Roll No. | | | | | | | | | | Total No. of Pages :03

Total No. of Questions :18

**B.Tech.(Artificial Intelligence & Machine Learning/Artificial Intelligence/ Artificial Intelligence & Data Science/ Computer Engg./Computer Science & Engg./Artificial Intelligence & Machine Learning/Computer Science & Engg.) (Cyber Security)/Computer Science & Engg.) (Data Science)/Computer Science & Engg.) (IOT)/Data Science/Electronics & Communication Engg./Information Technology/Mechanical Engg./CSE (Internet of Things and Cyber Security including Block Chain Technology)/B.Tech. (Computer Engg./CSE) (PIT) (Sem.–1)**

# MATHEMATICS-I

**Subject Code : BTAM-104-18**

**M.Code : 75362**

**Date of Examination : 01-07-22**

**Time : 3 Hrs.** **Max. Marks : 60**

**INSTRUCTIONS TO CANDIDATES :**

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B &C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B& C carrying EIGHT marks each.**
4. **Select atleast TWO questions EACH from SECTION - B &C.**

## SECTION-A



**Solve the following :**

1. Solve $\int_0^2 \sqrt{x}\,(4-x^2)^{-\frac{1}{4}}\,dx$

2. Write recurrence relation of Gamma function.

3. Find the area of the region enclosed by the curve $x^2 = 4y, y^2 = 4x$.

4. Evaluate $\lim\limits_{x\to\infty} \dfrac{\sin h^{-1}x}{\cos h^{-1}x}$.

5. Calculate approximate value of $\sqrt{24}$ to two decimal places by Taylor's theorem.

6. Find the value of K so that the set of vectors (K,1,1), (0,1,1), $(K,0,K)$ is linearly dependent?

7. Find a linear transformation T: $R^2 \to R^2$ such that $T(1,0) = (1,1)$ and $T(0,1)=(-1,2)$.

8. Show that two similar matrices A and B have same eigen values.

9. If A is an orthogonal matrix, then prove that $|A| = \pm 1$.

10. Define Vector space.

## SECTION-B

11. a) State and prove relation between Beta and Gamma functions.

b) Expand $f(x) = e^{\sin x}$ by Maclaurin's theorem.

12. a) Prove that the area of the region bounded by the curve $a^4 y^2 = x^5(2a - x)$ is to that of the circle whose radius is 'a' is 5:4.

b) Find absolute maximum and minimum value of $f(x) = x - \log x$ on $\left[\frac{1}{2}, 2\right]$.

13. a) Find rank of $A = \begin{bmatrix} 2 & -6 & -2 & -3 \\ -5 & -13 & -4 & -7 \\ -1 & 4 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

b) Solve by Gauss Jordan method the system of equations

$$x + 2y + z = 2,\ 3x + y - 2z = 1,\ \ 4x - 3y - z = 3,\ \ 2x + 4y + 2z = 4.$$

14. a) Find the inverse of a matrix $\begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 3 & 2 & -3 \\ -1 & 2 & 1 & -1 \\ 2 & -3 & -1 & 4 \end{bmatrix}$.

b) Using properties of determinants, evaluate $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$

## SECTION-C

15. a) Diagonalize $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

b) Find Eigen Values & Eigen Vectors of $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

16. a) Express the matrix A as sum of symmetric and skew symmetric matrix where $A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$

b) Prove that $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is orthogonal.

17. a) Let $T: R^3 \rightarrow R^3$ defined by

$$T(x,y,z) = (x + y + z,\ 2x + 2y + 2z,\ 3x + 3y + 3z)$$

Find the associated matrix corresponding to standardbasis.

b) Find the rank and nullity of the matrix $\begin{bmatrix} 1 & -2 & 2 & 3 & 6 \\ 0 & -1 & -3 & 1 & 1 \\ -2 & 4 & -3 & -6 & 11 \end{bmatrix}$.

18. Determine the coordinate vectors of $p = 4 - 2x + 3x^2$ relative to the following bases.

a) The standard basis for$P_2$, $S = \{1, x, x^2\}$.

b) The basis for $P_2$, $A = \{p_1, p_2, p_3\}$, where $p_1 = 2, p_2 = -4x, p_3 = 5x^2 - 1$.

**NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.**