AP Calculus Ch. 5 Test

1) $g(t) = t^2 e^t$

$g'(t) = t^2 e^t + e^t \cdot 2t$

$= e^t(t^2 + 2t)$.

2) $g(x) = e^{-2/x^2}$

$g'(x) = \dfrac{4}{x^3} \cdot e^{-2/x^2}$

$= \dfrac{4e^{-2/x^2}}{x^3}$.

3) $f(x) = \arctan 2x - \frac{1}{4} \ln(1 + 4x^2)$

$f'(x) = \dfrac{2}{4x^2+1} - \dfrac{1}{\cancel{4}} \cdot \dfrac{\cancel{8}x^2}{1+4x^2}$

$= \dfrac{2-2x}{4x^2+1}$

$= \dfrac{2(1-x)}{4x^2+1}$.

$\dfrac{d}{dx} \arctan(x) = \dfrac{1}{\sec^2(\tan^{-1}(x))}$

$= \dfrac{1}{x^2+1}$

4) $f(x) = x^2 \ln(x^2)$

$f'(x) = 2x \ln(x^2) + \dfrac{2x}{x^2} \cdot x^2$

$= 2x\left(\ln x^2 + 1\right)$.

5) $y \cos x = e^y$

$\dfrac{dy}{dx} \cdot \cos x + (-\sin x)\, y = \dfrac{dy}{dx} e^y$

$\dfrac{dy}{dx}(\cos x - e^y) = y \sin x$

$\dfrac{dy}{dx} = \dfrac{y \sin x}{\cos x - e^y}$

6) $h(x) = 5^{x-2}$

$h'(x) = \ln(5) \cdot 5^{x-2}$

$h'(2) = \ln(5)$

$\therefore$ The tangent line is $y - 1 = \ln(5)(x-2)$

7) $\displaystyle\int -2x^3 e^{-x^4} dx$ Let $u = -x^4$

$du = -4x^3\, dx$

$= \dfrac{1}{2}\displaystyle\int e^u\, du$

$= \dfrac{1}{2}\left(e^u + C\right)$

$= \dfrac{1}{2} e^{-x^4} + C.$

8) $\displaystyle\int \frac{4}{4+x^2}\, dx$

$= \dfrac{1}{2} \arctan\left(\dfrac{x}{2}\right) + C.$

9) $\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}}$

$= \arcsin\left(\dfrac{x}{2}\right)\Big|_0^1$

$= \arcsin\left(\dfrac{1}{2}\right) - \arcsin(0)$

$= \dfrac{\pi}{6}.$

10) $\int \frac{1}{x^2+4x+8}\,dx$

$= \int \frac{1}{x^2+4x+4+4}\,dx$

$= \int \frac{1}{(x+2)^2+4}\,dx$

$= \frac{1}{2}\arctan\left(\frac{x+2}{2}\right) + C.$

11) $\int \frac{x^3+2}{x+1}\,dx$

$$\begin{array}{r} x^2 - x + 1 \\ x+1\,\overline{)\,x^3+2} \\ \underline{x^3+x^2} \\ -x^2 \\ \underline{-(-x^2-x)} \\ x+2 \\ \underline{-(x+1)} \\ 1 \end{array}$$

$= \int x^2 - x + 1 + \frac{1}{x+1}\,dx$

$= \frac{x^3}{3} - \frac{x^2}{2} + x + \ln|x+1| + C.$

Bonus) $\int_{\pi/3}^{\pi/2} e^{\sec 2x} \cdot \sec 2x \cdot \tan 2x\,dx$

~~Let u = 2x~~
~~du = 2dx~~

Let $u = \sec 2x$
$du = 2\sec 2x \tan 2x\,dx$

$= \frac{1}{2}\int_{-2}^{-1} e^u\,du$

$= \frac{1}{2}\left[e^u\right]_{-2}^{-1}$

$= \frac{1}{2}\left[e^{-1} - e^{-2}\right] = \frac{1}{2}\left[\frac{1}{e} - \frac{1}{e^2}\right] = 0.116.$