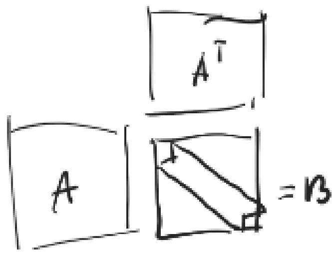
Bsz 12. gyak

9. gyak. 12. feladat



$B = A \cdot A^T$

$$\sum_{i=1}^{n} b_{ii} = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^2 = 2018$$

$$b_{ii} = \sum_{j=1}^{n} a_{ij} \cdot a^T_{ji} = \sum_{j=1}^{n} a_{ij}^2$$

①

$$A A^{-1} = E = A^{-1} A$$

$$\begin{array}{cc} A & E \\ \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 7 & 1 & 0 & 0 \\ -1 & 3 & -6 & 0 & 1 & 0 \\ 2 & -5 & 12 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -3 & 7 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & -2 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -3 & 7 & 1 & 0 & 0 \\ 0 & 1 & -2 & -2 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right)$$

Van inverze

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & -3 & 0 & -6 & -7 & 0 \\ 0 & 1 & -2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right) \sim \begin{array}{c} E \qquad A^{-1} \\ \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -6 & -1 & 3 \\ 0 & 1 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right) \end{array}$$

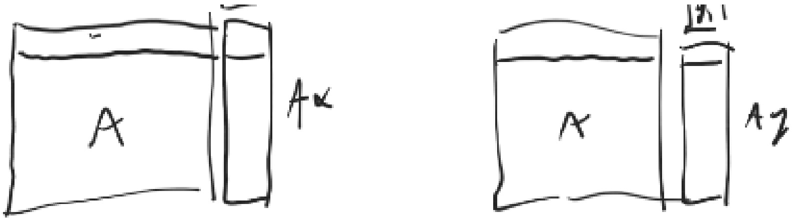
②

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 7 & 9 \\ 1 & 4 & 7 & 10 & 13 \\ 1 & 5 & 9 & 13 & 17 \end{pmatrix} \sim \begin{pmatrix} \quad \end{pmatrix} \sim \begin{pmatrix} \boxed{1} & 2 & 3 & 4 & 5 \\ 0 & \boxed{1} & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \text{rang } 2$$

⑤

$A \in \mathbb{R}^{n \times n}$ $\quad x, y \in \mathbb{R}^n$, $x \neq y$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \qquad \begin{bmatrix} y_1 \\ \vdots \end{bmatrix}$$



$$\sum_{i=1}^{n} x_i a_{ni} = \sum y_i a_{ni} \Rightarrow$$

- Tfh.: $\det A \neq 0 \Rightarrow \exists A^{-1}$

$$Ax = Ay \quad / A^{-1} \cdot (\,)$$
$$A^{-1}(Ax) = A^{-1}(Ay)$$
$$(A^{-1}A)x = (A^{-1}A)y$$
$$Ex = Ey$$
$$x = y \ ↯ \Rightarrow \det A = 0$$

- $Ax, Ay = \underline{b}$

$(A|\underline{b})$ egyértelműen megoldható $\Leftrightarrow \det A \neq 0$

↳ nem oldható meg egyértelműen $\Rightarrow \underline{\det A = 0}$

- $Ax - Ay = \underline{0}$

$$\left.A\underbrace{(x-y)}_{\neq \underline{0}} = \underline{0}\right\} \Rightarrow$$ a oszlopok összefüggőek $\Rightarrow \det A = 0$

4.b

$$\begin{pmatrix} 1 & 2 & 9 & 8 \\ 3 & 4 & 3 & 4 \\ 9 & 8 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & & & \\ 0 & & & \\ 0 & & & \end{pmatrix} \sim \begin{pmatrix} \boxed{1} & - & - & - \\ 0 & \boxed{1} & - & - \\ 0 & 0 & \boxed{1} & \frac{1}{4} \end{pmatrix}$$ 3 a rangja

6)

$$\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{pmatrix}$$

$$2x_1 + 4x_2 + 7x_3 = 1$$

$$\begin{pmatrix} 2 & 4 & 7 \\ 3 & 7 & 10 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$3x_1 + 7x_2 + 10x_3 = 0$$

$$\left(\begin{array}{ccc|cc} 2 & 4 & 7 & 1 & 0 \\ 3 & 7 & 10 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|cc} 1 & 2 & \frac{7}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{3}{2} & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|cc} 1 & 0 & 4{,}5 & 3{,}5 & -2 \\ 0 & 1 & -0{,}5 & -1{,}5 & 1 \end{array}\right)$$

$$\begin{pmatrix} 3{,}5 - 4{,}5x_3 & -2 - 4{,}5y_3 \\ -1{,}5 + 0{,}5x_3 & 1 + 0{,}5y_3 \\ x_3 & y_3 \end{pmatrix}$$