24/09/20 Ch.1 - DC Circuits

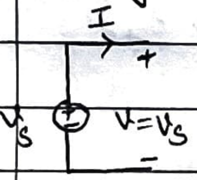
Circuit Elements

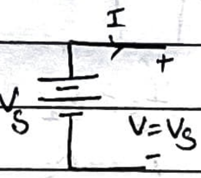
① Resistance ② Inductance ③ Capacitance ④ Source

Circuit Elements

- Active
  - Voltage Source
  - Current Source
- Passive (no capability to supply power)
  - Resistance
  - Inductance
  - Capacitance

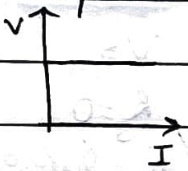
- **Voltage Source :-**

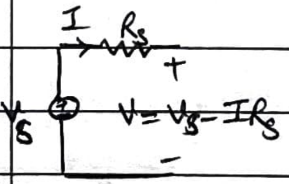


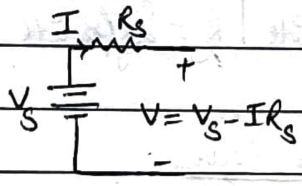


DC/AC

Ideal VS → it cont. to supply the same voltage irrespective of load.

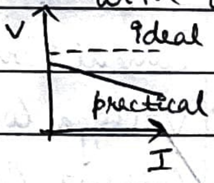






Practical VS → it is not continuous and starts decreasing with current.

★★ In case of ideal voltage source ⇒ $R_S=0$.



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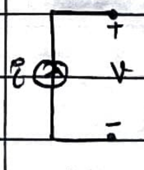
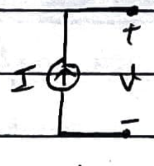
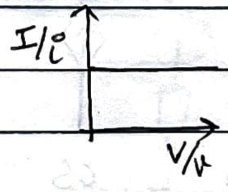
- **Ideal Current Source :-**



AC



DC



$\boxed{R_S=\infty}$

[$R_s \rightarrow$ internal resistance of current source]

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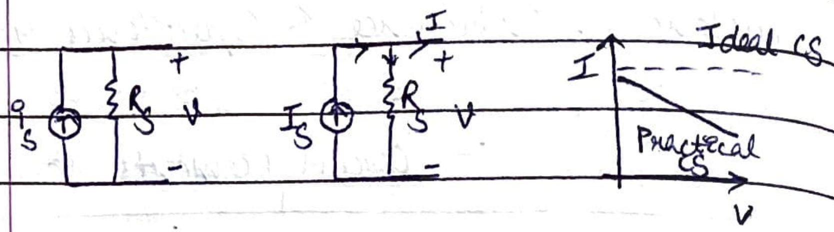
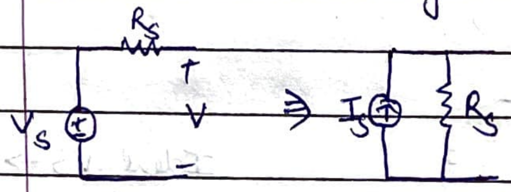
- **Practical Current Source :-**



$I = I_s - \frac{V}{R_s}$

Current doesn't depend on voltage ideally by practically it decreases.

- **Current Source dual of Voltage Source :**



$V_s = R_s I_s$ $\qquad$ $I_s = \frac{V_s}{R_s}$

	VS	CS
Ideal	$R_s = 0$	$R_s = \infty$
Practical	R_s = Definite	R_s = Definite

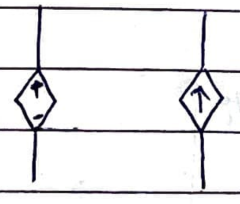
- **Dependent Source**

① Voltage Controlled Voltage Source (dependent) (Voltage supplied by VS is controlled by another voltage)

② Current Controlled Voltage Source

③ Voltage Controlled Current Source

④ Current Controlled Current Source



VS $\qquad$ CS

diamond-like shape for dependent source

① Voltage Controlled Voltage Source

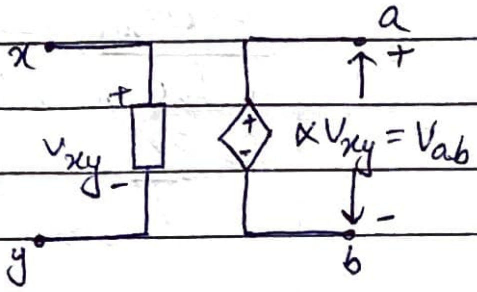
$V_{ab} = \alpha V_{xy}$

$\alpha = \dfrac{V_{ab}}{V_{xy}}$

$\alpha \rightarrow$ Voltage Ratio

eg. Transformer

$V_2 = \dfrac{N_2}{N_1} V_1$



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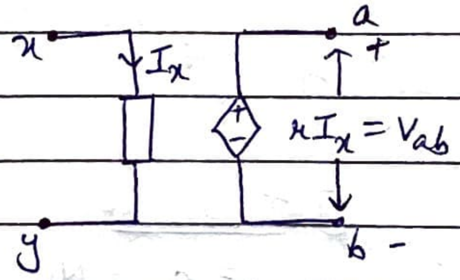
② Current Controlled Voltage Source

$V_{ab} = r I_x$

$r = \dfrac{V_{ab}}{I_x}$

$r \rightarrow$ Trans-resistance

eg. Generator (AC & DC)



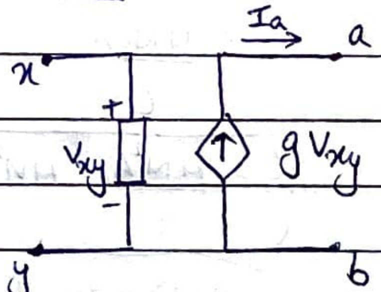
③ Voltage Controlled Current Source

$I_a = g V_{xy}$

$g = \dfrac{I_a}{V_{xy}}$

$g \rightarrow$ Trans-conductance

eg. MOSFET



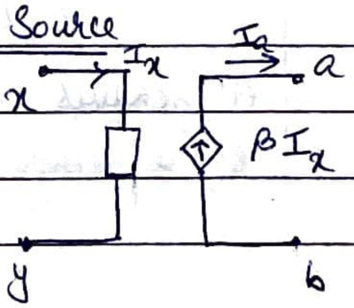
④ Current Controlled Current Source

$I_a = \beta I_x$

$\beta = \dfrac{I_a}{I_x}$

$\beta \rightarrow$ Current ratio

eg. Transistor



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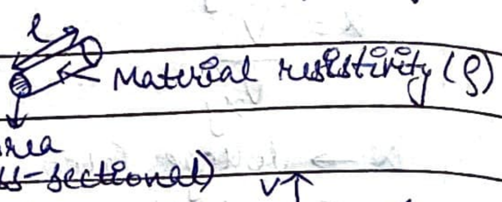
* Passive Circuit Elements

① Resistance (Ω)

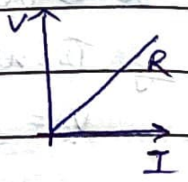
It has the ability to obstruct the flow of current.

$$R = \frac{\rho l}{A}$$



acc. to Ohm's Law ⇒ $V = IR$

$$P = VI = I^2R = \frac{V^2}{R} = GV^2$$



$G \rightarrow \frac{1}{R}$
(conductance)

$$W = \int_{-\infty}^{t} V(t)\, I(t)\, dt$$

$R_t = R_o\left(1 + \alpha\left(T_t - T_o\right)\right)$

$\alpha \rightarrow$ +ve for conductor

-ve for insulator & semi-conductor

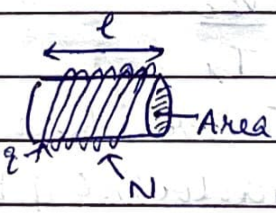
② Inductors

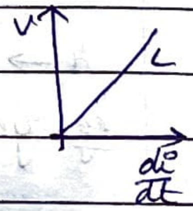
magnetic field intensity

$$L = \frac{\lambda}{i} = \frac{N\phi}{i}$$

$\oint H\, dl = Ni$

$H = \frac{Ni}{l}$

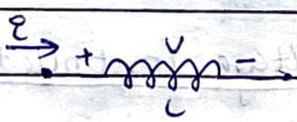




$\phi = BA$

$= \mu H A$

$= \frac{\mu N i A}{l}$



$$\boxed{L = \frac{\mu N^2 A}{l} = \mu n^2 A l}$$

$$\boxed{V = L\frac{di}{dt}}$$

$$\boxed{W = \frac{1}{2} L i^2}$$

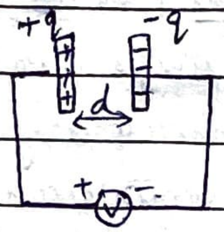
$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v\, dt$$

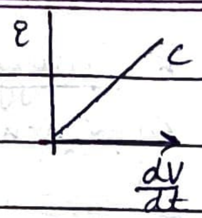
$$= \frac{1}{L}\int_{t_0}^{t} v(t) + i(t_0)$$

It behaves as short-circuit in case of DC.
(stores energy in the form of magnetic field)

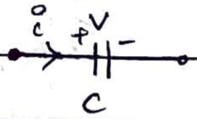
③ Capacitance

$C = \frac{A\varepsilon}{d}$





$C = \frac{q}{V}$



$i = C\frac{dv}{dt}$

$$\boxed{W = \frac{1}{2}CV^2}$$

$dv = \int_{-\infty}^{t} \frac{i\,dt}{C}$

(store energy in the form of electric field).

$\Rightarrow v(t) = \frac{1}{C}\int_{-\infty}^{t_0} i\,dt + \frac{1}{C}\int_{t_0}^{t} i\,dt$

$\Rightarrow v(t) = \underbrace{v(t_0)}_{\text{(if already charged)}} + \frac{1}{C}\int_{t_0}^{t} i\,dt$

It behaves as open-circuit in case of DC.

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Kirchhoff's Laws

① Kirchhoff's Current Law (KCL)

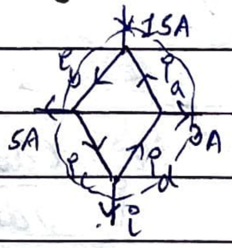
Algebraic sum of current on a junction (or node or close boundary) is 0.

It is application of conservation of charge. $\boxed{\sum_{n=1}^{N} i_n = 0}$

leaving → outgoing → +ve

entering → incoming → -ve

eg.



on a closed loop,

$i + 5 + 3 - 15 = 0$

$i = 7A$

* Topology Terms (Circuit)

(b) ① Branch – a simple circuit element representation.

(n) ② Node – interconnection of 2 or more circuit elements

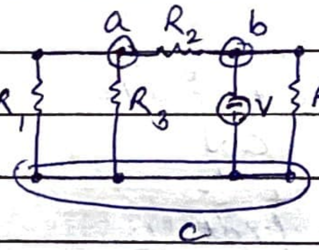
③ Junction – interconnection of 3 or more circuit elements.

④ Loop – closed path in circuit through which current passes.

⑤ Mesh – independent loop (l).

$$l = b - (n-1)$$

eg



3 nodes (n)

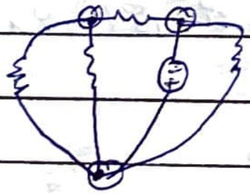
3 junctions

6 branches

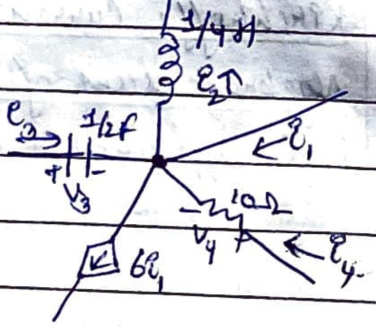
$l = 5-(3-1) = 3$

(loop)

⇓



Ques. Find v_2.



$i_1 = \frac{1}{2}e^{-2t}$

$v_3 = 2e^{-2t}$

$v_4 = 5e^{-2t}$ } given

$$v_2 = L\frac{di_2}{dt}$$

$$-i_1 + i_2 - i_3 + 6i_1 - i_4 = 0$$

$$i_2 = -5i_1 + i_3 + i_4$$

$$\therefore i_2 = -4e^{-2t}$$

So, $v_2 = \frac{1}{4} \times 8e^{-2t}$

$= 2e^{-2t}$

$i_3 = C\frac{dv_3}{dt} = -\frac{1}{2} \times 4e^{-2t}$

$= -2e^{-2t}$

$i_4 = \frac{v_4}{10} = 0.5e^{-2t}$

(2) Kirchhoff's Voltage Law

It is based on conservation of energy, and applied in a closed path / loop. $\boxed{\sum_{m=1}^{M} v_m = 0}$

voltage rise → −ve (−+)
voltage drop → +ve (+−)

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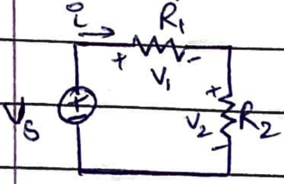
Series Equivalent of Resistance

- 

$v_1 = iR_1$; $v_2 = iR_2$

$v_s = v_1 + v_2$

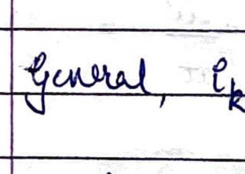
$v_s = i(R_1 + R_2)$

$i R_{eq} = i(R_1 + R_2)$

$\boxed{R_{eq} = R_1 + R_2}$

- General, $v_k = \dfrac{v_s R_k}{R_{eq}}$

Parallel Equivalent of Resistance

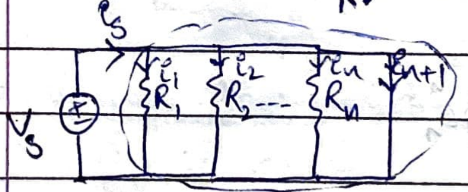
- 

$i_s = i_1 + i_2$

$\dfrac{V_s}{R_{eq}} = \dfrac{V_s}{R_1} + \dfrac{V_s}{R_2}$

$\boxed{\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}}$

- General, $i_k = \dfrac{i_s . R_{eq}}{R_k}$

- 

$R_{eq} = 0 \rightarrow$ Short circuit

$i_{n+1} = i_s$

$i_1 = i_2 - - - = i_n = 0$

- Open circuit → $R = \infty$; $i = 0$

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$\frac{18 \times 9}{27}$ **Ques.** Find R_{eq} b/w a & b.

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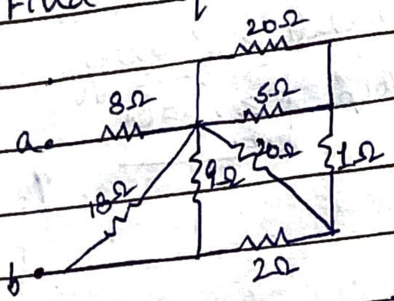


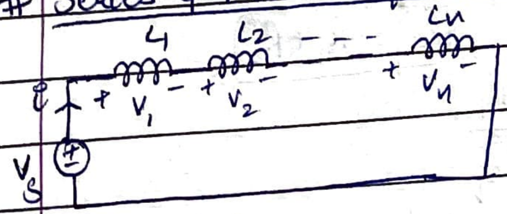
$$\left[\left[\left[(20 \| 5)+1\right] \| 20\right]+2\right] \| \left[9 \| 18\right] + 8$$

$$\Rightarrow \left[\left[5 \| 20\right]+2\right] \| \left[6\right] + 8$$

$$\Rightarrow 3+8$$

$$= 11\ \Omega$$

Series & Parallel Equivalent of Inductance



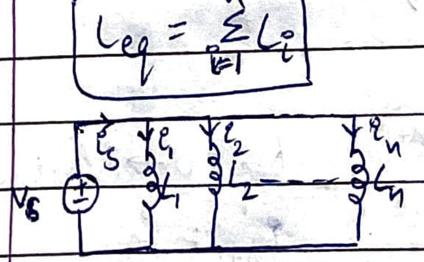
$$V = L\frac{di}{dt}$$

$$V_S = V_1 + V_2 + \dots + V_n$$

$$V_S = L_1\frac{di}{dt} + L_2\frac{di}{dt} + \dots + L_n\frac{di}{dt}$$

$$L_{eq}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt} + \dots + L_n\frac{di}{dt}$$

$$\boxed{L_{eq} = \sum_{i=1}^{n} L_i}$$

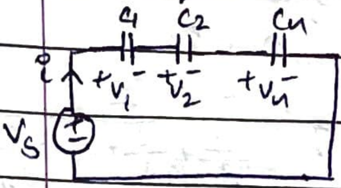


$$i_S = i_1 + i_2 + \dots + i_n$$

$$\frac{di_S}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \dots + \frac{di_n}{dt}$$

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}}$$

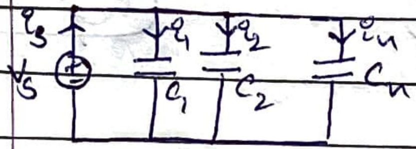
Series & Parallel Equivalent of Capacitance



$$V_s = V_1 + V_2 + \dots + V_n$$

$$\frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2} + \dots + \frac{q}{C_n}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$$



$$i_s = i_1 + i_2 + \dots + i_n$$

$$C_{eq}\frac{dV_s}{dt} = C_1\frac{dV_s}{dt} + C_2\frac{dV_s}{dt} + \dots + C_n\frac{dV_s}{dt}$$

$$\boxed{C_{eq} = \sum_{i=1}^{n} C_i}$$

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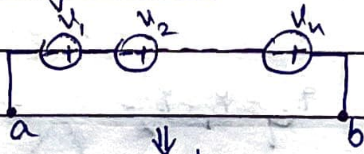
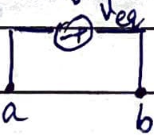
Series-Parallel Connection of Sources

★ Voltage Sources (Ideal)



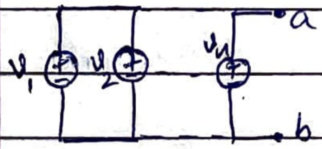
$$\boxed{V_{eq} = V_1 + V_2 + \dots + V_n}$$

⇓



★ Restriction
all VS should have same voltage polarity & magnitude

$$\boxed{V_{eq} = V_1 = V_2 = \dots = V_n}$$



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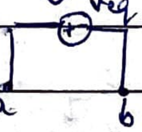
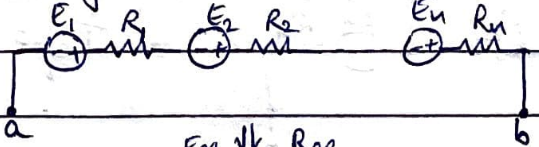
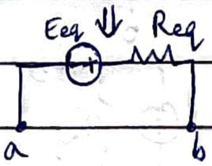


★ Voltage Sources (Practical)

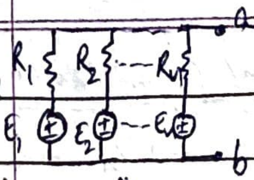


$$\boxed{\begin{aligned} E_{eq} &= E_1 + E_2 + \dots + E_n \\ R_{eq} &= R_1 + R_2 + \dots + R_n \end{aligned}}$$

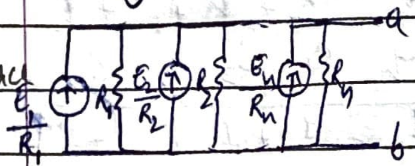
⇓



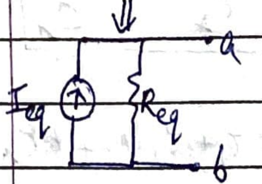
Voltage source
↓
Current source
↓
Voltage source



$$I_{eq} = \frac{E_1}{R_1} + \frac{E_2}{R_2} + \cdots + \frac{E_n}{R_n}$$



$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$



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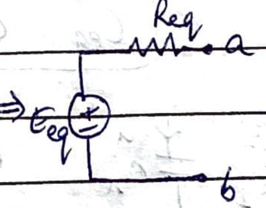
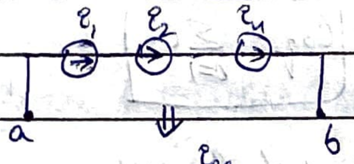


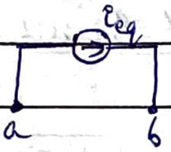
$$\boxed{E_{eq} = I_{eq} R_{eq}}$$

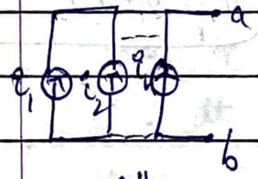
★ <u>Current Sources</u> (Ideal)



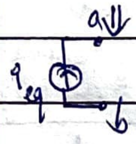
★ <u>Restriction</u> equal current

$$i_{eq} = i_1 = i_2 = \cdots = i_n$$

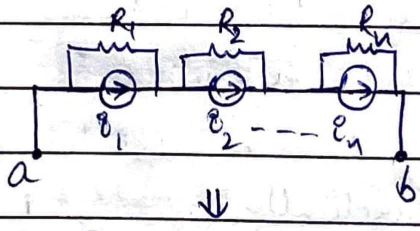




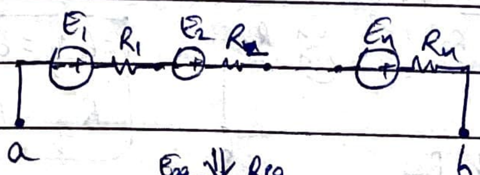
$$i_{eq} = i_1 + i_2 + \cdots + i_n$$



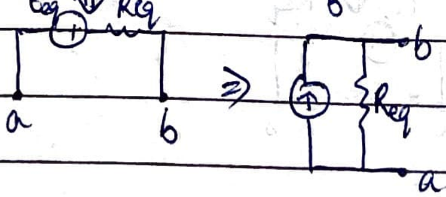
★ <u>Current Sources</u> (Practical)



$$E_{eq} = E_1 + E_2 + \cdots + E_n$$
$$R_{eq} = R_1 + R_2 + \cdots + R_n$$



$$I_{eq} = \frac{E_{eq}}{R_{eq}}$$



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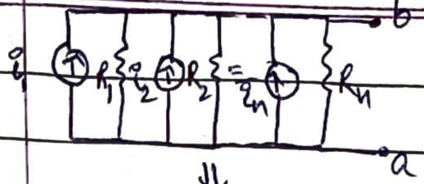
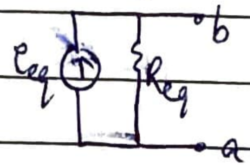


$$I_{eq} = I_1 + I_2 + \cdots + I_n$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$

⇓



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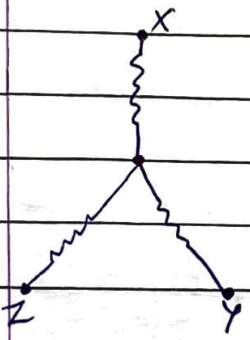
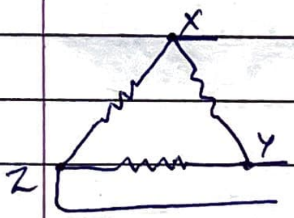
Star-Delta Transformation



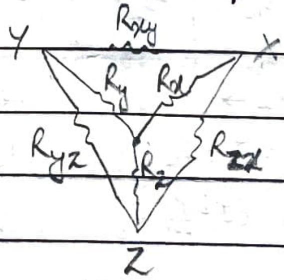
Star (Y)

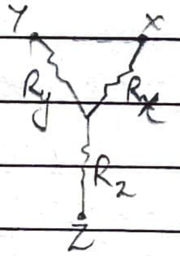


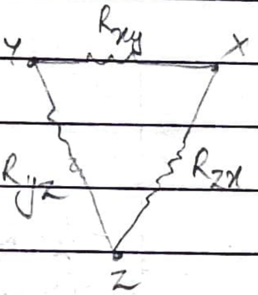
Delta (Δ)

Resistance between any pair of terminals is same in both arrangements. (3^{rd} terminal is open).

eg.







$$R_y + R_z = \frac{R_{yz}(R_{zx} + R_{xy})}{R_{xy} + R_{yz} + R_{zx}} \quad -(2)$$

$$R_x + R_z = \frac{R_{zx}(R_{xy} + R_{yz})}{R_{xy} + R_{yz} + R_{zx}} \quad -(3)$$

$$R_x = \frac{R_{xy} R_{zx}}{R_{xy} + R_{yz} + R_{zx}}$$

$$R_y = \frac{R_{xy} R_{yz}}{R_{xy} + R_{yz} + R_{zx}}$$

$$R_z = \frac{R_{zx} R_{yz}}{R_{xy} + R_{yz} + R_{zx}}$$

$R_{xy}(\text{star}) = R_{xy}(\text{delta})$

$R_{yz}(\text{star}) = R_{yz}(\text{delta})$

$R_{zx}(\text{star}) = R_{zx}(\text{delta})$

$R_{xy}(\text{star}) = R_x + R_y$

$$R_{xy}(\text{delta}) = R_{xy} \parallel (R_{yz} + R_{zx}) = \frac{R_{xy}(R_{yz} + R_{zx})}{R_{xy} + R_{yz} + R_{zx}}$$

$$\therefore R_x + R_y = \frac{R_{xy}(R_{yz} + R_{zx})}{R_{xy} + R_{yz} + R_{zx}} \quad -(1)$$

Voltage Drop → +ve
Voltage Rise → -ve

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$$R_{xy} = \frac{R_xR_y + R_yR_z + R_zR_x}{R_z}$$

$$R_{yz} = \frac{R_xR_y + R_yR_z + R_zR_x}{R_x}$$

$$R_{zx} = \frac{R_xR_y + R_yR_z + R_zR_x}{R_y}$$

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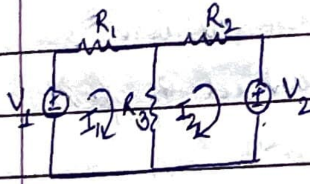
KVL application

Mesh Analysis (mesh currents)

Mesh is a loop that doesn't contain any other loop inside it. (Independent loop)



Mesh 1 →

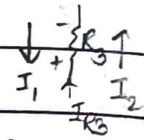
$I_{R3} = I_1 - I_2$

$V_{R3} = (I_1 - I_2)R_3$

Mesh 2 →

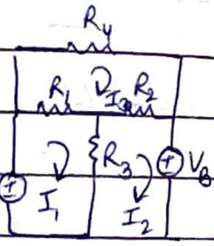
$I_{R3} = I_2 - I_1$

$V_{R3} = (I_2 - I_1)R_3$



① Identify meshes
② assign mesh currents (clockwise)
③ KVL equations for each mesh

Method for only independent voltage sources



$-V_A + R_1(I_1 - I_3) + R_3(I_1 - I_2) = 0$

$\Rightarrow (R_1 + R_3)I_1 - R_3I_2 - R_1I_3 = V_A$ ①

$+V_B + R_3(I_2 - I_1) + R_2(I_2 - I_3) = 0$

$\Rightarrow -R_3I_1 + (R_2 + R_3)I_2 - R_2I_3 = -V_B$ ②

$$\begin{bmatrix} R_1+R_3 & -R_3 & -R_1 \\ -R_3 & R_2+R_3 & -R_2 \\ -R_1 & -R_2 & R_1+R_2+R_4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_A \\ -V_B \\ 0 \end{bmatrix}$$

$$[R][I] = [V]$$

R_{ij} → mutual term; -ve of resistance b/w i^{th} & j^{th} mesh

R_{ii} → sum of resistances around i^{th} mesh

I_j → j^{th} mesh current

V_j → net voltage rise around the j^{th} mesh

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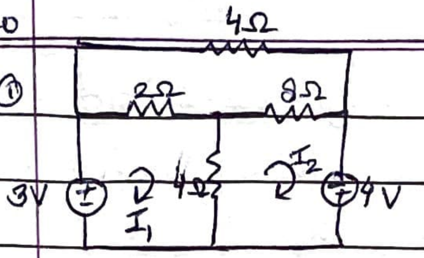
Ques. ①



$$\begin{bmatrix} 2+4 & -4 & -2 \\ -4 & 8+4 & -8 \\ -2 & -8 & 2+8+4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$$

$I_1 = 2.678A$, $I_2 = 2.393A$, $I_3 = 1.75A$

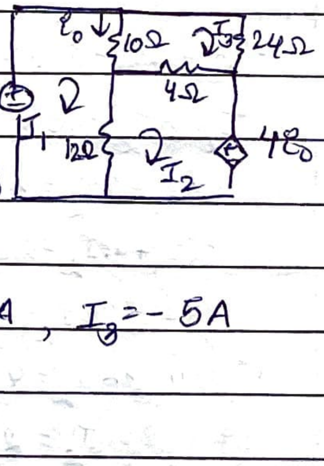
Ques. ② Find i_o using mesh analysis.

$-24 + 10(I_1 - I_3) + 12(I_1 - I_2) = 0$ - ①

$12(I_2 - I_1) + 4(I_2 - I_3) + 4i_o = 0$ - ②

$24I_3 + 4(I_3 - I_2) + 10(I_3 - I_1) = 0$ - ③

$i_o = I_1 - I_3$ - ④



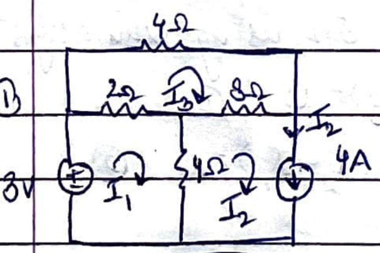
$I_1 = -3.2143A$, $I_2 = -9.642A$, $I_3 = -5A$

$i_o = 1.7857A$

★ Circuit with Current Source

① → CS is in outer loop

② → CS is in branch b/w two meshes (super mesh analysis)

① 

$-3 + 2(I_1 - I_3) + 4(I_1 - I_2) = 0$ - ①

$4I_3 + 8(I_3 - I_2) + 2(I_3 - I_1) = 0$ - ②

~~$4(I_2 - I_1) + 8(I_2 - I_3) = 0$ - ③~~

$\therefore \boxed{I_2 = 4A}$

Solve using ①, ② and $I_2 = 4A$.

$\Rightarrow -3 + 2I_1 - 2I_3 + 4I_1 - 16 = 0$

$\Rightarrow 19 = 6I_1 - 2I_3$

$96 = -6I_1 + 42I_3$

$115 = 40I_3$

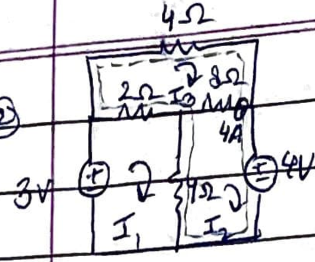
$I_3 = 2.875A$

$\Rightarrow 4I_3 + 8I_3 - 32 + 2I_3 - 2I_1 = 0$

$\Rightarrow (32 = 14I_3 - 2I_1) \times 3$

$I_1 = 4.125A$

(2)



$-3+2(I_1-I_3)+4(I_1-I_2)=0$ — (1)

Combine mesh 2 & mesh 3 to form super mesh.

$4I_3+4+4(I_2-I_1)+2(I_3-I_1)=0$ — (2)

$I_2-I_3=4$ — (3) (Constraint)

$I_1=2.917A$; $I_2=3.75A$; $I_3=-0.25A$

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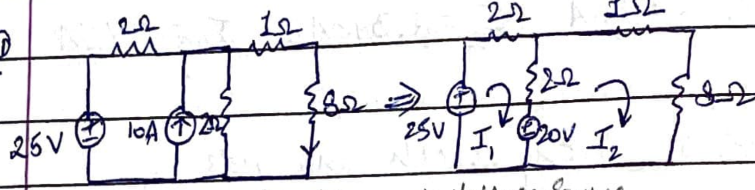
Ques. (1)



Current Source to Voltage Source

$-25+2I_1+2(I_1-I_2)+20=0$ — (1)

$I_2+8I_2-20+2(I_2-I_1)=0$ — (2)

$4I_1-2I_2=5$

$(-2I_1+11I_2=20)\times 2$

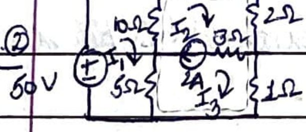
$-4I_1+22I_2=40$

$20I_2=45$

$I_2=2.25A$

$I_1=2.375A$

Ques. (2)



Determine the current through 5Ω using mesh analysis.

$\Rightarrow -50+10(I_1-I_2)+5(I_1-I_3)=0$ — (1)

mesh 2 + mesh 3 (super mesh) $\Rightarrow 2I_2+I_3+5(I_3-I_1)+10(I_2-I_1)=0$ — (2)

$\Rightarrow I_2-I_3=2$ — (3)

$I_1=20A$; $I_2=17.33A$; $I_3=15.33A$

$I_{5\Omega}=I_1-I_3=4.67A$

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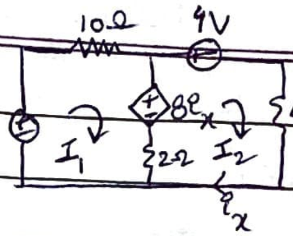
Ques. ③



Determine i_x using mesh analysis

$\Rightarrow -6 + 10I_1 + 8i_x + 2(I_1 - I_2) = 0$ — ①

$\Rightarrow +4 + 4I_2 + 2(I_2 - I_1) - 8i_x = 0$ — ②

$i_x = I_2$

$\Rightarrow -6 + 12I_1 + 6I_x = 0 \Rightarrow 2I_1 + I_x = 1$

$\Rightarrow +4 - 2I_1 - 2I_x = 0 \Rightarrow -2I_1 - 2I_x = 4$

$-I_x = -3$

$I_x = 3A$

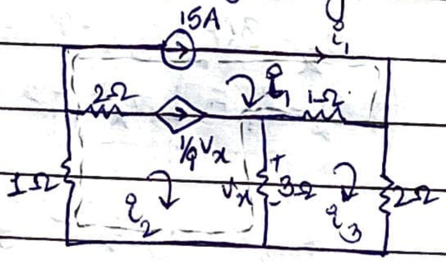
$I_1 = -1A$

Ques. ④ find i_1, i_2, i_3 using mesh analysis.

$i_1 = 15A$

$i_2 = 17A$

$i_3 = 11A$



we have a current source ∴ super mesh can be applied in mesh ① & mesh ②

as voltage across 15A is not known

$i_1 = 15A$

Mesh ③ →

$\Rightarrow +2i_3 + 3(i_3 - i_2) + (i_3 - i_1) = 0$ — ②

$\Rightarrow 6i_3 - 3i_2 - i_1 = 0$

$\Rightarrow 6i_3 - 3i_2 = 15$

$\Rightarrow 2i_3 - i_2 = 5$

$i_2 - i_1 = \frac{1}{9}V_x$ — ①

V_x 3Ω

$V_x = 3(i_2 - i_3)$ — ③

$\Rightarrow 3i_2 - 3i_3 = 9i_2 - 9i_1$

$\Rightarrow 9i_1 - 3i_3 = 6i_2$

$\Rightarrow 3i_1 = 2i_2 + i_3$

$\Rightarrow 45 = 2i_2 + i_3$

$45 = 2i_2 + i_3$

$10 = -2i_2 + 4i_3$

$55 = 5i_3$

$i_3 = 11A$

$i_2 = 17A$

** When we have a current source in the branch, we don't know the voltage across it, ∴ we use super mesh by combining individual meshes.

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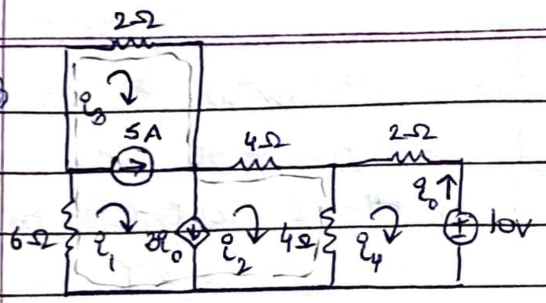
Ques. 5



(super mesh)

Mesh 4 →

$2i_4 + 10 + 4(i_4 - i_2) = 0$

$\Rightarrow 6i_4 - 4i_2 = -10$

$\Rightarrow 3i_4 - 2i_2 = -5$ — (5)

Mesh 1, 2, 3 →

$2i_3 + 4i_2 + 4(i_2 - i_4) + 6i_1 = 0$ — (1)

$i_1 - i_3 = 5$ — (2)

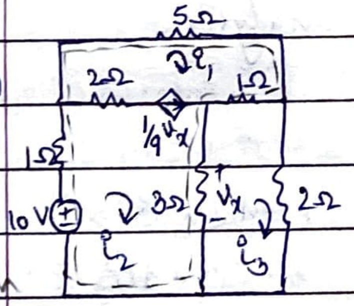
$i_1 - i_2 = 3i_0$ — (3)

$i_0 = -i_4$ — (4)

$i_1 = -8.75A$; $i_2 = 13.75A$; $i_3 = 13.75A$; $i_4 = 7.5A$

Ques 6

Mesh (1) + mesh (2) to make supermesh



$v_x = 3(i_2 - i_3)$ — (1)

$i_2 - i_1 = \frac{1}{9} v_x$ — (2)

$2i_3 + 3(i_3 - i_2) + i_3 - i_1 = 0$ — (3)

$5i_1 + 1(i_1 - i_3) + 3(i_2 - i_3) - 10 + i_2 = 0$ — (4)

$9i_2 - 9i_1 = 3i_2 - 3i_3$

$6i_2 = 9i_1 - 3i_3$

$2i_2 = 3i_1 - i_3$ — (5)

$6i_1 - 2i_2 + 2i_3 = 10$

$6i_1 - 4i_2 - 2i_3 = 0$

$2i_2 + 4i_3 = 10$

$6i_3 - 3i_2 - i_1 = 0$ — (3)

$6i_1 - 2i_2 + 2i_3 = 10$ — (4)

$6i_1 - 2i_2 + 2i_3 = 10$

$-6i_1 - 18i_2 + 36i_3 = 0$

$-20i_2 + 38i_3 = 10$

$20i_2 + 40i_3 = 100$

$78i_3 = 110$

$i_3 = 1.4A$

$i_2 = 2.2A$

$i_1 = 1.9A$

$i_1 = 1.32A$

$i_2 = 1.49A$

$i_3 = 0.97A$

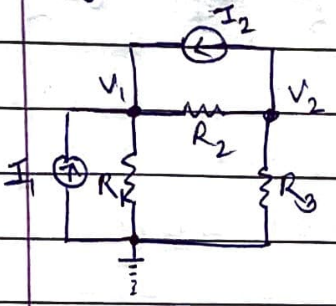
23/10/20

Nodal Analysis

- focussed approach
- KCL at diff. nodes

★ Steps in nodal analysis

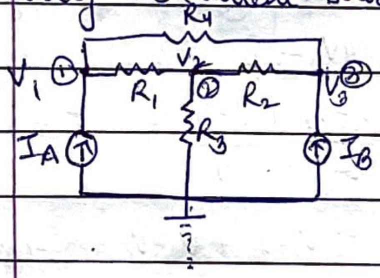
1. identify the nodes & select one as a reference.
2. assign voltage to each node w.r.t. ref.
3. apply KCL at all nodes except ref. node.



node 1 ⇒ $\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} - I_1 - I_2 = 0$

node 2 ⇒ $\frac{V_2}{R_3} + \frac{V_2 - V_1}{R_2} + I_2 = 0$

→ Only CS (current source)

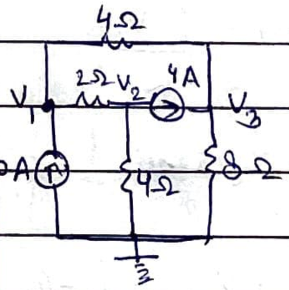


node 1 ⇒ $\frac{V_1 - V_2}{R_1} + \frac{V_1 - V_3}{R_4} - I_A = 0$

node 2 ⇒ $\frac{V_2 - V_1}{R_1} + \frac{V_2 - V_3}{R_2} + \frac{V_2}{R_3} = 0$

node 3 ⇒ $\frac{V_3 - V_2}{R_2} + \frac{V_3 - V_1}{R_4} - I_B = 0$

Ques ①



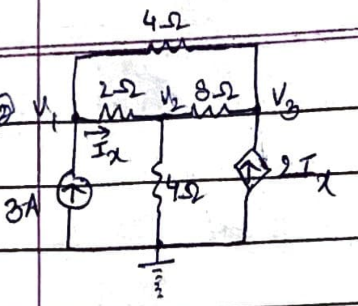
node 1 ⇒ $\frac{V_1 - V_3}{4} + \frac{V_1 - V_2}{2} - 10 = 0$

node 2 ⇒ $\frac{V_2 - V_1}{2} + \cancel{\frac{V_2 - V_3}{4}} + \frac{V_2}{4} + 4 = 0$

node 3 ⇒ $\frac{V_3 - V_1}{4} + \frac{V_3}{8} \cancel{-4} = 0$

$V_1 = 40V$; $V_2 = 21.33V$; $V_3 = 37.33V$

Ques. ②



$$I_x = \frac{V_1 - V_2}{2}$$

$$\text{node } 1 \Rightarrow \frac{V_1 - V_3}{4} + \frac{V_1 - V_2}{2} - 3 = 0$$

$$\text{node } 2 \Rightarrow \frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{8} + \frac{V_2}{4} = 0$$

$$\text{node } 3 \Rightarrow \frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} - 2\left(\frac{V_1 - V_2}{2}\right) = 0$$

$V_1 = -168V$

$V_2 = -132V$

$V_3 = -252V$

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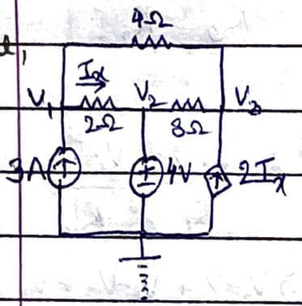
→ Voltage Source (VS)

- Ideal VS b/w a node & reference

node at which VS is there with ref. node, no need to apply KCL



$V_2 - 0 = 4V$

$V_2 = 4V$ — ①

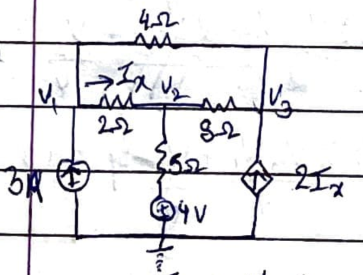
$$-3 + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 0 \quad \text{— ②}$$

$$-2I_x + \frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} = 0 \quad \text{— ③}$$

also, $I_x = \frac{V_1 - V_2}{2}$

- Practical VS b/w a node & reference

KCL at all the 3 nodes



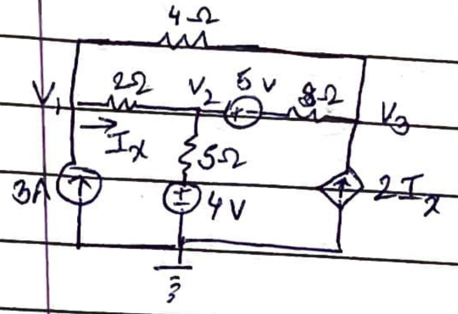
$$I_x = \frac{V_1 - V_2}{2}$$

$$-3 + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 0 \quad \text{— ①}$$

$$\frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} - 2I_x = 0 \quad \text{— ②}$$

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{8} + \frac{V_2 - 4 - 0}{5} = 0 \quad \text{— ③}$$

- Practical VS b/w two reference nodes

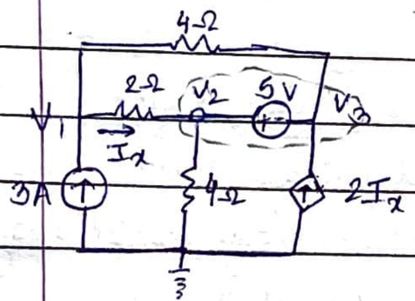


$$\frac{V_1-V_2}{2} + \frac{V_1-V_3}{4} - 3 = 0 \quad -(1)$$

$$\frac{V_2-V_1}{2} + \frac{V_2-4-0}{5} + \frac{V_2-5-V_3}{8} = 0 \quad -(2)$$

$$\frac{V_3-V_1}{4} - 2I_x + \frac{V_3-V_2+5}{8} = 0 \quad -(3)$$

- Ideal VS b/w two non-reference nodes. (super node)



$$\frac{V_1-V_2}{2} + \frac{V_1-V_3}{4} - 3 = 0 \quad -(1)$$

Combine node 2 & 3 to make super node and apply KCL at super node.

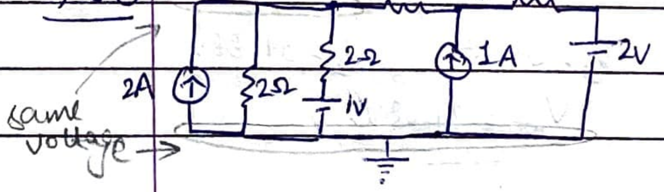
$$\frac{V_2-V_1}{2} + \frac{V_2}{4} + \frac{V_3-V_1}{4} - 2I_x = 0 \quad -(2)$$

$$I_x = \frac{V_1-V_2}{2}$$

Constraint:

$$V_2 - V_3 = 5 \quad -(3)$$

Ques. 1



find V_A & V_B using nodal analysis.

$$\frac{V_A-1-0}{2} + \frac{V_A-V_B}{1} - 2 + \frac{V_A}{2} = 0 \quad -(1)$$

$$\frac{V_B-V_A}{1} - 1 + \frac{V_B-2-0}{2} = 0 \quad -(2)$$

$$V_B = 3.25V$$

$$V_A = 2.875V$$

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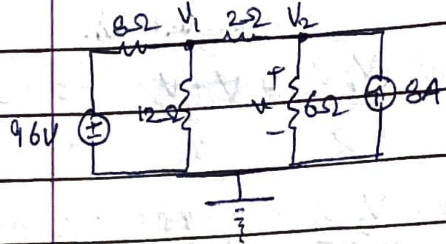
Ques. ① Determine voltage v in the network using nodal analysis



$\boxed{V = V_2}$

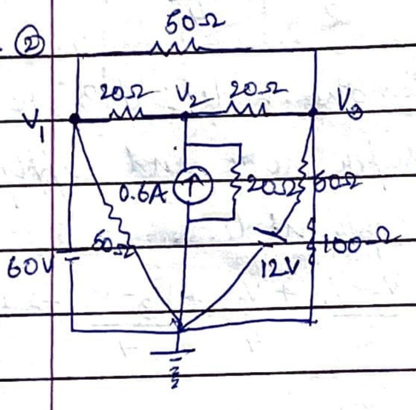
$$\frac{V_1-96-0}{8}+\frac{V_1}{12}+\frac{V_1-V_2}{2}=0 \quad -①$$

$$\frac{V_2-V_1}{2}-8+\frac{V_2}{6}=0 \quad -②$$

$$V_2 = v = 52.5\,V$$

$$V_1 = 54\,V$$

Ques. ②



Find voltage across 100Ω resistor using nodal analysis

$V_1 = 60V$

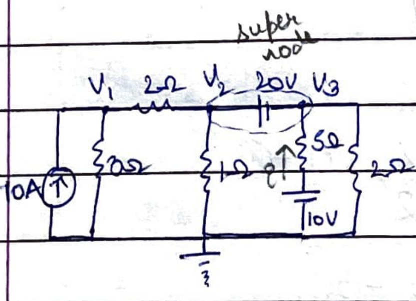
$$\frac{V_3-V_1}{50}+\frac{V_3-V_2}{20}+\frac{V_3}{100}+\frac{V_3-12}{50}=0 \quad -①$$

$$\frac{V_2-V_1}{20}+\frac{V_2-V_3}{20}-0.6+\frac{V_2}{20}=0 \quad -②$$

$V_3 = 31.68V \qquad V_2 = 34.56V$

$$V_{100} = 31.68V$$

Ques. ③



Find the current through 5Ω resistor using nodal analysis

$$-10+\frac{V_1}{3}+\frac{V_1-V_2}{2}=0 \quad -①$$

$$\cancel{\frac{V_2-V_1}{2}+\frac{V_2-20}{1}+\frac{V_3}{5}+\frac{V_3}{2}=0}$$

$$\cancel{\frac{V_3}{2}+\frac{V_3-10}{5}}$$

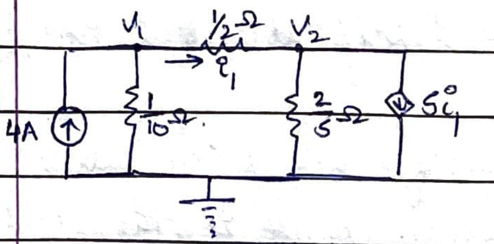
$$\frac{V_2-V_1}{2}+\frac{V_2}{1}+\frac{V_3-10}{5}+\frac{V_3}{2}=0 \quad -②$$

$$V_2-V_3=20 \quad -③$$

$V_1 = 18.94V \quad V_2 = 11.6V \quad V_3 = -8.4V$

$$I_{5\Omega} = \frac{V_3 - 10}{5} = \frac{-8.4-10}{5} = -3.68A \text{ (upward dir}^n\text{)}$$

Ques. 4



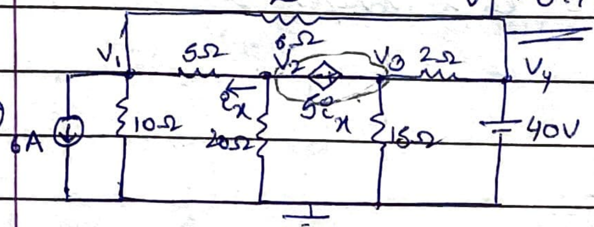
$$-4 + \frac{V_1}{1/10} + \frac{V_1 - V_2}{1/2} = 0 \quad -(1)$$

$$\frac{V_2 - V_1}{1/2} + \frac{V_2}{2/5} + 5i_1 = 0 \quad -(2)$$

$$i_1 = \frac{V_1 - V_2}{1/2}$$

$V_1 = 0.44V \; ; \; V_2 = 0.64V$

Ques. 5



$$6 + \frac{V_1 - V_4}{6} + \frac{V_1 - V_2}{5} + \frac{V_1}{10} = 0 \quad -(1)$$

$$\cancel{\frac{V_3 - V_4}{2} + \frac{V_3}{15} - 5i_x = 0 \quad -(3)}$$

$$\cancel{\frac{V_2 - V_1}{5} + \frac{V_2}{20} + 5i_x = 0 \quad -(2)}$$

$V_4 = 40V \quad -(4)$

Super Mesh (at V_2 & V_3)

$$V_3 - V_2 = 5i_x \quad -(2)$$

$$\frac{V_2 - V_1}{5} + \frac{V_2}{20} + \frac{V_3 - V_4}{2} + \frac{V_3}{15} = 0 \quad -(3)$$

$$i_x = \frac{V_2 - V_1}{5}$$

$V_1 = 10V \; ; \; V_2 = 20V \; ; \; V_3 = 30V$

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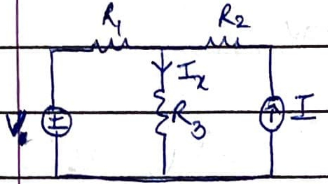
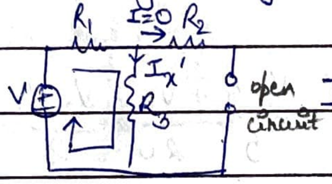
Superposition Theorem

- applicable to linear circuits (directly proportional relationship eg. $V = IR$)
- applicable to circuits having atleast 2 independent sources

→ It states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of voltages across (or currents through) that element due to each independent source acting alone.

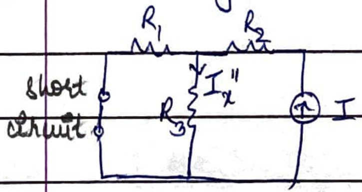


Case I When only voltage source V is acting



$$I_x' = \frac{V}{R_1+R_3}$$

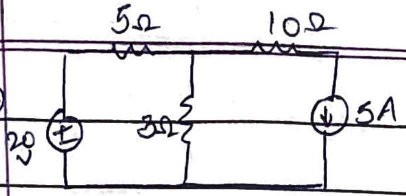
Case II When only current source I is acting



$$I_x'' = \frac{I R_1}{R_1+R_3}$$ current division rule

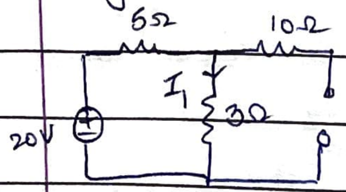
$$I_x = I_x' + I_x''$$

/19/20

Ques ① 

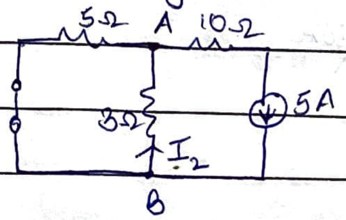
Find the current in 3Ω resistance using superposition theorem.

Case 1: only 20V is acting



$$I_1 = \frac{20}{5+3} = \frac{20}{8} = 2.5A$$

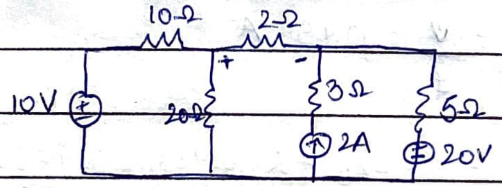
Case 2: When only 5A is acting



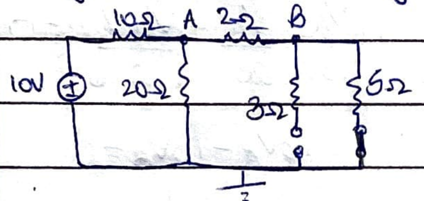
$$I_2 = \frac{5 \times 5}{5+3} = \frac{25}{8} = 3.125A$$

$$I_{net} = 3.15 - 2.5 = 0.625A \ (B \to A)$$

Ques ② Find voltage across 2Ω resistor using superposition theorem.



Case 1: only 10V is acting



$V_A = 10$

$$\frac{V_B - 10}{2} + \frac{V_B}{5} = 0$$

$$\frac{7V_B}{10} = 5$$

$V_B = 50/7$

$$\frac{V_A - 10}{10 \times 2} + \frac{V_A}{20} + \frac{V_A - V_B}{2 \times 10} = 0 \quad -①$$

$\Rightarrow 2V_A - 20 + V_A + 10V_A - 10V_B = 0$

$\Rightarrow 13V_A - 10V_B = 20$

$$I'_{2\Omega} = \frac{V_A - V_B}{2} = \frac{10 - 50/7}{2} = \frac{20}{7 \times 2}$$

$I'_{2\Omega} = 1.43A$

$\frac{V_B - V_A}{2} + \frac{V_B}{5} = 0$ — (2)

$\Rightarrow \frac{7V_B}{10} - \frac{5V_A}{10} = 0$

$\Rightarrow 7V_B = 5V_A$

$\Rightarrow V_A = 1.4V_B$

$13 \times 1.4V_B - 10V_B = 20$

$(18.2 - 10)V_B = 20$

$V_B = 2.44A$

$V_A = 3.416A$

$I'_{2\Omega} =$

Nodal Analysis

$\frac{V-10}{10} + \frac{V}{20} + \frac{V}{7} = 0$ — (1)

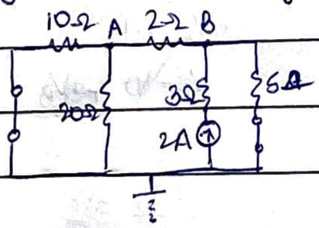
$0.293V = 1$

$V = 3.415V$

$I'_{AB} = \frac{V}{7} = 0.487 A$

$V_{AB}' = 2 \times I'_{AB} = 2 \times 0.487 = 0.976V$

Case 2: only 2A is acting



$\frac{V_A}{10\times2} + \frac{V_A}{20} + \frac{V_A - V_B}{2\times10} = 0$ — (1)

$\Rightarrow 13V_A - 10V_B = 0$

$\Rightarrow 1.3V_A = V_B$

current is fixed ∴ no $V_B/3$

$\frac{V_B - V_A}{2} + \frac{V_B}{5} + \cancel{\frac{V_B}{3}} - 2 = 0$ — (2)

$\cancel{\Rightarrow \frac{V_B - V_A}{2} + \frac{8V_B}{15} = 2}$

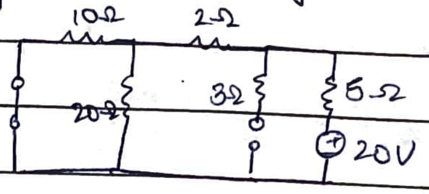
$\cancel{\Rightarrow 23V_B} \Rightarrow \cancel{31V_B - 15V_A = 60}$

$\Rightarrow 7V_B - 5V_A = 20$

$I'' = 0.732 A$

$V'_{BA} = 2 \times I'' = 1.464V$

case 3: only 20V is acting



$$I_{20V} = \frac{20}{\frac{20}{3}+7} = \frac{20}{41} \times 3 = 1.463A$$

$$V''_{BA} = 2 \times 1.463$$
$$= 2.926V$$

$$V_{2\Omega} = -0.976 + 1.464 + 2.926$$
$$= 3.414V \quad (B \to +ve \; ; \; A \to -ve)$$

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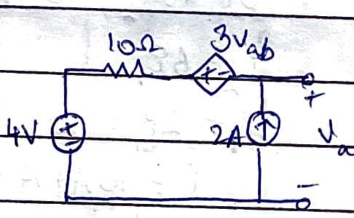
→ Superposition Theorem in presence of Dependent Source

- Dependent sources are kept as it is.
- No. of cases = No. of independent sources

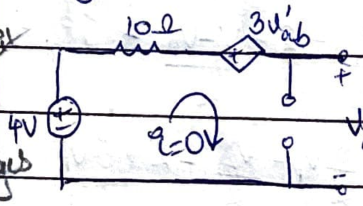
Ques. 6



find V_{ab} using superposition theorem.

Case 1: Only 4V source is acting

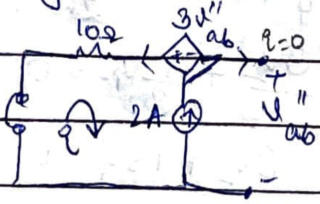
because of open circuit i=0 voltage also changes across ab



KVL ⇒

$$-4 + 10 \times i + 3V'_{ab} + V'_{ab} = 0$$
$$4V'_{ab} = 4.$$
$$V'_{ab} = 1V$$

Case 2: Only 2A source is acting



KCL ⇒

$$\frac{V''_{ab} - (-3V''_{ab})}{10} - 2 + 0 = 0$$

$$V''_{ab} = 5V$$

OR KVL ⇒

$10i + 3V_{ab}'' + V_{ab}'' = 0$

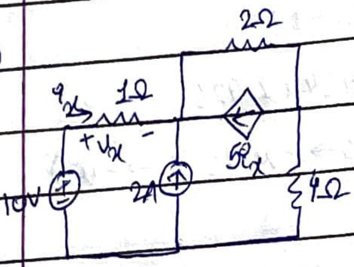
$i = -2A$

$V_{ab}'' = 5V$

According to Superposition theorem,

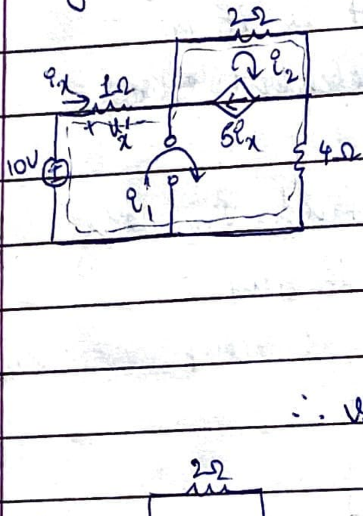
$V_{ab} = V_{ab}' + V_{ab}'' = 6V$

Ques ④



Find V_x using superposition theorem

Case 1: only 10V is acting



$i_x = i_1$

$i_2 - i_1 = 5i_x$ —(1)

$-10 + i_1 + 2i_2 + 4i_1 = 0$ —(2)

$5i_1 + 2i_2 = 10$

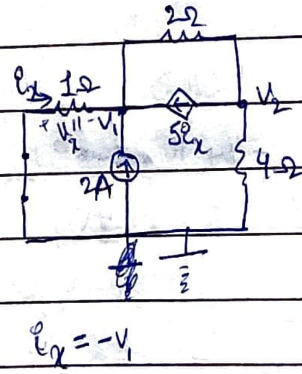
$i_2 = 6i_1$

$5i_1 + 12i_1 = 10$

$i_1 = 10/17$ A

$\therefore V_x' = \frac{10}{17} = 0.588V$

Case 2:



$i_x = -V_1$

$\frac{V_1}{1} - 2 + \frac{V_1 - V_2}{2} - 5i_x = 0$ —(1)

$\frac{3}{2}V_1 - \frac{V_2}{2} - 2 + 5V_1 = 0$

$\frac{V_2}{2} + 2 + \frac{13}{2}V_1 = 0$

$\frac{V_2}{4} + 5i_x + \frac{2\times V_2 - V_1}{2\times 2} = 0$ —(2)

$\frac{3V_2}{4} + \frac{9}{2\times 2}V_1 \times 2 = 0$

$3V_2 + 18V_1 = 0$

$V_2 = -6V_1$

$$-3V_1 + \frac{7}{10}V_1 + 2 = 0$$

$$2 = \frac{23}{10}V_1$$

$$V_1 = \frac{20}{23}$$

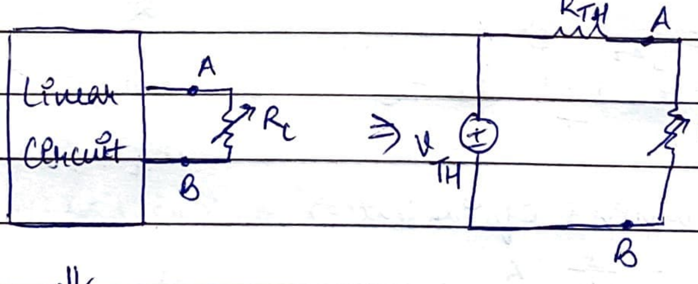
$$V_x'' = -0.706A$$

$$\text{Net } V_x = 0.588 - 0.706$$

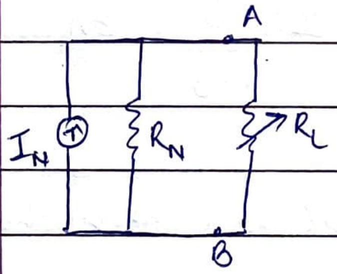
$$\underline{\underline{V_x = -0.118V}}$$

Thevenin's Theorem

- applicable when R_L is variable.
- calculate R_L for maximum power transfer to load.



⇓



Norton's Theorem ($R_N = R_{TH}$)

Thevenin's Theorem

→ It states that a linear two-terminal ~~linear~~ circuit can be represented by an equivalent circuit consisting of voltage source (V_{TH}) and a resistor (R_{TH}) in series.

V_{TH} : open circuit terminal voltage

R_{TH} : equivalent resistance looking into the terminals while load is disconnected & independent sources are inactive.

VS → short circuit ; CS → open circuit

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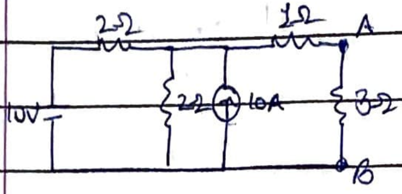
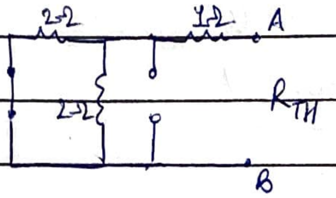
Ques. ① Find the current through 3Ω resistor using Thevenin's Theorem.



For R_{TH} ⇒

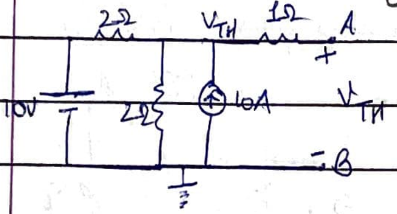


$R_{TH} = (2 \| 2) + 1 = 1 + 1$

$R_{TH} = 2\Omega$

For V_{TH} ⇒

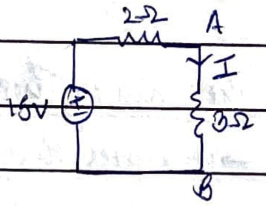
no V drop as I is 0 due to open circuit



$$\frac{V_{TH} - 10}{2} + \frac{V_{TH} - 10}{2} = 0$$

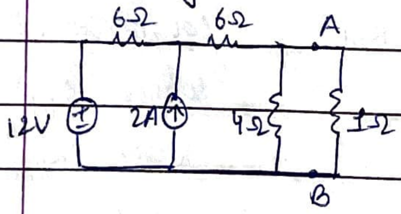
$V_{TH} = 15V$

Thevenin's Equivalent ⇒

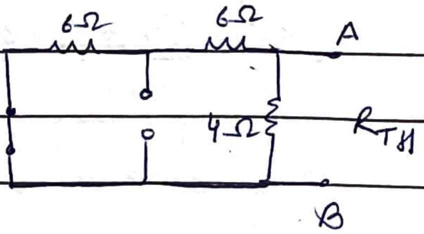


$$I = \frac{15}{2+3} = 3A$$

Ques. ② Find Thevenin's Equivalent and then the current I through 1Ω.



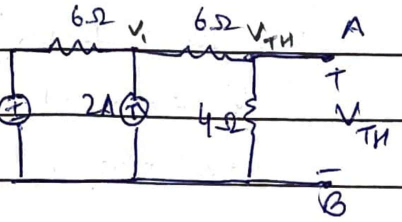
For R_{TH} ⇒



$R_{TH} = 3\Omega$

For. V_{TH} ⇒

V_{TH} is not a node because current through 6Ω & 4Ω remains same



$$\frac{-2+V_1-12}{6} + \frac{V_1}{6+4} = 0$$

$V_1 = 15V$

$$V_{TH} = 4 \times \frac{15}{10} = 6V$$

OR

$$\frac{V_1-12}{6} + \frac{V_1-V_{TH}}{6} - 2 = 0 \quad -(1)$$

$$2V_1 - V_{TH} = 24$$

$$\frac{V_{TH}-V_1}{6} + \frac{V_{TH}}{4} = 0 \quad -(2)$$

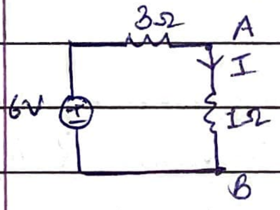
$$10V_{TH} - 4V_1 = 0$$

$$V_1 = \frac{5}{2}V_{TH}$$

$$\cancel{2} \times \frac{5}{\cancel{2}} V_{TH} - V_{TH} = 24$$

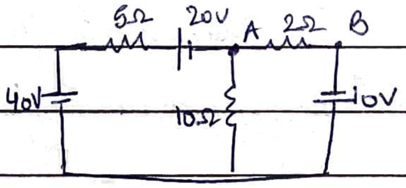
$$V_{TH} = 6V$$

Thevenin's Equivalent ⇒



$$I = \frac{6}{3+1} = 1.5A$$

Ques. (3) Calculate the current through 2Ω resistor using Thevenin's Theorem.



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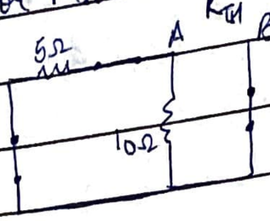
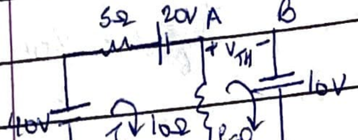
For R_{TH} ⇒



$R_{TH} = \frac{50}{15} = 3.33\Omega$

For V_{TH} ⇒



$V_{TH} - 10 - 10I = 0$ — ①

$V_{TH} = 10 + 10I$

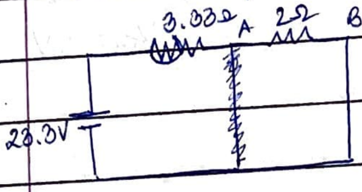
$-40 + 5I + 20 + 10I = 0$ — ②

$15I = 20$

$I = 4/3$

$V_{TH} = 10 + 40/3 = 70/3 = 23.3$

Thevenin's Equivalent ⇒



$I = \frac{23.33}{2 + 3.33} = 4.375A$

10/11/20 # Norton's Theorem

It states that a linear two-terminal circuit can be represented by an equivalent circuit consisting of a current source (I_N) and a resistor (R_N / R_{TH}) in parallel.

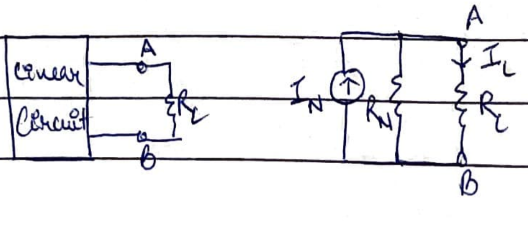
I_N → short circuit current through the terminals.

R_{TH} / R_N → same as R_{TH}.

Steps

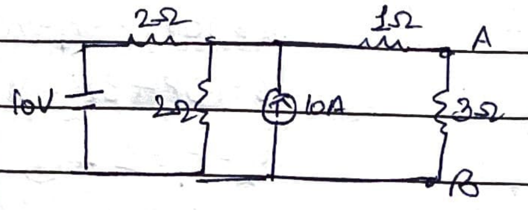
① Calculate I_{sc} or I_N

② Calculate R_N or R_{TH}



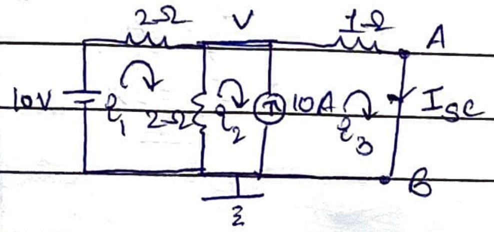
$I_L = \frac{I_N \times R_N}{R_N + R_L}$

Ques. ① Find the current through 3Ω resistor using Norton's Theorem.



$R_N = R_{TH} = 2\Omega$

for I_{sc} ⇒



$$-10+2i_1+2(i_1-i_2)=0 \quad -①$$
$$4i_1-2i_2=10$$
$$2i_1-i_2=5$$
$$i_3-i_2=10 \quad -②$$
$$i_2=i_1 \quad -③$$
$$i_1=i_2=5A$$
$$i_3=15$$

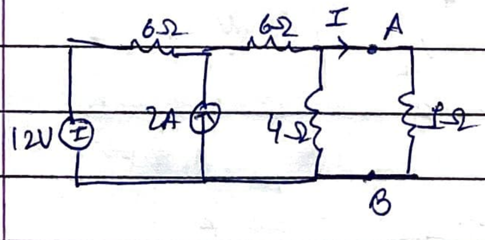
$$\frac{V-10}{2}+\frac{V}{2}+\frac{V}{1}-10=0$$

$$V = 7.5V$$

$$I_{sc} = \frac{7.5}{1} = 7.5A$$

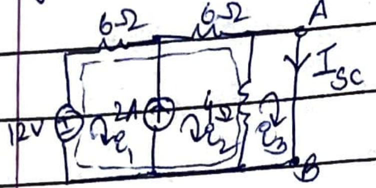
$$\therefore I_L = \frac{7.5\times 2}{2+3} = 3A$$

Ques. ② Find the current through 1Ω resistor using Norton's Theorem.



$R_N = R_{TH} = 3\Omega$

for $I_{sc} \Rightarrow$



$-12+6i_1+6i_2+4(i_2-i_3)=0$ — ①

$12=10i_2+6i_1-4i_3$

$6=3i_1+5i_2-2i_3$

$i_2-i_1=2$ — ②

$4(i_3-i_2)=0$ — ③

$i_3=i_2$

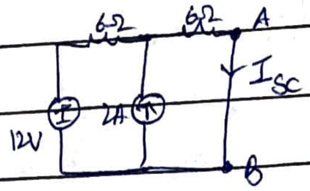
$6=3(i_3-2)+5i_3-2i_3$

$6=6i_3-6$

$i_3=2A$

$\therefore I_{sc}=2A$

OR



$\frac{4\times 0}{4+0}=0$

$\therefore$ short circuit in parallel with another resistance

$\frac{V-12}{6}+\frac{V}{6}-2=0$

$V=12V$

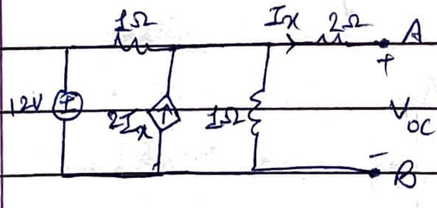
$I_{sc}=\frac{V}{6}=2A$

So, $I_L=\frac{2\times 3}{3+1}=1.5A$

→ Dependent Sources in Norton's Theorem

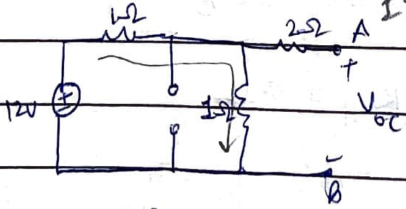
$R_{TH}=R_N=\frac{V_{oc}}{I_{sc}}$

Ques-③ Obtain Thevenin's & Norton's Equivalent of the circuit.



for $V_{oc} \Rightarrow$

$I_x=0$

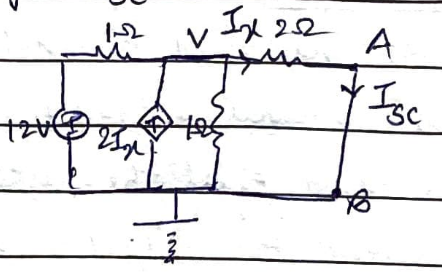


no V drop as I is 0.

$V_{oc}=\frac{12}{2}=6V$

series $1+1=2\Omega$

for I_{sc} ⇒



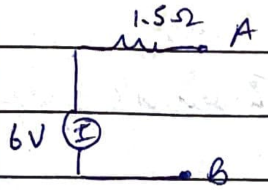
$I_{sc} = I_x = V/2$

$$\frac{V-12}{1} - 2I_x + \frac{V}{1} + \frac{V}{2} = 0$$

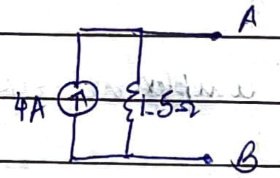
$V = 8V$

$I_x = 4A = I_{sc}$

$$R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{6}{4} = 1.5\Omega$$



Thevenin

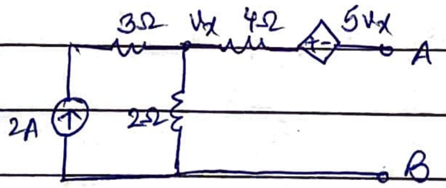


Norton

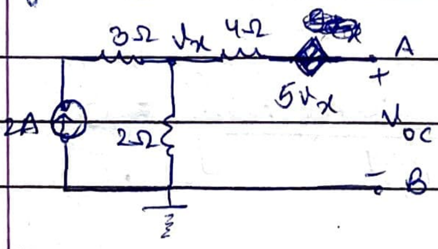
Ques. (4) Draw Thevenin's & Norton's Equivalent.

$V_{oc} = 16V$

$I_{sc} = 4A$



for V_{oc} ⇒



$$\frac{V_x}{2} - 2 + \frac{V_x}{4} = 0$$

$$\frac{3V_x}{4} = 2$$

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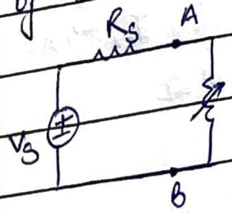
Maximum Power Transfer Theorem

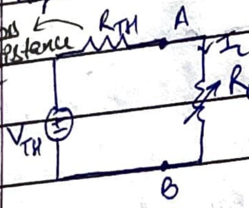
It states that to obtain maximum external power from a source with finite internal resistance, the resistance of the load must equal the resistance of source as viewed from its output terminals.



$R_L = R_S$

Replace the complex network by Thevenin's Equivalent.

power across ~~that~~ resistance is lost (50% lost)



$R_L = R_{TH}$

(50% gained)

Proof :

$$P = I_L^2 R_L = \left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2 R_L$$

$$\frac{dP}{dR_L} = 0$$

$$\Rightarrow V_{TH}^2 \frac{d}{dR_L}\left(\frac{R_L}{(R_{TH}+R_L)^2}\right)$$

$$\Rightarrow V_{TH}^2 \left[\frac{(R_{TH}+R_L)^2 . 1 - R_L . 2(R_{TH}+R_L) . 1}{(R_{TH}+R_L)^4}\right] = 0$$

$$\Rightarrow R_{TH} + R_L = 2R_L$$

$$\Rightarrow R_{TH} = R_L$$

$$\frac{d^2P}{dR_L^2} < 0$$

$$\frac{d^2P}{dR_L^2} = \frac{-V_{TH}^2}{(R_{TH}+R_L)^3}$$

$$P_{max} = I_L^2 R_{TH}$$

$$= \frac{V_{TH}^2}{(R_{TH}+R_{TH})^2} R_{TH}$$

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$$

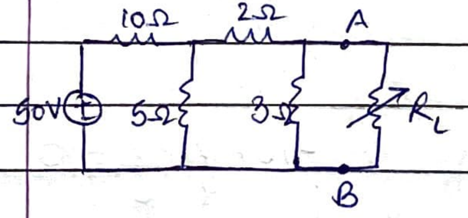
$$\eta = \frac{Output}{Input} = \frac{P_{max}}{P_{max}+P_{loss}}$$

$$P_{loss} = I_L^2 \cdot R_{TH}$$

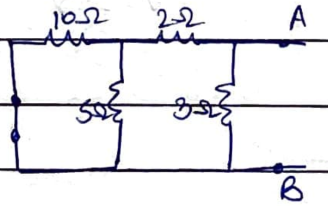
$$= \frac{V_{TH}^2}{4R_{TH}}$$

$$\therefore \eta = 0.5 \text{ or } 50\%.$$

Ques. ① Find the max. power that can be transferred to R_L.

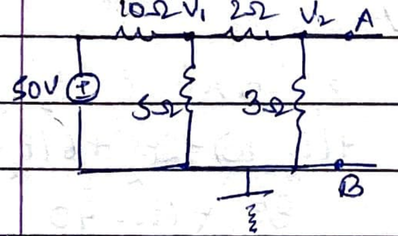


for R_{TH}



$R_{TH} = 1.92\Omega$

for V_{TH}



$$\frac{V_1-50}{10} + \frac{V_1\times 2}{5\times 2} + \frac{V_1-V_2\times 5}{2\times 5} = 0 \quad -①$$

$$\frac{8V_1}{10} - \frac{5V_2}{10} = 5$$

$$8V_1 - 5V_2 = 50$$

$$\frac{V_2-V_1}{2\times 3} + \frac{V_2}{3\times 2} = 0$$

$$\frac{5V_2}{6} - \frac{3V_1}{6} = 0$$

$$5V_2 = 3V_1$$

$$V_1 = 10V$$

$$V_2 = 6V$$

$$\therefore V_{TH} = V_2 = 6V$$

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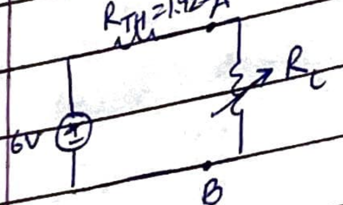
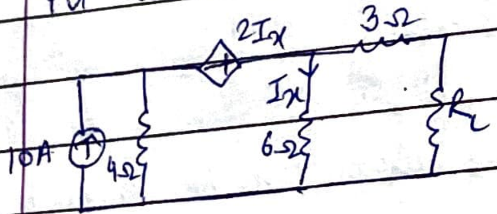
Thevenin's Equivalent ⇒

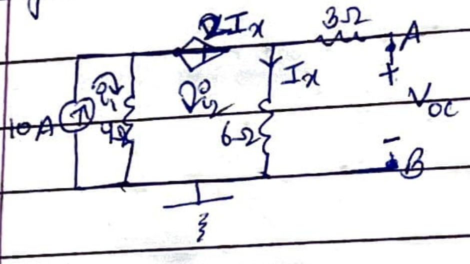


$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{6^2}{4\times1.92} = 4.6875\ W$$

Ques. ② Calculate the max. power that may be dissipated in load resistance R_L.



for V_{oc} ⇒



$-10 + \frac{V}{4} + \frac{V}{6} = 0$

$10 = \frac{10V}{24}$

$V = 24V$

$i_1 = 10A, \quad I_x = i_2$

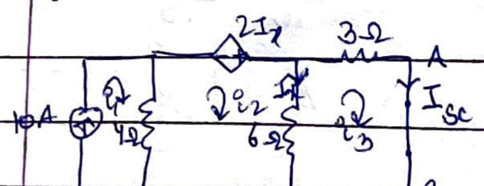
$4(i_2 - i_1) - 2I_x + 6I_x = 0$

$8I_x = 4\times10$

$I_x = 5A$

$\therefore V_{oc} = 6\times5 = 30V$

for I_{sc} ⇒



$i_1 = 10\,A, \quad i_x = i_2 - i_3$

$4(i_2 - 10) - 2(i_2 - i_3) + 6(i_2 - i_3) = 0$ -(1)

$8i_2 - 6i_3 = 40$

$3i_3 + 6(i_3 - i_2) = 0$ -(2)

$9i_3 = 6i_2$

$\frac{3}{2}i_3 = i_2$

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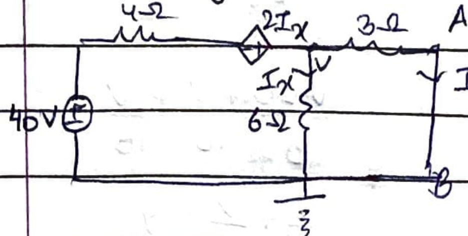
CS dual of VS →



$$\frac{V-2I_x-40}{4}+\frac{V}{6}+\frac{V\times 2}{3\times 2}=0$$

$$I_x=\frac{V}{6}$$

$$\frac{V}{4}-\frac{V}{12}+\frac{V}{2}=10$$

$$V=15V$$

$$\therefore I_{sc}=\frac{V}{3}=5A$$

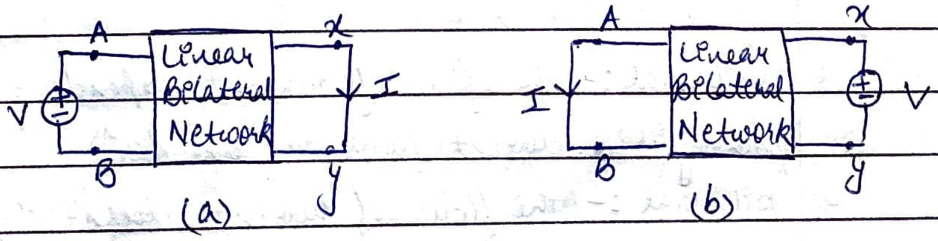
$$R_{TH}=\frac{V_{oc}}{I_{sc}}=\frac{30}{5}=6\Omega$$

$$P_{max}=\frac{V_{TH}^2}{4R_{TH}}=\frac{(30)^2}{4\times 6}=\underline{37.5W}$$

17/11/20

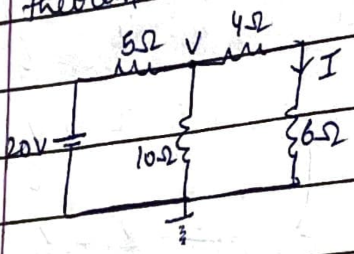
Reciprocity Theorem

It states that in a linear bilateral, active single source network, the ratio of excitation and response remains same when excitation and response positions are interchanged.



Trans-resistance → $\left.\frac{V}{I}\right|_{(a)}=\left.\frac{V}{I}\right|_{(b)}$ → Network is reciprocal

Ques. ① Calculate current I and verify reciprocity theorem.



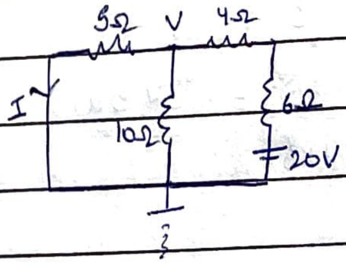
$$\frac{V-20}{5}+\frac{V}{10}+\frac{V}{10}=0$$

$$\frac{V}{5}+\frac{V}{5}=4$$

$$\frac{2V}{5}=4$$

$$V=10V$$

$$\therefore I=\frac{V}{10}=1A$$



$$\frac{V\times 2}{5\times 2}+\frac{V}{10}+\frac{V-20}{10}=0$$

$$\frac{3V}{10}+\frac{V}{10}=2$$

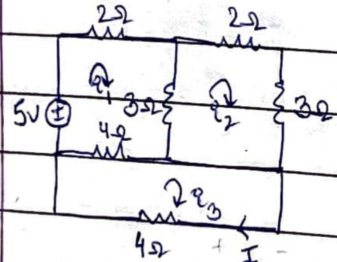
$$V=\frac{2\times 10}{4}=5V$$

$$\therefore I=\frac{V}{5}=1A$$

So, this network is reciprocal.

* Unilateral :- the flow of current depends on the polarity (i.e. current flows in one dirⁿ).
* Bilateral :- the flow of current doesn't depend on the terminal (i.e. current can flow in both dirⁿ)

Ques ② Find current I and validate the reciprocity theorem



$$i_3 = I$$

$$-5+2i_1+3i_1-3i_2+4i_1-4i_3=0 \text{ ①}$$

$$9i_1-3i_2-4i_3=5$$

$$2i_2+\cancel{3i_2}+3i_2-3i_1=0 \text{ ②}$$

$$8i_2=3i_1$$

$4i_3+4i_3-4i_1=0$ —③

$8i_3=4i_1$

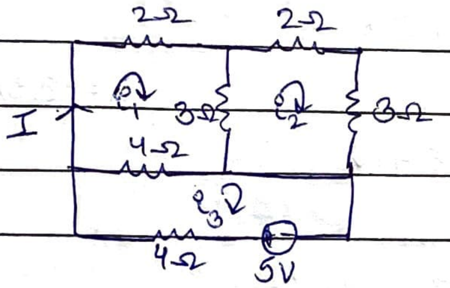
$2i_3=i_1$

$i_2=\frac{3i_1}{8}=\frac{3\times 2i_3}{\cancel{8}\,4}=\frac{3}{4}i_3$

$9\times 2i_3-3\times\frac{3}{4}i_3-4i_3=5$

$14i_3-\frac{9}{4}i_3=5$

$i_3=0.4255A=I$



$I=i_1$

$2i_1+3i_1-3i_2+4i_1-4i_3=0$ —①

$9i_1-3i_2-4i_3=0$

$5i_2+3i_2-3i_1=0$ —②

$8i_2=3i_1$

$5+4i_3=0$

$i_3=-1.25A$

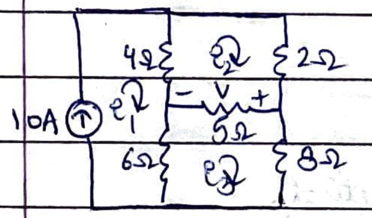
$i_2=\frac{3}{8}i_1$

$9i_1-\frac{9}{8}i_1+5=0$

$i_1=0.4255A=I$

∴ this network is reciprocal

Ques ⑤ Find voltage V and verify reciprocity theorem.



$i_1=10A$

$V=(i_2-i_3)5$

$4i_2-40+2i_2+5i_2-5i_3=0$ —①

$11i_2-5i_3=40$

$8i_3+6i_3-6i_1-5i_2+5i_3=0$ —②

$19i_3-5i_2=60$

$\cancel{25i_3+5i_2=200}$

$\cancel{16i_3=140}$

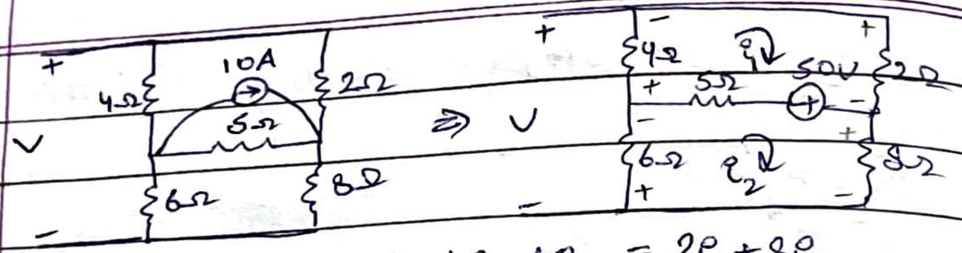
$\cancel{i_3=8.75A}$

$\cancel{i_2}$

$i_2=5.76A$

$i_3=4.67A$

∴ $V=5.45V$



$$V = -4i_1 - 6i_2 = 2i_1 + 8i_2$$

$$4i_1 + 2i_1 + 50 + 5i_1 - 5i_2 = 0 \quad -(1)$$

$$5i_2 - 11i_1 = 50$$

$$i_2 = \frac{50 + 11i_1}{5}$$

$$6i_2 + 5i_2 - 5i_1 - 50 + 8i_2 \quad (2)$$

$$19i_2 - 5i_1 = 50$$

$$19\left(10 + \frac{11i_1}{5}\right) - 5i_1 = 50$$

$$140 = 5i_1 - \frac{19 \times 11 i_1}{5}$$

$$i_2 = 1.64A \qquad i_1 = -3.8A$$

$$V = 5.44V$$

19/11/20

Negative Resistance in Thevenin (this is possible)

$V_{oc} = V_{TH}$

$I_{sc} = I_N$

$R_{TH} = \frac{V_{oc}}{I_{sc}} \rightarrow -ve$

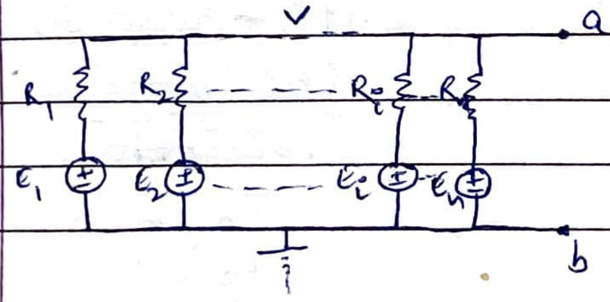
Load resistance → delivering power to circuit

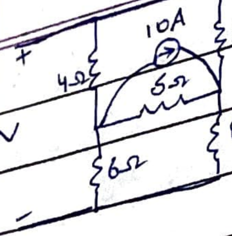
$I_L = \frac{V_{TH}}{R_{TH} + R_L}$

R_{TH} ↓ -ve value

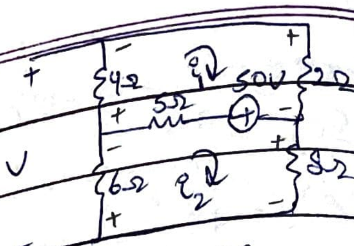
Milliman's Theorem

- application of KCL / nodal method.
- source transformation.





$\Rightarrow$



$V = -4i_1 - 6i_2 = 2i_1 + 8i_2$

$4i_1 + 2i_1 + 50 + 5i_1 - 5i_2 = 0$ — (1)

$5i_2 - 11i_1 = 50$

$i_2 = \frac{50 + 11i_1}{5}$

$6i_2 + 5i_2 - 5i_1 - 5...$

$19i_2 - 5i_1 = 5...$ — (2)

$19\left(\frac{10 + 11i_1}{5}\right) - 5i_1 = 5...$

$140 = 5i_1 - \frac{19 \times 11 i_1}{5}$

$i_1 = -3.8A$

$i_2 = 1.64A$

$V = 5.44V$

19/11/20

Negative Resistance in Thevenin (this is possible)

$V_{oc} = V_{TH}$

$I_{sc} = I_N$

$R_{TH} = \frac{V_{oc}}{I_{sc}} \rightarrow -ve$

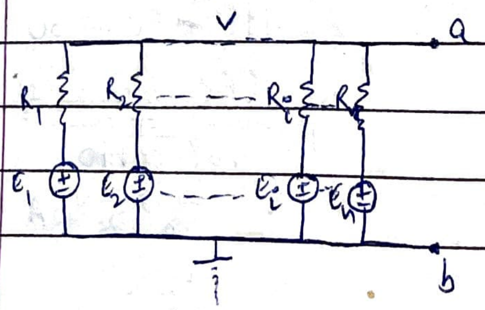
Load resistance $\rightarrow$ delivering power to circuit

$I_L = \frac{V_{TH}}{R_{TH} + R_L}$

R_{TH} ↓ -ve value

Millman's Theorem

- application of KCL / nodal method.
- source transformation.



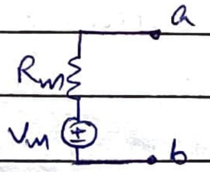
$$\frac{V-E_1}{R_1} + \frac{V-E_2}{R_2} + \cdots + \frac{V-E_i}{R_i} + \frac{V-E_n}{R_n} = 0 \quad \text{(nodal analysis)}$$

$$V = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \cdots + \frac{E_i}{R_i} + \cdots + \frac{E_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_i} + \cdots + \frac{1}{R_n}}$$

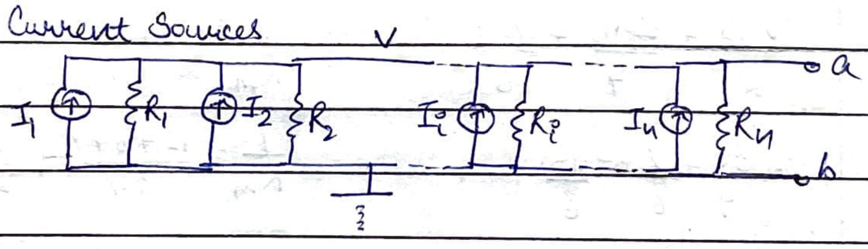
$V_M = V \rightarrow$ Milleman Voltage

$$\frac{1}{R_M} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

$R_M \rightarrow$ Milleman Resistance



→ Current Sources

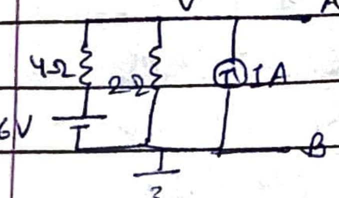


$$\frac{V}{R_1} - I_1 + \frac{V}{R_2} - I_2 + \cdots + \frac{V}{R_i} - I_i + \cdots + \frac{V}{R_n} - I_n = 0$$

$$V\left[\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right] = I_1 + I_2 + \cdots + I_i + \cdots + I_n$$

$$V = \frac{I_1 + I_2 + \cdots + I_1 + \cdots + I_n}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_i} + \cdots + \frac{1}{R_n}} \checkmark$$

Ques-① Find Millman's eq. b/w terminals AB.



$$\frac{V-6}{4} + \frac{V}{2} - 1 = 0$$

$$\frac{V}{4} + \frac{V\times 2}{2\times 2} = 1 + 3/2$$

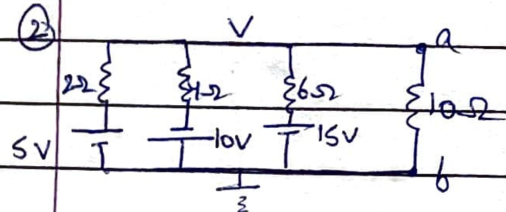
$$\frac{3V}{4} = \frac{5}{2}$$

$$V_m = \frac{5}{2} \times \frac{4}{3}^{2} = \frac{10}{3} = 3.33V$$

$$\frac{1}{R_m} = \frac{1}{4} + \frac{1\times 2}{2\times 2} = \frac{3}{4}$$

$$R_m = \frac{4}{3} = 1.33\Omega$$

② Find current through 10Ω resistor.



$$\frac{V-5}{2} + \frac{V+10}{4} + \frac{V-15}{6} = 0$$

$$\frac{1}{R_m} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6}$$

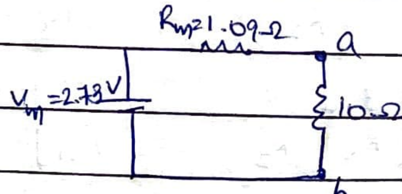
$$\frac{3V}{4} + \frac{V}{6} = \frac{15}{6}$$

$$\frac{1}{R_m} = \frac{12+6+4}{24} = \frac{22}{24}$$

$$\frac{22V}{24} = \frac{15}{6}$$

$$R_m = 1.09\Omega$$

$$V_m = 2.73V$$



$$I_{10\Omega} = \frac{2.73}{10+1.09} = 0.246A$$

$$V = \frac{\frac{5}{2} - \frac{10}{4} + \frac{15}{6}}{\frac{1}{2} + \frac{1}{4} + \frac{1}{6}}$$

$$= \frac{5}{2} \times \frac{24}{22}$$

$$= \frac{30}{11}$$

$$= 2.73V$$

11/20 **Ch.2 - Steady State Analysis of DC Circuits**

Transient Analysis

- Energy storage component
  1. inductor
  2. capacitor
- Switching operations

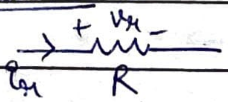
- Differential equations.

↓

Transient & Steady State solution.

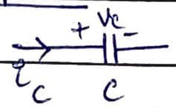
time dependent ↙ ↘ time independent

★ Resistance



$v_R = i_R R$

★ Capacitor



$$\boxed{i_c = C\frac{dv_c}{dt}}$$

it opposes instantaneous change in voltage.

$$v_c(0^-) = v_c(0^+)$$

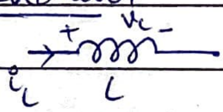
when switching is done at $t=0$ ⇒

0^- → before closing / opening

0^+ → after closing / opening

Under steady state, capacitor behaves as open circuit

★ Inductor



$$\boxed{v_L = L\frac{di_L}{dt}}$$

it opposes the instantaneous change in current.

$$i_L(0^-) = i_L(0^+)$$

Under steady state, inductor behaves as short circuit

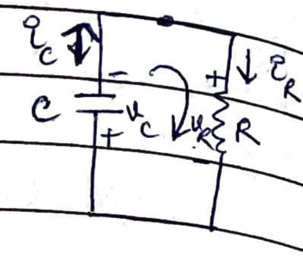
RL / RC → 1st order differential eqn

RLC / LC → 2nd order differential eqn

$$y = \underset{\text{(natural)}}{\text{transient}} + \underset{\text{(forced)}}{\text{steady state}}$$

Source free → discharging
with source → charging

Source Free RC Circuit



as R & C are in series

$$v_c(0) = V_0$$

$$i_R = i_c$$

$$v_c + v_R = 0$$

$$v_c + i_R R = 0$$

$$i_c = \frac{C\,dv_c}{dt} = i_R$$

$$v_c + \frac{CR\,dv_c}{dt} = 0$$

$$\frac{CR\,dv_c}{dt} = -v_c$$

$$\int \frac{dv_c}{v_c} = \int -\frac{1}{RC}\,dt$$

$$\ln v_c = \frac{-t}{RC} + k$$

at $t=0 \Rightarrow v_c(0) = V_0$

$$\ln V_0 = k$$

$$\ln v_c = \frac{-t}{RC} + \ln V_0$$

$$\ln v_c - \ln V_0 = \frac{-t}{RC}$$

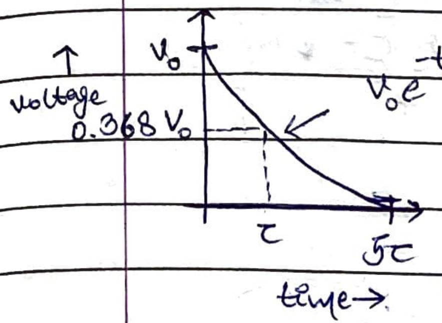
$$\ln \frac{v_c}{V_0} = \frac{-t}{RC}$$

$$\frac{v_c}{V_0} = e^{-t/RC}$$

$$v_c = V_0 e^{-t/\tau}$$

[where $\tau = RC$]
↓
time constant

$$\boxed{i_c = C\frac{dV_c}{dt} = -\frac{V_0}{R}e^{-t/\tau}}$$



$$V_c(t) = V_0 e^{-t/\tau}$$

$$i_c(t) = -\frac{V_0}{R}e^{-t/\tau}$$

$$\frac{V_c(t)}{V_0} = e^{-t/\tau}$$

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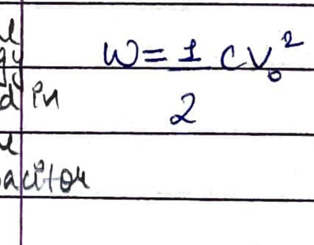
Rate of decay of voltage → $\frac{d}{dt}\left(\frac{V_c(t)}{V_0}\right) = \frac{-1}{\tau}e^{-t/\tau}$

Initial rate of decay, $t=0$

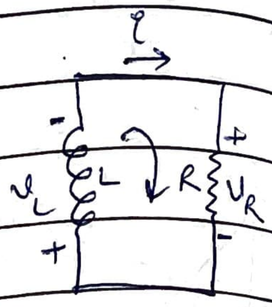
$$\frac{d}{dt}\left(\frac{V_c(t)}{V_0}\right) = \frac{-1}{\tau}$$

Smaller value of τ → fast decay



initial energy stored in the capacitor $W = \frac{1}{2}CV_0^2$

Source Free RL Circuit



$i_L(0) = I_0$

$w_L(0) = \frac{1}{2} L I_0^2$

$v_L + v_R = 0$

$L\frac{di}{dt} + iR = 0$

$L\frac{di}{dt} = -iR$

$\frac{di}{dt} = -\frac{R}{L} i$

$\int \frac{di}{i} = \int -\frac{R}{L} dt$

$\ln i = -\frac{R}{L} t + A$

at $t = 0 \Rightarrow i(0) = I_0$

$\ln I_0 = A$

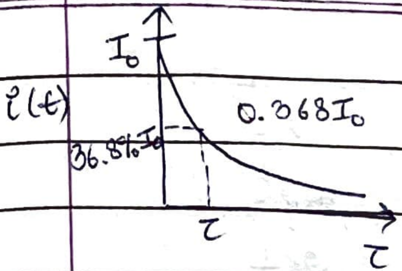
$\ln i - \ln I_0 = -\frac{Rt}{L}$

$\ln \frac{i}{I_0} = \frac{-Rt}{L}$

$\frac{i}{I_0} = e^{-\frac{R}{L}t}$

$i = I_0 e^{-t/\tau}$

[where $\tau = L/R$]



$$\frac{i}{I_0} = e^{-Rt/L}$$

$$\frac{d}{dt}\left(\frac{i}{I_0}\right) = -\frac{R}{L} e^{-Rt/L}$$

$$= -\frac{1}{\tau} e^{-t/\tau}$$

$$= -\frac{1}{\tau}\Big|_{t=0}$$

$$v_R(t) = iR = I_0 R e^{-t/\tau}$$

$$v_L(t) = L\frac{di}{dt} = -\frac{L I_0}{\tau} e^{-t/\tau} = -R I_0 e^{-t/\tau}$$

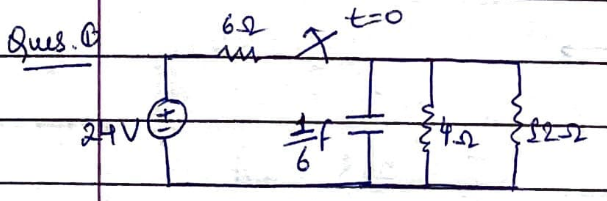
** **NOTE:** $R_{eq} \rightarrow$ across terminals of L/C.

$\tau = R_{eq} C$ $\qquad$ $\tau = \frac{L}{R_{eq}}$

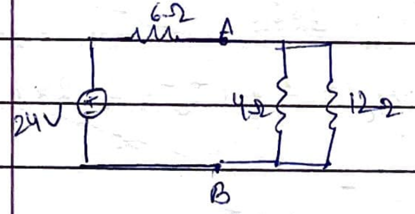
$\downarrow$ $\qquad$ $\downarrow$

RC circuit $\qquad$ RL circuit

Ques. 8



For capacitor, find $v(t)$, $i(t)$, $w_c(0)$.



$$\therefore V_{AB} = \frac{24 \times 3}{3+6} = 8\text{ V}$$

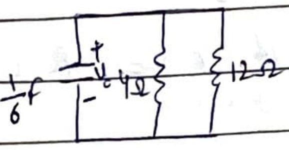
$v_0 = 8V$

$$w_c(0) = \frac{1}{2} C V_0^2$$

$$= \frac{1}{2} \times \frac{1}{6} \times 64$$

$$= 5.33\text{ J}$$

After switching →



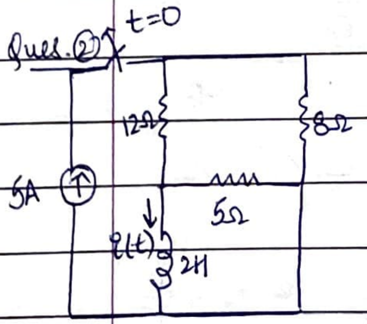
$$v_c(t) = v_o e^{-t/\tau}$$

$$\tau = R_{eq} C = 3 \times \frac{1}{6} = 0.5\,sec$$

$$v_c(t) = 8e^{-2t}\ V$$

$$i_c(t) = -\frac{V_o}{R_{eq}} e^{-t/\tau} = -\frac{8}{3} e^{-2t}\ A$$

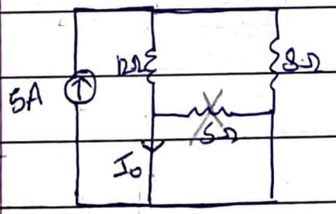
Ques. ② find $i(t)$ for $t > 0$.



$$i(t) = I_o e^{-t/\tau}$$

Before switching →

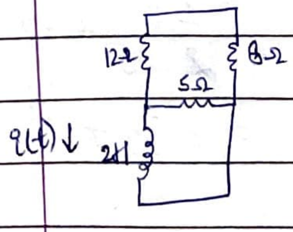
no current through 5Ω as there is short circuit (least resistive path)



$$I_o = \frac{5 \times 8}{12+8} = 2A$$

$$w_L(0) = \frac{1}{2} L I_o^2 = \frac{1}{2} \times 2 \times 4 = 4J$$

After switching →



$$\tau = \frac{L}{R_{eq}} = \frac{2}{\frac{20 \times 5}{20+5}} = \frac{2}{4} = \frac{1}{2}\ sec.$$

$$i(t) = 2e^{-2t}\ A$$

$$v_L(t) = -I_o R_{eq} e^{-t/\tau} = -8e^{-2t}\ V$$

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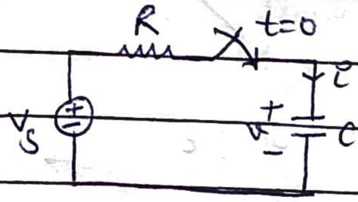
Step Response of RC Circuit



Initial voltage (v)

$v(0^-) = v(0^+) = V_0$

at $t=0$ switch is closed

$t > 0$

$-V_S + iR + v = 0 \qquad \because i = C\frac{dv}{dt}$

$$-V_S + RC\frac{dv}{dt} + v = 0$$

$$v - V_S = -RC\frac{dv}{dt}$$

$$\int \frac{dv}{v - V_S} = \frac{-1}{RC}\int dt$$

$$\ln(v - V_S) = \frac{-1}{RC}t + A$$

at $t=0$, $v(0^-) = v(0^+) = V_0$

$$\ln(V_0 - V_S) = A$$

$$\ln(v - V_S) = \frac{-t}{RC} + \ln(V_0 - V_S)$$

$$\ln\left(\frac{v - V_S}{V_0 - V_S}\right) = \frac{-t}{RC}$$

$$\frac{v - V_S}{V_0 - V_S} = e^{-t/RC}$$

$$v - V_S = (V_0 - V_S)e^{-t/RC}$$

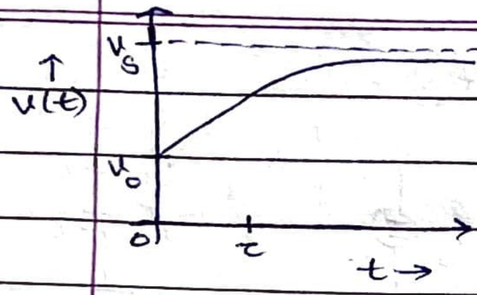
$$v = V_S + (V_0 - V_S)e^{-t/RC}$$

$$v(t) = V_S + (V_0 - V_S)e^{-t/\tau}$$

(steady state) forced response ← V_S ; $(V_0 - V_S)e^{-t/\tau}$ → Natural response (transient response)

[where $\tau \rightarrow$ time constant $= RC$]

$$v(t) = \begin{cases} V_0, & t \le 0 \\ V_S + (V_0 - V_S)e^{-t/\tau}, & t \ge 0 \end{cases}$$



$V_s = v(\infty)$, $V_o = v(0)$

$$\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}}$$

If capacitor is initially uncharged,

$$v(0) = 0$$

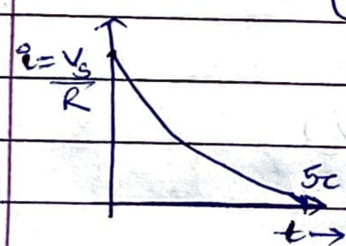
$$v(t) = V_s(1 - e^{-t/\tau})$$

$$i = C\frac{dv}{dt}$$

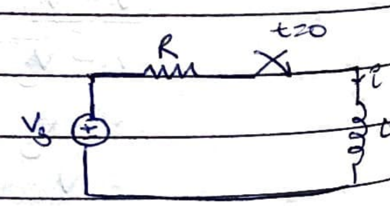
$$i = -C V_s \cdot e^{-t/\tau} \cdot \frac{-1}{\tau}$$

uncharged $\boxed{i = \frac{V_s e^{-t/\tau}}{R}}$

charged $\boxed{i = -\left(\frac{V_o - V_s}{R}\right)e^{-t/\tau}}$



Step Response of RL Circuit



Initial inductor current

$$i(0^-) = i(0^+) = I_0$$

$$t > 0$$

$$-V_s + Ri + L\frac{di}{dt} = 0$$

at $t = 0$, $i(0) = I_0$

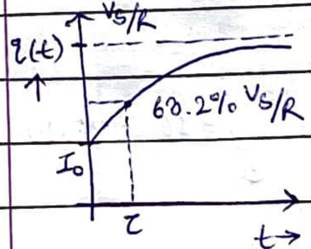
Page No.
Date

$$i(t) = \frac{V_S}{R} + \left(I_0 - \frac{V_S}{R}\right) e^{-t/\tau}$$

$[\tau = L/R]$

$i(\infty) = \frac{V_S}{R}$ = steady state value of current

$$\boxed{i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau}}$$



$$i(t) = \begin{cases} I_0, & t \le 0 \\ \frac{V_S}{R} + \left(I_0 - \frac{V_S}{R}\right) e^{-t/\tau}, & t \ge 0 \end{cases}$$

If inductor is initially uncharged,

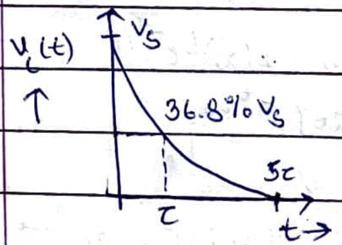
$I_0 = 0$

$$i(t) = \begin{cases} 0, & t \le 0 \\ \frac{V_S}{R}\left(1 - e^{-t/\tau}\right), & t \ge 0 \end{cases}$$

$$V_L(t) = L\frac{di}{dt}$$

$$V_L(t) = L \cdot \frac{V_S}{R\tau} e^{-t/\tau}$$

$$\boxed{V_L(t) = V_S e^{-t/\tau}}$$



★ NOTE: RC → $\tau = R_{TH} C$ $\quad$ R_{TH} → equivalent resistance seen from capacitor / inductor

RL → $\tau = \frac{L}{R_{TH}}$

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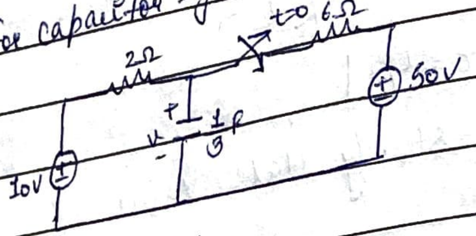
Ques. ① For capacitor find $v(t)$, $i_c(t)$, and $w_c(0)$.



Before switching →

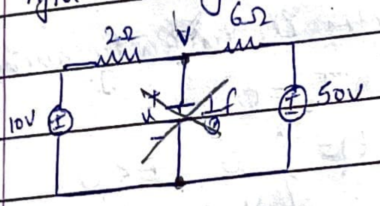
$w_c(0) = \frac{1}{2} C V_0^2$.

$v(0^-) = v(0^+) = 10V$

$w_c(0) = \frac{1}{2} \times \frac{1}{3} \times 10 \times 10 = \frac{100}{6} = \frac{50}{3} = 16.67$ Joules

After switching →

at steady state $t=\infty$ capacitor acts as open circuit



$\frac{V-10}{2} + \frac{V-50}{6} = 0$

$\frac{2V}{\cancel{3}} = 5 + \frac{25}{3} = \frac{40}{\cancel{3}}$

$V = 20V$

$\therefore v(\infty) = 20V$

$R_{eq} = \frac{6 \times 2}{6+2} = 1.5\Omega$ $\quad \tau = CR_{eq} = \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{2} = 0.5$

So, $v(t) = v(\infty) + [v(0) - v(\infty)] e^{-t/\tau}$

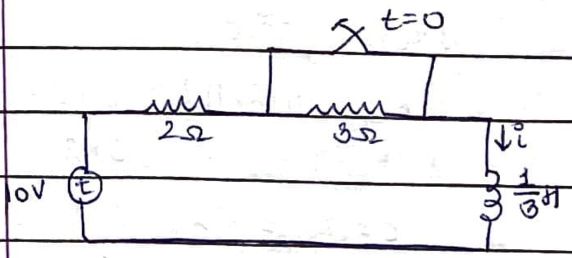
$= 20 + [10 - 20] e^{-t/0.5}$

$= 20 - 10e^{-2t}$ V

$i(t) = C\frac{dv}{dt} = \frac{1}{3}\left[-10e^{-2t} \cdot (-2)\right] = \frac{20}{3} e^{-2t}$

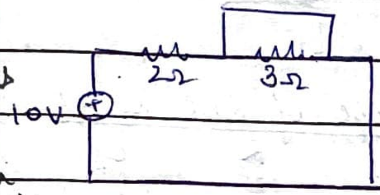
$i(t) = 6.67e^{-2t}$ A

Ques. ② Find current $i(t)$



Before switching →

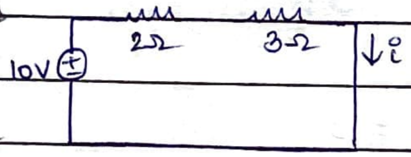
inductor behaves as short circuit as it is closed for long time



$$\therefore\ i = \frac{10}{2} = 5A = i(0^-) = i(0^+)$$

After switching →

inductor behaves as short circuit as it is opened for long time at $t=\infty$



$$i(\infty) = \frac{10}{5} = 2A$$

$$R_{eq} = 2+3 = 5\Omega$$

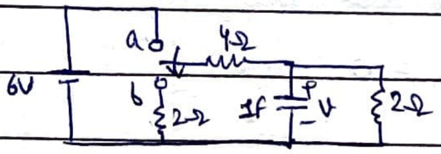
$$\tau = \frac{L}{R_{eq}} = \frac{1}{3\times5} = \frac{1}{15}\ s$$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$i(t) = 2 + [5-2]e^{-15t}$$

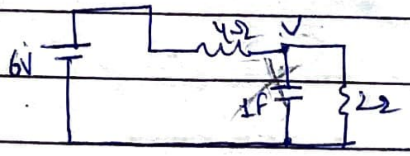
$$= 2 + 3e^{-15t}\ A$$

Ques. ⑬ In the network shown in fig., the switch is moved from a to b at $t=0$. Find $v(t)$.



Before switching →

capacitor is open circuit

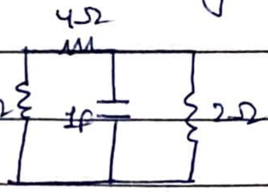


$$\frac{V-6}{4} + \frac{V}{2} = 0$$

$$\frac{3V}{4} = \frac{6}{4}$$

$$V = 2V \quad \therefore\ v(0^+) = v(0^-) = 2V$$

After switching →



source free RC circuit

$v(t) = V_0 e^{-t/\tau}$

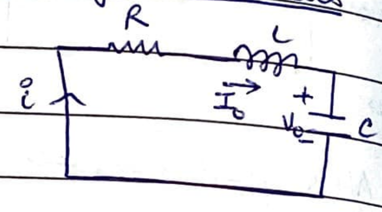
$R_{eq} = (4+2) \parallel 2 = \frac{6 \times 2}{6+2} = 1.5\Omega$

$\tau = C R_{eq} = 1.5\, s$

$v(t) = 2e^{-t/1.5}\ V$

1/12/20

Natural / Source free response of RLC series



at $t = 0^-$,

inductor current − I_0

capacitor voltage − V_0

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt = 0$$

at $t = 0^+$,

$i = I_0$, $V_c = V_0$

$$RI_0 + L\frac{di}{dt} + V_0 = 0$$

$$\left.\frac{di}{dt}\right|_{t=0} = -\frac{1}{L}(RI_0 + V_0)$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt = 0$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

characteristic equation

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

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Standard form →

$$s^2 + 2\alpha s + \omega_n^2 = 0$$

or $$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

omega ← ω_n → natural frequency

ζ → damping ratio

zeta

$$\alpha = \zeta\omega_n$$

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{4}{4LC}} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_n^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_n^2}$$

$$\omega_n = \frac{1}{\sqrt{LC}}, \quad \alpha = \frac{R}{2L}, \quad \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

Case ① Overdamped

$\alpha > \omega_n$ or $\zeta > 1$

Case ② Critically damped

$\alpha = \omega_n$ or $\zeta = 1$

Case ③ Under damped

$\alpha < \omega_n$ or $\zeta < 1$

★ Overdamped

$\alpha > \omega_n$ or $\zeta > 1$

s_1 & s_2 are real but negative

$$i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

★ Critically damped

$\alpha = \omega_n$ or $\zeta = 1$

s_1 & s_2 are equal and negative

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$$\frac{d^2i}{dt^2} + 2\alpha\frac{di}{dt} + \omega_n^2 i = 0$$

$\omega_n = \alpha$

$$\frac{d^2i}{dt^2} + 2\alpha\frac{di}{dt} + \alpha^2 i = 0$$

$$\Rightarrow \frac{d^2i}{dt^2} + \alpha\frac{di}{dt} + \alpha\frac{di}{dt} + \alpha^2 i = 0$$

$$\Rightarrow \frac{d}{dt}\left[\frac{di}{dt} + \alpha i\right] + \alpha\left[\frac{di}{dt} + \alpha i\right] = 0$$

$$\Rightarrow \underbrace{\left[\frac{di}{dt} + \alpha i\right]}_{f}\left[\frac{d}{dt} + \alpha\right] = 0$$

$$\Rightarrow \frac{df}{dt} + \alpha f = 0$$

$$f = k_1 e^{-\alpha t}$$

$$\frac{di}{dt} + \alpha i = f = k_1 e^{-\alpha t}$$

premultiply by $e^{\alpha t}$

$$e^{\alpha t}\frac{di}{dt} + \alpha i e^{\alpha t} = k_1$$

$$\int \frac{d}{dt}\left(ie^{\alpha t}\right) = \int k_1$$

$$\int d\left(ie^{\alpha t}\right) = \int k_1\,dt$$

$$ie^{\alpha t} = k_1 t + k_2$$

$$i(t) = (k_1 t + k_2)e^{-\alpha t}$$

★ **Underdamped**

$\alpha < \omega_n$ or $\xi < 1$

s_1 & s_2 are complex and conjugate

$$i(t) = e^{-\alpha t}\left[(k_1 + k_2)\cos\omega_d t + j(k_1 - k_2)\sin\omega_d t\right]$$

$$\omega_d = \sqrt{\omega_n^2 - \alpha^2} = \omega_n\sqrt{1-\xi^2}$$

- Inductor opposes the change in instantaneous current.
- capacitor opposes the instantaneous change in voltage.

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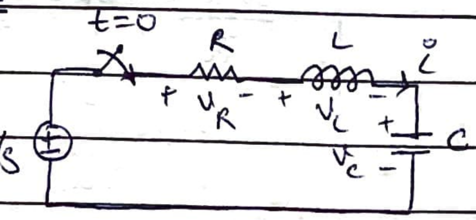
Step Response of RLC Series



$-V_s + V_R + V_L + V_c = 0$

$V_R + V_L + V_c = V_s$

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_s \quad -①$$

differentiating

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

i → variable

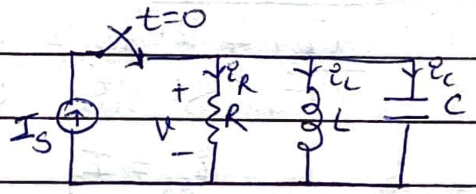
$$RC\frac{di}{dt} + LC\frac{d^2i}{dt^2} + i = 0 \qquad \because i = C\frac{dv_c}{dt}$$

value of i in ① −

V_c → variable

$$RC\frac{dv_c}{dt} + LC\frac{d^2v_c}{dt^2} + V_c = V_s$$

* Parallel RLC



$i_R + i_L + i_c = I_s$

differentiating

$$\frac{v}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt} = I_s \quad -①$$

v → variable

$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} + C\frac{d^2v}{dt^2} = 0 \qquad \because v = L\frac{di_L}{dt}$$

value of v in ① −

i_L → variable

$$\frac{L}{R}\frac{di_L}{dt} + LC\frac{d^2i_L}{dt^2} + i_L = I_s$$

$$a_2\frac{d^2x(t)}{dt^2} + a_1\frac{dx(t)}{dt} + a_0x(t) = b_0f(t)$$

$$\frac{d^2x(t)}{dt^2} + \frac{a_1}{a_2}\frac{dx(t)}{dt} + \frac{a_0}{a_2}x(t) = \frac{b_0f(t)}{a_2}$$

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$$\frac{d^2x(t)}{dt^2} + 2\xi\omega_n\frac{dx(t)}{dt} + \omega_n^2 x(t) = k_s f(t)$$

$$\omega_n = \sqrt{\frac{a_0}{a_2}} \quad , \quad 2\xi\omega_n = \frac{a_1}{a_2}$$

natural frequency

damping ratio $\xi = \frac{a_1}{2}\sqrt{\frac{1}{a_0 a_2}}$

$$k_s = \frac{b_0}{a_2} \rightarrow \text{DC gain}$$

characteristic equation $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

Total Response = Steady State Response + Transient Response

$$x(t) = x_{ss} + x_T(t)$$

→ for Parallel RLC

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I_s}{LC}$$

$$\boxed{\alpha = \frac{1}{2RC}}$$

$i_{ss} = I_s$ (steady state)

- Overdamped

  $\xi > 1$ or $\alpha > \omega_n$

  $i(t) = I_s + k_1 e^{s_1 t} + k_2 e^{s_2 t}$

- Critically damped

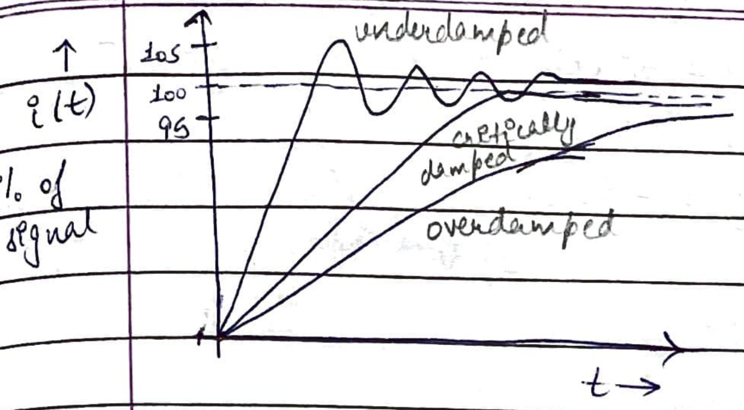
  $\xi = 1$ or $\alpha = \omega_n$

  $i(t) = I_s + (k_1 t + k_2)e^{-\alpha t}$

- Underdamped

  $\xi < 1$ or $\alpha < \omega_n$

  $i(t) = I_s + e^{-\alpha t}[(k_1+k_2)\cos\omega_d t + j(k_1-k_2)\sin\omega_d t]$



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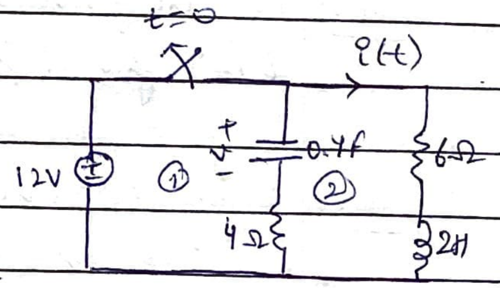
Ques.① For the circuit find-

i) $i(0^+)$ and $v(0^+)$

ii) $\frac{di}{dt}(0^+)$ and $\frac{dv}{dt}(0^+)$



iii) $i(\infty)$ and $v(\infty)$

Sol. Before switching operation –

at $t=0^-$ $\quad L \to$ short circuit

$C \to$ open circuit

$v(0^-) = 12V$

$i(0^-) = \frac{12}{6} = 2A$

$\therefore v(0^+) = v(0^-) = 12V$

$i(0^+) = i(0^-) = 2A$

After switch is open –

KVL in ② loop

$\Rightarrow 6i(0^+) + v_L(0^+) + 4i(0^+) - v_c(0^+) = 0$

$\Rightarrow -10\,i(0^+) + v_c(0^+) = L\frac{di}{dt}(0^+)$

$\Rightarrow \quad -10\times2 + 12 = 2\frac{di}{dt}(0^+)$

$\Rightarrow \quad \frac{di}{dt}(0^+) = -4A/sec$

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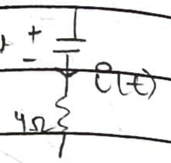
$$\frac{C\,dv_c(0^+)}{dt} = -i(t)$$

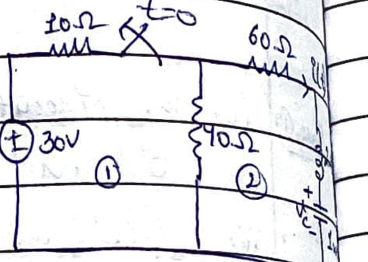


$$\frac{dv(0^+)}{dt} = -5\,V/sec$$

capacitor & inductor gets discharged

$$i(\infty) = 0$$
$$v(\infty) = 0$$

Que. ② For the given circuit find- $i(t)$ for $t > 0$.



Sol. Before switching -

C → open

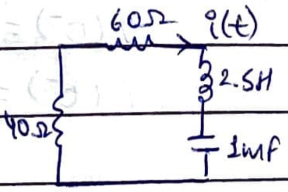
L → short

no current flows in ② as C is open circuit

$$i(0^-) = i(0^+) = 0$$

$$v_c(0^-) = v_c(0^+) = 40 \times \frac{30}{10+40} = 24V$$

After switching -



$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

$$\alpha = \frac{R}{2L} = \frac{100}{2\times 2.5} = 20$$

$$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.5\times10^{-3}}} = \frac{1}{\sqrt{25\times10^{-4}}} = \frac{1}{5\times10^{-2}} = 20$$

$$\alpha = \omega_n$$

∴ critically damped

$$i(t) = (k_1 t + k_2)e^{-\alpha t}$$

at $t = 0$, $i(0^+) = i(0^-) = 0$

$$0 = (0 + k_2)e^{-\alpha\times 0} = k_2$$

$$k_2 = 0$$

$$\frac{di}{dt} = e^{-\alpha t}k_1 + (k_1 t + k_2)(-\alpha)e^{-\alpha t}$$

KVL –

$\Rightarrow 100\, i(0^+) + v_L(0^+) + v_c(0^+) = 0$

$\Rightarrow 100\, i(0^+) + v_c(0^+) = -L\frac{di(0^+)}{dt}$

$\Rightarrow 100 \times 0 + 24 = -2.5\frac{di(0^+)}{dt}$

$\Rightarrow \frac{di(0^+)}{dt} = -9.6$ A/sec

at $t = 0^+$

$-9.6 = 0 + k_1$

$k_1 = -9.6$

So, $\underline{i(t) = -9.6\, t\, e^{-20t}}$

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AC Circuits

$V_m/I_m \rightarrow$ peak / maximum value

$V/I \rightarrow$ root mean square value

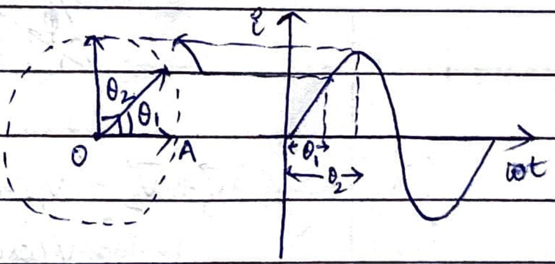
$V_{av}/I_{av} \rightarrow$ average value

$v/i \rightarrow$ instantaneous value

★ **Phasors**

$i = I_m \sin \omega t$

$\omega \rightarrow$ angular speed



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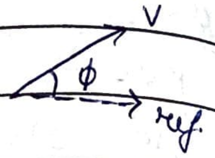
* Reference

$v = V_m \sin(\omega t + \phi)$



** NOTE: j-operator

$i = \sqrt{-1}$

$j = i = \sqrt{-1}$

as 'i' is representation of instantaneous current

$\therefore$ 'j' is used.

$e^{i\theta} = \cos\theta + i\sin\theta$

$V_m e^{j(\omega t+\phi)} = \underbrace{V_m \cos(\omega t+\phi)}_{Real} + j\underbrace{V_m \sin(\omega t+\phi)}_{Im}$

$V_{rms} = V = \frac{V_m}{\sqrt{2}}$

$v = V_m \sin(\omega t + \phi)$

$= Im[V_m e^{j(\omega t+\phi)}]$

$= Im[\sqrt{2} V e^{j\omega t} e^{j\phi}]$

$\vec{V} = V\angle\phi$ V_{real} $V_{imag/reactive}$

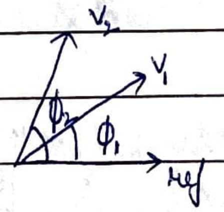
$\vec{V} = V\cos\phi + jV\sin\phi$

$|V| = \sqrt{V_{real}^2 + V_{reactive}^2}$

$\phi = \tan^{-1} \frac{V_{reactive}}{V_{real}}$

$v_1 = V_{m1} \sin(\omega t + \phi_1) \Rightarrow \vec{V_1} = V_1 \angle\phi_1$

$v_2 = V_{m2} \sin(\omega t + \phi_2) \Rightarrow \vec{V_2} = V_2 \angle\phi_2$



Addition $\vec{V_1} + \vec{V_2} = V_1\angle\phi_1 + V_2\angle\phi_2$

$= V_1\cos\phi_1 + jV_1\sin\phi_1 + V_2\cos\phi_2 + jV_2\sin\phi_2$

$= (V_1\cos\phi_1 + V_2\cos\phi_2) + j(V_1\sin\phi_1 + V_2\sin\phi_2)$

$\vec{V} = \vec{V_1} + \vec{V_2} = V\cos\phi + jV\sin\phi$

$a + jb = r\angle\theta$

$r = \sqrt{a^2+b^2}$

$\theta = \tan^{-1} b/a$

$$\left[\begin{array}{l} v(t) = \sqrt{2}V\sin(\omega t+\phi) \\ \text{or } v(t) = \sqrt{2}V\cos(\omega t+\phi) \end{array}\right]$$

Subtraction $\vec{V_1} - \vec{V_2} = V_1\angle\phi_1 - V_2\angle\phi_2$

$= (V_1\cos\phi_1 - V_2\cos\phi_2) + j(V_1\sin\phi_1 - V_2\sin\phi_2)$

Multiplication $\vec{V_1}\times\vec{V_2} = V_1V_2\angle\phi_1+\phi_2$

Division $\dfrac{\vec{V_1}}{\vec{V_2}} = \dfrac{V_1}{V_2}\angle\phi_1-\phi_2$

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Time derivative & Integral of a Phasor →

$v(t) = V_m\sin(\omega t+\phi)$

$v(t) = Im[\sqrt{2}Ve^{j\omega t}e^{j\phi}]$

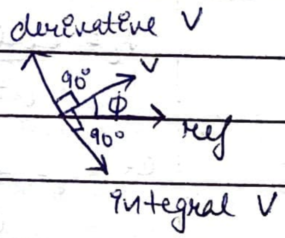
$\dfrac{d\,v(t)}{dt} = Im[\sqrt{2}Vj\omega e^{j\omega t}e^{j\phi}]$

$\dfrac{d\vec{V}}{dt} = j\omega Ve^{j\phi} = j\omega\vec{V}$

$\int v(t)\,dt = Im[\sqrt{2}V\dfrac{e^{j\omega t}}{j\omega}e^{j\phi}]$

$\int\vec{V}\,dt = \dfrac{V}{j\omega}e^{j\phi} = \dfrac{\vec{V}}{j\omega}$

$= \dfrac{-j^2\vec{V}}{j\omega} = \dfrac{-j\vec{V}}{\omega}$



eg. $\vec{V_1} = 3\angle 15^\circ$ $\quad\vec{V_2} = 2.5\angle 45^\circ$

cosine as reference

$\vec{V} = \vec{V_1} + \vec{V_2}$

$= 3\angle 15^\circ + 2.5\angle 45^\circ$

$= 3\cos 15^\circ + j3\sin 15^\circ + 2.5\cos 45^\circ + j2.5\sin 45^\circ$

$= (3\cos 15^\circ + 2.5\cos 45^\circ) + j(3\sin 15^\circ + 2.5\sin 45^\circ)$

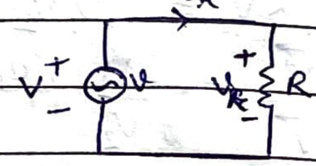
$\vec{V} = 4.66 + j2.54$

$= 5.31\angle 28.6^\circ$

Instantaneous form $v(t) = \underline{\underline{5.31\sqrt{2}\cos(\omega t + 28.6^\circ)}}$

★ <u>Circuit Parameters</u>

① Resistance, R



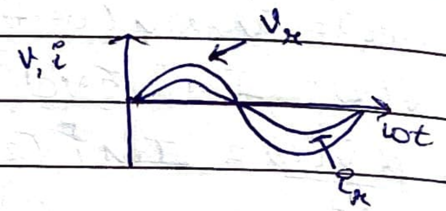
$v_R(t) = i_R(t) R$

$v_R = i_R R$

$V = V_m \sin\omega t$

$i_R = \dfrac{v_R}{R}$

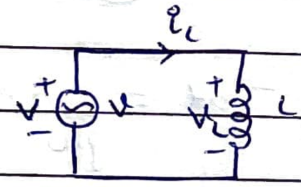
$= \dfrac{V_m \sin\omega t}{R}$



$\vec{V}_R = \vec{I}_R R$

no phase difference

② Inductance, L



$v_L = L\dfrac{di_L}{dt}$

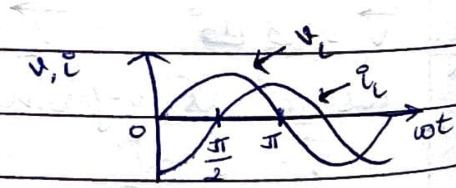
or $i_L = \dfrac{1}{L}\int v_L\, dt$

$V = V_m \sin\omega t$

$= \dfrac{1}{L}\int V_m \sin\omega t\, dt$

Current lags by 90°

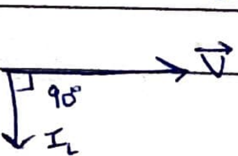
$= \dfrac{V_m}{\omega L}(-\cos\omega t)$



$i_L = \dfrac{V_m}{\omega L}\sin(\omega t - 90^\circ)$

$\omega L \rightarrow I_m$

$\vec{I}_L = \dfrac{\vec{V}}{\omega L}\angle{-90^\circ} = \dfrac{\vec{V}\cdot -j}{\omega L} = \dfrac{\vec{V}}{j\omega L} = \dfrac{\vec{V}}{jX_L}$



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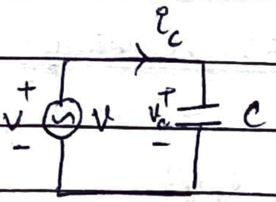
③ Capacitance, C



$$v_c = \frac{1}{C}\int i_c\,dt$$

$v = V_m \sin\omega t$

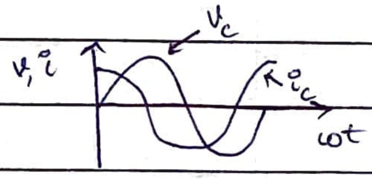
$\vec{V} = V\angle 0^\circ$

$$i_c = C\frac{dv_c}{dt}$$

$$= C\frac{d\,V_m\sin\omega t}{dt}$$

$$= C V_m \cos\omega t \cdot \omega$$

$$= V_m \omega C \sin(\omega t + 90^\circ)$$

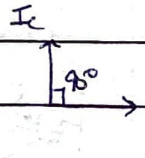


voltage lags by 90°

$$i_c = \frac{V_m}{1/\omega C}\sin(\omega t + 90^\circ)$$

$\frac{V_m}{1/\omega C} \rightarrow I_m$

$$\vec{I_c} = \frac{\vec{V}}{1/\omega C}\angle 90^\circ = \frac{\vec{V}j}{1/\omega C} = \vec{V}j\omega C$$



$$\vec{I_c} = \frac{\vec{V}}{-j\,1/\omega C} = \frac{\vec{V}}{-jX_c}$$

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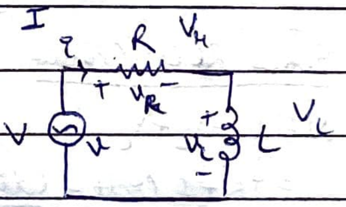
★ Series RL Circuit



$v = v_R + v_L$

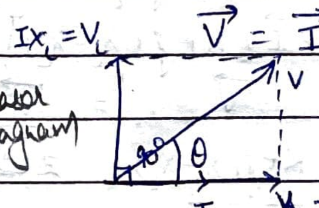
$$v = iR + L\frac{di}{dt}$$

Phasor relation → $\vec{V} = \vec{V_R} + \vec{V_L}$

$$= \vec{I}R + \vec{I}jX_L$$

$$\vec{V} = \vec{I}(R + jX_L)$$

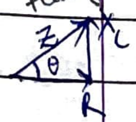
Phasor diagram



$$I = \frac{\vec{V}}{R + jX_L} = \frac{\vec{V}}{Z\angle\theta} = \frac{\vec{V}}{Z}\angle -\theta$$

$$Z = \sqrt{R^2 + X_L^2}$$

impedance triangle



$$\theta = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right)$$

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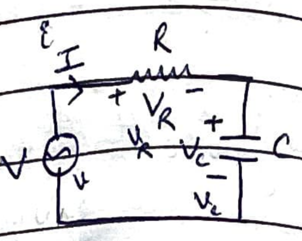
* Series RC Circuit



$V = V_R + V_C$

$= iR + \frac{1}{C}\int i\,dt$

Phasor Relation → $\vec{V} = \vec{V_R} + \vec{V_C}$

$= \vec{I}R + (-jX_C\vec{I})$

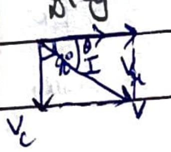
$\vec{V} = \vec{I}\underbrace{(R - jX_C)}_{Z}$

$Z = R - jX_C$

$|Z| = \sqrt{R^2 + X_C^2}$

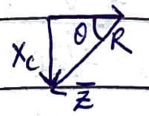
$\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$

Phasor Diagram



$\theta = \tan^{-1}\left(\frac{V_C}{V_R}\right)$

Impedance triangle

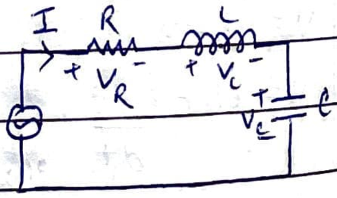


$I = \frac{\vec{V}}{Z\angle -\theta} = \frac{\vec{V}\angle\theta}{Z}$

* Series RLC Circuit

$V = V_R + V_L + V_C$

$= iR + \frac{1}{C}\int i\,dt + L\frac{di}{dt}$



Phasor Relation → $\vec{V} = \vec{V_R} + \vec{V_L} + \vec{V_C}$

$= \vec{I}R + \vec{I}jX_L + \vec{I}(-jX_C)$

$\vec{V} = \vec{I}[R + j(X_L - X_C)]$

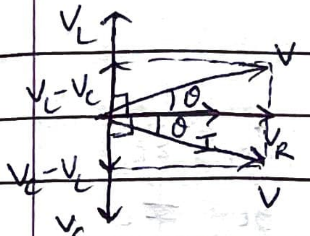
$\vec{I} = \frac{\vec{V}}{R + j(X_L - X_C)} = \frac{\vec{V}}{Z\angle\theta} = \frac{\vec{V}\angle -\theta}{Z}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

θ → +ve → inductive

θ → −ve → capacitive



$V_L = IX_L$

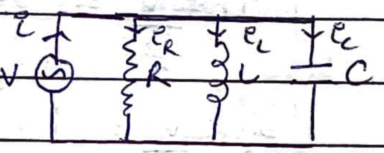
$V_C = IX_C$

① if $X_L = X_C \Rightarrow Z = R \rightarrow$ purely resistive

current lagging by θ ② if $X_L > X_C \Rightarrow V_L > V_C \rightarrow$ inductive

current leading by θ ③ if $X_C > X_L \Rightarrow V_C > V_L \rightarrow$ capacitive

* Parallel RLC Circuit



$i = i_R + i_L + i_C$

$$\vec{I} = \vec{I_R} + \vec{I_L} + \vec{I_C}$$

$$= \frac{\vec{V}}{R} + \frac{\vec{V}}{jX_L} + \frac{\vec{V}}{-jX_C}$$

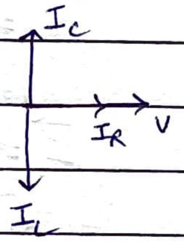
$$= \vec{V}\left[\frac{1}{R} + \frac{1}{jX_L} - \frac{1}{jX_C}\right]$$

$$= \vec{V}\left[\frac{1}{R} - \frac{j}{X_L} + \frac{j}{X_C}\right]$$

$$\boxed{\vec{I} = \vec{V}\,[G - jB_L + jB_C]}$$

$$\vec{I} = \vec{V}Y$$

Phasor diagram



where, $\boxed{Y = G + j(B_C - B_L)}$

conductance

admittance

inductive susceptance

capacitive susceptance

$Y = 1/Z$

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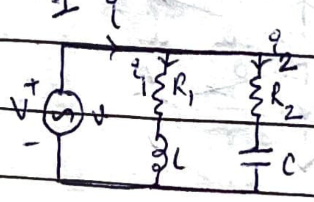
* Practical Parallel Circuit



$i = i_1 + i_2$

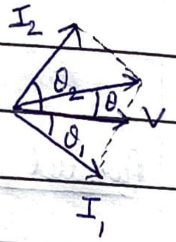
$v = v_{R_1} + v_L = v_{R_2} + v_C$

$\vec{V} = \vec{V}_{R_1} + \vec{V}_L = I_1 Z_1$

$= I_1 (R_1 + jX_L)$

$\vec{V} = \vec{V}_{R_2} + \vec{V}_C = I_2 Z_2$

$= I_2 (R_2 - jX_C)$



$\vec{I}_1 = \dfrac{V_0}{Z_1} = \dfrac{V_0}{R_1 + jX_L} = I_1 \angle -\theta_1$

$\vec{I}_2 = \dfrac{V_0}{Z_2} = \dfrac{V}{R_2 - jX_C} = I_2 \angle \theta_2$

$\vec{I} = \vec{I}_1 + \vec{I}_2$

$= I_1 \angle -\theta_1 + I_2 \angle \theta_2$

$= I_1 \cos -\theta_1 + j I_1 \sin -\theta_1 + I_2 \cos\theta_2 + j I_2 \sin\theta_2$

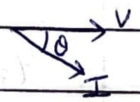
$\vec{I} = (I_1 \cos\theta_1 + I_2 \cos\theta_2) + j(-I_1 \sin\theta_1 + I_2 \sin\theta_2)$

$|I| = \sqrt{(I_1 \cos\theta_1 + I_2 \cos\theta_2)^2 + (-I_1 \sin\theta_1 + I_2 \sin\theta_2)^2}$

$\theta = \tan^{-1} \dfrac{(-I_1 \sin\theta_1 + I_2 \sin\theta_2)}{(I_1 \cos\theta_1 + I_2 \cos\theta_2)}$

* Power Components & Power factor

$p = vi$



$v = V_m \sin\omega t \qquad i = I_m \sin(\omega t - \theta)$

$p = V_m \sin\omega t . I_m \sin(\omega t - \theta)$

$= \dfrac{V_m I_m}{2} (2 \sin\omega t \sin(\omega t - \theta))$

$= \dfrac{V_m}{\sqrt{2}} . \dfrac{I_m}{\sqrt{2}} [\cos(\cancel{\omega t} - \cancel{\omega t} + \theta) - \cos(\omega t + \omega t - \theta)]$

$= VI [\cos\theta - \cos(2\omega t - \theta)]$

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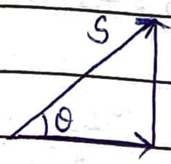
$$P_{av} = \frac{1}{\pi}\int_0^{\pi} p = \frac{1}{\pi}\int_0^{\pi} VI\left[\cos\theta - \underbrace{\cos(2\omega t-\theta)}_{0}\right]$$

$P = VI\cos\theta$ → active power (watts)

$S = VI$ → apparent power (VA)

$Q = VI\sin\theta$ → reactive power (VAR)



$Q = S\sin\theta$

$P = S\cos\theta$

Complex AC power

$$\begin{aligned}\vec{S} &= S\angle\theta \\ &= P + jQ \\ &= VI^{*} \rightarrow \text{conjugate} \\ &= V(I\angle\theta)^{*} \\ &= VI\angle\theta \\ &= \underbrace{VI\cos\theta}_{P} + j\underbrace{VI\sin\theta}_{Q}\end{aligned}$$

$\underbrace{\cos\theta}_{\text{always +ve}}$ → power factor $[0,1]$

lag RL circuit

lead RC circuit

Ques. (1) Let a series circuit consume 700 W at power factor of 0.707 lead with applied voltage $141.1\sin(314t+30)$. Find the circuit elements. Find current, voltage across each element in phasor and in instantaneous form. Calculate reactive power (VAR).

Sol. $v(t) = 141.1\sin(314t+30)$

$P = 700\,W$ $\quad pf = 0.707$ lead

$\cos\theta = 0.707 = \frac{1}{\sqrt{2}}$

$\therefore \boxed{\theta = 45^{\circ}}$

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$P = VI\cos\theta$ $\quad V_m = 141.1$

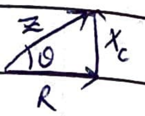
$\Rightarrow 700 = 99.84 I \times 0.707$ $\quad V = V_m/\sqrt{2} = 99.84\,V$

$I = 9.92\,A$

$Z = \frac{V}{I} = \frac{99.84}{9.92} = 10.06\,\Omega$

leading circuit ∴ Capacitor is present so, series RC circuit.

$R = Z\cos\theta = 10.06 \times 0.707 = 7.11\,\Omega$



$X_c = Z\sin\theta = 10.06 \times \sin 45^\circ = 7.11\,\Omega$

$X_c = \frac{1}{2\pi f C}$

$C = \frac{1}{2\pi f X_c} = \frac{1}{314 \times 7.11} = 4.47 \times 10^{-4}\,F$

$V_R = IR = \cancel{10.06}\,^{9.92} \times 7.11 = 70.53\,V$

$V_c = IX_c = 9.92 \times 7.11 = 70.53\,V$

Phasor form

$\vec{V} = 99.84 \angle 30^\circ$

$\vec{I} = 9.92 \angle 30+45^\circ = 9.92 \angle 75^\circ$

$\vec{Z} = 10.06 \angle -45^\circ$

$\vec{V_R} = 70.53 \angle 75^\circ$

$\vec{V_c} = -jIX_c = 70.53 \angle 75^\circ \angle -90^\circ = 70.53 \angle -15^\circ$

Instantaneous values

$i = \sqrt{2} \times 9.92 \sin(314t + 75^\circ)$

$v_R = \sqrt{2} \times 70.53 \sin(314t + 75^\circ)$

$v_c = \sqrt{2} \times 70.53 \sin(314t - 15^\circ)$

$Q = VI\sin\theta$

$= 99.84 \times 9.92 \times \sin 45^\circ$

$Q = 700.32$ VAR

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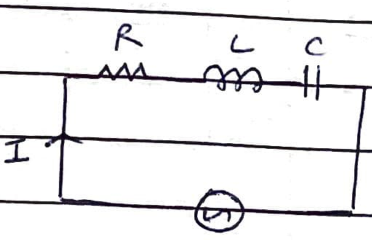
★ <u>Resonance in Series Circuit</u>



$$I = \frac{V}{R + j(X_L - X_C)}$$

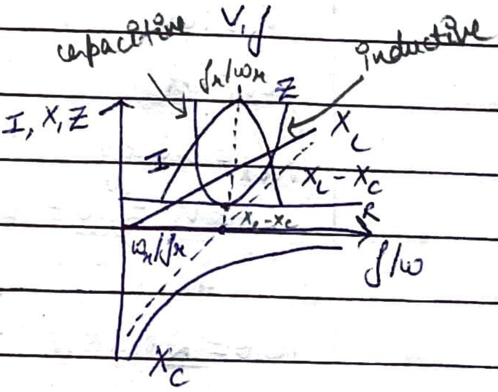
$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$X_L = \omega L = 2\pi f L$$

LC combination acts as short circuit.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



Max. current

$$X_L = X_C$$

Resistive circuit

$$\therefore Z = R$$

Least Impedance

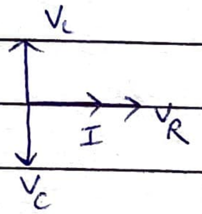
$$\theta = \tan^{-1}\frac{X_L - X_C}{R} = 0^\circ$$

$$pf = \cos\theta = \cos 0^\circ = 1$$

Circuit behaves as resistive circuit.

$$X_L = X_C$$

$$I X_L = I X_C$$

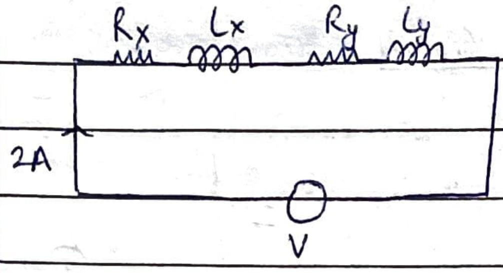


$$V_L = V_C$$

$$\therefore V = V_R$$

Ques. ① A coil 'X' takes 2A at pf of 0.8 lagging with an applied potential difference of 10V. Second coil 'Y' takes 2A with a pf of 0.7 lagging and applied potential of 5V. What voltage will be required to produce a total current of 2A with coils X and Y in series. Find out the pf. of the combination.

Sol.



$V_x = 10V$	$V_y = 5V$
$I_x = 2A$	$I_y = 2A$
$pf_x = 0.8$ lag	$pf_y = 0.7$ lag
$Z_x = \frac{V_x}{I_x} = \frac{10}{2} = 5\Omega$	$Z_y = \frac{V_y}{I_y} = \frac{5}{2} = 2.5\Omega$
$\cos\theta = \frac{R}{Z}$	$\cos\theta = \frac{R}{Z}$
$R_x = 5 \times 0.8 = 4\Omega$	$R_y = 2.5 \times 0.7 = 1.75\Omega$
$Z_x = \sqrt{R_x^2 + X_x^2}$	$Z_y = \sqrt{R_y^2 + X_y^2}$
$X_x^2 = Z_x^2 - R_x^2$	$X_y^2 = Z_y^2 - R_y^2$
$X_x = \sqrt{25-16} = 3\Omega$	$X_y = \sqrt{6.25 - 3.0625} = 1.785\Omega$

for series combination of 2 coils ⇒

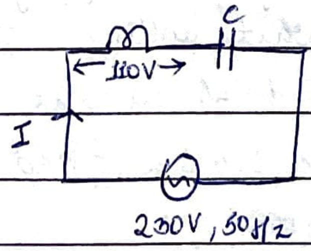
$$Z = \sqrt{(R_x+R_y)^2 + (X_x+X_y)^2}$$
$$= 7.48\Omega$$

$$V = IZ = 2 \times 7.48 = 14.96\,V$$

as it is an inductive circuit $(pf)\ \cos\theta = \frac{R_x+R_y}{Z} = \frac{5.75}{7.48} = 0.768$ lagging

(2) Find the capacitance which must be connected in series with a 100W, 110V lamp in order that the lamp may draw its normal current when the combination is connected to 230V, 50 Hz supply.

Sol.



Lamp → resistive load

$P = VI$

$I = \frac{100}{220} = 0.909A$

$C = 14.323\,\mu F$

$V^2 = V_R^2 + V_C^2$

$V_c = \sqrt{12100 + 52900} = 202V$

$X_c = \frac{V_c}{I} = \frac{202}{0.909} = 222.22\,\Omega$

$C = \frac{1}{2\pi f X_c} = \underline{\underline{14.323\,\mu F}}$

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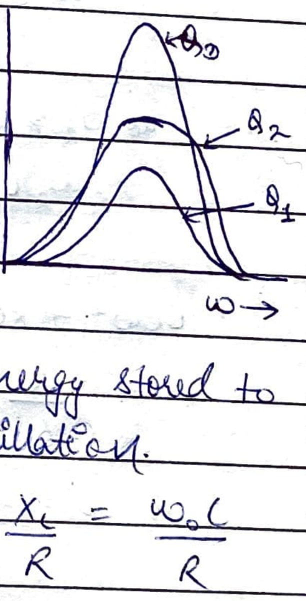
* **Quality Factor (Q-Factor)**

Q_1 → least selective
Q_2 → medium selectivity
Q_3 → highest selectivity



Q-factor relates the max. energy stored to energy dissipated per cycle of oscillation.

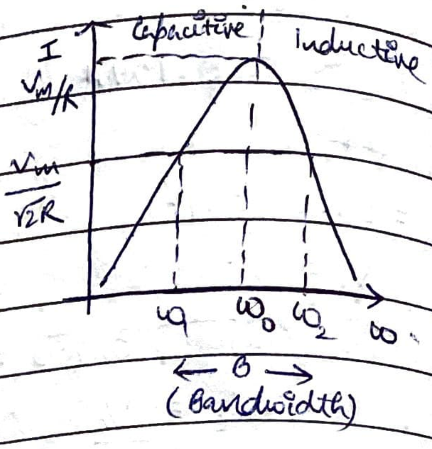
Voltage magnification

$Q = \frac{V_L}{V_R} = \frac{V_L}{V} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega_0 L}{R}$

$Q = \frac{V_C}{V_R} = \frac{V_C}{V} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{\omega_0 CR}$

or $Q = \frac{\cancel{2\pi f} \cdot \frac{1}{2} L I_0^2}{I^2 R} = \frac{\pi f L(\sqrt{2}I)^2}{I^2R} = \frac{2\pi f L}{R} = \frac{\omega_0 L}{R}$

Half Power Frequencies (ω_1, ω_2)



at ω_0,

$$I_{max} = \frac{V_m}{R}$$

at ω_1, ω_2,

$$I = \frac{V_m}{\sqrt{2}R} = \frac{I_{max}}{\sqrt{2}}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2}R$$

$$\therefore (X_L - X_C)^2 = R^2$$

$$X_L - X_C = \pm R \Rightarrow \omega L - \frac{1}{\omega C} = \pm R$$

at $\omega_1 < \omega_0 \Rightarrow$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

at $\omega_2 > \omega_0 \Rightarrow$

$$\omega_2 L - \frac{1}{\omega_2 C} = +R$$

$$LC\omega_1^2 + RC\omega_1 - 1 = 0$$

$$\omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

$$\omega_1 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2}$$

$$\omega_1 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

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$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

$$\omega_1\omega_2 = \frac{1}{LC} = \omega_0^2$$

$$\omega_0 = \sqrt{\omega_1\omega_2}$$

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{\omega_0}{B}}$$

$$\omega_1, \omega_2 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} \mp \frac{\omega}{2Q}$$

when $Q > 10 \rightarrow$ High Q circuit

then, $\omega_1 \simeq \omega_0 - \frac{\omega_0}{2Q} \simeq \omega_0 - \frac{B}{2}$ Symmetric about ω_0

$$\omega_2 \simeq \omega_0 + \frac{\omega_0}{2Q} \simeq \omega_0 + \frac{B}{2}$$

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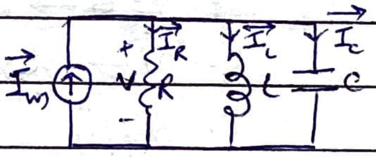
★ Resonance in RLC Parallel Circuit



$$\vec{I}_w = \vec{I}_R + \vec{I}_L + \vec{I}_c$$

$$\vec{I}_R = \frac{V}{R}$$

$$\vec{I}_L = \frac{V}{jX_L}$$

$$\vec{I}_c = \frac{V}{-jX_c}$$

$$\vec{I}_w = \frac{V}{R} + \frac{V}{jX_L} - \frac{V}{jX_c}$$

$$\vec{I}_w = \vec{V}[G + j(B_c - B_L)] = \vec{V}Y$$

$$B_c = \omega C$$

$$B_L = \frac{1}{\omega L}$$

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at resonance, pf = 1.

Circuit behaves as purely resistive.

Reactive component of admittance (Y) = 0

$$B_C - B_L = 0$$
$$B_C = B_L$$

at resonance, $\omega_0 C = \frac{1}{\omega_0 L}$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$\omega < \omega_0 \Rightarrow B_L > B_C$ (Inductive)

$\omega = \omega_0 \Rightarrow B_L = B_C$ (Resistive)

$\omega > \omega_0 \Rightarrow B_C > B_L$ (Capacitive)

at resonance,

$$V = V_{max} = I_m R$$

- $B_L = B_C$
- $Y = G \rightarrow$ minimum
- $V \rightarrow$ maximum
- Supply current passes through R.
- $I_L = I_C$ (but not zero)
- parallel LC combination acts like open circuit.
- $\vec{I_m} = \vec{I_R}$, $|\vec{I_L}| = |\vec{I_C}| \neq 0$
- $Q = \frac{\text{Current through L/C}}{\text{Current through resistance}}$

$$= \frac{I_L}{I_R} \text{ or } \frac{I_C}{I_R}$$

$$Q = \frac{V/X_L}{V/R} = \frac{R}{X_L} = \frac{R}{\omega_0 L}$$

$$\text{or } Q = \frac{V/X_C}{V/R} = \frac{R}{X_C} = R\omega_0 C$$

Half Power Frequency

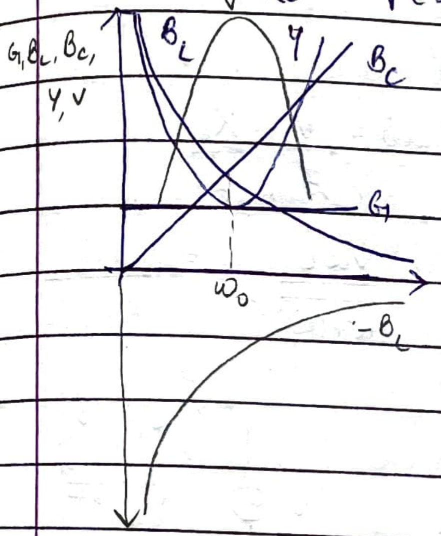
$$V = \frac{V_{max}}{\sqrt{2}}$$

~~$V = IY$~~ $I = VY$

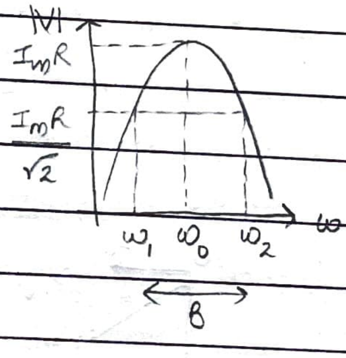
$$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = \frac{\sqrt{2}}{R}$$

$$\omega_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$



$$Y = \sqrt{G^2 + (B_C - B_L)^2}$$



$$B = \omega_2 - \omega_1 = \frac{1}{RC} = \frac{\omega_0}{Q}$$

$$\omega_1, \omega_2 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \mp \frac{\omega_0}{2Q}$$

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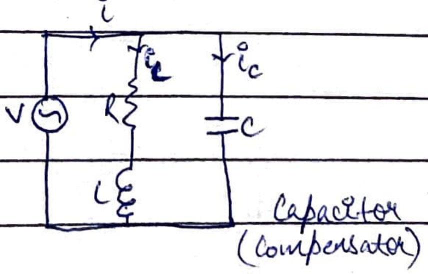
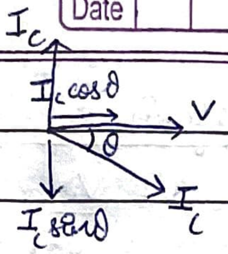
★ Power Factor Correction

Compensator is connected in parallel to load.

$$\vec{I} = \vec{I}_L + \vec{I}_C$$

$$\vec{I}_L = \frac{\vec{V}}{R + j\omega L} = \frac{\vec{V}}{R + jX_L}$$





$$\vec{I_l} = \frac{\vec{V}}{Z\angle\theta} = \frac{\vec{V}}{Z}\angle{-\theta}$$

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\vec{V} = V\angle 0$$

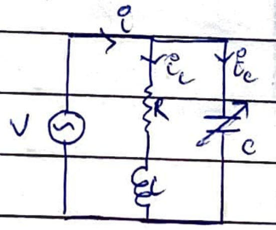
$$I_{real} = I_l\cos\theta = \frac{V}{Z}\cdot\frac{R}{Z}$$

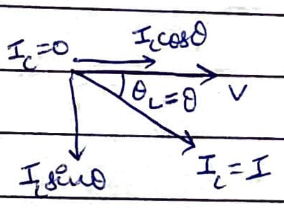
$$= \frac{V}{\sqrt{R^2+(\omega L)^2}}\cdot\frac{R}{\sqrt{R^2+(\omega L)^2}}$$

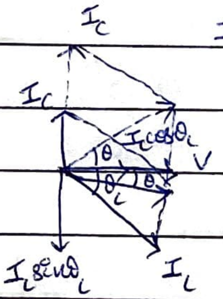
$$I_{real} = \frac{VR}{R^2+(\omega L)^2}$$

$$I_{reactive} = I_l\sin\theta = \frac{V}{Z}\cdot\frac{X_L}{Z}$$

$$I_{reactive} = \frac{V\omega L}{\sqrt{R^2+(\omega L)^2}^{\,2}} = \frac{V\omega L}{R^2+(\omega L)^2}$$







$I_c = 0$

$I \rightarrow$ total current $\rightarrow \theta$ with V

$I_l \rightarrow \theta_L$ with V

- $I_c < I_l\sin\theta_L$, $\theta < \theta_L$, pf $\rightarrow$ lagging
- $I_c = I_l\sin\theta_L$, $\theta = 0°$, $I = I_l\cos\theta_L$, pf $\rightarrow 1$
- $I_c > I_l\sin\theta_L$, pf $\rightarrow$ leading

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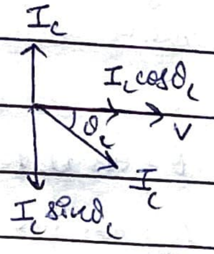
* Resonance in Tank circuit
- unity pf
- supply frequency is varied

$$I_c = I_L \sin\theta_L$$

$$I_c = \frac{V}{X_c} = V\omega C$$



$$I_L \sin\theta_L = \frac{V\omega L}{R^2+(\omega L)^2}$$

at resonance,

$$\cancel{V}\cancel{\omega_r}C = \frac{\cancel{V}\cancel{\omega_r}L}{R^2+(\omega_r L)^2}$$

$$R^2+(\omega_r L)^2 = \frac{L}{C}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

if R is very small → approaches to 0.

$$\omega_r = \frac{1}{\sqrt{LC}}$$

Supply current,

$$I = I_L\cos\theta_L$$

$$= \frac{VR}{R^2+(\omega_r L)^2}$$

$$= \frac{VR}{L/C}$$

$$= \frac{V}{L/RC}$$

for tank circuit only

$Z = \frac{L}{RC}$ → dynamic impedance

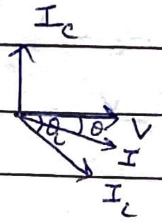
Q-factor,

ratio of reactive to real factor of current as $I_c = I_L\sin\theta_L$

$$Q = \frac{I_c}{I} = \frac{I_c}{I_L\cos\theta_L} = \frac{\omega_R L}{R}$$

Que.⑥ A single phase load of 5kW operates at a pf of 0.6 lagging. It is proposed to improve pf to 0.95 lagging by connecting a capacitor across the load. Calculate capacitor kVA rating.

Sol.



$\theta = \cos^{-1} 0.95$

$\theta_L = \cos^{-1} 0.6$

pf = 0.6

$VI_L \cos\theta_L = VI\cos\theta = 5kW$

$VI_L = \frac{5}{0.6} = 8.33 kVA$

Reactive power $= VI_L \sin\theta_L$

$= 8.33 \sin(\cos^{-1} 0.6)$

$= 6.667$ kVAR

pf = 0.95

New Reactive power demand $= VI\sin\theta$

$VI\cos\theta = 5$

$VI = \frac{5}{0.95} = 5.263$ kVA

New Reactive power $= 5.263 \sin(\cos^{-1} 0.95)$

$= 1.6434$ kVAR

Capacitor rating = Old reactive power demand − New reactive power demand

$= 6.667 - 1.6434$

$= 5.0236$ kVAR

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Three-Phase System

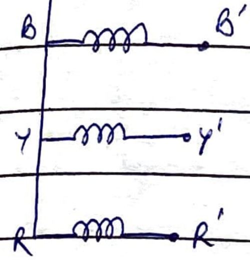
- A 3-phase system is considered as three separate single phase systems displaced from each other by 120°.

- $V_R = V_m \sin(\omega t)$

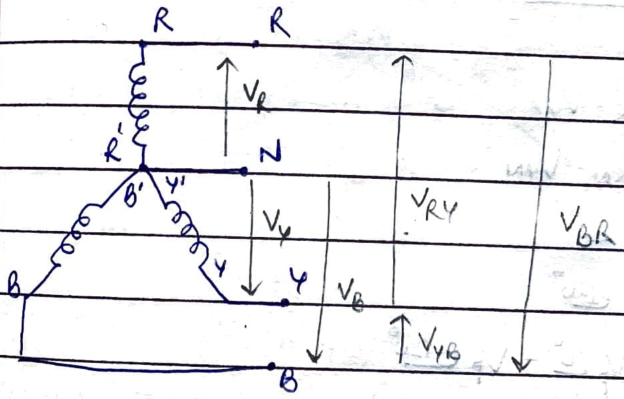
  $V_Y = V_m \sin(\omega t - 120^\circ)$

  $V_B = V_m \sin(\omega t - 240^\circ) = V_m \sin(\omega t + 120^\circ)$

* Interconnection of 3-phases



Star Connection (Y-connection)



Phase Voltage → voltage b/w neutral & coils

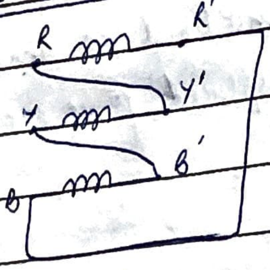
Line Voltage → voltage b/w two coils

$I_L = I_P$

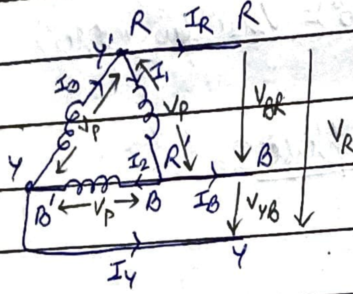
I_L ↙ line current

I_P ↘ phase current

Delta connection



$\Downarrow$



$V_p = V_L$

phase voltage ↙ ↘ line voltage

* Star connection

$V_{RY} = V_R - V_Y$

or $V_{RN} - V_{YN}$

$\vec{V_R} = V_p \angle 0^\circ$

$\vec{V_Y} = V_p \angle -120^\circ$

$V_{RY} = V_p \angle 0^\circ - V_p \angle -120^\circ$

$= V_p [\cos 0^\circ + j \sin 0^\circ - (\cos(-120^\circ) + j \sin(-120^\circ))]$

$= V_p [1 + 0 - (-1/2 - j\sqrt{3}/2)]$

$= V_p [3/2 + \frac{\sqrt{3}}{2} j]$

$= \sqrt{3} V_p \angle 30^\circ$

line voltage leads phase voltage by 30°

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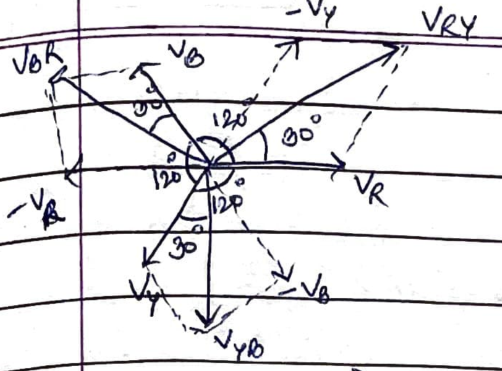

$V_{YB} = \vec{V_Y} - \vec{V_B} = \sqrt{3}\, V_p \angle -90^\circ$

$V_{BR} = \vec{V_B} - \vec{V_R} = \sqrt{3}\, V_p \angle -210^\circ$ or $\sqrt{3}\, V_p \angle 150^\circ$

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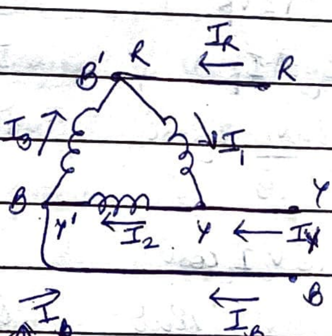
★ Delta Connection



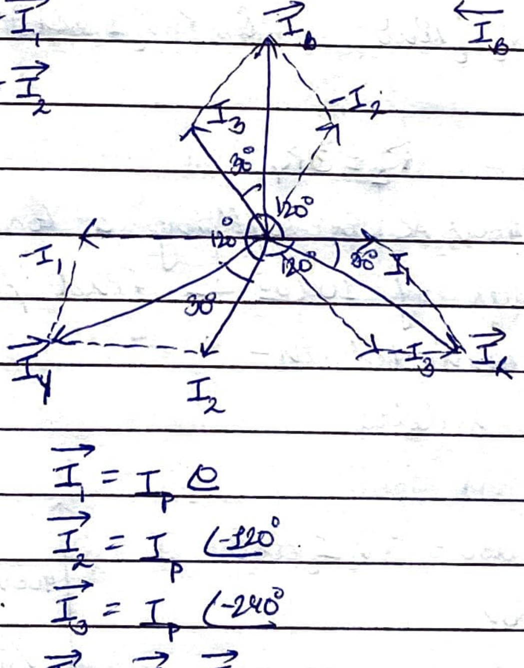
$\vec{I_1} = \vec{I_3} + \vec{I_R}$

$\vec{I_R} = \vec{I_1} - \vec{I_3}$

$\vec{I_2} = \vec{I_1} + \vec{I_Y}$

$\vec{I_Y} = \vec{I_2} - \vec{I_1}$

$\vec{I_B} = \vec{I_3} - \vec{I_2}$



line ~~phase~~ current lags behind ~~line~~ phase current by 30°

$\vec{I_1} = I_p \angle 0$

$\vec{I_2} = I_p \angle -120^\circ$

$\vec{I_3} = I_p \angle -240^\circ$

$\vec{I_R} = \vec{I_1} - \vec{I_3}$

$= I_p \angle 0 - I_p \angle -240$

$= I_p[\cos 0 + j\sin 0^\circ - (\cos(-240^\circ) + j\sin(-240^\circ))]$

$= I_p[1 + 0 - (-1/2 + 0.866j)]$

$= \sqrt{3} I_p \angle 30^\circ$

$\vec{I_B} = \sqrt{3} I_p \angle 90^\circ$

$\vec{I_Y} = \sqrt{3} I_p \angle -150^\circ = \sqrt{3} I_p \angle 210^\circ$

* Power in 3-Phase

Single phase →

$P = VI\cos\theta$ $\qquad$ $Q = VI\sin\theta$

Three-phase →

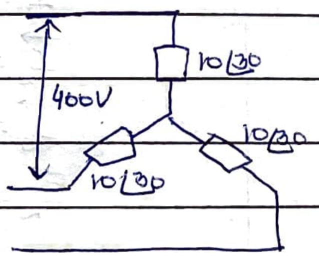
$P = 3V_pI_p\cos\theta$ $\qquad$ $Q = 3V_pI_p\sin\theta$ → angle b/w phase voltage and phase current.

Star	Delta
$V_L = \sqrt{3}V_p$	$V_L = V_p$
$I_L = I_p$	$I_L = \sqrt{3}I_p$
$P = 3.\frac{V_L}{\sqrt{3}}.I_L\cos\theta$	$P = 3.V_L.\frac{I_L}{\sqrt{3}}\cos\theta$
$= \sqrt{3}V_LI_L\cos\theta$	$= \sqrt{3}V_LI_L\cos\theta$
$Q = \sqrt{3}V_LI_L\sin\theta$	$Q = \sqrt{3}V_LI_L\sin\theta$

$$P_\Delta = 3P_Y$$

eg. A 3-ϕ, 400V AC is supplying a load formed by 3 impedances of $10\angle 30^\circ\ \Omega$. Find power when load is connected in-
a) star b) delta

STAR



$P = 3V_pI_p\cos\theta = \sqrt{3}V_LI_L\cos\theta$

$V_L = 400V$

$V_p = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230V$

$I_p = \frac{V_p}{Z} = \frac{230\angle 0^\circ}{10\angle 30^\circ} = 23\angle -30^\circ$

$P_Y = 3 \times 230 \times 23 \times \cos(-30^\circ)$

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DELTA

$V_L = V_P$

$I_P = \frac{V_P}{Z} = \frac{400}{\cancel{10}} \cdot \frac{1}{1\phi\angle 30^\circ} = 40\angle -30^\circ$

$P_\Delta = 3 \times 400 \times 40 \cos(-30^\circ)$

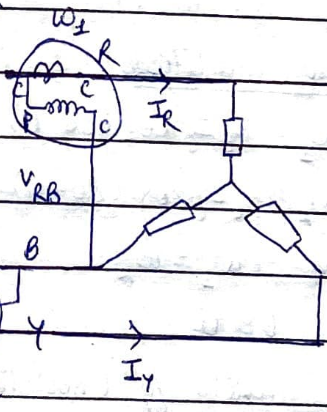
$\therefore P_\Delta = 3 P_Y$

Power Measurement

3 phase 4-wire system
(i) 3 wattmeter → balanced & unbalanced
(ii) 2 wattmeter → balanced & unbalanced
(iii) 1 wattmeter → balanced

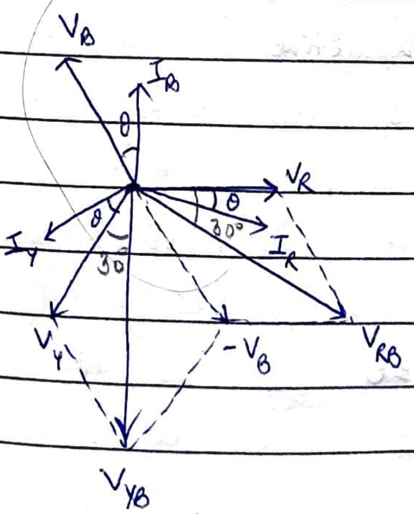
★ Two Wattmeter Method

Current Coil → series
Power Coil → parallel (shunt)



$W_1 = I_R V_{RB} \cos(\text{angle b/w } I_R \,\&\, V_{RB})$ → $30-\theta$

$W_2 = I_Y V_{YB} \cos(\text{angle b/w } I_Y \,\&\, V_{YB})$ → $30+\theta$



$W_1 = I_R V_{RB} \cos(30-\theta)$

$W_2 = I_Y V_{YB} \cos(30+\theta)$

$\begin{bmatrix} I_R = I_Y = I_P = I_L \\ V_{RB} = V_{YB} = V_L \end{bmatrix}$

$W_1 = V_L I_L \cos(30-\theta)$

$W_2 = V_L I_L \cos(30+\theta)$

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$W_1 + W_2 = V_L I_L [\cos 30^\circ \cos\theta + \cancel{\sin 30 \sin\theta} + \cos 30 \cos\theta - \cancel{\sin 30 \sin\theta}]$

$= \frac{\sqrt{3}}{\cancel{2}} V_L I_L \cancel{2} \cos\theta$

$$\boxed{W_1 + W_2 = \sqrt{3} V_L I_L \cos\theta}$$

$$\boxed{W_1 - W_2 = V_L I_L \sin\theta}$$

$$\frac{\tan\theta}{\sqrt{3}} = \frac{W_1 - W_2}{W_1 + W_2}$$

$$\theta = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$$

$$\cos\theta = pf = \cos\left[\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right]$$

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Special conditions

$W_1 = V_L I_L \cos(30 - \theta)$

$W_2 = V_L I_L \cos(30 + \theta)$

① when, $W_1 = W_2$

$\theta = 0^\circ$

$\therefore$ unity pf (Resistive load)

② when one-wattmeter will read zero

$\theta = 60^\circ$

$W_2 = 0 \rightarrow$ inductive

$W_1 = 0 \rightarrow$ capacitive

pf is 0.5

③ when $\theta = 90^\circ$

zero pf

$W_1 = -W_2$

$\therefore W_1 + W_2 = 0$

(4) when a wattmeter will read −ve.

$60 < \theta < 90$

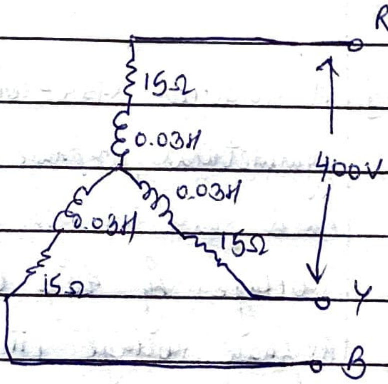
$0 < \cos\theta < 0.5$

Ques. (1) Three inductive coils each with a resistance of 15Ω and an inductance of 0.03H are connected in (i) star (ii) delta, to 3-phase, 400V; 50Hz supply. Calculate-

a) phase current and line current

b) total power absorbed

Sol. (i) STAR



$V_p = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94V$

$I_p = \frac{V_p}{Z_p}$

$Z_p = \sqrt{R_p^2 + X_p^2} = 17.72\Omega$

$\because [X_p = 2\pi f L]$

$\cos\theta = \frac{R_P}{Z_P} = 0.846$

$I_p = 13.03A$

$I_L = I_p = 13.03A$

$P = 3V_pI_p\cos\theta = \sqrt{3}V_LI_L\cos\theta = 7637.22\ W$

(ii) DELTA

$Z_P = 17.72\Omega$

$\cos\theta = 0.846$

$V_P = V_L = 400V$

$I_P = \frac{V_P}{Z_P} = \frac{400}{17.72} = 22.57A$

$I_L = \sqrt{3}I_P = 39.1A$

$P = 22913.1\ W$

(2) Two wattmeters connected to measure the power input to a 3-phase circuit indicated 15kW and 1.5 kW. respectively, the latter reading being obtained after reversing the current coil connections. Calculate the power and pf of load.

Sol. $W_1 = 15kW$

$W_2 = -1.5\ kW$

$P = W_1 + W_2 = 13.5\ kW$

$pf = \cos\theta = \cos\left[\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]\right] = 0.427$

(3) A balanced star-connected load is supplied from a symmetrical, 3-phase, 440V, 50Hz supply. The current in each phase is 20A and lags behind its phase voltage by 40°. Calculate:

(i) phase voltage (ii) load parameters (iii) total power (iv) reading of 2 wattmeter connected in load circuit to measure total power.

Sol. $V_L = 440V$

$I_p = 20A$

$\theta = 40°$

(i) $V_p = 254.04V$

(ii) $Z_p = \frac{V_p}{I_p} = 12.702\ \Omega$

$\cos 40° = \frac{R_p}{Z_p} \Rightarrow R_p = Z_p \cos 40° = 9.73\Omega$

$\sin 40° = \frac{X_p}{Z_p} \Rightarrow X_p = Z_p \sin 40° = 8.16\Omega$

$2\pi f L = X_p$

$L = \frac{8.16}{2 \times 3.14 \times 50} = 25.99\ mH$

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(iii) $P = 3 V_p I_p \cos\theta$

$= 11.675\ kW$

(iv) $W_1 + W_2 = 11.675\ kW$

$W_1 = V_L I_L \cos(30-\theta) = 8.668\ kW$

$W_2 = V_L I_L \cos(30+\theta) = 3.008\ kW$

8/1/21 Ch.3 – Magnetic Circuits

$$\oint B.dl = \mu I$$

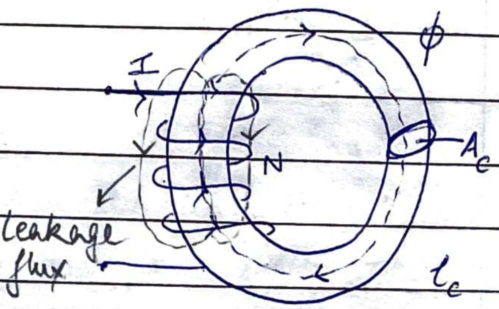
$$B = \mu H$$

$$\therefore \oint H.dl = I$$

Magnetic circuit consists of mainly-iron paths of specified geometry.

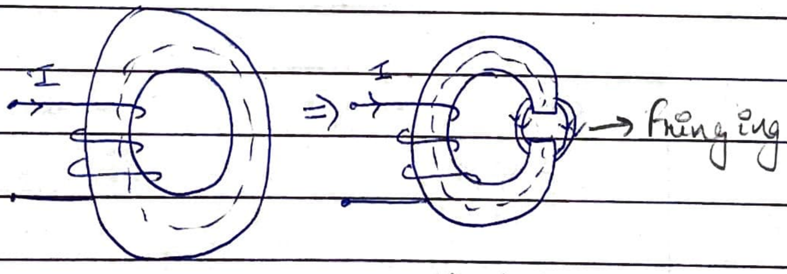
Core → ferromagnetic materials → high permeability

magnetic flux density ← $B = \dfrac{\phi}{A_c}$



Magnetomotive force (MMF) →

$MMF = NI = f$ → Ampere turns



No Airgap Air Gap

magnetic field strength ← $H = \dfrac{mmf}{l}$

Reluctance →

l_c → mean circumference

A_c → cross-sectional area

N → no. of turns

Flux = flux density × cross-sectional area

$\phi = BA_c$ — ①

$MMF = Hl_c$ — ②

$B = \mu H$

$= \mu_0 \mu_r H$

$$H = \frac{B}{\mu_0 \mu_r}$$

$$MMF = \frac{B}{\mu_0 \mu_r} l_c = \frac{\phi l_c}{A_c \mu_0 \mu_r}$$

$$\phi = \frac{MMF}{\frac{l_c}{A_c \mu_0 \mu_r}}$$

$\frac{l_c}{A_c \mu_0 \mu_r} \rightarrow$ Reluctance S/R

$$\phi = \frac{f}{R}$$

reluctance is the opposition offered to the flow of flux

$$\boxed{R = \frac{l_c}{\mu_0 \mu_r A_c}}$$

for air gap, $\mu_r = 1$

$\mu_0 = 4\pi \times 10^{-7}$

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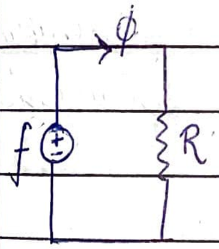
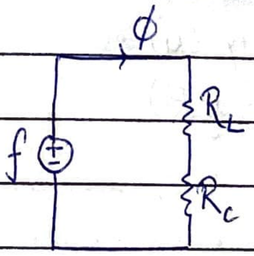
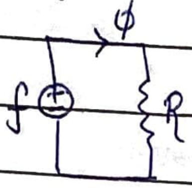
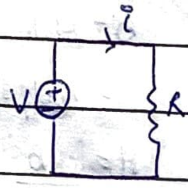


The equivalent if there is no air gap

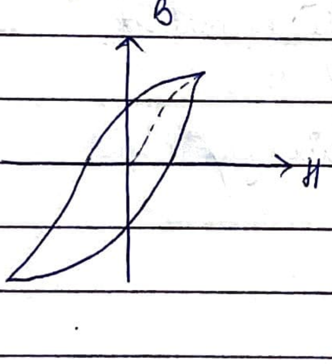


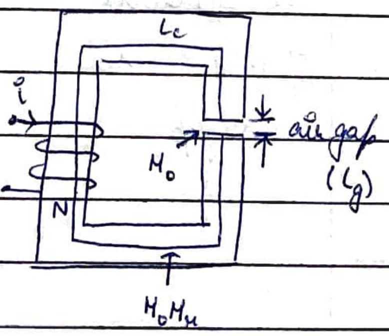
The equivalent if there is air gap

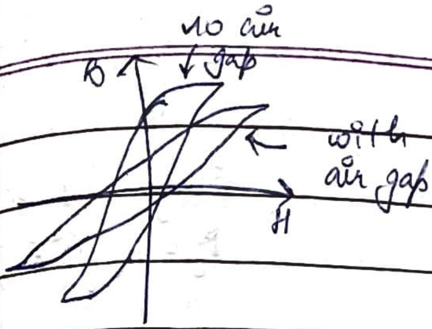
* Analogy

Magnetic Circuit	Electric Circuit
	
$\phi = \frac{f}{R}$	$i = \frac{V}{R}$
$R = \frac{l}{\mu A}$; $\mu = \mu_0 \mu_r$	$R = \frac{\rho l}{A}$
Source: Magnetomotive force	Electromotive force
Response: Flux	Current
Reluctance	Resistance
Magnetic flux density $B = \frac{\phi}{A}$	Current density $j = \frac{i}{A}$
Magnetic field (H) $B = \mu H$	Electric field (E) $j = \sigma E$
Permeability (μ)	Conductivity (σ)
Permeance (P)	Conductance (G)

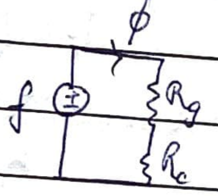
* Air gap

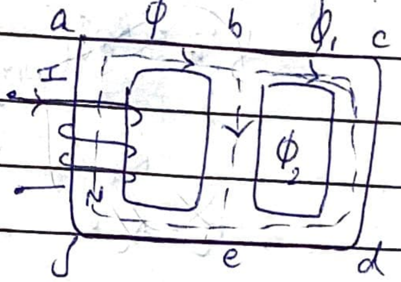






$$f = H_g L_g + H_c L_c$$
$$= \phi R_g + \phi R_c$$
$$= \phi [R_g + R_c]$$
$$R = R_g + R_c$$



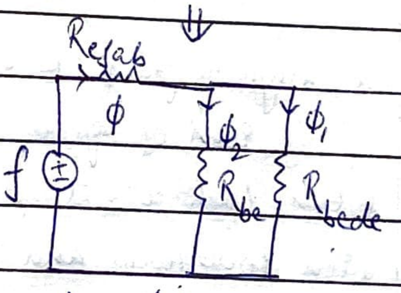


$$\phi = \phi_1 + \phi_2$$

$R_{efab} \rightarrow \phi$

$R_{be} \rightarrow \phi_2$

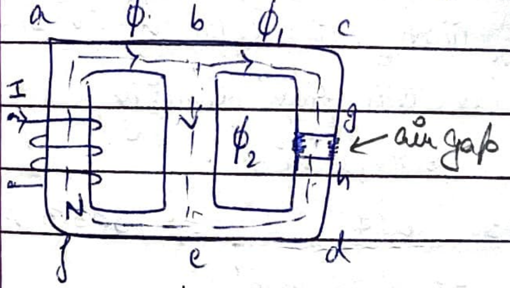
$R_{bcde} \rightarrow \phi_1$

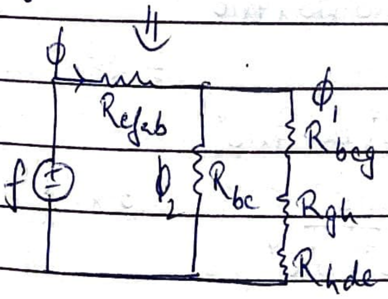


$$f = \phi R_{efab} + \phi_1 R_{bcde}$$

or

$$f = \phi R_{efab} + \phi_2 R_{be}$$





$$f = \phi R_{efab} + \phi_1 [R_{bcg} + R_{gh} + R_{hde}]$$

* **Core loss**

- Hysteresis loss

$P_h = k_h f B_m^n$ — Steinmetz's constant; B_m → max. flux density; f → frequency

- Eddy Current loss

$P_e = k_e f^2 B_m^2$

$P_i = P_h + P_e$

$V = 4.44 f N \phi_m = 4.44 f N B_m A_c$

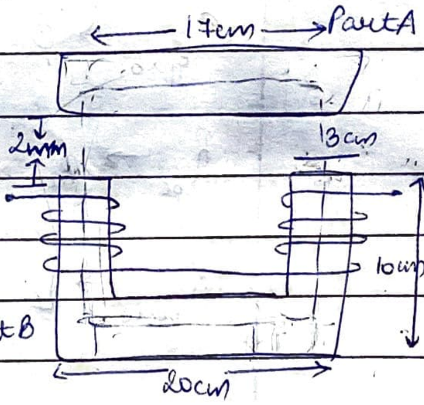
$B_m = \dfrac{V}{4.44 f N A_c}$

$P_h = k_h f \left(\dfrac{V}{4.44 f N A_c}\right)^n$

$= K_h f^{1-n} V^n$

$P_e = K_e V^2$

Ques. ① A magnetic frame is built up of iron of square cross-section. Each coil is wound with 1000 turns and the exciting current is 1A.



$\mu_r(A) = 1000$

$\mu_r(B) = 1200$

Calculate - (i) R_A (ii) R_B (iii) R_g (both) (iv) total R (v) MMF (vi) flux (vii) flux density

air gap → R_g

Sol. $A_c = 3 \times 3 = 9 cm^2 = 9 \times 10^{-4} m^2$

(i) $R_A = \dfrac{l}{\mu_0 \mu_r A} = \dfrac{0.17}{4\pi \times 10^{-7} \times 10^3 \times 9 \times 10^{-4}} = 150313$ AT/wb

(ii) $R_B = \dfrac{l}{\mu_0 \mu_r A} = \dfrac{0.34}{4\pi \times 10^{-7} \times 1200 \times 9 \times 10^{-4}} = 250521.6$ AT/wb

(iii) $R_g = \dfrac{l_g}{\mu_0 A_c} = \dfrac{4 \times 10^{-3}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}} = 3.5 \times 10^6$ AT/wb

(iv) $R = R_A + R_B + R_g = 3.94 \times 10^6$ AT/wb

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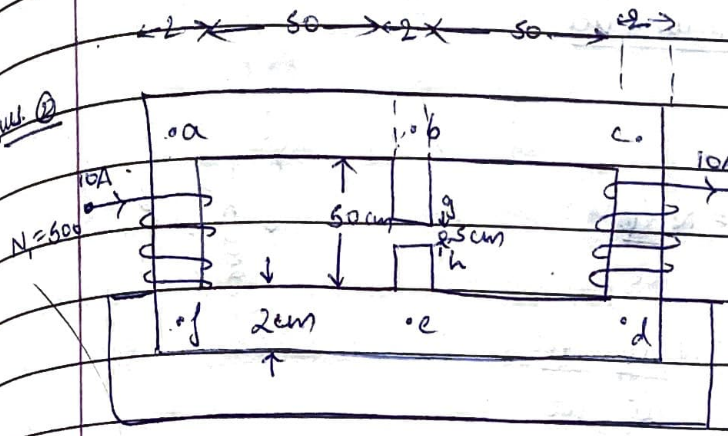
(v) $mmf = NI = 2000 \times 1 = 2000\ AT$

(vi) $flux,\ \phi = \dfrac{mmf}{R} = \dfrac{2000}{3.94 \times 10^{6}} = 5.08 \times 10^{-4}\ wb$

(vii) $B = \dfrac{\phi}{A_c} = \dfrac{5.08 \times 10^{-4}\ wb}{9 \times 10^{-4}} = 0.564\ wb/m^2$

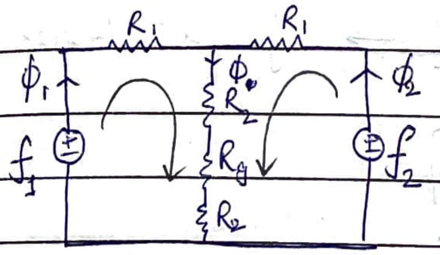
Que. ②



$\mu_r = 1200$

$\phi,\ B,\ H$

Sol.



$mmf = 500 \times 10 = 5000$

$A_c = 2 \times 2 = 4\ cm^2$

$R_1 = R_{bafe} = R_{bcde} = \dfrac{l}{\mu_0 \mu_r A_c} = \dfrac{156 \times 10^{-2}}{4\pi \times 10^{-7} \times 4 \times 10^{-4} \times 1200} = 2.5863 \times 10^{6}$

$R_2 = R_{bg} = \dfrac{25.75 \times 10^{-2}}{4\pi \times 10^{-7} \times 1200 \times 4 \times 10^{-4}} = \cancel{5.12 \times 10}\ 4.26 \times 10^{5}$

$R_g = R_{gh} = \dfrac{0.5 \times 10^{-2}}{4\pi \times 10^{-7} \times 1200 \times 4 \times 10^{-4}} = 9.94 \times 10^{6}$

$\phi = \phi_1 + \phi_2$

$f_1 = \phi_1 R_1 + \phi(2R_2 + R_g) = 5000$

$f_2 = \phi_2 R_1 + \phi(2R_2 + R_g) = 5000$

$\phi = \quad wb$

$B_g = \dfrac{\phi}{A} = \quad wb/m^2 \qquad H_g = \dfrac{B_g}{\mu_0} = \quad AT/m^2$

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* Electromagnetic Induction

$$v = -N\frac{d\phi}{dt}$$

* Self Inductance

$$\phi \propto i$$

$$v = N\frac{d\phi}{dt}$$

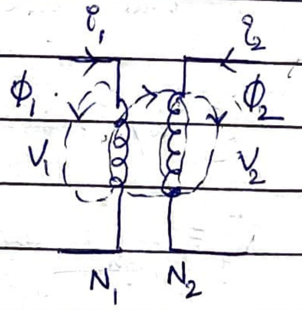
$$= N\frac{d\phi}{di}\cdot\frac{di}{dt}$$

$$v = L\frac{di}{dt}$$

as $L = N\frac{d\phi}{dt}$ → self inductance

or $\boxed{L = \frac{N\phi}{i}}$

* Mutual Inductance



$$V_1 = V_{1S} + V_{1m}$$

$$V_2 = V_{2S} + V_{2m}$$

$$V_{1S} = N_1\frac{d\phi_1}{dt} = L_1\frac{di_1}{dt}$$

$$V_{2S} = N_2\frac{d\phi_2}{dt} = L_2\frac{d\phi_2}{dt}$$

$V_{1m} = N_1\frac{d\phi_{21}}{dt}$ → flux produced by i_2 linking with coil 1

$$= M_{12}\frac{di_2}{dt}$$

$$\boxed{M_{12} = N_1 \frac{d\phi_{21}}{di_2}}$$

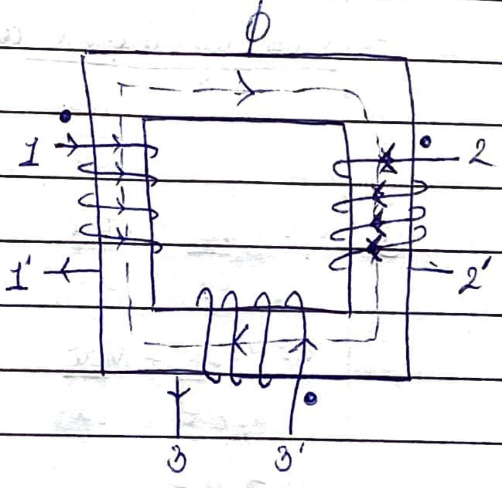
$$V_{2m} = N_2 \frac{d\phi_{12}}{dt}$$

$$= M_{21}\frac{di_1}{dt} = M\frac{di_1}{dt}$$

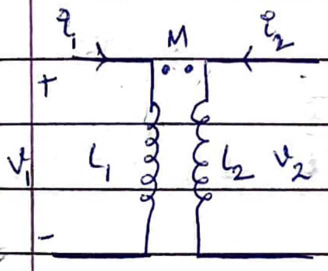
$$\boxed{M_{21} = N_2 \frac{d\phi_{12}}{di_1}}$$

$$\therefore \quad M_{12} = M_{21} = M$$

★ <u>Dot Convention</u>



Current is entering the dot



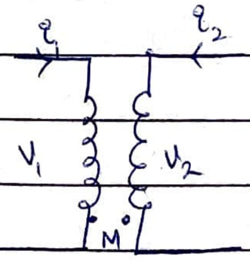
$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

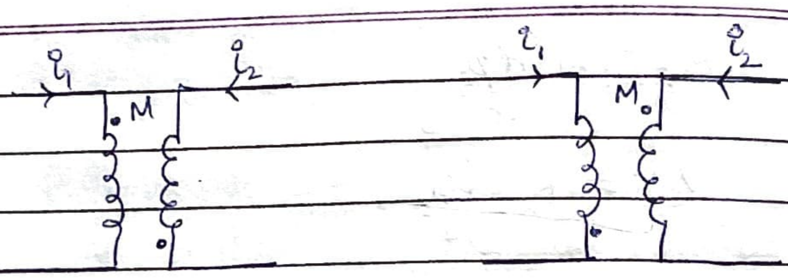
$$v_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

↓

(Same equations)

current is leaving the dot

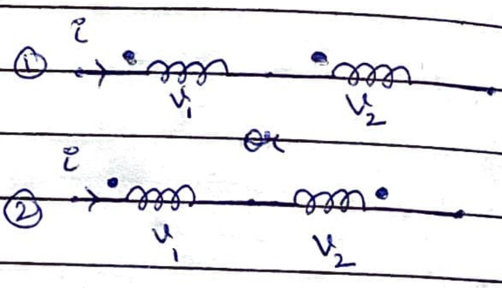




$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

* <u>Mutually Coupled coils connected in series</u>



① $v_1 = L_1 \frac{di}{dt} + M \frac{di}{dt}$

$v_2 = L_2 \frac{di}{dt} + M \frac{di}{dt}$

$v = v_1 + v_2$

$= L_1 \frac{di}{dt} + M \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt}$

$v = (L_1 + L_2 + 2M) \frac{di}{dt}$

$= L_{eq} \frac{di}{dt}$

$L_{eq} = L_1 + L_2 + 2M$

② $v = v_1 + v_2$

$v_1 = L_1 \frac{di}{dt} - M \frac{di}{dt}$

$v_2 = L_2 \frac{di}{dt} - M \frac{di}{dt}$

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$$v = (L_1 + L_2 - 2M)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 - 2M$$

$$\boxed{L_{eq} = L_1 + L_2 \pm 2M}$$

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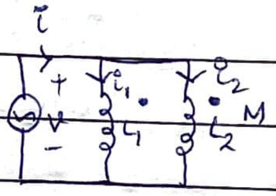
* <u>Mutually Coupled parallel coils</u>

→ Cumulative

→ Differential

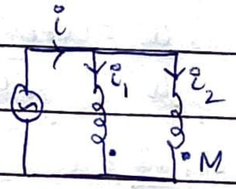
<u>Cumulative</u>

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$



$$i = i_1 + i_2 \quad -①$$

$$v = L_{eq}\frac{di}{dt}$$



$$v = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

differentiating -①

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad -②$$

$$L_{eq}\frac{di}{dt} = v = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$\Rightarrow L_1\frac{di_1}{dt} - M\frac{di_1}{dt} = L_2\frac{di_2}{dt} - M\frac{di_2}{dt}$$

$$\Rightarrow (L_1 - M)\frac{di_1}{dt} = (L_2 - M)\frac{di_2}{dt}$$

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$$\Rightarrow \frac{di_2}{dt} = \left(\frac{L_1 - M}{L_2 - M}\right)\frac{di_1}{dt}$$

or $\Rightarrow$ $$\frac{di_1}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt}$$

in ② $\rightarrow$ $$\frac{di}{dt} = \frac{di_1}{dt} + \left(\frac{L_1 - M}{L_2 - M}\right)\frac{di_1}{dt}$$

$$\Rightarrow \frac{di}{dt} = \frac{di_1}{dt}\left[1 + \frac{L_1 - M}{L_2 - M}\right]$$

$$\Rightarrow \frac{di}{dt} = \frac{di_1}{dt}\left[\frac{L_1 + L_2 - 2M}{L_2 - M}\right]$$

$$\Rightarrow \frac{di_1}{dt} = \left[\frac{L_2 - M}{L_1 + L_2 - 2M}\right]\frac{di}{dt}$$

$$\frac{di_2}{dt} = \left[\frac{L_1 - M}{L_1 + L_2 - 2M}\right]\frac{di}{dt}$$

$$L_{eq}\frac{di}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$L_{eq}\frac{\cancel{di}}{\cancel{dt}} = L_2\left[\frac{L_1 - M}{L_1 + L_2 - 2M}\right]\frac{\cancel{di}}{\cancel{dt}} + M\left[\frac{L_2 - M}{L_1 + L_2 - 2M}\right]\frac{\cancel{di}}{\cancel{dt}}$$

$$L_{eq} = \frac{L_2 L_1 - \cancel{ML_2} + \cancel{ML_2} - M^2}{L_1 + L_2 - 2M}$$

$$\therefore \boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$$

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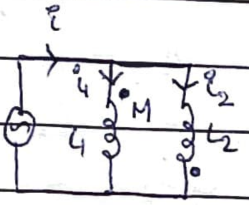
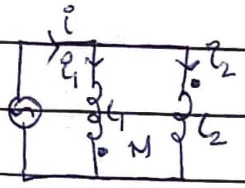
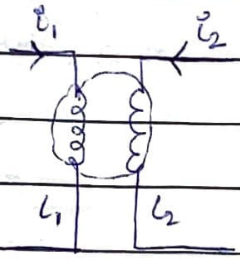
Differential

$$\boxed{L_{eq} = \frac{L_1L_2 - M^2}{L_1+L_2+2M}}$$





* Coefficient of Coupling (k)



$$M_{12} = N_1 \frac{d\phi_{21}}{di_2}$$

$$\Rightarrow M_{12} = N_1 \frac{\phi_{21}}{i_2}$$

$$M_{21} = N_2 \frac{\phi_{12}}{i_1}$$

$$\begin{bmatrix} \phi_{21} = k_2\phi_2 \\ \phi_{12} = k_1\phi_1 \end{bmatrix}$$

$$M_{12} = \frac{N_1k_2\phi_2}{i_2} \quad -(1) \qquad M_{21} = \frac{N_2k_1\phi_1}{i_1} \quad -(2)$$

$$M_{12} = M_{21} = M$$

$$(1) \times (2)$$

$$M^2 = \frac{N_1N_2k_1k_2\phi_1\phi_2}{i_1i_2}$$

$$= k_1k_2 \cdot \frac{N_1\phi_1}{i_1} \cdot \frac{N_2\phi_2}{i_2}$$

$$M^2 = k_1k_2 \cdot L_1 \cdot L_2$$

$$[k_1 = k_2 = k]$$

$$M^2 = k^2L_1L_2$$

$$k = \frac{M}{\sqrt{L_1L_2}}$$

Que. ①

$L_1 = 2H$ $\quad L_2 = 4H$ $\quad L_3 = 6H$

$M_{12} = 1H$ $\quad M_{23} = 1H$ $\quad M_{13} = 0.5H$

$L_{eq} = ?$

Sol.

② The combined inductance of 2 coils connected in series is 0.6H & 0.4H depending upon relative direction of currents in the coils. If one of the coils, when isolated has self inductance of 0.15H then find
a) mutual inductance
b) coefficient of coupling

Sol.

$0.6 = 0.15 + L_2 + 2M$

$0.4 = 0.15 + L_2 - 2M$

$1 = 0.3 + 2L_2$

$0.35 = L_2$

$0.6 = 0.5 + 2M$

$M = 0.05H$

$k = \frac{0.05}{\sqrt{0.35 \times 0.15}} = 0.218$

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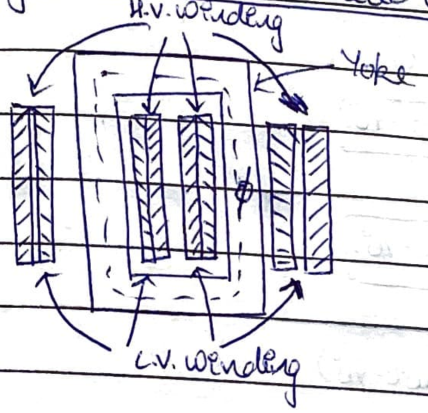
13/11/21 Ch. 4 - Transformers

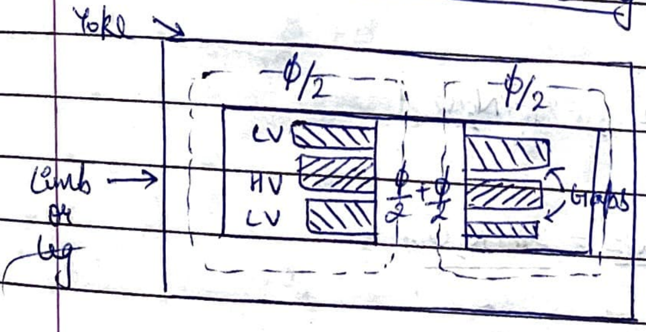
Transformer
- It is a static device
- Working Principle: Electromagnetic Induction
- Transfer of power from one electric circuit to another.

Windings of Transformer

Cylindrical or Concentric Winding (Core Type)



Sandwich or Disc Winding (Shell Type)



Ideal Transformer

1. The winding resistances are zero.
2. There is no leakage flux → leakage reactance is zero.
3. There is no core loss. [$P_h + P_e$ → neglected]

P_h → hysteresis loss; P_e → eddy current loss

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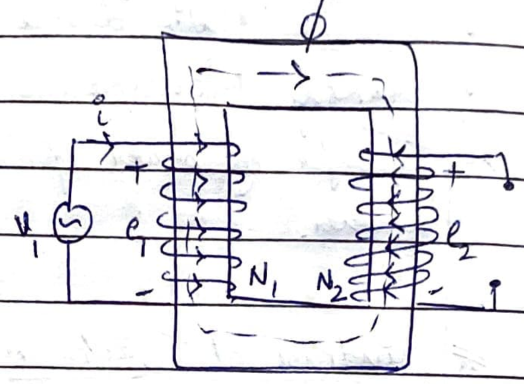
4. Neglegible magnetizing current to establish the working flux. (Permeability = ∞)

-ve due to Lenz's law

$$e_1 = -N_1 \frac{d\phi}{dt}$$



$$e_2 = -N_2 \frac{d\phi}{dt}$$

$$\boxed{\phi = \phi_m \sin \omega t}$$

$$e_1 = -N_1 \phi_m \omega \cos \omega t$$

$$e_1 = N_1 \phi_m \omega (-\cos \omega t)$$

$$\boxed{e_1 = N_1 \phi_m \omega \sin(\omega t - 90^\circ)}$$

$$\boxed{e_2 = N_2 \phi_m \omega \sin(\omega t - 90^\circ)}$$

$$e_2 = N_2 \phi_m 2\pi f \sin(\omega t - 90^\circ)$$
$$= 2\pi f N_2 \phi_m \sin(\omega t - 90^\circ)$$
$$e_2 = E_{m2} \sin(\omega t - 90^\circ)$$

$$\because [E_{max2} \text{ or } E_{m2} = 2\pi f N_2 \phi_m]$$

$$E_2 = \frac{E_{max2}}{\sqrt{2}} = \frac{2\pi f N_2 \phi_m}{\sqrt{2}}$$

rms value

$$\boxed{E_2 = 4.44 f N_2 \phi_m}$$

$$\boxed{E_1 = 4.44 f N_1 \phi_m}$$

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = a \rightarrow \text{turns ratio}$$

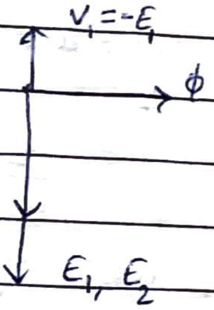
$$\because \begin{bmatrix} V_1 = E_1 \\ V_2 = E_2 \end{bmatrix}$$

* Phasor for Ideal Transformer on No Load

→ magnetizing current is negligible

$\mu_r \to \infty$

$V_1 = e_1$ (same magnitude, opp. dir^n)

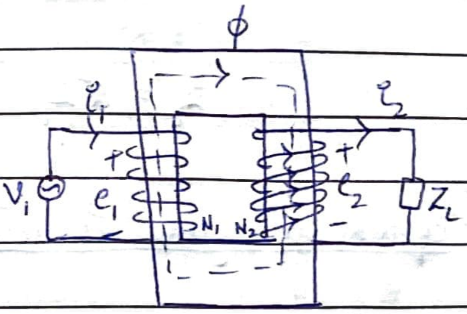


* Ideal Transformer on Load

$i_2 \to \phi_2$

$\phi_2 \to$ opposing ϕ.

$\phi_r \to \phi - \phi_2$



$I_2 N_2 = N_1 I_2'$

$N_1 I_1 \to$ mmf

$$\frac{N_1 I_1}{\text{core reluctance}} = \phi$$

$$\phi_2 = \frac{I_2 N_2}{R_c}$$

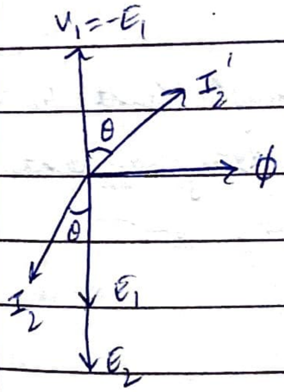
$$\frac{I_2}{I_2'} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

$$\phi_r = \phi \underbrace{- \cancel{\phi_2} + \cancel{\phi_1}}$$

equal in magnitude but opposite dir^n

in ideal cond^n

$I_2' = I_1$

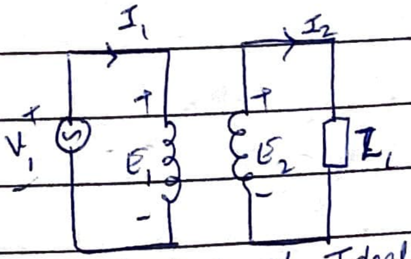


$$I_2' = I_2 \frac{N_2}{N_1}$$

In ideal Transformer,

$I_2' = I_1$

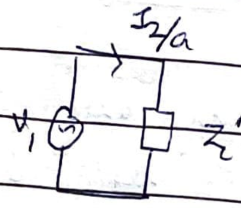
loss $= I_2^2 Z_L$



Equivalent of Ideal Transformer

Equivalent referred to primary

$$\frac{V_1}{V_2} = a = \frac{I_2}{I_1}$$



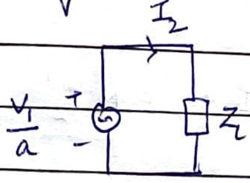
$$I_1 = \frac{I_2}{a}$$

$$\text{loss} = Z_L' \times \left(\frac{I_2}{a}\right)^2$$

$$\therefore Z_L'\left(\frac{I_2}{a}\right)^2 = Z_L I_2^2$$

$$Z_L' = a^2 Z_L$$

Equivalent referred to secondary



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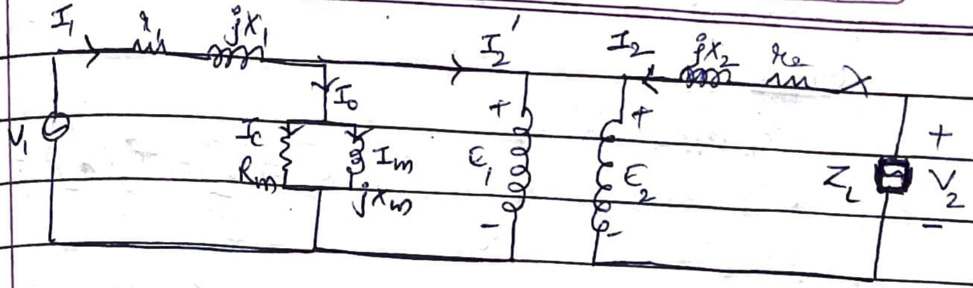
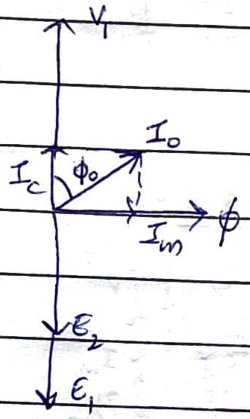
★ Practical Transformer

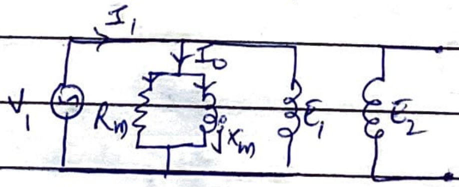
- Winding resistance is accounted as series resistance.
- The effect of leakage flux is accounted through series reactances.
- Core loss will be accounted through shunt resistance.
- Permeability a finite → magnetizing current to establish working flux → shunt element.



★ Phasor of Practical Transformer on no load

$I_2 = 0$
$I_2' = 0$
$I_1 = I_0$
$\phi_0 \to$ no load pf angle
at no load ϕ_0 is large
as $I_m > I_c$
no load pf of transformer is poor.





drop across $(r_1 + jX_1)$ is neglected as I_0 is just 2.5% of rated I_1

★ Phasor diagram of Transformer on load

Inductive, Resistive, Capacitive

Inductive ↓ I_2 lags V_2

Resistive ↓ I_2 & V_2 are in phase

Capacitive ↓ I_2 leads V_2

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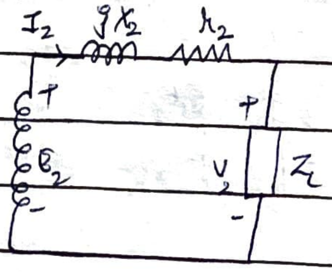
Inductive Load

Step 1

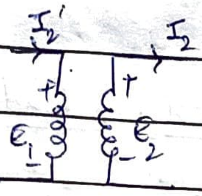
secondary winding



$$\vec{E_2} = \vec{V_2} + \vec{I_2}(r_2 + jX_2)$$

Step 2

both

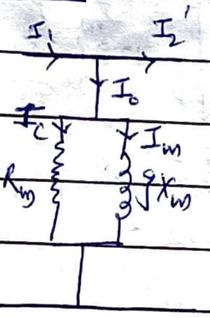


$$I_2' = \frac{I_2}{a}$$

$$\frac{E_1}{E_2} = a$$

Step 3

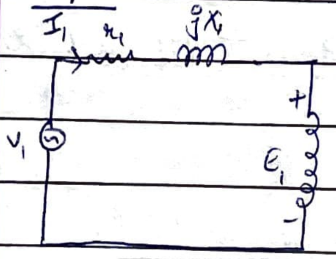
primary winding branch



$$\vec{I_1} = \vec{I_0} + \vec{I_2'}$$

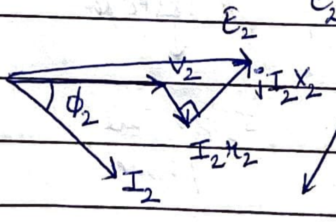
Step 4

I_0 is negligible $\therefore$ branch can be removed

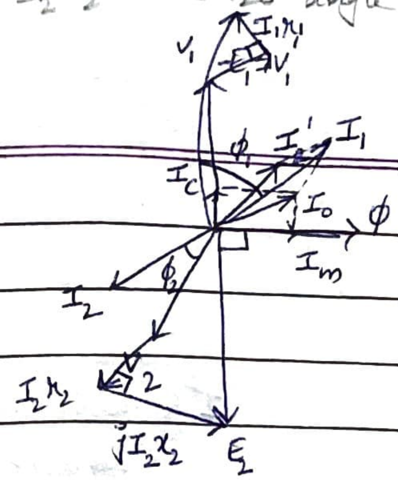


$$V_1 = \vec{E_1} + \vec{I_1}(r_1 + jX_1)$$

$$\vec{E_2} = \vec{V_2} + \vec{I_2} r_2 + jX_2 \vec{I_2}$$



I_2 I_2 have 120° angle b/w them.

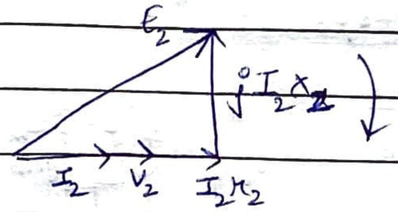


$I_m = I_0 \sin\phi_0$

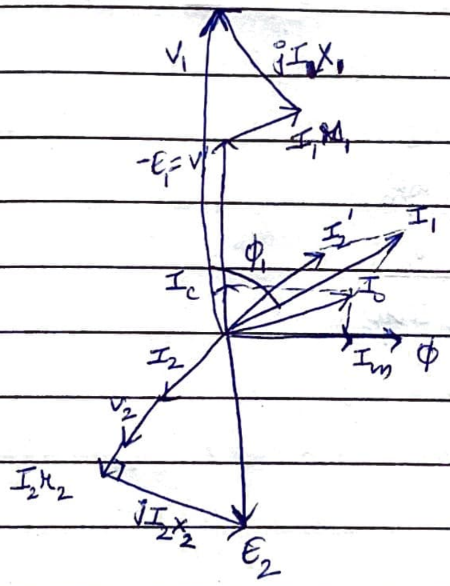
$I_c = I_0 \cos\phi_0$

Resistive Load

I_2 and V_2 are in same phase.



$\vec{E_2} = \vec{V_2} + \vec{I_2 r_2} + j\vec{I_2 X_2}$



$I_1 \rightarrow I_1 r_1$ same phase

$I_1 x_1$ lead I_1 by 90°

$I_2 \rightarrow I_2 r_2$ same phase

$I_2 x_2$ lead I_2 by 90°

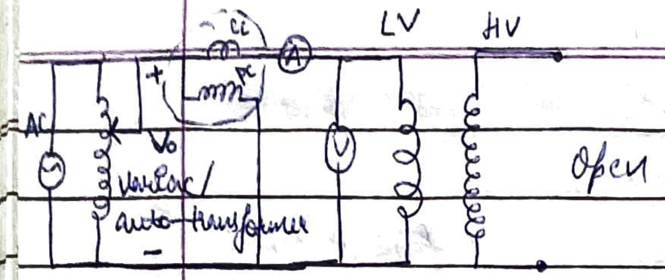
★ Tests of Transformer

① Open circuit Test

→ Low Voltage side

→ High Voltage side is open circuit.

→ Performed at rated voltage and frequency.



I^2R loss are negligible

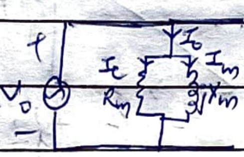
$V_0 = V_L^{\text{rated}}$

Power consumed → core loss P_c

→ R_m & X_m referred to LV (core loss)

$W_o = I_o V_o \cos\phi_o$

$\cos\phi_o = \dfrac{W_o}{I_o V_o}$



$I_c = I_o \cos\phi_o$

$I_m = I_o \sin\phi_o$

$R_m = \dfrac{V_o}{I_c} = \dfrac{V_o}{I_o\cos\phi_o}$

$X_m = \dfrac{V_o}{I_m} = \dfrac{V_o}{I_o\sin\phi_o}$

→ $R_m^{HV} = a^2 R_m$
$X_m^{HV} = a^2 X_m$ } referred to HV (secondary side)

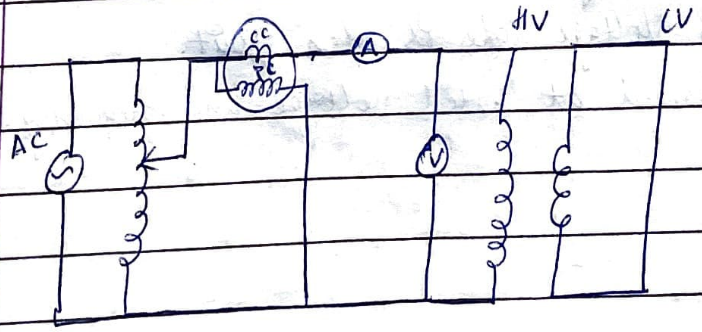
② Short Circuit Test

→ Performed at rated current

$I_{sc} = I_{fL} = I_{\text{rated}}$
→ full load

→ HV side

→ LV side is short-circuited



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$$P_{sc} = P_{cu}^{fL}$$

$$P_{sc} = I_{sc}^2 R_{sc}$$

$$R_{sc} = \frac{P_{sc}}{I_{sc}^2}$$

$$Z_{sc} = \frac{V_{sc}}{I_{sc}}$$

$$X_{sc} = \sqrt{Z_{sc}^2 - R_{sc}^2}$$

R_{sc} & $X_{sc} \rightarrow$ referred to HV

$$X_{HV} = X_{LV}' = \frac{X_{sc}}{2}$$

$\downarrow$

LV side reactance referred to HV side

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Losses → Core loss (P_i)
→ Copper loss / I^2R loss (P_{cu})

$P_i = P_e + P_h$

$P_{cu} = I_1 r_1 + I_2 r_2$
$= I_1^2 R_{e1} = I_2^2 R_{e2}$

R_{e1} → eq. resistance on primary side
R_{e2} → eq. resistance on secondary side

Power Output $= V_2 I_2 \cos\phi_2$ → angle b/w V_2 & I_2

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + \underbrace{P_{cu}}_{\to I_2^2 R_{e2}}}$$

$$\eta = \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + \frac{P_i}{I_2} + I_2 R_{e2}}$$

for max. efficiency -

$$\frac{d\eta}{dI_2} = 0$$

$\cancel{\Rightarrow 0}$

$$\Rightarrow P_i = R_{e2} I_2^2$$

Iron loss = ~~full load~~ copper loss

constant $\boxed{P_i = P_{cu}}$ variable

$$P_{cu}^{fl} = I_{2fl}^2 R_{e2}$$

$$\frac{I_2^2}{I_{2fl}^2} = \frac{P_i}{P_{cu}^{fl}} = k^2 \qquad \because k = \frac{I_2}{I_{2fl}}$$

$$k = \frac{I_2}{I_{2fl}} = \sqrt{\frac{P_i}{P_{cu}^{fl}}}$$

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{e2}}$$

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$$\eta = \frac{V_2 I_{2fl} k \cos\phi_2}{V_2 I_{2fl} k \cos\phi_2 + 2P_i}$$

$$= \frac{k V_2 I_{2fl} \cos\phi_2}{k V_2 I_{2fl} \cos\phi_2 + 2P_i}$$

$$\boxed{\eta_{max} = \frac{k S_{rated} \cos\phi_2}{k S_{rated} \cos\phi_2 + 2P_i}}$$

<u>Rating of Transformer</u>

$\underbrace{kVA / VA / MVA}_{\text{rated VA}}$; N_1/N_2 (V_1/V_2) ; frequency
(S_{rated})

eg. The following test results are obtained on a 20 kVA, 2000/200 V, 50Hz transformer.

OC Test : 200 V ; 4 A ; 120 W $\rightarrow P_i$ (LV)

SC Test : 60 V ; 10 A ; 300 W $\rightarrow P_{cu}^{fl}$ (HV)

Calculate the efficiency at -

(i) rated load and 0.8 lagging p.f.

(ii) at half load and 0.8 lagging p.f.

Sol. (i) $S_{rated} = 20 kVA$

$= I_{LV} V_{LV} = I_{HV} V_{HV}$

$20 \times 10^3 = I_{HV} \times 2000$

$I_{HV} = 10 A$

$$\eta = \left(\frac{20 \times 10^3 \times 0.8}{20 \times 10^3 \times 0.8 + 120 + 300} \right) \times 100$$

$\eta = 97.44\%$

(ii) half load $\rightarrow I_2 = \frac{I_{2fl}}{2}$

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + P_{cu}}$$

$$= \frac{V_2 \frac{I_{2fl}}{2} \cos\phi_2}{\frac{V_2 I_{2fl}}{2}\cos\phi_2 + P_i + \left(\frac{I_{2fl}}{2}\right)^2 R_{e2}}$$

$$= \frac{\frac{20\times10^3}{2}\times0.8}{\frac{20\times10^3}{2}\times0.8 + 120 + \frac{P_{cu}^{fl}}{4}}$$

$$= 97.62\ \%$$

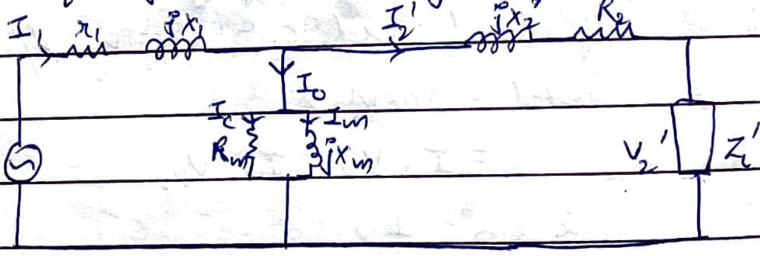
NOTE: at any fraction of load $(x) \rightarrow I_2 = x I_{2fl}$

Output $= x\, S_{rated}$

Copper Loss $= x^2 P_{cu}^{fl}$

* Equivalent

Exact eq. referred to primary side –



$$a = \frac{N_1}{N_2} = \frac{V_1}{V_2}$$

$V_2' = aV_2$

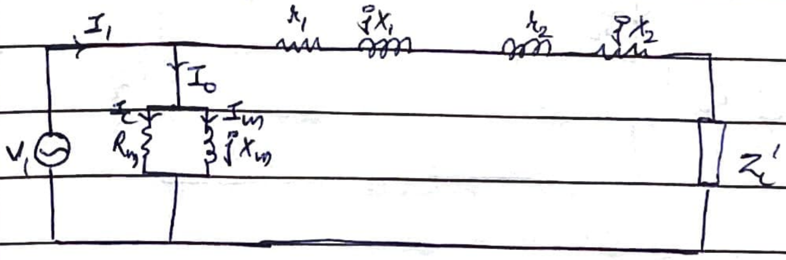
$I_2' = I_2/a$

$r_2' = a^2 r_2$

$X_2' = a^2 X_2$

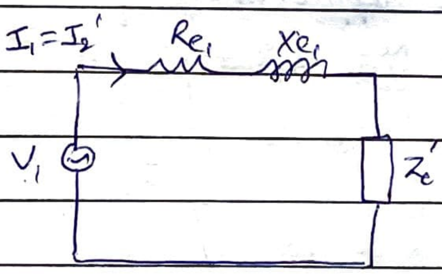
$Z_L' = a^2 Z_L$

Approximate equivalent circuit

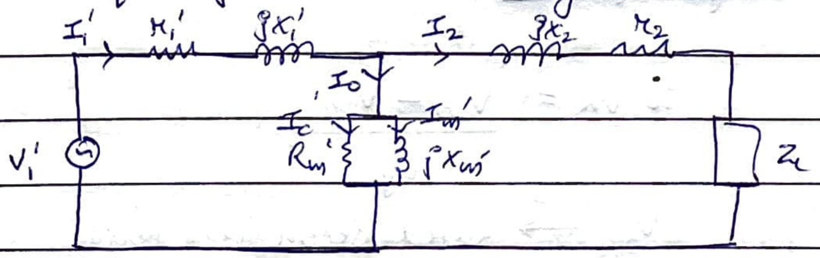


$$R_{eq_1} = r_1 + r_2' = r_1 + a^2 r_2$$

$$X_{eq_1} = X_1 + X_2' = X_1 + a^2 X_2$$



Exact eq. referred to secondary side -



$$\frac{N_1}{N_2} = a$$

$$V_1' = \frac{V_1}{a} \qquad I_0' = aI_0$$

$$I_c' = aI_c$$

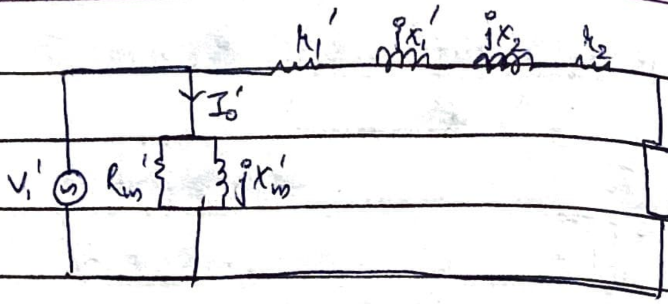
$$I_1' = aI_1 \qquad I_m' = aI_m$$

$$r_1' = r_1/a^2 \qquad \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

$$X_1' = X_1/a^2$$

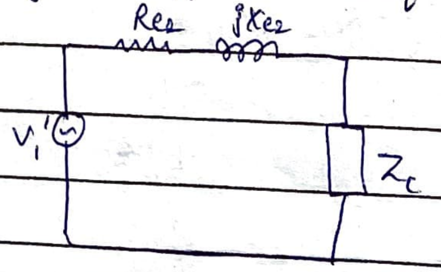
$$R_m' = R_m/a^2 \qquad \frac{I_1'}{I_1} = \frac{N_1}{N_2} \rightarrow a$$

$$X_m' = X_m/a^2$$



$$R_{e2} = r_1' + r_2 = r_1/a^2 + r_2$$
$$X_{e2} = x_1' + x_2 = x_1/a^2 + x_2$$

Simplified approximate eq. referred to secondary



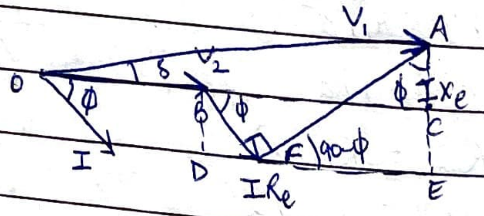
★ Voltage Regulation

$$\% VR = \frac{V_{20} - V_{2fl}}{V_{2fl}} \times 100$$

$V_{2fl} \rightarrow$ rated secondary voltage
$V_{20} \rightarrow$ no load secondary voltage.

- Use simplified approximate equivalent

$$\vec{V_1} = \vec{V_2} + \vec{I}\,(R_e + jX_e)$$



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Voltage drop across Z_e is very small as compared to rated V_2. $\delta \simeq 0$

$V_1 \simeq OC$

$OC = OB + Df + FE$

$V_1 = V_2 + IR_e\cos\phi + IX_e\sin\phi$

$V_1 - V_2 \simeq IR_e\cos\phi + IX_e\sin\phi$

for lagging pf (RL load) — $VR = \dfrac{IR_e\cos\phi + IX_e\sin\phi}{V_2} \times 100$

for leading pf (RC load) — $VR = \dfrac{IR_e\cos\phi - IX_e\sin\phi}{V_2} \times 100$

Ques. ⑥ A 100 kVA transformer has —

$N_1 = 400$; $N_2 = 100$

$r_1 = 0.3\Omega$; $r_2 = 0.015\Omega$

$x_1 = 1.1\Omega$; $x_2 = 0.055\Omega$

Supply voltage is 2400V. Calculate

- R_{e1}, X_{e1}
- VR at 0.8 lagging pf ; V_2
- VR at 0.8 leading pf ; V_2
- pf for zero VR

Sol. • $R_{e1} = r_1 + a^2 r_2$ $\qquad a = \dfrac{N_1}{N_2} = 4$

$= 0.3 + 16 \times 0.015$

$= 0.54\Omega$

$X_{e1} = x_1 + a^2 x_2$

$= 1.1 + 16 \times 0.055$

$= 1.98\Omega$

$S_{rated} = V_1 I_1 = V_2 I_2$

$100 \times 10^3 = 2400 \times I_1$

$I_1 = 41.67A$

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lagging • $VR = \frac{41.67 \times 0.54 \times 0.8 + 41.67 \times 1.98 \times 0.8}{2400} \times 100$

$= 2.81\%$

leading • $VR = \frac{41.67 \times 0.54 \times 0.8 - 41.67 \times 1.98 \times 0.8}{2400} \times 100$

$= -1.81\%$

lagging • $V_1 - V_2 = I_e R_e \cos\phi + I_e X_e \sin\phi$

$V_2 = V_1 - I_e R_e \cos\phi - I_e X_e \sin\phi$

$V_2' = 2332.99 \text{ V} = V_2'$

$[\text{as } V_1 = V_2' + I_1 (R_{e1} + jX_{e1})]$

$V_2 = \frac{V_2'}{a} = 583.12 \simeq 583 \text{ V}$

always (leading) for 0 VR • $\tan\phi = \frac{+R_e}{X_e} = 16.255$

$\cos\phi = 0.96$ leading

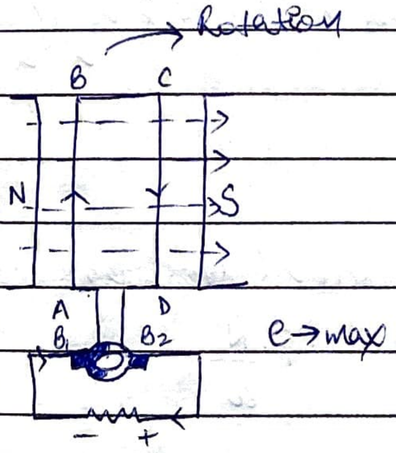
25/1/21 Ch. 5 - DC Machines

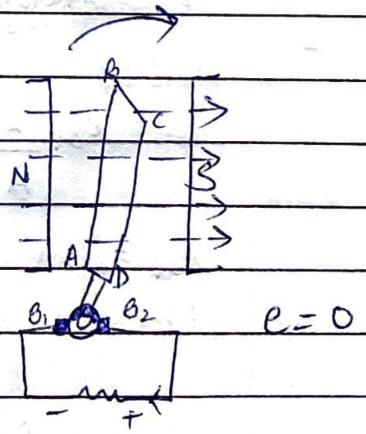
- Direct-current (DC) machines are divided into DC generators and DC motors.
- DC machines are similar to AC machines.
- DC machines have DC outputs - they have a mechanism converting AC voltages to DC voltages at their terminals.
- This mechanism is called a commutator, ∴ DC machines are also called commutating machines.

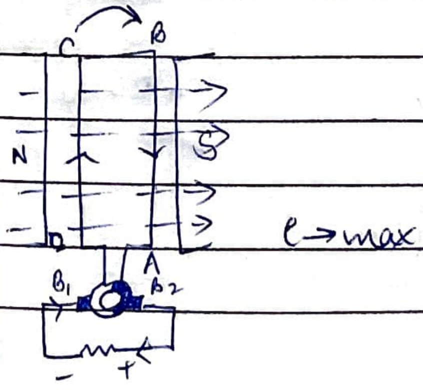
DC Generator

- Mechanical energy is converted to electrical energy.
- BF1 Induced current is found out using Fleming's Right Hand Rule.

* Working Principle







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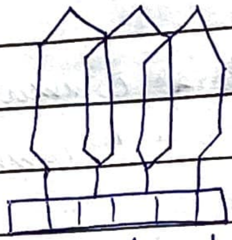
★ EMF Equation of DC Generator

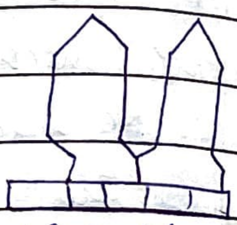
Rotor / Armature

- Lap wound $\rightarrow A = P$
- Wave wound $\rightarrow A = 2$

A : no. of parallel paths



Lap wound



Wave wound

$$e = -N\frac{d\phi}{dt}$$

EMF generated by one parallel path $\rightarrow E_g$

$P \rightarrow$ no. of poles

$\phi \rightarrow$ flux per pole

$Z \rightarrow$ total no. of conductors

$A \rightarrow$ no. of parallel paths

$N \rightarrow$ speed of machine in rpm

E_g = Average emf per conductor × no. of conductor in a parallel path

$$= e_{av}\frac{Z}{A}$$

e_{av} = flux cut per second (Faraday's law)

Flux cut per second $= P\phi.\frac{N}{60}$

$$e_{av} = \frac{P\phi N}{60}$$

$$E_g = \frac{P\phi N Z}{60A}\ V$$

$\phi, N \rightarrow$ variable

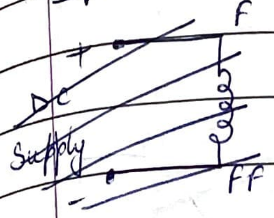
$P, Z, A \rightarrow$ constant for a machine

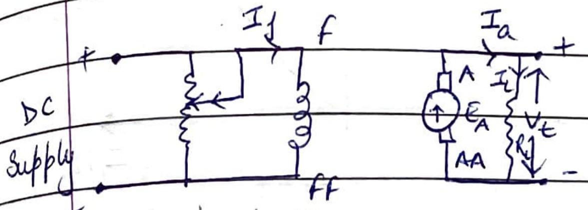
$$= \frac{PZ}{2\pi 60A}\ 2N\phi\pi$$

$$\boxed{E_g = K_a \phi \omega_m}$$

$$\because K_a = \frac{PZ}{2\pi \times 60A}$$

* Equivalent of DC Machine





field winding armature

$R_a \rightarrow$ armature resistance

Separately excited DC generator

$$\boxed{V_t = E_a - I_a R_A}$$

$$I_a = I_L$$

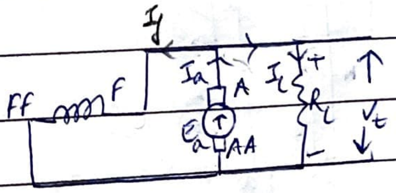
$$I_f = \frac{V_f}{R_f}$$

Shunt Generator

$$I_a = I_f + I_L$$

$$I_f = \frac{V_t}{R_{sh}}$$

$$I_L = \frac{V_t}{R_L}$$



$$\boxed{V_t = E_a - I_a R_a}$$

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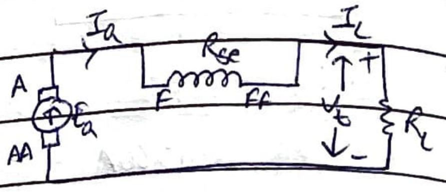
Series Generator



$$I_a = I_f = I_L$$

$$\boxed{V_t = E_a - I_a(R_a + R_{se})}$$

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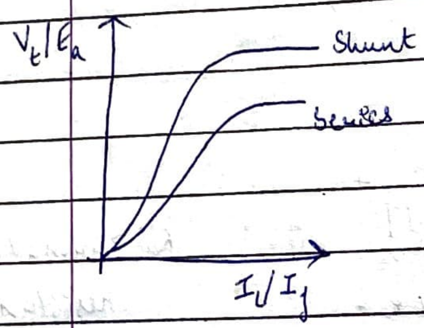
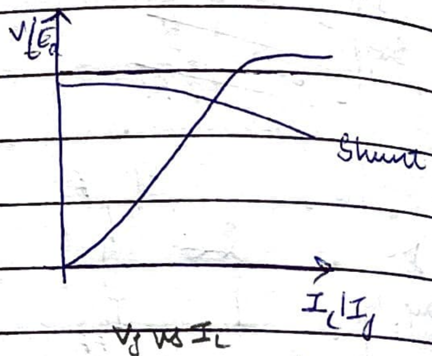




Shunt: Magnetization Curve / E_a vs I_f

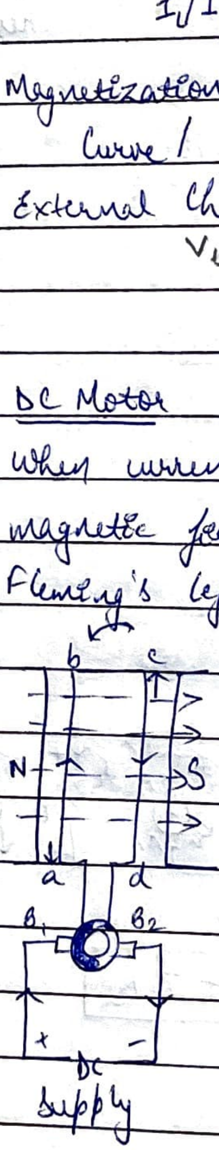
Series: External Characteristics V_t vs I_L

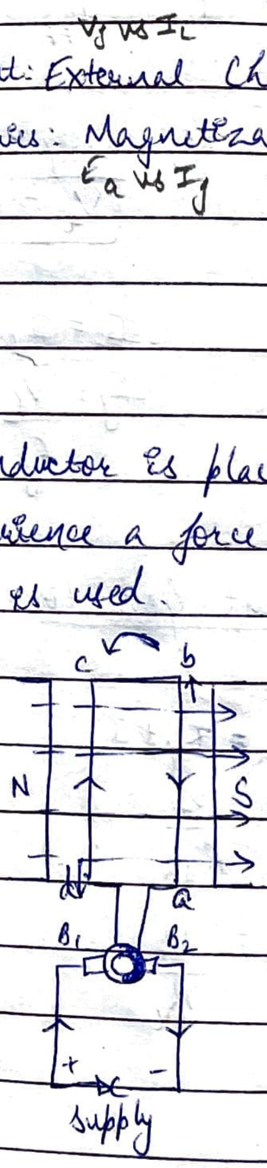
Shunt: External Characteristics V_t vs I_L

Series: Magnetization Curve E_a vs I_f

DC Motor

- When current carrying conductor is placed inside magnetic field, it will experience a force.
- Fleming's left Hand Rule is used.





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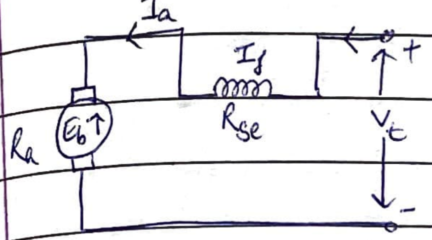
$T_e \rightarrow$ electromagnetic torque is experienced
$E_b \rightarrow$ back emf is ~~introduced~~ produced $= K_a \phi \omega_m$

* Electrical Equivalent of Motor

Series

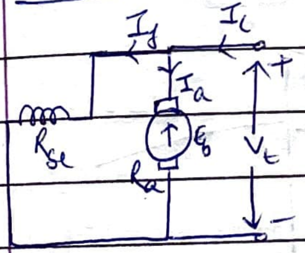


$I_f = I_a = I_L$

$V_t = I_f R_{se} + I_a R_a + E_b$

$$\boxed{V_t = E_b + I_a (R_a + R_{se})}$$

Shunt



$I_L = I_a + I_f$

$I_f = \dfrac{V_t}{R_{se}}$

$$\boxed{V_t = E_b + I_a R_a}$$

* Torque Equation

$V_t = E_b + I_a R_a$ or $V_t = E_b + I_a (R_a + R_{se})$

$V_t I_a = E_b I_a + I_a^2 R_a$

$V_t I_a$: total electrical power supplied to armature

$E_b I_a$: mechanical power

$I_a^2 R_a$: copper loss

Power = Torque × Angular Speed

$E_b I_a = T_e \omega_m$

$$T_e = \frac{E_b I_a}{\omega_m}$$

$$= \frac{K_a \phi \cancel{\omega_m} I_a}{\cancel{\omega_m}}$$

$$\boxed{T_e = K_a \phi I_a}$$

Series motor

$\phi \propto I_a$

$\phi = C I_a$

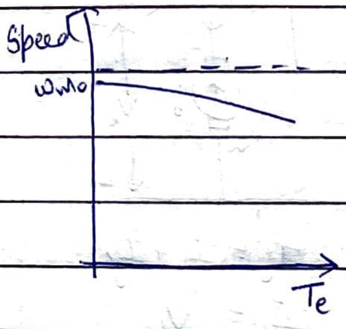
$T_e = K_a C I_a^2$

Shunt motor

$$\omega_m = \frac{E_b}{K_a\phi} = \frac{V_t - I_a R_a}{K_a\phi}$$

$T_e = K_a \phi I_a$

$$I_a = \frac{T_e}{K_a\phi}$$



$$\omega_m = \frac{V_t}{K_a\phi} - \frac{T_e R_a}{(K_a\phi)^2}$$

$$\omega_m = \underbrace{\omega_{m_0}}_{\text{no load speed}} - \frac{T_e R_a}{(K_a\phi)^2}$$

$$\because \quad \omega_m = \frac{V_t - I_a R}{K_a\phi} \quad \text{if } I_a \approx 0 \rightarrow \text{no load}$$

$$\omega_{m_0} = \frac{V_t}{K_a\phi}$$

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Series Motor

never operated at no load

$T_e = K_a C I_a^2$, $\phi = C I_a$, $I_a = \sqrt{\frac{T_e}{K_a C}}$

$V_t = E_b + I_a(R_a + R_{se})$

$E_b = K_a \phi \omega_m$

$$\omega_m = \frac{E_b}{K_a \phi}$$

$$= \frac{V_t - I_a(R_a + R_{se})}{K_a \phi}$$

$$= \frac{V_t}{K_a \phi} - \frac{I_a(R_a + R_{se})}{K_a \phi}$$

$$= \frac{V_t}{K_a C I_a} - \frac{\cancel{I_a}(R_a + R_{se})}{K_a C \cancel{I_a}}$$

$$= \frac{V_t}{K_a C \sqrt{\frac{T_e}{K_a C}}} - \frac{R_a + R_{se}}{K_a C}$$

$$\omega_m = \frac{V_t}{\sqrt{T_e K_a C}} - \frac{(R_a + R_{se})}{K_a C}$$