11. hét Anal gyak

$x_0 = e \qquad y(x)$

$x \ln y + y \ln x = 1 \qquad y'(e) = ?$

$P_0(e,1)$

$x_0 = e$

$y_0 = 1$

$\xrightarrow{()'} 1 \cdot \ln y + x \cdot \frac{1}{y} \cdot y' + y' \cdot \ln x + y \cdot \frac{1}{x} = 0$

$\xrightarrow{x=e,\ y=1} \cancel{\ln 1} + e y' + y' \cdot \cancel{\ln e} + \frac{1}{e} = 0$

$y' = \frac{-1}{e(e+1)}$

$y_E = y'(x-e)+1$

$y_E = \frac{-1}{e(e-1)}(x-e)+1$

érintő

$\boxed{y_E(x) = y'(e)(x-x_0)+y_0}$

$y^2 + 2y^5 + e^{2x-2} - (x-1)^4 = 0$

$x_0 = 1 \qquad y_0 = -1$

lokális szélsőérték? Inflexió?

$\downarrow ()'$

$2yy' + 10y^4y' + e^{2x-2} \cdot 2 - 4 \cdot (x-1)^3 \cdot 1 = 0$

$\downarrow x=1 \quad y=-1$

$-2y' + 10y' + 2 = 0$

$y' = \frac{-1}{4} \neq 0 \Rightarrow$ nincs szélsőérték

$\downarrow ()'$

$2y'y' + 2yy'' + 40y^3y'y' + 10y^4y'' + 4 \cdot e^{2x-2} - 12(x-1)^2 = 0$

$\downarrow y=-1,\ x=1,\ y' = -\frac{1}{4}$

$\frac{1}{8} - \frac{1}{2}y'' - \frac{40}{16} + 10y'' + 4 = 0$

$y''(1) = -\frac{13}{64} \neq 0 \Rightarrow$ nincs inflexiós pontja

$$\begin{cases} x = t + \sin 4t \\ y = t + \sin 2t \end{cases}$$

$y = f(x)$

$t_0 = \frac{\pi}{8}$

$x_0(t_0)$

$\Rightarrow$ invertálni kell: szig. mon.

$\hookrightarrow (\cdot)'$

$$\dot{x} = 1 + \cos(4t) \cdot 4$$

$$\dot{x}\left(\frac{\pi}{8}\right) = 1 + 4\cos\frac{\pi}{2} = 1 \neq 0 \Rightarrow \text{szig. mon.}$$

$$I\left(\frac{\pi}{8} - \delta;\ \frac{\pi}{8} + \delta\right) : x \text{ szig. monoton}$$

$$y = y(t(x))$$

$$f'(x_0) = \frac{\dot{y}(t_0)}{\dot{x}(t_0)} = \frac{1+\sqrt{2}}{1} = 1+\sqrt{2} \Longrightarrow \text{nincs szélsőérték}$$

$$\dot{y} = 1 + 2\cos(2t)$$

$$f''(x_0) = \frac{\ddot{y}\dot{x} - \dot{y}\ddot{x}}{(\dot{x})^2} = \frac{-2\sqrt{2} - (1+\sqrt{2})(-16)}{(1)^2} = \quad \begin{vmatrix} \ddot{x} = -\sin(4t)16 \\ \ddot{y} = -\sin(2t)4 \end{vmatrix}$$

$$= 16 + 14\sqrt{2} \neq 0 \Longrightarrow \text{nincs inflexiós pontja}$$

Integrálás

$$\int (ax+b)^n dx = \boxed{\frac{(ax+b)^{n+1}}{(n+1)a} + c}$$

$$\int (2x+3)^5 dx = \frac{(2x+3)^6}{12} + c$$

$$\boxed{\int \frac{1}{x^\alpha} \ldots \frac{1}{\ldots}^{\alpha+1}}$$

$$\int \frac{1}{2}\cdot 2(2x+3)^5\,dx = \frac{1}{2}\cdot\frac{(2x+3)^6}{6}+c$$

$$\boxed{\int f^{\alpha} f'\,dx = \frac{f^{\alpha+1}}{\alpha+1}+c}$$

$$\int \frac{1}{(2x+3)^5}\,dx = \int (2x+3)^{-5}\,dx = \frac{(2x+3)^{-4}}{-4\cdot 2}+c$$

$$\int \frac{2}{9x+3} = \frac{2}{9}\cdot \ln(|9x+3|)+c$$

$$\boxed{\int \frac{f'}{f}\,dx = \ln|f|+c}$$

$$\int \frac{2}{9x^2+1}\,dx = 2\int \frac{1}{1+9x^2}\,dx = \frac{2\,\mathrm{arctg}(3x)}{3}+c$$

$$\int \frac{2}{9x^2+3}\,dx = \frac{2}{3}\int \frac{1}{1+(\sqrt{3}x)^2}\,dx = \frac{2}{3}\cdot\frac{\mathrm{arctg}(\sqrt{3}x)}{\sqrt{3}}+c$$

$$\int \frac{2}{9x^2+6x+3}\,dx = \int \frac{1}{1+\left(\frac{3x+1}{\sqrt{2}}\right)^2}\,dx = \frac{\mathrm{arctg}\left(\frac{3x+1}{\sqrt{2}}\right)}{\frac{3}{\sqrt{2}}}+C$$

parciális
integrálás

$$(u\cdot v)' = u'v + uv'$$

$$\int (uv)' = \int (u'v + uv')$$

$$uv = \int u'v + \int uv'$$

$$\boxed{\int u'v = uv - \int uv'}$$

$$\int (3x-1)\sin(5x+3)\,dx = \overbrace{(3x-1)\left(-\frac{\cos(5x+3)}{5}\right)}^{A} -$$

$$- \int 3\cdot\left(-\frac{\cos(5x+3)}{5}\right)dx =$$

$$= A + \frac{3}{5}\cdot\frac{\sin(5x+3)}{5} + c$$

$$\int \ln x\,dx = \int 1\cdot \ln x\,dx = \ln x\cdot x - x = x(\ln x - 1) + c$$

$$\int \operatorname{arctg}(2x)\cdot 1\,dx = x\cdot\operatorname{arctg}(2x) - \int \frac{2x}{1+(2x)^2}\,dx =$$

$$= x\cdot\operatorname{arctg}(2x) - \frac{1}{4}\ln|4x^2+1| + 1$$