5. gyakorlat

$$\sum_1 \frac{3n^3-2n^2+n}{2n^4+5n^2+6} \geq \sum_1 \frac{3n^3-2n^3}{2n^4+5n^4+6n^4} = \sum_1 \frac{\cancel{n^3}}{13n^{\cancel{4}}} = \sum_1 \frac{1}{13}\cdot\frac{1}{n} \to \text{divergens}$$

$$\Downarrow$$

Divergens

$$\sum_1 \frac{n^3+3n-5}{2n^7+4n^3+5n} \leq \sum_1 \frac{n^3+3n^3}{2n^7} \leq$$

$$\leq \sum_1 2\cdot\frac{1}{n^4} \to \text{konvergens}$$

$$\Downarrow$$

Konvergens

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mértani sor

$$\sum_1 \underbrace{\frac{3^{n+1}+5^n}{2+7^{n-2}}}_{a_n} \leq \sum \frac{49\cdot 4\cdot 5^n}{7^n} = \sum_1 196\left(\frac{5}{7}\right)^n$$

$$\downarrow$$

konvergens

$$\Downarrow$$

konvergens

$$\left|\sum_{1}^{\infty} a_n - \sum_{1}^{100} a_n\right| = \sum_{101}^{\infty} a_n \le \sum_{101}^{\infty} 196\left(\frac{5}{7}\right)^n =$$

$$= \frac{196\cdot\left(\frac{5}{7}\right)^{101}}{\cancel{1-\frac{5}{7}}\ \frac{2}{7}} = \frac{7}{2}\cdot 196\cdot\left(\frac{5}{7}\right)^{101}$$

$$\sum (-1)^n \frac{n^2+2}{5n^4-n^3}$$

$\downarrow$ abs

$$\sum \frac{n^2+2}{5n^4-n^3} \le \sum \frac{n^2+2n^2}{5n^4} = \sum \frac{3}{5n^2}$$

konvergens

abszolút konvergens

$$\sum_{1}^{\infty} \overbrace{(-1)^n \frac{n^2+2}{5n^3-n}}^{a_n}$$

monoton csökkenve tart a 0-hoz

$$3\ 1 = \frac{n^2+2}{} = \frac{n^2}{} = \left(\frac{1}{}\cdot\frac{1}{}\right) > \ldots$$

$\frac{\cdot}{4}\frac{\cdot}{n}$ // $\frac{}{5n^3-n}$ // $5n^3$ - $5\ n$ → divergens

Leibniz kritérium ⟹ konvergens, feltételesen

$$\left|\sum_1^{\infty} a_n - \sum_1^{1000} a_n\right| < \left|\cancel{(-1)^{1001}}^{-1} \frac{1001^2+2}{5\cdot 1001^3-1001}\right| =$$

—

$$\sum \frac{7^{n-3}}{n!}$$

$$\lim \frac{\frac{7^{n-2}}{(n+1)!}}{\frac{7^{n-3}}{n!}} = \lim \frac{7}{n+1} = 0 < 1$$

⇓

abszolút konvergens

—

$$\sum \frac{3^{5n}}{\ldots}$$

n

$$\sqrt[n]{\left|\frac{3^{5n}}{n^3}\right|} = \frac{3^5}{(\underbrace{\sqrt[n]{n}}_{\to 1})^3}_{\to 1} \to 3^5 > 1$$

$$\Downarrow$$

divergens

$$\sum \frac{(n+2)!}{n^{n+1}}$$

$$\lim \frac{\dfrac{(n+3)!}{(n+1)^{n+2}}}{\dfrac{(n+2)!}{n^{n+1}}} = \lim \frac{n^{n+1}}{(n+1)^{n+2}} \cdot \overset{\nearrow \infty}{(n+3)} =$$

$$= \lim \overset{\nearrow 1}{\frac{n+3}{n+1}} \left(\frac{n}{n+1}\right)^{n+1} = \frac{1}{e} < 1$$

$$\left(1+\frac{1}{n}\right)^{-(n+1)} \to e \qquad \Downarrow$$

konvergens

$$\sum \left(\frac{n^2-1}{\quad}\right)^{n^2} = \sum \frac{\left(1-\frac{1}{n^2}\right)^{n^2}}{\quad} \to \frac{e^{-1}}{\quad} = \frac{1}{\quad}$$

$$\underbrace{(n^2+4)}_{a_n} \qquad \left(1+\frac{4}{n^2}\right)^{n^2} \to e^4 \neq e^5$$

$$\Leftarrow 0$$

divergens

$$\sum \left(\frac{n^2-1}{n^2+4}\right)^n = \sum \sqrt[n]{a_n} \to 1 \Rightarrow \text{divergens}$$

$$\sum \underbrace{\left(\frac{n^2-1}{n^2+4}\right)}_{b_n}^{n^3} = \sum a_n^n \to 0$$

$$\lim \sqrt[n]{b_n} = e^{-5} < 1 \Rightarrow \text{konvergens}$$

$$\sum \frac{3}{5n \ln n^2} = \frac{3}{10}\cdot\sum \frac{1}{n\cdot \ln n}$$

Integrál kritérium

$$\int^{\infty} 1 \qquad \sum \ldots 7\infty$$

$$\int_2 \frac{}{x \ln x} = \left[\ln \ln x \right]_2 = \infty$$

$\Downarrow$

divergens