Differential Calculus

Indeterminate Forms

$$\frac{0}{0},\ 0\times\infty,\ \frac{\infty}{\infty},\ \infty-\infty,\ 0^0,\ \infty^0,\ 1^\infty$$

Prepared by

Dr. Sunil

NIT Hamirpur (HP)

Indeterminate forms:

If the value of a function f(x) when x = a takes one of the following forms, i.e.

$$\frac{0}{0},\ 0\times\infty,\ \frac{\infty}{\infty},\ \infty-\infty,\ 0^0,\ \infty^0,\ 1^\infty,$$

then the function is said to be in an indeterminate form. The value of the function is obtained by finding the limit of f(x) as x approaches or tends to a. All the indeterminate forms, by a little arrangements or (simplification), can be brought to the form $\frac{0}{0}$.

(i) Problems solved by using different algebraic laws

First of all, we will discuss some problems which can be solved easily by simplifying the given function with the help of different algebraic laws:

Q.No.1.: Prove that $\underset{x\to\infty}{\mathrm{Lt}}\left[\sqrt{x^2+3x+1}-\sqrt{x^2-2x+8}\right]=\frac{5}{2}$ **.**

Sol.: L.H.S. $=\underset{x\to\infty}{\mathrm{Lt}}\left[\sqrt{x^2+3x+1}-\sqrt{x^2-2x+8}\right]$ $\qquad [\infty-\infty \text{ form}]$

Multiplying and dividing by $\left[\sqrt{x^2+3x+1}+\sqrt{x^2-2x+8}\right]$, we get

$$\text{L.H.S.} = \underset{x\to\infty}{\text{Lt}} \frac{\left[\sqrt{x^2+3x+1}-\sqrt{x^2-2x+8}\right]\left[\sqrt{x^2+3x+1}+\sqrt{x^2-2x+8}\right]}{\left[\sqrt{x^2+3x+1}+\sqrt{x^2-2x+8}\right]}$$

$$= \underset{x\to\infty}{\text{Lt}} \frac{\left(x^2+3x+1\right)-\left(x^2-2x+8\right)}{\sqrt{\left(x^2+3x+1\right)}+\sqrt{\left(x^2-2x+8\right)}}$$

$$= \underset{x\to\infty}{\text{Lt}} \frac{5x-7}{\sqrt{\left(x^2+3x+1\right)}+\sqrt{\left(x^2-2x+8\right)}}.$$

Divide the numerator and denominator by x, we get

$$\text{L.H.S.} = \underset{x\to\infty}{\text{Lt}} \frac{5-\frac{7}{x}}{\sqrt{1+\frac{3}{x}+\frac{1}{x^2}}+\sqrt{1-\frac{2}{x}+\frac{8}{x^2}}} = \frac{5-0}{\sqrt{1+0+0}+\sqrt{1-0+0}} = \frac{5}{1+1} = \frac{5}{2} = \text{R.H.S.}$$

This completes the proof.

Q.No.2.: Prove that $\lim\limits_{x\to 2} \dfrac{\sqrt{1+4x}-\sqrt{5+2x}}{x-2} = \dfrac{1}{3}$.

Sol.: L.H.S.= $\lim\limits_{x\to 2} \dfrac{\sqrt{1+4x}-\sqrt{5+2x}}{x-2}$. $\left[\dfrac{0}{0} \text{ form}\right]$

Multiplying and dividing by $\sqrt{1+4x}+\sqrt{5+2x}$, we get

$$\text{L.H.S.} = \lim_{x\to 2} \frac{\sqrt{1+4x}-\sqrt{5+2x}}{x-2} \times \frac{\sqrt{1+4x}+\sqrt{5+2x}}{\sqrt{1+4x}+\sqrt{5+2x}}$$

$$= \lim_{x\to 2} \frac{(1+4x)-(5+2x)}{(x-2)\left[\sqrt{1+4x}+\sqrt{5+2x}\right]} = \lim_{x\to 2} \frac{2(x-2)}{(x-2)\left[\sqrt{1+4x}+\sqrt{5+2x}\right]}$$

$$= \lim_{x\to 2} \frac{2}{\left[\sqrt{1+4x}+\sqrt{5+2x}\right]} = \frac{2}{6} = \frac{1}{3} = \text{R.H.S.}$$

This completes the proof.

Q.No.3.: Prove that $\lim\limits_{x\to 5} \dfrac{5-x}{\sqrt{6x-5}-\sqrt{4x+5}} = -5$.

Sol.: L.H.S.= $\lim\limits_{x\to 5} \dfrac{5-x}{\sqrt{6x-5}-\sqrt{4x+5}}$.

Multiplying and dividing by $\sqrt{6x-5}+\sqrt{4x+5}$, we get

$$\text{L.H.S.}= \lim_{x\to 5}\frac{5-x}{\sqrt{6x-5}-\sqrt{4x+5}}\times\frac{\sqrt{6x-5}+\sqrt{4x+5}}{\sqrt{6x-5}+\sqrt{4x+5}}$$

$$=\lim_{x\to 5}\frac{(5-x)\left[\sqrt{6x-5}+\sqrt{4x+5}\right]}{(6x-5)-(4x+5)}=\lim_{x\to 5}\frac{(5-x)\left[\sqrt{6x-5}+\sqrt{4x-5}\right]}{-2(5-x)}$$

$$=\lim_{x\to 5}\frac{\left[\sqrt{6x-5}+\sqrt{4x-5}\right]}{-2}=\frac{5+5}{-2}=-5=\text{R.H.S.}$$

This completes the proof**.**

Q.No.4.: Evaluate $\lim\limits_{x\to\infty}\dfrac{1^2+2^2+3^2+\ldots\ldots+x^2}{x^3}$**.**

Sol.: $$\lim_{x\to\infty}\frac{1^2+2^2+3^2+\ldots\ldots+x^2}{x^3}=\lim_{x\to\infty}\frac{x(x+1)(2x+1)}{6x^3}\qquad\left[\frac{\infty}{\infty}\text{form}\right]$$

$$=\lim_{x\to\infty}\frac{x^3\left(1+\frac{1}{x}\right)\left(2+\frac{1}{x}\right)}{6x^3}=\frac{1}{3}\text{. Ans.}$$

Q.No.5.: Evaluate $\lim\limits_{n\to\infty}\dfrac{\sum n^3}{n^4}$**.**

Sol.: $$\lim_{n\to\infty}\frac{\sum n^3}{n^4}=\lim_{n\to\infty}\frac{\left[\frac{n(n+1)}{2}\right]^2}{n^4}=\lim_{n\to\infty}\frac{n^2(n+1)^2}{4n^4}=\lim_{n\to\infty}\frac{n^4\left(1+\frac{1}{n}\right)^2}{4n^4}=\frac{1}{4}\text{. Ans.}$$

Q.No.6.: Evaluate $\lim\limits_{x\to a}\dfrac{x^3-a^3}{x^2-a^2}$**.**

Sol.: Since $$\lim_{x\to a}\frac{x^3-a^3}{x^2-a^2}=\lim_{x\to a}\frac{(x-a)\left(x^2+ax+a^2\right)}{(x-a)(x+a)}$$

$$=\lim_{x\to a}\frac{\left(x^2+ax+a^2\right)}{(x+a)}$$

$$=\frac{3a^2}{2a}=\frac{3}{2}a.\quad\text{Ans.}$$

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