Screening and Sorting: Hiring Schemes and Endogenous Applicant Pools*

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Abstract

I propose a simple model of hiring in which the applicant pool that a firm faces is endogenously determined by its hiring policy. Each applicant's quality is private information, which necessitates the firm to perform costly screening. The firm decides the optimal hiring criteria and wage such that the costs of false positive hires and false negative rejections are balanced off, taking into account that setting high standards discourages some unqualified job seekers from making an application in the first place. The model predicts that the equilibrium wage and hiring standards will be higher when the labor market is more competitive. The comparative statics exhibits quite different patterns in the monopsonistic and competitive labor markets. The paper also provides some extensions that speak to the possibility that firms may deliberately impose applicants extra application costs.

Keywords: Hiring, Recruitment, Employment Decision

JEL Codes: M51, D82, D83

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1 Introduction

Hiring quality workers is one of the most important managerial activities for firms. Since most technologies rely on labor as one of the principal production factors, firms cannot successfully operate without hiring the right workers. What makes hiring difficult is the worker heterogeneity and asymmetric information. Workers significantly differ in various dimensions such as ability, skills, and experience. However, due to asymmetric information, firms cannot perfectly observe those differences. This feature of the labor market necessitates firms to obtain a noisy signal of the worker's quality by using costly screening technologies in order to make a right hiring decision.

Typically, a firm screens applicants before giving them a job offer in order to avoid hiring workers whose net value to the firm is negative. Namely, applicants whose noisily observed quality is below a certain threshold are rejected as unqualified for the job. This can be interpreted as truncating the downside risk of the applicant quality distribution (Lazear, 1998). By raising the bar, the firm can enjoy an increase in the average quality of those accepted but must incur a decrease in the quantity of favorable applicants hired. Conversely, lowering the bar leads to a worsening of the average quality and an increase in the number of desired applicants hired. The optimal threshold is set such that it balances off the costs of mistakenly accepting bad applicants and rejecting good ones, respectively (Sah and Stiglitz, 1984). The two types of errors are respectively referred to as *false positive hires* and *false negative rejections* throughout the paper.

Another important feature of the hiring problem is the applicant decision making in job application. Applicants with a very low likelihood of getting an offer would not bother to make an application in the first place. For example, Google, one of the most famous companies in the world, is reported to be the most attractive employer in the world, but that does not imply that every job seeker applies for a Google job.[1] This is because Google adopts a very strict hiring policy, which makes it almost impossible for job seekers to get hired except for high competent ones. Inadequate applicants anticipate that it is going to be a waste of time and effort and are discouraged from applying. Namely, not only does a strict hiring scheme screen out actual applicants but also induces potential applicants to self-select before application, which will change the characteristics of the applicant pool that the firm faces.

Hiring policies thus may have an oft-ignored effect which serves as ex ante screening through strict criteria and costly application. Especially, when applicants face time-constraints as they usually do in the case of the job market for college graduates for example, risk of getting rejected and cost of wasting time become a serious issue. This observation motivates us to hypothesize that the applicant pool that a firm face depends on its hiring policy such as screening criteria and a wage that applicants must satisfy and expect to receive, respectively. In turn, the firm is expected to design the hiring scheme taking into account that the applicants will respond to it. This perspective illustrates that the applicant pool is endogenously determined by the firm's hiring policy.

In this paper, I propose a simple model in which the applicant pool that a firm faces is endogenously determined by its hiring scheme. A hiring scheme stipulates the hiring criteria and wage. Applicants choose whether to apply for a job position at the firm or one elsewhere

[1] See Whitler (2020).

in the labor market given the firm's hiring scheme. A stricter hiring criterion can be beneficial in three ways. First, it increases the average quality of eventual hires given the applicant pool by rejecting undesirable applicants. Second, it also improves that of the applicant pool by discouraging unwanted job seekers to apply in the first place. Third, by discouraging job seekers from making an application, it reduces the total screening cost. I refer to the first one and the last two as *screening effect* and *sorting effect*, respectively. These two distinct types of effect come with a cost of mistakenly rejecting favorable applicants. Given this structure, the firm designs its optimal hiring policy in terms of the hiring standards by balancing off the costs and benefits.

Specifically, the employer sets a threshold beforehand and then observes a noisy signal of a given applicant's quality. She accepts the applicant if the signal is above the threshold and rejects otherwise. Screening is costly in terms of both physical and opportunity costs. The employer's screening technology is described by the noise variance and the screening costs, which represent the inaccuracy and inexpensiveness. The firm's production function is assumed to be productive in the sense that it can map worker quality to the monetary output. The labor market offers a job opportunity in which worker quality does not matter and which produces a mediocre output. Applicants choose whether to apply for a job at the firm or elsewhere in the market considering the firm's hiring scheme, i.e., the threshold and wage. The employer decides the hiring scheme taking into account the applicant optimal response.

The model provides a basic framework that elucidates firms' hiring strategies in the labor market. I consider two distinct cases. One is the case where the labor market is monopsonistic in the sense that a firm faces no serious competitors but only the labor market that offers a mediocre outside option to the potential applicants. In this case, an increase in the screening cost, screening inaccuracy, and outside option value induces the firm to increase the threshold. An increase in the screening cost makes it more costly to maintain a lower threshold because it attracts too many applicants. An increase in the screening inaccuracy enlarges the chance of erroneously getting hired for job seekers in the lower tail in the quality distribution and thus attracts more applicants, which renders a lower threshold less affordable. When the outside option value increases in the monopsonistic labor market, it pushes up the firm's wage offer because any wage below the outside option value would result in zero applicant. If the wage goes up, then it becomes more costly to incorrectly accepting inadequate applicants, which forces the firm to increase the threshold to offset the increment in the mistaken hires. In contrast, an improvement in the upside risk of the potential applicant pool allows the firm to lower the threshold because it becomes more costly to incorrectly rejecting favorable applicants.

The other is the case where the labor market is competitive in the sense that the firm must compete for its targeted job seekers against many other firms possessing the symmetric technologies. In the competitive labor market, the effect direction of a change in the parameters flips. This is because a parameter change affects the competitive wage as well as the threshold while the monopsonistic wage sticks on the outside option value. Specifically, an increase in the screening cost allows the firm to lower the threshold. Higher screening costs weaken the market capacity to offer a higher wage, which results in a wage decrease. Because the wage is now smaller and because it makes false positive hires less costly, a lower threshold becomes affordable. An improvement in the upside risk of the potential applicant pool induces the firm to raise the threshold because the market becomes willing to offer a better wage offer, which makes it more costly to mistakenly accepting inadequate applicants. Regarding the effect of the screening inaccuracy, its sign depends on the size of the screening cost. When the screening cost is small, more inaccurate screening leads to a higher threshold, but lower when large. This is because the size of the screening cost magnifies the magnitude of the downward effect of an inaccuracy increase on the wage. When the screening cost is large, an increase in the inaccuracy brings about a sharp wage drop, which allows the firm to lower the threshold. In contrast, when the screening cost is small, the wage drop is not sharp enough to offset the direct downward effect of the inaccuracy increase on the threshold.

The key feature of the model is the endogenous applicant pool. Utility-maximizing applicants decide whether or not to apply to the firm given its hiring scheme and their private knowledge on their own quality. It follows that the characteristics of the applicants that the firm actually screens varies with the hiring scheme. Namely, the model demonstrates that screening induces sorting. This is one of the distinctive features of this model contrasted with the existing models in the literature, most of which assume an exogenous candidate pool.

I also extend the model to explore the economic role of the application cost. The model is extended such that it is more explicit about costly applications. Application processes usually cost applicants time, effort, and occasionally even money, which is also supposed to discourage potential applicants with low likelihood of being accepted. For example, some firms require applicants to submit a set of an elaborate resumé, state of purpose, and reference letter, which costs them much time and effort. Sometimes the application is required to submitted in hard copy and by mail, which costs applicants even more. Whether it is done intentionally or not, imposing a costly application procedure is also expected to change the quality of the applicants who actually make an application. The extended model suggests the possibility that the firm may deliberately impose applicants extra application costs to reduce the total screening cost.

The remainder of this paper proceeds as follows. Section 2 briefly reviews the related literature. Section 3 develops the model. Section 4 analyzes the equilibria . Section 5 extends the model to allow application costs. Section 6 concludes.

2 Literature Review

Hiring has traditionally been studied in terms of matching (Lazear and Oyer, 2013). For example, the learning model (Jovanovic, 1979), search model (Mortensen and Pissarides, 1994), and model of asymmetric information (Spence, 1973) are canonical frameworks that have been applied to hiring problems. These models are useful in understanding the workings of the labor market, but this stream of research is rather silent on firms' screening technology. As Oyer and Schaefer (2011) point out, those existing models treat firms as black-box production functions. Namely, they do not emphasize how different firms differently approach the hiring problem.

Of course, there is some recent work that partially addresses this issue. Lazear (1998) showed that hiring risky workers can be beneficial because firms may exploit their upside potential. Frankel (2021) characterized the contract in which biased agents are delegated to evaluate candidates based on the soft and hard information criteria imposed by the principal. Lee and Waddell (2021) illustrated how the agent's personal preference for diversity can unexpectedly result in a less pro-diversity hiring decision in the two-stage hiring process and empirically confirmed the result in a laboratory experiment.

This paper presents a model that fills the gap in the literature by characterizing the firms' optimal hiring schemes. The model is most relevant to Lazear (1998)'s model of hiring, in which he showed that risky workers are more profitable to firms because they can exploit the workers' upside potential. Lazear (1998) also mentions that firms collect information on candidates before hiring so that they can truncate the ability distribution of candidates, i.e., to reject bad applicants. Despite the astute observation, his model itself does not consider how firms collect information about applicants. With this as a starting point, I develop a model which explicitly incorporates this aspect and speaks to firms' decision making in hiring policies.

Another key feature of our model is that the applicant pool is endogenized into the firm's hiring policy decision making. Since applicants choose whether or not to apply for a job considering how likely they are to get hired given the firm's hiring policy, the firm also needs to take it into account in designing its hiring scheme. This feature is similar to Lazear (1986), who demonstrated that incentive schemes not only incentivizes exsisting employees but also induces sorting of employees and applicants through attrition and new entry. In our model, hiring schemes not only screens out applicants but also induces sorting before application. This relationship is analogous to the juxtaposition of incentive and sorting effects in Lazear (1986)'s model. Highlighting two distinct effects of hiring schemes also potentially contributes to studies on the dual-HR practices.[2]

This paper also partially contributes to the literature that studies the architecture of economic systems and its performance in making right decisions (Sah and Stiglitz, 1984, 1988; Koh, 1992, 2005b,a). In this stream of research, the objects of screening are supposed to be business projects, the quality distribution of which is exogenous. Although the screening technology in our model is a degenerate case of the three types of economic system, i.e., hierarchy, polyarchy, and committee, it partly extends the existing framework by introducing the endogeneity of the candidate pool.

3 Model

In this section, I develop a model in which potential applicants respond to a firm's hiring scheme and in which the firm designs its optimal hiring scheme taking into account that they respond to it. Both parties are assumed to be risk-neutral. Namely, the firm balances off the benefits of screening and sorting and the costs of screening applicants and rejecting favorable ones. The applicants have private information about their own productivity at the firm. The employer has her existing employee noisily observe the applicant's ability by the firm's screening technology. She accepts them if the signal is above a certain threshold set by the employer before the beginning of the screening process, and rejects otherwise.

[2] Dual-purpose HR practices are HR practices that have multiple effects, often in a rather unexpected way. For example, Lazear (2000)'s Safelite study shows that incentive schemes may not only more strongly incentivize existing employees but also induce sorting (through attrition and new entry). Friebel et al. (2019)'s experiment on the employee referral program demonstrates that a certain hiring policy may have a direct effect on the quality of new employees and also an indirect effect on the existing employees' job satisfaction. See Rebitzer and Taylor (2011) for a survey.

Screening technologies are exemplified by examining resumes, imposing aptitude tests, and conducting interviews, but I abstract away from particular methods. What is common to all those examples is that better applicants are more likely to be accepted. The screening technology in the model is described as a increasing function of applicants' quality the range of which is the probability of getting hired. Given this structure, the firm chooses the optimal passing threshold and wage.

3.1 Basic Setup

Assume that there is a unit mass of potential applicants and that their ability, denoted by X, is uniformly distributed, i.e., $X \sim U(a, b)$, where $b > a$. I refer to this as a worker's ability or quality for brevity. A worker's ability is assumed to be private information that the firm cannot directly observe. For the sake of simplicity, I assume this ability represents the monetary revenue that a given applicant provides to the firm. Namely, let $v(x)$ be the firm's production technology that maps workers' ability to the firm's monetary output. Namely, we are assuming $v(x) = x$ for all x.

The employer sets a certain threshold r before the application process starts. Once the application is completed, she observes s, a noisy signal about a given applicant's ability x in the screening process. The signal is constructed such that $s = x + \varepsilon$, where $\varepsilon \sim U(-\sigma, \sigma)$, where $\sigma > 0$. For brevity, I call σ simply the noise or the size of the noise because the standard deviation of ε is $\frac{\sqrt{3}}{3}\sigma$. Note that σ intuitively corresponds to the inaccuracy of the screening technology.

After observing the signal s, the employer accepts an applicant if $s > r$ and rejects otherwise. Knowledge of the distributions of applicants' ability and noise is assumed to be public. Let $P_H(x)$ denote the probability of an applicant with an ability of x getting hired, then it is given as $P_H(x) = \Pr(s > r) = \Pr(\varepsilon > r - x)$. Namely,

$$P_H(x) = \begin{cases} 1 & \text{if } x > r + \sigma \\ \frac{x - r + \sigma}{2\sigma} & \text{if } r + \sigma \geq x \geq r - \sigma \\ 0 & \text{if } x < r - \sigma. \end{cases} \tag{1}$$

I refer to $P_H(\cdot)$ as the hiring function. The hiring function describes the firm's screening technology.

The firm also chooses the amount of wage denoted by w. I refer to the tuple, (r, w), as the firm's *hiring scheme*. Assume that the firm's hiring scheme is publicly known. The assumption can be justified by claiming that firms usually announce their hiring policies such as the information about the ideal candidate profile, screening process, and work environments. Some firms also disclose the firm-level jobs-to-applicants ratio in the past hiring seasons. Applicants may infer each firm's threshold and compensation from this kind of information publicly available. It is also assumed that the firm can commit to its prespecified hiring scheme.

3.2 Applicants

The potential applicants have two options. One is to apply for a job elsewhere in the labor market. Assume that the applicants earn $\underline{u} > 0$ for doing this job. This job can be interpreted as a standardized one such that anyone can produce the same output regardless of his ability or as simply the market wage job. I call $\underline{u}$ the outside option value. The other is to apply for a job position at the firm. Assume that the applicants can still get a job elsewhere and receive $\underline{u}$ even when they are rejected. Accordingly, an applicant's expected payoff is $P_H w + (1 - P_H)\underline{u} - c_A$, where $c_A \geq 0$ is an exogenous parameter which represents the cost of application such as time and effort. Note that the application cost is not transferred but only lost. Assume $b > \underline{u}$ to make the problem nontrivial. To keep the discussion succinct, I also assume $\underline{u} > \frac{b+a}{2}$.

A Utility-maximizing applicant chooses the option with a higher expected payoff. His maximization problem is described as follows.

$$\max\{P_H(x)w + (1 - P_H(x))\underline{u} - c_A, \underline{u}\}. \tag{2}$$

This implies that he will apply for a job position at the firm if and only if

$$P_H(x)(w - \underline{u}) \geq c_A. \tag{3}$$

If we restrict our attention to the case where $w > \underline{u}$ and $1 > \frac{c_A}{w - \underline{u}}$ hold, then the inequality boils down to

$$x \geq \frac{2\sigma c_A}{w - \underline{u}} + r - \sigma. \tag{4}$$

Equating both sides of the inequality yields the lower limit of the applicants who actually make an application. Namely, let $\underline{x}$ denote the lower limit of the actual applicants' ability, then it is given as

$$\underline{x} = \frac{2\sigma c_A}{w - \underline{u}} + r - \sigma. \tag{5}$$

Note that $\underline{x} \geq r - \sigma$ always holds, which implies that applicants with a zero probability of getting hired will never apply. The applicant pool that the firm actually screens is $U(\underline{x}, b)$, not $U(a, b)$. Namely, the actual applicant pool is endogenized into the firm's hiring policy. I refer to such applicant pools as *endogenous applicant pools.* We can immediately establish the following basic results.

Proposition 1 (Quality of the endogenous applicant pool) *An increase in the threshold, application cost, or outside option value leads to an improvement in the applicant pool in the sense of first-order stochastic dominance. An increase in the wage leads to a worsening of the applicant pool in the sense of first-order stochastic dominance. An increase in the noise leads to an improvement in the applicant pool in the sense of first-order stochastic dominance if the application cost is sufficiently greater than the difference between the wage and outside option value, and a worsening otherwise.*

Proof. Note that, in general, if $Y \sim U(a, b)$, $Z \sim U(a', b)$, and $a < a'$, then Z first-order stochastically dominates Y. Also recall that the endogenous applicant pool given the firm's hiring policy is $U(\underline{x}, b)$. Taking the derivative of $\underline{x}$ with respect to each variable or parameter, we obtain $\frac{\partial \underline{x}}{\partial r} > 0$, $\frac{\partial \underline{x}}{\partial c_A} > 0$, $\frac{\partial \underline{x}}{\partial \underline{u}} > 0$, $\frac{\partial \underline{x}}{\partial w} < 0$, $\frac{\partial \underline{x}}{\partial \sigma} > 0$ if $\frac{c_A}{w-\underline{u}} > \frac{1}{2}$, and $\frac{\partial \underline{x}}{\partial \sigma} < 0$ if $\frac{c_A}{w-\underline{u}} < \frac{1}{2}$, which completes the proof. ■

The intuition is straightforward for r, c_A, w, and $\underline{u}$. For the effect of σ, the intuition is relatively unclear. Proposition 1 states that the sign of the effect depends on the value of $\frac{c_A}{w-\underline{u}}$, the intuitive sense of which quantity is the relative unattractiveness of the firm's job. Observe that the job is more attractive when $\frac{c_A}{w-\underline{u}}$ is smaller, and less when larger. When the job is not so attractive, the noisiness simply discourages inadequate applicants. In contrast, when the job is highly attractive, the noisiness encourages inadequate applicants to make an application because the inaccuracy enlarges the chance of getting hired for the applicants in the lower tail.

Besides the quality of the applicant pool, a similar conclusion can be drawn for the quantity. Since the initial number of potential applicants is fixed to be a unit mass, larger or smaller $\underline{x}$ implies that a smaller or larger proportion of them will apply, respectively. We can assert the following.

Corollary 1 (Quantity of the endogenous applicant pool) *An increase in the threshold leads to a smaller number of applicants who actually make an application. An increase in the noise leads to a larger number of applicants who actually make an application.*

3.3 Screening Costs and Errors

The number of applications matters to the firm's profit when the total screening cost is a function of it. Let c_S denote the screening cost per applicant. I also refer to it as simply the screening cost for brevity when it is clear from the context. The screening cost may represent the physical cost of screening technology and also the opportunity cost of existing employees engaging in hiring activities. The total screening cost for the firm is then given as follows.

$$\text{Total Screening Cost} = c_S \int_{\underline{x}}^{b} \frac{1}{b-a}\, dx. \tag{6}$$

Combining Corollary 1 and equation (6) gives the following corollary.

Corollary 2 (Screening cost) *An increase in the threshold, application cost, or outside option value leads to a reduction in the firm's total screening cost. An increase in the wage leads to an increase in the firm's total screening cost. An increase in the noise leads to a reduction in the firm's total screening cost if the application cost is sufficiently greater than the difference between the wage and outside option value, and an increase otherwise.*

Corollary 2 asserts that the firm can reduce the total screening cost by designing its hiring scheme. Since this result states that an increase in c_A reduces the screening cost, it suggests that an intentionally demanding application process observed in the real world can be rationalized. We treat c_A to be an exogenous parameter throughout our main analysis, but we will extend the model such that the firm can choose the level of c_A in Section 5.

Now, we turn to the firm's production. Recall that $v(x)$ denotes the firm's production technology that maps the labor of worker with an ability of x to the output in monetary terms. For the sake of simplicity, we are assuming $v(x) = x$ for all x, then the worker's net value to the firm is $x - w$. Applicants whose ability falls in the range where $x \geq w$ are the ones whose net value to the firm is positive. In contrast, the other applicants in the range where $x < w$ are the ones whose net value to the firm is negative. I call the former *positive applicants*, and the latter *negative applicants*. We can define the number of *false positive hires* and *false negative rejections* as follows.[3]

$$\# \text{ of False Positive Hires} = \int_{\underline{x}}^{w} \frac{1}{b-a} \frac{x - r + \sigma}{2\sigma} \, dx, \tag{7}$$

$$\# \text{ of False Negative Rejections} = \int_{w}^{b} \frac{1}{b-a} \frac{r - x + \sigma}{2\sigma} \, dx, \tag{8}$$

where we restrict our attention to the case when $w > \underline{x}$. We can establish the following.

Proposition 2 (False positive hires) *An increase in the threshold, application cost, or outside option value leads to a smaller number of false positive hires. An increase in the wage leads to a larger number of false positive hires. An increase in the noise can lead to a larger or smaller number of false positive hires depending on the values of the other variables and parameters. When the wage is smaller than the lowest ability of the actual applicants, the number of false positive hires is zero.*

Proof. See Appendix A. ■

The intuition is given as follows. Raising the bar (r) reduces the chance of hiring for all job seekers by vertically lowering the curve of $P_H(x)$, which directly decreases the number of false positive hires. In addition, it makes the job application less attractive, which discourages inadequate job seekers who are less likely to be hired from applying for it in the first place, i.e., $\underline{x}$ becomes larger. An increase in the application cost (c_A) directly decreases false positive hires by pulling up the lower limit $\underline{x}$. It makes the job application less attractive and discourages unwanted job seekers. An increase in the wage (w) affects both upper and lower limits of the integration. An wage increase directly implies an increase in the upper limit. The increase in the upper limit corresponds to the increase of applicants whose ability falls below the new wage. Namely, some of those who were more productive than the wage become less productive than the wage because the wage is increased. The lower limit decreases as the wage increases because the job becomes more attractive and because more people are willing to apply for it. In sum, a wage increase widens the range of the integration, which leads to an increase in the number of false negative hires.

[3] Some researchers employ the terms, "Type-I error" and "Type-II error," using an analogy from classical statistics, but this taxonomy seems to be confusing. For example, Sah and Stiglitz (1984) refer to the errors of accepting projects which should have been rejected and of rejecting projects which should have been accepted as Type-II and Type-I errors, respectively. On the other hand, Kuhn (2018)'s usage is the opposite way. He makes a different analogy that compares the Type-I error as the error of incorrectly rejecting the null hypothesis that the applicant is favorable to the error of mistakenly hiring an applicant whose net value is negative. Hence, I decided to use the more intelligible words, "false positive" and "false negative," for clarity.

As to the noise (σ), the implication is nuanced. For the sake of tractability, assume $c_A = 0$, then it turns out to be clear that an increase in the noise leads to more false positive hires as long as the threshold and the wage are not extremely distant from each other. An increase in the noise means that the slope of $P_H(x)$ flattens, results in a larger chance of accepting those below the threshold. In addition, note that $\underline{x}$ also decreases as the noise becomes larger because we are assuming $c_A = 0$. Accordingly, an increase in the noise causes an increase in the number of false positive hires. Now assume $c_A \geq 0$ again, then σ-derivative of $\underline{x}$ is $\frac{2c_A}{w-\underline{u}} - 1$. When $\frac{2c_A}{w-\underline{u}} < 1$, an increase in the noise continues to have an adverse effect on the number of false positive hires, and the same intuition applies. In contrast, when $\frac{2c_A}{w-\underline{u}} > 1$, the negative effect is offset, and the lower limit increases as the noise becomes large. This is because an increase in the noise reduces the chance of getting hired for those who are above the threshold. If the application is very large, an increase in the noise can discourage more job seekers than it can attract.

Proposition 3 (False negative rejections) *An increase in the threshold leads to a larger number of false negative rejections. An increase in the wage leads to a smaller number of false negative rejections. An increase in the noise leads to a smaller number of false negative rejections if the threshold is above the average ability of the positive applicants, and larger if below.*

Proof. See Appendix B. ■

The intuition is as follows. Increasing the threshold (r) corresponds to a downward shift of the curve of $P_H(x)$, which clearly inflates the chance of mistakenly rejecting positive applicants. If the wage (w) increases, some of those whose ability is just above the old wage fall below the new wage, which means that some of the false negative rejections turn to true negative rejections. The effect of a noise increase (σ) depends on the threshold level. This is because an increase in the noise translates to a flatter slope of $P_H(x)$. The slope flattens at the threshold as the pivot. Namely, the chance of getting hired increases for those below the threshold while it decreases for those above. The effect of a noise increase depends on the ratio of job seekers below and above the threshold in the range where $b > x > w$. For example, if the slope flattens at $\frac{b+w}{2}$, then the number of false negative rejections does not change because the decrease below and the increase above the threshold compensate each other.

Propositions 2 and 3 summarize the job seekers' optimal response to the parameter changes and the firm's hiring scheme. The firm is supposed to design its hiring scheme considering these results. Namely, the firm attempts to balance off the costs of false positive hires and false negative rejections taking into account that its hiring scheme can affect job seekers' application decisions.

The two propositions predicts that job seekers' application decisions are highly dependent on firms' hiring policy, which suggests that regarding the candidate pool as exogenous might be a quite strong assumption, especially in the context of hiring. The job seekers' optimal response is an important feature of the hiring problem and our model, which attempts to analyze the workings of the firms' hiring activities. The two propositions may also provide some managerial implications for practitioners in charge of designing hiring policies at firms because they offer a clear perspective on what affects the quality of hiring decisions and how they do.

3.4 Employer

Now, we consider the employer's maximization problem. Her expected profit is the following.

$$\Pi = \int_{\underline{x}}^{b} \frac{x-w}{b-a}\frac{x-r+\sigma}{2\sigma}\,dx - c_S\int_{\underline{x}}^{b}\frac{1}{b-a}\,dx. \tag{9}$$

An assumption that $r+\sigma > b$ is made to exclude the existence of those who get hired for sure to keep the analysis succinct. The firm maximizes the expected profit by designing its hiring scheme (r, w). The constraint on the wage varies with the market structure.

Let $y(x, r, w)$ and $n(x, r, w)$ denote the first and second integrands in equation (9), respectively, then we have

$$\frac{\partial \Pi}{\partial z} = \underbrace{\left[-y(\underline{x}, r, w) + c_S\, n(\underline{x}, r, w)\right]\frac{\partial \underline{x}}{\partial z}}_{\text{Sorting effect}} + \underbrace{\int_{\underline{x}}^{b}\frac{\partial}{\partial z} y(x, r, , w)\,dx}_{\text{Screening effect}}, \tag{10}$$

where $z \in \{r, w\}$. Equation (10) illustrates that the effect of an increase in one of the hiring scheme variables derives from two distinct channels. One is the effect from the change in the quality and quantity of the applicant pool, i.e., the *sorting effect*. The other is the effect from the change in the screening strictness given the applicant pool, i.e., the *screening effect*. Also observe that the sorting effect can be further decomposed into two effects. One is the effect from discouraging negative applicants from application, and the other comes from the reduced cost.

4 Equilibrium

We consider two distinct cases. One is the case where the labor market is monopsonistic in the sense that the firm confronts no serious competitors but only the labor market offering workers a wage of $\underline{u}$. The other is the case where the labor market is competitive in the sense the firm must compete for the targeted workers against other firms which also demand the same type of workers.

For tractability, we make an additional assumption that the application cost is infinitesimal, i.e., $c_A \to 0$. This assumption will be relaxed in Section 5. Under this assumption, equation (5) turns to the following.

$$\underline{x} = r - \sigma. \tag{11}$$

Namely, we are assuming that those with a zero probability of getting hired will not bother to apply. The profit function then collapses to

$$\Pi = \frac{(b-r+\sigma)^2(2b+r-\sigma-3w)}{12\sigma(b-a)} - \frac{c_S(b-r+\sigma)}{b-a}. \tag{12}$$

4.1 The Monopsonistic Labor Market

In the monopsonistic labor market, the firm only faces a constraint $w \geq \underline{u}$. If the wage is less than the outside option value, every applicant will avoid applying and seek a job elsewhere in the labor market. Taking the derivative of the profit function with respect to w, we get

$$\frac{\partial \Pi}{\partial w} = -\frac{(b-r+\sigma)^2}{4\sigma(b-a)} < 0. \tag{13}$$

The profit is monotonically decreasing in w, so the optimal wage is the minimal one that satisfies the constraint, which implies the following.

$$w^M = \underline{u}, \tag{14}$$

where superscript M is used to denote the optimal value under monopsony.

As to the threshold, solving the first and second order conditions yields the following result.

$$r^M = \underline{u} + \sigma - \sqrt{(b-\underline{u})^2 - 4\sigma c_S}. \tag{15}$$

When $\frac{(b-w)^2}{4\sigma} = c_S$, the profit becomes strictly negative. Intuitively, screening is too costly to make a profit. The firm will already have exited the market at this point, so we need not consider the case when $\frac{(b-w)^2}{4\sigma} < c_S$.

Both two endogenous variables are expressed in terms of exogenous parameters. To sum up, we have demonstrated that

$$\begin{cases} w^M = \underline{u} \\ \\ r^M = \underline{u} + \sigma - \sqrt{(b-\underline{u})^2 - 4\sigma c_S}. \end{cases} \tag{16}$$

By differentiating r^M with respect to each parameter, we obtain the following result.

$$\frac{\partial r^M}{\partial \underline{u}} = 1 + \frac{b-\underline{u}}{\sqrt{(b-\underline{u})^2 - 4\sigma c_S}} > 0, \tag{17}$$

$$\frac{\partial r^M}{\partial \sigma} = 1 + \frac{2c_S}{\sqrt{(b-\underline{u})^2 - 4\sigma c_S}} > 0, \tag{18}$$

$$\frac{\partial r^M}{\partial c_S} = \frac{2\sigma}{\sqrt{(b-\underline{u})^2 - 4\sigma c_S}} > 0, \tag{19}$$

and

$$\frac{\partial r^M}{\partial b} = -\frac{b-\underline{u}}{\sqrt{(b-\underline{u})^2 - 4\sigma c_S}} < 0. \tag{20}$$

The intuition is given as follows. An increase in the outside option value ($\underline{u}$) leads to an increase in the cost of false positive hires in terms of both extensive and intensive margins. In order to compensate the increased false positive cost, the firm wishes to increase the threshold so that inadequate applicants may be discouraged before application and screened out after application. An increase in the noise (σ) leads to a worsening of the applicant pool as can be seen from $\underline{x} = r - \sigma$. This is because the slope of $P_H(x)$ becomes flatter as the noise grows, which results in an increased chance of getting hired for those in the lower range. To push back the decreased lower limit of the pool, the firm increases the threshold. As the screening cost per applicant (c_S) increases, it becomes more costly to screen many applicants. The firm increases the threshold to discourage applicants from making an application to reduce the screening cost. An increase in the pool upper limit (b) makes it more costly to make false negative rejections, which induces the firm to lower the threshold to mitigate the risk of false negative errors.

4.2 The Competitive Labor Market

Now, we turn to the case when the labor market is more competitive. Specifically, we impose the zero-profit condition after solving the firm's maximization problem. Solving the first and second order conditions for the optimal threshold, we get

$$r^C = w + \sigma - \sqrt{(b-w)^2 - 4\sigma c_S}, \tag{21}$$

where superscript C is used to denote the optimal value under competition. Solving the equation $\Pi(r^C, w) = 0$ for w, we obtain the following result.

$$w^C = b - \frac{4\sqrt{3\sigma c_S}}{3}. \tag{22}$$

Substituting w^C into equation (21), we get

$$r^C = b + \sigma - 2\sqrt{3\sigma c_S}. \tag{23}$$

Both two endogenous variables are expressed in terms of exogenous parameters. In sum, we have shown that

$$\begin{cases} w^C = b - \frac{4\sqrt{3\sigma c_S}}{3} \\ \\ r^C = b + \sigma - 2\sqrt{3\sigma c_S}. \end{cases} \tag{24}$$

First, taking the derivative of w^C with respect to each parameter, we obtain the following result.

$$\frac{\partial w^C}{\partial c_S} = -\frac{2\sqrt{\sigma}}{\sqrt{3c_S}} < 0, \tag{25}$$

$$\frac{\partial w^C}{\partial \sigma} = -\frac{2\sqrt{c_S}}{\sqrt{3\sigma}} < 0, \tag{26}$$

and

$$\frac{\partial w^C}{\partial b} = 1 > 0. \tag{27}$$

The intuition is straightforward. An increase in the screening cost per applicant (c_S) decreases the market's capacity to offer a higher wage, which leads a decline in the competitive wage. An increase in the noise size (σ) pushes down the lower limit of actual applicants, which worsens the quality of the applicant pool. The worsening of the pool, in turn, demotivates firms to offer a higher wage. The opposite is true of the upper limit parameter (b).

Next, differentiating r^C with respect to each parameter yields the following result.

$$\frac{\partial r^C}{\partial c_S} = -\frac{\sqrt{3\sigma}}{\sqrt{c_S}} < 0, \tag{28}$$

$$\frac{\partial r^C}{\partial \sigma} = 1 - \frac{\sqrt{3c_S}}{\sqrt{\sigma}} \lesseqgtr 0 \quad \left(\text{if } \frac{\sigma}{3} \lesseqgtr c_S\right), \tag{29}$$

and

$$\frac{\partial r^C}{\partial b} = 1 > 0. \tag{30}$$

The intuition is as follows. An increase in the screening cost per applicant (c_S) induces firms to raise the threshold. In contrast with the monopsonistic case, where $\frac{\partial r^M}{\partial c_S} > 0$, the increased individual screening cost arouses a decline in the competitive wage. This decline makes it less costly to produce false positive hires, which allows firms to afford a more generous threshold. The effect of an increase in the noise (σ) can go in both directions depending on its relationship with the screening cost per applicant in terms of magnitude. When the screening is sufficiently costly, an increase in the noise significantly drops the competitive wage as can be seen from equation (26), which means that the marginal effect of the noise on the competitive wage is increasing in c_S. This sharp drop in the wage makes a lower threshold more affordable. The opposite explanation applies when the screening cost is relatively small. Finally, an increase in the upper limit (b) increases the competitive wage, which in turn induces firms to increase the threshold in order to reduce the cost of false positive hires.

4.3 Comparison between Monopsony and Competition

We have seen that the introduction of competition flips the direction of the effects of parameter changes on the optimal threshold. If the employer holds the wage-setting power, she adapts to changes in the parameters by adjusting her hiring scheme only and maintains the current wage level. In contrast, if the labor market is competitive, the wage does change as parameters change, and the employer cannot control it because she is only a price-taker. Now, she needs to adapt to parameter changes and also the change in the wage caused by them. This result implies that we may expect to observe different patterns in the consequences of a certain parameter change depending on the degree of competition of the labor market.

Proposition 4 (Effects of a change in the parameters) *An increase in the screening cost or the upper limit of applicant pool leads to a lower threshold in the competitive labor market while it leads to a higher threshold in the monopsonistic labor market. If the screening cost is large, an increase in the noise leads to a lower threshold in the competitive labor market while it leads to a higher threshold in the monopsonistic labor market. If the screening cost is small, an increase in the noise leads to a higher threshold under both competition and monopsony.*

Proof. See the discussion in Subsections 4.1 and 4.2. ■

In the competitive labor market, an increase in the screening cost or the noise size leads to an increase in the total screening cost and also a wage decrease. The firm's adjustment depends on whether or not the increase in the cost exceeds the benefit from the wage decline. Namely, the effect of a change in the screening cost or the noise have two distinct channels in the competitive labor market. One is the direct upward effect on the threshold. The other is the indirect downward effect mediated by the competitive wage. These two effects compensate each other. For the screening cost, the indirect effect dominates the direct effect. As to the size, which effect dominates depends on the magnitude of the screening cost per applicant.

By directly comparing the two markets, we can also establish the following result.

Proposition 5 (Wage and threshold) *As long as the firm can operate under monopsony, (i) the competitive wage is higher than the monopsonistic wage, and (ii) the competitive threshold is higher than the monopsonistic threshold.*

Proof. First, note that $\Pi(r^M, w^M) \geq 0 \Leftrightarrow \frac{3(b-\underline{u})^2}{16\sigma} \geq c_S$, and the last inequality is equivalent to $b - \underline{u} \geq \frac{4\sqrt{3\sigma c_S}}{3}$, which implies $w^C \geq w^M$. Likewise, $\frac{3(b-\underline{u})^2}{16\sigma} \geq c_S \Rightarrow r^C > r^M$. ■

Intuitively, once the employer loses its price-setting power in the labor market, she will be forced to adjust her hiring strategy to offset the labor cost increase. The competition forces firms to increase the wage, and the increased wage makes it more costly to produce false positive hires. As a result, firms become unable to maintain a lower threshold, which would decrease false negative rejections.

5 Extensions

We have so far assumed the zero application cost for the sake of tractability. In this section, we introduce the non-negative application cost as an additional element of the firm's hiring scheme. This explicit introduction of the application cost entails some technical difficulties. The aim of this section is thus to present some basic properties of the extended model rather than to fully analyze the equilibrium.

5.1 The Monopsonistic Labor Market

First, we consider the case when the labor market is monopsonistic. We focus on the relationship between the wage and the other two optimal responses to get an insight into how they work rather than the optimal wage solution, which requires an analytically challenging exercise to obtain. By solving the first and second order conditions, we obtain the following results. (For derivation, see Appendix C.)

$$\begin{cases} r^* = b + \sigma - \frac{2\sigma c_S}{b-w} \\ \\ c_A^* = \frac{c_S(w-\underline{u})}{b-w}. \end{cases} \tag{31}$$

Although c_A^* is not fully expressed in terms of exogenous parameters, this result suggests that a positive application cost can be chosen by the firm. It also connotes that the application cost and the wage have a positive association in the range where $b > w$ and that the application cost may be increased when the screening is more costly.

5.2 The Competitive Labor Market

Next, we consider the case where the zero-profit condition is imposed. We use an alternative approach to impose the zero-profit condition. Specifically, we first maximize the sum of hired workers' welfare and the firm's profit and then impose the zero-profit condition. Namely, letting W denote the hired workers' welfare, we solve the following equation for w.

$$\Pi(r^*, c_A^*, w) = 0, \tag{32}$$

where

$$\Pi(r^*, c_A^*, w) + W(r^*, c_A^*, w) = \max_{r, c_A} \Pi(r, c_A, w) + W(r, c_A, w). \tag{33}$$

Solving the maximization step, we get

$$c_A^* = -(w - \underline{u}) \left(\frac{\underline{u} - c_S}{b - w} - \frac{w - \underline{u}}{\sigma} \right). \tag{34}$$

This solution is valid only when $\underline{u} - \frac{(b+u-2w)(b-w)}{2\sigma} \geq c_S$ and will be an extraneous solution otherwise. There is another solution obtained if $c_S \geq \underline{u} + \frac{(b-w)(b-u)}{2\sigma}$ is assumed, but this results in zero hires, which makes the firm's profit and the workers' welfare each vanish at any w. The intuition is that the screening cost is prohibitively expensive. We focus on the former case. The application cost must be non-negative, so $c_S \geq \underline{u} - \frac{(b-w)(w-\underline{u})}{\sigma}$ must hold for c_A^* to be valid.

If the inequalities hold, i.e., the screening cost per applicant is moderately high, then we can obtain the following result.

$$\begin{cases} w^* = b - \sqrt{6\sigma c_S} \\ \\ r^* = \underline{u} + \sigma - (b - \underline{u}) - \sqrt{\frac{2\sigma}{3c_S}}(\underline{u} - c_S) \\ \\ c_A^* = \left(\frac{b-\underline{u}-\sqrt{6\sigma c_S}}{\sigma} + \frac{\underline{u}-c_S}{\sqrt{6\sigma c_S}} \right) \left(b - \underline{u} - \sqrt{6\sigma c_S} \right). \end{cases} \tag{35}$$

This result shows that it is possible that the firm imposes a non-zero application cost under a certain parameter setting.

Proposition 6 (Non-zero application) *If the screening cost per applicant is moderately high, it is possible that the firm deliberately imposes a non-zero application cost.*

Proof. See the above discussion. ■

The main message of this section is to show the possibility that the non-zero endogenous application cost may arise in the labor market. In order to reduce the total screening cost, firms may impose applicants extra application costs to induce positive sorting to application. Namely, the application cost might play a role as a costly signal. Due to technical difficulties, we have only presented some conjectures on and demonstrated the possibility of the non-zero endogenous application cost. A more elaborate analysis is left for future work.

6 Conclusion

We have developed a simple model of hiring and analyzed the workings of firms' hiring activities in the labor market. The model predicts that firms' hiring criteria will be set higher when the labor market is more competitive. A theoretical explanation for this result is that the competition forces firms to increase the wage and thus makes false positive hires more costly, which in turn induces them to raise the bar in order to offset the magnified cost of false positive hires. The comparative statics exhibits distinct patterns in the monopsonistic and competitive labor markets. This is because a change in the parameters affect not only the hiring standards but also the wage when the labor market is competitive while the wage sticks on the reservation utility in the monopsonistic market. These results imply that competition have a significant impact on firms' hiring policy decisions. We have also demonstrated the possibility that firms' intentionally impose applicants some extra application costs in order to induce sorting in the extend model.

Our analysis is subject to three major limitations. First, the key assumption driving our results is that job seekers can perfectly observe the firm's hiring scheme, which arouses self-selection into application and walkout. In reality, however, firms seldom disclose their hiring criteria explicitly either before or after the screening process. One way to justify this caveat is to claim that workers can at least infer the firm's hiring scheme from some available information or history, but this issue should be addressed by explicitly incorporating into the model the belief-updating behavior of the applicant side. Next, we assumed that the employer can commit to her prespecified hiring scheme even after the application process. A valid concern would be that the firm may be tempted to lower the threshold after the application process is over because now that the applicants are expected to be positively sorted. Namely, the optimal threshold ex ante is too strict ex post, and the firm should be tempted to modify the criteria. This issue may complicates the analysis especially together with the first limitation. Last but not least, our analysis on the application cost remains preliminary due to technical difficulties. Addressing these limitations is a potential direction for future research.

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Appendices

A Appendix A. Proof of Proposition 2

Recall that $\underline{x} = \frac{2\sigma c_A}{w-\underline{u}} + r - \sigma$. First, taking the derivative of the number of false negative positive hires with respect r, we get

$$\begin{aligned}\frac{\partial}{\partial r}\int_{\underline{x}}^{w}\frac{1}{b-a}\frac{x-r+\sigma}{2\sigma}\,dx &= -\frac{1}{b-a}\frac{\underline{x}-r+\sigma}{2\sigma}\frac{\partial \underline{x}}{\partial r} + \int_{\underline{x}}^{w}\frac{\partial}{\partial r}\frac{1}{b-a}\frac{x-r+\sigma}{2\sigma}\,dx \\ &= -\frac{1}{b-a}\frac{\underline{x}-r+\sigma}{2\sigma} - \frac{w-\underline{x}}{2\sigma(b-a)} \\ &= -\frac{c_A}{(b-a)(w-\underline{u})} + \frac{\underline{x}-w}{2\sigma(b-a)}.\end{aligned}$$

Since $c_A \geq 0$, $b > a$, and $w > \underline{u}$, the first term is negative. Since we are assuming $w > \underline{x}$, the second term is also negative. Hence, $\frac{\partial}{\partial r}\int_{\underline{x}}^{w}\frac{1}{b-a}\frac{x-r+\sigma}{2\sigma}\,dx < 0$. It is notable that the first term is the effect that comes from the change in the applicant pool while the second term is the one from the change in the actual screening. The former and the latter each correspond to sorting and screening effects.

Next, taking the derivative of the number of false negative positive hires with respect c_A, we get

$$\begin{aligned}\frac{\partial}{\partial c_A}\int_{\underline{x}}^{w}\frac{1}{b-a}\frac{x-r+\sigma}{2\sigma}\,dx &= -\frac{1}{b-a}\frac{\underline{x}-r+\sigma}{2\sigma}\frac{\partial \underline{x}}{\partial c_A} + \int_{\underline{x}}^{w}\frac{\partial}{\partial c_A}\frac{1}{b-a}\frac{x-r+\sigma}{2\sigma}\,dx \\ &= -\frac{1}{b-a}\frac{2\sigma c_A}{(w-\underline{u})^2} + 0.\end{aligned}$$

Clearly, this is negative. Notably, c_A affects the number of false positive hires only through sorting.

Similarly, for w, we have

$$\frac{\partial}{\partial w}\int_{\underline{x}}^{w}\frac{1}{b-a}\frac{x-r+\sigma}{2\sigma}\,dx = \frac{4c_A^2\sigma^2 + (r-w)(u-w)^3 - \sigma(u-w)^3}{2\sigma(a-b)(u-w)^3}.$$

It is easily shown that the w-derivative is positive as long as $r \leq w + \sigma + \frac{4c_A^2\sigma^2}{(w-\underline{u})^3}$, which our initial assumption assures to be true. Thus, an increase in w widens the range of false positive hires.

Also similarly, for $\underline{u}$, we have

$$\frac{\partial}{\partial \underline{u}}\int_{\underline{x}}^{w}\frac{1}{b-a}\frac{x-r+\sigma}{2\sigma}\,dx = -\frac{2\sigma c_A^2}{(b-a)(w-\underline{u})^3},$$

which is negative.

In addition, taking the derivative of the number of false negative positive hires with respect

σ, we get

$$\frac{\partial}{\partial \sigma}\int_{\underline{x}}^{w}\frac{1}{b-a}\frac{x-r+\sigma}{2\sigma}\,dx=\frac{1-\frac{4c_A^2}{(u-w)^2}-\frac{(r-w)^2}{\sigma^2}}{4(b-a)}.$$

The derivative is positive if $1>\frac{4c_A^2}{(u-w)^2}+\frac{(r-w)^2}{\sigma^2}$ and is negative otherwise. To make it tractable, assume $c_A=0$, then the inequality reduces to $\sigma^2>(r-w)^2$. Intuitively, when σ increases, the slope of $P_H(x)$ flattens, which results in a larger chance of accepting for job seekers below the threshold. In addition, a flatter curve means that the x-intercept moves to the left, i.e., $\underline{x}$ becomes smaller. The relationship between the threshold and the wage determines how these two forces affect the number of false positive hires, but as long as the threshold is set near the wage, an increase in the noise increases the number of false positive hires. When $c_A\neq 0$, then the attractiveness of the job also magnifies this effect.

Finally, when $\underline{x}\geq w$, there cannot be any false positive hires because no one with an ability lower than w makes an application in the first place. The proof is complete. ■

B Appendix B. Proof of Proposition 3

For the threshold, taking the derivative, we have

$$\frac{\partial}{\partial r}\int_{w}^{b}\frac{1}{b-a}\frac{r-x+\sigma}{2\sigma}\,dx=\frac{b-w}{2\sigma(b-a)}.$$

This is positive, so it has been shown that an increase in the threshold leads to a larger number of false negative rejections.

For the wage, taking the derivative, we have

$$\frac{\partial}{\partial w}\int_{w}^{b}\frac{1}{b-a}\frac{r-x+\sigma}{2\sigma}\,dx=-\frac{r-w+\sigma}{2\sigma(b-a)}.$$

Since we are assuming that $r+\sigma>b$, as long as $b>w$, the derivative is negative. Hence, an increase in the wage leads to a smaller number of false negative rejections.

For the threshold, taking the derivative, we have

$$\frac{\partial}{\partial \sigma}\int_{w}^{b}\frac{1}{b-a}\frac{r-x+\sigma}{2\sigma}\,dx=\frac{(b-w)(b-2r+w)}{4\sigma^2(b-a)}.$$

This is positive if $b-2r+w>0\Leftrightarrow\frac{b+w}{2}>r$. Hence, an increase in the noise leads to a smaller number of false negative rejections if the threshold is above the average ability of the positive applicants, and larger if below.

C Appendix C. Derivation of Equations (31)

We consider the case when $r+\sigma>b$.

FOC of r^*:

$$\frac{\partial \Pi}{\partial r} = \frac{-\frac{2\sigma^2 c_A^2}{(w-\underline{u})^2} + \frac{1}{6}(b-r+\sigma)^2 - \frac{1}{3}(b-r+\sigma)(2b+r-\sigma-3w)}{2\sigma(b-a)} + \frac{c_S}{b-a}$$
$$= 0.$$
$$\Rightarrow r^* = \sigma + w \pm \sigma\sqrt{\frac{4c_A^2}{(w-\underline{u})^2} + \frac{(b-w)^2 - 4\sigma c_S}{\sigma^2}}$$

SOC of r^*:

$$\frac{\partial^2 \Pi}{\partial r^2} = \frac{r-\sigma-w}{2\sigma(b-a)} < 0.$$

When $r = \sigma + w + \sigma\sqrt{\frac{4c_A^2}{(w-\underline{u})^2} + \frac{(b-w)^2-4\sigma c_S}{\sigma^2}}$,

$$\frac{\partial^2 \Pi}{\partial r^2} = \frac{\sqrt{\frac{4c_A^2}{(w-\underline{u})^2} + \frac{(b-w)^2-4\sigma c_S}{\sigma^2}}}{2(b-a)} > 0. \quad (\because b > a.)$$

When $r = \sigma + w - \sigma\sqrt{\frac{4c_A^2}{(w-\underline{u})^2} + \frac{(b-w)^2-4\sigma c_S}{\sigma^2}}$,

$$\frac{\partial^2 \Pi}{\partial r^2} = -\frac{\sqrt{\frac{4c_A^2}{(w-\underline{u})^2} + \frac{(b-w)^2-4\sigma c_S}{\sigma^2}}}{2(b-a)} < 0. \quad (\because b > a.)$$

Therefore,

$$r^* = \sigma + w - \sigma\sqrt{\frac{4c_A^2}{(w-\underline{u})^2} + \frac{(b-w)^2 - 4\sigma c_S}{\sigma^2}}. \tag{36}$$

FOC of c_A^*:

$$\frac{\partial \Pi}{\partial c_A} = \frac{-\frac{4\sigma^2 c_A(r-\sigma-w)}{(w-\underline{u})^2} - \frac{8\sigma^3 c_A^2}{(w-\underline{u})^3}}{2\sigma(b-a)} + \frac{2\sigma c_S}{(b-a)(w-\underline{u})}$$
$$= 0.$$
$$\Rightarrow c_A^* = \frac{(w-\underline{u})\left(\mp w\sqrt{\frac{8\sigma c_S + (-r+\sigma+w)^2}{(w-\underline{u})^2}} \pm \underline{u}\sqrt{\frac{8\sigma c_S + (-r+\sigma+w)^2}{(w-\underline{u})^2}} - r + \sigma + w\right)}{4\sigma}$$

SOC of c_A^*:

$$\frac{\partial^2 \Pi}{\partial c_A^2} = -\frac{2\sigma\left((w-\underline{u})(r-\sigma-w)+4\sigma c_A\right)}{(b-a)(w-\underline{u})^3} < 0.$$

When $c_A = \frac{(w-\underline{u})\left(-w\sqrt{\frac{8\sigma c_S+(-r+\sigma+w)^2}{(w-\underline{u})^2}}+\underline{u}\sqrt{\frac{8\sigma c_S+(-r+\sigma+w)^2}{(w-\underline{u})^2}}-r+\sigma+w\right)}{4\sigma}$,

$$\frac{\partial^2 \Pi}{\partial c_A^2} = \frac{2\sigma\sqrt{\frac{8\sigma c_S+(-r+\sigma+w)^2}{(w-\underline{u})^2}}}{(b-a)(w-\underline{u})} > 0. \quad (\because b > a.)$$

When $c_A = \frac{(w-\underline{u})\left(w\sqrt{\frac{8\sigma c_S+(-r+\sigma+w)^2}{(w-\underline{u})^2}}-\underline{u}\sqrt{\frac{8\sigma c_S+(-r+\sigma+w)^2}{(w-\underline{u})^2}}-r+\sigma+w\right)}{4\sigma}$,

$$\frac{\partial^2 \Pi}{\partial c_A^2} = -\frac{2\sigma\sqrt{\frac{8\sigma c_S+(-r+\sigma+w)^2}{(w-\underline{u})^2}}}{(b-a)(w-\underline{u})} < 0. \quad (\because b > a.)$$

Therefore,

$$\begin{aligned} c_A^* &= \frac{(w-\underline{u})\left(w\sqrt{\frac{8\sigma c_S+(-r+\sigma+w)^2}{(w-\underline{u})^2}}-\underline{u}\sqrt{\frac{8\sigma c_S+(-r+\sigma+w)^2}{(w-\underline{u})^2}}-r+\sigma+w\right)}{4\sigma} \\ &= \frac{(w-\underline{u})\left(\sqrt{8\sigma c_S+(-r+\sigma+w)^2}-r+\sigma+w\right)}{4\sigma} \end{aligned} \tag{37}$$

Now we have the following:

$$\begin{cases} r^* = \sigma + w - \sigma\sqrt{\frac{4c_A^2}{(w-\underline{u})^2}+\frac{(b-w)^2-4\sigma c_S}{\sigma^2}} \\ c_A^* = \frac{(w-\underline{u})\left(\sqrt{8\sigma c_S+(-r+\sigma+w)^2}-r+\sigma+w\right)}{4\sigma} \end{cases}$$

Substituting r^* into c_A^* yields

$$c_A^* = \frac{c_S\left(\pm w \mp \underline{u}\right)}{b-w}.$$

Negative application cost is not possible and $w > \underline{u}$, so

$$c_A^* = \frac{c_S\left(w-\underline{u}\right)}{b-w}. \tag{38}$$

This solution is only valid under the assumption that $\sqrt{2\sigma c_S} > b - w$. Otherwise, it becomes an extraneous solution. Substituting this into r^*, we get

$$r^* = \sigma + w - \frac{\sqrt{\Big((b-w)^2 - 2\sigma c_S\Big)^2}}{b-w}. \tag{39}$$