① SHEAR FORCE AND BENDING MOMENT

20/09/30

4, 5, 18, 20, 23, 24, 2
39, 40, 43, 44, 48, 47, 5
55, 56, 66, 71, 77, 78
89,

14/10/30

9. 22, 36, 43, 54,
77, 81, 84,

① Type of load
- Point Load
- Uniformly distributed load
- Uniformly Varying Load
- Couple

② Type of Support
- Roller
- Hinged
- Fixed

③ Type of Beams
- Simply Supported
- Cantilever Beam
- Fixed Beam
- Continous Beam.

Determinate and Indeterminate Beams.

Determinate :> it can be analyzed using static equations only (i.e. equilibrium in all direction), $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$. e.g. S.S. Beam, Cantilever, Single/ or double overhanging beam, 3-hinged arches.

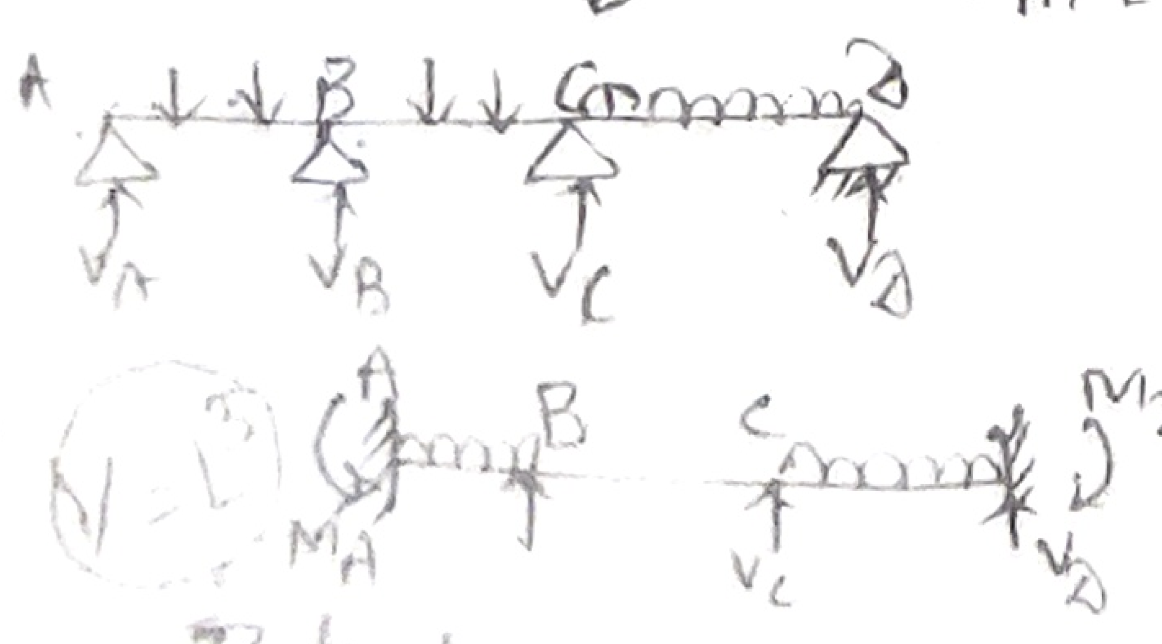
Indeterminate: it cannot be analyzed using static equations only, they require other material properties e.g. deformation. e.g. Fixed beam, continuous beam, Fixed arches, 2-hinged arches, multist frame

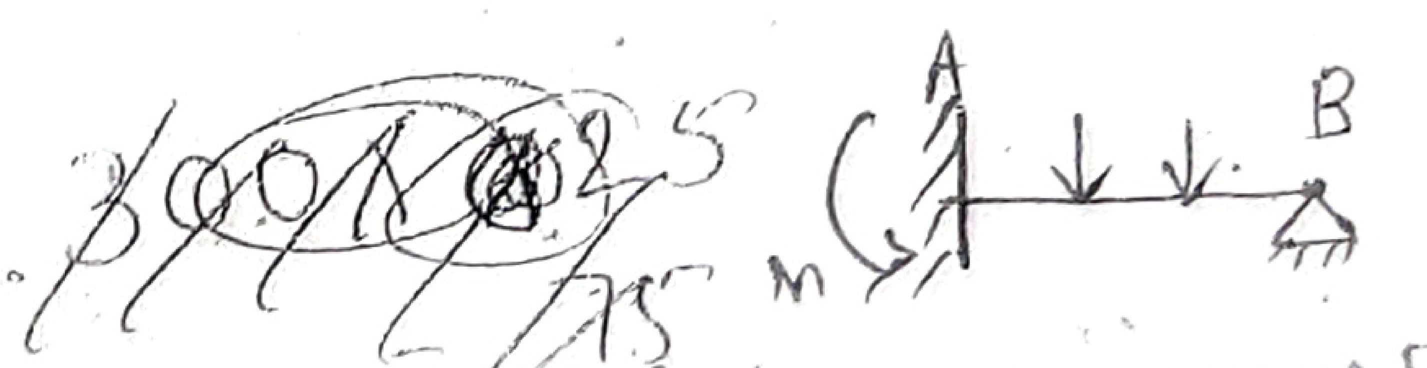
Shear Force :→ Shear force is define as a algebric sum of all the forces either to the left or to the right hand side of the section.

Bending Moment :→ Bending moment is define as the algebric sum of the moment of all the forces either to the left or to the right of a section.

① 37, 50, 54, 67, 81,

m = 4 - 2 = 2



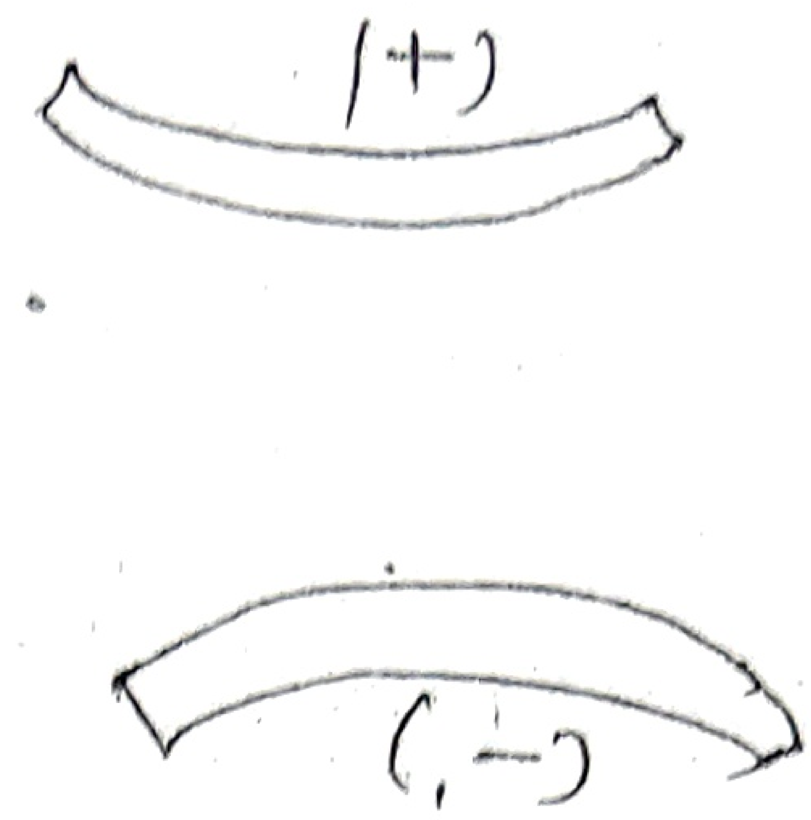


Redundancy: 3 unknown R
2 Static eq
= 3 - 2 1

Redundancy =

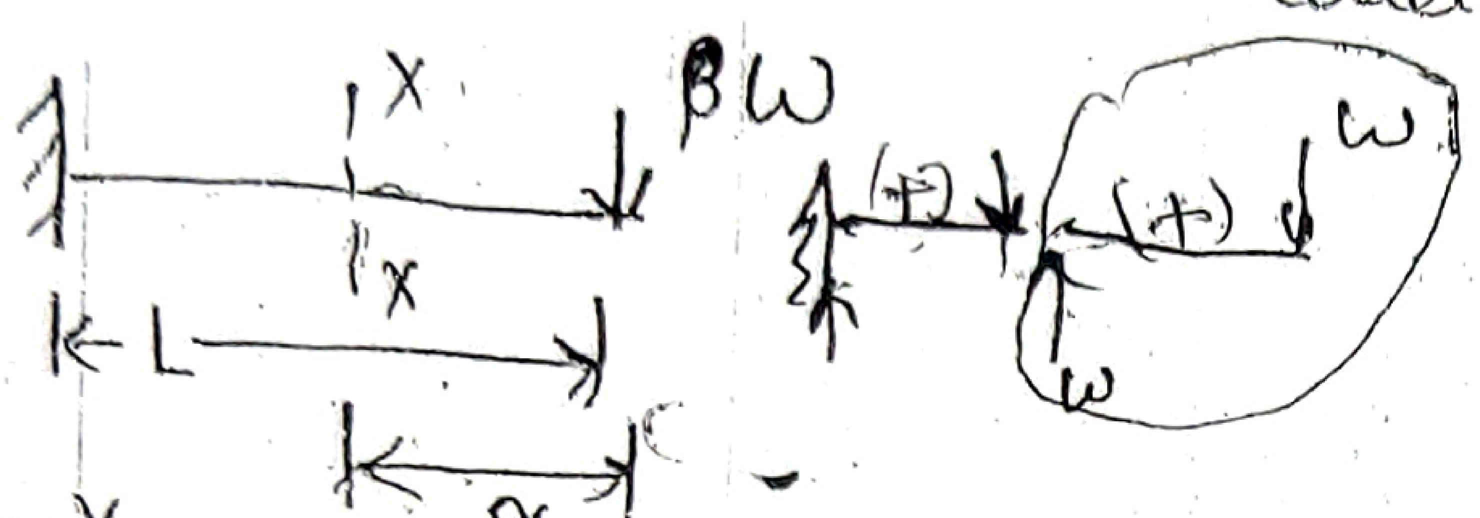
Sign convention :→

Shear force



Shear force and bending moment diagram :→

Cantilever Beam : Point load at one end



Section X – X

$F_x = +W$

$x = 0, \quad F_x = W$

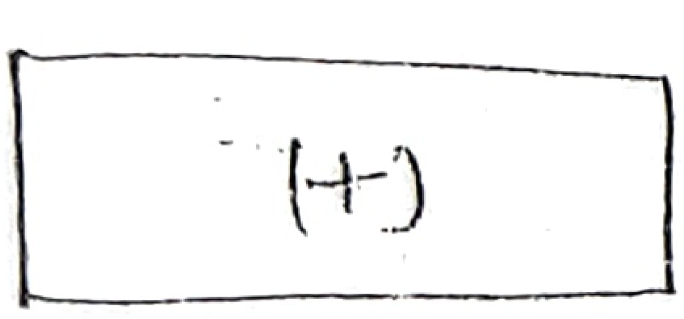
$x = L \quad F_x = W$

B.M

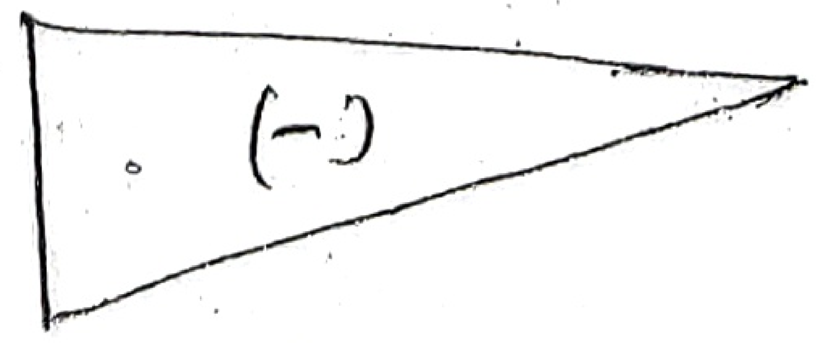
$M_x = -Wx$ (-tive, hogging)

$M_{x=0} = 0$

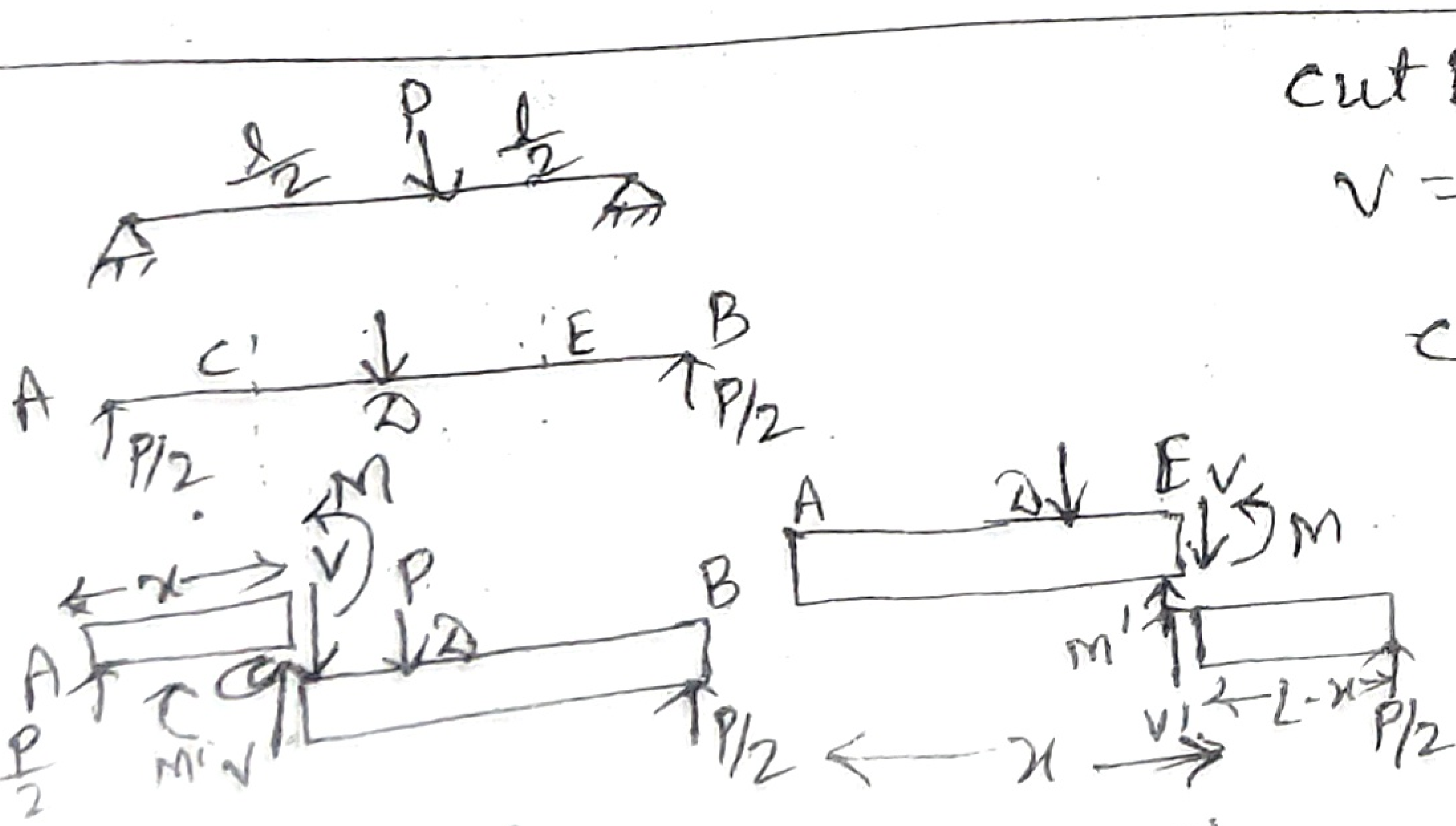
$M_{x=L} = -WL$ (variation is linear)



S.F.D.



B.M.D

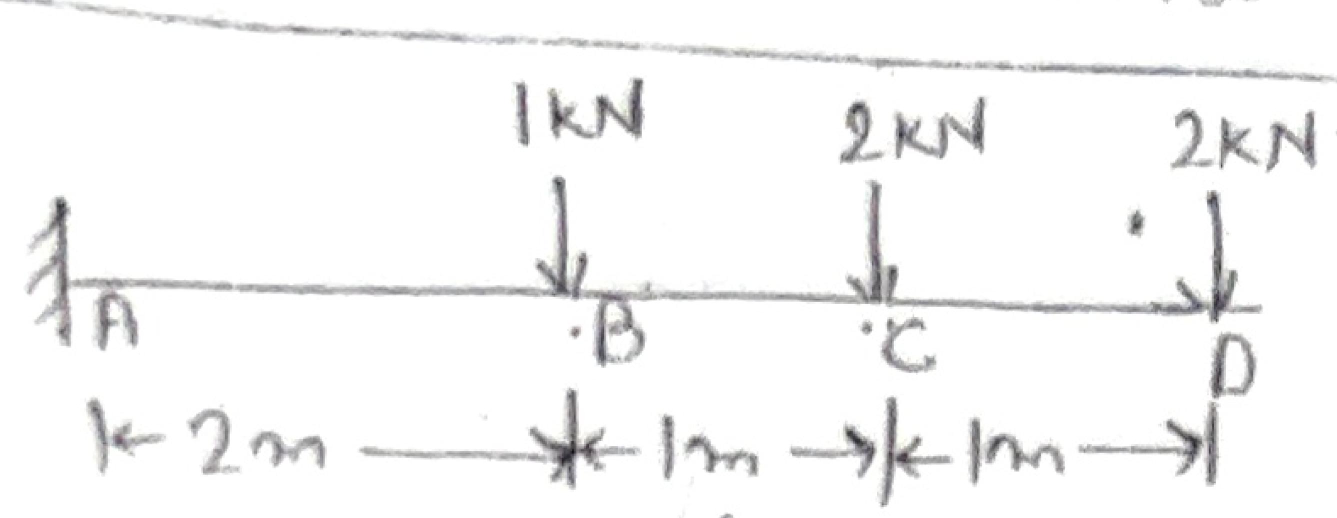


Cut beam at C and consider member AC

$V = +\frac{P}{2}$, $M = +\frac{Px}{2}$

Cut beam at E and consider member EB , $V = -\frac{P}{2}$, $M = +\frac{P(L-x)}{2}$

(2) Cantilever : Several Point Load

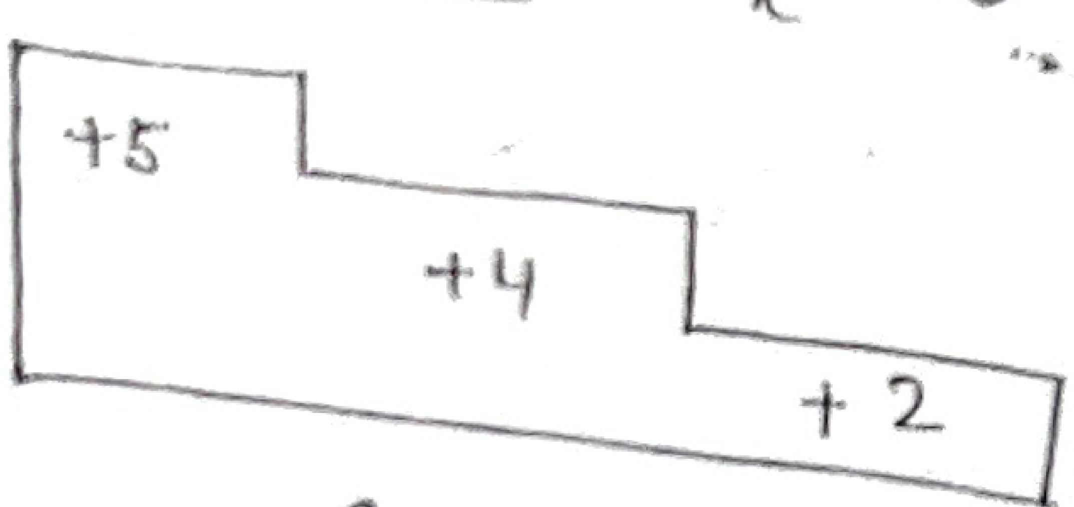


S.F. between CD

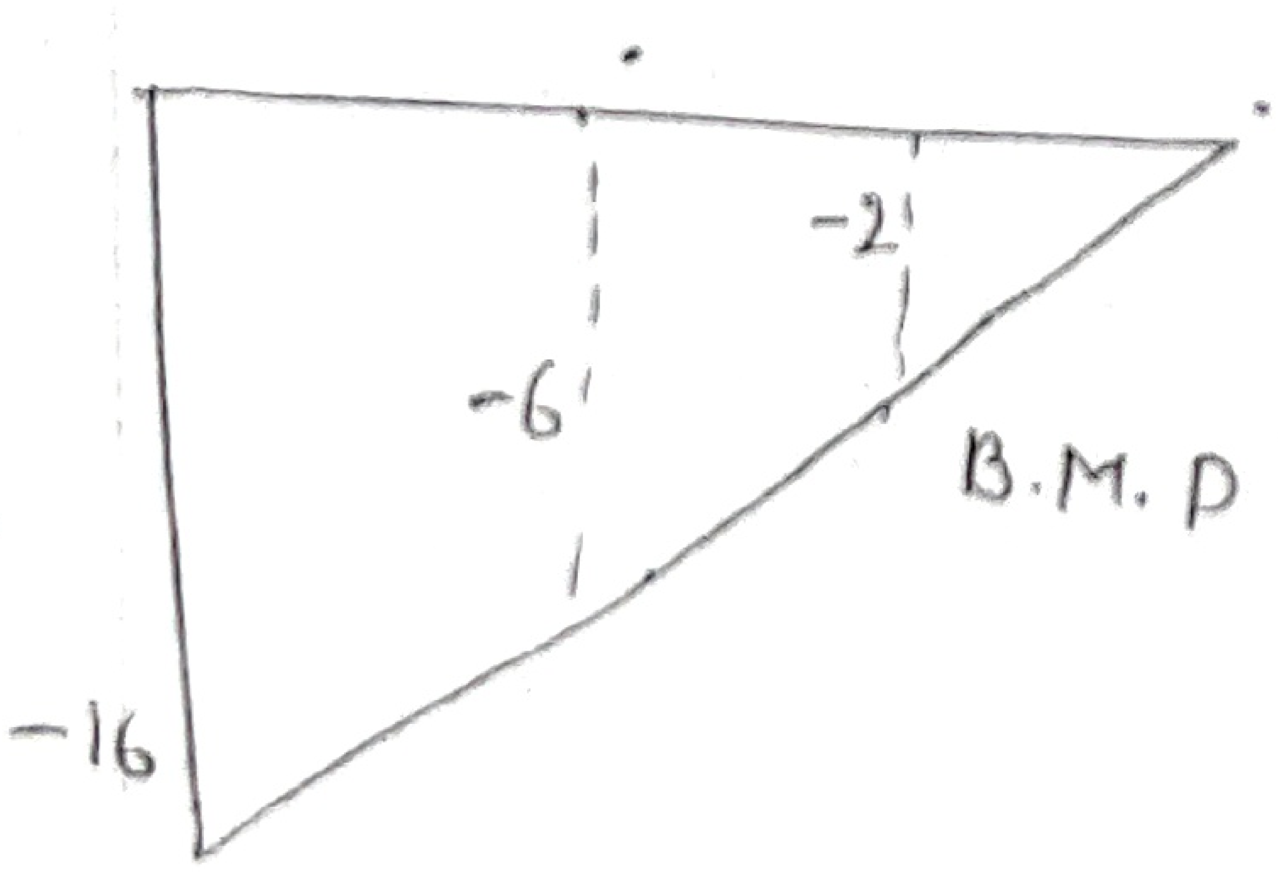
$F_x = +2$

Between BC $F_x = +4$

Between AB $F_x = +5$



S.F.D.



B.M. between C and D

$M_x = -2x$

at $x=0$, $M = 0$

at $x=1$, $M = -2$

B.M. b/w B and C

~~$M_x = 2(1+x)+2($~~

$M_x = -2(x) + 2(x-1)$

$= -4x+2$

at $x=1$, $M = -2$ kNm

at $x=2$, $M = 6$ kNm

B.M. b/w A-B

$M_x = -2x - 2(x-1) - 1(x-2)$

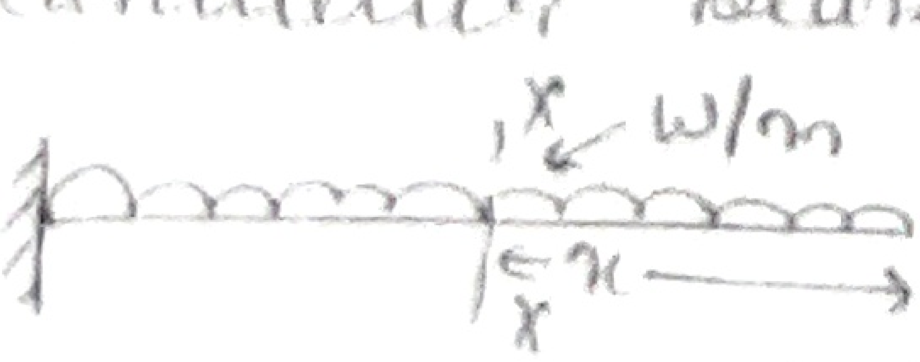
$M_x = -2x - 2x + 2 - x + 2$

$= -5x + 4$

at $x=2$, $M = -6$ kNm

at $x=4$, $M = -16$ kNm

(3) Cantilever Beam with U.D.L



S.F. at Section X-X

$F_x = +wx$

$F_{x=L} = +wL$

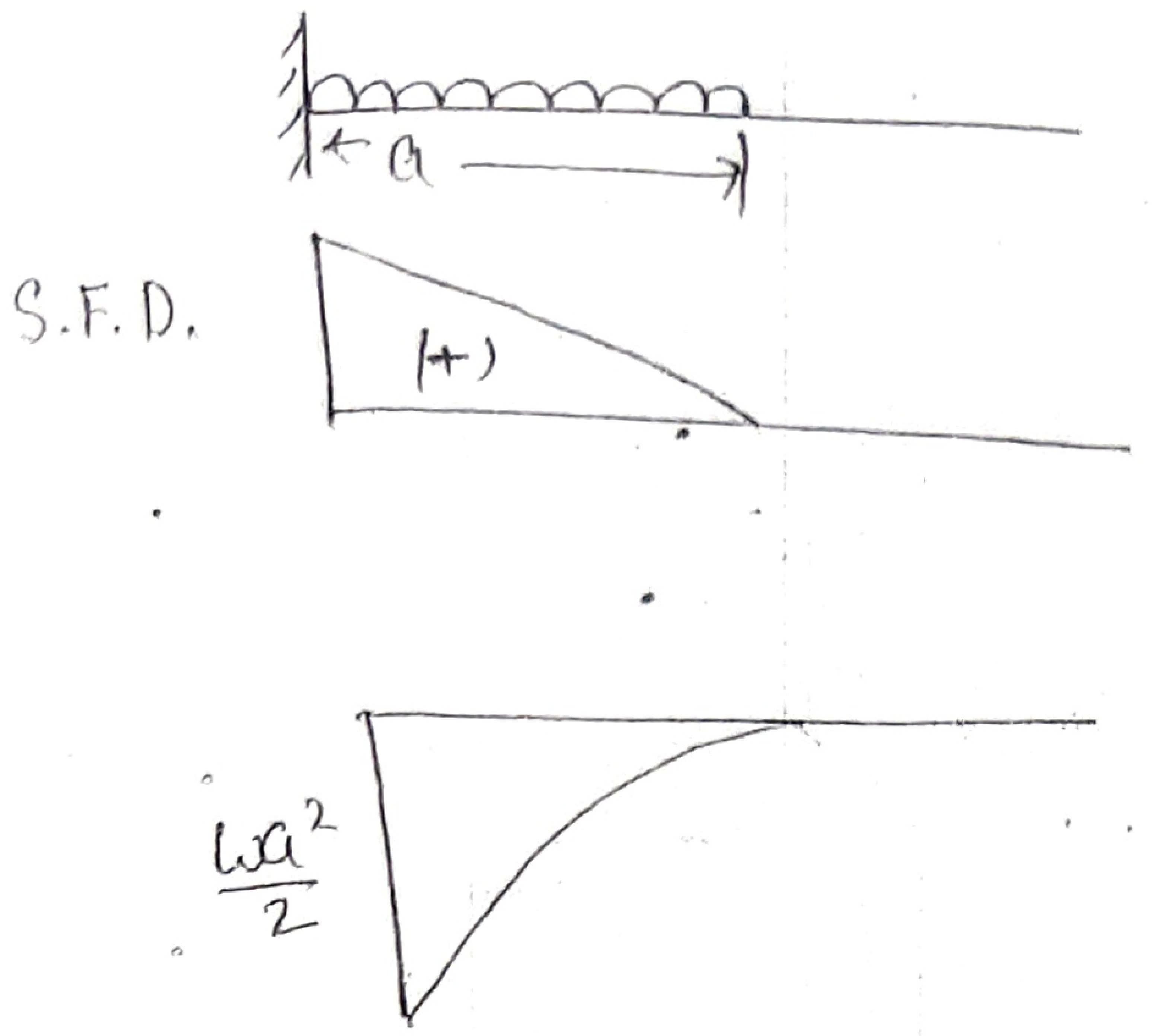
B.M. at Section X-X

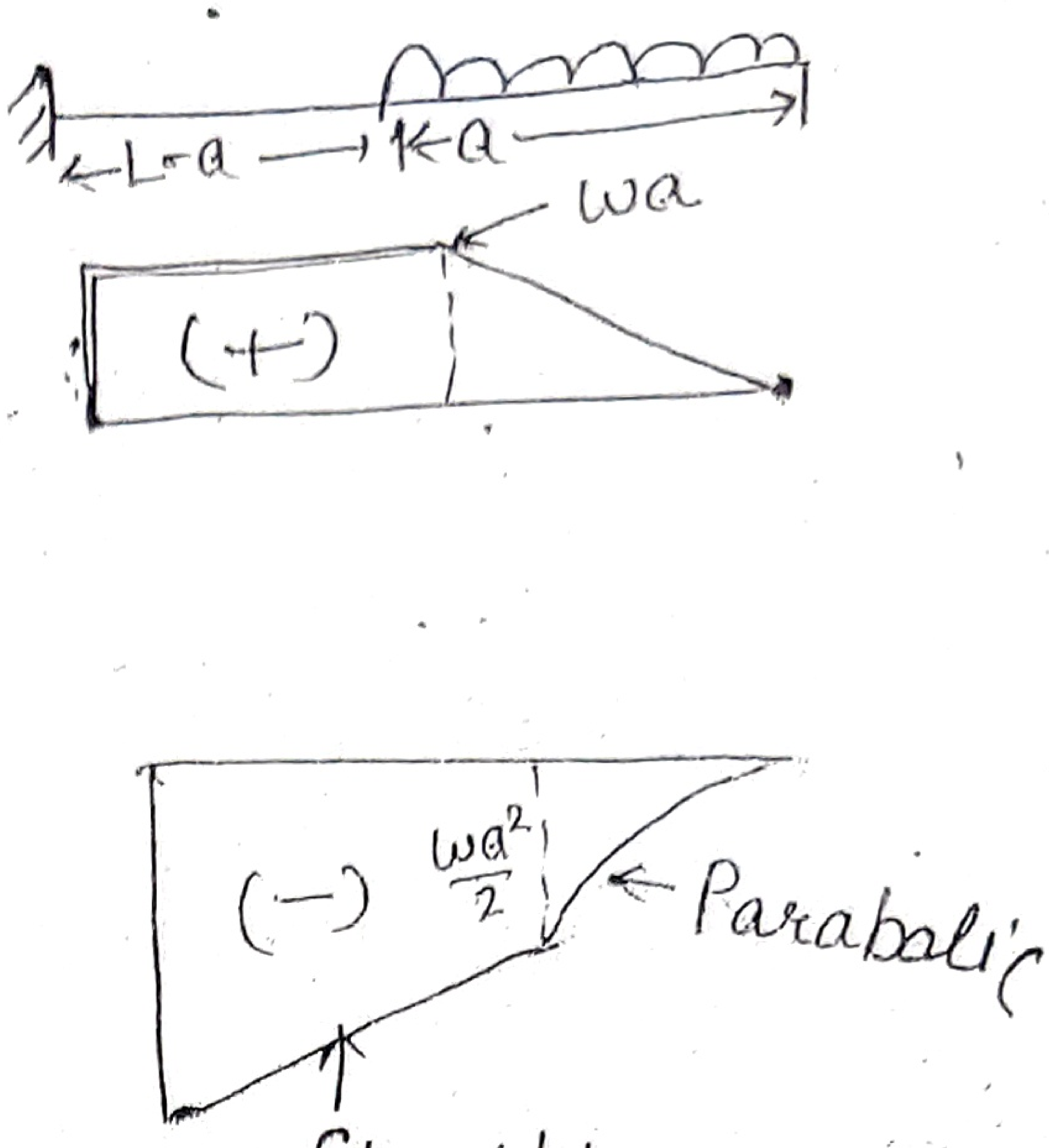
$M_x = -wx^2/2$

$M_L = -\frac{wL^2}{2}$

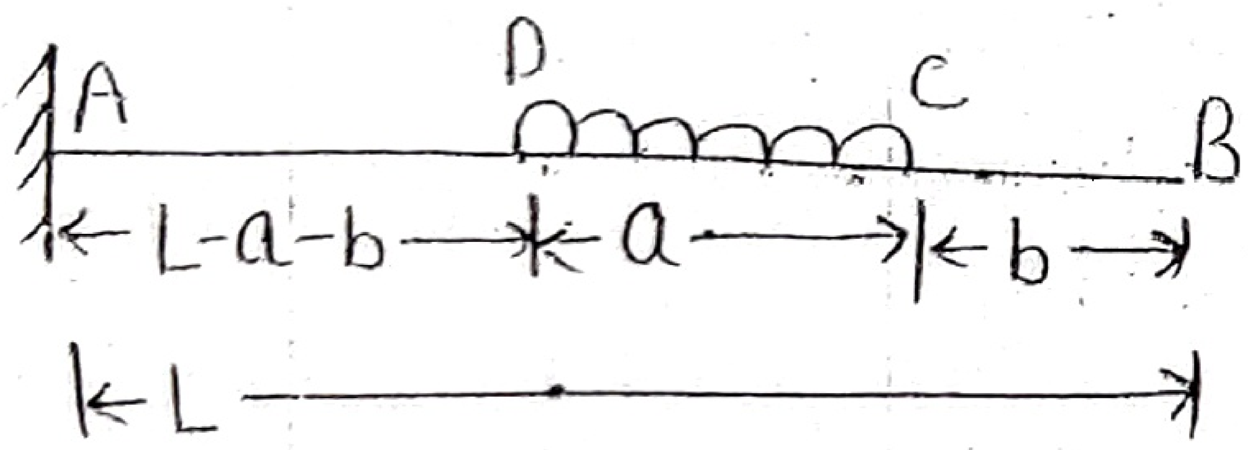
S.F.D.

(4) Cantilever : U.D.L on part Span from the Support :





(5) Cantilever U.D.L. Somewhere on the beam :



S.F. b/w C-B

$F_x = 0$

S.F. b/w C-D

$F_D = +wa$

S.F. b/w A-D

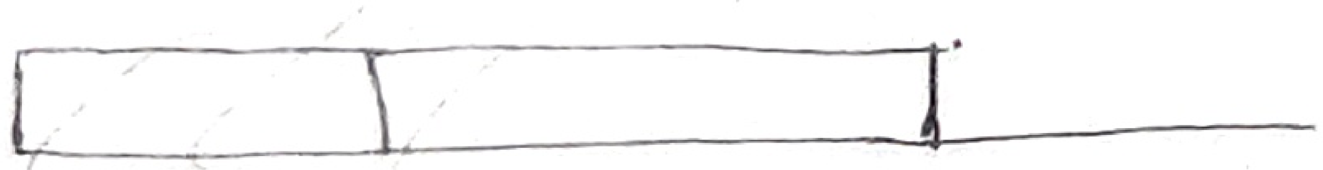
$F_A = +wa$

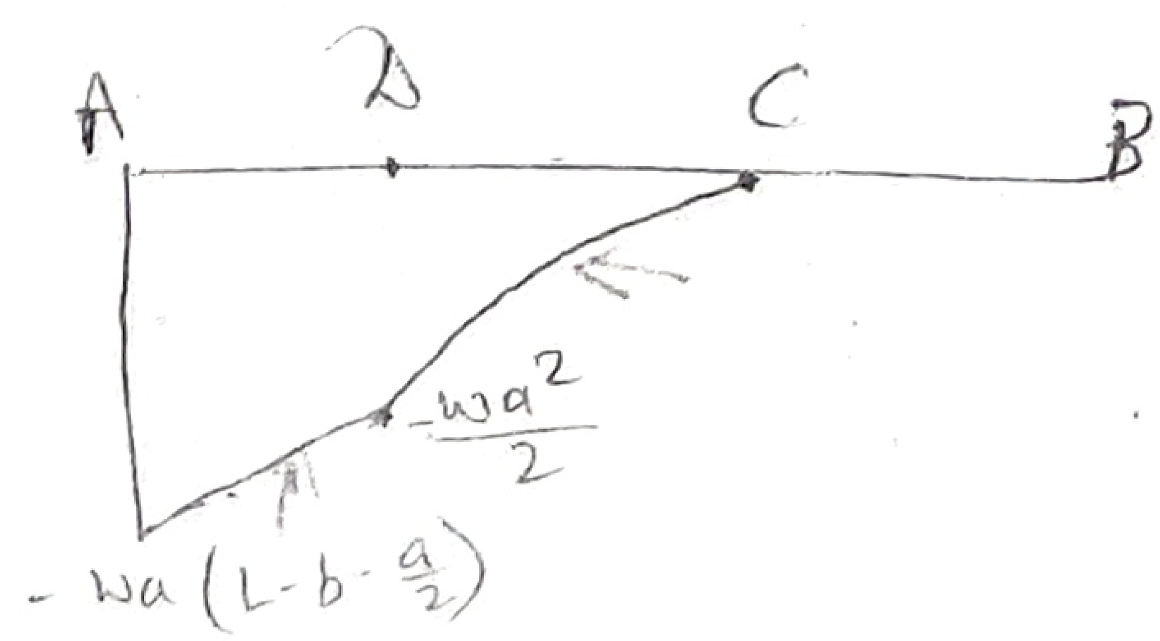
B.M. at C $= 0$

B.M at D $= -\frac{wa^2}{2}$

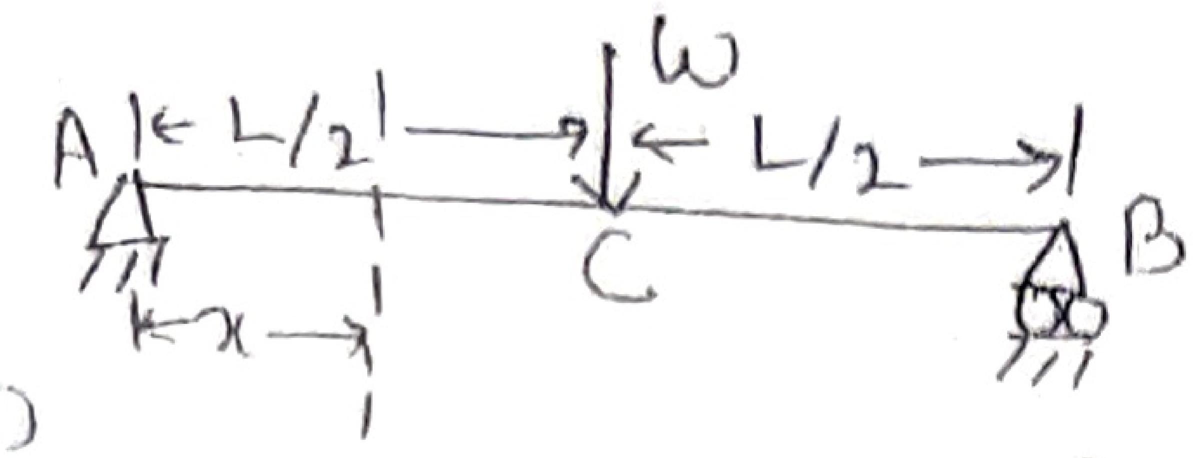
B.M. at A $= -wa\left(L-a-b+\frac{a}{2}\right)$

$= -wa\left(L-b-\frac{a}{2}\right)$





Simply Supported Beam: Point Load at the Centre



S.F.D

Due to Symmetry

$$R_A = R_B = \frac{W}{2} (\uparrow)$$

at distance x from A

$$F_x = +\frac{W}{2}$$

$$F_c = +\frac{W}{2}$$

for portion CB

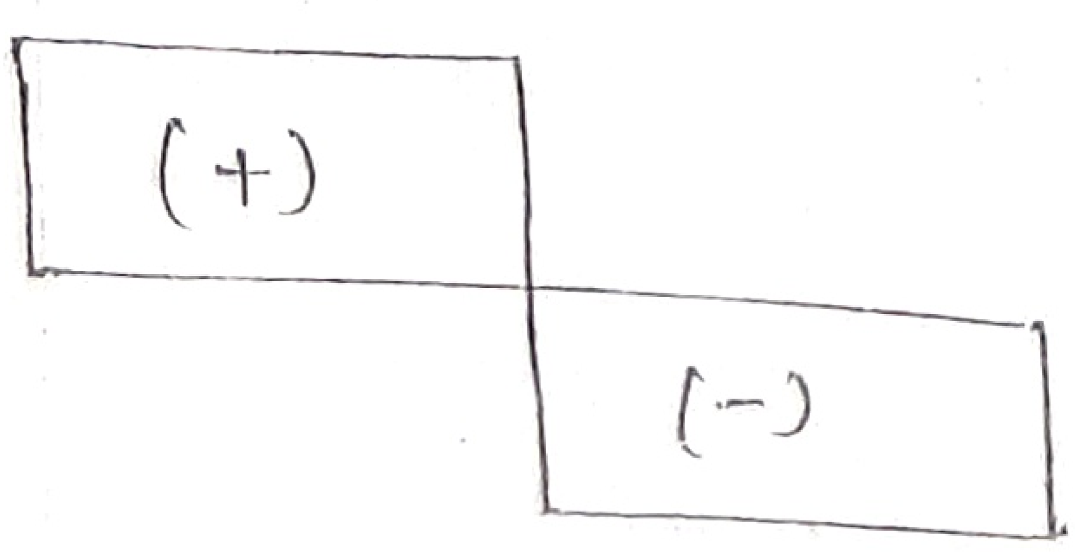
$$x > L/2$$

$$F_x = R_A - W$$

$$F_x = \frac{W}{2} - W = -\frac{W}{2}$$

$$F_c\,(\text{right}) = -\frac{W}{2}$$

$$F_c\,(\text{left}) = +W/2$$



S.F.D

B.M.D

at x,

$$M_x = +R_A\,x = \frac{W}{2}x \text{ (Linear)}$$

at $x = 0$, $\quad M = 0$

at $x = L/2$, $\quad M_c = \frac{W}{2} \times \frac{L}{2} = \frac{WL}{4}$

for portion CB

$$M_x = R_A\,x - W(x - L/2)$$

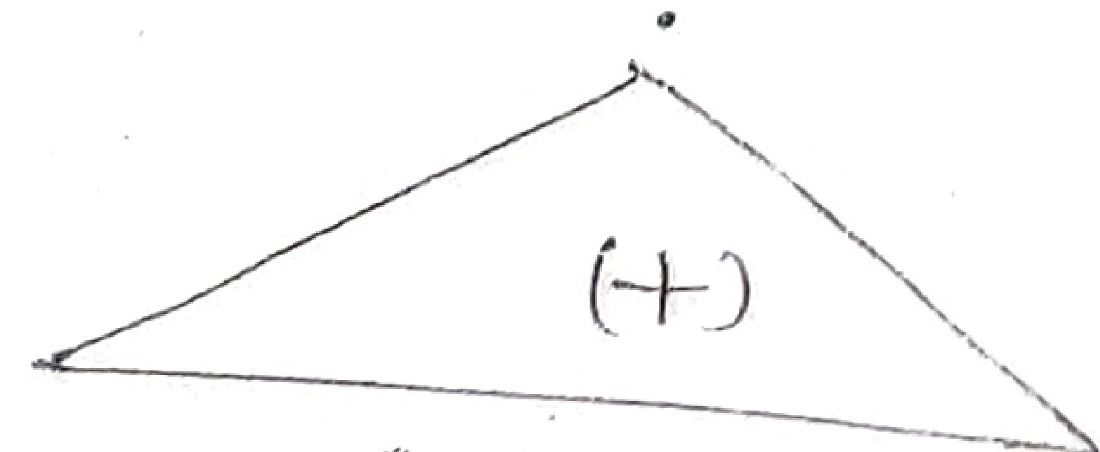
$$= \frac{W}{2}x - Wx + \frac{WL}{2}$$

$$= -\frac{Wx}{2} + \frac{WL}{2}$$

$x = L/2 \quad = +\frac{WL}{2}$

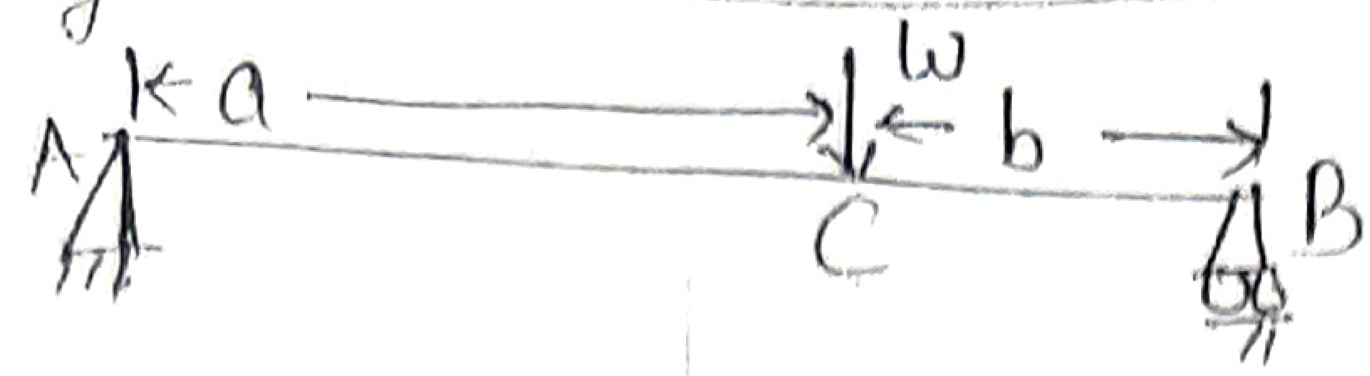
$x = L \quad = 0$

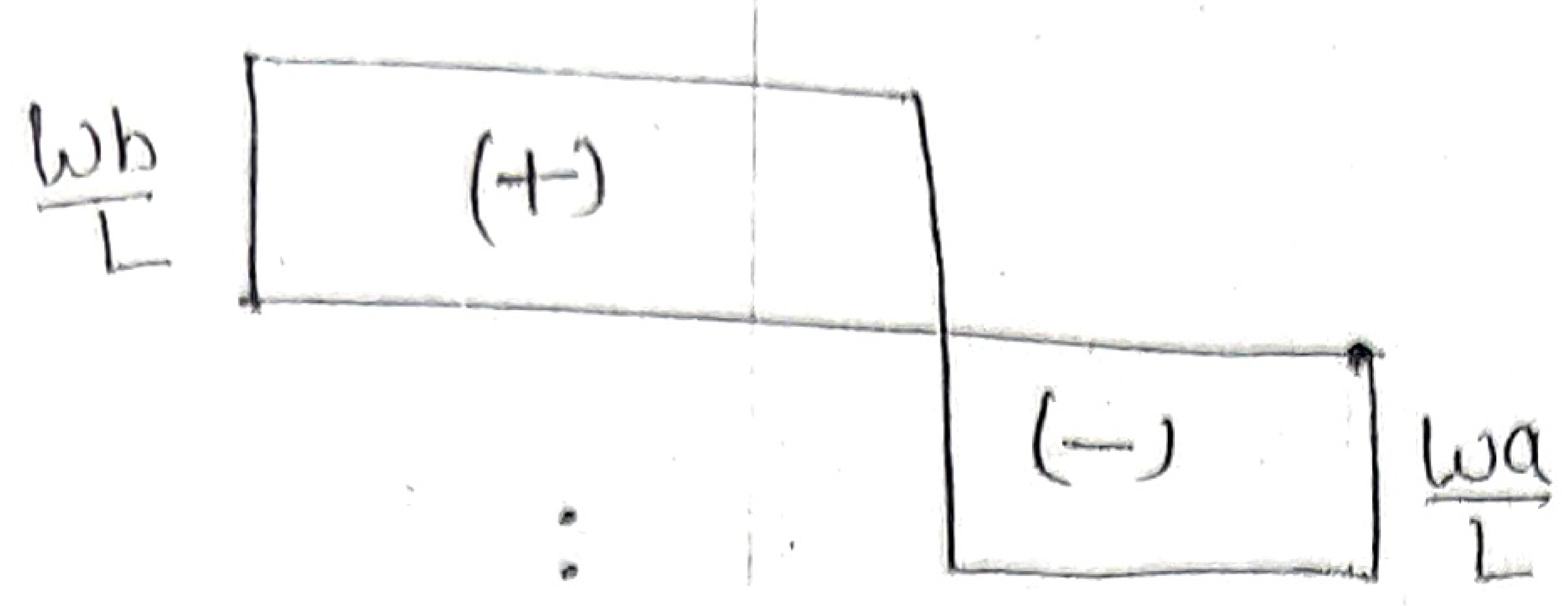
$$BM_{max} = \frac{WL}{4}$$



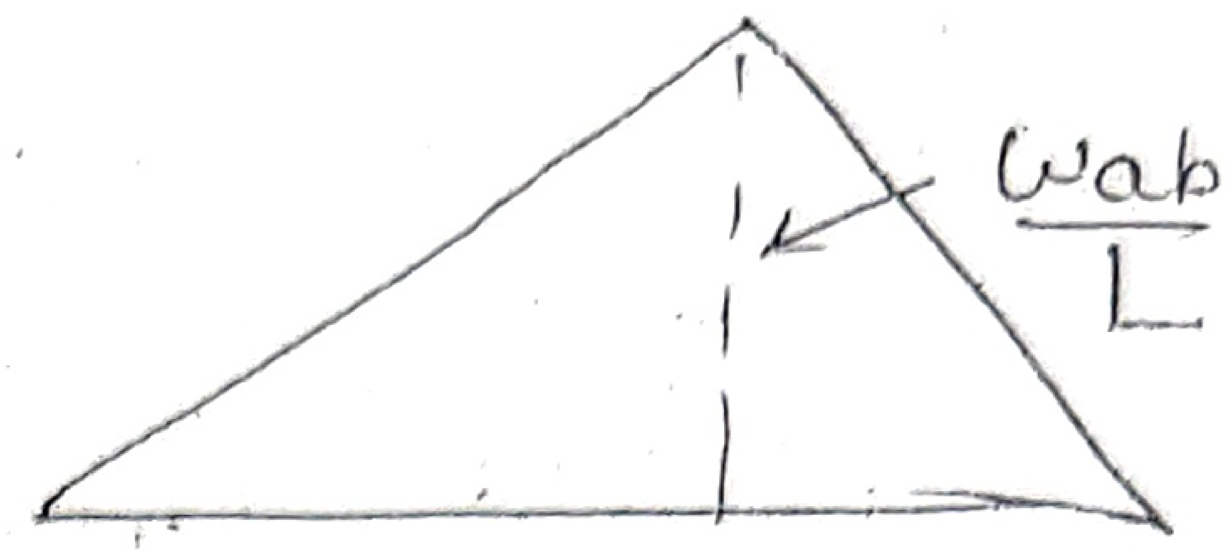
B.M.D

Simply Supported Beam : Eccentric load





S.F.D.



B.M.D.

$\underline{M_A = 0}$

$\therefore R_B \times L - W \times a = 0$

$R_B = \frac{Wa}{L}$

$R_A = W - \frac{Wa}{L}$

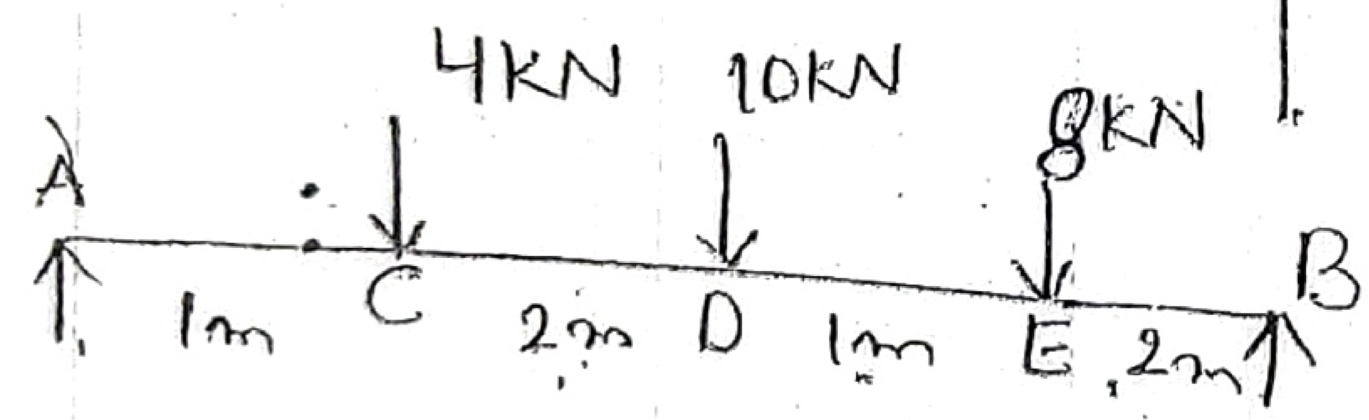
$= \frac{W(L-a)}{L} = \frac{Wb}{L}$

B.M.

B.M. at C

$= \frac{Wb}{L} \times a = \frac{Wba}{L}$

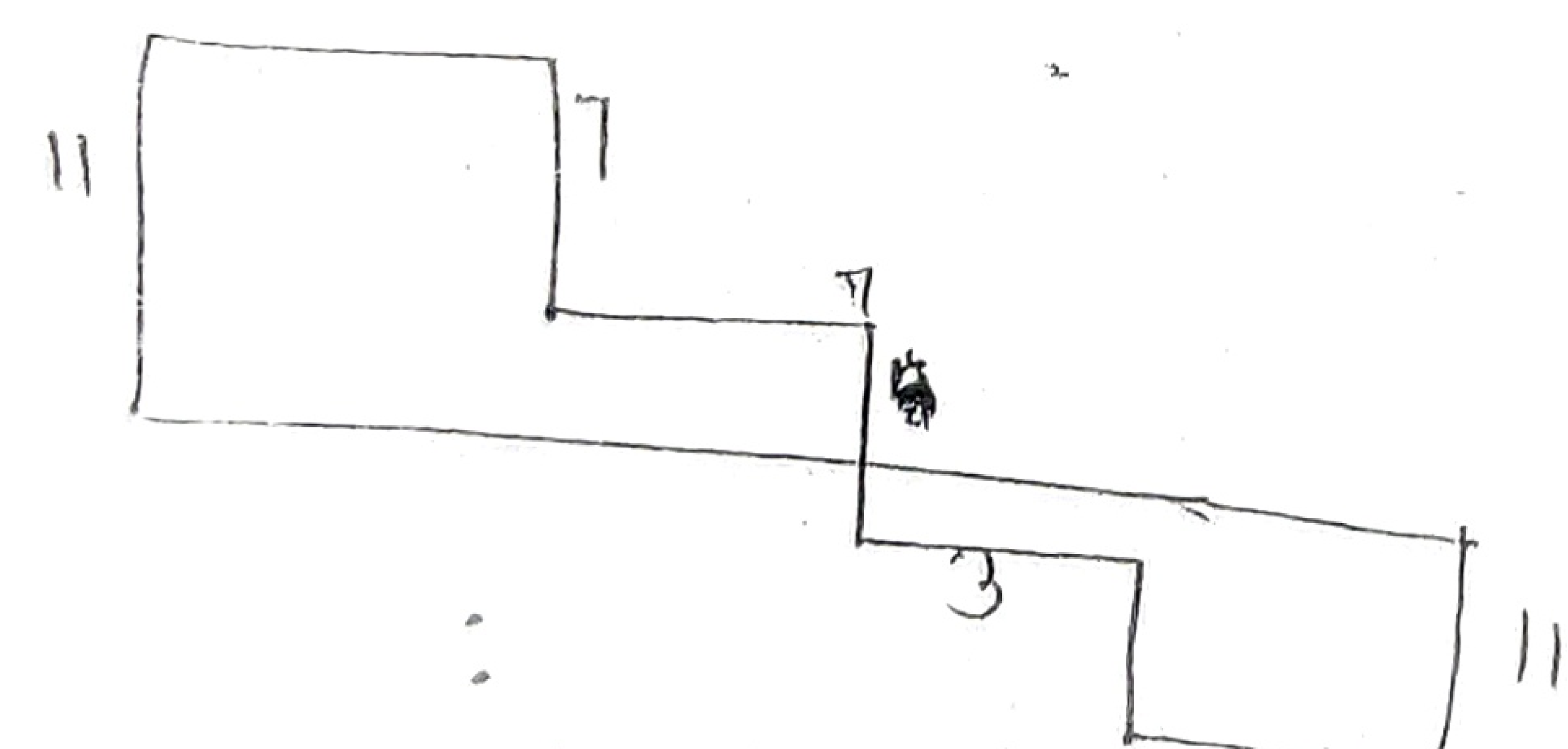
Q:



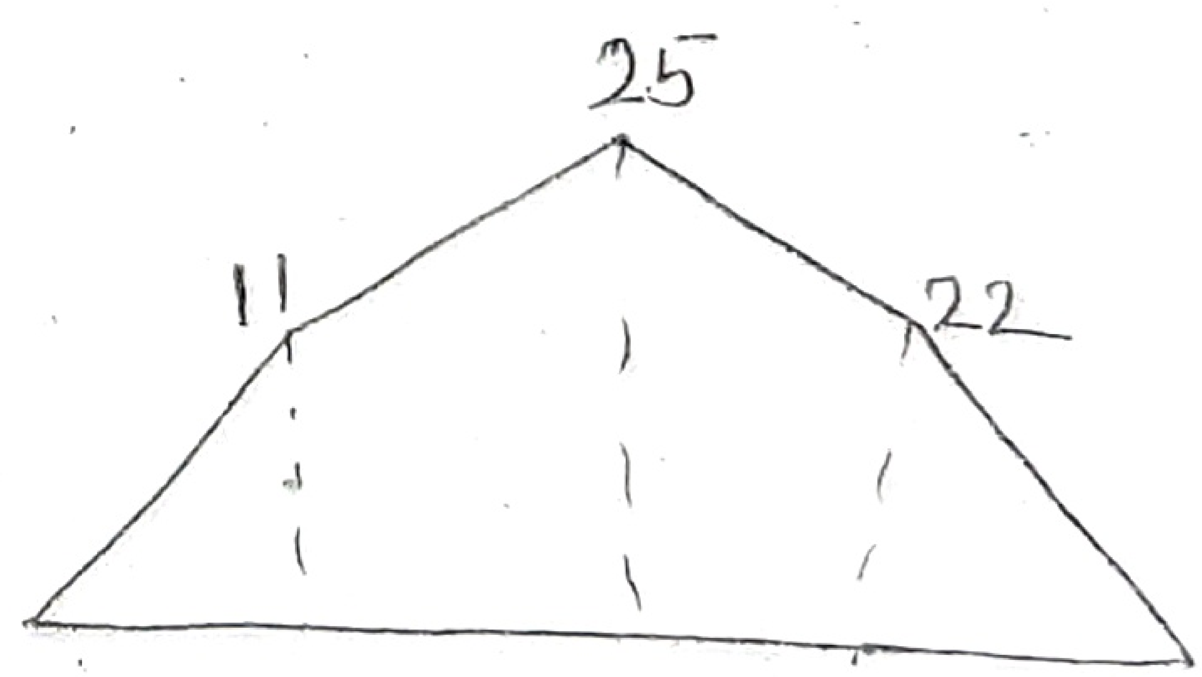
$R_B \times 6 = (8 \times 4) + (10 \times 3) + (4 \times 1)$

$R_B = \frac{32 + 30 + 4}{6} = \frac{66}{6} =$ ~~10.5KN~~ 11KN

$R_A =$ ~~5.5KN~~ 11KN

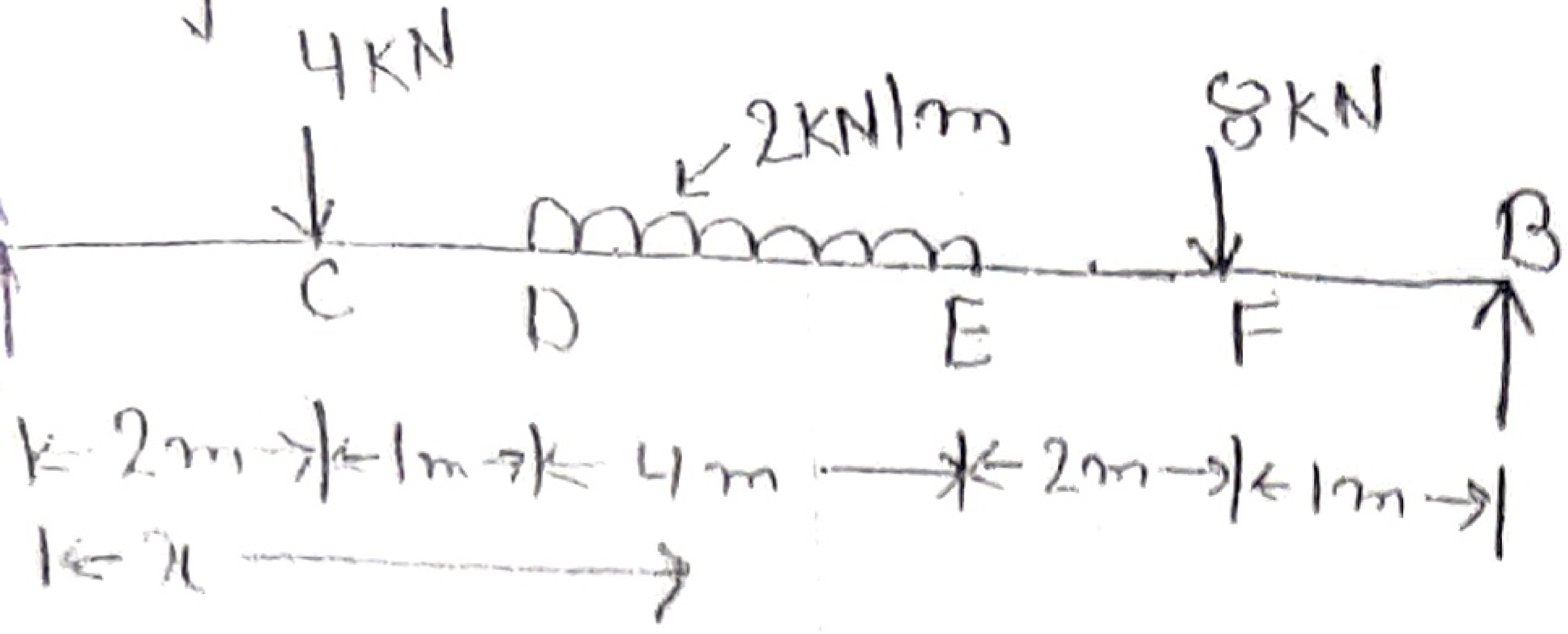


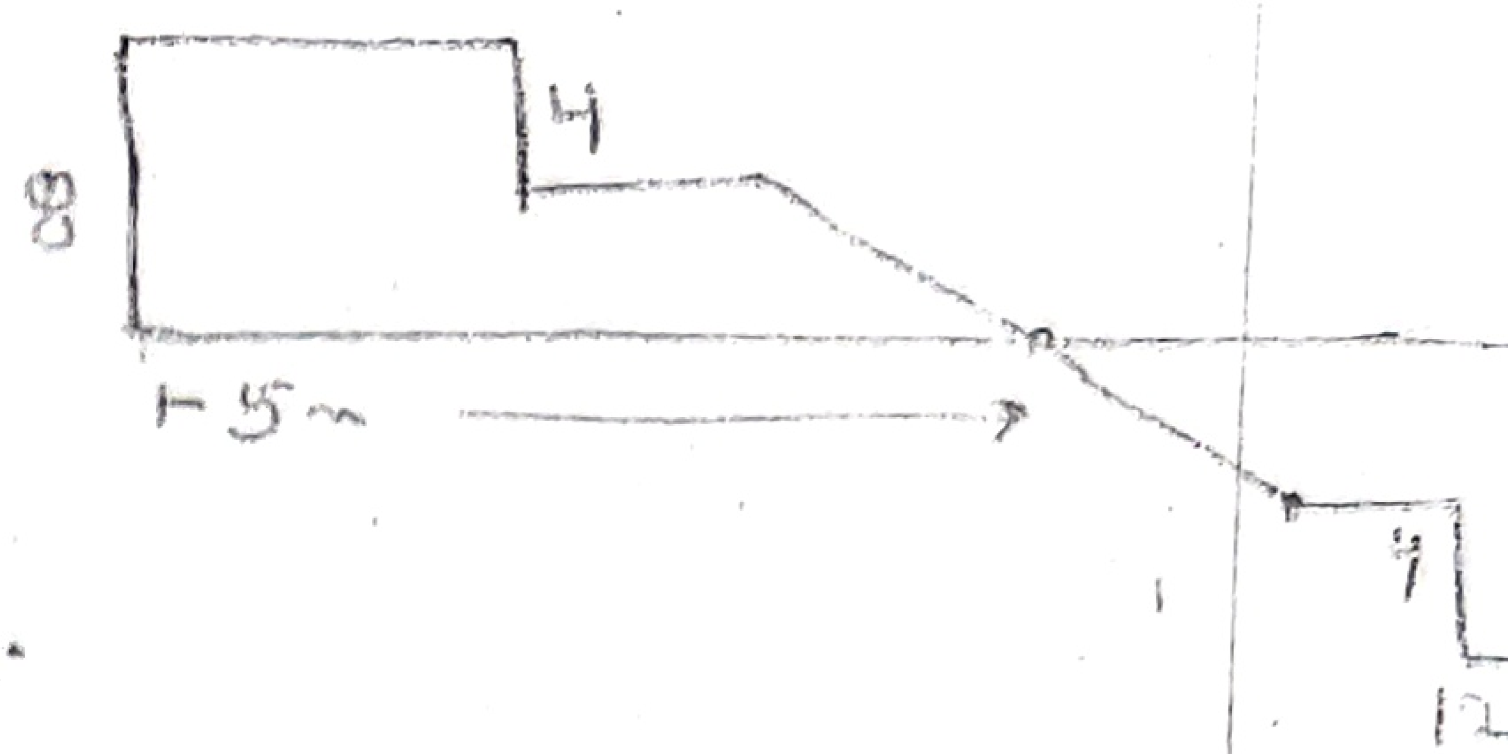
S.F.D



B.M.D.

...mply Supported Beam : Combination of Loads :→





$\ldots \times 10 - (4\times 8) - (2\times 4\times 5) - (8\times 1) = 0$

$R_A = 8\ \text{KN}$

∴ $R_B = 12\ \text{KN}$

S.F. for AC

∴ $F = +R_A = 8\ \text{KN}$

For CD

$F = 8 - 4 = +4\ \text{KN}$

For DE

$F_x = 8 - 4 - 2(x-3)$

$x = 3 \quad F_x = +4$

$x = 7 \quad F_x = 8 - 4 - 2(7-3)$
$= 8 - 4 - 8 = -4$

For $F_x = 0$

$8 - 4 - 2(x-3) = 0$

$4 - 2x + 6 = 0$

$-2x = -10$

$x = 5$

For EF

$F_x = 8 - 4 - 8 = -4$

For FB $= 8 - 4 - 8 - 8$
$= -12\ \text{KN}$

B.M.D :→

For AC

$M_x = R_A \cdot x$

$x = 0 \quad M = 0$

$x = 2 \quad M = 8\times 2 = 16$

For CD

$M_x = R_A x - 4(x-2)$

$M_C, \quad M = +16$

$M_D \quad M = 8\times 3 - 4(1)$
$= 24 - 4 = +20\ \text{KN}$

For DE

$M_x = R_A x - 4(x-2) - 2(x-\ldots$

~~$M_x = 8x - 4(x-2) - (x-3)^2$~~

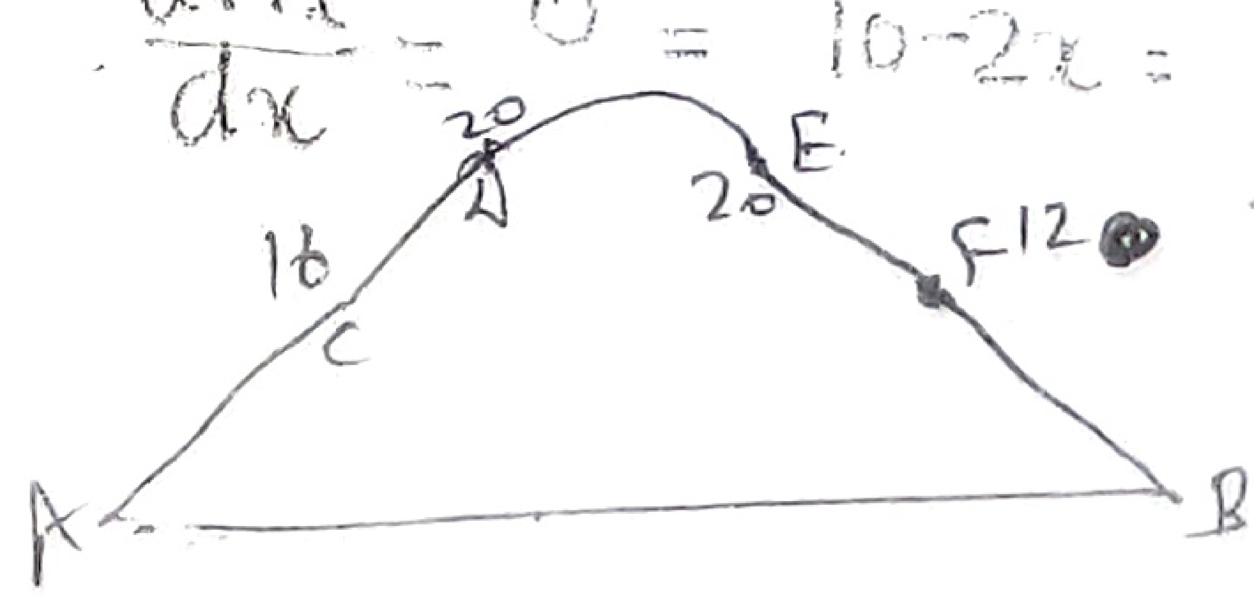
~~$x = 0 \quad M = +8 - 9$~~

$M_x = 10x - x^2 - 1$

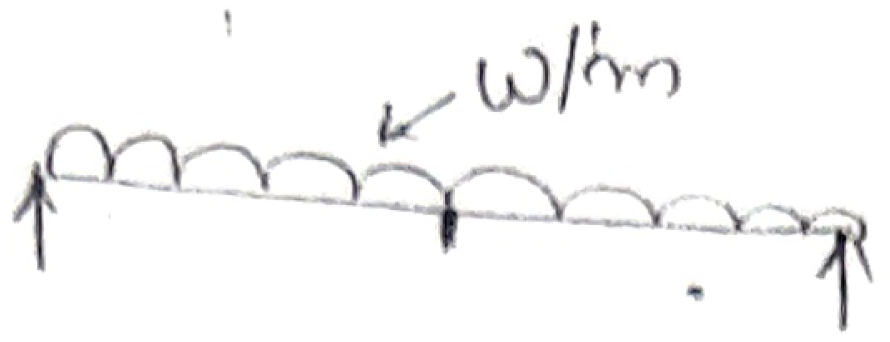
$M_D = 10\times 3 - 3^2 - 1 = 20\ \text{KN}$

$M_E = 10\times 7 - 7^2 - 1 = 20\ \text{KN}$

$\frac{dM_x}{dx} = 0 = 10 - 2x \quad x = 5\ \text{m}$



Q Simply Supported beam with U.D.L.



Calculate Reaction

Total Load = ωL

$R_A + R_B = \omega L$

$R_A = R_B = \frac{\omega L}{2}$

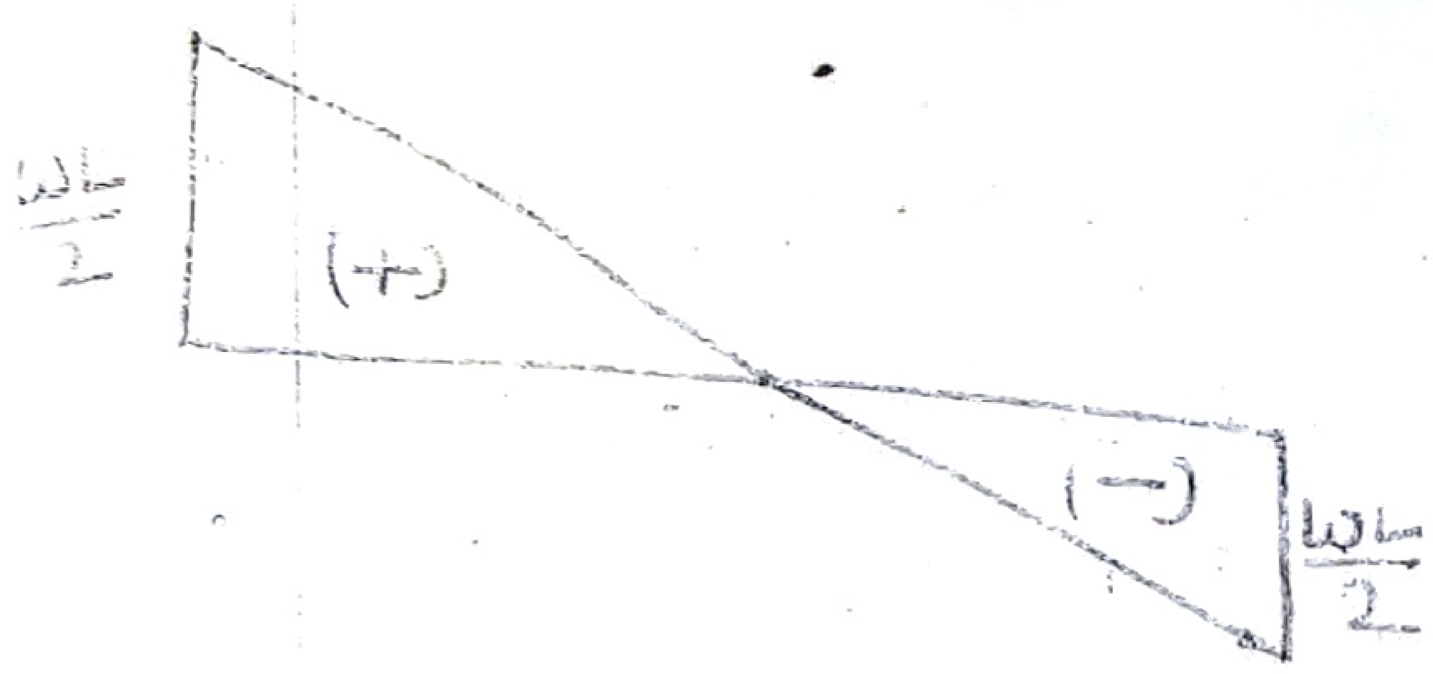
S.F.D. $F_x = R_A - \omega(x)$

$= \frac{\omega L}{2} - \omega x$ (Linear)

$x = 0 \quad = \frac{\omega L}{2}$

$x = L \quad = -\frac{\omega L}{2}$

$x = L/2 \quad = 0$



B.M.D

$M_x = R_A x - \omega x . \frac{x}{2}$

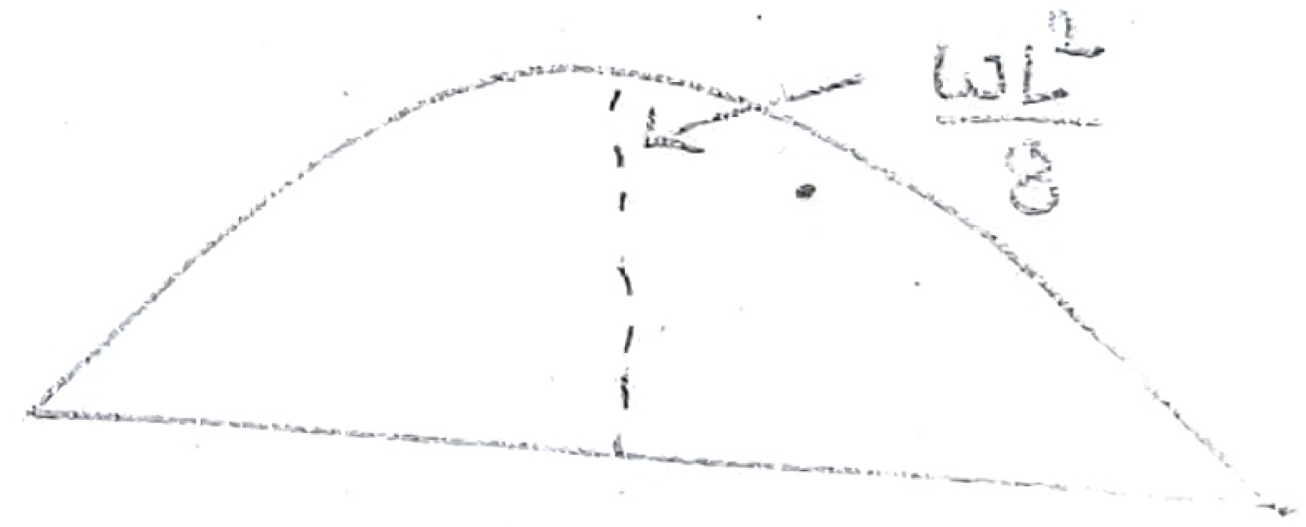
$= R_A x - \frac{\omega x^2}{2}$ (Parabolic

$x = 0 \quad = \frac{\omega L}{2}.(0) - \frac{\omega x^2}{2} = 0$

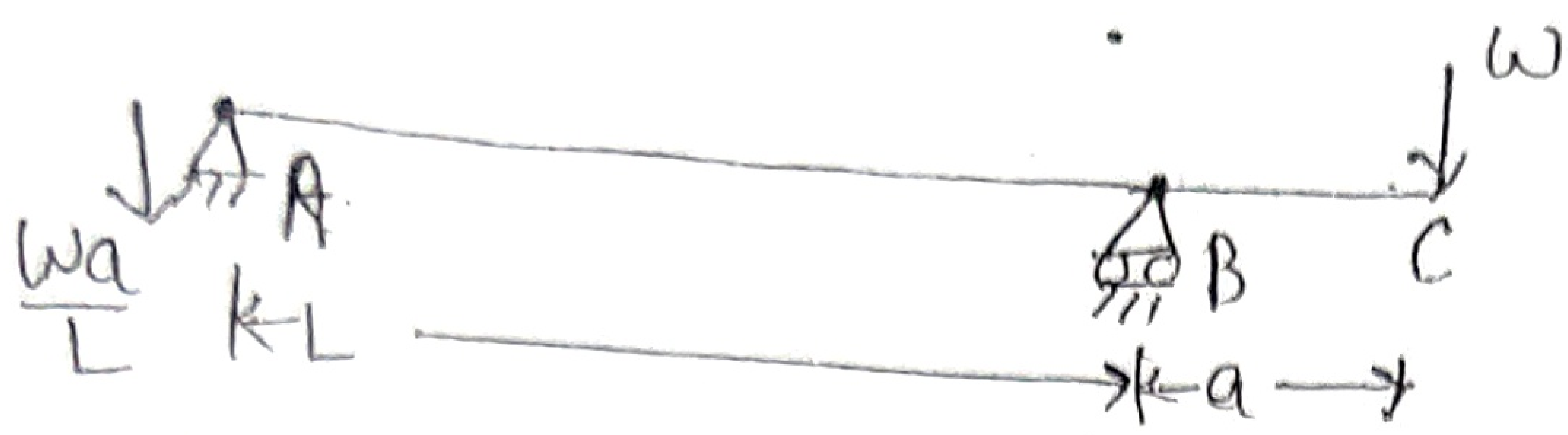
$x = L/2 = \frac{\omega L}{2}.\frac{L}{2} - \frac{\omega L^2}{8}$

$= \frac{\omega L^2}{4} - \frac{\omega L^2}{8} = + \frac{\omega L^2}{8}$

$x = L \quad = \frac{\omega L}{2}.\frac{L}{4} - \frac{\omega}{2}.\left(\frac{L^2}{4}\right) = 0$



Simply Supported Beam with one side overhang: Point Load



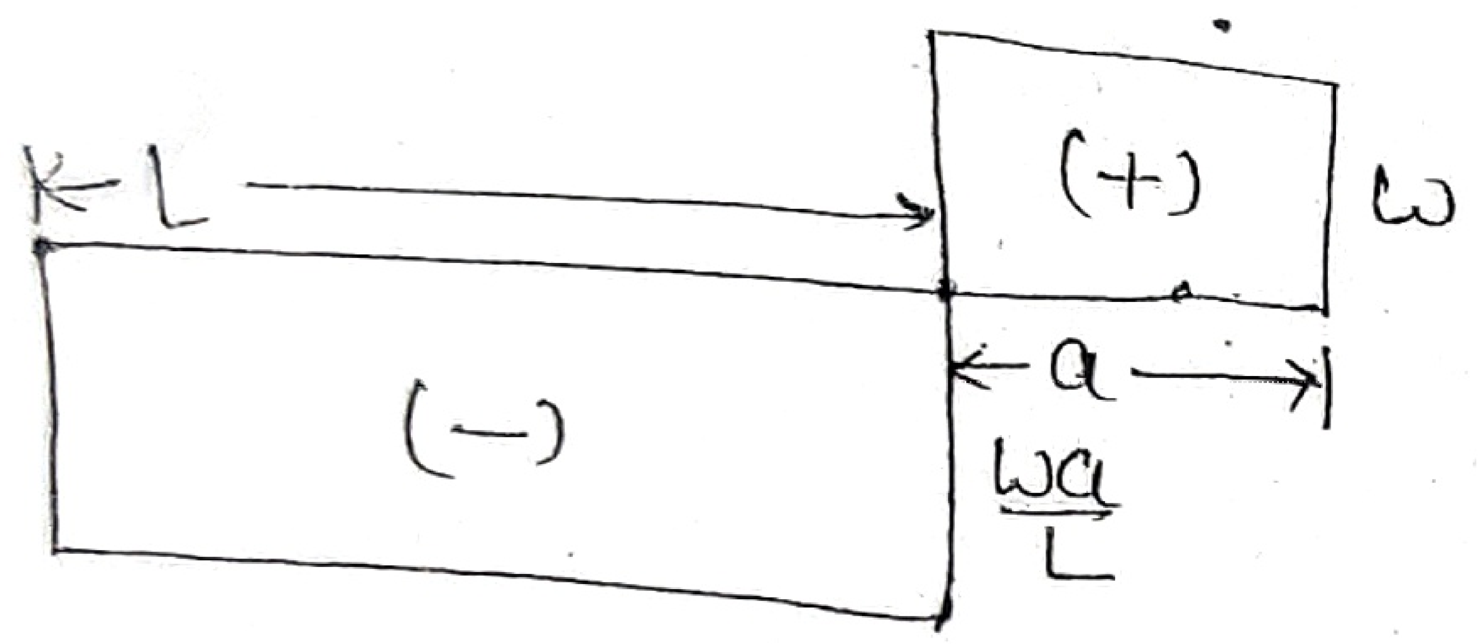
$w \times (L+a) - R_B \times L = 0$

$R_B = \frac{w(L+a)}{L}$

$R_A = w - R_B$

$= w - \frac{L+a}{L}$

$= \frac{wL - \cancel{wL} - wa}{L} = -\frac{wa}{L}$



B.M.D

For AB

$\frac{wa}{L} \times x = -\frac{wa}{L} x$ (hogging)

$x = 0 \quad = \frac{wa}{L} x = 0$

$x = L/2 \quad = -\frac{wa}{L} \times \frac{L}{2} = -\frac{wa}{2}$

$x = L \quad = -wa$

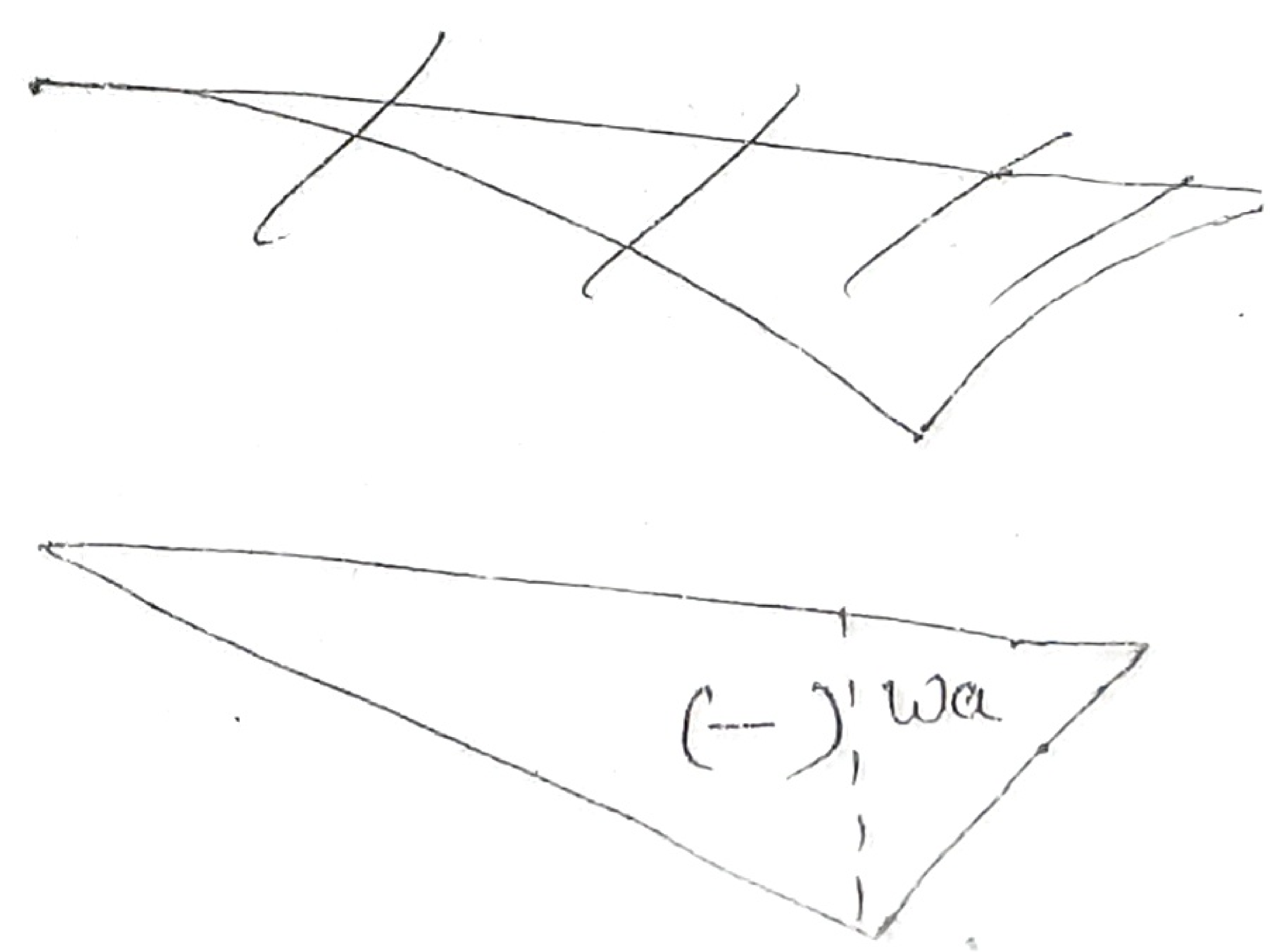
for BC $= -\frac{wa}{L} x + \frac{w(L+a)}{L}(x$

$M_B = -wa$

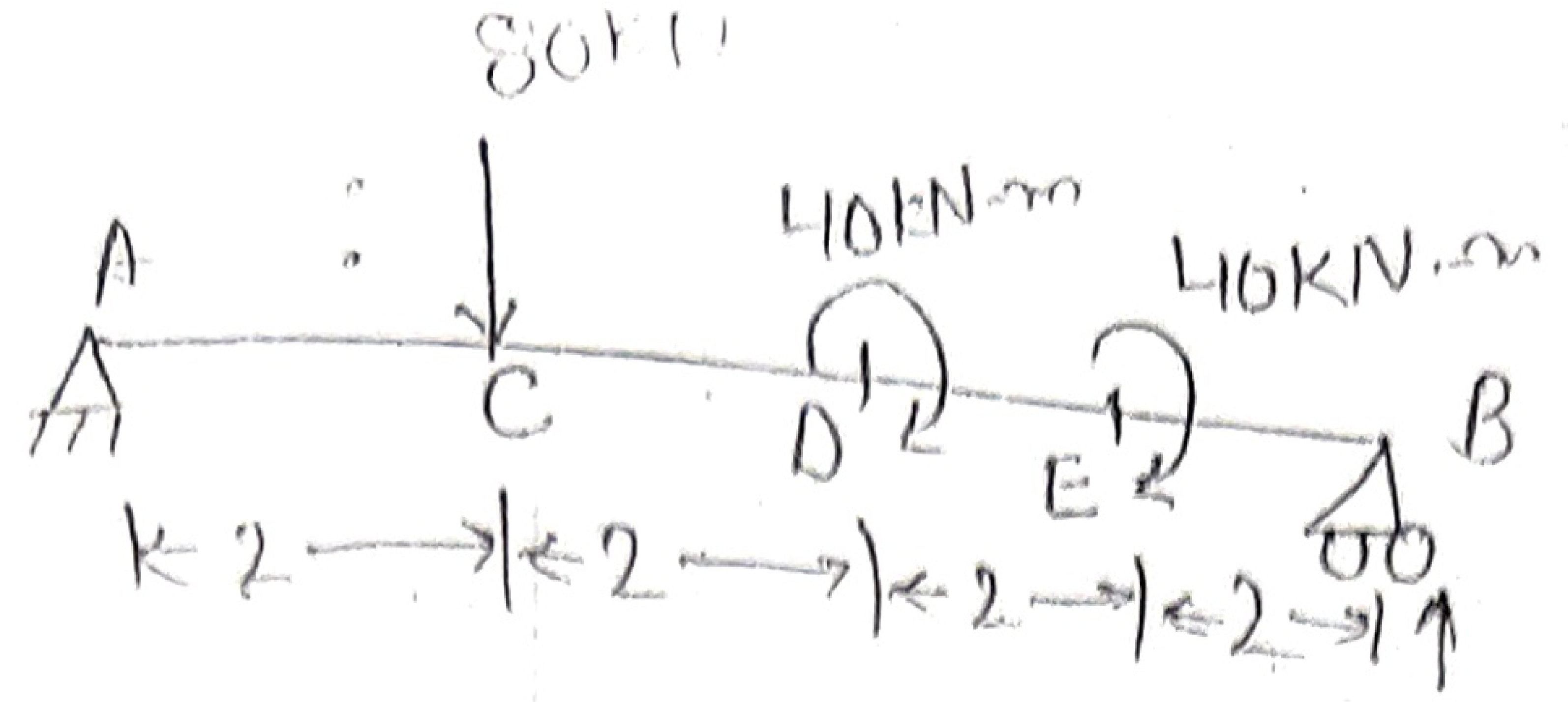
$M_C = -\frac{wa(L+a)}{L} + \frac{w(L+a)(x}{L}$

$= -\frac{wa(L+a)}{L} + \frac{w(L+a)}{L}$

$= 0$



Q

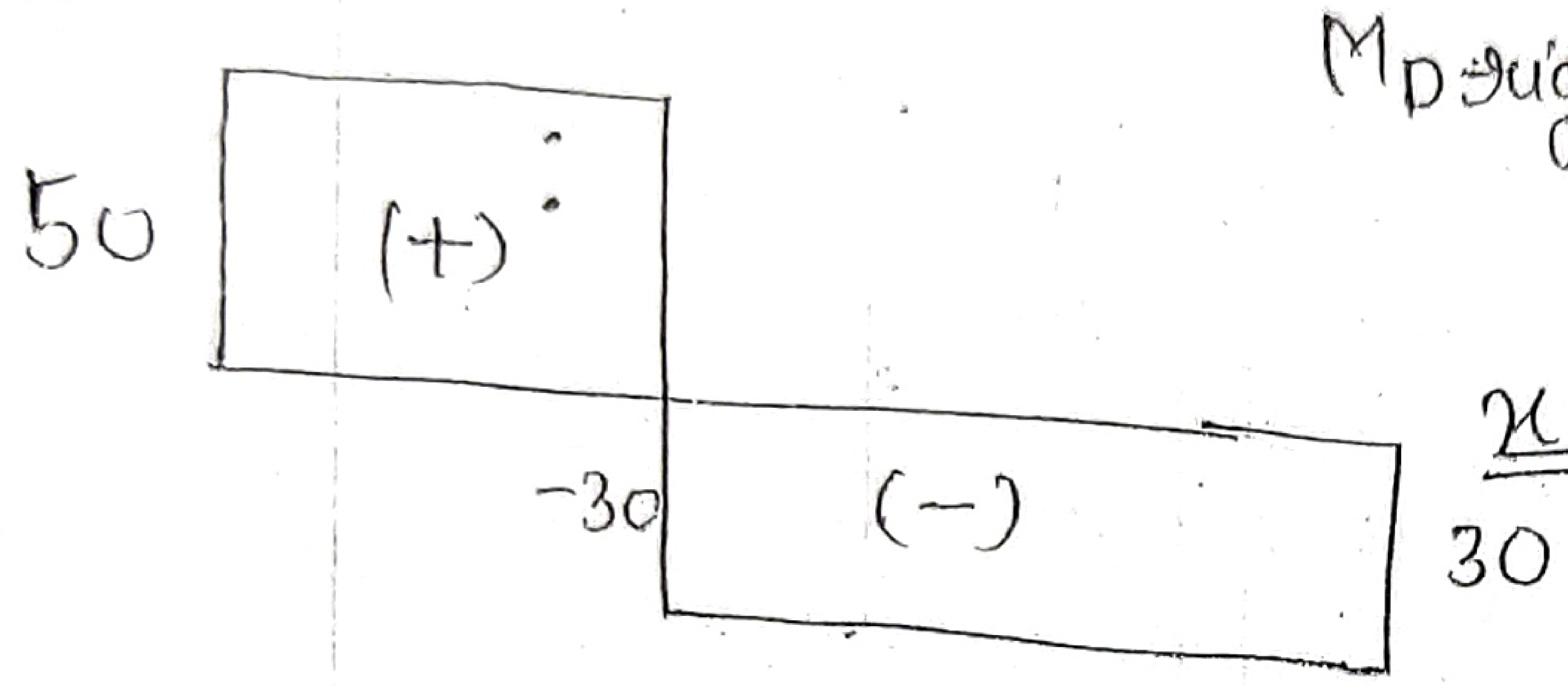


$R_B \times L - 40 - 40 - (80 \times 2) = 0$

$R_B = \frac{80+160}{8} = \frac{240}{8} = 30\,kN$

$R_A = 50\,kN$

S.F.D

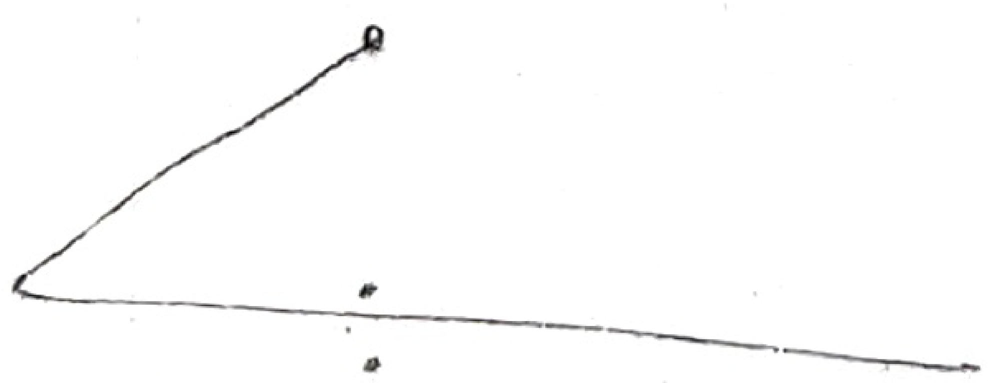


B.M.D

for AC $= 50x$

$x=0 \quad M_A = 0$

$x=2 \quad M_C = 100\,kN\text{-}m$



for CD

$= R_A \times x - 80(x-2)$

$x=2$: $50 \times 2 - 80(0) = 100$

$x=4 = (50 \times 4) - 80 \times 2$

$= 200 - 160 = 40$

For DE

$= R_A x - 80(x-2) + 40$

$x=4$

$M_{D\,right} = (50 \times 4) - 80(4-2) + 40 = 200 - 80(2)\,\cancel{} = +40$

M_D right

$= 50 \times x - 80 \times (x-2) + 40$

M_D right $= (50 \times 4) - (80 \times 2) + 40$

$= 200 - 160 + 40 = +8$

$x = 6$ (left)

$x = 6$ $= 50x - 80(x-2) + 40$

$= 50(6) - 80(4) + 40$

$= \cancel{300-4} + 20$

$x=6$ Right

$= 50x - 80(x-2) + 40 + 40$

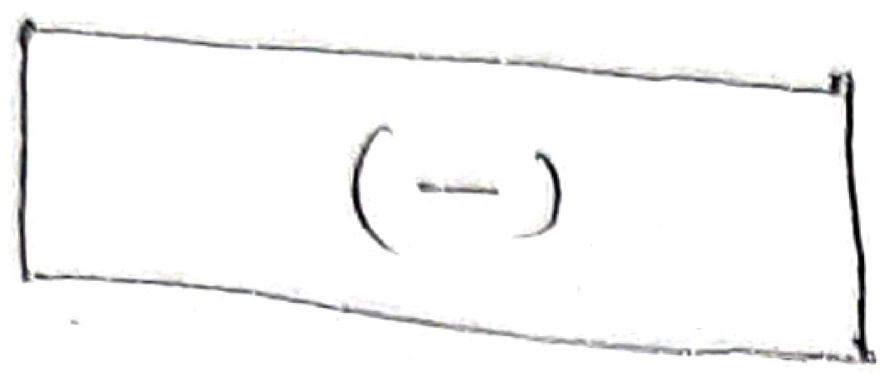
$= 300 - 320 + 40 + 40$

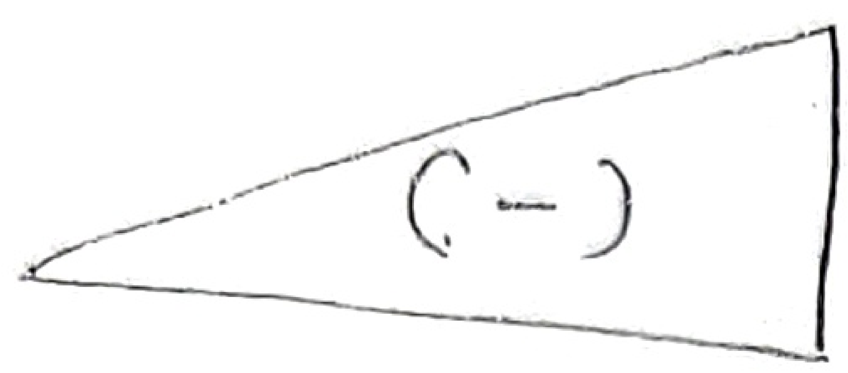
$= 340 - 320 = 20 + kN$

at $x=8$,

$M_B = 0$ expected

3





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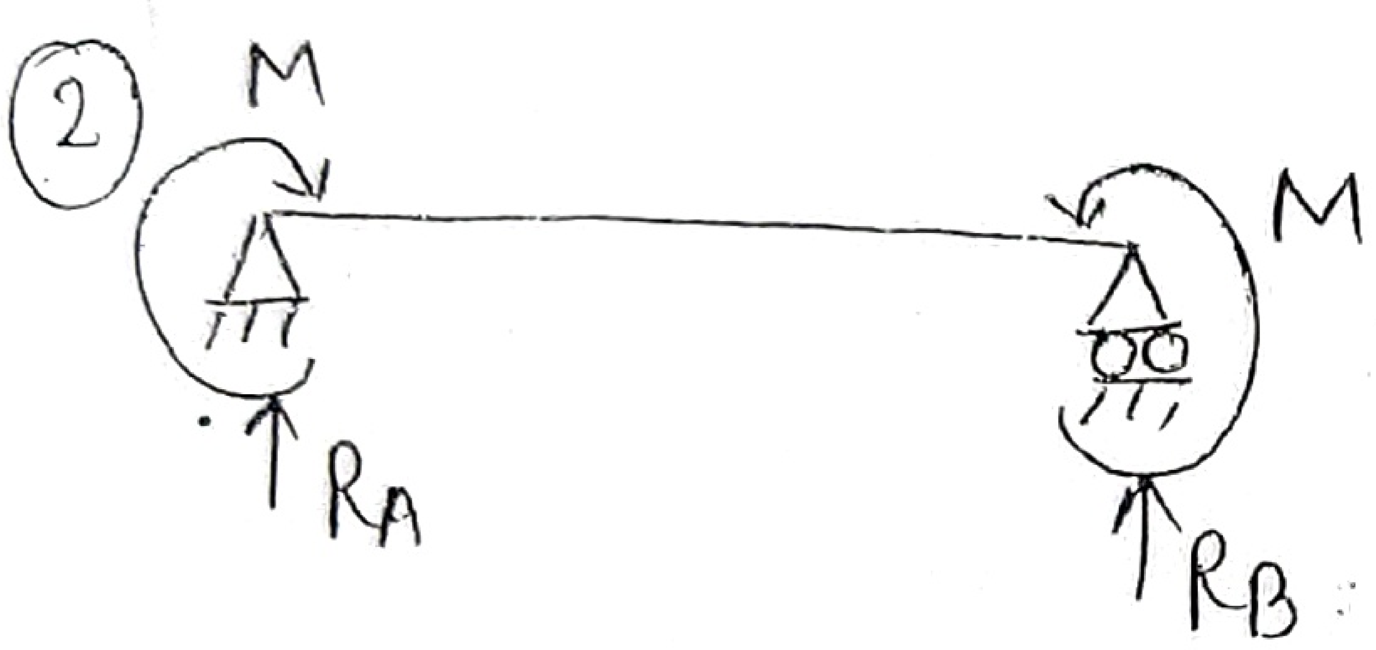
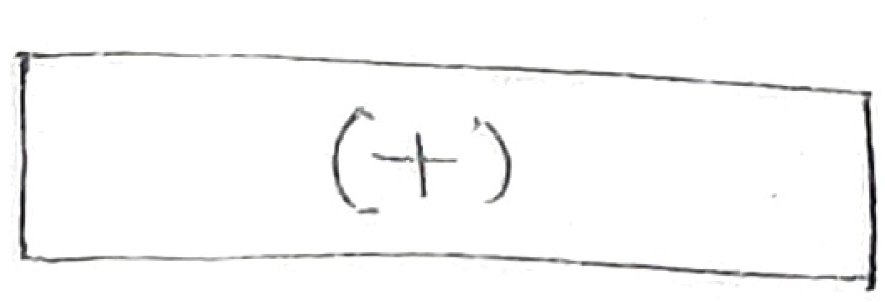


$R_B \times L + M - M = 0$

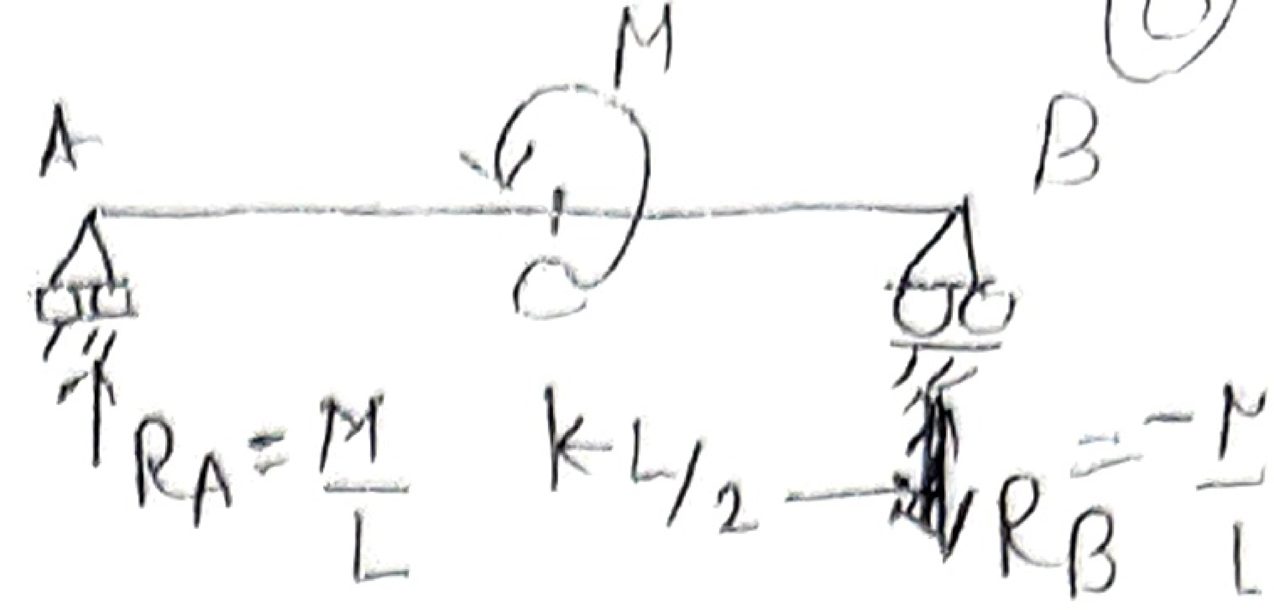
$R_B = 0$

$R_A = 0$

S.F.D.



4

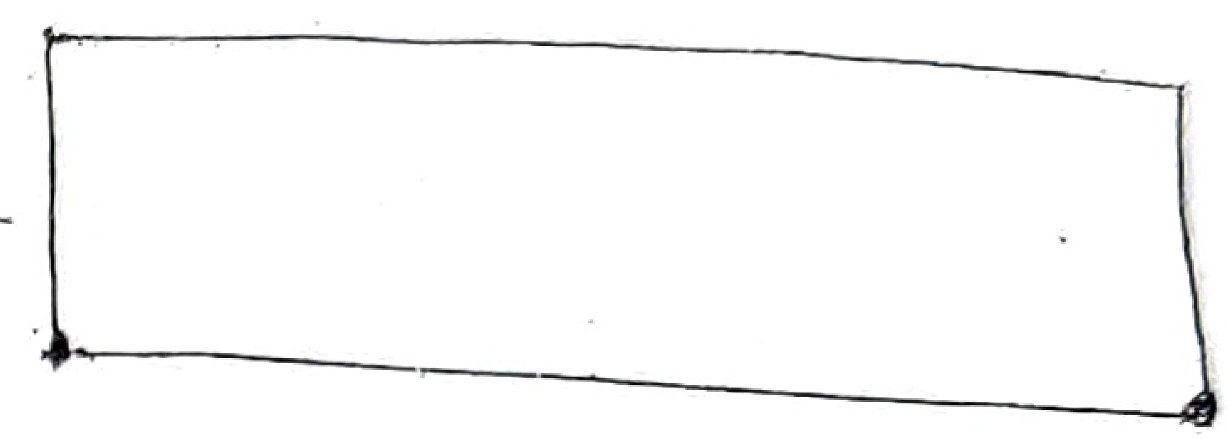


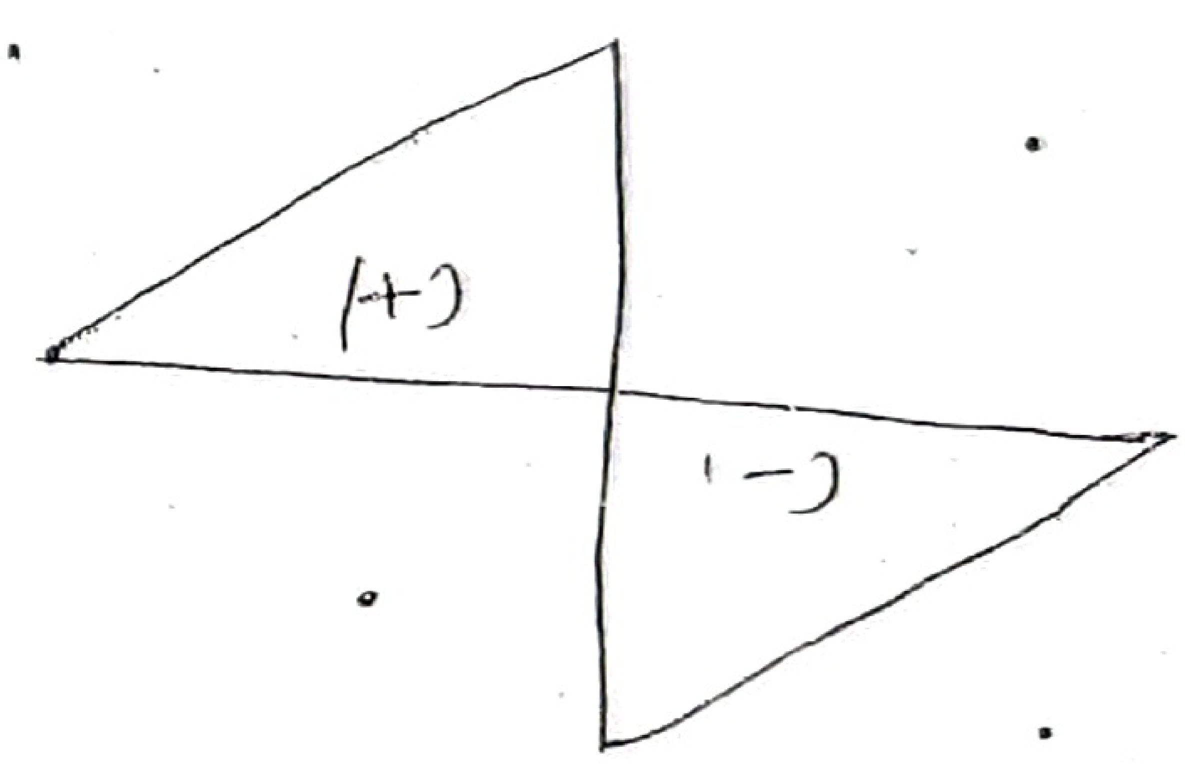
$R_B \times L + M = 0$

$R_B = -\frac{M}{L}$

$R_A \times L - M = 0$

$R_A = \frac{M}{L}$

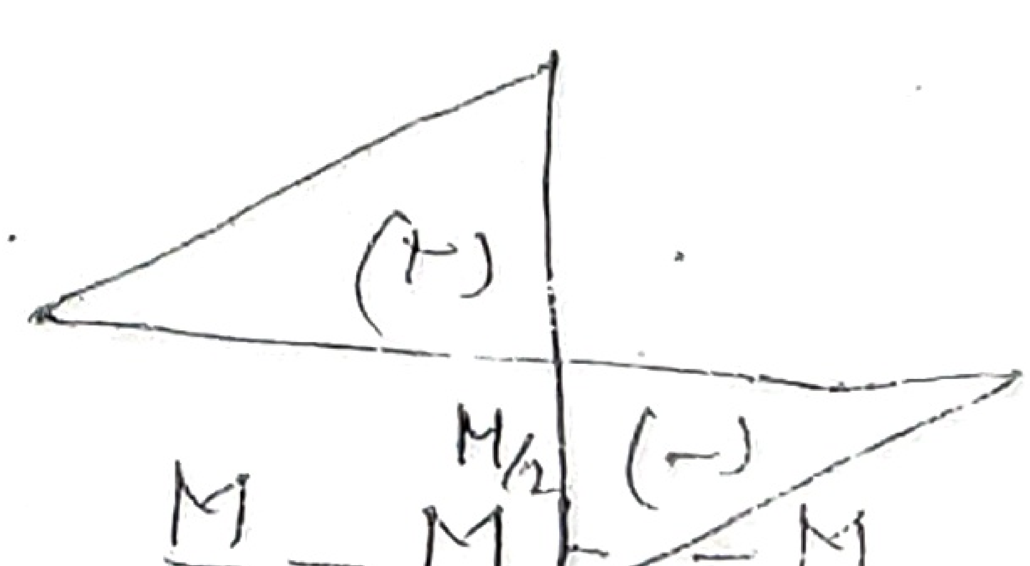




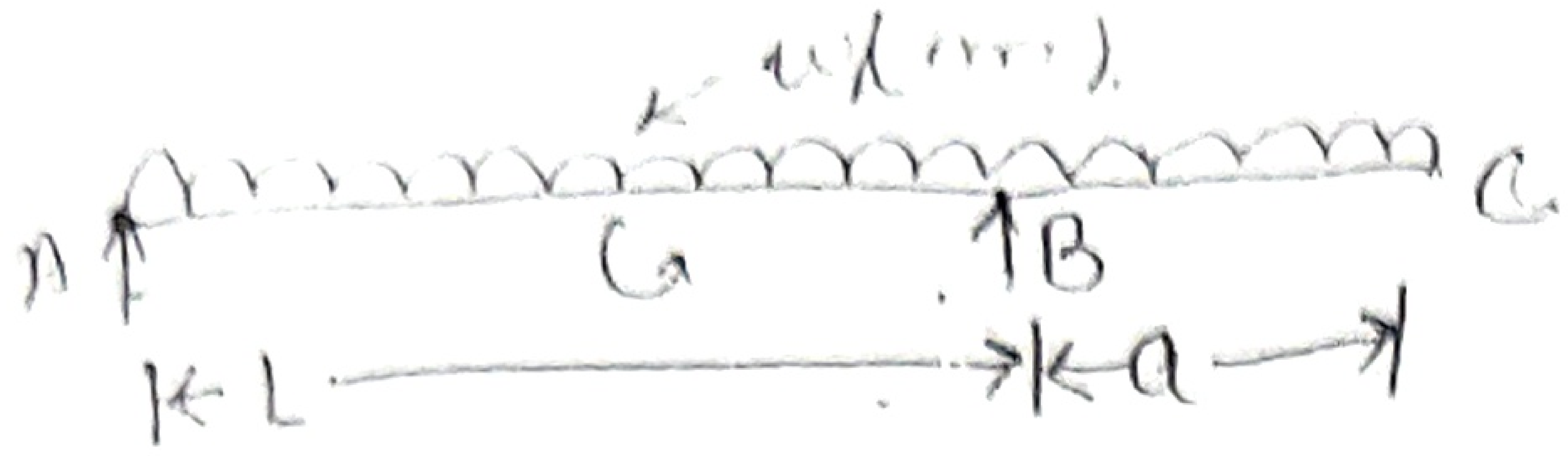
$M_x = \frac{M}{L} x$

AC

$\frac{M}{L} \cdot x = \frac{M}{L} \times \frac{L}{2} = \frac{M}{2}$



C $\frac{M}{2} - M = -\frac{M}{2}$



$M_A = 0$, $\frac{w(L+a)^2}{2} - R_B \times L = 0$

$R_B = \frac{w(L+a)^2}{2L}$

$M_B = 0$

$R_A \times L - \frac{wL^2}{2} + \frac{wa^2}{2} = 0$

$R_A = \frac{w(L^2 - a^2)}{2L}$

$= \frac{w}{2L}(L+a)(L-a)(\uparrow)$

if $L > a$, $R_A (\uparrow)$

$L < a$, $R_A (\downarrow)$

$L = a$, $R_A (0)$

if $L = 5m$, $a = 3m$, $w = 2 kN/m$

$R_A = \frac{2}{2\times 5}(8)(2) = \frac{16}{5} = 3.2 kN (\uparrow)$

$R_B = \frac{2(8)^2}{2\times 5} = \frac{2\times 8\times 8}{2\times 5} = \frac{64}{5} = 12.8 kN (\uparrow)$

S.F. = AB

$F_x = 3.2 - 2x$

$x = 0$ $= 3.2$

$x = 5$ $= 3.2 - 10 = -6.8 kN$

for S.F. D = 0 $3.2 - 2x = 0$

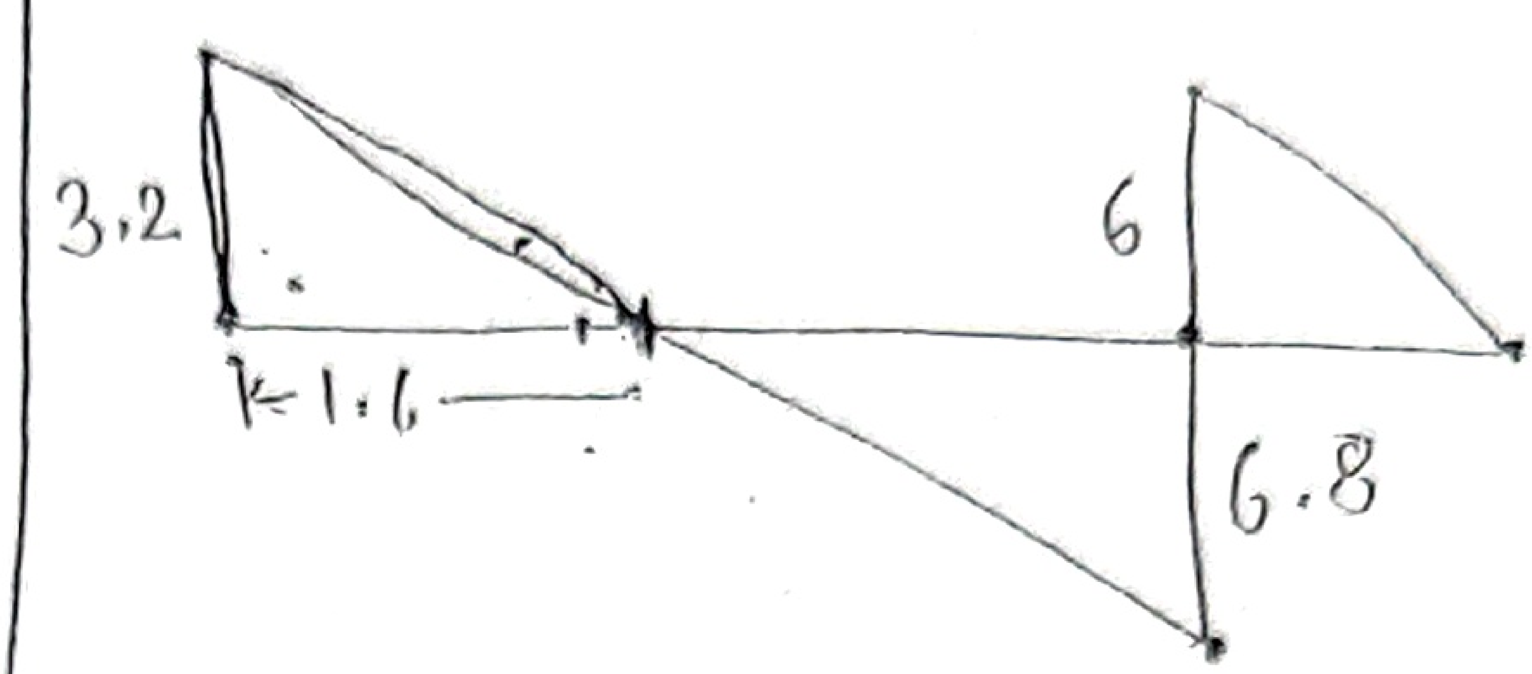
$x = \frac{3.2}{2} = 1.6 m$

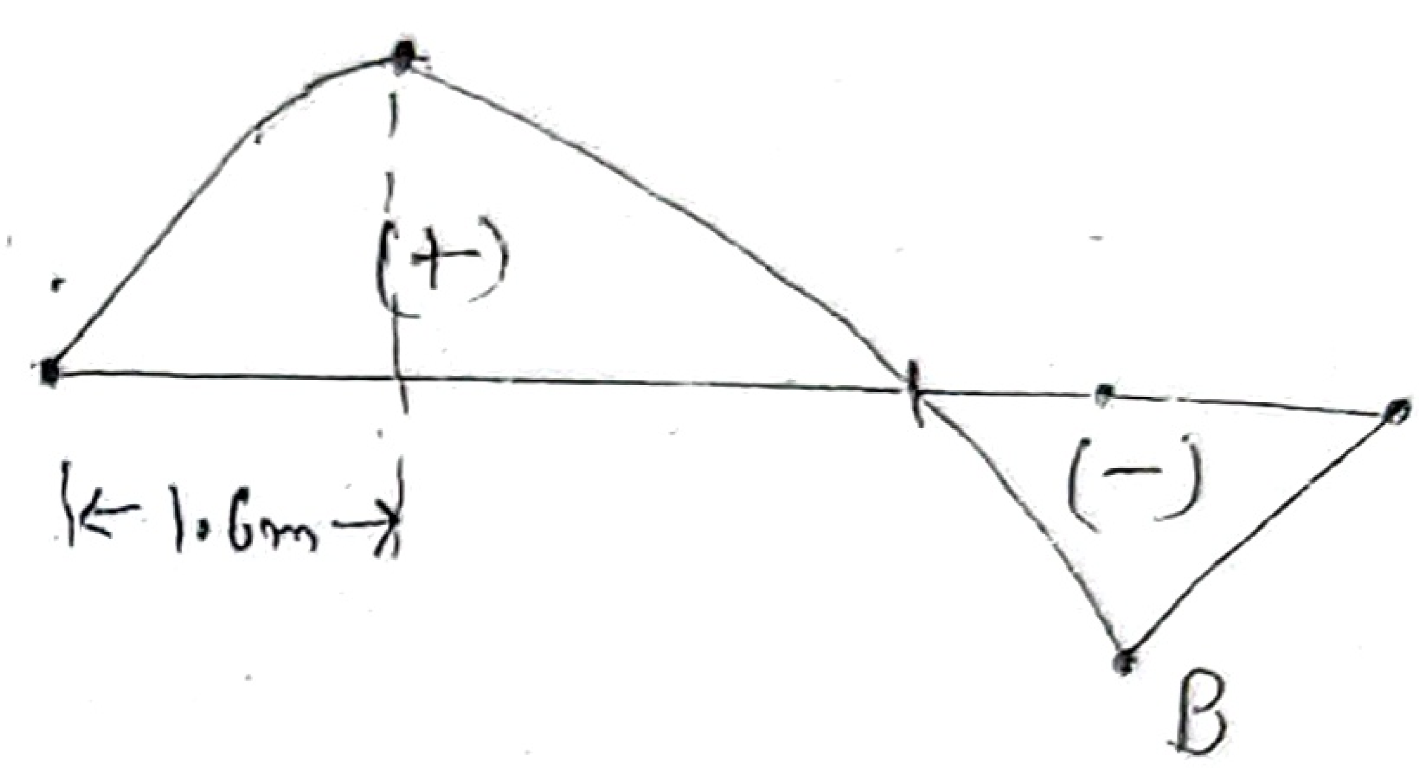
for BC $= 3.2 - 2x + 12.8$

$x = 5$ $= 16 - 2\times 5 = +6 kN$

$= 3.2 - 2x + 12.8$

$x = 8$ $= 3.2 - 16 + 12.8 = 0$





B.M.D

for AB

$= 3.2x - x^2$

$x = 0$ $M_A = 0$

$x = 1.6$ $M = 3.2(1.6) - 1.6$

$x = 5$ $= 3.2(5) - 5^2 = -9 kNm$

for B.M max

$\frac{dM}{dx} = 3.2x - x^2$

$= 3.2 - 2x = 0$

$x = \frac{3.2}{2} = 1$

for B.M. to be zero in AB

$3.2x - x^2 = 0$

$3.2 - x = 0$

$x = 3.2 m$