7. gyakorlat

$$f(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$

$$f(x_0) = a_0$$

$$f'(x_0) = a_1$$

$$f''(x_0) = 2a_2$$

$$\vdots$$

$$f^{(k)}(x_0) = k! \cdot a_k$$

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

$$x_0 = 0 : T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}}{n!} x^n$$

$$T_k(x) = \sum_{n=0}^{k} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

$$f(x) - T_k(x) = \frac{f^{(k+1)}(\xi)}{(k+1)!} (x - x_0)^{k+1}$$

(1)

$$f(x) = x^4 - 4x + \sin 3x$$

$x_0 = 0 : T(4)$

$$f(0) = 0$$

$$f'(x) = 4x^3 - 4 + 3\cos 3x \qquad f'(0) = -1$$

$$f''(x) = 12x^2 - 9\sin 3x \qquad f''(0) = 0$$

$f'''(x) = 24x - 27\cos 3x \qquad f'''(0) = -27$

$f^{(4)}(x) = 24 + 81\sin 3x \qquad f^{(4)}(0) = 24$

$f^{(5)}(x) = 243\cos 3x$

$$T_4(x) = 0 + \frac{-1}{1}x + 0 + \frac{-27}{6}x^3 + \frac{24}{24}x^4$$

$$T_4(x) = x^4 - \frac{9}{2}x^3 - x$$

$$f(x) - T_4(x) = \frac{243\cos 3\xi}{5!} \cdot x^5$$

②

$$y' = y^3 - 3x^2y^2 + 2 \quad , \quad y(-1) = 1$$

$x_0 = -1 \quad T_3(x)$

$$y(-1) = 1$$

$$y'(-1) = 0$$

$$y''(-1) = 6$$

$$y'''(-1) = -24$$

$I_3(x) = 1 \ldots$

(3)

$$f(x) = \frac{1}{x^2-5} = \frac{1}{-5}\,\frac{1}{1-\frac{x^2}{5}} = -\frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{x^2}{5}\right)^n$$

$$\cancel{-1} < \frac{x^2}{5} < 1 \qquad x^2 < 5$$

$$-\sqrt{5} < x < \sqrt{5}$$

$$g(x) = \frac{x^4}{x^2-5} = \sum_{n=0}^{\infty}\left(-\frac{1}{5}\right)^{n+1} x^{2n+4}$$

$$g^{(99)}(0) = a_{99}\cdot 99! = 0$$

$$g^{100}(0) = a_{100}\cdot 100! = \frac{1}{5^{49}}\cdot 100!$$

(5)

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \ldots = -\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{n}$$

$R=1$

$$\ln\left(2+\frac{x^3}{4}\right)$$

$$e^x = \sum_0^{\infty} \frac{x^n}{n!}$$

$$\sin x = \sum_1^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = \sum_0^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\operatorname{sh} x = \sum \frac{x^{2n+1}}{(2n+1)!}$$

$$\operatorname{ch} x = \sum \frac{x^{2n}}{(2n)!}$$