Wind Farm Layout Optimization (WindFLO): A User's Guide

Version: 1.0.0

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The WindFLO Model

This guide outlines the parameters and files needed to use the WindFLO program to analyze wind farm configurations. This theory/user guide is a supplement to the published article [1, 2] which outlines the numerics and the WindFLO algorithm. If you use WindFLO, please cite the following references

Sohail R. Reddy, "Wind Farm Layout Optimization (WindFLO): An Advanced Framework for Fast Wind Farm Analysis and Optimization," Applied Energy, Vol. 269, pp. 115090, 2020

Sohail R. Reddy, "Wind Farm Layout Optimization (WindFLO): A Framework for Fast Wind Farm Layout Optimization", DOI:10.5281/zenodo.3694675, 2019

WindFLO can be run (provided that both the WindFLO input and turbine input files are present) using

```
./WindFLO_OSX /path/to/the/input/file /path/to/the/result/file
```

```
./WindFLO_LINUX /path/to/the/input/file /path/to/the/result/file
```

Ambient Wind Models

Accurately modeling the contributions of undisturbed ambient wind conditions is an important aspect of wind farm modeling. The ambient atmospheric wind profile is traditionally defined using the constant, log-law, power-law or Deaves-Harris model.

The log-law is a semi-empirical relationship used to describe the horizontal mean wind speed as a function of elevation within the lower region of the atmospheric boundary layer. The log-law profile [3] is given by Eq. 1 where $\kappa \approx 0.4$ is the von-Karman constant, u^* is the friction velocity and z_0 is the surface roughness.

$$U\left(z\right) = \frac{u^*}{\kappa} \ln\left(\frac{z}{z_0}\right) \tag{1}$$

Between $10m$ and $50m$, the log-law is an accurate predictor of the mean wind speed. From $50m$ to the top of the atmospheric boundary layer, the power-law was shown to be more accurate [4, 5]. The power-law [3] is defined by Eq. 2, where U_{ref} is the known velocity at a reference height z_{ref} and α is a coefficient that is dependent on the atmospheric stability. For neutrally stable atmosphere, $\alpha \approx 0.143$.

$$U\left(z\right) = U_{ref}\left(\frac{z}{z_{ref}}\right)^{\alpha} \tag{2}$$

The Deaves-Harris atmospheric boundary layer (ABL) model combines the accuracy of the log-law at lower heights with the accuracy of power-law at moderate heights continuing up to the top of the ABL [5]. It is the only model of the three (log-law, power-law and Deaves-Harris) that 'recognizes' the top of the ABL. The Deaves-Harris model is given as

$$U(z) = \frac{u^*}{\kappa}\left(\ln\frac{z}{z_0} + 5.75\left(\frac{z}{z_{ref}}\right) - 1.88\left(\frac{z}{z_{ref}}\right)^2 - 1.33\left(\frac{z}{z_{ref}}\right)^3 + 0.25\left(\frac{z}{z_{ref}}\right)^4\right) \quad (3)$$

Analytical Wake Models

One of the most important parameters in wind farm turbine modeling is the velocity deficit downstream of the turbine. The relative velocity deficit in the wake region is given as

$$\frac{\Delta U}{U_\infty} = \frac{U_\infty - U_w}{U_\infty} \quad (4)$$

where U_w is the average velocity in the wake region and U_∞ is the ambient wind speed averaged over the rotor swept area. Although the wake is a 3D, time-dependent structure, the analytical models assume steady state about the equilibrium conditions to simplify the equations. The wake velocity and wake region is dictated by the choice of the wake model. When modeling wind farms, it is possible that multiple turbines influence a signal downstream turbine. In this case, wake merging methods must be used.

Jensen's Wake Model

The Jensen model (also sometimes called the Katic model) [6] assumes that the velocity deficit ($\Delta U(x)$) is only a function of the distance x downstream of the turbine. This leads to a conical, axis-symmetric wake region with a well defined edge. Since the velocity is only a function of x, the wind speed deficit is constant inside the wake disk and zero outside of it.

The Jensen model defines the wake diameter as

$$D_w(x) = D\left(1 + 2k_w\frac{x}{D}\right)^2 \quad (5)$$

where k_w is the wake expansion coefficient defined as $k_w = \partial D_w/\partial x$. It can be seen that the wake diameter is independent of the free stream wind velocity U_∞. The typically used values for k_w are 0.075 and 0.04 for onshore and offshore wind farms, respectively [7, 8].

The velocity deficit in the Jensen model is defined as

$$\frac{\Delta U}{U_\infty} = \frac{\left(1 - \sqrt{1 - C_T}\right)}{\left(1 + 2k_w\frac{x}{D}\right)^2} \quad (6)$$

where C_T is the coefficient of thrust, which is a function of wind speed ($C_T = C_T(U)$). The Jensen model is recommended for inter-turbine spacing greater than 3D [9, 10]. Although various modifications have been implemented for the Jensen model, such as assuming a sinusoidal rather than a top-hat shaped velocity deficit [11], the formulation used here is the most commonly used in literature.

Frandsen's Wake Model

The Frandsen wake model [12] was derived by applying the mass and momentum conservation to a control volume around a turbine. Like in the Jensen model, the velocity deficit in the wake is only dependent on the distance x downstream from the turbine. The Frandsen model defines the wake diameter as

$$D_w(x) = D\left(\beta^{k/2} + \alpha\frac{x}{D}\right)^{1/k} \tag{7}$$

where k is either 3 (Schlichting solution) or 2 (square root shape solution), α is the wake expansion factor and β is a function of C_T, i.e. $\beta(C_T)$, given by

$$\beta = \frac{1+\sqrt{1-C_T}}{2\sqrt{1-C_T}} \tag{8}$$

The wake expansion factor α can be computed using

$$\alpha = \beta^{k/2}\left[(1+2\alpha_{noj}x)^k - 1\right]x^{-1} \tag{9}$$

where $\alpha_{noj} \approx 0.05$. The velocity deficit in the wake is defined as

$$\frac{\Delta U}{U_\infty} = \frac{1}{2}\left(1 - \sqrt{1 - 2\frac{A}{A_w(x)}C_T}\right) \tag{10}$$

$A_w(x) = \pi D_w^2/4$ is the area of the wake disk. It should be noted, that the velocity deficit is constant at all points on this wake disk and zero outside of it. The wake expansion factor is typically of the order of $10k_w$. The Frandsen model was originally recommended for both small and large wind farms with inter-turbine spacing of lower than 10D [13].

Larsen's Wake Model

The Larsen wake model, proposed in 1988, is based on Prandtl's turbulent boundary layer equations. The model was later modified in 2009 [14]. This model assumes that the fluid is incompressible and therefore ignores the pressure gradient. The Larsen model again assumes a circular wake disk but, unlike the Jensen and Frandsen models, defines the velocity deficit in the wake as both a function of axial distance x and radial distance r.

The Larsen model defines the wake diameter as

$$D_w(x) = \left(\frac{105c_1^2}{2\pi}\right)^{1/5}(C_T A(x+x_0))^{1/3} \tag{11}$$

where c_1 is the non-dimensional mixing length defined as

$$c_1 = \left(\frac{105}{2\pi}\right)^{-1/2}\left(\frac{\sqrt{\beta}D}{2}\right)^{5/2}(C_T A x_0)^{-5/6} \tag{12}$$

In Eq. 12, x_0 is the non-dimensional reference distance given by

$$x_0 = \frac{9.6D}{\frac{2R_{9.6D}}{\left(\sqrt{\beta}D\right)^3} - 1} \tag{13}$$

in which $R_{9.6D}$, the wake radius at 9.6 times the rotor diameter downstream of the turbine, is defined as

$$R_{9.6D} = a_1 \exp\left(a_2 C_T^2 + a_3 C_T + a_4\right)\left(b_1 I_a + 1\right) D \tag{14}$$

and $a_1 = 0.435449861$, $a_2 = 0.797853685$, $a_3 = 0.124807893$, $a_4 = 0.136821858$, $b_1 = 15.6298$. In Eq. 14, I_a is the ambient turbulence intensity. The velocity deficit in the wake is defined as

$$\frac{\Delta U}{U_\infty} = -\frac{1}{9}\left[C_T A \left(x + x_0\right)^{-2}\right]^{1/3} \times \left\{ r^{3/2}\left[3c_1^2 C_T A \left(x + x0\right)\right]^{-1/2} - \left(\frac{35}{2\pi}\right)^{3/10} \left(3c_1^2\right)^{-1/5} \right\}^2 \tag{15}$$

The Larsen model is recommended for inter-turbine spacing lower than 9.6D, which is the distance used for its original calibration [15].

Ishihara's Wake Model

The Ishihara wake model [16] was developed in 2018 using data from a scaled model of a Mitsubishi-MWT1000 wind turbine. This model accounts for both incoming wind velocity and ambient turbulence intensity I_a when computing the wake properties. The model assumes the velocity deficit to follow a Gaussian distribution. The standard deviation of the wake region using this model is defined as

$$\frac{\sigma(x)}{D} = k^* \frac{x}{D} + \epsilon \tag{16}$$

where $k^* = 0.11 C_T^{1.07} I_a^{0.20}$ is the rate of wake expansion and $\epsilon = 0.23 C_T^{-0.25} I_a^{0.17}$. The wake diameter can be taken to be $D_w(x) = 2\sigma(x)$, which includes 95% of the data of the Gaussian distribution.

The velocity deficit in the wake region is defined as

$$\frac{\Delta U}{U_\infty} = \frac{1}{\left(a + bx/D + c\left(1 + x/D\right)^{-2}\right)^2} \exp\left(-\frac{y^2 + \left(z - z_h\right)^2}{2\sigma^2}\right) \tag{17}$$

where $a = 0.93 C_T^{-0.75} I_a^{0.17}$, $b = 0.42 C_T^{0.6} I_a^{0.2}$ and $c = 0.15 C_T^{-0.25} I_a^{-0.7}$. It can be seen that the velocity deficit is a function of the axial distance x, cross-stream direction y, and elevation z.

Bastankhah and Porte-Agel's Wake Model

The wake model proposed by Bastankhah and Porte-Agel [17] (termed the BP model), models the velocity deficit in the wake as a Gaussian distribution. The mass and momentum conservation principles are then applied to derive the definition of the wake diameter and velocity deficit.

The velocity deficit in the BP model is defined as

$$\frac{\Delta U}{U_\infty} = C(x) \exp\left(-\frac{y^2 + \left(z - z_h\right)^2}{2\sigma^2}\right) \tag{18}$$

where $C(x)$ is given by

$$C(x) = 1 - \sqrt{1 - \frac{C_T}{8\left(\frac{\sigma}{D}\right)^2}} \tag{19}$$

The standard deviation of the velocity deficit σ is defined by

$$\frac{\sigma(x)}{D} = k^* \frac{x}{D} + \epsilon \tag{20}$$

where $\epsilon = 0.2\sqrt{\beta}$, $k^* = \partial\sigma/\partial x$ is the wake growth rate (which is not the same as $k_w = \partial D/\partial x$). An expression for k^* is not given but was found to vary between 0.023 and 0.055 from fitting LES results [17]. Since the velocity is modeled as a continuous Gaussian distribution, the wake diameter, in theory, is infinite and does not have a well defined edge. Since σ is the standard deviation of the velocity deficit in the wake, the wake diameter can considered to be $D_w(x) = 2\sigma(x)$ which contains 95 percent of the data given by the Gaussian distribution. It should be noted that although the BP model defines the velocity deficit in the wake as a function of spanwise coordinate y and vertical coordinate z, the deficit is effectively axis-symmetric due to the Gaussian distribution assumption.

Xie and Archer's Wake Model

The XA wake model, developed by Xie and Archer [18], models the velocity deficit, not as axis-symmetric or conical, but as an ellipsoidal, which is a more realistic approximation [19]. The velocity deficit is defined as a bivariate Gaussian distribution to account for the wake growth in the y and z direction separately.

The XA model gives the velocity deficit in the wake as

$$\frac{\Delta U}{U_\infty} = \frac{\Delta U_{hub}}{U_\infty} \exp\left(-\left\{\frac{y^2}{2\sigma_y^2} + \frac{(z - z_h)^2}{2\sigma_z^2}\right\}\right) \tag{21}$$

where the velocity deficit at hub height is given by

$$\frac{\Delta U_{hub}}{U_\infty} = 1 - \sqrt{1 - \frac{C_T}{8\frac{\sigma_y \sigma_z}{D^2}}} \tag{22}$$

Notice that if $\sigma_y = \sigma_z$ the BP model is recovered. The standard deviations of the velocity deficit in the y and z directions are given by

$$\frac{\sigma_y(x)}{D} = k_y^* \frac{x}{D} + \epsilon, \quad \frac{\sigma_z(x)}{D} = k_z^* \frac{x}{D} + \epsilon \tag{23}$$

where $\epsilon = 0.2\sqrt{\beta}$, and k_y^* and k_z^* are the wake expansion rates in the horizontal and vertical direction. As was done in the BP model, the major axis of the wake ellipse in y and z are taken as $2\sigma_y(x)$ and $2\sigma_z(x)$, respectively.

Wake Merging Schemes

Having discussed the analytical wake models, the schemes for wake merging must now be discussed. Four different types of wake merging models based on superposition are used [13]. In the superposition concept, the total velocity deficit due to n upstream turbines can be expressed using the linear sum (Eq. 24), quadratic sum (Eq. 25), energy balance (Eq. 26) or Dynamic Wake Meandering (DWM) model (Eq. 27) [20].

$$U_\infty - U_i = \sum_{j=1}^{n} \zeta \left(U_\infty - U_{w,j}\right) \tag{24}$$

$$\left(U_\infty - U_i\right)^2 = \sum_{j=1}^{n} \zeta \left(U_\infty - U_{w,j}\right)^2 \tag{25}$$

$$U_\infty^2 - U_i^2 = \sum_{j=1}^{n} \zeta \left(U_\infty^2 - U_{w,j}^2\right) \tag{26}$$

$$U_\infty - U_i = \max_{j\in[1,n]} \left(\zeta \left(U_\infty - U_{w,j}\right)\right) \tag{27}$$

where $\zeta = \dfrac{\tilde{A}\left(D_{w,j}, D_i\right)}{A_i}$.

WindFLO Input File

Input Parameters Definition

The WindFLO program read inputs from the FORTRAN namelist. The main WindFLO input file belongs to the 'WindFLO_data' namelist and, therefore, must begin with '&WindFLO_data' and end with '/'. All parameters must be specified within this region.

The parameters for the main WindFLO input file are as follows:

- **rho** $[kg/m^3]$ (double): Fluid density
- **windModel** (string): Model for the ambient wind profile. Possible options are: 'constant', 'log', 'power', 'deaves-harris'.
- **modelVelocity** $[m/s]$ (double(3)): A double array of length three containing the ambient wind model velocity (x, y & z direction). For each of the 'windModel', modelVelocity would represent
  1. 'constant': constant wind velocity
  2. 'log': the friction velocity (u^*) used in the log-law wind profile
  3. 'power': the wind velocity (U_{ref}) at reference height
  4. 'deaves-harris': the friction velocity (u^*) used in the deaves-harris model
- **referenceHeight** $[m]$ (double): Reference height (z_{ref}) used in the 'power' and 'deaves-harris' ambient wind model.
- **surfaceRoughness** $[m]$ (double): Surface roughness (z_0) used in 'log' and 'deaves-harris' ambient wind model
- **wakeModel** (string): Wake model. Possible options are: 'Jensen', 'Frandsen', 'Larsen', 'Ishihara', 'BP' (Bastankhah & Porte-Agel) and 'XA' (Xie & Archer)

- **wakeMergeModel** (string): Wake merge/superposition model. Possible options are: 'Linear', 'Quadratic', 'Energy' and 'DWM' (Dynamic Wake Meandering)
- **wakeExpansionCoeff** (double(2)): Wake expansion coefficient (k_w or k^*) used in the wake models of 'Jensen', 'Frandsen', 'BP' and 'XA'. The 'XA' wake model requires that two wake expansion coefficients be specified (one for each direction k^*_y, k^*_z), where as the remaining wake models only require one wake expansion coefficient (since the wake is symmetric).
- **turbulenceIntensity** [%] (double): Turbulence intensity (I_a) for Larsen's and Ishihara's wake model
- **gaussOrder** (integer): Number of points to use for Gauss integration. Possible options are: 1,...,5
- **monteCarloPts** (integer): Number of points to use in Monte Carlo integration
- **coe** **[$/MW]** (double): Cost of energy
- **terrainModel** (string): Method for terrain interpolation. Possible options are: 'IDW' (inverse distance weighting) and 'RBF' (radial basis function)
- **terrainFile** (string): File containing a list of x, y and z coordinates of points defining the terrain. These points are used to construct the interpolation by the 'terrainModel'.
- **powerIDW** (integer): The exponent for the inverse-distance weighting interpolation
- **rbfKernel** (integer): The RBF kernel used for the interpolation. Possible options are: 1 = Multiquadrics RBF, 2 = Inverse Multiquadrics RBF and 3 = Gaussian RBF.
- **shapeFactor** (double): The shape parameter c in RBF kernels
- **octreeMaxPts** (integer): Maximum number of points in an octree leaf cell. Octree decomposition is used for space partitioning and fast interpolation
- **octreeDepth** (integer): Maximum depth of the octree decomposition
- **turbineFiles** (string(1000)): The path to the turbine input file. Each element of the array is a separate turbine. The maximum allowable number of turbines is fixed at 1000. Note: Turbine file names are case sensitive.

Sample WindFLO Input File

```
/*  Copyright 2019, Sohail R. Reddy
    email: sredd001@fiu.edu
    www.sohailreddy.com  */

&WindFLO_data    !NOTE THE STARTING POINT OF NAMELIST
```

```
  !Fluid density
  rho = 1.2d0,

!-- Ambient Wind Model Parameters --!
  !Ambient wind model velocity (3 components)
  modelVelocity = 10.0d0,0.0d0,0.0d0,

  !Wind profile model
  !choice of constant, log, power, deaves-harris
  windModel = 'constant',

  !Reference height for power, deaves-harris model
  referenceHeight = 0.1,

  !Surface roughness for log and deaves-harris model
  surfaceRoughness = 0.1d0 ,

!-- Wake Model Parameters --!
  !Turbulent intensity for Larsen's and Ishihara models
  turbulenceIntensity = 0.083d0,

  !Wake models... Available models are Jensen,
  !Frandsen, Larsen, Ishihara, BP, XA
  wakeModel = 'Frandsen',

  !Wake merge/superposition models
  !Available models are Linear, Quadratic, Energy, DWM
  wakeMergeModel = 'Quadratic',

  !Wake expansion coefficient/s for Jensen, Frandsen, BP
  !and XA models... XA model requires two coefficients
  wakeExpansionCoeff = 0.045d0, 0.0315d0,

!-- Numerics Parameters --!
  !Cost of energy
  coe = 1230.0d0,

  !Order of gauss integration
  gaussOrder = 4,

  !Number of points for Monte Carlo integration
  monteCarloPts = 1000,

!-- Terrain Parameters --!
  !Model for terrain interpolation
  terrainModel = 'RBF',

  !File containing terrain points to interpolate
```

```
  terrainFile = 'terrain.dat',

  !Exponent in IDW interpolation
  powerIDW = 4,

  !RBF kernel...see above
  rbfKernel = 1,

  !RBF shape parameter... controls interpolation
  !smoothness
  shapeFactor = 5.0d0,

!-- Octree Parameters --!
  !Maximum points in an octree leaf cell
  octreeMaxPts = 10

  !Maximum depth of octree decomposition
  octreeDepth = 10

!-- Turbine files
!   currently there are 9 turbines in farm

  turbineFiles ='turbine1.dat', 'turbine2.dat', 'turbine3.dat',
               'turbine4.dat', 'turbine5.dat', 'turbine6.dat',
               'turbine7.dat', 'turbine8.dat', 'turbine9.dat'

/     !NOTE THE ENDING POINT OF NAMELIST
```

Turbine Input File

Input Parameters Definition

The WindFLO program read inputs from the FORTRAN namelist. The main turbine input file belongs to the 'turbine_data' namelist and, therefore, must begin with '&turbine_data' and end with '/'. All parameters must be specified within this region.

- **turbineNum** (integer): The i^{th} turbine (not needed)
- **name** (string): Name of turbine (not needed)
- **position** $[m]$ (double(3)): Location (x, y & z) of the turbine in the farm.
- **orientation** (double(3)): Angle of rotation (yaw angle) about each axis (x, y & z) to orient the turbine
- **yaw** (logical): If true, the yaw angle is 0. i.e. turbine always faces the ambient wind direction. If false, the 'orientation' is held constant.
- **height** $[m]$ (double): The hub height of the turbine
- **diameter** $[m]$ (double): Rotor diameter of the turbine

- **ratedPower** $[W]$ (double): Rated power of the turbine. If the specified 'ratedPower' is negative, the rated power is computed from the C_p curve.
- **CpCurve** (double(2,5000)): The coefficient of power curve ($C_p vs.U$). The first (CpCurve(1,:)) and second (CpCurve(2,:)) rows store the velocity and C_p respectively. If 'CpCurve' is not provided, it is computed from the supplied 'CtCurve'.
- **CtCurve** (double(2,5000)): The coefficient of thrust curve ($C_t vs.U$). The first (CtCurve(1,:)) and second (CtCurve(2,:)) rows store the velocity and C_t respectively. If 'CtCurve' is not provided, it is computed from the supplied 'CpCurve'.

Sample Turbine Input File

```
/*  Copyright 2019, Sohail R. Reddy
    email: sredd001@fiu.edu
    www.sohailreddy.com  */

&turbine_data   !NOTE THE STARTING POINT OF NAMELIST

  !-- Optional Parameters --!
  turbineNum = 1,     !Turbine number
  name = 'V90-3 MW',  !Turbine name

  !-- Turbine Location --!
  position = 0.0,0.0,0.0,  !Position of turbine
  orientation = -1.0,0.0,0.0,  !Orientation of turbine

  !-- Turbine Parameters --!
  yaw = .true.        !Determine whether a turbine can yaw
  height = 105.0,     !Hub height
  diameter = 90.0,    !Rotor diameter
  ratedPower = 3.0d6, !Rated power of turbine

  !Cp curve... velocities
  CpCurve(1,:) = 3.5d0,4.0d0, 5.0d0, 6.0d0,
                 7.0d0, 8.0d0, 9.0d0, 10.0d0,
                 11.0d0, 12.0d0, 13.0d0, 14.0d0,
                 15.0d0, 16.0d0, 17.0d0, 18.0d0

  !Cp curve... Cp
  CpCurve(2,:) = 0.22745656d0,0.30876620d0,0.390088d0,
                 0.41941114d0,0.43471139d0,0.44410204d0,
                 0.44779433d0, 0.43525609d0, 0.40606852d0,
                 0.36965805d0,0.32672428d0,0.27599684d0,
                 0.22758936d0, 0.18790378d0, 0.15670884d0,
                 0.13201484d0
/    !NOTE THE ENDING POINT OF NAMELIST
```

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