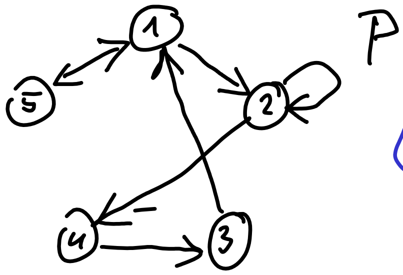
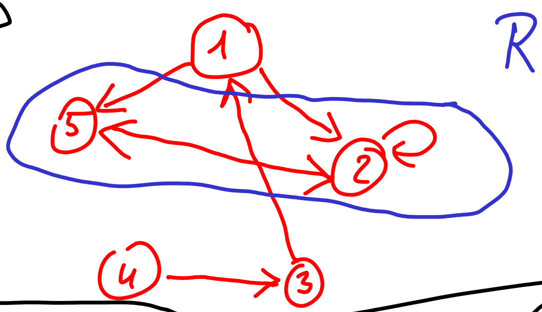
(1) $\begin{bmatrix} 0 & \times & 0 & 0 & \times \\ 0 & \times & 0 & \otimes & \otimes \\ \times & 0 & 0 & 0 & 0 \\ 0 & 0 & \times & 0 & 0 \\ \otimes & \otimes & 0 & 0 & 0 \end{bmatrix}$





(2)

$-7,\ 4,\ -4,\ -3$

$A^TA = A^2$

$7^2\ 4^2\ 3^2$

$\|A\|_2 = \sigma_1 = 7$

$\|A^{-1}\|_2 = \frac{1}{\sigma_4} = \frac{1}{3}$

(3) vl. párov

+ +

− +

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

1, 1, 1

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

1, 1, 0

(4)

$$\det\begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & \cos\alpha & & -\sin\alpha & \\ & & & \ddots & & \\ & & \sin\alpha & & \cos\alpha & \\ & & & & & 1 \\ & & & & & & \ddots \\ & & & & & & & 1 \end{bmatrix} = 1$$

$$A^TA = AA^T = I$$

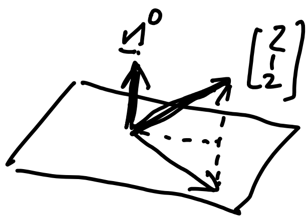
$$\rho(A^TA) = \rho(A)$$

$$Ax = b$$

$$A^TAx = A^Tb$$

(5) $A = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $A^+ = (A^TA)^{-1}A^T = \left(\begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right)^{-1}\begin{bmatrix} a & b & c \end{bmatrix} = \frac{1}{4}\begin{bmatrix} a & b & c \end{bmatrix}$

(5) $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ $\qquad \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \longmapsto \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$



$n = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$ $\qquad n^0 = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$

$$v - 2n^0(n^{0T}v) = (I - 2n^0n^{0T}) =$$

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} - \frac{2}{6}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}\begin{bmatrix} 1 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ -2 & 2 & 4 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & -2 \\ 2 & -2 & -1 \end{bmatrix}$$

⑥ $\begin{bmatrix}2&0&0\\2&2&0\\2&2&2\end{bmatrix}$ $\quad (2-\lambda)^3 \quad \lambda=2 \quad A-\lambda I=\begin{bmatrix}0&0&0\\2&0&0\\2&2&0\end{bmatrix}\xrightarrow{rref}\begin{bmatrix}1&0&0&0\\0&1&0&0\end{bmatrix}$

$\rightsquigarrow \quad x=0$

$y=0$

$z=t$

$\begin{bmatrix}0\\0\\t\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}t$

$\begin{bmatrix}0\\0\\4\end{bmatrix} \quad \begin{bmatrix}0\\2\\2\end{bmatrix} \quad \begin{bmatrix}1\\0\\0\end{bmatrix}$

$0 \leftarrow \bullet \leftarrow \bullet \xleftarrow{A-2I} \bullet$

$C=\begin{bmatrix}0&0&1\\0&2&0\\4&2&0\end{bmatrix} \qquad C^{-1}: \left[\begin{array}{ccc|ccc}0&0&1&1&&\\0&2&0&&1&\\4&2&0&&&1\end{array}\right]\to\left[\begin{array}{ccc|ccc}1&\frac12&0&0&0&\frac14\\0&1&0&0&\frac12&0\\0&0&1&1&0&0\end{array}\right]$

$\to\left[\begin{array}{ccc|ccc}1&&&0&-\frac14&\frac14\\&1&&0&\frac12&0\\&&1&1&0&0\end{array}\right]$

$4\ 2\ 0 \mid 0\ 0\ 1$

$1\ \frac12\ 0 \mid 0\ 0\ \frac14$

$J=\begin{bmatrix}2&1&\\&2&1\\&&2\end{bmatrix}$

$A=CJC^{-1} \qquad Ax=\lambda x \qquad AC=CJ$

⑦

$$\underset{B}{\begin{bmatrix}0&0&1\\1&0&0\\0&0&0\end{bmatrix}}=\underset{C}{\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}}\underset{J}{\begin{bmatrix}0&1&\\&0&1\\&&0\end{bmatrix}}\underset{C^{-1}}{\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}}$$

$J = A$

$f(B)=Cf(A)C^{-1}$

$$\cos(A)=\cos\begin{bmatrix}0&1&\\&0&1\\&&0\end{bmatrix}=\begin{bmatrix}\cos 0&\cos' 0&\frac{\cos''0}{2!}\\0&\cos 0&\cos' 0\\0&0&\cos 0\end{bmatrix}=\begin{bmatrix}1&0&-\frac12\\&1&0\\&&1\end{bmatrix}\to\begin{bmatrix}1&0&0\\0&1&-\frac12\\0&0&1\end{bmatrix}$$

$=\cos B$

⑧

$r=\rho(A)=1 \qquad \lim_{\infty}\left(\frac{A}{r}\right)^n=\frac{pq^T}{q^Tp}=\frac{\begin{bmatrix}1/5\\2/5\\2/5\end{bmatrix}\begin{bmatrix}1/3&1/3&1/3\end{bmatrix}}{\frac13}=\frac15\begin{bmatrix}1&1&1\\2&2&2\\2&2&2\end{bmatrix}$

$A\begin{bmatrix}1\\2\\2\end{bmatrix}=\begin{bmatrix}1\\2\\2\end{bmatrix} \qquad \begin{bmatrix}1&1&1\end{bmatrix}A=\begin{bmatrix}1&1&1\end{bmatrix}$

$p=\begin{bmatrix}1/5\\2/5\\2/5\end{bmatrix} \qquad q=\begin{bmatrix}1/3\\1/3\\1/3\end{bmatrix}$