6. gyakorlat

Hatványsorok

$$\sum_{n=0}^{\infty} a_n (x-x_0)^n$$

konvergenciasugár: $R = \dfrac{1}{\limsup \sqrt[n]{|a_n|}}$

ha $\exists$, $R = \dfrac{1}{\lim \sqrt[n]{|a_n|}}$

$$R = \left| \frac{a_n}{a_{n+1}} \right|$$

konvergencia tartomány: $KT = (x_0 - R \, ; \, x_0 + R)$

(1) $\displaystyle\sum_{1}^{\infty} \frac{(-1)^n}{n \cdot 3^n} (x+2)^n \implies x_0 = -2$

$$R = \frac{1}{\lim \sqrt[n]{\left|\frac{(-1)^n}{n \cdot 3^n}\right|}} = \frac{1}{\lim \frac{1}{3 \cdot \underbrace{\sqrt[n]{n}}_{\to 1}}} = 3$$

$x = 1$: konvergens $\qquad x = -5$: divergens

$$KT =]-5 \, ; \, 1]$$

② $\sum (-1)^n \frac{3n-2}{(3n)!} (x-5)^n$

$$R = \lim \left| \frac{\cancel{(-1)^n} \frac{3n-2}{(3n)!}}{\cancel{(-1)^{n+1}} \frac{3n+1}{(3n+3)!}} \right| = \lim \frac{3n-2}{\cancel{(3n)!}} \cdot \frac{\overbrace{\cancel{(3n+3)!}}^{(3n+3)(3n+2)(3\cancel{n+1})}}{3\cancel{n+1}} =$$

$$= \lim (3n-2)(3n+2)(3n+3) = \infty$$

③ $\sum \frac{(3x-6)^n}{n^3 \cdot 4^n} = \sum \frac{3^n}{n^3 \cdot 4^n} \cdot (x-2)^n$

④ $\sum \frac{n^2}{5^n} \cdot x^{4n}$

$$a_n = \begin{cases} 0, \text{ ha } 4 \nmid n \\ \frac{\left(\frac{n}{4}\right)^2}{5^{\frac{n}{4}}}, \text{ ha } 4 \mid n \end{cases}$$

$$\sqrt[n]{a_n} = \begin{cases} 0, \text{ ha } 4 \nmid n \\ \sqrt[n]{n^2} \end{cases}$$

$$\left(\frac{\sqrt[n]{n^2}}{5^{\frac{1}{4}}}\right) \to 5^{-\frac{1}{4}}$$

$$\limsup \sqrt[n]{a_n} = 5^{-\frac{1}{4}}$$

$$\Downarrow$$

$$R = 5^{\frac{1}{4}}$$

$u := x^4$

$$\sum \frac{n^2}{5^n} u^n$$

$$R_u = \frac{1}{\lim \sqrt[n]{\frac{n^2}{5^n}}} = 5$$

$$\Updownarrow$$

$$R_x = 5^{\frac{1}{4}}$$

(5)

$$\sum \frac{n-2}{16^n} (x+3)^{4n}$$

$u := (x+3)^4$

$$R_u = \lim \frac{n-2}{16^n} = \lim \frac{n-2}{\quad} \cdot 16 = 16$$

$$\frac{\frac{n-1}{16^{n+4}}}{}\qquad n-1$$

$$|u| < 16 \iff (x+3)^4 < 16 \iff |x+3| < 2$$

$$-5 < x < -1$$

$x = -5$: divergens

$x = -1$: divergens

$$K_r =]-5\,;-1[$$

⑥ $$\sum_{n=1}^{\infty} \frac{x^{n+1}}{n} = x \sum \frac{x^n}{n} = f(x)$$

$$x \neq 0 \quad \left(\frac{f(x)}{x}\right)' = \left(\sum_1^{\infty} \frac{x^n}{n}\right)' = \sum x^{n-1} = \frac{1}{1-x} \qquad |x| < 1$$

$$\left.\begin{array}{l} \dfrac{f(x)}{x} = \displaystyle\int \frac{1}{1-x}\,dx = -\ln|1-x| + c \\ x = 0 \text{ va} \end{array}\right\} c = 0$$

$$f(x) = -x \ln|1-x| \xrightarrow{|x|<1} -x \cdot \ln(1-x)$$

$$-1 \leq x < 1$$

$x = a$:

$x = 0,$

(7)

$$\sum_{n=1}^{\infty} \frac{n-1}{n} \cdot x^{n+1} = \sum \left(1 - \frac{1}{n}\right) x^{n+1} =$$

$$= \underbrace{\underbrace{\sum x^{n+1}}_{\frac{x^2}{1-x}} - \underbrace{\frac{x^{n+1}}{n}}_{-x \cdot \ln(1-x)}}_{\frac{x^2}{1-x} + x \ln(1-x)} \qquad |x| < 1$$

(8)

$$\sum_{1}^{\infty} n\, x^{n+3} = x^4 \sum n\, x^{n-1} = x^4 \sum (x^n)' = x^4 \left(\sum x^n\right)'$$

$$= x^4 \left(\frac{x}{1-x}\right)' = x^4 \frac{1-x+x}{(1-x)^2} = \underline{\frac{x^4}{(1-x)^2}} \qquad -1 < x < 1$$