Social Network Analysis

Positive & Negative Relationships

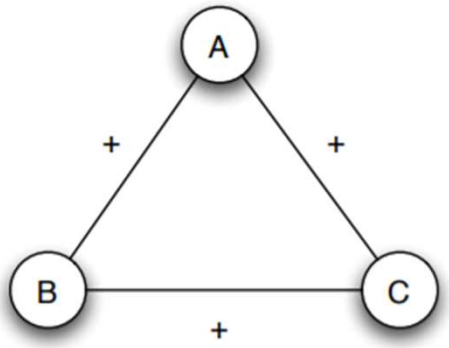
Course Outline

- **Graph Theory and Social Networks**
  - **Positive and Negative Relationships**
- Visualizing Social Networks
- Game Theory
- Information Networks and the World Wide Web
- Network Dynamics
- Applications of SNA in various domains

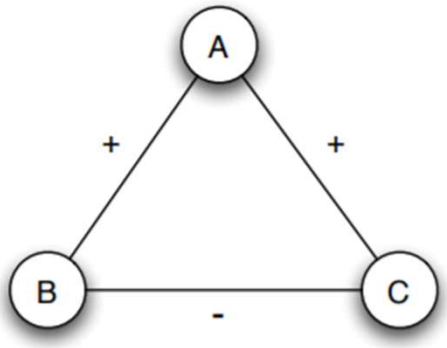
Positive & Negative Relationships

- Required in many scenarios
  - Friends/Enemies
  - International Relations
  - Opinion

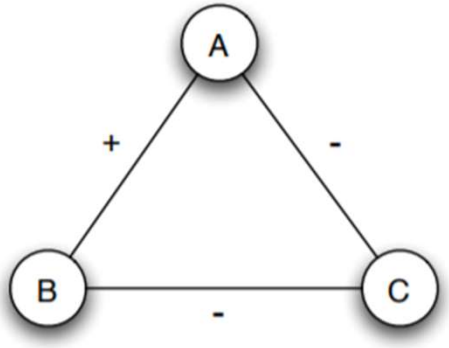
3 Nodes – 4 Scenarios



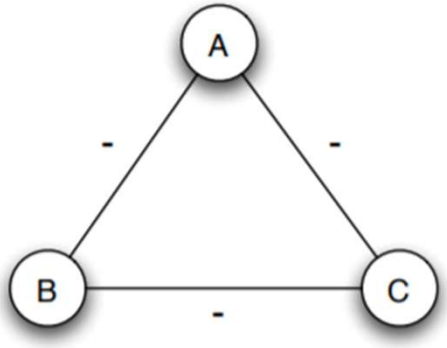
(a) *A, B, and C are mutual friends: balanced.*



(b) *A is friends with B and C, but they don't get along with each other: not balanced.*



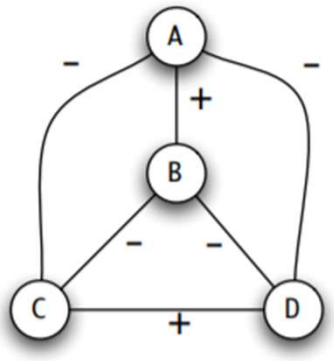
(c) *A and B are friends with C as a mutual enemy: balanced.*



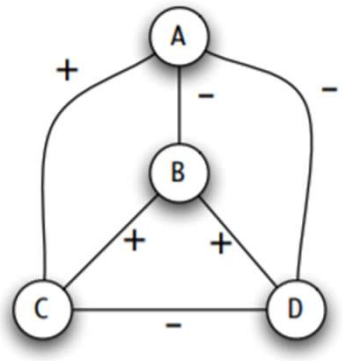
(d) *A, B, and C are mutual enemies: not balanced.*

Structural Balance Property

- A labeled complete graph is balanced if every one of its triangles is balanced — that is, if it obeys the Structural Balance Property
  - For every set of three nodes, if we consider the three edges connecting them, either all three of these edges are labeled +, or else exactly one of them is labeled +.



balanced



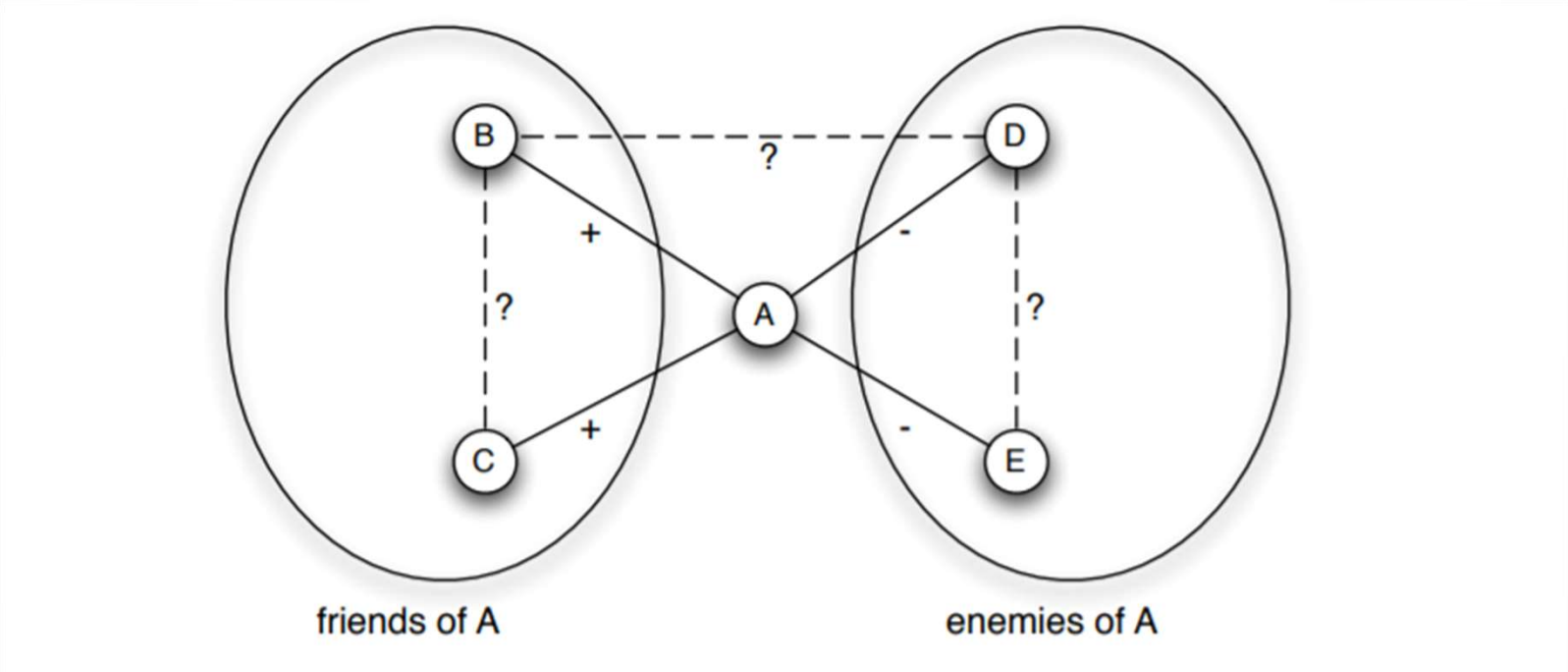
not balanced

Balance Theorem

- If a labeled complete graph is balanced, then:
  - either all pairs of nodes are friends, or
  - else the nodes can be divided into two groups, X and Y , such that every pair of nodes in X like each other, every pair of nodes in Y like each other, and everyone in X is the enemy of everyone in Y

- Another example of:
  - Local effects — phenomena involving only a few nodes at a time — can have global consequences that are observable at the level of the network as a whole.
  - A recurring issue in the analysis of networked systems

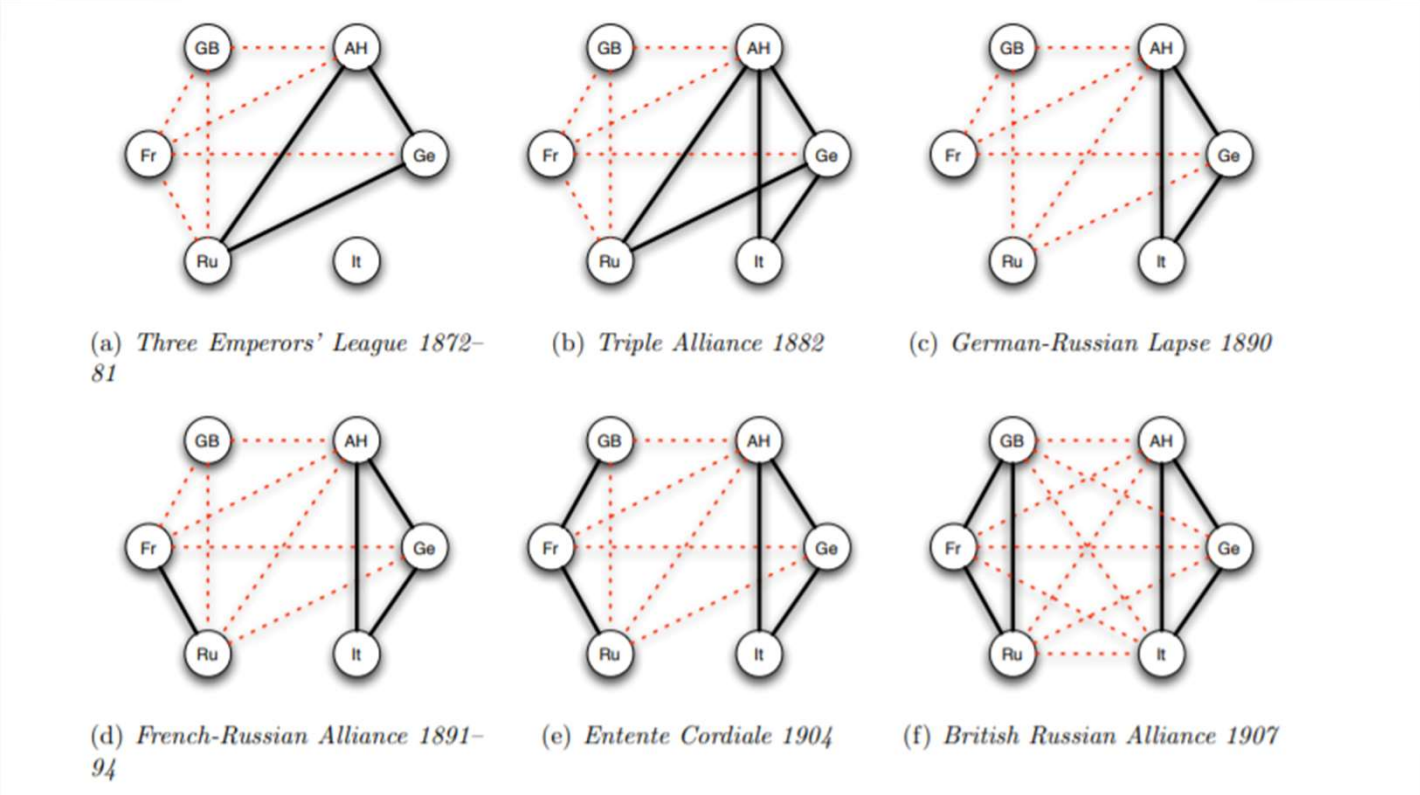
Proof



- To Prove

i. Every two nodes in X are friends
ii. Every two nodes in Y are friends
iii. Every node in X is an enemy of every node in Y

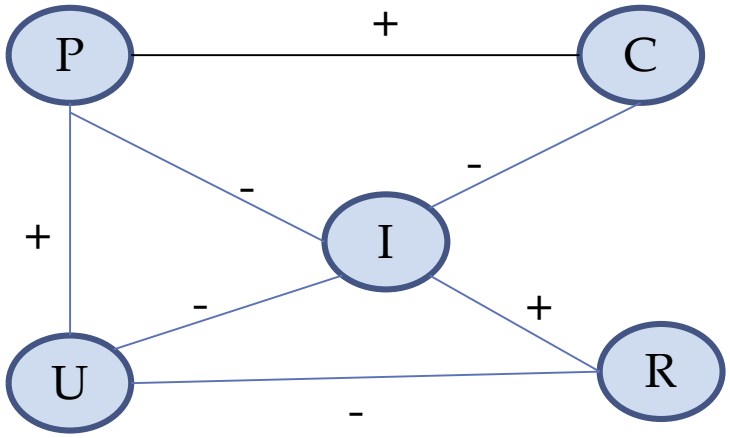
Application of Structural Balance – World War I



(a) *Three Emperors' League 1872–81*
(b) *Triple Alliance 1882*
(c) *German-Russian Lapse 1890*
(d) *French-Russian Alliance 1891–94*
(e) *Entente Cordiale 1904*
(f) *British Russian Alliance 1907*

The evolution of alliances in Europe, 1872-1907 leading to World War I: Positive and Negative relations between Great Britain, France, Russia, Italy, Germany, and Austria-Hungary formed to ensure balance

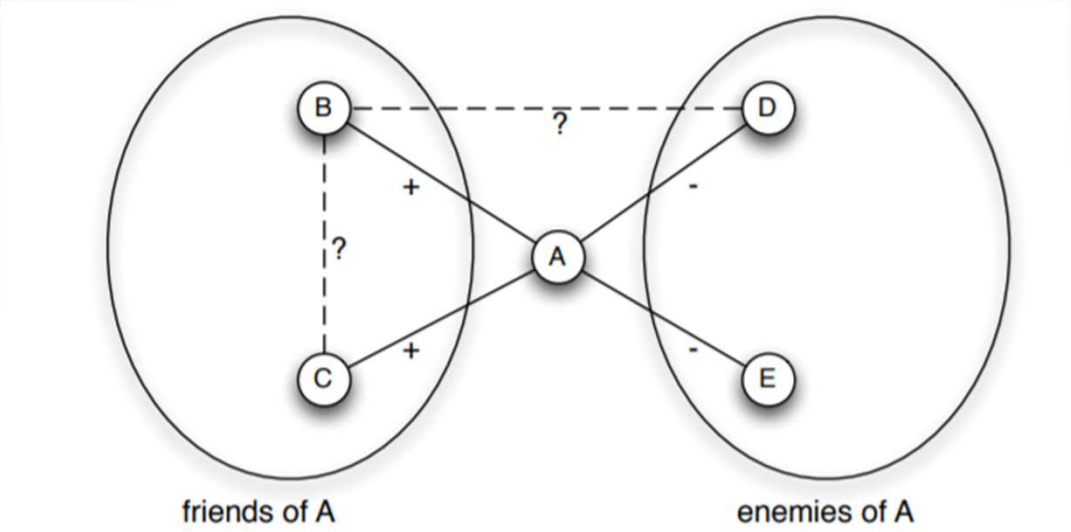
Application of Structural Balance – Formation of Bangladesh



Weakly Balanced Network

- **Weak Structural Balance Property**: There is no set of three nodes such that the edges among them consist of exactly two positive edges and one negative edge
- Characterization of Weakly Balanced Networks: If a labeled complete graph is weakly balanced, then its nodes can be divided into groups in such a way that:
  - Every two nodes belonging to the same group are friend, and
  - Every two nodes belonging to different groups are enemies

Proof



i. All of A's friends are friends with each other. (This way, we have indeed produced a group of mutual friends).

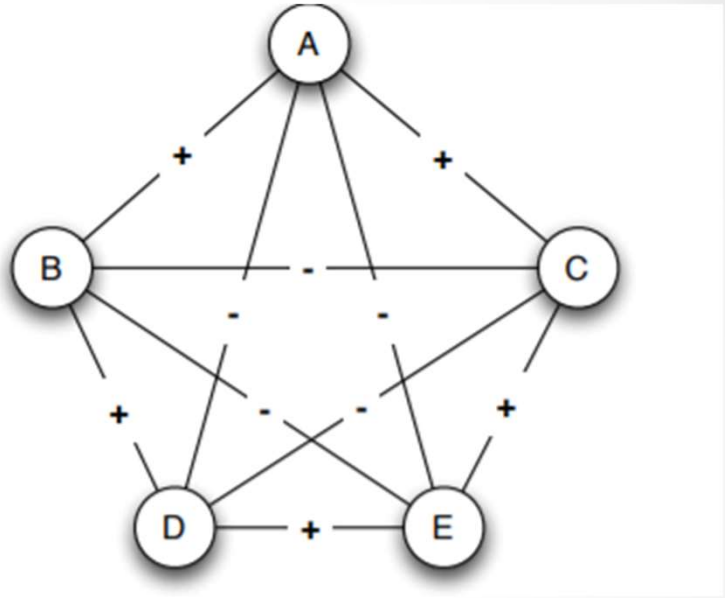
ii. A and all his friends are enemies with everyone else in the graph. (This way, the people in this group will be enemies with everyone in other groups, however we divide up the rest of the graph).

Exercises

- Suppose that a team of anthropologists is studying a set of three small villages that neighbor one another. Each village has 30 people, consisting of 2-3 extended families. Everyone in each village knows all the people in their own village, as well as the people in the other villages. When the anthropologists build the social network on the people in all three villages taken together, they find that each person is friends with all the other people in their own village, and enemies with everyone in the two other villages. This gives them a network on 90 people (i.e., 30 in each village), with positive and negative signs on its edges. According to the definitions in this chapter, is this network on 90 people balanced? Give a brief explanation for your answer.

Exercises

- Each edge in this network participates in three triangles: one formed by each of the additional nodes who is not already an endpoint of the edge. (For example, the A-B edge participates in a triangle on A, B, and C, a triangle on A, B, and D, and a triangle on A, B, and E. We can list triangles for the other edges in a similar way.) For each edge, how many of the triangles it participates in are balanced, and how many are unbalanced.



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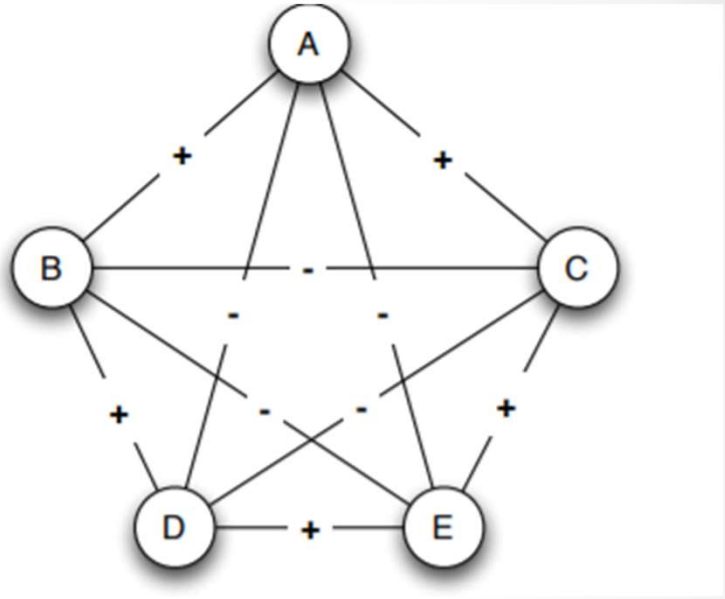

ABC: N ABD: N ABE: Y
ACD: Y ACE: N ADE: Y
BCD: N BCE: Y BDE: Y
CDE: N

References

1. https://www.cs.cornell.edu/home/kleinber/networks-book/
   - **Chapter 5**