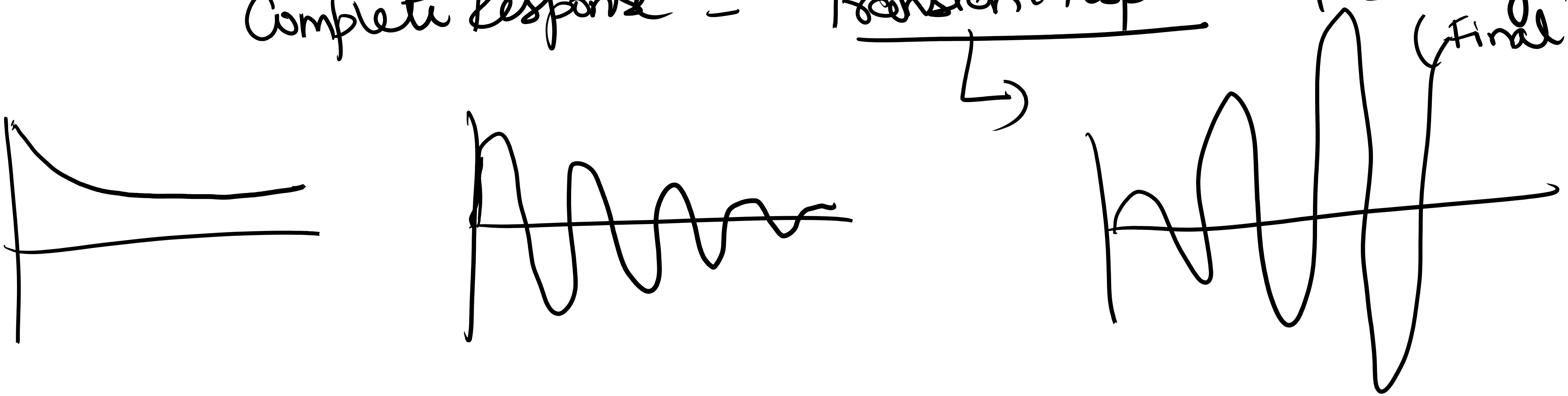
DC Transient Analysis

→ 'Transitory' ⇒ Small period of time

- Excitation ⇒ Input
- Response ⇒ Output

Complete Response = Transient Response + Steady State (Final)



* 1800's electricity | magnetism

→ Oersted's → Current carrying conductor produces magnetic field.

Faradays, maxwell ⇒ Electromagnetism

L ⇒ Inductor

C ⇒ Capacitor

DC Transients → Transitory

$L \rightarrow$; $C \rightarrow$

* Faraday & Henry → Changing magnetic field could induce a voltage in a neighbouring circuit

Induced voltage $\propto$ time rate of change of current which produced the magnetic field

$$v \propto \frac{di}{dt}$$

$$v = K\frac{di}{dt}$$

↳ 'L' → Inductor

$$\boxed{v = L\frac{di}{dt}}$$

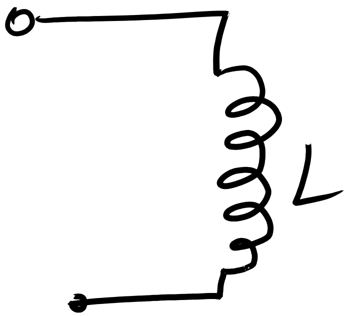
Inductance:

Inductor (Coil/ Reactor)

pure Inductance

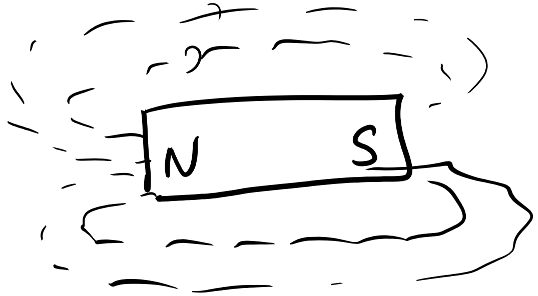
μ_0 = permeabilit

ε_0 = permitt



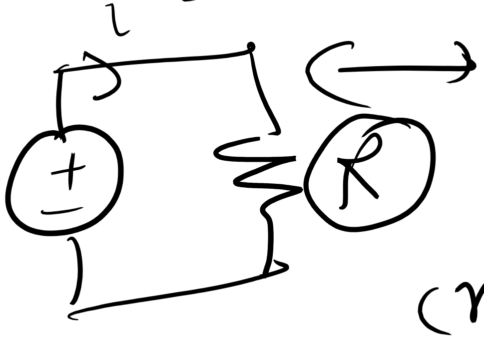
$$L = N\frac{d\phi}{di} = \frac{N\phi}{i} \quad — (1)$$

$$\boxed{L = \frac{N^2 \mu A}{l}}$$



ϕ = Magnetic flux

N = No. of turns



Reluctance

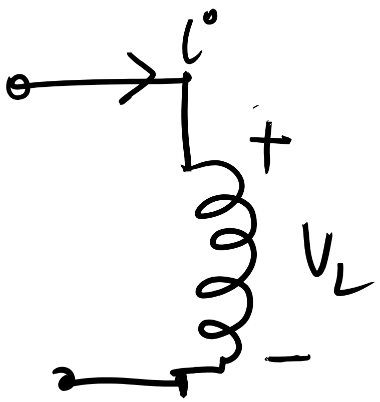
$$\phi = \frac{mmf}{Reluctance} = \frac{Ni}{(l/\mu A)} \quad — (2)$$

electromotive force (emf)

(mmf) → magnetomotive force

$$L = \frac{N^2 \cancel{i} \mu A}{l \cancel{i}} = \frac{N^2 \mu A}{l}$$

$$V_L = L\frac{di}{dt}$$



Unit of L

→ Henry (H)

No changing current;

$$\frac{di}{dt} = 0$$

$$\boxed{V_L = 0}$$

In case of DC; $\frac{di}{dt} = 0$; $V_L = 0$;

Inductor behaves as an ~~open~~ short circuit

* Capacitance :

$$\boxed{q = CV}$$

$q \rightarrow$ charge

$C \rightarrow$ Capacitance

$V \rightarrow$ potential

$$i = \frac{dq}{dt} = \frac{C\,dv}{dt}$$

$$\boxed{i = \frac{C\,dv}{dt}}$$

$\downarrow$

OC → DC

	L	C
DC	Short (SC)	Open (OC)

* if there is no changing potential ;

i.e. $\frac{dv}{dt} = 0$; $i = 0$;

In DC ; $dv/dt = 0$; Capacitance will behave as open circuit

$$i = C\frac{dv}{\boxed{dt}}$$

$dt = 0$; $\frac{dv}{dt} = \infty$; $i = \infty$ × Not possible

→ Capacitor voltage cannot change instantaneously

× Capacitor

$$C = \frac{\varepsilon A}{d}$$

$$i = C\frac{dv}{dt}$$

$$\frac{i}{C}\,dt = dv$$

× $\boxed{v_c = \frac{1}{C}\int i\,dt + V_c(0)}$ ×

* Energy stored in Inductance & Capacitance

$W_L = ?\quad \frac{1}{2} L i^2$

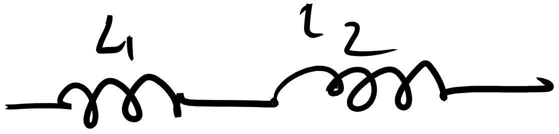
$W_C = \frac{1}{2} C V^2$

DC

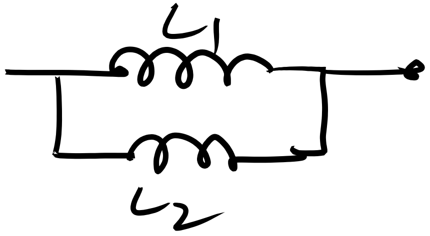
$L \rightarrow$ (SC)

$C \rightarrow$ (OC)

* Inductors in Series / Parallel

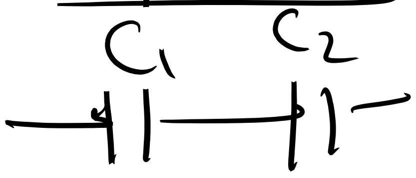


$\Rightarrow L_{eq} = L_1 + L_2$

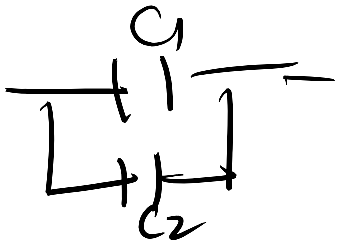


$\Rightarrow L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$

* Capacitors:



$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$



$C_{eq} = C_1 + C_2$

* Homogenous Linear Differential Equations

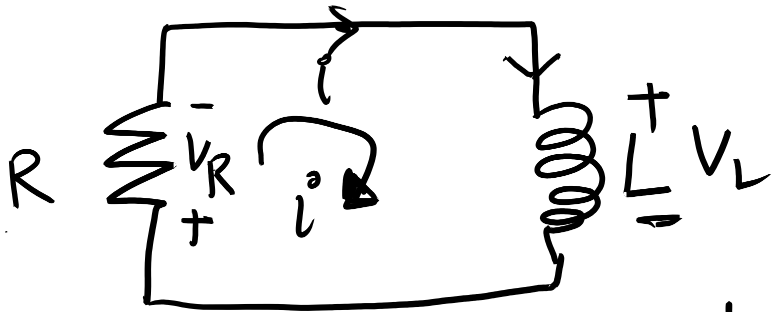
↳ Variable Separation Method

* Response = Transient Response + Steady State Response

(Natural Response) (Forced Response)

* Source Free RL circuit

Assume inductor to be precharged.



Apply KVL in the given circuit;

$$+V_R + V_L = 0 \longrightarrow (1)$$

$$iR + L\frac{di}{dt} = 0 \longrightarrow (2)$$

$$L\frac{di}{dt} = -iR$$

$$\frac{di}{i} = -\frac{R}{L}dt \longrightarrow (3)$$

Eq 3
Integrating both sides
w.r.t. time

At $t=0 \rightarrow I_0$
At $t=t \rightarrow i$

$$\int_{I_0}^{i} \frac{di}{i} = -\frac{R}{L}\int_{0}^{t} dt$$

$$\boxed{i = I_0 e^{-\frac{Rt}{L}}}$$

$$\boxed{i = I_0 e^{-t/\tau}} \rightarrow$$

Where $\boxed{\tau = \frac{L}{R}} \rightarrow$ Time constant

$$i = I_0 e^{-t/\tau}$$

$$\frac{i}{I_0} = e^{-t/\tau}$$

$t=0;\ \frac{i}{I_0} = 1$ $\rightarrow$ 63%

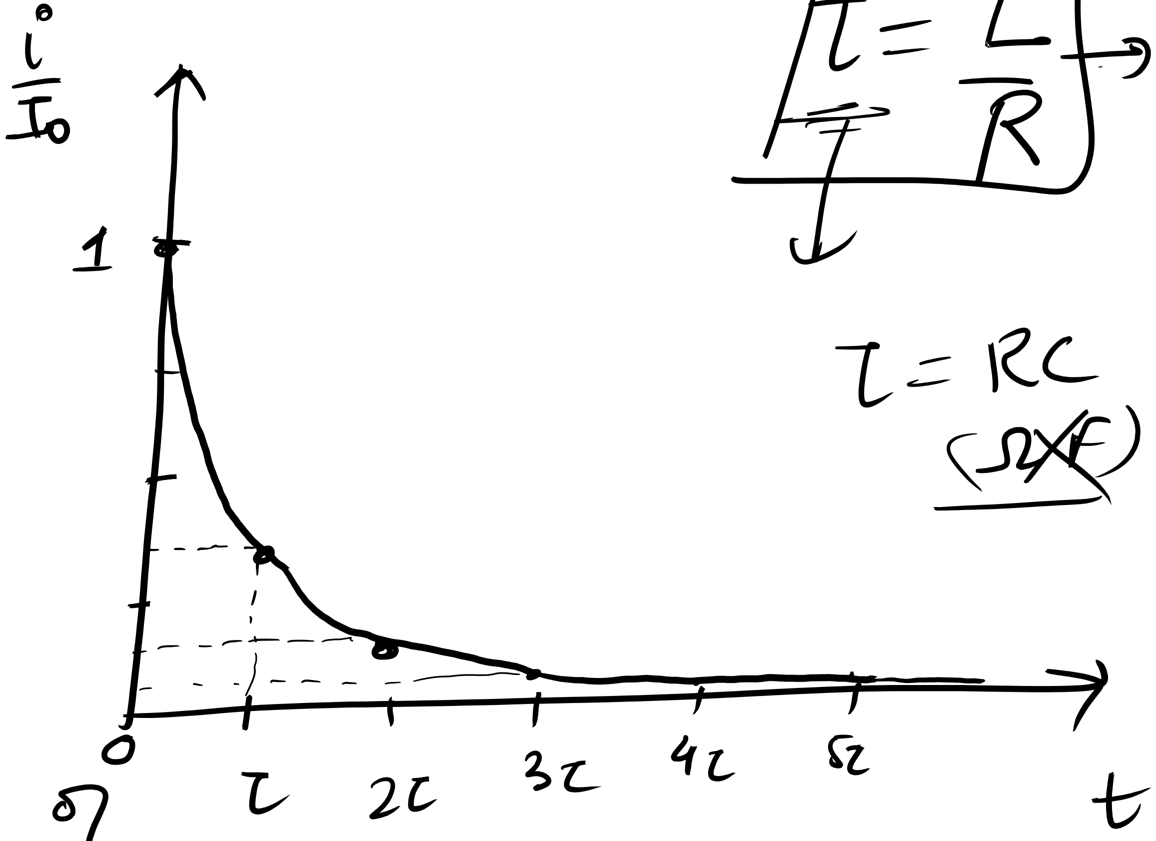
$t=\tau;\ \frac{i}{I_0} = \frac{1}{e} = 0.37$ 87%

$t=2\tau;\ \frac{i}{I_0} = \frac{1}{e^2} = 0.13$ 95%

$t=3\tau;\ 1/e^3 = 0.049$

$t=4\tau;\ = 0.018 \approx \Rightarrow$ 98.2%

$t=5\tau;\ \approx 0.006$~



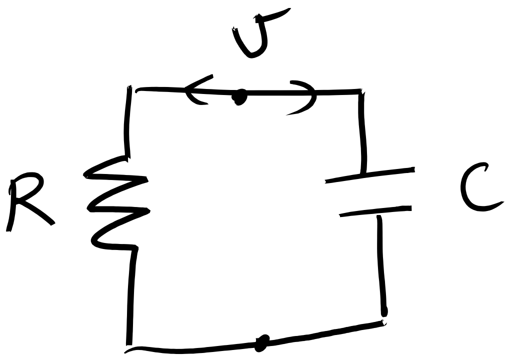
$t = 4\tau \Rightarrow$ Settling time

$$\boxed{\tau = \frac{L}{R}} \rightarrow \text{(seconds)}$$

$\tau = RC$ (s)

$(\Omega \times F)$

* Source Free RC Circuit: (Assume that capacitor is precharged)



Applying KCL at node;

$$\frac{v-0}{R} + \frac{c\,d(v-0)}{dt} = 0$$

$$\frac{v}{RC} + \frac{dv}{dt} = 0 \quad \text{——— (1)}$$

for solving H.L.D.E; -

$$\frac{v}{RC} = -\frac{dv}{dt}$$

$$\int \frac{dv}{v} = -\int \frac{dt}{RC}$$

Integrating both sides w.r.t.

$t=0 \rightarrow V_0$

$t=t \rightarrow v$

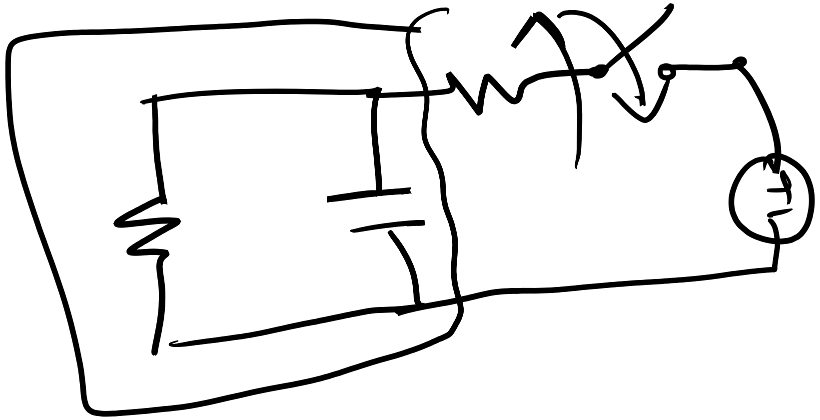
$$\boxed{v = V_0 e^{-t/RC}}$$

$$v = V_0 e^{-t/RC}$$

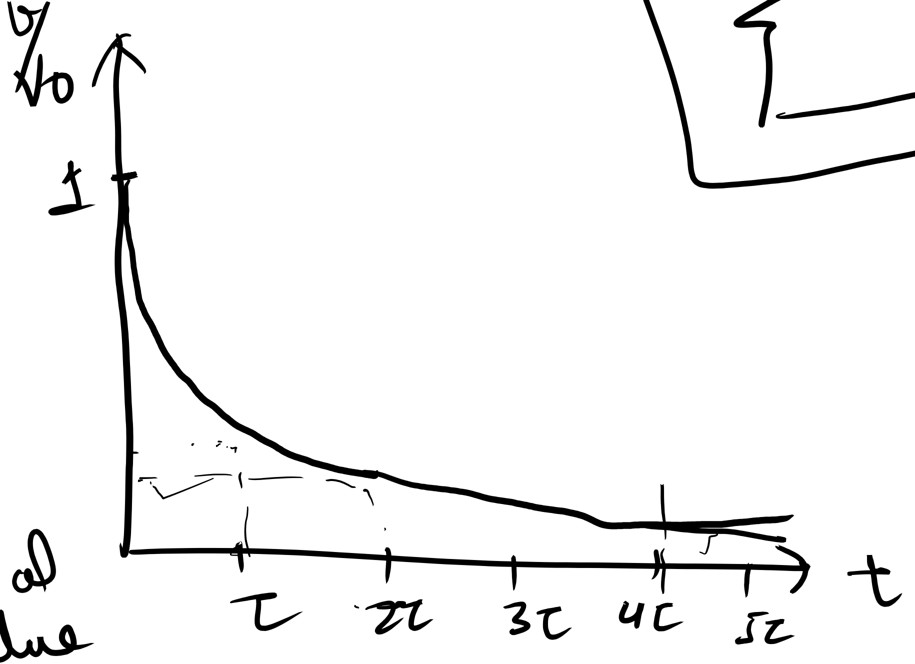
τ = time constant for RC ckt = RC

$$\boxed{v = V_0 e^{-t/\tau}}$$

	τ
• RL	L/R
• RC	RC



* Plot the response;



⇒ Settling time = 4τ

(98.2%)

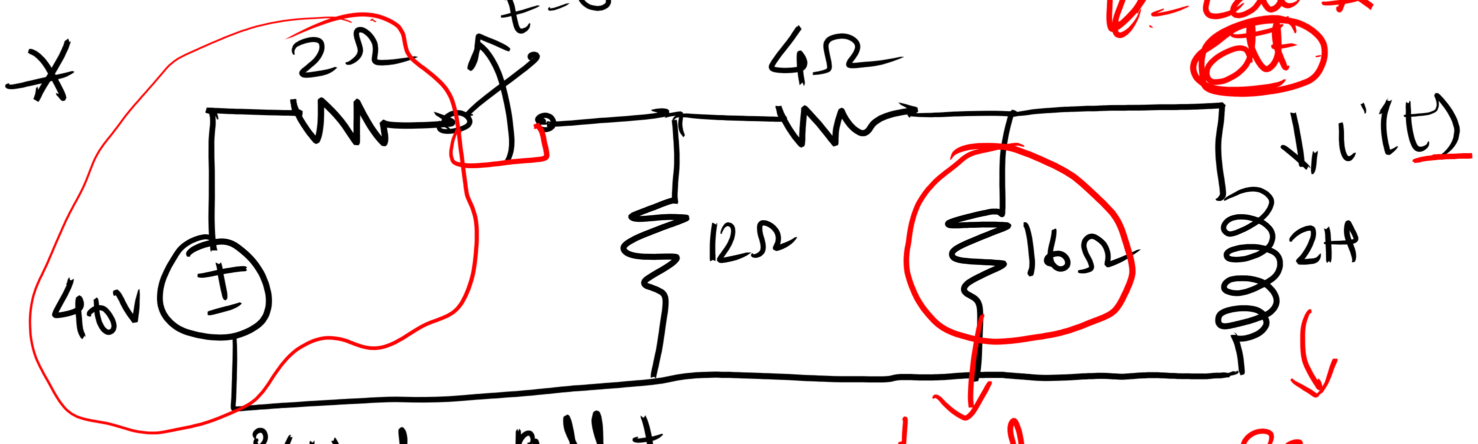
↳ time at which the response has decayed to 98.2% of its initial value

$RL \Rightarrow \underline{i = I_0 e^{-t/\tau}}$ $[\tau = L/R]$

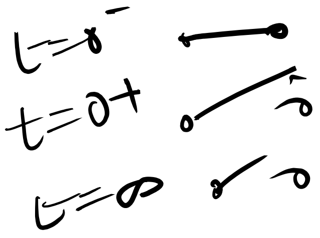
$RC \Rightarrow \underline{v = V_0 e^{-t/\tau}}$ $[\tau = RC]$

$v = L\dfrac{di}{dt}$ ✗

*

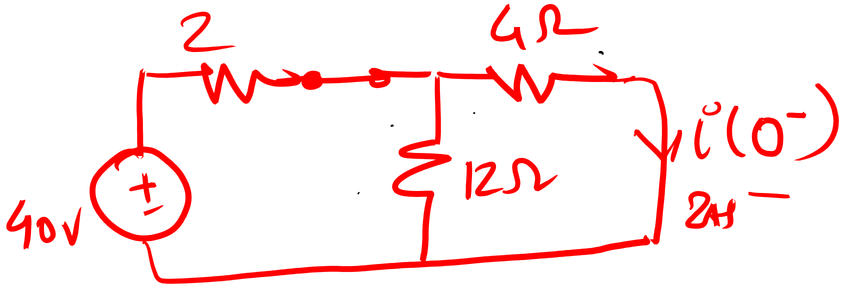


$i(t)$ for all t



$\tau = \dfrac{L}{R} = \tau = \dfrac{L}{R_{eq}}$

(i) $t < 0$ ($t = 0^-$) (SW → Closed)

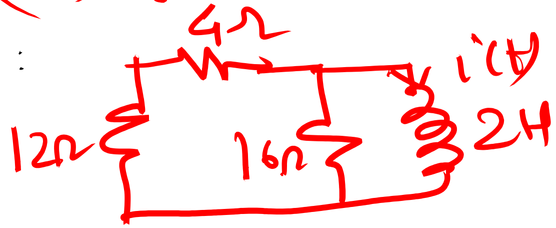


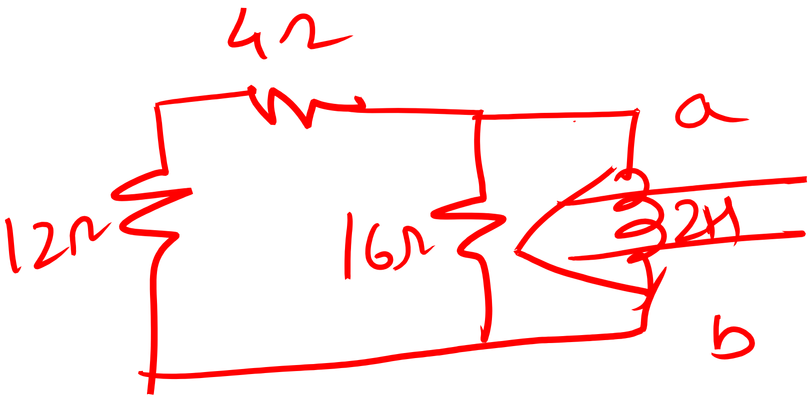
$i(0^-) = 6A$

(ii) Since inductor current cannot change instantaneously

$i(0^-) = i(0) = i(0^+) = 6A$

(iii) for $t > 0$ (SW → open)





$$R_{eq} \Rightarrow ((12+4) \parallel 16) = 8\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}\,s$$

$$i = I_0 e^{-t/\tau}$$

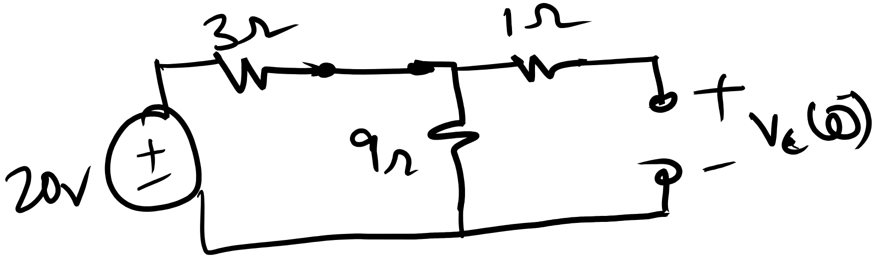
$$i(t) = 6e^{-t/(1/4)}$$

$$\boxed{i(t) = 6e^{-4t}} \text{ (Amp)}$$

*

⇒ $v_c(t)$ for $t>0$ & initial Energy stored in capacitor

(i) $t<0$ (SW → closed)



$v_c(0^-) = 15V$

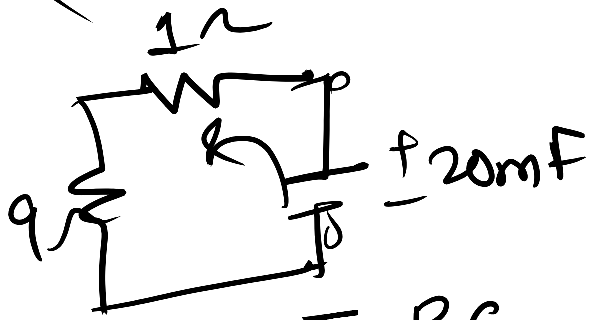
$W_E = \frac{1}{2} C V^2$

$= \frac{1}{2} \times 20 \times 10^{-3} \times (15)^2$

(ii) Since Cap. voltage cannot change instantaneously;

$v_c(0^-) = v_c(0) = v_c(0^+) = \underline{15V}$

(iii) At $t>0$



$\tau = RC$

$\tau = R_{eq} C$

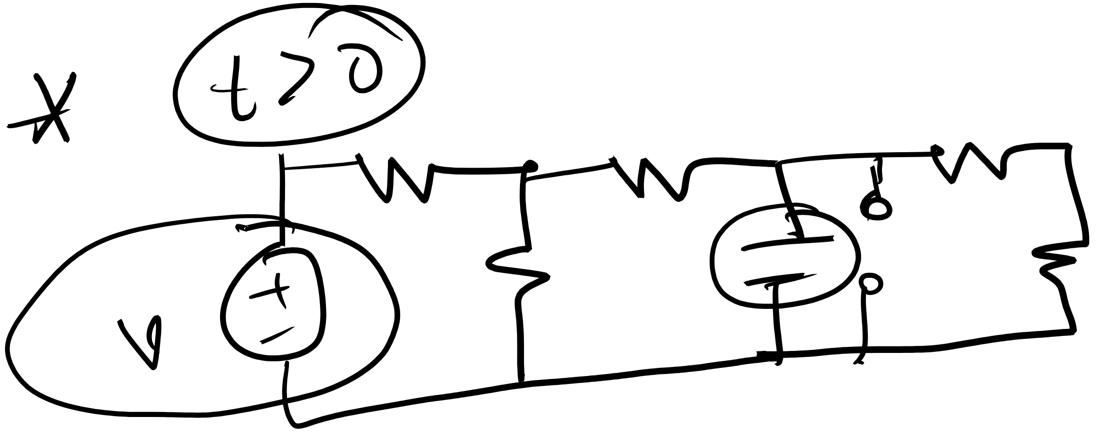
$R_{eq} = 10\Omega$

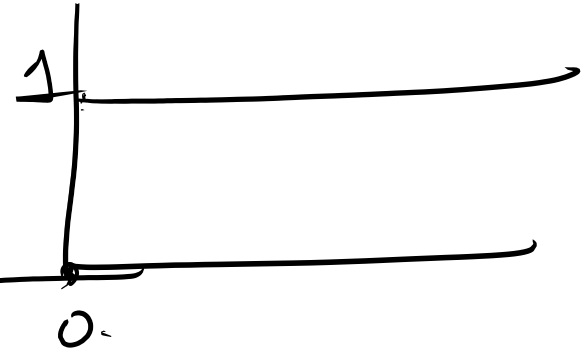
$\tau = 10 \times 20 \times 10^{-3}$ s

$= 0.2$ s

$\boxed{V_c(t) = 15 e^{-t/0.2}}$ V.

*





→ **Sourced RL & RC circuit**

Test input

1. Ramp signal
2. Step signal
3. delta signal

→4 Sinusoida

→ $$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$