Lógica e Sistemas Digitais

Mapas de Karnaugh

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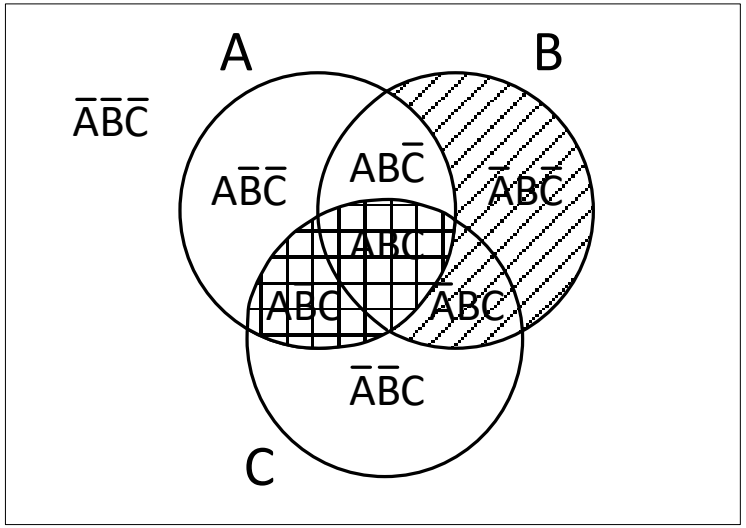
Diagrama de Venn

- Considerando a função $F = \bar{A}B + AC + BC$ representada num diagrama de Venn



- Este tipo de diagrama evidencia termos adjacentes: $\bar{A}BC + \bar{A}B\bar{C} = \bar{A}B(C + \bar{C}) = \bar{A}B$
- Evidencia igualmente termos redundantes (termo BC)
- Por análise do diagrama de Venn, extrai-se a função simplificada $F = \bar{A}B + AC$, sendo que a interceção BC já está incluída na união anterior
- A representação na forma deste diagrama torna-se impraticável para funções com mais do que três variáveis

Mapa de Karnaugh

- Mapa de Karnaugh com três variáveis

F =

A \ B	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
1	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$AB\bar{C}$

C

F =

A \ B	00	01	11	10
0	0	1	3	2
1	4	5	7	6

C

- Para $F_{(A,B,C)} = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$

F =

A \ B	00	01	11	10
0	0	0	1	1
1	0	1	1	0

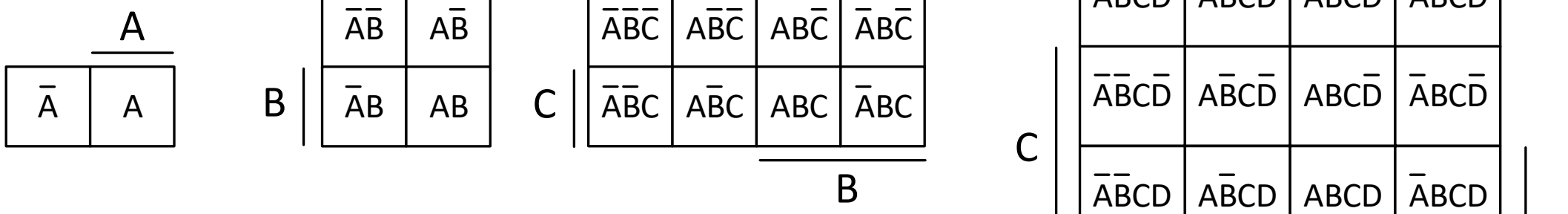
C

- Função simplificada $F_{(A,B,C)} = \bar{A}B + AC$

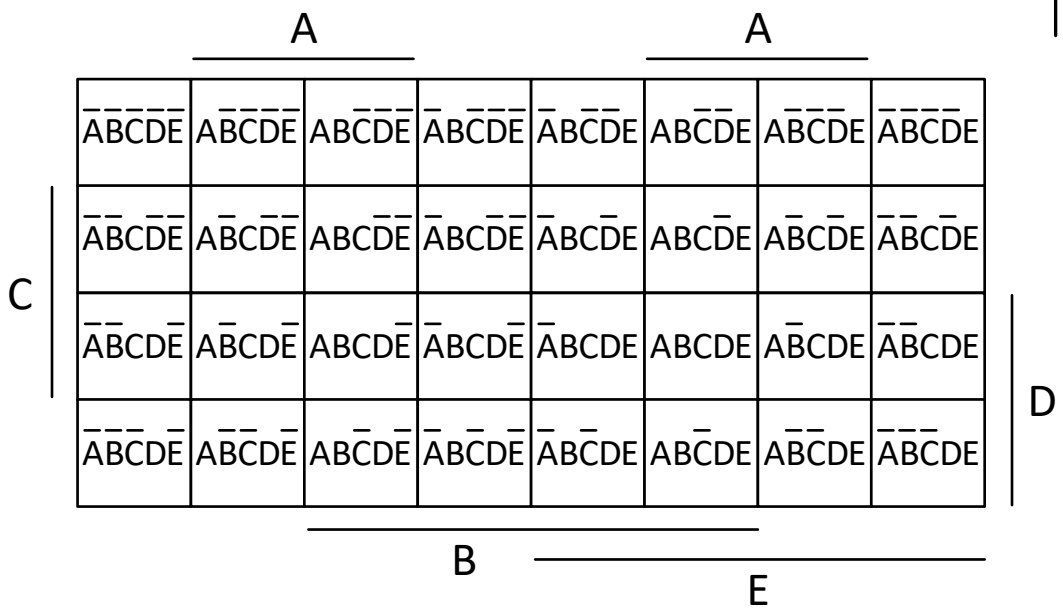
	A	B	C	$F_{(A.B.C)}$
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

Mapa de Karnaugh com diferente número de variáveis

- Uma, duas, três e quatro variáveis



- Cinco variáveis

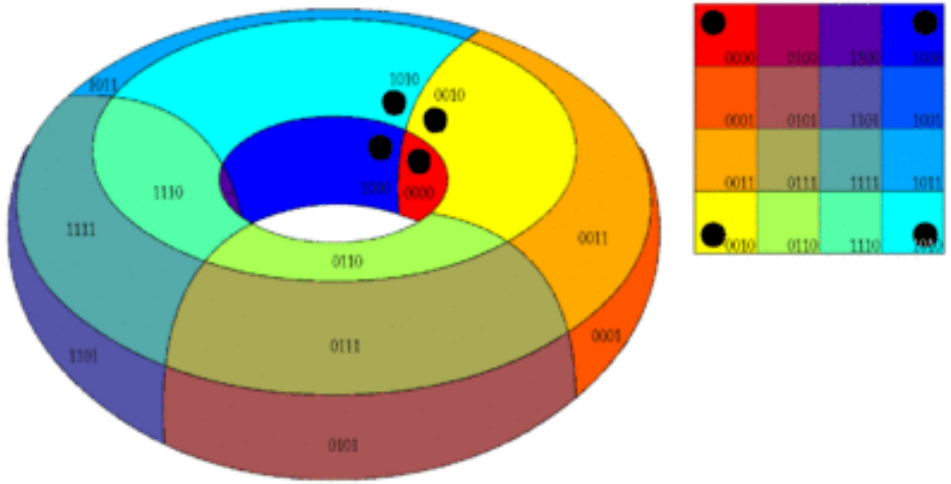


Regras para formação de grupos

- As regras são as mesmas para tirar pelos 0s ou pelos 1s
- Pelos 0s, a expressão terá de ser negada
- Formar grupos com células vizinhas
- Entenda-se como vizinhas as células nas mesmas linhas e/ou colunas; células nas diagonais não são vizinhas
- O grupo formado tem de corresponder a uma potência inteira de 2 (1, 2, 4, …)
- Uma ou mais células podem pertencer a vários grupos
- Para resultar na expressão mais simples, tem de se formar o menor número de grupos e cada grupo deverá incluir o maior número de células possível

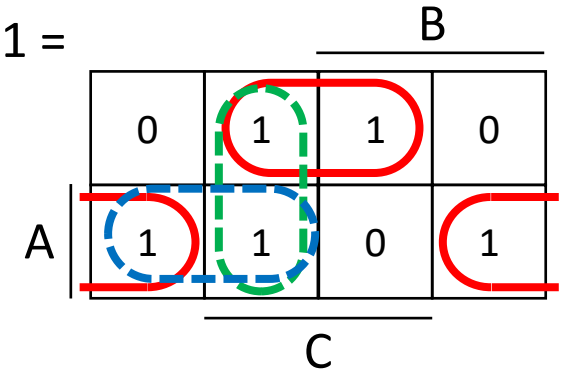
	A			
	$\bar{A}\bar{B}\bar{C}$	$A\bar{B}\bar{C}$	$AB\bar{C}$	$\bar{A}B\bar{C}$
C	$\bar{A}\bar{B}C$	$A\bar{B}C$	ABC	$\bar{A}BC$
				B

- O mapa Karnaug é uma representação planar de um Torus (figura geométrica) (donuts), logo são células vizinhas as células da primeira linha com as células da última linha e da primeira coluna com as células da última coluna. Inclusive, as células dos cantos são igualmente vizinhas (pontos a negro no torus e no mapa)



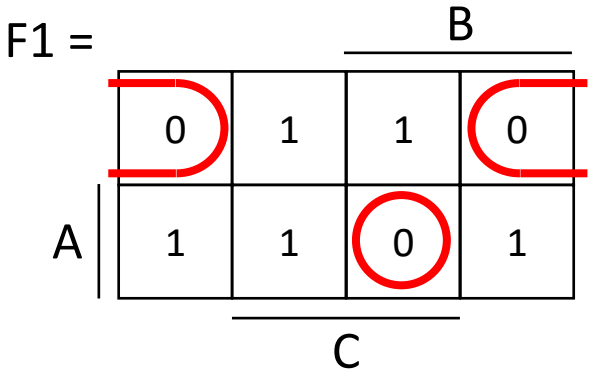
Exemplo 1 – forma AND-OR e OR-AND

F1 =



$$F1 = A\bar{C} + \bar{A}C + A\bar{B} \Leftrightarrow$$
$$A\bar{C} + \bar{A}C + \bar{B}C$$

F1 =

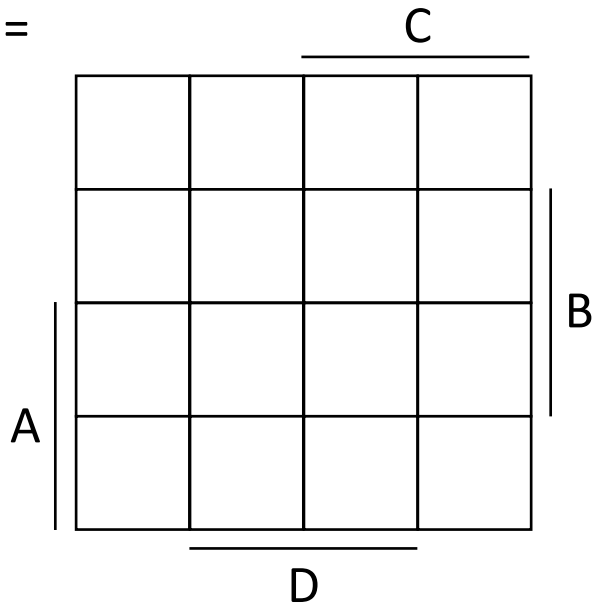


$$\overline{F1} = \bar{A}\bar{C} + ABC$$
$$F1 = \overline{\bar{A}\bar{C} + ABC}$$
$$F1 = (A + C)(\bar{A} + \bar{B} + \bar{C})$$

Exemplo 2 (1 de 13)

$$F2 = \bar{A}D + CD + BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

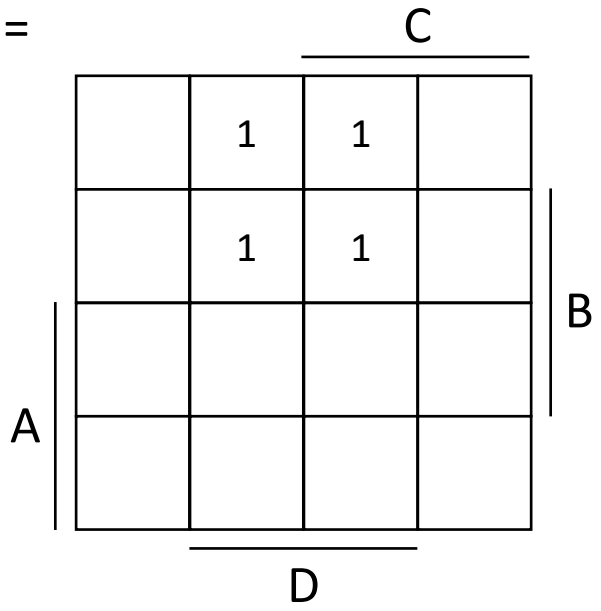
F2 =



Exemplo 2 (2 de 13)

$$F2 = \boxed{\bar{A}D} + CD + BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

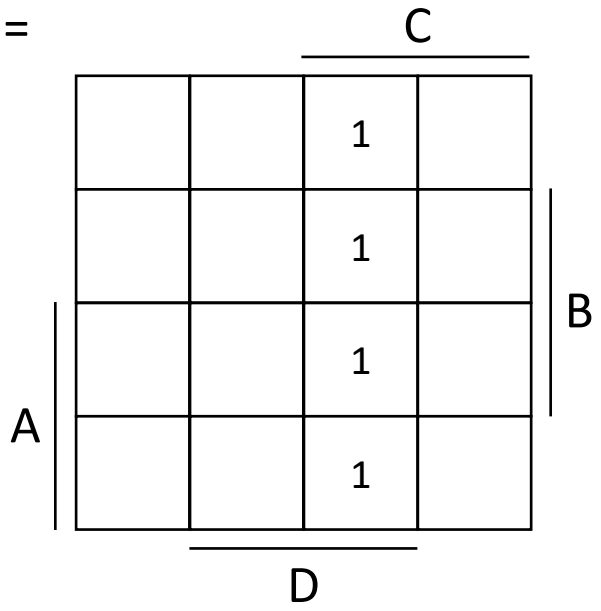
F2 =



Exemplo 2 (3 de 13)

$$F2 = \bar{A}D + \boxed{CD} + BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

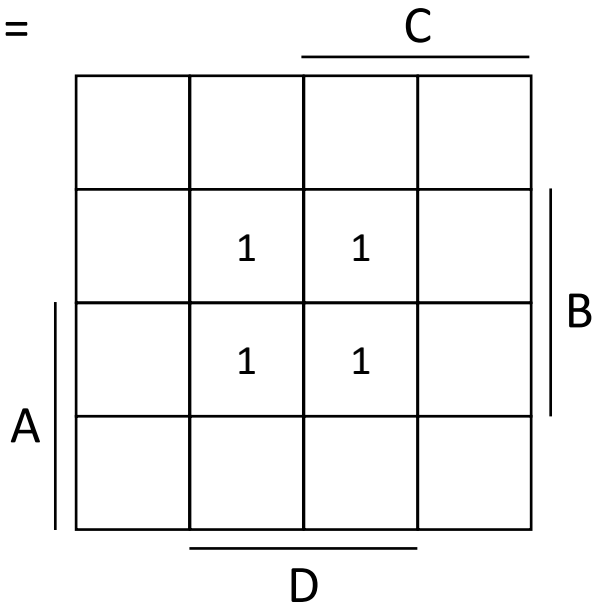
F2 =



Exemplo 2 (4 de 13)

$$F2 = \bar{A}D + CD + \boxed{BD} + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

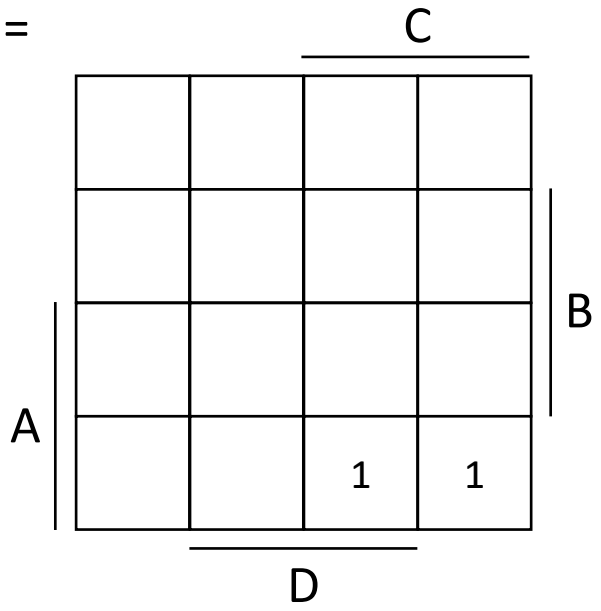
F2 =



Exemplo 2 (5 de 13)

$$F2 = \bar{A}D + CD + BD + \boxed{A\bar{B}C} + \bar{A}\bar{B}\bar{C}$$

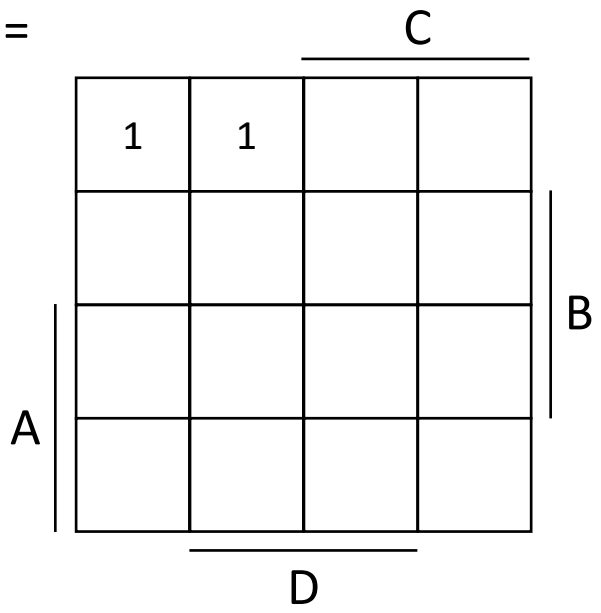
F2 =



Exemplo 2 (6 de 13)

$$F2 = \bar{A}D + CD + BD + A\bar{B}C + \boxed{\bar{A}\bar{B}\bar{C}}$$

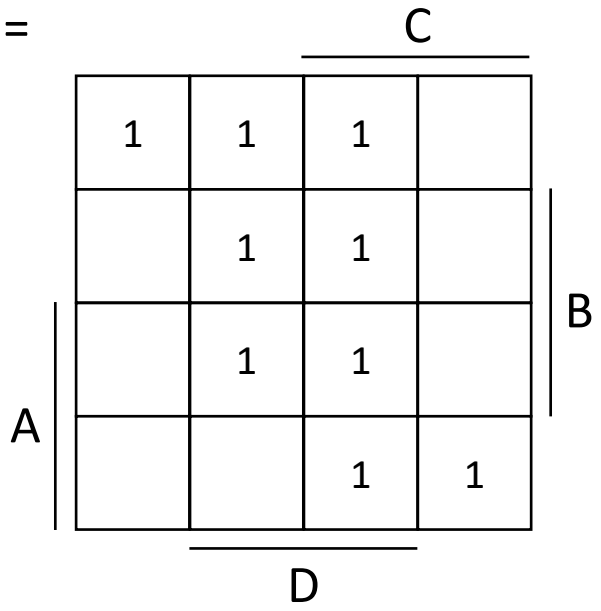
F2 =



Exemplo 2 (7 de 13)

$$F2 = \bar{A}D + CD + BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

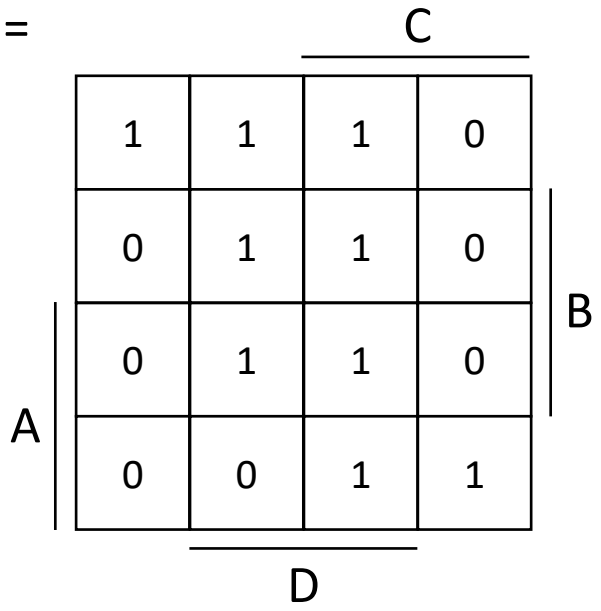
F2 =



Exemplo 2 (8 de 13)

$$F2 = \bar{A}D + CD + BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

F2 =



Exemplo 2 (9 de 13)

$$F2 = \bar{A}D + CD + BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

F2 =

A \ CD	00	01	11	10
00	0	0	1	1
01	0	1	1	0
11	0	1	1	0
10	1	1	1	0

C

B

A

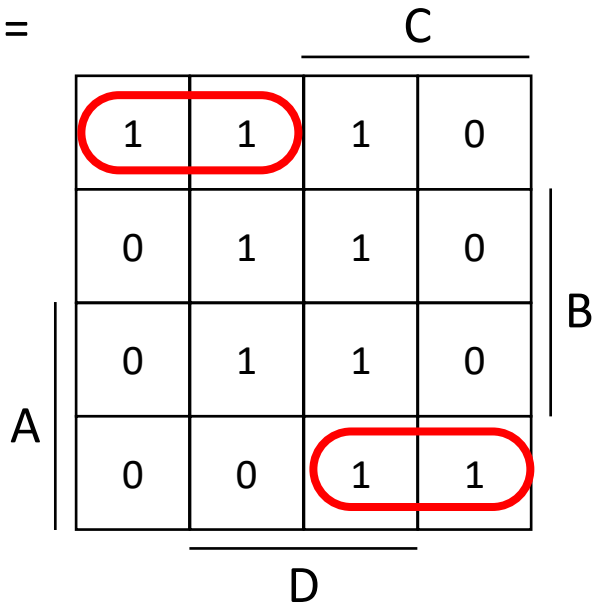
D

$$F2 = \bar{A}\bar{B}\bar{C}$$

Exemplo 2 (10 de 13)

$$F2 = \bar{A}D + CD + BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

F2 =

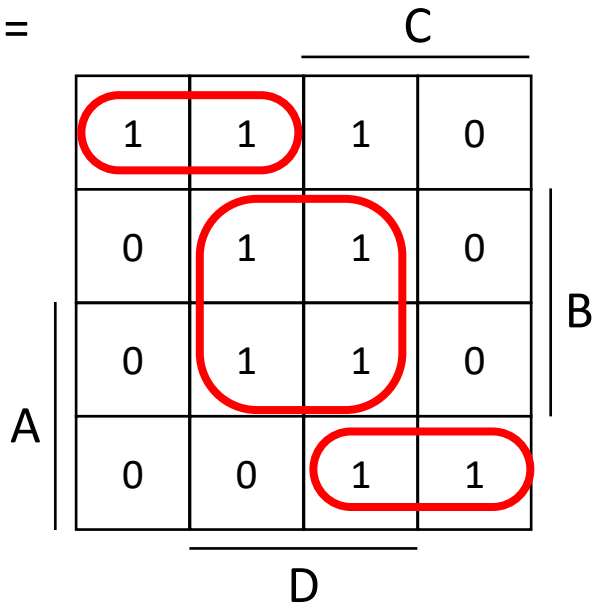


$$F2 = A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

Exemplo 2 (11 de 13)

$$F2 = \bar{A}D + CD + BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

F2 =

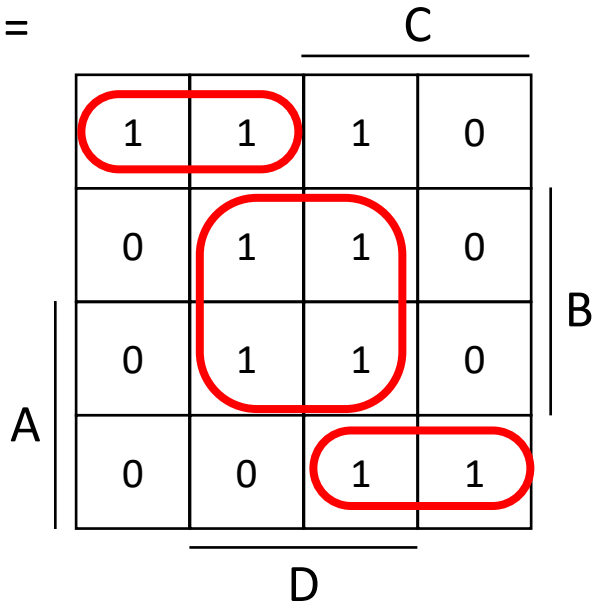


$$F2 = BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

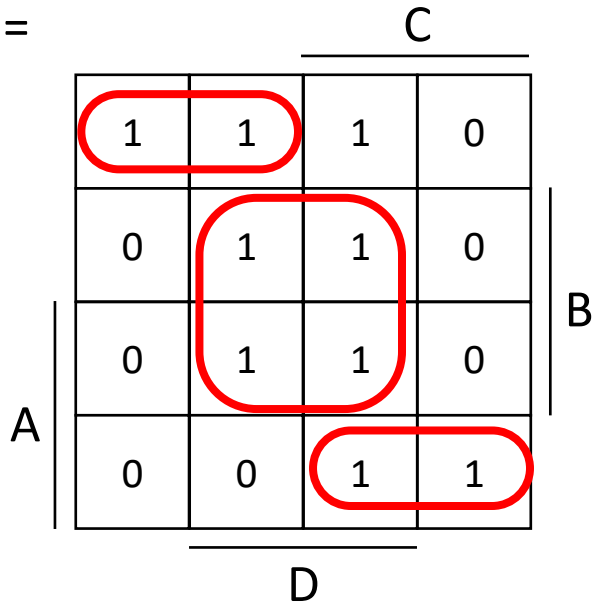
Exemplo 2 (12 de 13)

$$F2 = \bar{A}D + CD + BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

F2 =



F2 =



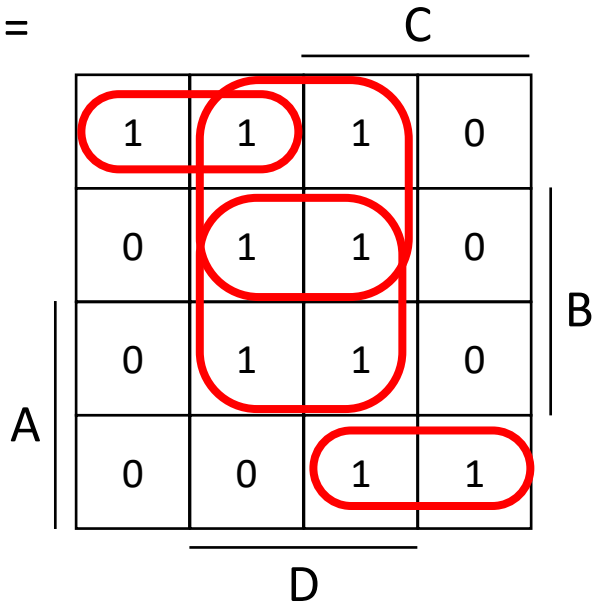
$$F2 = BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

$$F2 = BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

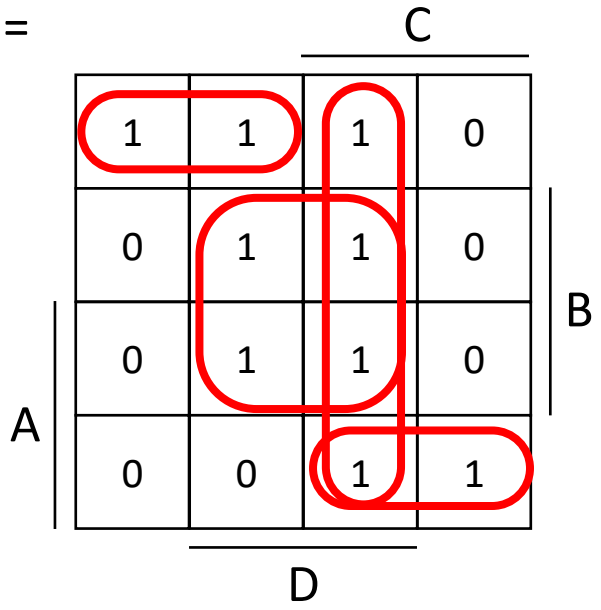
Exemplo 2 (13 de 13)

$$F2 = \bar{A}D + CD + BD + A\bar{B}C + \bar{A}\bar{B}\bar{C}$$

F2 =



F2 =



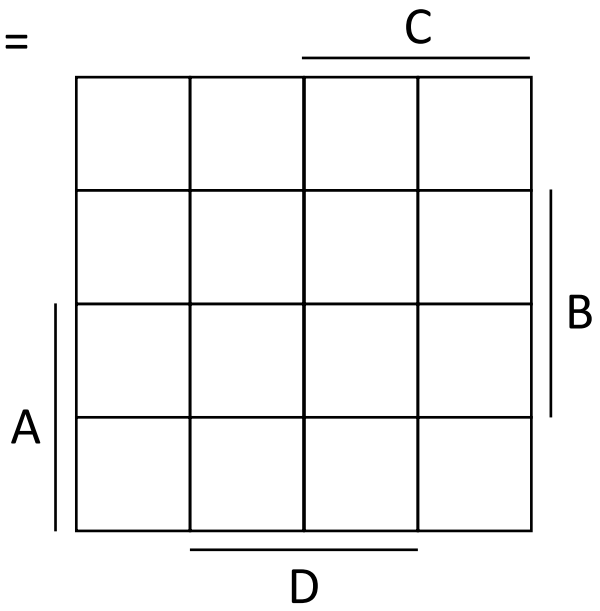
$$F2 = BD + A\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}D$$

$$F2 = BD + A\bar{B}C + \bar{A}\bar{B}\bar{C} + CD$$

Exemplo 3 (1 de 10)

$$F3 = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + A\bar{D} + \bar{B}\bar{D}$$

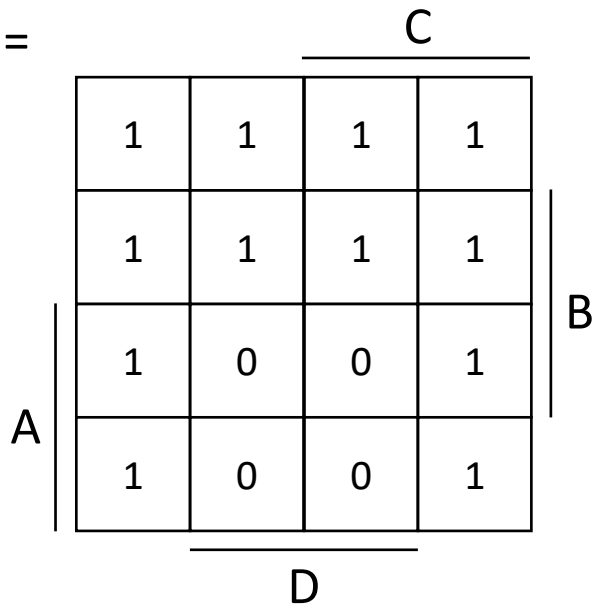
F3 =



Exemplo 3 (2 de 10)

$$F3 = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + A\overline{D} + \overline{B}\,\overline{D}$$

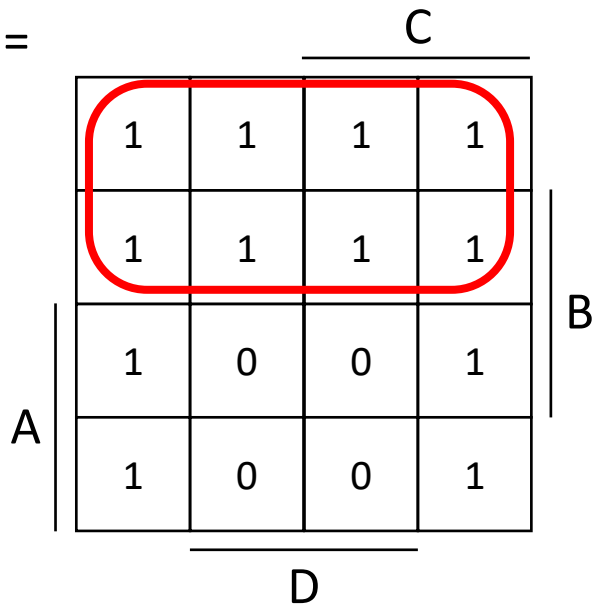
F3 =



Exemplo 3 (3 de 10)

$$F3 = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + A\bar{D} + \bar{B}\bar{D}$$

F3 =

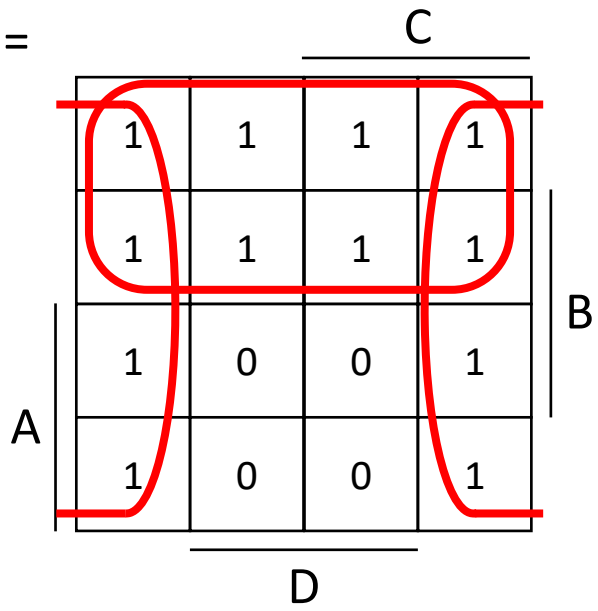


$$F3 = \bar{A}$$

Exemplo 3 (4 de 10)

$$F3 = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + A\overline{D} + \bar{B}\overline{D}$$

F3 =



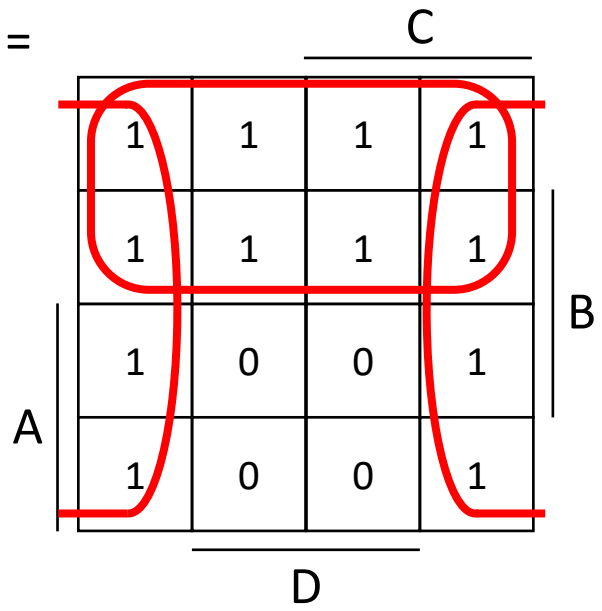
$$F3 = \bar{A} + \overline{D}$$

Exemplo 3 (5 de 10)

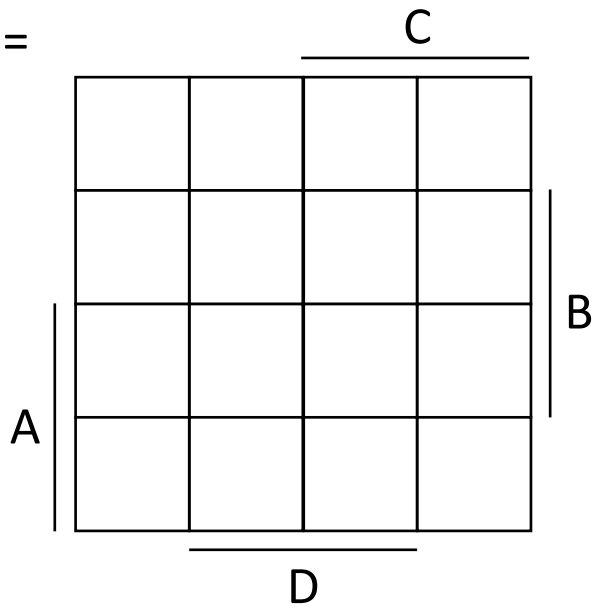
$$F3 = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + A\bar{D} + \bar{B}\bar{D}$$

$$F4 = \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D} + BC + BD + \bar{A}\bar{C}\bar{D}$$

F3 =



F4 =



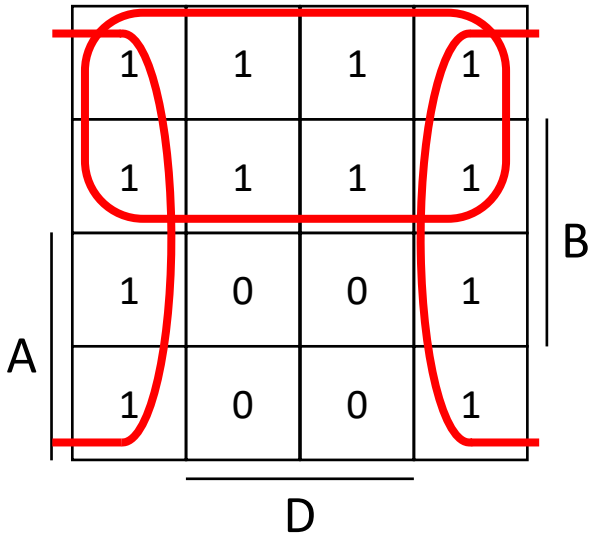
$$F3 = \bar{A} + \bar{D}$$

Exemplo 3 (6 de 10)

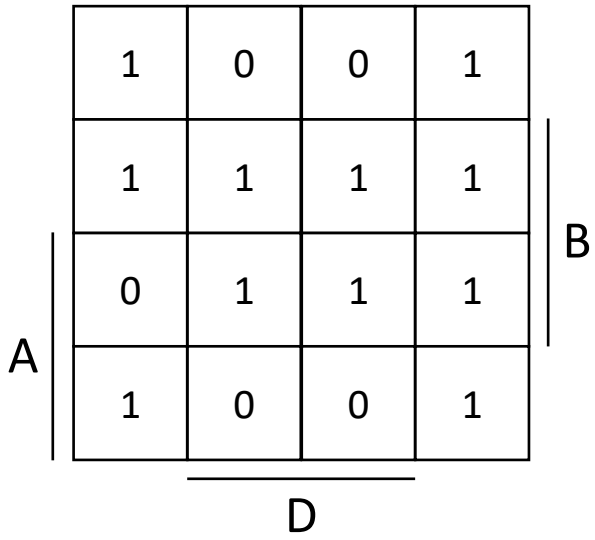
$$F3 = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + A\bar{D} + \bar{B}\bar{D}$$

$$F4 = \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D} + BC + BD + \bar{A}\bar{C}\bar{D}$$

F3 =



F4 =



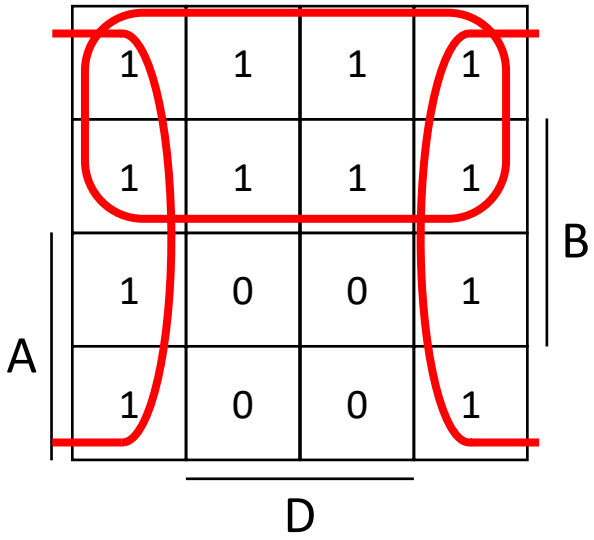
$$F3 = \bar{A} + \bar{D}$$

Exemplo 3 (7 de 10)

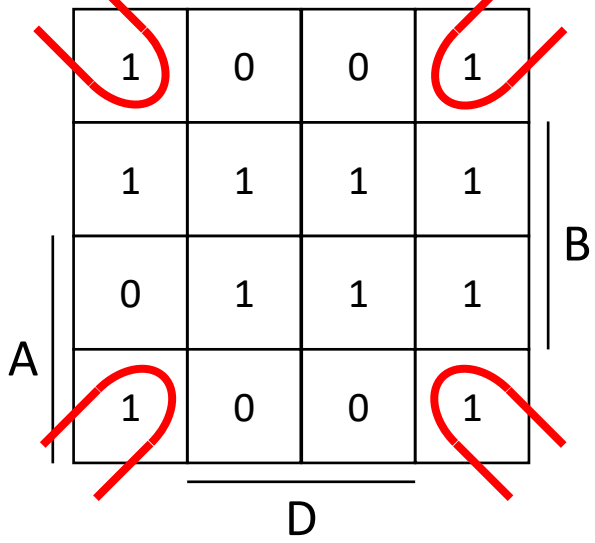
$$F3 = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + A\bar{D} + \bar{B}\bar{D}$$

$$F4 = \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D} + BC + BD + \bar{A}\bar{C}\bar{D}$$

F3 =



F4 =



$$F3 = \bar{A} + \bar{D}$$

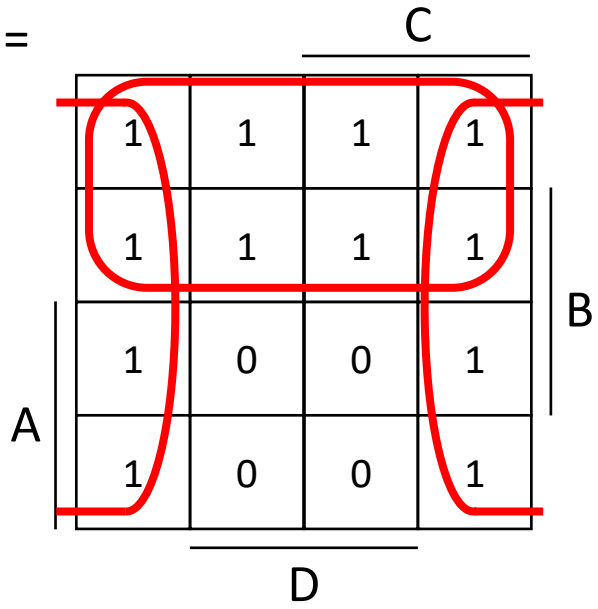
$$F4 = \bar{B}\bar{D}$$

Exemplo 3 (8 de 10)

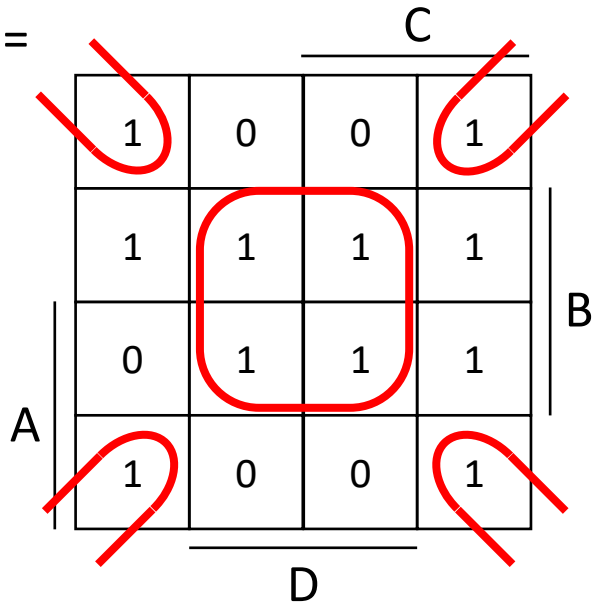
$$F3 = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + A\bar{D} + \bar{B}\bar{D}$$

$$F4 = \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D} + BC + BD + \bar{A}\bar{C}\bar{D}$$

F3 =



F4 =



$$F3 = \bar{A} + \bar{D}$$

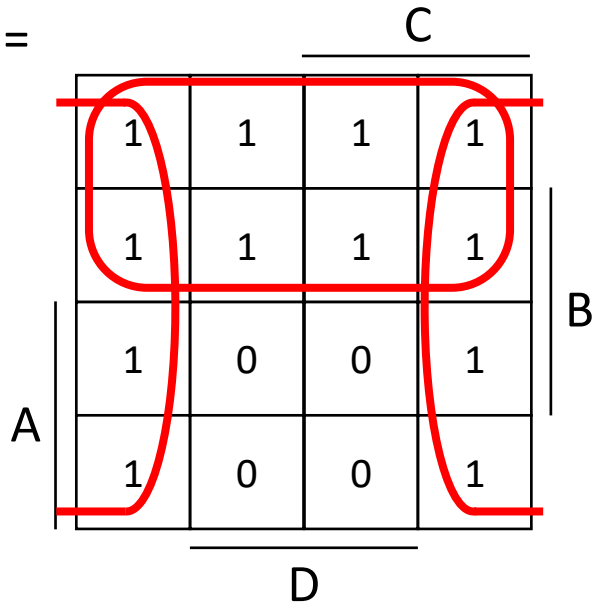
$$F4 = \bar{B}\bar{D} + BD$$

Exemplo 3 (9 de 10)

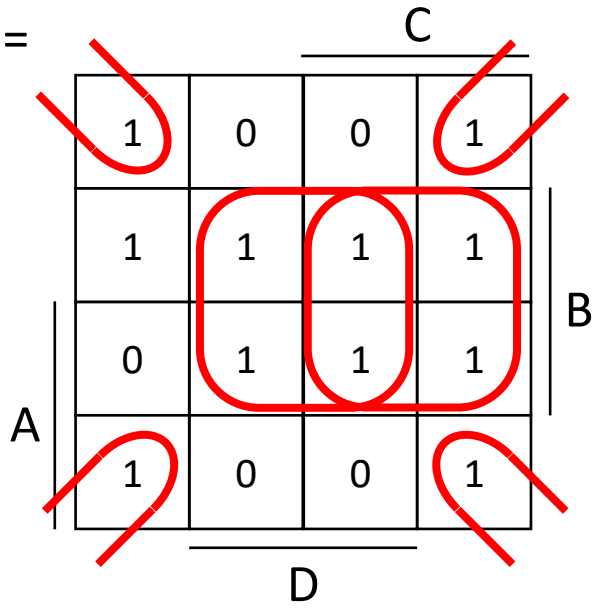
$$F3 = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + A\bar{D} + \bar{B}\bar{D}$$

$$F4 = \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D} + BC + BD + \bar{A}\bar{C}\bar{D}$$

F3 =



F4 =



$$F3 = \bar{A} + \bar{D}$$

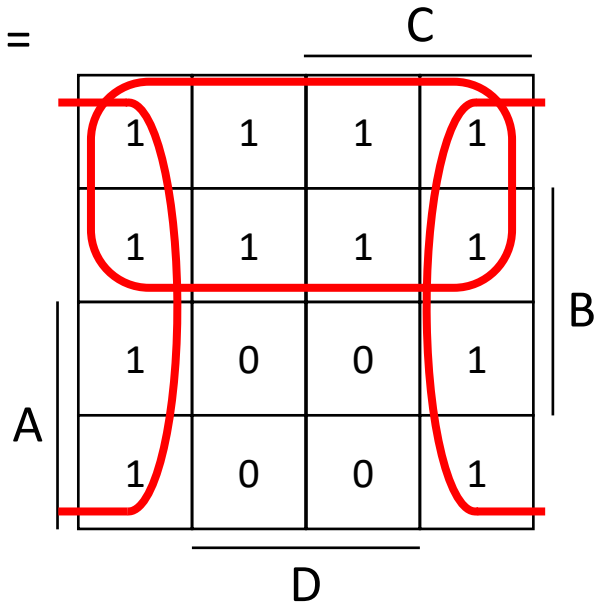
$$F4 = \bar{B}\bar{D} + BD + BC$$

Exemplo 3 (10 de 10)

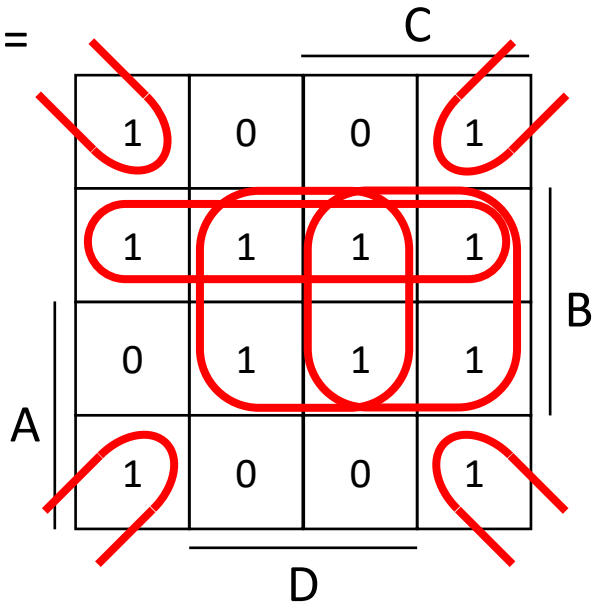
$$F3 = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + A\bar{D} + \bar{B}\bar{D}$$

$$F4 = \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D} + BC + BD + \bar{A}\bar{C}\bar{D}$$

F3 =



F4 =



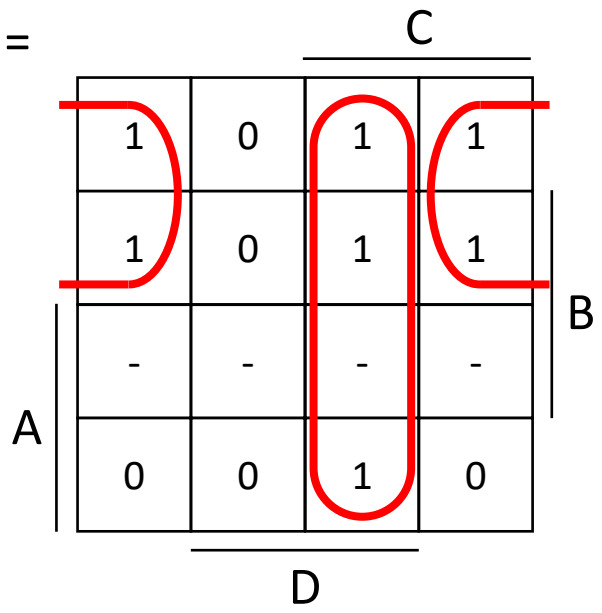
$$F3 = \bar{A} + \bar{D}$$

$$F4 = \bar{B}\bar{D} + BD + BC + \bar{A}B$$

Exemplo 4 – indiferenças (*don't care*)

- Por vezes, podem existir combinações entre variáveis da função que nunca ocorrem
- O resultado da função para essas combinações é indiferente
- Essas combinações são usadas como 0 ou 1 com o objetivo de simplificar a função

F5 =



$$F5 = CD + \bar{A}\bar{D}$$

Exemplo 5 – funções complexas

- Considere a função $F = (B\bar{C} + A(B \oplus C)).(A + B.\overline{C + D})$
- Para reduzir a possibilidade de engano, constrói-se dois mapas para representar termos parciais da função

F' = $B\bar{C} + A(B \oplus C)$

A \ CD	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	1	1	0	0
10	0	0	1	1

B

and

F'' = $A + B.\overline{C + D}$

A \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	0
11	1	1	1	1
10	1	1	1	1

B

=

F = $AB\bar{C} + A\bar{B}C + B\bar{C}\bar{D}$

A \ CD	00	01	11	10
00	0	0	0	0
01	1	0	0	0
11	1	1	0	0
10	0	0	1	1

B

Exercícios

Simplificação de funções por mapa de Karnaugh

1. $F1_{(A,B,C)} = (\bar{A}B + A.B.\bar{C}) \oplus (A.B.C + \bar{C}.D)$

2. $F2_{(A,B,C,D)} = (\bar{A} + B.\bar{D}).(\bar{A}.B.C + \bar{C}.D)$

3. $F3_{(W,X,Y,Z)} = \overline{(W + \bar{X}).(Z\bar{Y} + X).(W + Y + Z).(W + \bar{Y} + \bar{Z})}$

Soluções

1. Realize um mapa de quatro variáveis para cada um dos termos do XOR e um terceiro mapa com o resultado do XOR entre os dois primeiros mapas

$$F1_{(A,B,C,D)} = B\overline{D} + B.C + \bar{B}.\bar{C}.D$$

2. Adote a mesma técnica do exercício anterior ou, em alternativa, aplique a propriedade distributiva antes de passar a mapa

$$F2_{(A,B,C,D)} = \bar{A}.B.C + \bar{A}.\bar{C}.D$$

3. Antes de passar a mapa, aplique a lei de De Morgan

$$F3_{(W,X,Y,Z)} = \overline{W}.X + \bar{X}.\bar{Z} + \bar{X}.Y$$