

Angular Kinematics

1a. $28^\circ \times \frac{\pi}{180^\circ} = 0.49 \text{ rad.}$

b. $50^\circ \times \frac{\pi}{180^\circ} = 0.87 \text{ rad}$

c. $145^\circ \times \frac{\pi}{180^\circ} = 2.53 \text{ rad}$

d. $3 \text{ rev} \times \frac{2\pi}{1 \text{ rev}} = 6\pi \text{ rad.}$

e. $0.25 \frac{\text{rev}}{\text{s}} \cdot \frac{2\pi}{1 \text{ rev}} = \frac{1}{2} \pi \frac{\text{rad}}{\text{s}}$

f. $60 \text{ rev} \cdot \frac{2\pi}{1 \text{ rev}} = 120\pi \text{ rad.}$

g. $0.35 \frac{\text{rev}}{\text{s}^2} \times \frac{2\pi}{1 \text{ rev}} = 0.70\pi \text{ rad/s}^2$

2a. $\Delta x = r \Delta \theta$
 $\theta = \frac{\Delta x}{r} = \frac{3 \text{ m}}{2 \text{ m}} = 1.5 \text{ rad.}$

b. $r = \frac{\Delta x}{\theta} = \frac{8 \text{ in}}{4 \text{ rad}} = 2 \text{ in.}$

c. $\Delta x = r \theta$
 $= (4 \text{ ft}) (25^\circ \cdot \frac{\pi}{180^\circ})$
 $= \frac{5\pi}{9} \text{ ft} \approx 1.745 \text{ ft.}$

d. $r = \frac{\Delta x}{\theta} = \frac{30 \text{ in.}}{260^\circ \cdot \frac{\pi}{180^\circ}} = 6.6 \text{ in}$

| 3. | θ_{rev} | $\theta (\text{rad})$ | $r (\text{m})$ | $\Delta x (\text{m})$ |
|----|-------------------------------|-----------------------|----------------|-----------------------|
| | 17 | $34\pi \approx 106.8$ | 0.5 | $17\pi \approx 53.4$ |
| | $\frac{1}{\pi} \approx 0.318$ | 2 | 0.25 | 0.5 m |
| | $\frac{2}{\pi} \approx 0.636$ | 4 | 3 | 12 |
| | $\frac{5}{\pi} \approx 1.59$ | 10 | 4.5 | 45 |
| | 8 | $16\pi \approx 50.3$ | 0.398 | 20 |

| ω (RPM) | ω (rad/s) | r (m) | v ($\frac{m}{s}$) |
|----------------|---|---------|-------------------------------|
| 40 | $\frac{2\pi}{60} = \frac{4}{3}\pi \approx 4.18$ | 2 | $\frac{8}{3}\pi \approx 8.38$ |
| 238.7 | 25 | 0.25 | 6.25 |
| 22.9 | $12/5 = 2.4$ | 5 | 12 |
| 143.2 | 15 | 2.66 | 40 |
| 22 | 2.30 | 3.47 | 7.8 |

4. $7200 \text{ RPM} \times \frac{2\pi}{60} = 240\pi \frac{\text{rad}}{s} \approx 754 \frac{\text{rad}}{s}$

$v = (0.0889 \text{ m})(240\pi \frac{\text{rad}}{s})$
 $= 67.0 \frac{m}{s}$

5. $v = r\omega$
 $= r(\text{RPM} \times \frac{2\pi}{60})$
 $= (0.085 \text{ m})(33.33 \text{ RPM} \times \frac{2\pi}{60})$
 $= 0.297 \frac{m}{s}$

6a. $15 \text{ RPM} \times 3 \text{ min} = 45 \text{ rev.}$

b. $15 \text{ RPM} \times \frac{10 \text{ s}}{60 \text{ s}} = 2.5 \text{ rev.}$

c. $1 \text{ rev} \times \frac{1 \text{ min}}{15 \text{ rev}} \times \frac{60 \text{ s}}{1 \text{ min}} = 4 \text{ s.}$

d. $v = (1.75 \text{ m})(15 \text{ RPM} \times \frac{2\pi}{60})$
 $= \frac{7}{2}\pi$
 $\approx 2.75 \frac{m}{s}$

7a. $\frac{1 \text{ rev}}{24 \text{ hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \frac{1}{1440} \text{ RPM} = 6.94 \cdot 10^{-4} \text{ RPM}$

b. $v = r\omega$
 $= (6.378 \times 10^6 \text{ m})(6.94 \cdot 10^{-4} \text{ RPM} \times \frac{2\pi}{60})$
 $= 463 \frac{m}{s}$

$$8. \quad \omega = \frac{110 \frac{\text{m}}{\text{s}}}{0.184 \text{ m}} = 597 \frac{\text{rad}}{\text{s}} \times \frac{60 \frac{\text{s}}{\text{min}}}{2\pi \frac{\text{rad}}{\text{rev}}} = 5709 \text{ RPM}$$

$$9. \quad \omega = \frac{343 \frac{\text{m}}{\text{s}}}{0.25 \text{ m}} = 1372 \frac{\text{rad}}{\text{s}} \times \frac{60}{2\pi} = 13,101 \text{ RPM}$$

$$10a. \quad \frac{1 \text{ rev}}{12 \text{ hr}} = \frac{2\pi}{12 \text{ hr} \cdot 3600 \frac{\text{s}}{\text{hr}}} = 1.45 \cdot 10^{-4} \frac{\text{rad}}{\text{s}} \times \frac{60}{2\pi} = 0.00138 \text{ RPM}$$

$$b. \quad \frac{1 \text{ rev}}{60 \text{ min}} = 0.016 \text{ RPM} \times \frac{2\pi}{60} = 1.74 \cdot 10^{-3} \frac{\text{rad}}{\text{s}}$$

$$c. \quad 1 \text{ RPM} = 0.105 \frac{\text{rad}}{\text{s}}$$

$$11a. \quad v = (0.05 \text{ m})(1.45 \cdot 10^{-4} \frac{\text{rad}}{\text{s}}) = 7.25 \cdot 10^{-6} \frac{\text{m}}{\text{s}}$$

$$b. \quad v = (0.15 \text{ m})(1.74 \cdot 10^{-3} \frac{\text{rad}}{\text{s}}) = 2.61 \cdot 10^{-4} \frac{\text{m}}{\text{s}}$$

$$c. \quad v = (0.12 \text{ m})(0.105 \frac{\text{rad}}{\text{s}}) = 0.0126 \frac{\text{m}}{\text{s}}$$

$$12a. \quad 5 \frac{\text{rad}}{\text{s}} \cdot 20 \text{ s} = 100 \text{ rad}$$

$$b. \quad \frac{100 \text{ rad}}{2\pi} = 15.9 \text{ rev}$$

$$13a. \quad \omega = \frac{600 \text{ rad}}{10 \text{ s}} = 60 \frac{\text{rad}}{\text{s}}$$

$$14. \quad \frac{200 \text{ rad}}{60 \frac{\text{rad}}{\text{s}}} = 3.3 \text{ s}$$

$$15a. \quad \omega = 25 \text{ RPM} \times \frac{2\pi}{60} = 2.6 \frac{\text{rad}}{\text{s}}$$

$$b. \quad 26 \frac{\text{rad}}{\text{s}} \cdot 4 \text{ s} = 104 \text{ rad}$$

16. $HS \text{ RPM} \times \frac{2\pi}{60} = 4.71 \text{ rad/s}$

They are equal.

17. $r_1 \omega_1 = r_2 \omega_2$
 $\omega_2 = \frac{(0.35 \text{ m})(6.5 \frac{\text{rad}}{\text{s}})}{0.12 \text{ m}} = 18.9 \frac{\text{rad}}{\text{s}}$

18. $\omega_f = \omega_i + at$
 $a = \frac{\omega_f - \omega_i}{t} = \frac{-5 \text{ rad/s}}{30 \text{ s}} = -\frac{1}{6} \frac{\text{rad}}{\text{s}^2}$

19a. $\omega_f = 0 + (0.75 \text{ rad/s}^2)(60 \text{ s})$
 $= 45 \text{ rad/s}$

b. $\theta = \frac{1}{2}(0.75 \text{ rad/s}^2)(60 \text{ s})^2$
 $= 1350 \text{ rad}$

c. $v = (45 \text{ rad/s})(7.3 \text{ m})$
 $= 328.5 \text{ m/s}$

20a. $\omega_f^2 = \omega_i^2 + 2\alpha\theta$
 $\alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta}$
 $= \frac{-20^2 + 8^2}{2 \cdot 500 \cdot \frac{2\pi}{360}} = -0.053 \frac{\text{rad}}{\text{s}^2}$

★ answer key forgot to convert 500 rev to rad on 20.

b. $8 \text{ rad/s} = 20 \text{ rad/s} + (-0.053 \frac{\text{rad}}{\text{s}^2}) \cdot t$
 $t = 226 \text{ s}$

21a. $200 \text{ rad} = \frac{1}{2}(2.5 \text{ rad/s}^2)(t^2)$
 $t = 12.6 \text{ s}$

b. $\omega_f = (2.5 \text{ rad/s}^2)(12.6 \text{ s})$
 $= 31.5 \text{ rad/s}$

c. $v = (31.5 \text{ rad/s})(0.05 \text{ m})$
 $= 1.6 \frac{\text{m}}{\text{s}}$

$$22a \quad \Delta\theta = \frac{1}{2}(0.4 \frac{\text{rad}}{\text{s}^2})(9 \text{ s})^2$$

$$= 16.2 \text{ rad.}$$

$$b. \quad \frac{16.2 \text{ rad}}{2\pi} = 2.57 \text{ rev.}$$

$$23a \quad 0 = 27 \frac{\text{rad}}{\text{s}} - (0.15 \frac{\text{rad}}{\text{s}^2})t$$

$$t = 180 \text{ s}$$

$$b. \quad \theta = \cancel{0.15 \text{ rad}} \cdot 27 \frac{\text{rad}}{\text{s}} \cdot 180 \text{ s} - \frac{1}{2}(0.15 \frac{\text{rad}}{\text{s}^2})(180 \text{ s})^2$$

$$= 2430 \text{ rad}$$

$$= 387 \text{ rev.}$$