Presented By:

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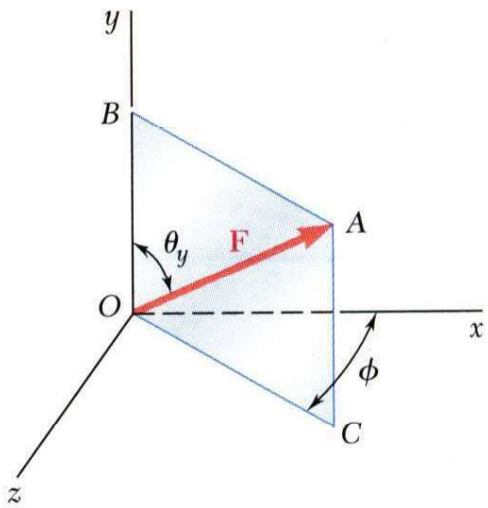
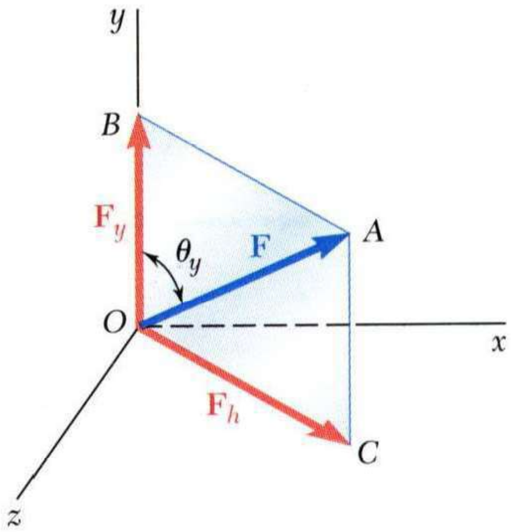
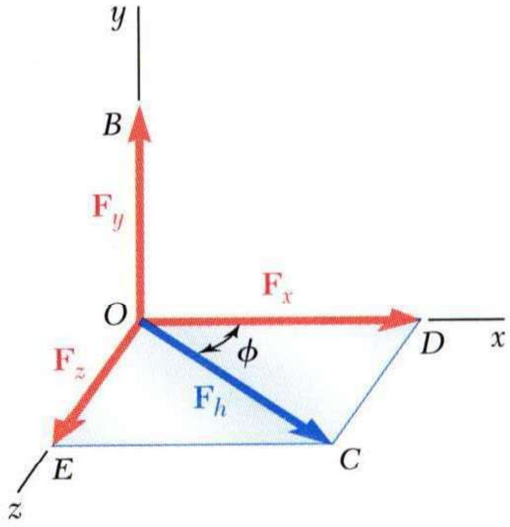
Rectangular Components in Space







- The vector $\vec{F}$ is contained in the plane *OBAC*.

- Resolve $\vec{F}$ into horizontal and vertical components.

$$F_y = F\cos\theta_y$$

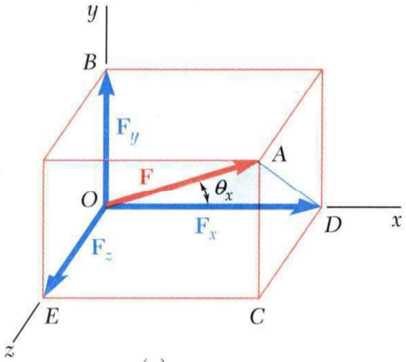
$$F_h = F\sin\theta_y$$

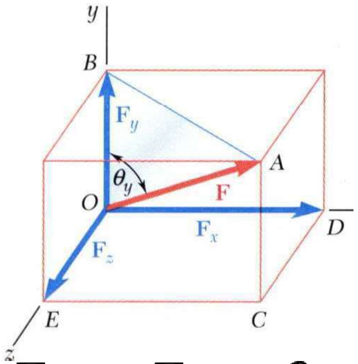
- Resolve F_h into rectangular components

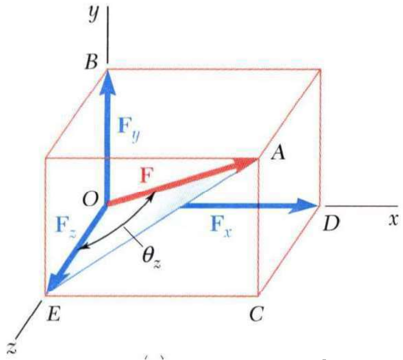
$$F_x = F_h\cos\phi$$
$$= F\sin\theta_y\cos\phi$$

$$F_z = F_h\sin\phi$$
$$= F\sin\theta_y\sin\phi$$

Rectangular Components in Space







$F_x = F cos\theta_x$ $\quad$ $F_y = F cos\theta_y$ $\quad$ $F_z = F cos\theta_z$

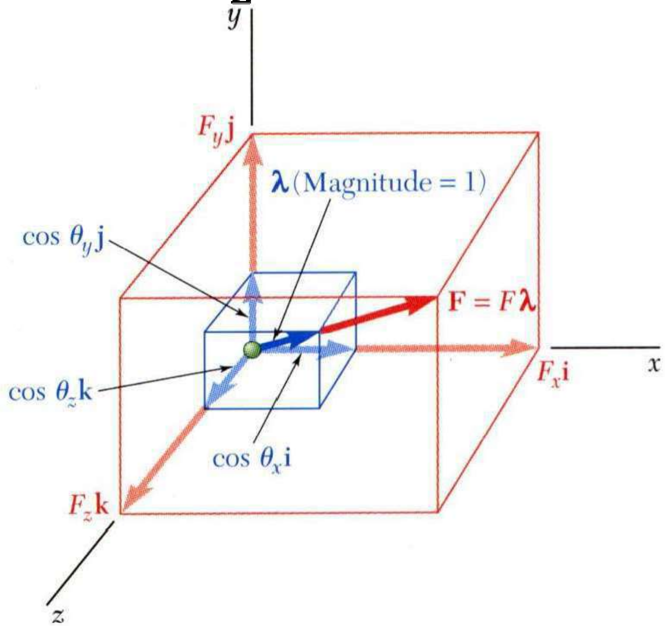
$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{F} = F cos\theta_x\boldsymbol{i} + F cos\theta_y\boldsymbol{j} + F cos\theta_z\boldsymbol{k}$$

$$\boldsymbol{F} = F\left(cos\theta_x\boldsymbol{i} + cos\theta_y\boldsymbol{j} + cos\theta_z\boldsymbol{k}\right)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

Where $\boldsymbol{\lambda} = cos\theta_x\boldsymbol{i} + cos\theta_y\boldsymbol{j} + cos\theta_z\boldsymbol{k}$

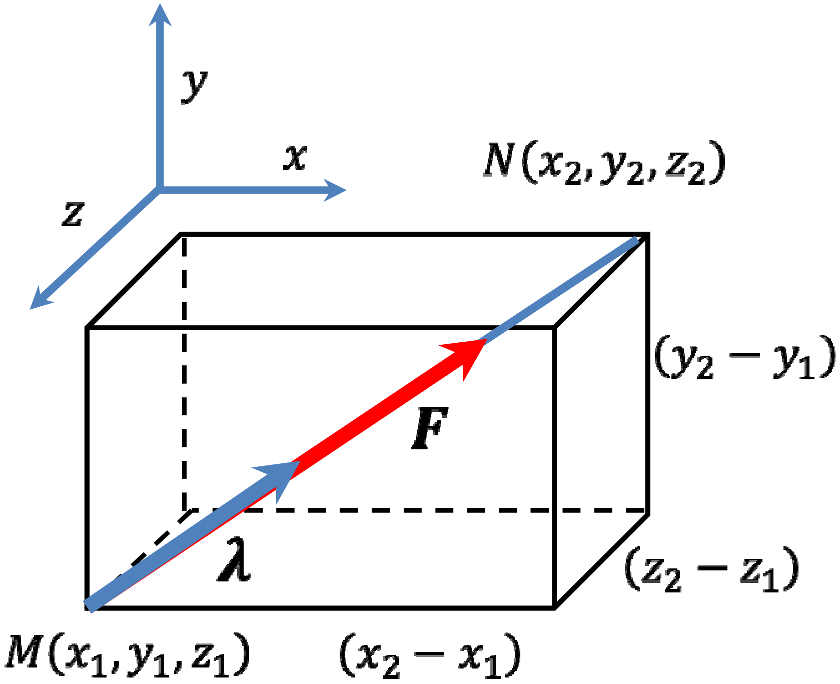


$\boldsymbol{\lambda}$ is a unit vector along the line of action of $\boldsymbol{F}$ and $cos\theta_x$, $cos\theta_y$ and $cos\theta_z$ are the direction cosine for $\boldsymbol{F}$

Rectangular Components in Space

Direction of the force is defined by the location of two points

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$\boldsymbol{d}$ is the vector joining M and N

$$\boldsymbol{d} = d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}$$

$$d_x = (x_2 - x_1) \qquad d_y = (y_2 - y_1)$$

$$d_z = (z_2 - z_1)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

$$= F\left(\frac{d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}}{d}\right)$$

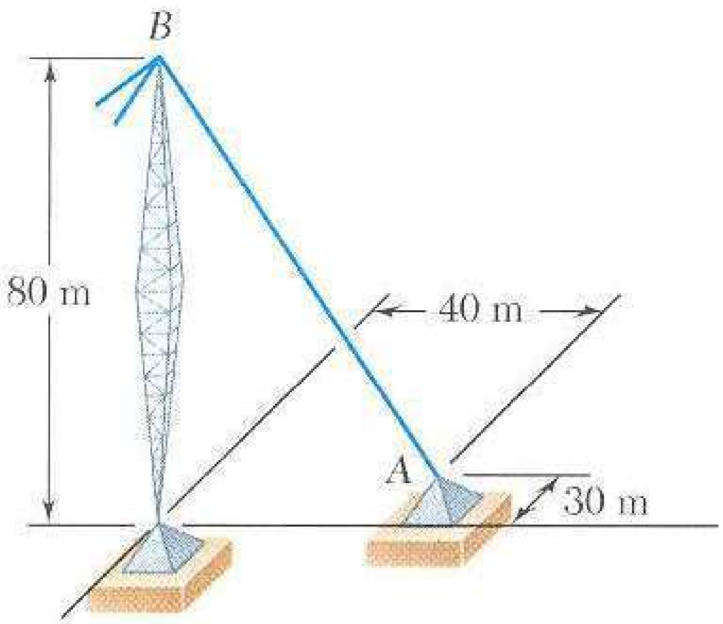
$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

Rectangular Components in Space

Example: The tension in the guy wire is 2500 N. Determine:

a) components F_x, F_y, F_z of the force acting on the bolt at A,

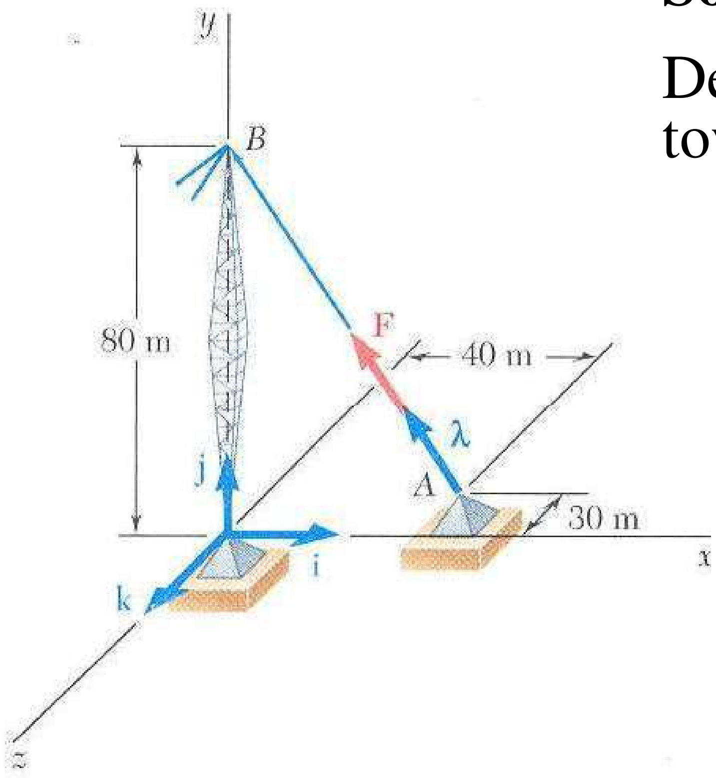
b) the angles q_x, q_y, q_z defining the direction of the force



SOLUTION:

- Based on the relative locations of the points A and B, determine the unit vector pointing from A towards B.
- Apply the unit vector to determine the components of the force acting on A.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

Rectangular Components in Space



Solution

Determine the unit vector pointing from *A* towards *B*.

$$\boldsymbol{AB} = -40\boldsymbol{i} + 80\boldsymbol{j} + 30\boldsymbol{k}$$

$$AB = \sqrt{(-40)^2 + (80)^2 + (30)^2} = 94.3$$

$$\boldsymbol{\lambda} = \frac{\boldsymbol{AB}}{AB} = \frac{-40\boldsymbol{i} + 80\boldsymbol{j} + 30\boldsymbol{k}}{94.3}$$

$$= -0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k}$$

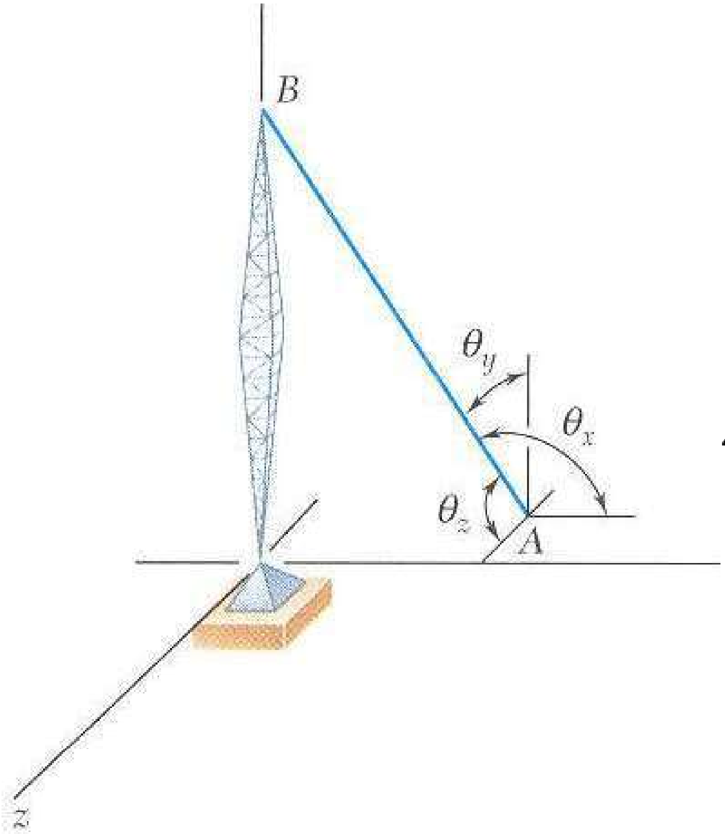
Determine the components of the force.

$$\boldsymbol{F} = F\boldsymbol{\lambda} \qquad = 2500(-0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k})$$

$$= -1060\boldsymbol{i} + 2120\boldsymbol{j} + 795\boldsymbol{k}$$

$$\boxed{\begin{aligned} F_x &= -1060\ N \\ F_y &= 2120\ N \\ F_z &= 795\ N \end{aligned}}$$

Rectangular Components in Space



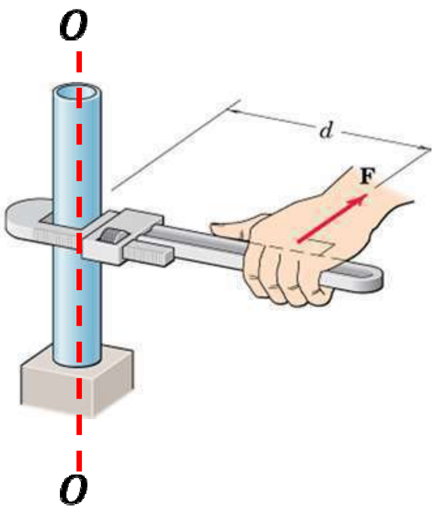
Solution

Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\boldsymbol{\lambda} = cos\theta_x \boldsymbol{i} + cos\theta_y \boldsymbol{j} + cos\theta_z \boldsymbol{k}$$
$$= -0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k}$$

$$\boxed{\begin{aligned} \theta_x &= 115.1^\circ \\ \theta_y &= 32.0^\circ \\ \theta_z &= 71.5^\circ \end{aligned}}$$

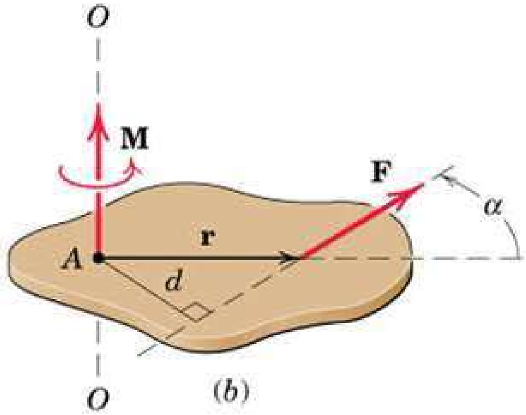
Moment of a Force (Torque)



Moment about axis O-O is $\boldsymbol{M_o} = Fd$

Magnitude of $\boldsymbol{M_o}$ measures tendency of **F** to cause rotation of the body about an axis along $\boldsymbol{M_o}$.

Moment about axis O-O is $\boldsymbol{M_o} = Frsin\alpha$

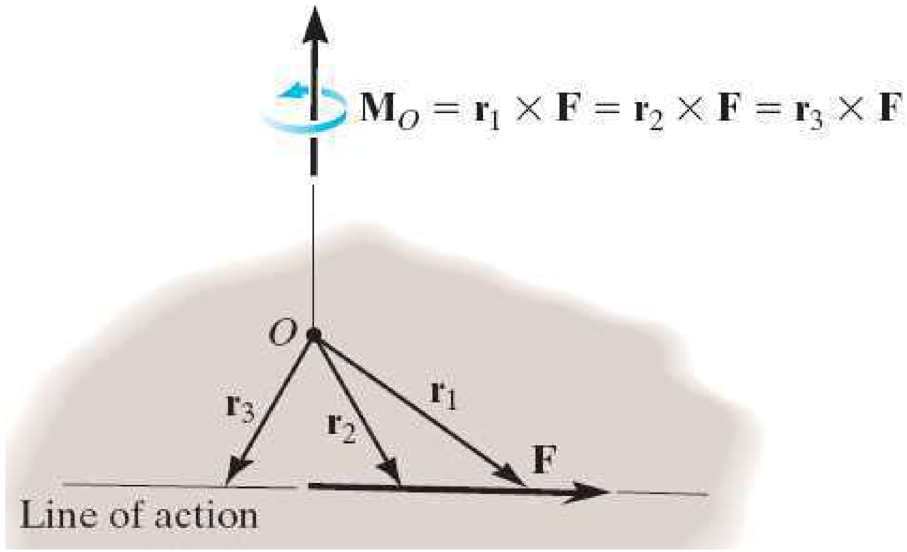


$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F}$$

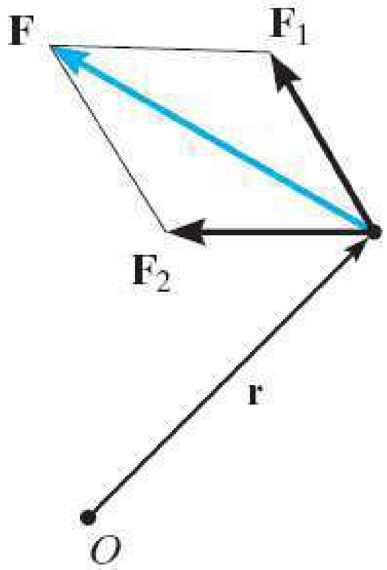
Sense of the moment may be determined by the right-hand rule

Moment of a Force

Principle of Transmissibility
Any force that has the same magnitude and direction as **F**, is *equivalent* if it also has the same line of action and therefore, produces the same moment.



Varignon's Theorem (Principle of Moments)



Moment of a Force about a point is equal to the sum of the moments of the force's components about the point.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

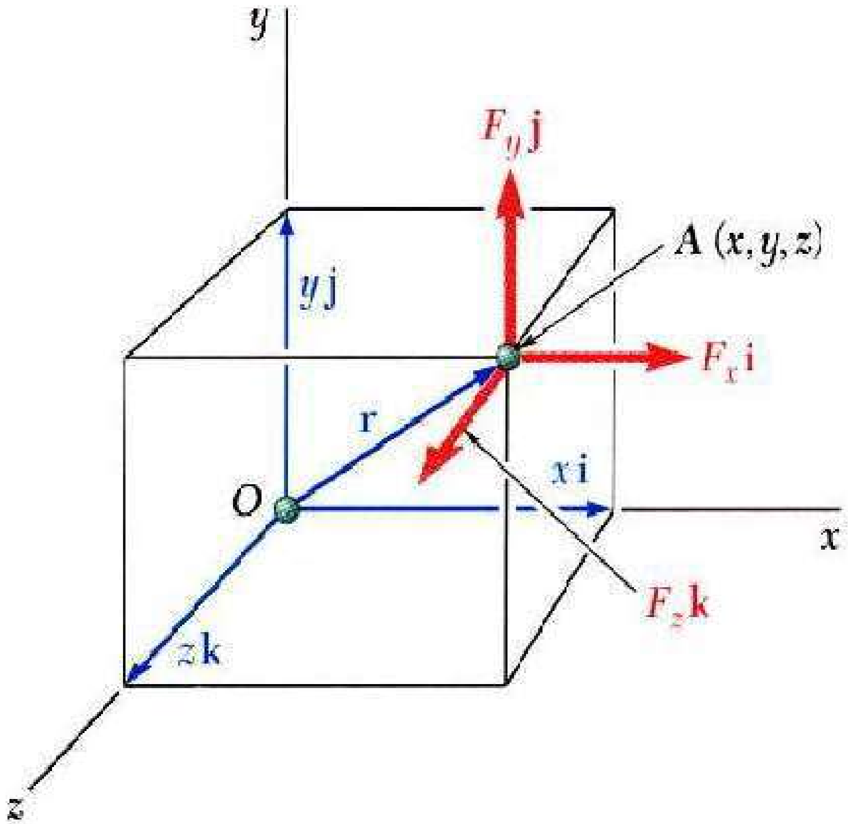
Rectangular Components of a Moment

The moment of $\boldsymbol{F}$ about O,

$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$



$$\boldsymbol{M_o} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (yF_z - zF_y)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{j} + (xF_y - yF_x)\boldsymbol{k}$$

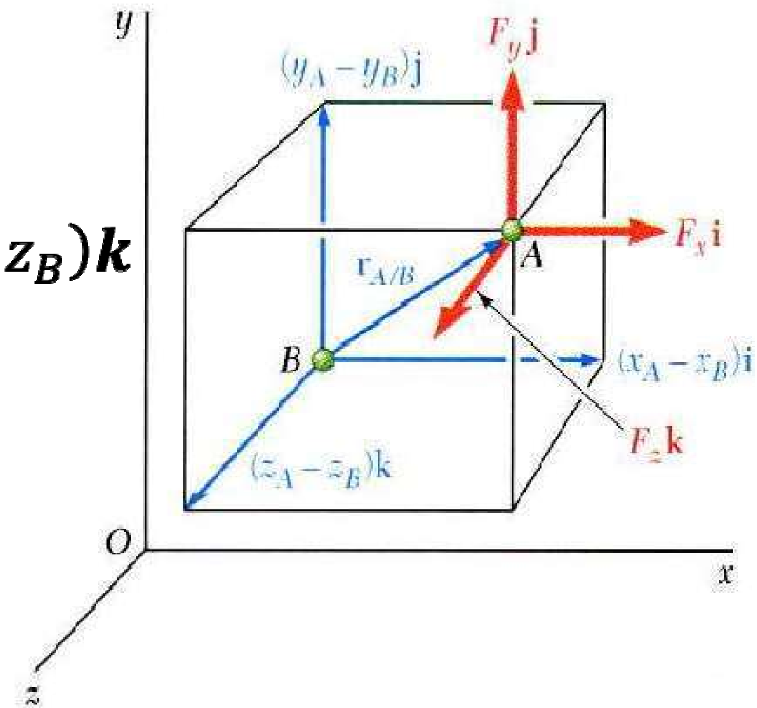
Rectangular Components of the Moment

The moment of $\boldsymbol{F}$ about B,

$$\boldsymbol{M_B} = \boldsymbol{r}_{AB} \times \boldsymbol{F}$$

$$\boldsymbol{r}_{AB} = (x_A - x_B)\boldsymbol{i} + (y_A - y_B)\boldsymbol{j} + (z_A - z_B)\boldsymbol{k}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

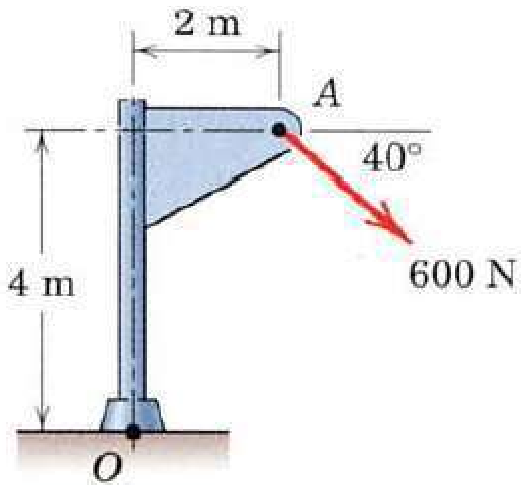


$$\boldsymbol{M_B} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_A - x_B & y_A - y_B & z_A - z_B \\ F_x & F_y & F_z \end{vmatrix}$$

Moment: Example

Calculate the magnitude of the moment about the base point O of the 600 N force in different ways

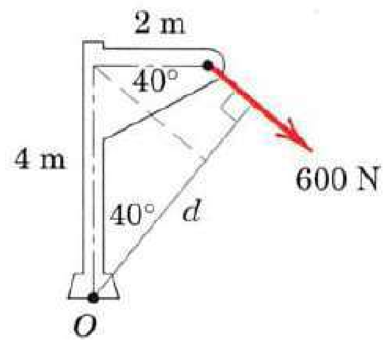


Solution 1.

Moment about O is

$$M_o = dF \qquad d = 4cos40^o + 2sin40^o = 4.35m$$

$$M_o = 600(4.35) = 2610\, N.m \;\; \textbf{Ans}$$

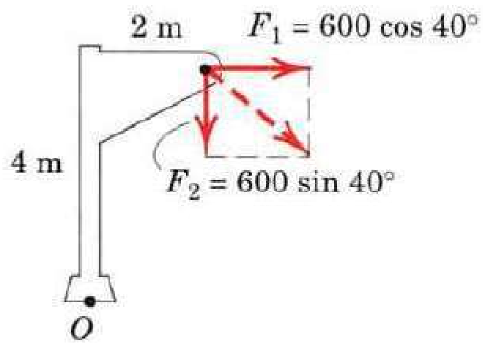


Solution 2.

$$F_x = 600cos40^o = 460\, N$$

$$F_x = 600sin40^o = 386\, N$$

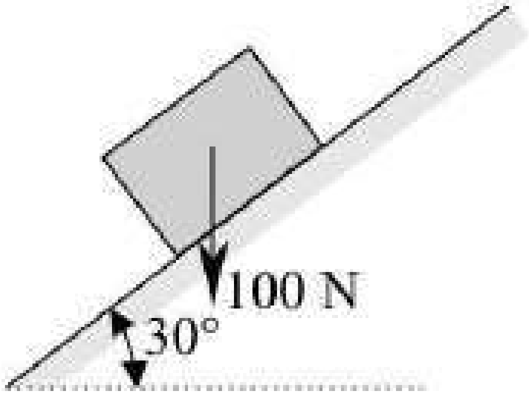
$$M_o = 460(4.00) + 386(2.00) = 2610\, N.m \; \textbf{Ans}$$

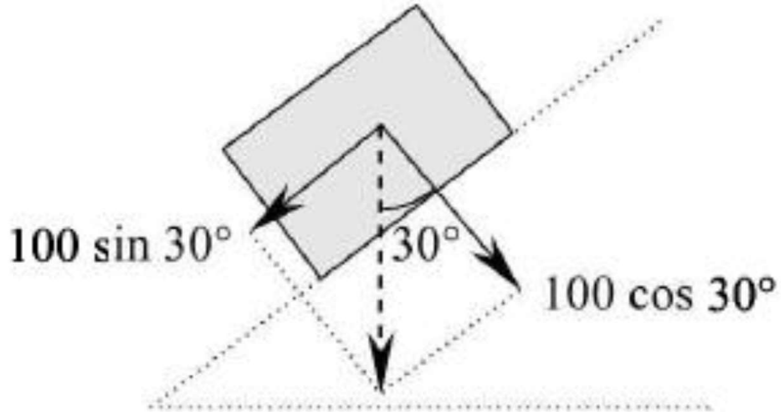


FBD : “Free body diagram” and “Equations of Equilibrium”

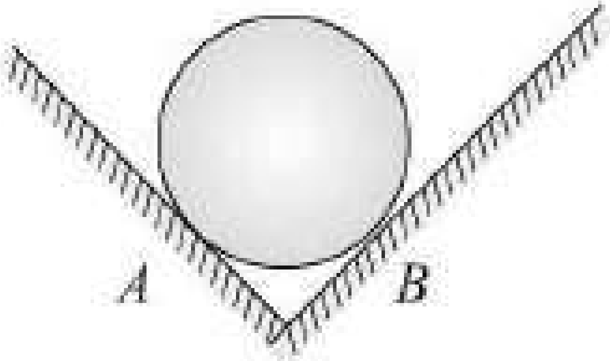
Graphical / Pictorial representation used to visualize the applied forces, moments and resultant reaction on a body in a given condition

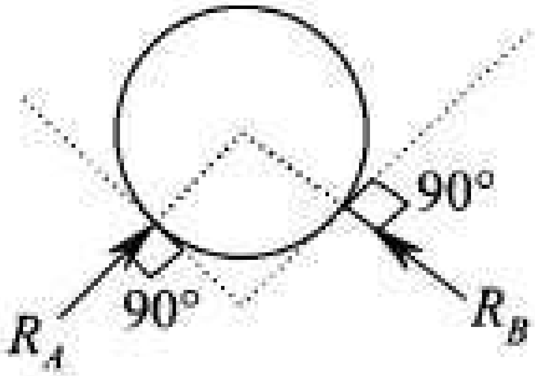
Example 1



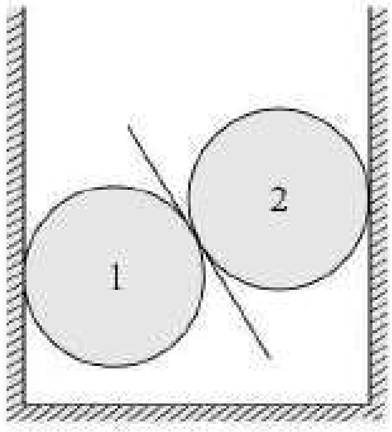


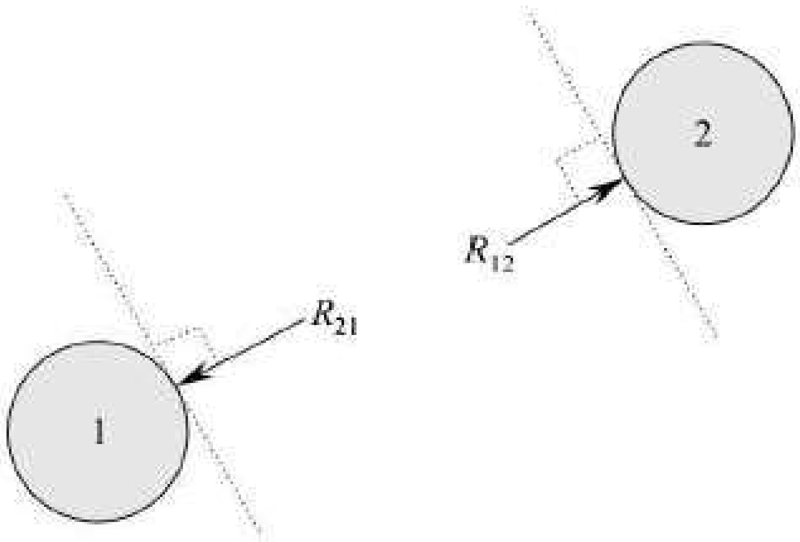
Example 2



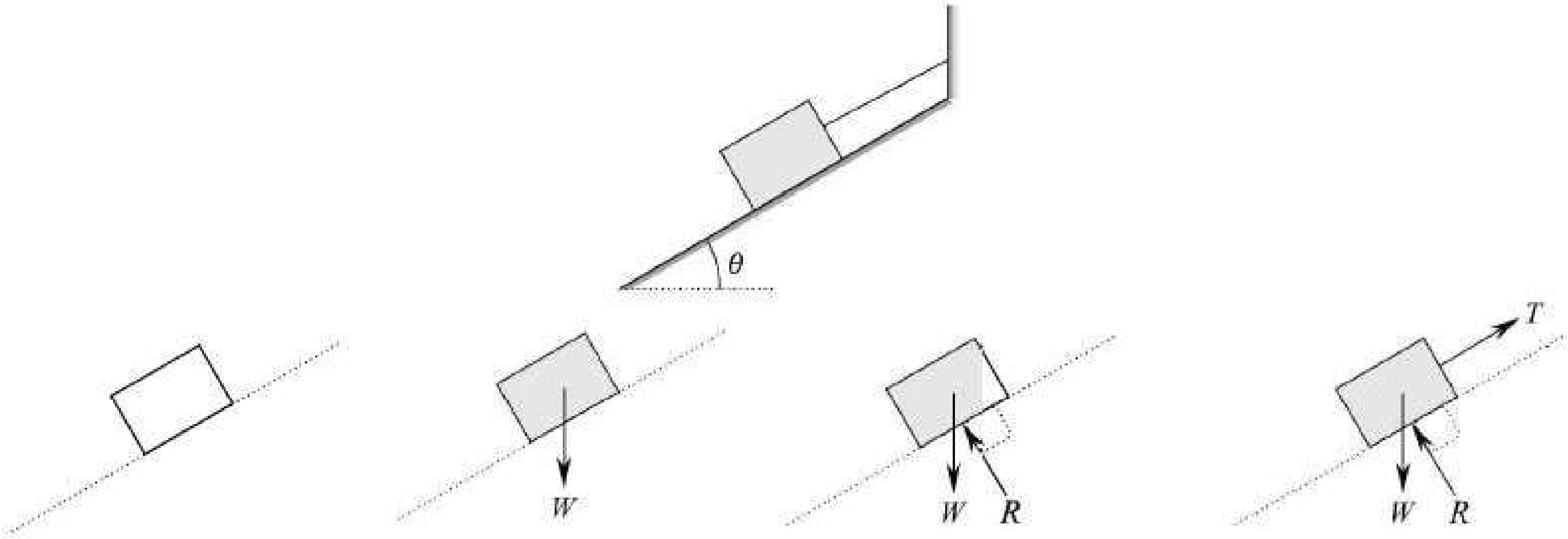


Example 3

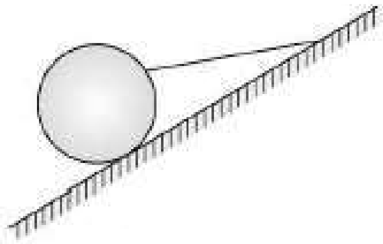


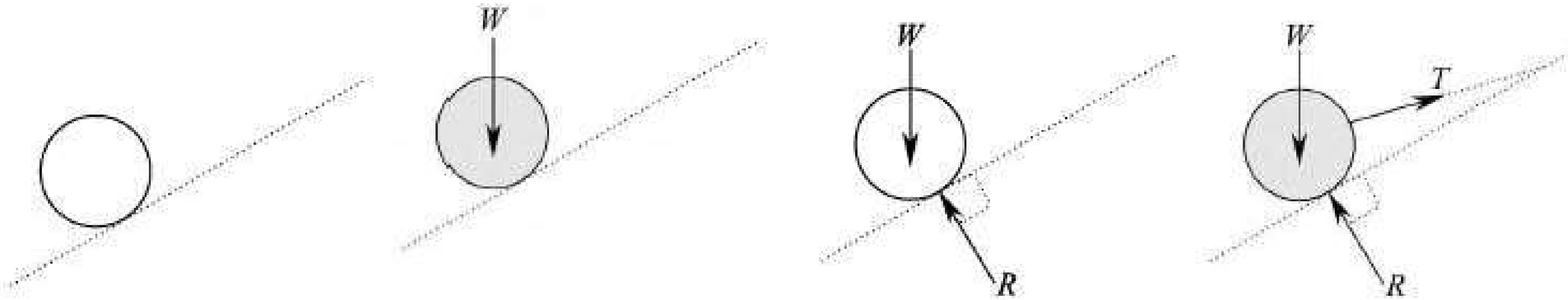


Example 4



Example 5





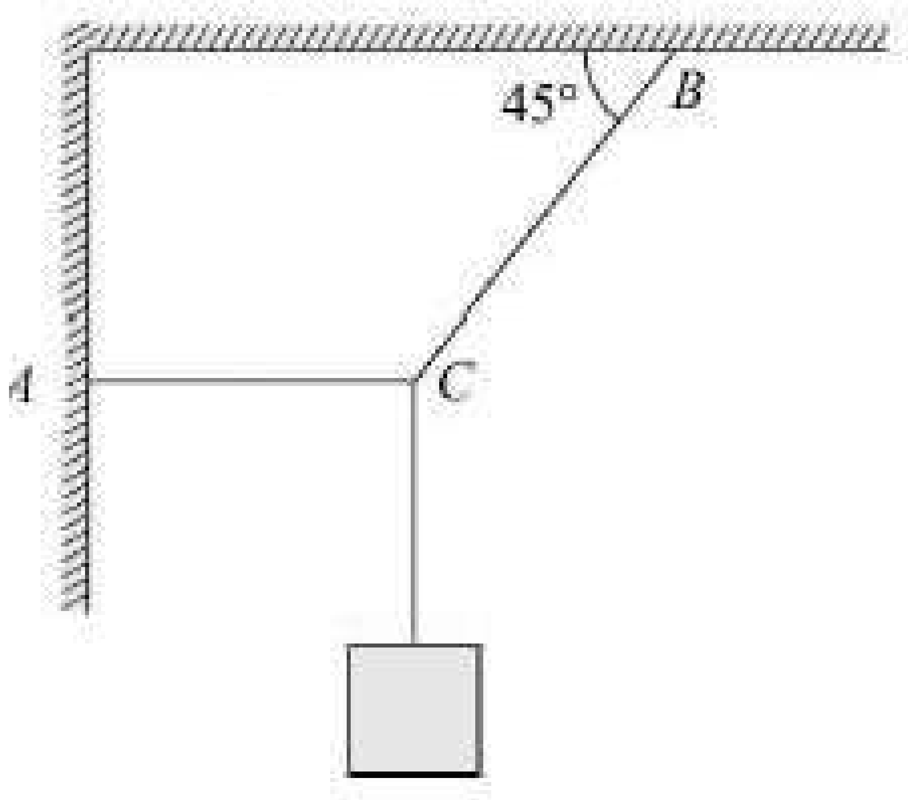
Draw a sketch of the sphere

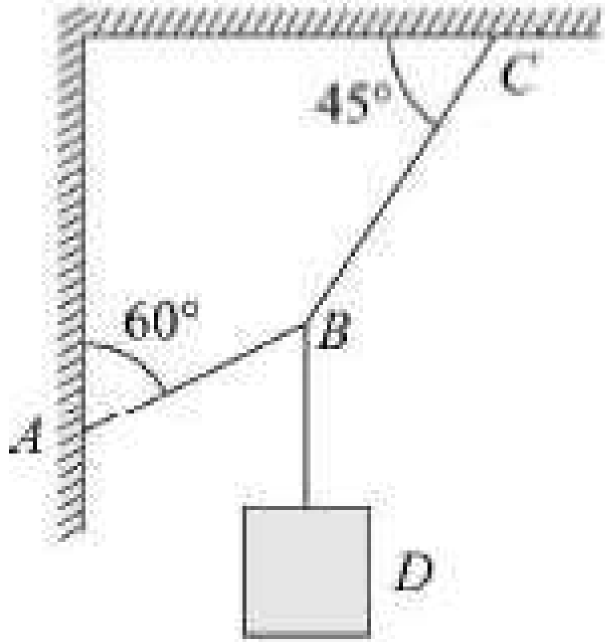
Show the weight of the sphere

Show the normal reaction

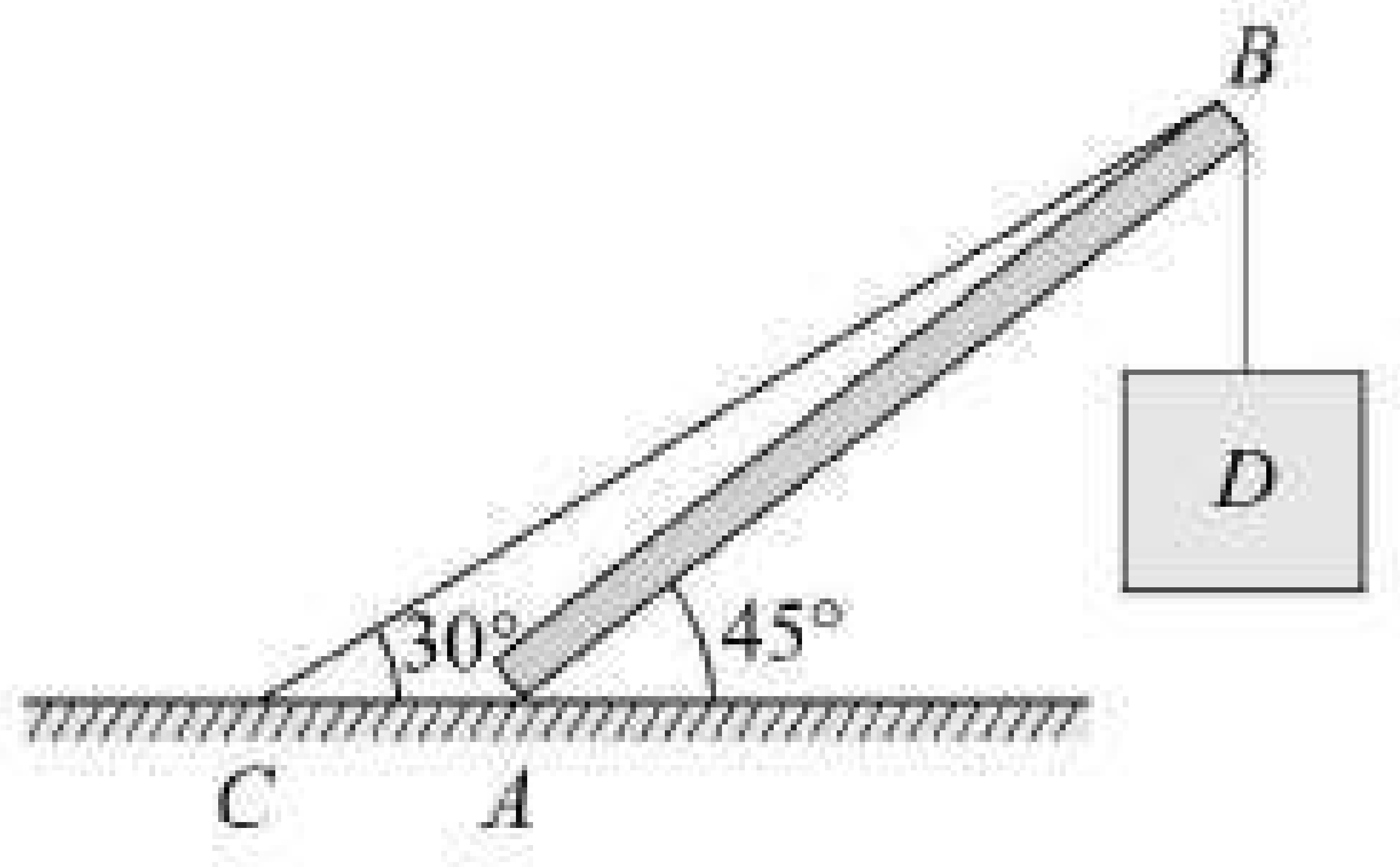
Show the tension in the string

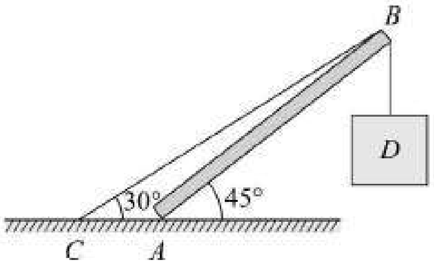
Example 6



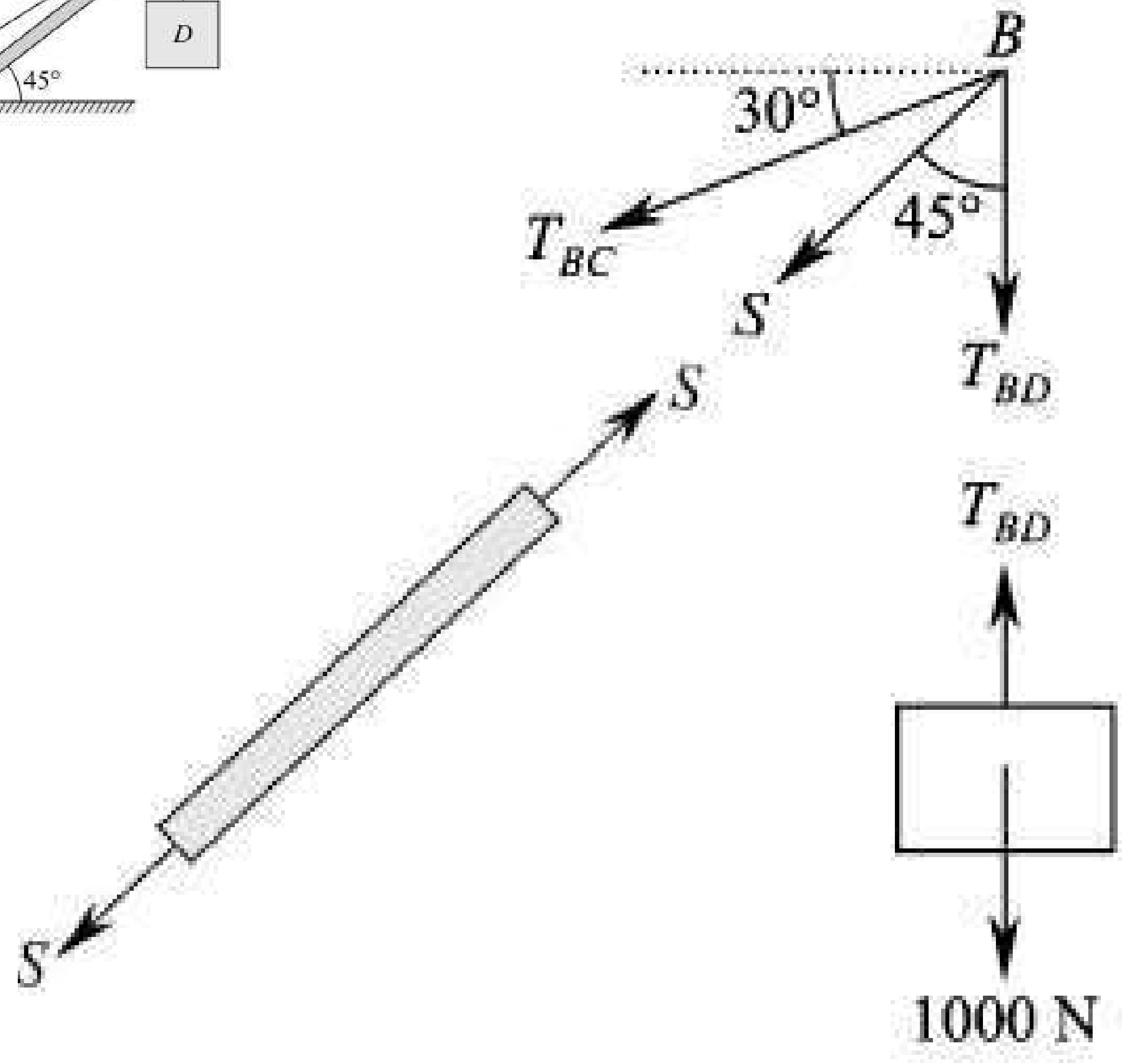


Example 8

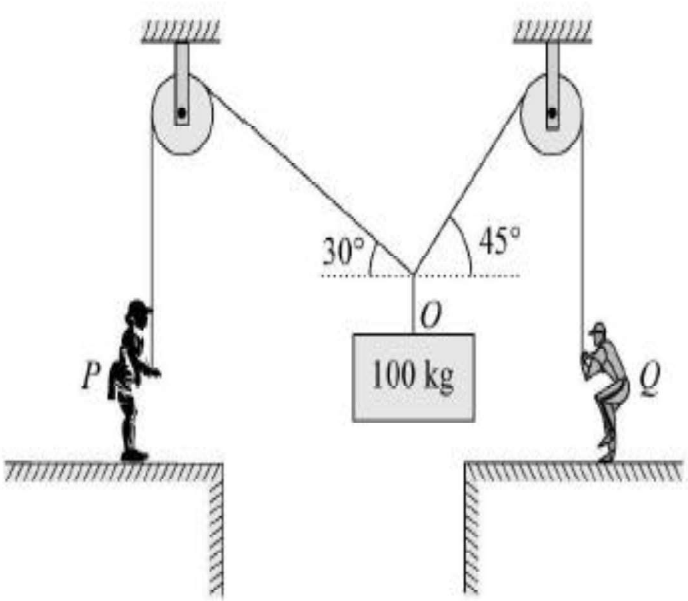


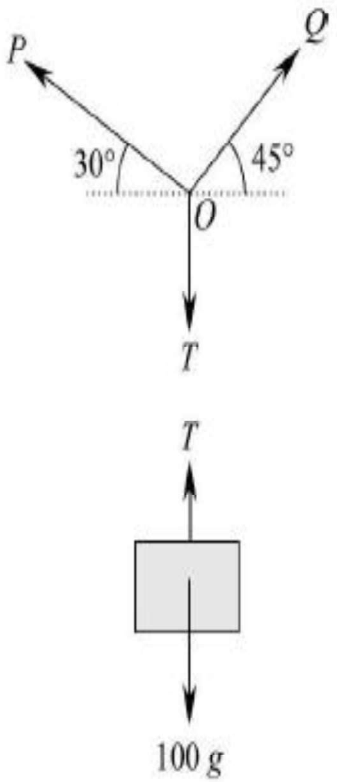


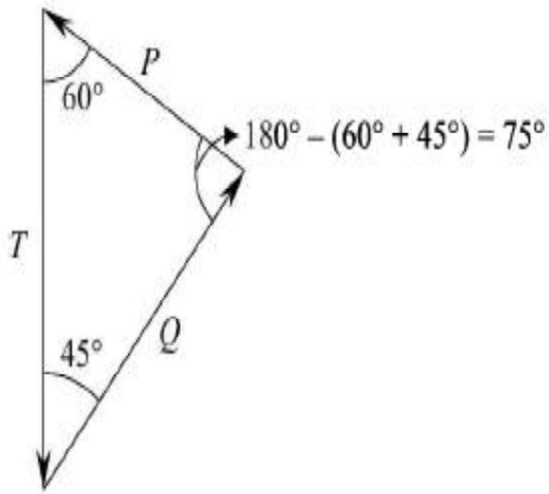
Solution 8



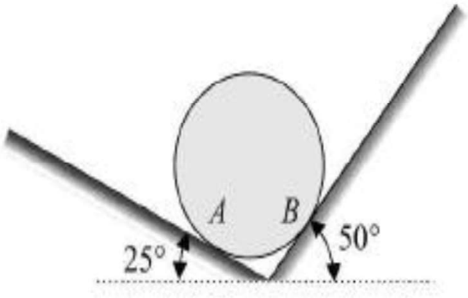
Example 9

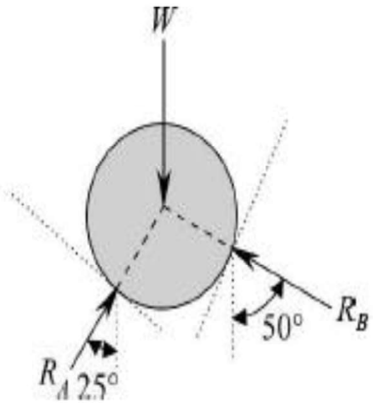


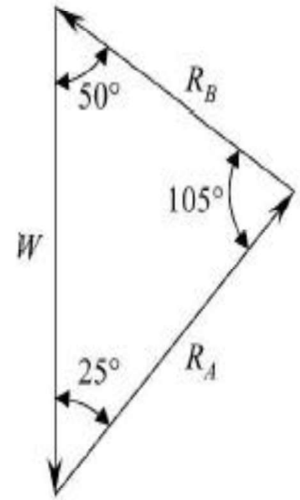




Example 10

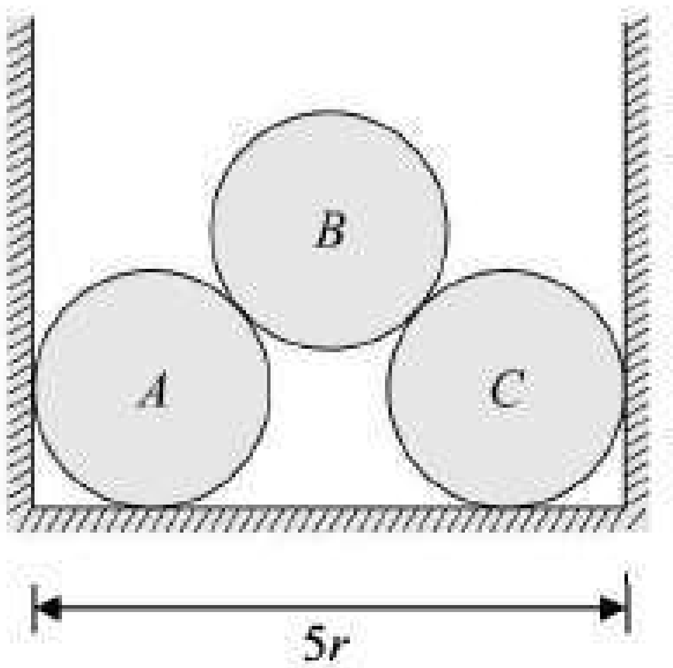




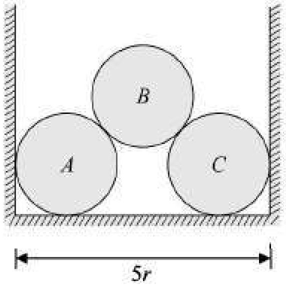


Example 11

Problem Statement: Three smooth cylinders, each of radius *r* and Weight *W* are placed in rectangular channel of width *5r*. Determine the reaction at all contact surfaces



Solution 11 (*cont..*)



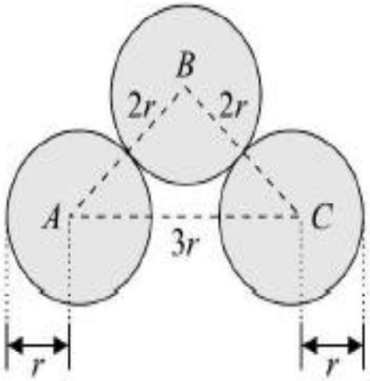


Fig. 5.36(a)

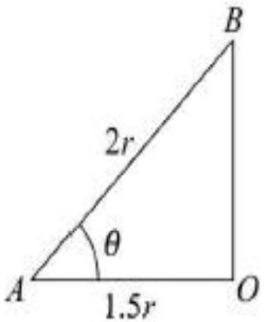


Fig. 5.36(b)

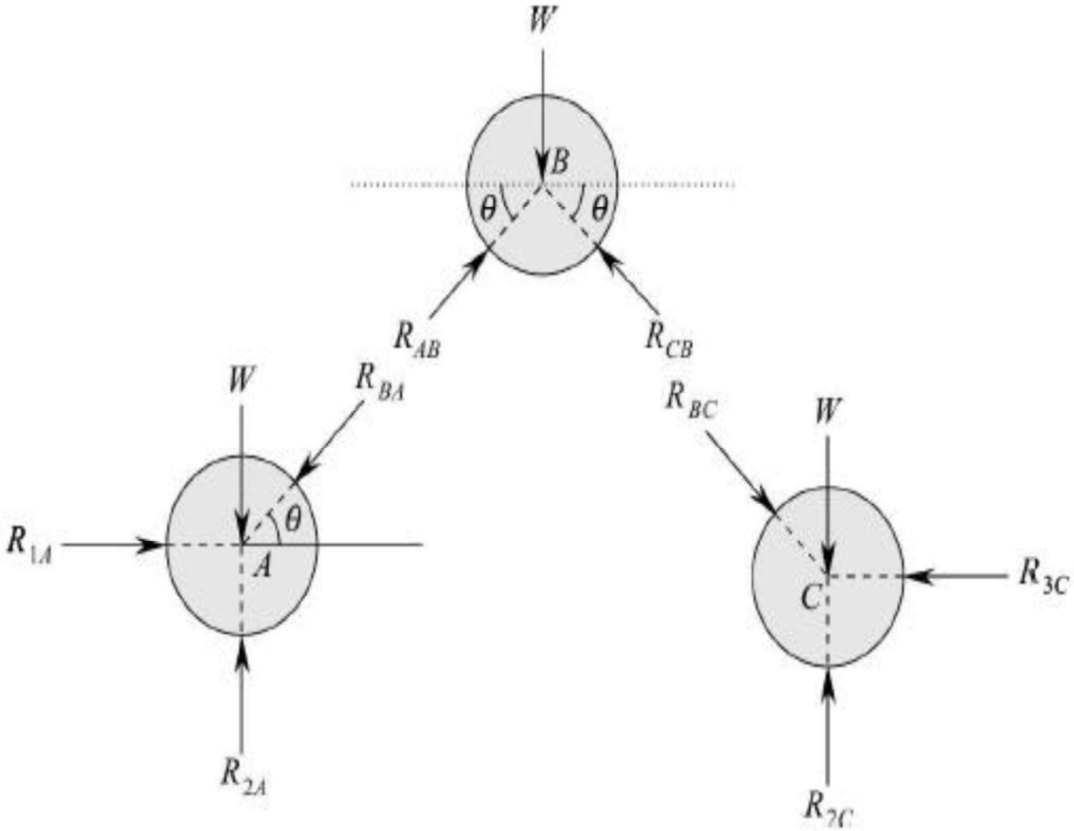


Fig. 5.36(c)

Solution 11 (*cont..*)

$$\cos\theta = \frac{1.5r}{2r} = \frac{3}{4}$$

$\Rightarrow$ $$\theta = \cos^{-1}(3/4) = 41.41°$$

The free-body diagrams of the cylinders are shown in Fig. 5.36(c): R_{1A} is the force exerted by the left wall on the cylinder A, R_{2A} is the force exerted by the base on the cylinder A, and so on.

Cylinder A

Applying the conditions of equilibrium for the cylinder A,

$\sum F_x = 0 \Rightarrow$

$$R_{1A} - R_{BA}\cos\theta = 0$$

$\therefore$ $$R_{1A} = R_{BA}\cos\theta \qquad \text{(a)}$$

$\sum F_y = 0 \Rightarrow$

$$R_{2A} - W - R_{BA}\sin\theta = 0$$

$\therefore$ $$R_{2A} = W + R_{BA}\sin\theta \qquad \text{(b)}$$

Cylinder B

As three concurrent forces are acting on the cylinder B, we can apply Lami's theorem to the force triangle shown in Fig. 5.36(d). Note that due to symmetry, the inclination of the reactions R_{AB} and R_{CB} must be same equal to θ.

$$\frac{W}{\sin 2\theta} = \frac{R_{AB}}{\sin(90° - \theta)} = \frac{R_{CB}}{\sin(90° - \theta)}$$

$$\frac{W}{\sin 2\theta} = \frac{R_{AB}}{\cos\theta} = \frac{R_{CB}}{\cos\theta}$$

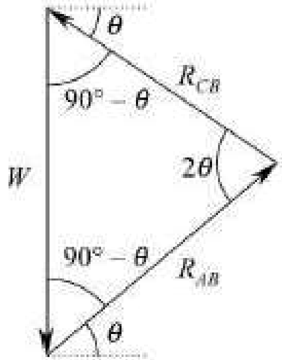


Fig. 5.36(d)

Solution 11 (*cont..*)

Due to symmetry, we can see that $R_{AB} = R_{CB}$. Hence, we have

$$R_{AB} = R_{CB} = W \frac{\cos 41.41°}{\sin(2 \times 41.41°)} = 0.756 \text{ W} \tag{c}$$

By Newton's third law, we know that $R_{AB} = R_{BA}$. Substituting the value of R_{AB} from equation (c) in equations (a) and (b), we get,

$$\begin{aligned} R_{1A} &= R_{BA} \cos\theta \\ &= (0.756\, W) \cos 41.41° \\ &= 0.567\, W \end{aligned}$$

and

$$\begin{aligned} R_{2A} &= W + R_{BA} \sin\theta \\ &= W + (0.756\, W) \sin 41.41° \\ &= 1.5 \text{ W} \end{aligned}$$

Due to symmetry, we know that

$$R_{1A} = R_{3C},\ R_{AB} = R_{CB} \text{ and } R_{2A} = R_{2C}$$

Example 12

Problem Statement: Three smooth cylinders are placed in a channel as shown in Fig. 5.37. Their respective diameters are indicated in figure. The weight of the smaller cylinder is ***W*** and that of the larger cylinder is ***3W***. Determine contact forces at point A, B, C and D. Take W = 10 kN

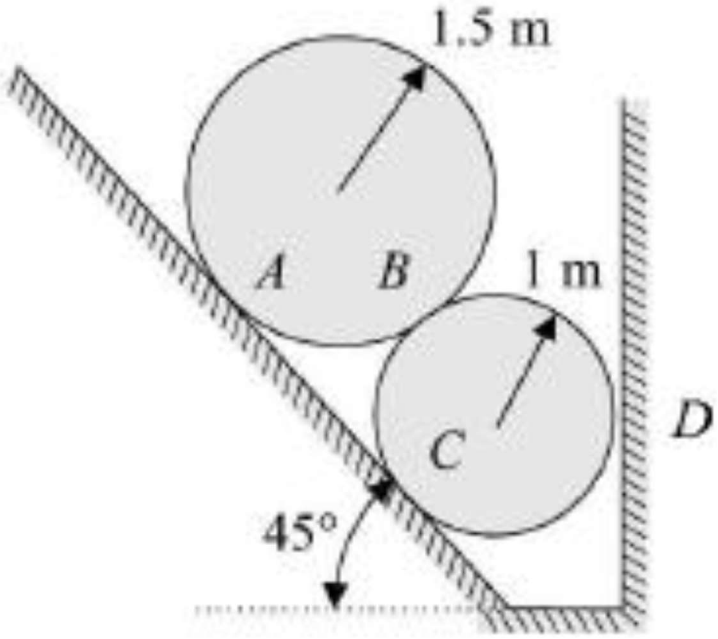


Fig. 5.37

Solution 12(*cont..*)

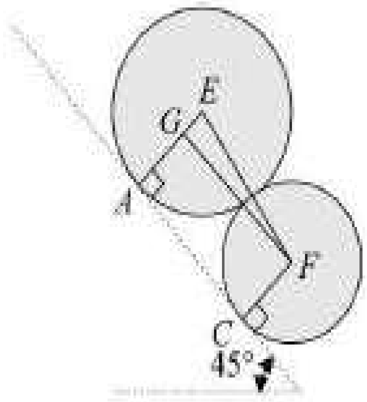


Fig. 5.37(a)

EF = sum of radii of the two cylinders

$= 1 + 1.5 = 2.5$ m

$GE = AE - AG = AE - CF$ [since $AG = CF$]

$= 1.5 - 1 = 0.5$ m

$\angle EFG = \sin^{-1}(GE/EF)$

$= \sin^{-1}(0.5/2.5) = 11.54°$

Therefore, inclination of EF with the horizontal is $45° + 11.54° = 56.54°$

The free-body diagrams of the cylinders can then be drawn as shown below.

As the number of unknowns is smaller in the case of larger cylinder than that of the smaller cylinder, we proceed with the equilibrium of larger cylinder first.

Larger cylinder

As the forces are concurrent at the centre of the cylinder, we can apply Lami's theorem to the force triangle [see Fig. 5.37(c)].

Solution 12(*cont..*)

$$\frac{3W}{\sin 101.54^\circ} = \frac{R_A}{\sin 33.46^\circ} = \frac{R_B}{\sin 45^\circ}$$

$$\Rightarrow \quad R_A = 3W\left[\frac{\sin 33.46^\circ}{\sin 101.54^\circ}\right]$$

$$= 3(10)\frac{\sin 33.46^\circ}{\sin 101.54^\circ}$$

$$= 16.88 \text{ kN}$$

$$\Rightarrow \quad R_B = 3W\frac{\sin 45^\circ}{\sin 101.54^\circ}$$

$$= 3(10)\frac{\sin 45^\circ}{\sin 101.54^\circ}$$

$$= 21.65 \text{ kN}$$

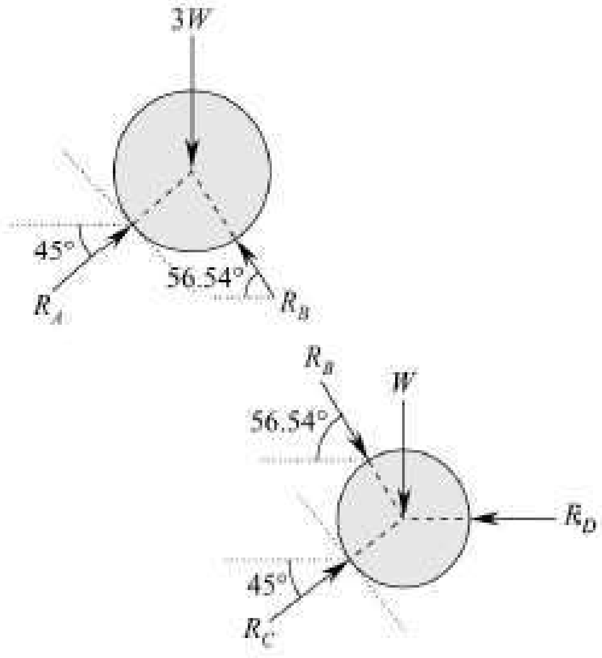


Fig. 5.37(b)

Solution 12 (*cont..*)

Smaller cylinder

Since there are four forces concurrent at the centre of the cylinder, we use equilibrium conditions to solve the unknowns. Applying the conditions of equilibrium along X and Y directions,

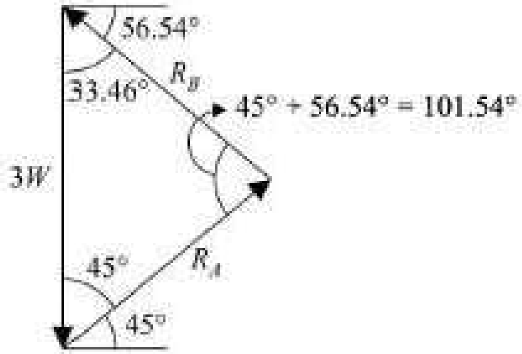


Fig. 5.37(c)

$$\sum F_x = 0 \Rightarrow$$

$$R_B \cos 56.54° + R_C \cos 45° - R_D = 0$$

$$\therefore \quad R_D - R_C \cos 45° = R_B \cos 56.54°$$

$$= 11.94 \text{ kN} \qquad \text{(a)}$$

$$\sum F_y = 0 \Rightarrow$$

$$R_C \sin 45° - W - R_B \sin 56.54° = 0$$

$$\therefore \quad R_C = [W + R_B \sin 56.54°]/\sin 45°$$

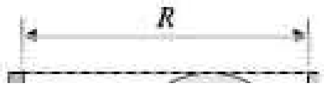
$$= [10 + (21.65) \sin 56.54°]/\sin 45°$$

$$= 39.69 \text{ kN}$$

Substituting this value in equation (a), we get

$$R_D - R_C \cos 45° = 11.94 \text{ kN}$$

$$\Rightarrow \quad R_D = (39.69) \cos 45° + 11.94 = 40 \text{ kN}$$



Example 13

Problem statement: Two identical smooth cylinder each of weight ***W*** and radius ***r*** are placed in a quarter circular cross sectional channel of radius ***R*** as shown in Fig. 5.38. such that they just fit in the channel. Determine the reactions at the contact surfaces A, B, C and D.

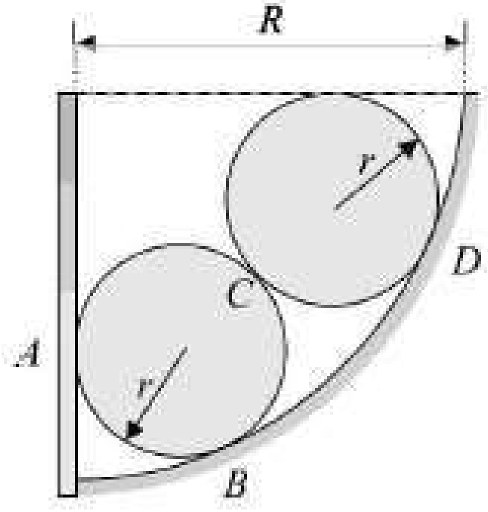


Fig. 5.38

Solution 13 (*cont..*)

Solution Due to symmetry, we see that the angle made by the radial line *OB* with the vertical wall is 90°/4 =22.5° as shown in Fig. 5.38(a).

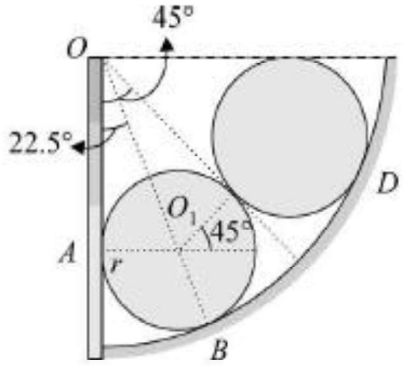


Fig. 5.38(a)

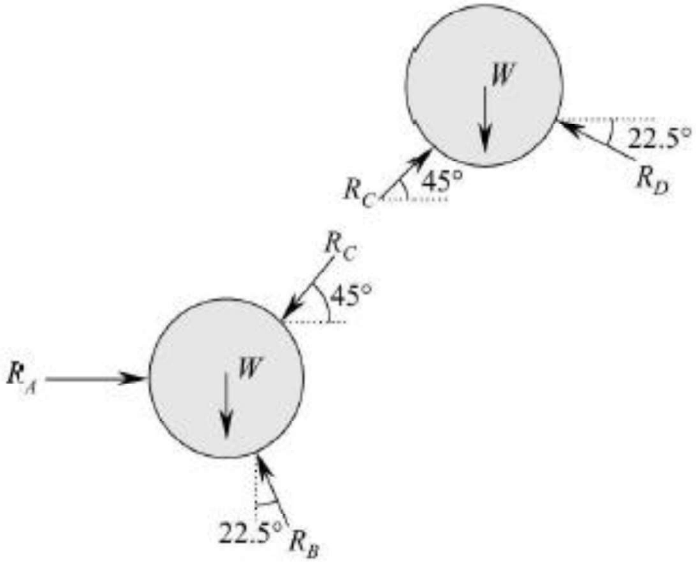


Fig. 5.38(b)

$$\sin 22.5° = \frac{O_1A}{OO_1} = \frac{O_1A}{OB - O_1B} = \frac{r}{R-r}$$

$$\Rightarrow \quad R - r = \frac{r}{\sin 22.5°}$$

$$\therefore \quad R = 3.613r$$

Solution 13 (*cont..*)

As the number of unknowns is smaller in the case of upper cylinder than that of the lower cylinder, we proceed with the equilibrium of upper cylinder first.

Upper cylinder

Applying the conditions of equilibrium along the X and Y directions,

$\sum F_y = 0 \Rightarrow$

$$R_C \sin 45° + R_D \sin 22.5° - W = 0 \tag{a}$$

$\sum F_x = 0 \Rightarrow$

$$R_C \cos 45° - R_D \cos 22.5° = 0 \tag{b}$$

Solving for R_C and R_D from the above two equations, we get

$$R_D = 0.765\ W \quad \text{and} \quad R_C = W$$

Lower cylinder

Applying the conditions of equilibrium along the X and Y directions,

$\sum F_y = 0 \Rightarrow$

$$R_B \cos 22.5° - W - R_C \sin 45° = 0 \tag{c}$$

$\sum F_x = 0 \Rightarrow$

$$R_A - R_C \cos 45° - R_B \sin 22.5° = 0 \tag{d}$$

Substituting the value of R_C in the above two equations, we get

$$R_B = 1.848\ W \quad \text{and} \quad R_A = 1.414\ W$$

Problem on “Application of Unit vector”

Example 1

Problem statement: A vertical load of **50 kg** is supported by three rods positioned as shown in Fig. 5.40. Determine the forces in each rod.

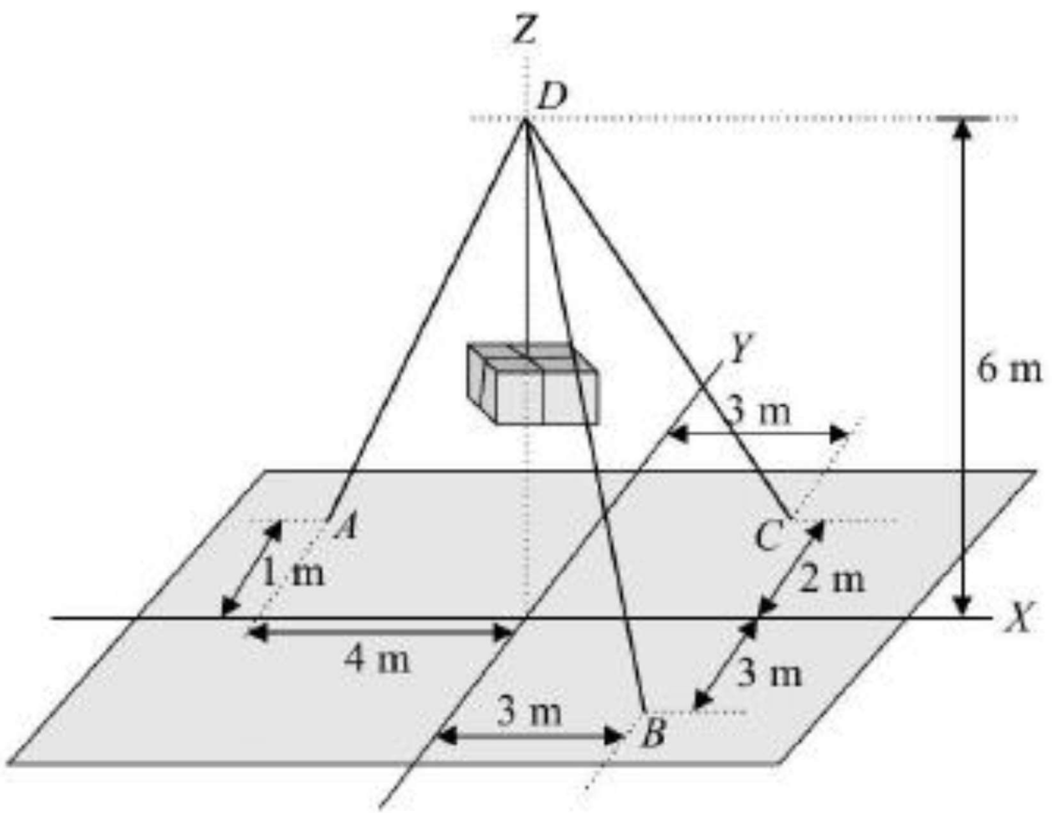


Fig. 5.40

Solution 1

Solution From the figure, we see that the coordinates of points A, B, C and D are

$$A(-4, 1, 0),\ B(3, -3, 0),\ C(3, 2, 0) \text{ and } D(0, 0, 6)$$

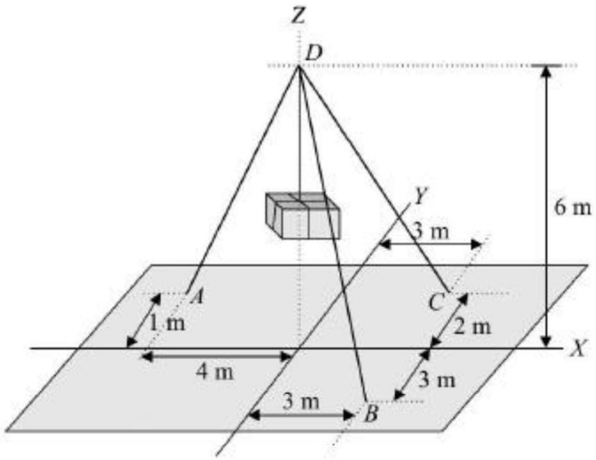


Fig. 5.40

Determination of unit vectors along DA, DB and DC

$$\overrightarrow{DA} = \overrightarrow{OA} - \overrightarrow{OD}$$

$$= -4\vec{i} + \vec{j} - 6\vec{k}$$

$$\therefore \quad \hat{n}_{DA} = \frac{-4\vec{i} + \vec{j} - 6\vec{k}}{\sqrt{(-4)^2 + (1)^2 + (-6)^2}} = \frac{-4\vec{i} + \vec{j} - 6\vec{k}}{\sqrt{53}}$$

$$= -0.549\vec{i} + 0.137\vec{j} - 0.824\vec{k}$$

$$\overrightarrow{DB} = \overrightarrow{OB} - \overrightarrow{OD}$$

$$= 3\vec{i} - 3\vec{j} - 6\vec{k}$$

$$\therefore \quad \hat{n}_{DB} = \frac{3\vec{i} - 3\vec{j} - 6\vec{k}}{\sqrt{(3)^2 + (-3)^2 + (-6)^2}} = \frac{3\vec{i} - 3\vec{j} - 6\vec{k}}{\sqrt{54}}$$

$$= 0.408\vec{i} - 0.408\vec{j} - 0.816\vec{k}$$

$$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$$

$$= 3\vec{i} + 2\vec{j} - 6\vec{k}$$

$$\therefore \quad \hat{n}_{DC} = \frac{3\vec{i} + 2\vec{j} - 6\vec{k}}{\sqrt{(3)^2 + (2)^2 + (-6)^2}} = \frac{3\vec{i} + 2\vec{j} - 6\vec{k}}{7}$$

$$= 0.429\vec{i} + 0.286\vec{j} - 0.857\vec{k}$$

Solution 1(*cont..*)

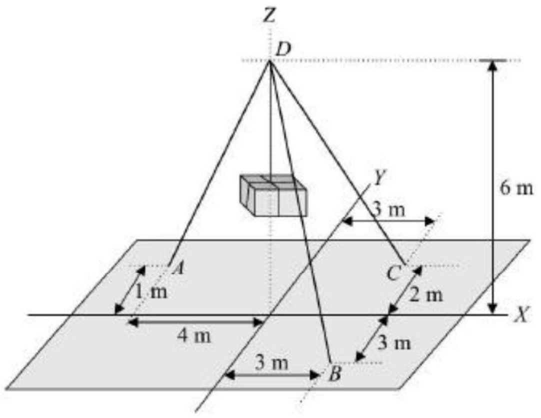


Fig. 5.40

Let S_{DA}, S_{DB} and S_{DC} be the magnitudes of the forces in the rods DA, DB and DC respectively. Then the force vectors can be represented as

$$\vec{S}_{DA} = S_{DA}\hat{n}_{DA}$$
$$= S_{DA}[-0.549\vec{i} + 0.137\vec{j} - 0.824\vec{k}]$$
$$\vec{S}_{DB} = S_{DB}\hat{n}_{DB}$$
$$= S_{DB}[0.408\vec{i} - 0.408\vec{j} - 0.816\vec{k}]$$
$$\vec{S}_{DC} = S_{DC}\hat{n}_{DC}$$
$$= S_{DC}[0.429\vec{i} + 0.286\vec{j} - 0.857\vec{k}]$$

The load at D can be represented in vector form as

$$\vec{W} = 50 \times 9.81[-\vec{k}] \quad \text{(Note that the weight is directed along the negative } Z\text{-axis.)}$$
$$= -490.5\vec{k}$$

Solution 1(*cont..*)

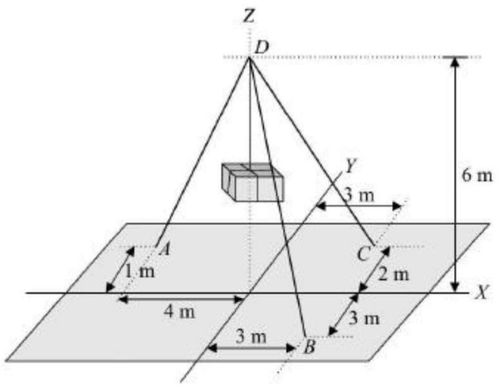


Fig. 5.40

Since the forces are concurrent at the point D, the resultant of the system of forces is given as

$$\begin{aligned}\vec{R} &= \vec{S}_{DA} + \vec{S}_{DB} + \vec{S}_{DC} + \vec{W}\\ &= [-0.549S_{DA} + 0.408S_{DB} + 0.429S_{DC}]\vec{i}\\ &\quad + [0.137S_{DA} - 0.408S_{DB} + 0.286S_{DC}]\vec{j}\\ &\quad + [-0.824S_{DA} - 0.816S_{DB} - 0.857S_{DC} - 490.5]\vec{k}\end{aligned}$$

Applying the condition of equilibrium, $\vec{R} = \vec{O}$, we get three independent simultaneous equations:

$$-0.549S_{DA} + 0.408S_{DB} + 0.429S_{DC} = 0$$

$$0.137S_{DA} - 0.408S_{DB} + 0.286S_{DC} = 0$$

$$-0.824S_{DA} - 0.816S_{DB} - 0.857S_{DC} = 490.5$$

Solving the simultaneous equations, we get

$$S_{DA} = -255.14 \text{ N}, S_{DB} = -188.9 \text{ N and } S_{DC} = -147.02 \text{ N}$$

(Note that the negative sign in the above values indicates that the members are under compression.)

Example 2

Problem statement: A box of mass **200 kg** is supported by three cables as shown in Fig. 5.42. Determine the tension in each cable.

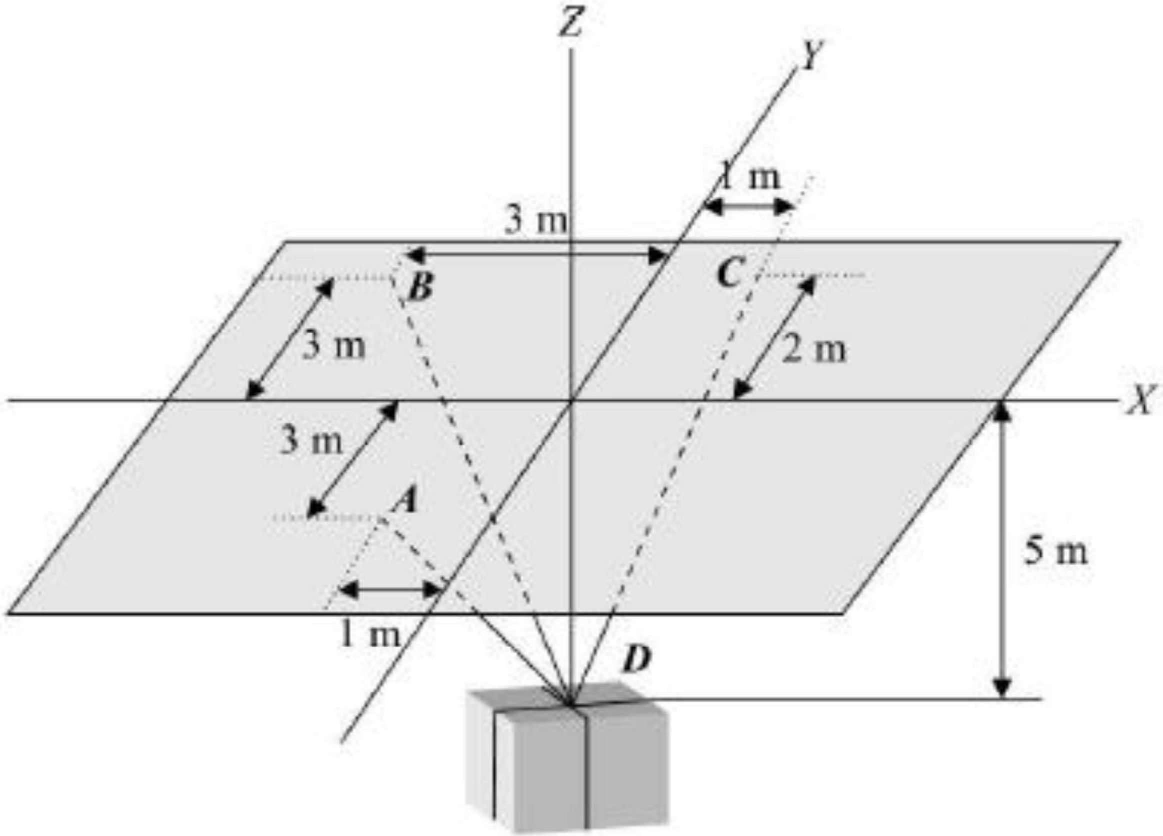


Fig. 5.42

Solution 2 (*cont..*)

Solution From the figure, we see that the coordinates of points A, B, C and D are

$$A(-1, -3, 0),\ B(-3, 3, 0),\ C(1, 2, 0) \text{ and } D(0, 0, -5)$$

Determination of unit vectors along DA, DB and DC

$$\overrightarrow{DA} = \overrightarrow{OA} - \overrightarrow{OD}$$

$$= -\vec{i} - 3\vec{j} + 5\vec{k}$$

$$\therefore \qquad \hat{n}_{DA} = \frac{-\vec{i} - 3\vec{j} + 5\vec{k}}{\sqrt{(-1)^2 + (-3)^2 + (5)^2}} = \frac{-\vec{i} - 3\vec{j} + 5\vec{k}}{\sqrt{35}}$$

$$= -0.169\vec{i} - 0.507\vec{j} + 0.845\vec{k}$$

$$\overrightarrow{DB} = \overrightarrow{OB} - \overrightarrow{OD}$$

$$= -3\vec{i} + 3\vec{j} + 5\vec{k}$$

$$\therefore \qquad \hat{n}_{DB} = \frac{-3\vec{i} + 3\vec{j} + 5\vec{k}}{\sqrt{(-3)^2 + (3)^2 + (5)^2}} = \frac{-3\vec{i} + 3\vec{j} + 5\vec{k}}{\sqrt{43}}$$

$$= -0.457\vec{i} + 0.457\vec{j} + 0.762\vec{k}$$

$$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$$

$$= \vec{i} + 2\vec{j} + 5\vec{k}$$

$$\therefore \qquad \hat{n}_{DC} = \frac{\vec{i} + 2\vec{j} + 5\vec{k}}{\sqrt{(1)^2 + (2)^2 + (5)^2}} = \frac{\vec{i} + 2\vec{j} + 5\vec{k}}{\sqrt{30}}$$

$$= 0.183\vec{i} + 0.365\vec{j} + 0.913\vec{k}$$

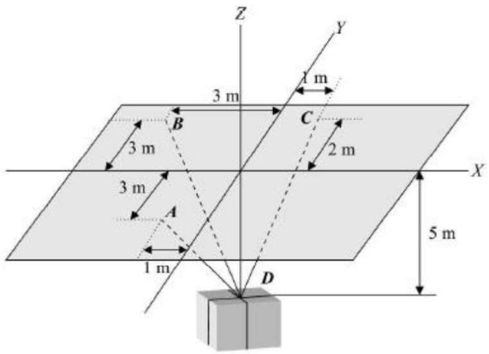


Fig. 5.42

Solution 2 (*cont..*)

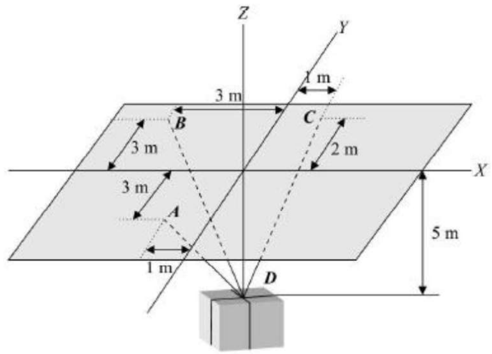


Fig. 5.42

Let T_{DA}, T_{DB} and T_{DC} be the tensions in the cables *DA*, *DB* and *DC* respectively. Then the tension vectors can be represented as

$$\vec{T}_{DA} = T_{DA}\hat{n}_{DA}$$
$$= T_{DA}[-0.169\vec{i} - 0.507\vec{j} + 0.845\vec{k}] \quad \text{(a)}$$
$$\vec{T}_{DB} = T_{DB}\hat{n}_{DB}$$
$$= T_{DB}[-0.457\vec{i} + 0.457\vec{j} + 0762\vec{k}] \quad \text{(b)}$$
$$\vec{T}_{DC} = T_{DC}\hat{n}_{DC}$$
$$= T_{DC}[0.183\vec{i} + 0.365\vec{j} + 0.913\vec{k}] \quad \text{(c)}$$

As the mass of the box is 200 kg, its weight can be represented in vector form as

$$\vec{W} = 200 \times 9.81[-\vec{k}]$$
$$= -1962\vec{k} \quad \text{(d)}$$

Solution 2 (*cont..*)

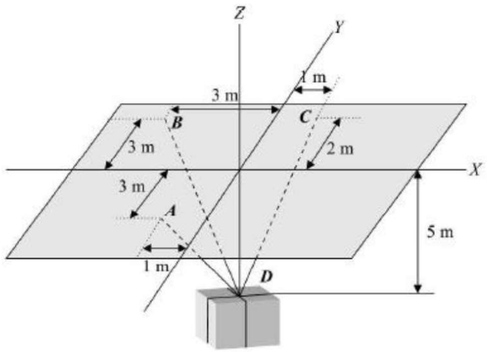


Fig. 5.42

Note that the weight is directed along the negative Z-axis. Since the tension and weight vectors are concurrent at point D, the resultant of the system of forces is given as the vector addition of individual forces.

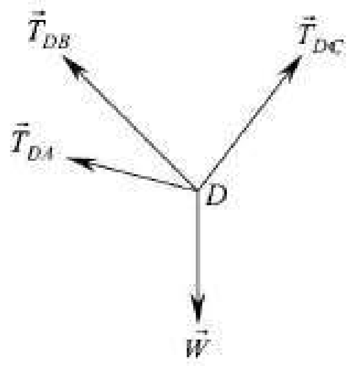


Fig. 5.42(a)

$$\begin{aligned}\vec{R} &= \vec{T}_{DA} + \vec{T}_{DB} + \vec{T}_{DC} + \vec{W}\\ &= [-0.169T_{DA} - 0.457T_{DB} + 0.183T_{DC}]\vec{i}\\ &\quad + [-0.507T_{DA} + 0.457T_{DB} + 0.365T_{DC}]\vec{j}\\ &\quad + [0.845T_{DA} + 0.762T_{DB} + 0.913T_{DC} - 1962]\vec{k}\end{aligned}$$

Applying the condition of equilibrium, $\vec{R} = \vec{O}$, we get three independent simultaneous equations:

$$-0.169T_{DA} - 0.457T_{DB} + 0.183T_{DC} = 0 \quad \text{(e)}$$

$$-0.507T_{DA} + 0.457T_{DB} + 0.365T_{DC} = 0 \quad \text{(f)}$$

$$0.845T_{DA} + 0.762T_{DB} + 0.913T_{DC} = 1962 \quad \text{(g)}$$

Solving the above simultaneous equations, we get

$$T_{DA} = 950.02 \text{ N},\ T_{DB} = 117.26 \text{ N and } T_{DC} = 1171.92 \text{ N}$$

Thank you