1. gyakorlat

① $y = e^{-x}\int_0^x e^{t^4}\,dt - 2e^{-x}$ megoldása-e

az $y' + y = e^{x^4-x}$

$$y' = -e^{-x}\int_0^x e^{t^4}\,dt + e^{-x}e^{x^4} + 2e^{-x}$$

$$\cancel{-e^{-x}\int_0^x e^{t^4}\,dt} + e^{-x}e^{x^4} + \cancel{2e^{-x}} + \cancel{e^{-x}\int_0^x e^{t^4}\,dt} \cancel{-2e^{-x}} =$$

$$= e^{x^4-x}$$

② $y'' = sh(2x) - x^2$

$$y' = \frac{ch(2x)}{2} - \frac{x^3}{3} + C_1$$

$$y = \frac{sh(2x)}{4} - \frac{x^4}{12} + C_1 x + C_2$$

$y(0) = 2$ $\qquad \frac{sh(0)}{4} - \frac{0}{12} + C_1 \cdot 0 + C_2 = 2 \Rightarrow C_2 = 2$

$y'(0) = 1$ $\qquad \frac{ch(0)}{2} - \frac{0}{3} + C_1 = 1 \Rightarrow C_1 = \frac{1}{2}$

$$y(x) = \frac{sh(2x)}{4} - \frac{x^4}{12} + \frac{1}{2}x + 2$$

$y'(x) = f(x, y(x))$ elsőrendű explicit diff. egyenlet

Szeparábilis $\quad y' = f(x) \cdot g(y)$

$g(y) \neq 0 \qquad\qquad g(y) = 0$

$$\frac{y'}{g(y)} = f(x)$$

$$\int \frac{1}{g(y)}\,dy = \int \frac{y'}{g(y)}\,dx = \int f(x)\,dx$$

$$G(y) = F(x) + c \quad \Rightarrow y = G^{-1}(F(x)+c)$$

(3)
$$y' = \frac{x}{y} e^{4x^2-5y} = x e^{4x^2} \cdot \frac{1}{y e^{5y}}$$

$$\int y e^{5y}\,dy = \int x e^{4x^2}\,dx$$

$$y\,\frac{e^{5y}}{5} - \frac{e^{5y}}{25} = \frac{1}{8} e^{4x^2} + c$$

(4)
$$y' = \frac{y+1}{(x-1)y} = \frac{y+1}{y} \cdot \frac{1}{x-1}$$

$$\int \frac{y+1-1}{\;}\,dy = \int \frac{1}{\;}\,dx$$

$$\int \frac{\ }{y+1}\,dy = \int \frac{\ }{x-1}$$

$$y - \ln|y+1| = \ln|x-1| + C$$

ha $y = -1$ $\quad \frac{y+1}{y} = 0$

$y \equiv -1$ megoldása

$y(2) = -1 \qquad y \equiv -1$

$y(2) = 2 \qquad 2 - \ln|3| = \ln|1| + C$

$$2 - \ln|3| = C$$

$$y(x) - \ln(y(x)+1) = \ln(x-1) + 2 - \ln(3)$$

$y(2) = -2 \qquad -2 - \ln|-1| = \ln|1| + C$

$$C = -2$$

$$y - \ln|y+1| = \ln|x-1| - 2$$

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$$y' = \frac{y^2+2y+5}{(x-2)(x+4)}$$

$$\int \frac{1}{\quad}\,dy = \int \frac{1}{\quad}\,dx \qquad \frac{A}{x-2} + \frac{B}{x+4}$$

$$\int \frac{1}{y^2+2y+5}\,dy = \int \frac{1}{(x-2)(x+4)}\,dx \qquad 1 = A(x+4)+B(x-2)$$

$$A+B=0$$
$$4A-2B=1$$
$$6A=1$$

$$\frac{1}{4}\int \frac{1}{\left(\frac{y+1}{2}\right)^2+1}\,dy = \int \frac{\frac{1}{6}}{x-2}+\frac{-\frac{1}{6}}{x+4}\,dx$$

$$c_1+\frac{1}{2}\operatorname{arctg}\left(\frac{y+1}{2}\right) = \frac{1}{6}\ln|x-2| - \frac{1}{6}\ln|x+4| + c_2$$

$$\operatorname{arctg}\left(\frac{y+1}{2}\right) = \frac{1}{3}\ln\left|\frac{x-2}{x+4}\right|$$

$$\frac{y+1}{2} = \operatorname{tg}\left(\frac{1}{3}\ln\left|\frac{x-2}{x+4}\right| + c\right)$$

$$y = 2\operatorname{tg}\left(\frac{1}{3}\ln\left|\frac{x-2}{x+4}\right| + c\right) - 1$$

(6)

$y(t)$ polonium mennyisége az idő függvényében

$$1-\frac{y(200)}{y(0)}$$

$$\frac{y(2900)}{y(0)} = \frac{1}{2}$$

$$\frac{y'(t)}{y(t)} = -\lambda$$

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$$\dot{y} = -\lambda y$$

$$\int \frac{1}{y}\,dy = \int -\lambda\,dt$$

$$\ln|y| = -\lambda t + c$$

$$y = e^{-\lambda t + c}$$

$$\frac{e^{-2900\lambda + c}}{e^{c}} = \frac{1}{2}$$

$$e^{-2900\lambda} = \frac{1}{2}$$

$$\lambda = -\frac{\ln(\frac{1}{2})}{2900} = \frac{\ln 2}{2900}$$

$$1 - \frac{y(200)}{y(0)} = 1 - \frac{e^{-\frac{\ln 2}{29\cancel{00}} \cdot 2\cancel{00} + \cancel{c}}}{\cancel{e^{c}}} = 1 - e^{-\frac{2}{29}\ln 2}$$