## 12. hét anal gyak

**(4)**

$$\int \frac{x+1}{x^2+3x}\,dx = \int \frac{\frac{1}{3}}{x} + \frac{\frac{2}{3}}{x+3}\,dx = \frac{1}{3}\ln|x| + \frac{2}{3}\ln|x+3| + C$$

$$\frac{x+1}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3}$$

$$x+1 = A(x+3) + Bx$$

$$x+1 = x(A+B) + 3A \implies \begin{array}{l} A+B=1 \\ 3A=1 \end{array} \implies A=\frac{1}{3}$$

$$B\frac{2}{3}$$

**(2)**

$$\int \frac{2x+1}{x^2-5x+6}\,dx = \int \frac{5}{2-x} + \frac{7}{x-3}\,dx = -5\ln|x-2| + 7\ln|x-3| + C$$

$$\frac{2x+1}{x^2-5x+6} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$$

$$2x+1 = (x-3)A + (x-2)B$$

$$\begin{cases} A+B = 2 \longrightarrow A = 2-B \\ -3A-2B = 1 \end{cases}$$

$$-6+3B-2B=1$$
$$B = 7$$
$$A = -5$$

**(3)**

$$\int \frac{1}{x^3+2x^2}\,dx = \int \frac{\frac{1}{2}}{x^2} + \frac{\frac{1}{4}}{x+2} + \frac{-\frac{1}{4}}{x}\,dx = \frac{1}{2}\cdot -x^{-1} + \frac{1}{4}\ln|x+2| - \frac{1}{4}\ln|x|$$

$$+C$$

$$\frac{1}{x^2(x+2)} = \frac{A}{x^2} + \frac{B}{x+2} + \frac{C}{x}$$

$$1 = \qquad\qquad \implies \begin{array}{l} B = \frac{1}{4} \\ C = -\frac{1}{4} \end{array}$$

$$A = \frac{11}{2}$$

$$\int \frac{x+1}{(x-1)^2(x-3)}\, dx = \int \frac{-1}{(x-1)^2} + \frac{-1}{x-1} + \frac{1}{x-3}\, dx = \frac{1}{x-1} - \ln|x-1| + \ln|x-3| + c$$

$$\frac{x+1}{(x-1)^2(x-3)} = \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{C}{x-3}$$

$$x+1 = A(x-3) + B(x-1)(x-3) + C(x-1)^2$$
$$x+1 = x^2(B+C) + x(A-4B-2C) - 3A + 3B + C$$

$$\begin{cases} B+C=0 \quad \Rightarrow B=-C \\ A-4B-2C=1 \longrightarrow A+2C=1 \Rightarrow A=1-2C \\ -3A+3B+C=1 \longrightarrow -3+6C-3C+C=1 \end{cases}$$

$$C=1$$
$$B=-1$$
$$A=-1$$

$$\int \frac{x^5 - 15x}{x^4 - 16}\, dx = \int x +$$

$$\frac{x^5 - 15x}{x^4 - 16} = x + \frac{x}{x^4 - 16}$$

$$\frac{x}{x^4 - 16} = \frac{x}{(x-2)(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{Cx+D}{x^2+4}$$

$$x = A(x-2)(x^2+4) + B(x+2)(x^2+4) + C(x(x^2+4) + D(x^2 + x))$$

$$x = -2$$
$$-2 = -32A \Rightarrow A = \frac{1}{16}$$

$$\cdots\cdots\cdots\cdots 16$$

$x = 2$

$\quad 2 = 32B \implies B = \dfrac{1}{16}$

$x = 0$

$\quad 0 = -1 + 1 - 4D \implies D = 0$

$x = 1$

$\qquad\qquad\qquad\qquad C = -\dfrac{1}{8}$