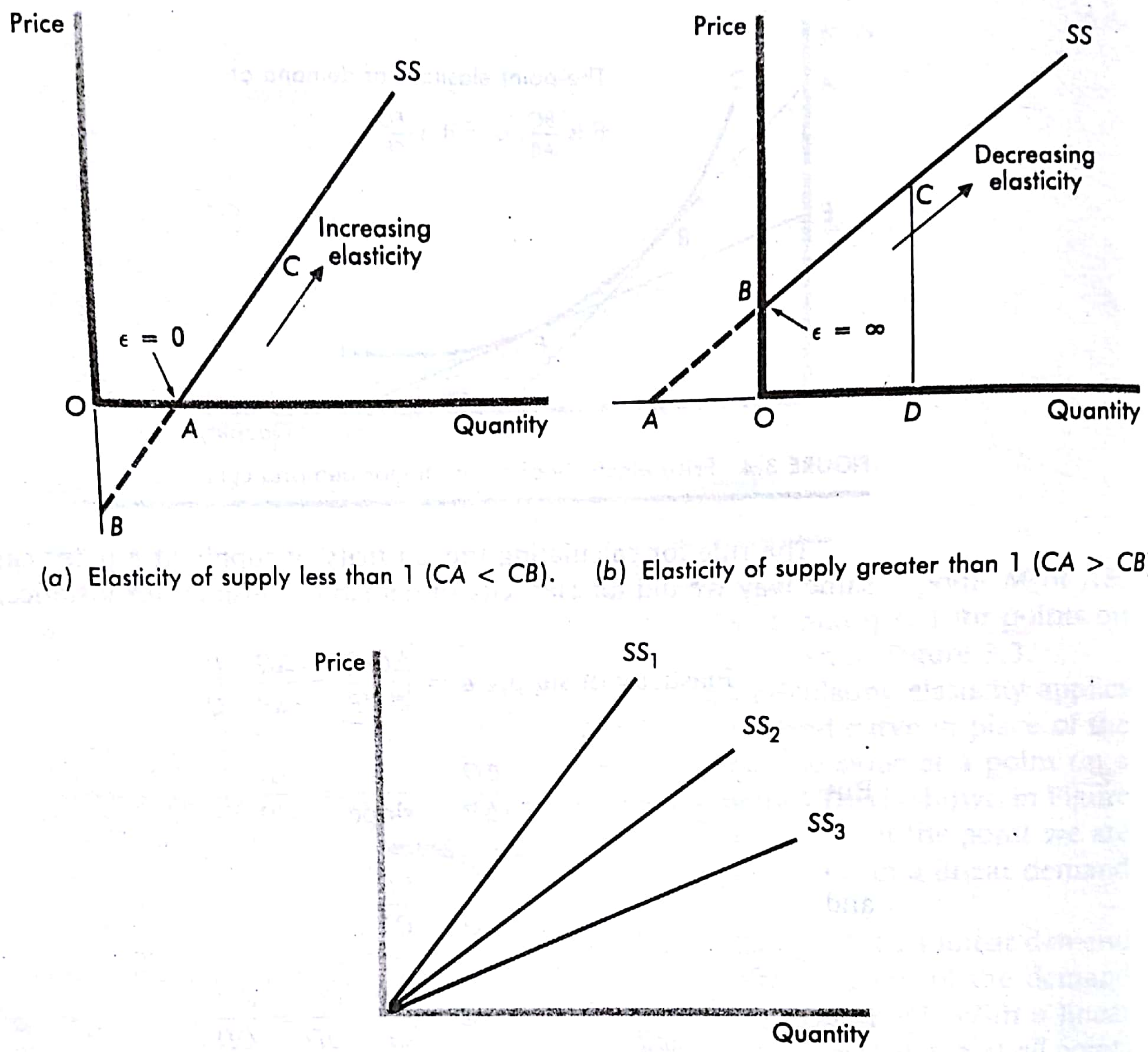
but
$$\frac{AD}{OD} = \frac{AC}{BC} = \frac{\text{Distance of } C \text{ from the } Q \text{ axis}}{\text{Distance of } C \text{ from the } P \text{ axis}}$$

Both distances are measured *along the supply curve.*

Note that the elasticity of supply is less than 1 if the supply curve has a negative intercept as in Figure 3.5(*a*). Take any point C on the supply curve. Its distance *along the supply curve* from the quantity axis is CA, and its distance from the price axis is CB. Since $CA < CB$, the elasticity of supply is less than 1. If the supply curve has a positive intercept as in Figure 3.5(*b*), then we have $CA > CB$, and the elasticity of supply is greater than 1.

Finally, if the intercept is zero, so that the supply curve passes through the origin, the distance of the point, along a supply curve, is the same from both the quantity and price axes. Hence, elasticity of supply equals 1. Thus, all the supply curves shown in Figure 3.5(*c*) have elasticity of supply equal to 1.

Note that there are no linear demand curves with elasticity equal to 1 at all points, but several linear supply curves have elasticity equal to 1 at all points.



(a) Elasticity of supply less than 1 (CA < CB). (b) Elasticity of supply greater than 1 (CA > CB).

(c) Elasticity of supply equals 1 for all the curves.

FIGURE 3.5 Elasticity of supply for linear supply curves.

Similar is the case with elasticity less than 1 or elasticity greater than 1. There are no linear demand curves with such elasticities at all points, but there are several linear supply curves whose elasticities are either less than 1 or greater than 1 at all points.

Also, for nonlinear supply curves, we just draw a tangent to the supply curve at the point we are considering and then use the same rule as for a linear supply curve. Since the tangent line can have a positive or negative intercept at different points, we will have the elasticity of supply greater than 1 at some and less than 1 at other points. The elasticity of supply declines from ∞ to 0 along the nonlinear supply curve the way it declines for the linear demand curve as shown earlier in Figure 3.3. The decline of elasticity for the supply curve is shown in Figure 3.6.

Finally, we will show what a demand curve with elasticity equal to 1 at all points looks like. This is a curve for which $P \cdot Q$ is constant and is called a rectangular hyperbola. This is shown in Figure 3.7. Note that for a supply curve with elasticity

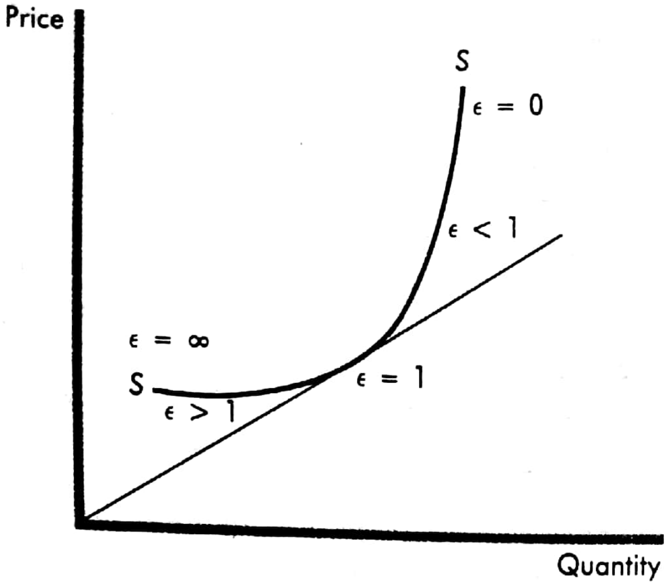


FIGURE 3.6 Changes in the elasticity of supply along a curvilinear supply curve.

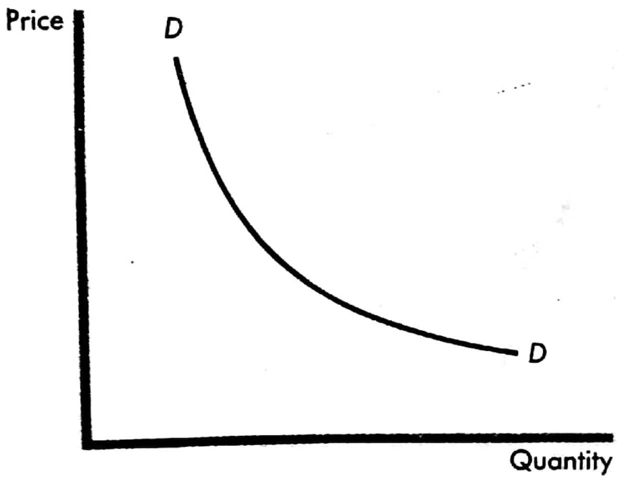
equal to 1, *P/Q* is constant and for a demand curve with elasticity equal to 1, *PQ* is constant.[5]

3.5 SHORT-RUN AND LONG-RUN PRICE ELASTICITIES

When the price of a product changes, it takes some time for consumers to fully respond. For instance, the effect of a rise in the price of heating oil results in consumers switching to alternative fuels only after some time. Consumers initially will try to economize on the usage with existing appliances, but eventually they also switch to alternative fuels. With a rise in the price of gasoline, consumers will

[5]Mathematically the equation $Q = AP^{\alpha}$ is a curve with constant elasticity α, since $(d \log Q)/(d \log P) = \alpha$. For demand curves, α is negative. For supply curves, α is positive.

FIGURE 3.7 A demand curve with elasticity equal to 1 at all points: A rectangular hyperbola.



try to economize on the use of gasoline with their gas guzzlers but will eventually get rid of their gas guzzlers and buy fuel-efficient cars or maybe join a car pool.

Figure 3.8 illustrates some typical demand curves in the short run, intermediate run, and long run. A_1B_1 is the short-run demand curve, A_2B_2 is the intermediate-run demand curve, and A_3B_3 is a long-run demand curve. If the price of the product increases from P_1 to P_2, quantity demanded immediately drops from Q_1 to Q_2. But as consumers have time to adjust to the higher price, quantity drops to Q_3 and eventually to Q_4 in the long run. Clearly, the long-run demand curve is more elastic than the short-run or intermediate-run curves.

How long is the long run? Its length depends on the goods being considered. Clearly it is longer when durable goods and appliances are involved as in the case of gasoline or heating oil. It is also longer when it comes to long-standing habits. Suppose the price of coffee goes up. Although theoretically it is easy to switch from coffee to tea (there are no appliances involved as in the switching from natural gas to heating oil), we do not observe many people making such a switch immediately. For many individuals drinking tea is not the same as drinking coffee. It takes time for them to change their drinking habits.

In actual practice there is no such thing as complete adjustment. Suppose the price of gasoline goes up 10 percent and *stays there*. Then, ceteris paribus we can ask by what percentage the quantity demanded of gasoline goes down. Suppose the initial quantity demanded is 100 (million gallons). Then we observe the demand in successive years until the decline peters off. In Table 3.1 we show the decline over the first 10 years.

The figures in Table 3.1 are, of course, hypothetical because, in practice, it is not possible to maintain the ceteris paribus assumption, nor is it possible to maintain the price of gasoline at the 10 percent increased level. To compute the elasticities in practice we have to use multiple-regression analysis to control for the other factors and estimate appropriate dynamic demand functions to disentangle the short-run and long-run effects. We have chosen a hypothetical example to illustrate the difference between short-run and long-run elasticities.

FIGURE 3.8 Short-run and long-run demand curves.

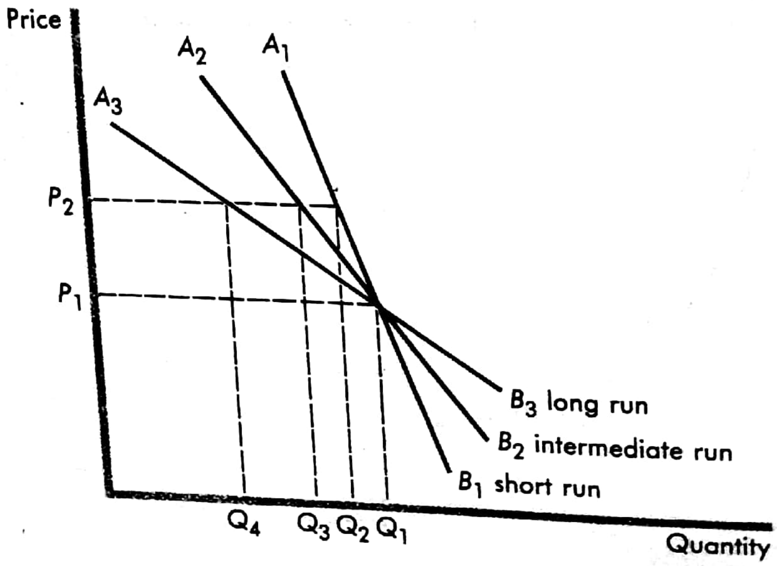


TABLE 3.1 Effect of a Price Increase on the Quantity of Gasoline Demanded
Initial demand = 100 million gallons
Increase in price = 10 percent

Year	Quantity Demanded	Cumulative Decline	Price Elasticity	
1	98.0	2.00	0.200	← Short run
2	97.0	3.00	0.300	
3	96.5	3.50	0.350	
4	96.0	4.00	0.400	
5	95.7	4.30	0.430	← Intermediate run
6	95.4	4.60	0.460	
7	95.2	4.80	0.480	
8	95.05	4.95	0.495	
9	95.01	4.99	0.499	
10	95.00	5.00	0.500	← Long run

Note: Initial quantity was 100 million gallons, and increase in price was 10 percent.

In Table 3.1 we have stopped at 10 years because the decline in quantity demanded seems to have almost petered out in 10 years. If we went another 10 years, we would have observed a little more decline, but for all practical purposes 10 years is enough. If we treat 1 year as short run, 5 years as intermediate run, and 10 years as long run, we have the respective elasticities as 0.2, 0.43, and 0.5.

Note that the definition of what constitutes short run and intermediate run is somewhat arbitrary. For the definition of long run we have used the idea that it is the time taken for the decline in quantity to peter out (almost). In any case, it is important to note that the short-run elasticity is less than the intermediate-run elasticity, which, in turn, is less than the long-run elasticity.

The distinction between short-run and long-run elasticities becomes a bit complicated when we discuss the demand for *durable goods.* Again, if we consider the demand for *services* provided by durable goods, the long-run elasticity would be higher than the short-run elasticity. However, if we look at the *current purchases* of durable goods, the short-run price elasticity may appear higher than the long-run price elasticity. Consider, for instance, the case of automobiles. We buy automobiles for the services they provide. Assume that the *flow of services* is proportional to the *stock* of automobiles. Suppose the current stock of automobiles is 80 million. If the long-run price elasticity of demand for automobiles (services they provide) is 0.5, then a 10 percent decrease in the price of automobiles results in an ultimate increase in the desired stock by 4 million. However, if consumers initially increase their *current purchases* by, say, 3 million, and the normal level of yearly purchases is 10 million, we observe a 30 percent increase in current purchases of automobiles. This would imply a short-run price elasticity of 3.0, and one might be tempted to conclude that the short-run elasticity is higher than the long-run elasticity. However, this is not true. When we computed the long-run elasticity we considered the flows of services or the stock of automobiles. When we computed the short-term elasticity, we considered the *current purchases* of automobiles, not the current stock. If we define the variables consistently, we would find that the short-run increase in purchases is 3 million, whereas the long-run increase in purchases is 4 million, so that the short-run elasticity of purchases is 3, which is less than the long-run elasticity of 4. Similarly, the short-run increase in the stock is 3.75%, whereas the long-run increase is 5 percent, making the long-run elasticity of the stock greater than the corresponding short-run elasticity. In both cases the usual pattern in short-run and long-run elasticities holds.

When we consider the demand for automobiles, we have to make sure what we are talking about. Is it the demand for services of automobiles (which we have assumed to be proportional to the stock of automobiles) or is it the current purchases of automobiles (addition to the stock)? From the consumers' point of view the former is the relevant concept. On the other hand, from the automobile manufacturers' point of view, what counts is the elasticity of current purchases. In both cases, if the elasticities are properly defined, the short-run elasticities will be lower than the long-run elasticities.

Houthakker and Taylor estimated the following short-run and long-run elasticities for three energy uses[6]:

Good	*Elasticity* Short Run	Long Run
Natural gas	0.15	10.74
Gasoline and oil	0.14	0.48
Electricity (residential)	0.13	1.90

Note that the short-run elasticities are all about the same but the long-run elasticities differ considerably. The results imply that gasoline has very few substitutes, electricity has some substitutes, and natural gas has many substitutes.

Actually, many estimates of the demand elasticity of energy sources derived during the days of the "energy crisis" of the 1970s varied considerably, although one could see that the short-run elasticities were all considerably below 1 and the long-run elasticities generally were above 1.

Suppliers also require time to react to price changes. For example, it might take considerable time to interview the necessary additional labor required to produce a larger output in response to a higher price. Thus, supply curves also tend to be more elastic, the longer the time frame considered. This point will be considered further in future chapters when we look behind a supply curve.

3.6 OTHER ELASTICITY CONCEPTS

Elasticities can be defined with respect to any two variables. Next we will examine the income elasticity of demand and the cross-price elasticity of demand.

[6]H. S. Houthakker and L. D. Taylor, *Consumer Demand in the United States: Analysis and Projections*, 2d ed., Harvard University Press, Cambridge, Mass., 1970.

Income elasticity

The income elasticity of demand, u, is defined as

$$u = \frac{\Delta Q}{\Delta Y} \cdot \frac{Y}{Q}$$

where Y stands for income and Q denotes quantity demanded. Again, the assumption of ceteris paribus applies. But this time we are assuming that everything except consumer incomes remains the same. In particular, the price of the given good and also the prices of all related goods are assumed to remain constant.

Income elasticity tells us how responsive quantity demanded is to a change in income. If income elasticity of demand equals 2.3, then a 1 percent increase in income leads to a 2.3 percent increase in quantity demanded, ceteris paribus.

You will notice that when we defined the income elasticity of demand, we did not take the absolute value. That is because the sign is of interest. For most goods, when income increases, the quantity demanded of the good also increases. In this case, the income elasticity of demand is positive, and the good is a normal good at the level of income considered. However, if the quantity demanded of the good falls when income increases, the good is an inferior good at the income level considered, and, in this case, income elasticity of demand is negative. Remember that no good is inferior at all income levels. Almost all goods are normal at sufficiently low levels of income and are inferior at sufficiently high levels of income. For example, at lower levels of income, the demand for poultry increases as the income level rises, but at sufficiently high levels of income, quality cuts of beef will be substituted for poultry, causing the quantity demanded for poultry to decrease. Thus, poultry is a normal good at lower levels of income and an inferior good at higher levels of income.

Normal goods are further classified as necessities and luxuries. A good is called a *necessity* if its income elasticity of demand is positive and less than 1. Thus when income rises, demand for the product increases, but less than proportionately. Similarly, if income elasticity of demand exceeds 1, the good is called a *luxury*. A good can be a necessity at high levels of income and a luxury at low levels of income. In empirical studies, however, economists generally either assume a constant elasticity or report the elasticity at the mean (or median) level of income. In Table 3.2, we present some estimates of income elasticity computed for the United States on the basis of quarterly data for the years 1967 through 1979.

As with price elasticities, we can talk of short-run and long-run income elasticities. Because consumers can make the necessary adjustments in the long run, we would expect the long-run income elasticities to be higher than short-run income elasticities in absolute value. However, as mentioned earlier in connection with short-run and long-run price elasticities, in the case of durable goods we must be again careful whether we are discussing stocks or flows.

Are there *any* cases where the short-run elasticity is higher than the long-run elasticity? Yes, if there is *over-shooting*.[7] In models of flexible exchange rates, an increase in money supply ceteris paribus would have an impact on domestic price

[7]This terminology is commonly used in models of flexible exchange rates, where the short-run effect is higher than the long-term effect.

TABLE 3.2 Estimates of Income Elasticity of Demand

Commodity Group	Elasticity	Commodity Group	Elasticity
Beef	0.94	Housing	0.41
Pork	0.32	Other services	0.72
Broilers	0.65	Transportation	0.64
Milk	0.24	Household operation	0.03
Eggs	0.52	Electricity and gas	0.56
Clothing and shoes	1.72	Gasoline and oil	0.36
Other nondurables	0.91	Fuel oil and coal	0.27

Source: Dale Heien, "Seasonality in U.S. Consumer Demand," Journal of Business and Economic Statistics, vol. 1, no. 4, October 1983, p. 283.

level and the exchange rate. However, if prices are sticky and exchange rates are not, then initially the brunt of the adjustment falls on the exchange rate and it would have to overadjust or "overshoot." The short-run effect on the exchange rate is consequently higher than the long-run effect. Similarly, a decrease in consumers' income would eventually be felt on *all* commodities. However, in the short run, consumers would overadjust (cut down drastically) the expenditures on commodities whose purchase they can easily postpone. These are usually big-ticket items such as houses, cars, or some appliances. These adjustments would produce an effect that is higher in the short run than the long run. This would be an argument why the demand for automobiles (or other durables or owner-occupied housing) can be more income elastic in the short run than in the long run.

Cross-price elasticities

Another useful concept is the *cross-price elasticity of demand.* (Price elasticity of demand discussed in Section 3.1 is sometimes referred to as *own* price elasticity.) The elasticity of demand for good Y with respect to the price of good X measures responsiveness of demand for Y to a change in the price of X and is defined (with obvious notation) by

$$\eta_{Y,P_x} = \frac{\Delta Q_y}{\Delta P_x} \cdot \frac{P_x}{Q_y}$$

Again, the assumption of ceteris paribus applies, and this time we assume that everything remains the same except the price of X.

A cross-price elasticity of -1.4 means that a 1 percent increase in the price of good X leads to a 1.4 percent reduction in the demand for good Y. Again, the sign of the elasticity is important. A positive cross-price elasticity means that an increase in P_x leads to an increase in Q_y, and the two goods are substitutes. Fuel oil and natural gas are examples of substitutes for heating. However, a negative cross-price elasticity implies that an increase in P_x causes a reduction in Q_y so that products X and Y are complements. Gasoline and motor oil are complements. If the price of gasoline rises, the quantity demanded for gasoline falls but so does the demand for motor oil. If the cross-price elasticity is 0, the goods are independent or unrelated.

With respect to two goods X and Y there are two cross-price elasticities.

$$\eta_{Y,P_x} = \text{elasticity of demand for } Y \text{ with respect to the price of } X$$

and

$$\eta_{X,P_y} = \text{elasticity of demand for } X \text{ with respect to the price of } Y$$

These two elasticities need not be equal.[8]

Note that cross-price elasticities frequently tell us something about the magnitude of the own price elasticity of demand. Why? Because an important determinant of own price elasticity is the availability and closeness of substitutes. Consider the demand for Pepsi. If the price of Pepsi increases, many consumers quickly switch to Coke or similar cola products. What does this tell us about the cross-price elasticity of demand for these products? It should be positive and fairly large, indicating that these two goods are close substitutes. In turn, what does the high cross-price elasticity imply about own price elasticity of demand for Pepsi? Demand should be fairly elastic. So, in general, when close substitutes are available, own price elasticity will be larger.

EXAMPLE 3.3 Use of Cross-Price Elasticity in an Antitrust Case

The Antitrust Division of the U.S. Department of Justice brought suit against the Du Pont Company for monopolizing the sale of cellophane. Du Pont sold 75 percent of the cellophane used in the United States. However, Du Pont argued that the relevant market was that of packaging materials that included aluminum foil, waxed paper, polyethylene, and so on. In fact, cellophane accounted for only 20 percent of this market. To prove its point, Du Pont produced cross-price elasticities between cellophane and the other substitute products. In 1956 the U.S. Supreme Court agreed with Du Pont and dismissed the case.[9]

EXAMPLE 3.4 Demand for Liquor

Estimates of price and income elasticities vary a lot depending on the time periods, type of data, country for which the data apply, and so on. Of particular importance are elasticities of demand for some items whose consumption is considered detrimental to individuals' health or has adverse social consequences (e.g., drunken driving).

Hogarty and Elzinga estimated the price elasticity of demand for beer as 1.13.[10] McGuiness finds with U.K. data that consumption of alcohol is not sensitive to

[8] In Chapter 5 we will discuss the income and substitution effects of a price change. At that time we will discuss what is known as an income-compensated demand curve. For such demand curves these two elasticities are equal.

[9] See *U.S. Reports*, vol. 351, U.S. Government Printing Office, Washington, D.C., 1956, p. 400. See also the analysis in G. W. Stocking and W. F. Mueller, "The Cellophane Case and the New Competition," *The American Economic Review*, March 1955, pp. 29–63.

[10] T. F. Hogarty and K. G. Elzinga, "The Demand for Beer," *Review of Economics and Statistics*, May 1972, pp. 195–198.

price changes although it goes up with income.[11] He also finds large elasticity of consumption with respect to number of licensed premises. The two studies give different answers to the question of whether an increase in the taxes on liquor restricts alcohol consumption.

3.7 THE SELLERS' VIEW: TOTAL REVENUES, AVERAGE REVENUES, AND MARGINAL REVENUES

In Chapter 2, when we discussed the concept of the demand curve, we said that the demand curve gives the *hypothetical* quantities that a consumer would buy at different prices. In this view, quantity is the dependent variable, and strictly speaking, we should measure quantity on the vertical axis and price on the horizontal axis. However, we followed the usual convention of measuring price on the vertical axis and quantity on the horizontal axis, a procedure that is justifiable if we view the demand curve from the seller's point of view. We will now discuss this view.

From the seller's point of view, the demand curve tells what price the sale of a number of units will fetch. Note that we are not talking of selling one unit of the commodity at a time. What we are asking is a hypothetical question as to what price sellers would get if they *block-auctioned* different quantities.

If a seller offers 10 units of a commodity for sale and gets \$100, then \$100 is called the *total revenue. Average revenue* or per-unit revenue is total revenue divided by the number of units sold or \$10. This is actually the *bid price* when 10 units are offered for sale.

Table 3.3 shows a demand curve from the buyers' point of view and the sellers' point of view. Note that the average revenue curve is the demand curve. For the data in Table 3.3 we can compute the total and average revenue.

Also listed in Table 3.3 is marginal revenue. *Marginal revenue* of the nth unit is the *extra* revenue that the seller gets by offering n units instead of $(n - 1)$ units for sale. Since marginal revenue is the increase in total revenue for a one unit increase in the quantity Q offered for sale, we can write

$$MR = \frac{\text{Change in } TR}{\text{Change in } Q} = \frac{\Delta TR}{\Delta Q}$$

where MR denotes marginal revenue and TR denotes total revenue.[12] Graphically, the MR at a point on the TR curve is the slope of the TR curve at that point.

Henceforth, we will use the abbreviations TR, AR, and MR to denote total revenues, average revenue, and marginal revenue, respectively.

Note that like the demand curve (which is also the AR curve from the sellers' point of view), the TR and MR curves are hypothetical. Transactions do not take place at all the points. This does not mean they are not useful. We discussed this point in Chapter 2 in connection with demand curves.

[11] T. W. McGuiness, "The Demand for Beer, Spirits and Wine in the U.K. 1956–79," in M. Grant, M. Plant, and A. Williams, eds., *Welfare Economics and Alcohol*, Croom Helm, London, 1983.

[12] For an arbitrarily small change in Q, $MR = d(TR)/dQ$.

TABLE 3.3 A Demand Curve from the Buyers' and Sellers' Points of View

Buyers' View			Sellers' View			
Price	Quantity Demanded	Total Expenditures	Quantity Offered for Sale	AR (Bid Price)	Total Revenue	Marginal Revenue
12	1	12	1	12	12	12 − 0 = 12
10	2	20	2	10	20	20 − 12 = 8
8	3	24	3	8	24	24 − 20 = 4
6	4	24	4	6	24	24 − 24 = 0
4	5	20	5	4	20	20 − 24 = −4

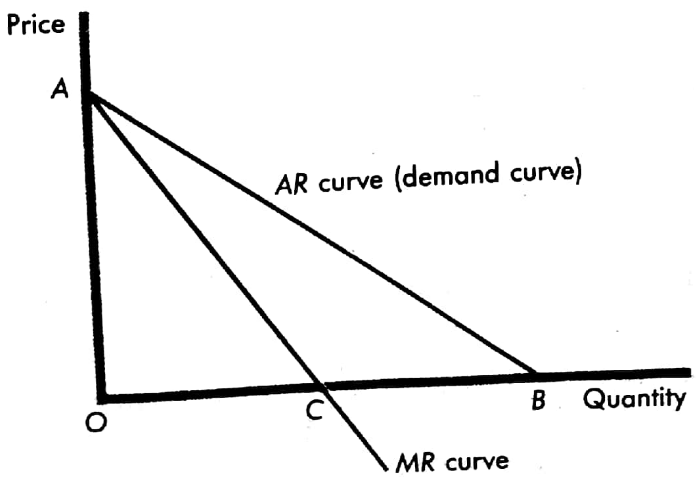
Examining Table 3.3, we see that the *AR* declines at a rate of 2 per unit, whereas the *MR* declines at the rate of 4 per unit, and *MR* is equal to 0 when *TR* is maximum.

Figure 3.9 shows typical average and marginal revenue curves when the demand curve is linear. Note that in Figure 3.9, $OC = (1/2)OB$. Since the *MR* and demand curves have the same vertical intercept, this means that the *MR* curve is twice as steep as a corresponding linear demand curve. We will prove this result in the next section.[13]

Figure 3.10 shows a typical *TR* curve for a linear demand curve. With a linear demand curve, *MR* is strictly diminishing as *Q* increases. And since *MR* at any point on the *TR* curve is the slope of the *TR* curve at that point, diminishing marginal revenue means that the slope of the *TR* curve is diminishing. This implies a humped shape for the *TR* curve. In Figure 3.10, the slope diminishes as we go from *A* to *B* to *C*. At the point *C*, or at maximum total revenue, the slope is 0. After that, the slope turns negative as it is at the point *D* and continues to diminish or become increasingly negative.

[13]Mathematically we can derive this as follows: $p = \alpha - \beta q$ is a linear demand curve. $TR = pq = \alpha q - \beta q^2$. Hence, $MR = d(TR)/(dq) = \alpha - 2\beta q$. Thus, the *MR* curve is linear. The intercept on the quantity axis (obtained by setting $p = 0$) is $-\alpha/\beta$ for the demand curve. For the *MR* curve (at $MR = 0$) it is $-\alpha/2\beta$. Thus, $OC = (1/2)OB$ in Figure 3.9.

FIGURE 3.9 Linear *AR* and *MR* curves.



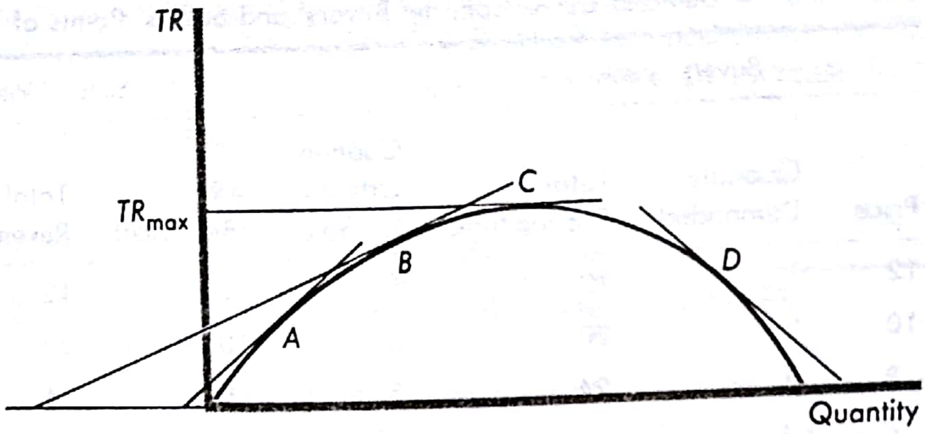


FIGURE 3.10 A typical *TR* curve for a linear demand curve. Diminishing marginal revenue implies that the slope of the *TR* curve diminishes as quantity offered for sale increases.

We will now discuss how the shapes of the *TR*, *AR*, and *MR* curves are related to the elasticity of the demand curve.

3.8 RELATIONSHIP BETWEEN ELASTICITY OF DEMAND, PRICE (*AR*), *TR*, AND *MR*

The elasticity of demand plays a crucial role in the relationship between *TR*, *AR*, and *MR* and in shaping these curves.

To examine these relationships, let us start with an initial price of P and quantity Q on the demand curve. Then total revenue $TR = P \cdot Q$. Now increase Q to $Q + \Delta Q$, where ΔQ is very small. With a downward-sloping demand curve, the price falls to $P - \Delta P$. (Note that we are taking the absolute values of ΔP.) Since ΔQ is very small, ΔP will also be. The total revenue now is $(Q + \Delta Q)\ (P - \Delta P) = PQ + P(\Delta Q) - Q(\Delta P)$. We have omitted the term $\Delta Q \cdot \Delta P$, since if ΔQ and ΔP are very small, their product will be negligible. Subtracting the initial total revenue of PQ we get:

$$\Delta TR = P(\Delta Q) - Q(\Delta P)$$

hence

$$MR = \frac{\Delta TR}{\Delta Q} = P - Q \cdot \frac{\Delta P}{\Delta Q} = P\left(1 - \frac{Q}{P} \cdot \frac{\Delta P}{\Delta Q}\right)$$

but

$\dfrac{\Delta Q}{\Delta P} \cdot \dfrac{P}{Q}$ is the elasticity of demand η

So finally, $MR = P(1 - [1/\eta])$. *This is an important relationship which we will use repeatedly in this and future chapters in the book.* Since P is the same as *AR*, we can also state

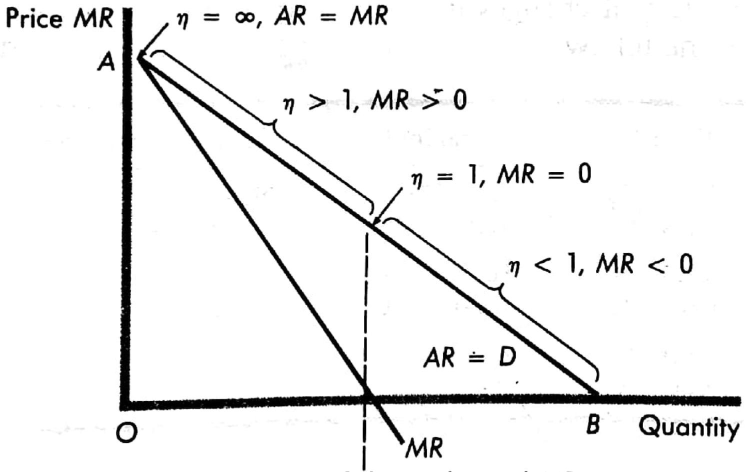
$$MR = AR\,[1 - (1/\eta)]$$

This establishes the relationship between MR and AR. We can note the following results:

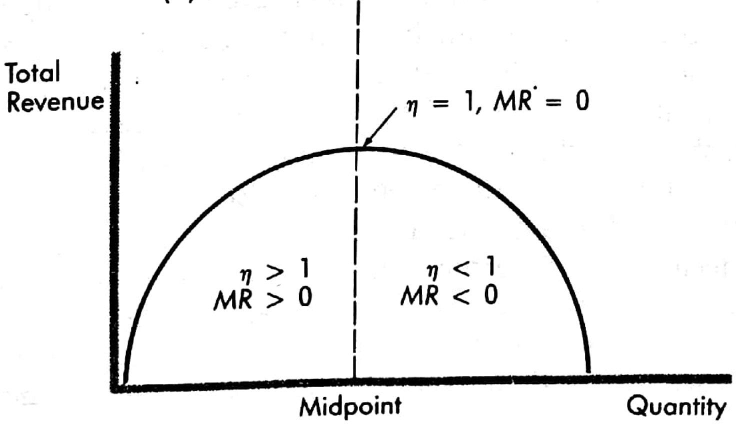
1. Since we have defined elasticity of demand as a positive number, $1 - (1/\eta)$ is less than 1, and, hence, $MR \leq AR$.
2. As η keeps decreasing, $1/\eta$ keeps increasing, and MR keeps falling.
3. If $\eta > 1$, $[1 - (1/\eta)]$ is positive. Hence, $MR > 0$, or TR will be increasing as Q increases.
4. If $\eta < 1$, $[1 - (1/\eta)]$ is negative. Hence, $MR < 0$, or TR will be decreasing as Q increases.
5. If $\eta = 1$, $[1 - (1/\eta)] = 0$ and $MR = 0$. Thus, the TR will be constant.
6. If $\eta = \infty$, $[1 - (1/\eta)] = 1$ and, hence, $MR = AR$.

We can now study these results in relation to the linear demand curve. (Figure 3.3 showed how the elasticity η changes along the demand curve.) We will now superimpose the results for MR. This is shown in Figure 3.11.

FIGURE 3.11 Elasticity of demand, MR, and TR for a linear demand curve.



(a) Elasticity of demand η and MR



(b) Elasticity of demand η and TR.

The behavior of TR is shown in Figure 3.11(b). When $MR > 0$, TR will be increasing, and when $MR < 0$, TR will be decreasing. At the maximum value for TR, $MR = 0$. Why? (Note that for the hyperbolic demand curve shown in Figure 3.7, $\eta = 1$ at all points. Hence, $MR = 0$ and $TR =$ constant. In fact, the demand curve with a constant $P \cdot Q$ means TR is constant.)

The hump-shaped TR curve shown in Figure 3.11(b) is for a linear demand curve. For a curvilinear demand curve with elasticity less than 1 at all points, the total revenue will be falling steadily, and for a curvilinear demand curve with elasticity greater than 1 at all points, TR will be rising steadily.

We can get a hump-shaped TR curve even for a curvilinear demand curve if η is steadily falling as Q increases. However, the peak of the hump need not be at the midpoint. This is true only for a linear demand curve because $\eta = 1$ at the midpoint of a linear demand curve.

We have until now considered changes in total revenue TR when quantity changes. The price elasticity of demand will also tell us whether total revenue will increase or decrease with changes in price. To see what happens to total revenue all we have to do is note that the elasticity of demand is defined as

$$\left| \frac{\text{Percentage change in quantity}}{\text{Percentage change in price}} \right|$$

and see the effect of changes in price on quantity and hence on total revenue. Thus, we get the following results for a 1 percent change in price:

	Price P	Quantity Q	Total Revenue PQ
$\eta = 1$	Rises 1%	Falls 1%	No change
	Falls 1%	Rises 1%	No change
$\eta > 1$	Rises 1%	Falls >1%	Falls
	Falls 1%	Rises >1%	Rises
$\eta < 1$	Rises 1%	Falls <1%	Rises
	Falls 1%	Rises <1%	Falls

Thus if $\eta < 1$, the seller can get more total revenue by raising prices. If $\eta > 1$, the seller can get more total revenue by cutting prices. We consider only a small price change of 1 percent—similar results hold for moderately higher percentages of, say, 5 percent. If we consider a 20 percent change, then of course we cannot say that TR will show no change if $\eta = 1$. Suppose initially that $P = 10$ and $Q = 10$ so that $TR = 100$. With a 20 percent price rise, P will rise to 12, Q will fall to 8, and $TR = 96$, which is a 4 percent decline. Similarly, for $\eta = 0.9$, if P rises by 20 percent, Q declines by 18 percent. Thus $P = 12$ and $Q = 8.2$ and $PQ = (12)\ (8.2) = 98.4$. Thus total revenue, instead of rising, has fallen.

The reason for this can be seen by looking at the expression for the change in TR. If P rises by ΔP, then Q falls by ΔQ. Hence the change in total revenue is given by

$$\Delta TR = (P + \Delta P)\,(Q - \Delta Q) - PQ$$
$$= Q\Delta P - P\Delta Q$$

if we ignore the term $\Delta P \cdot \Delta Q$. Thus

$$\frac{\Delta TR}{\Delta P} = Q - P\frac{\Delta Q}{\Delta P}$$
$$= Q - Q\left(\frac{P}{Q} \cdot \frac{\Delta Q}{\Delta P}\right) = Q(1 - \eta)$$

Hence if $\eta < 1$, this expression is positive and ΔTR is positive, and if $\eta > 1$, this expression is negative and ΔTR is negative, as we obtained earlier. However, all this assumes that ΔP and ΔQ are small and hence their product is much smaller in magnitude and can be ignored. If they are not small, the cross product $\Delta P \cdot \Delta Q$ cannot be ignored.

EXAMPLE 3.5 Demand for Shakespearean Plays

Nonprofit lively arts often lose money and depend on patronage to cover losses.[14] An estimation of price elasticities of demand would help us to see whether prices could have been raised to increase revenues. Gapinski estimated demand functions for performances by Britain's Royal Shakespeare Company (RSC) during the 1965 to 1980 period and for the Aldwych Theater in London and the Shakespeare Memorial Theater in Stratford-upon-Avon.[15] He obtained a price elasticity of 0.657 (implying that the RSC could have raised price and thereby revenue) and an income elasticity of 1.327 (implying that Shakespeare performances were a luxury item). Gapinski estimates that profit maximization would require prices to more than double at Aldwych and exactly double at Stratford. According to his calculations, real profit would rise from −£505,000 to £5,000 at Aldwych and from −£214,000 to £317,000 at Stratford.

3.9 APPLICATIONS OF THE ELASTICITY CONCEPTS

We will now return to the problems of taxes, subsidies, tariffs, and quotas, that we discussed in Chapter 2 and show how the elasticities of demand and supply determine who bears the burden of taxes, who benefits more from subsidies, and so on. First, we will start with the excise tax discussed in Section 2.7.

3.9.1 Who bears the burden of excise taxes?

As we discussed in Section 2.7 we can analyze a sales tax (a percentage tax) the same way as a per unit tax with minor changes. Hence, we will analyze only a per unit tax here.

[14] W. J. Baumol and W. G. Bowen, *Performing Arts—The Economic Dilemma*, Twentieth Century Fund, New York, 1966.
[15] J. H. Gapinski, "The Economics of Performing Shakespeare," *The American Economic Review*, June 1984, pp. 458–466.

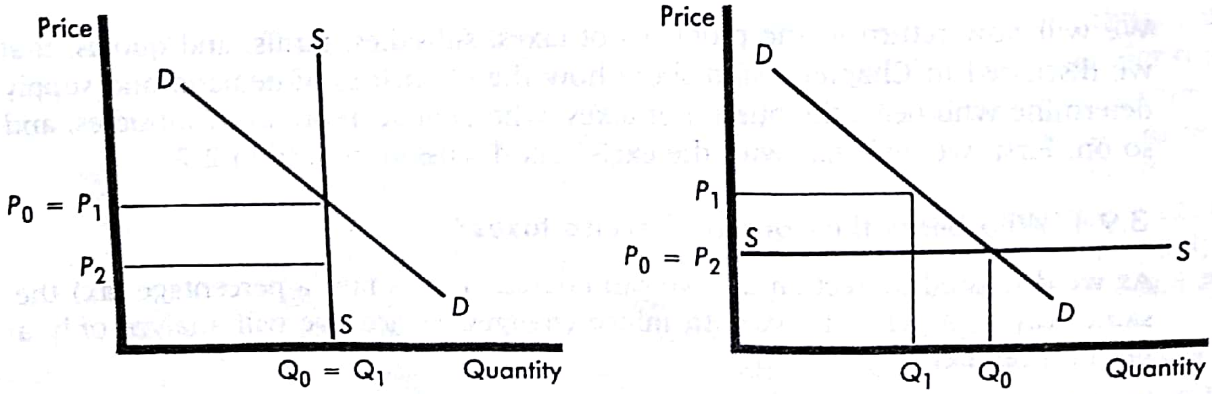
In our discussion, we said that the effect of a per unit tax is to raise the prices the buyers pay and reduce the prices sellers get. Thus, the tax is "shared" by buyers and sellers. The question is whose share is larger? The answer depends on the elasticities of demand and supply. The general rule is that if supply is less elastic than demand, then the suppliers pay the greater portion of the tax. If demand is less elastic than supply, then the buyers pay a greater portion of the tax. We illustrate the extreme cases of completely inelastic and perfectly elastic supply curves in Figure 3.12. We follow the same notation as in Section 2.7 of Chapter 2. *DD* is the demand curve, *SS* the supply curve, and P_0 the market equilibrium price without the tax. With the tax, P_1 is the price buyers pay, and P_2 is the price sellers get. $P_1 - P_2 = T$, the tax per unit.

As we did in Section 2.7 of Chapter 2, we have to find P_1 and P_2 such that $P_1 - P_2 = T$, the tax, and quantity demanded at the price P_1 is equal to the quantity supplied at the price P_2. In Figure 3.12(*a*) when supply is completely inelastic, quantity supplied is the same at all prices. Hence, P_1 stays at the price P_0 (the buyers pay the pretax price), and P_2 falls by the *full* amount of the tax (the price suppliers get falls by the full amount of the tax). Thus, suppliers bear the total tax burden.

In Figure 3.12(*b*) when supply is perfectly elastic, the price suppliers get cannot change and so stays at P_0. Hence, the price buyers pay has to rise by the full amount of the tax. Buyers bear the full burden of the tax. One can illustrate the cases of perfectly elastic and completely inelastic demand curves in a similar fashion. This will be left as an exercise. Now we must consider the case where the demand and supply curves are neither perfectly elastic nor completely inelastic but with elasticities very low or very high. This is shown in Figure 3.13. The figure is self-explanatory, and the conclusions are straightforward.

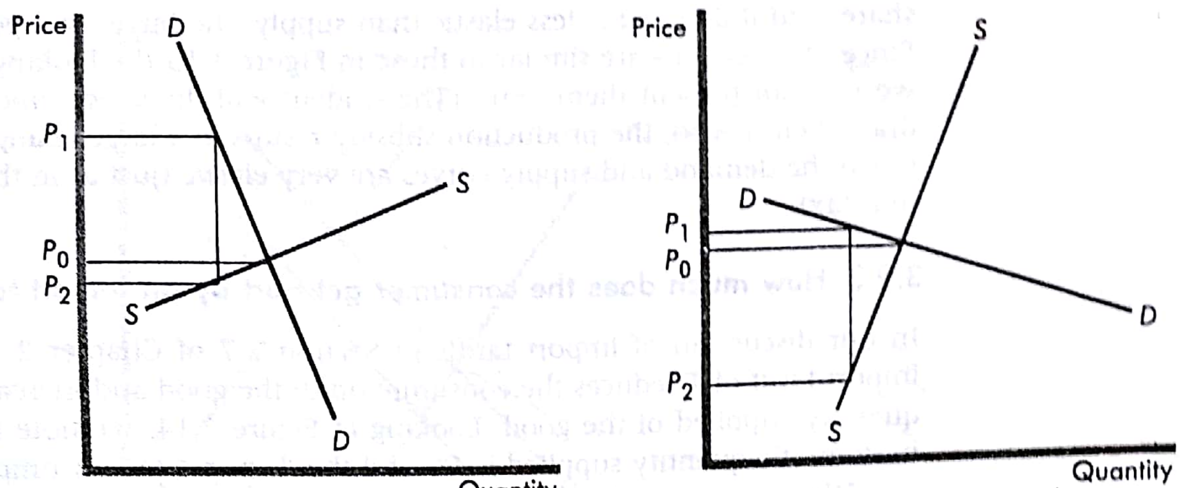
One additional result concerns the impact of the tax on output. If both the demand and supply curves are inelastic, the effect of the tax on quantity is minimal, but if both the curves are highly elastic, then there will be a drastic reduction in output. This result is obvious, since if the elasticities are high, a given percentage

FIGURE 3.12 Effect of a per unit tax with completely inelastic and completely elastic supply.



(a) Supply completely inelastic—tax is completely borne by the suppliers.

(b) Supply perfectly elastic—tax is completely borne by buyers.



(a) Elastic supply curve, inelastic demand curve—tax mostly borne by buyers.

(b) Elastic demand curve and inelastic supply curve—tax mostly borne by sellers.

FIGURE 3.13 Effect of per unit tax with an elastic supply curve (with an inelastic demand curve) and an inelastic supply curve (with an elastic demand curve).

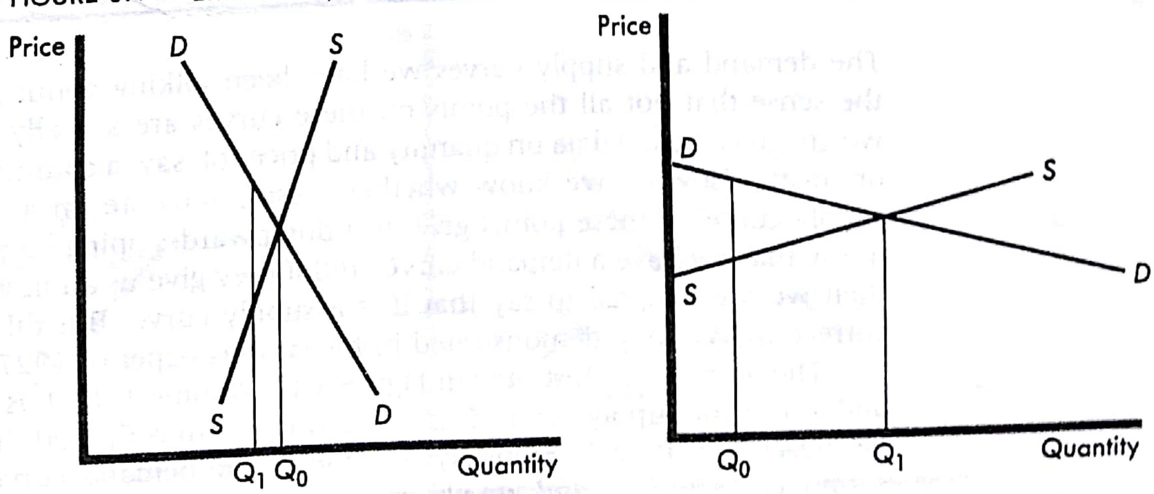
increase in price results in a higher percentage reduction in output. We show this in Figure 3.14.

3.9.2 Who benefits from production subsidies?

A production subsidy is similar to the excise tax. We said in Section 2.7 of Chapter 2 that a production subsidy raises the price for the producer and reduces the price for the buyer. Thus, both share the benefit. The question is: Who gets the bigger share?

Again, if supply is less elastic than demand, then the supplier gets the bigger

FIGURE 3.14 Effect of a per unit tax on output.



(a) Both demand and supply inelastic—output reduction is small.

(b) Both demand and supply highly elastic—output reduction substantial.

share, and if demand is less elastic than supply, the buyer will get a bigger share. Since the diagrams are similar to those in Figure 3.13 (by looking at Figure 2.13), we will not present them here. (The student will, however, find it instructive to draw them.) Also, the production subsidy results in a large change in output only when the demand and supply curves are very elastic (just as in the case of the per unit tax).

3.9.3 How much does the consumer get hurt by an import tariff?

In our discussion of import tariffs in Section 2.7 of Chapter 2, we said that an import tariff of T reduces the consumption of the good and increases the domestic quantity supplied of the good. Looking at Figure 2.14, we note that the increase in domestic quantity supplied is EG and the decrease in consumption is HF.

We can now see how the elasticities of the demand and supply determine the magnitudes of these changes. We can infer the following:

1. If supply is inelastic and demand very elastic, there will be very little increase in domestic quantity supplied (EG will be very small). The reduction in imports will come almost entirely from reduction in consumption (it will come almost entirely from HF). The effect is merely a rise in domestic price to benefit the suppliers.
2. The opposite will be our conclusion if demand is inelastic and supply is very elastic.
3. If both demand and supply are very elastic, we would need only a small tariff to reduce imports.

All these conclusions follow directly from the definitions of elasticity and from Figure 2.14.

3.10 IS IT A DEMAND CURVE OR A SUPPLY CURVE?

The demand and supply curves we have been talking about are hypothetical in the sense that not all the points on these curves are actually observed. Suppose we are given actual data on quantity and price (of, say, a commodity such as wheat or sugar). How do we know whether these points are on a demand curve or a supply curve? If these points give us a downward-sloping curve, we are tempted to say that we have a demand curve, and if they give us an upward-sloping curve, then we are tempted to say that it is a supply curve. But this conclusion is not correct, as Working demonstrated in his famous paper of 1927.[16]

The problem is illustrated in Figure 3.15. At time 1, D_1D_1 is the demand curve, and S_1S_1 is the supply curve. The equilibrium point is C_1, and this is the only point we observe at time 1. Similarly, at time 2, the demand curve is D_2D_2, and the supply curve is S_2S_2, and we observe the point C_2. The points we observe are C_1

[16] E. J. Working, "What Do Statistical Demand Curves Show?" *Quarterly Journal of Economics*, February 1927. Reprinted in American Economic Association, *Readings in Price Theory*, Irwin Publishers, Homeward, Illinois, 1953, pp. 97–118.

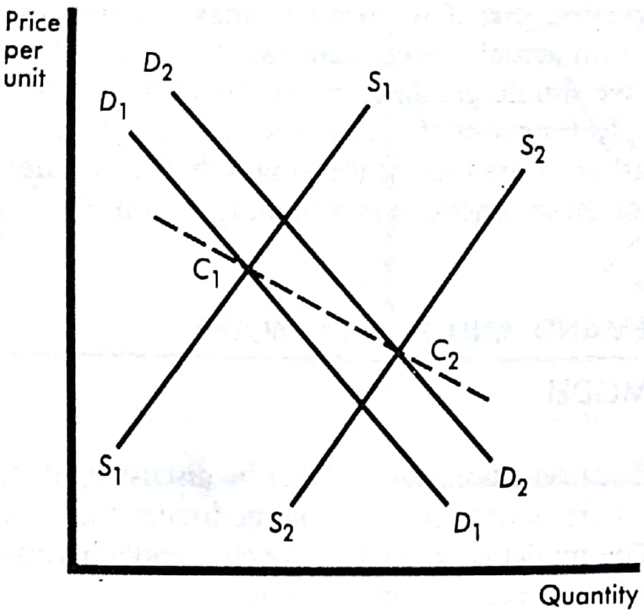
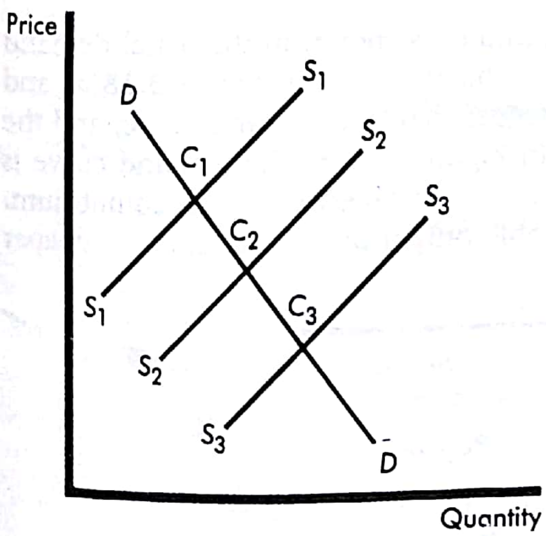


FIGURE 3.15 Observations on quantity and price when both the demand and supply curves shift over time.

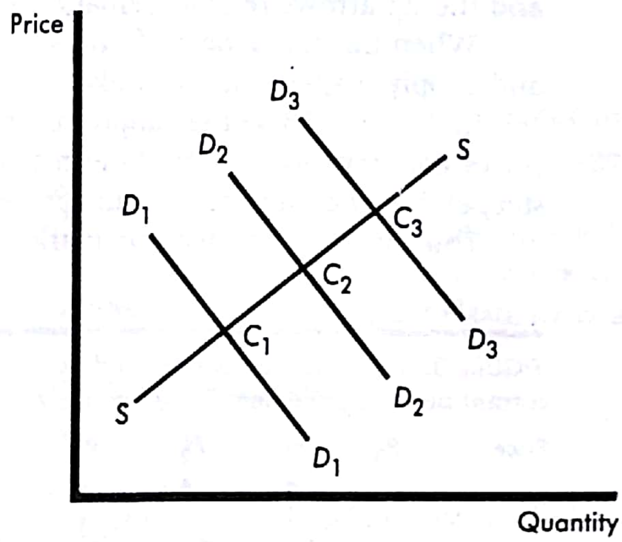
and C_2 alone, and if we join them we get a downward-sloping line. But C_1C_2 is neither a demand curve nor a supply curve. This is called the *identification problem.* We cannot *identify* the observed curve as a demand curve or a supply curve. We can do this only if one of the two curves is stable.

Suppose the demand curve is stable over time but the supply curve shifts as shown in Figure 3.16(*a*). Then the observations we have will all be on the same demand curve. Similarly, if the supply curve is stable but the demand curve shifts, as shown in Figure 3.16(*b*), then the observed points will trace out a supply curve. Thus, we can identify from the observed data the curve that is stable.

FIGURE 3.16 Identification of the demand or supply curves from market data on quantity and price.



(a) Demand curve stable; demand curve identified.



(b) Supply curve stable; supply curve identified.

Working suggested that if we want to make inferences about demand and supply functions from actual market data (which consist of equilibrium quantities and prices only), we should get data on variables that account for the shifts in the demand and supply functions, for example, income in the case of the demand function and weather in the case of the supply function. Then we can control for the shifts in one of these functions and identify it from the market data.

3.11 A DYNAMIC DEMAND AND SUPPLY MODEL: THE COBWEB MODEL

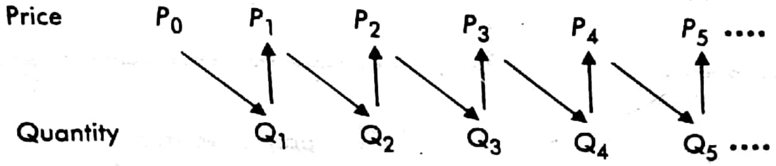
As we said in the introduction, we will not be discussing dynamic models in this book. However, there is one simple dynamic model that is easy to explain: the cobweb model. The model is called a "cobweb" model because the path taken by the observed price and quantity form a cobweb.

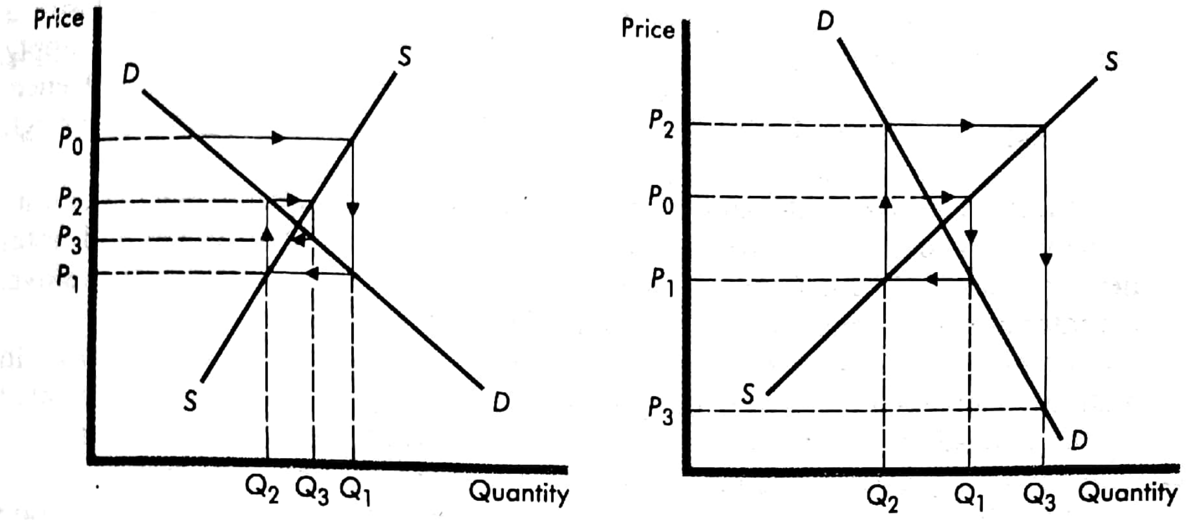
In a dynamic model every variable has to be dated. Let P_t, Q_t be the price and quantity transacted at time t. We consider a market where supply decisions have to be made one time period (say a year or 6 months) in advance. This is a reasonable assumption in the case of many agricultural commodities. The suppliers are assumed to be very naive, and they look at last period's price and make their supply decisions (assuming that price to prevail this year as well). Once they produce the output, they have to sell it for whatever price it fetches. Since the quantity is given, the price they get is determined by the demand curve.

Suppose we start with some price P_0. Suppliers look at this and decide how much to produce for the next period. Thus, this determines Q_1, the quantity brought into the market at time 1. The price they get, P_1, is determined by the demand curve. The suppliers go back and change their production based on P_1. Next time period they come to the market with output Q_2. Again, the demand curve determines the price P_2 they get. This process keeps on going. The situation can be depicted by the arrows in Figure 3.17. The down arrows refer to supply response, and the up arrows refer to demand response.

When this movement of prices and quantities is shown in the usual demand and supply diagram, it looks like a cobweb. This is shown in Figure 3.18(a) and (b). In Figure 3.18(a) the supply curve is steeper than the demand curve, and the prices converge to the equilibrium point. In Figure 3.18(b) the demand curve is steeper than the supply curve, and prices progressively diverge from the equilibrium. Thus, in this example, the market is stable only if the supply curve is steeper

FIGURE 3.17 Production determined by last period's price and current period's price determined by current output supplied.





(a) Supply curve steeper than the demand curve: prices converge to an equilibrium.

(b) Demand curve steeper than the supply curve: prices are explosive.

FIGURE 3.18

than the demand curve. If both the demand and supply curves have the same slope, one can easily check that prices continually oscillate between two limits. Prices neither converge to an equilibrium, nor diverge from the equilibrium.

3.12 SUMMARY AND CONCLUSIONS

Elasticity measures the responsiveness of quantity to changes in some other variable. Price elasticity of demand is defined as the ratio of percentage change in quantity demanded to percentage change in price. It is given by

$$\eta = \left|\frac{\Delta Q/Q}{\Delta P/P}\right| = \left|\frac{\Delta Q}{\Delta P}\cdot\frac{P}{Q}\right| = \left|\frac{1}{\text{slope}}\cdot\frac{P}{Q}\right|$$

Point elasticity measures the elasticity at a given point on the demand curve, or equivalently, at a given price. Arc elasticity is an approximate average of the elasticities at two points on the demand curve.

For a linear demand curve the slope is constant, and since P/Q falls as we move down the demand curve, price elasticity declines as we move down the demand curve. A convenient formula to use to compute elasticity at a point C is

$$\eta = \frac{\text{Distance of the point } C \text{ from the } Q \text{ axis}}{\text{Distance of the point } C \text{ from the } P \text{ axis}}$$

with both distances being measured along the demand curve in the case of a linear demand curve and along a tangent to the demand curve at the point C, if the demand curve is nonlinear.

The same convenient formula applies for supply elasticity as well. Since a supply curve is positively sloped, it can pass through the origin. For this supply curve, the distance of any point from the Q axis or the P axis is the same. Hence, the supply elasticity equals 1 at all points for any linear supply curve passing through the origin.

Income elasticity of demand is defined as the proportionate change in quantity demanded divided by the proportionate change in income, with prices and tastes held constant. A normal good is one for which the income elasticity is positive. An inferior good is one for which income elasticity is negative.

Cross-price elasticity of demand is defined as the proportionate change in quantity demanded divided by the proportionate change in the price of a related good. A substitute good is one for which the cross-price elasticity is positive. A complementary good is one for which the cross-price elasticity is negative.

Total revenue is the product of the price and quantity demanded of the good at that price. Equivalently, it is the product of the quantity offered for sale and the price the offered quantity fetches in the market place. Average revenue is total revenue divided by the quantity sold. Marginal revenue is the change in the total revenue divided by the change in the quantity sold.

Total revenue (TR), marginal revenue (MR), and average revenue (AR) are related as follows:

1. MR is the slope of the TR curve.
2. TR is increasing if $MR > 0$ and decreasing if $MR < 0$. TR is maximum when $MR = 0$.
3. $MR = AR[1 - (1/\eta)]$ where η is the elasticity of the demand curve at that point.

The effects of excise taxes, subsidies, import tariffs, and so forth on price and output depend on the elasticities of the demand and supply curves. Other things remaining the same, the effect on quantity is greater if the elasticities of the demand and supply curves are high. If the supply curve is perfectly elastic but the demand curve is not, the taxes are entirely borne by the buyers. However, if the demand curve is perfectly elastic but the supply curve is not, the taxes are entirely borne by the suppliers. Thus, the distribution of the burden depends on the elasticities of the two curves.

In practice, from given data on quantities and prices we cannot determine whether we have a demand curve or a supply curve. If the demand curve is stable and the supply curve shifts, then the observed data on quantities and prices will give us a demand curve. If the supply curve is stable and the demand curve shifts, the observed data will give us a supply curve. If both the curves shift, then we do not know what we have. This is called the identification problem.

If quantity demanded depends on current price and quantity supplied depends on last year's price, we have the cobweb model. Prices converge to an equilibrium point if the supply curve is steeper than the demand curve. Prices move away from the equilibrium level if the demand curve is steeper than the supply curve.

KEY TERMS

Arc Elasticity
Cobweb Model
Complementary Goods
Cross-Price Elasticity of Demand
Identification Problem
Income Elasticity of Demand
Inferior Goods
Intermediate-run Elasticity
Long-run Elasticity
Luxury Goods
Necessities
Normal Goods
Point Elasticity
Price Elasticity of Supply
Price (Own) Elasticity of Demand
Short-run Elasticity
Substitute Goods
Total Revenue

QUESTIONS

1. In Table 3.2, we presented estimates of the income elasticity of demand for certain commodity groups. Which commodities are necessities? Which are luxuries? Which are inferior? Explain your answer.

2. Market analysts often use cross-price elasticities to determine a measure of the "competitiveness" of a particular good in a market. How might cross-price elasticities be used in this manner? What would you expect the cross-price elasticity coefficient to be if the market for a good was highly competitive? Why?

3. Suppose that the demand curve for product X is

$$Q = 100 - 5P$$

where Q is the number of units of X demanded, and P is the per-unit price of X in dollars. Express the total revenue and marginal revenue functions for product X. Graph both the demand and MR curves and determine over what range of output demand is elastic and inelastic. Calculate elasticity at an output of 50 both algebraically and graphically? What is MR at an output of 50?

4. Suppose that you are the president of a firm that produces and sells four products: apples, oranges, grapefruit, and kiwi fruit. Each product has the following price elasticity of demand:

Product	Price Elasticity
Apples	2.50
Oranges	1.00
Grapefruit	1.75
Kiwi fruit	0.65

Because the company is experiencing serious cash flow problems, your immediate objective is to increase total revenue. What is your pricing strategy for each product? Why? Would it help to know cross-price elasticities? Why?

5. The price elasticity of demand for table salt is very small. Why is this the case? Could this explain why table salt is seldom advertised at a "special price" by grocers?

6. Suppose that when the price of pork chops in a certain town was $2.20 per pound, the quantity of chicken sold was 1,200 pounds per week. But when the price of pork chops rose to $2.75 per pound, the quantity of chicken sold increased to 1,800 pounds per week. Nothing changed over this period except the price of the pork chops. Calculate and interpret the cross-price elasticity of demand. How are these two products related?

7. Underdeveloped countries frequently argue that unless they industrialize, they will remain forever poor relative to the rest of the world. Does this argument make sense in light of the small (but positive) income elasticities for the agricultural commodities typically now produced by many of these countries? Explain.

8. During the summer of 1986, the southeastern United States experienced one of the worst droughts of this century. It was argued that farm relief programs were essential because the reduced agricultural output meant that farmers would not have the revenues needed to meet their mortgage payments. Do you agree with this conclusion? Does the price elasticity of demand enter into your answer?

9. Over the past few years, college and university administrators have been increasing tuition rates (sometimes quite substantially), even though there have been significant declines in student enrollment. Is this a rational decision on the administrators' part? What assumptions do they make about the price elasticity of demand for higher education?

10. Using the appropriate diagrams, graphically show that a linear supply curve intercepting the vertical axis at some point above the origin is elastic at every price and that the elasticity of supply declines as price increases.

11. Would you expect the demand for Skippy Peanut Butter to be more or less elastic than the demand for peanut butter in general? Why? What general conclusion can you draw about the elasticity of a specific product or brand name as opposed to a product class?

12. Suppose that products A and B are produced by different firms and are substitutes. Do you think that a change in the price of product A will affect the marginal revenue of the firm producing B? Explain your answer with diagrams.

13. Using the demand curve in Figure 3.2, show that the price elasticity of demand is also equal to FB/OF.

14. When the federal government increased the excise tax on whiskey and other distilled spirits, many retailers complained that the tax would cause a reduction in their sales. On whom do you think that the burden of the tax fell? Were retailers able to pass the tax on to consumers? Why or why not?

15. A diabetic individual must take a prescribed amount of insulin per time period to avoid severe health risks. Draw the individual's demand curve for insulin. What is the price elasticity of demand? Who would bear the burden of a tax on insulin?

16. Explain, intuitively, why the law of demand implies that $MR \leq P$ for all output levels.

17. Consider the following demand curve for product Y:

$$Q = 100 - 10P$$

where Q is the number of units of Y demanded and P is the per unit price of Y in dollars. Demonstrate graphically and mathematically that the demand for Y is more elastic than the demand for X (from problem 3) at any price.

18. Show that commodities which take up a large percentage of a consumer's budget generally have relatively small income elasticities of demand.

19. The price elasticity of demand for a given commodity is alleged to be greater:

    a. The more numerous and closer the substitutes *b*. In the long run as opposed to the short run

c. At high prices rather than low prices

Give supporting arguments in each case.

20. An airline is considering introducing an advance purchase fare to supplement its existing economy fare. It conducts a study to assess the patronage of such a fare. The following table summarizes the projected weekly sales for various advance purchase sales. The economy class fare is $200.

Advance Purchase Fare ($)	Number of Advance Purchase Tickets	Number of Economy Tickets
50	2,000	200
100	1,200	400
120	900	500
150	600	600
180	200	1,000

a. What is the own-price elasticity of advance purchase tickets when the fare rises from $100 to $180?

b. What is the cross-price elasticity of economy tickets in response to advance purchase fares when the advance fare increases from $50 to $150?

c. Would you expect the cross-price elasticity of advance purchase tickets to economy fares to be lower or higher than your answer to question *b*?