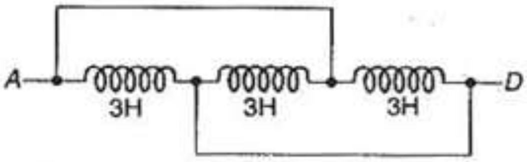
1. The inductance between A and D is :



(a) 3.66 H (b) 9 H
(c) 0.66 H (d) 1 H

2. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be :

(a) E (b) $E/\sqrt{2}$
(c) $E/2$ (d) zero

3. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then :

(a) $v_B > v_A$
(b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses

4. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest ?

(a) 1 cm (b) 2 cm
(c) 3 cm (d) 4 cm

5. If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, then the satellite will :

(a) continue to move in its orbit with same velocity
(b) move tangentially to the original orbit with the same velocity
(c) become stationary in its orbit
(d) move towards the earth

6. If an ammeter is to be used in place of a voltmeter, then we must connect with the ammeter a :

(a) low resistance in parallel
(b) high resistance in parallel
(c) high resistance in series
(d) low resistance in series

7. If in a circular coil A of radius R, current i is flowing and in another coil B of radius $2R$ a current $2i$ is flowing, then the ratio of the magnetic fields, B_A and B_B produced by them will be :

(a) 1 (b) 2
(c) 1/2 (d) 4

8. If two mirrors are kept at 60° to each other, then the number of images formed by them is :

(a) 5 (b) 6
(c) 7 (d) 8

9. A wire when connected to 220 V mains supply has power dissipation P_1. Now the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2. Then $P_2 : P_1$ is :

(a) 1 (b) 4
(c) 2 (d) 3

10. If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from $n = 2$ is :

(a) 10.2 eV (b) 0 eV
(c) 3.4 eV (d) 6.8 eV

11. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tubes A and B is :

(a) 1 : 2 (b) 1 : 4
(c) 2 : 1 (d) 4 : 1

12. A tuning fork arrangement (pair) produces 4 beats/s with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/s. The frequency of the unknown fork is :

(a) 286 cps (b) 292 cps
(c) 294 cps (d) 288 cps

13. A wave $y = a \sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is :
(a) $y = a \sin(\omega t + kx)$
(b) $y = -a \sin(\omega t + kx)$
(c) $y = a \sin(\omega t - kx)$
(d) $y = -a \sin(\omega t - kx)$

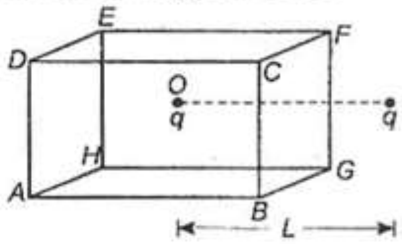
14. On moving a charge of 20 C by 2 cm, 2 J of work is done, then the potential difference between the points is :
(a) 0.1 V (b) 8 V
(c) 2 V (d) 0.5 V

15. If an electron and a proton having same momenta enter perpendicularly to a magnetic field, then :
(a) curved path of electron and proton will be same (ignoring the sense of revolution)
(b) they will move undeflected
(c) curved path of electron is more curved than that of proton
(d) path of proton is more curved

16. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is :
(a) $GMm/12R^2$ (b) $GMm/3R^2$
(c) $GMm/8R$ (d) $GMm/6R$

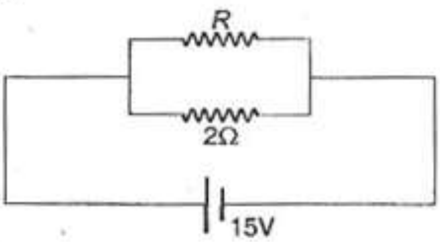
17. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be :
(a) $T\sqrt{n}$ (b) $\dfrac{T}{\sqrt{n}}$
(c) nT (d) T

18. A charged particle q is placed at the centre O of cube of length L ($ABCDEFGH$). Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is :



(a) $q/4\pi\varepsilon_0 L$ (b) zero
(c) $q/2\pi\varepsilon_0 L$ (d) $q/3\pi\varepsilon_0 L$

19. If in the circuit, power dissipation is 150 W, then R is :



(a) 2Ω (b) 6Ω
(c) 5Ω (d) 4Ω

20. Wavelength of light used in an optical instrument are $\lambda_1 = 4000$ Å and $\lambda_2 = 5000$ Å, then ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is :
(a) 16 : 25 (b) 9 : 1
(c) 4 : 5 (d) 5 : 4

21. Two identical particles move towards each other with velocity $2v$ and v respectively. The velocity of centre of mass is :
(a) v (b) $v/3$
(c) $v/2$ (d) zero

22. If a current is passed through a spring then the spring will :
(a) expand (b) compress
(c) remain same (d) none of these

23. Heat given to a body which raises its temperature by 1°C is :
(a) water equivalent
(b) thermal capacity
(c) specific heat
(d) temperature gradient

24. At absolute zero, Si acts as :
(a) non-metal (b) metal
(c) insulator (d) none of these

25. Electromagnetic waves are transverse in nature is evident by :
(a) polarization (b) interference
(c) reflection (d) diffraction

26. Which of the following is used in optical fibres ?
(a) Total internal reflection
(b) Scattering
(c) Diffraction
(d) Refraction

27. The escape velocity of a body depends upon mass as :
(a) m^0 (b) m^1
(c) m^2 (d) m^3

28. Which of the following are not electromagnetic waves ?
(a) Cosmic-rays (b) γ-rays
(c) β-rays (d) X-rays

29. Identify the pair whose dimensions are equal :
(a) Torque and work (b) Stress and energy
(c) Force and stress (d) Force and work

30. If θ_i is the inversion temperature, θ_n is the neutral temperature, θ_c is the temperature of the cold junction then :
(a) $\theta_i + \theta_c = \theta_n$ (b) $\theta_i - \theta_c = 2\theta_n$
(c) $\dfrac{\theta_i + \theta_c}{2} = \theta_n$ (d) $\theta_c - \theta_i = 2\theta_n$

31. Infrared radiations are detected by :
(a) spectrometer (b) pyrometer
(c) nanometer (d) photometer

32. If N_0 is the original mass of the substance of half-life period $t_{1/2} = 5$ years, then the amount of substance left after 15 years is :
(a) $\frac{N_0}{8}$ (b) $\frac{N_0}{16}$
(c) $\frac{N_0}{2}$ (d) $\frac{N_0}{4}$

33. By increasing the temperature, the specific resistance of a conductor and a semiconductor :
(a) increases for both
(b) decreases for both
(c) increases, decreases respectively
(d) decreases, increases respectively

34. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to :
(a) CV (b) $\frac{1}{2}nCV^2$
(c) CV^2 (d) $\frac{1}{2n}CV^2$

35. Which of the following is more close to a black body ?
(a) Black board paint (b) Green leaves
(c) Black holes (d) Red roses

36. Which statement is incorrect ?
(a) All reversible cycles have same efficiency
(b) Reversible cycle has more efficiency than an irreversible one
(c) Carnot cycle is a reversible one
(d) Carnot cycle has the maximum efficiency in all cycles

37. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it, is :
(a) 20 (b) 80
(c) 40 (d) 120

38. The power factor of an AC circuit having resistance R and inductance L (connected in series) and an angular velocity ω is :
(a) $\frac{R}{\omega L}$ (b) $\frac{R}{(R^2 + \omega^2 L^2)^{1/2}}$
(c) $\frac{\omega L}{R}$ (d) $\frac{R}{(R^2 - \omega^2 L^2)^{1/2}}$

39. An astronomical telescope has a large aperture to :
(a) reduce spherical aberration
(b) have high resolution
(c) increase span of observation
(d) have low dispersion

40. The kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity is :
(a) $\frac{mgR}{2}$ (b) $2mgR$
(c) mgR (d) $\frac{mgR}{4}$

41. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will :
(a) increase
(b) decrease
(c) remain same
(d) decrease for some, while increase for others

42. When temperature increases, the frequency of a tuning fork :
(a) increases
(b) decreases
(c) remains same
(d) increases or decreases depending on the material

43. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should :
(a) increase
(b) remain unchanged
(c) decrease
(d) first increase then decrease

44. The energy band gap is maximum in :
(a) metals (b) superconductors
(c) insulators (d) semiconductors

45. The part of a transistor which is most heavily doped to produce large number of majority carriers is :
(a) emitter
(b) base
(c) collector
(d) can be any of the above three

46. In a simple harmonic oscillator, at the mean position :
(a) kinetic energy is minimum, potential energy is maximum
(b) both kinetic and potential energies are maximum
(c) kinetic energy is maximum, potential energy is minimum
(d) both kinetic and potential energies are minimum

47. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc ?

(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$

(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$

48. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is :

(a) 60 (b) 30
(c) 15 (d) 25

49. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom, is :

(a) 10 (b) 20
(c) 25.5 (d) 5

50. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is :

(a) 16 J (b) 8 J
(c) 32 J (d) 24 J

51. A child swinging on a swing in sitting position, stands up, then the time period of the swing will :

(a) increase
(b) decrease
(c) remain same
(d) increase if the child is long and decrease if the child is short

52. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively :

(a) g, g (b) $g-a, g-a$
(c) $g-a, g$ (d) a, g

53. The mass of a product liberated on anode in an electrochemical cell depends on :

(a) $(It)^{1/2}$ (b) It
(c) I/t (d) I^2t

(where t is the time period for which the current is passed)

54. At what temperature is the rms velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C ?

(a) 80 K (b) –73 K
(c) 3 K (d) 20 K

55. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its :

(a) speed (b) mass
(c) charge (d) magnetic induction

56. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling) :

(a) solid sphere (b) hollow sphere
(c) ring (d) all same

57. In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4 A, then that in the secondary is :

(a) 4 A (b) 2 A
(c) 6 A (d) 10 A

58. Even Carnot engine cannot give 100% efficiency because we cannot :

(a) prevent radiation
(b) find ideal sources
(c) reach absolute zero temperature
(d) eliminate friction

59. Moment of inertia of a circular wire of mass M and radius R about its diameter is :

(a) $MR^2/2$ (b) MR^2
(c) $2MR^2$ (d) $MR^2/4$

60. When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed then the acceleration of the particle is :

(a) F_1/m (b) F_2F_3/mF_1
(c) $(F_2-F_3)/m$ (d) F_2/m

61. Two forces are such that the sum of their magnitudes is 18 N and their resultant which has magnitude 12 N, is perpendicular to the smaller force. Then the magnitudes of the forces are :

(a) 12 N, 6 N (b) 13 N, 5 N
(c) 10 N, 8 N (d) 16 N, 2 N

62. Speeds of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances at which the two cars are stopped from that instant is :

(a) 1 : 1 (b) 1 : 4
(c) 1 : 8 (d) 1 : 16

63. 1 mole of a gas with $\gamma = 7/5$ is mixed with 1 mole of a gas with $\gamma = 5/3$, then the value of γ for the resulting mixture is :

(a) 7/5 (b) 2/5
(c) 24/16 (d) 12/7

64. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium then the value of q is :

(a) $Q/2$ (b) $-Q/2$
(c) $Q/4$ (d) $-Q/4$

65. Capacitance (in F) of a spherical conductor having radius 1 m, is :

(a) 1.1×10^{-10} (b) 10^{-6}
(c) 9×10^{-9} (d) 10^{-3}

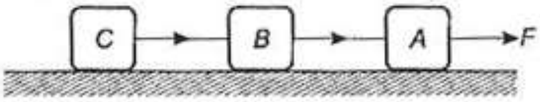
66. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is :

(a) 8 : 1 (b) 9 : 7
(c) 4 : 3 (d) 5 : 3

67. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is :

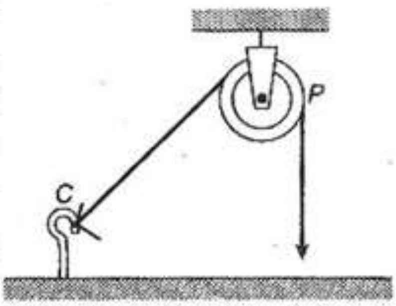
(a) 1 : 1 (b) 16 : 1
(c) 4 : 1 (d) 1 : 9

68. Three identical blocks of masses $m = 2$ kg are drawn by a force $F = 10.2$ N with an acceleration of 0.6 ms^{-2} on a frictionless surface, then what is the tension (in N) in the string between the blocks B and C ?



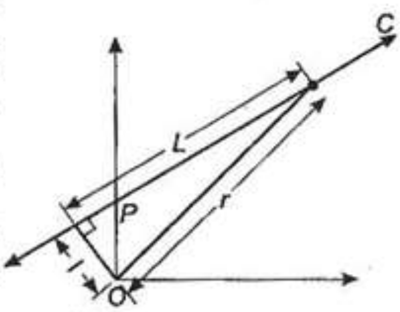
(a) 9.2 (b) 7.8
(c) 4 (d) 9.8

69. One end of massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 360 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 60 kg climb on the rope ?



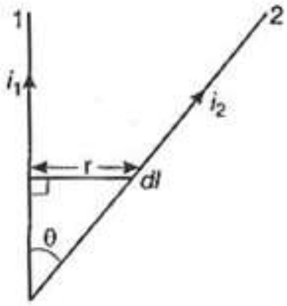
(a) 16 (b) 6
(c) 4 (d) 8

70. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about O ?



(a) mvL (b) mvl
(c) mvr (d) Zero

71. Wires 1 and 2 carrying currents i_1 and i_2 respectively are inclined at an angle θ to each other. What is the force on a small element dl of wire 2 at a distance r from wire 1 (as shown in figure) due to the magnetic field of wire 1 ?



(a) $\frac{\mu_0}{2\pi r} i_1 i_2 \, dl \tan\theta$ (b) $\frac{\mu_0}{2\pi r} i_1 i_2 \, dl \sin\theta$
(c) $\frac{\mu_0}{2\pi r} i_1 i_2 \, dl \cos\theta$ (d) $\frac{\mu_0}{4\pi r} i_1 i_2 \, dl \sin\theta$

72. At a specific instant emission of radioactive compound is deflected in a magnetic field. The compound can emit :

(i) electrons (ii) protons
(iii) He^{2+} (iv) neutrons

The emission at the instant can be :

(a) i, ii, iii (b) i, ii, iii, iv
(c) iv (d) ii, iii

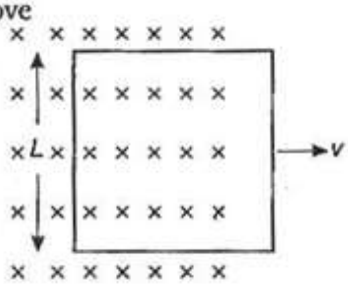
73. Sodium and copper have work functions 2.3 eV and 4.5 eV respectively. Then the ratio of the wavelengths is nearest to :

(a) 1 : 2 (b) 4 : 1
(c) 2 : 1 (d) 1 : 4

74. Formation of covalent bonds in compounds exhibits :

(a) wave nature of electron
(b) particle nature of electron
(c) both wave and particle nature of electron
(d) none of the above

75. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is :



(a) zero (b) RvB
(c) $\frac{vBL}{R}$ (d) vBL

76. Which of the following is a redox reaction ?
(a) $NaCl + KNO_3 \longrightarrow NaNO_3 + KCl$
(b) $CaC_2O_4 + 2HCl \longrightarrow CaCl_2 + H_2C_2O_4$
(c) $Ca(OH)_2 + 2NH_4Cl \longrightarrow CaCl_2 + 2NH_3 + 2H_2O$
(d) $2K[Ag(CN)_2] + Zn \longrightarrow 2Ag + K_2[Zn(CN)_4]$

77. For an ideal gas, number of mol per litre in terms of its pressure P, temperature T and gas constant R is :
(a) $\frac{PT}{R}$ (b) PRT
(c) $\frac{P}{RT}$ (d) $\frac{RT}{P}$

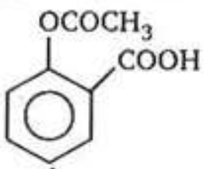
78. Number of P—O bonds in P_4O_{10} is :
(a) 17 (b) 16
(c) 15 (d) 6

79. KO_2 is used in space and submarines because it :
(a) absorbs CO_2 and increases O_2 concentration
(b) absorbs moisture
(c) absorbs CO_2
(d) produces ozone

80. Which of the following ions has the maximum magnetic moment ?
(a) Mn^{2+} (b) Fe^{2+}
(c) Ti^{2+} (d) Cr^{2+}

81. Acetylene does not react with :
(a) Na (b) ammoniacal $AgNO_3$
(c) HCl (d) NaOH

82. Compound A given below is :



A

(a) antiseptic (b) antibiotic
(c) analgesic (d) pesticide

83. For the following cell with hydrogen electrodes at two different pressures p_1 and p_2

$$\underset{p_1}{Pt(H_2)} \mid \underset{1M}{H^+(aq)} \mid \underset{p_2}{Pt(H_2)}$$

emf is given by :
(a) $\frac{RT}{F}\log_e\frac{p_1}{p_2}$ (b) $\frac{RT}{2F}\log_e\frac{p_1}{p_2}$
(c) $\frac{RT}{F}\log_e\frac{p_2}{p_1}$ (d) $\frac{RT}{2F}\log_e\frac{p_2}{p_1}$

84. Acetylene reacts with hypochlorous acid to form :
(a) Cl_2CHCHO (b) $ClCH_2COOH$
(c) CH_3COCl (d) $ClCH_2CHO$

85. On heating benzyl amine with chloroform and ethanolic KOH, product obtained is :
(a) benzyl alcohol (b) benzaldehyde
(c) benzonitrile (d) benzyl isocyanide

86. Which of the following reaction is possible at anode ?
(a) $F_2 + 2e^- \longrightarrow 2F^-$
(b) $2H^+ + \frac{1}{2}O_2 + 2e^- \longrightarrow H_2O$
(c) $2Cr_2^{3+} + 7H_2O \longrightarrow Cr_2O_7^{2-} + 14H^+ + 6e^-$
(d) $Fe^{2+} \longrightarrow Fe^{3+} + e^-$

87. Which of the following concentration factor is affected by change in temperature ?
(a) Molarity (b) Molality
(c) Mole fraction (d) Weight fraction

88. Cyanide process is used for the extraction of :
(a) barium (b) silver
(c) boron (d) zinc

89. Following reaction

$$(CH_3)_3CBr + H_2O \longrightarrow (CH_3)_3COH + HBr$$

is an example of :
(a) elimination reaction
(b) free radical substitution
(c) nucleophilic substitution
(d) electrophilic substitution

90. A metal M forms water soluble MSO_4 and inert MO. MO in aqueous solution forms insoluble $M(OH)_2$ soluble in NaOH. Metal M is :
(a) Be (b) Mg
(c) Ca (d) Si

91. Half-life of a substance A following first order kinetics is 5 days. Starting with 100g of A, amount left after 15 days is :
(a) 25 g (b) 50 g
(c) 12.5 g (d) 6.25 g

92. The most stable ion is :
(a) $[Fe(OH)_5]^{3-}$ (b) $[FeCl_6]^{3-}$
(c) $[Fe(CN)_6]^{3-}$ (d) $[Fe(H_2O)_6]^{3+}$

93. A substance forms Zwitter ion. It can have functional groups :
(a) $-NH_2$, $-COOH$ (b) $-NH_2$, $-SO_3H$
(c) both (a) and (b) (d) none of these

94. If Fe^{3+} and Cr^{3+} both are present in group III of qualitative analysis, then distinction can be made by :
(a) addition of NH_4OH in presence of NH_4Cl when only $Fe(OH)_3$ is precipitated
(b) addition of NH_4OH in presence of NH_4Cl when $Cr(OH)_3$ and $Fe(OH)_3$ both are precipitated and on adding Br_2 water and NaOH, $Cr(OH)_3$ dissolves
(c) precipitate of $Cr(OH)_3$ and $Fe(OH)_3$ as obtained in (b) are treated with conc. HCl when only $Fe(OH)_3$ dissolves
(d) both (b) and (c)

95. In an organic compound of molar mass 108 g mol^{-1} C, H and N atoms are present in 9 : 1 : 3.5 by weight. Molecular formula can be :
(a) $C_6H_8N_2$ (b) $C_7H_{10}N$
(c) $C_5H_6N_3$ (d) $C_4H_{18}N_3$

96. Solubility of $Ca(OH)_2$ is s mol L^{-1}. The solubility product (K_{sp}) under the same condition is :
(a) $4s^3$ (b) $3s^4$
(c) $4s^2$ (d) s^3

97. Heat required to raise the temperature of 1 mole of a substance by 1° is called :
(a) specific heat
(b) molar heat capacity
(c) water equivalent
(d) specific gravity

98. β-particle is emitted in a radioactive reaction when :
(a) a proton changes to neutron
(b) a neutron changes to proton
(c) a neutron changes to electron
(d) an electron changes to neutron

99. In a mixture of A and B, components show negative deviation when :
(a) A—B interaction is stronger than A—A and B—B interaction
(b) A—B interaction is weaker than A—A and B—B interaction
(c) $\Delta V_{mix} > 0$, $\Delta S_{mix} > 0$
(d) $\Delta V_{mix} = 0$, $\Delta S_{mix} > 0$

100. Refining of impure copper with zinc impurity is to be done by electrolysis using electrodes as :

	Cathode	Anode
(a)	pure copper	pure zinc
(b)	pure zinc	pure copper
(c)	pure copper	impure copper
(d)	pure zinc	impure zinc

101. Aluminium is extracted by the electrolysis of :
(a) alumina
(b) bauxite
(c) molten cryolite
(d) alumina mixed with molten cryolite

102. For an aqueous solution, freezing point is – 0.186°C. Elevation of the boiling point of the same solution is ($K_f = 1.86°$ mol^{-1} kg and $K_b = 0.512°$ mol^{-1} kg) :
(a) 0.186° (b) 0.0512°
(c) 1.86° (d) 5.12°

103. Underlined carbon is sp^3 hybridised in :
(a) $CH_3\underline{C}H{=}CH_2$ (b) $CH_3\underline{C}H_2NH_2$
(c) $CH_3\underline{C}ONH_2$ (d) $CH_3CH_2\underline{C}N$

104. Bond angle of 109°28′ is found in :
(a) NH_3 (b) H_2O
(c) $\overset{\oplus}{C}H_5$ (d) $\overset{\oplus}{N}H_4$

105. For a reaction $A + 2B \longrightarrow C$, rate is given by $+\frac{d[C]}{dt} = k[A][B]$, hence the order of the reaction is :
(a) 3 (b) 2
(c) 1 (d) 0

106. CH_3MgI is an organometallic compound due to :
(a) Mg—I bond (b) C—I bond
(c) C—Mg bond (d) C—H bond

107. One of the following species acts as both Bronsted acid and base :
(a) $H_2PO_2^-$ (b) HPO_3^{2-}
(c) HPO_4^{2-} (d) all of these

108. Hybridisation of the underline atom changes in :
(a) $\underline{Al}H_3$ changes to AlH_4^-
(b) $H_2\underline{O}$ changes to H_3O^+
(c) $\underline{N}H_3$ changes to NH_4^+
(d) in all cases

109. Racemic mixture is formed by mixing two :
(a) isomeric compounds
(b) chiral compounds
(c) meso compounds
(d) enantiomers with chiral carbon

110. The number of lone pairs on Xe in XeF_2, XeF_4 and XeF_6 respectively are :
(a) 3, 2, 1 (b) 2, 4, 6
(c) 1, 2, 3 (d) 6, 4, 2

111. An aqueous solution of 1M NaCl and 1M HCl is :
(a) not a buffer but pH < 7
(b) not a buffer but pH > 7
(c) a buffer with pH < 7
(d) a buffer with pH > 7

112. Consider following two reactions

$$A \longrightarrow \text{Product} \quad -\frac{d[A]}{dt} = k_1 [A]^0$$

$$B \longrightarrow \text{Product} \quad -\frac{d[B]}{dt} = k_2 [B]$$

k_1 and k_2 are expressed in terms of molarity (mol L^{-1}) and time (s^{-1}) as :

(a) s^{-1}, $M\ s^{-1}\ L^{-1}$ (b) $M\ s^{-1}$, $M\ s^{-1}$
(c) s^{-1}, $M^{-1}\ s^{-1}$ (d) $M\ s^{-1}$, s^{-1}

113. RNA contains :
(a) ribose sugar and thymine
(b) ribose sugar and uracil
(c) deoxyribose sugar and uracil
(d) deoxyribose sugar and thymine

114. For a cell given below :

$$Ag \mid Ag^+ \mid\mid Cu^{2+} \mid Cu$$
$$- \qquad\qquad +$$

$$Ag^+ + e^- \longrightarrow Ag \qquad E^\circ = x$$
$$Cu^{2+} + 2e^- \longrightarrow Cu, \qquad E^\circ = y$$

E° cell is :
(a) $x + 2y$ (b) $2x + y$
(c) $y - x$ (d) $y - 2x$

115. Based on kinetic theory of gases following laws can be proved :
(a) Boyle's law (b) Charles' law
(c) Avogadro's law (d) all of these

116. MnO_4^- is a good oxidising agent in different medium changing to

$$MnO_4^- \longrightarrow Mn^{2+}$$
$$\longrightarrow MnO_4^{2-}$$
$$\longrightarrow MnO_2$$
$$\longrightarrow Mn_2O_3$$

Changes in oxidation number respectively are :
(a) 1, 3, 4, 5 (b) 5, 4, 3, 2
(c) 5, 1, 3, 4 (d) 2, 6, 4, 3

117. For the reaction : $H_2 + I_2 \longrightarrow 2HI$, the differential rate law is :

(a) $-\frac{d[H_2]}{dt} = -\frac{d[I_2]}{dt} = 2\frac{d[HI]}{dt}$

(b) $-2\frac{d[H_2]}{dt} = -2\frac{d[I_2]}{dt} = \frac{d[HI]}{dt}$

(c) $-\frac{d[H_2]}{dt} = -\frac{d[I_2]}{dt} = \frac{d[HI]}{dt}$

(d) $-\frac{d[H_2]}{2dt} = -\frac{d[I_2]}{2dt} = \frac{d[HI]}{dt}$

118. Number of atoms in 560g of Fe (atomic mass 56 g mol^{-1}) is :
(a) twice that of 70 g N
(b) half that of 20 g H
(c) both (a) and (b)
(d) none of the above

119. Geometrical isomerism is not shown by :
(a) 1, 1-dichloro-1-pentene
(b) 1, 2-dichloro-1-pentene
(c) 1, 3-dichloro-2-pentene
(d) 1, 4-dichloro-2-pentene

120. Number of atoms in the unit cell of Na (BCC type crystal) and Mg (FCC type crystal) are respectively :
(a) 4, 4 (b) 4, 2
(c) 2, 4 (d) 1, 1

121. Which of the following compounds has incorrect IUPAC nomenclature ?

(a) $CH_3CH_2CH_2COC_2H_5$ (C=O on the carbonyl carbon)
(Ethylbutanoate)

(b) CH_3CHCH_2CHO with CH_3 on the second carbon
3-methyl butanal

(c) $CH_3CHCCH_2CH_3$ with CH_3 on the second carbon and C=O on the third carbon
2-methyl-3-pentanone

(d) $CH_3CHCHCH_3$ with H_3C and OH on the second and third carbons
2-methyl-3-butanol

122. End product of the following reaction is :

$$CH_3CH_2COOH \xrightarrow[\text{red P}]{Cl_2} \xrightarrow{\text{alcoholic KOH}}$$

(a) $CH_3CH(OH)COOH$ (b) $CH_2(OH)CH_2COOH$
(c) $CH_2{=}CHCOOH$ (d) $CH_2(Cl)CH(OH)COOH$

123. For the following reaction in gaseous phase

$$CO + \frac{1}{2}O_2 \longrightarrow CO_2 \qquad K_c/K_p \text{ is :}$$

(a) $(RT)^{1/2}$ (b) $(RT)^{-1/2}$
(c) (RT) (d) $(RT)^{-1}$

124. Energy of H-atom in the ground state is –13.6 eV, hence energy in the second excited state is :
(a) – 6.8 eV (b) – 3.4 eV
(c) – 1.51 eV (d) – 4.53 eV

125. A square planar complex is formed by hybridisation of the following atomic orbitals :

(a) s, p_x, p_y, p_z

(b) s, p_x, p_y, p_z, d

(c) d, s, p_x, p_y

(d) s, p_x, p_y, p_z, d, d

126. Type of isomerism shown by $[Cr(NH_3)_5NO_2]Cl_2$ is :

(a) optical (b) ionisation

(c) geometrical (d) linkage

127. One of the following equilibria is not affected by change in volume of the flask :

(a) $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

(b) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

(c) $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

(d) $SO_2Cl_2(g) \rightleftharpoons SO_2(g) + Cl_2(g)$

128. Uncertainty in position of a particle of 25 g in space is 10^{-5} m. Hence, uncertainty in velocity (ms^{-1}) is (Planck's constant $h = 6.6 \times 10^{-34}$ Js) :

(a) 2.1×10^{-28} (b) 2.1×10^{-34}

(c) 0.5×10^{-34} (d) 5.0×10^{-24}

129. Consider the following reactions at 1100°C

(I) $2C + O_2 \longrightarrow 2CO, \Delta G° = -460$ k J mol^{-1}

(II) $2Zn + O_2 \longrightarrow 2ZnO,$
$\Delta G° = -360$ k J mol^{-1}

Based on these, select correct alternate :

(a) zinc can be oxidised by CO

(b) zinc oxide can be reduced by carbon

(c) both (a) and (b)

(d) none is the correct

130. A reaction is non-spontaneous at the freezing point of water but is spontaneous at the boiling point of water then :

	ΔH	ΔS
(a)	+ve	+ve
(b)	– ve	– ve
(c)	– ve	+ve
(d)	+ve	– ve

131. Monomers are converted to polymer by :

(a) hydrolysis of monomers

(b) condensation reaction between monomers

(c) protonation of monomers

(d) none of the above

132. Increasing order of bond strength of O_2, O_2^-, O_2^{2-} and O_2^+ is :

(a) $O_2^+ < O_2 < O_2^- < O_2^{2-}$

(b) $O_2 < O_2^+ < O_2^- < O_2^{2-}$

(c) $O_2^- < O_2^{2-} < O_2^+ < O_2$

(d) $O_2^{2-} < O_2^- < O_2 < O_2^+$

133. Most common oxidation states of Ce (Cerium) are :

(a) +3, + 4 (b) +2, +3

(c) +2, +4 (d) +3, +5

134. Ce^{3+}, La^{3+}, Pm^{3+} and Yb^{3+} have ionic radii in the increasing order as :

(a) $La^{3+} < Ce^{3+} < Pm^{3+} < Yb^{3+}$

(b) $Yb^{3+} < Pm^{3+} < Ce^{3+} < La^{3+}$

(c) $La^{3+} = Ce^{3+} < Pm^{3+} < Yb^{3+}$

(d) $Yb^{3+} < Pm^{3+} < La^{3+} < Ce^{3+}$

135. pH of 0.005 M calcium acetate (pK_a of $CH_3COOH = 4.74$) is :

(a) 7.04 (b) 9.37

(c) 9.26 (d) 8.37

136. H_2 gas is absorbed on the metal surface like tungsten. This follows . . . . . . order reaction.

(a) third (b) second

(c) zero (d) first

137. Rate constant k of the first order reaction when initial concentration C_0 and concentration C_t at time t is given by equation

$$kt = \log C_0 - \log C_t$$

Graph is a straight line if we plot :

(a) t vs $\log C_0$

(b) t vs $\log C_t$

(c) t^{-1} vs $\log C_t$

(d) $\log C_0$ vs $\log C_t$

138. Alum is widely used to purify water since :

(a) it forms complex with clay particles

(b) it coagulates the mud particles

(c) it exchanges Ca^{2+} and Mg^{2+} ions present in hard water

(d) its sulphate ion is water purifier

139. On vigorous oxidation by permangnate solution $(CH_3)_2C{=}CHCH_2CHO$ gives :

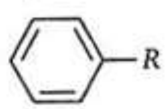
(a) $(CH_3)_2CO$ and $OHCCH_2CHO$

(b) $(CH_3)_2C(OH){-}CH(OH)CH_2CHO$

(c) $(CH_3)_2CO$ and $OHCCH_2COOH$

(d) $(CH_3)_2CO$ and $CH_2(COOH)_2$

140. In the following benzyl/allyl system

$R{-}CH{=}CH_2$ or 

(R is alkyl group)

decreasing order of inductive effect is :

(a) $(CH_3)_3C- > (CH_3)_2CH- > CH_3CH_2-$
(b) $CH_3CH_2- > (CH_3)_2CH- > (CH_3)_3C-$
(c) $(CH_3)_2CH- > CH_3CH_2- > (CH_3)_3C-$
(d) $(CH_3)_2C- > CH_3CH_2- > (CH_3)_3CH-$

141. PCl_3 and PCl_5 both exist; NCl_3 exists but NCl_5 does not exist. It is due to :
(a) lower electronegativity of P than N
(b) lower tendency of N to form covalent bond
(c) availability of vacant *d*-obital in P but not in N
(d) statement is itself incorrect

142. Following types of compounds (as I, II)
(I) $CH_3CH=CHCH_3$ (II) $CH_3-\underset{\underset{CH_2CH_3}{|}}{CH}-OH$
are studied in terms of isomerism in :
(a) chain isomerism (b) position isomerism
(c) conformers (d) stereoisomerism

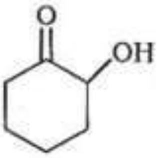
143. Conductivity (Seimen's S) is directly proportional to area of the vessel and the concentration of the solution in it and is inversely proportional to the length of the vessel, then constant of proportionality is expressed in :
(a) $S\ m\ mol^{-1}$ (b) $S^2\ m^2\ mol^{-2}$
(c) $S\ m^2\ mol^{-1}$ (d) $S^2\ m^2\ mol$

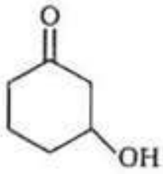
144. A heat engine absorbs heat q_1 from a source at temperature T_1 and heat q_2 from a source at temperature T_2. Work done is found to be $J(q_1+q_2)$. This is in accordance with :
(a) first law of thermodynamics
(b) second law of thermodynamics
(c) Joules equivalent law
(d) none of the above

145. Select correct statement :
(a) when a covalent bond is formed, transfer of electrons takes place
(b) pure H_2O does not contain any ion
(c) a bond is formed when attractive forces overcome repulsive forces
(d) HF is less polar than HBr

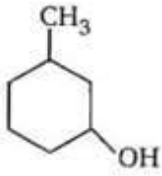
146. The metallic sodium dissolves in liquid ammonia to form a deep blue coloured solution. The deep blue colour is due to formation of :
(a) solvated electron, $e^-(NH_3)_x^-$
(b) solvated atomic sodium, $Na(NH_3)_y$
(c) $(Na^+ + Na^-)$
(d) $NaNH_2 + H_2$

147. Maximum dehydration takes place that of :

(a) 

(b) 

(c) 

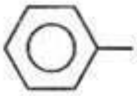
(d) 

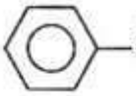
148. SN^1 reaction is feasible in :

(a)  Cl + KOH $\longrightarrow$

(b) 

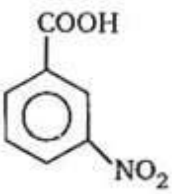
+ KOH $\longrightarrow$

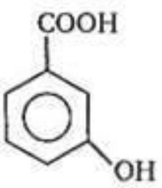
(c)  Cl + KOH $\longrightarrow$

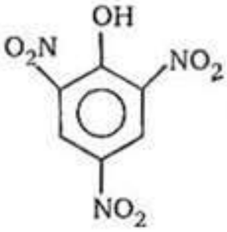
(d)  CH_2CH_2Cl + KOH $\longrightarrow$

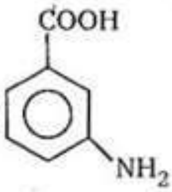
149. Oxidation number of Cl in $CaOCl_2$ (bleaching powder) is :
(a) zero, since it contains Cl_2
(b) -1, since it contains Cl^-
(c) $+1$, since it contains ClO^-
(d) $+1$ and -1 since it contains ClO^- and Cl^-

150. Picric acid is :

(a) 

(b) 

(c) 

(d) 

1. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the equation having α/β and β/α as its roots, is :
(a) $3x^2 + 19x + 3 = 0$ (b) $3x^2 - 19x + 3 = 0$
(c) $3x^2 - 19x - 3 = 0$ (d) $x^2 - 16x + 1 = 0$

2. If $y = (x + \sqrt{1 + x^2})^n$, then
$(1 + x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is :
(a) n^2y (b) $-n^2y$
(c) $-y$ (d) $2x^2y$

3. If 1, $\log_3\sqrt{(3^{1-x} + 2)}$, $\log_3(4 \cdot 3^x - 1)$ are in AP, then x equals :
(a) $\log_3 4$ (b) $1 - \log_3 4$
(c) $1 - \log_4 3$ (d) $\log_4 3$

4. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved, is :
(a) 3/4 (b) 1/2
(c) 2/3 (d) 1/3

5. The period of $\sin^2\theta$ is :
(a) π^2 (b) π
(c) 2π (d) $\pi/2$

6. l, m, n are the pth, qth and rth term of an GP and all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals :
(a) 3 (b) 2
(c) 1 (d) zero

7. $\lim\limits_{x \to 0} \dfrac{\sqrt{1 - \cos 2x}}{\sqrt{2}\,x}$ is :
(a) λ (b) -1
(c) zero (d) does not exist

8. A triangle with vertices (4, 0), (–1, –1), (3, 5) is :
(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles

9. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls?
(a) 73 (b) 65
(c) 68 (d) 74

10. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x$ is equal to :
(a) $\tan^2\left(\dfrac{\alpha}{2}\right)$ (b) $\cot^2\left(\dfrac{\alpha}{2}\right)$
(c) $\tan\alpha$ (d) $\cot\left(\dfrac{\alpha}{2}\right)$

11. The order and degree of the differential equation $\left(1 + 3\dfrac{dy}{dx}\right)^{2/3} = 4\dfrac{d^3y}{dx^3}$ are :
(a) $\left(1, \dfrac{2}{3}\right)$ (b) (3, 1)
(c) (3, 3) (d) (1, 2)

12. A plane which passes through the point (3, 2, 0) and the line $\dfrac{x-4}{1} = \dfrac{y-7}{5} = \dfrac{z-4}{4}$ is :
(a) $x - y + z = 1$ (b) $x + y + z = 5$
(c) $x + 2y - z = 1$ (d) $2x - y + z = 5$

13. The solution of the equation $\dfrac{d^2y}{dx^2} = e^{-2x}$ is :
(a) $\dfrac{e^{-2x}}{4}$ (b) $\dfrac{e^{-2x}}{4} + cx + d$
(c) $\dfrac{1}{4}e^{-2x} + cx^2 + d$ (d) $\dfrac{1}{4}e^{-2x} + c + d$

14. $\lim\limits_{x \to \infty}\left(\dfrac{x^2 + 5x + 3}{x^2 + x + 2}\right)^x$ is equal to :
(a) e^4 (b) e^2
(c) e^3 (d) e

15. The domain of $\sin^{-1}[\log_3(x/3)]$ is :
(a) [1, 9] (b) [–1, 9]
(c) [–9, 1] (d) [–9, –1]

16. The value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \ldots \infty$ is :
(a) 1 (b) 2
(c) 3/2 (d) 4

17. Fifth term of a GP is 2, then the product of its 9 terms is :
(a) 256 (b) 512
(c) 1024 (d) none of these

18. $\int_0^{10\pi} |\sin x|\, dx$ is :
(a) 20 (b) 8
(c) 10 (d) 18

19. $I_n = \int_0^{\pi/4} \tan^n x\, dx$, then $\lim_{n\to\infty} n[I_n + I_{n+2}]$ equals :

(a) $\frac{1}{2}$ (b) 1

(c) ∞ (d) zero

20. $\int_0^2 [x^2]\, dx$ is :

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$

(c) $\sqrt{2}-1$ (d) $-\sqrt{2}-\sqrt{3}+5$

21. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\, dx$ is :

(a) $\frac{\pi^2}{4}$ (b) π^2

(c) zero (d) $\frac{\pi}{2}$

22. The period of the function $f(x) = \sin^4 x + \cos^4 x$ is :

(a) π (b) $\frac{\pi}{2}$

(c) 2π (d) none of these

23. The domain of definition of the function

$$f(x) = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)} \text{ is :}$$

(a) [1, 4] (b) [1, 0]

(c) [0, 5] (d) [5, 0]

24. If $\sin y = x \sin(a+y)$, then $\frac{dy}{dx}$ is :

(a) $\frac{\sin a}{\sin^2(a+y)}$ (b) $\frac{\sin^2(a+y)}{\sin a}$

(c) $\sin a \sin^2(a+y)$ (d) $\frac{\sin^2(a-y)}{\sin a}$

25. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is :

(a) $\frac{1+x}{1+\log x}$ (b) $\frac{1-\log x}{1+\log x}$

(c) not defined (d) $\frac{\log x}{(1+\log x)^2}$

26. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$:

(a) cut at right angle (b) touch each other

(c) cut at an angle $\frac{\pi}{3}$ (d) cut at an angle $\frac{\pi}{4}$

27. The function $f(x) = \cot^{-1} x + x$ increases in the interval :

(a) $(1, \infty)$ (b) $(-1, \infty)$

(c) $(-\infty, \infty)$ (d) $(0, \infty)$

28. The greatest value of $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$ on [0, 1] is :

(a) 1 (b) 2

(c) 3 (d) 1/3

29. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) zero (d) 1

30. $\int \frac{dx}{x(x^n+1)}$ is equal to :

(a) $\frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right)+c$

(b) $\frac{1}{n}\log\left(\frac{x^n+1}{x^n}\right)+c$

(c) $\log\left(\frac{x^n}{x^n+1}\right)+c$

(d) none of these

31. The area bounded by the curve $y = 2x - x^2$ and the straight line $y = -x$ is given by :

(a) $\frac{9}{2}$ sq unit (b) $\frac{43}{6}$ sq unit

(c) $\frac{35}{6}$ sq unit (d) none of these

32. The differential equation of all non-vertical lines in a plane is :

(a) $\frac{d^2y}{dx^2} = 0$ (b) $\frac{d^2x}{dy^2} = 0$

(c) $\frac{dy}{dx} = 0$ (d) $\frac{dx}{dy} = 0$

33. Given two vectors are $\hat{i} - \hat{j}$ and $\hat{i} + 2\hat{j}$ the unit vector coplanar with the two vectors and perpendicular to first is :

(a) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (b) $\frac{1}{\sqrt{5}}(2\hat{i}+\hat{j})$

(c) $\pm\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$ (d) none of these

34. The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x-2)\hat{j} + 2\hat{k}$. The value of x are :

(a) $\left\{-\frac{2}{3}, 2\right\}$ (b) $\left\{\frac{1}{3}, 2\right\}$

(c) $\left\{\frac{2}{3}, 0\right\}$ (d) $\{2, 7\}$

35. A parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7), parallel to the co-ordinate planes. The length of a diagonal of the parallelopiped is :
(a) 7 unit (b) $\sqrt{38}$ unit
(c) $\sqrt{155}$ unit (d) none of these

36. The equation of the plane containing the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where :
(a) $ax_1+by_1+cz_1=0$
(b) $al+bm+cn=0$
(c) $\frac{a}{l}=\frac{b}{m}=\frac{c}{n}$
(d) $lx_1+my_1+nz_1=0$

37. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial, is :
(a) $\frac{1}{25}$ (b) $\frac{24}{25}$
(c) $\frac{2}{25}$ (d) none of these

38. If A and B are two mutually exclusive events, then :
(a) $P(A)<P(\bar{B})$ (b) $P(A)>P(\bar{B})$
(c) $P(A)<P(B)$ (d) none of these

39. The equation of the directrix of the parabola $y^2+4y+4x+2=0$ is :
(a) $x=-1$ (b) $x=1$
(c) $x=-3/2$ (d) $x=3/2$

40. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1}-T_n=21$, then n equals :
(a) 5 (b) 7
(c) 6 (d) 4

41. In a triangle ABC, $2ca\sin\frac{A-B+C}{2}$ is equal to :
(a) $a^2+b^2-c^2$ (b) $c^2+a^2-b^2$
(c) $b^2-c^2-a^2$ (d) $c^2-a^2-b^2$

42. For $x\in R$, $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is equal to :
(a) e (b) e^{-1}
(c) e^{-5} (d) e^5

43. The incentre of the triangle with vertices $(1,\sqrt{3})$, $(0,0)$ and $(2,0)$ is :
(a) $\left(1,\frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3},\frac{1}{\sqrt{3}}\right)$
(c) $\left(\frac{2}{3},\frac{\sqrt{3}}{2}\right)$ (d) $\left(1,\frac{1}{\sqrt{3}}\right)$

44. If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ from the sides BC, CA and AB respectively of a triangle ABC, then :
(a) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{b}=0$
(b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$
(c) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$
(d) $\vec{a}\times\vec{a}+\vec{a}\times\vec{c}+\vec{c}\times\vec{a}=0$

45. If ω is an imaginary cube root of unity, then $(1+\omega-\omega^2)^7$ equals :
(a) $128\,\omega$ (b) $-128\,\omega$
(c) $128\,\omega^2$ (d) $-128\,\omega^2$

46. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}=x+iy$, then :
(a) $x=3,\ y=1$ (b) $x=1,\ y=3$
(c) $x=0,\ y=3$ (d) $x=0,\ y=0$

47. $\sin^2\theta=\frac{4xy}{(x+y)^2}$ is true if and only if :
(a) $x+y\neq 0$ (b) $x=y,\ x\neq 0, y\neq 0$
(c) $x=y$ (d) $x\neq 0,\ y\neq 0$

48. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having its centre at (0, 3), is :
(a) 4 unit (b) 3 unit
(c) $\sqrt{12}$ unit (d) $\frac{7}{2}$ unit

49. The probability of India winning a test match against West-Indies is 1/2 assuming independence from match to match. The probability that in a match series India's second win occurs at the third test is :
(a) $\frac{1}{8}$ (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

50. If $(\omega\neq 1)$ is a cubic root of unity, then
$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix}$ equals :
(a) zero (b) 1
(c) i (d) ω

51. A biased coin with probability p, $0<p<1$, of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even, is 2/5, then p equals :

(a) 1/3 (b) 2/3
(c) 2/5 (d) 3/5

52. A fair die is tossed eight times. The probability that a third six is observed on the eight throw, is :

(a) $\frac{^7C_2 \times 5^5}{6^7}$ (b) $\frac{^7C_2 \times 5^5}{6^8}$

(c) $\frac{^7C_2 \times 5^5}{6^6}$ (d) none of these

53. Let $f(2) = 4$ and $f'(2) = 4$. Then $\lim_{x \to 2} \frac{x f(2) - 2f(x)}{x - 2}$ is given by :

(a) 2 (b) – 2
(c) – 4 (d) 3

54. Three straight lines $2x + 11y - 5 = 0$, $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$:

(a) form a triangle
(b) are only concurrent
(c) are concurrent with one line bisecting the angle between the other two
(d) none of the above

55. A straight line through the point (2, 2) intersects the lines $\sqrt{3}x + y = 0$ and $\sqrt{3}x - y = 0$ at the points A and B. The equation to the line AB so that the triangle OAB is equilateral, is :

(a) $x - 2 = 0$ (b) $y - 2 = 0$

(c) $x + y - 4 = 0$ (d) none of these

56. The greatest distance of the point $P(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is :

(a) 10 unit (b) 15 unit
(c) 5 unit (d) none of these

57. The equation of the tangent to the circle $x^2 + y^2 + 4x - 4y + 4 = 0$ which make equal intercepts on the positive co-ordinate axes, is :

(a) $x + y = 2$ (b) $x + y = 2\sqrt{2}$
(c) $x + y = 4$ (d) $x + y = 8$

58. The equation of the ellipse whose foci are $(\pm 2, 0)$ and eccentricity is 1/2, is :

(a) $\frac{x^2}{12} + \frac{y^2}{16} = 1$ (b) $\frac{x^2}{16} + \frac{y^2}{12} = 1$

(c) $\frac{x^2}{16} + \frac{y^2}{8} = 1$ (d) none of these

59. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is :

(a) $\frac{x}{x_1 + x_2} + \frac{y}{y_1 + y_2} = 1$

(b) $\frac{x}{x_1 - x_2} + \frac{y}{y_1 - y_2} = 1$

(c) $\frac{x}{y_1 + y_2} + \frac{y}{x_1 + x_2} = 1$

(d) $\frac{x}{y_1 - y_2} + \frac{y}{x_1 - x_2} = 1$

60. If the vectors $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and such that $\vec{a}$, $\vec{c}$ and $\vec{b}$ form a right handed system, then $\vec{c}$ is :

(a) $z\hat{i} - x\hat{k}$ (b) $\vec{0}$

(c) $y\hat{j}$ (d) $-z\hat{i} + x\hat{k}$

61. The centre of the circle given by $\vec{r} \cdot (\hat{i} + 2\hat{j} + 2\hat{k}) = 15$ and $|\vec{r} - (\hat{j} + 2\hat{k})| = 4$ is :

(a) (0, 1, 2) (b) (1, 3, 4)
(c) (–1, 3, 4) (d) none of these

62. The value of $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$ is :

(a) 1 (b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) 2

63. If $\tan\theta = -\frac{4}{3}$, then $\sin\theta$ is :

(a) $-\frac{4}{5}$ but not $\frac{4}{5}$ (b) $-\frac{4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) none of these

64. If $\sin(\alpha + \beta) = 1$, $\sin(\alpha - \beta) = \frac{1}{2}$, then $\tan(\alpha + 2\beta)\tan(2\alpha + \beta)$ is equal to :

(a) 1 (b) –1
(c) zero (d) none of these

65. If $y = \sin^2\theta + \text{cosec}^2\theta$, $\theta \neq 0$, then :

(a) $y = 0$ (b) $y \leq 2$
(c) $y \geq -2$ (d) $y > 2$

66. In a triangle ABC, $a = 4$, $b = 3$, $\angle A = 60^\circ$, then c is the root of the equation :

(a) $c^2 - 3c - 7 = 0$ (b) $c^2 + 3c + 7 = 0$
(c) $c^2 - 3c + 7 = 0$ (d) $c^2 + 3c - 7 = 0$

67. In a ΔABC, $\tan\frac{A}{2} = \frac{5}{6}$, $\tan\frac{C}{2} = \frac{2}{5}$, then :

(a) a, c, b are in AP (b) a, b, c are in AP
(c) b, a, c are in AP (d) a, b, c are in GP

68. The equation $a\sin x + b\cos x = c$ where $|c| > \sqrt{a^2 + b^2}$ has :

(a) a unique solution
(b) infinite number of solutions
(c) no solution
(d) none of the above

69. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$ $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to :

(a) $\frac{24}{25}$ (b) $-\frac{24}{25}$

(c) $\frac{13}{18}$ (d) $-\frac{13}{18}$

70. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to :

(a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$

(c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

71. $\sum_{n=0}^{\infty} \frac{(\log_e x)^n}{n!}$ is equal to :

(a) $\log_e x$ (b) x

(c) $\log_x e$ (d) none of these

72. $e^{(x-1) - \frac{1}{2}(x-1)^2 + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots}$ equal to :

(a) $\log(x-1)$ (b) $\log x$

(c) x (d) none of these

73. The coefficient of x^5 in $(1 + 2x + 3x^2 + \dots)^{-3/2}$ is :

(a) 21 (b) 25

(c) 26 (d) none of these

74. If $|x| < 1$, then the coefficient of x^n in expansion of $(1 + x + x^2 + x^3 + \dots)^2$ is :

(a) n (b) $n - 1$

(c) $n + 2$ (d) $n + 1$

75. The number of real roots of $3^{2x^2 - 7x + 7} = 9$ is :

(a) zero

(b) 2

(c) 1

(d) 4

➡ PHYSICS AND CHEMISTRY

1.	(d)	2.	(c)	3.	(b)	4.	(a)	5.	(b)	6.	(a)	7.	(a)	8.	(a)
9.	(b)	10.	(c)	11.	(c)	12.	(b)	13.	(b)	14.	(a)	15.	(a)	16.	(d)
17.	(b)	18.	(b)	19.	(b)	20.	(d)	21.	(c)	22.	(b)	23.	(b)	24.	(c)
25.	(a)	26.	(a)	27.	(a)	28.	(c)	29.	(a)	30.	(c)	31.	(b)	32.	(a)
33.	(c)	34.	(b)	35.	(a)	36.	(a)	37.	(b)	38.	(b)	39.	(b)	40.	(c)
41.	(c)	42.	(b)	43.	(a)	44.	(c)	45.	(a)	46.	(c)	47.	(c)	48.	(b)
49.	(b)	50.	(b)	51.	(b)	52.	(c)	53.	(b)	54.	(d)	55.	(a)	56.	(d)
57.	(b)	58.	(c)	59.	(a)	60.	(a)	61.	(b)	62.	(d)	63.	(c)	64.	(d)
65.	(a)	66.	(b)	67.	(a)	68.	(b)	69.	(c)	70.	(b)	71.	(c)	72.	(a)
73.	(c)	74.	(a)	75.	(d)	76.	(d)	77.	(c)	78.	(b)	79.	(a)	80.	(a)
81.	(d)	82.	(c)	83.	(b)	84.	(a)	85.	(b)	86.	(b)	87.	(a)	88.	(b)
89.	(c)	90.	(a)	91.	(c)	92.	(c)	93.	(c)	94.	(b)	95.	(a)	96.	(a)
97.	(b)	98.	(b)	99.	(a)	100.	(c)	101.	(d)	102.	(b)	103.	(b)	104.	(d)
105.	(b)	106.	(c)	107.	(c)	108.	(a)	109.	(d)	110.	(a)	111.	(a)	112.	(d)
113.	(b)	114.	(c)	115.	(d)	116.	(c)	117.	(b)	118.	(c)	119.	(a)	120.	(c)
121.	(d)	122.	(c)	123.	(a)	124.	(c)	125.	(c)	126.	(d)	127.	(c)	128.	(a)
129.	(b)	130.	(a)	131.	(b)	132.	(d)	133.	(a)	134.	(b)	135.	(d)	136.	(c)
137.	(b)	138.	(b)	139.	(d)	140.	(a)	141.	(c)	142.	(d)	143.	(c)	144.	(c)
145.	(c)	146.	(a)	147.	(b)	148.	(a)	149.	(d)	150.	(c)				

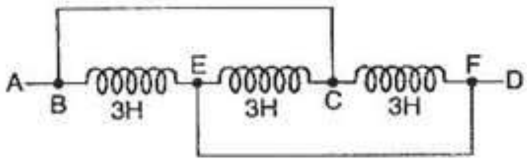
➡ MATHEMATICS

1.	(b)	2.	(a)	3.	(b)	4.	(a)	5.	(b)	6.	(d)	7.	(d)	8.	(a)
9.	(b)	10.	(a)	11.	(c)	12.	(a)	13	(b)	14.	(a)	15.	(a)	16.	(b)
17.	(b)	18.	(a)	19.	(b)	20.	(d)	21.	(b)	22.	(b)	23.	(a)	24.	(b)
25.	(d)	26.	(a)	27.	(c)	28.	(b)	29.	(a)	30.	(a)	31.	(a)	32.	(a)
33.	(a)	34.	(a)	35.	(a)	36.	(b)	37.	(b)	38.	(a)	39.	(d)	40.	(b)
41.	(b)	42.	(c)	43.	(d)	44.	(b)	45.	(d)	46.	(d)	47.	(a)	48.	(a)
49.	(b)	50.	(a)	51.	(a)	52.	(b)	53.	(c)	54.	(c)	55.	(b)	56.	(b)
57.	(b)	58.	(b)	59.	(a)	60.	(a)	61.	(b)	62.	(c)	63.	(b)	64.	(a)
65.	(d)	66.	(a)	67.	(b)	68.	(c)	69.	(b)	70.	(d)	71.	(b)	72.	(c)
73.	(d)	74.	(d)	75.	(b)										

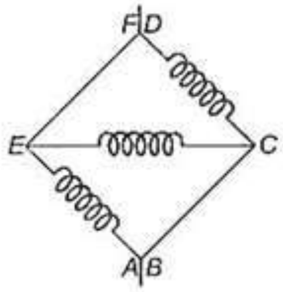
HINTS & SOLUTIONS

Physics

1.



Here, inductors are in parallel



$$\therefore \quad \frac{1}{L} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$L = 1$$

2. At the highest point of its flight, vertical component of velocity is zero and only horizontal component is left which is

$$u_x = u \cos \theta$$

Given : $\theta = 45°$

$$\therefore \quad u_x = u \cos 45° = \frac{u}{\sqrt{2}}$$

Hence, at the highest point kinetic energy

$$E' = \frac{1}{2} m u_x^2 = \frac{1}{2} m \left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2} m \left(\frac{u^2}{2}\right)$$

$$= \frac{E}{2} \qquad \left(\because \frac{1}{2} m u^2 = E\right)$$

3. From conservation of energy, potential energy at height h

= KE at ground

Therefore, at height h, PE of ball A

$$\text{PE} = m_A g h$$

$$\text{KE at ground} = \frac{1}{2} m_A v_A^2$$

So, $$m_A g h = \frac{1}{2} m_A v_A^2$$

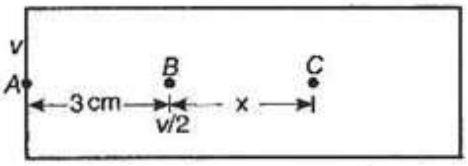
$$v_A = \sqrt{2gh}$$

Similarly, $$v_B = \sqrt{2gh}$$

Therefore, $$v_A = v_B$$

Note : In question it is not mentioned that magnitude of thrown velocity of both balls are same which is assumed in solution.

4. Let initial velocity of body at point A is v, AB is 3 cm.



From $$v^2 = u^2 - 2as$$

$$\left(\frac{v}{2}\right)^2 = v^2 - 2a \times 3$$

$$a = \frac{v^2}{8}$$

Let on penetrating 3 cm in a wooden block, the body moves x distance from B to C.

So, for B to C

$$u = \frac{v}{2}, \quad v = 0,$$

$$s = x, \quad a = \frac{v^2}{8} \qquad \text{(deceleration)}$$

$$\therefore \quad (0)^2 = \left(\frac{v}{2}\right)^2 - 2 \cdot \frac{v^2}{8} \cdot x$$

$$x = 1 \text{ cm}$$

Note : Here, it is assumed that retardation is uniform.

5. When gravitational force becomes zero, then centripetal force required can not be provided. So the satellite will move with the velocity as it has at the instant when gravitational force becomes zero, *i.e.*, it moves tangentially to the original orbit.

6. A voltmeter is a high resistance device and is always connected in parallel with the circuit. While an ammeter is a low resistance device and is always connected in series with the circuit. So, to use ammeter in place of voltmeter a low resistance must be connected in parallel with the ammeter to make its resistance small.

7. Magnetic field in circular coil A is

$$B_A = \frac{\mu_0 N i}{2R}$$

where R is radius and i is current flowing in coil.

Similarly $$B_B = \frac{\mu_0 N(2i)}{2\cdot(2R)} = \frac{\mu_0 Ni}{2R}$$

$\therefore$ $$\frac{B_A}{B_B} = \frac{1}{1} = 1$$

8. Number of images $n = \frac{360°}{\theta} - 1$

where θ is angle between mirrors.

Thus, $\theta = 60°$ (given)

So, number of images

$$n = \frac{360°}{60°} - 1 = 5$$

9. In 1st case :

Using the formula $P = \frac{V^2}{R}$ ...(1)

where R is resistance of wire, V is voltage across wire and P is power dissipation in wire and

$$R = \frac{\rho l}{A} \quad ...(2)$$

From Eqs. (1) and (2)

$$P_1 = \frac{V^2}{\rho l/A} = \frac{V^2}{\rho l}\cdot A$$

$$P_1 = \frac{V^2}{\rho l}\cdot A \quad ...(3)$$

In 2nd case :

Let R_2 is net resistance.

$$R_2 = \frac{R\times R}{R+R} = \frac{R}{2}$$

where, R is the resistance of half wire.

$\therefore$ $$R_2 = \frac{\rho\cdot\left(\frac{l}{2}\right)}{A\cdot 2} = \frac{\rho l}{4A}$$

$\therefore$ $$P_2 = \frac{V^2}{\rho l}\cdot 4A \quad ...(4)$$

Hence, from Eqs. (3) and (4)

$$\frac{P_1}{P_2} = \frac{1}{4}$$

$\Rightarrow$ $$\frac{P_2}{P_1} = \frac{4}{1}$$

10. Energy required to remove an electron from nth orbit is,

$$E_n = -\frac{13.6}{n^2}$$

Here, $n = 2$

Therefore, $$E_2 = -\frac{13.6}{2^2} = -3.4\text{ V}$$

11. Let the tubes A and B have equal length called as l. Since, tube A is opened at both the ends, therefore, its fundamental frequency

$$n_A = \frac{v}{2l} \quad ...(1)$$

Since, tube B is closed at one end, therefore, its fundamental frequency

$$n_B = \frac{v}{4l} \quad ...(2)$$

From Eqs. (1) and (2), we get

$$\frac{n_A}{n_B} = \frac{v/2l}{v/4l} = \frac{4}{2} = 2:1$$

12. The tuning fork of frequency 288 Hz is producing 4 beats/s with the unknown tuning fork *i.e.*, the frequency difference between them is 4. Therefore, the frequency of unknown tuning fork

$$= 288 \pm 4 = 292 \text{ or } 284$$

On placing a little wax on unknown tuning fork, its frequency decreases but now the number of beats produced per second is 2 *i.e.*, the frequency difference now decreases. It is possible only when before placing the wax, the frequency of unknown fork is greater than the frequency of given tuning fork. Hence, the frequency of unknown tuning fork = 292 Hz.

13. Equation of a wave

$$y = a\sin(\omega t - kx) \quad ...(1)$$

Let equations of another wave may be,

$$y = a\sin(\omega t + kx) \quad ...(2)$$

$$y = -a\sin(\omega t + kx) \quad ...(3)$$

If Eq. (1) propagates with Eq. (2), then we get

$$y = 2a\cos kx \sin\omega t \quad ...(4)$$

If Eq. (1), propagates with Eq. (3), then we get

$$y = -2a\sin kx\cos\omega t \quad ...(5)$$

After putting $x = 0$ in Eqs. (4) and (5) respectively, we get

$$y = 2a\sin\omega t \quad\text{and}\quad y = 0$$

Hence, Eq. (3) is a equation of unknown wave.

14. Potential difference between two points in an electric field is,

$$V_A - V_B = \frac{W}{q_0}$$

where, W is work done by moving charge q_0 from point A to B.

(Here : $W = 2\text{J}, q_0 = 20\text{C}$)

So, $$V_A - V_B = \frac{2}{20}$$

$$= 0.1\text{ V}$$

15. Since momenta are same and masses for electron and proton are different, so they will attain different velocities and hence experience different forces. But radius of circular path is depending on momenta so both will be moving on same trajectory (curved path).

16. Gravitational potential energy of body will be

$$E = -\frac{GMm}{r}$$

where, M = mass of earth,

m = mass of the body,

R = radius of earth

At $r = 2R$,

$$E_1 = -\frac{GMm}{(2R)}$$

At $r = 3R$,

$$E_2 = -\frac{GMm}{(3R)}$$

Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

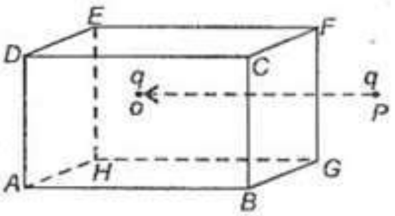
$$\Delta E = \frac{GMm}{R}\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{GMm}{6R}$$

17. As we know that spring constant of spring is inversely proportional to length of spring, so new spring constant for each part is given by $k' = nk$ where, k is the spring constant of whole spring. From the theory of spring pendulum, we know that time period of spring pendulum is inversely proportional to square root of spring constant *i.e.*, $T \propto \frac{1}{\sqrt{k}}$

$$T' \propto \frac{1}{\sqrt{nk}}$$

So, $$T' = \frac{T}{\sqrt{n}}$$

18. Electric flux for any surface is defined as $\phi = \int \vec{E} \cdot \vec{ds}$



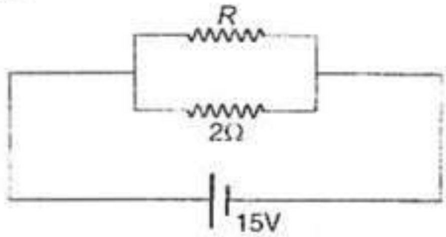
The flux through $ABCD$ can be calculated, by first taking a small elemental surface and then writing the $\vec{E} \cdot \vec{ds}$ for this element, keep in mind that electric field at the location of this element is the resultant of both the charges.

It is quite obvious the flux through $ABCD$ comes out to be non-zero because at every point of the surface, the angle between $\vec{E}$ and $\vec{ds}$ is less than 90° giving a positive non-zero value for the entire surface. So, option (b) cannot be the answer.

The options (a), (c) and (d) are dimensionally incorrect, so they cannot be answer.

Note : In this particular question flux through $BCFG$ would be zero because component of electric field along the area vector is zero at every point on this surface.

19. $P = \frac{V^2}{R_{net}}$



$$150 = \frac{(15)^2}{\frac{R \cdot 2}{2+R}} \Rightarrow R = 6\,\Omega$$

20. Resolving power of an optical instrument is inversely proportional to λ *i.e.*, RP $\propto \frac{1}{\lambda}$

$$\therefore \frac{\text{Resolving power at } \lambda_1}{\text{Resolving power at } \lambda_2} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{4000} = 5:4$$

21. Let mass of each body is m. Their motion is represented as shown in figure.

From $$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

$$v_{CM} = \frac{m \times 2v - mv}{m + m} = \frac{v}{2}$$

[The direction of motion of first particle is taken as positive]

So, velocity of centre of mass of the system is $\frac{v}{2}$ in the direction of motion of particle having larger speed.

22. Due to flow of current in same direction in two adjacent sides, an attractive magnetic force will be produced due to which spring will get compressed.

23. As we know that thermal capacity of a substance is defined as the amount of heat required to raise its temperature by 1°C.

24. At absolute zero, the energy band is large and hence, Si acts as insulator or in other words, we can say that there is no free charge carriers :

26. Optical fibres works on the principle of total internal reflection.

27. Escape velocity $= \sqrt{2gR_e}$

So, escape velocity is independent of m. So, it depends upon mass as m°.

29. The dimensions of torque and work are $[ML^2T^{-2}]$.

30. In Seeback (thermoelectric) effect, the temperature of hot junction at which the thermo emf is maximum is called the neutral temperature (θ_n), and the temperature at which thermo emf changes its sign is called inversion temperature (θ_i). If θ_c is the temperature of cold junction then these three are related by the expression.

$$\theta_n - \theta_c = \theta_i - \theta_n$$

$$\therefore \qquad \theta_n = \frac{\theta_i + \theta_c}{2}$$

31. Spectrometer is an instrument which is used to obtain a pure spectrum of white light. It is used to determine the wavelength of different colours of white light. (or wavelength of monochromatic light source), the refractive index of the material of the prism and the dispersive power of the material of the prism.

Pyrometers is infrared sensitive devices, so these are used to detect infrared radiations.

Nanometer is the small unit of distance and is not a device.

Photometer is used to measure luminous intensity, illuminance and other photometric quantities.

32. N_0 is the initial amount of substance and N is the amount left after decay.

Thus, $$N = N_0 \left(\frac{1}{2}\right)^n$$

$$n = \text{no. of half-lives} = \frac{t}{t_{1/2}} = \frac{15}{5} = 3$$

Therefore, $$N = N_0 \left(\frac{1}{2}\right)^3 = \frac{N_0}{8}$$

33. If we increase the temperature, the specific resistance (resistivity) of conductor increases while that of semiconductor decreases.

34. Energy stored by any system of capacitors is

$$= \frac{1}{2} C_{\text{net}} V^2$$

where, V is source voltage

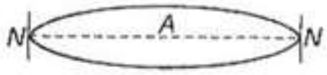
Thus, n capacitors are connected in parallel.

so, $$C_{\text{net}} = nC$$

$$\therefore \qquad E_{\text{net}} = \frac{1}{2} nCV^2$$

36. Efficiency of all reversible cycles depends upon temperature of source and sink which will be different.

37. When the string is plucked in the middle, it vibrates in one loop with nodes at fixed ends and an antinode in the middle.



So that, $$\frac{\lambda_1}{2} = l$$

$$\lambda_1 = 2 \times 40 = 80 \text{ cm}$$

38. From the relation, $\tan\phi = \dfrac{\omega L}{R}$

$$\text{Power factor, } \cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}} = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

40. The minimum kinetic energy required to project a body of mass m from earth's surface to infinity is known as escape energy. Therefore,

$$\text{KE} = \frac{GM_e m}{R} = mgR \qquad \left(\because gR = \frac{GM_e}{R}\right)$$

41. Temperature of a gas is determined by the total translational KE measured with respect to the centre of mass of the gas. Therefore, the motion of centre of mass of the gas does not affect the temperature. Hence, the temperature of gas will remain same.

43. According to the mass-energy equivalence, mass and energy remain conserved. So, when water is cooled to form ice, water loses its energy so, change in energy increases the mass of water.

44. Metals have minimum energy gap and insulators have maximum energy gap and semiconductors have energy gap lying between insulators and metals.

46. Kinetic energy of particle of mass m in SHM at any point is,

$$= \frac{1}{2} m\omega^2 (a^2 - x^2)$$

and potential energy $= \frac{1}{2} m\omega^2 x^2$

where, a is amplitude of particle and x is the distance from mean position.

So, at mean position, $x = 0$

$\therefore \quad KE = \frac{1}{2} m\omega^2 a^2 \quad$ (maximum)

$PE = 0 \quad$ (minimum)

47. Conservation of angular momentum gives

$$\frac{1}{2} MR^2\omega_1 = \left(\frac{1}{2} MR^2 + 2mR^2\right) \omega_2$$

$$\Rightarrow \quad \frac{1}{2} MR^2 \omega_1 = \frac{1}{2} R^2 (M + 4m) \omega_2$$

$$\therefore \quad \omega_2 = \left(\frac{M}{M + 4m}\right) \omega_1$$

48. Using the relation

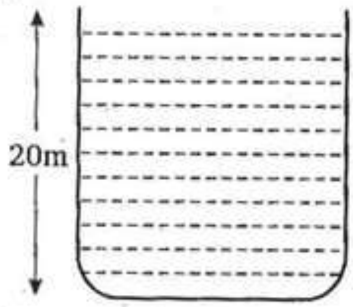
$$\frac{mv^2}{r} = \mu R, \qquad R = mg$$

$$\frac{mv^2}{r} = \mu mg \quad \text{or} \quad v^2 = \mu rg$$

or $\quad v^2 = 0.6 \times 150 \times 10$

$\Rightarrow \quad v = 30 \text{ m/s}$

49. Apply the Bernoulli's theorem just inside and outside the hole. Take reference line for gravitational potential energy at the bottom of the vessel.



Let P_0 is the atmospheric pressure, ρ the density of liquid and v the velocity at which water is coming out.

$$P_{\text{inside}} + \rho gh + 0 = P_{\text{outside}} + \frac{\rho v^2}{2}$$

$$\Rightarrow \quad P_0 + \rho gh = P_0 + \frac{\rho v^2}{2}$$

$$\Rightarrow \quad v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20 \text{ m/s}$$

50. The work is stored as the PE of the body and is given by,

$$U = \int_{x_1}^{x_2} F_{\text{external}}\, dx$$

$$\text{or } U = \int_{x_1}^{x_2} kx\, dx = \frac{1}{2} k(x_2^2 - x_1^2)$$

$$= \frac{800}{2} [(0.15)^2 - (0.05)^2][k = 800 \text{ (given)}]$$

$$= 400[0.2 \times 0.1] = 8 \text{ J}$$

51. As the child stands up, the effective length of pendulum decreases due to the reason that the centre of gravity rises up. Hence, according to

$$T = 2\pi \sqrt{\frac{l}{g}}$$

T will decrease.

52. Apparent weight of ball

$$w' = w - R$$

$$R = ma \text{ (acts upward)}$$

$$w' = mg - ma = m(g - a)$$

Hence, apparent acceleration in the lift is $g - a$. Now if the man is standing stationary on the ground, then the apparent acceleration of the falling ball is g.

53. In electrochemical cell, the anode produces the charge q which depends on It.

54. The rms velocity of the molecule of a gas of molecular weight M at Kelvin temperature T is given by,

$$C_{\text{rms}} = \sqrt{\left(\frac{3RT}{M}\right)}$$

Let M_O and M_H are molecular weights of oxygen and hydrogen and T_O and T_H the corresponding Kelvin temperatures at which C_{rms} is same for both gases.

That is, $\quad C_{\text{rms (O)}} = C_{\text{rms (H)}}$

$$\sqrt{\left(\frac{3RT_O}{M_O}\right)} = \sqrt{\left(\frac{3RT_H}{M_H}\right)}$$

Hence, $\quad \frac{T_O}{M_O} = \frac{T_H}{M_H}$

Given, $\quad T_O = 273 + 47 = 320 \text{ K}$

$M_O = 32, M_H = 2$

$\therefore \quad T_H = \frac{2}{32} \times 320 = 20 \text{ K}$

55. In a circular motion in a uniform magnetic field, the necessary centripetal force to the charged particle is provided by the magnetic force, *i.e.*,

$$\frac{mv^2}{r} = qvB$$

or $\quad r = \frac{mv}{qB}$

Thus, the time period T is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\left(\frac{mv}{qB}\right) = \frac{2\pi m}{qB}$$

So, T is independent of its speed.

56. Since, the inclined plane is frictionless, then there will be no rolling and the mass will only slide down.

Hence, acceleration is same for all the given bodies.

57. Given : $i_p = 4\,A$, $N_p = 140$, $N_s = 280$

From the formula

$$\frac{i_p}{i_s} = \frac{N_s}{N_p}$$

or $$\frac{4}{i_s} = \frac{280}{140}$$

So, $$i_s = 2\,A$$

58. The efficiency of Carnot engine is,

$$\eta = 1 - \frac{T_2}{T_1}$$

where, T_1 is the temperature of the source and T_2 that of sink.

Since, $$\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$$

So, $$\eta = 1 - \frac{Q_2}{Q_1}$$

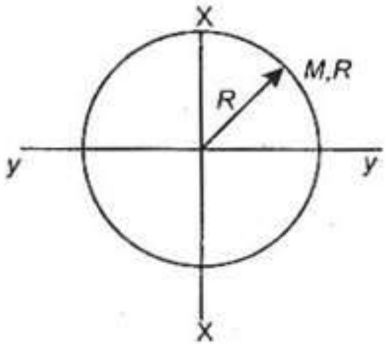
To obtain 100% efficiency (*i.e.*, $\eta = 1$), Q_2 must be zero that is, if a sink at absolute zero would be available, all the heat taken from the source would have been converted into work.

The temperature of sink means a negative temperature on the absolute scale at which the efficiency of engine is greater than unity. This would be a violation of the 2nd law of thermodynamics. Hence, a negative temperature on the absolute scale is impossible. Hence, we cannot reach absolute zero temperature.

59. Moment of inertia of circular wire about its axis is MR^2. Consider two diameters XX and YY moment of inertia about any of this diameter is same let us say I.

From perpendicular axis theorem, $I + I = MR^2$

$$I = \frac{MR^2}{2}$$



60. The particle is remains stationary under the action of three forces $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$, it means resultant force is zero.

$$\vec{F_1} = -(\vec{F_2} + \vec{F_3})$$

Since, in second case F_1 is removed (in terms of magnitude we are talking now), the forces acting are F_2 and F_3 the resultant of which has the magnitude as F_1, so acceleration of particle is $\frac{F_1}{m}$ in the direction opposite to that of $\vec{F_1}$.

61. $A + B = 18$ ...(1)

$$12 = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad ...(2)$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\Rightarrow \tan 90° = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow \cos\theta = \frac{-A}{B} \quad ...(3)$$

Solving Eqs. (1), (2) and (3), $A = 5N$, $B = 13N$

62. In this question the cars are identical means coefficient of friction between the tyre and the ground is same for both the cars, as a result retardation is same for both the cars equal to μg.

Let first car travel distance s_1, before stopping while second car travel distance s_2, then from

$$v^2 = u^2 - 2as$$

$$\Rightarrow \quad 0 = u^2 - 2\mu g \times s_1 \Rightarrow s_1 = \frac{u^2}{2\mu g}$$

and $$0 = (4u)^2 - 2\mu g \times s_2$$

$$\Rightarrow \quad s_2 = \frac{16u^2}{2\mu g} = 16 s_1$$

$$\Rightarrow \quad \frac{s_1}{s_2} = \frac{1}{16}$$

63. Using the relation

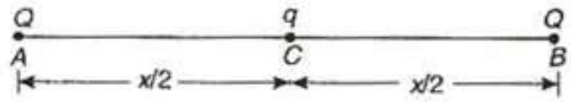
$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$$\Rightarrow \quad \frac{1 + 1}{\gamma - 1} = \frac{1}{(5/3 - 1)} + \frac{1}{(7/5 - 1)}$$

$$\Rightarrow \quad \frac{2}{\gamma - 1} = \frac{3}{2} + \frac{5}{2} = 4$$

$$\Rightarrow \quad \gamma = \frac{3}{2} = \frac{24}{16}$$

64. Let charge q is placed at mid point of line AB as shown below.



Also $$AB = x \quad \text{(say)}$$

$$\therefore \quad AC = \frac{x}{2}, BC = \frac{x}{2}$$

For the system to be in equilibrium

$$F_{Qq} + F_{QQ} = 0$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{(x/2)^2}+\frac{1}{4\pi\varepsilon_0}\frac{QQ}{x^2}=0$$

$$\Rightarrow \quad q=-\frac{Q}{4}$$

65. Capacitance of spherical conductor $= 4\pi\varepsilon_0 a$ where, a is radius of conductor.

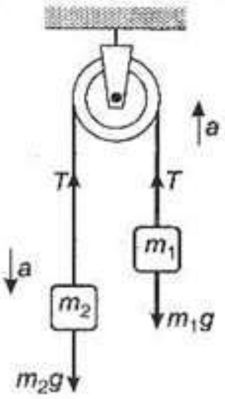
Therefore, $C=\frac{1}{9\times10^9}\times 1$

$$=\frac{1}{9}\times10^{-9}$$

$$=0.11\times10^{-9}\text{ F}$$

$$=1.1\times10^{-10}\text{ F}$$

66. As the string is inextensible, both masses have the same acceleration a. Also, the pulley is massless and frictionless, hence, the tension at both ends of the string is the same. Suppose the mass m_2 is greater than mass m_1, so, the heavier mass m_2 is accelerated downward and the lighter mass m_1 is accelerated upwards.



Therefore, by Newton's 2nd law

$$T-m_1g=m_1a \quad ...(1)$$

$$m_2g-T=m_2a \quad ...(2)$$

After solving Eqs. (1) and (2)

$$a=\frac{(m_2-m_1)}{(m_1+m_2)}\cdot g=\frac{g}{8} \quad \text{(given)}$$

So, $$\frac{g}{8}=\frac{m_2\,(1-m_1/m_2)}{m_2\,(1+m_1/m_2)}\cdot g \quad ...(3)$$

Let $$\frac{m_1}{m_2}=x$$

Thus Eq. (3) becomes

$$\frac{1-x}{1+x}=\frac{1}{8} \quad \text{or} \quad x=\frac{7}{9} \quad \text{or} \quad \frac{m_2}{m_1}=\frac{9}{7}$$

So, the ratio of the masses is 9 : 7.

67. Energy radiated per second by a body which has surface area A at temperature T is given by Stefen's law,

$$E=\sigma AT^4$$

Therefore, $\frac{E_1}{E_2}=\left(\frac{r_1}{r_2}\right)^2\left(\frac{T_1}{T_2}\right)^4=\left(\frac{1}{4}\right)^2\left(\frac{4000}{2000}\right)^4$

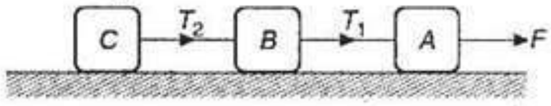
[Since bodies are of same material $e_1=e_2$]

$$\therefore \quad \frac{E_1}{E_2}=\frac{16}{16}=\frac{1}{1}$$

$$=1:1$$

68. The system of masses is shown below.

From the figure.



$$F-T_1=ma \quad ...(1)$$

and $$T_1-T_2=ma \quad ...(2)$$

Eq. (1) gives

$$10.2-T_1=2\times0.6$$

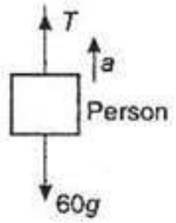
$$\Rightarrow \quad T_1=10.2-1.2=9\text{ N}$$

Again from Eq. (2), we get

$$9-T_2=2\times0.6$$

$$\Rightarrow \quad T_2=9-1.2=7.8\text{ N}$$

69. The free body diagram of the person can be drawn as



Let the person moves up with an acceleration 'a' then $T-60g=60a$

$$\Rightarrow \quad a_{max}=\frac{T_{max}-60g}{60}$$

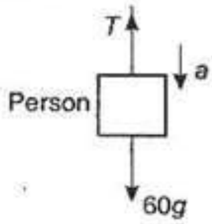
$$=\frac{360-60g}{60}=-\text{ve}$$

Which means it is not possible to climb up on the rope.

Even in this problem it is not possible to remain at rest on rope.

No option is right.

But if they will ask for the acceleration of climbing down then

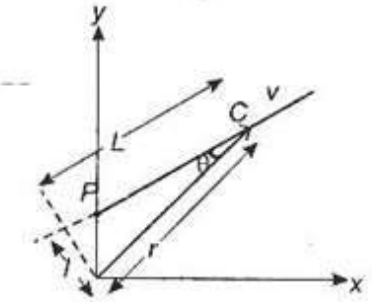


$$60g-T=60a$$

$\Rightarrow \quad 60g - T_{max} = 60a_{min}$

$\therefore \quad a_{min} = \dfrac{60g - 360}{60} = 4 \text{ m/s}^2$

70. Angular momentum of particle about O



$$\vec{\mathbf{L}} = m(\vec{\mathbf{r}} \times \vec{\mathbf{v}})$$

$$|\vec{\mathbf{L}}| = mrv \sin\theta$$
$$= mv(r\sin\theta)$$
$$= mvl$$

71. The component $dl\cos\theta$ of element dl is parallel to the length of the wire 1. Hence, force on this elemental component

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{r}(dl\cos\theta)$$
$$\frac{\mu_0 i_1 i_2 dl\cos\theta}{2\pi r}$$

72. Electrons, protons, and helium atoms are deflected in magnetic field, so the compound can emit electrons, protons and He^{2+}.

73. Work function $W = \dfrac{hc}{\lambda}$

[Here they are interested in asking threshold wavelength]

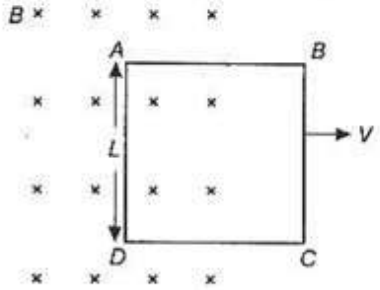
where, h = Planck's constant, c = velocity of light.

Therefore, $\dfrac{W_{Na}}{W_{Cu}} = \dfrac{\lambda_{Cu}}{\lambda_{Na}}$

$$\frac{\lambda_{Na}}{\lambda_{Cu}} = \frac{W_{Cu}}{W_{Na}} = \frac{4.5}{2.3} = 2 \text{ (nearly)}$$

74. Formation of covalent bonds due to the wave nature of particles is done in compounds.

75. As the side BC is outside the field no emf is induced across BC. Since, AB and CD are not cutting any flux the emf induced across these two sides will also be zero.



The side AD is cutting the flux and emf induced across this side is BvL with corner A at higher potential.

Chemistry

76. (a) $\overset{+1\,-1}{NaCl} + \overset{+1\,-1}{KNO_3} \longrightarrow \overset{+1\,-1}{NaNO_3} + \overset{+1\,-1}{KCl}$

(b) $\overset{+2\,-2}{CaC_2O_4} + 2\overset{+1\,-1}{HCl} \longrightarrow \overset{+2\,-1}{CaCl_2} + \overset{+1\,-2}{H_2C_2O_4}$

(c) $\overset{+2\,-1}{Ca(OH)_2} + 2\overset{-3\,+1\,-1}{NH_4Cl} \longrightarrow \overset{+2\,-1}{CaCl_2} + 2\overset{-3\,+1}{NH_3} + 2\overset{+1\,-2}{H_2O}$

in all these cases during reaction, there is no change in oxidation state of ion or molecule or constituent atom, these are simply ionic reactions.

(d) $2K[Ag(CN)_2] + Zn \rightarrow 2Ag + K_2[Zn(CN)_4]$

$Ag^+ \rightarrow Ag$ gaining of e^-, reduction

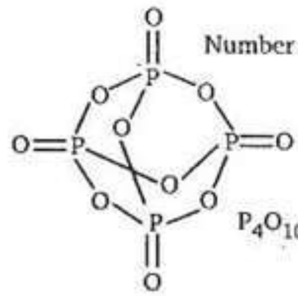
$Zn \rightarrow Zn^{2+}$ loss of e^-, oxidation

77. According to ideal gas equation

$PV = nRT$

Number of mole $n = \dfrac{PV}{RT}$ or $\dfrac{P \times 1}{RT} = \dfrac{P}{RT}$

78.



Number of P—O bonds = 16

Actual bonds = 20

P_4O_{10}

79. $4KO_2 + 2CO_2 \longrightarrow 2K_2CO_3 + 3O_2$

80. Larger the number of unpaired electrons, greater the magnetic moment $\mu = \sqrt{n(n+2)}$

n = number of unpaired electrons

$$Mn_{25}[Ar]\ 4s^2 3d^5$$
$$Mn^{2+}[Ar]\ 3d^5 4s^0$$

↑	↑	↑	↑	↑

number of unpaired electrons = 5

$$Fe_{26}[Ar]\ 4s^2 3d^6$$
$$Fe^{2+}[Ar]\ 3d^6 4s^0$$

↑↓	↑	↑	↑	↑

number of unpaired electrons = 4

$$Ti_{22}[Ar]\ 4s^2 3d^2$$
$$Ti^{2+}[Ar]\ 3d^2 4s^0$$

↑	↑			

number of unpaired electrons = 2

$$Cr_{24}[Ar]\ 3d^5 4s^1$$
$$Cr^{2+}[Ar]\ 3d^4 4s^0$$

↑	↑	↑	↑	

number of unpaired electrons = 4

Mn^{2+} : (Ar) $3d^5$ with five unpaired electrons hence maximum magnetic moment.

81. $CH{\equiv}CH + Na \longrightarrow CH{\equiv}C^-Na^+ + \frac{1}{2}H_2$

$CH{\equiv}CH$ + ammoniacal $AgNO_3 \longrightarrow$ $CH{\equiv}C^-Ag^+$ white ppt.,

$CH{\equiv}CH + HCl \longrightarrow CH_2{=}CH{-}Cl$

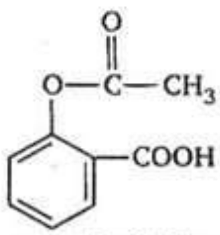
usually occurs in the presence of Hg^{2+}

$CH{\equiv}CH + NaOH \longrightarrow$ no reaction

Because of hard acid hard base soft acid soft base (HASB) theory

$CH{\equiv}CH + NaNH_2 \longrightarrow CH{\equiv}C^-Na^+$ feasible.

82.



o-acetyl salicylic acid
Aspirin (Analgesic)

83. LHS half cell :

$$\underset{p_1}{H_2(g)} \longrightarrow 2H^+\ (1M) + 2e^-$$

RHS half cell :

$$2H^+\ (1M) + 2e^- \longrightarrow \underset{p_2}{H_2(g)}$$

$$\underset{p_1}{H_2(g)} \longrightarrow \underset{p_2}{H_2(g)}$$

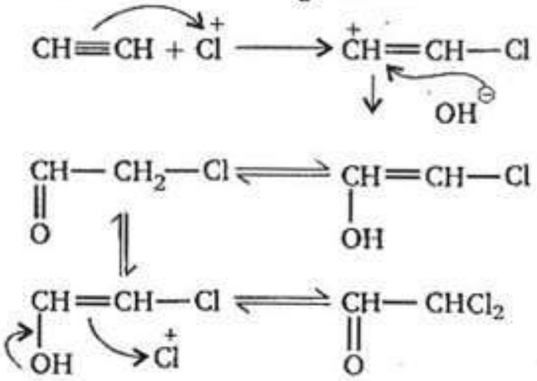
$$E^\circ_{cell} = 0.00\text{ V},\ K = \frac{p_2}{p_1},\ n = 2$$

$$E_{cell} = E^\circ_{cell} - \frac{RT}{nF}\log_e K$$

$$= 0 - \frac{RT}{2F}\log_e \frac{p_2}{p_1}$$

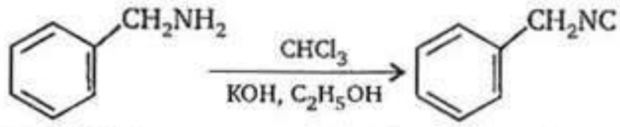
$$E_{cell} = \frac{RT}{2F}\log_e \frac{p_1}{p_2}$$

84. $CH{\equiv}CH \xrightarrow{HOCl} Cl_2CHCHO$



tautomerising electrophile is Cl^+ mainly.

85. It is carbylamine reaction.

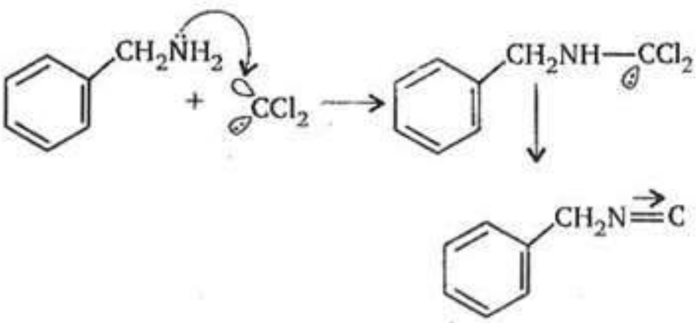


Benzyl amine Benzyl isocyanide

Carbylamine reaction (test for 1° amine)

$$CHCl_3 + KOH \longrightarrow \overset{\ominus}{C}Cl_3 \longrightarrow :CCl_2$$

Dichloro carbene



86. Oxidation takes place at anode. Hence (d)

(c) is not feasible , *i.e.*, Cr^{3+} is not oxidised to $Cr_2O_7^{2-}$ under given condition.

87. Molarity gets affected as number of moles per unit volume (volume increases with increase of temperature)

88. Cyanide process or Mc. Arthur-Forest cyanide process

$$2Ag_2S + 8NaCN + O_2 + 2H_2O \longrightarrow 4Na[Ag(CN)_2] + 4NaOH + 2S$$

Soluble silver complex is filtered and treated with zinc dust and silver gets precipitated.

$$2Na[Ag(CN)_2] + Zn \longrightarrow Na_2[Zn(CN)_4] + 2Ag\downarrow$$

89. $(CH_3)_3—CBr + H_2O \longrightarrow (CH_3)_3—C—OH + HBr$

Br is substituted by $-OH^-$ (nucleophile)
S_N1 (unimolecular substitution reaction)

90. On moving down water solubility of alkaline earth metals decreases :

Oxides and hydroxides of alkaline earth metals are basic, except Be, which gives amphoteric.

Hence, metal is Be

91. $C = C_0\left(\frac{1}{2}\right)^y$

$$y = \frac{\text{total time}}{\text{half-life}} = 3$$

$$= 100\left(\frac{1}{2}\right)^3 = 12.5 \text{ g}$$

92. Stability of complex increases with increase in charge on the central metal ion increase with basic strength of ligand.
In $[Fe(CN)_6]^{3-}$

oxidation state of Fe is +3 and basicity of CN^- is higher than OH^-, Cl^- and H_2O.

93. For the formation of Zwitter ion, basic part and acidic part both should be stronger one.

94. If Fe^{3+} and Cr^{3+} both are present then very first solid ammonium chloride and ammonium hydroxide is added slowly till the solution smells of ammonia. Fe^{3+} and Cr^{3+} precipitates in the hydroxide form

For identification precipitate is treated with NaOH and Br_2 water, yellow colouration confirms Cr^{3+} ion.

$$2NaOH + Br_2 \longrightarrow NaBrO + NaBr + H_2O$$

$$NaBrO \longrightarrow NaBr + O$$

$$2Cr(OH)_3 + 4NaOH + 3O \longrightarrow 2Na_2CrO_4 + 5H_2O$$

Solution is acidified and treated with lead acetate solution

$$Na_2CrO_4 + Pb(CH_3COO)_2 \longrightarrow \underset{\text{(yellow ppt.)}}{PbCrO_4} + 2CH_3COONa$$

95. Molar mass 108g

Total part by weight = 9 + 1 + 3.5 = 13.5

$$\text{Weight of carbon} = \frac{9}{13.5} \times 108 = 72 \text{ g}$$

$$\text{Number of carbon atoms} = \frac{72}{12} = 6$$

$$\text{Weight of hydrogen} = \frac{1}{13.5} \times 108 = 8 \text{ g}$$

$$\text{Number of hydrogen atoms} = \frac{8}{1} = 8$$

$$\text{Weight of nitrogen} = \frac{3.5}{13.5} \times 108 = 28 \text{ g}$$

$$\text{Number of nitrogen atom} = \frac{28}{14} = 2$$

Hence, molecular formula = $C_6H_8N_2$

96. $Ca(OH)_2 \rightleftharpoons Ca^{2+} + 2OH^-$

$$K_{sp} = [Ca^{2+}][OH^-]^2$$

$$= (s)(2s)^2 = 4s^3$$

97. The amount of heat required to raise the temperature of one mole of substance through 1°C is called **molar heat capacity**.

$$C = \frac{q}{T_2 - T_1}$$

98. ${}^1_0n \longrightarrow {}^1_1H + {}^{0}_{-1}e\,(\beta)$

99. Negative deviation means lower vapour pressure, it suggests high boiling point, thus resultant intermolecular force should be stronger than individual one.

100. For purification

impure gets oxidised (deelectronation) falling into solution with mud, that's why impure at anode.

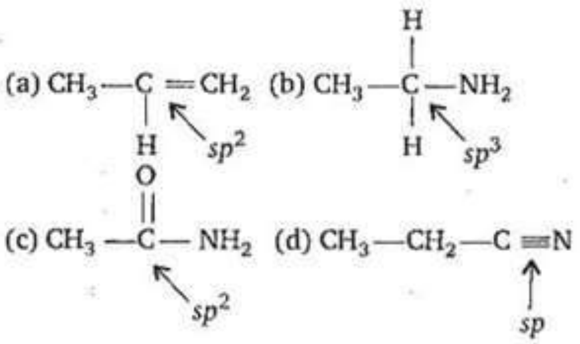
101. Aluminium is extracted by electrolysis of a fused mixture of alumina (2–8%) synthetic cryolite Na_3AlF_6 (80–85%), AlF_3 and fluorspar. This makes alumina good conductor and lowers the fusion temperature also.

102. $\Delta T_b = m K_b$

$$\Delta T_f = m K_f$$

$$\frac{\Delta T_b}{\Delta T_f} = \frac{K_b}{K_f} = \frac{0.512}{1.86}$$

$$\Delta T_b = \frac{0.512}{1.86} \times 0.186 = 0.0512°$$

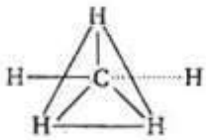
103. 

104. 109° 28′, it means pure regular tetrahedral.

(a) NH_3 sp^3 pyramidal

(b) H_2O sp^3 bent

(c) CH_5^+



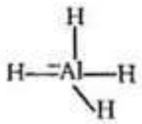
(d) NH_4^+ sp^3 tetrahedral

107. $HPO_4^{2-} + H_2O \rightleftharpoons H_2PO_4^{2-} + OH^-$

$HPO_4^{2-} + H_2O \rightleftharpoons PO_4^{3-} + H_3O^+$

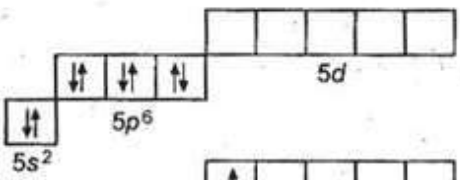
$H_2PO_2^-$ is a conjugate base of H_3PO_2 (a monobasic acid) and does not give H^+, HPO_3^{2-} is a conjugate base of $H_2PO_3^-$ and does not ionise further, since H_3PO_3 is a dibasic acid.

108.

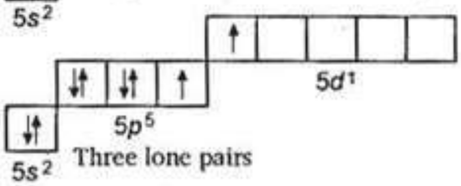
(a)	AlH_3 sp^2		Change in hybridisation as well as in shape
(b)	H_2O sp^3 (2 bp + 2 lp)	H_3O^+ sp^3 (3 bp + 1 lp)	Change in shape only, bent to pyramidal
(c)	NH_3 sp^3 (3 bp + 1 lp)	NH_4^+ sp^3 (4 bp)	Change in shape only, pyramidal to tetrahedral

109. Racemic mixture is an equimolar mixture of enantiomers.

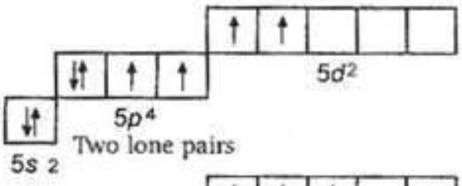
110. Xe



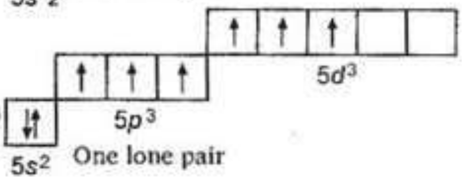
Xe (first excited state) which forms XeF_2



Xe (second excited state) which forms XeF_4



Xe (third excited state) which forms XeF_6



111. NaCl is salt of strong acid and strong base. Its not the case of buffer.

$$NaCl + H_2O \rightleftharpoons NaOH + HCl$$

aqueous NaCl, itself exact neutral solution

$$HCl + H_2O \rightleftharpoons H_3O^+ + Cl^-$$

makes solution acidic.

112. Unit of rate constant $k = \frac{1}{\text{Time}} \cdot \frac{1}{(\text{conc.})^{n-1}}$

For k_1 $n = 0$ $k = \frac{1}{t} \cdot \frac{1}{(\text{conc})^{0-1}}$ hence

$\text{conc} \cdot t^{-1} \equiv M \cdot \sec^{-1}$

k_2 $n = 1$ $k = \frac{1}{t} \cdot \frac{1}{(\text{conc})^{1-1}} = \frac{1}{t} = s^{-1}$

113. RNA Ribonucleic acid

Sugar present in RNA is D (–) ribose and consists of cytosine and uracil as pyrimidine and guanine and adenine as purine bases.

114. At LHS (oxidation) $2 \times (Ag \longrightarrow Ag^+ + e^-)$

$$E^\circ_{ox} = -x$$

At RHS (reduction) $Cu^{2+} + 2e^- \longrightarrow Cu$

$$E^\circ_{red} = +y$$

$$2Ag + Cu^{2+} \longrightarrow Cu + 2Ag^+, E^\circ_{red} = (y - x)$$

Note : E° values remain constant when half-cell equation is multipled/divided.

115. On the basis of kinetic theory of gases

$$PV = \frac{1}{2} N_A m\bar{v}^2$$

and $\frac{1}{2} m\bar{v}^2 = \frac{3}{2} KT$

$$P = \frac{1}{3}\left(\frac{N}{V}\right) m\bar{v}^2$$

or $P = \frac{2}{3}\left(\frac{N}{V}\right)\frac{1}{2} m\bar{v}^2$

$$= \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{3}{2}\right) KT$$

or $PV = NKT$

116. $\overset{+7}{MnO_4^-} \longrightarrow Mn^{2+}$ change 5

$\longrightarrow \overset{+6}{MnO_4^{2-}}$ change 1

$\longrightarrow \overset{+4}{MnO_2}$ change 3

$\longrightarrow \overset{+3}{Mn_2O_3}$ change 4

117. $H_2 + I_2 \longrightarrow 2HI$

Rate of reaction =

$$\frac{-d[H_2]}{dt} = \frac{-d[I_2]}{dt} = \frac{1}{2}\frac{d[HI]}{dt}$$

or $$\frac{-2d[H_2]}{dt} = \frac{-2d[I_2]}{dt} = \frac{d[HI]}{dt}$$

118. 560 g of Fe

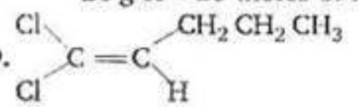
No. of moles = $\frac{560\text{ g}}{56\text{ g}} = 10$ mole

70 g of N

$14\text{ g} = 1$ mole of N

$70\text{ g} = 5$ moles of N

$20\text{ g H} = 20$ moles of H-atoms

119. 

At least two groups or atoms attached to doubly bonded carbon atoms should be different.

120. Na unit cell (body centred cubic)

8 corner atoms $\times \frac{1}{8} = 1$

1 atom placed inside the unit cell = 1

Total atoms = 2

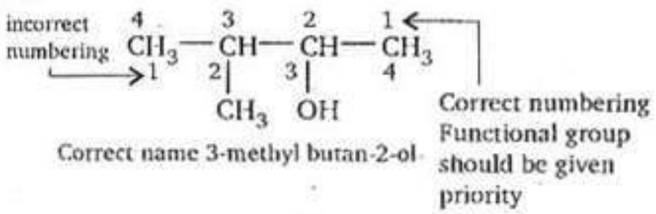
Mg unit cell (Face centred cubic)

8 corner atoms $\times \frac{1}{8} = 1$

6 face atoms $\times \frac{1}{2} = 3$

Total atoms = 4

121.



Correct name 3-methyl butan-2-ol

122. $CH_3CH_2COOH \xrightarrow[\text{red P (HVZ reaction)}]{Cl_2} CH_3CH(Cl)COOH \xrightarrow[\text{(elimination)}]{\text{alcoholic KOH}} CH_2{=}CHCOOH$ Acrylic acid

123. $K_p = K_c\,(RT)^{\Delta n_g}$

$$\Delta n_g = 1 - 1.5 = -0.5$$

$$K_p = K_c\,(RT)^{-1/2} = \frac{K_c}{(RT)^{1/2}}$$

$$\therefore \quad \frac{K_c}{K_p} = (RT)^{1/2}$$

124. $E_n = -\frac{13.6}{n^2}$ eV

For second excited state $n = 3$,

$$E_3 = -\frac{13.6}{9} = -1.51 \text{ eV}.$$

125. A square planar geometry is the result of dsp^2 hybridisation where inner d sub-shell (($d_{x^2-y^2}$ orbital) participates.

126. $[Cr(NH_3)_5NO_2]Cl_2$

Nitropentaamine chromium III chloride exhibits linkage isomerism as $-NO_2$ is ambidentate ligand

Isomer is $[Cr(NH_3)_5ONO]Cl_2$.

127. Change in volume affects number of mole per unit volume and move in the direction which undo the change.

$$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$$

no. of moles of reactants and products are equal.

128. Heisenberg uncertainty principle

$$\Delta x \cdot m\Delta v \approx \frac{h}{4\pi}$$

uncertainty in velocity

$$\Delta v = \frac{h}{4 \times 3.14 \times \Delta x \times m}$$

$$= \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 0.025 \times 10^{-5}}$$

$$= 2.1 \times 10^{-28}$$

129. $ZnO + C \longrightarrow Zn + CO$, $\Delta G^\circ = -$ ve.

Hence, this is spontaneous.

130. For spontaneity $\Delta G = -$ ve

$$\Delta G = \Delta H - T\Delta S$$

ΔH, for endothermic process +ve

At lower temperature, ΔS +ve hence ΔG = + ve

But at high temperature $T\Delta S$ will be greater than ΔH, hence $\Delta G = -$ ve, spontaneous

131. Condensation is the process of aggregation of more than one molecule without loosing any atom or group (sometimes smaller group or atoms H_2O, R—OH etc. are released)

$$\underset{\text{(Monomer)}}{n\,CH_2{=}CH_2} \longrightarrow \underset{\text{Polymer}}{-CH_2-(CH_2-CH_2)-CH_2-}$$

132. Higher the bond order greater the bond strength

Species	Bond order
O_2^{2-}	1
O_2^-	1.5
O_2	2.0
O_2^+	2.5

133. Cerium Ce_{58} [Xe] $4f^1\ 5d^1\ 6s^2$

Its most stable oxidation state is +3 but +4 is also existing.

134. r_n (radius) $\propto \frac{1}{Z}$

135. 0.005 M calcium acetate $(CH_3COO)_2Ca$

$$\underset{0.005\,M}{(CH_3COO)_2Ca} \longrightarrow Ca^{2+} + \underset{(2\times 0.005 = 0.01)}{2CH_3COO^-}$$

$\therefore \qquad [CH_3COO^-] = 0.01$ M

$$CH_3COO^- + H_2O \rightleftharpoons CH_3COOH + \underset{\text{Alkaline}}{OH^-}$$

$$pH = 7 + \frac{pK_a}{2} + \frac{\log C}{2}$$

$$= 7 + 2.37 + \frac{\log 0.01}{2}$$

$$= 7 + 2.37 - 1 = 8.37$$

136. Most of the reaction which takes place on the surface of hetrogenous catalyst proceeds being independent of concentration.

137. $Kt = -\log_e C_t + \log_e C_o$

$$t = -\frac{1}{K}\log_e C_t + \log_e C_o$$

$y = mx + c$ straight line, –ve slope

138. Mud is colloidal charged solution which on treatment with alum (Al^{3+}) gets neutralised or coagulated.

139. C=C bond is cleaved and oxidised to —COOH, —CHO group is also oxidised to —COOH.

$$(CH_3)_2C{=}CHCH_2CHO$$
$$(CH_3)C{=}O + HOOC{-}CH_2{-}COOH$$

140. Both where R is attached to vinylic ($\approx$ arylic) system

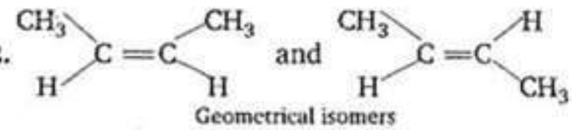
Positive inductive effect occurs in the following order :

$$-O^- > -COO^- > (CH_3)_3C- > (CH_3)_2CH-$$
$$> CH_3CH_2- > CH_3-$$

141. Phosphorus (3rd period element) can raise covalency facilitating vacant d-orbitals :

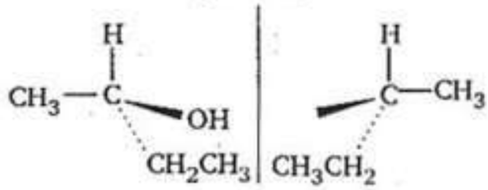
P (ground) [Ne] $3s^2\ \ 3p^3\ \ 3d^0$

P(excited) [Ne] $3s^1\ \ 3p^3\ \ 3d^1$

142. 

Geometrical isomers

(*cis–trans*)



Molecule is optically active enantiomers

Geometrical isomers and enantiomers both are stereoisomers.

143. $S \propto$ area (m^2)

$\propto$ conc. (mol/m^3)

$\propto \frac{1}{\text{length}}\ (m^{-1})$

$S = k\,m^2\,\frac{mol}{m^3}\,m^{-1}$

$k = S\ mol^{-1}\ m^2$

144. Joules law suggests

$$J = \frac{\text{Mechanical work done by the system, } w}{\text{Net heat given to the system } Q}$$

hence $J = \frac{w}{q_1 + q_2}$

Hence, $w = J(q_1 + q_2)$

is consant with Joules law of equivalence.

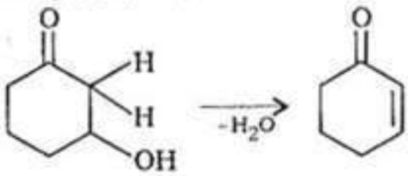
145. A bond is formed when attractive forces $(n_1 \leftrightarrow e_2, n_2 \leftrightarrow e_1)$ overcomes or balance is established with repulsive force $(n_1 \leftrightarrow n_2, e_1 \leftrightarrow e_2)$

146. All alkali metals dissolve in liquid ammonia

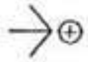
$$M + (x+y)NH_3 \rightarrow M^+(NH_3)_x + e^-(NH_3)_y$$

These electrons are excited to higher energy levels and the absorption of photons occurs in the red region of the spectrum. Thus solution appears blue.

147. Dehydrated product will be conjugated with —C=O that's why more tendency and as carbocation is more stable further to C=O that's why (b)

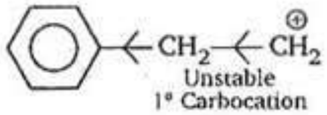


148. S_N1 unimolecular nucleophilic substitution reaction is favoured with stability of carbocation.

(a)  3° Carbocation

(b)  1° Carbocation

(c)  Arylic carbocation

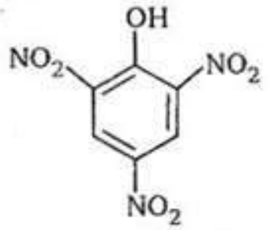
(d) 
Unstable
1° Carbocation

149. $CaOCl_2$ is written basically as $Ca(OCl)Cl$

$OCl^- \rightarrow Cl$ has $+1$ oxidation state

$Cl^- \rightarrow Cl$ has -1 oxidation state

150. Picric acid, 2, 4, 6 trinitro phenol



Strongly acidic

Mathematics

1. Key Idea : *The equation having α and β as its roots, is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.*

Since, $\alpha^2 = 5\alpha - 3 \Rightarrow \alpha^2 - 5\alpha + 3 = 0$

and $\beta^2 = 5\beta - 3 \Rightarrow \beta^2 - 5\beta + 3 = 0$

These two equations shows that α and β are the roots of the equation

$$x^2 - 5x + 3 = 0.$$

$$\therefore \quad \alpha + \beta = 5 \text{ and } \alpha\beta = 3$$

Now $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = \dfrac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$$= \frac{25 - 6}{3} = \frac{19}{3}$$

and $\qquad \dfrac{\alpha}{\beta} \cdot \dfrac{\beta}{\alpha} = 1$

Thus the equation having $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$ as its roots is given by

$$x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 0$$

$$\Rightarrow \quad x^2 - \frac{19}{3}x + 1 = 0$$

$$\Rightarrow \quad 3x^2 - 19x + 3 = 0$$

2. $\because \; y = (x + \sqrt{1 + x^2})^n$

$\therefore$ On differentiating with respect to x, we get

$$\frac{dy}{dx} = n(x + \sqrt{1 + x^2})^{n-1} \left(1 + \frac{2x}{2\sqrt{1 + x^2}}\right)$$

$$= \frac{n(x + \sqrt{1 + x^2})^n}{\sqrt{1 + x^2}}$$

$$\Rightarrow \quad (1 + x^2)\left(\frac{dy}{dx}\right)^2 = n^2y^2$$

On again differentiating with respect to x, we get

$$(1 + x^2) \cdot 2\frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + 2x\left(\frac{dy}{dx}\right)^2 = n^2 2y\frac{dy}{dx}$$

$$\Rightarrow \quad (1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$$

3. $\because$ 1, $\log_3 \sqrt{3^{1-x} + 2}$, $\log_3(4 \cdot 3^x - 1)$ are in AP

$$\therefore \quad 2\log_3(3^{1-x} + 2)^{1/2} = \log_3 3 + \log_3(4 \cdot 3^x - 1)$$

$$\Rightarrow \quad \log_3(3^{1-x} + 2) = \log_3 3(4 \cdot 3^x - 1)$$

$$\Rightarrow \quad 3^{1-x} + 2 = 12 \cdot 3^x - 3$$

Let $3^x = t$

$$\therefore \quad \frac{3}{t} + 2 = 12t - 3$$

$$\Rightarrow \quad 12t^2 - 5t - 3 = 0$$

$$\Rightarrow \quad (3t + 1)(4t - 3) = 0$$

$$\Rightarrow \quad t = -\frac{1}{3}, \frac{3}{4}$$

$$\Rightarrow \quad 3^x = \frac{3}{4}$$

$$\Rightarrow \quad \log_3\left(\frac{3}{4}\right) = x$$

$$\Rightarrow \quad x = \log_3 3 - \log_3 4$$

$$\Rightarrow \quad x = 1 - \log_3 4$$

4. Probabilities of solving the problem by A, B and C are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively.

$\therefore$ Probability that the problem is not solved

$$= \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

Hence the probability that the problem is solved

$$= 1 - \frac{1}{4} = \frac{3}{4}.$$

5. Key Idea : *Period of* $\sin\theta$ *and* $\cos\theta$ *is* 2π.

Since, $\sin^2\theta = \frac{1-\cos 2\theta}{2} = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

$\therefore$ Period of $\sin^2\theta = \frac{2\pi}{2} = \pi$

6. $\because$ l, m and n are the pth, qth and rth term of an GP whose first term is A and common ratio is R.

$\therefore$ $l = AR^{p-1}$

$\Rightarrow$ $\log l = \log A + (p-1)\log R$

Similarly, $\log m = \log A + (q-1)\log R$

and $\log n = \log A + (r-1)\log R$

Now
$$\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix}$$

$[C_1 \to C_1 - (C_3 \log A + (C_2 - C_3)\log R)]$

$= 0$

7. Key Idea : *Limit of a function exists only, if* *LHL = RHL.*

$$\lim_{x\to 0} \frac{\sqrt{1-\cos 2x}}{\sqrt{2}x} = \lim_{x\to 0} \frac{\sqrt{1-1+2\sin^2 x}}{\sqrt{2}x}$$

$$= \lim_{x\to 0} \frac{\sqrt{2}|\sin x|}{\sqrt{2}x} = \lim_{x\to 0} \frac{|\sin x|}{x}$$

Let $f(x) = \frac{|\sin x|}{x}$

Now, LHL $= \lim_{h\to 0} \frac{|\sin(0-h)|}{0-h}$

$$= \lim_{h\to 0} \frac{\sin h}{-h} = -1$$

and RHL $= \lim_{h\to 0} \frac{|\sin(0+h)|}{0+h}$

$$= \lim_{h\to 0} \frac{\sin h}{h} = 1$$

$\because$ LHL $\neq$ RHL

$\therefore$ $\lim_{x\to 0} \frac{|\sin x|}{x}$ does not exist.

8. Let $A(4, 0)$, $B(-1, -1)$ and $C(3, 5)$ be the vertices of a ΔABC.

$\therefore$ $AB = \sqrt{(-1-4)^2 + (-1-0)^2}$

$= \sqrt{25+1} = \sqrt{26}$

$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{4^2 + 6^2}$

$= \sqrt{16+36} = \sqrt{52}$

and $CA = \sqrt{(4-3)^2 + (0-5)^2}$

$= \sqrt{1+25} = \sqrt{26}$

and $CA^2 + AB^2 = (\sqrt{26})^2 + (\sqrt{26})^2$

$= 26 + 26 = 52$

$= BC^2$

$\Rightarrow$ $CA^2 + AB^2 = BC^2$

Thus, the triangle is isosceles and right angled triangle.

9. $\because$ Total number of students $= 100$

and number of boys $= 70$

$\therefore$ Number of girls $= (100 - 70) = 30$

Now the total marks of 100 students

$= 100 \times 72 = 7200$

and total marks of 70 boys $= 70 \times 75 = 5250$

$\Rightarrow$ Total marks of 30 girls

$= 7200 - 5250 = 1950$

$\therefore$ Average marks of 30 girls $= \frac{1950}{30} = 65$

10. We know that

$$\cot^{-1}(\sqrt{\cos\alpha}) + \tan^{-1}(\sqrt{\cos\alpha}) = \frac{\pi}{2} \quad \text{...(i)}$$

and given that

$$\cot^{-1}\sqrt{\cos\alpha} - \tan^{-1}\sqrt{\cos\alpha} = x \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2\cot^{-1}(\sqrt{\cos\alpha}) = \frac{\pi}{2} + x$$

$$\Rightarrow \quad \sqrt{\cos\alpha} = \cot\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

$$\Rightarrow \quad \sqrt{\cos\alpha} = \frac{\cot\frac{x}{2} - 1}{1 + \cot\frac{x}{2}}$$

$$\Rightarrow \quad \sqrt{\cos\alpha} = \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$$

$$\Rightarrow \quad \cos\alpha = \frac{1-\sin x}{1+\sin x}$$

$$\Rightarrow \quad \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{1-\sin x}{1+\sin x}$$

$$\Rightarrow \quad \sin x = \tan^2\frac{\alpha}{2}$$

Alternate Solution

$$\because \quad \cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1+\frac{1}{\sqrt{\cos\alpha}}\cdot\sqrt{\cos\alpha}}\right) = x$$

$$\Rightarrow \quad \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} = \tan x$$

$$\Rightarrow \quad \cot x = \frac{2\sqrt{\cos\alpha}}{1-\cos\alpha}$$

$$\because \quad \operatorname{cosec} x = \sqrt{1+\cot^2 x}$$

$$\therefore \quad \operatorname{cosec} x = \frac{1+\cos\alpha}{1-\cos\alpha}$$

$$\Rightarrow \quad \sin x = \frac{1-\cos\alpha}{1+\cos\alpha} = \tan^2\frac{\alpha}{2}$$

11. The given differential equation is

$$\left(1+3\frac{dy}{dx}\right)^{2/3} = 4\left(\frac{d^3y}{dx^3}\right)$$

This equation can be rewritten as

$$\left(1+3\frac{dy}{dx}\right)^{2} = 4\left(\frac{d^3y}{dx^3}\right)^3$$

This shows that the order and degree of given equation are 3 and 3 respectively.

12. Key Idea : *A line will be a plane, iff*

(a) the normal to the plane is perpendicular to the line.

(b) a point on the line lies on the plane.

In the given options equation $x - y + z = 1$ is satisfied by (3, 2, 0) and (4, 7, 4). Thus the equation of plane which passes through (3, 2, 0) and the line $\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$ is $x - y + z = 1$.

13. $\because$

$$\frac{d^2y}{dx^2} = e^{-2x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$$

$$\Rightarrow \quad y = \frac{e^{-2x}}{4} + cx + d$$

14. Key Idea : $\lim_{x\to\infty}(1+\lambda x)^{1/x} = e^{\lambda}$

$$\text{Now, } \lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$$

$$= \lim_{x\to\infty}\left(1+\frac{4x+1}{x^2+x+2}\right)^x$$

$$= \lim_{x\to\infty}\left[\left(1+\frac{4x+1}{x^2+x+2}\right)^{1/\frac{(4x+1)}{x^2+x+2}}\right]^{\frac{(4x+1)x}{x^2+x+2}}$$

$$= e^{\lim_{x\to\infty}\frac{\left(4+\frac{1}{x}\right)}{1+\frac{1}{x}+\frac{2}{x^2}}}$$

$$= e^4$$

15. Key Idea : *Domain of* $\sin^{-1} x = [-1, 1]$ *and range of* $\sin^{-1} x = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Since, domain of $\sin^{-1} x = [-1, 1]$

$$\therefore \quad -1 \le \log_3\left(\frac{x}{3}\right) \le 1$$

$$\Rightarrow \quad 3^{-1} \le \frac{x}{3} \le 3$$

$$\Rightarrow \quad 1 \le x \le 9$$

$\therefore$ Domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is [1, 9].

16. $2^{1/4}\cdot 4^{1/8}\cdot 8^{1/16}\ldots$

$$= 2^{1/4}\cdot 2^{2/8}\cdot 2^{3/16}\ldots$$

$$= 2^{\frac{1}{4}\left[1+\frac{2}{2}+\frac{3}{2^2}+\ldots\right]}$$

$$= 2^{\frac{1}{4}\left[\frac{1}{1-\frac{1}{2}}+\frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2}\right]}$$

$$= 2^{\frac{1}{4}[2+2]} = 2$$

17. Since fifth term of a GP = 2

$$\therefore \quad ar^4 = 2$$

Where a and r are first term and common ratio of a GP.

Now required product

$$= a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8$$

$$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$$

18. $\because |\sin x|$ is a periodic function with period π.

$$\therefore \quad \int_0^{10\pi}|\sin x|\,dx = 10\int_0^{\pi}|\sin x|\,dx$$

$$= 10\int_0^{\pi} \sin x\, dx$$
$$= 10[-\cos x]_0^{\pi}$$
$$= 10[-\cos\pi + \cos 0]$$
$$= 10[1+1] = 20$$

19. $\because I_n = \int_0^{\pi/4} \tan^n x\, dx$

$\therefore \quad I_{n+2} = \int_0^{\pi/4} \tan^{n+2} x\, dx$

Now $I_n + I_{n+2} = \int_0^{\pi/4} \tan^n x\,(1+\tan^2 x)\, dx$

$$= \int_0^{\pi/4} \sec^2 x \tan^n x\, dx$$

Let $\tan x = t$

$\Rightarrow \quad \sec^2 x\, dx = dt$

$\therefore \quad I_n + I_{n+2} = \int_0^1 t^n\, dt$

$$= \left[\frac{t^{n+1}}{n+1}\right]_0^1 = \frac{1}{n+1}$$

Hence $\lim_{n\to\infty} [I_n + I_{n+2}] = \lim_{n\to\infty} \frac{n}{n+1}$

$$= \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}}$$
$$= 1$$

20. $\int_0^2 [x^2]\, dx$

$$= \int_0^1 [x^2]\, dx + \int_1^{\sqrt{2}} [x^2]\, dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]\, dx + \int_{\sqrt{3}}^2 [x^2]\, dx$$

$$= \int_0^1 0\, dx + \int_1^{\sqrt{2}} 1\, dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\, dx + \int_{\sqrt{3}}^2 3\, dx$$

$$= [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^2$$
$$= \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$$
$$= 5 - \sqrt{3} - \sqrt{2}$$

21. Key Idea :

$$\int_{-a}^{a} f(x)\, dx = \begin{cases} 2\int_0^a f(x)dx, & f(-x) = f(x) \\ 0, & f(-x) = -f(x) \end{cases}$$

Let $\quad I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\, dx$

$$= \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x}\, dx + \int_{-\pi}^{\pi} \frac{2x \sin x}{1+\cos^2 x}\, dx$$

$$= 0 + 4\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$$

$\left(\because \frac{2x}{1+\cos^2 x} \text{ is an odd function}\right)$

$\therefore \quad I = 4\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\, dx$

$\Rightarrow \quad I = 4\int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}\, dx$

$\Rightarrow \quad I = 4\int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x}\, dx - 4\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\, dx$

$\Rightarrow I = 4\pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\, dx - I$

$\Rightarrow I = 2\pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\, dx$

Let $\cos x = t \Rightarrow -\sin x\, dx = dt$

$\therefore \quad I = -2\pi \int_1^{-1} \frac{1}{1+t^2}\, dt$

$$= 2\pi [\tan^{-1} t]_{-1}^{1}$$
$$= 2\pi \left[\frac{\pi}{4} + \frac{\pi}{4}\right] = 2\pi \cdot \frac{\pi}{2} = \pi^2$$

22. $\because f(x) = \sin^4 x + \cos^4 x$

$$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$$
$$= 1 - \frac{1}{2}(2\sin x \cos x)^2$$
$$= 1 - \frac{1}{2}(\sin 2x)^2 = \frac{3}{4} + \frac{1}{4}\cos 4x$$

$\because \cos x$ is periodic with period 2π.

$\therefore$ The period of $f(x) = \frac{2\pi}{4} = \frac{\pi}{2}$.

23. $\because f(x) = \sqrt{\log_{10}\left(\frac{5x - x^2}{4}\right)}$

$\therefore f(x)$ exists only for

$$\log_{10}\left(\frac{5x - x^2}{4}\right) \geq 0$$

$\Rightarrow \quad \frac{5x - x^2}{4} \geq 1$

$\Rightarrow \quad x^2 - 5x + 4 \leq 0$

$\Rightarrow \quad (x-1)(x-4) \leq 0$

$\Rightarrow \quad x \in [1, 4]$

24. $\because \sin y = x \sin(a + y)$

$\Rightarrow \quad x = \frac{\sin y}{\sin(a+y)}$

On differentiating with respect to y, we get

$$\frac{dx}{dy}=\frac{\sin(a+y)\cos y-\sin y\cos(a+y)}{\sin^2(a+y)}$$
$$=\frac{\sin(a+y-y)}{\sin^2(a+y)}$$
$$\Rightarrow \quad \frac{dx}{dy}=\frac{\sin a}{\sin^2(a+y)}$$
$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$$

25. $\because x^y=e^{x-y}$

Taking log on both sides, we get
$$y\log x=(x-y)\log_e e$$
$$\Rightarrow \quad y=\frac{x}{1+\log x}$$
On differentiating with respect to x, we get
$$\frac{dy}{dx}=\frac{(1+\log x)-x\cdot\frac{1}{x}}{(1+\log x)^2}$$
$$=\frac{\log x}{(1+\log x)^2}$$

26. The equations of two curves are
$$x^3-3xy^2+2=0 \quad \ldots\text{(i)}$$
and
$$3x^2y-y^3-2=0 \quad \ldots\text{(ii)}$$
On differentiating Eqs. (i) and (ii) with respect to x, we get
$$\left(\frac{dy}{dx}\right)_{C_1}=\frac{x^2-y^2}{2xy}$$
and
$$\left(\frac{dy}{dx}\right)_{C_2}=\frac{-2xy}{x^2-y^2}$$
$$\because \left(\frac{dy}{dx}\right)_{C_1}\times\left(\frac{dy}{dx}\right)_{C_2}=\left(\frac{x^2-y^2}{2xy}\right)\left(\frac{-2xy}{x^2-y^2}\right)$$
$$=-1$$
Hence, the two curves cut at right angle.

27. Key Idea : *A function $f(x)$ is said to be increasing function, if $f'(x)>0$.*

$$\because \quad f(x)=\cot^{-1}x+x$$
$$\therefore \quad f'(x)=-\frac{1}{1+x^2}+1=\frac{x^2}{1+x^2}$$
Hence, $f(x)$ is increasing function since $f'(x)>0$ for all x.

28. We have
$$f(x)=(x+1)^{1/3}-(x-1)^{1/3}$$
$$\therefore \quad f'(x)=\frac{1}{3}\left[\frac{1}{(x+1)^{2/3}}-\frac{1}{(x-1)^{2/3}}\right]$$
$$=\frac{(x-1)^{2/3}-(x+1)^{2/3}}{3(x^2-1)^{2/3}}$$

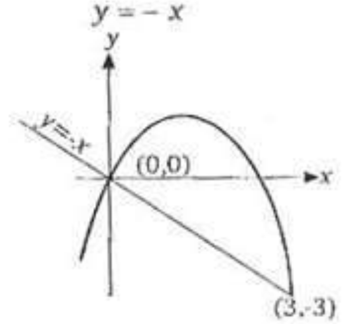
Clearly, $f'(x)$ does not exists at $x=\pm 1$.
Now, $f'(x)=0$, then $(x-1)^{2/3}=(x+1)^{2/3}$
$$\Rightarrow \quad x=0$$
Clearly, $f'(x)\neq 0$ for any other value of $x\in[0,1]$. The value of $f(x)$ at $x=0$ is 2.
Hence, the greatest value of $f(x)$ is 2

29. Let, $I=\int_0^{\pi/2}\frac{\sqrt{\sin x}\,dx}{\sqrt{\cos x}+\sqrt{\sin x}} \quad \ldots\text{(i)}$
$$I=\int_0^{\pi/2}\frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\,dx$$
$$I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad \ldots\text{(ii)}$$
On adding Eqs. (i) and (ii)
$$2I=\int_0^{\pi/2}1\,dx \Rightarrow I=\frac{\pi}{4}$$

30. Let $I=\int\frac{dx}{x(x^n+1)}$

Let $x^n+1=t$
$$\Rightarrow \quad nx^{n-1}\,dx=dt$$
$$\therefore \quad I=\frac{1}{n}\int\frac{dt}{t(t-1)}=\frac{1}{n}\int\left(\frac{1}{t-1}-\frac{1}{t}\right)dt$$
$$=\frac{1}{n}\log\left(\frac{t-1}{t}\right)+c=\frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right)+c$$

31. The equations of given curve and a line are
$$y=2x-x^2 \quad \ldots\text{(i)}$$
and
$$y=-x \quad \ldots\text{(ii)}$$



On solving Eqs. (i) and (ii), we get the points of intersection of curves which are (0, 0) and (3, −3).
$$\therefore \text{ Required area}=\int_0^3\{(2x-x^2)-(-x)\}\,dx$$
$$=\int_0^3(3x-x^2)\,dx$$
$$=\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^3$$
$$=\frac{27}{2}-\frac{27}{3}=\frac{27}{2}-9$$
$$=\frac{9}{2}\text{ sq unit}$$

32. The general equation of all non-vertical lines in a plane is $ax + by = 1$, where $b \neq 0$

$$\therefore \quad a + b\frac{dy}{dx} = 0$$

$$\Rightarrow \quad b\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 0$$

Which is the required differential equation.

33. Given two vectors lie in xy-plane. Hence a vector coplanar with them is

$$\vec{\mathbf{a}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$\because \quad \vec{\mathbf{a}} \perp (\hat{\mathbf{i}} - \hat{\mathbf{j}}) \Rightarrow \vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}}) = 0$$

$$\Rightarrow \quad (x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}}) = 0 \Rightarrow x - y = 0$$

$$\Rightarrow \quad x = y$$

$$\therefore \quad \vec{\mathbf{a}} = x\hat{\mathbf{i}} + x\hat{\mathbf{j}} \text{ and } |\vec{\mathbf{a}}| = \sqrt{x^2 + x^2} = x\sqrt{2}$$

$$\therefore \quad \text{Required unit vector} = \frac{\vec{\mathbf{a}}}{|\vec{\mathbf{a}}|} = \frac{x(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{x\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

34. The vector $\hat{\mathbf{i}} + x\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ doubled in magnitude, then it becomes $4\hat{\mathbf{i}} + (4x - 2)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$$\therefore \quad 2|\hat{\mathbf{i}} + x\hat{\mathbf{j}} + 3\hat{\mathbf{k}}| = |4\hat{\mathbf{i}} + (4x - 2)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}|$$

$$\Rightarrow \quad 2\sqrt{1 + x^2 + 9} = \sqrt{16 + (4x - 2)^2 + 4}$$

$$\Rightarrow \quad 40 + 4x^2 = 20 + (4x - 2)^2$$

$$\Rightarrow \quad 3x^2 - 4x - 4 = 0$$

$$\Rightarrow \quad 3x^2 - 6x + 2x - 4 = 0$$

$$\Rightarrow \quad 3x(x - 2) + 2(x - 2) = 0$$

$$\Rightarrow \quad (x - 2)(3x + 2) = 0$$

$$\Rightarrow \quad x = 2, -\frac{2}{3}$$

35. A parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7), parallel to the co-ordinate planes.

Let a, b, c be the lengths of edges, then

$a = 5 - 2 = 3$, $b = 9 - 3 = 6$, and $c = 7 - 5 = 2$

So the length of diagonal of a parallelopiped

$$= \sqrt{a^2 + b^2 + c^2}$$

$$= \sqrt{9 + 36 + 4}$$

$$= \sqrt{49} = 7 \text{ unit}$$

36. Key Idea : *A line* $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ *lie in the plane* $ax + by + cz + d = 0$, *if*

(a) $al + bm + cn = 0$, *and*

(b) $ax_1 + by_1 + cz_1 + d = 0$.

The equation of plane containing the line

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} \text{ is}$$

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0, \text{ if}$$

$$al + bm + cn = 0$$

37. The total number of ways in which numbers can be choosed $= 25 \times 25 = 625$

The number of ways in which either players can choose same numbers $= 25$

$$\therefore \quad \text{Probability that they win a prize} = \frac{25}{625} = \frac{1}{25}$$

Thus the probability that they will not win a prize $= 1 - \frac{1}{25} = \frac{24}{25}$.

38. $\because$ A and B are two mutually exclusive events

$$\therefore \quad A \cap B = \phi \Rightarrow A \subseteq \bar{B} \text{ and } B \subseteq \bar{A}$$

$$\Rightarrow \quad P(A) \leq P(\bar{B}) \text{ and } P(B) \leq P(\bar{A}).$$

Note : Two events A and B are said to be mutually exclusive events if $A \cap B = \phi$.

39. The equation of parabola is

$$y^2 + 4y + 4x + 2 = 0$$

$$\Rightarrow \quad y^2 + 4y + 4 = -4x - 2 + 4$$

$$\Rightarrow \quad (y + 2)^2 = -4\left(x - \frac{1}{2}\right)$$

Let $y + 2 = Y$ and $x - \frac{1}{2} = X$

$$\therefore \quad Y^2 = -4X$$

Here $a = 1$

$\therefore$ Equation of directrix is $X = 1$

$$\Rightarrow \quad x - \frac{1}{2} = 1$$

$$\Rightarrow \quad x = \frac{3}{2}$$

40. Key Idea : *The number of triangles those can be formed using n points* $= {}^nC_3$

$$\because \quad T_n = {}^nC_3$$

$$\therefore \quad T_{n+1} - T_n = 21$$

$$\Rightarrow \quad {}^{n+1}C_3 - {}^nC_3 = 21$$

$$\Rightarrow \quad {}^nC_2 + {}^nC_3 - {}^nC_3 = 21$$

$$(\because {}^nC_2 + {}^nC_3 = {}^{n+1}C_3)$$

$$\Rightarrow \quad {}^nC_2 = 21$$

$$\Rightarrow \quad \frac{n(n-1)}{2} = 21 \Rightarrow n^2 - n - 42 = 0$$

$$\Rightarrow \quad (n-7)(n+6) = 0$$

$$\Rightarrow \quad n = 7 \qquad (\because n \neq -6)$$

41. We know that

$$A + B + C = \pi$$

$$\Rightarrow \quad A + C = \pi - B$$

$$\Rightarrow \quad \frac{A - B + C}{2} = \frac{\pi}{2} - B$$

$$\therefore \quad 2ca \sin\left(\frac{A-B+C}{2}\right) = 2ca \sin\left(\frac{\pi}{2} - B\right)$$

$$= 2ac \cos B = 2ac\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

$$= a^2 + c^2 - b^2$$

42. $\lim_{x \to \infty} \left(\frac{x-3}{x+2}\right)^x$

$$= \lim_{x \to \infty} \left[1 - \frac{5}{x+2}\right]^x$$

$$= \lim_{x \to \infty} \left[\left(1 + \left(\frac{-5}{x+2}\right)\right)^{1/\left(\frac{-5}{x+2}\right)}\right]^{\left(\frac{-5x}{x+2}\right)}$$

$$= e^{\lim_{x \to \infty}\left(-\frac{5}{1+2/x}\right)}$$

$$= e^{-5}$$

Alternative Solution :

$$\lim_{x \to \infty} \left(\frac{x-3}{x+2}\right)^x$$

$$= \lim_{x \to \infty} \frac{\left(1 - \frac{3}{x}\right)^x}{\left(1 + \frac{2}{x}\right)^x}$$

$$= \frac{e^{-3}}{e^2} = e^{-5}$$

43. Key Idea : *If the triangle is equilateral, then incentre is coincide with centroid of the triangle.*

Let $A(1, \sqrt{3}) B(0, 0), C(2, 0)$ be the vertices of a triangle ABC.

$$\therefore \quad a = BC = \sqrt{(2-0)^2 + (0-0)^2} = 2$$

$$b = AC = \sqrt{(2-1)^2 + (0-\sqrt{3})^2} = 2$$

and $c = AB = \sqrt{(0-1)^2 + (0-\sqrt{3})^2} = 2$

$\therefore$ The triangle is an equilateral triangle.

$\therefore$ Incentre is same as centroid of the triangle.

$\Rightarrow$ Co-ordinates of incentre are

$$\left(\frac{1+0+2}{3}, \frac{\sqrt{3}+0+0}{3}\right) \text{ i.e., } \left(1, \frac{1}{\sqrt{3}}\right).$$

44. Key Idea : *If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the sides of a triangle, then $\vec{a} + \vec{b} + \vec{c} = 0$*

$$\because \quad \vec{a} + \vec{b} + \vec{c} = 0$$

$$\Rightarrow \quad \vec{a} + \vec{b} = -\vec{c}$$

$$\Rightarrow \quad (\vec{a} + \vec{b}) \times \vec{c} = -\vec{c} \times \vec{c}$$

$$\Rightarrow \quad \vec{a} \times \vec{c} + \vec{b} \times \vec{c} = 0$$

$$\Rightarrow \quad \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$$

Similarly, $\quad \vec{a} \times \vec{b} = \vec{b} \times \vec{c}$

Hence $\quad \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

45. Key Idea : *If ω is a cube root of unity, then $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$*

$$(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7$$

$$(\because 1 + \omega + \omega^2 = 0)$$

$$= (-2\omega^2)^7$$

$$= -2^7 \cdot \omega^{14}$$

$$= -128\,(\omega^3)^4 \omega^2$$

$$= -128\,\omega^2 \qquad (\because \omega^3 = 1)$$

46. $\because \begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$

$$= \begin{vmatrix} 6i+4 & 0 & 0 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} \quad (R_1 \to R_1 + R_2)$$

$$= (6i + 4)\begin{vmatrix} 3i & -1 \\ 3 & i \end{vmatrix}$$

$$= (6i + 4)(3i^2 + 3) = 0$$

Put $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$

$$\Rightarrow \quad 0 + 0i = x + iy$$

$$\Rightarrow \quad x = 0, \quad y = 0$$

47. $\because \sin\theta \le 1$

$$\sin^2\theta \le 1$$

$$\Rightarrow \quad \frac{4xy}{(x+y)^2} \le 1$$

$$\Rightarrow \quad 0 \le (x+y)^2 - 4xy$$

$$\Rightarrow \quad x^2 + y^2 + 2xy - 4xy \ge 0$$

$$\Rightarrow \quad (x-y)^2 \ge 0$$

Which is true for all real x and y provided $x+y \neq 0$, otherwise $\frac{4xy}{(x+y)^2}$ will be meaningless.

Note : $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is possible only, if $x = y$.

48. The equation of an ellipse is

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

Here $a = 4$, $b = 3$

$$\therefore \quad e=\sqrt{1-\frac{b^2}{a^2}}$$

$$=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$$

$\therefore$ Foci of an ellipse are $(\pm\sqrt{7}, 0)$.

$\therefore$ Radius of required circle

$$=\sqrt{(\sqrt{7}-0)^2+(0-3)^2}$$

$$=\sqrt{7+9}=\sqrt{16}=4 \text{ unit}$$

49. Required probability

$$=P(A_1 A_2' A_3)+P(A_1' A_2 A_3)$$

$$=P(A_1)P(A_2')P(A_3)+P(A_1')P(A_2)P(A_3)$$

$$=\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^3=\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$$

50. $$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix} \quad (R_1 \to R_1+R_3)$$

$$=\begin{vmatrix} 1-i & -1 & \omega^2-1 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix}$$

$$=0$$

($\because$ Two rows are identical).

51. Let $q = 1-p$. Since, head appears first time in an even throw 2 or 4 or 6... .

$$\therefore \quad \frac{2}{5}=qp+q^3p+q^5p+\dots$$

$$\Rightarrow \quad \frac{2}{5}=\frac{qp}{1-q^2}$$

$$\Rightarrow \quad \frac{2}{5}=\frac{(1-p)p}{1-(1-p)^2}$$

$$\Rightarrow \quad \frac{2}{5}=\frac{1-p}{2-p}$$

$$\Rightarrow \quad 4-2p=5-5p$$

$$\Rightarrow \quad 3p=1$$

$$\Rightarrow \quad p=\frac{1}{3}$$

52. Required probability $={}^7C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^5\times\frac{1}{6}$

$$=\frac{{}^7C_2\times 5^5}{6^8}$$

53. $\lim_{x\to 2}\frac{x f(2)-2f(x)}{x-2}$

$$=\lim_{x\to 2}\frac{xf(2)-2f(2)+2f(2)-2f(x)}{x-2}$$

$$=\lim_{x\to 2}\frac{f(2)(x-2)-2\{f(x)-f(2)\}}{x-2}$$

$$=f(2)-2\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$$

$$=f(2)-2f'(2)=4-2\times 4=-4$$

Alternative Solution :

$$\lim_{x\to 2}\left\{\frac{xf(2)-2f(x)}{x-2}\right\}$$

$=\lim_{x\to 2}\{f(2)-2f'(x)\}$ (by L' Hospital's Rule)

$=f(2)-2f'(2)$

$=4-2\times 4=-4$

54. Key Idea : *Equations of angle bisectors of lines* $a_1x+b_1y+c_1=0$ *and* $a_2x+b_2y+c_2=0$ *are*

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}=\pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}.$$

For the two lines $24x+7y-20=0$ and $4x-3y-2=0$, the angle bisectors are given by

$$\frac{24x+7y-20}{25}=\pm\frac{4x-3y-2}{5}$$

Taking positive sign, we get

$$2x+11y-5=0$$

Therefore the given three lines are concurrent with one line bisecting the angle between the other two.

55. $\sqrt{3}x+y=0$ makes an angle of $120°$ with OX and $\sqrt{3}x-y=0$ makes an angle of $60°$ with OX. So, the required line is $y-2=0$.

56. Equation of circle is

$$x^2+y^2-4x-2y-20=0.$$

Whose centre is $C(2,1)$ and radius is 5 unit.

Since $S_1=10^2+7^2-4\times 10-2\times 7-20>0$

So P lies outside the circle.

Now, $PC=\sqrt{(2-10)^2+(1-7)^2}$

$$=\sqrt{8^2+6^2}=\sqrt{10^2}=10$$

$\therefore$ Greatest distance between circle and the point $P=10+5=15$ unit.

57. Equation of circle is $x^2+y^2+4x-4y+4=0$ whose centre is $(-2, 2)$.

58. Key Idea : *The foci of an ellipse* $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ *is given by* $(\pm ae, 0)$.

Since, $e=\frac{1}{2}, ae=2$

$\Rightarrow \quad a=4$

$\because \quad b^2=a^2(1-e^2)$

$=16\left(1-\frac{1}{4}\right)=12$

Thus, the equation of an ellipse is $\frac{x^2}{16}+\frac{y^2}{12}=1$.

59. The mid-point of the chord is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. The equation of the chord in terms of its mid-point is $T=S_1$

or $\quad x\left(\frac{y_1+y_2}{2}\right)+y\left(\frac{x_1+x_2}{2}\right)$

$=2\left(\frac{x_1+x_2}{2}\right)\left(\frac{y_1+y_2}{2}\right)$

$\Rightarrow \quad x(y_1+y_2)+y(x_1+x_2)$

$=(x_1+x_2)(y_1+y_2)$

$\Rightarrow \quad \frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$

60. Given that $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\vec{b}=\hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ form a right handed system.

$$\therefore \quad \vec{c}=\vec{b}\times\vec{a}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 0\\ x & y & z\end{vmatrix}$$

$=\hat{i}z-x\hat{k}$

61. The equation of a line through the centre $\hat{j}+2\hat{k}$ and normal to the given plane is

$\vec{r}=\hat{j}+2\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k}) \quad \ldots(i)$

This meets the plane at a point for which we must have

$[(\hat{j}+2\hat{k})+\lambda(\hat{i}+2\hat{j}+2\hat{k})]\cdot(\hat{i}+2\hat{j}+2\hat{k})=15$

$\Rightarrow \quad 6+9\lambda=15$

$\Rightarrow \quad \lambda=1$

On putting $\lambda=1$ in Eq. (i), weget

$\vec{r}=\hat{i}+3\hat{j}+4\hat{k}$

$\therefore$ Centre of the circle is $(1, 3, 4)$.

62. Key Idea : $\cos 2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\therefore \quad \frac{1-\tan^2 15°}{1+\tan^2 15°}=\cos 30°$

$=\frac{\sqrt{3}}{2}$

63. Key Idea : $\tan\theta$ *is negative in second and fourth quadrants.*

$\because \quad \tan\theta=-\frac{4}{3}$

Here $p=4$ and $b=3$

$\therefore \quad h=\sqrt{p^2+b^2}=\sqrt{16+9}=\sqrt{25}$

$\Rightarrow \quad h=5$

$\therefore \quad \sin\theta=\frac{p}{h}=\frac{4}{5}$

But $\tan\theta$ is negative which is possible only if θ lie in second and fourth quadrants.

$\therefore \quad \sin\theta$ may be $\frac{4}{5}$ or $-\frac{4}{5}$.

64. $\because \sin(\alpha+\beta)=1$

$\Rightarrow \quad \sin(\alpha+\beta)=\sin\frac{\pi}{2}$

$\Rightarrow \quad \alpha+\beta=\frac{\pi}{2} \quad \ldots(i)$

and $\quad \sin(\alpha-\beta)=\frac{1}{2}$

$\Rightarrow \quad \sin(\alpha-\beta)=\sin\frac{\pi}{6}$

$\Rightarrow \quad \alpha-\beta=\frac{\pi}{6} \quad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$\alpha=\frac{\pi}{3}, \beta=\frac{\pi}{6}$

$\therefore \quad \tan(\alpha+2\beta)\tan(2\alpha+\beta)$

$=\tan\left(\frac{2\pi}{3}\right)\tan\left(\frac{5\pi}{6}\right)$

$=\left(-\cot\frac{\pi}{6}\right)\left(-\cot\frac{\pi}{3}\right)$

$=\sqrt{3}\times\frac{1}{\sqrt{3}}=1$

65. $\because y=\sin^2\theta+\operatorname{cosec}^2\theta$

$=(\sin\theta-\operatorname{cosec}\theta)^2+2$

$\Rightarrow \quad y\geq 2$

But at $\theta=0$, y becomes meaningless.

$\therefore \quad y>2$

66. Key Idea : $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

We have $a = 4, b = 3$ and $\angle A = 60°$

$\therefore \quad \cos 60° = \dfrac{c^2 + 9 - 16}{2 \times 3 \times c}$

$\Rightarrow \quad \dfrac{1}{2} = \dfrac{c^2 - 7}{2 \times 3c}$

$\Rightarrow \quad c^2 - 7 = 3c$

$\Rightarrow \quad c^2 - 3c - 7 = 0$

Thus, c is the root of above equation.

67. Key Idea : $\tan \dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

$\because \quad \tan \dfrac{A}{2} = \dfrac{5}{6}$ and $\tan \dfrac{C}{2} = \dfrac{2}{5}$

Now, $\tan \dfrac{A}{2} \tan \dfrac{C}{2} = \dfrac{5}{6} \times \dfrac{2}{5}$

$\Rightarrow \quad \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}} \cdot \sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}} = \dfrac{1}{3}$

$\Rightarrow \quad \dfrac{s-b}{s} = \dfrac{1}{3}$

$\Rightarrow \quad 3s - 3b = s$

$\Rightarrow \quad 2s = 3b$

$\Rightarrow \quad a + b + c = 3b$

$\Rightarrow \quad a + c = 2b$

$\Rightarrow a, b, c$ are in AP.

68. Key Idea :
$-\sqrt{a^2 + b^2} \le a \sin x + b \cos x \le \sqrt{a^2 + b^2}$.

Since, $\quad |c| > \sqrt{a^2 + b^2}$

but $-\sqrt{a^2 + b^2} \le a \sin x + b \cos x \le \sqrt{a^2 + b^2}$

Thus, LHS $\ne$ RHS. Then no solution exist for $a \sin x + b \cos x = c$.

69. $\because \; \alpha$ is a root of $25 \cos^2 \theta + 5 \cos \theta - 12 = 0$

$\therefore \quad 25 \cos^2 \alpha + 5 \cos \alpha - 12 = 0$

$\Rightarrow \; 25 \cos^2 \alpha + 20 \cos \alpha - 15 \cos \alpha - 12 = 0$

$\Rightarrow 5 \cos \alpha \,(5 \cos \alpha + 4) - 3 (5 \cos \alpha + 4) = 0$

$\Rightarrow \quad \cos \alpha = -\dfrac{4}{5}, \dfrac{3}{5}$

But $\quad \dfrac{\pi}{2} < \alpha < \pi$

$\therefore \quad \cos \alpha = -\dfrac{4}{5} \qquad (\because \cos \alpha < 0)$

$\Rightarrow \quad \sin \alpha = \dfrac{3}{5}$

$\therefore \sin 2\alpha = 2 \sin \alpha \cos \alpha = -2 \times \dfrac{3}{5} \times \dfrac{4}{5}$

$= -\dfrac{24}{25}$

70. Key Idea : $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$

$\therefore \quad \tan^{-1} \dfrac{1}{4} + \tan^{-1} \dfrac{2}{9} = \tan^{-1}\left(\dfrac{\dfrac{1}{4} + \dfrac{2}{9}}{1 - \dfrac{1}{4} \times \dfrac{2}{9}}\right)$

$= \tan^{-1}\left(\dfrac{1}{2}\right)$

71. $\sum_{n=0}^{\infty} \dfrac{(\log_e x)^n}{n!} = e^{\log_e x} = x$

72. $e^{(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 \dots}$

$= e^{\log(1 + x - 1)} = x$

73. $\because (1 + 2x + 3x^2 + \dots)^{-3/2} = [(1-x)^{-2}]^{-3/2}$

$= (1-x)^3$

So, coefficient of x^5 in $(1 + 2x + 3x^2 + \dots)^{-3/2}$

$=$ coefficient of x^5 in $(1-x)^3 = 0$

74. $\because (1 + x + x^2 + x^3 + \dots)^2 = [(1-x)^{-1}]^2$

$= (1-x)^{-2}$

Coefficient of x^n in $(1 + x + x^2 + \dots)^2$

$=$ coefficient of x^n in $(1-x)^{-2}$

$= {}^{n+2-1}C_{2-1} = {}^{n+1}C_1$

$= n + 1$

75. Key Idea : *If the discriminant of $ax^2 + bx + c = 0$ is positive, then this equation has two real roots.*

We have $\quad 3^{2x^2 - 7x + 7} = 3^2$

$\Rightarrow \quad 2x^2 - 7x + 7 = 2$

$\Rightarrow \quad 2x^2 - 7x + 5 = 0$

$\therefore \quad D = b^2 - 4ac$

$= (-7)^2 - 4 \times 2 \times 5$

$= 49 - 40$

$= 9$

The discriminant of this equation is positive. Hence, it has two real roots.