Circuit Analysis

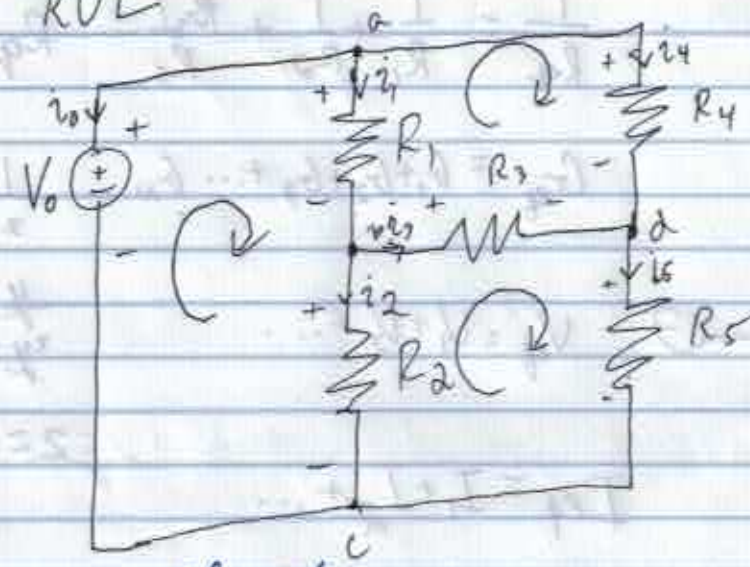
KCL KVL

Method #1

① Element eq. $V = IR$, $q = CV$

② KCL

③ KVL



Element

$v_0 = V_0$ ∓

$V_1 = i_1 R_1$

$V_2 = i_2 R_2$

⋮

KCL (Junction Rule)

$-i_0 - i_1 - i_4 = 0$

$i_1 - i_3 - i_2 = 0$

$i_4 - i_5 + i_3 = 0$

$i_2 + i_5 + i_0 = 0$

KVL (Loop Rule)

$-v_0 + v_1 + v_2 = 0$

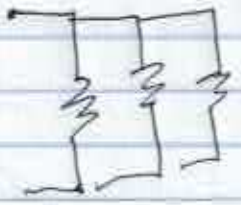
$-v_1 + v_4 - v_3 = 0$

$v_2 - v_3 - v_5 = 0$

Method #2

Element Combination Laws

$R_1 + R_2 + R_3 + \ldots +$



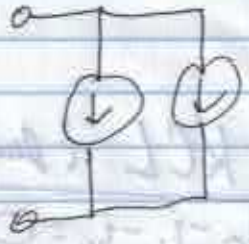
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$G_{eq} = G_1 + G_2 + G_3 + \ldots G_N \qquad \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

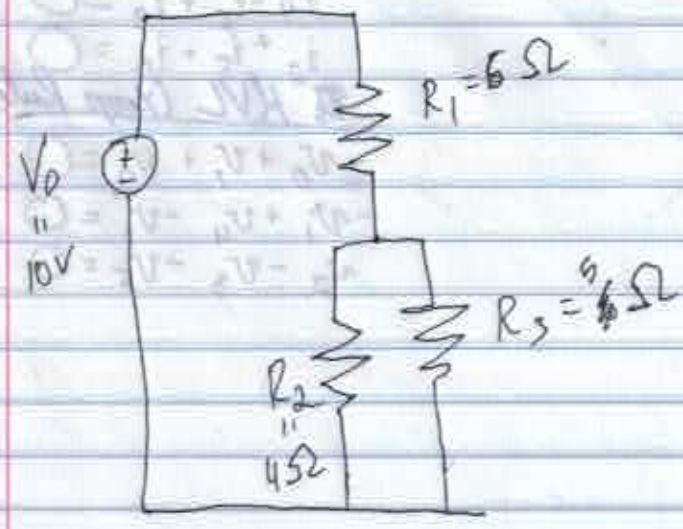
$V_{eq} = V_1 + V_2 + \ldots$

$$\frac{y}{zy} + \frac{z}{zy} = \frac{y+z}{xy}$$

$$z = \frac{xy}{x+y}$$



$I_{eq} = I_1 + I_2 + \ldots$

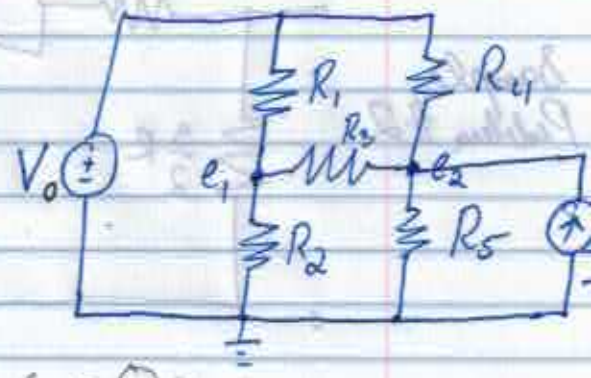


Method #3 – Node Analysis

① Select $\perp$ $\frac{1}{=}$ (Ground)

② Label voltages $e_1 \dots e_N$

③ KCL, substituting in element eqs.



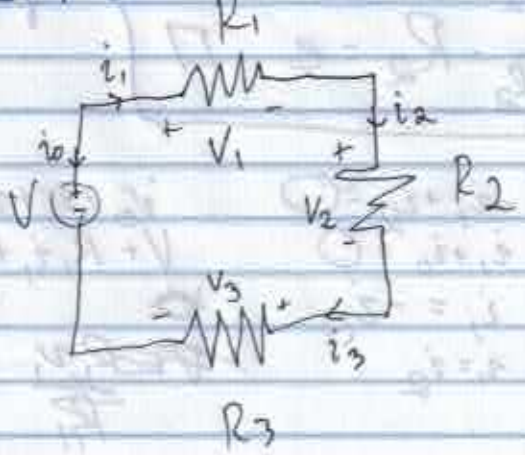
$$(e_1 - V_0)G_1 + (e_1 - e_2)G_3 + e_1 G_2 = 0$$

$$(e_2 - e_1)G_3 + (e_2 - V_0)G_4 + e_2 G_5 - I_1 = 0$$

$$e_1(G_1 + G_3 + G_2) + e_2(-G_3) = V_0 G_1$$

$$e_1(-G_3) + e_2(G_3 + G_4 + G_5) = V_0 G_4 + I_1$$

Sample Problems



$$R_{eq} = R_1 + R_2 + R_3$$

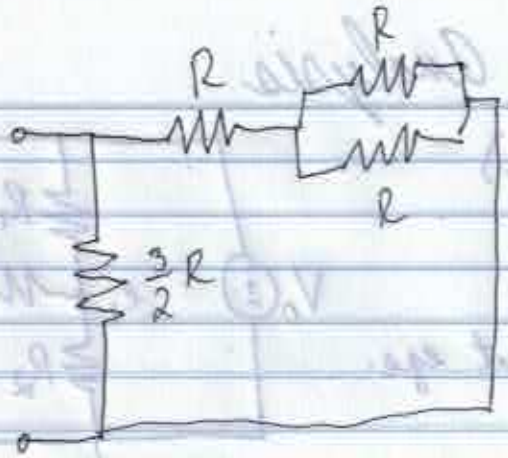
$$i_1 = i_2 = i_3 = -i_0 = \frac{V}{R_{eq}}$$

$$V_1 = i_1 R_1 \qquad P_{R_1} = i_1 V_1$$

$$= i_1^2 R_1$$

Power $= \frac{V^2}{R_{eq}^2} R_1$

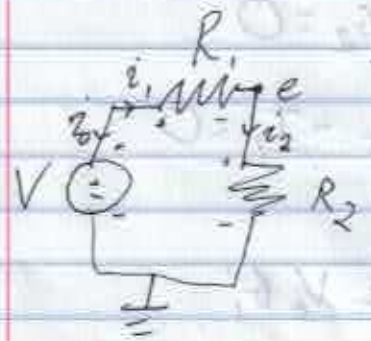
Sample Problem #2



$$R_{eq} = \left(\frac{R^2}{2R} + R\right) \| \frac{3}{2}R$$

$$R_{eq} = \frac{\frac{3}{2}R\left(\frac{R^2}{2R} + R\right)}{\frac{3}{2}R + R + \frac{R^2}{2R}}$$

Voltage Divider: Derivation



$(e - V)R_1 + eR_2 = 0$

$e(R_1 + R_2) = VR_1$

$\boxed{e = \frac{VR_1}{R_1 + R_2}}$ □

Node Method

INCORRECT!!

Eq. Res.

$R_{eq} = R_1 + R_2$

$i = \frac{V}{R_1 + R_2} = i_1 = i_2 = -i_0$

$v_2 = \frac{V}{R_1 + R_2} \cdot R_2 = e$ □

Eq. Resistance

KCL/KVL

$i_0 = V$ $\quad -i_1 + i_2 = 0$ $\quad i_0 + v_1 + v_2 = 0$

$v_1 = R_1 i_1 = R_1 i$ $\quad i_1 + i_0 = 0$ $\quad V + R_1 i_1 + R_2 i_2 = 0$

$v_2 = R_2 i_2 = R_2 i$ $\quad i_1 = -i_0$

$i_1 = i_2$

~~$V = \frac{R_1 v_1}{R_1}$~~

$V = -i(R_1 + R_2)$

Current Divider

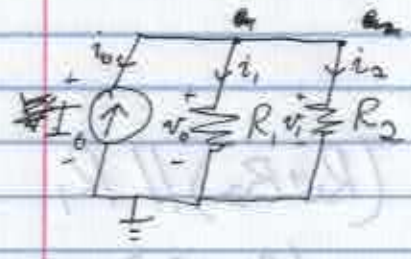
Correct Voltage Divider!!

$\frac{(e - V)}{R_1} + \frac{e}{R_2} = 0$

$e\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_1}$

$e = \frac{V}{R_1} \cdot \frac{R_1 R_2}{R_1 + R_2} = \boxed{\frac{VR_2}{R_1 + R_2}}$

Current Divider Derivation



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

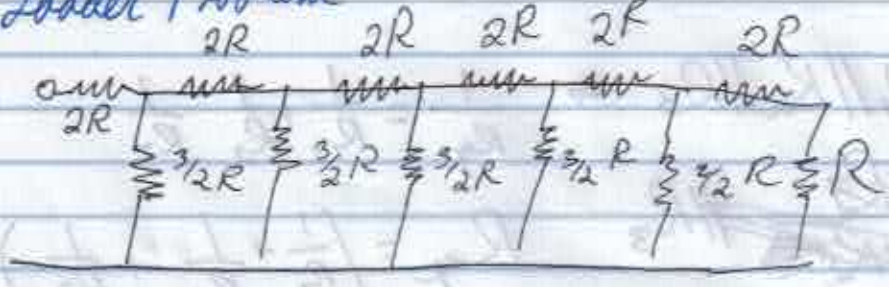
$$v_0 = v_1 = I_0 R_{eq}$$

$$i_{0,1} = \frac{v_0}{R_1} = \frac{I_0 R_{eq}}{R_1}$$

$$= \frac{1}{R_1} \frac{R_1 R_2}{R_1 + R_2} \cdot I_0$$

$$= I_0 \frac{R_2}{R_1 + R_2}$$

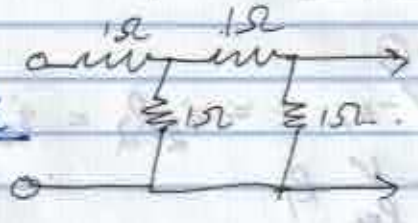
Partial Ladder Problem



let $R = 1$

$3R \,//\, \frac{3}{2}R = 1$ ✓ $\boxed{3R}$

Inf. Ladder

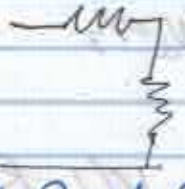


$a_1 = 2\,\Omega$

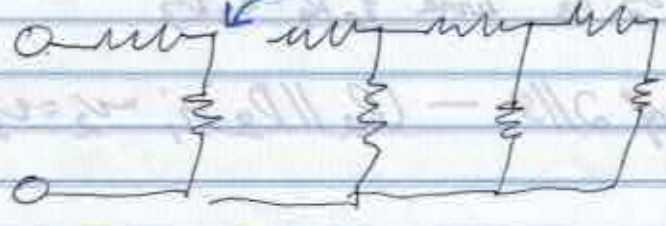
$a_2 = \frac{5}{3}\,\Omega$

$a_3 = \frac{5}{8}\,\Omega$

$a_4 =$



$a_n = a_{n-1} \,//\, 2\Omega$ Insert Break Here



$x = x \,//\, R + R$

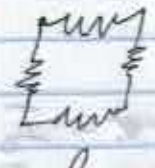
$x = \frac{x}{x+1} + 1$

$x - 1 = \frac{x}{x+1}$

$x^2 - 1 = x$

$\boxed{x = \frac{1 \pm \sqrt{5}}{2}}$

Sola to HW # from MIT



2.1 $R_2 // R_3 + R_1$ $\quad (R_2+R_3) // R_1$

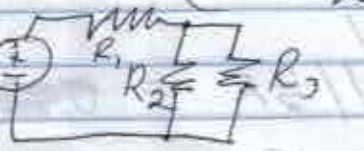
$\frac{R_2 R_3}{R_2+R_3} + R_1$ $\quad \frac{(R_2+R_3) R_1}{R_1+R_2+R_3}$

$R_1+R_2+R_3$

~~R_1+R_2~~ $R_1 // R_2 // R_3 \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \frac{1}{R_3}$

~~$\frac{R_1 R_2}{R_1+R_2} // R_3$~~ $\quad R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$

2.2 $R_{eq} = \frac{R_1 (R_2+R_3)}{R_1+R_2+R_3}$



$i_1 = \frac{V}{R_{eq}}$ $\quad i_2 + i_3 - i_1 = 0$

$v_1 = i_1 \cdot R_1$ $\quad v_2 = v_3 = V - v_1 = i_2 R_2 = i_3 R_3$

$= \frac{V}{R_{eq}} R_1$ $\quad = V - \frac{V}{R_{eq}} R_1$

$= V\left(1 - \frac{R_1}{R_{eq}}\right)$

$(R_2 // R_3) i_1 = v_2 = v_3$

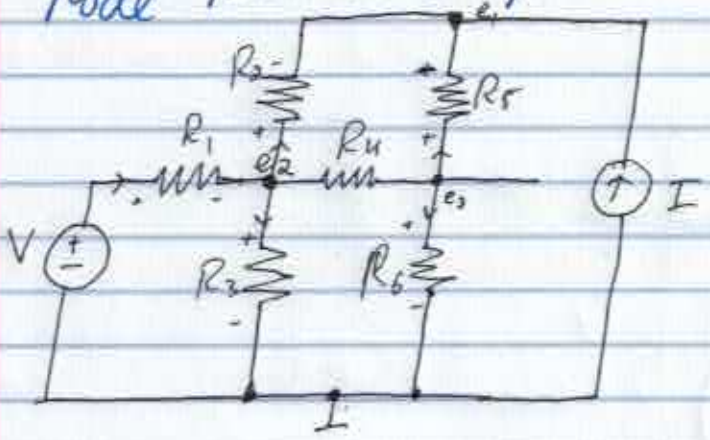
3 approaches

① KVL $\rightarrow V\left(1 - \frac{R_1}{R_{eq}}\right)$

② KCL $i_1 = i_2 + i_3$ with $i_2 R_2 = i_3 R_3$

③ Eq. resistance of 2//3 $\rightarrow (R_2 // R_3) i_1 = v_2 = v_3$

Node Method Sample



$$\frac{(e_2-e_1)}{R_2}+\frac{(e_2-V)}{R_1}+\frac{(e_2)}{R_3}+\frac{(e_2-e_3)}{R_4}=0$$

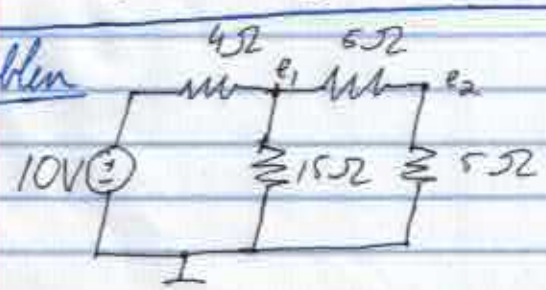
$$\frac{(e_3-e_1)}{R_5}+\frac{(e_3-0)}{R_6}+\frac{e_3-e_2}{R_4}=0$$

$$\frac{e_1-e_2}{R_2}+\frac{e_1-e_3}{R_5}-I=0$$

$$e_1(-G_2)+e_2(G_2+G_1+G_3+G_4)+e_3(-G_4)=VG_1$$
$$e_1(-G_5)+e_2(-G_4)+e_3(G_5+G_6)=0$$
$$e_1(+G_2+G_5)+e_2(-G_2)+e_3(-G_5)=I$$

$$\begin{bmatrix} -G_2 & G_T & -G_4 \\ -G_5 & -G_4 & G_5+G_6 \\ G_2+G_5 & -G_2 & -G_5 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}=\begin{bmatrix} VG_1 \\ 0 \\ I \end{bmatrix}$$

Power Problem



$$\frac{e_1-10}{4}+\frac{e_1-e_2}{5}+\frac{e_1}{15}=0$$

$$\frac{e_2-e_1}{5}+\frac{e_2}{5}=0$$

$$\frac{2e_2}{5}=\frac{e_1}{5}$$

$$i_{15\Omega}=\frac{e_1}{15}=\frac{6}{15} \qquad e_2=3 \qquad e_1=6$$

$$\frac{2e_2-10}{4}+\frac{e_2}{5}+\frac{2e_3}{15}=0$$

$P=6.6=24W$