Circular Motion FRQs

1a) $K_i = \frac{1}{2}(0.50\,kg)(1.5\,\frac{m}{s})^2 = 0.5625\,J$

$U_i = (0.50\,kg)(9.8\,\frac{N}{kg})(2.0\,m) = 9.8\,J$

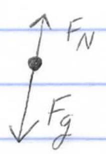
$E = K_i + U_i = 10.3625\,J$

$U_f = (0.50\,kg)(9.8\,\frac{N}{kg})(1.9\,m) = 9.31\,J$

$\therefore K_f = E - U_f = 10.3625 - 9.31 = 1.0525\,J$

$\therefore v_f = \sqrt{\frac{2K_f}{m}} = 2.05\,\frac{m}{s}$

b)



c)

$$\sum F = F_c$$

$$F_g - F_N = \frac{mv^2}{r}$$

(F_N is negative because F_c is in the same direction ~~at~~ as F_c)

$$F_N = mg - \frac{mv^2}{r}$$

$$= (0.50\,kg)\left(9.8\,\frac{m}{s^2} - \frac{(2.05\,\frac{m}{s})^2}{0.95\,m}\right)$$

$$= 2.69\,N$$

d) The work needed to stop the car equals the kinetic energy it would otherwise have. From (a), $W = K_f = 1.05\,J$.

e) The track would have to be modified so that $F_N = 0$ at A, ~~in~~ so $F_c = F_g$. This could be achieved by reducing the radius, which would increase F_c until it equals F_g.

Assuming A is kept at the same height, $F_g = F_c$ when

$$g = \frac{v^2}{r}$$

$$r = \frac{v^2}{g} = \frac{(2.05\,\frac{m}{s})^2}{9.8\,\frac{m}{s^2}} = 0.43\,m.$$

2a)

$$\sum_{m_1} F = F_c$$

$$F_T = \frac{m_1 v^2}{r} \quad (1)$$

But m_2 is stationary, and F_T is uniform in a string.

$$\sum_{m_2} F = 0$$

$$F_T - F_g = 0$$

$$F_T = F_g$$

$$= m_2 g \quad (2)$$

Substitute (2) into (1).

$$m_2 g = \frac{m_1 v^2}{r}$$

$$g = \frac{v^2}{r} \cdot \frac{m_1}{m_2}$$

$$v^2 = g r \frac{m_2}{m_1}$$

By the kinematic equations,

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi r}{\sqrt{g r \frac{m_2}{m_1}}}$$

$$= 2\pi \cdot \frac{r}{\sqrt{r}} \cdot \sqrt{\frac{m_1}{m_2 g}}$$

$$T = 2\pi \sqrt{\frac{m_1 r}{m_2 g}}$$ as required.

b) From (a), $T = 2\pi\sqrt{\dfrac{m_1 r}{m_2 g}}$

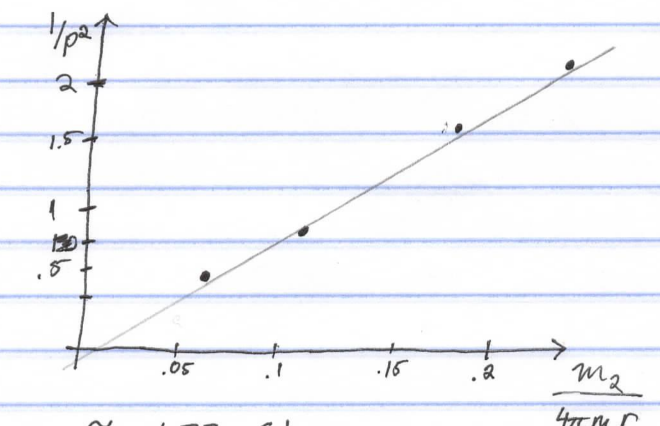
$$T^2 = \frac{4\pi^2 m_1 \cdot r}{m_2 \cdot g}$$

$$\frac{1}{T^2} = g\left[\frac{m_2}{4\pi^2 m_1 \cdot r}\right]$$

The graph of $1/P^2$ against $\dfrac{m^2}{4\pi^2 m_1 r}$ should yield a straight line with slope g.

c)

m_2	0.020	.040	.060	.080
P	1.40	1.05	0.80	.75
$1/P^2$	.51	.91	1.56	1.77
$\frac{m_2}{4\pi^2 m_1 \cdot r}$	.0528	.1055	.1583	.211



d) $g \approx \dfrac{1.77 - .51}{.211 - .0528} = 7.96\ \frac{m}{s^2}$