Minor I (COL 702)

- Write brief answers. You can assume any result which was used or proved in the class.

1. (**5 marks**) You are given a directed graph $G = (V, E)$. A vertex v is called nice if there is a directed path from v to each of the other vertices in G. Suppose we run DFS on G starting from some vertex s and let w be the vertex with the highest finish time $F[w]$. Prove that if G has a nice vertex, then w is a nice vertex.
2. (**4 marks**) Recall the coin changing problem: you are given (infinite supply of) coins of denomination$c_1, c_2, \ldots, c_k$. Assume $c_1 = 1$ and c_i divides c_{i+1} for $i = 1, \ldots, k-1$. Suppose we want to have a change for an integer amount M and would like to use as few coins as possible. Let c_i be the largest coin denomination which is at most M. Prove that there is an optimal solution which chooses c_i.
3. (**6 marks**) You have n workers. Worker i has skill s_i. You are given a target T. You need to divide the workers into teams of two workers each (assume n is even, and so there will be $n/2$ teams). A team consisting of workers i and j is said to be good if $s_i + s_j \geq T$. Devise an efficient greedy algorithm to divide the workers into teams of two workers each such that the number of good teams is maximized. Prove its correctness in two steps: (i) prove that there is an optimal solution which agrees with the greedy algorithm on the first choice, and (ii) use induction for rest of the input.
4. (**5 marks**) You are given an undirected connected graph G with n vertices and $n + 10$ edges. Each edge has a positive weight, and you can assume that the weights are distinct. Give an $O(n)$ time algorithm to find the minimum spanning tree of G. Justify why your algorithm is correct.