Quantum Physics

* Schrodinger's equation

$$E = KE + PE$$

$$E = \frac{p^2}{2m} + U$$

$$\therefore E\psi(x) = \frac{p^2}{2m}\psi(x) + U\psi(x) \quad ---(i)$$

$$\psi(x) = Ae^{i(kx-\omega t)}$$

$$\frac{d\psi(x)}{dx} = ik\psi(x)$$

$$\frac{d^2\psi(x)}{dx^2} = i^2k^2\psi(x) \Rightarrow \frac{d^2\psi(x)}{dx^2} = -k^2\psi(x) \quad ---(ii)$$

$$k = \frac{2\pi}{\lambda} \qquad \lambda = \frac{h}{p}$$

$$\therefore k = \frac{2\pi p}{h} \Rightarrow \frac{p}{h/2\pi} = \frac{p}{\hbar}$$

$$\therefore k = \frac{p}{\hbar} \quad ---(iii)$$

$$\frac{d^2\psi(x)}{dx^2} = -\frac{p^2}{\hbar^2}\psi(x) \qquad \text{from (ii) \& (iii)}$$

$$p^2\psi(x) = -\hbar^2\frac{d^2}{dx^2}\psi(x) \quad ----(iv)$$

$$\therefore E\psi(x) = \frac{1}{2m} \times -\hbar^2 \frac{d^2 \psi(x)}{dx^2} + U\psi(x)$$

$$\boxed{(E - U)\psi(x) = \frac{-\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2}}$$

Time independent Schrödinger's Equation

$$E = \hbar\omega = \frac{h}{2\pi} \times 2\pi f = hf$$

$$\psi(x,t) = Ae^{i(kx - \omega t)}$$

$$\frac{d}{dt}\psi(x,t) = -i\omega\,\psi(x,t) \quad \text{---(i)}$$

$$\frac{E}{\hbar} = \omega \Rightarrow \frac{Ei}{\hbar} = i\omega$$

$$-\frac{Ei}{\hbar}\psi(x,t) = -i\omega\,\psi(x,t) \quad \text{---(ii)}$$

from (i) & (ii)

$$-\frac{Ei}{\hbar}\psi(x,t) = \frac{d}{dt}\psi(x,t)$$

$$\therefore E\psi(x,t) = -\frac{\hbar}{i}\frac{d}{dt}\psi(x,t)$$

$$\boxed{\therefore E\psi(x,t) = i\hbar \frac{d}{dt}\psi(x,t)}$$

Putting $E\psi(x,t)$ in TISE

$$\boxed{i\hbar \frac{d}{dt}\psi(x,t) - \frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} - U\psi(x) = 0}$$

TDSE

* Solving Schrödinger's equation for a hydrogen atom

$$E\psi(x) = \frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U\psi(x)$$

for a hydrogen atom

$$U = \frac{-ne^2}{4\pi\epsilon_0 r}$$

$$\boxed{\therefore E\psi = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) - \frac{ze^2}{4\pi\epsilon_0 r}\psi}$$

① $E = KE + PE$

$$E = \frac{1}{2}mv^2 - \frac{ze^2}{4\pi\epsilon_0 r}$$

② $$\frac{mv^2}{r} = \frac{z}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

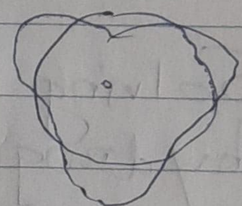
(3) $\Psi = Ae^{ikx}$
$= Ae^{ik(x+L)}$ for a stationary wave

$e^{ikL} = 1$ for stationary wave with single node

$kL = 2\pi$

$kL = n2\pi$

$L = 2\pi r$



$\therefore 2\pi r k = 2\pi n$

$rk = n$

$r\frac{p}{\hbar} = n$

$rp = n\hbar$

$\boxed{mvr = n\hbar}$ Angular momentum is quantized

$m^2v^2r^2 = n^2\hbar^2 = \frac{n^2h^2}{4\pi^2}$

$v^2 = \frac{n^2h^2}{4\pi^2 m^2 r^2}$

Put in (2)

$\frac{\cancel{m}}{r}\frac{n^2h^2}{\cancel{4}\pi^2 m^{\cancel{2}} r^{\cancel{2}}} = \frac{Z}{\cancel{4\pi}\epsilon}\frac{e^2}{\cancel{r^2}}$

$\therefore r = \frac{n^2h^2\epsilon_0}{Ze^2\pi m}$

$$r = \left(\frac{\epsilon_0 h^2}{\pi m e^2}\right)\frac{n^2}{z} \Rightarrow r = a_0 \frac{n^2}{z}$$

Bohr's constant

$$\frac{mv^2}{r} = \frac{z}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

$$\therefore \frac{mv^2}{2} = \frac{z}{8\pi\epsilon_0}\frac{e^2}{r} \quad \text{Put in } (1)$$

$$E = \frac{ze^2}{8\pi\epsilon_0 r} - \frac{ze^2}{4\pi\epsilon_0 r}$$

$$\boxed{\therefore E = -\frac{ze^2}{8\pi\epsilon_0 r}}$$

$$\therefore E = -\frac{ze^2}{8\pi\epsilon_0}\frac{\pi m e^2 z}{\epsilon_0 n^2 h^2}$$

$$\therefore E = -\frac{me^4 z^2}{8\epsilon_0^2 n^2 h^2}$$

$$\therefore E = -\left(\frac{me^4}{8\epsilon_0^2 h^2}\right)\frac{z^2}{n^2} \quad \therefore R\frac{z^2}{n^2}$$

R = Rydberg constant ~13.6 eV

* Schrödinger's equation for a free particle

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

for a free particle $V=0$

$$\therefore -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

$$\therefore \frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}E\psi$$

$$E = \frac{p^2}{2m} \qquad p = \hbar k \qquad k^2 = \frac{p^2}{\hbar^2}$$

$$p^2 = 2mE \Rightarrow k^2\hbar^2 = 2mE$$

$$\therefore k^2 = \frac{2mE}{\hbar^2}$$

$$\frac{d^2\psi}{dx^2} = -k^2\psi$$

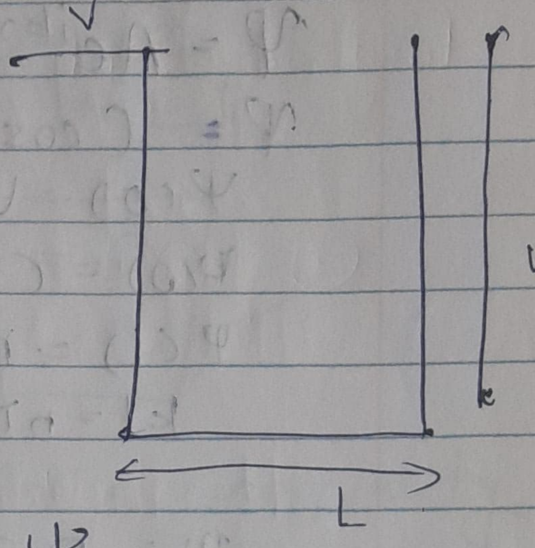
$$\boxed{\frac{d^2\psi}{dx^2} + k^2\psi = 0}$$

solving $\psi = Ae^{ikx} + Be^{-ikx}$

$$p = \frac{nh}{L} \qquad E = \frac{p^2}{2m} \Rightarrow E = \left(\frac{h^2}{2mL^2}\right)n^2$$

$$E \propto n^2$$

* Schrödinger's eqn for particle in a box



$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = (E-V)\psi$$

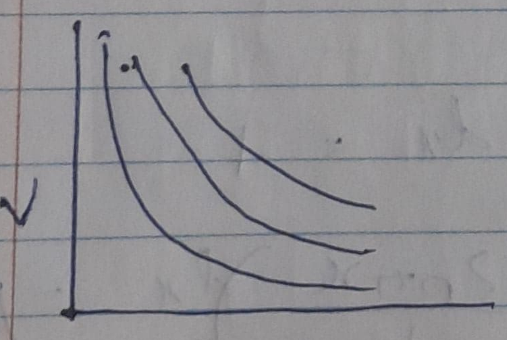
$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = (V-E)\psi$$

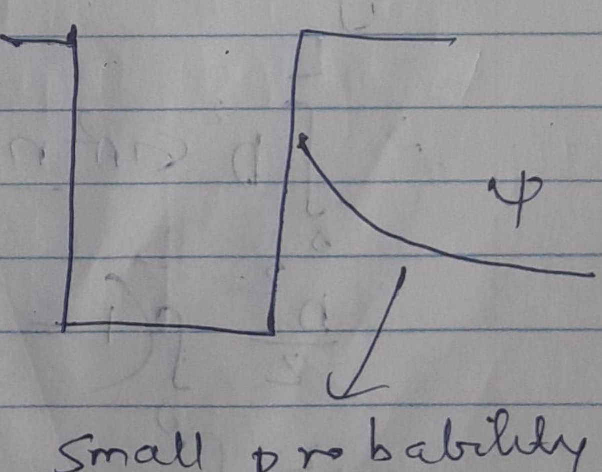
$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi$$

$$\frac{d^2\psi}{dx^2} = k^2\psi \quad \text{where } k^2 = \frac{2m}{\hbar^2}(V-E)$$

$$\psi = Ae^{kx} + Be^{-kx}$$

$\therefore \psi = Be^{-kx}$ since Ae^{kx} leads to exponential growth of ψ





Small probability of finding a particle outside the box

Wave function inside the box

$\psi = Ae^{ikx} + Be^{-ikx}$

$\psi = C\cos kx + D\sin kx$

$\psi(0) = \psi(L) = 0$

$\psi(0) = C = 0$

$\psi(L) = D\sin kL$

$kL = n\pi \Rightarrow k = \frac{n\pi}{L}$

$\therefore \psi = D\sin\frac{n\pi}{L}x$

$E = \frac{p^2}{2m} = \frac{k^2\hbar^2}{2m} = \frac{\hbar^2}{2m}\cdot\frac{n^2\pi^2}{L^2}$

$E = \left(\frac{\hbar^2\pi^2}{2mL^2}\right)n^2$

$\int |\psi|^2 dx = 1$

$\int_0^L D^2\sin^2\frac{n\pi x}{L}\,dx = 1$

$\frac{D^2}{2}\int_0^L\left(1 - \cos\frac{2n\pi x}{L}\right)dx = 1$

$\frac{D^2}{2}\left[L - \left|\frac{\sin\frac{2n\pi}{L}x}{\frac{2n\pi}{L}}\right|_0^L\right] = 1$

$$\therefore \frac{D^2}{2} L = 1$$

$$D^2 = \frac{2}{L} \implies D = \sqrt{\frac{2}{L}}$$

$$\boxed{\therefore \psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n \pi x}{L}\right)}$$

EPR pardox $\longrightarrow$ Bell's inequality