What will we learn today?

- Why Geometric Vision Matters
- Geometric Primitives in 2D & 3D
- 2D & 3D Transformations

Images are 2D projections of the 3D world

Simplified Image Formation

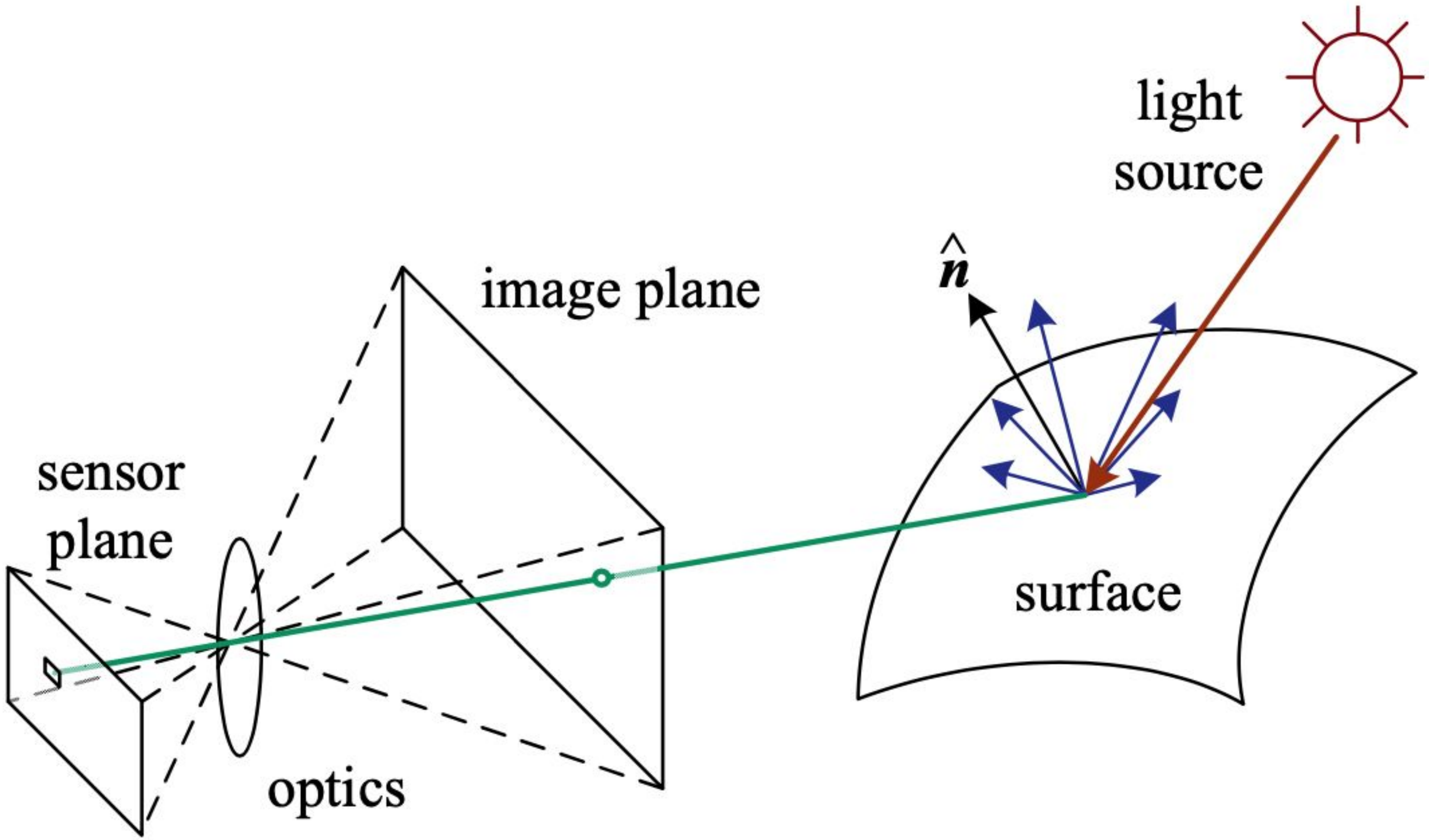
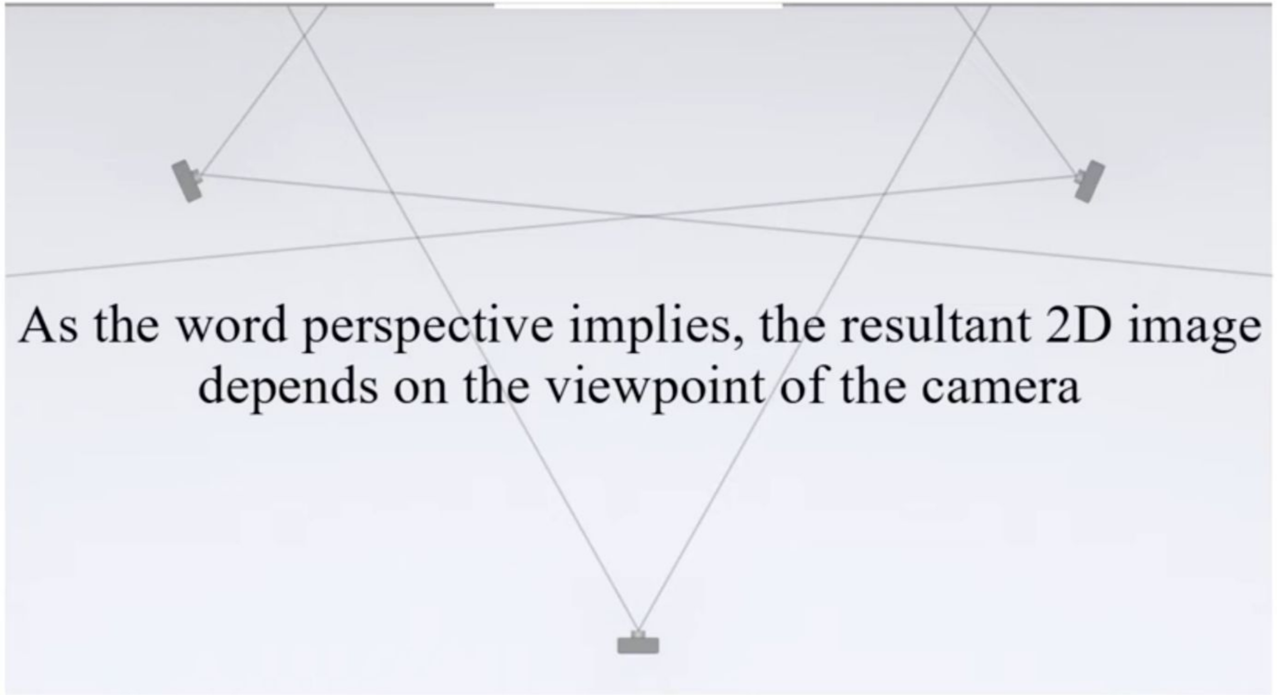


Figure: R. Szeliski

Perspective Projection



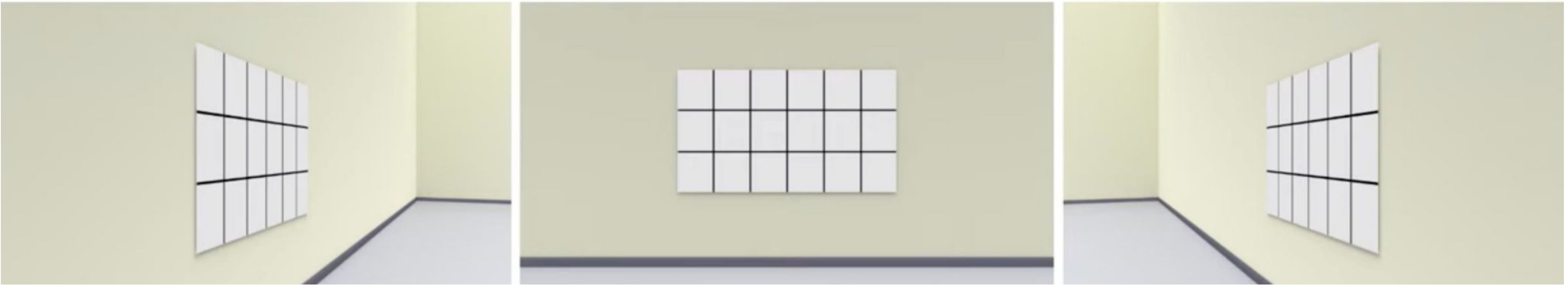
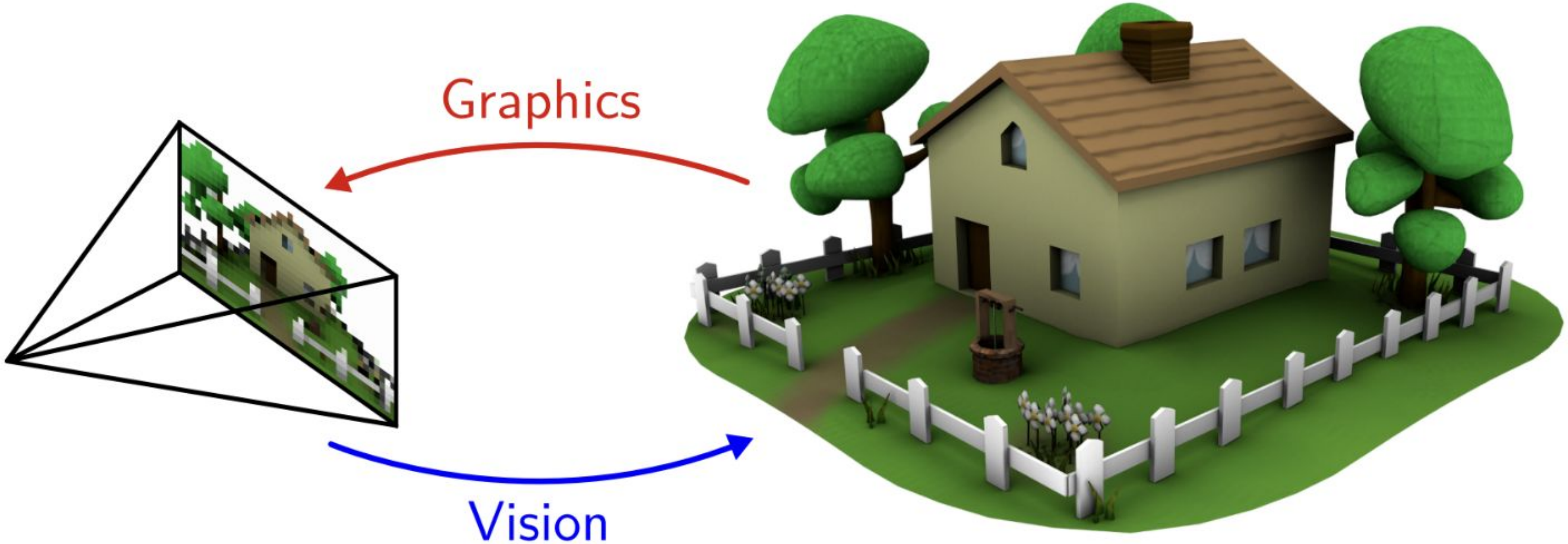


Figure: https://www.youtube.com/@huseyin_ozde...

Can we understand the 3D world from 2D images?

CV is an ill-posed inverse problem

2D Image **3D Scene**



Pixel Matrix

217	191	252	255	239
102	80	200	146	138
159	94	91	121	138
179	106	136	85	41
115	129	83	112	67
94	114	105	111	89

Objects Material

Shape/Geometry Motion

Semantics 3D Pose

Slide credit: Andreas Geiger

What will we learn today?

Why Geometric Vision Matters

Geometric Primitives in 2D & 3D

2D & 3D Transformations

Points in Cartesian and Homogeneous Coordinates

2D points: $\mathbf{x} = (x, y) \in \mathcal{R}^2$ or column vector $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

3D points: $\mathbf{x} = (x, y, z) \in \mathcal{R}^3$(often noted $\mathbf{X}$ or $\mathbf{P}$)

Homogeneous coordinates: append a 1

$$\bar{\mathbf{x}} = (x, y, 1) \qquad \bar{\mathbf{x}} = (x, y, z, 1)$$

Why?

Homogeneous coordinates in 2D

2D Projective Space: $\mathcal{P}^2 = \mathcal{R}^3 - (0,0,0)$ (same story in 3D with $\mathcal{P}^3$)

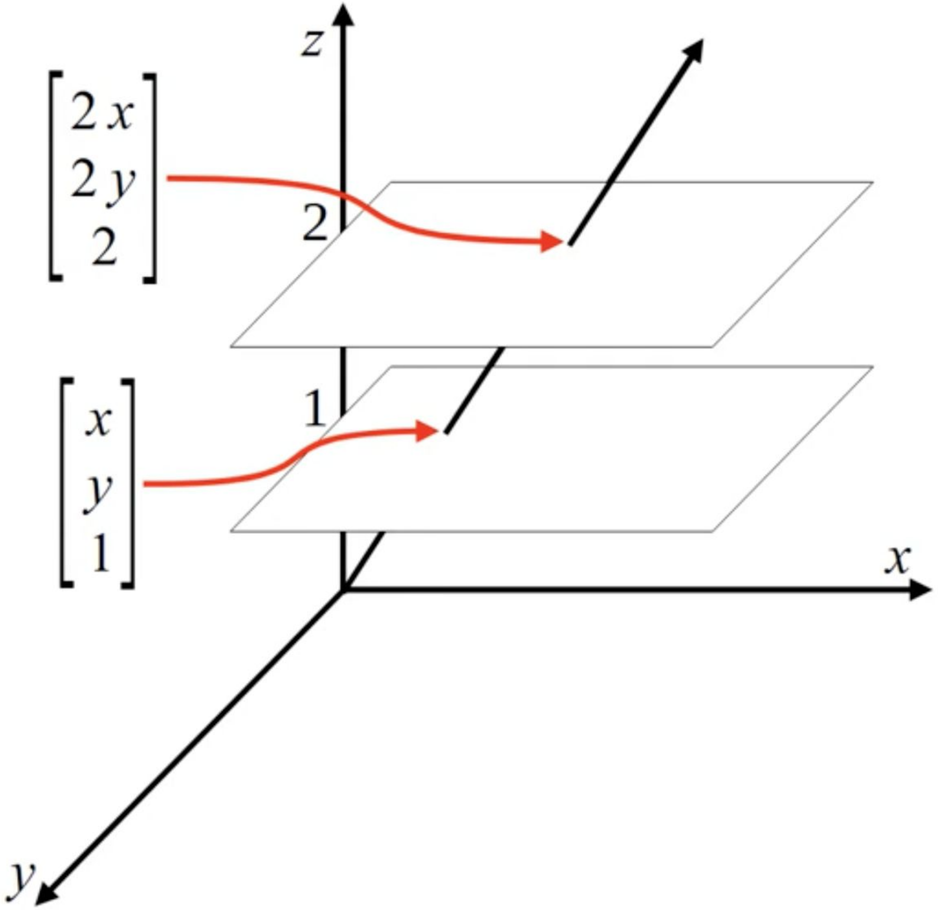
- heterogeneous → homogeneous $\begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

- homogeneous → heterogeneous $\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow \begin{bmatrix} x/w \\ y/w \end{bmatrix}$

- points differing only by scale are *equivalent*: $(x, y, w) \sim \lambda\,(x, y, w)$

$$\tilde{\mathbf{x}} = (\tilde{x}, \tilde{y}, \tilde{w}) = \tilde{w}(x, y, 1) = \tilde{w}\bar{\mathbf{x}},$$

Homogeneous coordinates in 2D



In homogeneous coordinates, a point and its scaled versions are same

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = w \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} \qquad w \neq 0$$

Figure: https://www.youtube.com/@huseyin_ozde...

Everything is easier in Projective Space

2D Lines:

Representation: $l = (a, b, c)$

Equation: $ax + by + c = 0$

In homogeneous coordinates: $\bar{x}^T l = 0$

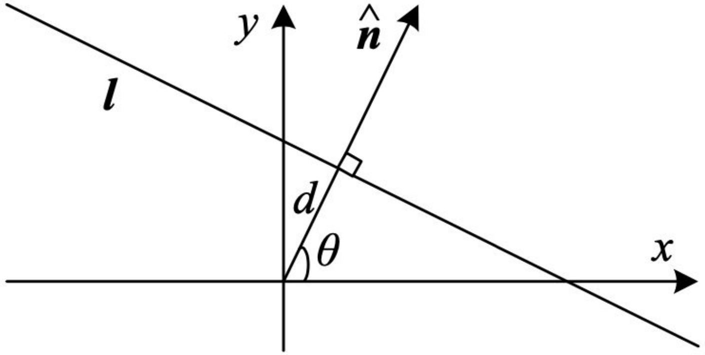
General idea: homogenous coordinates unlock the full power of linear algebra!

Everything is easier in Projective Space

2D Lines:

$$\tilde{\mathrm{x}}^{\mathrm{T}}\mathrm{l} = 0, \forall \tilde{\mathrm{x}} = (x, y, w) \in P^2$$

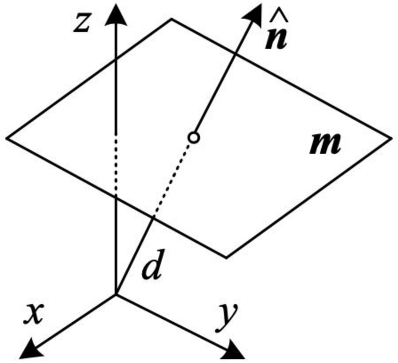
$$\mathbf{l} = (\hat{n}_x, \hat{n}_y, d) = (\hat{\mathbf{n}}, d) \text{ with } \|\hat{\mathbf{n}}\| = 1$$



3D planes: same!

$$\tilde{\mathrm{x}}^{\mathrm{T}}\mathrm{m} = 0, \forall \tilde{\mathrm{x}} = (x, y, z, w) \in P^3$$

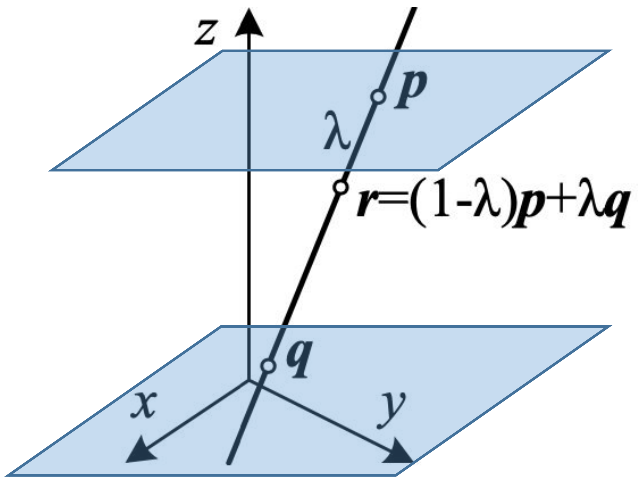
$$\mathbf{m} = (\hat{n}_x, \hat{n}_y, \hat{n}_z, d) = (\hat{\mathbf{n}}, d) \text{ with } \|\hat{\mathbf{n}}\| = 1$$



Lines in 3D

Two-point parametrization:

$$\mathbf{r} = (1-\lambda)\mathbf{p} + \lambda\mathbf{q} \qquad \tilde{\mathbf{r}} = \mu\tilde{\mathbf{p}} + \lambda\tilde{\mathbf{q}}$$



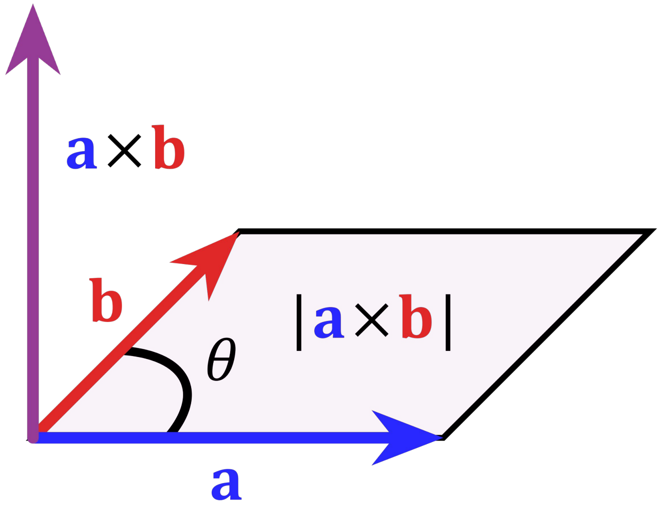
Two-plane parametrization:

coordinates (x_0, y_0) & (x_1, y_1) of intersection

with planes at $z = 0, 1$ (or other planes)

Cross-product quick reminder

$$\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \sin(\theta) \, \mathbf{n}$$



$$\mathbf{a} \times \mathbf{b} = [\mathbf{a}]_\times \mathbf{b} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Benefits of Homogeneous Coordinates

- Line – Point duality:
  - line between two 2D points: $\tilde{\mathbf{l}} = \tilde{\mathbf{x}}_1 \times \tilde{\mathbf{x}}_2$
  - intersection of two 2D lines: $\tilde{\mathbf{x}} = \tilde{\mathbf{l}}_1 \times \tilde{\mathbf{l}}_2$
- Representation of Infinity:
  - points at infinity: $(x, y, \mathbf{0})$; line at infinity: $(\mathbf{0,0,1})$
- Parallel & vertical lines are easy (take-home: intersect //)
- Makes 2D & 3D transformations linear!

Questions?

What will we learn today?

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2D & 3D Transformations

The camera as a coordinate transformation

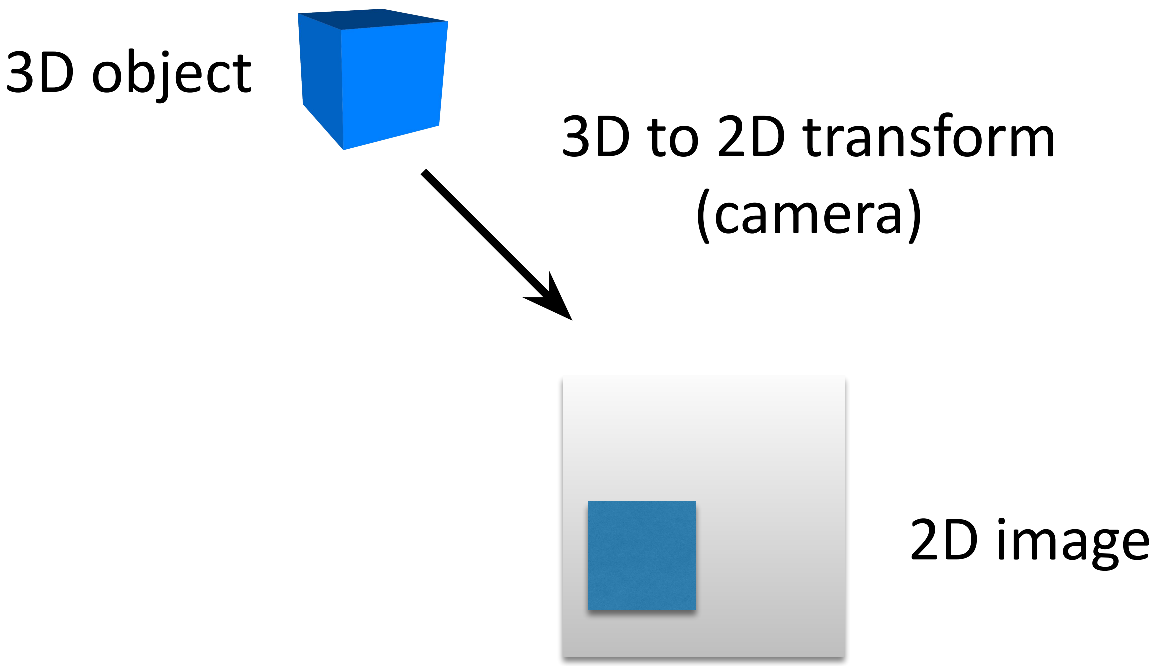
A camera is a mapping

from:

the 3D world

to:

a 2D image



Source: K. Kitani

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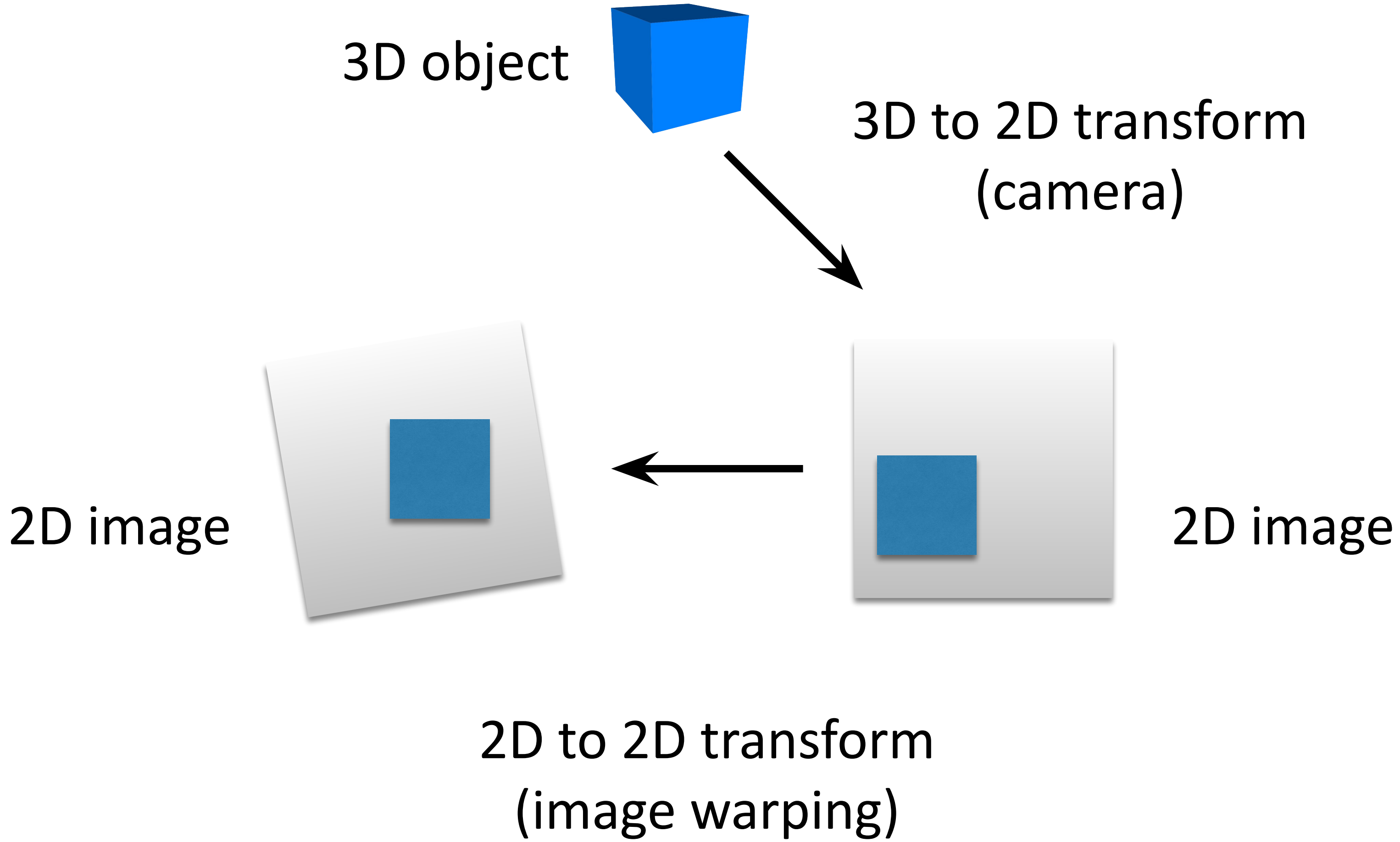


Source: K. Kitani

Cameras and objects can move!

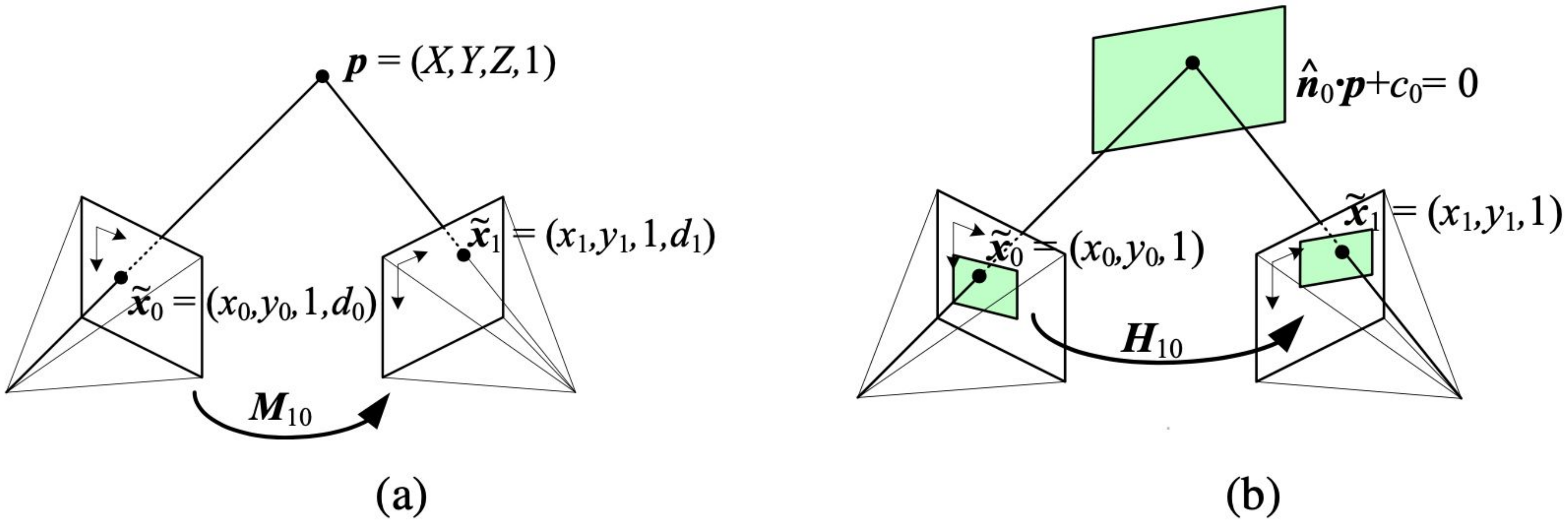


Figure 2.12 *A point is projected into two images: (a) relationship between the 3D point coordinate $(X, Y, Z, 1)$ and the 2D projected point $(x, y, 1, d)$; (b) planar homography induced by points all lying on a common plane $\hat{\mathbf{n}}_0 \cdot \mathbf{p} + c_0 = 0$.*

Figure: R. Szeliski

2D Transformations Zoo

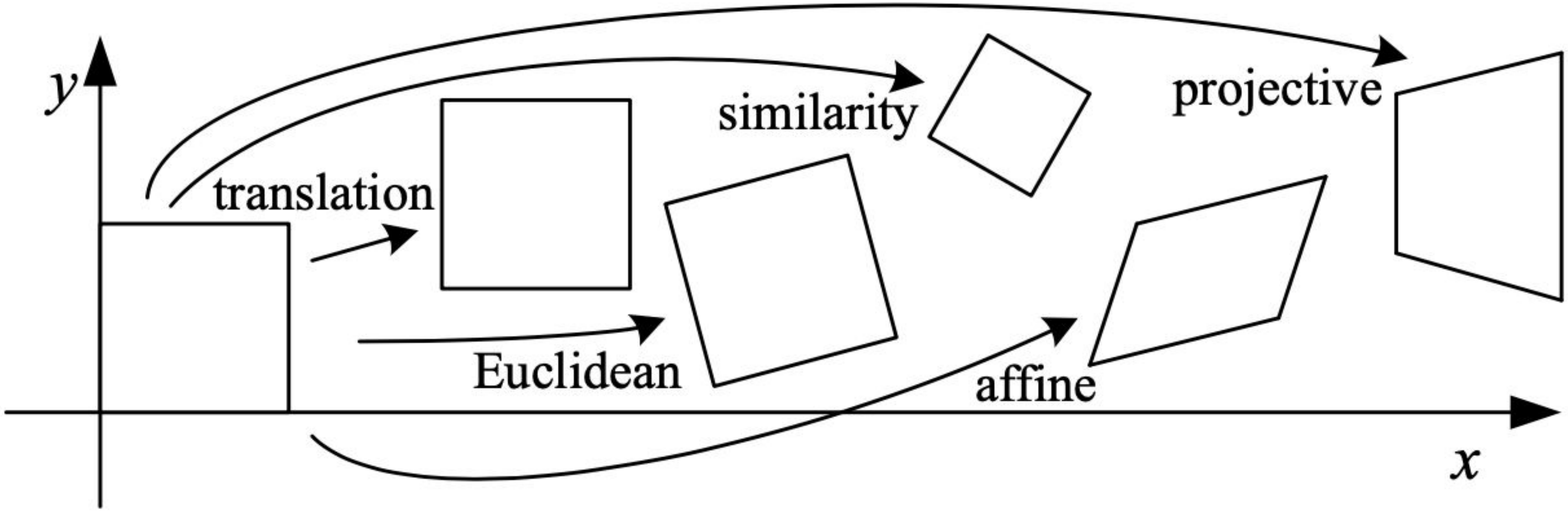


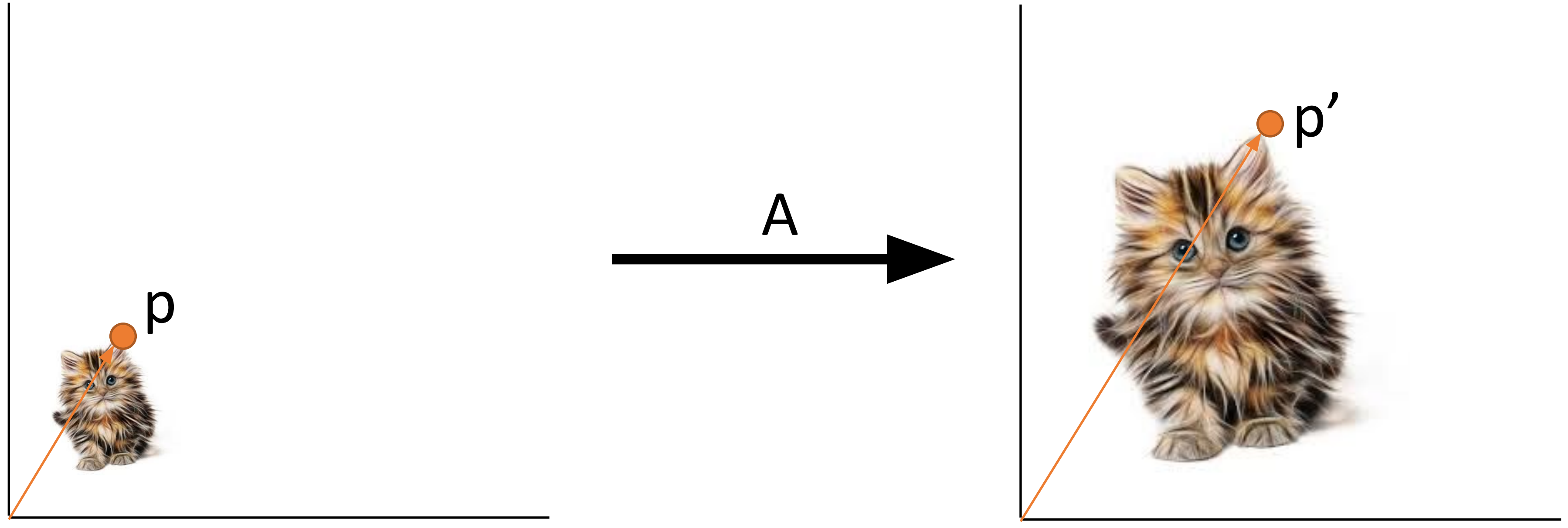
Figure: R. Szeliski

Transformation = Matrix Multiplication

Scale $$\mathbf{M} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$	Flip across y $$\mathbf{M} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$
Rotate $$\mathbf{M} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$	Flip across origin $$\mathbf{M} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$
Shear $$\mathbf{M} = \begin{bmatrix} 1 & s_x \\ s_y & 1 \end{bmatrix}$$	Identity $$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

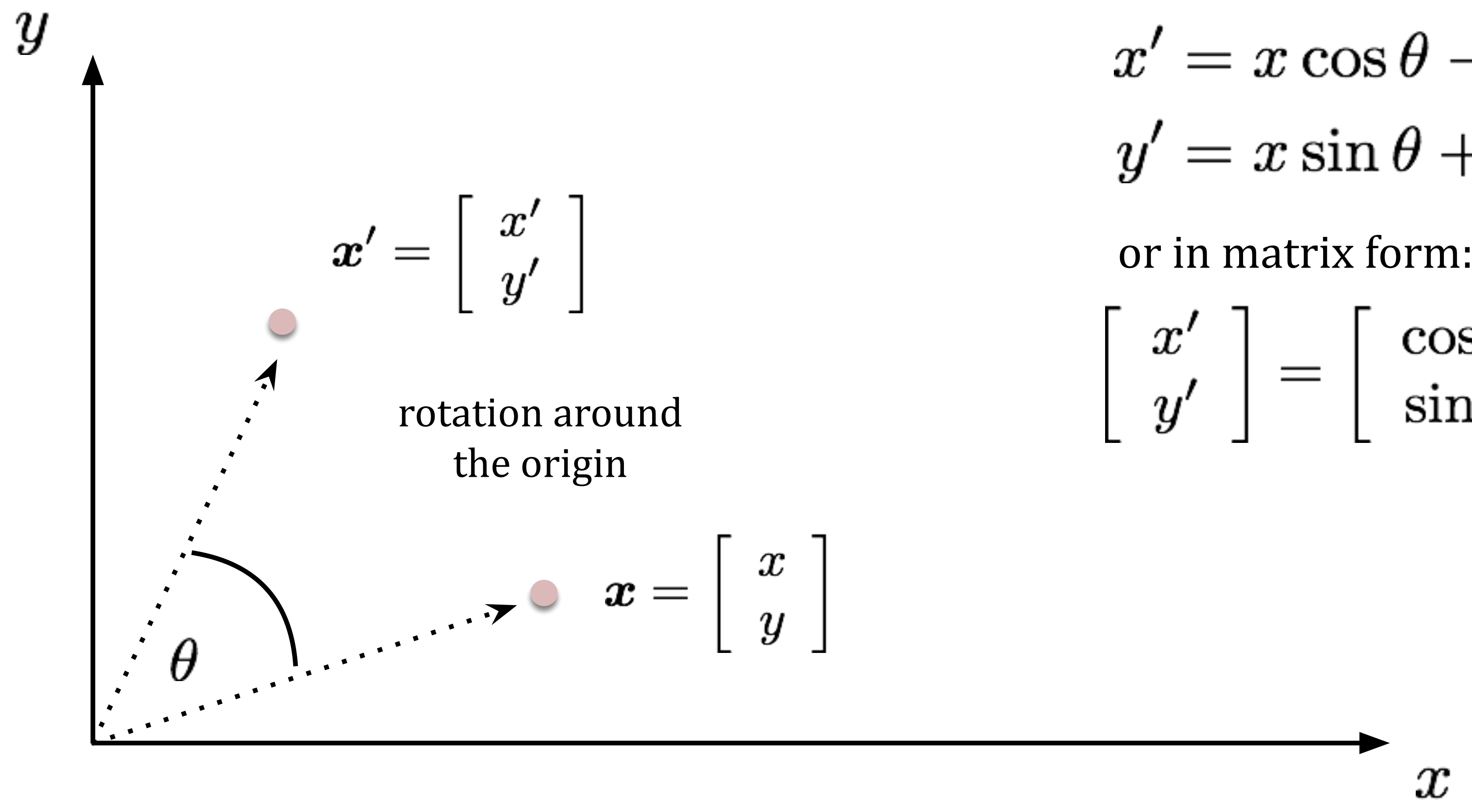
Scaling

$$\underbrace{\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{p} = \underbrace{\begin{bmatrix} s_x x \\ s_y y \end{bmatrix}}_{p'}$$



Slide: JC. Niebles

Rotation



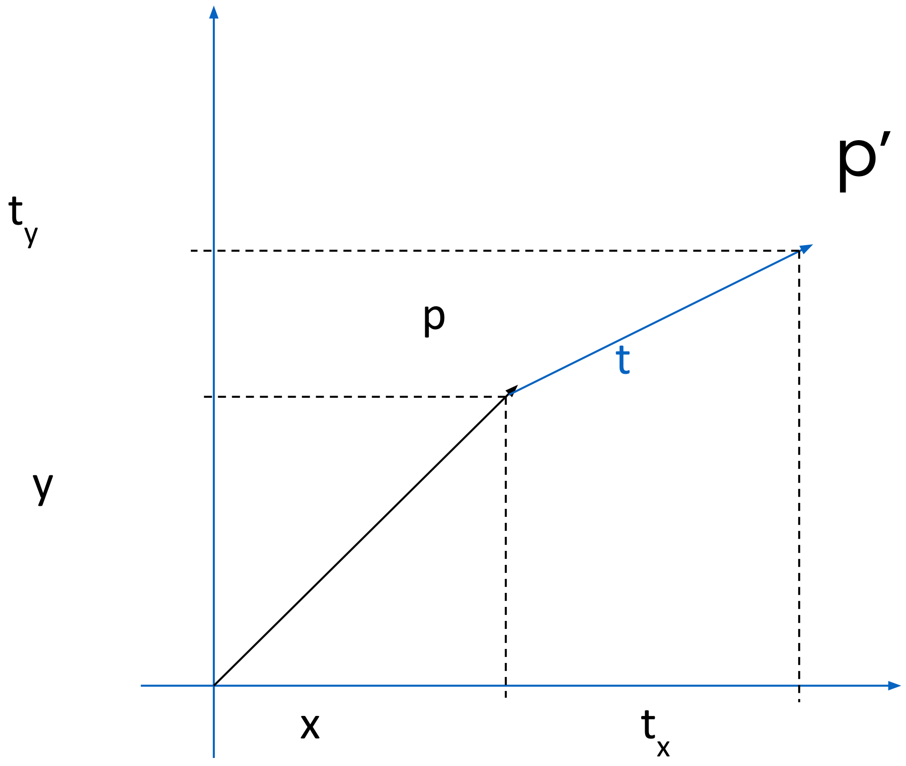
$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

or in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Slide: K. Kitani

Translation

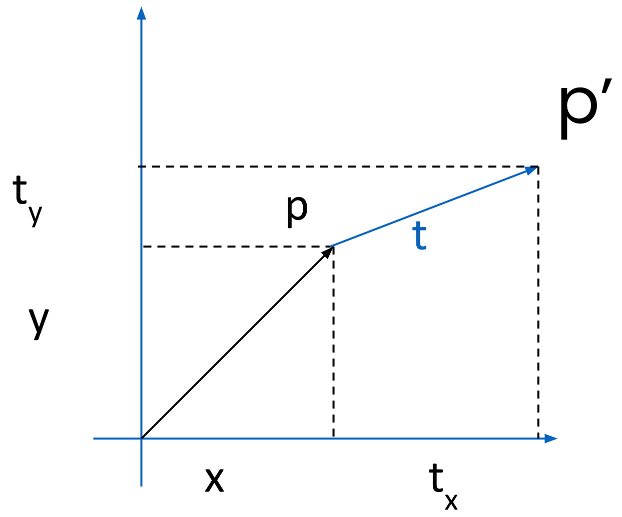


$$x' = x + t_x$$
$$y' = y + t_y$$

As a matrix?

Slide: JC. Niebles

Translation with homogeneous coordinates



$$p = \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$t = \begin{bmatrix} t_x \\ t_y \end{bmatrix} \rightarrow \begin{bmatrix} t_x \\ t_y \\ 1 \end{bmatrix}$$

$$p' = Tp$$

$$p' \rightarrow \begin{bmatrix} x + tx \\ y + ty \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{t} \\ \boldsymbol{0} & 1 \end{bmatrix} p = Tp$$

Slide: JC. Niebles

2D Transformations with homogeneous coordinates

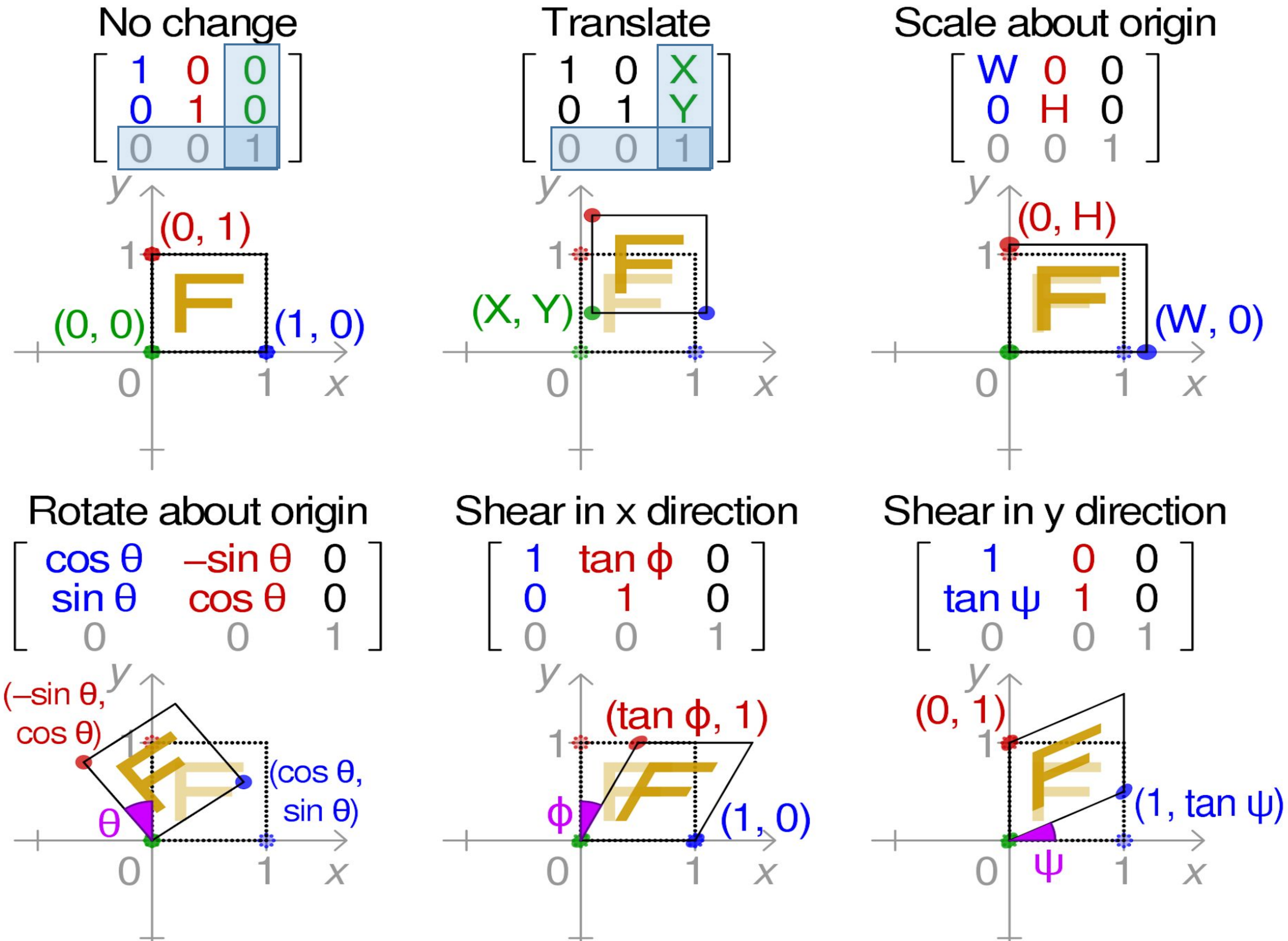


Figure: Wikipedia

Questions?

2D Transformations Zoo

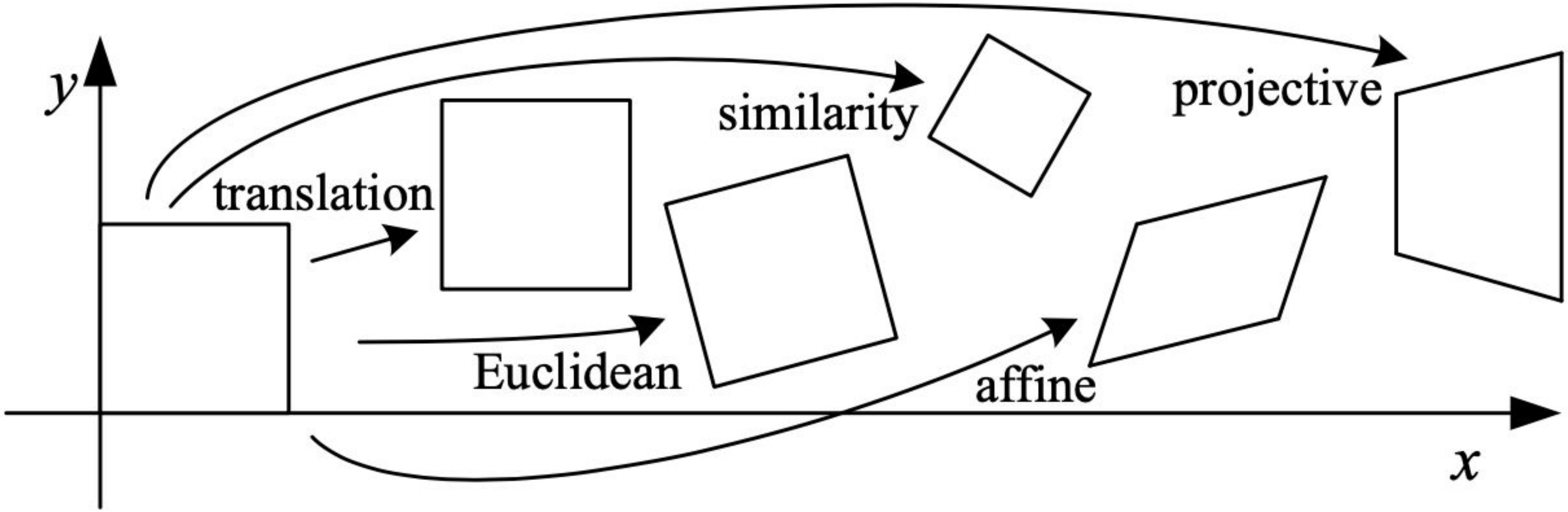


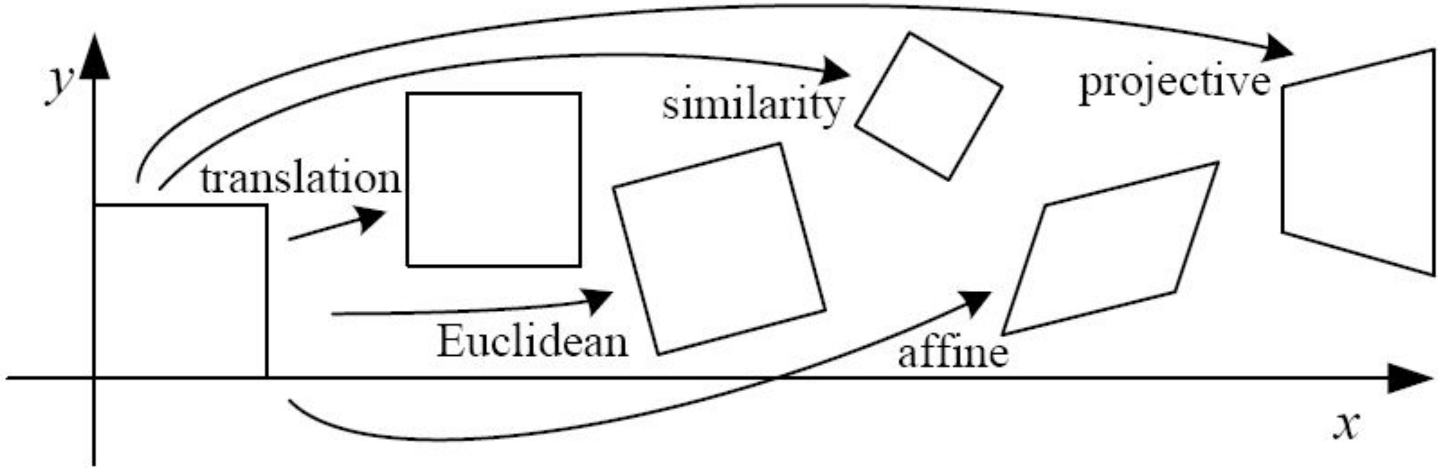
Figure: R. Szeliski

Euclidean / Rigid Transformation

Euclidean (rigid): rotation + translation

$$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

How many degrees of freedom?

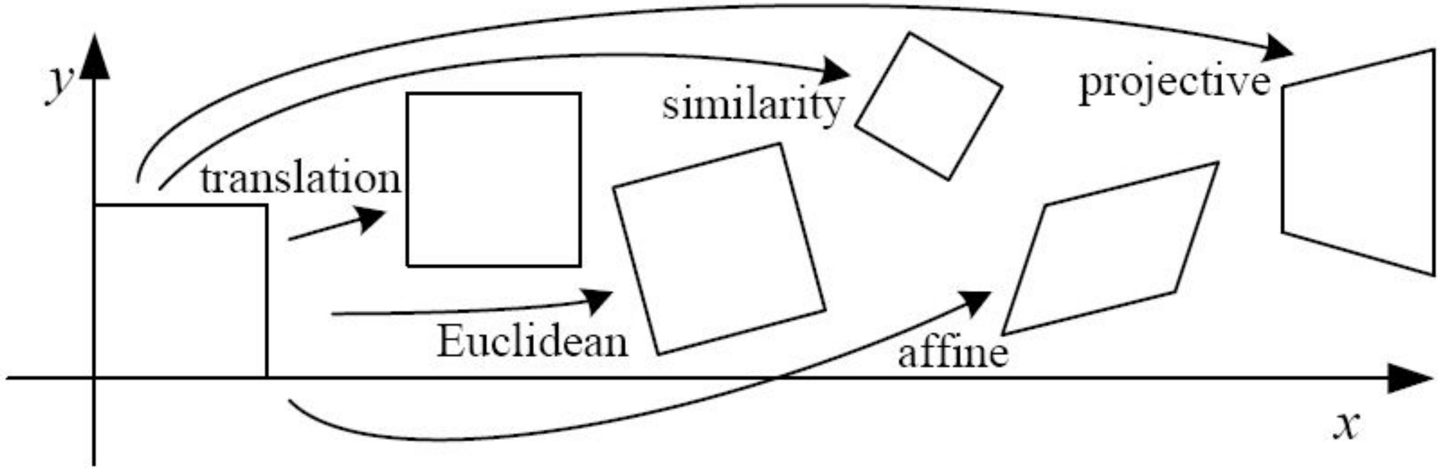


Similarity Transformation

Similarity: Scaling + rotation + translation

$$\begin{bmatrix} a & -b & t_x \\ b & a & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

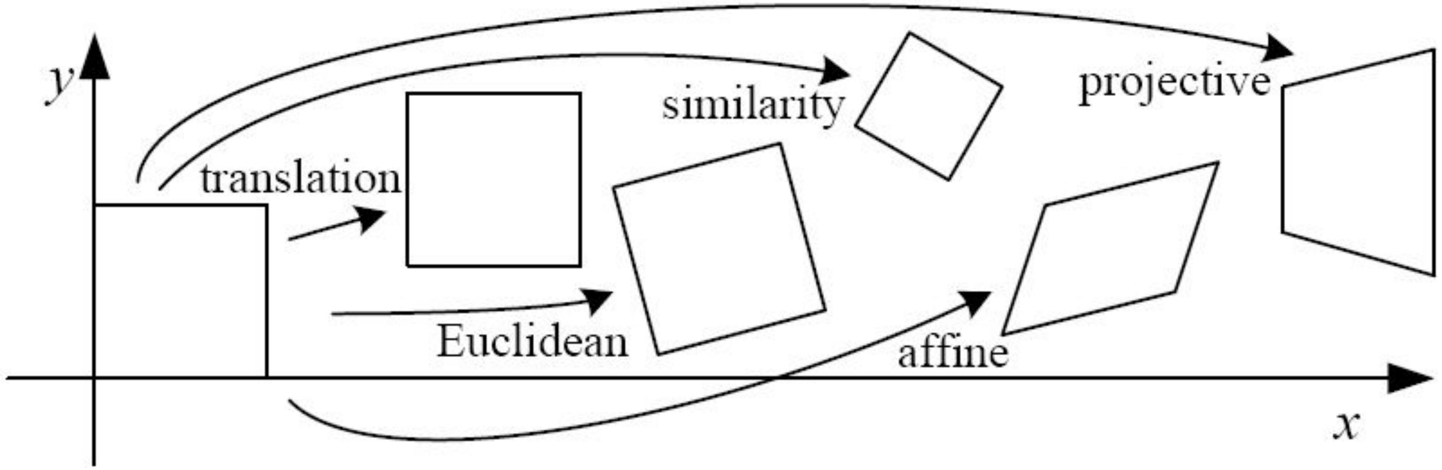
How many degrees of freedom?



Similarity Transformation

Similarity: Scaling + rotation + translation
$$\begin{bmatrix} a & -b & t_x \\ b & a & t_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha\cos\theta & -\alpha\sin\theta & b_0 \\ \alpha\sin\theta & \alpha\cos\theta & b_1 \\ 0 & 0 & 1 \end{bmatrix}$$

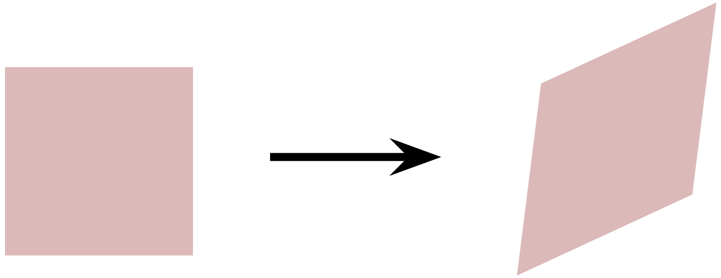
How many degrees of freedom?



Affine Transformation

Affine transformations are combinations of

- Arbitrary (4-DOF) linear transformations + translations



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \quad \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Cartesian coordinates

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Homogeneous coordinates

$$\longrightarrow \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A_{00} & A_{01} & b_0 \\ A_{10} & A_{11} & b_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Source: K. Kitani

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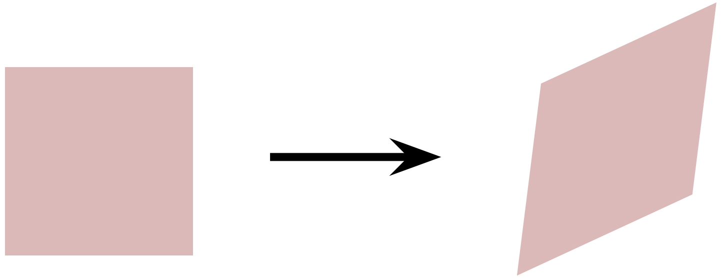

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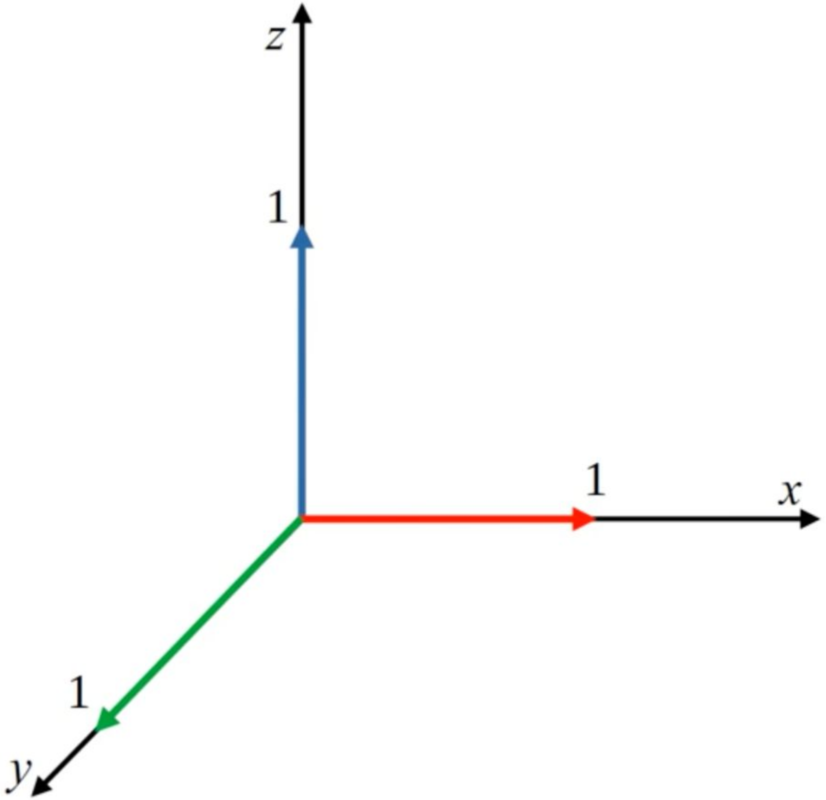
How many degrees of freedom?

Source: K. Kitani

Affine Transformation

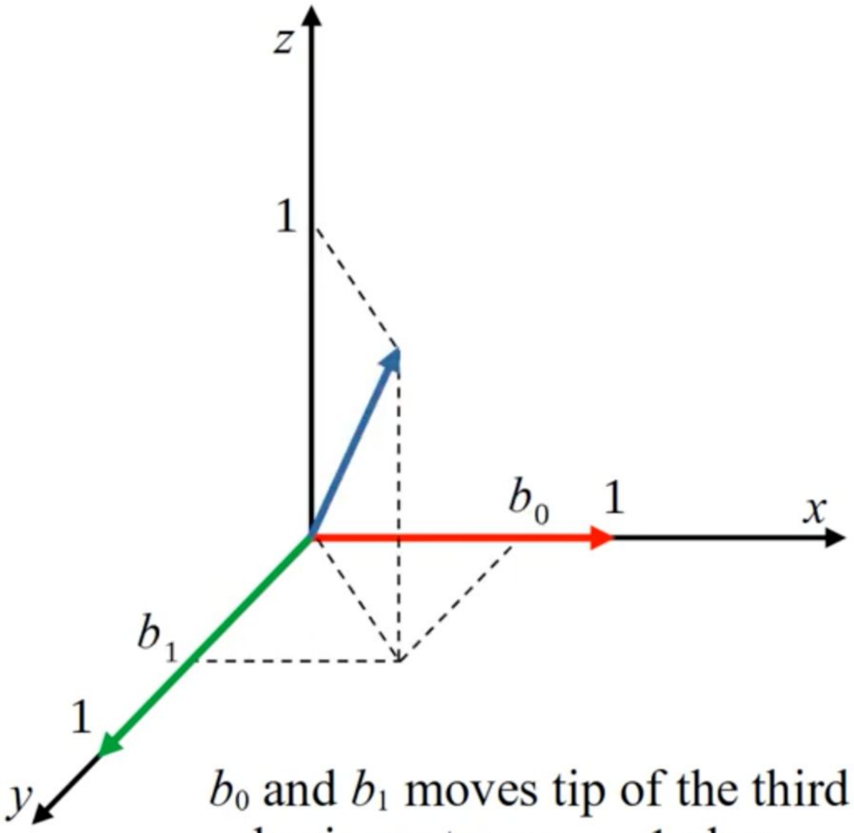


This matrix is a linear transformation matrix in 3D

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} A_{00} & A_{01} & b_0 \\ A_{10} & A_{11} & b_1 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Figure: https://www.youtube.com/@huseyin_ozde...

Affine Transformation



b_0 and b_1 moves tip of the third basis vector on $z=1$ plane

This matrix is a linear transformation matrix in 3D

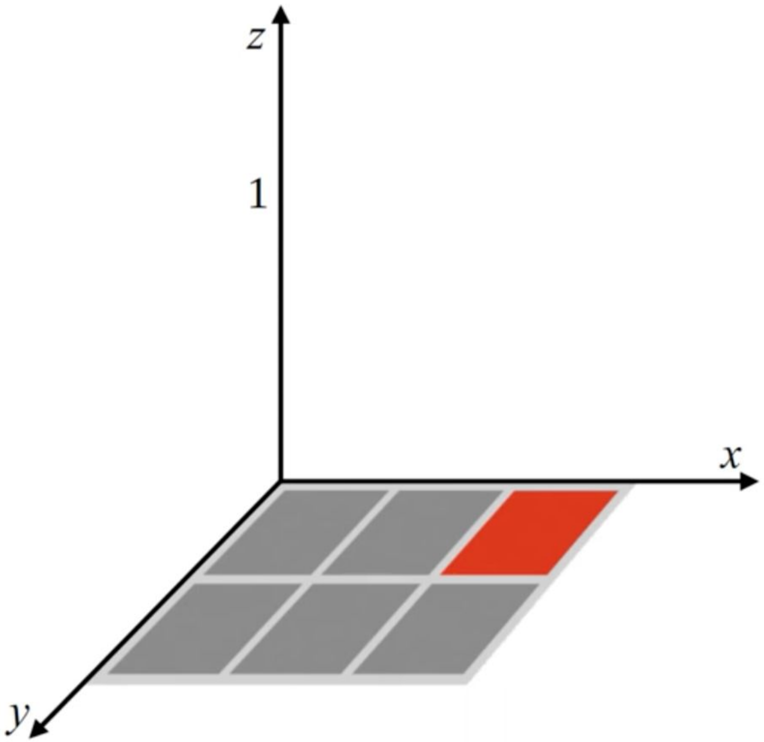
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A_{00} & A_{01} & b_0 \\ A_{10} & A_{11} & b_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Then this column is the third basis vector of transformed vector space

And what b_0 and b_1 do is to change the orientation of that basis vector

Figure: https://www.youtube.com/@huseyin_ozde...

Affine Transformation



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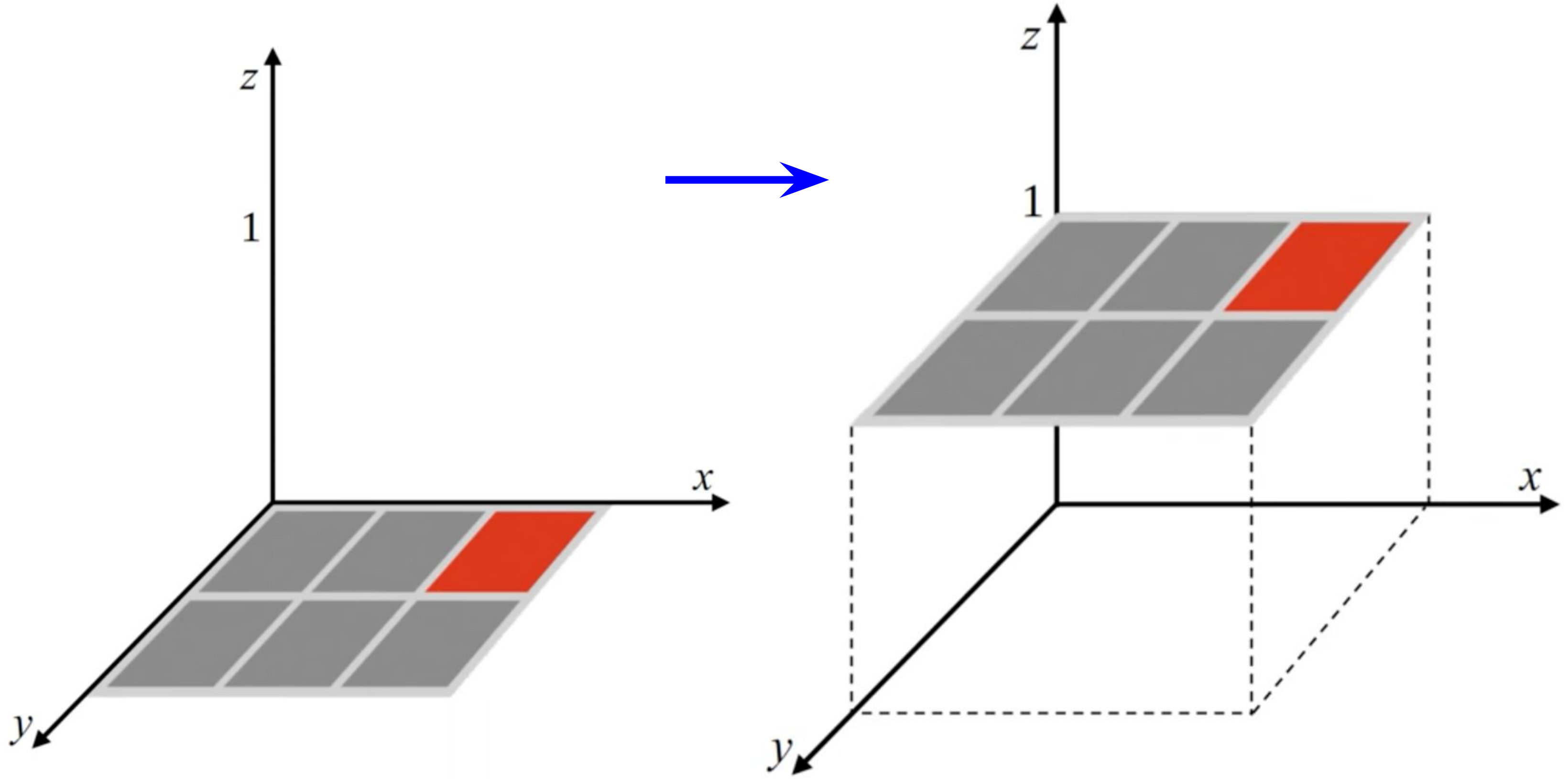
Affine Transformation



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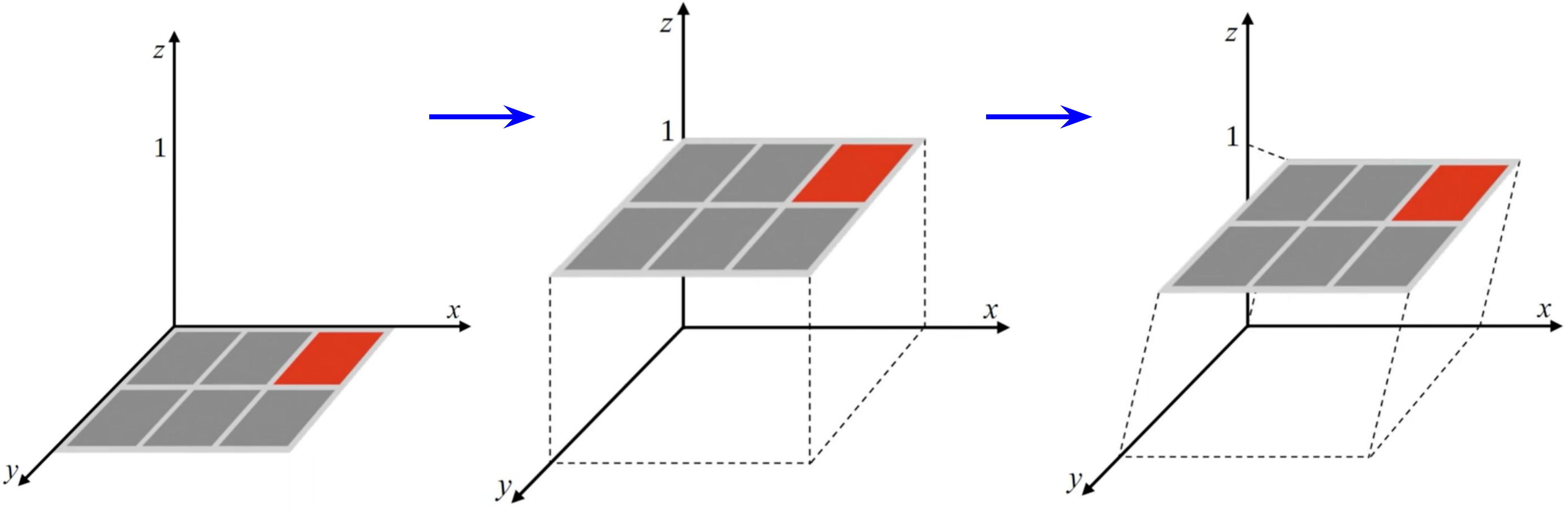
Affine Transformation



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Affine Transformation

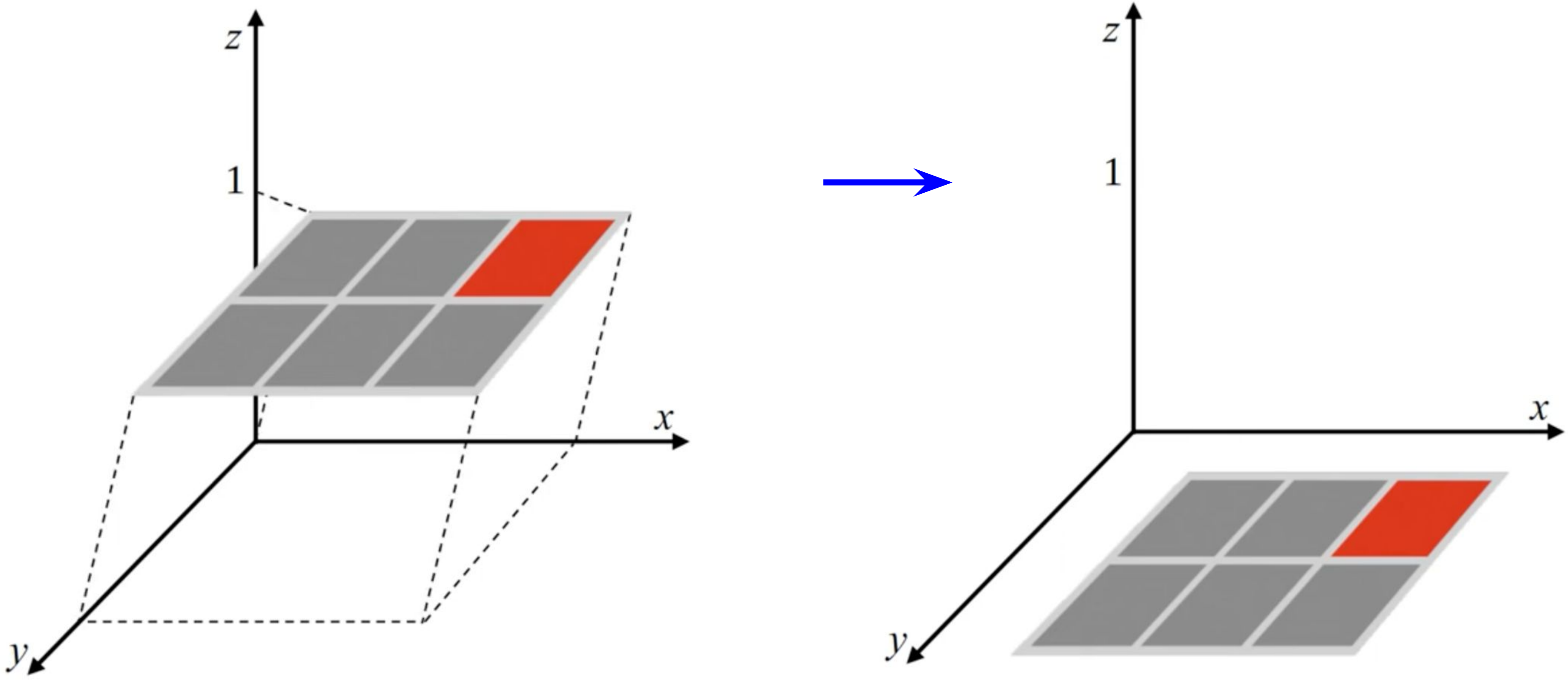
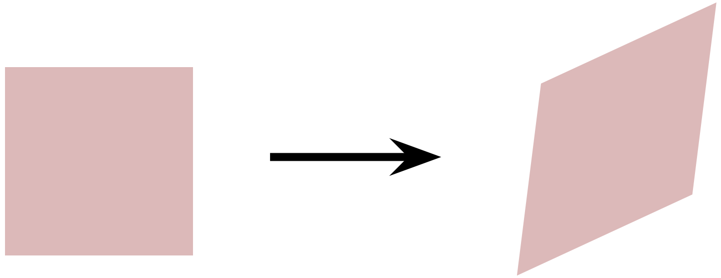


Figure: https://www.youtube.com/@huseyin_ozde...

Affine Transformation

Affine transformations are combinations of

- Arbitrary (4-DOF) linear transformations + translations



Properties of affine transformations:

- origin does not necessarily map to origin
- lines map to lines
- parallel lines map to parallel lines
- ratios are preserved

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A_{00} & A_{01} & b_0 \\ A_{10} & A_{11} & b_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Source: K. Kitani

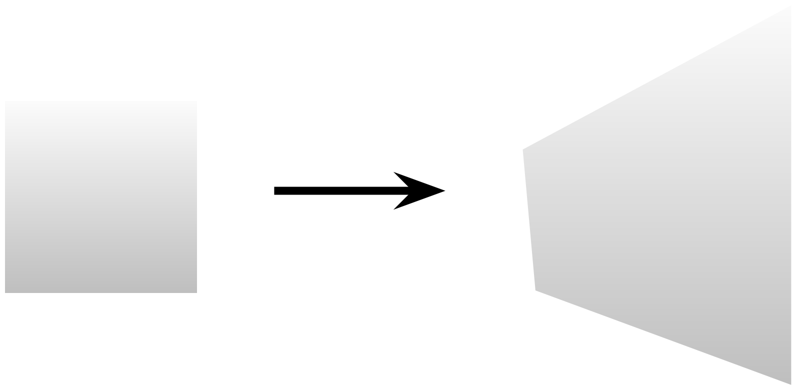
Projective Transformation (homography)

Projective transformations are combinations of

- Affine transformations + projective warps

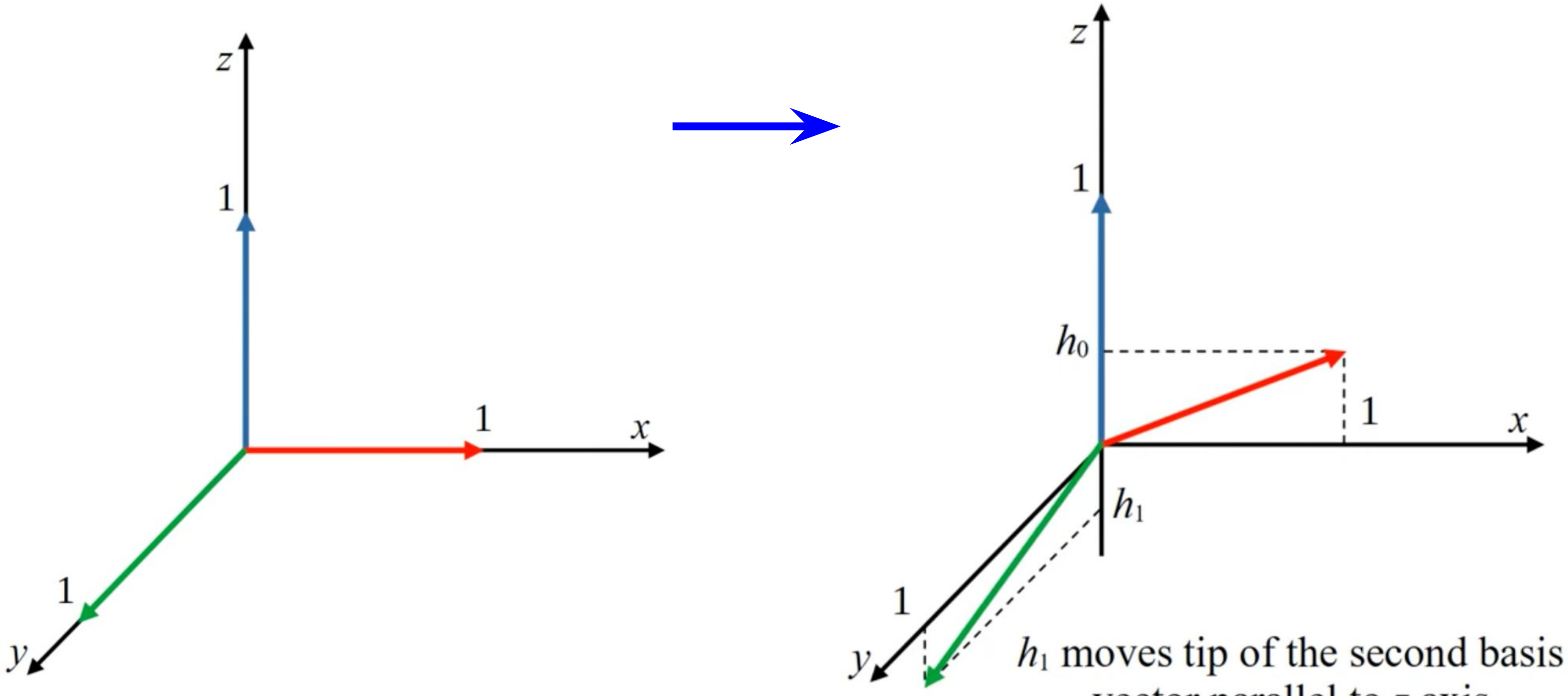
$$w\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A_{00} & A_{01} & b_0 \\ A_{10} & A_{11} & b_1 \\ h_0 & h_1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

How many degrees of freedom?



Source: K. Kitani

Projective Transformation (homography)



h_1 moves tip of the second basis vector parallel to z axis

This matrix is a linear transformation matrix in 3D

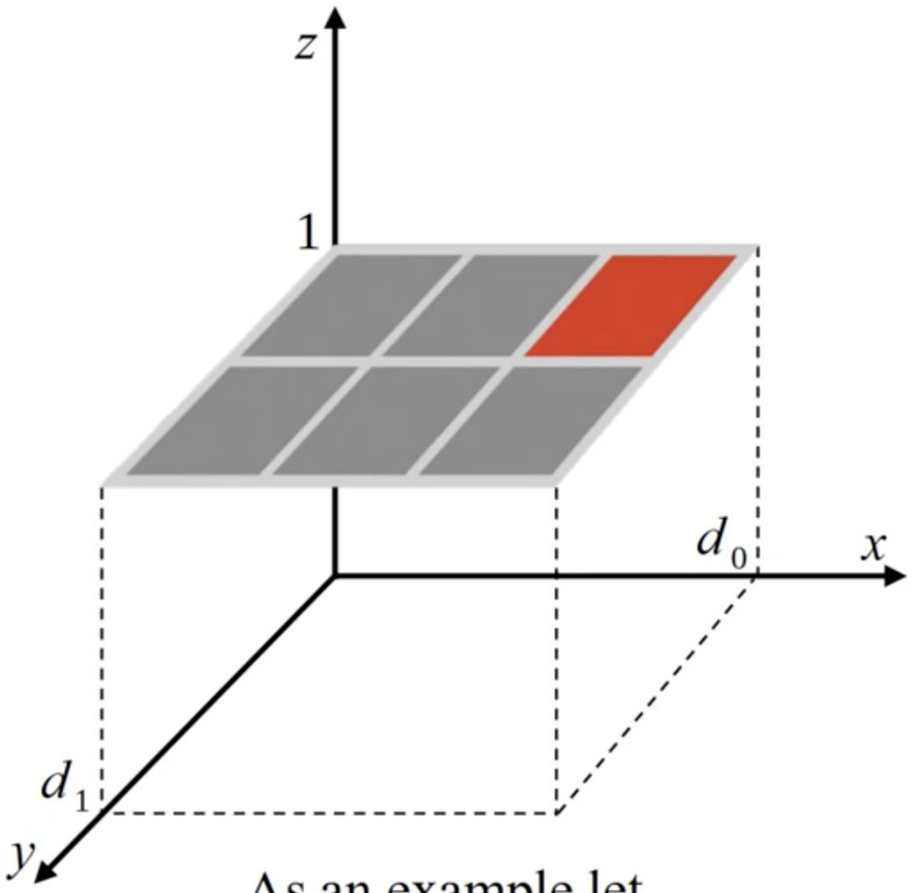
$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ h_0 & h_1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Then these two columns are the first and the second basis vectors of transformed vector space

And what h_0 and h_1 do is to change the orientation of those basis vectors

Figure: https://www.youtube.com/@huseyin_ozde...

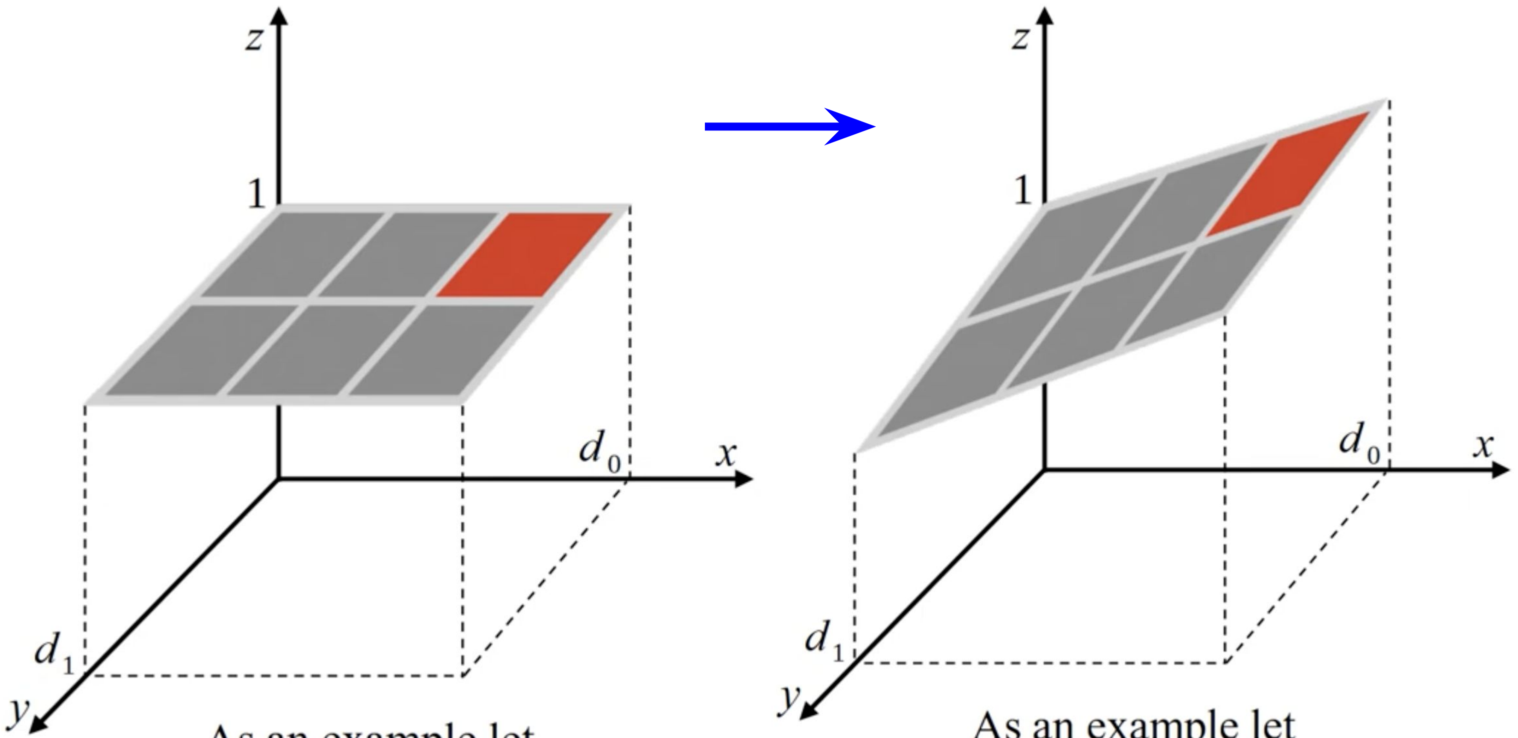
Projective Transformation (homography)



As an example let
$h_0>0$, $h_1<0$ and $|h_0|>|h_1|$

Figure: https://www.youtube.com/@huseyin_ozde...

Projective Transformation (homography)



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Projective Transformation (homography)

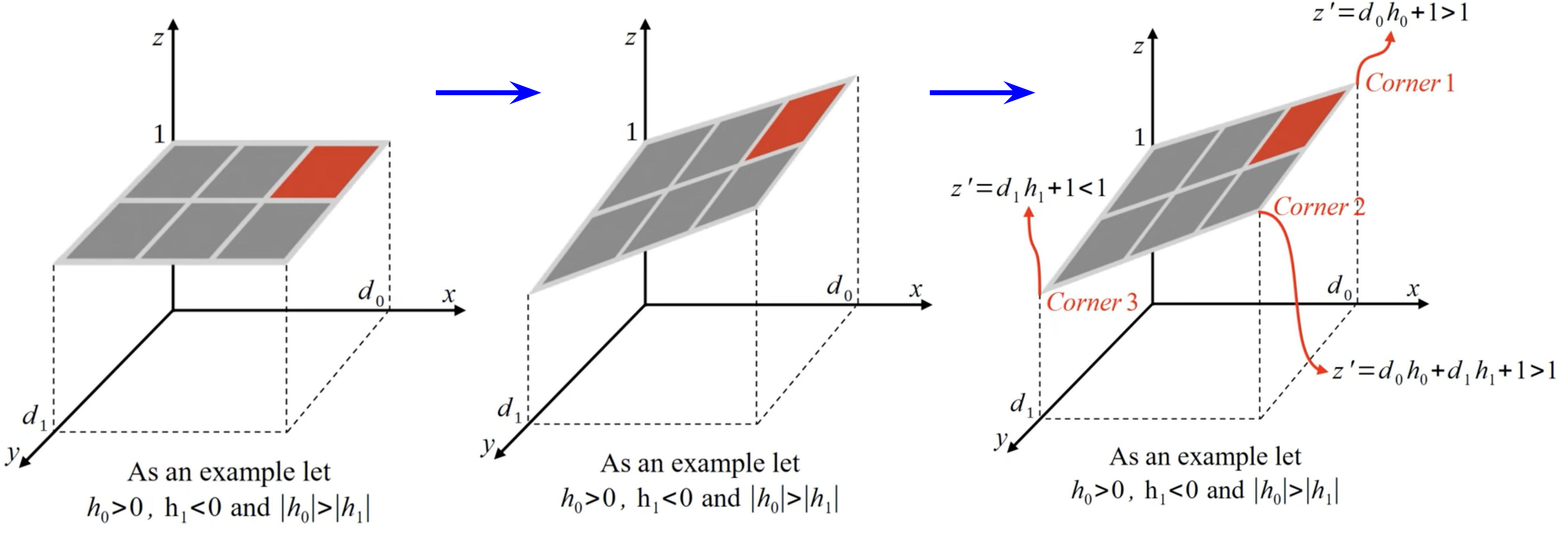


Figure: https://www.youtube.com/@huseyin_ozde...

Projective Transformation (homography)

When going back to Cartesian coordinates

$$\frac{x'}{z'}$$

$$\frac{y'}{z'}$$

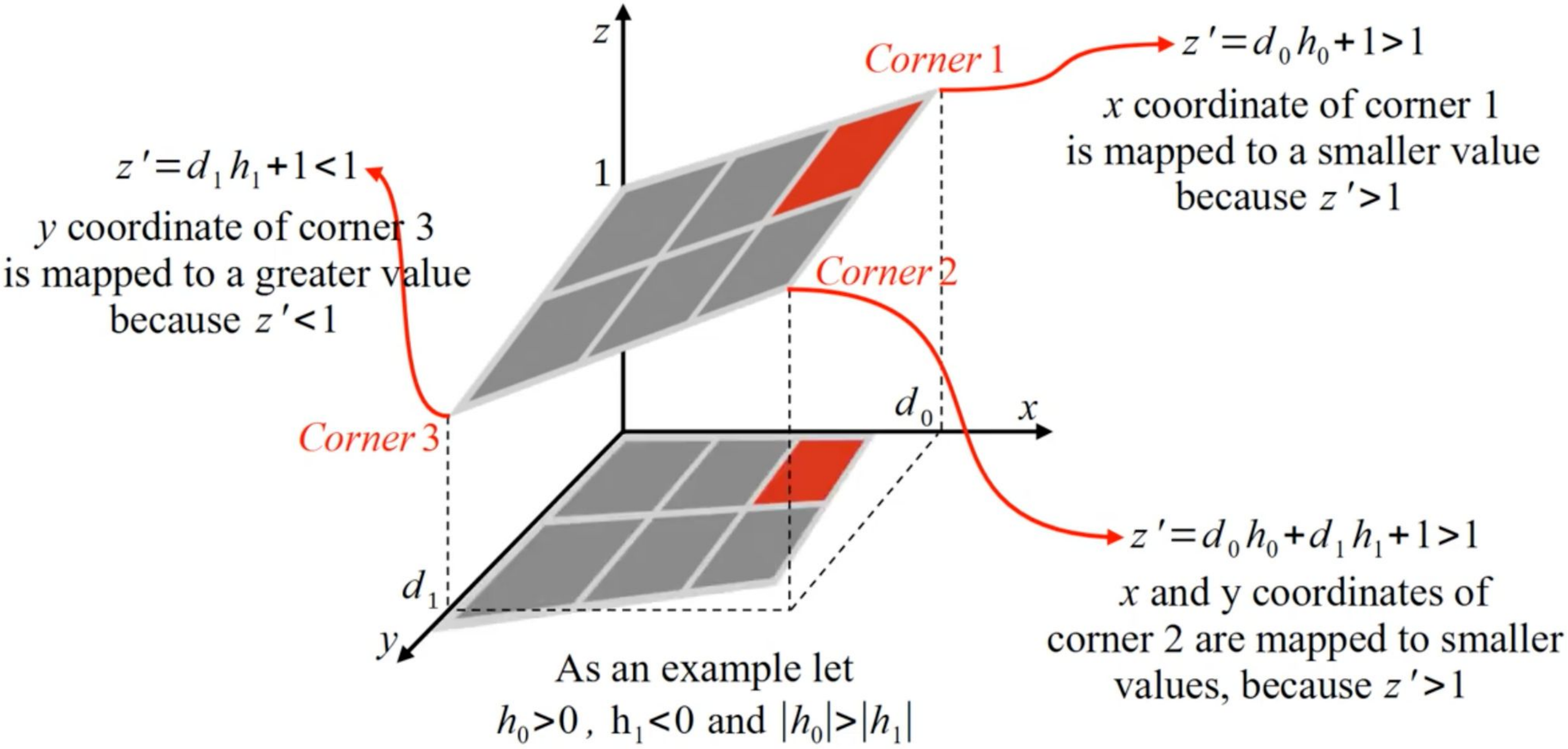
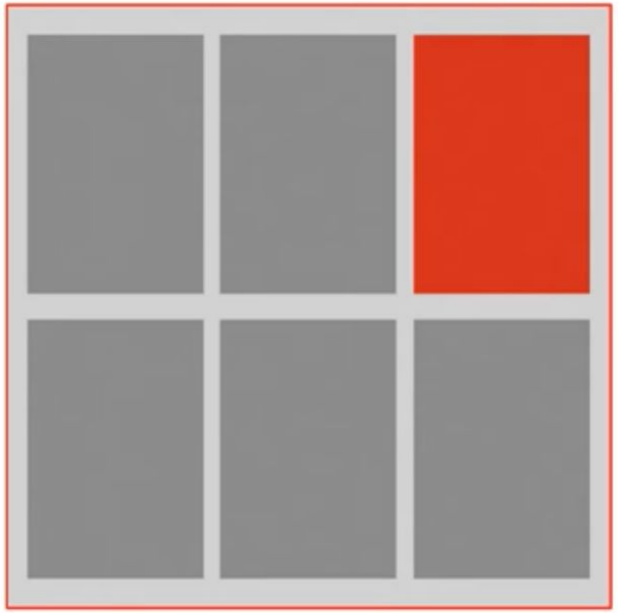


Figure: https://www.youtube.com/@huseyin_ozde...

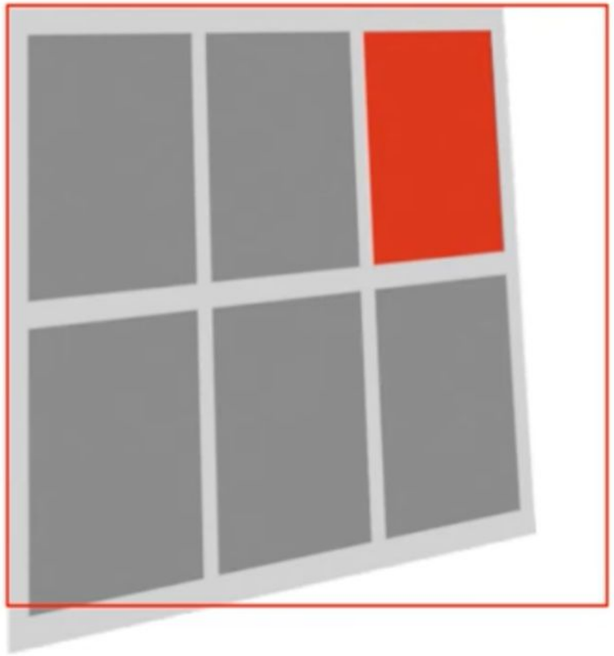
Projective Transformation (homography)



Original Image

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ h_0 & h_1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$h_0 > 0$, $h_1 < 0$ and $|h_0| > |h_1|$



Warped Image

Figure: https://www.youtube.com/@huseyin_ozde...

Projective Transformation (homography)

Projective transformations are combinations of

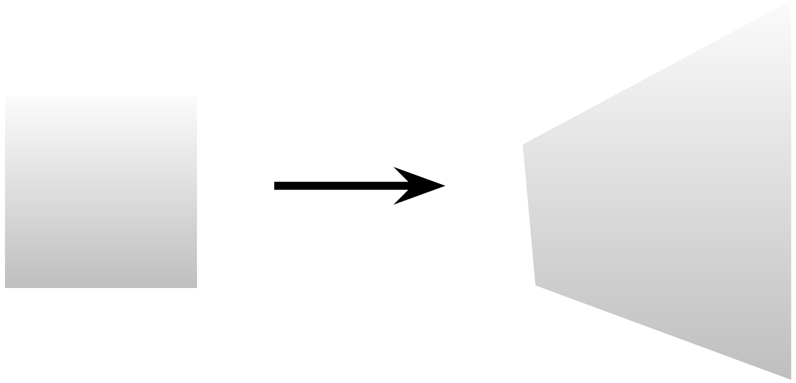
- Affine transformations + projective warps

$$w\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A_{00} & A_{01} & b_0 \\ A_{10} & A_{11} & b_1 \\ h_0 & h_1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

How many degrees of freedom?

Properties of projective transformations:

- origin does not necessarily map to origin
- lines map to lines
- parallel lines do not necessarily map to parallel lines
- ratios are not necessarily preserved



Source: K. Kitani

Questions?

Composing Transformations

Transformations = Matrices => Composition by Multiplication!

$$p' = R_2 R_1 S p$$

In the example above, the result is equivalent to

$$p' = R_2(R_1(Sp))$$

Equivalent to multiply the matrices into single transformation matrix:

$$p' = (R_2 R_1 S)p$$

Order Matters! Transformations from ***right to left***.

Scaling & Translating != Translating & Scaling

$$p'' = TSp = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & t_x \\ 0 & s_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x x + t_x \\ s_y y + t_y \\ 1 \end{bmatrix}$$

$$p''' = STp = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & s_x t_x \\ 0 & s_y & s_y t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x x + s_x t_x \\ s_y y + s_y t_y \\ 1 \end{bmatrix}$$

Similarity: Translation + Rotation + Scaling

p'= (T R S) p

$$p' = TRSp = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} S & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} RS & t \\ 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

This is the form of the general-purpose transformation matrix

2D Transforms = Matrix Multiplication

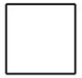
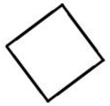
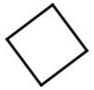
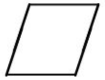
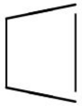
Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{2\times 3}$	3	lengths	
similarity	$\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{2\times 3}$	4	angles	
affine	$\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2\times 3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	

Table 2.1 *Hierarchy of 2D coordinate transformations, listing the transformation name, its matrix form, the number of degrees of freedom, what geometric properties it preserves, and a mnemonic icon. Each transformation also preserves the properties listed in the rows below it, i.e., similarity preserves not only angles but also parallelism and straight lines. The 2 $\times$ 3 matrices are extended with a third $[\mathbf{0}^T\ 1]$ row to form a full 3 $\times$ 3 matrix for homogeneous coordinate transformations.*

Figure: R. Szeliski

3D Transforms = Matrix Multiplication

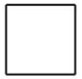
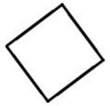
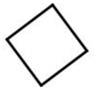
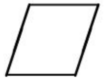
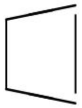
Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{3\times 4}$	3	orientation	
rigid (Euclidean)	$\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{3\times 4}$	6	lengths	
similarity	$\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{3\times 4}$	7	angles	
affine	$\begin{bmatrix} \mathbf{A} \end{bmatrix}_{3\times 4}$	12	parallelism	
projective	$\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{4\times 4}$	15	straight lines	

Table 2.2 *Hierarchy of 3D coordinate transformations. Each transformation also preserves the properties listed in the rows below it, i.e., similarity preserves not only angles but also parallelism and straight lines. The 3 $\times$ 4 matrices are extended with a fourth* $[\mathbf{0}^T \; 1]$ *row to form a full 4 $\times$ 4 matrix for homogeneous coordinate transformations. The mnemonic icons are drawn in 2D but are meant to suggest transformations occurring in a full 3D cube.*

Figure: R. Szeliski