13. gyak

Furie - sor

$$\phi(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \dots =$$

$$= \sum_{k=0}^{\infty} a_k \cos kx + b_k \sin kx$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx \, dx$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx \, dx$$

$$f(x) = \left. \begin{cases} -3, & x \in [-\frac{\pi}{2}, 0[\\ 3, & x \in [0, \frac{\pi}{2}[\\ 0, & x \in \text{más a } [-\pi, \pi[\end{cases} \right\} \text{páratlan}$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{f(x) \cos kx}_{\text{páratlan}} \, dx = \frac{1}{\pi} \left[\int_{-\pi}^{-\frac{\pi}{2}} 0 \, dx + \int_{-\frac{\pi}{2}}^{0} -3 \cos kx \, dx + \int_{0}^{\frac{\pi}{2}} 3 \sin kx \, dx + \int 0 \, d \right]$$

$$\Downarrow$$

$$a_k = 0$$

$$b_k = \frac{1}{\pi} \int \underbrace{f(x) \sin kx}_{\text{páros}} \, dx = 2 \cdot \frac{1}{\pi} \cdot \int_0^{\pi} f(x) \sin kx \, dx =$$

$$= 2 \cdot \frac{1}{\pi} \int_0^{\frac{\pi}{2}} 3 \sin kx \, dx + 0 =$$

$$= 2 \cdot \frac{1}{\pi} \left[\frac{-3 \cos kx}{k} \right]_0^{\frac{\pi}{2}} =$$

$$= b_k = \begin{cases} \\ \\ \end{cases}$$

akkor $f(x)$ folytonos $\quad \phi(x_0) = f(x_0)$

akkor véges ugrás van $\quad \phi(x_0) = \dfrac{\lim\limits_{x \to x_0 - 0} f(x) + \lim\limits_{x \to x_0 + 0} f(x)}{2}$