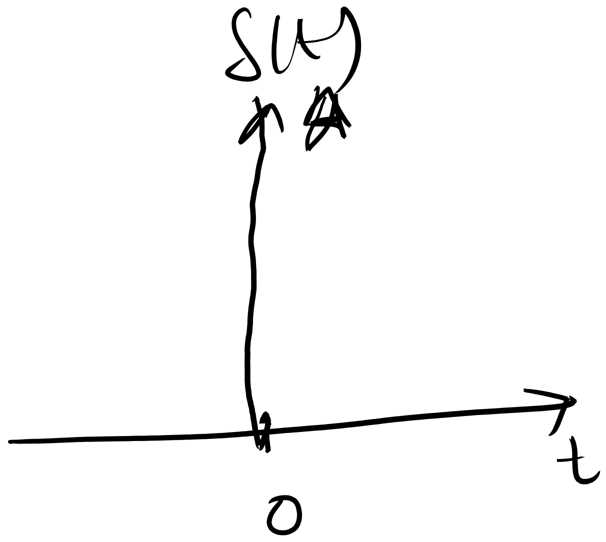
* Test Signals:

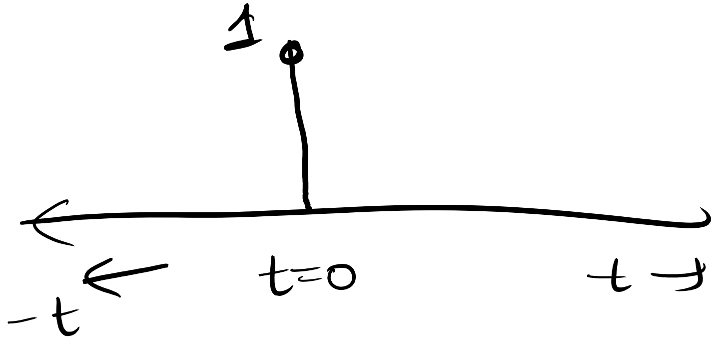
① Impulse function: (δ)

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & \text{otherwise} \end{cases}$$



Unit Impulse function:

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$$



② Step function : (u)

$$u(t) = \begin{cases} A & t > 0 \\ 0 & t < 0 \end{cases}$$

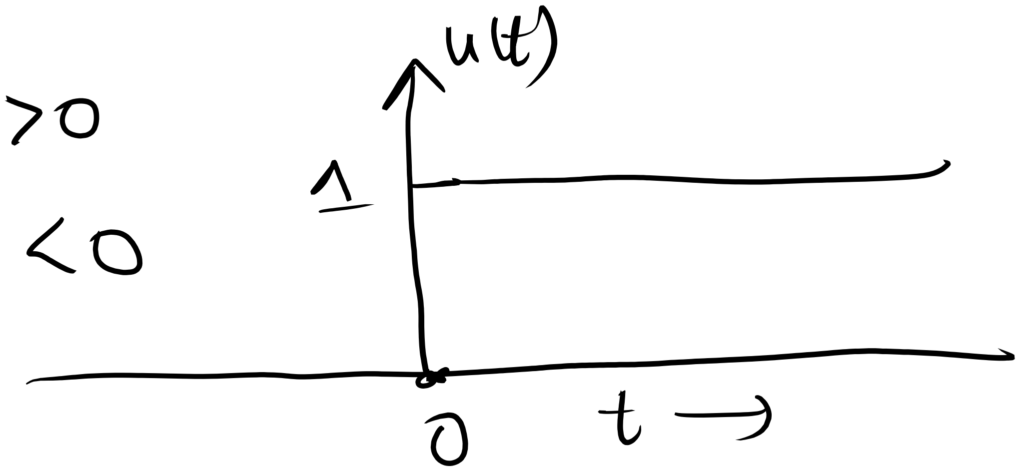
At $t = 0$; step function is undefined.

At $t = 0 \rightarrow$ discontinous

Unit step function: $(A = 1)$

①

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

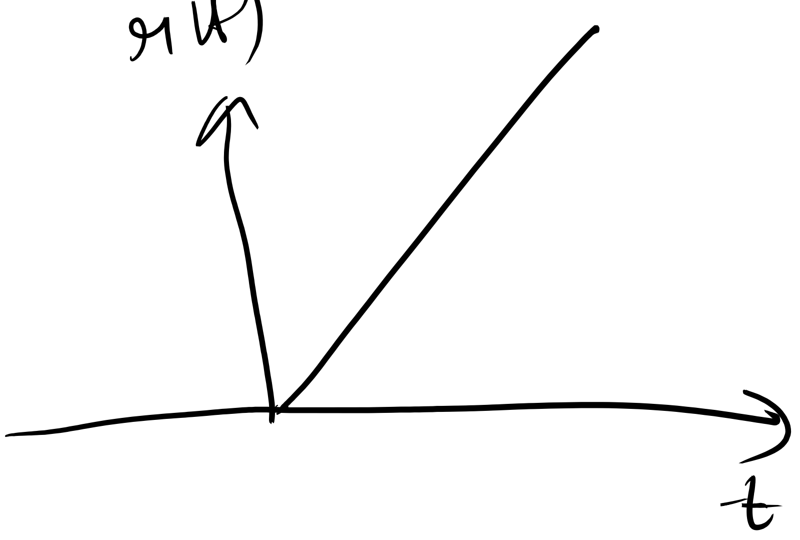


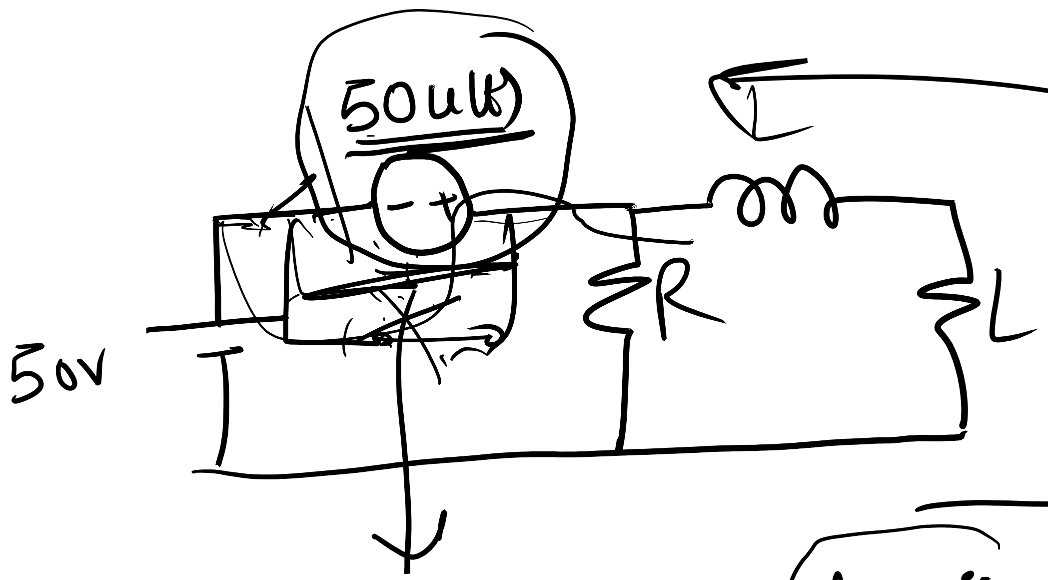
③ Ramp function (r(t))

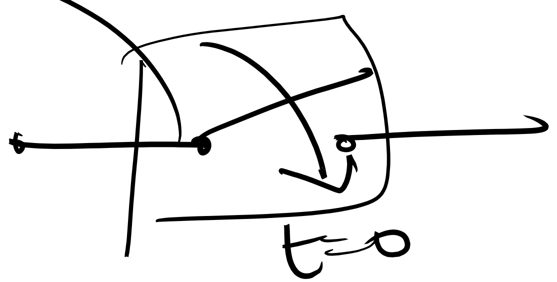
$$r(t) = \begin{cases} At & t > 0 \\ 0 & t < 0 \end{cases}$$

Unit ramp function:

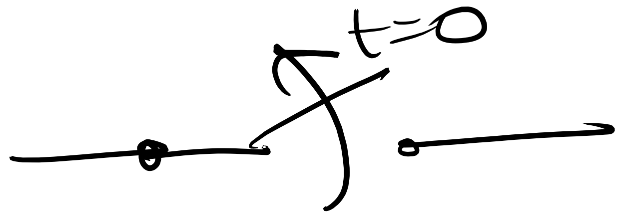
$$\delta(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$







output response

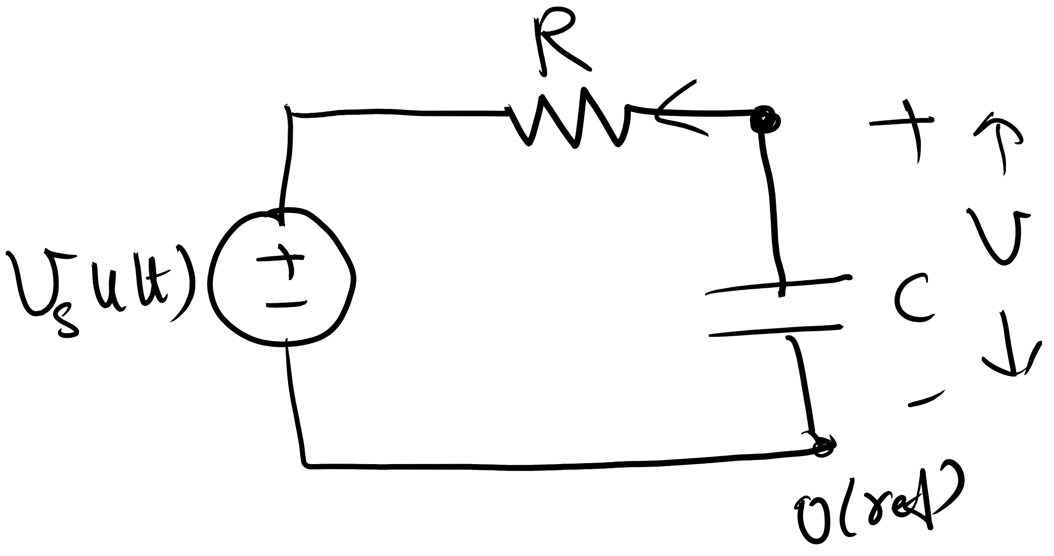


Voltage Step forcing function

$$50u(t) = \begin{cases} 50 & t > 0 \\ 0 & t < 0 \end{cases}$$

excitation
forcing function
input

* Step response of RC Circuit



$$V_s u(t) = \begin{cases} V_s & t>0 \\ 0 & t<0 \end{cases}$$

$$\frac{v - V_s u(t)}{R} + \frac{C\,d(v-0)}{dt} = 0$$

$$\frac{v}{R} - \frac{V_s u(t)}{R} + \frac{C\,dv}{dt} = 0 \longrightarrow (1)$$

For $t>0$

$$\frac{v}{R} - \frac{V_s}{R} + \frac{C\,dv}{dt} = 0 \longrightarrow (2)$$

$$\frac{v}{R} + \frac{C\,dv}{dt} = \frac{V_s}{R}$$

$$\frac{v}{RC} + \frac{dv}{dt} = \frac{V_s}{RC}$$

$$\frac{dv}{dt} = \frac{V_s}{RC} - \frac{v}{RC}$$

$t=0 \longrightarrow V_0$

$t=t \longrightarrow v(t)$

* $v(t) = V_s + (v_0 - v_s)e^{-t/RC}$

let $\tau = RC$;

$$\boxed{v(t) = \underline{v_s} + (v_0 - v_s)e^{-t/\tau}}$$

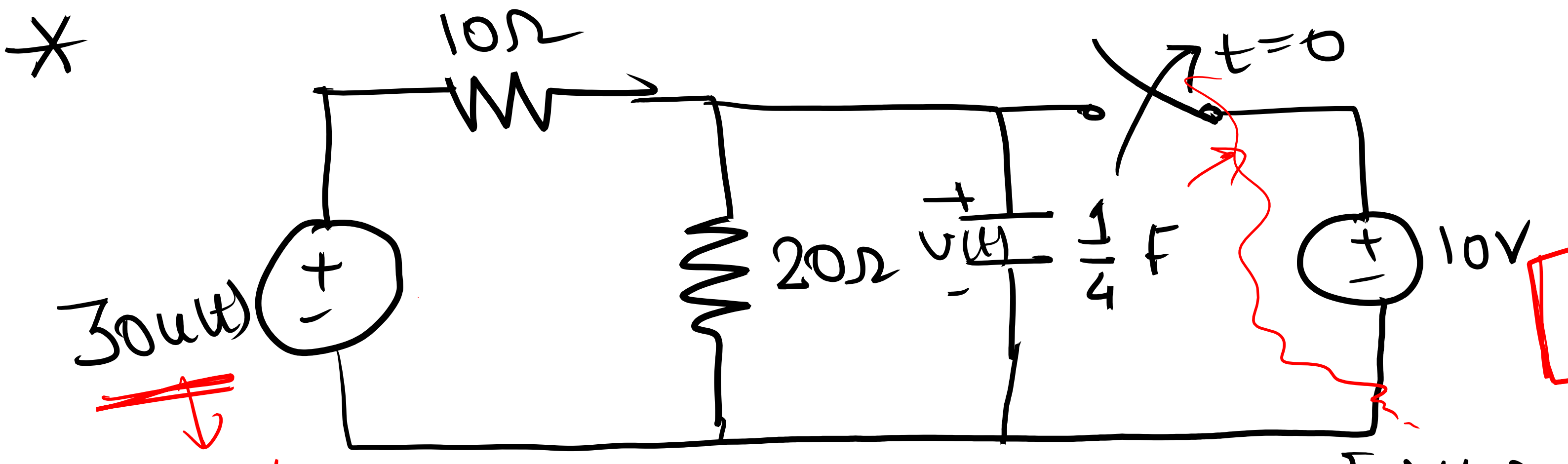
for source free circuits (RC)

$v_s = 0$

$$\boxed{v(t) = V_0 e^{-t/\tau}}$$

$$\boxed{v(t) = \underbrace{V(\infty)}_{\text{Steady state}} + (\overbrace{V(0)}^{\text{initial}} - V(\infty))e^{-t/\tau}}$$

*



v(t) for all time

$$\boxed{v(t) = 20 + [10-20]\,e^{-3t/5}} \text{ Volts}$$

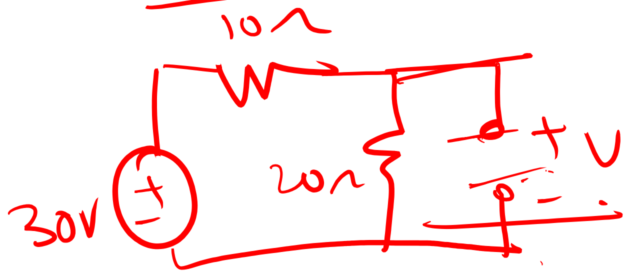
$$v(t) = \underbrace{v(\infty)}_{t=\infty} + [\underbrace{v(0)}_{t=0^-} - v(\infty)]\,e^{-t/\tau}$$

$\tau = R_{eq} \cdot C$

Sol: At $t<0$

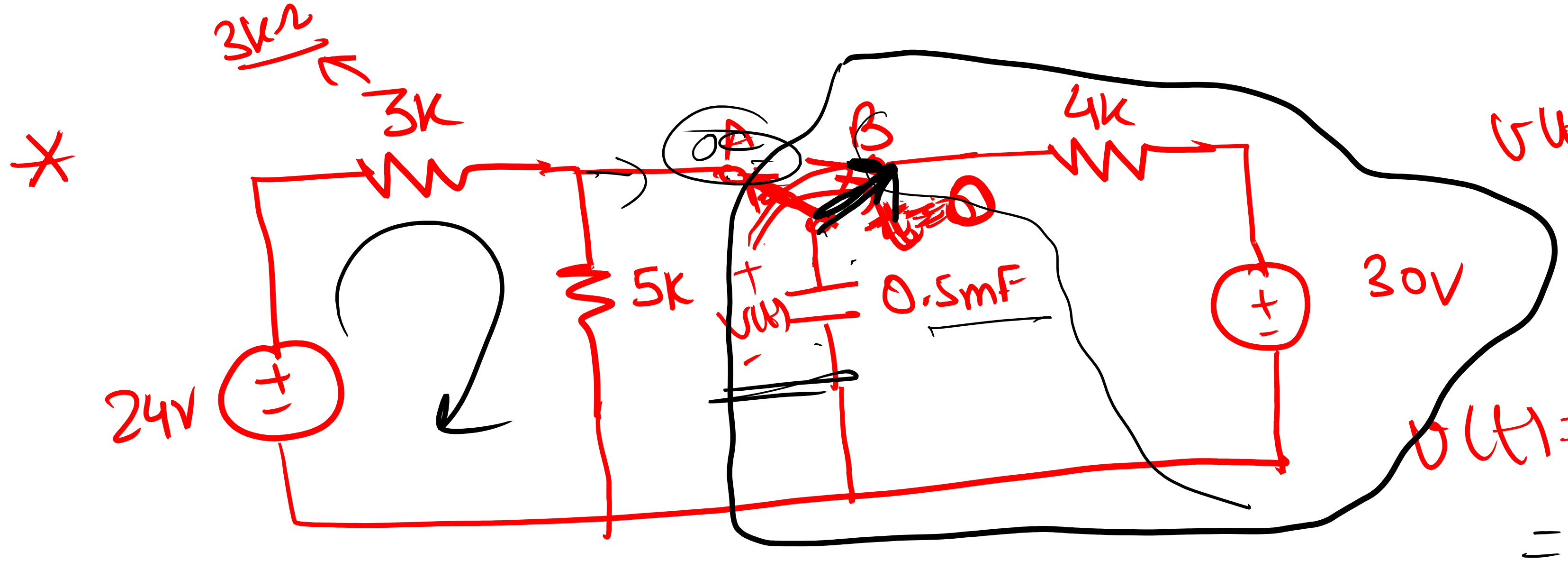
$v(0) = 10\text{ V}$

For $t>0$

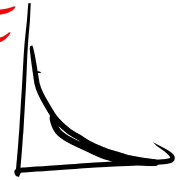


$+ \; v(\infty) = 20\text{ V}$

$$\tau = R_{eq} \cdot C = \frac{20}{3} \times \frac{1}{4} = \frac{5}{3}\text{ s}$$



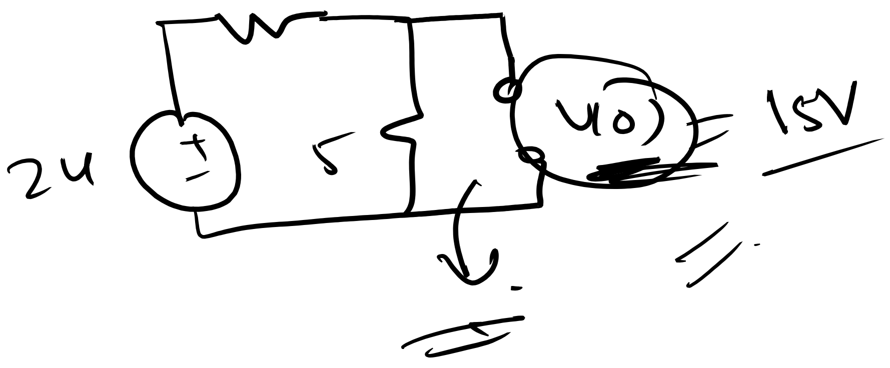
v(t) for all t



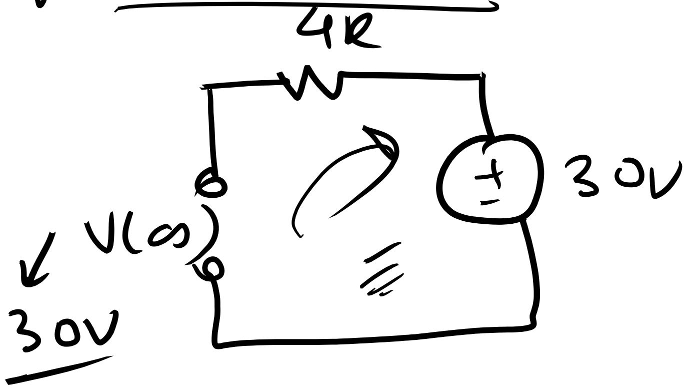
$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

$$= (30 - 15e^{-t/2}) \text{ Volt}$$

① for t<0



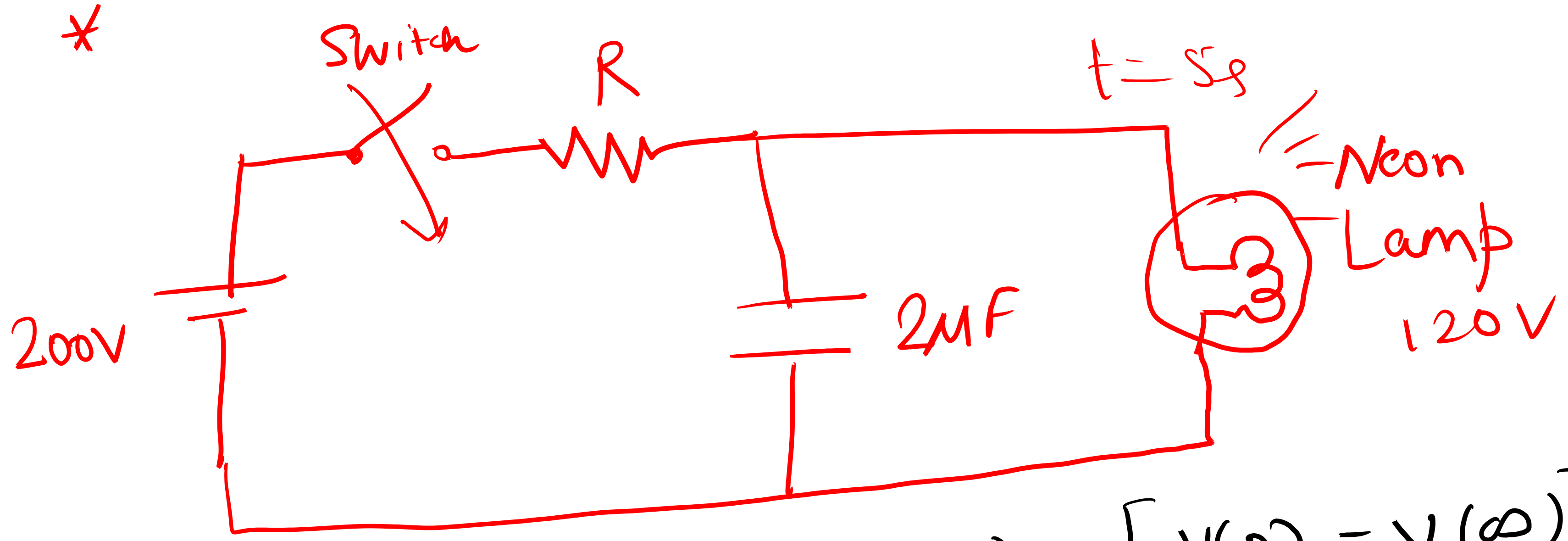
② for t>0 v(∞)



$R_{eq} \Rightarrow 4K$

$= 4000 \times 0.5 \times 10^{-3}$

*



$$R = \cancel{2.79\,M\Omega}$$

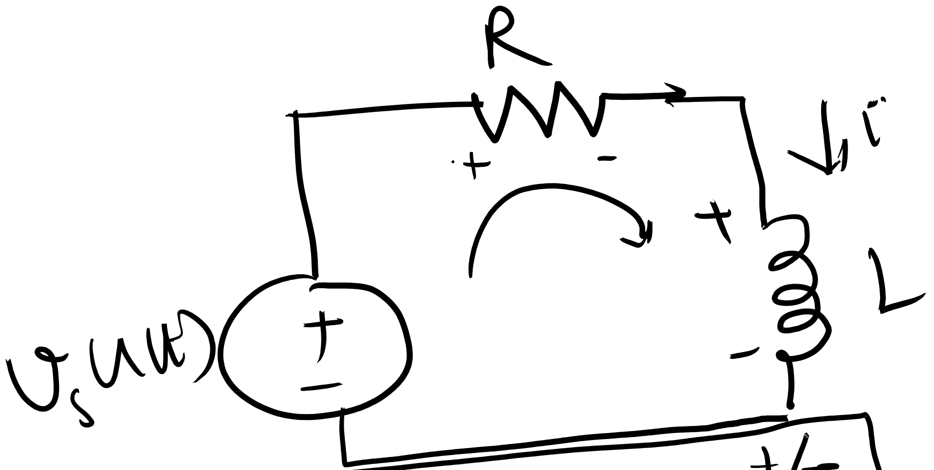
$$2.72/2.73\,M\Omega$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]\, e^{-t/\tau}$$

$$120 = 200 + [0 - 200]\, e^{-5/\tau}$$

$$\tau = RC, \quad C = 2\times10^{-6}$$

* Step Response of RL circuit



$$\boxed{i(t) = \frac{V_s}{R} + \left(i(0) - \frac{V_s}{R}\right)e^{-t/\tau}}$$

$$-V_s u(t) + iR + L\frac{di}{dt} = 0$$

for $t > 0$; $u(t) = 1$

$$-V_s + iR + L\frac{di}{dt} = 0$$

$$iR + L\frac{di}{dt} = V_s \quad \text{—— (1)}$$

Homogenous linear Differential Equation.

$$i + \frac{L}{R}\frac{di}{dt} = \frac{V_s}{R}.$$

$$\int_{i=i(0)}^{i(t)} \frac{L\,di}{V_s - iR} = \int_{t=0}^{t=t} dt$$

$$\boxed{i(t) = \frac{V_s}{R} + \left(i(0) - \frac{V_s}{R}\right)e^{-t/\tau}}$$

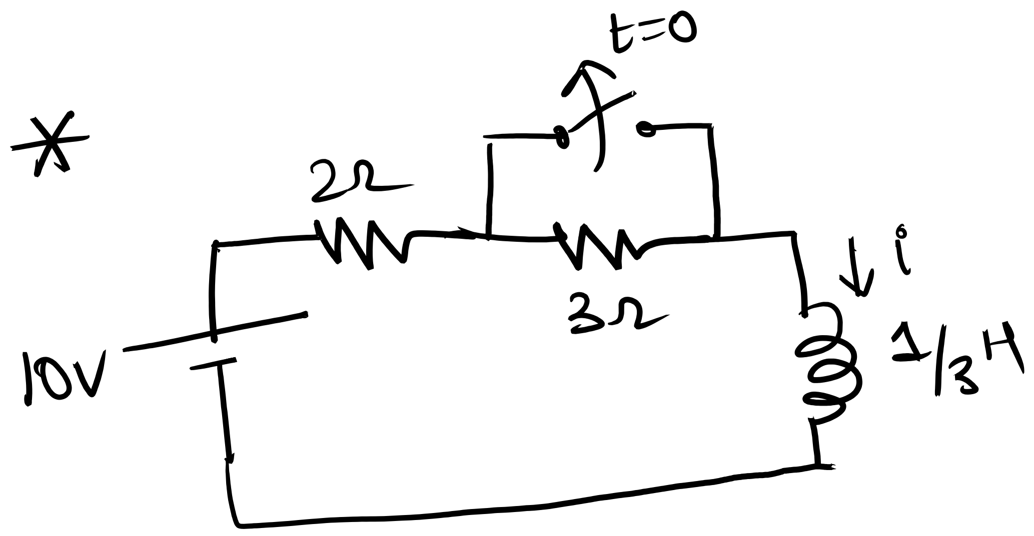
where $\boxed{\tau = \frac{L}{R}}$

$$i(t) = i_{ss} + \left(i(0) - i_{ss}\right)e^{-t/\tau}$$

where $i_{ss} = V_s/R$

$$\boxed{i(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{-t/\tau}}$$ Amp

*



i(t) for t>0

$$\boxed{i(t) = 2 + 3e^{-15t}}$$ (Amp) Ans

(i) $t<0$

$i(0) = 5A$

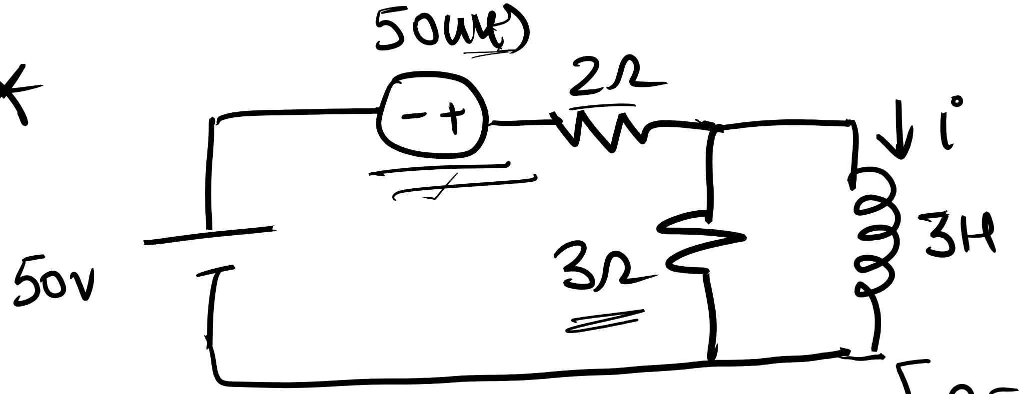
$$\boxed{i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}}$$

(ii) $t>0$

$i(\infty) = 2A$

$\tau = \frac{L}{R} = \frac{1}{15}$ s

*



i(t) for all t

$\cancel{i(t) = 20 - 10e^{-t/2.5}\ A}$

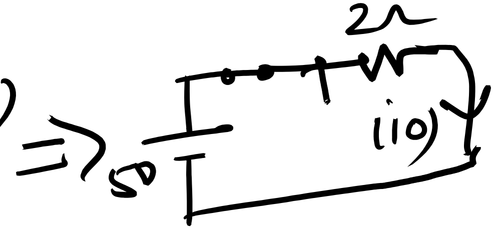
(i) $\boxed{t < 0}$ $i(0) = X$

$i(t) = 50 + [25 - 50]e^{-t/2.5}$

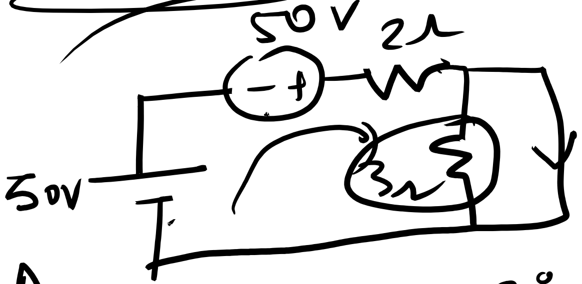
$\boxed{i(t) = 50 - 25e^{-t/2.5}\ \text{Amp}}$

$i(0) = 25A$

$\cancel{25 - 15e^{-t/c}}$



(ii) $t > 0$



$i(\infty) = 50A$

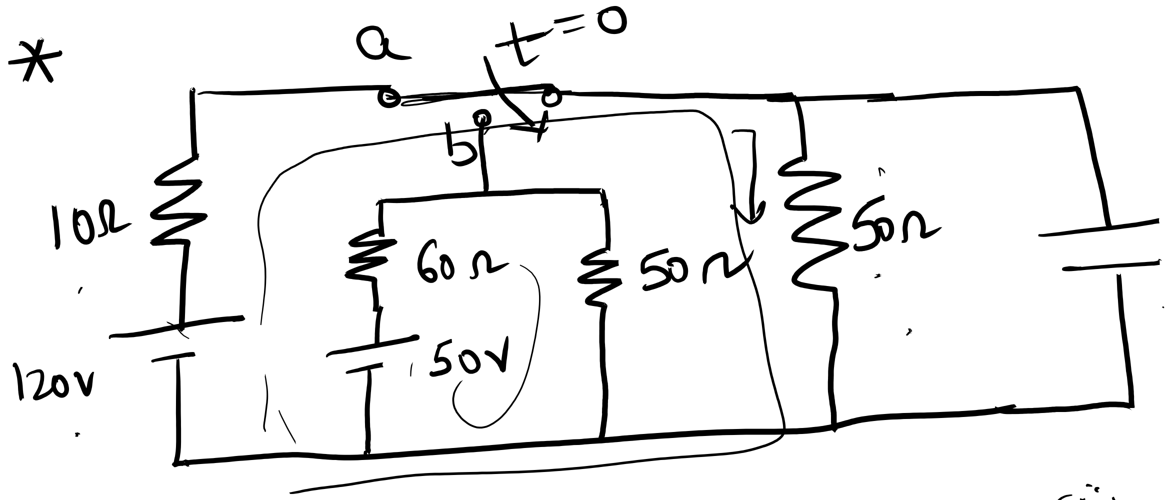
$-50 - 50 + 2i = 0$

$\boxed{i = 50A}$

$R_{eq} = 6/5\ \Omega$

$\tau = \frac{L}{R_{eq}} = \frac{\cancel{3} \times 5}{\cancel{6}\,2} = 2.5s$

*



$^{+}V_c(t)$ → for $t>0$

(i) $t<0$

$V_c(0^-) = 100V$ ✓

(ii) $t>0$

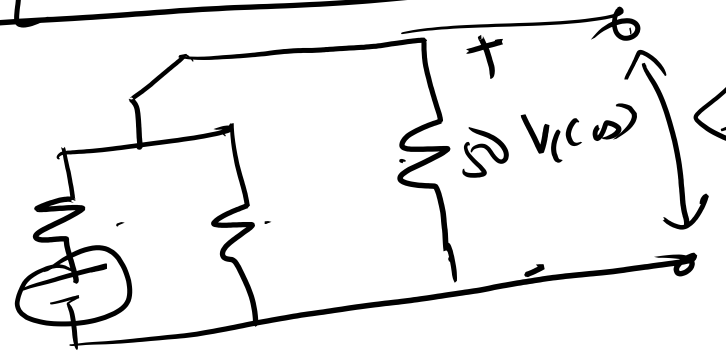
$V(\infty) = 14.7V$

$\tau = R_{eq} \cdot C = 0.88\mu s$

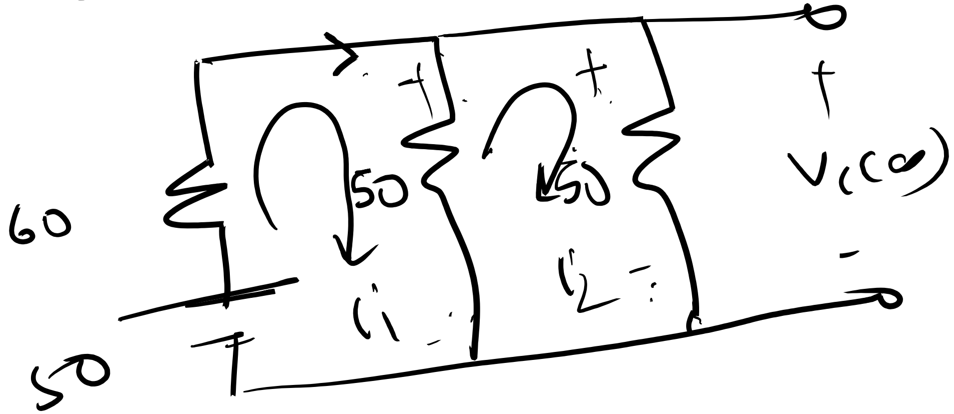
$R_{eq} = 60 \| 50 \| 50$

$= \frac{300}{17}\Omega$

$$\boxed{V_c(t) = V_c(\infty) + [V_c(0) - V_c(\infty)]e^{-t/\tau}}$$



for $t > 0$



$V_c(\infty) = \frac{25}{85} \times 50 = \underline{14.7A}$

$-50 + 60i_1 + 50(i_1 - i_2) = 0 = -50 + 110i_1 - 50i_2 = 0$

$50i_2 + 50(i_2 - i_1) = 0$

$\cancel{100}i_2 = \cancel{50}i_1$

$\underline{i_2 = i_1/2}$

$i_2 = \frac{25}{85} A.$

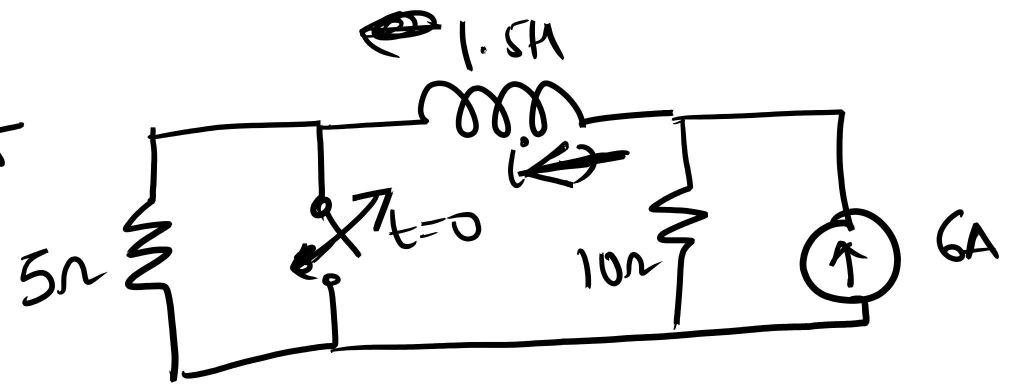
$110i_1 - 50\left(\frac{i_1}{2}\right) = 50$

$110i_1 - 25i_1 = 50$

$i_1 = \frac{50}{85} A$

-

*



$i(t)$ for $t>0$ $= \underline{\underline{4+2e^{-10t}}}$ (Amp)

(i) for $t<0$

$\underline{i(0^-)} = \underline{6A}$ ✓

$\underline{i(0^-) = i(0) = i(0^+)}$

(ii) $t>0$

$i(\infty) = 4A$

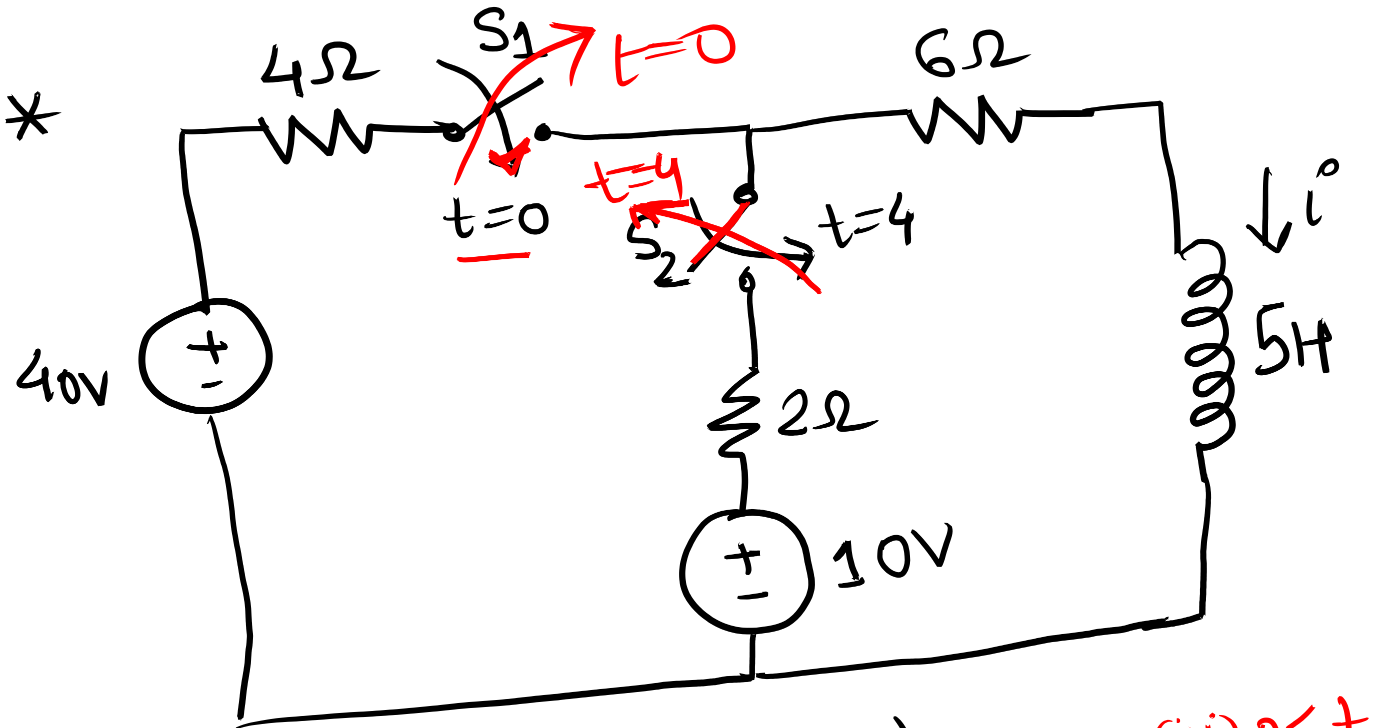
$R_{eq} = 15\Omega$

$\tau = \frac{L}{R_{eq}} = \frac{1.5}{15} = \underline{\frac{1}{10}s}$

$i(t) = i(\infty) + [\underline{i(0)} - i(\infty)]e^{-t/\tau}$

$$\boxed{i(t) = 4 + 2e^{-10t} \text{ Amp}}$$

*



(i) $t<0$

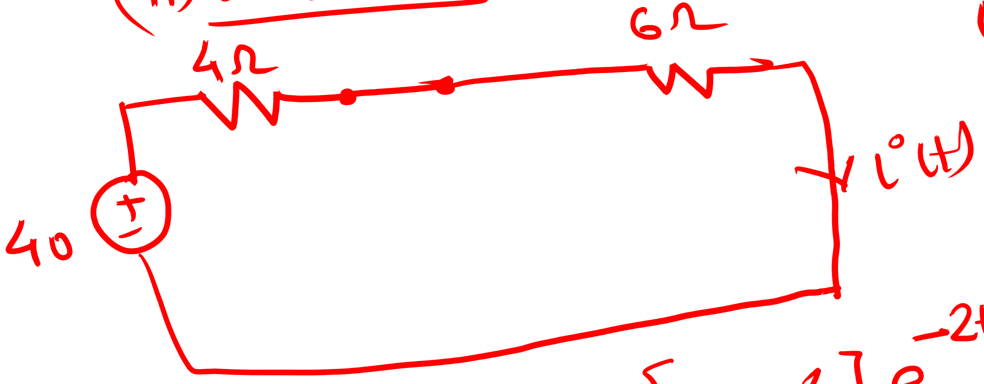
(ii) $0<t<4$

(iii) $t>4$

(i) for $t<0$ ($S_1, S_2 \rightarrow$ open)

$$i(0^-) = 0A = i(0) = i(0^+)$$

(ii) $0<t<4$ ($S_1 \rightarrow$ closed ; $S_2 \rightarrow$ open)



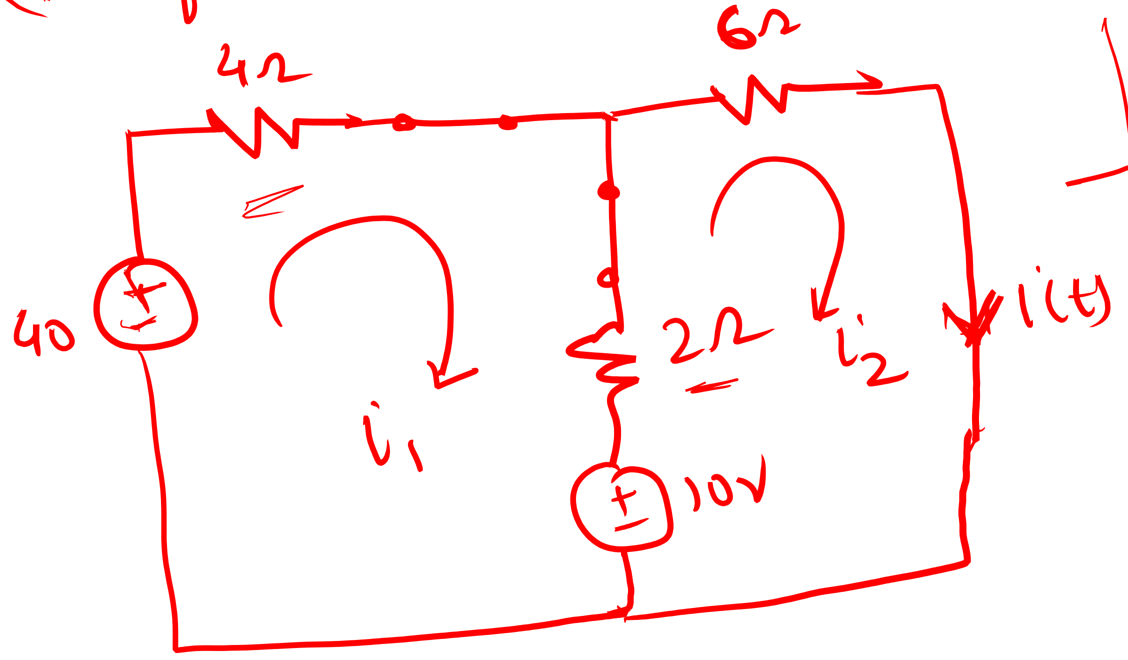
$$i(\infty) = 4A$$

$$\tau = \frac{L}{R_{eq}} = \frac{5}{10} = 0.5s$$

$$i(t) = 4 + [0-4]e^{-2t}$$

$$\boxed{i(t) = 4(1-e^{-2t})} \text{ Amp}$$

(iii) for t>0 ($S_1, S_2 \rightarrow$ closed)



$-40 + 4i_1 + 2(i_1 - i_2) = 0$

$+10$

$-10 + 2(i_2 - i_1) + 6i_2 = 0$

$i_2 = i(\infty) = 4.09\,A$

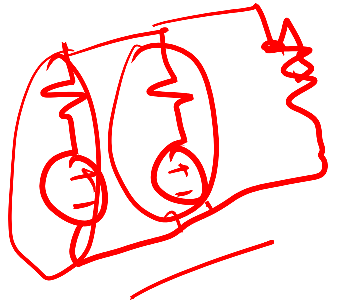
$$\boxed{i(t) = i(\infty) + [i(4) - i(\infty)]e^{-t/\tau}}$$

$i(t) = 4(1 - e^{-2t})$ $\{0 < t < 4\}$

$i(4) = 4(1 - e^{-2 \times 4})\,A$

$= 4(1 - e^{-8})\,A$

$= 4A$



$R_{eq} = (4 \| 2) + 6 = 9.33\,\Omega$

$\tau = \frac{L}{R_{eq}} = \frac{5}{7.33}\,s$