MATHEMATICAL MODELLING AND SIMULATION

MSMA- 213

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PROJECT REPORT

World War Z: An Alternate Ending Using SEIR Model

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1. ABSTRACT

This report uses the SEIR model to explore what happens in a zombie outbreak when there is no vaccine, presenting an alternative perspective to the movie "World War Z". We use the SQM model, considering the affected region, population, and resources. Our main question is: "What happens if there is no vaccine?". The results show that having enough essential resources like food and medicine can reduce the number of infected individuals in such a scenario, offering an alternative survival strategy.

However, our model has limitations and doesn't precisely mimic a real zombie outbreak, with factors like a brief latent period impacting results. To address this, the present model can be extended to more complex models while preserving important aspects of the scenario. ed society can find a different path to survival in such dire circumstances.

2. INTRODUCTION

2.1. Background

In recent years, the application of mathematical models in epidemiology has found its way into the vivid realms of pop culture. One such scenario is the concept of a zombie outbreak, as popularized by the 2013 film "World War Z". In this cinematic world, a viral pathogen revives the dead, causing a global crisis.

This report applies the Susceptibles-Exposed-Infectives-Recovered (SEIR) model to the hypothetical scenario of a zombie apocalypse depicted in "World War Z". The SEIR model is an extended form of the Susceptibles-Infectives-Recovered (SIR) model. While the SIR model is traditionally used to study the spread of infectious diseases and population dynamics in the real world, its adaptation to the concept of zombie outbreak offers an interesting perspective.

In this report, our aim is to explore the dynamics of the said zombie outbreak in the absence of a vaccination strategy. While the film shows that the characters ultimately discovered a vaccine, our objective is to simulate an alternative scenario where no vaccine exists. By doing so, we will explore the dynamics of the disease's spread and attempt to create an alternative ending to the film.

2.2. SQM Modeling

A mathematical model is a triplet (S, Q, M) where S is a system, Q is a question relating to S, and M is a set of mathematical statements $M = \{\Sigma_1, \Sigma_2, \ldots, \Sigma_n\}$ which can be used to answer Q.

In this report, system S comprises the geographical location of the spread of the infection, the population at this location and the resources they have. The question Q to be answered is- ‘What happens if there is no vaccine?’. And, the mathematical statements M will be modeled using the SEIR model in section 3.

3. MODEL FORMULATION

3.1. Model assumptions

For modeling the zombie attack (without vaccination) the population can be divided into four classes: Susceptibles S(t), Exposed E(t), Infectives I(t) and Recovered R(t).

S(t): Population which is susceptible to the infection but is in a safe closed quarter.

E(t): Population which is physically exposed to the infectives, i.e., these individuals do not have a safe closed quarter to hide.

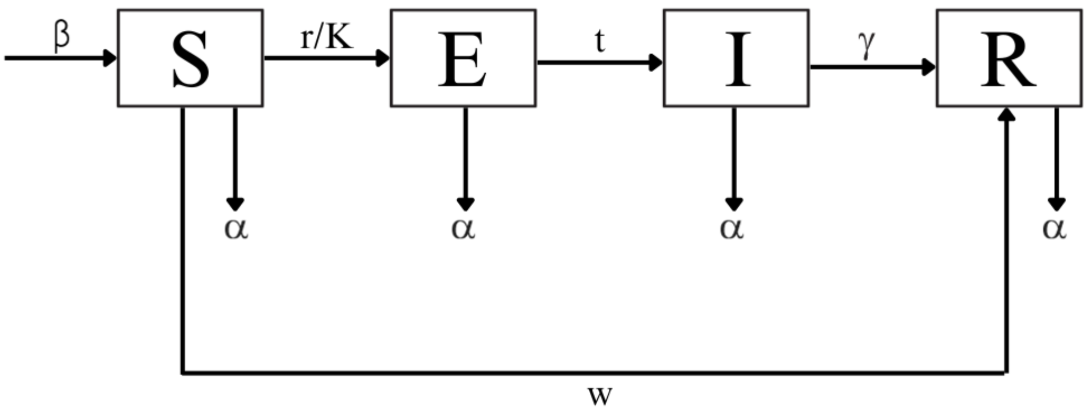
I(t): Population which is infected by the virus and now has turned into zombies.

R(t): Population which is immune (weak hosts) to the infection or zombie population which has recovered. These individuals cannot become susceptibles.

Following are the assumptions made to build the model:

1. Population of every compartment is so large that random differences between individuals can be neglected.
2. The infection is spread only through direct contact.
3. The latent period for the disease is 12 seconds which is much smaller than the time period considered for the model, thus we will take it as zero.
4. We assume that a weak host will never turn into a healthy host.
5. All infectives are symptomatic.
6. A zombie is said to have recovered if it loses its ability to spread the infection, i.e., it becomes immobile or loses all its teeth.
7. At any point within the time period, no individual makes any noise.
8. A susceptible individual only becomes exposed if they run out of essential resources.
9. If not due to infection, an individual may die of any reason (say, murdered, accident, any other disease etc.) which will be termed as ‘natural death’.
10. At any time in the time period, the population is homogeneously mixed, i.e., the population of every compartment is always randomly distributed over the geographical area in which the disease is spread.

3.2. Compartmental Model



N(t): total population at time t (units: persons)
S(t): population of susceptible at time t (units: persons)
E(t): population of exposed at time t (units: persons)
I(t): population of infectives at time t (units: persons)
R(t): population of recovered at time t (units: persons)
β: birth rate (units: day^{-1})
α : natural death rate (units: day^{-1})
w : weakness constant : fraction of population which is weak (units: day^{-1})
r = β-α : growth rate (units: day^{-1})
K: carrying capacity of essential resources (units: persons)
t: transmission rate of infection (units: $persons^{-1}$ day^{-1})
γ: recovery rate of infectives (units: day^{-1})

3.3. Balance Laws

$$\left\{\begin{array}{c}\text{Rate of change}\\ \text{of population of}\\ \text{susceptible}\end{array}\right\}=\left\{\begin{array}{c}\text{Rate}\\ \text{of birth}\end{array}\right\}-\left\{\begin{array}{c}\text{Rate at which}\\ \text{essential resources}\\ \text{get exhausted}\end{array}\right\}-\left\{\begin{array}{c}\text{Proportion of}\\ \text{weak hosts}\end{array}\right\}-\left\{\begin{array}{c}\text{Rate of}\\ \text{natural death}\end{array}\right\}$$

$$\left\{\begin{array}{c}\text{Rate of change}\\ \text{of population of}\\ \text{exposed}\end{array}\right\}=\left\{\begin{array}{c}\text{Rate at which essential}\\ \text{resources get exhausted}\\ \text{for susceptibles}\end{array}\right\}-\left\{\begin{array}{c}\text{Rate of}\\ \text{Transmission}\\ \text{of infection}\end{array}\right\}-\left\{\begin{array}{c}\text{Rate of}\\ \text{natural death}\end{array}\right\}$$

$$\left\{\begin{array}{c}\text{Rate of change}\\ \text{of population of}\\ \text{infectives}\end{array}\right\}=\left\{\begin{array}{c}\text{Rate of transmission}\\ \text{of infection to}\\ \text{exposed}\end{array}\right\}-\left\{\begin{array}{c}\text{Rate of}\\ \text{recovery}\end{array}\right\}-\left\{\begin{array}{c}\text{Rate of}\\ \text{natural death}\end{array}\right\}$$

$$\left\{\begin{array}{c}\text{Rate of change}\\ \text{of population of}\\ \text{recovered}\end{array}\right\}=\left\{\begin{array}{c}\text{Rate of}\\ \text{recovery of}\\ \text{infectives}\end{array}\right\}+\left\{\begin{array}{c}\text{Proportion of}\\ \text{weak hosts}\end{array}\right\}-\left\{\begin{array}{c}\text{Rate of}\\ \text{natural death}\end{array}\right\}$$

3.4. Model using Differential Equations

$$\frac{dS}{dt} = \beta N(t) - (\alpha + w)S(t) - \frac{r}{K}S^2(t)$$

$$\frac{dE}{dt} = \frac{r}{K}S^2(t) - tE(t)I(t) - \alpha E(t)$$

$$\frac{dI}{dt} = tE(t)I(t) - rI(t) - \alpha I(t)$$

$$\frac{dR}{dt} = rI(t) + wS(t) - \alpha R(t)$$

3.5. SQM Model

Thus, the mathematical model (S, Q, M) is such that,

S: The geographical location of the spread of the infection, the population at this location and the resources they have.

Q: What happens if there is no vaccine?

M: $\{ \frac{dS}{dt} = \beta N(t) - (\alpha + w)S(t) - \frac{r}{K}S^2(t),$

$\frac{dE}{dt} = \frac{r}{K}S^2(t) - tE(t)I(t) - \alpha E(t),$

$\frac{dI}{dt} = tE(t)I(t) - rI(t) - \alpha I(t),$

$\frac{dR}{dt} = rI(t) + wS(t) - \alpha R(t) \}$

Where, the variables and their units are as defined in section 3.2.

4. NUMERICAL SIMULATION

4.1. No varying parameter

We will simulate a hypothetical population with the following parameters:

N = 500
S(0) = 199
E(0) = 300
I(0) = 100
β = 0.5 day^{-1}
α = 0.2 day^{-1}
w = 0.3 day^{-1}
K = 10 persons
t = 10 $persons^{-1}$ day^{-1}
γ = 0.05 day^{-1}

zombie_attack.m

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N = 500; beta = 0.5; alpha = 0.2;
w = 0.3; K = 10; tr = 10; gamma = 0.05;

S0=199;E0=300;I0=100;R0=0;

tspan = 0:0.1:30;

dydt = @(t,y) [(beta*N - (alpha+w)*y(1) - ((beta-alpha)/K)*(y(1)*y(1)));
    (((beta-alpha)/K)*(y(1)*y(1)) - y(2)*(tr*y(3)+alpha));
    (tr*y(2)*y(3) - (gamma+alpha)*y(3));
    (gamma*y(3) + w*y(1) - alpha*y(4))];

ode45(dydt,tspan,[S0 E0 I0 R0]);

legend('S(t)','E(t)','I(t)','R(t)')
title('Zombie Outbreak Model')
xlabel('Time in days')
ylabel('Population')
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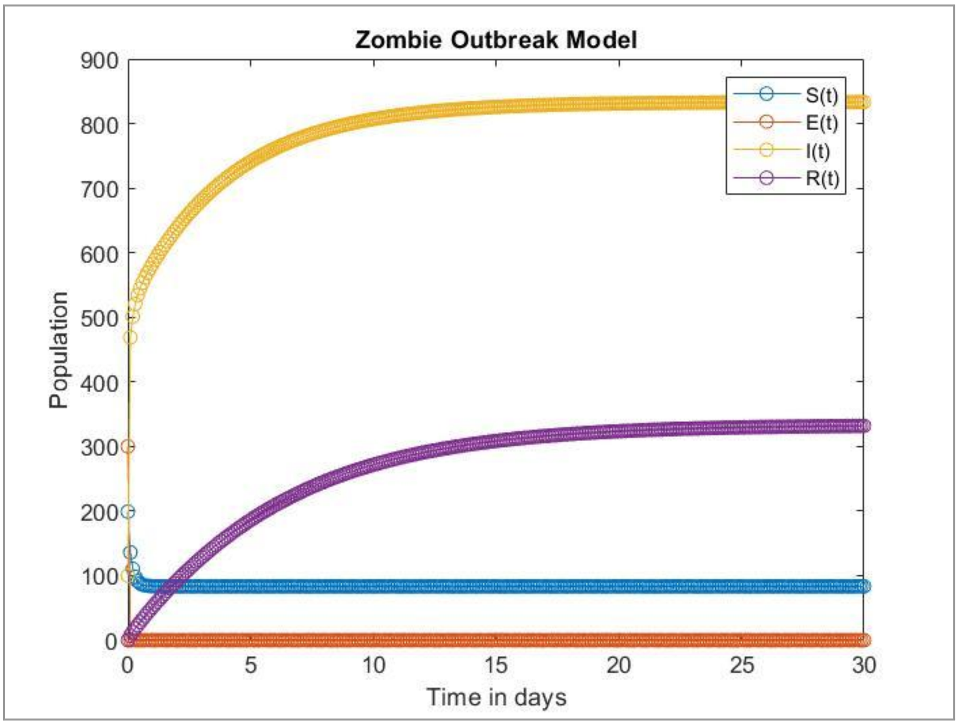
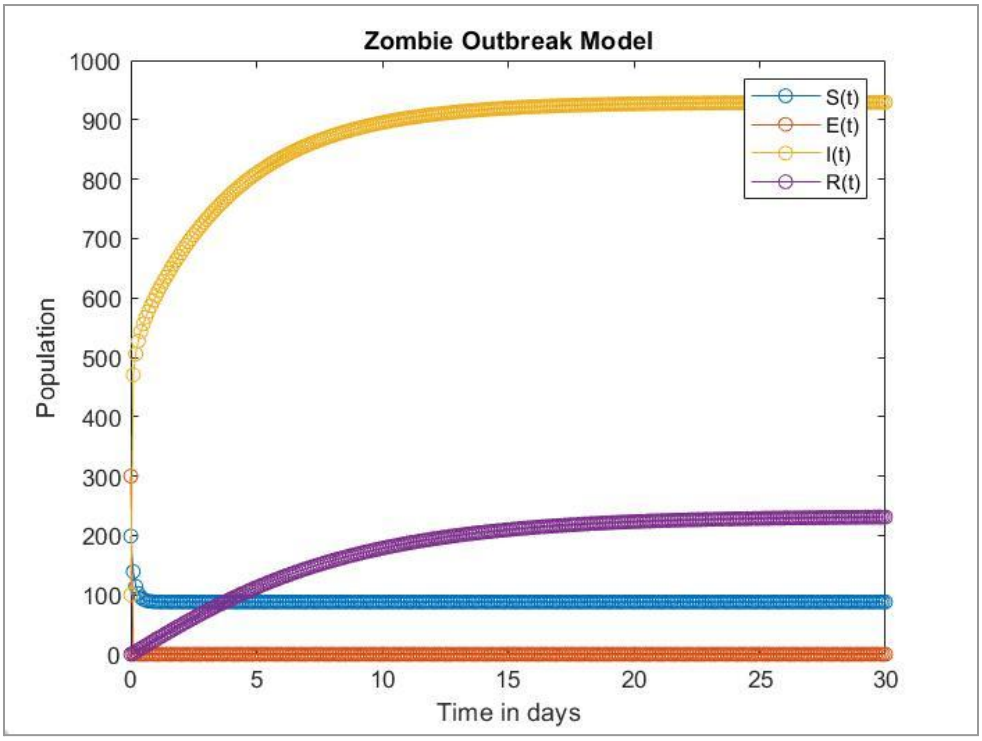
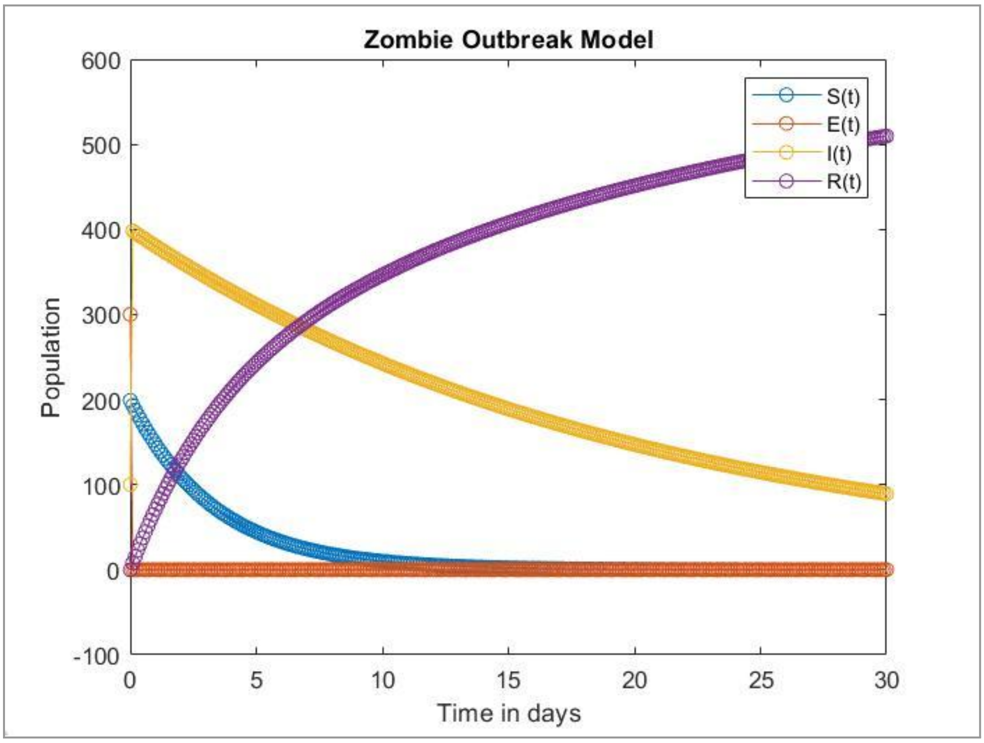


4.2. Single varying parameter

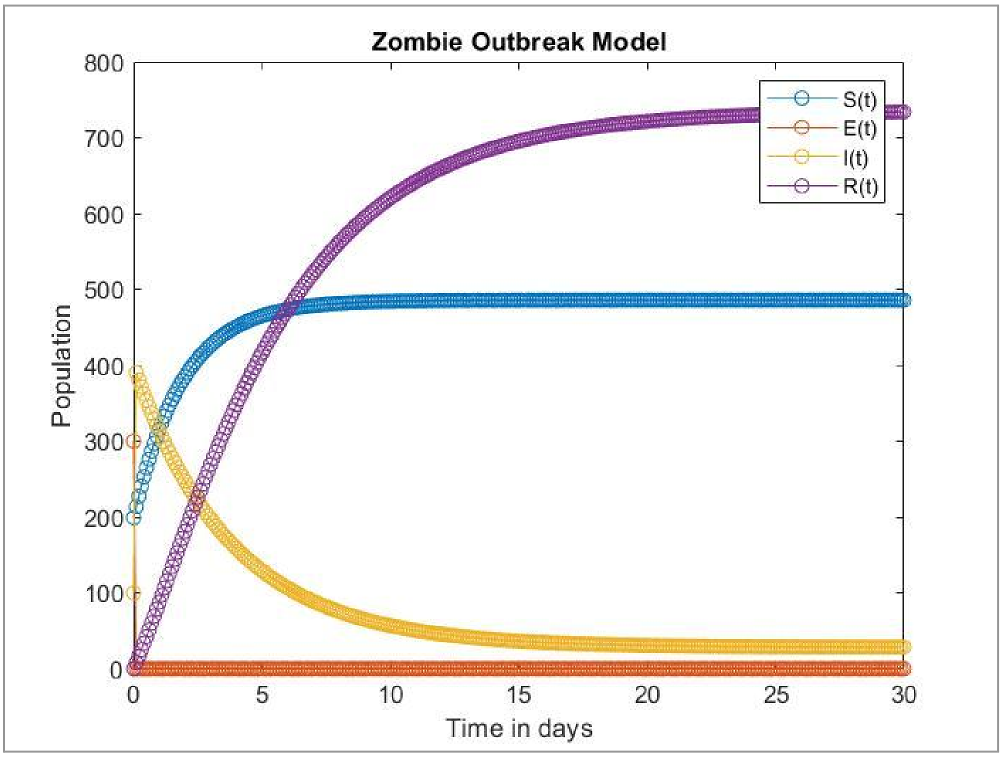
Case 1: No Weak Host, i.e., w = 0 day^{-1}



Case 2: No births and no deaths, i.e., β = 0 = α

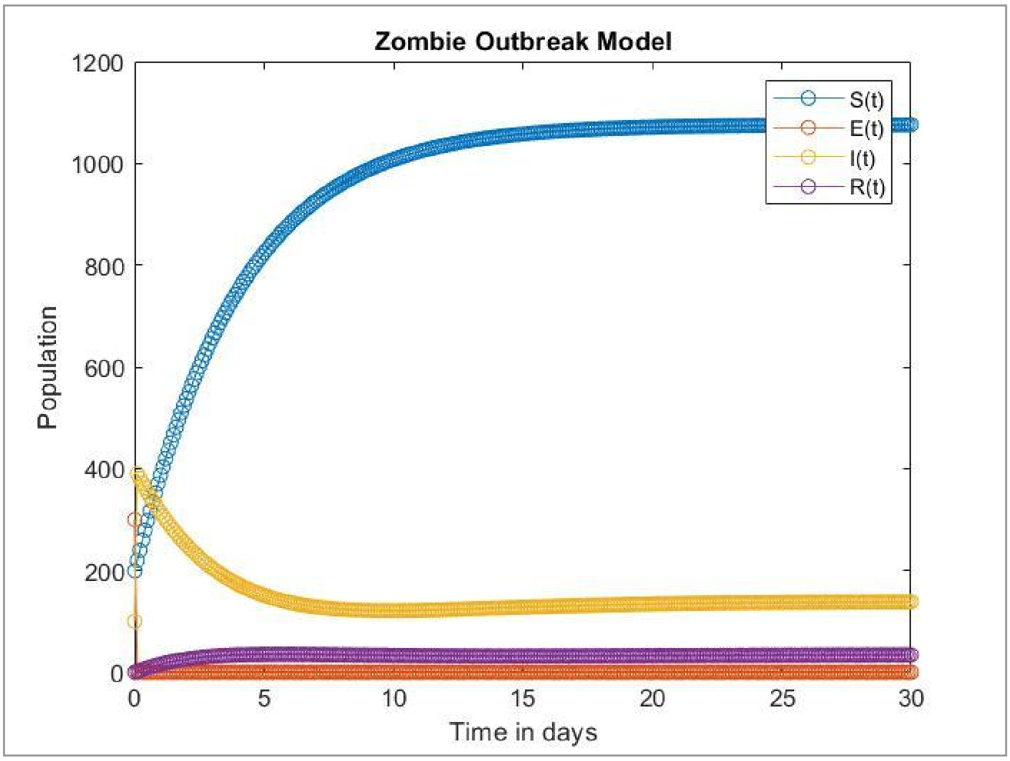


Case 3: Unlimited resources, i.e., K → ∞

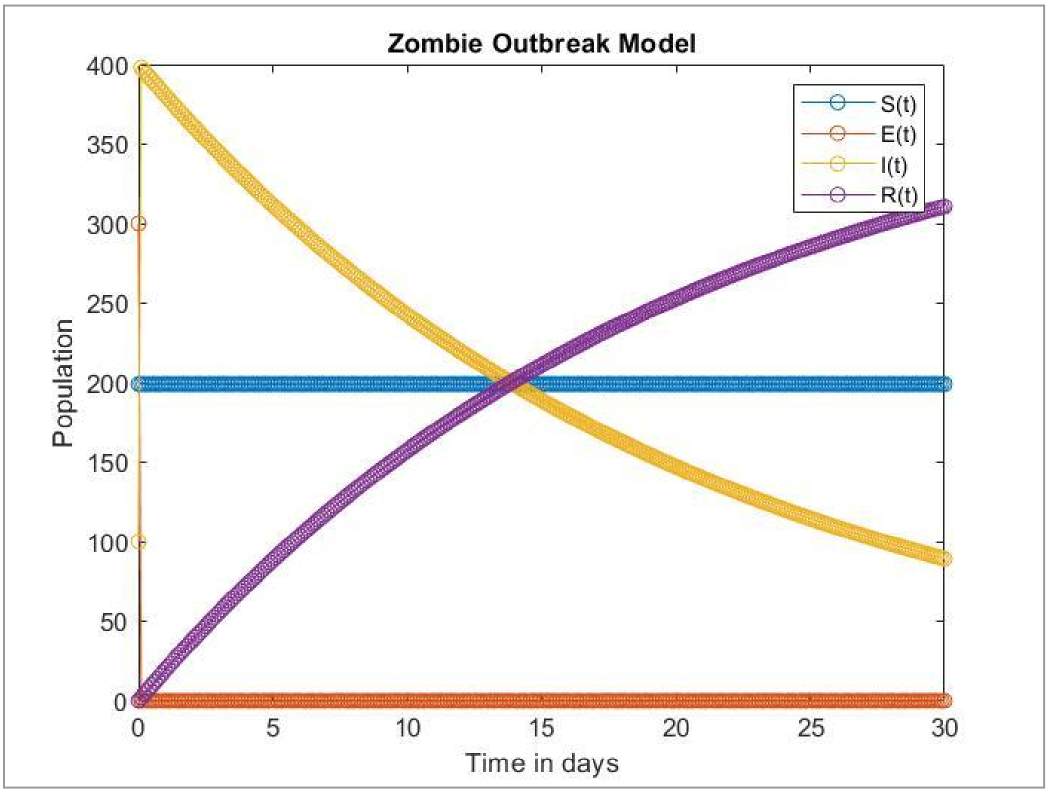


4.3. Multiple varying parameters

Case 1: No weak host and unlimited resources



Case 2: No birth or death, unlimited resources and no weak host



5. DISCUSSIONS

From the few cases simulated above, it can be observed that the population of infectives decreases in the presence of unlimited resources or in the case of no birth and deaths. This gives way to an alternate ending of the movie 'World War Z' where the only way to survive without a vaccine is to ensure unlimited supply of essential resources such as food and medicines.

Regardless of what the simulation concludes, our model is based on 10 assumptions most of which do not mimic the actual zombie outbreak to the fullest. For instance, taking the latent period (12 seconds) into consideration might give slightly different results. This is our model's limitation. We can overcome this by increasing the complexity of our model while ensuring that no important information is overshadowed.

6. REFERENCES

1. Mathematical Modeling and Simulation: Introduction for Scientists and Engineers, Kai Velten.
2. Mathematical Modelling with Case Studies, Belinda Barnes, Glenn R. Fulford.
3. World War Z, 2013 (movie)