14. hét gyak

$$\int \sqrt{x^2-4}\, dx \qquad \frac{x}{2} = \operatorname{ch} t \Rightarrow x = 2\operatorname{ch} t$$

$$1dx = 2\operatorname{sh} t\, dt$$

$$\int \sqrt{4(\operatorname{ch} t)^2 - 4}\, dx = \int 2\underbrace{\sqrt{\operatorname{ch}^2 t - 1}}_{\operatorname{sh} t} \cdot 2\operatorname{sh} t\, dt =$$

$$= 4 \cdot \int \operatorname{sh}^2 t\, dt = 2 \cdot \int \operatorname{ch} 2t - 1\, dt = \underline{2 \cdot \left(\frac{\operatorname{sh} 2t}{2} - t\right) + C} =$$

$$= 2(\operatorname{sh} t \cdot \operatorname{ch} t - t) + C = 2\left(\sqrt{\operatorname{ch}^2 t - 1} \cdot \operatorname{ch} t - t\right) + C \to$$

$$\to 2\left(\sqrt{\left(\frac{x}{2}\right)^2 - 1} \cdot \frac{x}{2} - \operatorname{arch} \frac{x}{2}\right) + C$$

$$\int \frac{e^{2x}}{e^{2x}+1}\, dx = \frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+1}\, dx = \frac{1}{2}\ln\left|e^{2x}+1\right| + C$$

$$\int \frac{e^{6x}}{e^{2x}+1}\, dx = \frac{1}{2}\int \frac{e^{4x}\overbrace{2e^{2x}\, dx}^{dt}}{e^{2x}+1} = \frac{1}{2}\int \frac{t^2}{t+1}\, dt =$$

$$t = e^{2x}$$

$$t' = 2e^{2x}$$

$$= \frac{1}{2}\int t - 1 + \frac{1}{t+1}\, dt =$$

$$= \frac{1}{2}\left(\frac{t^2}{2} - t + \ln|t+1|\right) + C \to$$

$$\to \frac{1}{2}\left(\frac{e^{4x}}{2} - e^{2x} + \ln\left|e^{2x}+1\right|\right) + C$$

$$\int \frac{9x}{\sqrt{2-3x}+1}\,dx = \int \frac{3(2-t^2)}{t+1}\,dx = 3\int \frac{-t^2+2}{t+1}\cdot -\frac{2}{3}t\,dt =$$

$t(t^2-2)$

$t = \sqrt{2-3x}$

$x = \frac{2-t^2}{3}$

$$= 2\int \frac{t^3-2t}{t+1}\,dt = 2\cdot\int t^2-t-1+\frac{1}{t+1}\,dt$$

$t^3-2t : t+1 = t^2-t-1$

$\ominus\ t^3 - t^2$

t^2-2t

$\ominus\ t^2+2t$

$-t$

$\ominus\ -t-1$

1

$$= 2\cdot\left(\frac{t^3}{3}-\frac{t^2}{2}-t+\ln|t+1|\right)+C =$$

$= \ t = \sqrt{x}\ldots\ !$

$(t^2-1)+2$

$$\int \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x^2}+x}\,dx = \int \frac{t^2+1}{t^2+t^3}\cdot 3t^2\,dt = 3\int \frac{t^2+1}{1+t}\,dt =$$

$t = \sqrt[3]{x}$

$t^3 = x$

$$= 3\int t-1+\frac{2}{1+t}\,dt = 3\left(\frac{t^2}{2}-t+2\ln|1+t|\right)$$

$$= 3\left(\frac{\sqrt[3]{x^2}}{2}-\sqrt[3]{x}+2\ln\left|1+\sqrt[3]{x}\right|\right)+C$$

Impropius integrál

$$\int_{-1}^{\infty} \frac{4}{x^2+2x+5}\,dx = \int_{-1}^{\infty} \frac{1}{1+\left(\frac{x+1}{2}\right)^2}\,dx = \lim_{\omega\to\infty}\int_{-1}^{\omega} \frac{1}{1+\left(\frac{x+1}{2}\right)^2}\,dx =$$

$$= \lim_{\omega\to\infty}\left[2\operatorname{arctg}\left(\frac{x+1}{2}\right)\right]_{-1}^{\omega} =$$

$\to 0$

$$= \lim_{\omega \to \infty} \left(2 \operatorname{arctg} \frac{\omega+1}{2} - 2 \operatorname{arctg} \frac{-1+\omega}{2} \right) = \pi$$

$$\int_1^{\infty} \frac{1}{x} \, dx = \lim_{\omega \to \infty} \left[\ln|x| \right]_1^{\omega} = \lim_{\omega \to \infty} \left(\ln|\omega| - \overset{0}{\overset{\shortparallel}{\ln|1|}} \right) = \infty$$

$$\int_1^{\infty} \frac{1}{x^2} \, dx = \lim_{\omega \to \infty} \left[-x^{-1} \right]_1^{\omega} = \lim_{\omega \to \infty} \left(-\frac{1}{\omega} - (-1) \right) = 1$$

$$\int_{-\infty}^{\infty} \frac{\operatorname{arctg}^2 2x}{1+4x^2} \, dx = \frac{1}{2} \lim_{\substack{\omega_2 \to \infty \\ \omega_1 \to -\infty}} \left[\frac{\operatorname{arctg}^3 2x}{3} \right]_{\omega_1}^{\omega_2} = \frac{1}{6} \lim_{\substack{\omega_2 \to \infty \\ \omega_1 \to -\infty}} \left(\operatorname{arctg}^3 2\omega_2 - \operatorname{arctg}^3 2\omega_1 \right) =$$

$$= \frac{1}{6} \cdot \left(\left(\frac{\pi}{2}\right)^3 - \left(-\frac{\pi}{2}\right)^3 \right) = \frac{\pi^3}{24}$$

$$\int_{-2}^{0} \frac{6}{\sqrt{4+2x}} \, dx = \qquad = 3 \int_{-2}^{0} 2 \cdot \sqrt{4+2x}^{\,-\frac{1}{2}} =$$

$t = \sqrt{4+2x}$

$x = \frac{t^2-4}{2}$

$$\int_0^4$$

$$= \lim_{\omega \to 0+0} 6 \left[\sqrt{4+2x} \right]_{-2+\omega}^{0} =$$

$$= \lim_{\omega \to 0+0} 6 \left(\overset{2}{\sqrt{4}} - \sqrt{4+2(-2+\omega)} \right) =$$

$$= 6 \cdot (2-0) = 12$$