4. gyakorlat

$$y'' + y' - 2y = 3e^{-2x}, \quad y(0) = 2$$
$$y'(0) = 0$$

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda_1 = -2 \qquad \lambda_2 = 1$$

$$y_{há} = c_1 e^{-2x} + c_2 e^{x}$$

$$y = Axe^{-2x}$$

$$-2Ae^{-2x} + 4Axe^{-2x} - 2Ae^{-2x} - 2Ae^{-2x} + Ae^{-2x} - 2Axe^{-2x} =$$
$$= 3e^{-2x}$$

$$\left.\begin{aligned} xe^{-2x} &: 4A - 2A - 2A = 0 \\ e^{-2x} &: -2A - 2A + 1 = 3 \end{aligned}\right\} A = 1$$

$$y_{iá} = -xe^{-2x} + c_1 e^{-2x} + c_2 e^{x}$$

$$2 = C_1 + C_2 \qquad y(0) = 2$$

$$0 = D = -e^{-2x} + 2xe^{-2x} - 2c_1 e^{-2x} + C_2 e^{x}\Big|_0 = -1 - 2C_1 + C_2$$

$$C_1 = \underline{1}$$

3

$c_2 = \frac{5}{3}$

$$y(x) = -x e^{-2x} + \frac{1}{3} e^{-2x} + \frac{5}{3} e^{x}$$

$$y''' + 2y'' + y' + 2y = sh\,2x$$

$$\lambda^3 + 2\lambda^2 + \lambda + 2 = 0$$

$\lambda_1 = -2$

$\lambda_{2,3} = \pm i$

$$y_{hc} = C_1 e^{-2x} + C_2 e^{0x} \cos(x) + C_3 e^{0x} \sin(x)$$

$$y = A e^{2x} + Bxe^{-2x}$$

$e^{2x}: \; 20A = \frac{1}{2}$

$e^{-2x}: \; 5B = -\frac{1}{2}$

$xe^{-2x}: \; 0B = 0$

$$y_{PC} = \frac{1}{\;} e^{2x} - \frac{1}{\;} x e^{-2x} + C_1 e^{-2x} + C_1 \cos x + C_2 \sin x$$

40 10

$$f(n) = 3f(n-1) - 2f(n-2)$$

$$f(n) := q^n$$

$$q^n = 3q^{n-1} - 2q^{n-2} \quad / : q^{n-2}$$

$$q^2 = 3q - 2 \qquad q_{1/2} \begin{matrix} 1 \\ 2 \end{matrix}$$

$$f(n) = c_1 \cdot 1^n + c_2 2^n$$

$$f(0) = 1, \quad f(1) = 5, \quad \ldots$$

$$\left.\begin{aligned} 1 &= f(0) = c_1 + c_2 2^0 \\ 5 &= f(1) = c_1 + c_2 2 \end{aligned}\right\} \begin{aligned} c_1 &= -3 \\ c_2 &= 4 \end{aligned}$$

$$f(n) = -3 + 4 \cdot 2^n$$

$O(1)$ $\quad |f(n)| < A \cdot 1 \Rightarrow c_2 = 0$

$\theta(1) \quad | \ldots | \ldots \Rightarrow c_2 = 0$

$$f(n+1) = \frac{10}{3} f(n) - f(n-1)$$

$$f(n) := q^n$$

$$q^{n+1} = \frac{10}{3} q^n - q^{n-1} \quad / q^{n-1}$$

$$q^2 = \frac{10}{3} q - 1 \quad \begin{cases} \frac{1}{3} \\ 3 \end{cases}$$

$$f(n) = c_1 \, 3^n + c_2 \, 3^{-n}$$

$$\theta(1) \quad |f(n)| \leq A \Rightarrow c_1 = 0$$

$$\left.\begin{aligned} f(0) &= -2 = c_1 3^0 + c_2 3^{-0} \\ f(1) &= 2 = c_1 3^1 + c_2 3^{-1} \end{aligned}\right\} \begin{aligned} c_1 &= 1 \\ c_2 &= -3 \end{aligned}$$

$$f(n) = 3^n - 3 \, 3^{-n}$$

$$5' \quad \frac{3^{2n}}{\ } = \frac{91}{\ } = \frac{9}{114}$$

$$\sum_{1} \quad (-11)^{n+3} \qquad 1-q \qquad 1+\frac{9}{11}$$

$$\hookrightarrow \; = \frac{9^n}{11^3 \cdot (-11)^n} = \frac{1}{11^3} \cdot \left(-\frac{9}{11}\right)^n \Rightarrow q = -\frac{9}{11}$$

$$\sum_{1} \frac{2^{3n+4} + (-7)^{n+2}}{3^{2n-1}} = \sum_{1} \frac{2^{3n+4}}{3^{2n-1}} + \sum_{1} \frac{(-7)^{n+2}}{3^{2n-1}} =$$

$$q_1 = \frac{8}{9} \qquad q_2 = -\frac{7}{9}$$

$$= \frac{48 \cdot \frac{8}{9}}{1-\frac{8}{9}} + \frac{-147 \cdot \frac{7}{9}}{1+\frac{7}{9}}$$

$$\sum_{1} (-1)^n \frac{1}{\sqrt[3]{n}+4}$$

Leibniz Vor

$\Downarrow$

konvergend

$$\sum_{1} (-1)^n \frac{1}{\left(\sqrt[4]{n^3}\right)^4 + 4}$$

$\downarrow$

$1 \quad > \frac{1}{8} \Rightarrow$ divergens

$$\sum_{1}^{99} \overbrace{(-1)^{n+1} \frac{7^n}{3^n + 8^n}}^{a_n} = s_{99} \approx \sum_{1}^{\infty} (-1)^{n+1} \frac{7^n}{3^n + 8^n}$$

s

$$|s - s_{99}| \leq |a_{100}| = \frac{7^{100}}{3^{100} + 8^{100}}$$