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Tut-4

Q1 $R_X = \{-1, 0, 1, 2, 3\}$

$P_X(-1) = P_X(0) = P_X(1) = P_X(2) = P_X(3) = 1/5$

& $Y = 2|X| \Rightarrow R_Y = \{2|x| : x \in R_X\}$

$\therefore R_Y = \{2, 0, 2, 4, 6\}$
$= \{0, 2, 4, 6\}$

To find $P_Y(y)$ we need to find $P(Y=y)$ for $y = 0, 2, 4, 6$

$P_Y(0) = P(Y=0) = P(2|X| = 0) = 1/5$

$P_Y(2) = P(Y=2) = P(2|X| = 2)$
$= P(X=-1)$ or $P(X=1)$
$= P_X(-1) + P_X(+1)$
$= 1/5 + 1/5 = 2/5$

$P_Y(4) = P(Y=4) = P(2|X| = 4)$
$= P(X=2) + P(X=-2)$
$= P(X=2)$
$= 1/5$

||ly $P_Y(6) = 1/5$.

X	=	-1	0	1	2	3
$P(X)$	=	$1/5$	$1/5$	$1/5$	$1/5$	$1/5$ (sum =

$Y = 2\|X\|$	=	0	2	4	6
	$\rightarrow$	$1/5$	$2/5$	$1/5$	$1/5$ (sum = 1)

Law of unconscious statistics (LOTUS) for discrete random variables

$$E[g(x)] = \sum g(x_k) \cdot P_X(x_k)$$

$$\Rightarrow E(X) = -1 \times (1/5) + 0 \times (1/5) + 1 \times (1/5) + 2 \times (1/5) + 3 \times (1/5)$$

$$= 1$$

$$\therefore E(2|X|) = 2 \times |-1| \times 1/5 + 2 \times |0| \times 1/5 + 2 \times |1| \times 1/5 + 2 \times |2| \times 1/5 + 2 \times |3| \times 1/5$$

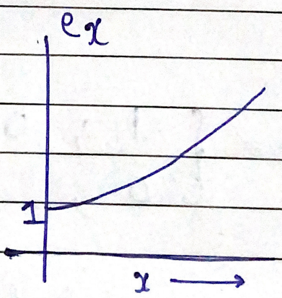
$$= 2/5 + 2/5 + 4/5 + 6/5$$

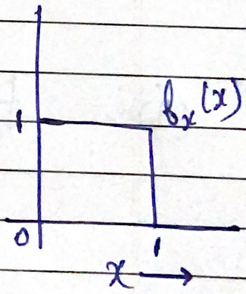
$$E(2|X|) = 14/5 \text{ Ans.}$$

Q2 $X \sim U[0,1]$

$$f_x(x) = \begin{cases} 1 & ; \ 0 \le x \le 1 \\ 0 & ; \ \text{else} \end{cases}$$

Now, $Y = e^x$





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(i) CDF of y

$$R_X = [0,1]$$
$$R_Y = [1,e]$$

Now, $F_Y(y) = P(Y \leq y) = 0$; for $y < 1$

$F_Y(y) = P(Y \leq y) = 1$; for $y \geq e$

To find $F_Y(y)$ for $y \in [1,0]$ we can write

$$F_Y(y) = P(Y \leq y)$$
$$= P(e^x \leq y)$$

$$F_Y(-y) = P(x \leq \log_e y)$$
$$= F_X(\log_e y)$$
$$= \log_e y.$$

So, $F_Y(y) = \begin{cases} 0 & ; y < 1 \\ \log_e y & ; 1 \leq y \leq e \\ 1 & ; y \geq e \end{cases}$

(ii) $f_X(x) = \begin{cases} 1 & ; 0 < x < 1 \\ 0 & ; \text{else} \end{cases}$

$$f_Y(y) = f_X(x) \left|\frac{dx}{dy}\right|$$

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Here, $y = e^{x} \Rightarrow x = \log_e y$

$$\left|\frac{dx}{dy}\right| = 1/y.$$

So, $f_Y(y) \Rightarrow \begin{cases} 1/y \; ; & 0 < x < 1 \\ & 0 < \log_e y < 1 \\ & 0 < y < e \\ 0 \; ; & \text{else} \end{cases}$

(iii) $E[Y] = E[e^{x}] = \int_{-\infty}^{0} e^{x} f_X(x)\, dx$

$$= \int_0^1 e^{x} dx = \boxed{e-1} \text{ Ans}$$

Q3 $Z = X + Y$

Now,

$$F_Z(z) = P(Z \leq 2)$$
$$= P((X+Y) \leq 2)$$
$$= \iint_{D_2} f(x,y)\, dx\, dy$$

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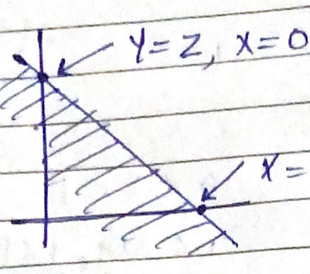
We need to find area D_z



$Z = X + Y$

if $X=0, Y=Z$
if $Y=0, X=Z$

our area of interest is defined as $X+Y \leq Z$

Locus of D_z will be this straight line and D_z will be area below this line.

$$E_z(z) = \int_{y=-\infty}^{0} \int_{x=-\infty}^{z-y} f_{xy}(x,y)\, d_x\, dy \quad \text{— (1)}$$

Now the Pdf is defined as

$$f_z(z) = \frac{d}{dz} \int_{-\infty}^{0} \int_{-\infty}^{z-y} f_{xy}(x,y)\, dx\, dy$$

$$= \int_{y=-\infty}^{0} f_x(z-y)\, f_y(y)\, dy$$

If X and Y are independent and $Z = X+Y$ then $f_z(z)$ = Convolution of PDF of X & Y $\Rightarrow f_X(x) \times f_Y(y)$.

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Q4 $f_{xy}(x,y) = (cxy\ ;\ 0 \le x, y \le 1)$

$$\int_0^1\int_0^1 f_{xy}(x,y)\,dx\,dy = 1$$

$$\Rightarrow \int_{x=0}^{1}\int_{y=0}^{1} cxy\,dx\,dy = 1$$

$$\Rightarrow c\int_0^1 x\,dx \int_0^1 y\,dy = 1$$

$$\Rightarrow \frac{c}{4} = 1 \longrightarrow \boxed{c=4}$$. Ans.

Q5 $c \ge 0$

$$\int_0^1\int_x^1 cxy\,dy\,dx = \int_0^1 cx \int_x^1 y\,dy\,dx$$

$$= \int_0^1 \frac{cx}{2}(1-x^2)\,dx = \frac{c}{8} = 1$$

$$\therefore \boxed{c = 8}$$ Ans

Q6 $p(0,4) = P(U=0, V=4)$

$$= P(\{2,2\}) = \frac{1}{9}$$

$\Rightarrow$ Array of Possibilities

V					
6	0	0	1/9	0	0
5	0	1/9	0	1/9	0
4	1/9	0	1/9	0	1/9
3	0	1/9	0	1/9	0
2	0	0	1/9	0	0
	-2	1	0	1	2 U

x — x — x