

Merge Sort :-

<https://leetcode.com/problems/sort-an-array/>

two merge Sorted array :-

$$\begin{bmatrix} 1, 3, 5, 7 \end{bmatrix} + \begin{bmatrix} 2, 4, 6, 8 \end{bmatrix} = \begin{bmatrix} 1, 2, 3, 4, 5, 6, 7, 8 \end{bmatrix}$$

A B C

A = [1, 3, 5, 7] C = [1, 2, 3, 4, 5, 6, 7, 8, 10, 12]

B = [2, 4, 6, 8]

i j k

$i = 1 \text{ OR } 2$ $j = 0 \text{ OR } 2$ $k = 0 \text{ OR } 2$

$\text{if } (a[i] < b[j]) \{$
 $c[k] = a[i];$
 $i++;$
 $\}$

$\text{else } \{$
 $c[k] = b[j];$
 $j++;$
 $\}$

a = [1, 3, 5, 7] $a[i] = 1 \text{ OR } 3 \text{ OR } 5$
b = [2, 4, 6, 8] $b[i] = 2 \text{ OR } 4 \text{ OR } 6$
 $k = 0 \text{ OR } 2 \text{ OR } 4$

C = [1, 2, 3, 4, 5, 6, 7, 8]

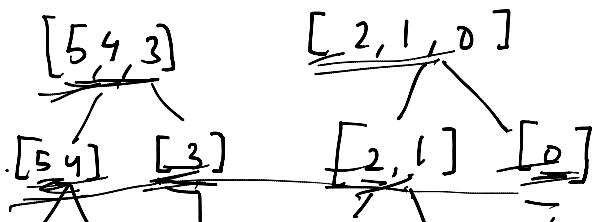
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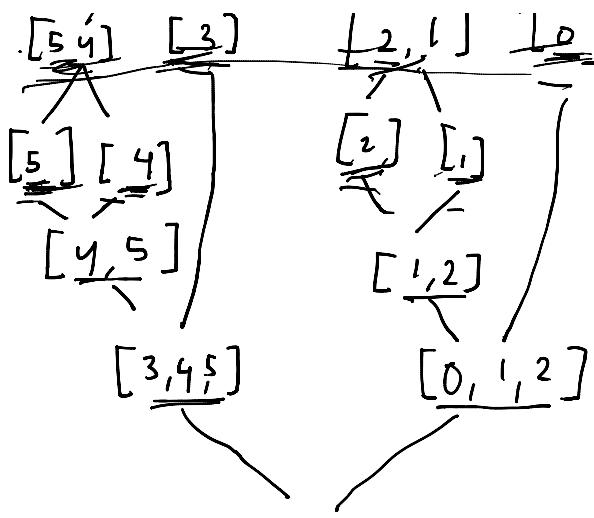
1 while (ai < a.length && bi < b.length) {
    if (a[ai] < b[bi]) {
        c[k] = a[ai];
        ai++;
        k++;
    } else {
        c[k] = b[bi];
        bi++;
        k++;
    }
}
2 while (ai < a.length) {
    c[k] = a[ai];
    ai++;
    k++;
}
3 while (bi < b.length) {
    c[k] = b[bi];
    bi++;
    k++;
}

return c;

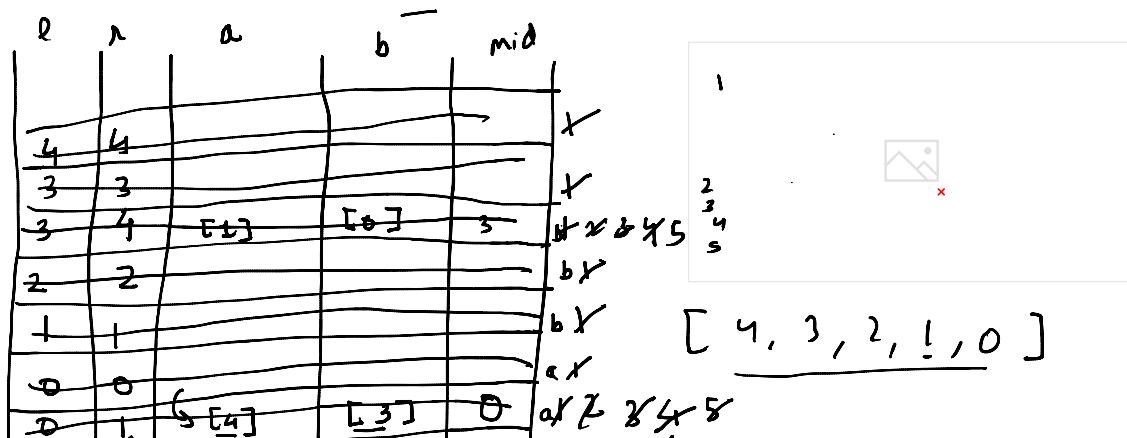
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[5, 4, 3, 2, 1, 0]





$\underline{[0, 1, 2, 3, 4, 5]}$
 l int[] mergeSort(int[], l, r){
 $5 = \underline{2}$ mid $y(l == r) \text{ return new int[] \{ arr[l]\};}$
 $4 = \underline{2}$ $mid = (l+r)/2;$
 $3 = \underline{2}$ int[] a = mergeSort(arr, l, mid);
 $2 = \underline{2}$ int[] b = mergeSort(arr, mid+1, r);
 $1 = \underline{2}$ $\}$ return mergeTwoSortedArry(a, b)



0	0	$\{ \underline{4} \}$	$\underline{\underline{3}} \}$	0	$\begin{matrix} ax \\ \underline{ax} \\ \underline{ax} \end{matrix}$	$\begin{matrix} 1 \\ 1 \\ 1 \end{matrix} = 1^0 \end{matrix}$
0	1	$\{ \underline{3,4} \}$	$\underline{\underline{2}} \}$	1	$\begin{matrix} ax \\ \underline{ax} \\ \underline{ax} \end{matrix}$	$\begin{matrix} 1 \\ 1 \\ 1 \end{matrix} = 1^1 \end{matrix}$
0	2	$\{ \underline{3,4,2} \}$	$\underline{\underline{1}} \}$	2	$\begin{matrix} ax \\ \underline{ax} \\ \underline{ax} \end{matrix}$	$\begin{matrix} 1 \\ 1 \\ 1 \end{matrix} = 1^2 \end{matrix}$
0	4	$\{ \underline{2,3,4} \}$	$\underline{\underline{0,1}} \}$	4	$\begin{matrix} ax \\ \underline{ax} \\ \underline{ax} \end{matrix}$	$\begin{matrix} 1 \\ 1 \\ 1 \end{matrix} = 1^4 \end{matrix}$

$[0, 1, 2, 3, 4]$

Quick Sort :-

recursion \ partition index

1, 5 4, 2, 3

Partition Index :-

$[7, 13, 12, \underline{16}, 5^4]$

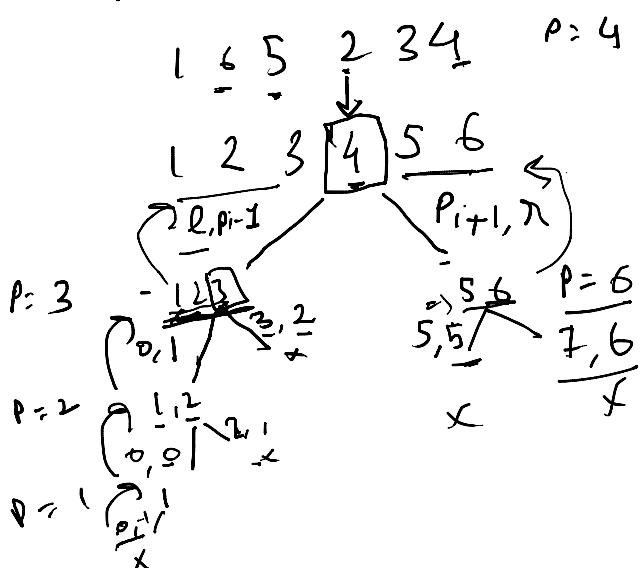
$p_{\text{pivot}} = 16$

$i = 5$

$j = 4$

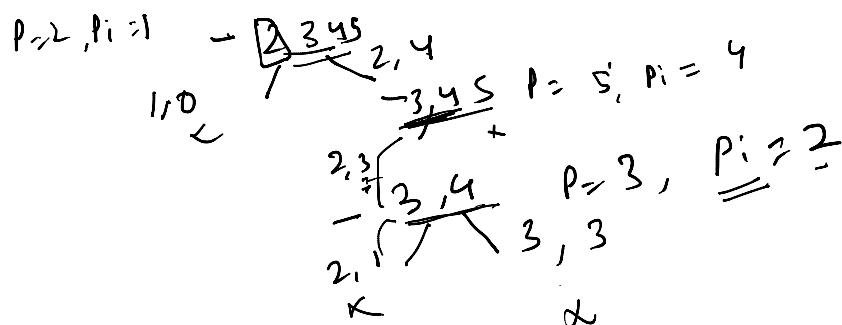
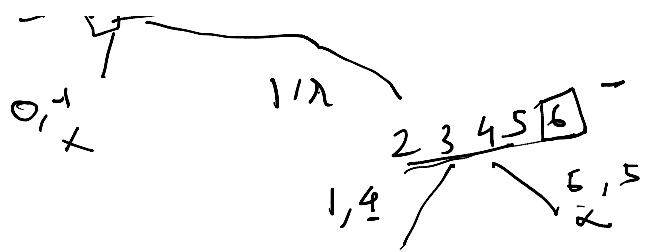
$\{$ if ($\text{arr}[i] \leq p_{\text{pivot}}$) {
swap(i, j); j++;
i++;
 $\}$ }
 $\{$ - else {
i++;
 $\}$

quickSort :- (Take the last element as pivot)



$qS(\text{arr}, l, r)$
if ($l \geq r$) returns





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void quickSort(arr, l, r) {
    if (l >= r) return;
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pivot = arr[r];

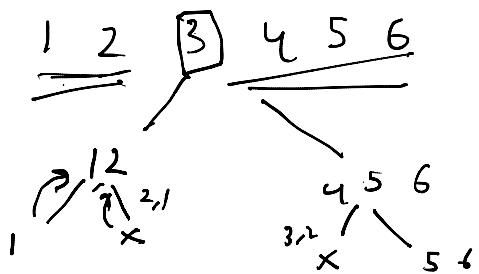
pi = partitionIndex(arr, pivot, l, r);

quickSort(arr, l, pi - 1);

quickSort(arr, pi + 1, r);

l	r	Pivot	p_i
6	5	5	✓
4	4	4	✓
4	5	6	✓
3	2	—	✓
3	5	4	X
2	1	—	✓
0	0	2	X
0	1	—	X
0	5	3	X
0	3	2	X

3



Time Complexity:

$$\underline{n} + \underline{n} = \underline{2n} = O(n)$$

$O(n)$:

$$n = s.nextInt(); \quad \frac{n}{\cancel{n}} \xrightarrow{\text{[10]}} n + n = \underline{2n} = O(n)$$

for (i to $n-1$) { $\sim n(n) = 2x/n$

$\text{for } \{ \text{from } 0 \text{ to } n-1 \} \{$ $\Rightarrow O(n) \quad 2 \times n$

$$\text{sum} = \frac{n(n+1)}{2} = \frac{n^2 + n}{2} = \frac{1}{2}n^2 + \frac{1}{2}n = O(n)$$

$\text{for } \{ \text{from } i=0; i < n; i=i+2 \} \{$ $i = 0, 2, 4, 6, 8$
 $\frac{n}{2} = \frac{n}{2}, n, \log n, \sqrt{n}$

$$O(n) = \frac{n}{4} = \frac{1}{4}n =$$

$$\frac{n}{n} = 1$$

$O(n^2)$

$\text{for } \{ \text{from } i=0 \text{ to } n-1 \} \{$
 $\quad \text{for } \{ \text{from } j=0 \text{ to } n-1 \} \{$

$$\begin{array}{l} n=5 \\ 0=01234 \\ 1=01234 \\ 2=01234 \\ 3=01234 \\ 4=01234 \end{array} \Rightarrow 2S = 5^2 = \underline{n^2}$$

$n \times n$
 $\text{for } \{ \text{from } i=0 \text{ to } n-1 \} \{$
 $\quad \text{for } \{ \text{from } j=i \text{ to } n-1 \} \{$
 $\quad \quad \quad O(n^2) = \underline{\frac{(n)(n+1)}{2}}$

$$\begin{array}{l} n=5 \\ 0=01234 \\ 1=1234 \\ 2=234 \\ 3=34 \\ 4=4 \end{array} \quad O(n^3)$$

$$n=5$$

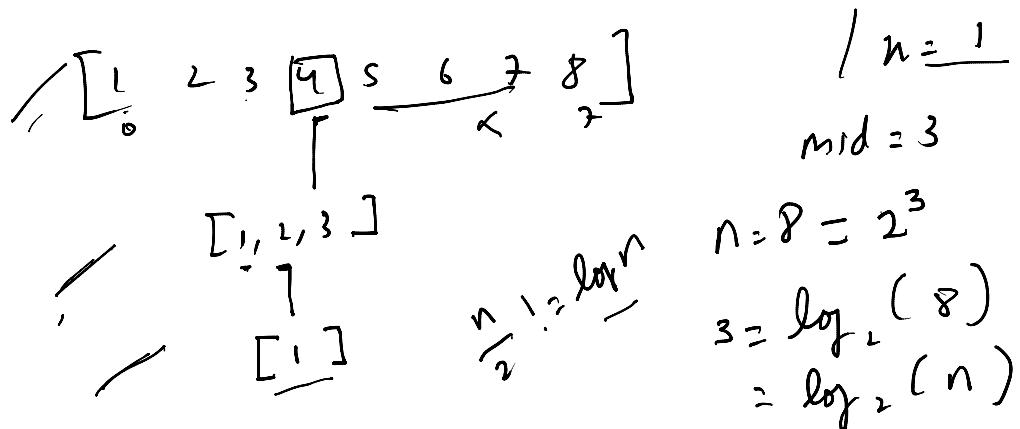
$$10n = O(n)$$

$\log(n)$

$\text{for } \{ \text{from } i=2; i < n; i=2 \times i \} \{$
 $\quad \quad \quad \underline{3 \ 2}$
 $\quad \quad \quad n, 2^{2^n}, \sqrt{n}, \log n$

$$\underline{2, 4, 8, 16, 32} = 5 \text{ times}$$

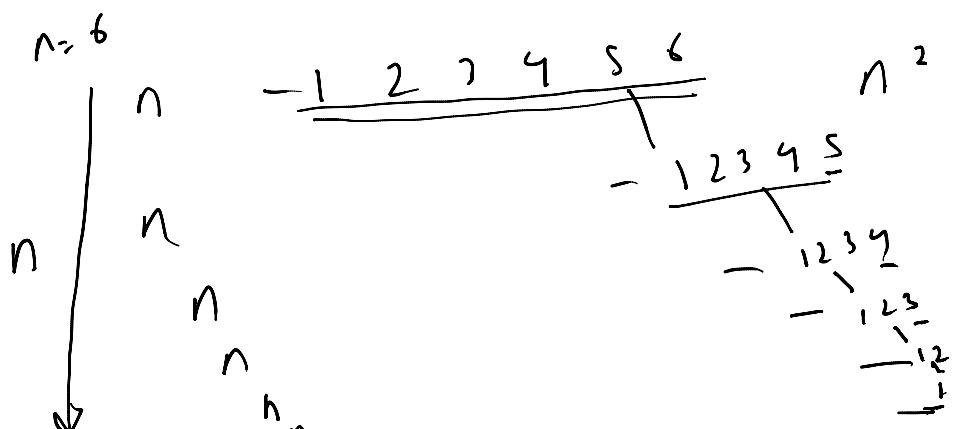
$$\begin{aligned} n=2 & \log_2(2) = 1 \\ \underline{\underline{32}} & = \underline{\underline{2}} = \\ \log_2(\underline{\underline{32}}) & = 5 \\ \underline{\underline{\log_2(\underline{\underline{2^k}})}} & = \underline{\underline{k}} \end{aligned}$$



$$\begin{aligned} n \log(n) &= [5 \ 4 \ 3 \ 2 \ 1 \ 0] \frac{n}{2} = \frac{n}{2} \\ &= \frac{5+4+3}{2} = \frac{12}{2} = 6 \\ &= \frac{5+4+3}{2} = \frac{12}{2} = 6 \\ &\xrightarrow{\log(n)} (n)(\log(n)) \quad \frac{n}{2} = \frac{2^k}{\log(n)} \end{aligned}$$

Merge Sort = always $(n)(\log(n))$

Quick Sort = $\Rightarrow n \log(n)$





n
 n_n

$\frac{1}{n+1}$

$$\text{Best Case} = \frac{n^2}{n^2}$$

$$\text{Worst Case} = \underline{n^2}$$
$$\text{Avg } " = \underline{n} \log \underline{(n)}$$

$O(1)$