①
$$\begin{bmatrix}3&0&0\\3&3&3\\0&0&3\end{bmatrix} \qquad \begin{vmatrix}3-\lambda&0&0\\3&3-\lambda&3\\0&0&3-\lambda\end{vmatrix} = (3-\lambda)^3$$

$$\lambda=3:\ \begin{bmatrix}0&0&0\\3&0&3\\0&0&0\end{bmatrix} \xrightarrow{rref} \begin{bmatrix}1&0&1\end{bmatrix} \rightsquigarrow x+\overset{t}{z}=0 \to \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-t\\s\\t\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}s+\begin{bmatrix}-1\\0\\1\end{bmatrix}t$$

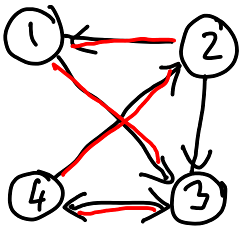
$$\begin{bmatrix}0\\3\\0\end{bmatrix}\bullet \xleftarrow{A-3I} \bullet \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}-1\\0\\1\end{bmatrix}\bullet$$

$$A=CJC^{-1}=\begin{bmatrix}0&1&-1\\3&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}3&1&\\&3&\\&&3\end{bmatrix}\begin{bmatrix}0&\frac{1}{3}&0\\1&0&1\\0&0&1\end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc}0&1&-1&1&&\\3&0&0&&1&\\3&0&1&&&1\end{array}\right] \to \left[\begin{array}{ccc|ccc}1&0&0&0&\frac{1}{3}&0\\0&1&0&1&0&1\\0&0&1&0&0&1\end{array}\right]$$

②
$$\begin{bmatrix}0&0&1&0\\1&0&1&0\\0&0&0&1\\0&1&1&0\end{bmatrix}$$



$$\underset{2.}{\begin{bmatrix}0&0&0&1\\0&0&1&1\\0&1&1&0\\1&0&1&1\end{bmatrix}} \quad \underset{4.}{\begin{bmatrix}1&0&1&1\\1&1&1&1\\0&1&1&1\\1&1&1&1\end{bmatrix}} \quad \underset{8.}{\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}} \quad P$$

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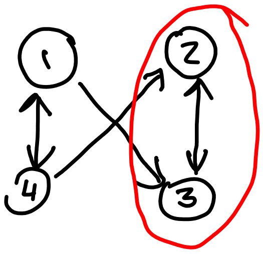
$$\begin{bmatrix}0&0&1&1\\0&0&1&0\\0&1&0&0\\1&1&0&0\end{bmatrix}$$

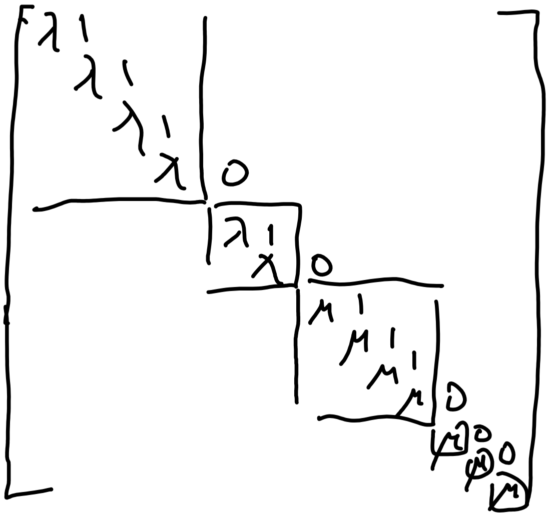


$$\begin{bmatrix}0&0&1&1\\1&1&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix} \to \left[\begin{array}{cc|cc}0&1&1&0\\1&0&0&1\\\hline \boxed{\begin{matrix}0&0\\0&0\end{matrix}}&&0&1\\&&0&0\end{array}\right] \qquad R$$

②

0	1	2	3	4
13	11	9	8	7
2	2	1	1	0
0	1	0	1	

0	1	2	3	4
13	9	8	7	6
4	1	1	1	0
3	0	0	1	



1. mo 111010111000

null(A−λI)

2. mo

2 4 5 6

$$\bullet \xleftarrow{A-\lambda I} \bullet \leftarrow \bullet \leftarrow \bullet$$

$$\bullet \leftarrow \bullet$$

4 5 6 7

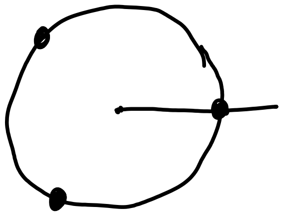
$$\bullet \leftarrow \bullet \leftarrow \bullet \leftarrow \bullet$$

$$\bullet$$

$$\bullet$$

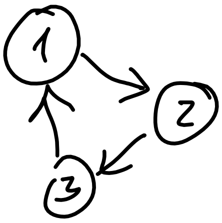
$$\bullet$$

③ $\det\begin{bmatrix}0-\lambda & 1 & 0\\ 0 & 0-\lambda & 2\\ 4 & 0 & 0-\lambda\end{bmatrix} = (-\lambda)^3 + 8$ $\quad x^3 = 8$



$2, \ -1 \pm \sqrt{3}i$

$\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 2\\ 4 & 0 & 0\end{bmatrix}$



$1 + 3k$
$2 + 3\ell$

infiniti

$$\frac{\begin{bmatrix}\frac{1}{5}\\ \frac{2}{5}\\ \frac{2}{5}\end{bmatrix}\begin{bmatrix}\frac{2}{4} & \frac{1}{4} & \frac{1}{4}\end{bmatrix}}{\begin{bmatrix}\frac{2}{4} & \frac{1}{4} & \frac{1}{4}\end{bmatrix}\frac{1}{5}\begin{bmatrix}1\\2\\2\end{bmatrix}} = \frac{\begin{bmatrix}1\\2\\2\end{bmatrix}\begin{bmatrix}2 & 1 & 1\end{bmatrix}}{\begin{pmatrix}2 & 1 & 1\end{pmatrix}\begin{bmatrix}1\\2\\2\end{bmatrix}} = \frac{1}{6}\begin{bmatrix}2 & 1 & 1\\ 4 & 2 & 2\\ 4 & 2 & 2\end{bmatrix}$$

⑤ $\begin{bmatrix}2 & 0 & 0\\ 0 & 1 & 2\\ 0 & -1 & 4\end{bmatrix}$ $\quad \begin{vmatrix}2-\lambda & 0 & 0\\ 0 & 1-\lambda & 2\\ 0 & -1 & 4-\lambda\end{vmatrix} = (2-\lambda)\Big[(1-\lambda)(4-\lambda)+2\Big]$

$\lambda^2 - 5\lambda + 6$

$= (2-\lambda)^2(3-\lambda)$

$\lambda = 2$: $\begin{bmatrix}0 & & \\ & -1 & 2\\ & -1 & 2\end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix}0 & 1 & -2\end{bmatrix} \rightsquigarrow \begin{bmatrix}s\\2t\\t\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}s + \begin{bmatrix}0\\2\\1\end{bmatrix}t$

$\lambda = 3$ $\begin{bmatrix}-1 & & \\ & -2 & 2\\ & -1 & 1\end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -1\end{bmatrix} \rightsquigarrow \begin{bmatrix}0\\t\\t\end{bmatrix} = \begin{bmatrix}0\\1\\1\end{bmatrix}t$

$A = C\Lambda C^{-1} = \left[\begin{array}{cc|c}1 & 0 & 0\\ 0 & 2 & 1\\ 0 & 1 & 1\end{array}\right]\left[\begin{array}{cc|c}2 & & \\ & 2 & \\ \hline & & 3\end{array}\right]\left[\begin{array}{ccc}1 & 0 & 0\\ 0 & 1 & -1\\ \hline 0 & -1 & 2\end{array}\right]$

$\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & & \\ 0 & 2 & 1 & & 1 & \\ 0 & 1 & 1 & & & 1\end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 0 & 0\\ & 1 & & 0 & 1 & -1\\ & & 1 & 0 & -1 & 2\end{array}\right]$

$\begin{matrix}1 & 0\\ 1 & 1\end{matrix} \qquad \begin{matrix}1 & -1\\ 0 & 1\end{matrix}$

$2\begin{bmatrix}1 & 0\\ 0 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -1\end{bmatrix} + 3\begin{bmatrix}0\\1\\1\end{bmatrix}\begin{bmatrix}0 & -1 & 2\end{bmatrix} = 2\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & -2\\ 0 & 1 & -1\end{bmatrix} + 3\begin{bmatrix}0 & 0 & 0\\ 0 & -1 & 2\\ 0 & -1 & 2\end{bmatrix}$

2.uw.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & 4 \end{bmatrix} = 2P_1 + 3P_2$$

$$I = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} = P_1 + P_2 \quad /(-2)$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & -1 & 2 \end{bmatrix} = P_2 \qquad P_1 = \begin{bmatrix} 1 & 0 & 0. \\ 0 & +2 & -2 \\ 0 & 1 & -1 \end{bmatrix}$$

⑧ $A = QR$ $\qquad QRx = b \longrightarrow \qquad Rx = Q^Tb$

$$Rx = Q^Tb$$

$$\begin{bmatrix} 2 & * & * \\ & 3 & * \\ & & 4 \end{bmatrix} x = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}$$