Euler Path And It's Application

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Abstract*—The objective of this paper is to study Euler graph and its various applications in real life. The Euler concepts and his theorems are used in representing any problem involving discrete arrangements of objects where concern is not with internal property but also the relationship between them. Also Fleury algorithm is mostly used to solve the problems for shortest path in Euler graph. Along with that Depth First Search algorithm is often used to solve a real life problem. In this paper we discussed about one such problem of cleaning roads of National Institute of Technology, Karnataka by travelling minimum possible distance.

***Keywords*—Euler path, fleury's algorithm, DFS algorithm**

I. Introduction

In graph theory, an Eulerian Path is a path in a finite graph which while traversing travels through each edge only once however it can visit every vertex more than once. An Eulerian circuit is an Eulerian Path that starts and ends on the same vertex. Leonhard Euler first discussed these while solving the famous Seven Bridges of Königsberg problem in 1736.

It was proved by Euler that the condition necessary for a path to be Eulerian circuit is that all vertices in the graph have an even degree, and he stated without proof that connected graphs with all even vertices have an Eulerian circuit. The first complete proof of this claim was published in 1873 by Carl Hierholzer. This is known as Euler's Theorem.

Thus a connected graph has an Euler cycle or is called an Eulerian if and only if every vertex has even degree.

The term Eulerian graph has two meanings in reference to graph theory. One meaning is that of a graph with an Eulerian circuit, and the other is that a graph which has all vertices of even degree. These definitions concur for connected graphs. For a path to be an Eulerian Path it is mandatory that zero or two vertices are of odd degree. If there are no vertices whose degree is odd, all Eulerian Paths are circuits. If there are exactly two vertices of odd degree, all Eulerian Paths start at one of them and end at the other. This graph would not have an Eulerian circuit.Such a graph that has an Eulerian Path but not an Eulerian circuit is called semi-Eulerian.

Fleury's algorithm

This algorithm is found in 1883. Consider a graph and it must be Euler graph and you want to "find the traversal in that graph such that in your path each of edges are included only once" for this problem statement following algorithm is used. The traversal starts at any vertex if graph has 0(zero) odd degree vertices and it starts from any one odd vertex if graph has 2 odd vertices. At each vertex it will choose next edge such that after removing that edge graph will not become disconnected. If there are many such edges than choose any one of them. So at the end of the procedure none of the edge will left and the traversed path will be Euler trail if graph has 2 odd degree vertices and it will be Euler cycle if graph has 0(zero) odd degree vertices.

A. Advantages of Fleury algorithm

There are many advantages of this algorithm. First is Fleury's algorithm is very important in allowing us to be able to construct an Eulerian path given an Eulerian graph G. Also this algorithm is completely optimal enough for all possible cases. Efficiency takes into account the number of calculations needed to run the algorithm as the size of the problem grows. An algorithm is considered efficient in run time grows at roughly the same speed as the size of the graph increases.

B. Problem Statement

As you are wondering how to find that the given graph is Euler graph or not? The problem is the same to this line "Is it possible to draw a given graph without lifting pencil from the paper and without tracing any of the edges more than once".
To implement Fleury's algorithm in a diagramatic representation of the map of National Institute of Technology, Karnataka and find the shortest distance travelled by the garbage truck in collecting garbage across National Institute of Technology, Karnataka by using depth first search algorithm.

C. Objectives

- To learn how Euler Graph is used in real life problem.
- How Fleury algorithm works on Euler Graph.
- How Depth First Search works to find shortest path in Graph.
- Implementation of this concept on real life problem of National Institute of Technology, Karnataka.

II. Literature Survey

The following papers were an inspiration to this project, and have therefore been listed here:

1. "A study on Euler Graph and its applications" written by Ashish Kumar, MSc Mathematics department, Shauts allahabad, UP., India.

The paper explains all the concept of Graph theory, Euler Graph, Euler theorem and Famous Chinese Postman Problem.This paper clears the concept of above mentioned topics.

2. "A study of fleury algorithm and its application on Euler Graph" taken from MM322 Graphs and Networks.

The paper explains all concept of implementation of fleury algorithm on Euler Graph. Also it gives Euler path or cycle based on graph in which every graph is traversed only once.

III. Methodology

Given below Fleury's Algorithm for finding Eulerian Path or cycle:

1. The graph must have either zero or two odd vertices.
2. If the graph has zero odd vertices, then we can start from anywhere. If the graph has two odd vertices, then start at any one of the two odd vertices.
3. Choose one edge at a time. Given a choice between a bridge edge and a non-bridge edge, always pick the non-bridge edge.
4. Stop when there are no more edges.

The main concept is, **"**don't burn bridges**"** so that you will not be disconnected from your graph and able to move through the remaining edges. We will consider an example in the below euler graph (Figure 1 to 1.4).

This graph contains two odd vertices , '2' and '3', we may start from any of the given two vertices. suppose that we started from vertex '2'.

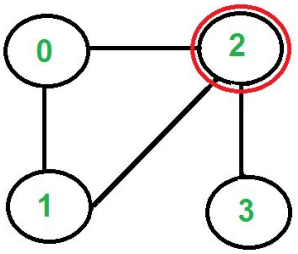

Figure 1.An euler graph showing the idea of **"**don't burn bridges**"**

The graph has three edges going outwards from vertex '2', we have to choose one of them. We will not choose the edge '2-3' because that is a bridge edge (we will not able to return to '3' that is because we have to choose no bridge edge according to our algorithm we have to choose non bridge). We can choose any of the two edges. Let us consider we chosen '2-0'. We will remove the edge and move towards vertex '0'.

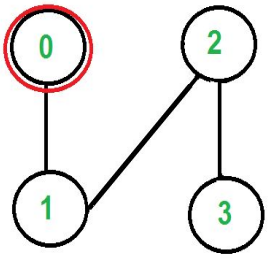

Figure 1.1. An euler graph showing the idea of **"**don't burn bridges**"**

There is only one edge remaining from vertex '0', so we will choose it, and delete it and move towards vertex '1'. Our Euler path will be '2-0 0-1'.

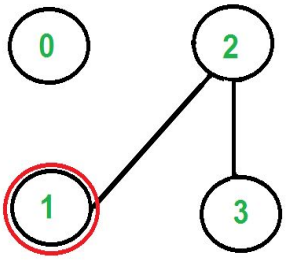

Figure 1.2.An euler graph showing the idea of **"**don't burn bridges**"**

There is only one edge from vertex '1', so we will choose it, and delete it and move towards vertex '2'. Our Euler path will be '2-0 0-1 1-2'

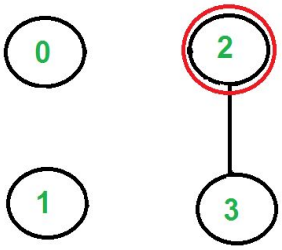

Figure 1.3.An euler graph showing the idea of "don't burn bridges"

Now, there is only one edge remaining from vertex 2, so we will choose it, delete it and move towards vertex 3. Our Euler path will be '2-0 0-1 1-2 2-3'

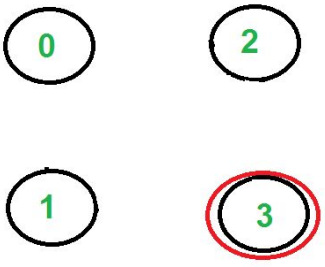

Figure 1.4.An euler graph showing the idea of "don't burn bridges"

Now, no edge is remaining, so our Euler path will be '2-0 0-1 1-2 2-3'.

After this implementation we will talk about Depth First Search algorithm which is used to travel the shortest path in Euler Graph. Depth-first search (DFS) is an algorithm which is used for searching or traversing graph or tree data structures. The algorithm starts from the root node and moves through or traverses as far as possible along each branch until the end is reached before backtracking. Now, the explanation of the algorithm is shown in below example.

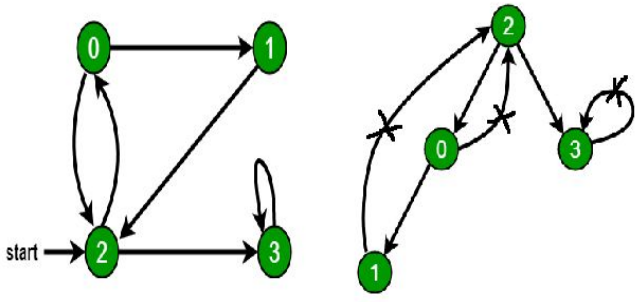

Figure 2.A graph showing the implementation of Depth-first search (DFS) algorithm

Here, start from vertex 2, then we come to vertex 0 then check for all adjacent sides of vertex 0. Then we noticed that 2 is also adjacent to 0 but it is already visited once so we will not visit that vertex again. Now go to vertex which is not visited. So the final Depth First Search traversal is 2-0-1-3.

Map of National Institute of Technology, Karnataka

The map shown below is a map of Eastern Campus of National Institute of Technology, Karnataka. We are finding shortest path by which a road cleaner can travel so that every road in the above map will be cleaned.

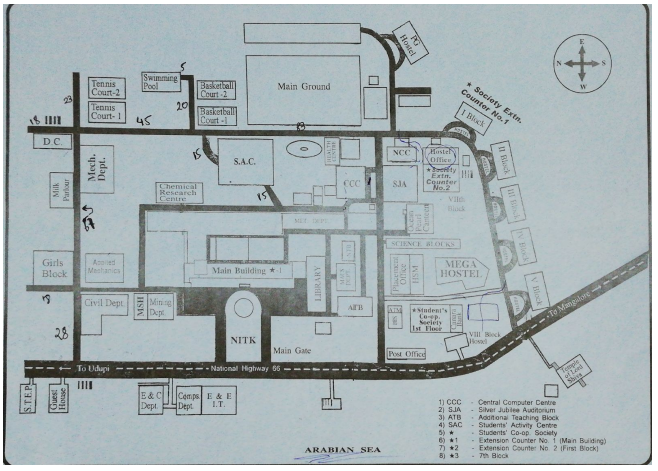

Figure 3. Map of National Institute of Technology, Karnataka.

Representation of Map in Euler Graph

Map of NITK in Euler format having the representation of graph with numbered vertices and showing the edges between them.

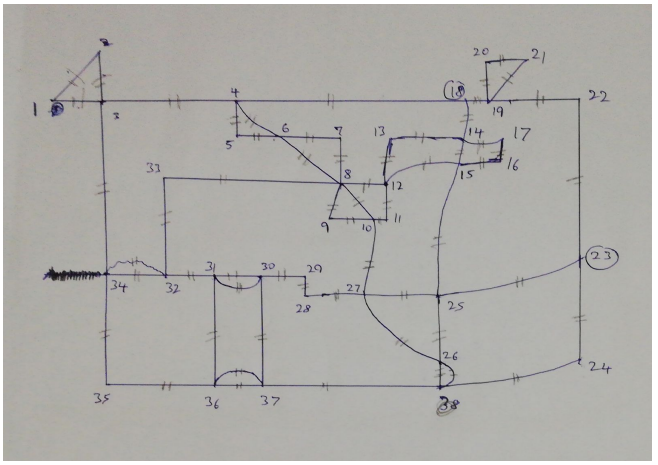

Figure 4. Euler graph of National Institute of Technology, Karnataka.

This map is inputted in a c++ program which has implementation of Fleury algorithm and Depth First Search algorithm and the output will be the shortest possible path which a cleaner can travel to clean all the roads in the eastern campus of National Institute of Technology, Karnataka.

III. Disadvantages

Just like any other algorithm, Fleury's algorithm comes with it's own set of disadvantages, for Fleury's algorithm to be applicable the necessary condition is that the graph must be Eulerian, if not the algorithm cannot be applied to the given graph. This significantly reduces the number of cases Fleury's algorithm can be applied to.

IV. Conclusion

As mentioned earlier Eulerian graph is used in many real life applications. Euler circuits are mostly used now a days in the deployment of the street sweepers, snowplows, buses and mail carries. In all of the above mentioned examples a person may have to travel a particular path more than once which will cost wastage of resources. Thus by following the Euler method we can save a lot of resources. Above discussed all the methods are now implemented and have resulted in a lot of savings to the municipalities involved.

Implementing Fleury's algorithm will assist the garbage collectors in selecting the best possible path while collecting trash in the campus.

Acknowledgement

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