9. gyakorlat

$$\lim_{(x,y)\to 0} \frac{3xy^3}{4x^2+5y^2} = \lim_{r\to 0} \frac{3r\cos\varphi \cdot r^3\sin^3\varphi}{4r^2\cos^2\varphi + 5r^2\sin^2\varphi} =$$

$$= \lim_{r\to 0} \overset{\to 0}{3r^2} \, \overset{\leq \frac{1}{4}}{\frac{\cos\varphi \cdot \sin^3}{4\cos^2\varphi + 5\sin^2\varphi}} \Rightarrow \text{korlátos}$$

$$\lim_{(x,y)\to 0} \frac{4x^2+7y^2}{3x^2+5y^2}$$

$$\left.\begin{array}{l} \displaystyle\lim_{x\to 0} \frac{4x^2}{3x^2} = \frac{4}{3} \\ \qquad\qquad\qquad \neq \\ \displaystyle\lim_{y\to 0} \frac{7y^2}{5y^2} = \frac{7}{5} \end{array}\right\} \Rightarrow \text{nincs határérték az origóban}$$

V

$$\lim_{\substack{y=mx \\ x\to 0}} \frac{4\cancel{x^2} + 7m^2\cancel{x^2}}{3\cancel{x^2} + 5m^2\cancel{x^2}} = \lim \frac{4+7m^2}{3+5m^2} \Rightarrow \text{nem létezik határérték}$$

$$f'_x(x_0,y_0) = D_1 f(x_0,y_0) = \lim_{x\to x_0} \frac{f(x,y_0) - f(x_0,y_0)}{x-x_0}$$

$$f(x,y) = \frac{y^3 e^{x^3+y}}{4y^3+2} + \ln(y^6+4) - (3x-1)^4$$

$$f'_x(x,y) = \frac{y^3 e^{x^3+y} \cdot 3x^2}{4y^3+2} + 0 - 12(3x-1)^3$$

$$f'_y(x,y) =$$

$$f(x,y) = \sqrt{3x^2+7(y+1)^4}$$

$$f'_x(x,y) = \begin{cases} \dfrac{1}{2\sqrt{3x^2+7(y+1)^4}} \cdot 6x, & \text{ha } (x,y) \neq (0,-1) \\[2ex] \lim\limits_{x \to 0} \dfrac{\sqrt{3x^2} - 0}{x - 0} = \pm\sqrt{3}, & \text{ha } (x,y) = (0,-1) \end{cases}$$

$\hookrightarrow$ nem létezik

$\downarrow$ nincsen x szerinti parciális derivált

$$f'_y(x,y) = \begin{cases} \dfrac{1}{2\sqrt{3x^2+7(y+1)^4}} \cdot 28(y+1)^3, & \text{ha } (x;y) \neq (0;-1) \\ \lim\limits_{y\to -1} \dfrac{\sqrt{7(y+1)^4} - 0}{y+1} = \sqrt{7}(y+1) = 0, & \text{ha } (x;y) = (0;-1) \end{cases}$$

Totális derivált

$$f: D \subset \mathbb{R}^n \to \mathbb{R}^m$$

$$f'(a) = A \in \mathbb{R}^{n \times m} \iff \lim \frac{|f(x) - f(a) - A(x-a)|}{|x-a|} = 0$$

$$f(x,y) = \begin{cases} \dfrac{x^2(y+3)}{x^2+y^2} - 5x + 4y, & \text{ha } (x;y) \neq (0;0) \\ 0, & \text{ha } (x;y) = 0 \end{cases}$$

$$\underset{(x;y)\neq(0;0)}{f'_x(x;y)} = \frac{2x(y+3)(x^2+y^2) - x^2(y+3)2x}{(x^2+y^2)^2} - 5$$

$$f'_x(0;0) = \lim_{x\to 0} \frac{\frac{3\cancel{x^2}}{\cancel{x^2}} - 5x}{x} = \pm\infty \Rightarrow \text{nem létezik}$$

$$f(x;y) = 4(x-3y)^3 - 6x + 4y^2$$

$$f'_x(x;y) = 12(x-3y)^2 - 6$$

$$f'_y(x;y) = -36(x-3y)^2 + 8y$$