2. gyakorlat

Elsőfokú lineáris de

$$y' + g(x)\,y = h(x)$$

homogén: $y' + g(x)\,y = 0$

$$y_{há} = c\,e^{-G(x)}$$

konstans variálása:

keressük az inhomogén mo-t

$$y_{ip} = c(x)\,e^{-G(x)}$$

$$c'(x)e^{-G(x)} + \cancel{c(x)e^{-G(x)}(-g(x)) + g(x)c(x)e^{-G(x)}} = h(x)$$

$$c'(x) = e^{G(x)}h(x) \Rightarrow c(x) = \int \dots dx$$

$$y_{iá} = y_{ip} + y_{há}$$

(1) $y' + 2x^3 y = \underset{f(x)}{8x^2}$

homogén: $y' + \overset{g(x)}{2x^3}\,y = 0$ $\qquad G(x) = \frac{x^4}{2}$

$$y_{há} = c\,e^{-\frac{x^4}{2}}$$

konstans variálása:

$$c'(x) = e^{\frac{x^4}{2}} \cdot 8x^3$$

$$C(x) = \int e^{\frac{x^4}{2}} \cdot 8x^3 \, dx = 4e^{\frac{x^4}{2}}$$

$y_{ip} = 4$

$$y_{iá} = 4 + ce^{-\frac{x^4}{2}}$$

② $y' + 3\cos x \, y = 4e^{-3\sin x}$

homogén: $y' + 3\cos x \, y = 0$ $\qquad \int 3\cos x \, dx = 3\sin(x)$

$$y_{há} = ce^{-3\sin x}$$

konstans variálás: $c'(x) = e^{3\sin x} \cdot 4e^{-3\sin x}$

$$c'(x) = 4$$

$$c(x) = 4x$$

$$y_{iá} = 4x^{-3\sin x} + ce^{-3\sin x}$$

③ $y' + \frac{4}{x} y = e^{-x} x^{-2}$

$y = 0$ ✓

$$y' + \frac{4}{x} y = 0$$

$$y' \quad 4 \quad = \quad \int y' \quad \int 4$$

$$\frac{y'}{y} = -\frac{4}{x} \quad \int \frac{y'}{y}\,dx = \int -\frac{4}{x}\,dx$$

$$\int \frac{1}{y}\,dy = -4\ln|x| + c$$

$$y_{hó} = c\,e^{\ln x^{-4}} = c \cdot x^{-4} \qquad \ln(y) = \ln|x^{-4}| + c$$

$$y_{hó} = c x^{-4}$$

$$y_{ip} = c \cdot e^{-x^{-4}}$$

$$c' e^{-x^{-4}} + c \cdot e^{-x^{-4}} \cdot 4x^{-5} + c \cdot e^{-x^{-4}} \cdot \frac{4}{x} = e^{-x} x^{2}$$

$$c' = e^{-x} \cdot x^2$$

$$c = \int e^{-x} \cdot x^2\,dx = -e^{-x} \cdot x^2 - \int -e^{-x} \cdot 2x\,dx$$

$$-e^{-x} \cdot x^2 - \left(e^{-x} \cdot 2x - \int e^{-x} \cdot 2\,dx\right) =$$

$$= -e^{-x}\left(x^2 + 2x + 2\right)$$

$$y_{iá} = \frac{-e^{-x}(x^2 + 2x + 2)}{x^4} + \frac{c}{x^4}$$

⑦ $xy' = y \cdot (1 - \ln y + \ln x)$

$y' = \frac{y}{x}\,(\quad)$

$$\frac{y}{x} := u \qquad u'x + u = u\,(1 - \ln u)$$

$$u' = -\frac{u \ln u}{x} \qquad \boxed{u = 1}$$

$$\int \frac{1}{u \ln u}\, du = -\int \frac{1}{x}\, dx$$

$$\ln|\ln u| = -\ln x + c$$

$$u = e^{\frac{c}{x}}$$

$$y = x\, e^{\frac{c}{x}}$$

⑧ $$y' = \frac{4}{2x - y}$$

$$u := 2x - y \Rightarrow u' = 2 - y'$$

$$2 - u' = \frac{4}{u}$$

$$u' = 2 - \frac{4}{u}$$

$$\frac{u}{2u - 4}\, u' = 1$$

$$\int \frac{u}{\quad}\, du = x$$

$\checkmark$ 2a-4

(9) Bernoulli $\quad u := y^\alpha$

$$\underbrace{5y^4 y'}_{u'} - 3y^5 = e^{4x} + 1, \quad y(0) = 1$$

$u := y^5 \qquad u' - 3u = e^{4x} + 1$

$$u' - 3u = 0$$

$$u_{hom} = c e^{3x}$$

$u = c(x) e^{3x}: \quad c'(x) \cancel{e^{3x}} + \cancel{c 3 e^{3x}} - \cancel{3 c e^{3x}} = e^{4x} + 1$

$$c'(x) = e^{x} + e^{-3x}$$

$$c = e^{x} + - \frac{e^{-3x}}{3}$$

$$u_{ia} = \left(e^{x} - \frac{e^{-3x}}{3}\right) e^{3x} + c e^{3x} =$$

$$= e^{4x} - \frac{1}{3} + c e^{3x}$$

$$y = \sqrt[5]{e^{4x} - \frac{1}{3} + c e^{3x}}$$

$y(0) = 1$

$$1 = \sqrt[5]{e^{0} - \frac{1}{3} + c e^{0}}$$

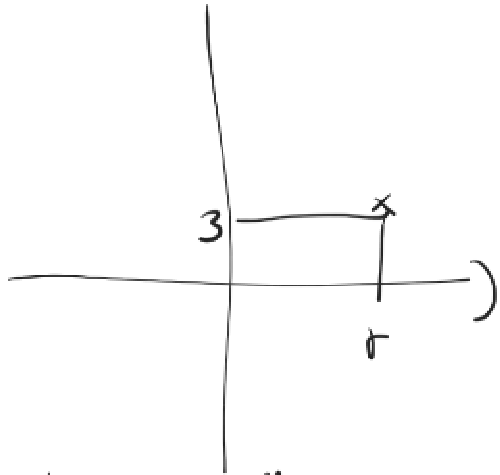
$$1 = c + \frac{2}{3}$$

$$c = \frac{1}{3}$$

$$y = \sqrt[3]{e^{4x} - \frac{1}{3} + \frac{1}{3}e^{3x}}$$

16 (a) $$y' = e^{y-3} + x$$

0 - izoklina



$$y'(5) = e^{y(5)-3} + 5 = 6$$