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25/09/20 Ch. 1

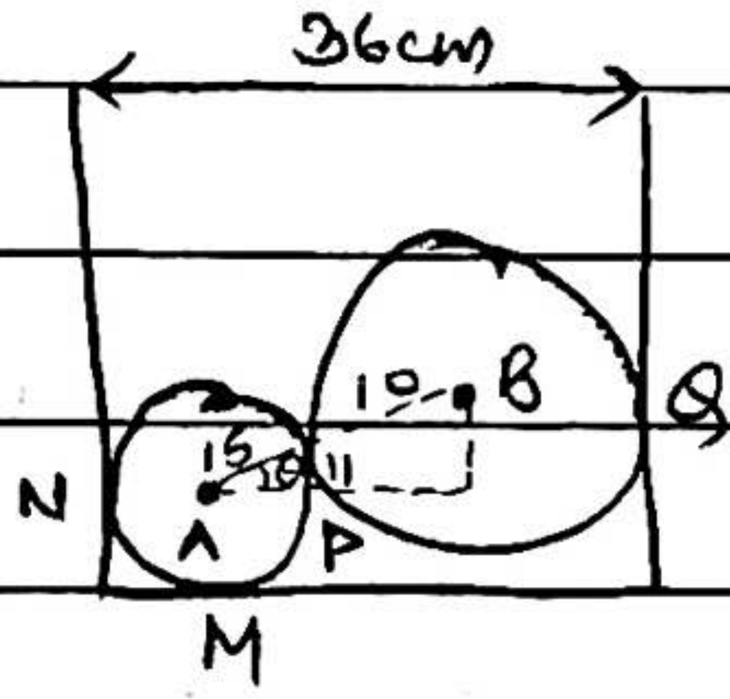
Static Equilibrium

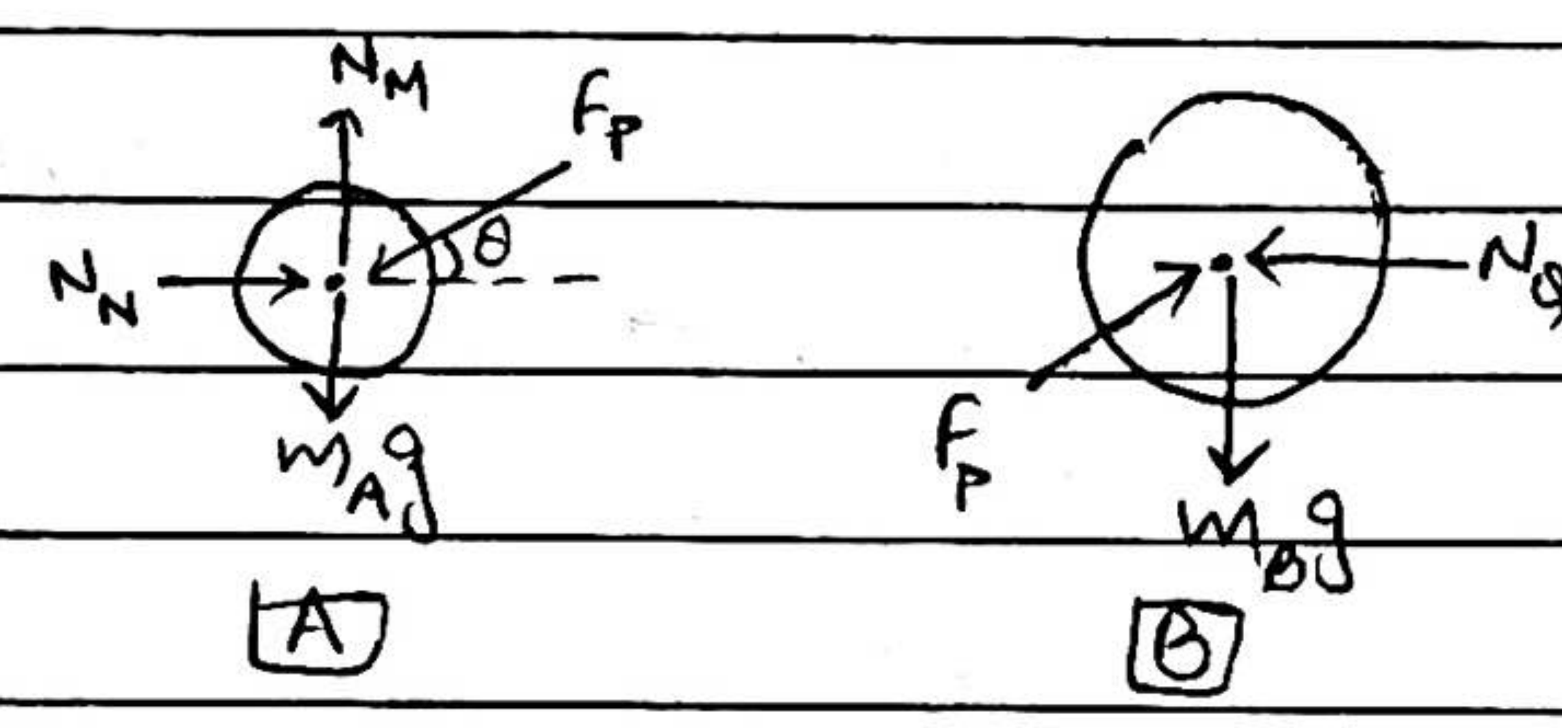
- Equilibrium means an unchanging state - a state of balance.
- Net force acting on an object is 0.
- Equilibrium equations are used to determine unknown forces acting on an object in equilibrium.
- First step is to draw a free body diagram.

* FBD:

Diagram showing the complete system of applied & reactive forces acting on a body.

eg.





assumptions: $w_A = 19N$; $w_B = 10N$; $r_A = 15cm$; $r_B = 10cm$

Find the normal force at all the contact points.

$\cos\theta = \frac{11}{25}$, $\theta = \cos^{-1}\left(\frac{11}{25}\right)$

for A: $N_N - F_P\cos\theta = 0$ - ①

$N_M - W_A - F_P\sin\theta = 0$ - ②

$N_N = W_B\cot\theta$

$N_M = W_A + W_B$

for B: $F_P\cos\theta' - N_Q = 0$ - ③

$F_P\sin\theta - W_B = 0$ - ④

$F_P = \frac{W_B}{\sin\theta}$

$N_Q = F_P\cos\theta = W_B\cot$

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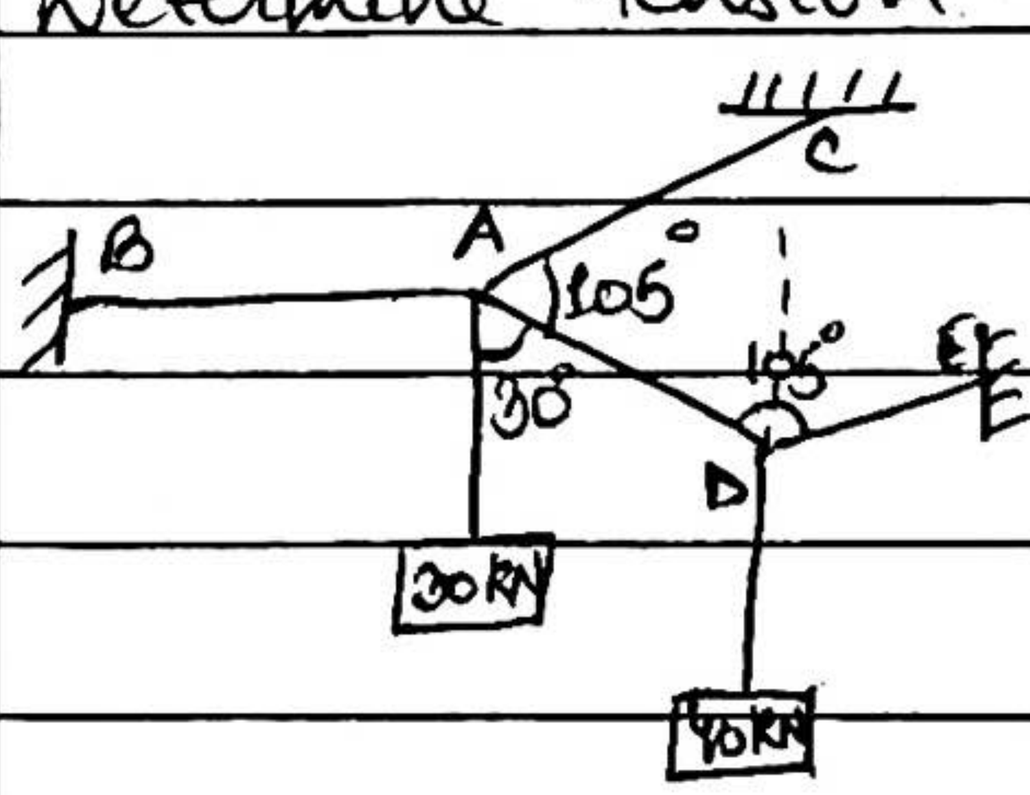
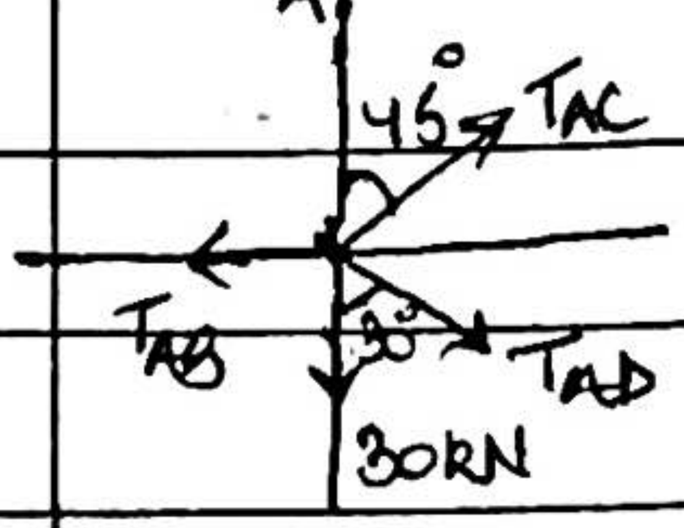
Ques. ① Determine tension in each cable.



$(FBD)_A$



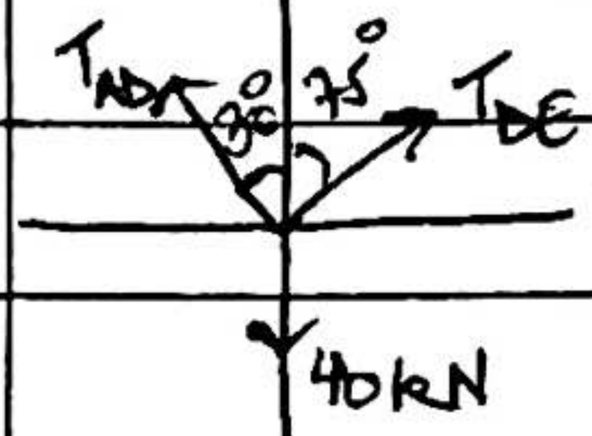
$T_{AC}\sin 45° + T_{AD}\sin 30° = T_{AB}$ — ③

$T_{AC}\cos 45° = T_{AD}\cos 30° + 30$ — ④

$T_{AC} = 91.42\,kN$

$T_{AB} = 84.65\,kN$

$(FBD)_D$

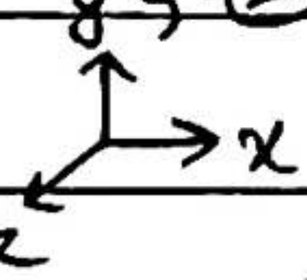


$T_{DE}\sin 75° = T_{AD}\sin 30°$ — ①

$T_{DE}\cos 75° + T_{AD}\cos 30° = 40$ — ②

$T_{AD} = 1.932\,T_{DE}$

$T_{DE} = 20.71\,kN \quad T_{AD} = 40\,kN$



Q.② Find force in each cable.

Step (i) → coordinates

A (0, 10, 0)

B (−4, 0, 6)

C (8, 0, 6)

D (0, 0, −8)

Step (ii) → force vector

$$\vec{F}_{AB} = F_{AB}\left[\frac{-4\hat{i} - 10\hat{j} + 6\hat{k}}{\sqrt{152}}\right]$$

$$\vec{F}_{AC} = F_{AC}\left[\frac{8\hat{i} - 10\hat{j} + 6\hat{k}}{\sqrt{200}}\right]$$

$$\vec{F}_{AD} = F_{AD}\left[\frac{-10\hat{j} - 8\hat{k}}{\sqrt{164}}\right]$$

Step (iii) → force equations

$\Sigma F_x = 0 \Rightarrow -0.324 F_{AB} + 0.566 F_{AC} = 0$ — ①

$\Sigma F_y = 0 \Rightarrow -0.811\,F_{AB} - 0.707 F_{AC} - 0.781 F_{AD} = 2000$ — ②

$\Sigma F_z = 0 \Rightarrow 0.487 F_{AB} + 0.424\,F_{AC} - 0.625\,F_{AD} = 0$ — ③

eq. ① → $F_{AB} = 1.747\,F_{AC}$

$F_{AB} = 1097.8\,kN$

$F_{AC} = 537.9\,kN$

$F_{AD} = 939.2\,kN$

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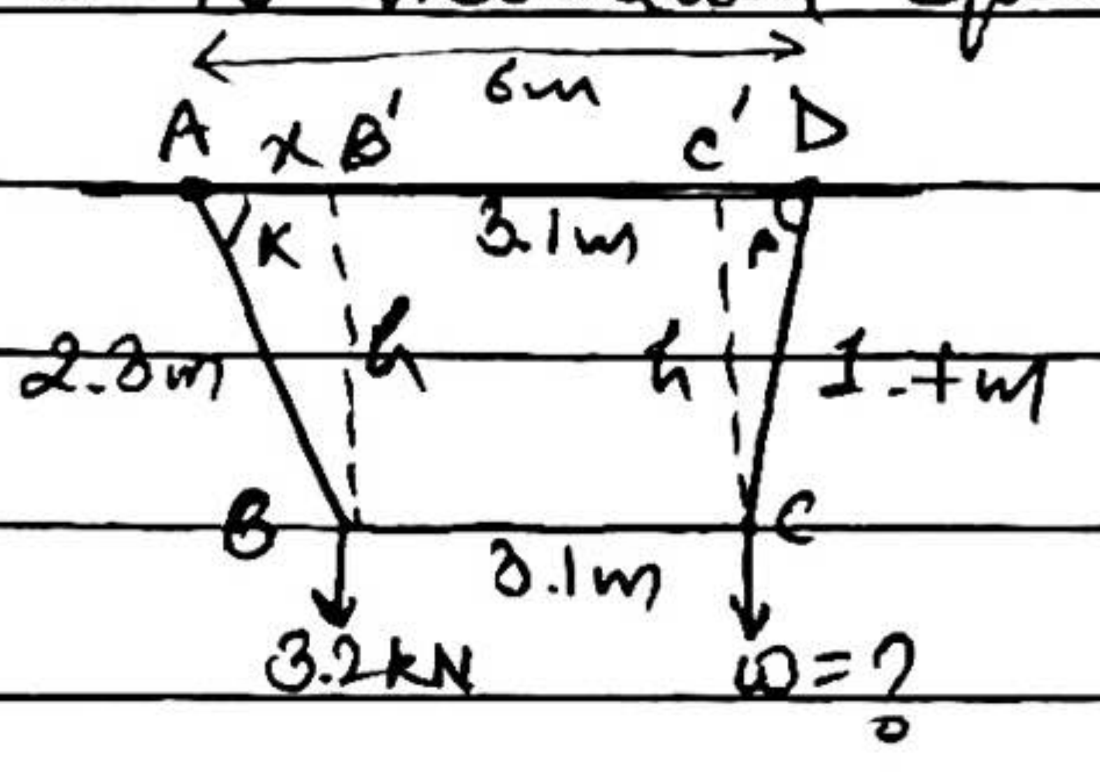
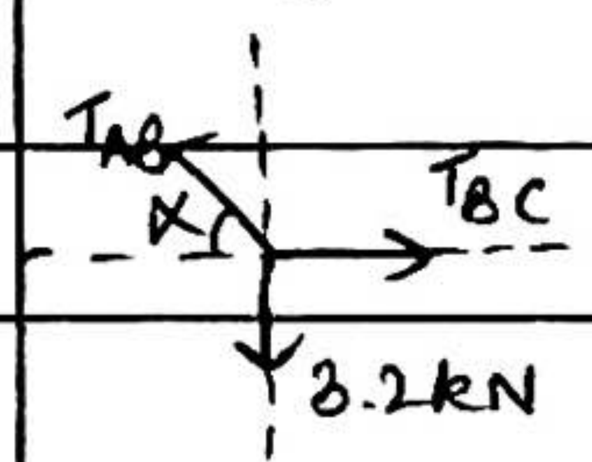
Q. (3) Two weights are suspended from 'B' & 'C' pts. of a rope. If distance AD is 6m, how much will be the magnitude of 'ω' to maintain equilibrium.

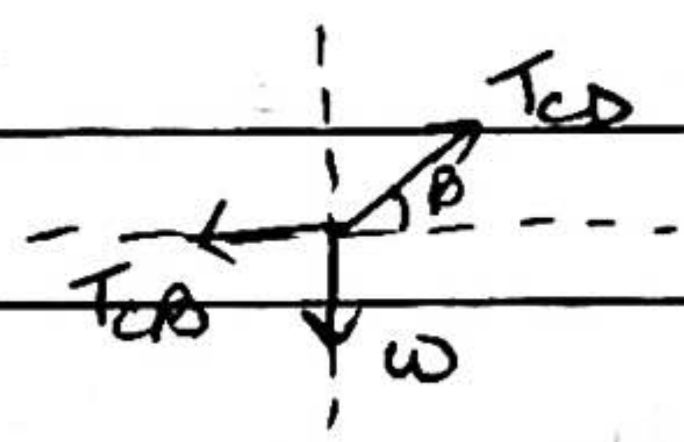


$C'D = 6 - 3.1 - x = 2.9 - x$

$(FBD)_B$



$(FBD)_C$



$T_{BC} = T_{BA}\cos\alpha$ — (1)

$3.2 = T_{BA}\sin\alpha$ — (2)

$T_{CB} = T_{CD}\cos\beta$ — (3)

$\omega = T_{CD}\sin\beta$ — (4)

$\Delta CDC'$

$\Rightarrow h^2 + (2.9-x)^2 = (1.7)^2$

$\Rightarrow h^2 + (2.9)^2 + x^2 - 2(2.9)x = (1.7)^2$

$\Rightarrow (2.3)^2 + (2.9)^2 - 2(2.9)x = (1.7)^2$

$\Rightarrow 5.29 + 8.41 - 5.8x = 2.89$

$\Rightarrow x = 1.864m$

$\Delta ABB'$

$\Rightarrow x^2 + h^2 = (2.3)^2$

$\cos\alpha = \frac{1.864}{2.3}$

$\alpha = 35.86°$

$\cos\beta = \frac{1.036}{1.7}$

$\beta = 52.45°$

$T_{BA} = \frac{3.2}{\sin\alpha} = 5.483kN$

$T_{BC} = 4.427 kN$

$T_{CD} = 7.264 kN$

$\omega = 5.295$

Q. (4) Given tension in cable AD = 540N
Determine wt. of plate.

Step (1) → O(0,0,0)
A(0,1.2,0)
B(0.65,0,−0.9)
C(1.15,0,0.9)
D(−0.8,0,0.9)

Step (2) → $\bar{F}_{AB} = |F_{AB}|\left(\dfrac{0.65\hat{i} - 1.2\hat{j} - 0.9\hat{k}}{1.685}\right)$

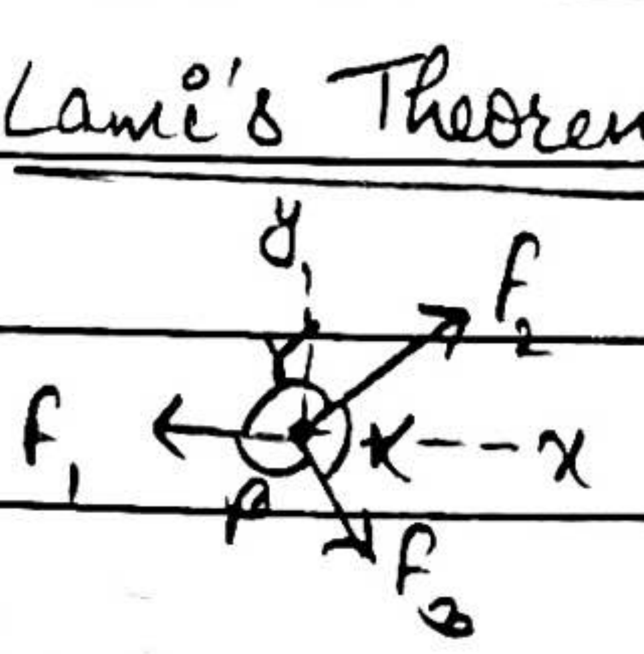
$\bar{F}_{AD} = 540\left(\dfrac{-0.8\hat{i} - 1.2\hat{j} + 0.9\hat{k}}{1.7}\right)$

$\bar{F}_{AC} = |F_{AC}|\left(\dfrac{1.15\hat{i} - 1.2\hat{j} + 0.9\hat{k}}{1.89}\right)$

Step (3) → $\Sigma F_x = 0 \Rightarrow 0.398 F_{AB} - 254.11 + 0.608 F_{AC} = 0$ — (1)
$\Sigma F_y = 0 \Rightarrow -0.733 F_{AB} - 381.18 - 0.636 F_{AC} = W$ — (2)
$\Sigma F_z = 0 \Rightarrow -0.551 F_{AB} + 285.882 + 0.476 F_{AC} = 0$ — (3)

$F_{AB} = 562.06N$
$F_{AC} = 50.02N$
$W = 823.24N$

** Lami's Theorem



$\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$

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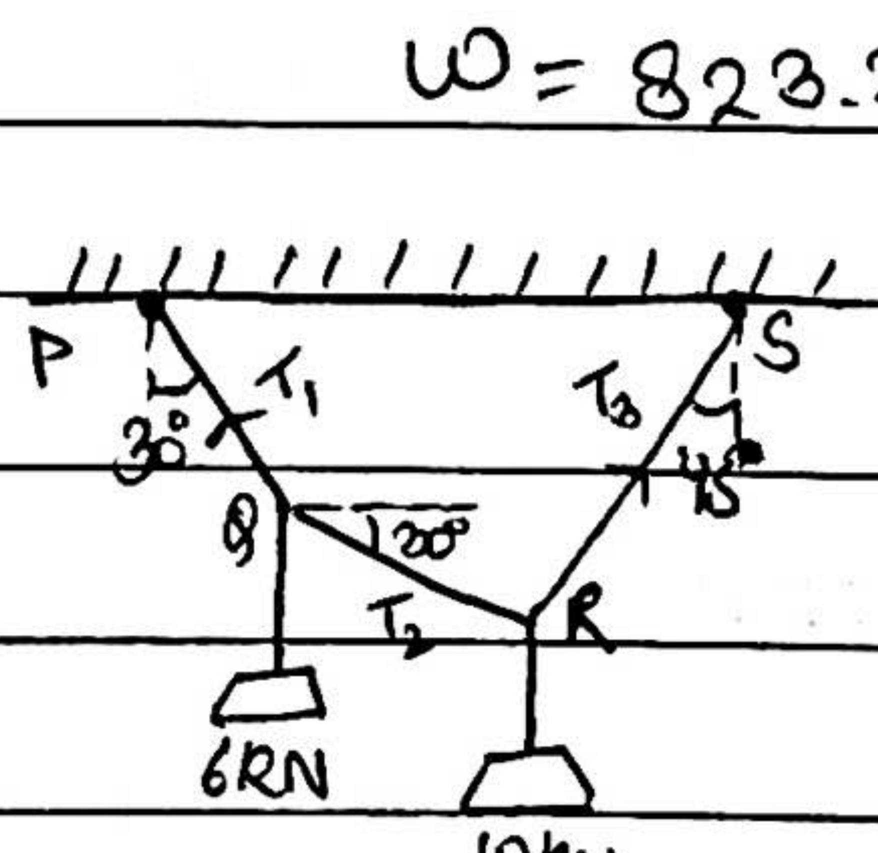
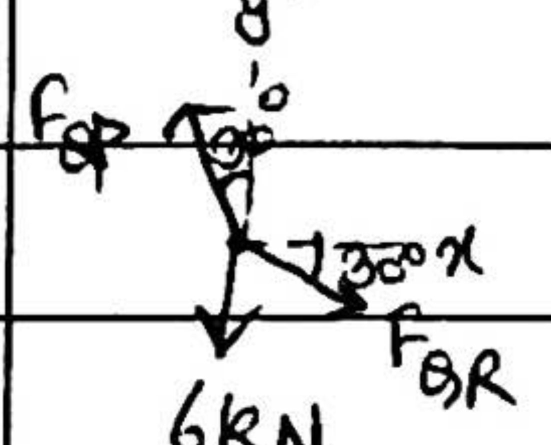
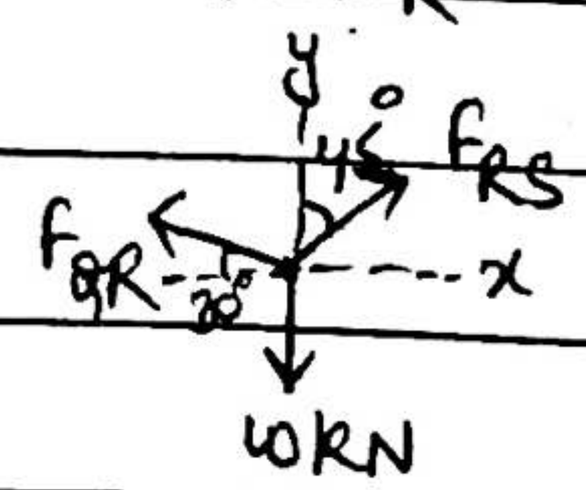
Q. (5)



$(FBD)_Q$



$(FBD)_R$



$$\frac{F_{QP}}{\sin 60^\circ} = \frac{F_{QR}}{\sin 150^\circ} = \frac{6000}{\sin 150^\circ}$$

$$\frac{\cancel{2}F_{QP}}{\sqrt{3}} = \frac{6000 \times \cancel{2}}{1} \Rightarrow F_{QP} = 6000\sqrt{3}\ N = 10.39\ kN$$

$F_{QR} = 6000N$

$$\frac{F_{RS}}{\sin 120^\circ} = \frac{F_{QR}}{\sin 135^\circ} = \frac{W}{\sin 105^\circ} = \frac{6000 \times \sqrt{2}}{1}$$

$$F_{RS} = 6000\sqrt{2} \times \frac{\sqrt{3}}{2} = \frac{6000\sqrt{3}}{\sqrt{2}} = 7.348\ kN$$

$$W = 6000\sqrt{2}\sin 105^\circ = \underline{\underline{8.196\ kN}}$$

Q.(6) $PQ = \sqrt{320^2 + 440^2} = 544.06\ mm$ } Stretched
$PS = \sqrt{320^2 + 600^2} = 680\ mm$ } scenario

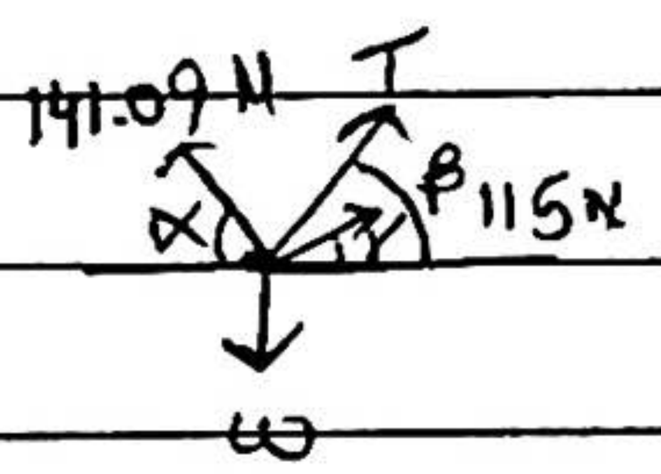
$\therefore$ Stretched lengths (x),

$PQ = 544.06 - 450 = 94.06\ mm$

$PS = 680 - 450 = 230\ mm$

$F_{PQ} = 1500 \times 94.06 \times 10^{-3} = 141.09\ N$

$F_{PS} = 230 \times 10^{-3} \times 500 = 115\ N$



$\alpha = \tan^{-1}\left(\frac{320}{440}\right)$ $\beta = \tan^{-1}\left(\frac{480}{140}\right)$ $\gamma = \tan^{-1}\left(\frac{320}{600}\right)$

$\alpha = 36.02^\circ$ $\beta = 73.73^\circ$ $\gamma = 28.07^\circ$

$T\cos\beta + 115\cos\gamma - 141.09\cos\alpha = 0$ — (1)

$\Rightarrow 0.28T + 101.47 - 114.11 = 0$

$\Rightarrow T = \underline{\underline{45.14\ N}}$

$T\sin\beta + 115\sin\gamma + 141.09\sin\alpha = W$ — (2)

$\Rightarrow 43.33 + 54.11 + 82.97 = W$

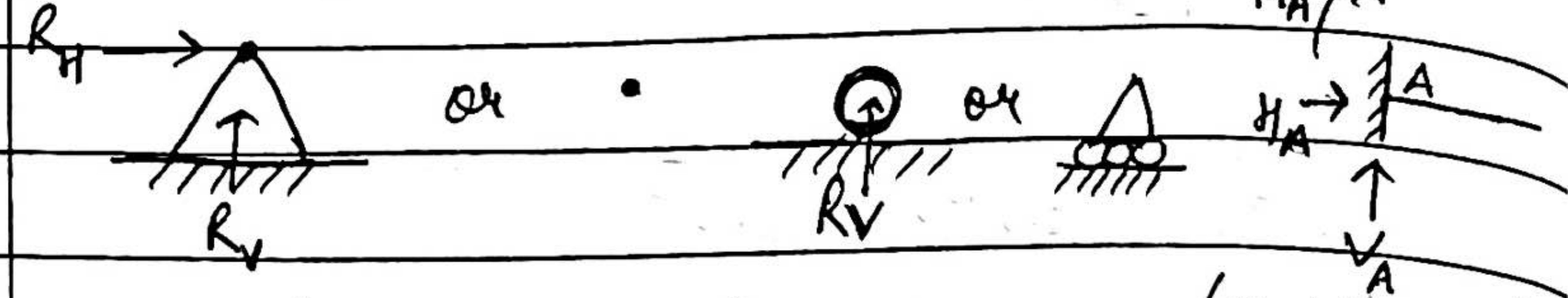
$\Rightarrow W = \underline{\underline{180.41\ N}}$

9/10/20

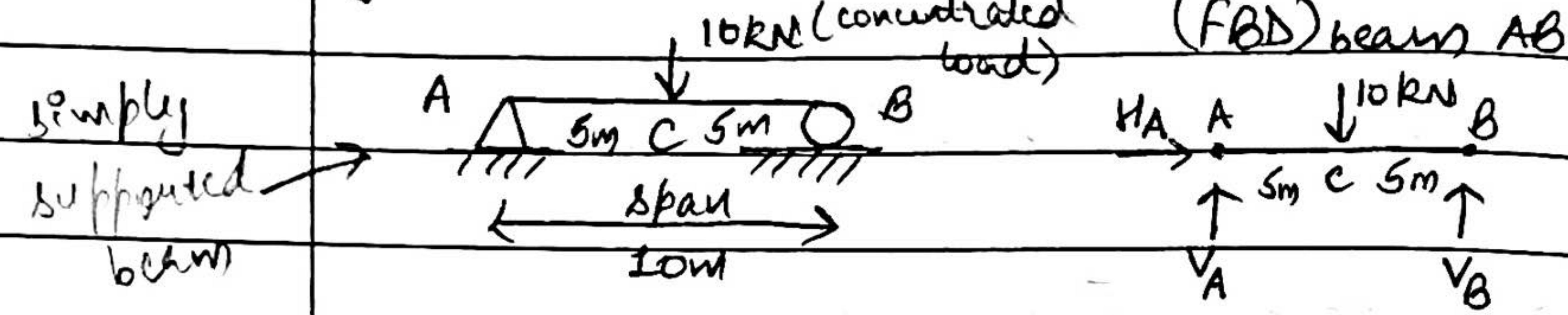
Equilibrium of Rigid Bodies

- Beam / Connections
- 3 types of connections (fixed support)

① Pin / Hinge ② Roller ③ Fixed / In-



eg. support reaction?



$\Sigma F_x = 0 \Rightarrow H_A = 0$ —①

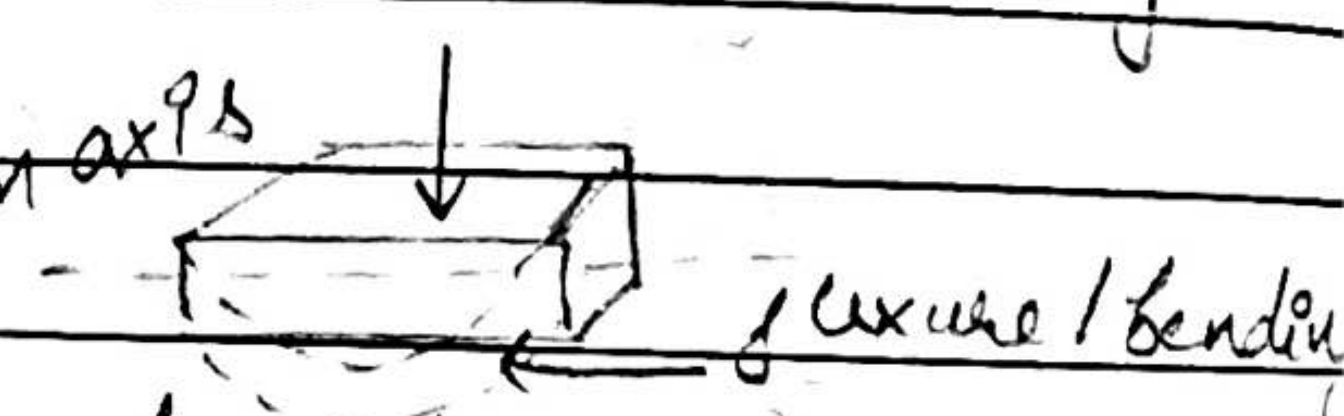
$\Sigma F_y = 0 \Rightarrow V_A + V_B = 10kN$ —②

$\Sigma M_A = 0 \Rightarrow (\cancel{10})5 = V_B \times \cancel{10}$ —③

$V_B = 5kN$

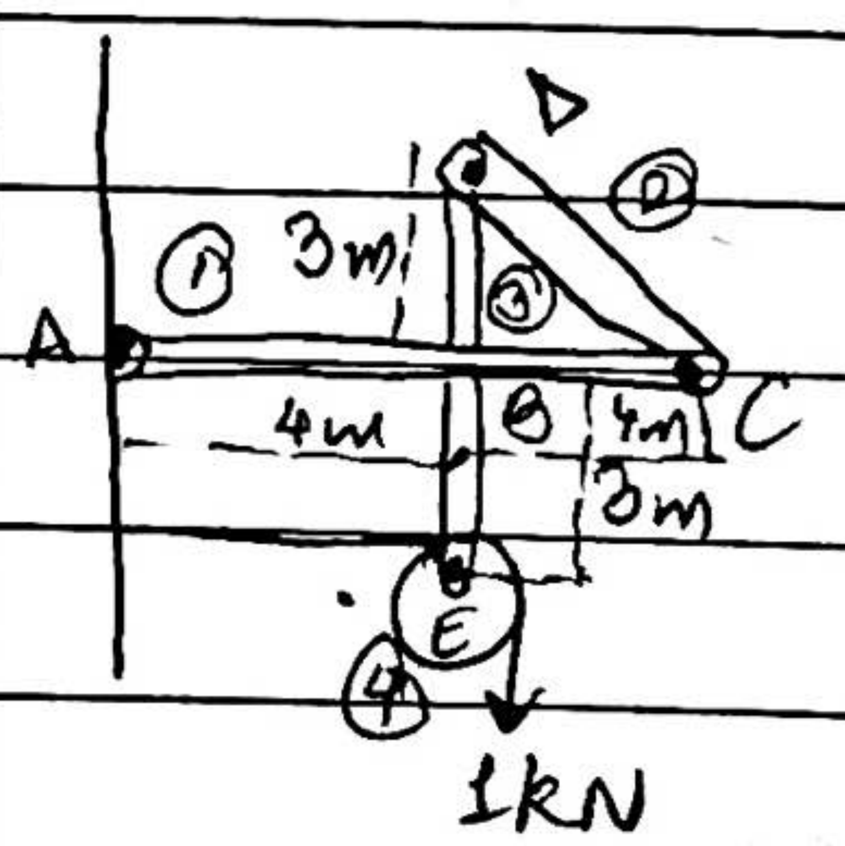
$\therefore V_A = 5kN$

15/10/20 ④ simple support



beam is a structural member

Ques ①

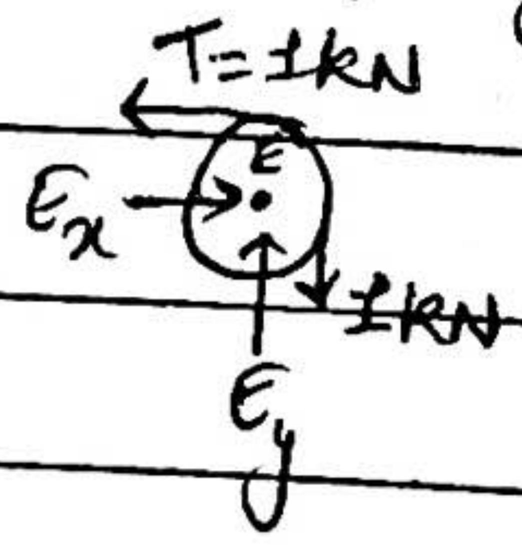


Step 1 → Identify members

Step 2 → FBD

Step 3 → Eqⁿ of equilibrium.

(FBD) ④ pulley



$\Sigma F_x = 0 \Rightarrow E_x = 1\,kN$

$\Sigma F_y = 0 \Rightarrow E_y = 1\,kN$

Members →

① ABC

② DC

③ DBE

④ Pulley

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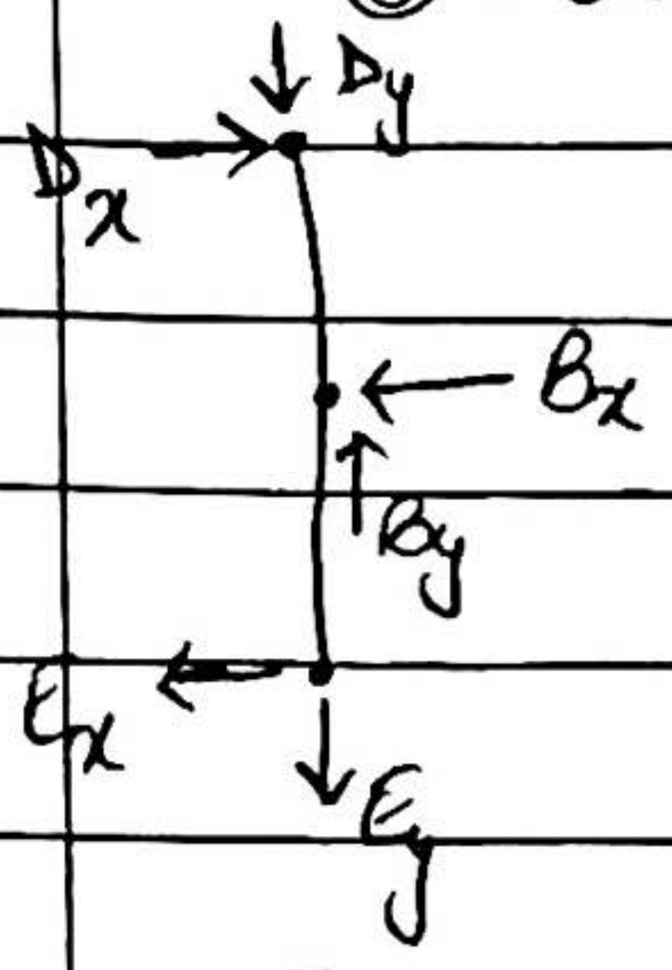
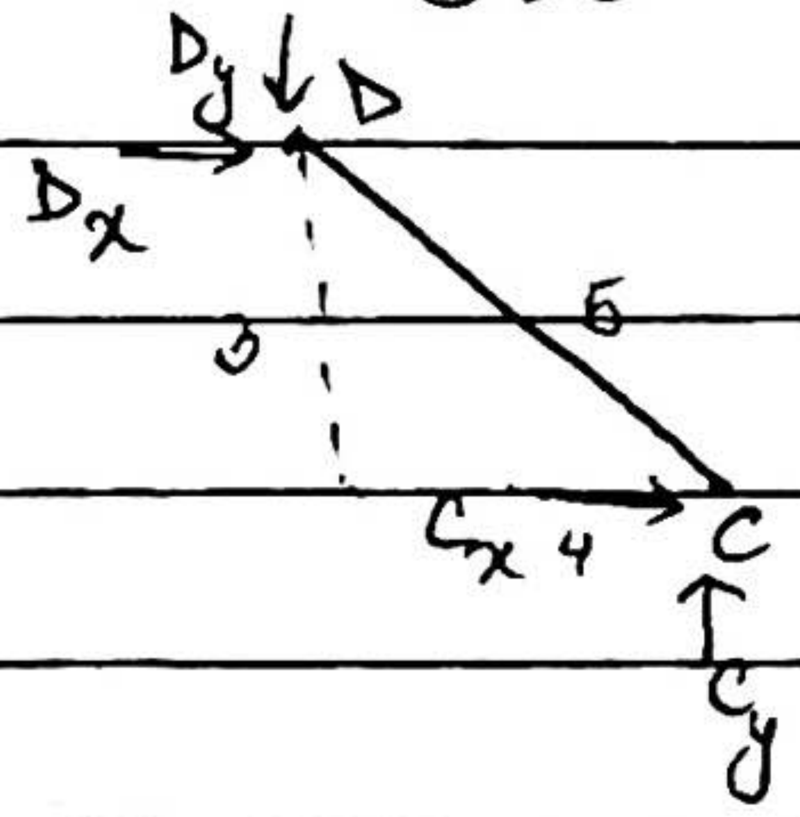
(FBD)③ DBE



(FBD)② DC



$\Sigma M_C = 0$

$\Rightarrow 3D_x = 4D_y \Rightarrow \boxed{D_y = \frac{3}{4} kN}$

$F_{DC} = \sqrt{1^2 + (3/4)^2}$

$F_{DC} = \sqrt{1 + 9/16} = 1.25 kN$

$\Sigma M_D = 0$

$\Rightarrow 6E_x + 3B_x = 0$

$\Rightarrow 6 \times 1 kN = -3B_x$

$\Rightarrow -2kN = B_x$

$\Sigma F_x = 0$

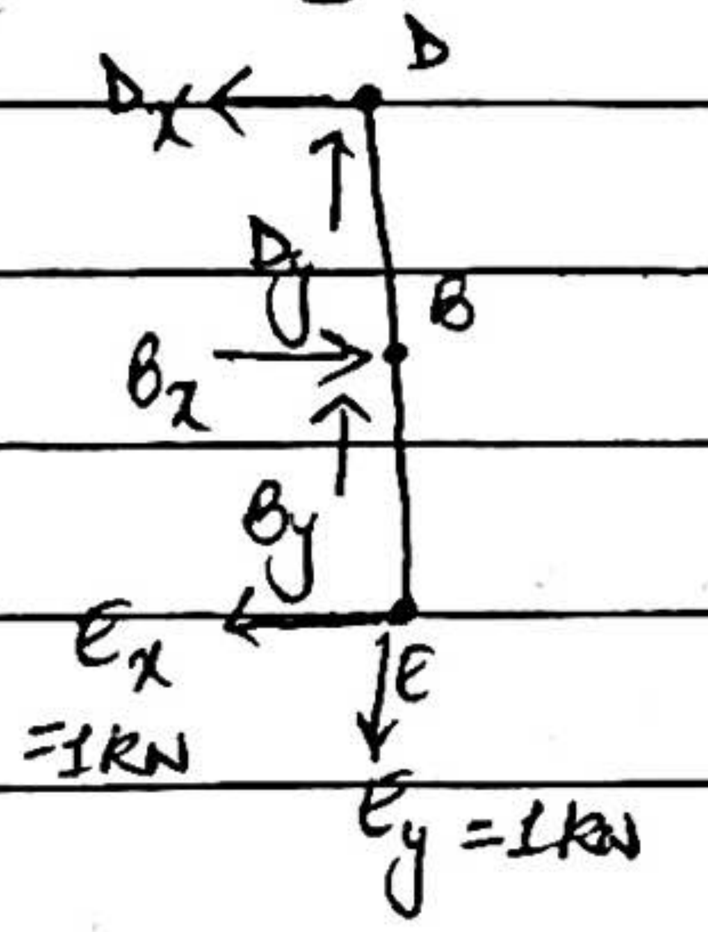
$\Rightarrow D_x = B_x + E_x$

$\Rightarrow D_x = -2kN + 1kN$

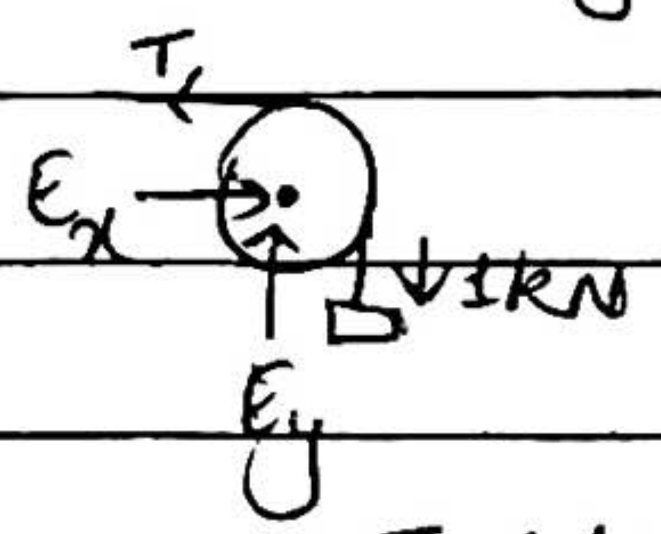
$\Rightarrow D_x = -1kN$

~~(FBD)①~~

(FBD)③ DBE



(FBD)④ Pulley



$T = 1 kN$

$\boxed{E_x = 1kN}$

$\boxed{E_y = 1kN}$

$\Sigma M_B = 0$

$\Rightarrow 3B_x - 6E_x = 0$

$\Rightarrow \boxed{B_x = 2kN}$

$\Sigma F_x = 0$

$\Rightarrow D_x + E_x - B_x = 0$

$\Rightarrow \boxed{D_x = 1 kN}$

$\Sigma F_y = 0$

$\Rightarrow D_y + B_y - E_y = 0$

$\Rightarrow \boxed{B_y = 1 - 3/4 = 1/4 kN}$

$F_B = \sqrt{2^2 + (1/4)^2} = \sqrt{4 + 1/16} = \frac{\sqrt{65}}{\sqrt{16}}$ ~~= 1.14kN~~

$F_B = 2.01 kN$

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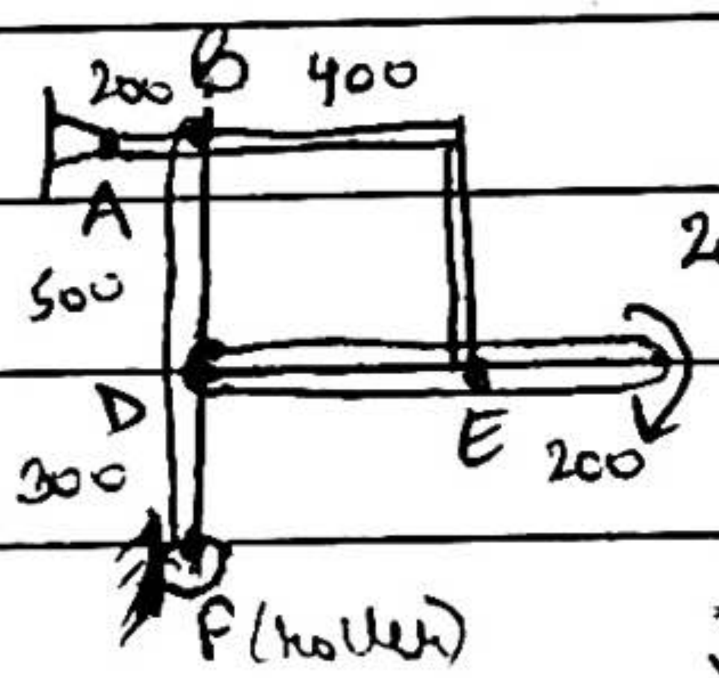
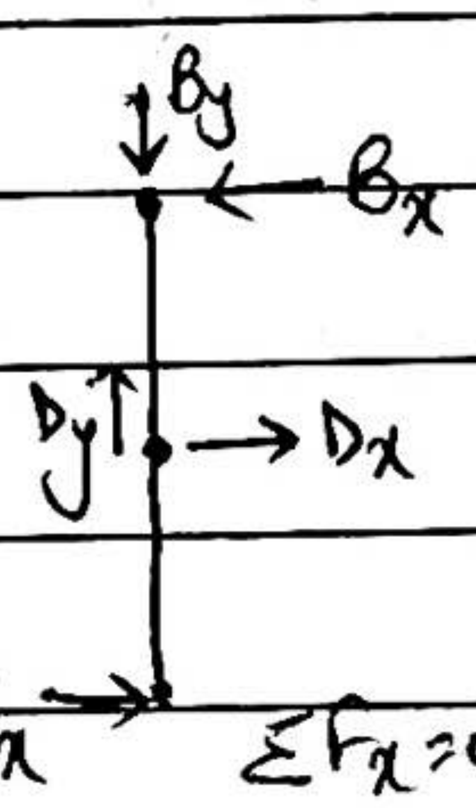
Ques ②



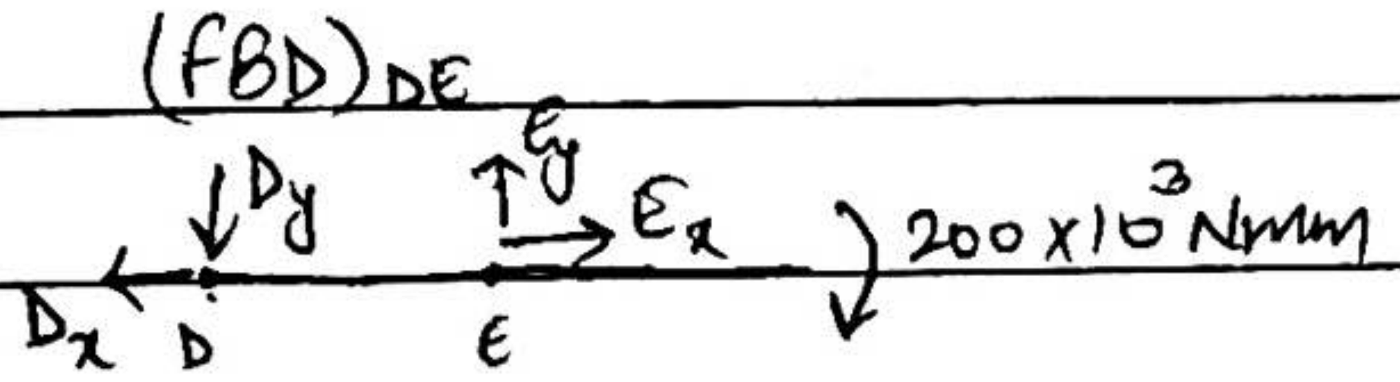
$200 Nm = 200 \times 10^3 Nmm$

Determine pin reactions at B, D, E.

(FBD)$_{BDF}$



(FBD)$_{DE}$



$\Sigma M_D = 0 \Rightarrow E_y \times 400 - 200 \times 10^3 = 0$

$\Rightarrow E_y = 500 N$

$\Sigma F_y = 0 \Rightarrow B_y - D_y = 0$

$\Sigma F_x = 0 \Rightarrow F_x + D_x = B_x$

$\Sigma F_y = 0$

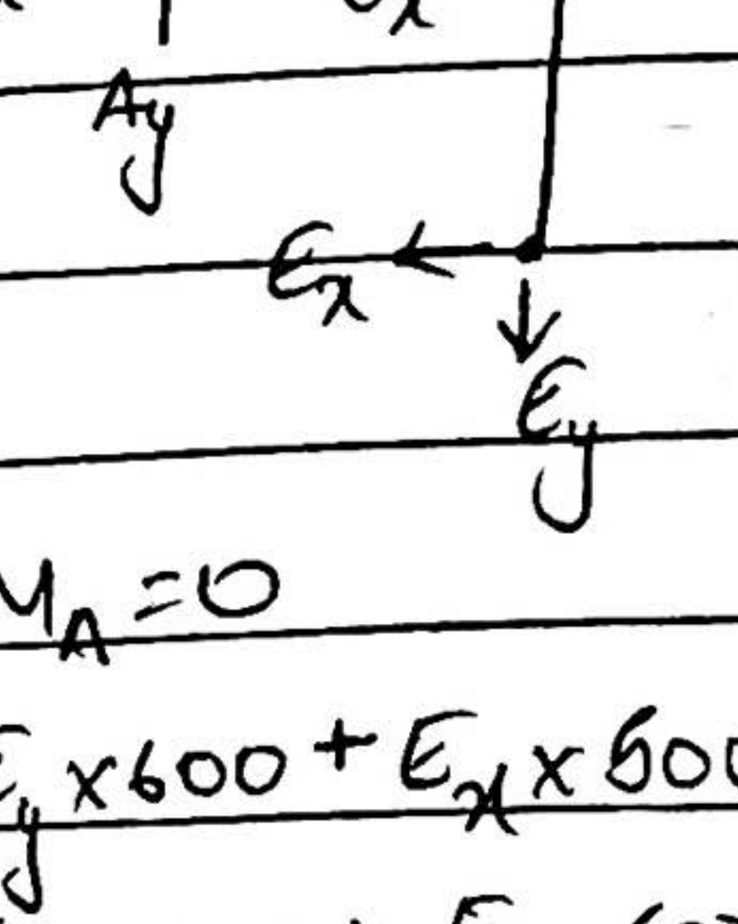
$\Rightarrow E_y - D_y = 0$

$\Rightarrow E_y = D_y = 500N$

$\Sigma F_x = 0$

$\Rightarrow D_x = E_x = -400N$

$(FBD)_{ABE}$



$\Sigma M_A = 0$

$\Rightarrow E_y \times 600 + E_x \times 500 - B_y \times 200 = 0$

$\Rightarrow 500 \times 600 + E_x \times 500 - 500 \times 200 = 0$

$\Rightarrow E_x = \frac{-\cancel{500} \times 400}{\cancel{600}} = -400 N$

In ①st FBD $\rightarrow$

$\Sigma M_C = 0 \Rightarrow -B_x \times 800 + D_x \times 300 = 0$

$\Rightarrow B_x = \frac{-\cancel{400} \times 300}{\cancel{800}_{2}} = -150 N$

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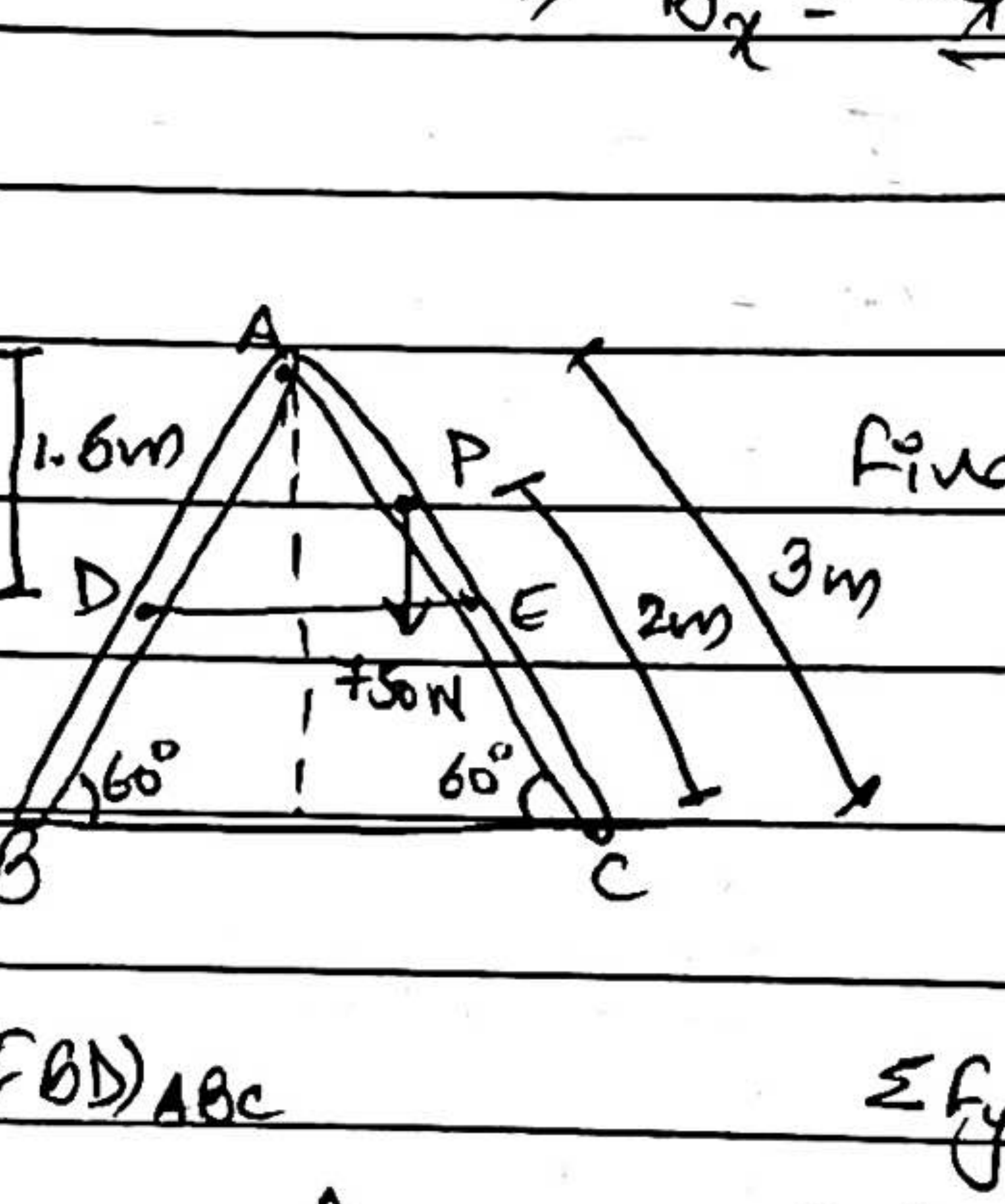
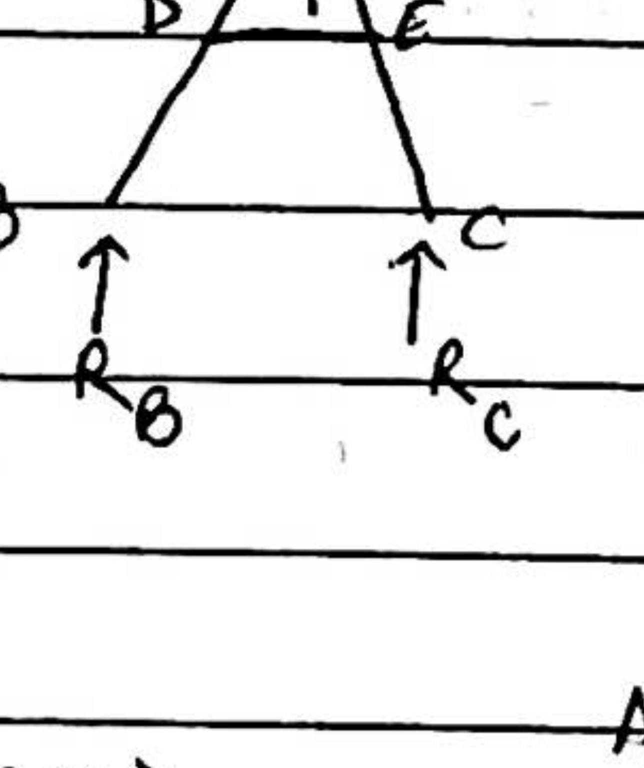
Ques. ③



Find Tension in string DE.

$(FBD)_{ABC}$



$\Sigma F_y = 0$

$\Rightarrow R_B + R_C = 750N$ —①

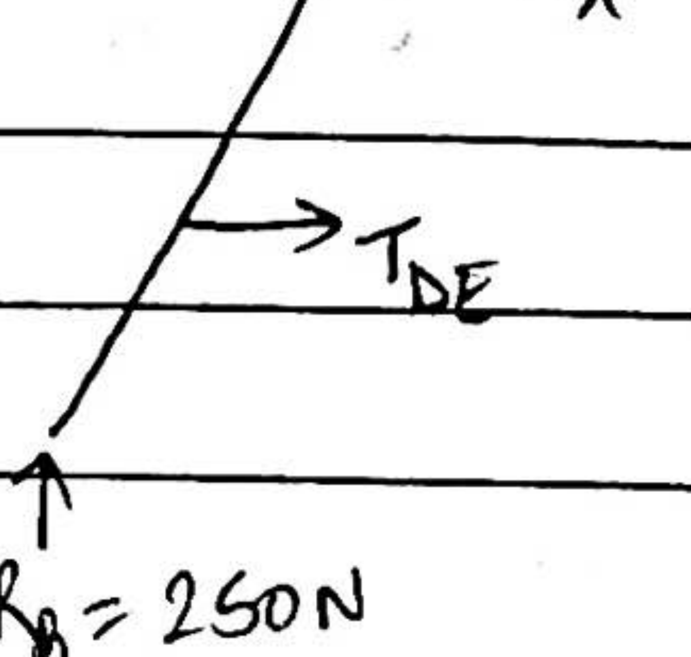
$\Sigma M_C = 0$

$\Rightarrow -750 \times 2\cos 60° + R_B \times 3 = 0$ —②

$\Rightarrow \frac{750 \times \cancel{2} \times 1}{3 \times \cancel{2}} = R_B = 250N$

$R_C = 500N$

$(FBD)_{AB}$



$\Sigma M_A = 0$

$\Rightarrow 250 \times 1.5 - T_{DE} \times 1.5 = 0$

$\Rightarrow \underline{\underline{T_{DE} = 250N}}$

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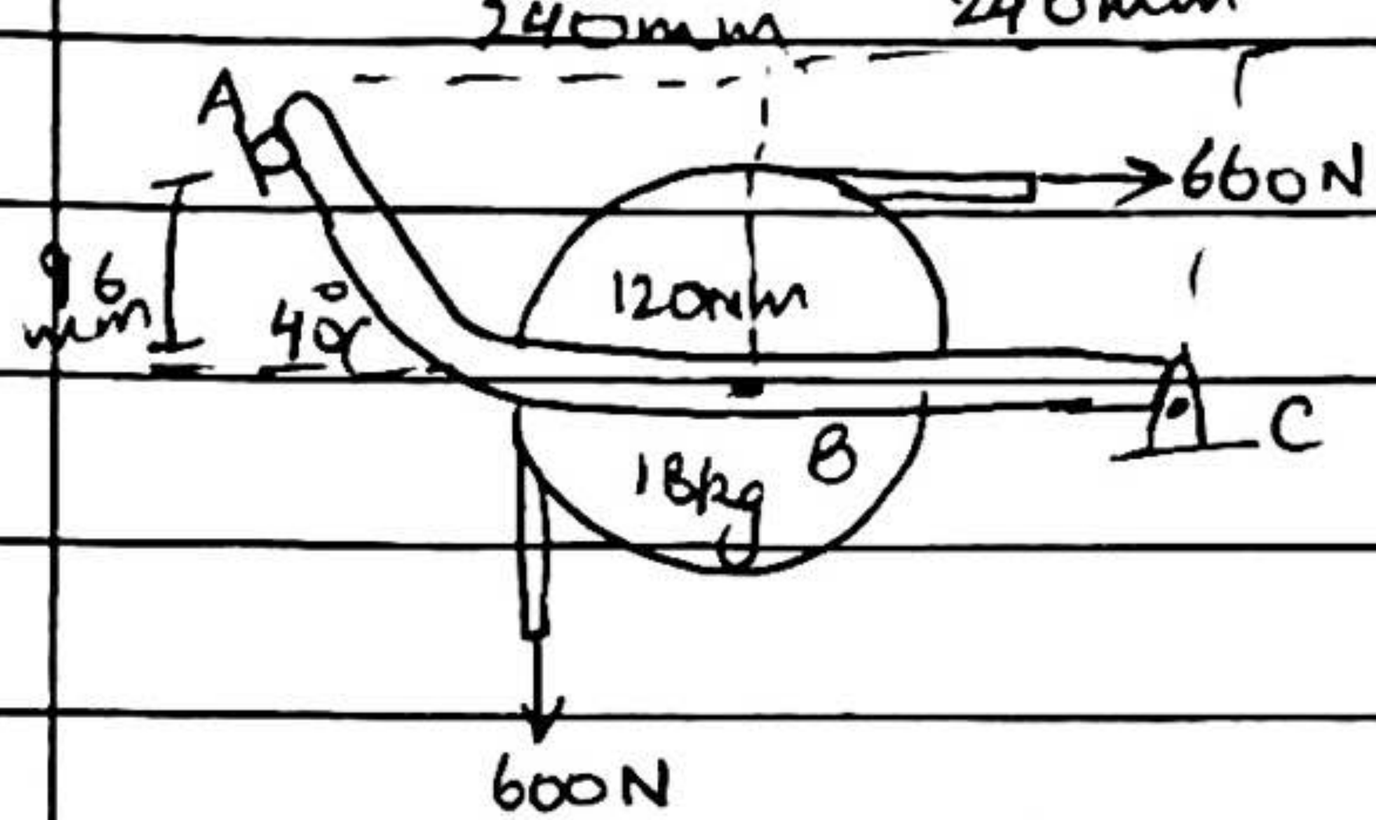
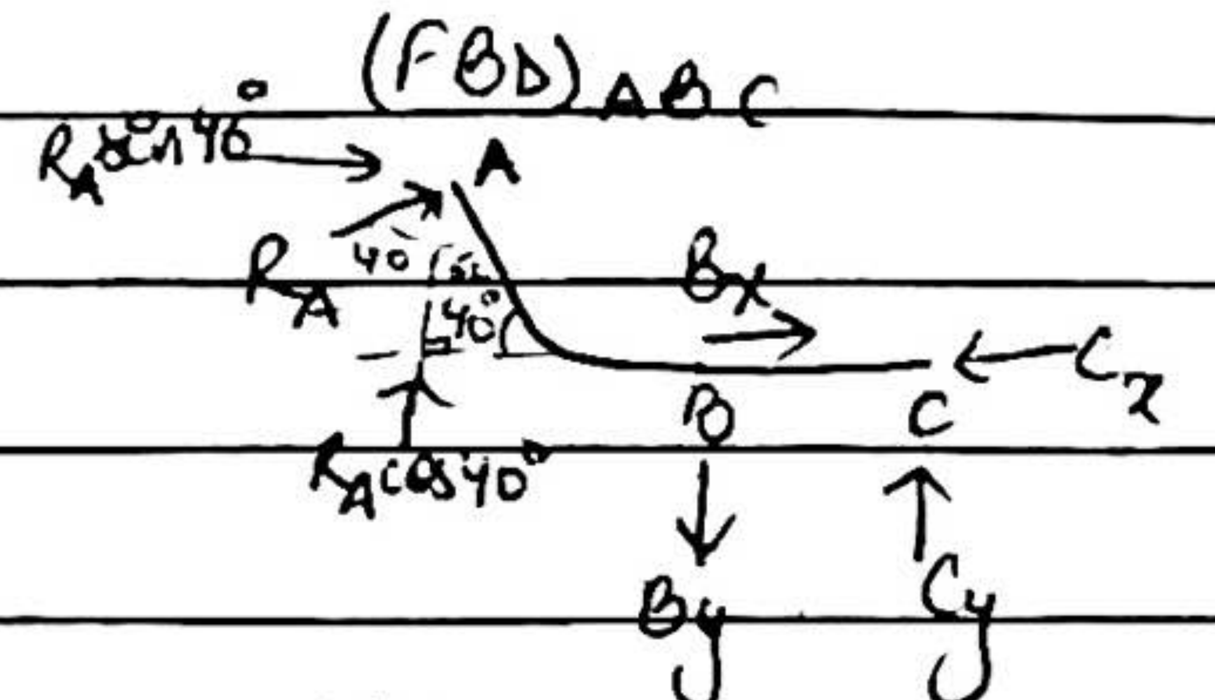
Que ④ Find support reactions at 'A' (roller) and 'C' (hinge).





$\Sigma M_C = 0$

$\Rightarrow -B_y \times 240 + R_A \sin 40° \times 96 + R_A \cos 40° \times 480 = 0$

$\Rightarrow -186379.2 + 61.44 R_A + 367.68 R_A = 0$

$\Rightarrow \dfrac{186379.2}{429.12} = R_A$

$\Rightarrow 434.03\text{N} = R_A$

$\Sigma F_y = 0$

$\Rightarrow R_A \cos 40° - 776.58 + C_y = 0$

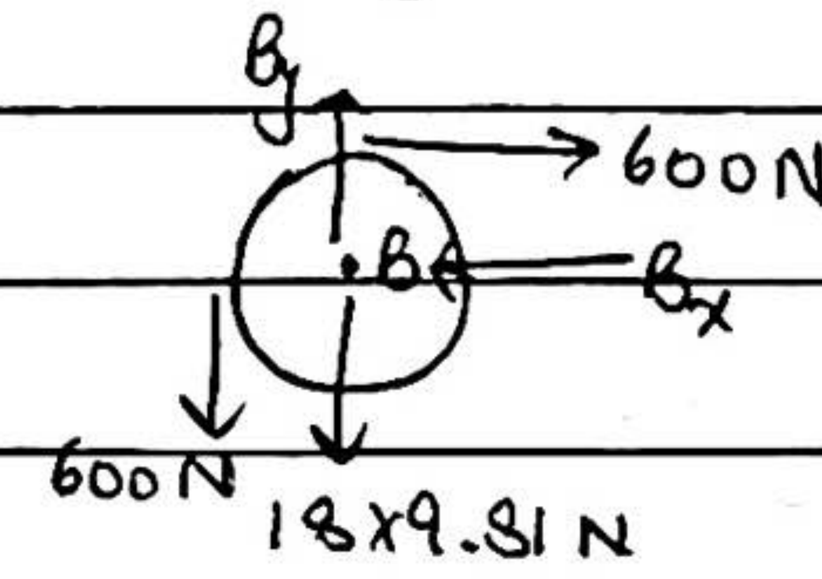
$\Rightarrow C_y = 444.09\text{N}$

$\Sigma F_x = 0$

$\Rightarrow R_A \sin 40° + B_x - C_x = 0$

$\Rightarrow C_x \approx 879\text{N} \quad (878.99\text{N})$

(FBD) pulley



$\Sigma F_x = 0$

$\Rightarrow B_x = 600\text{N}$

$\Sigma F_y = 0$

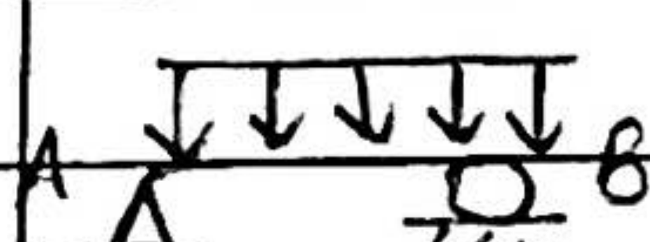
$\Rightarrow B_y = 600 + 18 \times 9.81$

$B_y = 776.58\text{N}$

29/10/20

★ Beam Questions

Distributed Loading
- ① Uniformly Distributed Loading (or Loading Intensity) (UDL) (ω kN/m)
- ② Uniformly Varying Loading (UVL) (linearly varying)

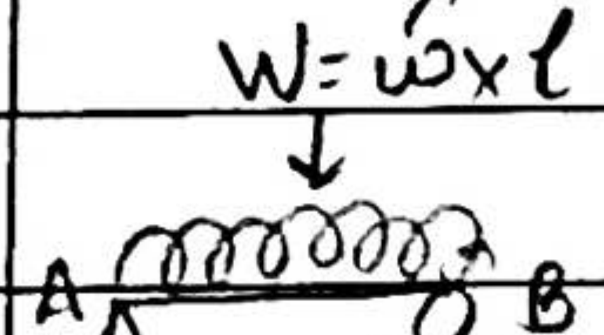


(UDL)

2kN/m

★★ UDL or UVL is replaced with a single conc. load or equivalent load

$W = \omega \times l = 2\text{kN/m} \times 2\text{m} = 4\text{kN}$



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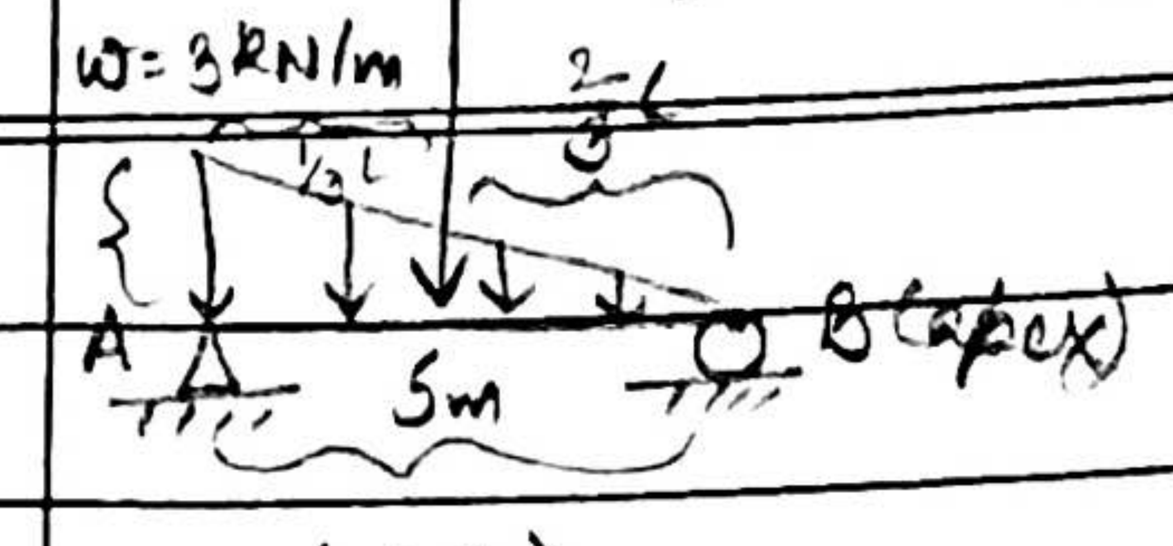
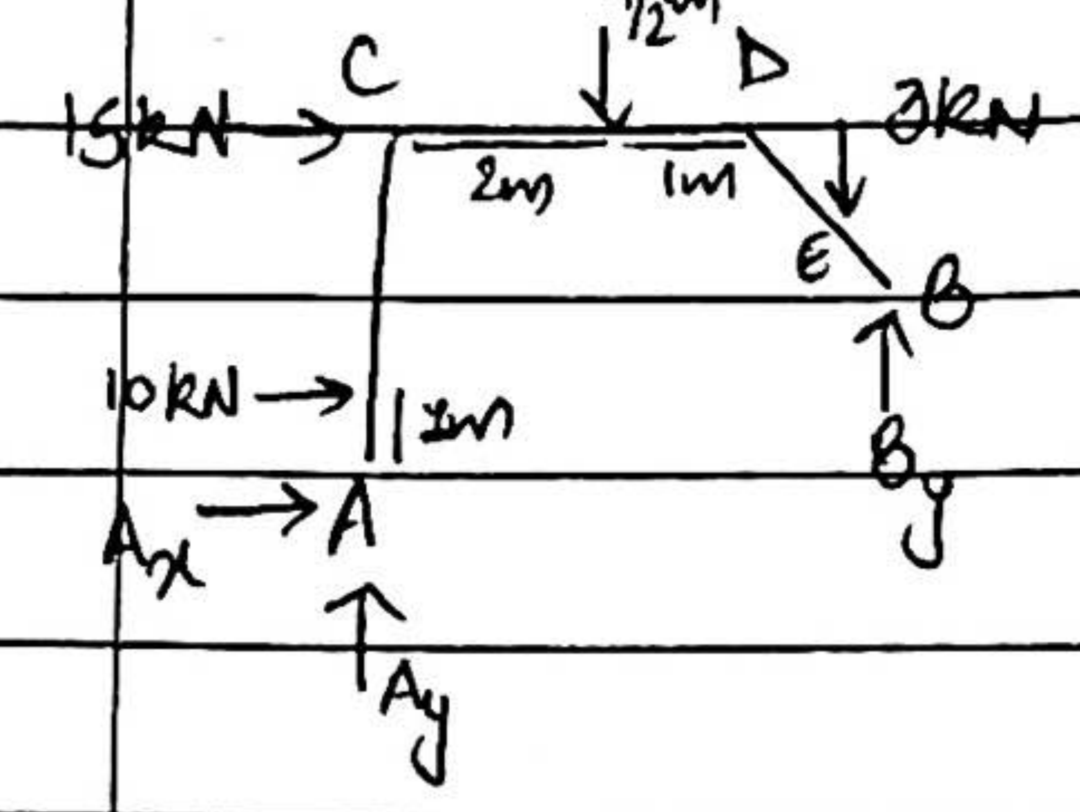
$\omega = \frac{1}{2} \times 5 \times 3 = \frac{15}{2}$ kN $\left(\frac{1}{2} wL\right)$



(UVL)
(triangular)

Ques. ① Determine support reactions at hinged support A and roller support B.



$\Sigma F_x = 0$

$\Rightarrow A_x + 10 + 15 = 0$ — ①

$\Rightarrow A_x = -25$ kN

$\Sigma F_y = 0$

$\Rightarrow A_y + B_y - 9/2 - 3 = 0$

$\Rightarrow A_y + B_y = 7.5$ kN ②

$\Sigma M_B = 0$

$\Rightarrow A_y \times 5 - A_x \times 2 - 10 \times 1 + 15 \times 2 - \frac{9}{2} \times 3 - 3 \times 1 = 0$

$\Rightarrow 5A_y + 50 - 10 + 30 - 13.5 - 3 = 0$

$\Rightarrow A_y = -10.7$ kN

~~$\therefore A_y = 3.2$ kN~~

$B_y = 18.2$ kN

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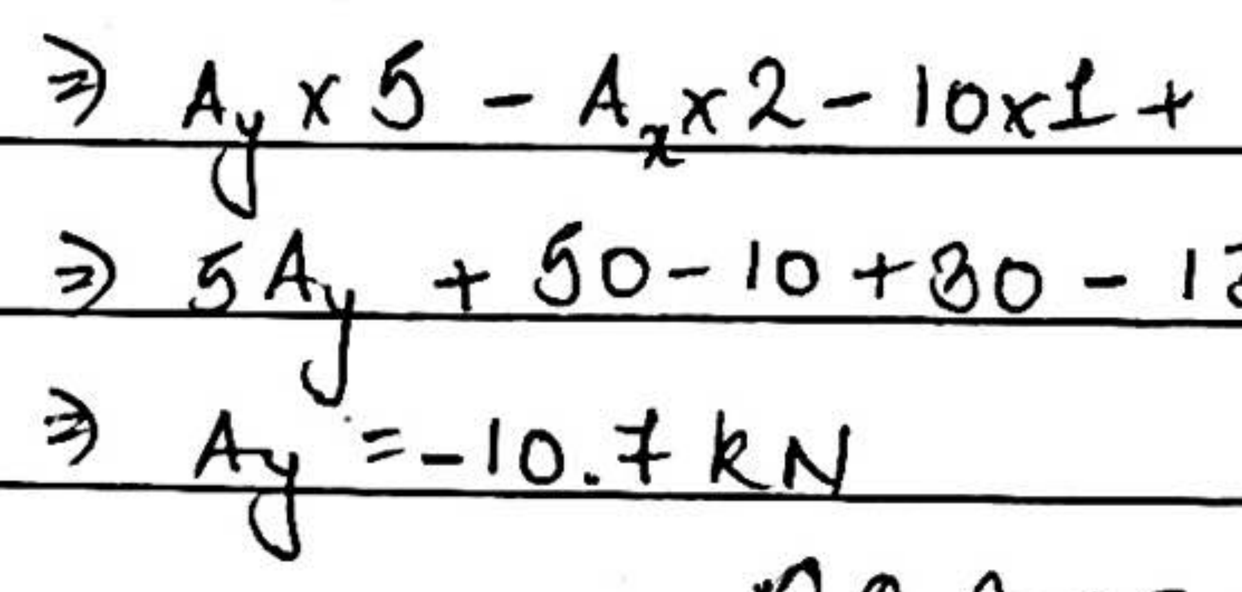
★ Couple (equal & opposite force)

① Rotational Effect $\longrightarrow F$, $\longleftarrow -F$

② $\Sigma F = 0$

Ques. ①

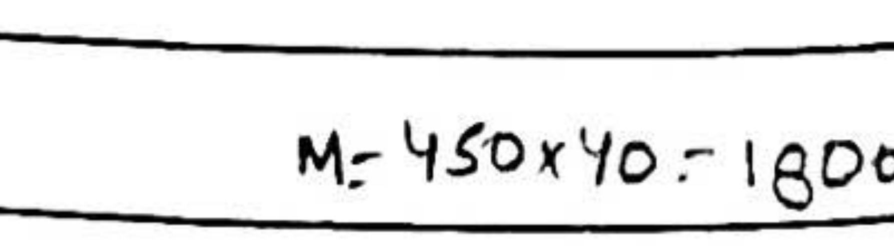


couple is cw. acc. to -150 kN

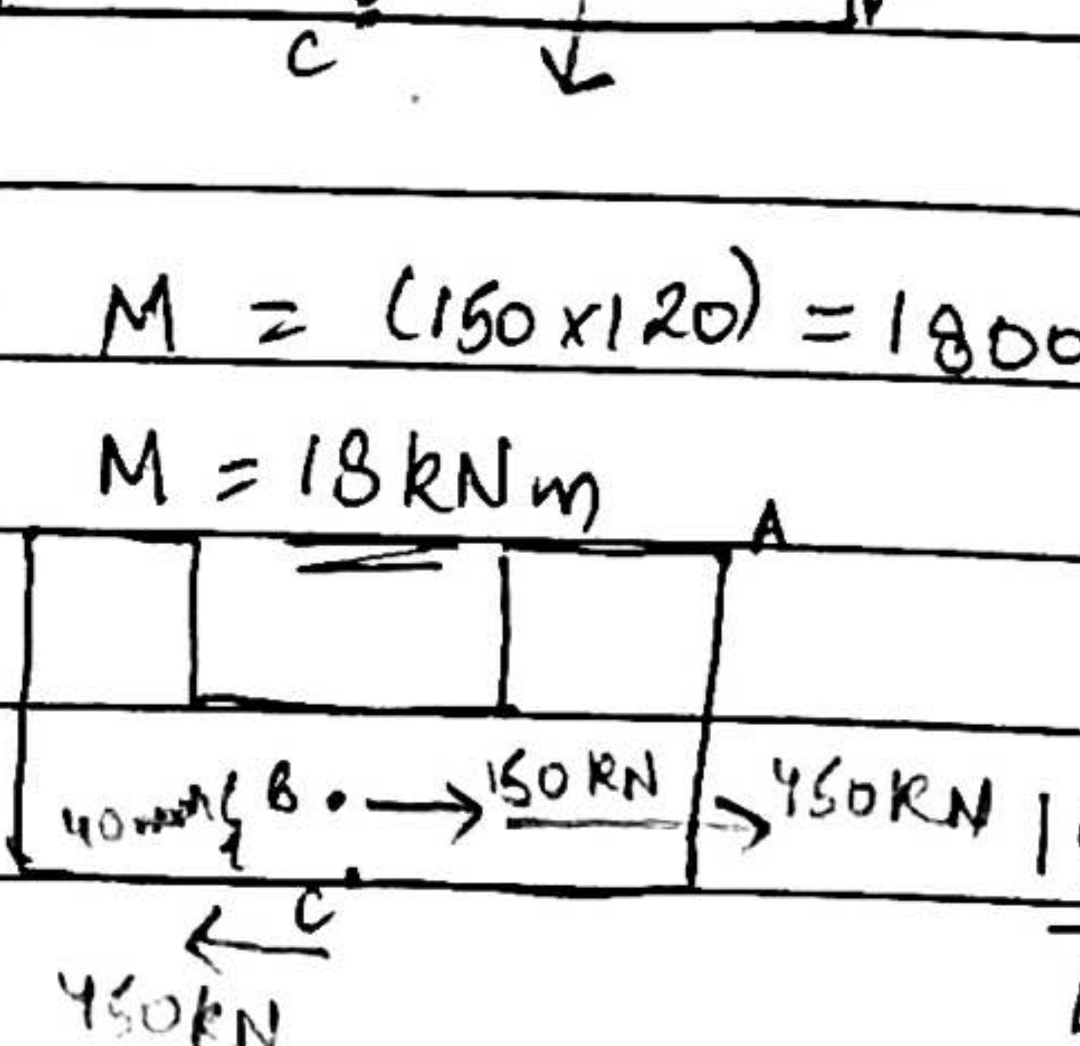
(1) $M = (150 \times 120) = 18000$ kN mm

$M = 18$ kNm

$M = 450 \times 40 = 18000$ kN mm

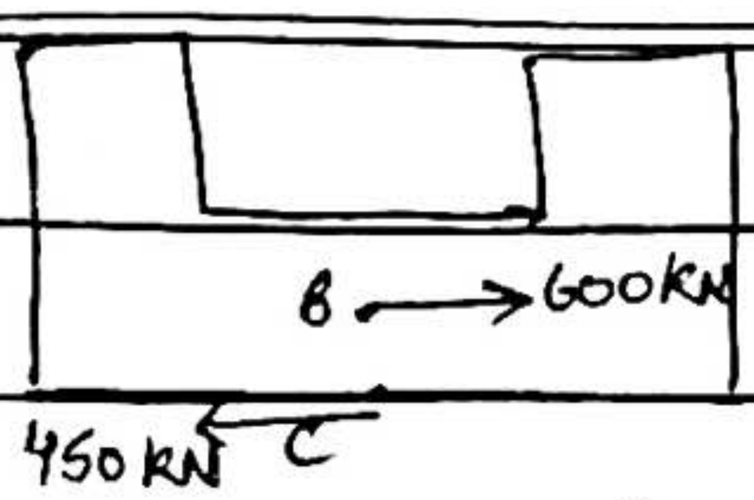


(2)

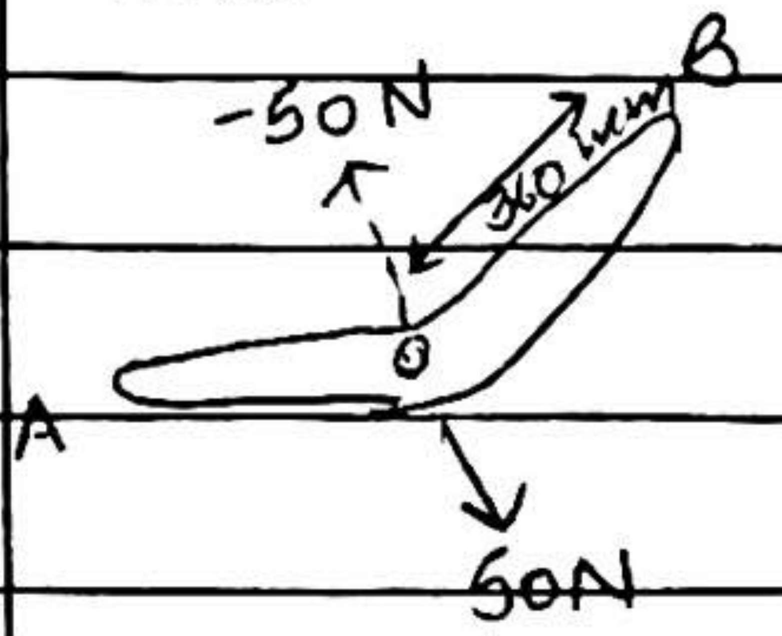


$M = Fd$

$\frac{18 \text{ kNm}}{40 \text{ mm}} = F = \frac{18000 \text{ kN mm}}{40 \text{ mm}} = 450$ kN

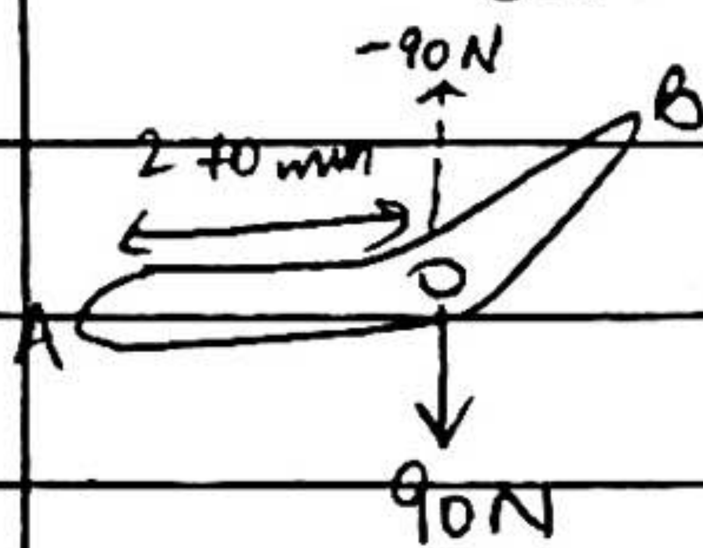


Que. 8



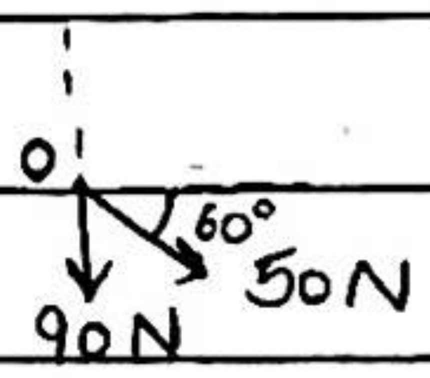
$M_B = 50 \times 360$

$= 18000 \text{ N mm}$



$M_A = 90 \times 270$

$= -24300 \text{ Nmm}$



Net Moment $= -6300$ N mm

$\Sigma f_x = 50 \cos 60° = 25$ N mm

$\Sigma f_y = 90 + 50 \sin 60°$

$= 133.3$ N mm

$F_o = 25\hat{i} - 133.3\hat{j}$

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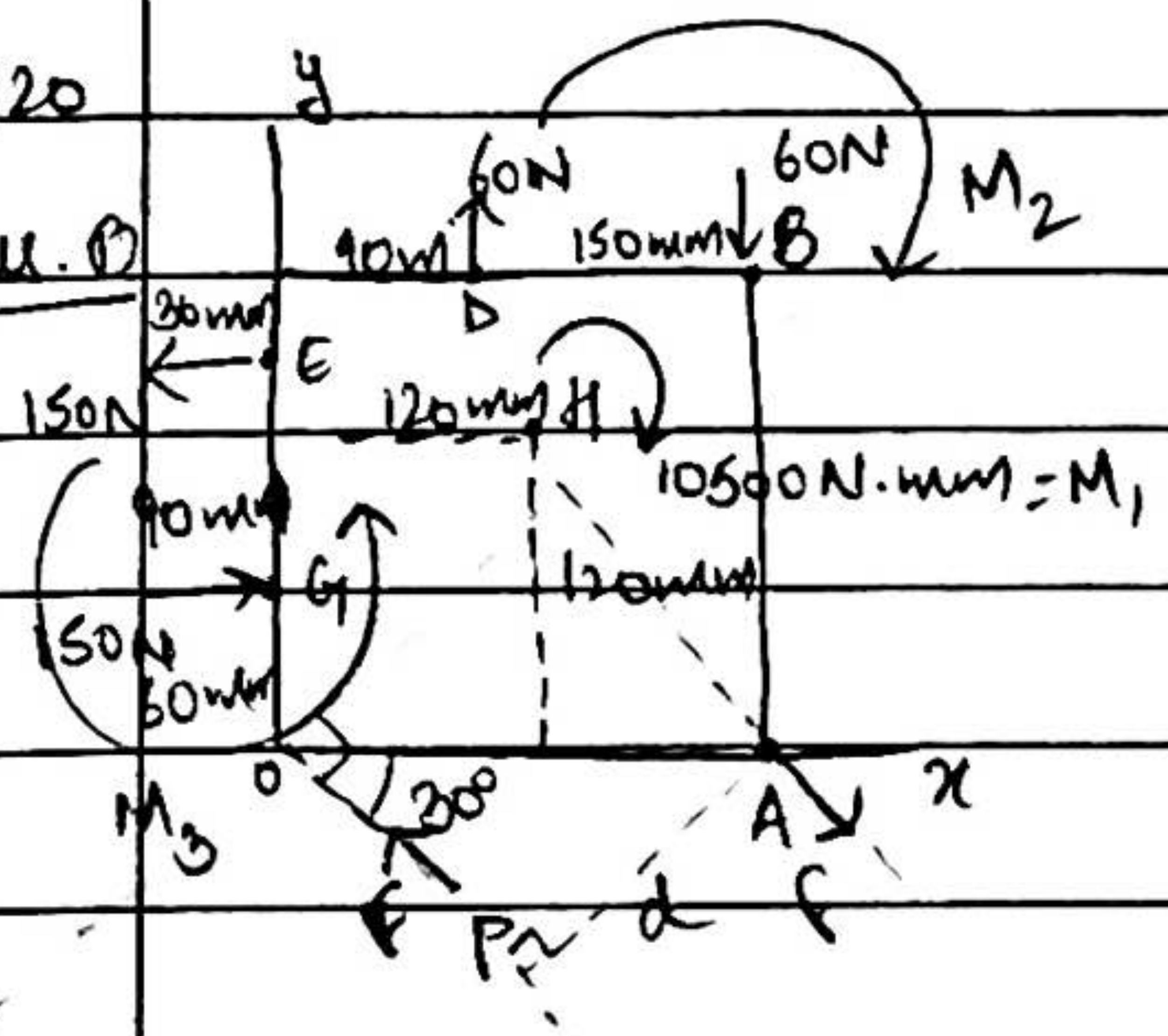
Que. 9



$M_1 = 10{,}500$ Nmm

$M_2 = 60 \times 150 = 9{,}000$ Nmm

$M_3 = 150 \times 90 = 13{,}500$ Nmm

→ find a) net couple acting on the plate and b) find the eq. force across OP.

as net moment is clockwise the force couple at OP will also be cw, & so the dir of F at O and A

a) Net Couple $= M_1 + M_2 - M_3$

$= 10{,}500 + 9000 - 13{,}500$

$= 6{,}000$ Nmm

$= 6$ Nm

b) $\frac{d}{240} = \sin 30° \Rightarrow d = 120$ mm

$M = Fd \Rightarrow F = \frac{M}{d} = \frac{600\cancel{0}}{12\cancel{0}} = 50\text{N}$

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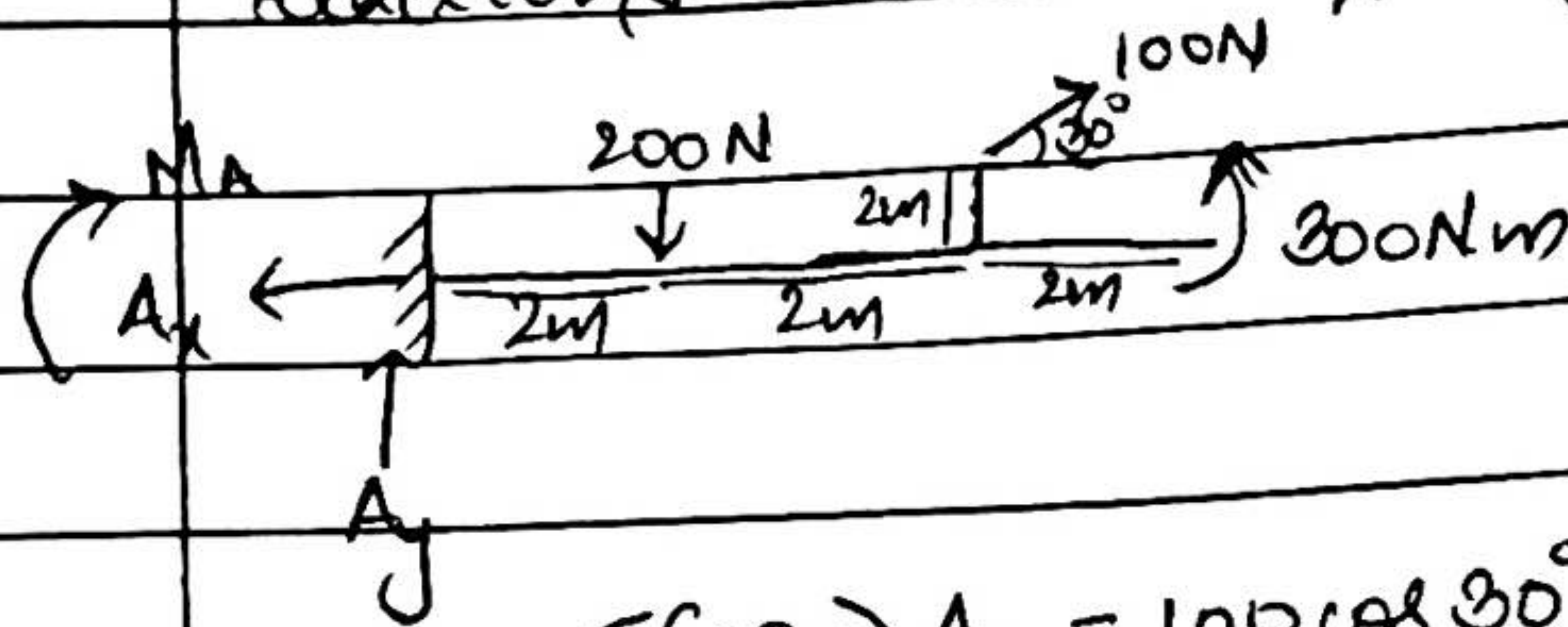
Ques. (2) The object has a fixed support at A and is subjected to two forces and a couple. What are the reactions at the support?



$\Sigma f_x = 0 \Rightarrow A_x = 100\cos 30^\circ N = 50\sqrt{3} N = 86.60 N$

$\Sigma f_y = 0 \Rightarrow A_y + 100\sin 30^\circ = 200$

$A_y = 150 N$

$M_A = 200 \times 2 - 300 - 100\sin 30^\circ \times 4 + 100\cos 30^\circ \times 2$

$= 400 - 300 - 200 + 1.732 \times 100$

$= 73.2 Nm$

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6/11/20 Ch.2

TRUSS

- Truss is a load bearing structure compressed of slender members that are fastened together by idealized hinged joints to support loads.
- It is a pin-connected framework of straight members arranged in the shape of triangles.

★ Elements of a truss

1. Chords (horizontals)
2. Verticals
3. Diagonals
4. End posts

★ Assumptions of Truss Analysis

1. Truss members are connected with frictionless pins.
2. Truss members are straight.
3. The displacement of the truss is small.
4. Loads are applied at the joints.

- Necessary condition of statically determinacy ⇒

members ← $m + r = 2j$ → joints

- If $m + r > 2j$ (statically indeterminate truss) support reactions
- If $m + r < 2j$ (unstable truss)

★ in $m + r > 2j$ ⇒ if m is more then internally indeterminate

if r is more then externally indeterminate

★ ideal condⁿ for r is 1 roller & 1 hinge ($\therefore r = 1 + 2 = 3$)

• Axial force → the force acting along the axis of the truss is called as Axial force (tensile / compression)

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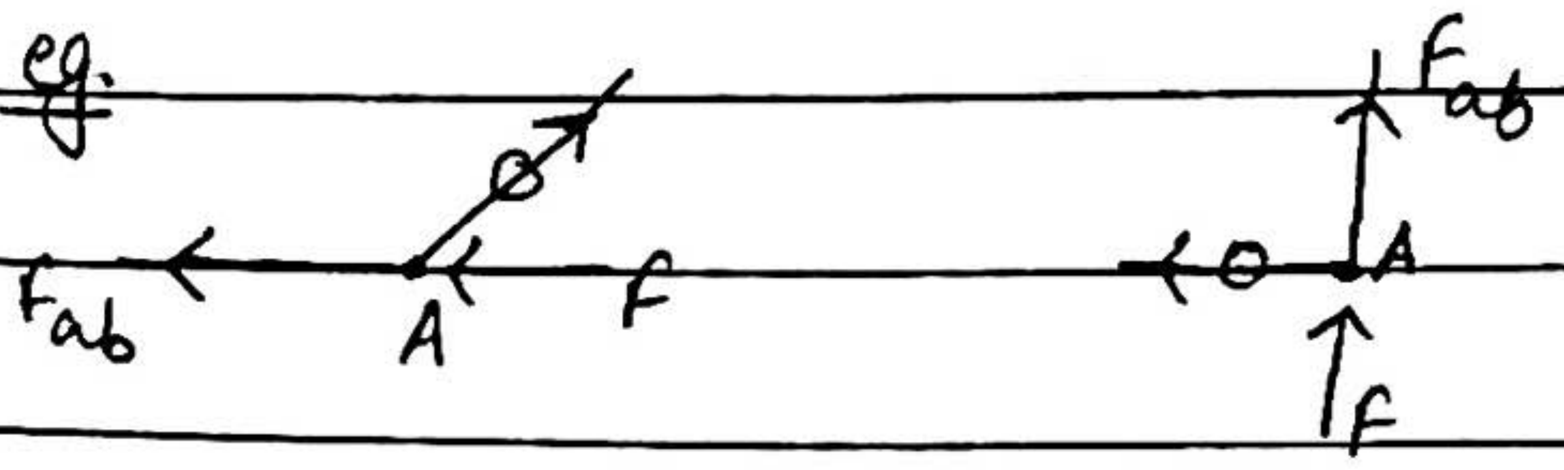
Zero Force Members

They do not carry any force and are provided only for stability.

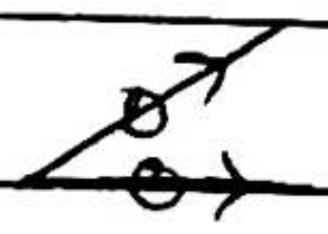
Rule 1: On a loaded joint, barring the members on which any external force is applied, all others is zero force member. (2 members)

eg.



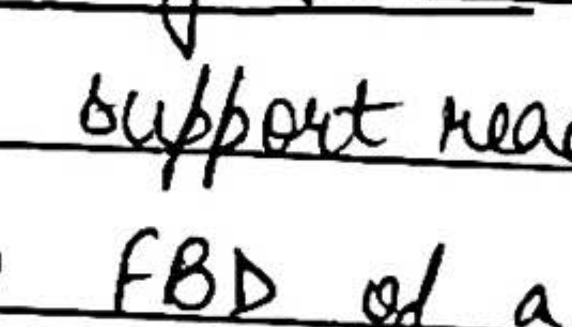
Rule 2: On an unloaded joint, the members are zero force members if they are not collinear. (only 2 members)

eg



Rule 3: On an unloaded joint, if there are two collinear members then the other members on that joint are zero force members. (3 members)

eg



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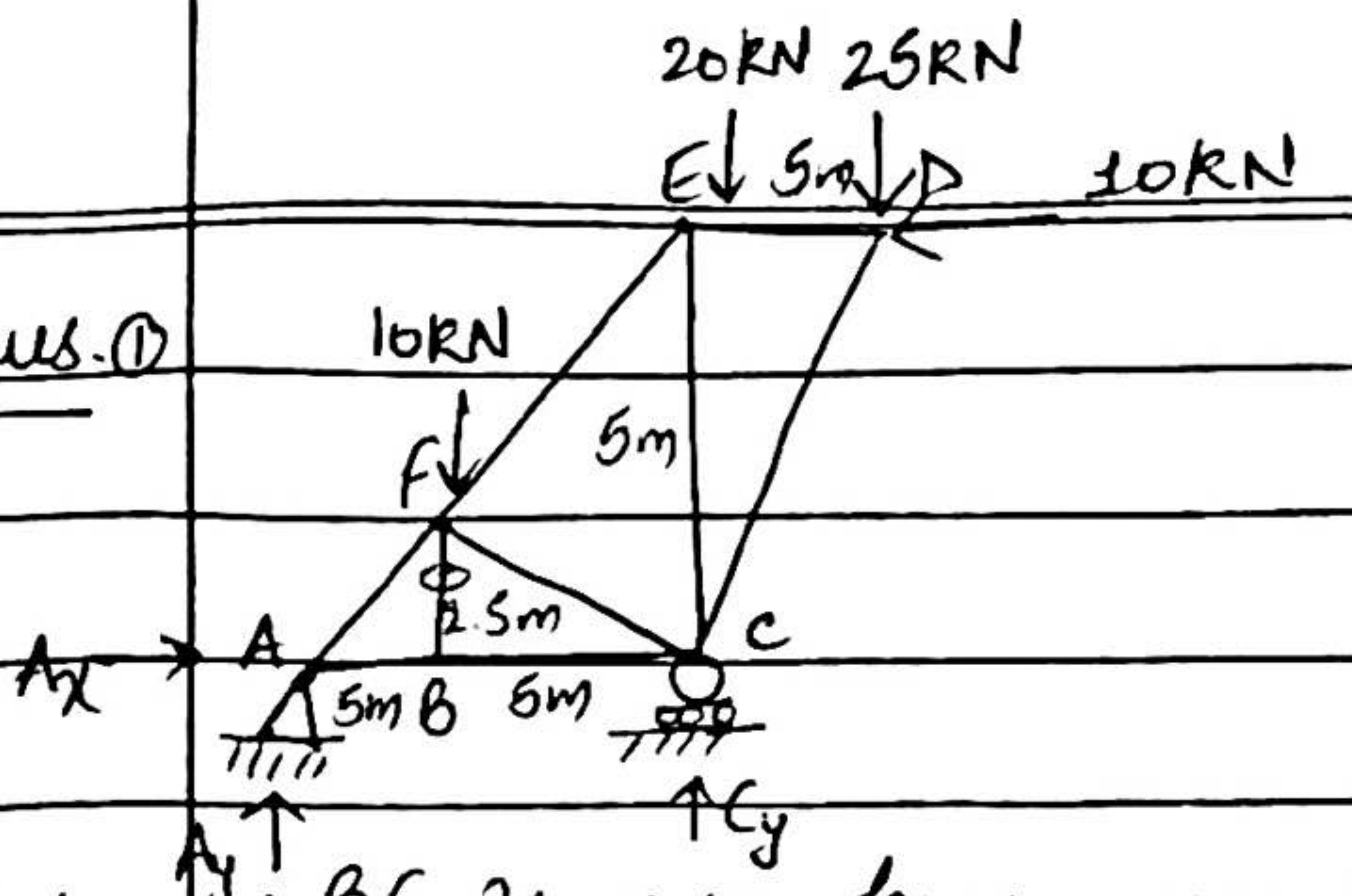
Method of Joints

① Find support reactions.

② Draw FBD of a joint which has not more than 2 unknowns and apply equations of equilibrium.

③ Consider FBD of next joint and repeat the same, until all the forces in members are obtained.

Ques. ①



Analyse the truss using method of joints.

Sol. 1. BF is zero force member by Rule 3 at joint B

2. $\Sigma F_x = 0 \Rightarrow A_x = 10kN$

$\Sigma F_y = 0 \Rightarrow A_y + C_y = 10+20+25 = 55 kN$

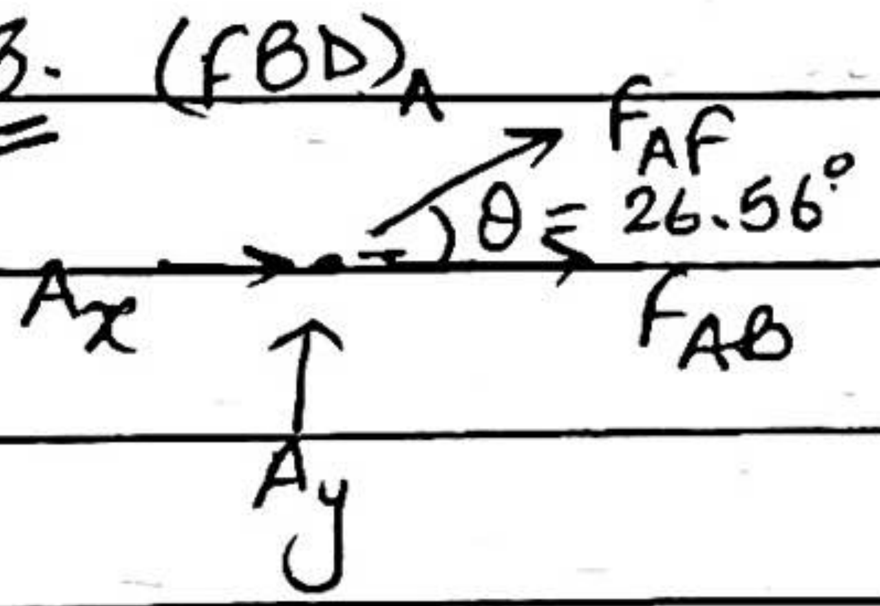
$\Sigma M_A = 0 \Rightarrow 10\times5 + 20\times10 + 25\times15 - 10\times5 - C_y\times10 = 0$

$\Rightarrow \cancel{50} + 200 + 375 - \cancel{50} = 10C_y$

$\therefore C_y = 57.5 kN$

$A_y = -2.5 kN$

3. $(FBD)_A$



$\Sigma F_y = 0 \Rightarrow A_y + F_{AF}\sin\theta = 0$

$\Rightarrow F_{AF}\times0.45 = 2.5$

$\Rightarrow F_{AF} = 5.56 kN$

$\Sigma F_x = 0 \Rightarrow A_x + F_{AB} + F_{AF}\cos\theta = 0$

$\Rightarrow 10 + F_{AB} + 5.56\times0.9 = 0$

$\Rightarrow F_{AB} = -15 kN$

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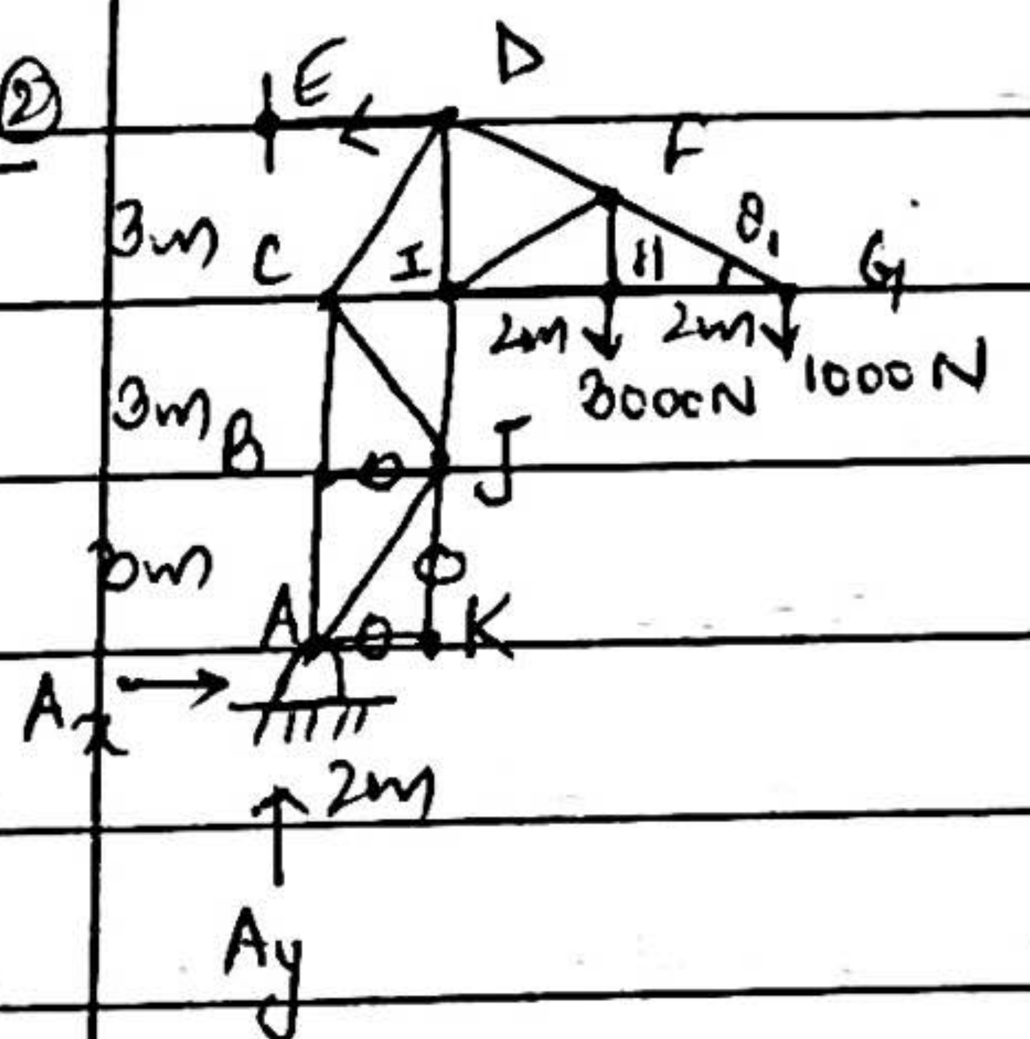
Ques. ②



Zero Members → BJ, AK, KJ

at A → (eq.)

$A_x = F_{DE}$

$A_y = 3000 + 1000 = 4000$

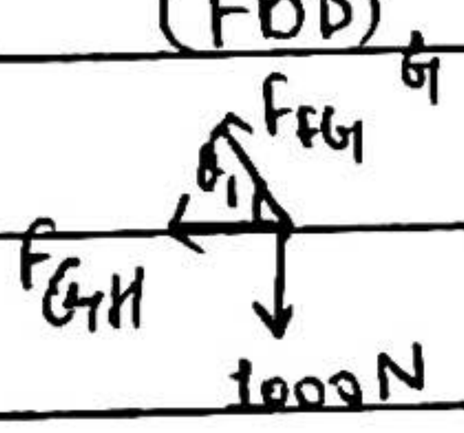
$\Sigma M_A = 0$

$\Rightarrow 3000\times4 + 1000\times6 - F_{DE}\times9 = 0$

$\Rightarrow F_{DE} = 2000N$

$\theta_1 = 36.87°$

$(FBD)_G$



$F_{FG}\sin\theta_1 = 1000$

$F_{FG} = 1666.67N$ (T)

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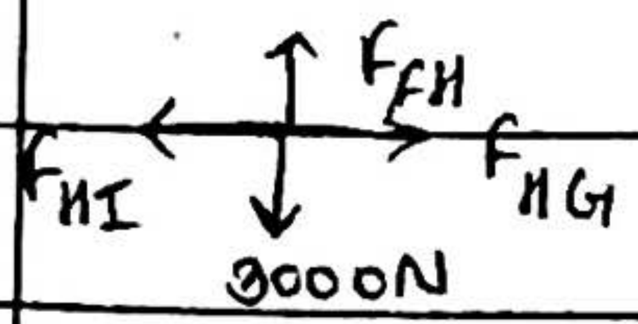
$F_{GH} + F_{FG}\cos\theta_1 = 0$

$F_{GH} = -1333.33N$ (C)

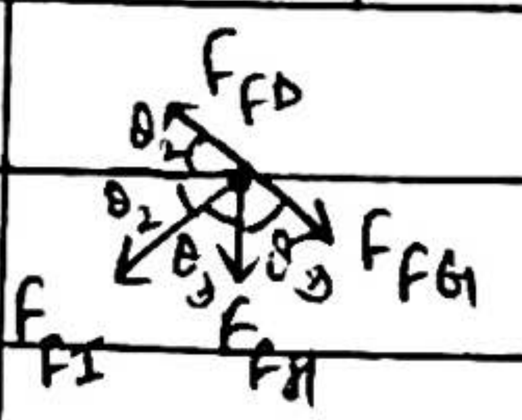
$(FBD)_H$



$F_{FH} = 3000N$

$F_{GH} = F_{HI} = -1333.33N$

$(FBD)_F$



$\theta_3 = 53.13^\circ$

$\theta_2 = 36.87^\circ$

$\Sigma F_x = 0 \Rightarrow F_{FG}\sin 53.13^\circ - F_{FI}\cos 36.87^\circ - F_{FD}\cos 36.87^\circ = 0$

$\Rightarrow 1333.34 - 0.8F_{FI} - 0.8F_{FD} = 0$

$\Rightarrow 1666.67 = F_{FI} + F_{FD}$

$\Sigma F_y = 0 \Rightarrow F_{FD}\sin 36.87^\circ - F_{FI}\sin 36.87^\circ - F_{FG}\cos 53.13^\circ - 3000 = 0$

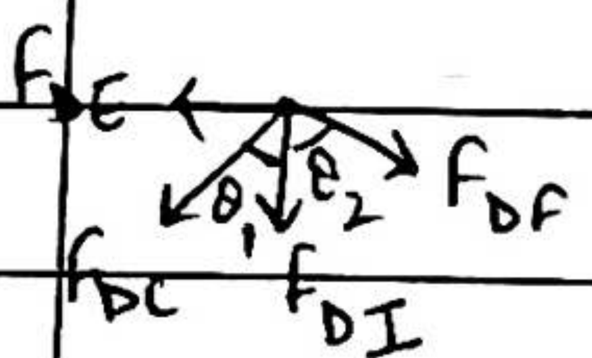
$\Rightarrow 0.6F_{FD} - 0.6F_{FI} - 1000 - 3000 = 0$

$\Rightarrow F_{FD} - F_{FI} = 6666.67$

$\therefore F_{FD} = 4166.67N$ (T)

$F_{FI} = -2500N$ (C)

$(FBD)_D$



$\theta_1 = 33.82^\circ$

$\theta_2 = 53.06^\circ$

$\Sigma F_x = 0 \Rightarrow F_{DF}\sin 53.06^\circ - F_{DE} - F_{DC}\sin 33.82^\circ = 0$

$\Rightarrow 3333.34 - 2000 - 0.56F_{DC} = 0$

$\Rightarrow F_{DC} = \cancel{2381N}$ 2403.07N (T)

$\Sigma F_y = 0 \Rightarrow F_{DI} + F_{DF}\cos 53.06^\circ + F_{DC}\cos 33.82^\circ = 0$

$\Rightarrow F_{DI} + 2500 + 1976.23 = 0$

$\Rightarrow F_{DI} = \cancel{-4476.23N}$ -4500N (C)

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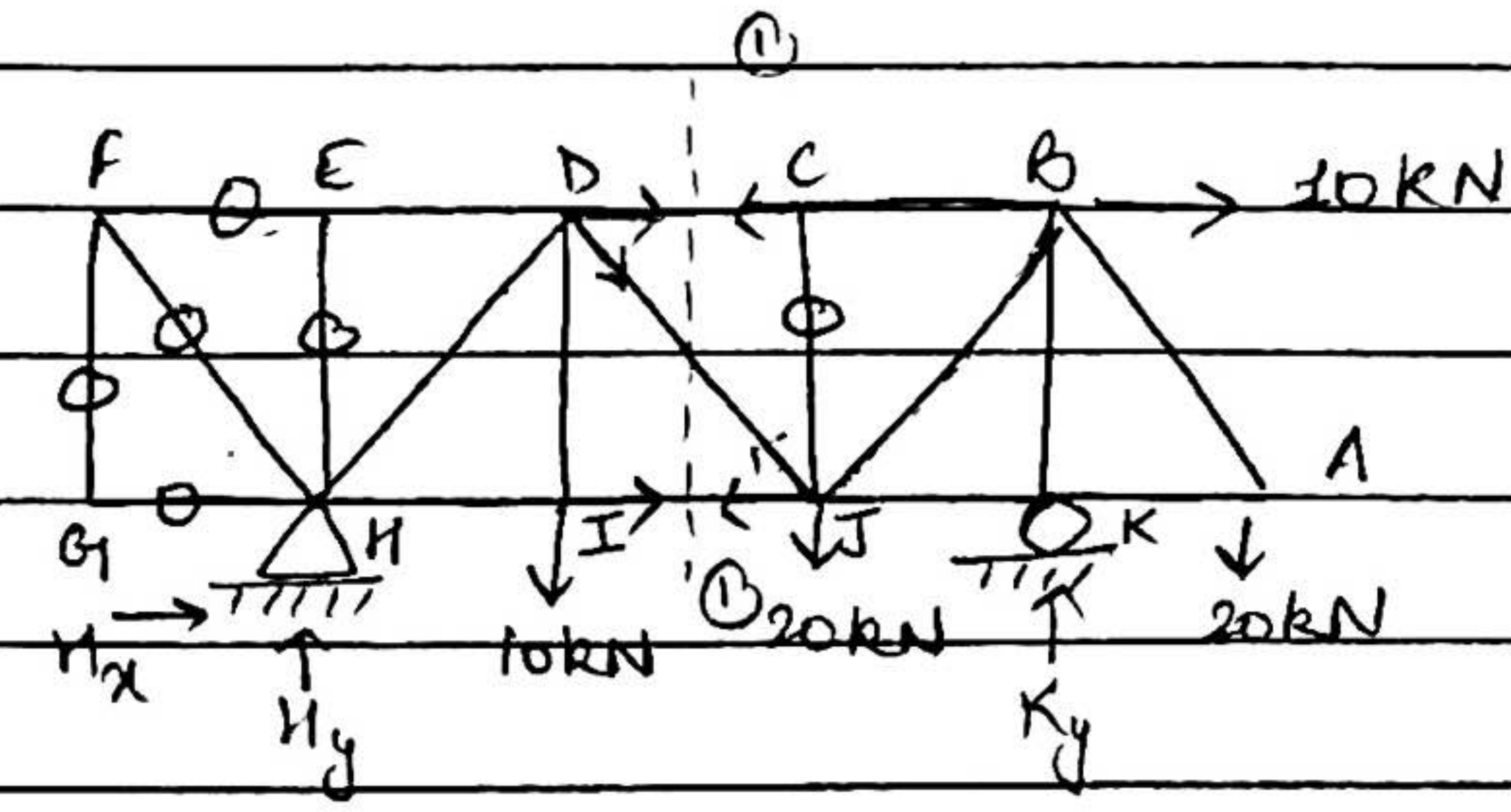
Method of Sections

1. Determine the support reactions.
2. Identify the members of interest to draw a section through them, this will divide truss in two parts.
3. Consider equilibrium of either left or right side of the section to solve for the unknown forces.

Ques. (1)



H: 3m each
V: 4m each

Find F_{CD}, F_{BJ} and F_{IJ}

Total eq. of truss

$$H_x = -10\,kN$$

$$H_y + K_y = 50\,kN$$

$$\Sigma M_H = 0 \Rightarrow 10\times3 + 20\times6 + 20\times12 + 10\times4 - K_y\times9 = 0$$

$$\Rightarrow \frac{30+120+240+40}{9} = K_y$$

$$\Rightarrow \frac{430}{9} = K_y$$

$$K_y = 47.78\,kN$$

$$\therefore H_y = 2.22\,kN$$

eq. of ~~left~~ right side of section 1-1 →

$$\Sigma M_J = 0 \Rightarrow 10\times4 + 20\times6 - 47.78\times3 - F_{DC}\times4 = 0$$

$$\Rightarrow \frac{40+120-143.34}{4} = F_{DC}$$

$$F_{DC} = 4.165\,kN$$

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$\Sigma M_D = 0 \Rightarrow F_{J1} \times 4 + 20 \times 8 - 47.78 \times 6 + 20 \times 9 = 0$

$\Rightarrow F_{J1} = \dfrac{286.68 - 60 - 180}{4} = 11.67\ kN$

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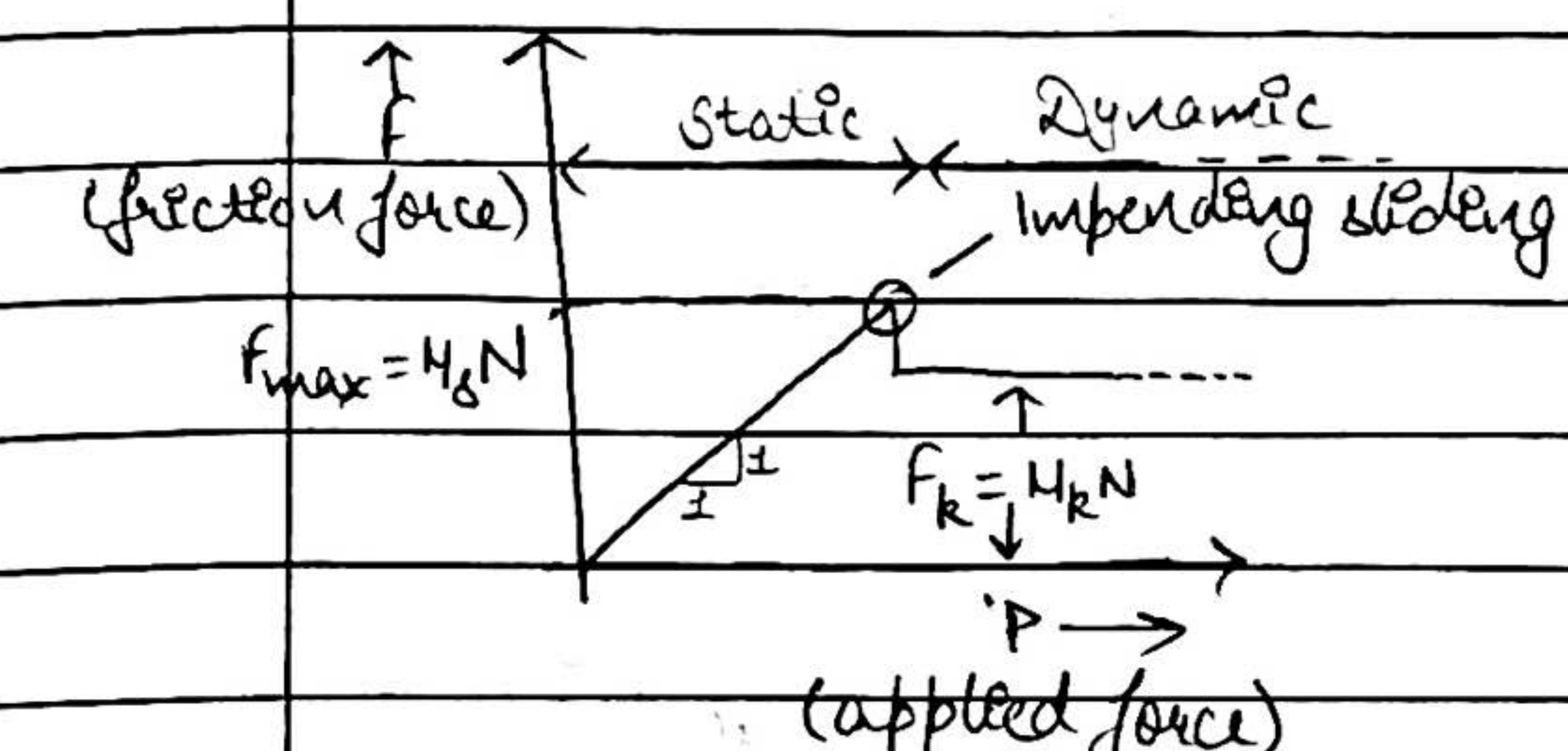
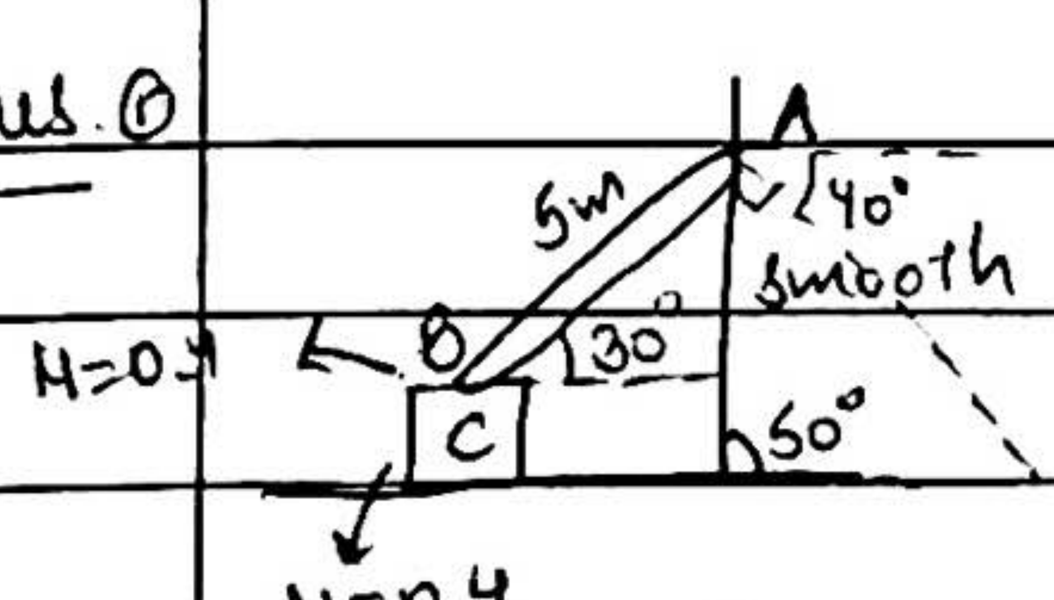
FRICTION

- It is a force that resists the movement of two contacting surfaces that slide relative to one another.
- Static case : $F \leq F_{max} = \mu_s N$
- Impending sliding : $F = F_{max} = \mu_s N$
- Dynamic case : $F = F_k = \mu_k N$

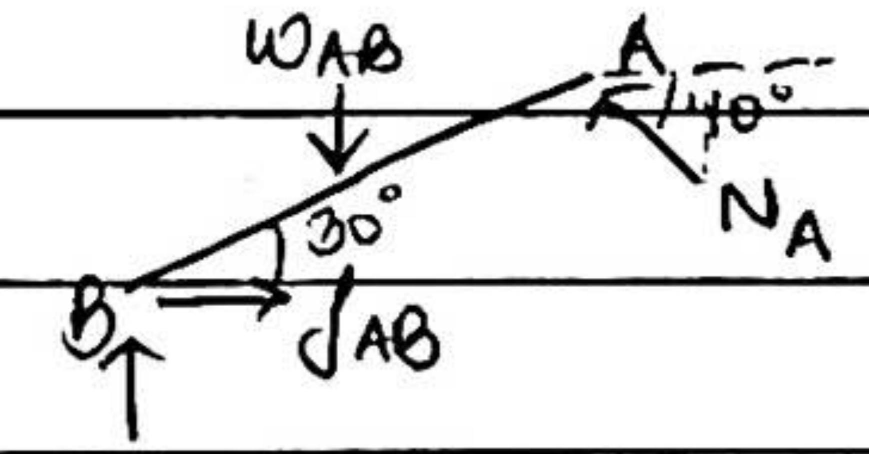


Ques. ① Is the system in static equilibrium?



$W_{AB} = 500 \times 9.81 = 4905 N$

$(FBD)_{AB}$



$\Sigma M_B = 0 \Rightarrow -N_A \cos 40° \times 5 \sin 30°$
$+ N_A \sin 40° \times 5 \cos 30°$
$+ 4905 \times 2.5 \cos 30° = 0$

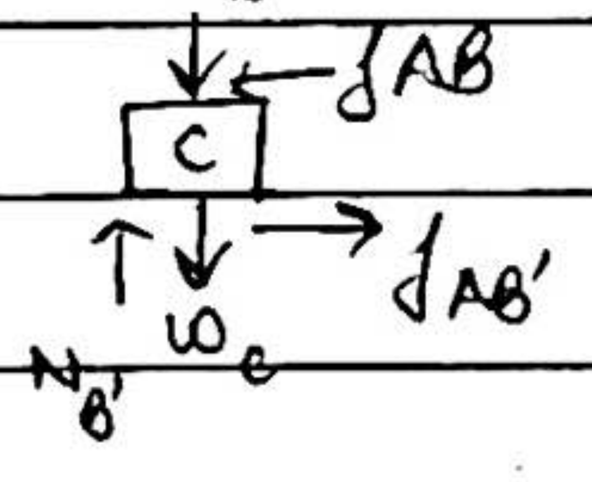
$N_A = 2260.24N$

$N_B = N_A \sin 40° + 4905$

$N_B = 3452.15N$

$N_A \cos 40° = f_{AB} = 1731.44N$

$(FBD)_C$



$W_c = 300 \times 9.81 = 2943 N$

$f_{AB} = f_{AB'} = 1731.44N$

$N_{B'} = N_B + 2943 = 6395.15N$

for static eq. $\rightarrow f_{AB} = \mu N_B = 0.4 \times 3452.15$
$= 1380.86 N$

$f_{AB'} = \mu N_{B'} = 2558.06N$

$\therefore$ system is not in static equilibrium.

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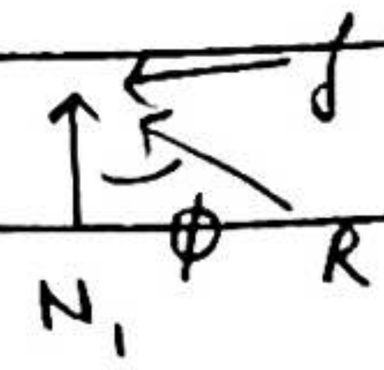
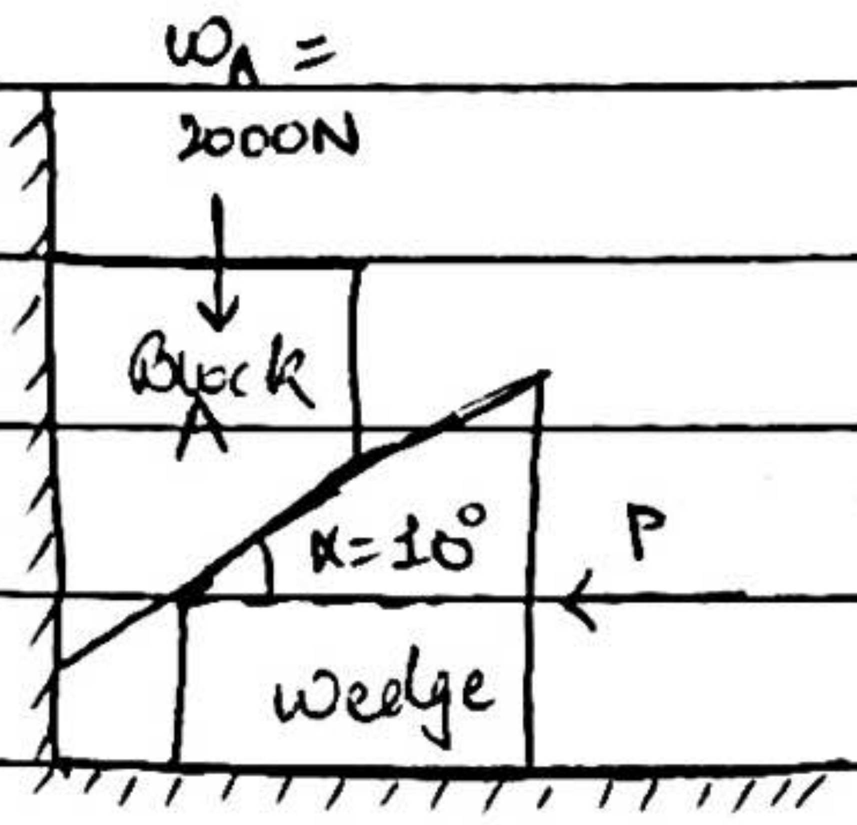
Friction - Wedge

Angle of static friction is the angle between the resultant of normal force and friction, and normal.



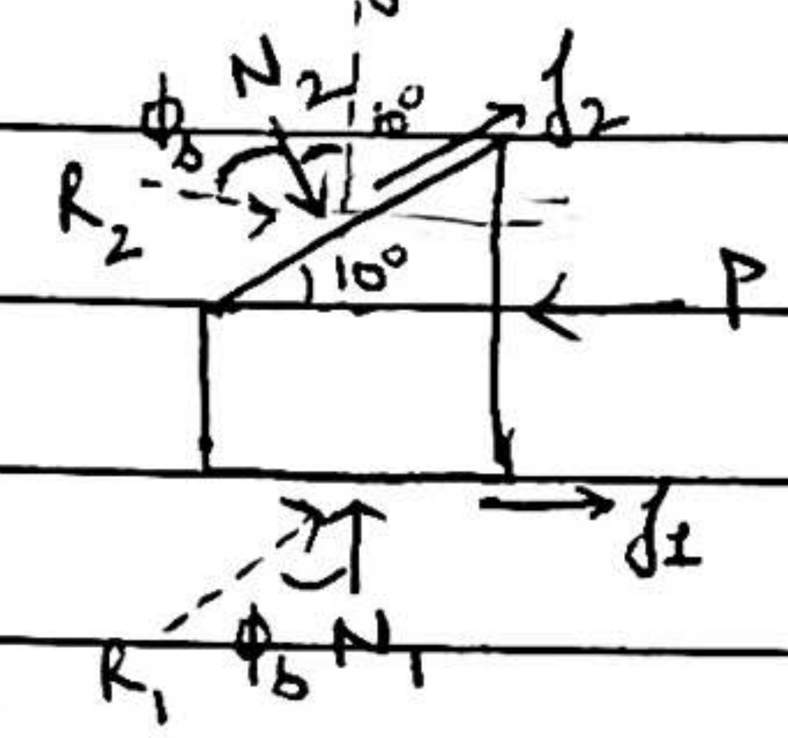
Ques. ①



$\mu_s = 0.25$

Find min. value of force P to raise the Block A.

(FBD) wedge



$\tan\phi_s = \frac{f_1}{N_1} = \frac{\mu_s \cancel{N_1}}{\cancel{N_1}} = \mu_s$

$\phi_s = \tan^{-1}(\mu_s)$

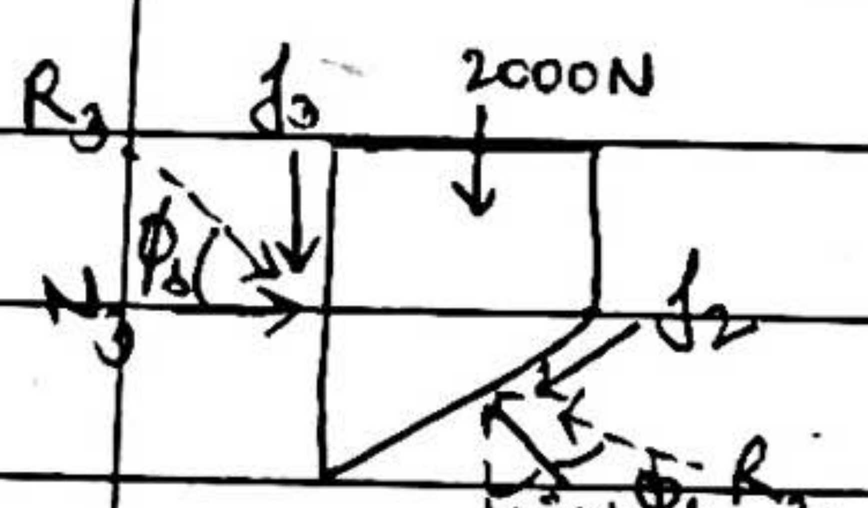
$\phi_s = 14.04°$

$\Sigma F_x = 0 \Rightarrow R_1 \sin\phi_s - P + R_2 \sin(\phi_s + 10) =$

$\Sigma F_y = 0 \Rightarrow R_1 \cos\phi_s - R_2 \cos(\phi_s + 10) = 0$

$R_1 = 2324.05\text{N}$

(FBD) Block A



$\Sigma F_x = 0 \Rightarrow R_3 \cos\phi_s - R_2 \sin(\phi_s + 10) = 0$

$\Sigma F_y = 0 \Rightarrow R_2 \cos(\phi_s + 10) - R_3 \sin\phi_s - 2000 = 0$

$R_2 = 2468.76\text{N}$

$R_3 = 1041.67\text{N}$

$\therefore\ \underline{\underline{P = 1563.48\text{N}}}$

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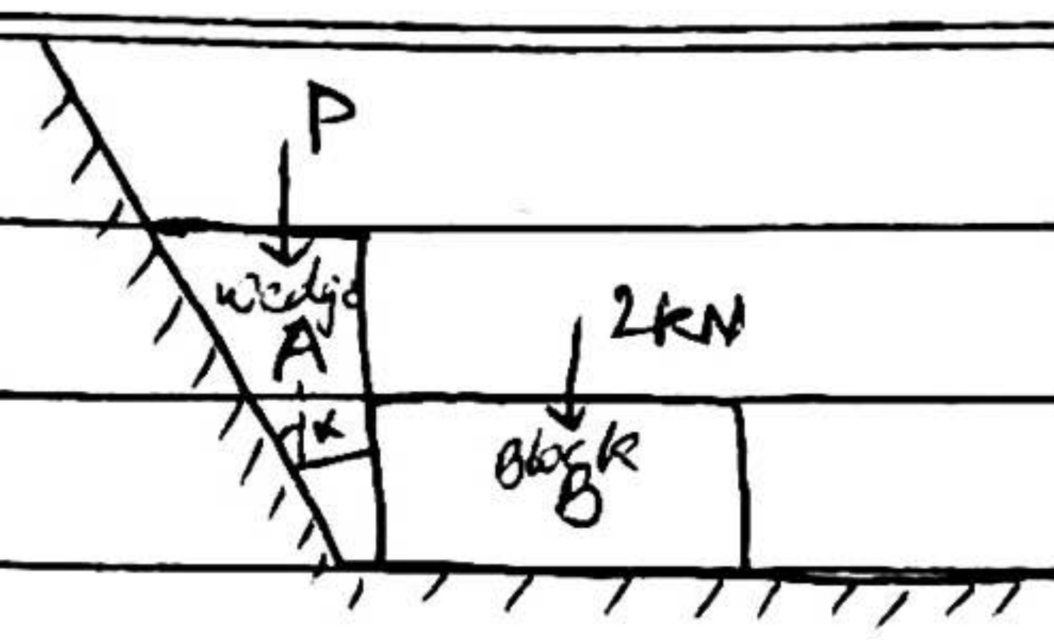
Ques.-(2)

P=1774.14N

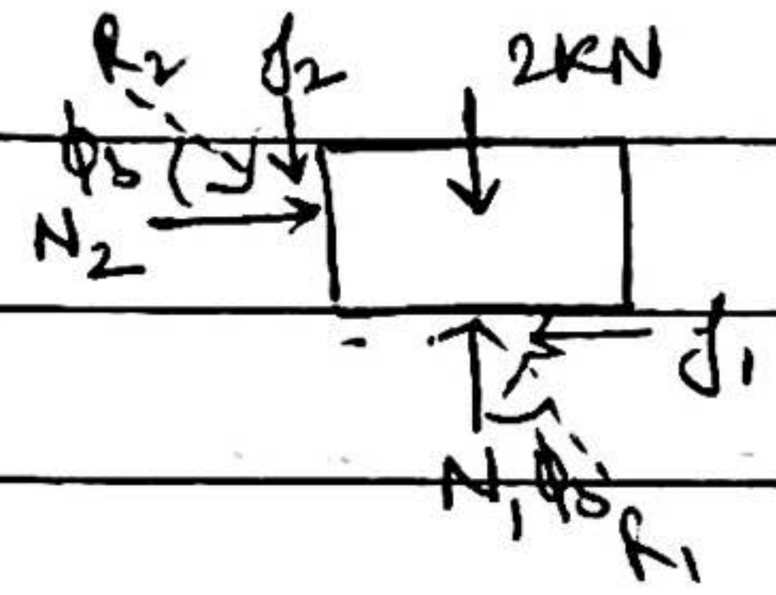


$\alpha = 21^\circ$

$\phi_s = 21^\circ$

find P to impend the motion of Block 'B'.

(FBD) Block B



$\Sigma F_x = 0 \Rightarrow -R_1 \sin\phi_s + R_2 \cos\phi_s = 0$

$\frac{R_2}{R_1} = \tan\phi_s = 0.38$

$\Sigma F_y = 0 \Rightarrow 2 + R_2 \sin\phi_s - R_1 \cos\phi_s = 0$

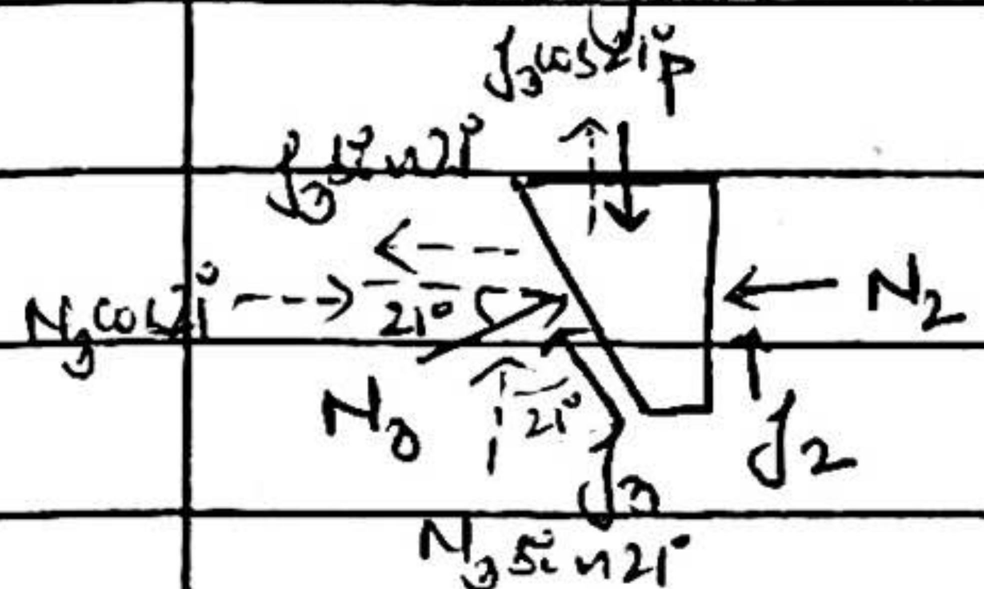
$\Rightarrow 2 + 0.1368 R_1 - 0.93 R_1 = 0$

$\Rightarrow 2 = 0.8 R_1$

$R_1 = 2.5 kN$

$R_2 = 0.95 kN$

(FBD) wedge A



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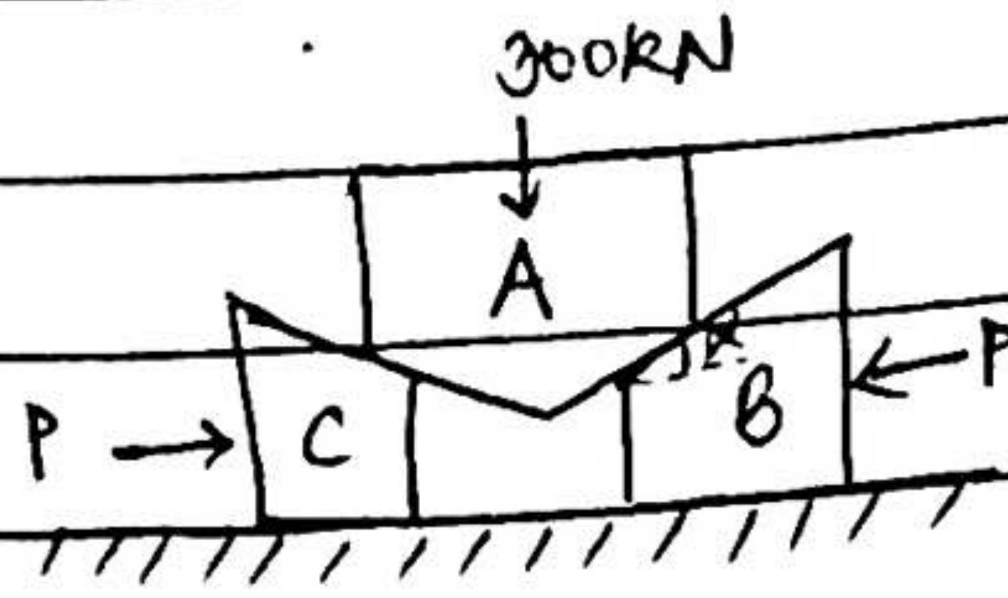
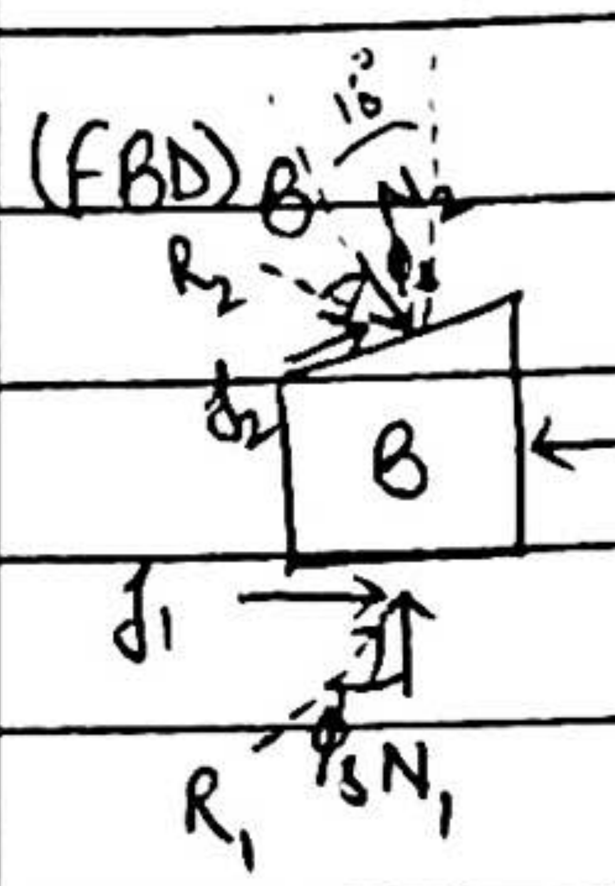
Qus. (3)



$\mu_s = 0.20$
$\alpha = 18^\circ$

$\phi_s = \tan^{-1}(\mu_s)$
$= \tan^{-1}(0.2)$
$= 11.31^\circ$

(FBD) B

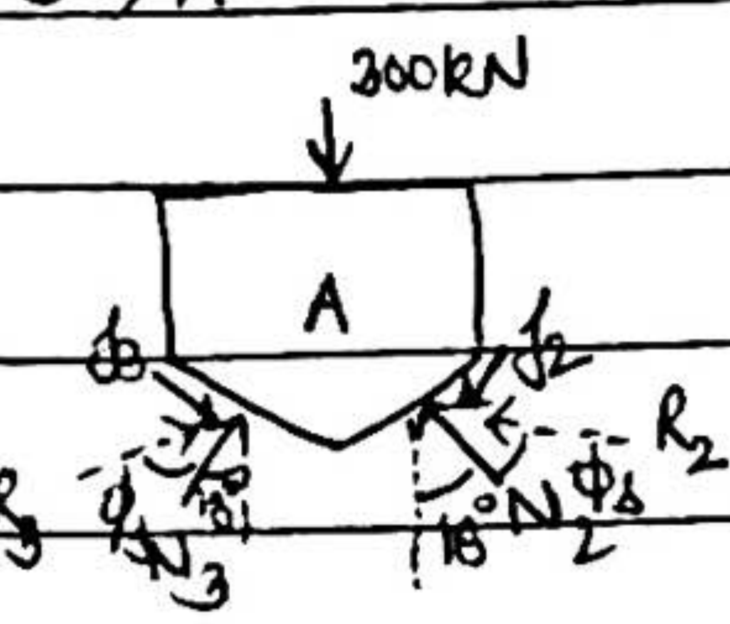


$\Sigma F_x = 0 \Rightarrow R_1 \sin\phi_s + R_2 \sin(\phi_s + 18^\circ) = P$

$\Sigma F_y = 0 \Rightarrow R_2 \cos(\phi_s + 18^\circ) = R_1 \cos\phi_s$

$R_1 = 153.59\ kN$

(FBD) A



$\Sigma F_x = 0 \Rightarrow R_2 \sin(\phi_s + 18^\circ) = R_3 \sin(\phi_s + 18^\circ)$

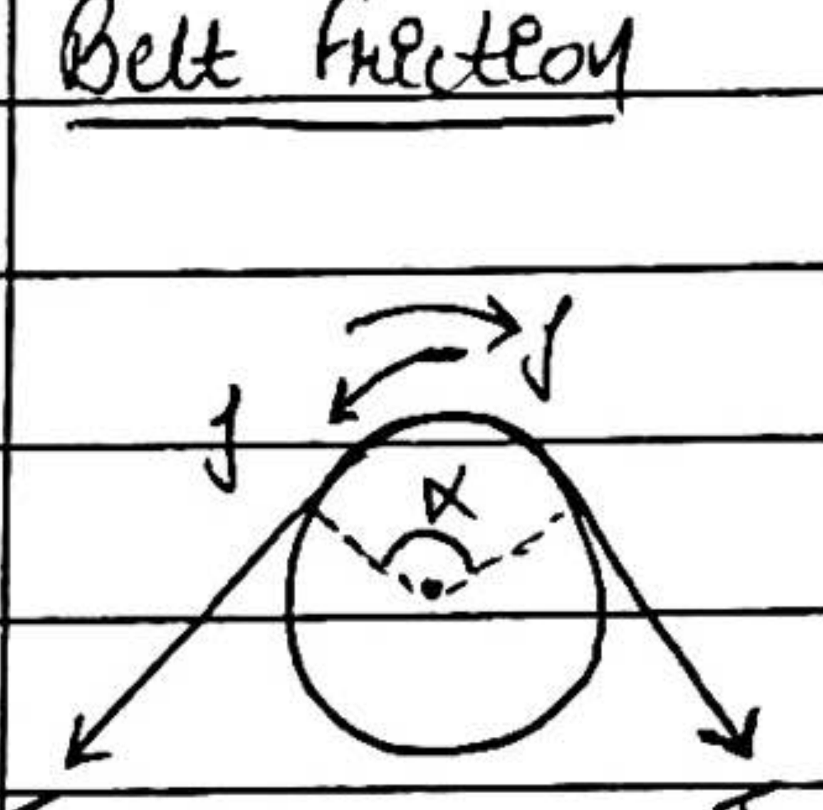
$R_3 = R_2$

$\Sigma F_y = 0 \Rightarrow 300 = R_3 \cos(\phi_s + 18^\circ) + R_2 \cos(\phi_s + 18^\circ)$

$\Rightarrow \dfrac{300}{2 \times 0.871} = R_2 = R_3 = 172.02\ kN$

$\therefore\ \underline{\underline{P = 114.27\ kN}}$

Belt friction



let, $T_1 > T_2$

$\therefore\ \boxed{T_1 = T_2 e^{\mu_s \alpha}}$

'α' is always in radians and is called wrap angle.

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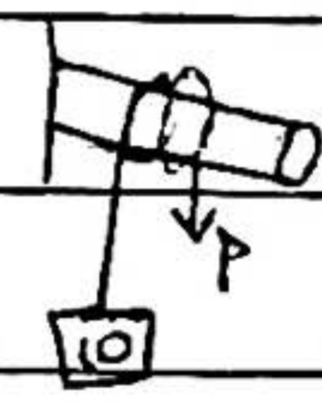
Ques ① $\mu = 0.2$. The rope is making $1\frac{1}{2}$ turns on the circular peg. Find the range of values of P for which system is in equilibrium. $W = 10kg$.

Sol. $\mu_s = 0.2$

$\theta = 2\pi + \pi = 3\pi$



largest value of $P \Rightarrow$

$P_l = We^{\mu_s \theta}$

$P_l = 10 \times 9.81\, e^{(0.2)(3\pi)}$

$P_l = 98.1\, e^{(0.2)(3\pi)}$

smallest value of $P \Rightarrow$

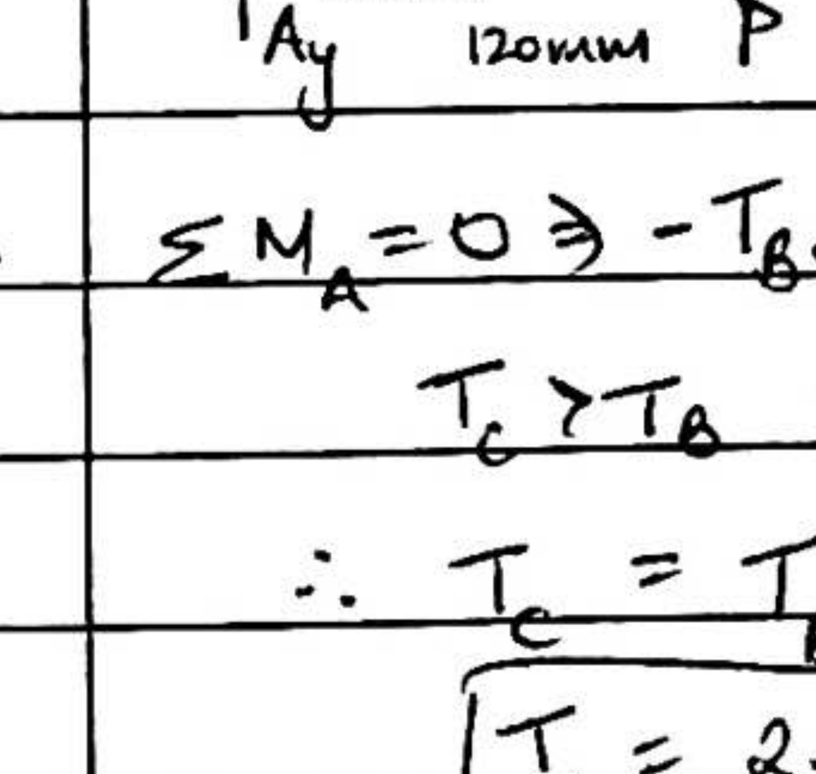
$P_s e^{\mu_s \theta} = W$

$P_s = \dfrac{98.1}{e^{(0.2)(3\pi)}}$

$\therefore \dfrac{98.1\, \cancel{e^{0.2(3\pi)}}}{e^{(0.2)(3\pi)}} \leq P \leq 98.1 e^{(0.2)(3\pi)}$

$14.9\,N \leq P \leq 646.49\,N$

Ques. ②



Find value of P.

$\mu = 0.2$

$\therefore \theta = 240°$ (360-120)

Sol. $\Sigma M_A = 0 \Rightarrow -T_B \sin 60° \times 60 - T_C \sin 60° \times 180 + P \times 240 = 0$ —①

$T_C > T_B$ (as motion is in counter cw $\therefore$ to stop the motion T_C is more)

$\therefore T_C = T_B e^{(0.2)(\frac{4\pi}{3})}$

$\boxed{T_C = 2.31 T_B}$ —②

$(T_C - T_B) \times r =$ Braking Torque

$(T_C - T_B) \times \cancel{120} = \cancel{1200}0$

$\boxed{T_C - T_B = 100}$ —③

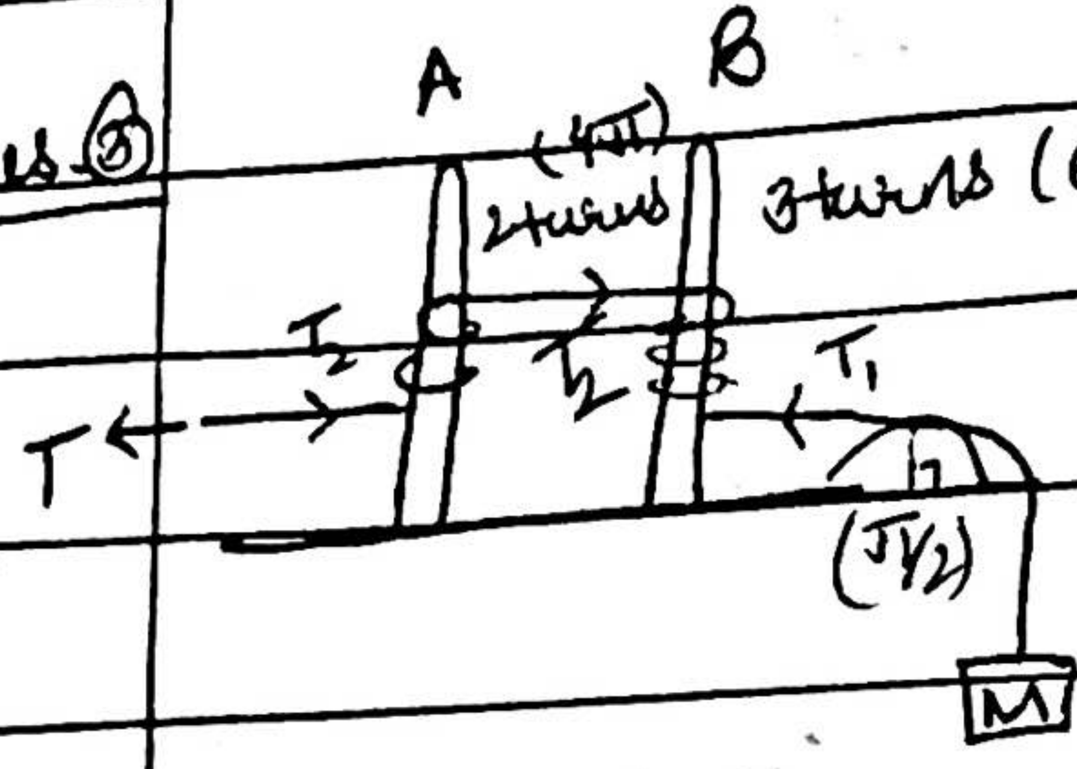
$T_B = 76.33\,N$

$T_C = 176.33\,N$

$\therefore P = \dfrac{3961.53 + 27404.58}{240}$

$P = 130.9\,N$

Ques 8



$\mu = 0.3$

$M = 500$ kg

find T to maintain equilibrium.

$M = T_1 e^{\mu(\pi/2)}$ $(M > T)$

$T_1 = T_2 e^{\mu(6\pi)}$

$T_2 = T e^{\mu(4\pi)}$

$M = T e^{(0.3)(\pi/2 + 6\pi + 4\pi)}$

$500 \times 9.81 = T e^{(0.3)(21\pi/2)}$

$\underline{\underline{T = 0.248\,N}}$

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23/12/20 Ch 4

Properties of Plane Surfaces

* Physical Properties

① Centroid ③ Centre of gravity

② Centre of Mass ④ Second moment area (moment of inertia)

Area × (distance)^2

Centroid → geometric centre of an area

* Centroid of Shaded Area

$$\overline{X} = \frac{\Sigma \bar{x} A}{\Sigma A}, \quad \overline{Y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

7/1/21

* Parallel-axis Theorem

$$I_x = \overline{I}_x + A\bar{y}^2$$

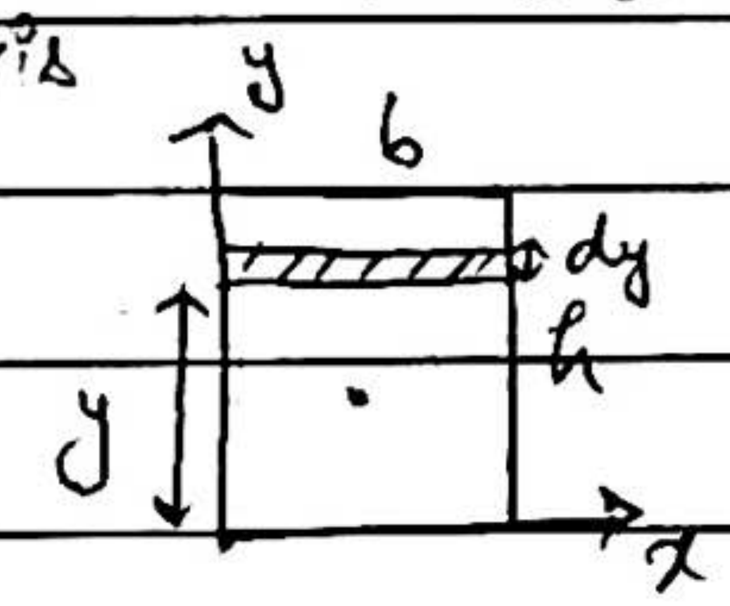
I_x → second moment area about axis

$\overline{I}_x$ → second moment area about centroid

$$I_y = \overline{I}_y + A\bar{x}^2$$

$\bar{x}$ → distance from axis to centroid

eg



$$I_x = \int y^2 dA$$

$$= \int_0^h y^2 \, b \, dy$$

$$= \left. \frac{by^3}{3} \right|_0^h$$

$$= \frac{bh^3}{3}$$

$$\overline{I}_x = \frac{bh^3}{3} - bh\left(\frac{h}{2}\right)^2 = \frac{bh^3}{12}$$

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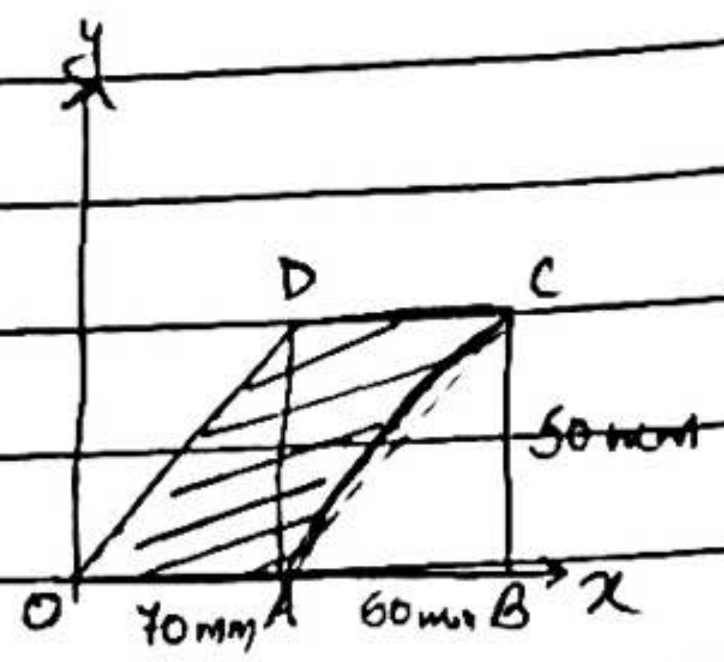
Ques Find the second moment area of the shaded region about centroidal axis.



Sol

	Shape	Area (mm^2)	x_i	y_i	$A_i x_i$	$A_i y_i$
+	OAD (△)	$\frac{1}{2} \times 70 \times 50 = 1750$	$\frac{2}{3} \times 70 = 46.67$	$\frac{1}{3} \times 50 = 16.67$	81672.5	29172.5
+	ABCD (□)	$50 \times 60 = 2600$	$70 + 50/2 = 95$	$50/2 = 25$	237500	62500
−	ADC (◠)	$\frac{\pi (50)^2}{4} = 1962.5$	$70 + 50 - \frac{4(60)}{3\pi} = 98.77$	$\frac{4(60)}{3\pi} = 21.23$	193836.125	41663.875
		~~2~~287.5			126336.375	50008.625

$\therefore \bar{x} = \frac{126336.375}{2287.5} = 54.79$ mm

$\bar{y} = \frac{50008.625}{2287.5} = 21.86$ mm

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second moment area about centroidal axis of shape (given individually) | second moment area about centroidal axis of shaded region

I_{xi} (mm^4)	$A_i (y_i - \bar{y})^2$	I_x
$\frac{70(50)^3}{36} \left(\frac{bh^3}{36}\right) = 243055.56$	47138.175	~~195917.335~~ 290193.735
$\frac{(50)^4}{12} = 520833.33 \left(\frac{b^4}{12}\right)$	24649	~~496184.33~~ 545482.33
341937.5 $\left(\frac{\pi r^4}{16} - \frac{\pi r^2}{4}\left(\frac{4r}{3\pi}\right)^2\right)$	778.91625	342716.416
		492959.649 mm^4

I_{yi} (mm^4)	$A_i (x_i - \bar{x})^2$	I_y
476388.89	~~1153862~~ ✓ ~~2542986.2~~ ✗	~~1630240.89~~ 591774.09
520833.33	~~2218610.25~~ ✗ 4042110.25 ✓	4562943.58
341937.5	3795946.78	4137884.28
		1016833.38 mm^4

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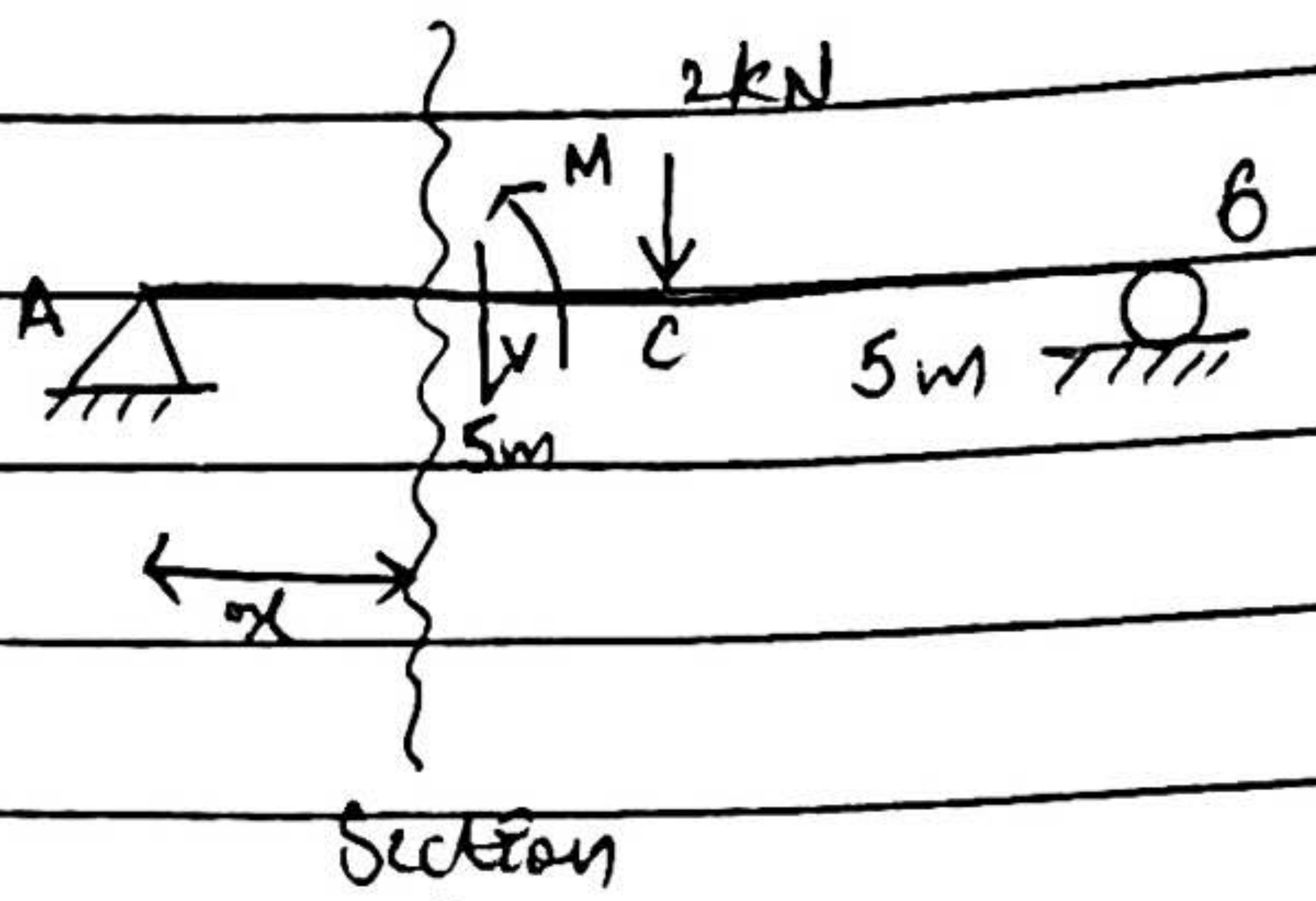
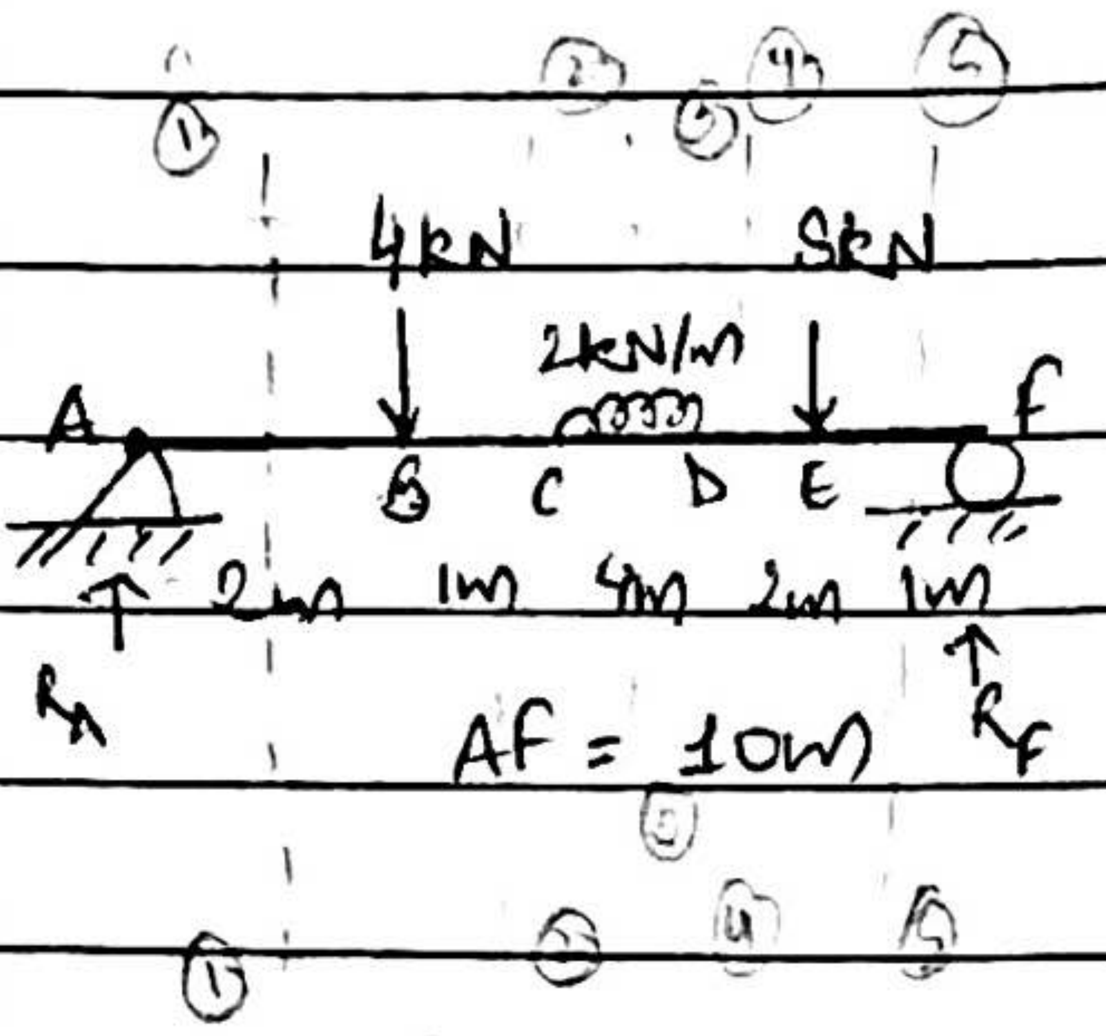
★ Shearing Force Diagram (SFD) & Bending Moment Diagram (BMD)

eg



eg



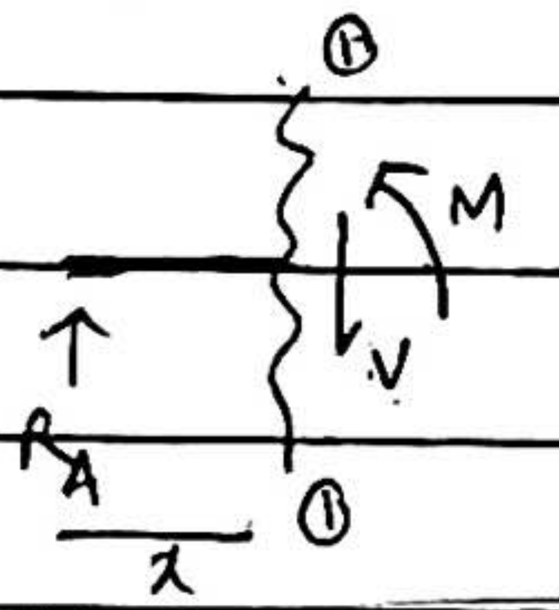
$R_A + R_F = 20\,kN$

$\Sigma M_A = 0$

$\Rightarrow 8 + 40 + 72 - 10R_F = 0$

$\Rightarrow R_F = 12\,kN$

$R_A = 8\,kN$

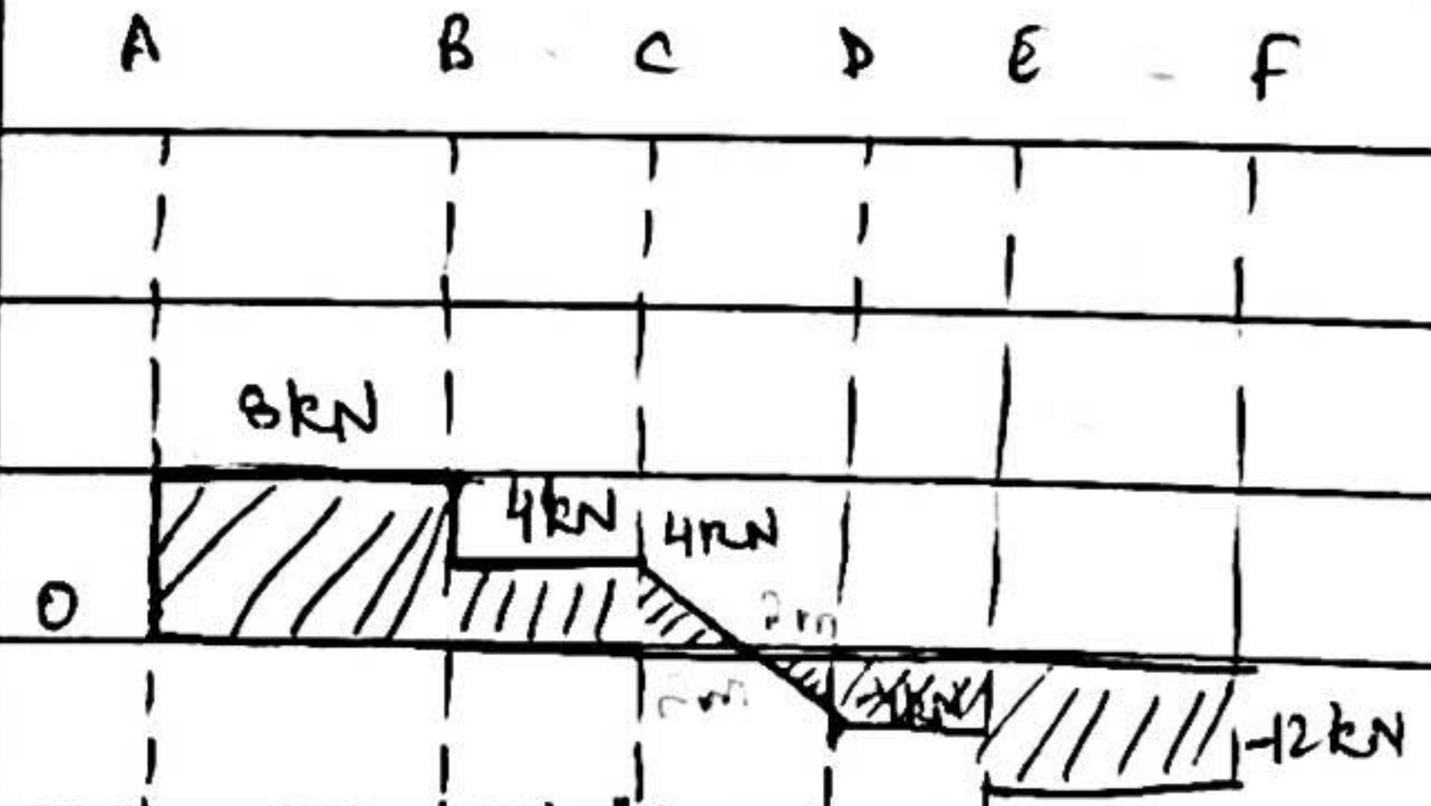


$\boxed{V = R_A = 8\,kN}$ $\quad x = 0;\ M_1 = 0$

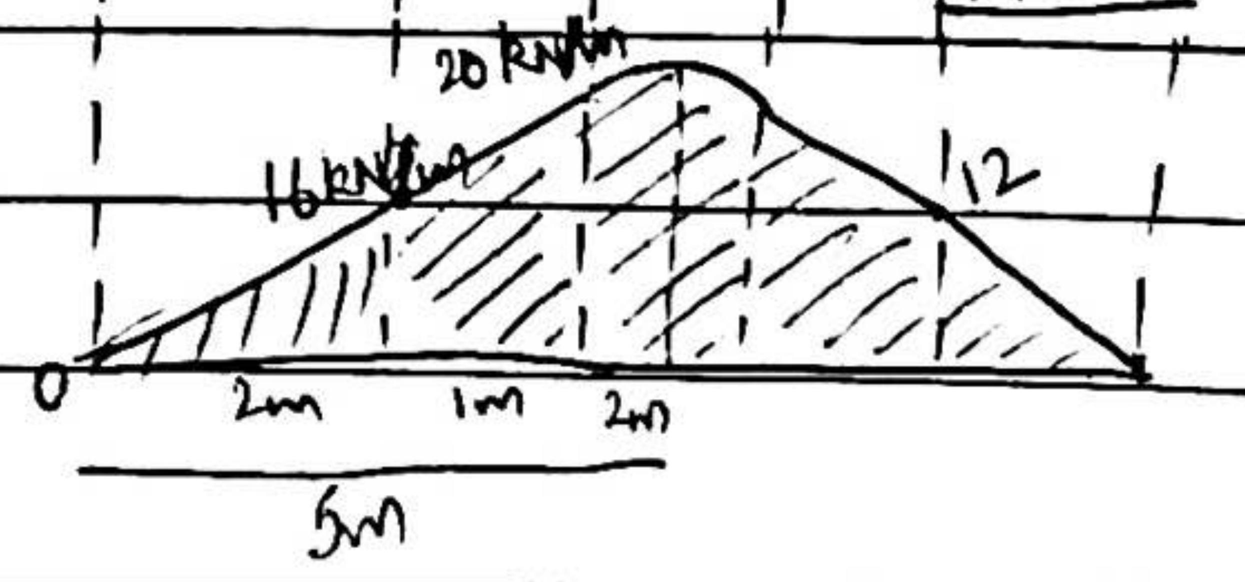
$M = R_A x$ $\quad x = 2;\ M_1 = 16$

$= 8x$

SFD



BMD



** SF = 0 ⇒ BM = max.

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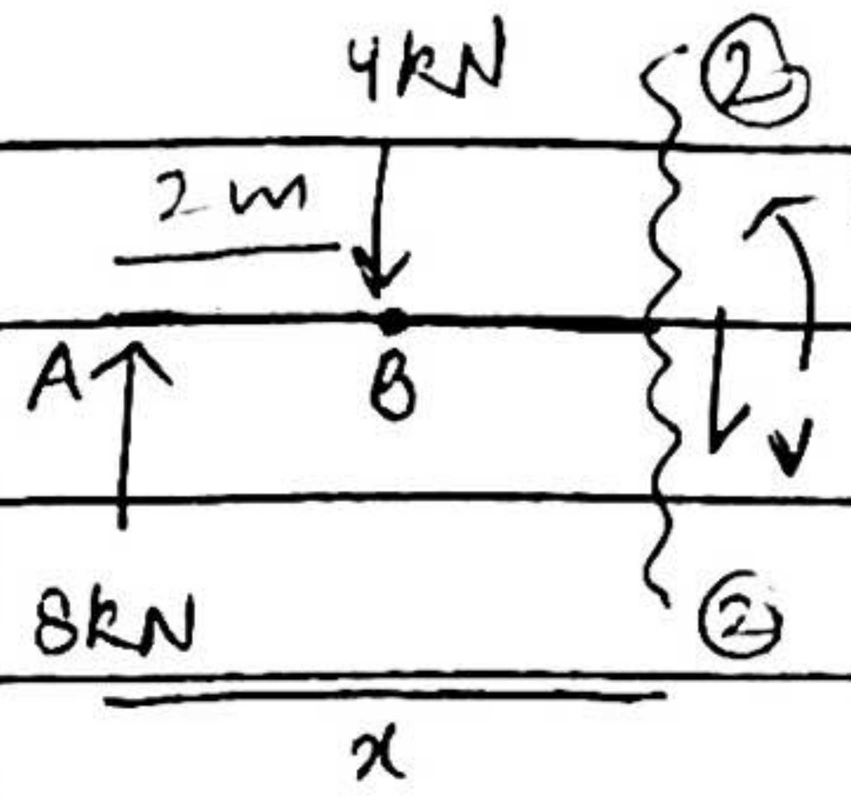


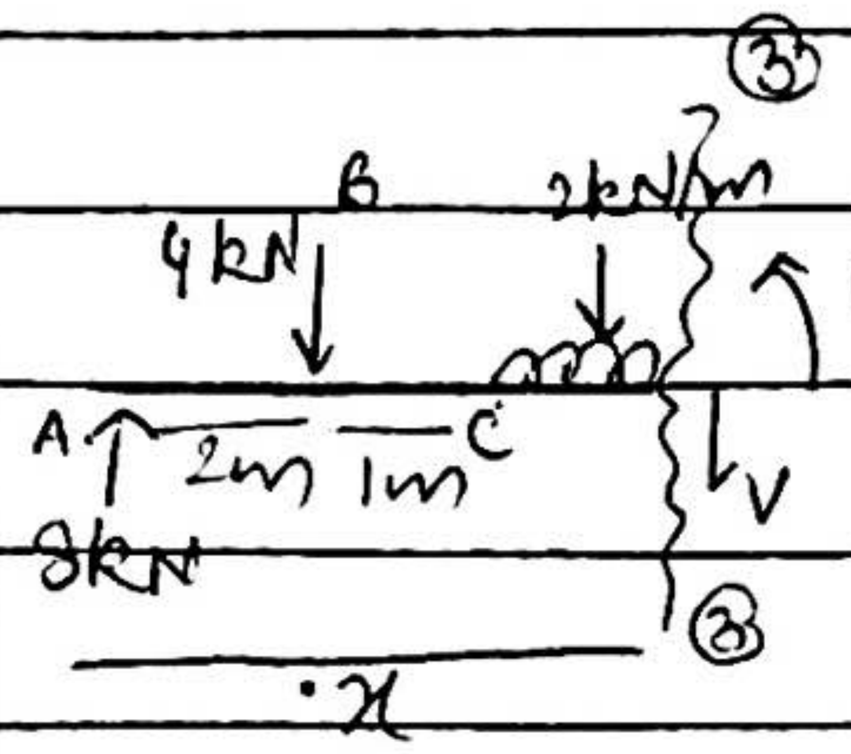
$V = 8 - 4 = 4 kN$

$M = 8x - 4(x-2)$

$= 4x + 8$

$x = 2;\ M = 16$

$x = 3;\ M = 20$



$V = 8 - 4 - 2(x-3) = 10 - 2x$

$x = 3;\ M = 4$

$x = 7;\ M = -4$

as in UDL force acts half of x-3.

$M = 8x - 4(x-2) - 2(x-3)\frac{(x-3)}{2}$

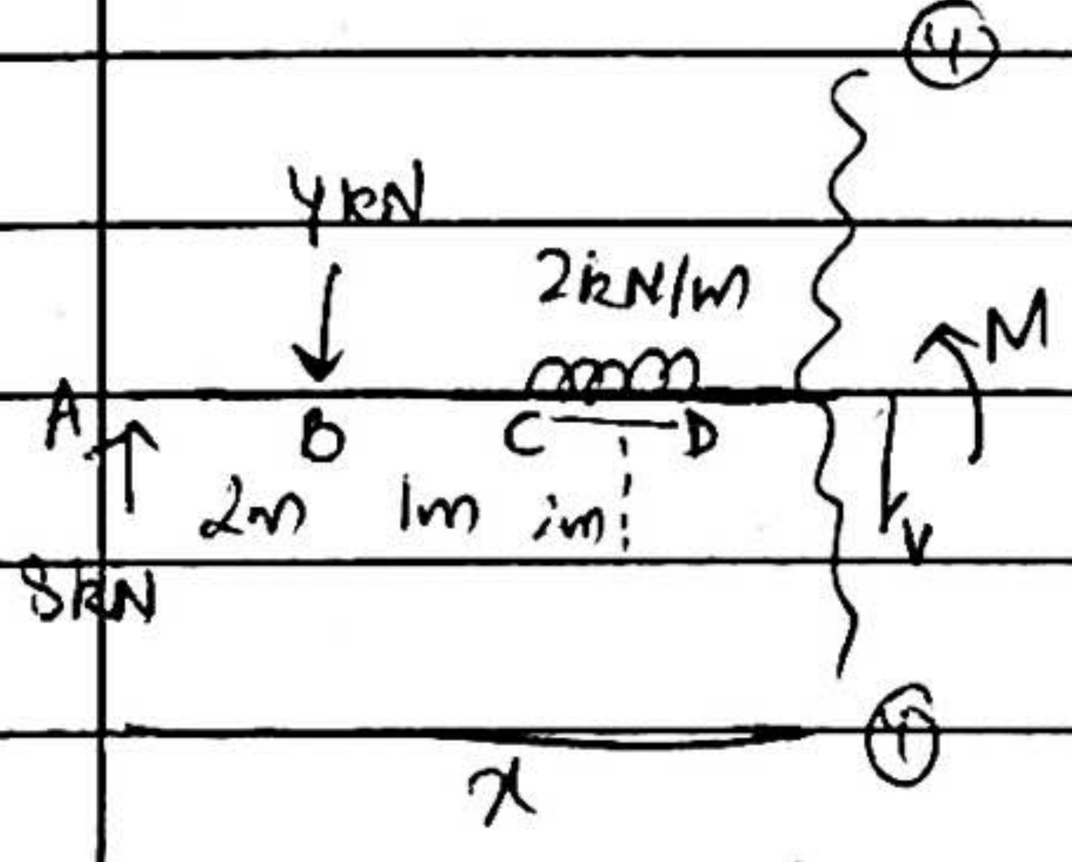
$= 8x - 4x + 8 - \frac{\cancel{2}(x^2 + 9 - 6x)}{\cancel{2}}$

$= -x^2 + 10x - 1$

$x = 3;\ M = -9 + 30 - 1 = 20$

$x = 7;\ M = -49 + 70 - 1 = 20$

$\left[\begin{array}{l} x = 5;\ M = -25 + 50 - 1 = 24\ kNm\ (max.) \\ x = 5;\ SF = 0 \end{array} \right]$



$V = 8 - 4 + 8 = -4 kN$

$M = 8x - 4x - 8(x-5)$

$= -4x + 40$

$x = 7;\ M = 12$

$x = 9;\ M = 4$

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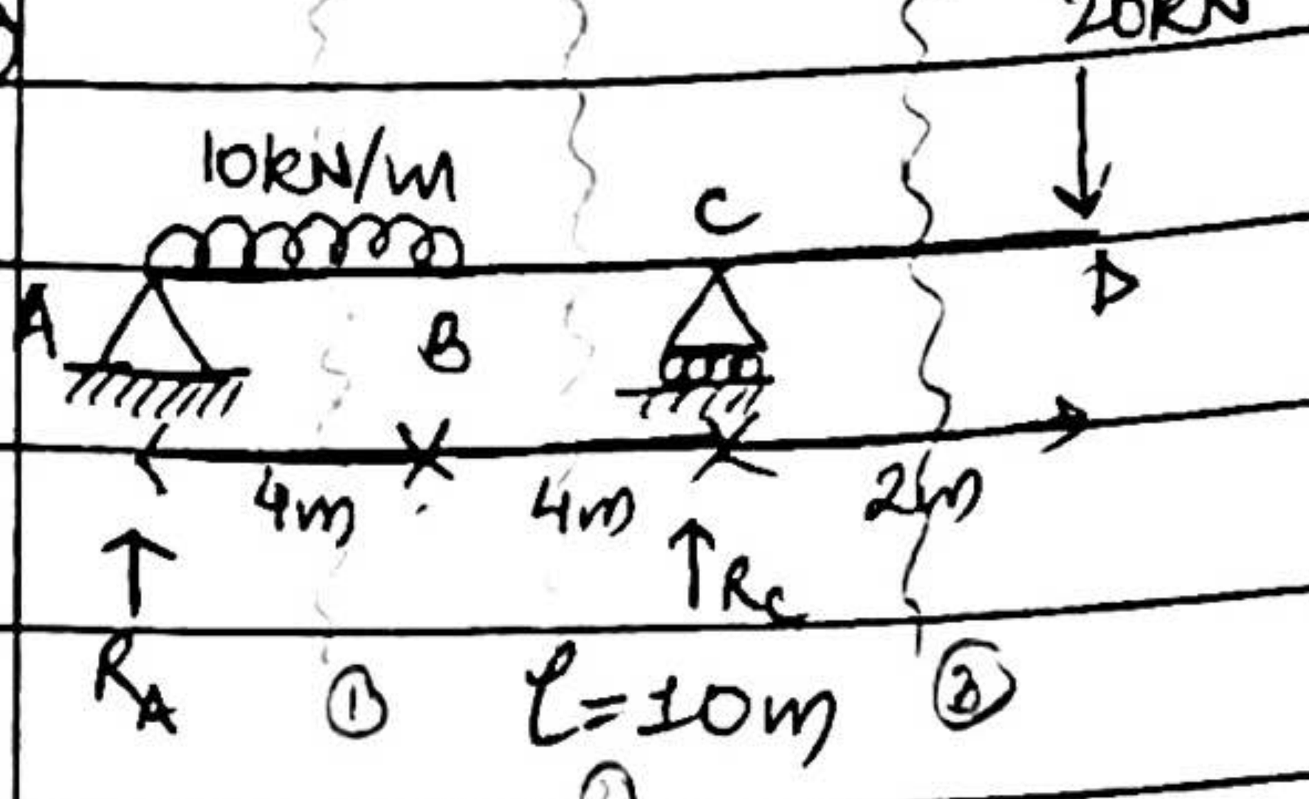
Ques. ① Draw SFD, BMD and find pt. of contre-flexure. [where M=0]

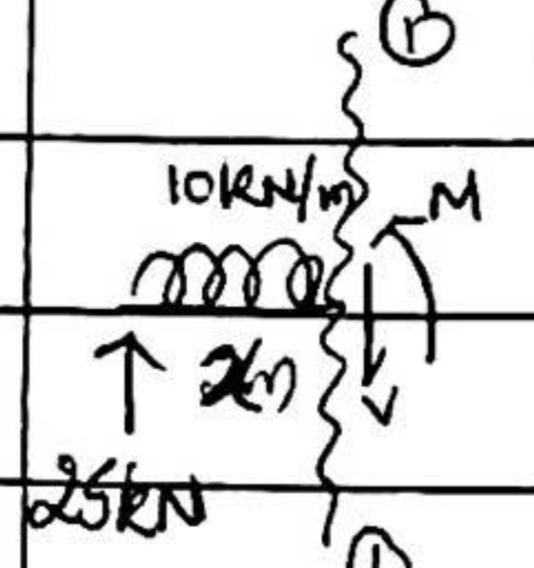


Sol. $\Sigma F_y = 0 \Rightarrow R_A + R_C = 20 + 10\times4 = 60kN$

$\Sigma M_A = 0 \Rightarrow -8R_C + 200 + \cancel{1}80 = 0$

$R_C = \cancel{3}5kN$

$R_A = 2\cancel{5}kN$



$25 = 10x + V$

$V = 25 - 10x$

$x = 0;\ V = 25\ kN$

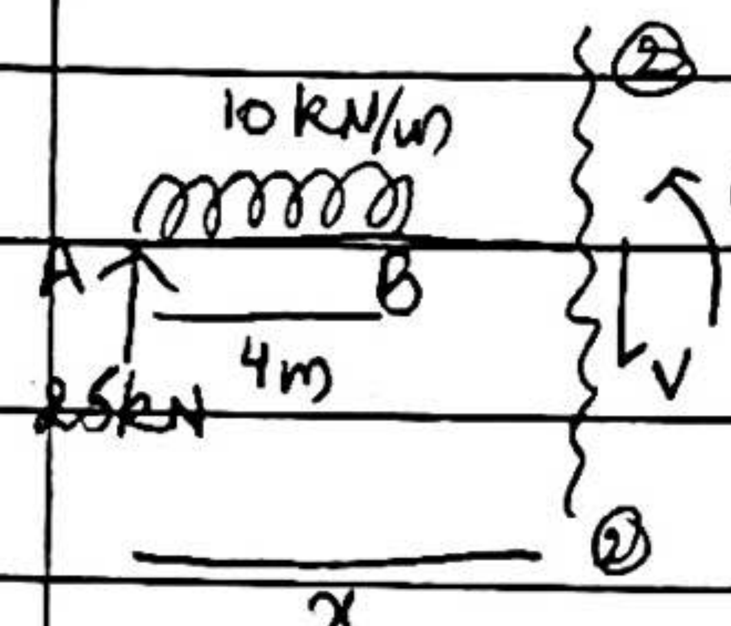
$x = 4;\ V = -15\ kN$

$M = 25x - 10x\left(\frac{x}{2}\right)$

$= 25x - 5x^2$

$x = 0;\ M = 0$

$x = 4;\ M = 20\ kN\cdot m$



$25 = 40 + V$

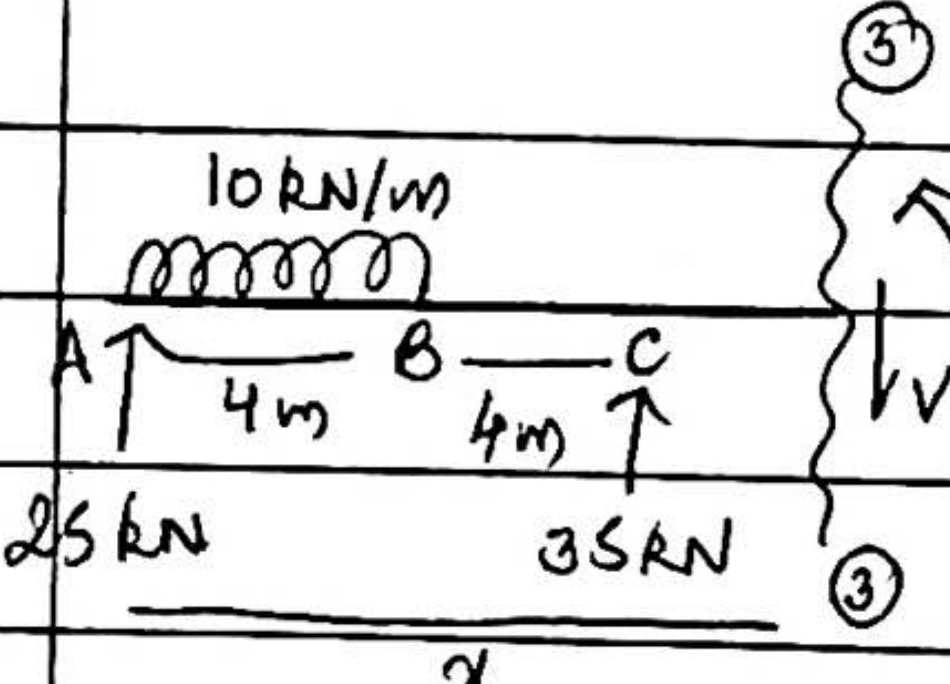
$V = -15kN$

$M = 25x - 40(x-2)$

$= 80 - 15x$

$x = 4;\ M = 20kNm$

$x = 8;\ M = -40\ kNm$



$25 + 35 = V + 40$

$V = 20\ kN$

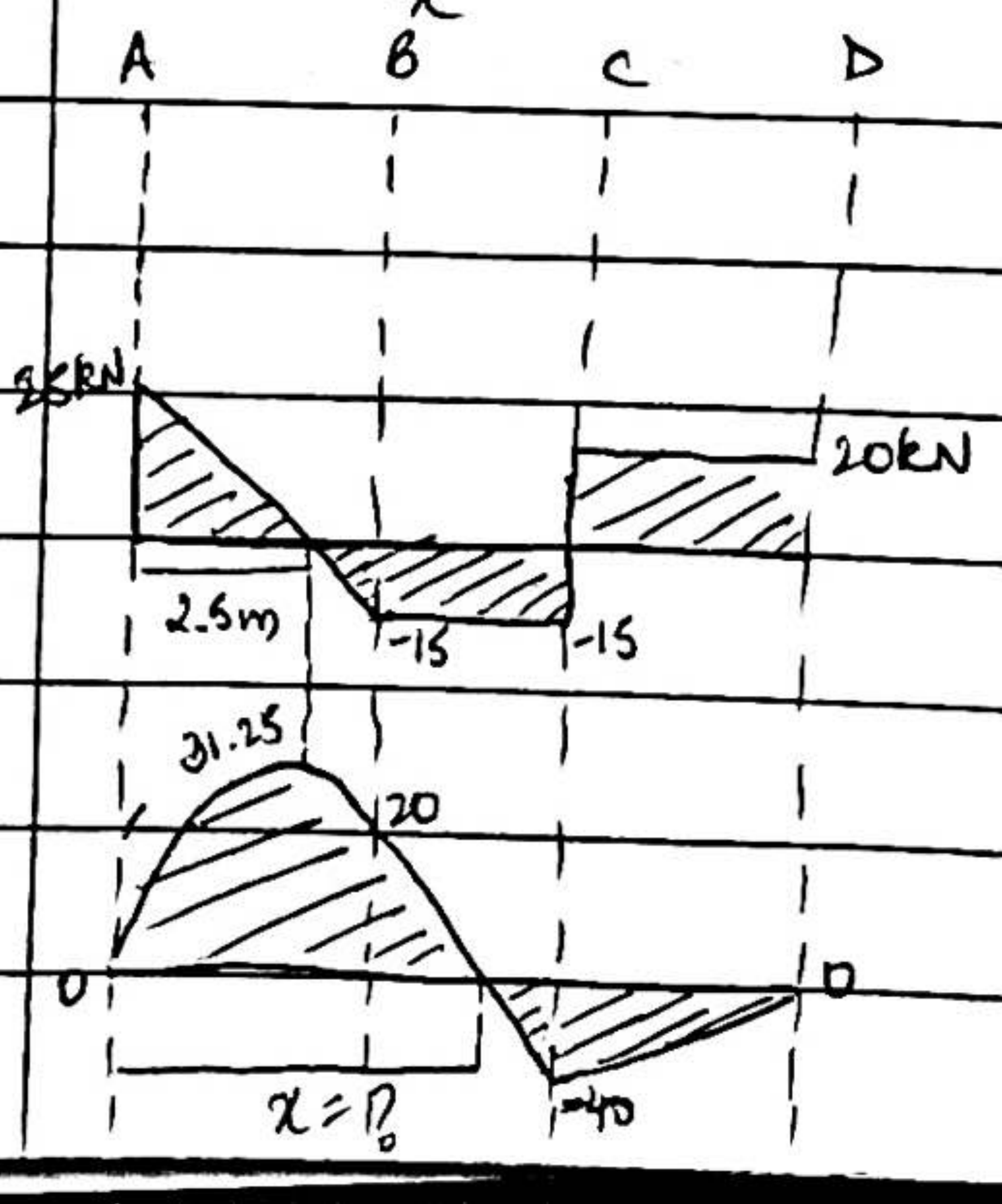
$M = 25x + 35(x-8) - 40(x-2)$

$x = 8;\ M = 40$

$x = 10;\ M = \cancel{0}0$

SFD

BMD



at $x = 2.5$ in AB

$SF = 0$

$\therefore BM = \max.$

$= 25(2.5) - 5(2.5)^2$

$= 31.25\ kNm$

Pt. of contre flexure $= \frac{80}{15}$

$\therefore x = 5.33m$