CHAPTER 6

Relational Algebra

Abstract

This chapter discusses the relational algebra operations that underlie both database design and SQL. The chapter emphasizes the procedural nature of relational algebra, and that the operations are used primarily by SQL command processors, rather than issued directly by a user.

Keywords

relational algebra
relational algebra project
relational algebra select
relational algebra restrict
relational algebra join
relational algebra difference
relational algebra intersect
relational algebra divide
relational algebra union

When we use SQL to manipulate data in a database, we are actually using something known as the *relational calculus*, a method for using a single command to instruct the DBMS to perform one or more actions. The DBMS must then break down the SQL command into a set of operations that it can perform one after the other to produce the requested result. These single operations are taken from the *relational algebra*.

Why look at relational algebra here, rather than when we look at SQL in depth? Because relational algebra also underlies some of the theory of relational database design, which we will discuss in Chapter 7.

Note: Don't panic. Although both the relational calculus and the relational algebra can be expressed in the notation of formal logic, there is no need for us to do so. You won't see anything remotely like mathematic notation in this chapter.

In this chapter we will look at seven relational algebra operations. The first five—restrict,[1] project, join, union, and difference—are fundamental to SQL and database design operations. In fact, any DBMS that supports them is said to be *relationally complete*. The remaining operations (product and intersect) are useful for helping us understand how SQL processes queries.

It is possible to design relational databases and use SQL without understanding much about relational algebra. However, you will find it easier to formulate effective, efficient queries if you have an understanding of what the SQL syntax is asking the DBMS to do. There is often more than one way to write a SQL command to obtain a specific result. The commands will often differ in the underlying relational algebra operations required to generate results and therefore may differ significantly in performance. Understanding relational algebra can also help you to resolve some difficult relational database design problems.

Note: The bottom line is: You really do need to read this chapter. Once you've finished reading Chapter 7, where relational algebra gives us the tools to create well designed databases, you can put it aside until we get to material about using SQL to retrieve data. (Notice that the preceding doesn't give you permission to forget about it entirely…)

The most important thing to understand about relational algebra is that each operation does one thing and one thing only. For example, one operation extracts columns while another extracts rows. The DBMS must do the operations one at a time, in a step-by-step sequence. We therefore say that relational algebra is *procedural*. SQL, on the other hand, lets us formulate our queries in a logical way, without necessarily specifying the order in which relational operations should be performed. SQL is, therefore, *non-procedural*.

There is no official syntax for relational algebra. What you will see in this chapter, however, is a relatively consistent way of expressing the operations, without resorting to mathematical symbols. In general, each command has the following format:

```
OPERATION parameters FROM source_table_name(s)
   GIVING result_table_name
```

The parameters vary, depending on the specific operation. They may specify a second table that is part of the operation or they may specify which attributes are to be included in the result.

The result of every relational algebra operation is another table. In most cases, the table will be a relation, but some operations—such as the outer join—may have nulls in primary key columns (preventing the table from having a unique primary key) and therefore will not be legal relations. The result tables are virtual tables that can be used as input to another relational algebra operation.

The Relational Algebra and SQL Example Database: Rare Books

The relational algebra examples in this chapter and most of the SQL examples in this book are taken from a portion of a relational database that supports a rare book dealer. You can find the ER diagram in Figure 6.1. The rare book dealer handles rare fiction editions and some modern fiction. Because many of books are one-of-a-kind, he tracks each volume individually.

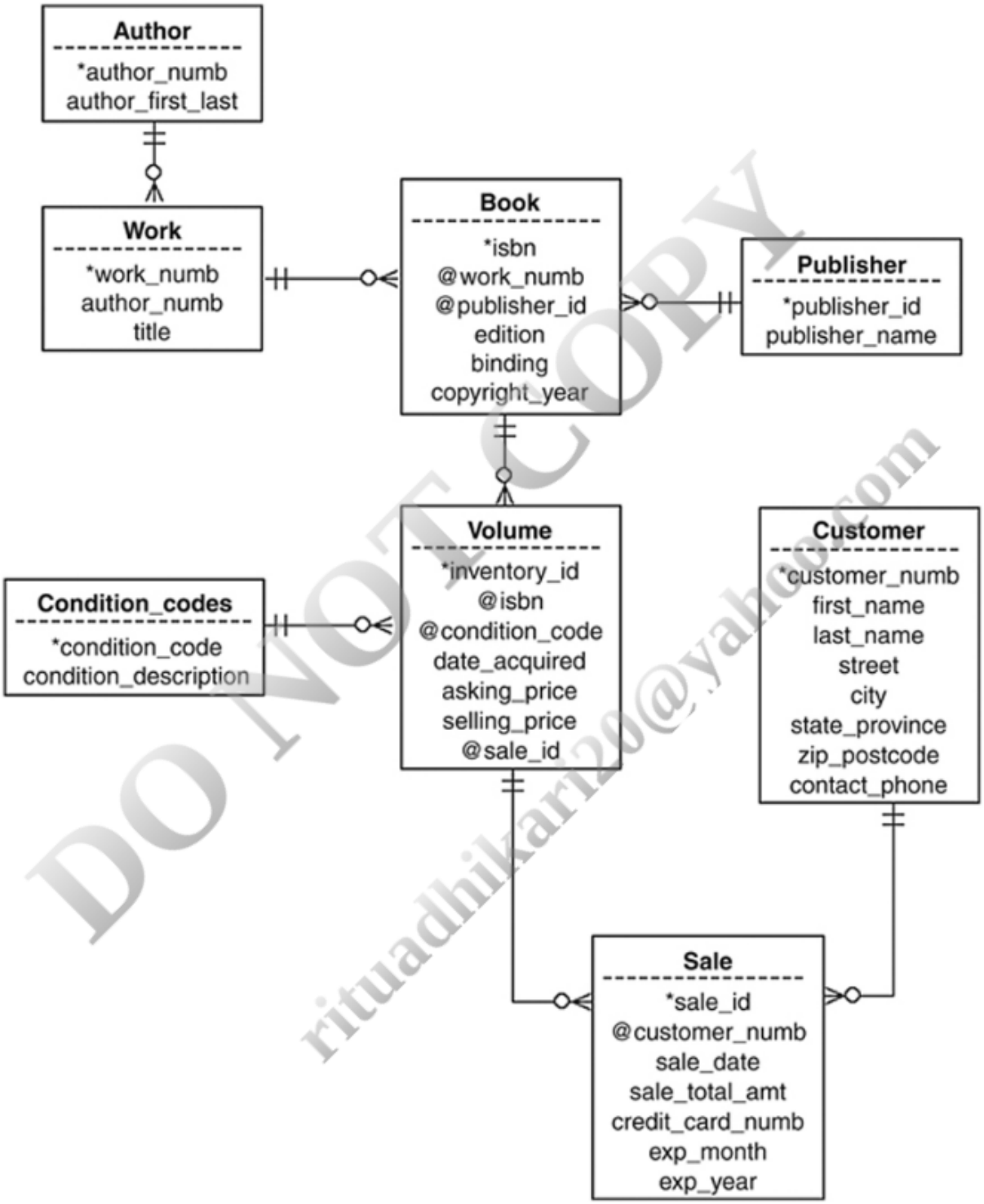


FIGURE 6.1 **The conceptual design of the rare book store database.**

The portion of the database we will be using contains data on customers and the volumes that they purchase. Notice, however, that it really takes three entities to describe a physical volume that is sold. The *work* entity describes a text written by one author, with one title. A *book* is a specific published version of a work (an edition); it is identified by an ISBN.[2] A *volume* is one physical copy of a book. This is the unit that is being sold.

Notice also that many of the text attributes of a *work*/*book*/*volume* are represented by numeric codes that act as foreign keys. For example, a book has a *work_id* that relates it to the work. It also has a *publisher_id* that connects to a table that contains the text of publisher names. Why

design the database this way? Because it is very easy to make mistakes when typing text, especially long text, such as names and titles. Not only is a pain in the neck to type the same text repeatedly, but the chance of typos is fairly high. As you might expect, this is a very undesirable situation: Queries may not return accurate results. For this particular business, we store the text just once, and relate it to *works/books/volume* using integer codes.

The Sample Data

To make it easier to understand the relational algebra and SQL examples, we have loaded the rare bookstore database with tables. The data can be found in Tables 6.1–6.8.

Table 6.1
Publisher

publisher_id	publisher_name
1	Wiley
2	Simon & Schuster
3	Macmillan
4	Tor
5	DAW

Table 6.2
Author

author_numb	author_last_first
1	Bronte, Charlotte
2	Doyle, Sir Arthur Conan
3	Twain, Mark
4	Stevenson, Robert Louis
5	Rand, Ayn
6	Barrie, James
7	Ludlum, Robert
8	Barth, John
9	Herbert, Frank
10	Asimov, Isaac
11	Funke, Cornelia
12	Stephenson, Neal

Table 6.3
Condition Codes

condition_code	condition_description

1	New
2	Excellent
3	Fine
4	Good
5	Poor

Table 6.4

Work

work_numb	author_numb	title
1	1	Jane Eyre
2	1	Villette
3	2	Hound of the Baskervilles
4	2	Lost World, The
5	2	Complete Sherlock Holmes
7	3	Prince and the Pauper
8	3	Tom Sawyer
9	3	Adventures of Huckleberry Finn, The
6	3	Connecticut Yankee in King Arthur's Court, A
13	5	Fountainhead, The
14	5	Atlas Shrugged
15	6	Peter Pan
10	7	Bourne Identity, The
11	7	Matarese Circle, The
12	7	Bourne Supremacy, The
16	4	Kidnapped
17	4	Treasure Island
18	8	Sot Weed Factor, The
19	8	Lost in the Funhouse
20	8	Giles Goat Boy
21	9	Dune
22	9	Dune Messiah
23	10	Foundation
24	10	Last Foundation
25	10	I, Robot
26	11	Inkheart
27	11	Inkdeath
28	12	Anathem
29	12	Snow Crash
30	5	Anthem
31	12	Cryptonomicon

Table 6.5

Books

isbn	work_numb	publisher_id	edition	binding	copyright_year
978-1-11111-111-1	1	1	2	Board	1857
978-1-11111-112-1	1	1	1	Board	1847
978-1-11111-113-1	2	4	1	Board	1842
978-1-11111-114-1	3	4	1	Board	1801
978-1-11111-115-1	3	4	10	Leather	1925
978-1-11111-116-1	4	3	1	Board	1805
978-1-11111-117-1	5	5	1	Board	1808
978-1-11111-118-1	5	2	19	Leather	1956
978-1-11111-120-1	8	4	5	Board	1906
978-1-11111-119-1	6	2	3	Board	1956
978-1-11111-121-1	8	1	12	Leather	1982
978-1-11111-122-1	9	1	12	Leather	1982
978-1-11111-123-1	11	2	1	Board	1998
978-1-11111-124-1	12	2	1	Board	1989
978-1-11111-125-1	13	2	3	Board	1965
978-1-11111-126-1	13	2	9	Leather	2001
978-1-11111-127-1	14	2	1	Board	1960
978-1-11111-128-1	16	2	12	Board	1960
978-1-11111-129-1	16	2	14	Leather	2002
978-1-11111-130-1	17	3	6	Leather	1905
978-1-11111-131-1	18	4	6	Board	1957
978-1-11111-132-1	19	4	1	Board	1962
978-1-11111-133-1	20	4	1	Board	1964
978-1-11111-134-1	21	5	1	Board	1964
978-1-11111-135-1	23	5	1	Board	1962
978-1-11111-136-1	23	5	4	Leather	2001
978-1-11111-137-1	24	5	4	Leather	2001
978-1-11111-138-1	23	5	4	Leather	2001
978-1-11111-139-1	25	5	4	Leather	2001
978-1-11111-140-1	26	5	1	Board	2001
978-1-11111-141-1	27	5	1	Board	2005
978-1-11111-142-1	28	5	1	Board	2008
978-1-11111-143-1	29	5	1	Board	1992
978-1-11111-144-1	30	1	1	Board	1952
978-1-11111-145-1	30	5	1	Board	2001
978-1-11111-146-1	31	5	1	Board	1999

Table 6.6

Volume

inventory_id	isbn	condition_ code	date_acquired	asking_price	selling_price	sale_id
1	978-1-11111-111-1	3	12-JUN-19 00:00:00	175.00	175.00	1
2	978-1-11111-131-1	4	23-JAN-20 00:00:00	50.00	50.00	1
7	978-1-11111-137-1	2	20-JUN-19 00:00:00	80.00		
3	978-1-11111-133-1	2	05-APR-18 00:00:00	300.00	285.00	1
4	978-1-11111-142-1	1	05-APR-18 00:00:00	25.95	25.95	2
5	978-1-11111-146-1	1	05-APR-18 00:00:00	22.95	22.95	2
6	978-1-11111-144-1	2	15-MAY-19 00:00:00	80.00	76.10	2
8	978-1-11111-137-1	3	20-JUN-20 00:00:00	50.00		
9	978-1-11111-136-1	1	20-DEC-19 00:00:00	75.00		
10	978-1-11111-136-1	2	15-DEC-19 00:00:00	50.00		
11	978-1-11111-143-1	1	05-APR-20 00:00:00	25.00	25.00	3
12	978-1-11111-132-1	1	12-JUN-20 00:00:00	15.00	15.00	3
13	978-1-11111-133-1	3	20-APR-20 00:00:00	18.00	18.00	3
15	978-1-11111-121-1	2	20-APR-20 00:00:00	110.00	110.00	5
14	978-1-11111-121-1	2	20-APR-20 00:00:00	110.00	110.00	4
16	978-1-11111-121-1	2	20-APR-20 00:00:00	110.00		
17	978-1-11111-124-1	2	12-JAN-21 00:00:00	75.00		
18	978-1-11111-146-1	1	11-MAY-20 00:00:00	30.00	30.00	6
19	978-1-11111-122-1	2	06-MAY-20 00:00:00	75.00	75.00	6
20	978-1-11111-130-1	2	20-APR-20 00:00:00	150.00	120.00	6
21	978-1-11111-126-1	2	20-APR-20 00:00:00	110.00	110.00	6
22	978-1-11111-139-1	2	16-MAY-20 00:00:00	200.00	170.00	6
23	978-1-11111-125-1	2	16-MAY-20 00:00:00	45.00	45.00	7
24	978-1-11111-131-1	3	20-APR-20 00:00:00	35.00	35.00	7
25	978-1-11111-126-1	2	16-NOV-20 00:00:00	75.00	75.00	8
26	978-1-11111-133-1	3	16-NOV-20 00:00:00	35.00	55.00	8
27	978-1-11111-141-1	1	06-NOV-20 00:00:00	24.95		

28	978-1-11111-141-1	1	06-NOV-20 00:00:00	24.95		
29	978-1-11111-141-1	1	06-NOV-20 00:00:00	24.95		
30	978-1-11111-145-1	1	06-NOV-20 00:00:00	27.95		
31	978-1-11111-145-1	1	06-NOV-20 00:00:00	27.95		
32	978-1-11111-145-1	1	06-NOV-20 00:00:00	27.95		
33	978-1-11111-139-1	2	06-OCT-20 00:00:00	75.00	50.00	9
34	978-1-11111-133-1	1	16-NOV-20 00:00:00	125.00	125.00	10
35	978-1-11111-126-1	1	06-OCT-20 00:00:00	75.00	75.00	11
36	978-1-11111-130-1	3	06-DEC-19 00:00:00	50.00	50.00	11
37	978-1-11111-136-1	3	06-DEC-19 00:00:00	75.00	75.00	11
38	978-1-11111-130-1	2	06-APR-20 00:00:00	200.00	150.00	12
39	978-1-11111-132-1	3	06-APR-20 00:00:00	75.00	75.00	12
40	978-1-11111-129-1	1	06-APR-20 00:00:00	25.95	25.95	13
41	978-1-11111-141-1	1	16-MAY-20 00:00:00	40.00	40.00	14
42	978-1-11111-141-1	1	16-MAY-20 00:00:00	40.00	40.00	14
43	978-1-11111-132-1	1	12-NOV-20 00:00:00	17.95		
44	978-1-11111-138-1	1	12-NOV-20 00:00:00	75.95		
45	978-1-11111-138-1	1	12-NOV-20 00:00:00	75.95		
46	978-1-11111-131-1	3	12-NOV-20 00:00:00	15.95		
47	978-1-11111-140-1	3	12-NOV-20 00:00:00	25.95		
48	978-1-11111-123-1	2	16-AUG-20 00:00:00	24.95		
49	978-1-11111-127-1	2	16-AUG-20 00:00:00	27.95		
50	978-1-11111-127-1	2	06-JAN-21 00:00:00	50.00	50.00	15
51	978-1-11111-141-1	2	06-JAN-21 00:00:00	50.00	50.00	15
52	978-1-11111-141-1	2	06-JAN-21 00:00:00	50.00	50.00	16
53	978-1-11111-123-1	2	06-JAN-21 00:00:00	40.00	40.00	16
54	978-1-11111-127-1	2	06-JAN-21 00:00:00	40.00	40.00	16
55	978-1-11111-133-1	2	06-FEB-21 00:00:00	60.00	60.00	17
56	978-1-11111-127-1	2	16-FEB-21 00:00:00	40.00	40.00	17
57	978-1-11111-135-1	2	16-FEB-21 00:00:00	40.00	40.00	18
59	978-1-11111-127-1	2	25-FEB-21 00:00:00	35.00	35.00	18
58	978-1-11111-131-1	2	16-FEB-21 00:00:00	25.00	25.00	18
60	978-1-11111-128-1	2	16-DEC-20 00:00:00	50.00	45.00	
61	978-1-11111-136-1	3	22-OCT-20 00:00:00	50.00	50.00	19
62	978-1-11111-115-1	2	22-OCT-20 00:00:00	75.00	75.00	20
63	978-1-11111-130-1	2	16-JUL-20 00:00:00	500.00		
64	978-1-11111-136-1	2	06-MAR-20 00:00:00	125.00		
65	978-1-11111-136-1	2	06-MAR-20 00:00:00	125.00		
66	978-1-11111-137-1	2	06-MAR-20 00:00:00	125.00		
67	978-1-11111-137-1	2	06-MAR-20 00:00:00	125.00		
68	978-1-11111-138-1	2	06-MAR-20 00:00:00	125.00		
69	978-1-11111-138-1	2	06-MAR-20 00:00:00	125.00		
70	978-1-11111-139-1	2	06-MAR-20 00:00:00	125.00		
71	978-1-11111-139-1	2	06-MAR-20 00:00:00	125.00		

Table 6.7

Customer

customer_numb	first_name	last_name	street	city	state_province	zip_postcode	contact_phone
1	Janice	Jones	125 Center Road	Anytown	NY	11111	518-555-1111
2	Jon	Jones	25 Elm Road	Next Town	NJ	18888	209-555-2222
3	John	Doe	821 Elm Street	Next Town	NJ	18888	209-555-3333
4	Jane	Doe	852 Main Street	Anytown	NY	11111	518-555-4444
5	Jane	Smith	1919 Main Street	New Village	NY	13333	518-555-5555
6	Janice	Smith	800 Center Road	Anytown	NY	11111	518-555-6666
7	Helen	Brown	25 Front Street	Anytown	NY	11111	518-555-7777
8	Helen	Jerry	16 Main Street	Newtown	NJ	18886	518-555-8888
9	Mary	Collins	301 Pine Road, Apt. 12	Newtown	NJ	18886	518-555-9999
10	Peter	Collins	18 Main Street	Newtown	NJ	18886	518-555-1010
11	Edna	Hayes	209 Circle Road	Anytown	NY	11111	518-555-1110
12	Franklin	Hayes	615 Circle Road	Anytown	NY	11111	518-555-1212
13	Peter	Johnson	22 Rose Court	Next Town	NJ	18888	209-555-1212
14	Peter	Johnson	881 Front Street	Next Town	NJ	18888	209-555-1414
15	John	Smith	881 Manor Lane	Next Town	NJ	18888	209-555-1515

Table 6.8

Table 6.8

Sale

sale_id	customer_numb	sale_date	sale_total_amt	credit_card_numb	exp_month	exp_year
3	1	15-JUN-21 00:00:00	58.00	1234 5678 9101 1121	10	18
4	4	30-JUN-21 00:00:00	110.00	1234 5678 9101 5555	7	17
5	6	30-JUN-21 00:00:00	110.00	1234 5678 9101 6666	12	17
6	12	05-JUL-21 00:00:00	505.00	1234 5678 9101 7777	7	16
7	8	05-JUL-21 00:00:00	80.00	1234 5678 9101 8888	8	16
8	5	07-JUL-21 00:00:00	90.00	1234 5678 9101 9999	9	15
9	8	07-JUL-21 00:00:00	50.00	1234 5678 9101 8888	8	16
10	11	10-JUL-21 00:00:00	125.00	1234 5678 9101 1010	11	16
11	9	10-JUL-21 00:00:00	200.00	1234 5678 9101 0909	11	15
12	10	10-JUL-21 00:00:00	200.00	1234 5678 9101 0101	10	15
13	2	10-JUL-21 00:00:00	25.95	1234 5678 9101 2222	2	15
14	6	10-JUL-21 00:00:00	80.00	1234 5678 9101 6666	12	17
15	11	12-JUL-21 00:00:00	75.00	1234 5678 9101 1231	11	17
16	2	25-JUL-21 00:00:00	130.00	1234 5678 9101 2222	2	15
17	1	25-JUL-21 00:00:00	100.00	1234 5678 9101 1121	10	18
18	5	22-AUG-21 00:00:00	100.00	1234 5678 9101 9999	9	15
2	1	05-JUN-21 00:00:00	125.00	1234 5678 9101 1121	10	18
1	1	29-MAY-21 00:00:00	510.00	1234 5678 9101 1121	10	18
19	6	01-SEP-21 00:00:00	95.00	1234 5678 9101 7777	7	16
20	2	01-SEP-21 00:00:00	75.00	1234 5678 9101 2222	2	15

Making Vertical Subsets: Project

The first relational algebra operation we will consider is one that is used by every SQL query that retrieves data: project. A *projection* of a relation is a new relation created by copying one or more the columns from the source relation into a new table. As an example, consider Figure 6.2. The result table (arbitrarily called *names_and_numbers*) is a projection of the *customer* relation, containing the attributes *customer_numb*, *first_name*, and *last_name*.

customer_numb	first_name	last_name	street	city	state_province	zip_postcode	contact_phone
1	Janice	Jones	125 Center Road	Anytown	NY	11111	518-555-1111
2	Jon	Jones	25 Elm Road	Next Town	NJ	18888	209-555-2222
3	John	Doe	821 Elm Street	Next Town	NJ	18888	209-555-3333
4	Jane	Doe	852 Main Street	Anytown	NY	11111	518-555-4444
5	Jane	Smith	1919 Main Street	New Village	NY	13333	518-555-5555
6	Janice	Smith	800 Center Road	Anytown	NY	11111	518-555-6666
7	Helen	Brown	25 Front Street	Anytown	NY	11111	518-555-7777
8	Helen	Jerry	16 Main Street	Newtown	NJ	18886	518-555-8888
9	Mary	Collins	301 Pine Road, Apt. 12	Newtown	NJ	18886	518-555-9999
10	Peter	Collins	18 Main Street	Newtown	NJ	18886	518-555-1010
11	Edna	Hayes	209 Circle Road	Anytown	NY	11111	518-555-1110
12	Franklin	Hayes	615 Circle Road	Anytown	NY	11111	518-555-1212
13	Peter	Johnson	22 Rose Court	Next Town	NJ	18888	209-555-1212
14	Peter	Johnson	881 Front Street	Next Town	NJ	18888	209-555-1414

```
PROJECT customer_numb, first_name, last_name
FROM customer GIVING Names_and_numbers
```

customer_numb	first_name	last_name
1	Janice	Jones
2	Jon	Jones
3	John	Doe
4	Jane	Doe
5	Jane	Smith
6	Janice	Smith
7	Helen	Brown
8	Helen	Jerry
9	Mary	Collins
10	Peter	Collins
11	Edna	Hayes
12	Franklin	Hayes
13	Peter	Johnson
14	Peter	Johnson

```
14 | Peter     | Johnson |
```

FIGURE 6.2 **Taking a projection.**

Using the syntax for relational algebra, the projection in Figure 6.2 is written:

```
PROJECT customer_rows, first_name, last_name
FROM customer GIVING names_and_numbers
```

The order of the columns in the result table is based on the order in which the column names appear in the project statement; the order in which they are defined in the source table has no effect on the result. Rows appear in the order in which they are stored in the source table; project does not include sorting or ordering the data in any way. As with all relational algebra operations, duplicate rows are removed.

Note: It is important to keep in mind that relational algebra is first and foremost a set of theoretical operations. A DBMS may not implement an operation the same way that it is described in theory. For example, most DBMSs don't remove duplicate rows from result tables unless the user requests it explicitly. Why? Because to remove duplicates the DBMS must sort the result table by every column (so that duplicate rows will be next to one another), and then scan the table from top to bottom, looking for the duplicates. This can be a very slow process if the result table is large.

Whenever you issue a SQL command that asks for specific columns—just about every retrieval command—you are asking the DBMS to perform the *project* operation. Project is a very fast operation because the DBMS does not need to evaluate any of the data in the table.

There is one issue with project with which you need to be concerned. A DBMS will project any columns that you request. It makes no judgment as to whether the selected columns produce a meaningful result. For example, consider the following operation:

```
PROJECT sale_total_amt, exp_month FROM sale GIVING invalid
```

In theory, there is absolutely nothing wrong with this projection. However, it probably doesn't mean much to associate a dollar amount with a credit card expiration month. Notice in Figure 6.3 that because there is more than one sale with the same total cost (for example, $110), the same sale value is associated with more than one expiration month. We could create a concatenated primary key for the result table using both columns, but that still would not make the resulting table meaningful in the context of the database environment. There is no set of rules as to what constitutes a meaningful projection. Judgments as to the usefulness of projections depend solely on the meaning of the data the database is trying to capture.

sale_id	customer_numb	sale_date	sale_total_amt	credit_card_numb	exp_month	exp_year
3	1	15-JUN-13	58.00	1234 5678 9101 1121	10	18
4	4	30-JUN-13	110.00	1234 5678 9101 5555	7	17
5	6	30-JUN-13	110.00	1234 5678 9101 6666	12	17
6	12	05-JUL-13	505.00	1234 5678 9101 7777	7	16
7	8	05-JUL-13	80.00	1234 5678 9101 8888	8	16
8	5	07-JUL-13	90.00	1234 5678 9101 9999	9	15
9	8	07-JUL-13	50.00	1234 5678 9101 8888	8	16
10	11	10-JUL-13	125.00	1234 5678 9101 1010	11	16
11	9	10-JUL-13	200.00	1234 5678 9101 0909	11	15
12	10	10-JUL-13	200.00	1234 5678 9101 0101	10	15
13	2	10-JUL-13	25.95	1234 5678 9101 2222	2	15
14	6	10-JUL-13	80.00	1234 5678 9101 6666	12	17
15	11	12-JUL-13	75.00	1234 5678 9101 1231	11	17
16	2	25-JUL-13	130.00	1234 5678 9101 2222	2	15
17	1	25-JUL-13	100.00	1234 5678 9101 1121	10	18
18	5	22-AUG-13	100.00	1234 5678 9101 9999	9	15
2	1	05-JUN-13	125.00	1234 5678 9101 1121	10	18
1	1	29-MAY-13	510.00	1234 5678 9101 1121	10	18
19	6	01-SEP-13	95.00	1234 5678 9101 7777	7	16
20	2	01-SEP-13	75.00	1234 5678 9101 2222	2	15

```
PROJECT sale_total_amt, exp_month
FROM sale GIVING invalid
```

sale_total_amt	exp_month
58.00	10
110.00	7
110.00	12
505.00	7
80.00	8
90.00	9
50.00	8
125.00	11
200.00	11
200.00	10
25.95	2
80.00	12
75.00	11
130.00	2

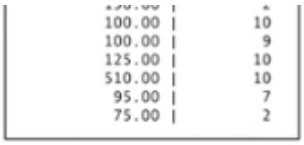


FIGURE 6.3 **An invalid projection.**

Making Horizontal Subsets: Restrict

The *restrict* operation asks a DBMS to choose rows that meet some logical criteria. As defined in the relational algebra, restrict copies rows from the source relation into a result table. Restrict copies all attributes; it has no way to specify which attributes should be included in the result table.

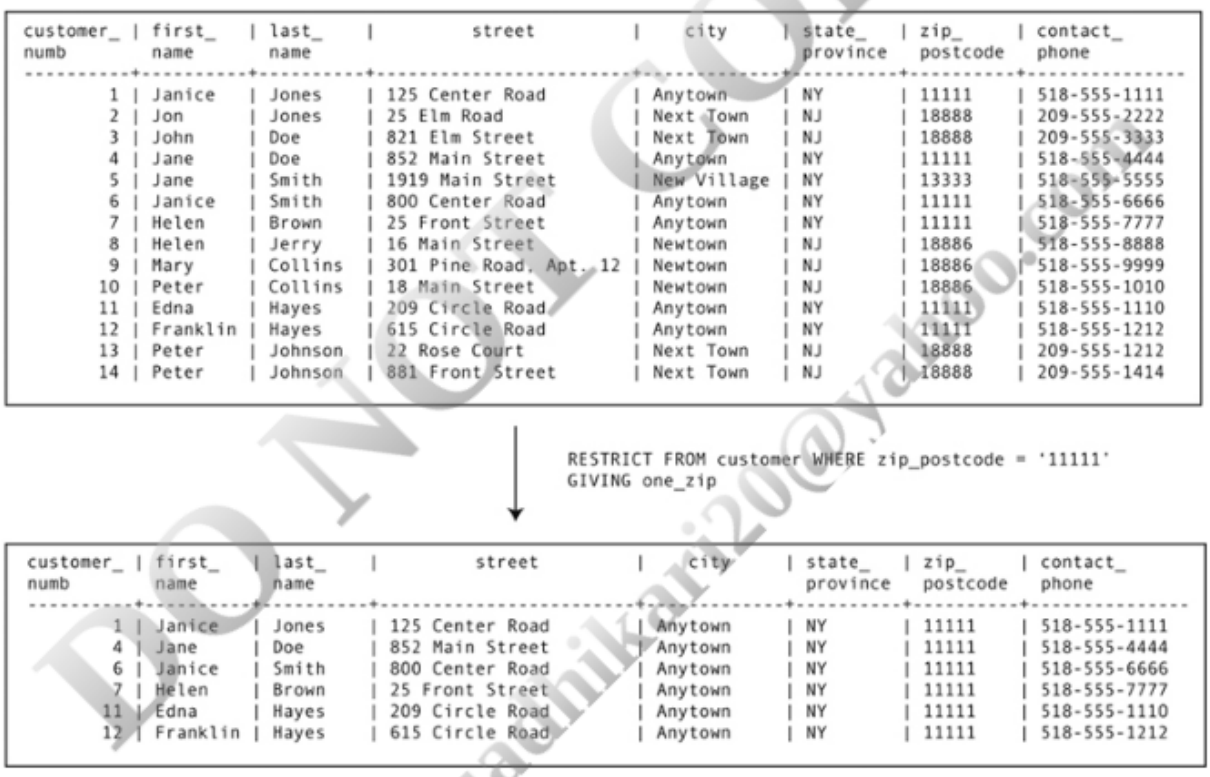
Restrict identifies which rows are to be included in the result table, with a logical expression known as a *predicate*. The operation, therefore, takes the following general form:

```
RESTRICT FROM source_table_name
  WHERE predicate GIVING result_table_name
```

For example, suppose we want to retrieve data about customers who live in zip code 11111. The operation might be expressed as

```
RESTRICT FROM customer WHERE zip_postcode = '11111'
   GIVING one_zip
```

The operation appears in Figure 6.4. The result table includes the entire row for each customer that has a value of 11111 in the *zip_postcode* column.



customer_numb	first_name	last_name	street	city	state_province	zip_postcode	contact_phone
1	Janice	Jones	125 Center Road	Anytown	NY	11111	518-555-1111
2	Jon	Jones	25 Elm Road	Next Town	NJ	18888	209-555-2222
3	John	Doe	821 Elm Street	Next Town	NJ	18888	209-555-3333
4	Jane	Doe	852 Main Street	Anytown	NY	11111	518-555-4444
5	Jane	Smith	1919 Main Street	New Village	NY	13333	518-555-5555
6	Janice	Smith	800 Center Road	Anytown	NY	11111	518-555-6666
7	Helen	Brown	25 Front Street	Anytown	NY	11111	518-555-7777
8	Helen	Jerry	16 Main Street	Newtown	NJ	18886	518-555-8888
9	Mary	Collins	301 Pine Road, Apt. 12	Newtown	NJ	18886	518-555-9999
10	Peter	Collins	18 Main Street	Newtown	NJ	18886	518-555-1010
11	Edna	Hayes	209 Circle Road	Anytown	NY	11111	518-555-1110
12	Franklin	Hayes	615 Circle Road	Anytown	NY	11111	518-555-1212
13	Peter	Johnson	22 Rose Court	Next Town	NJ	18888	209-555-1212
14	Peter	Johnson	881 Front Street	Next Town	NJ	18888	209-555-1414

customer_numb	first_name	last_name	street	city	state_province	zip_postcode	contact_phone
1	Janice	Jones	125 Center Road	Anytown	NY	11111	518-555-1111
4	Jane	Doe	852 Main Street	Anytown	NY	11111	518-555-4444
6	Janice	Smith	800 Center Road	Anytown	NY	11111	518-555-6666
7	Helen	Brown	25 Front Street	Anytown	NY	11111	518-555-7777
11	Edna	Hayes	209 Circle Road	Anytown	NY	11111	518-555-1110
12	Franklin	Hayes	615 Circle Road	Anytown	NY	11111	518-555-1212

FIGURE 6.4 **Restricting rows from a relation.**

Note: There are many operators that can be used to create a restrict predicate, some of which are unique to SQL. You will begin to read about constructing predicates in Chapter 16.

Choosing Columns and Rows: Restrict and Then Project

As we said at the beginning of this chapter, most SQL queries require more than one relational algebra operation. We might, for example, want to see just the names of the customers that live in zip code 11111. Because such a query requires both a *restrict* and a *project*, it takes two steps:

1. Restrict the rows to those with customers that live in zip code 1111.
2. Project the first and last name columns.

In some cases, the order of the restrict and project may not matter. However, in this particular example, the restrict must be performed first. Why? Because the project removes the column needed for the restrict predicate from the intermediate result table, which would make it impossible to perform the restrict.

It is up to a DBMS to determine the order in which it will perform relational algebra operations to obtain a requested result. A *query optimizer* takes care of making the decisions. When more than one series of operations will generate the same result, the query optimizer attempts to determine which will provide the best performance, and will then execute that strategy.

Note: There is a major tradeoff for a DBMS when it comes to query optimization. Picking the most efficient query strategy can result in the shortest query execution time, but it is also possible for the DBMS to spend so much time figuring out which strategy is best that it consumes any

shortest query execution time, but it is also possible for the DBMS to spend so much time figuring out which strategy is best that it consumes any performance advantage that might be had by executing the best strategy. Therefore, the query strategy used by a DBMS may not be the theoretically most efficient strategy, but it is the most efficient strategy that can be identified relatively quickly.

Union

The *union* operation creates a new table by placing all rows from two source tables into a single result table, placing the rows on top of one another. As an example of how a union works, assume that you have the two tables at the top of Figure 6.5. The operation

```
in_print_books UNION out_of_print_books GIVING union_result
```

produces the result table at the bottom of Figure 6.5.

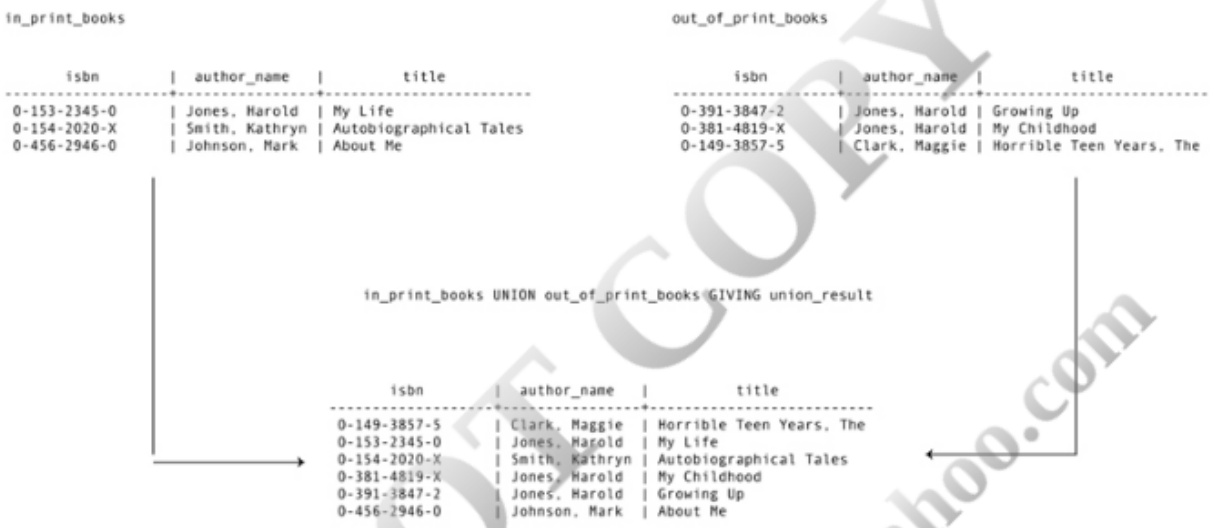


FIGURE 6.5 **The union operation.**

For a union operation to be possible, the two source tables must be *union compatible*. In the relational algebra sense, this means that their columns must be defined over the same domains. The tables must have the same columns, but the columns do not necessarily need to be in the same order or be the same size.

In practice, however, the rules for union compatibility are stricter. The two source tables on which the union is performed must have columns with the same data types and sizes, in the same order. As you will see, in SQL the two source tables are actually virtual tables created by two independent retrieval statements, which are then combined by the union operation.

Join

Join is arguably the most useful relational algebra operation, because it combines two tables into one, usually via a primary key–foreign key relationship. Unfortunately, a join can also be an enormous drain on database performance.

A Non-Database Example

To help you understand how a join works, we will begin with an example that has absolutely nothing to do with relations. Assume that you have been given the task of creating manufacturing part assemblies by connecting two individual parts. The parts are classified as either A parts or B parts.

There are many types of A parts (A1 through A*n*, where *n* is the total number of types of A parts) and many types of B parts (B1 through B*n*). Each B is to be matched to the A part with the same number; conversely, an A part is to be matched to a B part with the same number.

The assembly process requires four bins. One contains the A parts, one contains the B parts, and one will hold the completed assemblies. The remaining bin will hold parts that cannot be matched. (The unmatched parts bin is not strictly necessary; it is simply for your convenience.)

You begin by extracting a part from the B bin. You look at the part to determine the A part to which it should be connected. Then, you search the A bin for the correct part. If you can find a matching A part, you connect the two pieces and toss them into the completed assemblies bin. If you cannot find a matching A part, then you toss the unmatched B part into the bin that holds unmatched B parts. You repeat this process until the B bin is empty. Any unmatched A parts will be left in their original location.

Note: You could just as easily have started with the bin containing the A parts. The contents of the bin holding the completed assemblies will be the same.

As you might guess, the A bins and B bins are analogous to tables that have a primary to foreign key relationship. This matching of part numbers is very much like the matching of data that occurs when you perform a join. The completed assembly bin corresponds to the result table of the operation. As you read about the operation of a join, keep in mind that the parts that could not be matched were left out of the completed assemblies bin.

The Equi-Join

In its most common form, a join forms new rows when data in the two source tables match. Because we are looking for rows with equal values, this type of join is known as an *equi-join* (or a *natural equi-join*). It is also often called an *inner join*. As an example, consider the join in Figure 6.6.

customer_data

customer_numb	first_name	last_name
1	Janice	Jones
2	Jon	Jones
3	John	Doe
4	Jane	Doe
5	Jane	Smith
6	Janice	Smith
7	Helen	Brown
8	Helen	Jerry
9	Mary	Collins
10	Peter	Collins
11	Edna	Hayes
12	Franklin	Hayes
13	Peter	Johnson
14	Peter	Johnson
15	John	Smith

sale_data

sale_id	customer_numb	sale_date	sale_total_amt
3	1	15-JUN-13 00:00:00	58.00
4	4	30-JUN-13 00:00:00	110.00
5	6	30-JUN-13 00:00:00	110.00
6	12	05-JUL-13 00:00:00	505.00
7	8	05-JUL-13 00:00:00	80.00
8	5	07-JUL-13 00:00:00	90.00
9	8	07-JUL-13 00:00:00	50.00
10	11	10-JUL-13 00:00:00	125.00
11	9	10-JUL-13 00:00:00	200.00
12	10	10-JUL-13 00:00:00	200.00
13	2	10-JUL-13 00:00:00	25.95
14	6	10-JUL-13 00:00:00	80.00
15	11	12-JUL-13 00:00:00	75.00
16	2	25-JUL-13 00:00:00	130.00
17	1	25-JUL-13 00:00:00	100.00
18	5	22-AUG-13 00:00:00	100.00
2	1	05-JUN-13 00:00:00	125.00
1	1	29-MAY-13 00:00:00	510.00
19	6	01-SEP-13 00:00:00	95.00
20	2	01-SEP-13 00:00:00	75.00

JOIN cusstomer_data TO sale_data OVER customer_numb GIVING joined_table

joined_table

customer_numb	first_name	last_name	sale_id	sale_date	sale_total_amt
1	Janice	Jones	3	15-JUN-13 00:00:00	58.00
4	Jane	Doe	4	30-JUN-13 00:00:00	110.00
6	Janice	Smith	5	30-JUN-13 00:00:00	110.00
12	Franklin	Hayes	6	05-JUL-13 00:00:00	505.00
8	Helen	Jerry	7	05-JUL-13 00:00:00	80.00
5	Jane	Smith	8	07-JUL-13 00:00:00	90.00
8	Helen	Jerry	9	07-JUL-13 00:00:00	50.00
11	Edna	Hayes	10	10-JUL-13 00:00:00	125.00
9	Mary	Collins	11	10-JUL-13 00:00:00	200.00
10	Peter	Collins	12	10-JUL-13 00:00:00	200.00
2	Jon	Jones	13	10-JUL-13 00:00:00	25.95
6	Janice	Smith	14	10-JUL-13 00:00:00	80.00
11	Edna	Hayes	15	12-JUL-13 00:00:00	75.00
2	Jon	Jones	16	25-JUL-13 00:00:00	130.00
1	Janice	Jones	17	25-JUL-13 00:00:00	100.00
5	Jane	Smith	18	22-AUG-13 00:00:00	100.00
1	Janice	Jones	2	05-JUN-13 00:00:00	125.00
1	Janice	Jones	1	29-MAY-13 00:00:00	510.00
6	Janice	Smith	19	01-SEP-13 00:00:00	95.00
2	Jon	Jones	20	01-SEP-13 00:00:00	75.00

FIGURE 6.6 **An equi-join.**

Notice that the *customer_numb* column is the primary key of the *customer_data* table, and that the same column is a foreign key in the *sale_data* table. The *customer_numb* column in *sale_data* therefore serves to relate sales to the customers to which they belong.

Assume that you want to see the names of the customers who placed each order. To do so, you must join the two tables, creating combined rows wherever there is a matching *customer_numb*. In database terminology, we are joining the two tables *over customer_numb*. The result table, *joined_table*, can be found at the bottom of Figure 6.6.

An equi-join can begin with either source table. (The result should be the same, regardless of the direction in which the join is performed.) The join compares each row in one source table with the rows in the second. For each row in the first source table that matches data in the second source table in the column or columns over which the join is being performed, a new row is placed in the result table.

Assume that we are using the *customer_data* table as the first source table, producing the result table in Figure 6.6. The join might, therefore, proceed conceptually as follows:

1. Search *sale_data* for rows with a *customer_numb* of 1. There are four matching rows in *sale_data*. Create four new rows in the result table, placing the same customer information in each row, along with the data from *sale_data*.
2. Search *sale_data* for rows with a *customer_numb* of 2. Because there are three rows for customer 2 in *sale_data*, add three rows to the result table.
3. Search *sale_data* for rows with a *customer_numb* of 3. Because there are no matching rows in *sale_data*, do not place a row in the result table.
4. Continue as established until all rows from *customer_data* have been compared to *sale_data*.

If the value from the *customer_numb* column does not appear in both tables, then no row is placed in the result table. This behavior categorizes this type of join as an inner join. (Yes, there is such a thing as an outer join. You will read about it shortly.)

What's Really Going On: Product and Restrict

From a relational algebra point of view, a join can be implemented using two other operations: product and restrict. As you will see, this sequence

From a relational algebra point of view, a join can be implemented using two other operations: product and restrict. As you will see, this sequence of operations requires the manipulation of a great deal of data and, if implemented by a DBMS, can result in slow query performance. Many of today's DBMSs therefore use alternative techniques for processing joins. Nonetheless, the concept of using product followed by restrict underlies the original SQL join syntax.

The *product* operation (the mathematical Cartesian product) makes every possible pairing of rows from two source tables. The product of the tables in Figure 6.6 produces a result table with 300 rows (the 15 rows in *customer_data* times the 20 rows in *sale_data*), the first 60 of which appear in Figure 6.7.

customer_numb	first_name	last_name	sale_id	customer_numb	sale_date	sale_total_amt
1	Janice	Jones	3	1	15-JUN-13 00:00:00	58.00
2	Jon	Jones	3	1	15-JUN-13 00:00:00	58.00
3	John	Doe	3	1	15-JUN-13 00:00:00	58.00
4	Jane	Doe	3	1	15-JUN-13 00:00:00	58.00
5	Jane	Smith	3	1	15-JUN-13 00:00:00	58.00
6	Janice	Smith	3	1	15-JUN-13 00:00:00	58.00
7	Helen	Brown	3	1	15-JUN-13 00:00:00	58.00
8	Helen	Jerry	3	1	15-JUN-13 00:00:00	58.00
9	Mary	Collins	3	1	15-JUN-13 00:00:00	58.00
10	Peter	Collins	3	1	15-JUN-13 00:00:00	58.00
11	Edna	Hayes	3	1	15-JUN-13 00:00:00	58.00
12	Franklin	Hayes	3	1	15-JUN-13 00:00:00	58.00
13	Peter	Johnson	3	1	15-JUN-13 00:00:00	58.00
14	Peter	Johnson	3	1	15-JUN-13 00:00:00	58.00
15	John	Smith	3	1	15-JUN-13 00:00:00	58.00
1	Janice	Jones	4	4	30-JUN-13 00:00:00	110.00
2	Jon	Jones	4	4	30-JUN-13 00:00:00	110.00
3	John	Doe	4	4	30-JUN-13 00:00:00	110.00
4	Jane	Doe	4	4	30-JUN-13 00:00:00	110.00
5	Jane	Smith	4	4	30-JUN-13 00:00:00	110.00
6	Janice	Smith	4	4	30-JUN-13 00:00:00	110.00
7	Helen	Brown	4	4	30-JUN-13 00:00:00	110.00
8	Helen	Jerry	4	4	30-JUN-13 00:00:00	110.00
9	Mary	Collins	4	4	30-JUN-13 00:00:00	110.00
10	Peter	Collins	4	4	30-JUN-13 00:00:00	110.00
11	Edna	Hayes	4	4	30-JUN-13 00:00:00	110.00
12	Franklin	Hayes	4	4	30-JUN-13 00:00:00	110.00
13	Peter	Johnson	4	4	30-JUN-13 00:00:00	110.00
14	Peter	Johnson	4	4	30-JUN-13 00:00:00	110.00
15	John	Smith	4	4	30-JUN-13 00:00:00	110.00
1	Janice	Jones	5	6	30-JUN-13 00:00:00	110.00
2	Jon	Jones	5	6	30-JUN-13 00:00:00	110.00
3	John	Doe	5	6	30-JUN-13 00:00:00	110.00
4	Jane	Doe	5	6	30-JUN-13 00:00:00	110.00
5	Jane	Smith	5	6	30-JUN-13 00:00:00	110.00
6	Janice	Smith	5	6	30-JUN-13 00:00:00	110.00
7	Helen	Brown	5	6	30-JUN-13 00:00:00	110.00
8	Helen	Jerry	5	6	30-JUN-13 00:00:00	110.00
9	Mary	Collins	5	6	30-JUN-13 00:00:00	110.00
10	Peter	Collins	5	6	30-JUN-13 00:00:00	110.00
11	Edna	Hayes	5	6	30-JUN-13 00:00:00	110.00
12	Franklin	Hayes	5	6	30-JUN-13 00:00:00	110.00
13	Peter	Johnson	5	6	30-JUN-13 00:00:00	110.00
14	Peter	Johnson	5	6	30-JUN-13 00:00:00	110.00
15	John	Smith	5	6	30-JUN-13 00:00:00	110.00
1	Janice	Jones	6	12	05-JUL-13 00:00:00	505.00
2	Jon	Jones	6	12	05-JUL-13 00:00:00	505.00
3	John	Doe	6	12	05-JUL-13 00:00:00	505.00
4	Jane	Doe	6	12	05-JUL-13 00:00:00	505.00
5	Jane	Smith	6	12	05-JUL-13 00:00:00	505.00
6	Janice	Smith	6	12	05-JUL-13 00:00:00	505.00
7	Helen	Brown	6	12	05-JUL-13 00:00:00	505.00
8	Helen	Jerry	6	12	05-JUL-13 00:00:00	505.00
9	Mary	Collins	6	12	05-JUL-13 00:00:00	505.00
10	Peter	Collins	6	12	05-JUL-13 00:00:00	505.00
11	Edna	Hayes	6	12	05-JUL-13 00:00:00	505.00
12	Franklin	Hayes	6	12	05-JUL-13 00:00:00	505.00
13	Peter	Johnson	6	12	05-JUL-13 00:00:00	505.00
14	Peter	Johnson	6	12	05-JUL-13 00:00:00	505.00
15	John	Smith	6	12	05-JUL-13 00:00:00	505.00

FIGURE 6.7 **The first 60 rows of a 300 row product table.**

Note: Although 300 rows may not seem like a lot, consider the size of a product table created from tables with 10,000 and 100,000 rows! The manipulation of a table of this size can tie up a lot of disk I/O and CPU time.

Notice first that the *customer_numb* is included twice in the result table, once from each source table. Second, notice that in some rows the *customer_numb* is the same. These are the rows that would have been included in a join. We can therefore apply a restrict predicate (a *join condition*) to the product table to end up with same table provided by the join you saw earlier. The predicate can be written:

```
customer.customer_numb = sale.customer_numb
```

The rows that are selected by this predicate from the first 60 rows in the product table appear in black in Figure 6.8; those eliminated by the predicate are gray.

customer_numb	first_name	last_name	sale_id	customer_numb	sale_date	sale_total_amt
1	Janice	Jones	3	1	15-JUN-13 00:00:00	58.00
2	Jon	Jones	3	1	15-JUN-13 00:00:00	58.00
3	John	Doe	3	1	15-JUN-13 00:00:00	58.00
4	Jane	Doe	3	1	15-JUN-13 00:00:00	58.00
5	Jane	Smith	3	1	15-JUN-13 00:00:00	58.00
6	Janice	Smith	3	1	15-JUN-13 00:00:00	58.00
7	Helen	Brown	3	1	15-JUN-13 00:00:00	58.00
8	Helen	Jerry	3	1	15-JUN-13 00:00:00	58.00
9	Mary	Collins	3	1	15-JUN-13 00:00:00	58.00
10	Peter	Collins	3	1	15-JUN-13 00:00:00	58.00
11	Edna	Hayes	3	1	15-JUN-13 00:00:00	58.00
12	Franklin	Hayes	3	1	15-JUN-13 00:00:00	58.00
13	Peter	Johnson	3	1	15-JUN-13 00:00:00	58.00
14	Peter	Johnson	3	1	15-JUN-13 00:00:00	58.00
15	John	Smith	3	1	15-JUN-13 00:00:00	58.00
1	Janice	Jones	4	4	30-JUN-13 00:00:00	110.00
2	Jon	Jones	4	4	30-JUN-13 00:00:00	110.00
3	John	Doe	4	4	30-JUN-13 00:00:00	110.00
4	Jane	Doe	4	4	30-JUN-13 00:00:00	110.00
5	Jane	Smith	4	4	30-JUN-13 00:00:00	110.00
6	Janice	Smith	4	4	30-JUN-13 00:00:00	110.00
7	Helen	Brown	4	4	30-JUN-13 00:00:00	110.00
8	Helen	Jerry	4	4	30-JUN-13 00:00:00	110.00
9	Mary	Collins	4	4	30-JUN-13 00:00:00	110.00
10	Peter	Collins	4	4	30-JUN-13 00:00:00	110.00
11	Edna	Hayes	4	4	30-JUN-13 00:00:00	110.00
12	Franklin	Hayes	4	4	30-JUN-13 00:00:00	110.00
13	Peter	Johnson	4	4	30-JUN-13 00:00:00	110.00
14	Peter	Johnson	4	4	30-JUN-13 00:00:00	110.00
15	John	Smith	4	4	30-JUN-13 00:00:00	110.00
1	Janice	Jones	5	6	30-JUN-13 00:00:00	110.00
2	Jon	Jones	5	6	30-JUN-13 00:00:00	110.00
3	John	Doe	5	6	30-JUN-13 00:00:00	110.00
4	Jane	Doe	5	6	30-JUN-13 00:00:00	110.00
5	Jane	Smith	5	6	30-JUN-13 00:00:00	110.00
6	Janice	Smith	5	6	30-JUN-13 00:00:00	110.00
7	Helen	Brown	5	6	30-JUN-13 00:00:00	110.00
8	Helen	Jerry	5	6	30-JUN-13 00:00:00	110.00
9	Mary	Collins	5	6	30-JUN-13 00:00:00	110.00
10	Peter	Collins	5	6	30-JUN-13 00:00:00	110.00
11	Edna	Hayes	5	6	30-JUN-13 00:00:00	110.00
12	Franklin	Hayes	5	6	30-JUN-13 00:00:00	110.00
13	Peter	Johnson	5	6	30-JUN-13 00:00:00	110.00
14	Peter	Johnson	5	6	30-JUN-13 00:00:00	110.00
15	John	Smith	5	6	30-JUN-13 00:00:00	110.00
1	Janice	Jones	6	12	05-JUL-13 00:00:00	505.00
2	Jon	Jones	6	12	05-JUL-13 00:00:00	505.00
3	John	Doe	6	12	05-JUL-13 00:00:00	505.00
4	Jane	Doe	6	12	05-JUL-13 00:00:00	505.00
5	Jane	Smith	6	12	05-JUL-13 00:00:00	505.00
6	Janice	Smith	6	12	05-JUL-13 00:00:00	505.00
7	Helen	Brown	6	12	05-JUL-13 00:00:00	505.00
8	Helen	Jerry	6	12	05-JUL-13 00:00:00	505.00
9	Mary	Collins	6	12	05-JUL-13 00:00:00	505.00
10	Peter	Collins	6	12	05-JUL-13 00:00:00	505.00
11	Edna	Hayes	6	12	05-JUL-13 00:00:00	505.00
12	Franklin	Hayes	6	12	05-JUL-13 00:00:00	505.00
13	Peter	Johnson	6	12	05-JUL-13 00:00:00	505.00
14	Peter	Johnson	6	12	05-JUL-13 00:00:00	505.00
15	John	Smith	6	12	05-JUL-13 00:00:00	505.00

FIGURE 6.8 **The four rows of the product in Figure 6.6 that are returned by the join condition in a restrict predicate.**

Note: The "dot notation" that you see in the preceding join condition is used throughout SQL. The table name is followed by a dot, which is followed by the column name. This makes it possible to have the same column name in more than one table and yet be able to distinguish among them.

It is important that you keep in mind the implication of this sequence of two relational algebra operations when you are writing SQL joins. If you are using the traditional SQL syntax for a join, and you forget the predicate for the join condition, you will end up with a product. The product

you are using the traditional SQL syntax for a join, and you forget the predicate for the join condition, you will end up with a product. The product table contains bad information; it implies facts that are not actually stored in the database. It is therefore potentially harmful, in that a user who does not understand how the result table came to be might assume that it is correct and make business decisions based on the bad data.

Equi-Joins over Concatenated Keys

The joins you have seen so far have used a single-column primary key and a single-column foreign key. There is no reason, however, that the values used in a join can't be concatenated. As an example, let us look again at the four relations from the accounting firm database that was used as an example in Chapter 5:

```
accountant (acct_first_name, acct_last_name, date_hired,
     office_ext)

customer (customer_numb, first_name, last_name, street,
     city, state_province, zip_postcode, contact_phone)

job (tax_year, customer_numb, acct_first_name,
     acct_last_name)

form (tax_year, customer_numb, form_id, is_complete)
```

Suppose we want to see all the forms, and the year that the forms were completed for the customer named Peter Jones by the accountant named

Edgar Smith. The sequence of relational operations would go something like this:

1. Restrict from the customer table to find the single row for Peter Jones. Because some customers have duplicated names, the restrict predicate would probably contain the name and the phone number. (We are assuming that the person setting up the query doesn't know the customer number.)
2. Join the table created in Step 1 to the *job* table over the customer number. The result table contains one row for each job that the accounting firm has performed for Peter Jones. The primary key of the result table will be the customer number and the tax year.
3. Restrict from the table created in Step 2 to find the jobs for Peter Jones that were handled by the accountant Edgar Smith. (Because a restrict does not specify which columns appear in the result table, the result table will have the same primary key as the result table from Step 2.)
4. Now, we need to get the data about which forms appear on the jobs identified in Step 3. We therefore need to join the table created in Step 3 to the *form* table. There is a concatenated foreign key in the *form* table—the tax year and customer number—which just happens to match the primary key of the result table produced by Step 3. The join is therefore over the concatenation of the tax year and customer number, rather than over the individual values. When making its determination whether to include a row in the result table, the DBMS puts the tax year and customer number together for each row and treats the combined value as if it were one.
5. Project the tax year and form ID to present the specific data requested in the query.

To see why treating a concatenated foreign key as a single unit when comparing to a concatenated foreign key is required, take a look at Figure 6.9. The two tables at the top of the illustration are the original *job* and *form* tables created for this example. We are interested in customer number 18 (our friend Peter Jones), who has had jobs handled by Edgar Smith in 2006 and 2007.

job

tax_year	customer_numb	acct_first_name	acct_last_name
2006	12	Jon	Johnson
2007	18	Edgar	Smith
2006	18	Edgar	Smith
2007	6	Edgar	Smith

form

tax_year	customer_numb	form_id	is_complete
2006	12	1040	t
2006	12	Sch. A	t
2006	12	Sch. B	t
2007	18	1040	t
2007	18	Sch. A	t
2007	18	Sch. B	t
2006	18	1040	t
2006	18	Sch. A	t
2007	6	1040	t
2007	6	Sch. A	t

(a) project JOIN form OVER tax_year GIVING invalid_1

tax_year	customer_numb	acct_first_name	acct_last_name	tax_year	customer_numb	form_id	is_complete
2006	18	Edgar	Smith	2006	12	1040	t
2006	12	Jon	Johnson	2006	12	1040	t
2006	18	Edgar	Smith	2006	12	Sch. A	t
2006	12	Jon	Johnson	2006	12	Sch. A	t
2006	18	Edgar	Smith	2006	12	Sch. B	t
2006	12	Jon	Johnson	2006	12	Sch. B	t
2007	6	Edgar	Smith	2007	18	1040	t
2007	18	Edgar	Smith	2007	18	1040	t
2007	6	Edgar	Smith	2007	18	Sch. A	t
2007	18	Edgar	Smith	2007	18	Sch. A	t
2007	6	Edgar	Smith	2007	18	Sch. B	t
2007	18	Edgar	Smith	2007	18	Sch. B	t
2006	18	Edgar	Smith	2006	18	1040	t
2006	12	Jon	Johnson	2006	18	1040	t
2006	18	Edgar	Smith	2006	18	Sch. A	t
2006	12	Jon	Johnson	2006	18	Sch. A	t
2007	6	Edgar	Smith	2007	6	1040	t
2007	18	Edgar	Smith	2007	6	1040	t
2007	6	Edgar	Smith	2007	6	Sch. A	t
2007	18	Edgar	Smith	2007	6	Sch. A	t

(b) job JOIN form OVER tax_year GIVING invalid_2

tax_year	customer_numb	acct_first_name	acct_last_name	tax_year	customer_numb	form_id	is_complete
2006	12	Jon	Johnson	2006	12	1040	t
2006	12	Jon	Johnson	2006	12	Sch. A	t
2006	12	Jon	Johnson	2006	12	Sch. B	t
2006	18	Edgar	Smith	2007	18	1040	t
2007	18	Edgar	Smith	2007	18	1040	t
2006	18	Edgar	Smith	2007	18	Sch. A	t
2007	18	Edgar	Smith	2007	18	Sch. A	t
2006	18	Edgar	Smith	2007	18	Sch. B	t
2007	18	Edgar	Smith	2007	18	Sch. B	t
2006	18	Edgar	Smith	2006	18	1040	t
2007	18	Edgar	Smith	2006	18	1040	t
2006	18	Edgar	Smith	2006	18	Sch. A	t
2007	18	Edgar	Smith	2006	18	Sch. A	t
2007	6	Edgar	Smith	2007	6	1040	t
2007	6	Edgar	Smith	2007	6	Sch. A	t

```
 2007 |           6 | Edgar          | Smith          |     2007 |           6 | Sch. A  | t(
```

(c) job JOIN form OVER tax_year + customer_numb GIVING correct_result

tax_year	customer_numb	acct_first_name	acct_last_name	tax_year	customer_numb	form_id	is_complete
2006	12	Jon	Johnson	2006	12	1040	t
2006	12	Jon	Johnson	2006	12	Sch. A	t
2006	12	Jon	Johnson	2006	12	Sch. B	t
2006	18	Edgar	Smith	2006	18	1040	t
2006	18	Edgar	Smith	2006	18	Sch. A	t
2007	18	Edgar	Smith	2007	18	Sch. B	t
2007	18	Edgar	Smith	2007	18	1040	t
2007	18	Edgar	Smith	2007	18	Sch. A	t
2007	6	Edgar	Smith	2007	6	1040	t
2007	6	Edgar	Smith	2007	6	Sch. A	t

FIGURE 6.9 **Joining using concatenated keys.**

Result table (a) is what happens if you join the tables (without restricting for customer 18) only over the tax year. This incorrect join expands the 10 row *form* table to 20 rows. The data imply that the same customer had the same form prepared by more than one accountant in the same year.

Result table (b) is the result of joining the two tables just over the customer number. This time, the incorrect result table implies that, in some cases, the same form was completed in two years, which may or may not be correct.

The correct join appears in result table (c) in Figure 6.9. It has the correct 10 rows, one for each form. Notice that *both* the tax year and customer number are the same in each row, as we intended them to be.

Note: The examples you have seen so far involve two concatenated columns. There is no reason, however, that the concatenation cannot involve more than two columns if necessary.

Θ-Joins

An equi-join is a specific example of a more general class of join known as a Θ-*join* (*theta-join*). A Θ-join combines two tables on some condition, which may be equality or may be something else. To make it easier to understand why you might want to join on something other than equality and how such joins work, assume that you're on vacation at a resort that offers both biking and hiking. Each outing runs a half day, but the times at which the outings start and end differ. The tables that hold the outing schedules appear in Figure 6.10. As you look at the data, you will see that some ending and starting times overlap, which means that if you want to engage in two outings on the same day, only some pairings of hiking and biking will work.

hiking

tour_numb	start_time	end_time
6	01:00:00	16:00:00
8	09:00:00	11:30:00
9	10:00:00	14:00:00
10	09:00:00	12:00:00
7	12:00:00	15:30:00

biking

tour_numb	start_time	end_time
1	09:00:00	12:00:00
2	09:00:00	11:30:00
3	09:00:00	12:30:00
4	12:00:00	15:00:00
5	13:00:00	17:00:00

FIGURE 6.10 **Source tables for the Θ-join examples.**

To determine which pairs of outings you could do on the same day, you need to find pairs of outings that satisfy either of the following conditions:

```
hiking.end_time < biking.start_time

biking.end_time < hiking.start_time
```

A Θ-join over either of those conditions will do the trick, producing the result tables in Figure 6.11. The top result table contains pairs of outings where hiking is done first; the middle result table contains pairs of outings where biking is done first. If you want all the possibilities in the same table, a union operation will combine them, as in the bottom result table. Another way to generate the combined table is to use a complex join condition in the Θ-join:

```
hiking.end_time < biking.start_time OR
   biking.end_time < hiking.start_time
```

Note: As with the more restrictive equi-join, the "start" table for a Θ-join does not matter. The result will be the same, either way.

```
hiking                                    biking

 tour_numb | start_time | end_time          tour_numb | start_time | end_time
-----------+------------+----------        -----------+------------+----------
         6 | 01:00:00   | 16:00:00                 1 | 09:00:00   | 12:00:00
         8 | 09:00:00   | 11:30:00                 2 | 09:00:00   | 11:30:00
         9 | 10:00:00   | 14:00:00                 3 | 09:00:00   | 12:30:00
        10 | 09:00:00   | 12:00:00                 4 | 12:00:00   | 15:00:00
         7 | 12:00:00   | 15:30:00                 5 | 13:00:00   | 17:00:00

hiking JOIN biking OVER hiking.end_time < biking.start_time GIVING hiking_first

 tour_numb | start_time | end_time | tour_numb | start_time | end_time
-----------+------------+----------+-----------+------------+----------
         4 | 12:00:00   | 15:00:00 |         8 | 09:00:00   | 11:30:00
         5 | 13:00:00   | 17:00:00 |         8 | 09:00:00   | 11:30:00
         5 | 13:00:00   | 17:00:00 |        10 | 09:00:00   | 12:00:00

hiking JOIN biking OVER biking.end_time < hiking.start_time gIVING biking_first

 tour_numb | start_time | end_time | tour_numb | start_time | end_time
-----------+------------+----------+-----------+------------+----------
         2 | 09:00:00   | 11:30:00 |         7 | 12:00:00   | 15:30:00


hiking JOIN biking OVER hiking.end_time < biking.start_time GIVING hiking_first
UNION
hiking JOIN biking OVER biking.end_time < hiking.start_time GIVING biking_first
 tour_numb | start_time | end_time | tour_numb | start_time | end_time
-----------+------------+----------+-----------+------------+----------
         4 | 12:00:00   | 15:00:00 |         8 | 09:00:00   | 11:30:00
         5 | 13:00:00   | 17:00:00 |         8 | 09:00:00   | 11:30:00
         5 | 13:00:00   | 17:00:00 |        10 | 09:00:00   | 12:00:00
         7 | 12:00:00   | 15:30:00 |         2 | 09:00:00   | 11:30:00
```

FIGURE 6.11 **The results of Θ-joins of the tables in Figure 6.10.**

Outer Joins

An *outer join* (as opposed to the inner joins we have been considering so far) is a join that includes rows in a result table even though there may not be a match between rows in the two tables being joined. Wherever the DBMS can't match rows, it places nulls in the columns for which no data exist. The result may therefore not be a legal relation because it may not have a primary key. However, because they query's result table is a virtual table that is never stored in the database, having no primary key does not present a data integrity problem.

Why might someone want to perform an outer join? An employee of the rare book store, for example, might want to see the names of all customers along with the books ordered in the last week. An inner join of the customer table to the sale table would eliminate those customers who had not purchased anything during the previous week. However, an outer join will include all customers, placing nulls in the sale data columns for the customers who have not ordered. An outer join, therefore, not only shows you matching data, but also tells you where matching data *do not* exist.

There are really three types of outer join, which vary depending on the table or tables from which you want to include rows that have no matches.

The Left Outer Join

The *left outer join* includes all rows from the first table in the join expression, along with rows with matching foreign keys from the second:

```
Table1 LEFT OUTER JOIN table2 GIVING result_table
```

For example, if we use the data from the tables in Figure 6.6 and perform the *left outer join* as

```
customer LEFT OUTER JOIN sale GIVING left_outer_join_result
```

then the result will appear as in Figure 6.12: There is a row for every row in *customer*. For the rows that don't have orders, the columns that come from sale have been filled with nulls.

The Right Outer Join

The *right outer join* is the precise opposite of the left outer join. It includes all rows from the table on the right of the outer join operator. If you perform

```
customer RIGHT OUTER JOIN sale GIVING right_outer_join_result
```

customer_numb	first_name	last_name	sale_id	customer_numb	sale_date	sale_total_amt
1	Janice	Jones	1	1	29-MAY-13 00:00:00	510.00
1	Janice	Jones	2	1	05-JUN-13 00:00:00	125.00
1	Janice	Jones	17	1	25-JUL-13 00:00:00	100.00
1	Janice	Jones	3	1	15-JUN-13 00:00:00	58.00
2	Jon	Jones	20	2	01-SEP-13 00:00:00	75.00
2	Jon	Jones	16	2	25-JUL-13 00:00:00	130.00
2	Jon	Jones	13	2	10-JUL-13 00:00:00	25.95
3	John	Doe	null	null	null	null
4	Jane	Doe	4	4	30-JUN-13 00:00:00	110.00
5	Jane	Smith	18	5	22-AUG-13 00:00:00	100.00
5	Jane	Smith	8	5	07-JUL-13 00:00:00	90.00
6	Janice	Smith	19	6	01-SEP-13 00:00:00	95.00
6	Janice	Smith	14	6	10-JUL-13 00:00:00	80.00
6	Janice	Smith	5	6	30-JUN-13 00:00:00	110.00
7	Helen	Brown	null	null	null	null
8	Helen	Jerry	9	8	07-JUL-13 00:00:00	50.00
8	Helen	Jerry	7	8	05-JUL-13 00:00:00	80.00
9	Mary	Collins	11	9	10-JUL-13 00:00:00	200.00
10	Peter	Collins	12	10	10-JUL-13 00:00:00	200.00
11	Edna	Hayes	15	11	12-JUL-13 00:00:00	75.00
11	Edna	Hayes	10	11	10-JUL-13 00:00:00	125.00
12	Franklin	Hayes	6	12	05-JUL-13 00:00:00	505.00
13	Peter	Johnson	null	null	null	null
14	Peter	Johnson	null	null	null	null
15	John	Smith	null	null	null	null

FIGURE 6.12 **The result of a left outer join.**

using the data from Figure 6.6, the result will be the same as an inner join of the two tables. This occurs because there are no rows in sale that don't appear in customer. However, if you reverse the order of the tables, as in

```
sale RIGHT OUTER JOIN customer GIVING right_outer_join_result
```

you end up with the same data as Figure 6.12.

Choosing a Right versus Left Outer Join

As you have just read, outer joins are directional: The result depends on the order of the tables in the command. (This is in direct contrast to an inner join, which produces the same result regardless of the order of the tables.) Assuming that you are performing an outer join on two tables that have a primary key–foreign key relationship, then the result of left and right outer joins on those tables is predictable (Table 6.9). Referential integrity ensures that no rows from a table containing a foreign key will ever be omitted from a join with the table that contains the referenced primary key. Therefore, a left outer join where the foreign key table is on the left of the operator and a right outer join where the foreign key table is on the right of the operator are no different from an inner join.

Table 6.9

The Effect of Left and Right Outer Joins on Tables with a Primary Key–Foreign Key Relationship

Outer Join Format	Outer Join Result
`primary_key_table LEFT OUTER JOIN foreign_key_table`	All rows from primary key table retained; matching rows from foreign key table included
`foreign_key_table LEFT OUTER JOIN primary_key_table`	Same as inner join
`primary_key_table RIGHT OUTER JOIN foreign_key_table`	Same as inner join
`foreign_key_table RIGHT OUTER JOIN primary_key_table`	All rows from primary key table retained; matching rows from foreign key table included

When choosing between a left and a right outer join, you therefore need to pay attention to which table will appear on which side of the operator. If the outer join is to produce a result different from that of an inner join, then the table containing the primary key must appear on the side that matches the name of the operator.

The Full Outer Join

A full outer join includes all rows from both tables, filling in rows with nulls where necessary. If the two tables have a primary key–foreign key relationship, then the result will be the same as that of either a left outer join, when the primary key table is on the left of the operator or a right outer join, when the primary key table is on the right side of the operator. In the case of the full outer join, it does not matter on which side of the operator the primary key table appears; all rows from the primary key table will be retained.

Valid Versus Invalid Joins

To this point, all of the joins you have seen (with the exception of some outer joins) have involved tables with a primary key–foreign key relationship. These are the most typical types of join and always produce valid result tables. In contrast, most joins (other than outer joins) between tables that do not have a primary key–foreign key relationship are not valid. This means that the result tables contain information that is not represented in the database, conveying misinformation to the user. Invalid joins are therefore far more dangerous than meaningless projections.

As an example, let us temporarily add a table to the rare book store database. The purpose of the table is to indicate the source from which the store acquired a volume. Over time, the same book (different physical volumes) may come from more than one source. The table has the following structure:

```
book_sources (isbn, source_name)
```

Someone looking at this table and the *book* table might conclude that, because the two tables have a matching column (*isbn*), it makes sense to join the tables to find out the source of every volume that the store has ever had in inventory. Unfortunately, this is not the information that the result table will contain.

To keep the result table to a reasonable length, we will work with an abbreviated *book_sources* table that does not contain sources for all volumes (Figure 6.13). Let's assume that we go ahead and join the tables over the ISBN. The result table (without columns that are not of interest to the join itself) can be found in Figure 6.14.

```
        isbn          |     source_name
----------------------+----------------------
 978-1-11111-111-1 | Tom Anderson
 978-1-11111-111-1 | Church rummage sale
 978-1-11111-118-1 | South Street Market
 978-1-11111-118-1 | Church rummage sale
 978-1-11111-118-1 | Betty Jones
```

978-1-11111-120-1	Tom Anderson
978-1-11111-120-1	Betty Jones
978-1-11111-126-1	Church rummage sale
978-1-11111-126-1	Betty Jones
978-1-11111-125-1	Tom Anderson
978-1-11111-125-1	South Street Market
978-1-11111-125-1	Hendersons
978-1-11111-125-1	Neverland Books
978-1-11111-130-1	Tom Anderson
978-1-11111-130-1	Hendersons

FIGURE 6.13 The *book_source* table.

inventory_id	isbn	sale_id	source_name
1	978-1-11111-111-1	1	Church rummage sale
1	978-1-11111-111-1	1	Tom Anderson
20	978-1-11111-130-1	6	Hendersons
20	978-1-11111-130-1	6	Tom Anderson
21	978-1-11111-126-1	6	Betty Jones
21	978-1-11111-126-1	6	Church rummage sale
23	978-1-11111-125-1	7	Neverland Books
23	978-1-11111-125-1	7	Hendersons
23	978-1-11111-125-1	7	South Street Market
23	978-1-11111-125-1	7	Tom Anderson
25	978-1-11111-126-1	8	Betty Jones
25	978-1-11111-126-1	8	Church rummage sale
35	978-1-11111-126-1	11	Betty Jones
35	978-1-11111-126-1	11	Church rummage sale
36	978-1-11111-130-1	11	Hendersons
36	978-1-11111-130-1	11	Tom Anderson
38	978-1-11111-130-1	12	Hendersons
38	978-1-11111-130-1	12	Tom Anderson
63	978-1-11111-130-1		Hendersons
63	978-1-11111-130-1		Tom Anderson

FIGURE 6.14 An invalid join result.

If the store has ever obtained volumes with the same ISBN from different sources, there will be multiple rows for that ISBN in the *book_sources* table. Although this doesn't give us a great deal of meaningful information, in and of itself, the table is valid. However, when we look at the result of the join with the volume table, the data in the result table contradict what is in *book_sources*. For example, the first two rows in the result table have the same inventory ID number yet come from different sources. How can the same physical volume come from two places? That is just impossible. This invalid join therefore implies facts that simply cannot be true.

The reason this join is invalid is that the two columns over which the join is performed are not in a primary key–foreign key relationship. In fact, in both tables, the *isbn* column is a foreign key that references the primary key of the *book* table.

Are joins between tables that do not have a primary key–foreign key relationship ever valid? On occasion, they are, in particular if you are joining two tables with the same primary key. You will see an example of this type of join when we discuss joining a table to itself, when a predicate requires that multiple rows exist before any are placed in a result table.

For another example, assume that you want to create a table to store data about your employees:

```
employees (id_numb, first_name, last_name,
  department, job_title, salary, hire_date)
```

Some of the employees are managers. For those individuals, you also want to store data about the project they are currently managing and the date they began managing that project. (A manager handles only one project at a time.) You could add the columns to the employees table and let them contain nulls for employees who are not managers. An alternative is to create a second table just for the managers:

```
managers (id_numb, current_project, project_start_date)
```

When you want to see all the information about a manager, you must join the two tables over the *id_numb* column. The result table will contain rows only for the manager because employees without rows in the managers table will be left out of the join. There will be no spurious rows such as those we got when we joined the *volume* and *book_sources* tables. This join therefore is valid.

Note: Although the id_numb column in the managers table technically is not a foreign key referencing employees, most databases using such a design would nonetheless include a constraint that forced the presence of a matching row in employees for every manager.

The bottom line is that you need to be very careful when performing joins between tables that do not have a primary key–foreign key relationship. Although such joins are not always invalid, in most cases they will be.

Difference

Among the most powerful database queries are those phrased in the negative, such as "show me all the customers who have not purchased from us in the past year." This type of query is particularly tricky because it is asking for data that are not in the database. The rare book store has data about customers who *have* purchased but not those who have not, for example. The only way to perform such a query is to request the DBMS to use the *difference* operation.

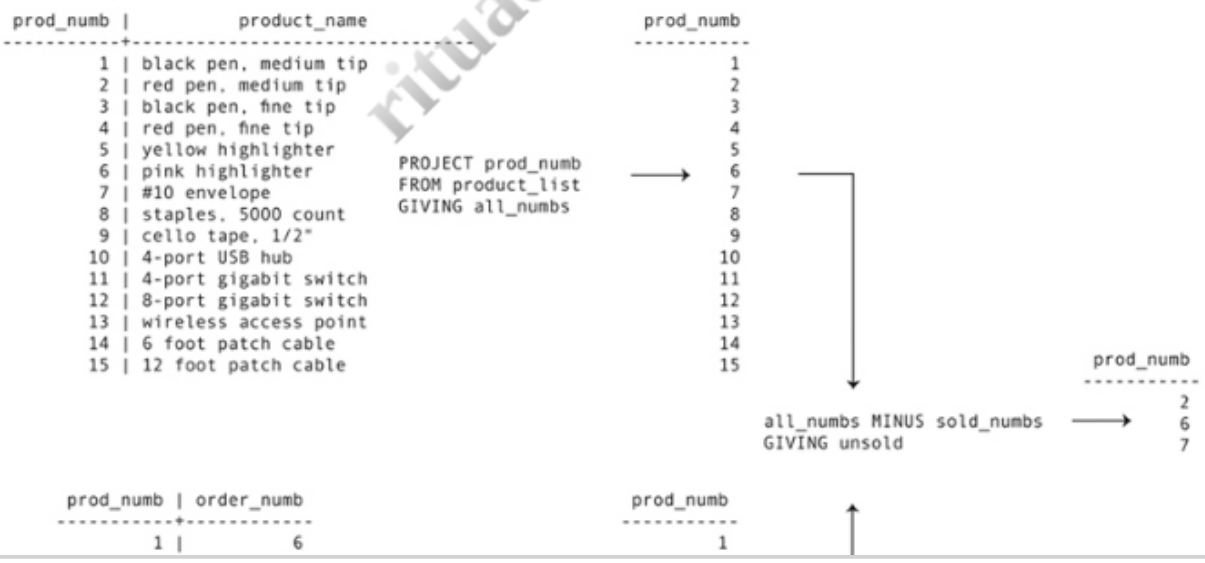
Difference retrieves all rows that are in one table but not in another. For example, if you have a table that contains all your products and another that contains products that have been purchased, the expression—

```
all_products MINUS products_that_have_been_purchased
   GIVING not_purchased
```

—is the products that have *not* been purchased. When you remove the products that *have* been purchased from all products, what are left are the products that *have not* been purchased.

The difference operation looks at entire rows when it makes the decision whether to include a row in the result table. This means that the two source tables must be union compatible. Assume that the *all_products* table has two columns—*prod_numb* and *product_name*—and the *products_that_have_been_purchased* table also has two columns—*prod_numb* and *order_numb*. Because they don't have the same columns, the tables aren't union-compatible.

As you can see from Figure 6.15, this means that a DBMS must first perform two projections to generate the union-compatible tables before it can perform the difference. In this case, the operation needs to retain the product number. Once the projections into union-compatible tables exist, the DBMS can perform the difference.



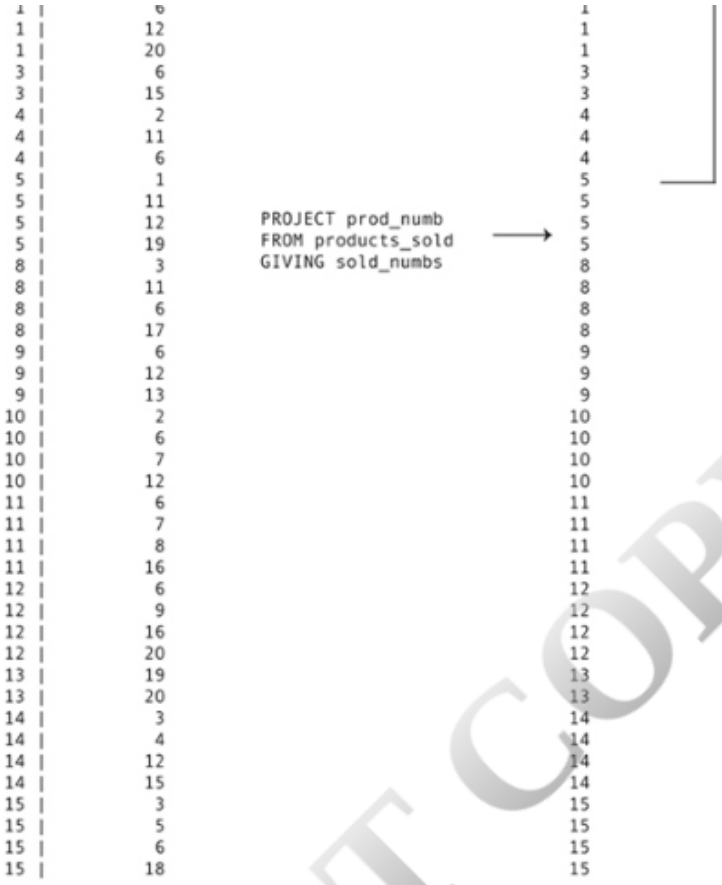


FIGURE 6.15 **The difference operation.**

Intersect

As mentioned earlier in this chapter, to be considered relationally complete, a DBMS must support restrict, project, join, union, and difference. Virtually every query can be satisfied using a sequence of those five operations. However, one other operation is usually included in the relational algebra specification: *intersect*.

In one sense, the intersect operation is the opposite of union. Union produces a result containing all rows that appear in either relation, while intersect produces a result containing all rows that appear in both relations. Intersection can therefore only be performed on two union-compatible relations.

Assume, for example, that the rare book store receives data listing volumes in a private collection that are being offered for sale. We can find out which volumes are already in the store's inventory using an intersect operation:

```
books_in_inventory INTERSECT books_for_sale
  GIVING already_have
```

As you can see in Figure 6.16, the first step in the process is to use the project operation to create union-compatible operations. Then, an intersect will provide the required result.

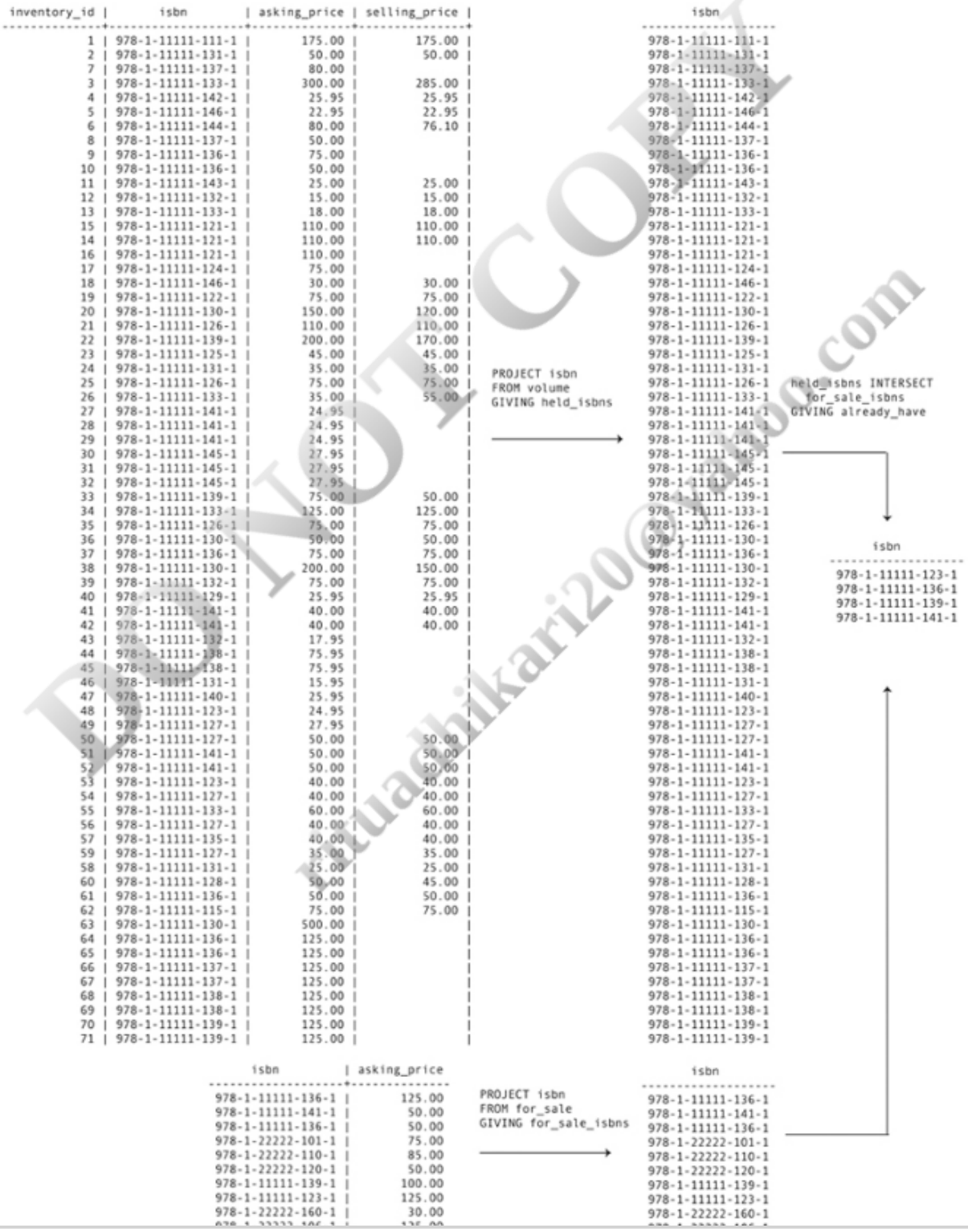


FIGURE 6.16 **The intersect operation.**

Note: A join over the concatenation of all the columns in the two tables produces the same result as an intersect.

Divide

An eighth relational algebra operation—*divide*—is often included with the operations you have seen in this chapter. It can be used for queries that need to have multiple rows in the same source table for a row to be included in the result table. Assume, for example, that the rare book store wants a list of sales on which two specific volumes have appeared.

There are many forms of the divide operation, all of which except the simplest are extremely complex. To set up the simplest form you need two relations, one with two columns (a *binary* relation) and one with a single column (a *unary* relation). The binary relation has a column that contains the values that will be placed in the result of the query (in our example, a sale ID) and a column for the values to be queried (in our example, the ISBN of the volume). This relation is created by taking a projection from the source table (in this case, the *volume* table).

The unary relation has the column being queried (the ISBN). It is loaded with a row for each value that must be matched in the binary table. A sale ID will be placed in the result table for all sales that contain ISBNs that match all of the values in the unary table. If there are two ISBNs in the unary table, then there must be a row for each of them with the same sale ID in the binary table to include the sale ID in the result. If we were to load the unary table with three ISBNs, then three matching rows would be required.

You can get the same result as a divide using multiple restricts and joins. In our example, you would restrict from the volume table twice, once for the first ISBN and once for the second. Then, you would join the tables over the sale ID. Only those sales that had rows in both of the tables being joined would end up in the result table.

Because divide can be performed fairly easily with restrict and join, DBMSs generally do not implement it directly.

For Further Reading

deHaan L. *Applied Mathematics for Database Professionals*. Apress; 2007.
Maddux RD. *Relation Algebras*. Elsevier Science; 2006.
Molkova L. *Theory and Practice of Relational Algebra: Transforming Relational Algebra to SQL*. LAMBERT Academic Publishing; 2012.

[1] Restrict is a renaming of the operation that was originally called “select.” Because SQL’s main retrieval command is SELECT (a relational calculus command), restrict was introduced by C. J. Date for the relational algebra operation to provide clarity. It is used in this book to help avoid confusion.

[2] It is true that some very old books do not have ISBNs. If that occurs, the rare book store gives the book a unique, arbitrary ID number.