HDSO Circuit Analysis

Ritoban Roy-Chowdhury

February 11, 2021

$$\frac{v_5 - v_2}{20\Omega} + \frac{v_5}{10\Omega} + \frac{v_5 - v_4}{16\Omega} = 0$$

(1)

$$\frac{v_4 - v_5}{16\Omega} + \frac{v_4}{160\Omega} + \frac{v_4 - v_3}{20\Omega} = 0$$

(2)

$$\frac{v_3 - v_4}{20\Omega} + \frac{v_3}{30\Omega} = 0$$

(3) Plugging into Wolram Alpha (see next page):

$$v_3 = -1.25 \quad v_4 = -2.09 \quad v_5 = -2.97$$

Solving the node equations for #9 Rearrange (3)

$$\begin{aligned} &\tfrac{v_3 - v_4}{20\Omega} + \tfrac{v_3}{30\Omega} = 0 \\ &v_3 \left[\tfrac{1}{20} + \tfrac{1}{30}\right] + v_4 \left[-\tfrac{1}{20}\right] = 0 \\ &v_3 = \tfrac{v_4}{20} \left(\tfrac{1}{20} + \tfrac{1}{30}\right) = \tfrac{v_4}{2\theta} \cdot 1^3 2 = \tfrac{3}{5} v_4 \end{aligned}$$

Substitute into (2)

$$v_3 \left[-\frac{1}{20}\right] + v_4 \left[\frac{1}{16} + \frac{1}{160} + \frac{1}{20}\right] + v_5 \left[-\frac{1}{16}\right] = 0$$

& Rearrange

$$\begin{aligned} &-\tfrac{3}{5} \tfrac{v_4}{20} + v_4 \cdot \tfrac{19}{160} + \tfrac{-v_5}{16} = 0 \\ &v_4 \left[\tfrac{19}{160} - \tfrac{3}{100}\right] = \tfrac{v_5}{16} \\ &v_4 = \tfrac{50}{71} v_5 \end{aligned}$$

$$\begin{aligned} v_5 &= -2.97 \text{ V} \\ v_4 &= -2.09 \text{ V} \\ v_3 &= -1.25 \text{ V} \end{aligned}$$

10. A continuation of #9. Skip (d), we'll come back to it.

    (e) $i_{R_1} = \frac{v_5 - v_2}{R_1} = \frac{(-2.97 \text{ V}) - (-10 \text{ V})}{20\Omega} = 0.35 \text{ A} = 350 \text{ mA}$

    (f) $P_{R_7} = i_{R_7}^2 \cdot R_7$

$$\begin{aligned}
V_{\text{out}} &= v_4\left(\frac{P_6}{R_6+R_7}\right) = -2.97\text{ V}\left(\frac{10\Omega}{160\Omega}\right)\\
&= -0.1856\text{ V}\\
\therefore P_{R_7} &= \left(\frac{-2.97-(-0.1856)}{150\Omega}\right)^2\cdot 150\Omega\\
&= 0.0255\text{ W}\\
&= 26\text{ mW}
\end{aligned}$$

(d) The equivalent resistance is whatever the resistance "appears" to the voltage source, i.e. the current through the source divided by its voltage.

Since R_1 and the source are connected in series, the current

$$i_S = i_{R_5} = \frac{v_5 - v_2}{20\Omega} = \frac{(-2.97)-(-10\text{v})}{20\Omega} = 0.3515\text{ A}$$

$$R_{eq} = \frac{10V}{0.3515A} = 28.44\Omega = 28\Omega$$

Alt. Using Matrices to solve the node equations

(1) O$v_3 + \left[\frac{1}{20}+\frac{1}{10}+\frac{1}{16}\right]v_5 + \left[-\frac{1}{16}\right]v_4 = -\frac{1}{2}$
(2) $\left[-\frac{1}{20}\right]v_3 + \left[-\frac{1}{16}\right]v_5 + \left[\frac{1}{16}+\frac{1}{100}+\frac{1}{80}\right]v_4 = 0$
(3) $\left[\frac{1}{20}+\frac{1}{30}\right]v_3 + Ov_5 + \left[-\frac{1}{20}\right]v_4 = 0$ $$

$$\begin{bmatrix} 0 & -1/16 & 17/80 \\ -1/20 & +19/160 & -1/16 \\ 1/12 & -1/20 & 0 \end{bmatrix}\begin{bmatrix} v_3 \\ v_4 \\ v_6 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 0 \\ 0 \end{bmatrix}$$

$v_3 = -1.25$ V $\quad v_4 = -2.09$ V $\quad v_5 = -2.97$ V

\end{document}

Question 11

(1) $\frac{v_1-v_3}{1.5k} + \frac{v_1-(-5v)}{1.6k} = 0$
(2) $\frac{v_3-v_1}{1.5k} + \frac{v_3}{6.3k} - 5mA = 0$
$v_2 = v_{R_4} = (-5mA)(0.500k\Omega) = -2.5V$
Rearrange (1)

$$v_1 = \frac{-5}{1600} + \frac{v_3}{1000}$$

$$\frac{1}{1500} + \frac{1}{1600}$$

Substitute into (2) & Solve: $v_3 = 7.037$ V

$v_1 = 1.213$ V

(a) $v_{12} = v_2 - v_1 = (-2.5v) - (1.213 \text{ V})$

$= -3.71$ V

$\approx 4V$

(b) $i_{R1} = \dfrac{v_3}{R_1} = \dfrac{7.037V}{6.3k\Omega} = 1.117$ mA

$\approx 1mA$