Anshika 10 2003183 3 COE8

Assignment – 2

Ques 1 Given, $P(i) = Ki \quad [i = 1, 2, 3, 4, 5, 6]$

(a) Since one of the faces must occur,

$P(S) = 1$

i.e. $k + 2k + 3k + 4k + 5k + 6k = 1$

$$\frac{6(6+1)}{2} k = 1 \qquad \left[1 + 2 + \dots n = \frac{n(n+1)}{2}\right]$$

$$21k = 1$$

$$k = \frac{1}{21}$$

(b) $P(\text{no. greater than 3 shows up}) = P(4) + P(5) + P(6)$

$= 4k + 5k + 6k$

$= 15k = \frac{15}{21} = \frac{5}{7}$

Ques 2 Number of questions = 9

No. of ways to answer a question = 2

∴ Total no. of ways to answer 9 questions $= 2^9 = 512$

Ques 3 Digits = {0, 1, 2, 3, 4, 5, 6}

One digit can only be used once.

(a)

×	×	×
↓	↓	↓
6	6	5

(leading 0 cannot be used)

∴ No. of possible 3-digit nos. $= 6 \times 6 \times 5 = 180$

(b) For odd digits, we have 3 options for unit's place,

i.e. (1, 3, 5)

×	×	×
↓	↓	↓
5	5	3

(no leading 0's) (left

No. of 3-digit odd nos. $= 5 \times 5 \times 3$

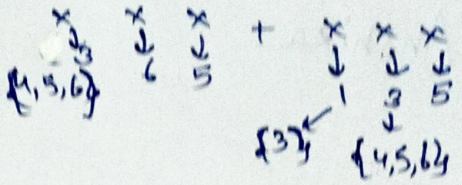
$= 75$

(c) No. greater than 330

$= (3 \times 6 \times 5) + (1 \times 3 \times 5)$

$= 90 + 15$

$= 105$



Ques 4

(a) Sample space for no. of gates open in each direction

$\{(0,0), (0,1), (0,2), (0,3), (0,4), (0,5), (0,6),$
$(1,0), (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,0), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,0), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,0), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,0), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
$(6,0), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$n = 49$

(b) X - At most one gate is open in each direction -

$\{(0,0), (0,1), (1,0), (1,1)\}$

$P(X) = \frac{4}{49}$

(c) X - At least one gate is open in each direction.

$P(X) = \frac{36}{49}$

(d) X - No. of gates open on each side is same.

$P(X) = \frac{7}{49} = \frac{1}{7}$

(e) X - Total no. of gates open is 6

$\{(0,6), (6,0), (1,5), (5,1), (2,4), (4,2), (3,3)\}$

$P(X) = \frac{7}{49}$

$= \frac{1}{7}$

Ques 5

Total no. of envelopes = 500

No. of envelopes containing $100 = 75

No. of envelopes containing $25 = 150

No. of envelopes containing $10 = 275

Cost of each envelope = $25

Sample space :- {$100, $25, $10}

$P(\$100 \text{ envelope}) = \frac{75}{500} = \frac{3}{20}$

$P(\$25 \text{ envelope}) = \frac{150}{500} = \frac{3}{10}$

$P(\$10 \text{ envelope}) = \frac{275}{500} = \frac{11}{20}$

$P(\text{less than } \$100) = 1 - P(\$100)$

$= 1 - \frac{3}{20} = \frac{17}{20}$

Ques 6

(a) $P(A \cap B) = P(A) + P(B) - 1$

We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Let $A \cup B$ be exhaustive

$\downarrow$

$P(S)$

we know $P(S) = 1$

$\therefore \quad 1 \geq P(A) + P(B) - P(A \cap B)$

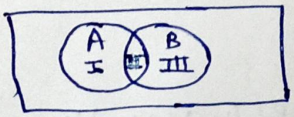
$P(A \cap B) \geq P(A) + P(B) - 1$

(b) $P(A \cup B) \leq P(A) + P(B)$

$P(A \cup B) = P(I) + P(II) + P(III)$

$= (P(A) - P(A \cap B)) + P(A \cap B) + (P(B) - P(A \cap B))$

$= P(A) + P(B) - P(A \cap B)$



$P(A \cap B) \geq 0$

$\therefore \quad P(A \cup B) \leq P(A) + P(B)$

Ques 7

$P(A) = 0.24$, $P(B) = 0.67$, $P(A \cap B) = 0.09$

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.67 + 0.24 - 0.09$

$= 0.82$

(b) $P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - 0.82 = 0.18$

(c) $P(A' \cup B') = P(A \cap B)'$

$= 1 - P(A \cap B)$

$= 1 - 0.09 = 0.91$

(d) $P(A \cap B)' = 1 - P(A \cap B)$

$= 1 - 0.09 = 0.91$

(e) $P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - 0.82 = 0.18$

Ques 8

Total exhaustive events $= {}^{52}C_5$

(a) $P(4 \text{ aces}) = \dfrac{{}^4C_4 \times {}^{48}C_1}{{}^{52}C_5}$

$= \dfrac{\frac{4!}{0!\,4!} \times \frac{48!}{47!\,1!}}{\frac{52!}{47!\,5!}} = \dfrac{1 \times 48 \times 5 \times 4 \times 3 \times 2 \times 1}{\cancel{52} \times \cancel{51} \times \cancel{50} \times 49 \times \cancel{48}}$

$\cancel{52}^{13} \times \cancel{51}^{17} \times \cancel{50}^{5}$

$= \dfrac{1}{54145} = 0.000018$

(b) $P(4 \text{ aces}, 1 \text{ king}) = \dfrac{{}^4C_4 \times {}^4C_1}{{}^{52}C_5}$

$= \dfrac{1 \times \cancel{4} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}{\cancel{52} \times \cancel{51} \times \cancel{50} \times 49 \times \cancel{48}}$

$\cancel{52}^{13} \times \cancel{51}^{17} \times \cancel{50}^{5} \times \cancel{48}^{12}$

$= \dfrac{1}{649740} = 0.0000015$

(c) $P(3 \text{ tens}, 2 \text{ jacks}) = \dfrac{{}^4C_3 \times {}^4C_2}{{}^{52}C_5}$

$= \dfrac{4 \times 4 \times 3 \times 5 \times 4 \times 3 \times 2 \times 1}{52 \times 51 \times 50 \times 49 \times 48} = \dfrac{1}{108290}$

$= 0.0000092$

(d) $P(3 \text{ of one suit}, 2 \text{ of another}) = \dfrac{{}^{13}C_3 \times {}^{13}C_2 \times 4 \times 3}{{}^{52}C_2}$

$= \dfrac{267696}{2598960} = 0.1$

(e) $P(\text{at least 1 ace}) = 1 - \dfrac{{}^{48}C_5}{{}^{52}C_5}$

$= 1 - \dfrac{48 \times 47 \times 46 \times 45 \times 44}{52 \times 51 \times 50 \times 49 \times 48}$

$= 1 - \dfrac{4280760}{6497400} = 0.341$

Ques → 9

20 apples + 5 oranges = 25 fruits in basket

$P(\text{both fruits are apple}) = \dfrac{{}^{20}C_2}{{}^{25}C_2}$

$= \dfrac{20 \times 19}{25 \times 24} = \dfrac{380}{600} = \dfrac{19}{30} = 0.63$

Ques → 10

$P(\text{Brake defect}) = 0.25 = P(B)$

$P(\text{Transmission defect}) = 0.18 = P(T)$

$P(\text{Fuel defect}) = 0.17 = P(F)$

$P(\text{other defect}) = 0.40 = P(O)$

(a) $P(B \cup F) = P(B) + P(F) - P(B \cap F)$

$= 0.25 + 0.17 - 0.15$

$= 0.27$

(b) $P(B \cup F)' = 1 - P(B \cup F)$

$= 1 - 0.27 = 0.73$

Ques 11

P(getting 4,5,6 on 1st toss and 1,2,3 or 4 on 2nd toss)

$= \frac{1}{2} \times \frac{4}{6} \times \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{3}$

Ques 12

Box - {6R, 4W, 5B}

X - drawn in the order red, white and blue

(a) P(X with replacement) $= \frac{6}{15} \times \frac{4}{15} \times \frac{5}{15}$

$= 0.036$

(b) P(X with no replacement) $= \frac{6}{15} \times \frac{4}{14} \times \frac{5}{13}$

$= 0.044$

Ques 13

$P(A) > 0, \quad P(B) > 0$

$P(A) < P(A|B)$

$\Rightarrow \frac{P(A)}{P(A|B)} < 1$

$\frac{P(A|B)}{P(A)} > 1 \quad \text{---(1)}$

$P(B|A) = \frac{P(A|B)}{P(A)} \cdot P(B)$

$P(B|A) > P(B)$ [From (1)]

Ques → 14

Education	Male	Female	
Elementary	38	45	83
Secondary	28	50	78
College	22	17	39
	88	112	200

(a) $P(\text{Male} | \text{Secondary}) = \frac{\frac{28}{200}}{\frac{78}{200}} = \frac{28}{78} = 0.36$

(b) $P(\overline{\text{College}} | \text{Female}) = \frac{\frac{95}{200}}{\frac{112}{200}} = \frac{95}{112} = 0.85$

Ques → 15

$P(\text{transmits 0s}) = 0.6$

$P(\text{transmits 1s}) = 0.4$

$P(\text{distortion}) = 0.01$

$P(\text{receiving 1}) = 0.4 \times 0.99 + 0.6 \times 0.01$

$= 0.402$

Ques → 16

$P(\text{correct diagnosis}) = 0.7$

$P(\text{files lawsuit} | \text{incorrect diagnosis}) = 0.9$

$P(\text{incorrect diagnosis} \cap \text{files lawsuit}) = 0.3 \times 0.9$

$= 0.27$

Ques → 17

$P(\text{Male}) = 0.7$ $\quad P(\text{Female}) = 0.3$

$P(\text{Male} \cap \text{A grade}) = 0.7 \times 0.05 = 0.035$

$P(\text{Female} \cap \text{A grade}) = 0.3 \times 0.1 = 0.030$

$P(\text{A grade}) = 0.065$

$P(\text{Male} | \text{A grade}) = \frac{0.035}{0.065} = \frac{7}{13}$

Ques 18

P(purchase latex pain) = 0.75

P(purchase latex paint ∩ rollers) = 0.60 × 0.75
= 0.45

P(purchase semi-gloss) = 0.25

[Assuming, P(latex ∪ semi gloss) = 1 And these two are disjoint]

P(purchase semi gloss ∩ rollers) = 0.30 × 0.25
= 0.075

$$P(\text{latex} \mid \text{roller purchases}) = \frac{0.45}{0.45 + 0.075}$$

$$= \frac{0.45}{0.525} = 0.857$$