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| Experiment No. | 5 |

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| AIM: | Implementation of Matrix Chain Multiplication |
| Program 1 | |
| PROBLEM STATEMENT : | Implementation of Matrix Chain Multiplication |
| ALGORITHM/ THEORY: | <ol style="list-style-type: none"> 1. $n \leftarrow \text{length}[p]-1$ 2. for $i \leftarrow 1$ to n 3. do $m[i, i] \leftarrow 0$ 4. for $l \leftarrow 2$ to n // l is the chain length 5. do for $i \leftarrow 1$ to $n-l + 1$ 6. do $j \leftarrow i+ l -1$ 7. $m[i,j] \leftarrow \infty$ 8. for $k \leftarrow i$ to $j-1$ 9. do $q \leftarrow m[i, k] + m[k + 1, j] + p_{i-1} p_k p_j$ 10. If $q < m[i,j]$ 11. then $m[i,j] \leftarrow q$ 12. $s[i,j] \leftarrow k$ 13. return m and s. |

PROGRAM:

```
#include <stdio.h>
#include<stdlib.h>

Bracketing(int* N, int a, int b, int n){
    int m=(N+a*n+b);
    if(m-a<=1 && b-m-1>1){
        if(m-a==0){
            printf("M%d*(",a+1);
            Bracketing(N,m+1,b,n);
            printf(")");
        }
        else if(m-a==1){
            printf("(M%d*M%d)",a+1,m+1);
            Bracketing(N,m+1,b,n);
            printf(")");
        }
    }
    else if(m-a>1 && b-m-1<=1){
        if(b-m-1==0){
            printf("(");
            Bracketing(N,a,m,n);
            printf(")*M%d",b+1);
        }
        else if(b-m-1==1){
            printf("(",a,m);
            Bracketing(N,a,m,n);
            printf(")(M%d*M%d)",m+2,b+1);
        }
    }
    else if(m-a<=1 && b-m-1<=1){
        if(m-a==1 && b-m-1==0)
            printf("(M%d*M%d)*M%d",a+1,m+1,m+2);
        else if(m-a==0 && b-m-1==1)
            printf("M%d*(M%d*M%d)",a+1,m+2,b+1);
        else if(m-a==1 && b-m-1==1)
            printf("(M%d*M%d)(M%d*M%d)",a+1,m+1,m+2,b+1);
    }
    else{
        printf("(");
        Bracketing(N,a,m,n);
        printf(")(");
        Bracketing(N,m+1,b,n);
        printf(")");
    }
}
```

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}

Chain(int* A, int* M, int *N, int m, int n){
    int i,l;
    for(i=0;i<m;i++){
        printf("\n  }\n\nM[%d,%d]= min
of\n  {\n      M[%d,%d] + M[%d,%d] + (d%d*d%d*d%d) = ",i+1,n-
m+i+1,i+1,i+1,i+2,n-m+i+1,i+1,i+2,n-m+i+2);
        printf("%d + %d + (%d*d*d) = 
",*(M+i*n+i),*(M+(i+1)*n+n-m+i),*(A+i),*(A+i+1),*(A+n-m+i+1));
        *(M+n*i+n-m+i)=*(M+i*n+i)+*(M+(i+1)*n+n-
m+i)+*(A+i))**(A+i+1))**(A+n-m+i+1));
        *(N+n*i+n-m+i)=i;
        printf("%d",*(M+n*i+n-m+i));
        for(l=i+1;l<=n-m+i-1;l++){
            if(*(M+n*i+n-m+i)>*(M+i*n+l)+*(M+(l+1)*n+n-
m+i)+*(A+i))**(A+l+1))**(A+n-m+i+1)))){
                printf("\n      M[%d,%d] + M[%d,%d] +
(d%d*d*d*d) = ",i+1,l+1,l+2,n-m+i+1,i+1,l+2,n-m+i+2);
                printf("%d + %d + (%d*d*d) = 
",*(M+i*n+l),*(M+(l+1)*n+n-m+i),*(A+i),*(A+l+1),*(A+n-m+i+1));
                *(M+n*i+n-m+i)=*(M+i*n+l)+*(M+(l+1)*n+n-
m+i)+*(A+i))**(A+l+1))**(A+n-m+i+1));
                *(N+n*i+n-m+i)=l;
                printf("%d",*(M+n*i+n-m+i));
            }
            else{
                printf("\n      M[%d,%d] + M[%d,%d] +
(d%d*d*d*d) = ",i+1,l+1,l+2,n-m+i+1,i+1,l+2,n-m+i+2);
                printf("%d + %d + (%d*d*d) = 
%d",*(M+i*n+l),*(M+(l+1)*n+n-m+i),*(A+i),*(A+l+1),*(A+n-
m+i+1),(*(M+i*n+l)+*(M+(l+1)*n+n-
m+i)+*(A+i))**(A+l+1))**(A+n-m+i+1)));
            }
        }
    }
    if(m>=2)
        Chain(A,M,N,m-1,n);
}

main()
{
    int i,j,k,l,n;

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    printf("Please enter the total number of matrix dimension
sequence, you wanna multiply:");
    scanf("%d",&n);
    printf("\nNow enter all the order of matrices maintaining a
proper sequence:");
    int A[n],M[n-1][n-1],N[n-1][n-1];
    for(i=0;i<n;i++)
    {
        // scanf("%d",&A[i]);
        A[i] = (rand() % (46 - 15 + 1) + 15);
    }
    for(i=0;i<n-1;i++)
        for(j=0;j<n-1;j++){
            M[i][j]=0;
            N[i][j]=0;
        }

    Chain(A,M,N,n-2,n-1);

    printf("\n  }\n\nThe matrix having minimum number of
multiplication is:\n");
    for(i=0;i<n-1;i++){
        printf("\n\n");
        for(j=0;j<n-1;j++){
            if(i>j)
                printf("\t-");
            else
                printf("\t%d",M[i][j]);
        }
    }
    printf("\n\nSo the minimum number of multiplication
required is: %d.",M[0][n-2]);
    printf("\n\nThe parenthesization would be like:\t");
    Bracketing(N,0,n-2,n-1);
    printf("\n\n\n");
}

```

RESULT:

```
M[1,2]= min of
{
    M[1,1] + M[2,2] + (d1*d2*d3) = 0 + 0 + (24*18*45) = 19440
}

M[2,3]= min of
{
    M[2,2] + M[3,3] + (d2*d3*d4) = 0 + 0 + (18*45*19) = 15390
}

M[3,4]= min of
{
    M[3,3] + M[4,4] + (d3*d4*d5) = 0 + 0 + (45*19*16) = 13680
}

M[4,5]= min of
{
    M[4,4] + M[5,5] + (d4*d5*d6) = 0 + 0 + (19*16*27) = 8208
}

M[5,6]= min of
{
    M[5,5] + M[6,6] + (d5*d6*d7) = 0 + 0 + (16*27*37) = 15984
}

M[6,7]= min of
{
    M[6,6] + M[7,7] + (d6*d7*d8) = 0 + 0 + (27*37*29) = 28971
}

M[7,8]= min of
{
    M[7,7] + M[8,8] + (d7*d8*d9) = 0 + 0 + (37*29*33) = 35409
}

M[8,9]= min of
{
    M[8,8] + M[9,9] + (d8*d9*d10) = 0 + 0 + (29*33*31) = 29667
}

M[1,3]= min of
{
    M[1,1] + M[2,3] + (d1*d2*d4) = 0 + 15390 + (24*18*19) = 23598
    M[1,2] + M[3,3] + (d1*d3*d4) = 19440 + 0 + (24*45*19) = 39960
}

M[2,4]= min of
{
    M[2,2] + M[3,4] + (d2*d3*d5) = 0 + 13680 + (18*45*16) = 26640
    M[2,3] + M[4,4] + (d2*d4*d5) = 15390 + 0 + (18*19*16) = 20862
}

M[3,5]= min of
{
    M[3,3] + M[4,5] + (d3*d4*d6) = 0 + 8208 + (45*19*27) = 31293
    M[3,4] + M[5,5] + (d3*d5*d6) = 13680 + 0 + (45*16*27) = 33120
}
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$M[6,9] = \min \text{ of}$
 $\{$
 $M[6,6] + M[7,9] + (d6*d7*d10) = 0 + 62930 + (27*37*31) = 93899$
 $M[6,7] + M[8,9] + (d6*d8*d10) = 28971 + 29667 + (27*29*31) = 82911$
 $M[6,8] + M[9,9] + (d6*d9*d10) = 54810 + 0 + (27*33*31) = 82431$
 $\}$

$M[1,5] = \min \text{ of}$
 $\{$
 $M[1,1] + M[2,5] + (d1*d2*d6) = 0 + 28638 + (24*18*27) = 40302$
 $M[1,2] + M[3,5] + (d1*d3*d6) = 19440 + 31293 + (24*45*27) = 79893$
 $M[1,3] + M[4,5] + (d1*d4*d6) = 23598 + 8208 + (24*19*27) = 44118$
 $M[1,4] + M[5,5] + (d1*d5*d6) = 27774 + 0 + (24*16*27) = 38142$
 $\}$

$M[2,6] = \min \text{ of}$
 $\{$
 $M[2,2] + M[3,6] + (d2*d3*d7) = 0 + 56304 + (18*45*37) = 86274$
 $M[2,3] + M[4,6] + (d2*d4*d7) = 15390 + 27189 + (18*19*37) = 55233$
 $M[2,4] + M[5,6] + (d2*d5*d7) = 20862 + 15984 + (18*16*37) = 47502$
 $M[2,5] + M[6,6] + (d2*d6*d7) = 28638 + 0 + (18*27*37) = 46620$
 $\}$

$M[3,7] = \min \text{ of}$
 $\{$
 $M[3,3] + M[4,7] + (d3*d4*d8) = 0 + 41968 + (45*19*29) = 66763$
 $M[3,4] + M[5,7] + (d3*d5*d8) = 13680 + 33152 + (45*16*29) = 67712$
 $M[3,5] + M[6,7] + (d3*d6*d8) = 31293 + 28971 + (45*27*29) = 95499$
 $M[3,6] + M[7,7] + (d3*d7*d8) = 56304 + 0 + (45*37*29) = 104589$
 $\}$

$M[2,5] = \min \text{ of}$
 $\{$
 $M[2,2] + M[3,5] + (d2*d3*d6) = 0 + 31293 + (18*45*27) = 53163$
 $M[2,3] + M[4,5] + (d2*d4*d6) = 15390 + 8208 + (18*19*27) = 32832$
 $M[2,4] + M[5,5] + (d2*d5*d6) = 20862 + 0 + (18*16*27) = 28638$
 $\}$

$M[3,6] = \min \text{ of}$
 $\{$
 $M[3,3] + M[4,6] + (d3*d4*d7) = 0 + 27189 + (45*19*37) = 58824$
 $M[3,4] + M[5,6] + (d3*d5*d7) = 13680 + 15984 + (45*16*37) = 56304$
 $M[3,5] + M[6,6] + (d3*d6*d7) = 31293 + 0 + (45*27*37) = 76248$
 $\}$

$M[4,7] = \min \text{ of}$
 $\{$
 $M[4,4] + M[5,7] + (d4*d5*d8) = 0 + 33152 + (19*16*29) = 41968$
 $M[4,5] + M[6,7] + (d4*d6*d8) = 8208 + 28971 + (19*27*29) = 52056$
 $M[4,6] + M[7,7] + (d4*d7*d8) = 27189 + 0 + (19*37*29) = 47576$
 $\}$

$M[5,8] = \min \text{ of}$
 $\{$
 $M[5,5] + M[6,8] + (d5*d6*d9) = 0 + 54810 + (16*27*33) = 69066$
 $M[5,6] + M[7,8] + (d5*d7*d9) = 15984 + 35409 + (16*37*33) = 70929$
 $M[5,7] + M[8,8] + (d5*d8*d9) = 33152 + 0 + (16*29*33) = 48464$
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M[6,9]= min of
{
    M[6,6] + M[7,9] + (d6*d7*d10) = 0 + 62930 + (27*37*31) = 93899
    M[6,7] + M[8,9] + (d6*d8*d10) = 28971 + 29667 + (27*29*31) = 82911
    M[6,8] + M[9,9] + (d6*d9*d10) = 54810 + 0 + (27*33*31) = 82431
}

M[1,5]= min of
{
    M[1,1] + M[2,5] + (d1*d2*d6) = 0 + 28638 + (24*18*27) = 40302
    M[1,2] + M[3,5] + (d1*d3*d6) = 19440 + 31293 + (24*45*27) = 79893
    M[1,3] + M[4,5] + (d1*d4*d6) = 23598 + 8208 + (24*19*27) = 44118
    M[1,4] + M[5,5] + (d1*d5*d6) = 27774 + 0 + (24*16*27) = 38142
}

M[2,6]= min of
{
    M[2,2] + M[3,6] + (d2*d3*d7) = 0 + 56304 + (18*45*37) = 86274
    M[2,3] + M[4,6] + (d2*d4*d7) = 15390 + 27189 + (18*19*37) = 55233
    M[2,4] + M[5,6] + (d2*d5*d7) = 20862 + 15984 + (18*16*37) = 47502
    M[2,5] + M[6,6] + (d2*d6*d7) = 28638 + 0 + (18*27*37) = 46620
}

M[3,7]= min of
{
    M[3,3] + M[4,7] + (d3*d4*d8) = 0 + 41968 + (45*19*29) = 66763
    M[3,4] + M[5,7] + (d3*d5*d8) = 13680 + 33152 + (45*16*29) = 67712
    M[3,5] + M[6,7] + (d3*d6*d8) = 31293 + 28971 + (45*27*29) = 95499
    M[3,6] + M[7,7] + (d3*d7*d8) = 56304 + 0 + (45*37*29) = 104589
}

M[4,8]= min of
{
    M[4,4] + M[5,8] + (d4*d5*d9) = 0 + 48464 + (19*16*33) = 58496
    M[4,5] + M[6,8] + (d4*d6*d9) = 8208 + 54810 + (19*27*33) = 79947
    M[4,6] + M[7,8] + (d4*d7*d9) = 27189 + 35409 + (19*37*33) = 85797
    M[4,7] + M[8,8] + (d4*d8*d9) = 41968 + 0 + (19*29*33) = 60151
}

M[5,9]= min of
{
    M[5,5] + M[6,9] + (d5*d6*d10) = 0 + 82431 + (16*27*31) = 95823
    M[5,6] + M[7,9] + (d5*d7*d10) = 15984 + 62930 + (16*37*31) = 97266
    M[5,7] + M[8,9] + (d5*d8*d10) = 33152 + 29667 + (16*29*31) = 77203
    M[5,8] + M[9,9] + (d5*d9*d10) = 48464 + 0 + (16*33*31) = 64832
}

M[1,6]= min of
{
    M[1,1] + M[2,6] + (d1*d2*d7) = 0 + 46620 + (24*18*37) = 62604
    M[1,2] + M[3,6] + (d1*d3*d7) = 19440 + 56304 + (24*45*37) = 115704
    M[1,3] + M[4,6] + (d1*d4*d7) = 23598 + 27189 + (24*19*37) = 67659
    M[1,4] + M[5,6] + (d1*d5*d7) = 27774 + 15984 + (24*16*37) = 57966
    M[1,5] + M[6,6] + (d1*d6*d7) = 38142 + 0 + (24*27*37) = 62118
}

M[2,7]= min of
{
    M[2,2] + M[3,7] + (d2*d3*d8) = 0 + 66763 + (18*45*29) = 90253
    M[2,3] + M[4,7] + (d2*d4*d8) = 15390 + 41968 + (18*19*29) = 67276
    M[2,4] + M[5,7] + (d2*d5*d8) = 20862 + 33152 + (18*16*29) = 62366
    M[2,5] + M[6,7] + (d2*d6*d8) = 28638 + 28971 + (18*27*29) = 71703
    M[2,6] + M[7,7] + (d2*d7*d8) = 46620 + 0 + (18*37*29) = 65934
}

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M[3,8]= min of

$$\begin{aligned} & \{ \\ & \quad M[3,3] + M[4,8] + (d3*d4*d9) = 0 + 58496 + (45*19*33) = 86711 \\ & \quad M[3,4] + M[5,8] + (d3*d5*d9) = 13680 + 48464 + (45*16*33) = 85904 \\ & \quad M[3,5] + M[6,8] + (d3*d6*d9) = 31293 + 54810 + (45*27*33) = 126198 \\ & \quad M[3,6] + M[7,8] + (d3*d7*d9) = 56304 + 35409 + (45*37*33) = 146658 \\ & \quad M[3,7] + M[8,8] + (d3*d8*d9) = 66763 + 0 + (45*29*33) = 109828 \\ & \} \end{aligned}$$

M[4,9]= min of

$$\begin{aligned} & \{ \\ & \quad M[4,4] + M[5,9] + (d4*d5*d10) = 0 + 64832 + (19*16*31) = 74256 \\ & \quad M[4,5] + M[6,9] + (d4*d6*d10) = 8208 + 82431 + (19*27*31) = 106542 \\ & \quad M[4,6] + M[7,9] + (d4*d7*d10) = 27189 + 62930 + (19*37*31) = 111912 \\ & \quad M[4,7] + M[8,9] + (d4*d8*d10) = 41968 + 29667 + (19*29*31) = 88716 \\ & \quad M[4,8] + M[9,9] + (d4*d9*d10) = 58496 + 0 + (19*33*31) = 77933 \\ & \} \end{aligned}$$

M[1,7]= min of

$$\begin{aligned} & \{ \\ & \quad M[1,1] + M[2,7] + (d1*d2*d8) = 0 + 62366 + (24*18*29) = 74894 \\ & \quad M[1,2] + M[3,7] + (d1*d3*d8) = 19440 + 66763 + (24*45*29) = 117523 \\ & \quad M[1,3] + M[4,7] + (d1*d4*d8) = 23598 + 41968 + (24*19*29) = 78790 \\ & \quad M[1,4] + M[5,7] + (d1*d5*d8) = 27774 + 33152 + (24*16*29) = 72062 \\ & \quad M[1,5] + M[6,7] + (d1*d6*d8) = 38142 + 28971 + (24*27*29) = 85905 \\ & \quad M[1,6] + M[7,7] + (d1*d7*d8) = 57966 + 0 + (24*37*29) = 83718 \\ & \} \end{aligned}$$

M[2,8]= min of

$$\begin{aligned} & \{ \\ & \quad M[2,2] + M[3,8] + (d2*d3*d9) = 0 + 85904 + (18*45*33) = 112634 \\ & \quad M[2,3] + M[4,8] + (d2*d4*d9) = 15390 + 58496 + (18*19*33) = 85172 \\ & \quad M[2,4] + M[5,8] + (d2*d5*d9) = 20862 + 48464 + (18*16*33) = 78830 \\ & \quad M[2,5] + M[6,8] + (d2*d6*d9) = 28638 + 54810 + (18*27*33) = 99486 \\ & \quad M[2,6] + M[7,8] + (d2*d7*d9) = 46620 + 35409 + (18*37*33) = 104007 \\ & \quad M[2,7] + M[8,8] + (d2*d8*d9) = 62366 + 0 + (18*29*33) = 79592 \\ & \} \end{aligned}$$

M[3,9]= min of

$$\begin{aligned} & \{ \\ & \quad M[3,3] + M[4,9] + (d3*d4*d10) = 0 + 74256 + (45*19*31) = 100761 \\ & \quad M[3,4] + M[5,9] + (d3*d5*d10) = 13680 + 64832 + (45*16*31) = 100832 \\ & \quad M[3,5] + M[6,9] + (d3*d6*d10) = 31293 + 82431 + (45*27*31) = 151389 \\ & \quad M[3,6] + M[7,9] + (d3*d7*d10) = 56304 + 62930 + (45*37*31) = 170849 \\ & \quad M[3,7] + M[8,9] + (d3*d8*d10) = 66763 + 29667 + (45*29*31) = 136885 \\ & \quad M[3,8] + M[9,9] + (d3*d9*d10) = 85904 + 0 + (45*33*31) = 131939 \\ & \} \end{aligned}$$

M[1,8]= min of

$$\begin{aligned} & \{ \\ & \quad M[1,1] + M[2,8] + (d1*d2*d9) = 0 + 78830 + (24*18*33) = 93086 \\ & \quad M[1,2] + M[3,8] + (d1*d3*d9) = 19440 + 85904 + (24*45*33) = 140984 \\ & \quad M[1,3] + M[4,8] + (d1*d4*d9) = 23598 + 58496 + (24*19*33) = 97142 \\ & \quad M[1,4] + M[5,8] + (d1*d5*d9) = 27774 + 48464 + (24*16*33) = 88910 \\ & \quad M[1,5] + M[6,8] + (d1*d6*d9) = 38142 + 54810 + (24*27*33) = 114336 \\ & \quad M[1,6] + M[7,8] + (d1*d7*d9) = 57966 + 35409 + (24*37*33) = 122679 \\ & \quad M[1,7] + M[8,8] + (d1*d8*d9) = 72062 + 0 + (24*29*33) = 95030 \\ & \} \end{aligned}$$

$M[2,9] = \min$ of

{

$$\begin{aligned} M[2,2] + M[3,9] + (d_2*d_3*d_{10}) &= 0 + 100761 + (18*45*31) = 125871 \\ M[2,3] + M[4,9] + (d_2*d_4*d_{10}) &= 15390 + 74256 + (18*19*31) = 100248 \\ M[2,4] + M[5,9] + (d_2*d_5*d_{10}) &= 20862 + 64832 + (18*16*31) = 94622 \\ M[2,5] + M[6,9] + (d_2*d_6*d_{10}) &= 28638 + 82431 + (18*27*31) = 126135 \\ M[2,6] + M[7,9] + (d_2*d_7*d_{10}) &= 46620 + 62930 + (18*37*31) = 130196 \\ M[2,7] + M[8,9] + (d_2*d_8*d_{10}) &= 62366 + 29667 + (18*29*31) = 108215 \\ M[2,8] + M[9,9] + (d_2*d_9*d_{10}) &= 78830 + 0 + (18*33*31) = 97244 \end{aligned}$$

}

$M[1,9] = \min$ of

{

$$\begin{aligned} M[1,1] + M[2,9] + (d_1*d_2*d_{10}) &= 0 + 94622 + (24*18*31) = 108014 \\ M[1,2] + M[3,9] + (d_1*d_3*d_{10}) &= 19440 + 100761 + (24*45*31) = 153681 \\ M[1,3] + M[4,9] + (d_1*d_4*d_{10}) &= 23598 + 74256 + (24*19*31) = 111990 \\ M[1,4] + M[5,9] + (d_1*d_5*d_{10}) &= 27774 + 64832 + (24*16*31) = 104510 \\ M[1,5] + M[6,9] + (d_1*d_6*d_{10}) &= 38142 + 82431 + (24*27*31) = 140661 \\ M[1,6] + M[7,9] + (d_1*d_7*d_{10}) &= 57966 + 62930 + (24*37*31) = 148424 \\ M[1,7] + M[8,9] + (d_1*d_8*d_{10}) &= 72062 + 29667 + (24*29*31) = 123305 \\ M[1,8] + M[9,9] + (d_1*d_9*d_{10}) &= 88910 + 0 + (24*33*31) = 113462 \end{aligned}$$

}

The matrix having minimum number of multiplication is:

| | | | | | | | | |
|---|-------|-------|-------|-------|-------|-------|-------|--------|
| 0 | 19440 | 23598 | 27774 | 38142 | 57966 | 72062 | 88910 | 104510 |
| - | 0 | 15390 | 20862 | 28638 | 46620 | 62366 | 78830 | 94622 |
| - | - | 0 | 13680 | 31293 | 56304 | 66763 | 85904 | 100761 |
| - | - | - | 0 | 8208 | 27189 | 41968 | 58496 | 74256 |
| - | - | - | - | 0 | 15984 | 33152 | 48464 | 64832 |
| - | - | - | - | - | 0 | 28971 | 54810 | 82431 |
| - | - | - | - | - | - | 0 | 35409 | 62930 |
| - | - | - | - | - | - | - | 0 | 29667 |
| - | - | - | - | - | - | - | - | 0 |

So the minimum number of multiplication required is: 104510.

The parenthesization would be like: $(M1*((M2*M3)*M4))(((M5*M6)*M7)*M8)*M9$

| | |
|--------------------|--|
| CONCLUSION: | Implemented and Learned the Matrix Chain Multiplication Algorithm. |
|--------------------|--|