Quantum Series

ENGINEERING

QUANTUM Series

Semester - 6 **Electrical & Electronics Engineering**

Power System-II

- Topic-wise coverage of entire syllabus in Question-Answer form.
- Short Questions (2 Marks)

Includes solution of following AKTU Question Papers:
2015-16 • 2016-17 • 2017-18 • 2018-19

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KEE 601 : Power System-II

1

UNIT

Fault Analysis in Power System

CONTENTS

PART-1

One Line Diagram, Impedance and Reactance Diagram, Per Unit System Changing the Base of Per Unit Qualities, Advantages of Per Unit System.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

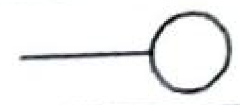
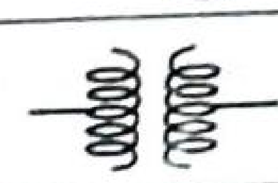
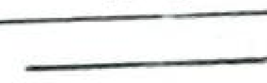
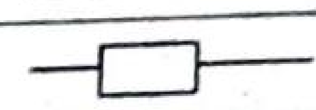
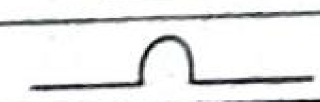
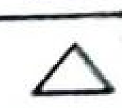
Que 1.1. **What do you mean by "SINGLE LINE DIAGRAM" in power system analysis ? Also mention its importance in Power System Analysis.**

Answer

A. **Single line diagram :**

1. The single line diagram of a power system network shows the main connections and arrangements of the system components along with their data (such as output rating, voltage, resistance and reactance etc.).
2. In a single line diagram, the system components are usually drawn in the form of their symbols.

Table 1.1.1.

S. No.	Components	Symbol
1.	Motor or generator	
2.	Two winding transformer	
3.	Transmission line	
4.	Liquid (oil) circuit breaker	
5.	Air circuit breaker	
6.	Delta connection	
7.	Y-connection, ungrounded	
8.	Y- connection, grounded	

B. Importance of single line diagram :

1. Single line diagram (SLD) helps to locate fault or problem by drawing SLD of an area.
2. Single line diagram makes updation of electrical network very easy even on regular basis.
3. The information from one line diagram can be widely used to enhance the performance of service activities.

Que 1.2. **What do you understand by impedance and reactance diagram ?**

OR

What do you understand by reactance diagram ? Discuss in detail.

AKTU 2015-16, Marks 07

Answer

Impedance and reactance diagram :

1. In the impedance diagram, the different components of the power system are replaced by their equivalent circuits.
2. The synchronous generator is replaced by a constant voltage source behind proper impedance.
3. The transformer is replaced by its equivalent circuit.
4. The transmission line is replaced by nominal π-equivalent circuit.

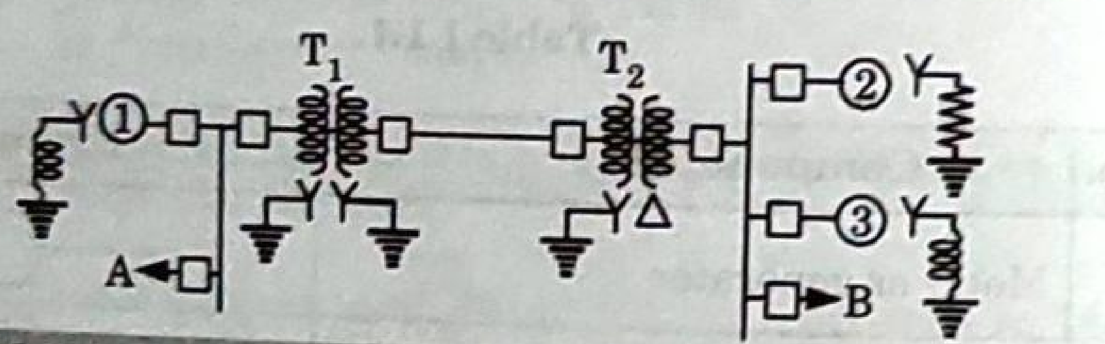


Fig. 1.2.1. One line representation of a simple power system.

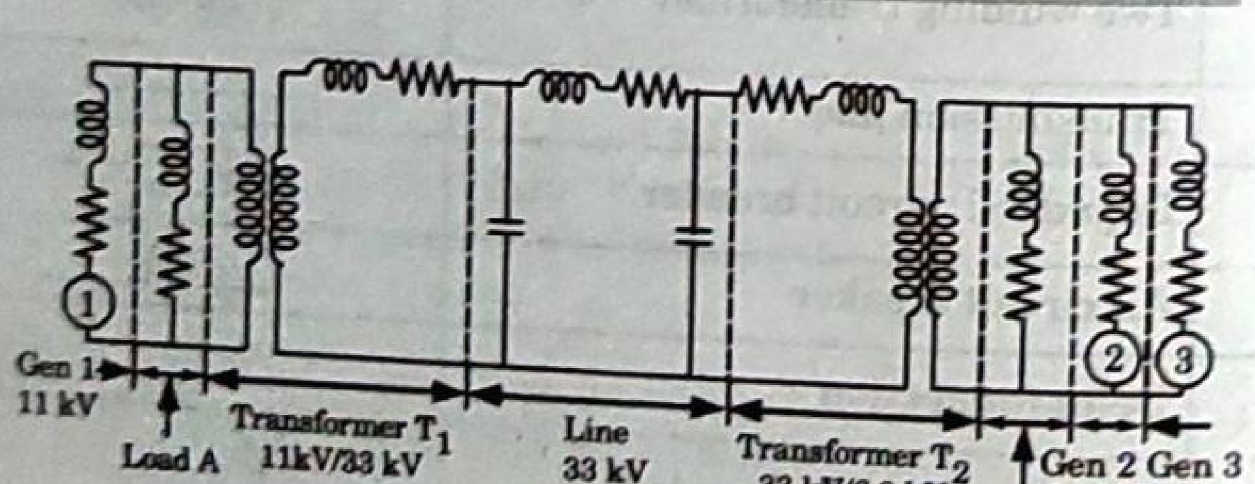


Fig. 1.2.2. Impedance diagram of the power system.

5. This is only true for normal operation (*i.e.*, for steady-state analysis) when both excitation and three-phase network are balanced and they can be analysed on per phase basis.
6. In many power system studies the synchronous generator resistance, resistance of transformer windings, resistance of transmission lines, line charging and magnetizing circuits of transformer are neglected. The impedance diagram then becomes the reactance diagram.

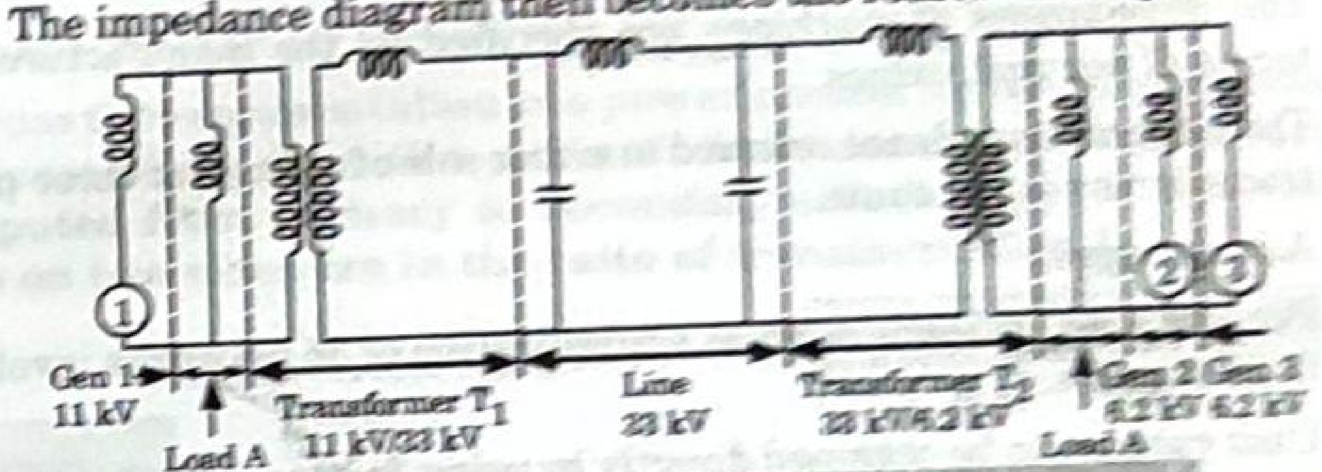


Fig. 1.2.3. Reactance diagram of the power system.

Que 1.3. **What do you understand by "PER UNIT SYSTEM" ? What are the significances in power system analysis ? And give its advantages.**

Answer

A. Per Unit :

1. The numerical per unit (pu) value of any quantity is defined as the ratio of its actual value to another arbitrarily chosen value of quantity of the same dimensions, assumed as the base or reference.

$$\text{Per unit value} = \frac{\text{Actual value of quantity}}{\text{Base value of quantity}}$$

2. The following relationship holds on a per phase basis :

$$\text{Base current} = \frac{\text{Base volt amperes}}{\text{Base voltage}} \text{ (in amperes)}$$

$$\text{Base impedance} = \frac{\text{Base voltage}}{\text{Base current}} \text{ (in ohm)}$$

3. In a three phase system the base kVA may be chosen as the three phase kVA and the base voltage as the line-to-line voltage or the base values may be taken as the phase quantities.
4. **Change of base :**

$$(\text{per unit impedance})_{\text{new base}} = \frac{(VA)_{\text{new base}}(kV)^2_{\text{old base}}}{(VA)_{\text{old base}}(kV)^2_{\text{new base}}} \times (\text{pu impedance})_{\text{old base}}$$

B. Significance :

1. The ordinary parameters (current, impedance, losses, etc.) vary considerably with the variation of physical size, terminal voltage, power

rating, etc., while the per unit parameters are independent of these quantities over a wide range of the same type of apparatus.

2. Per unit values provide more meaningful information.
3. A per unit phase quantity has the same numerical value as the corresponding per unit line quantity regardless of the three-phase connection whether star or delta.
4. The impedances of machines are specified by the manufacturers in terms of per unit values.
5. The per unit impedance referred to either side of a single or three-phase transformer is the same.

C. Advantages :

1. Per unit system leads to great simplification of 3ϕ networks involving transformers.
2. Unit values can be obtained directly by using 3ϕ base quantities.

Que 1.4. **Define the terms per unit voltage, per unit impedance and per unit volt-amperes. Express per unit impedance in terms of base MVA and kV for a three-phase system.**

AKTU 2016-17, Marks 10

Answer

1. **Per unit voltage :** It is the ratio of the actual voltage to the base voltage.

$$V_{pu} = \frac{V_{\text{actual}}}{V_{\text{base}}}$$

2. **Per unit impedance :** It is the ratio of the actual impedance to the base impedance.

$$Z_{pu} = \frac{Z_{\text{actual}}}{Z_{\text{base}}}$$

3. **Per unit volt-ampere :** It is the ratio of the actual volt-ampere to the base volt-ampere.

$$VA_{pu} = \frac{VA_{\text{actual}}}{VA_{\text{base}}}$$

4. **Per unit current :** It is the ratio of the actual current to the base current

$$I_{pu} = \frac{I_{\text{actual}}}{I_{\text{base}}}$$

5. Z_{pu} in terms of base MVA and kV

$$Z_{pu} = Z\frac{[(MVA)_b]_{3\phi}}{[(KV_l)_b]^2}$$

Que 1.5. **Prove the per unit impedance of the transformer is independent of side it is referred to.**
A generator is rated at 30 MVA, 11 kV and has a reactance of 20 %. Calculate its per unit reactance for 50 MVA, 10 kV base.

AKTU 2015-16, Marks 08

OR

Discuss the representation of a power system network by reactance diagram show that the per unit impedance of a transformer computed from primary or secondary side is same if the voltage base on two sides are in the ratio of transformation ?

AKTU 2018-19, Marks 07

Answer

A. Reactance diagram : Refer Q. 1.2, Page 1–3B, Unit-1.

B. Proof :

1. Consider a single-phase transformer in which the total series impedance of the two windings referred to the primary is Z_{1e}.
2. Suppose that the rated values are taken as the base quantities :
Base current in the primary (I_1), Base voltage in the primary (V_1) and
Base impedance in the primary $\left(Z_{b1} = \frac{V_1}{I_1}\right)$.
3. Per unit impedance of the transformer referred to the primary

$$Z_{1epu} = \frac{Z_{1e}}{Z_{b1}} = \frac{Z_{1e}}{V_1 / I_1} = \frac{Z_{e1}\, I_1}{V_1} \quad ...(1.5.1)$$

4. The total series impedance of the two windings referred to the secondary

$$Z_{2e} = Z_{1e}\left(\frac{N_2}{N_1}\right)^2 \quad ...(1.5.2)$$

where N_1 and N_2 represent primary and secondary turns respectively.

5. On the secondary side the base quantities are as follows :
Base current (I_2), Base voltage (V_2) and
Base impedance, $\left(Z_{b2} = \frac{V_2}{I_2}\right)$
6. Per unit impedance of the transformer referred to the secondary

$$Z_{2epu} = \frac{Z_{2e}}{V_2 / I_2} = \frac{Z_{2e}\, I_2}{V_2} \quad ...(1.5.3)$$

7. Now, $$I_2 = I_1\frac{N_1}{N_2} \quad ...(1.5.4)$$

and $V_2 = \frac{N_2}{N_1} V_1$ ...(1.5.5)

8. From eq. (1.5.2) to eq. (1.5.5)

$$Z_{2epu} = Z_{1e}\left(\frac{N_2}{N_1}\right)^2 \frac{I_1 N_1}{N_2} \frac{N_1}{N_2 V_1} = \frac{Z_{1e} I_1}{V_1} \quad ...(1.5.6)$$

9. From eq. (1.5.1) and eq. (1.5.6)

$$Z_{2epu} = Z_{1epu}$$

10. Hence, the per unit equivalent impedance of a 2-winding transformer is the same whether the calculation is made from the high voltage side or the low voltage side.

C. Numerical :

Given : Base volt-amperes = 30 MVA, Base voltage = 11 kV, X = 20 %
To Find : Per unit reactance.

1. Base current = $\frac{30 \times 10^6}{11 \times 10^3} = 2.73 \times 10^3$ A
2. Base reactance = $\frac{11 \times 10^3}{2.73 \times 10^3} = 4.03\ \Omega$
3. So, per unit reactance = $\frac{0.20}{4.03} = 0.05$ pu
4. For 50 MVA, 10 kV base we obtain

Per unit reactance = $0.05\left(\frac{10}{11}\right)^2\left(\frac{30}{50}\right) = 0.025$ pu

Que 1.6. **Draw the zero sequence network for the system shown below in Fig. 1.6.1.**

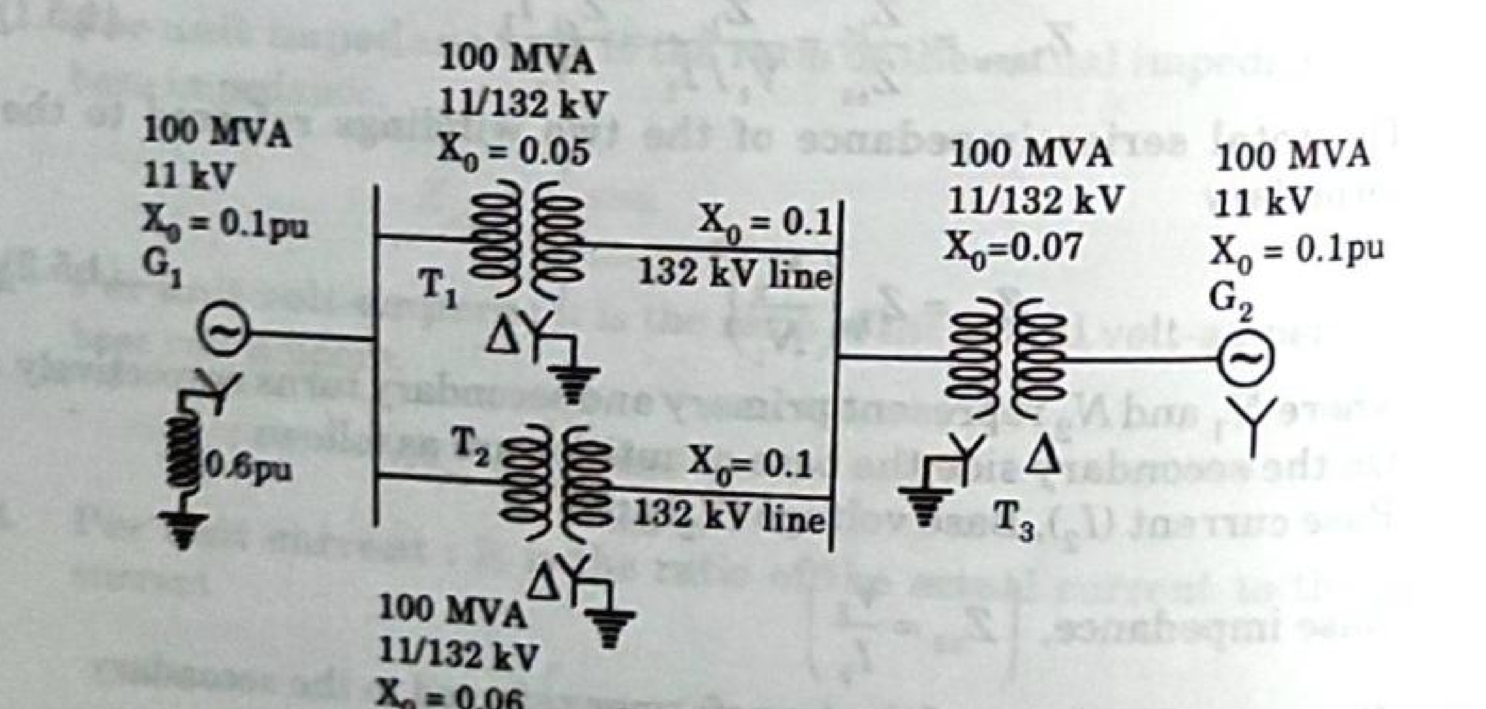


Fig. 1.6.1.

Answer

Given : T_1 : Rating = 100 MVA, N = 11/132 kV, X_0 = 0.05
G_1 : Rating = 100 MVA, V = 11 kV, X_0 = 0.1
G_2 : Rating = 100 MVA, V = 11 kV, X_0 = 0.1
T_1 : Rating = 100 MVA, N = 11/132 kV, X_0 = 0.05
T_2 : Rating = 100 MVA, N = 11/132 kV, X_0 = 0.06;
T_3 : Rating = 100 MVA, N = 11/132 kV, X_0 = 0.07;
TL_1 : V = 132 kV, X_0 = 0.1 and TL_2 : V = 132 kV, X_0 = 0.1
To Draw : Zero sequence network.

1. Choosing base values as,

Base MVA = 100 MVA
Base kV = 11 kV

2. **For G_1 (Generation) :**

$$X_{\text{pu old}} = 0.1 \text{ pu}$$

$$X_{\text{pu new}} = 0.1 \times \frac{100}{100} \times \left(\frac{11}{11}\right)^2 = 0.1 \text{ pu}$$

3. **For Transformer T_1 :**

(On low voltage side) $X_{\text{pu new}} = 0.05 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{100}{100}\right) = 0.05$ pu

(On high voltage side) $X_{\text{pu new}} = 0.05 \times \left(\frac{132}{132}\right)^2 \times \left(\frac{100}{100}\right) = 0.05$ pu

4. **For Transformer T_2 :**

$$X_{\text{pu new}} = 0.06 \times \left(\frac{100}{100}\right) \times \left(\frac{11}{11}\right)^2 = 0.06 \text{ pu}$$

5. **For Transmission line TL_1 :**

$$X_{\text{pu new}} = 0.1 \times \left(\frac{132}{132}\right)^2 \times \left(\frac{100}{100}\right) = 0.1 \text{ pu}$$

6. **For Transmission line TL_2 :**

$$X_{\text{pu new}} = 0.1 \times \left(\frac{132}{132}\right)^2 \times \left(\frac{100}{100}\right) = 0.1 \text{ pu}$$

7. **For Transformer T_3 :**

$$X_{\text{pu new}} = 0.07 \times \left(\frac{132}{132}\right)^2 \times \left(\frac{100}{100}\right) = 0.07 \text{ pu}$$

8. **For Generator G_2 :**

$$X_{\text{pu new}} = 0.12 \times \frac{100}{100} \times \left(\frac{11}{11}\right)^2 = 0.12 \text{ pu}$$

9. **Zero sequence network :**

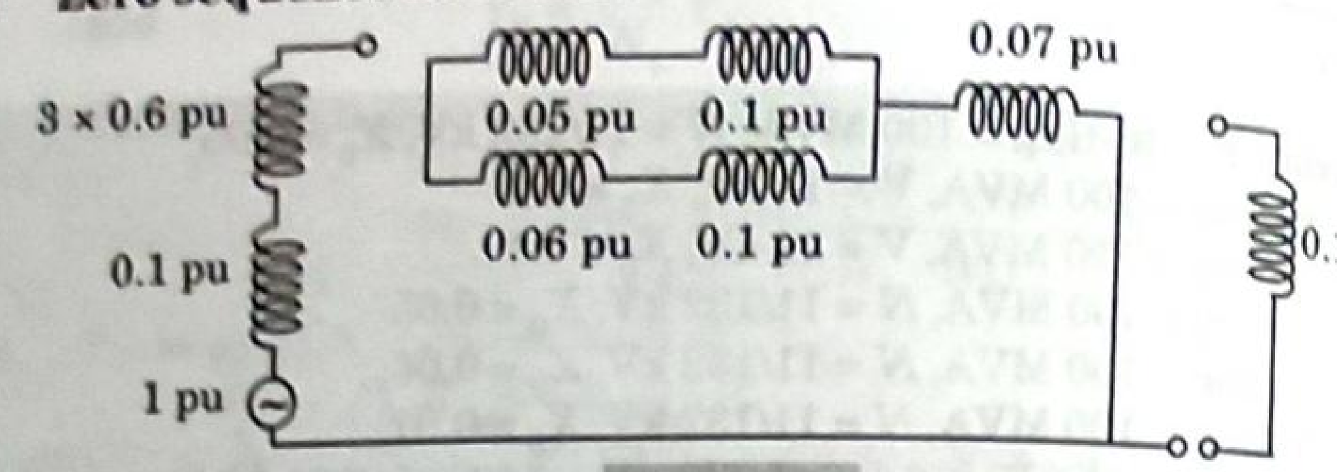


Fig. 1.6.2.

Que 1.7. **Two generators rated at 10 MVA, 13.2 kV and 15 MVA, 13.2 kV respectively are connected in parallel to a bus. The bus feeds two motors rated at 8 MVA and 12 MVA respectively. The rated voltage of motors is 12.5 kV. The reactance of each generator is 15 % and that of each motor is 20 % on its own rating. Assume 50 MVA, 13.8 kV base and draw reactance diagram.**

AKTU 2016-17, Marks 10

Answer

Given : G_1 **:** Rating = 10 MVA, V = 13.2 kV, X = 15 %;
G_2 **:** Rating = 15 MVA, V = 13.2 kV, X = 15 %;
M_1 **:** Rating = 8 MVA, V = 12.5 kV, X' = 20 %;
M_2 **:** Rating = 12 MVA, V = 12.5 kV, X' = 20 %,
Base value : 50 MVA and 13.8 kV
To Draw : Reactance diagram.

1. $$X_{pu\,(new)} = Z_{pu\,(old)} \times \frac{MVA_{new}}{MVA_{old}} \times \left(\frac{kV_{old}}{kV_{new}}\right)^2$$
2. Per unit reactance of generator 1,
$$X_{pu\ G1} = \frac{15}{100} \times \frac{50}{10} \times \left(\frac{13.2}{13.8}\right)^2 = 0.6862 \text{ pu}$$
3. Per unit reactance of generator 2,
$$X_{pu\ G2} = \frac{15}{100} \times \frac{50}{15} \times \left(\frac{13.2}{13.8}\right)^2 = 0.4575 \text{ pu}$$
4. Per unit reactance of motor 1,
$$X_{pu\ M1} = \frac{20}{100} \times \frac{50}{8} \times \left(\frac{12.5}{13.8}\right)^2 = 1.0256 \text{ pu}$$
5. Per unit reactance of motor 2,
$$X_{pu\ M2} = \frac{20}{100} \times \frac{50}{12} \times \left(\frac{12.5}{13.8}\right)^2 = 0.6837 \text{ pu}$$

6. **Reactance diagram :**

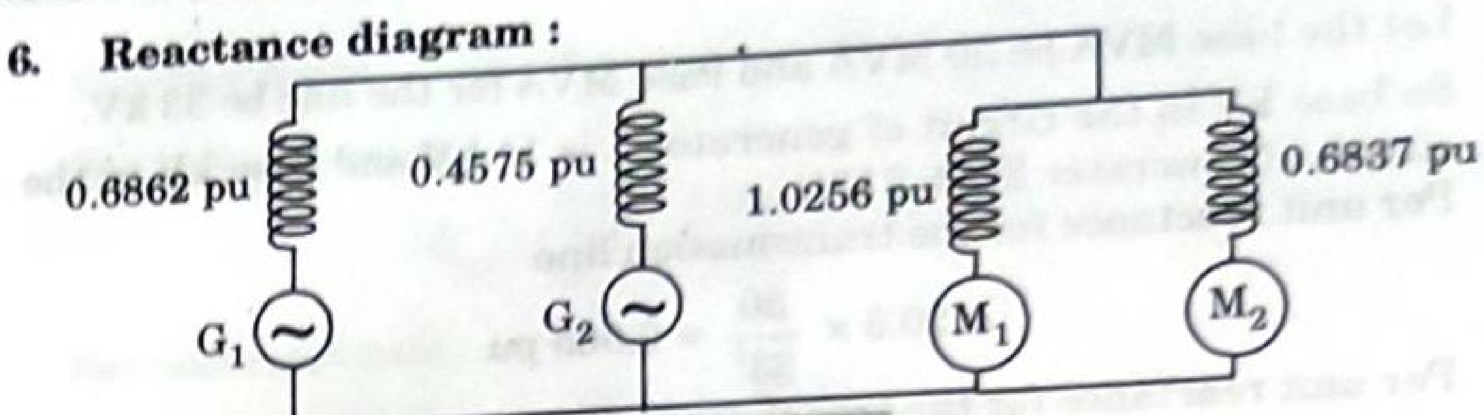


Fig. 1.7.1.

Que 1.8. **Obtain per unit reactance diagram of the power system shown in Fig. 1.8.1. The reactance data of the elements are given as :**

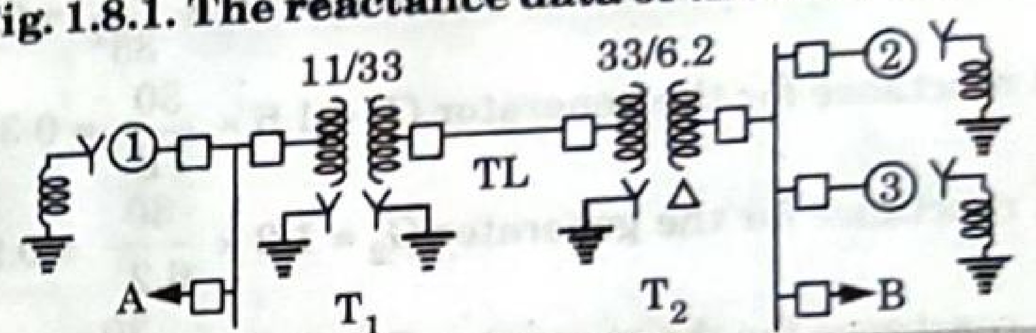


Fig. 1.8.1. One line representation of a simple power system.

G_1 **: 30 MVA**	**10.5 kV**	X'' **= 1.6 Ω**
G_2 **: 15 MVA**	**6.6 kV**	X'' **= 1.2 Ω**
G_3 **: 25 MVA**	**6.6 kV**	X'' **= 0.56 Ω**
T_1 **: 15 MVA**	**33/11 kV**	X **= 15.2 Ω per phase on** ***hv*** **side**
T_2 **: 15 MVA**	**33/6.2 kV**	X **= 1.2 Ω per phase on** ***hv*** **side**
Transmission line	**20.5 Ω per phase**	
Load A, 40 MW	**11 kV (L-L)**	**0.9 lagging pf**
Load B, 40 MW	**6.6 kV (L-L)**	**0.85 lagging pf**

AKTU 2017-18, Marks 10

Answer

Given : G_1 : 30 MVA, 10.5 kV, X'' = 1.6 Ω ;
G_2 : 15 MVA, 6.6 kV, X'' = 1.2 Ω ;
G_3 : 25 MVA, 6.6 kV, X'' = 1.56 Ω;
T_1 : 15 MVA, 33/11 kV,
X = 15.2 Ω per phase on *hv* side;
T_2 : 15 MVA, 33/6.2 kV, X = 1.2 Ω per phase on *hv* side transmission line 2.5 Ω **per phase,**
Load A : 40 MW, 11 kV (L-L), 0.9 lagging pf,
Load B : 40 MW, 6.6 kV (L-L), 0.85 lagging pf.
To Draw : Per unit reactance diagram.

A.

1. Let the base MVA be 30 MVA and base MVA for the line be 33 kV.
2. So base kV in the circuit of generator 1 is 11 kV and base kV in the circuit of generator 2 is 6.2 kV.
3. Per unit reactance for the transmission line

$$= 20.5 \times \frac{30}{33^2} = 0.565 \text{ pu}$$

4. Per unit reactance for the transformer T_1

$$= 15.2 \times \frac{30}{33^2} = 0.419 \text{ pu}$$

5. Per unit reactance for the transformer $T_2 = 16 \times \frac{30}{33^2} = 0.44$ pu
6. Per unit reactance for the generator $G_1 = 1.6 \times \frac{30}{11^2} = 0.397$ pu
7. Per unit reactance for the generator $G_2 = 1.2 \times \frac{30}{6.2^2} = 0.937$ pu
8. Per unit reactance for the generator $G_3 = 0.56 \times \frac{30}{6.2^2} = 0.437$ pu

B. For load A :

1. $$\text{Voltage} = \frac{11}{11} = 1 \text{ pu}$$
2. Power factor, $\cos\phi = 0.9$ lag

$$\sin\phi = \sqrt{1-0.9^2} = 0.436$$

3. Load, $$P = \frac{40}{30} = 1.333 \text{ pu}$$
4. Reactive power, $$Q = P \times \tan\phi = 1.333 \times \frac{0.436}{0.9} = 0.6456 \text{ pu}$$
5. Per unit resistance,

$$R_{pu} = \frac{V_{pu}^2}{P_{pu}} = \frac{1^2}{1.333} = 0.75 \text{ pu}$$

6. Per unit reactance,

$$X_{pu} = \frac{V_{pu}^2}{Q_{pu}} = \frac{1^2}{0.6456} = 1.549 \text{ pu}$$

C. For load B :

1. $$\text{Voltage} = \frac{6.6}{6.2} = 1.065 \text{ pu}$$
2. Power, $$P = \frac{40}{30} = 1.333 \text{ pu}$$
3. Power factor, $\cos\phi = 0.85$ lag

$$\sin\phi = \sqrt{1-0.85} = 0.53$$

4. Reactive power, $$Q = P \times \tan\phi = 1.333 \times \frac{0.53}{0.85} = 0.83 \text{ pu}$$
5. Per unit resistance,

$$R_{pu} = \frac{V_{pu}^2}{P_{pu}} = \frac{1.065^2}{1.333} = 0.85 \text{ pu}$$

6. Per unit reactance,

$$X_{pu} = \frac{V_{pu}^2}{Q_{pu}} = \frac{1.065^2}{0.83} = 1.36 \text{ pu}$$

D. Per unit reactance diagram :

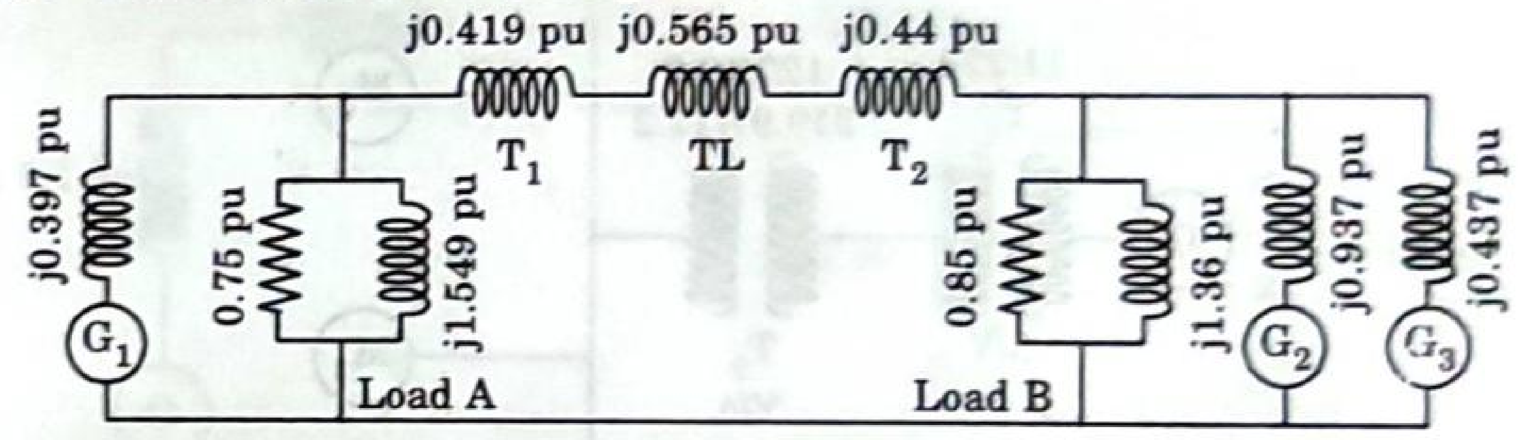


Fig. 1.8.2.

Que 1.9. **A 200 MVA, 11 kV, 3-phase generator has a sub-transient reactance of 10 %. The generator supplies a number of synchronous motors over a 34 km transmission line having transformers at both end in one line diagram. The motor all rated 12.2 kV, represented by just two equivalent motors. Rated inputs to the motor are 300 MVA and 200 MVA for M_1 and M_2 respectively. For both motors $X'' = 30$ %. The three phase transformers T_1 is rated 350 MVA, 220(delta)/11(star grounded) kV with leakage reactance 10 %. Transformer T_2 is composed of three single phase transformers each rated 127 (star grounded)/12.2(delta) kV, 300 MVA with leakage reactance of 10 %. Series reactance of transmission line is 0.2 Ω/km. Draw the reactance diagram with all reactance's marked in per unit. Select the generator rating as base in generator circuit.**

AKTU 2018-19, Marks 07

Answer

Given :
G = 200 MVA, V = 11 kV, X = 10 %
T_1 = 350 MVA, N = 11/220, X = 10 %
T_2 = 300 MVA, N = 127/12.2, X = 10 %
M_1 = 300 MVA, V = 12.2 kV, X = 30 %
M_2 = 200 MVA, V = 12.2 kV, X = 30 %
Series reactance of transmission line = 0.2 Ω/km
To Draw : Reactance diagram.

1. Base value 200 MVA, 11 kV
2. Base kV on *hv* side = Base kV on *lv* side × $\frac{(kV)_{hv}}{(kV)_{lv}}$

   or Base kV on *lv* side = Base kV and *hv* side × $\frac{(kV)_{lv}}{(kV)_{hv}}$
3. Base voltage of the transmission line

$$= \text{Base voltage on the secondary side } T_1$$

$$= 11 \times \frac{220}{11} = 220 \text{ kV}$$

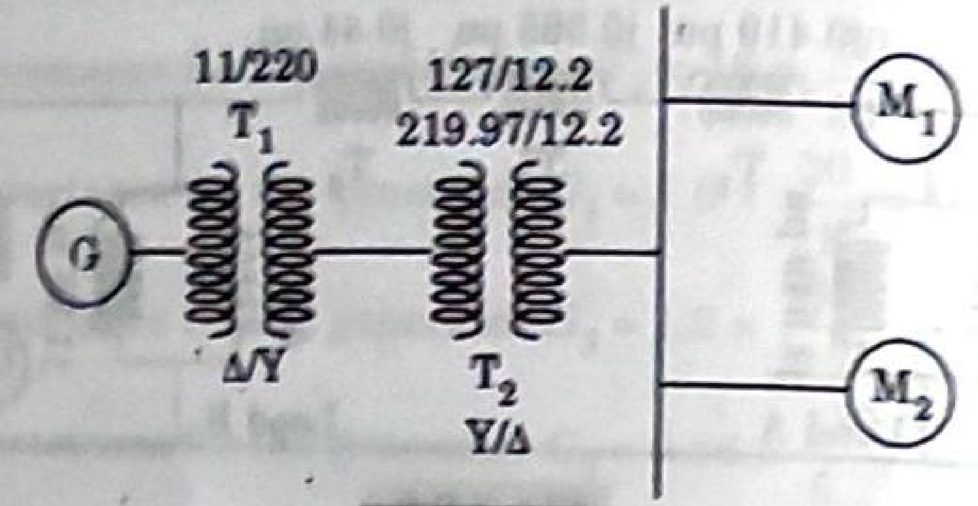


Fig. 1.9.1.

4. Base voltage in the motor circuit

$$= (\text{Base kV on primary of } T_2) \times (\text{Turns ratio on } T_2)$$

$$= 220 \times \frac{12.2}{219.97} = 12.20 \text{ kV}$$

5. For a phase voltage of 127 kV line voltage is

$$= \sqrt{3} \times 127 = 219.97 \text{ kV}$$

6. Calculation of per unit reactance of generator

$$Z_{2pu} = Z_{1pu} \frac{S_{b_2}}{S_{b_1}} \times \left(\frac{V_{b_1}}{V_{b_2}}\right)^2$$

$$= j\,0.1 \times \frac{200}{200} \times \left(\frac{11}{11}\right)^2 = j\,0.1 \text{ pu}$$

7. Calculation of per unit reactance of T_1

$$= j\,0.1 \times \frac{200}{350} \times \left(\frac{11}{11}\right)^2 = j\,0.057 \text{ pu}$$

8. Calculation of per unit reactance of T_2

$$= j\,0.1 \times \frac{200}{900} \times \left(\frac{12.2}{12.20}\right)^2 = j\,0.022 \text{ pu}$$

9. Calculation of per unit reactance of transmission line

$$Z_{pu} = Z_{\Omega} \frac{(MVA)_b}{(kV_l)_b^2}$$

$$= 6.8 \times \frac{200}{220^2} = 0.028 \text{ pu}$$

10. Calculation of per unit reactance of motor

For $M_1 = j\,0.30 \times \frac{200}{300} \times \left(\frac{12.20}{12.20}\right)^2 = j\,0.2 \text{ pu}$

For $M_2 = j\,0.30 \times \frac{200}{200} \times \left(\frac{12.20}{12.20}\right)^2 = j\,0.30 \text{ pu}$

Reactance diagram :

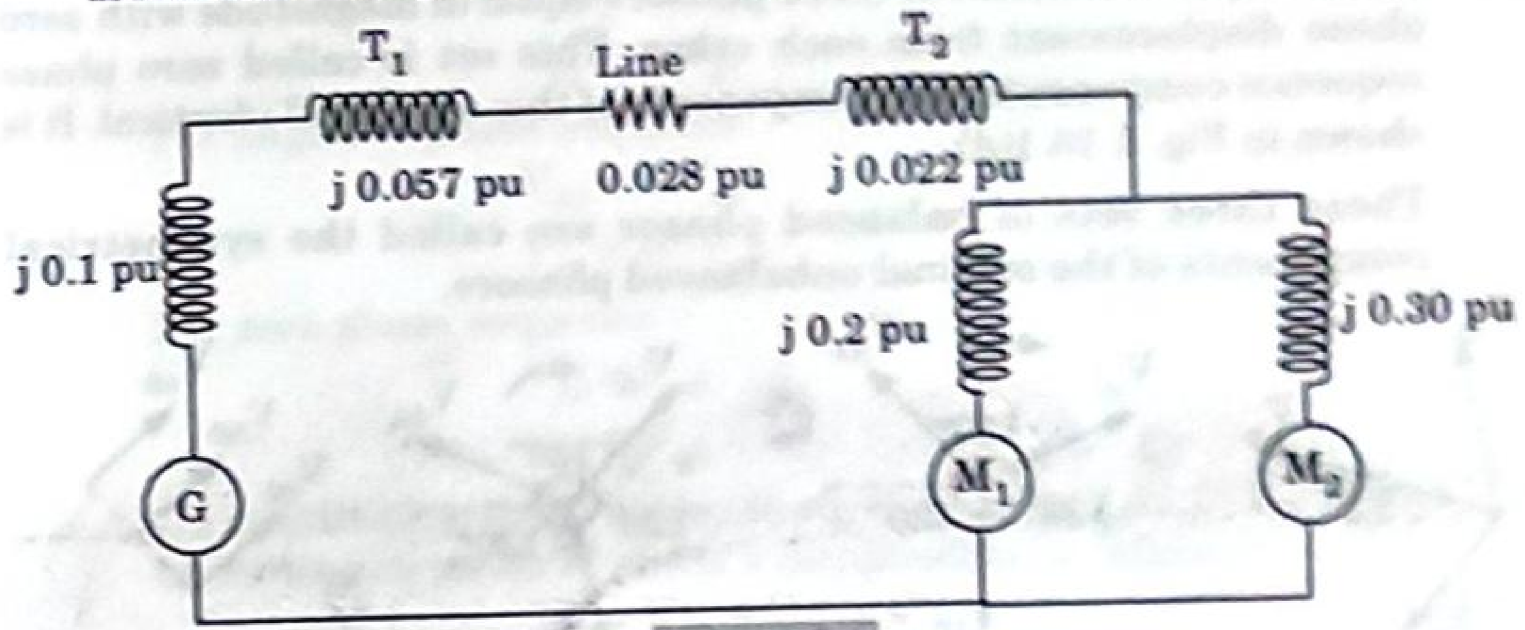


Fig. 1.9.2.

PART-2

Symmetrical Components : Significance of Positive, Negative and Zero Sequence Components, Average 3-Phase Power in Terms of Symmetrical Components.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.10. **Explain the positive, negative and zero sequence components. Also mention its significance in Power System Analysis.**

Answer

Symmetrical components : Three unsymmetrical and unbalanced phasors (voltages or currents) of a three-phase system can be resolved into following three component sets of balanced phasors which process certain symmetry :

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1. **Positive sequence :** A set of three phasor equal in magnitude, disp from each other by 120° in phase, and having the same phase seq as the original unbalanced phasors. This set of balanced phasor called positive phase-sequence components. It is shown in Fig. 1.10.1
2. **Negative sequence :** A set of three phasors equal in magnitu displaced from each other by 120° in phase, and having the ph sequence opposite to that of the original phasors. This set of balan phasors is called negative phase sequence components. It is shown Fig. 1.10.1(c).
3. **Zero sequence :** A set of three phasors equal in magnitude with ze phase displacement from each other. This set is called zero pha sequence components. The components of this set are all identical. It shown in Fig. 1.10.1(d).

These three sets of balanced phasor are called the symmetric components of the original unbalanced phasors.

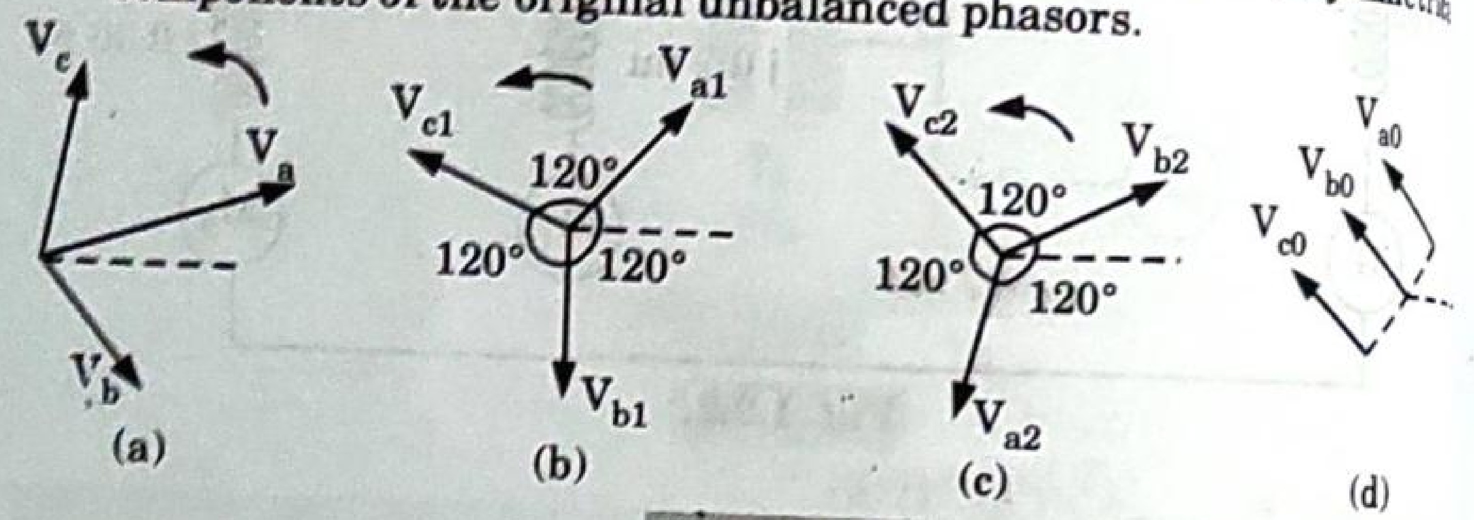


Fig. 1.10.1.

Significance :

1. **Positive sequence :** If the direction of rotation of the stator field i the same as that of the rotor, the set of voltages are positive sequen voltages.
2. **Negative sequence :** If the direction of rotation of the stator field i opposite to that of the rotor, the set of voltages are negative sequenc voltages.
3. **Zero sequence :** The zero sequence voltages are single phase voltage and, therefore, they give rise to an alternating field in space.

Que 1.11. **Discuss the principle of the symmetrical components Derive the necessary equation to convert phase quantities into symmetrical components.**

AKTU 2016-17, Marks 10

Answer

A. **Symmetrical components :** Refer Q. 1.10, Page 1–14B, Unit-1.

B. **Derivation :**

1. Unbalanced phasor can be expressed as the sum of its symmetrical components

$$V_a = V_{a0} + V_{a1} + V_{a2} \quad ...(1.11.1)$$
$$V_b = V_{b0} + V_{b1} + V_{b2} \quad ...(1.11.2)$$
$$V_c = V_{c0} + V_{c1} + V_{c2} \quad ...(1.11.3)$$

2. Here, we use the operator α to express each component of V_b and V_c in terms of the components of V_a.
3. Thus, for a balanced position phase sequence $a\ b\ c$, we may write the following relations :

For positive phase sequence
$$V_{b1} = \alpha^2 V_{a1}$$
$$V_{c1} = \alpha V_{a1}$$
For negative phase sequence
$$V_{b2} = \alpha V_{a2}$$
$$V_{c2} = \alpha^2 V_{a2}$$
For zero phase sequence
$$V_{b0} = V_{a0}$$
$$V_{c0} = V_{a0}$$

4. On substituting these values in eq. (1.11.1) to (1.11.3) V_a, V_b and V_c can be written in terms of phase a components as follows :

$$V_a = V_{a0} + V_{a1} + V_{a2} \quad ...(1.11.4)$$
$$V_b = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2} \quad ...(1.11.5)$$
$$V_c = V_{a0} + \alpha V_{a1} + \alpha^2 V_{c2} \quad ...(1.11.6)$$

5. Eq. (1.11.4) to (1.11.6) can be put in matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \quad ....(1.11.7)$$

6. Eq. (1.11.7) can be put in a more compact form

$$V_p = AV_s$$

where, $$V_p = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \text{Phase voltage vector}$$

$$V_s = \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \text{Symmetrical sequence voltage vector}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$(\alpha = 1 \angle 120°, \alpha^2 = 1 \angle 240°, \alpha^3 = 1)$

7. The voltage vectors V_p and V_s represent the actual phase voltages and the symmetrical voltage components respectively.
8. A is called the symmetrical components transform matrix which transforms the phase voltage V_p into components voltage V_s.

Que 1.12. **What do you understand by symmetrical components of unbalanced phasors ? Deduce the expression for symmetrical components.**

Answer

A. Symmetrical Component : Refer Q. 1.10, Page 1–14B, Unit-1.

B. Derivation :

1. These equations can be expressed in the matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_p = AV_s \qquad \text{...(1.12.1)}$$

2. We can write eq. (1.12.1) as

$$V_s = A^{-1}V_p$$

3. Computing A^{-1}, we get

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

4. $$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad \text{...(1.12.2)}$$

5. In explained form we can write eq. (1.12.2) as

$$V_{a1} = \frac{1}{3}(V_a + \alpha V_b + \alpha^2 V_c)$$

$$V_{a2} = \frac{1}{3}(V_a + \alpha^2 V_b + \alpha V_c)$$

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$$

Que 1.13. **Deduce the expression of power in terms of symmetrical components.**

Answer

1. In a single phase system voltampere S is given by

$$S_{1\phi} = P + jQ = VI^* \qquad \text{...(1.13.1)}$$

where V and I are phase voltage and phasor current and I^* is the conjugate of I.

2. Similarly in a three phase system

$$S_{abc} = S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^* \qquad \text{...(1.13.2)}$$

where V_a, V_b and V_c are the phase voltages and I_a^*, I_b^* and I_c^* are the conjugates of phase currents I_a, I_b and I_c.

3. In matrix form eq. (1.13.2) may be written as

$$S_{abc} = [V_a V_b V_c] \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

$$= \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = V_{abc}^T I_{abc}^*$$

where the superscript T stands for transpose.

4. Now, $$V_{abc} = AV_{012}$$

$$I_{abc} = AI_{012}$$

$$S_{abc} = [AV_{012}]^T [AI_{012}]^* = V_{012}^T A^T A^* I_{012}^* \qquad \text{...(1.13.3)}$$

5. Because the transpose of the product of two matrices is equal to the product of the transpose of the matrices in the reverse order.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} = A$$

$$A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^{2*} & \alpha^* \\ 1 & \alpha^* & \alpha^{2*} \end{bmatrix}$$

6. But $\alpha^* = \alpha^2$, $\alpha^{2*} = \alpha$

$$A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

7. And, $$A^T A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3U$$

where, $U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = Unit matrix

8. Substituting the value of $A^T A^*$ in eq. (1.13.3)

$$S_{abc} = 3V_{012}^T \, U I_{012}^*$$

$$= 3V_{012}^T \, I_{012}^* = 3S_{012}$$

$$= 3[V_{a0} \; V_{a1} \; V_{a2}] \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

$$= 3V_{a0}\, I_{a0}^* + 3\, V_{a1}\, I_{a1}^* + 3V_{a2}\, I_{a2}^* \quad \text{...(1.13.4)}$$

Eq. (1.13.4) shows that the total complex power in the unbalanced system is equal to the sum of the complex powers of the three symmetrical components.

9. Hence we can say that the symmetrical component transform is power invariant.

Que 1.14. **The line to ground voltage on the high voltage side of a transformer are 100 kV, 33 kV and 38 kV on phase *a*, *b*, and *c*. The voltage of phases *a* leads by phase *b* by 100° and lags that of phase *c* by 176.5°, determine analytically the symmetrical components of voltage.**

AKTU 2016-17, Marks 10

Answer

Given : V_a = 100 kV, V_b = 33 kV, V_c = 38 kV, V_a leads V_b by 100° and V_a lags V_c by 176.5°.
To Find : V_{a0}, V_{a1} and V_{a2}.

1. Zero sequence component :

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$$

$$= \frac{1}{3}(100\angle 0° + 33\angle{-100°} + 38\angle 176.5°)$$

$$= \frac{1}{3}(56.34 - j30.17) = 18.78 - j10.06 \text{ kV}$$

2. Positive sequence component :

$$V_{a1} = \frac{1}{3}(V_a + \alpha V_b + \alpha^2 V_c)$$

$$= \frac{1}{3}[(100\angle 0° + (1\angle 120°)(33\angle{-100°}) + (1\angle{-120°})(38\angle 176.5°))]$$

$$= \frac{1}{3}(100\angle 0° + 33\angle 20° + 38\angle 56.5°)$$

$$= \frac{1}{3}(151.98 + j42.97) = 50.06 + j14.3 \text{ kV}$$

3. Negative sequence component :

$$V_{a2} = \frac{1}{3}(V_a + \alpha^2 V_b + \alpha V_c)$$

$$= \frac{1}{3}[(100\angle 0° + (1\angle{-120})(33\angle{-100°}) + (1\angle 120°)(38\angle 176.5))]$$

$$= \frac{1}{3}(100\angle 0° + 33\angle{-220°} + 38\angle 296.5°)$$

$$= \frac{1}{3}(91.67 - j12.79) = 30.05 - j4.2 \text{ kV}$$

Que 1.15. **One conductor of a 3-phase line is open. The current flowing to the delta connected load through line '*a*' is 10 A. With the current in the line '*a*' as reference and assuming that line '*c*' is open. Find the symmetrical components of the line current.**

AKTU 2017-18, Marks 10

Answer

Given : I_a = 10 A and I_c = 0
To Find : Symmetrical components.

1. As $I_a = 10\angle 0°$ so $I_b = 10\angle 180°$

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3}(10 + 10\angle 180° + 0) = 0$$

2. $$I_{a1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c)$$

$$= \frac{1}{3}[10 + (1\angle 120°)(10\angle 180°) + 0] = 5.77\angle{-30°}$$

3. $$I_{a2} = \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c)$$

$$= \frac{1}{3}[10 + (1\angle{-120°})(10\angle 180°) + 0] = 5.77\angle 30°$$

4. $I_{b0} = I_{a0} = 0$

5. $I_{b1} = \alpha^2 I_{a1} = (1 \angle -120°)(5.77 \angle -30°) = -5 - j2.8$

6. $I_{b2} = \alpha I_{a2} = (1 \angle 120°)(5.77 \angle 30°) = -5 + j2.8$

7. $I_{c0} = I_{a0} = 0$

8. $I_{c1} = \alpha I_{a1} = (1 \angle 120°)(5.77 \angle -30°) = j5.77$

9. $I_{c2} = \alpha^2 I_{a2} = (1 \angle -120°)(5.77 \angle 30°) = -j5.77$

Que 1.16. **A generator supplying an unbalanced load measu the following phase to ground voltages. $V_a = 16.0\angle 0°$ k $V_b = 15.0\angle -152°$ kV, $V_c = 10.0\angle 100°$ kV. Find the symmetri components of the set of phase voltages.** **AKTU 2018-19, Marks**

Answer

Given : $V_a = 16.0 \angle 0°$ kV, $V_b = 15.0 \angle -152°$ kV, $V_c = 10.0 \angle 100°$ kV

To Find : V_{a0}, V_{a1}, V_{a2}.

1.
$$V_{a0} = \frac{1}{3}[V_a + V_b + V_c]$$
$$= \frac{1}{3}[16 \angle 0° + 15 \angle -152° + 10 \angle 100°]$$
$$= 0.995 \angle 70.04° \text{ kV}$$

2.
$$V_{a1} = \frac{1}{3}[V_a + \alpha V_b + \alpha^2 V_c]$$
$$= \frac{1}{3}[16 \angle 0° + \alpha \times 15 \angle -152° + \alpha^2 \times 10 \angle 100°]$$
$$= \frac{1}{3}[16 \angle 0° + 1 \angle 120° \times 15 \angle -152° + 1 \angle 240° \times 10 \angle 100°]$$
$$= \frac{1}{3}[16 \angle 0° + 15 \angle -32° + 10 \angle 340°] = 13.25 \angle -16.60° \text{ kV}$$

3.
$$V_{a2} = \frac{1}{3}[V_a + \alpha^2 V_b + \alpha V_c]$$
$$= \frac{1}{3}[16 \angle 0° + \alpha^2 \times 15 \angle -152° + \alpha \times 10 \angle 100°]$$
$$= \frac{1}{3}[16 \angle 0° + 1 \angle 240° \times 15 \angle -152° + 1 \angle 120° \times 10 \angle 100°]$$
$$= \frac{1}{3}[16 \angle 0° + 15 \angle 88° + 10 \angle 220°]$$

$$= 4.10 \angle 44.01° \text{ kV}$$

PART-3

Sequence Impedances and Sequence Networks.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.17. **Explain sequence impedances.**

OR

Prove that for the transmission lines zero sequence impedance is much larger than the positive sequence impedance. Also discuss the effect of mutual inductances.

Answer

A. Sequence impedances :

1. The sequence impedance of the network describes the behaviour of the system under asymmetrical fault conditions.
2. The performance of the system determines by calculating the impedance offered by the different element of the power system to the flow of the different phase sequence component of the current.
3. Every power system component (static or rotating) has three values of impedance one for each symmetrical value of current.

B. Transmission line :

1. The Fig. 1.17.1 shows the circuit of fully transposed line carrying unbalanced currents.
2. The neutral is kept sufficiently away so that mutual inductive effect of I_n can be ignored.
3. Let X_s = Self reactance of each line

   X_m = Mutual reactance of any line pair
4. From Fig. 1.17.1, following KVL equations can be conclusively written :

$$V_a - V_a' = jX_s I_a + jX_m I_b + jX_m I_c$$
$$V_b - V_b' = jX_m I_a + jX_s I_b + jX_m I_c$$
$$V_c - V_c' = jX_m I_a + jX_m I_b + jX_s I_c$$

Now,
$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \begin{bmatrix} V_a' \\ V_b' \\ V_c' \end{bmatrix} = j \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad ...(1.17.1)$$

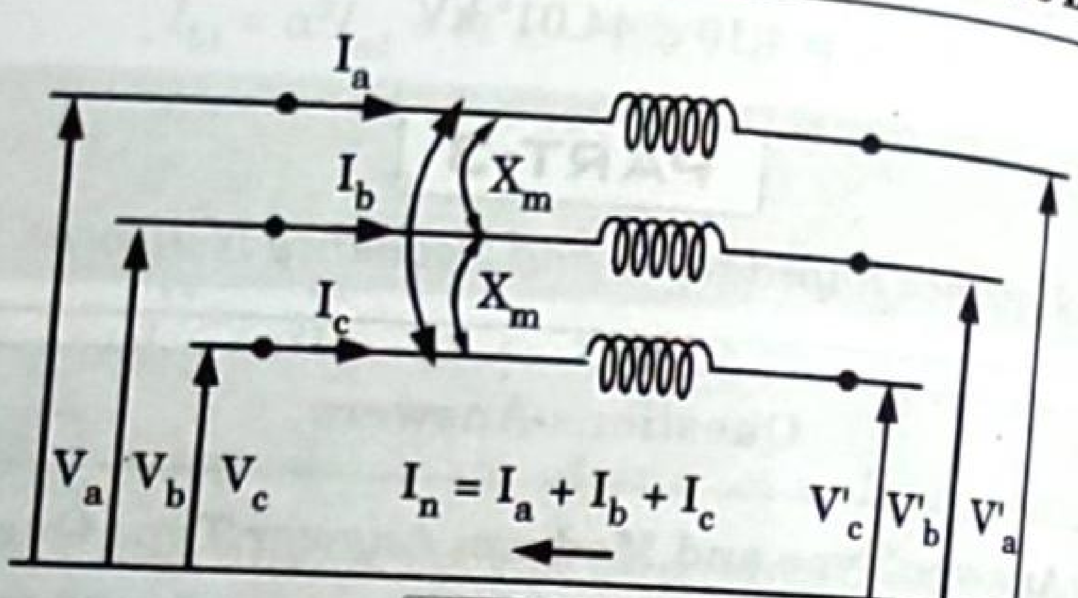


Fig. 1.17.1.

5. Also, $V_p - V_p' = ZI_p$

Multiplying A on both side,

$$A(V_s - V_s') = ZAI_s \qquad [V_p = AV_s \text{ and } I_p = AI_s]$$

$$V_s - V_s' = A^{-1} ZA\, I_s \qquad ...(1.17.2)$$

6. Now,

$$A^{-1}ZA = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} jX_s & jX_m & jX_m \\ jX_m & jX_s & jX_m \\ jX_m & jX_m & jX_s \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix}$$

$$= j\begin{bmatrix} X_s - X_m & 0 & 0 \\ 0 & X_s - X_m & 0 \\ 0 & 0 & X_s + 2X_m \end{bmatrix} \qquad ...(1.17.3)$$

7. Putting eq. (1.17.3) in eq. (1.17.2), we get

$$\begin{bmatrix} V_1 \\ V_2 \\ V_0 \end{bmatrix} - \begin{bmatrix} V_1' \\ V_2' \\ V_0' \end{bmatrix} = j\begin{bmatrix} X_s - X_m & 0 & 0 \\ 0 & X_s - X_m & 0 \\ 0 & 0 & X_s + 2X_m \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_0 \end{bmatrix} \qquad ...(1.17.4)$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_0 \end{bmatrix} - \begin{bmatrix} V_1' \\ V_2' \\ V_0' \end{bmatrix} = \begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_2 & 0 \\ 0 & 0 & Z_0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_0 \end{bmatrix}$$

where,

$Z_1 = j(X_s - X_m)$ = Positive Sequence Impedance

$Z_2 = j(X_s - X_m)$ = Negative Sequence Impedance

$Z_0 = j(X_s + 2X_m)$ = Zero Sequence Impedance

$$Z_1 = Z_2 = j(X_s - X_m) \qquad ...(1.17.5)$$

$$Z_0 = j(X_s + 2X_m) \qquad ...(1.17.6)$$

8. Comparing eq. (1.17.5) and (1.17.6),

$$Z_0 >> Z_1\ (= Z_2)$$

Thus, zero sequence impedance is much larger than the positive/negative sequence impedance.

9. If the mutual inductances are neglected, then the zero sequence impedance will be equal to positive sequence impedance, which in turn equals to negative sequence impedance of the transmission lines.

Que 1.18. **Explain the sequence impedances. Define balanced star connected load and sequence impedances of transmission lines.**

AKTU 2017-18, Marks 10

Answer

A. Sequence Impedance : Refer Q. 1.17, Page 1–22B, Unit-1.

B. Sequence impedance of transmission line : Refer Q. 1.17, Page 1–22B, Unit-1.

C. Balanced star connected load :

1. Consider the circuit in Fig. 1.18.1.

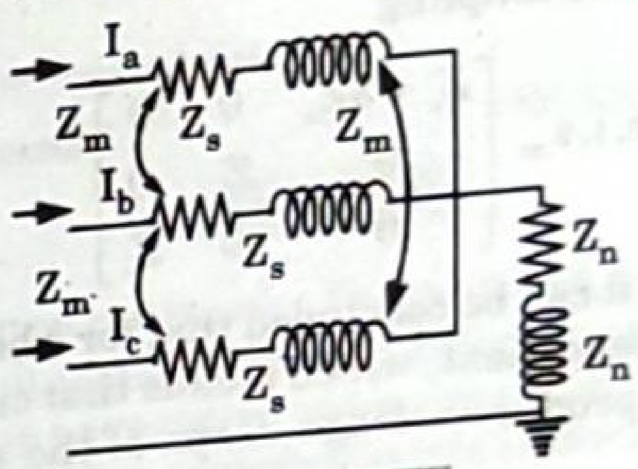


Fig. 1.18.1.

A three phase balanced load with self and mutual impedances Z_s and Z_m respectively drawn current I_a, I_b and I_c as shown in Fig. 1.18.1. Z_n is the impedance in the neutral circuit which is grounded and draws the current I_n.

2. The line-to-ground voltages are given by

$$V_a = Z_s I_a + Z_m I_b + Z_m I_c + Z_n I_n \qquad ...(1.18.1)$$

$$V_b = Z_m I_a + Z_s I_b + Z_m I_c + Z_n I_m$$

$$V_c = Z_m I_a + Z_m I_b + Z_s I_c + Z_n I_n$$

3. Since $I_a + I_b + I_c = I_n$

Eliminating I_n from eq. (1.18.1)

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_s + Z_n & Z_m + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_s + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_m + Z_n & Z_s + Z_n \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad ...(1.18.2)$$

4. Put in compact matrix notation

$$[V_{abc}] = [Z_{abc}][I_{abc}] \qquad ...(1.18.3)$$

$$V_{abc} = [A]\, V_a^{0,1,2} \quad ...(1.18.4)$$

$$I_{abc} = [A]\, I_a^{0,1,2} \quad ....(1.18.5)$$

Premultiplying eq. (1.18.3) by $[A]^{-1}$ and using eq. (1.18.4) and (1.18.5)

$$V_a^{0,1,2} = [A]^{-1}[Z_{abc}][A]\, I_a^{0,1,2}$$

5. Defining $\quad Z]^{0,1,2} = [A^{-1}][Z_{abc}][A]$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & 1 \end{bmatrix}\begin{bmatrix} Z_s + Z_n & Z_m + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_s + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_m + Z_n & Z_s + Z_n \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} (Z_s + 3Z_n + 2Z_m) & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}$$

6. If there is no mutual coupling

$$[Z]^{0,1,2} = \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \quad ...(1.18.6)$$

7. From eq. (1.18.6), it can be concluded that for a balanced load the three sequences are independent, which means that currents flowing in one sequence will be produce voltage drops of the same phase sequence only.

Que 1.19. **Consider a balanced 3ϕ system shown in Fig. 1.19.1.**

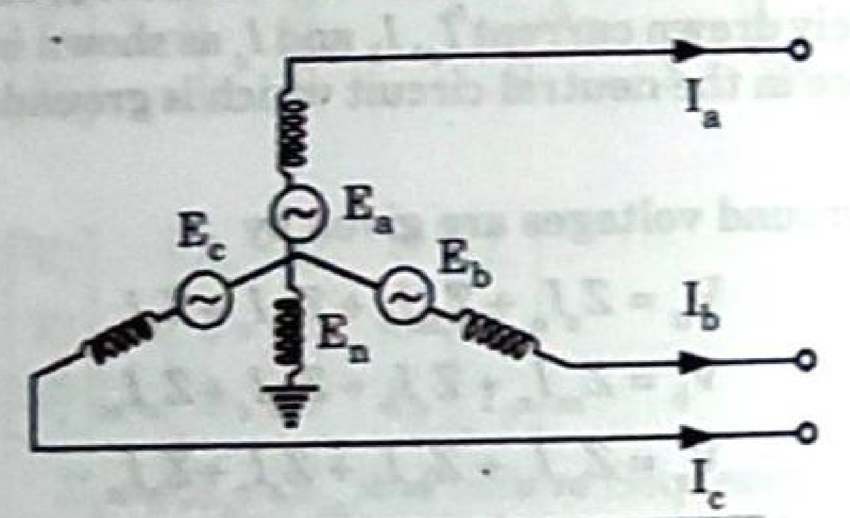


Fig. 1.19.1. 3ϕ balanced system.

Draw and explain the following :

i. Positive sequence network

ii. Negative sequence network

iii. Zero sequence network.

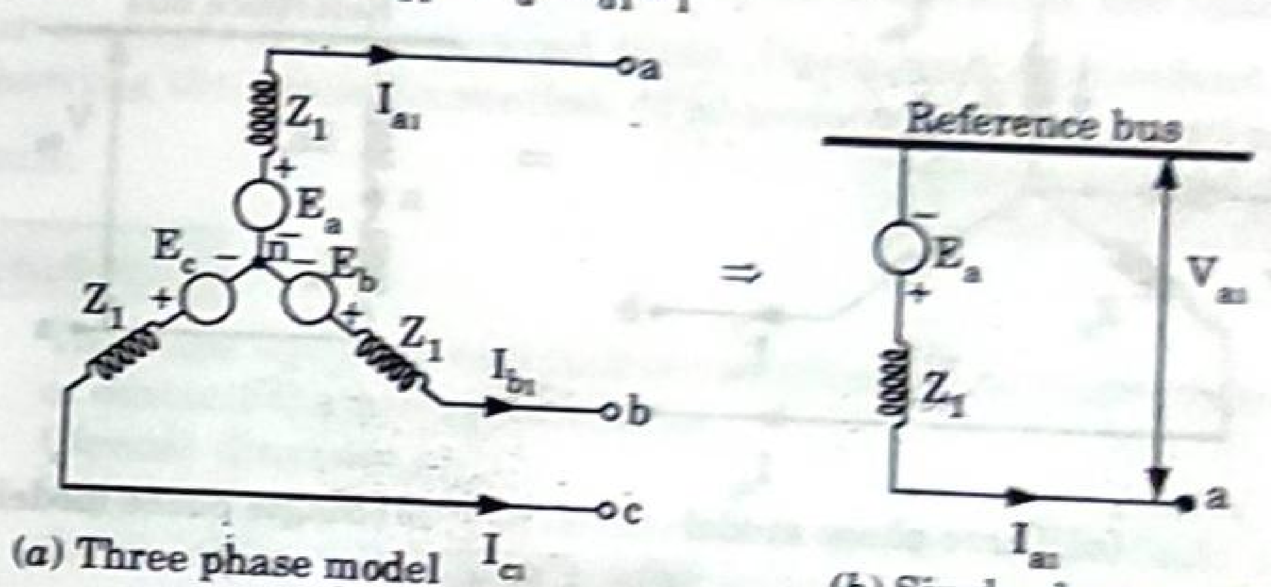
OR

Show that zero sequence impedance of a generator with neutral grounded impedance Z_0 is $(Z_s + 3Z_n)$ where Z_s is the impedance of synchronous generator.

Answer

i. **Positive sequence network :** The positive sequence component of voltage at fault point is the positive sequence generated voltage minus the drop to positive sequence current in positive sequence impedance (As positive sequence current does not produce drop in negative or zero sequence impedances).

$$V_{a1} = E_a - I_{a1} Z_1$$



(a) Three phase model (b) Single phase model

Fig. 1.19.2.

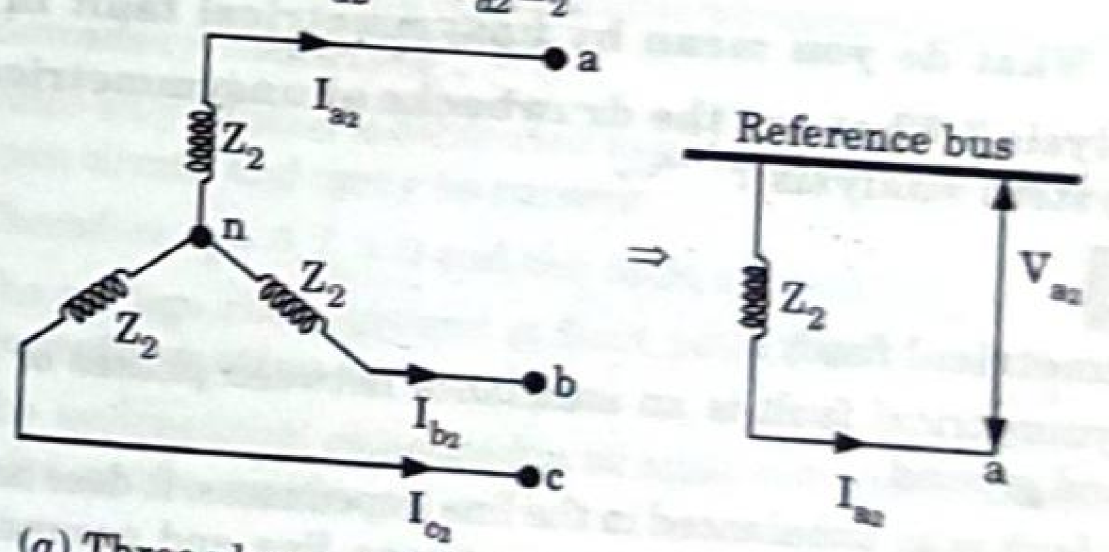
ii. **Negative sequence network :** The negative sequence component of voltage at the fault point is the generated negative sequence voltage minus the drop due to negative sequence current in negative sequence impedance (as negative sequence current does not produce drop in positive or zero sequence impedances)

$$V_{a2} = E_{a2} - I_{a2} Z_2$$

Since the negative sequence voltage generated is zero, therefore,

$$E_{a2} = 0$$

$$V_{a2} = -I_{a2} Z_2$$



(a) Three phase model (b) Single phase model

Fig. 1.19.3.

iii. **Zero sequence network :** For sequence voltages

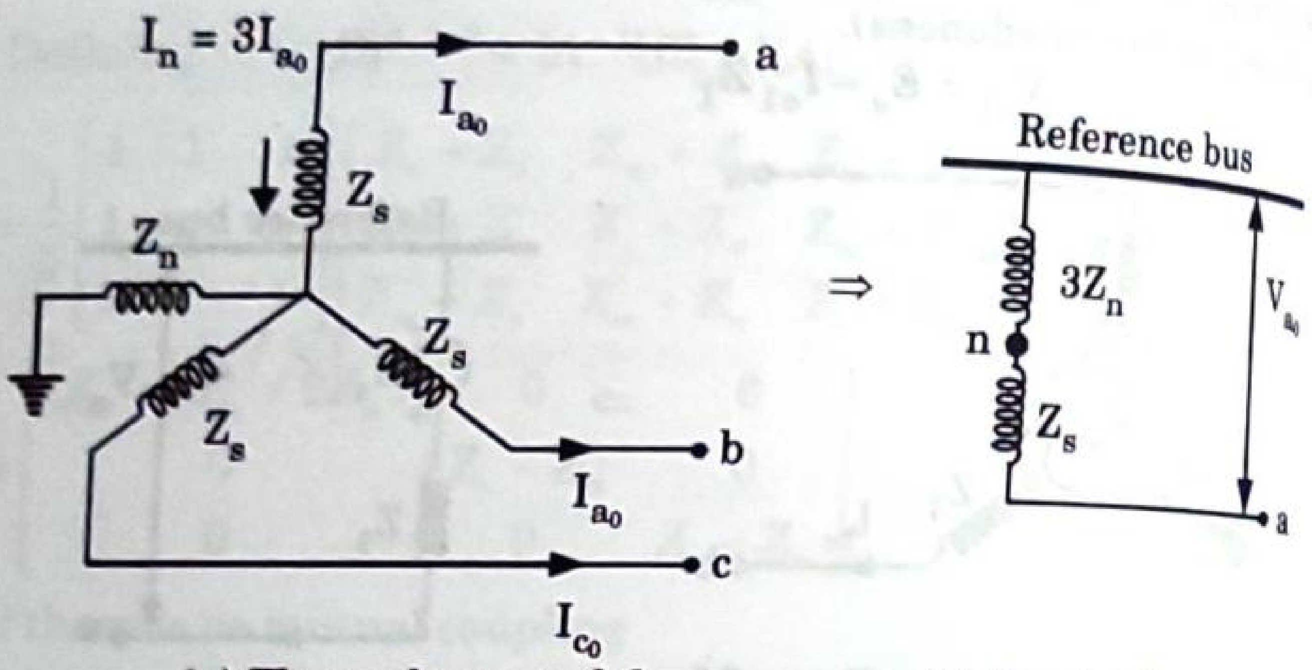
$$E_{a0} = 0$$

$$V_{a0} = V_n - I_{a0} Z_s = -3I_{a0} Z_n - I_{a0} Z_s$$

$$= -I_{a0}(Z_s + 3Z_n) = -I_a Z_0$$

(Pro...

where Z_0 is the zero sequence impedance of the generator and Z... the neutral impedance.



(*a*) Three phase model (*b*) Single phase model

Fig. 1.19.4.

PART-4

Fault Calculations : Fault Calculations, Sequence Network Equations, Single Line to Ground Fault, Line to Line Fault.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.20. **What do you mean by unsymmetrical fault in pow... system analysis ? What are the drawbacks of unsymmetrical fau... in power system analysis ?**

Answer

A. Unsymmetrical fault :

1. An unsymmetrical fault is an unbalance between phases or betwe... phase and ground.
2. A series fault is an unbalanced in the line impedances. It does not invol... any connections between lines or between line and ground at fau... point.

B. Classification of fault :

1. Single line to ground (LG) fault

2. Line to line (LL) fault
3. Double line to ground (LLG) fault
4. Three phase short circuit (LLL) fault
5. Three phase to ground (LLLG) fault.

C. Drawback : Unsymmetrical faults causes unbalance in power system analysis.

Que 1.21. **Derive the relationship to determine the fault current for a single line to ground fault. Draw and equivalent network showing the interconnection of sequence network to stimulate LG fault.**

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Answer

The single line-to-ground fault occurs when one conductor falls to ground or contacts the neutral conductor.

A. Circuit diagram :

1. Suppose that phase a is connected to ground at the fault point F as shown in Fig. 1.21.1. The fault impedance is Z_f.

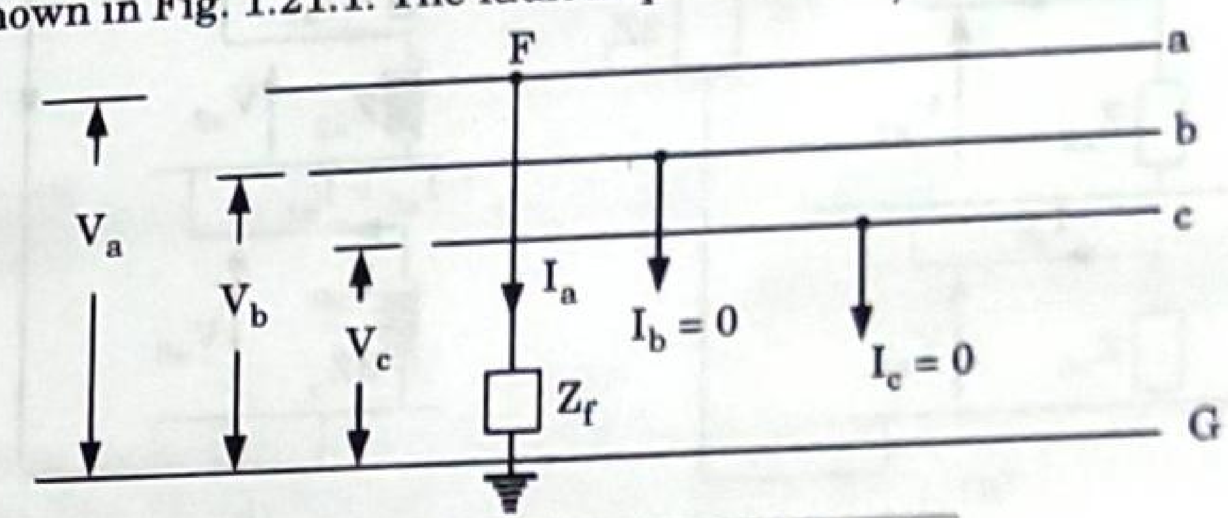

Fig. 1.21.1. Line to ground fault.

2. The fault current is $I_{af} = I_a$ by convention.
3. Fault current is taken to be positive when directed out of the fault point.

B. Boundary condition :

1. Since only phase a is connected to ground at the fault, phases b and c are open circuit and carry no current.
2. Therefore $I_b = 0$, $I_c = 0$ and the fault current is I_a.
3. The voltage above ground at fault point F is $V_a = Z_f I_a$.

C. Transformation :

1. The symmetrical components of fault current in phase a at the fault point can be written as

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c)$$

$$= \frac{1}{3}(I_a + 0 + 0) = \frac{1}{3} I_a$$

$$I_{a1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c)$$

$$= \frac{1}{3}(I_a + 0 + 0) = \frac{1}{3}I_a$$

$$I_{a2} = \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c)$$

$$= \frac{1}{3}(I_a + 0 + 0) = \frac{1}{3}I_a$$

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{3}I_a \quad ...(1.21.1)$$

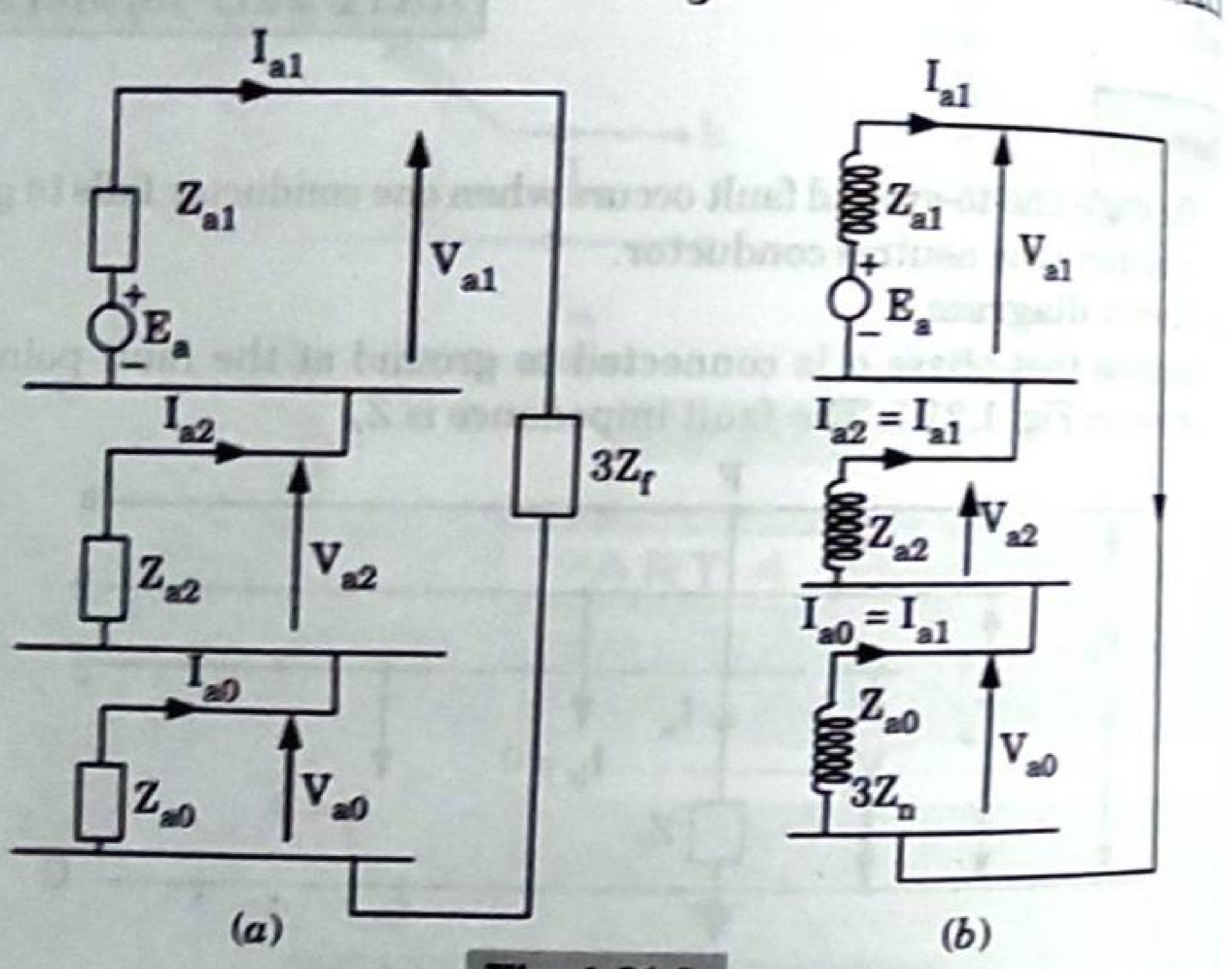

Fig. 1.21.2.

2. Hence we conclude that in the case of a single line-to-ground fault, the sequence currents are equal.
3. The sequence voltages at the fault point are found as follows :

$$V_{a0} = E_{a0} - Z_{a0} I_{a0}$$
$$V_{a1} = E_{a1} - Z_{a1} I_{a1}$$
$$V_{a2} = E_{a2} - Z_{a2} I_{a2}$$

where E_{a0}, E_{a1} and E_{a2} are the sequence generated voltages of phase a and Z_{a0}, Z_{a1}, Z_{a2} are the sequence impedances to the flow of current I_{a0}, I_{a1} and I_{a2} respectively.

4. For a balanced system

$$E_{a0} = 0, E_{a2} = 0, E_{a1} = V_f$$

5. We know that

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$Z_f I_a = -Z_{a0} I_{a0} + V_f - Z_{a1} I_{a1} - Z_{a2} I_{a2} \quad ...(1.21.2)$$

6. Combination of eq. (1.21.1) and (1.21.2) gives

$$Z_f I_a = V_f - \frac{I_a}{3}(Z_{a0} + Z_{a1} + Z_{a2})$$

$$I_a = \frac{V_f}{Z_f + \frac{1}{3}(Z_{a0} + Z_{a1} + Z_{a2})} \quad ...(1.21.3)$$

7. Since all the impedances and the voltages V_f at the fault point are known, the fault current can be determined by combining eq. (1.21.1) and (1.21.3).
8. We get the sequence current as

$$I_{a0} = I_{a1} = I_{a2} = \frac{V_f}{Z_{a0} + Z_{a1} + Z_{a2} + 3Z_f}$$

9. The phase voltages V_a, V_b and V_c at the fault point F can be found from the following relations :

$$V_a = V_{a0} + V_{a1} + V_{a2} = Z_f I_a$$
$$V_b = V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}$$
$$V_c = V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}$$

Que 1.22. **Write short note on double line fault.**

AKTU 2015-16, Marks 05

OR

Prove that zero sequence network is absent in case of occurrence of double line fault through certain impedance in a power system network.

Draw the zero sequence network for the system shown in Fig. 1.22.1.

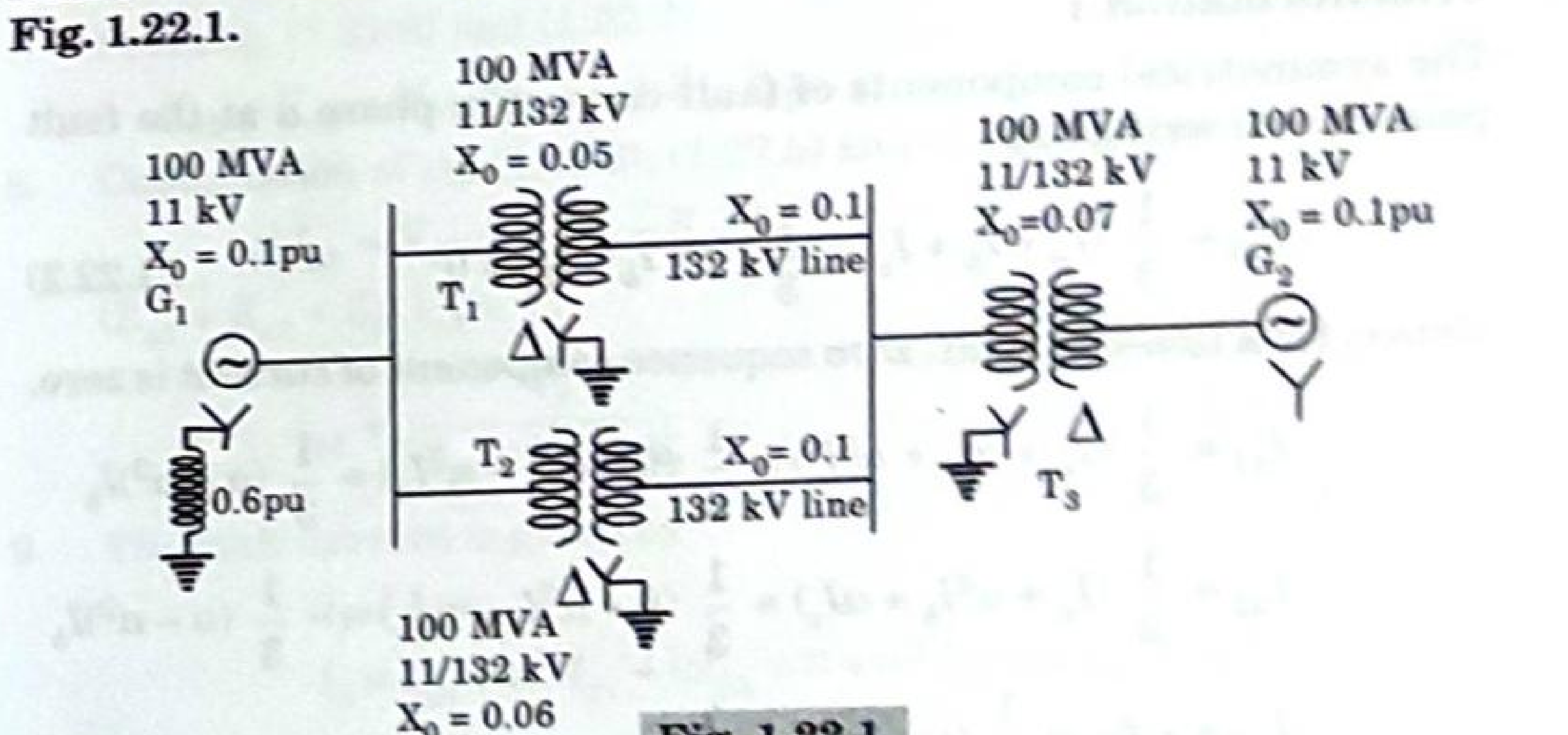

Fig. 1.22.1.

AKTU 2015-16, Marks 10

Answer

A. **Numerical :** Refer Q. 1.6, Page 1–7B, Unit-1.

B. **Double line fault (Line-to-line fault) :**

1. A line-to-line fault occurs when two conductors are short circuited. Fig. 1.22.2 shows a three phase system with a line-to-line (L-L) fault between phasors b and c.
2. The fault impedance is assumed to be Z_f.

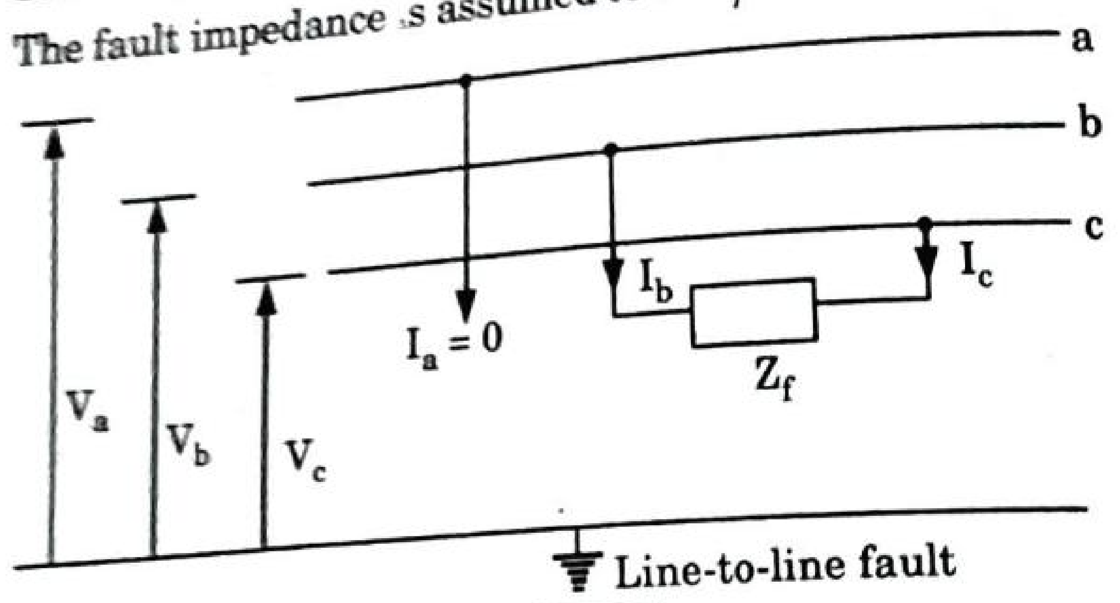

Fig. 1.22.2.

3. The L-L fault is placed between lines b and c in order that the fault is symmetrical with respect to the reference phase a which is unfaulted.

C. **Boundary condition :** The boundary conditions are :

$$I_a = 0$$
$$I_b = -I_c$$
$$V_b - V_c = Z_f I_b \quad \text{...(1.22.1)}$$

Fault current, $\quad I_f = I_b$

D. **Transformation :**

1. The symmetrical components of fault current in phase a at the fault point can be written as

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3}(0 + I_b - I_b) = 0 \quad \text{...(1.22.2)}$$

Hence, for a line-line fault, zero sequence component of current is zero.

2. $$I_{a1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c) = \frac{1}{3}(0 + \alpha I_b - \alpha^2 I_b) = \frac{1}{3}(\alpha - \alpha^2)I_b$$

$$I_{a2} = \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c) = \frac{1}{3}(0 + \alpha^2 I_b - \alpha I_b) = -\frac{1}{3}(\alpha - \alpha^2)I_b \quad \text{...(1.22.3)}$$

$$I_{a1} = -I_{a2} = \frac{1}{3}(\alpha - \alpha^2)I_b$$

The positive-sequence component of current is equal in magnitude but opposite in phase to the negative-sequence component of current.

3. The sequence currents can also be found by matrix method as follows :

$$I_{012} = A^{-1} I_{abc}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$\therefore \quad I_{a0} = 0$, and $I_{a1} = -I_{a2}$

4. Expressing V_b, V_c and I_b in terms of their sequence components, eq. (1.22.1) can be written as

$$(V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}) - (V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}) = Z_f(I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}) \quad \text{...(1.22.4)}$$

5. Combination of eq. (1.22.2), (1.22.3) and (1.22.4) gives

$$(\alpha^2 - \alpha)V_{a1} - (\alpha^2 - \alpha)V_{a2} = Z_f(\alpha^2 - \alpha)I_{a1}$$

or $V_{a1} - V_{a2} = Z_f I_{a1}$ ...(1.22.5)

6. The sequence components of voltages at the fault point are found by the relations given by

$$V_{012} = E - Z_{012} I_{012}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_f \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{a0} & 0 & 0 \\ 0 & Z_{a1} & 0 \\ 0 & 0 & Z_{a2} \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$V_{a0} = -Z_{a0} I_{a0}$$
$$V_{a1} = V_f - Z_{a1} I_{a1}$$
$$V_{a2} = -Z_{a2} I_{a2} \quad \text{...(1.22.6)}$$

7. From eq. (1.22.6) and (1.22.7) ...(1.22.7)

$$V_{a1} - V_{a2} = V_f - Z_{a1} I_{a1} + Z_{a2} I_{a2} \quad \text{...(1.22.8)}$$

8. Combination of eq. (1.22.3), (1.22.5) and (1.22.8) gives

$$Z_f I_{a1} = V_f - Z_{a1} I_{a1} - Z_{a2} I_{a2}$$
$$(Z_{a1} + Z_{a2} + Z_f) I_{a1} = V_f$$
$$I_{a1} = \frac{V_f}{Z_{a1} + Z_{a2} + Z_f}$$

9. The fault current is given by

$$I_f = I_b = -I_c$$
$$I_b = I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2} = 0 + \alpha^2 I_{a1} - \alpha I_{a1} = (\alpha^2 - \alpha) I_{a1}$$

$$= \frac{(\alpha^2-\alpha)V_f}{Z_{a1}+Z_{a2}+Z_f} = \frac{-j\sqrt{3}\,V_f}{Z_{a1}+Z_{a2}+Z_f}$$

PART-5

Double Line to Ground Fault, Three Phase Faults, Faults on Power Systems, and Faults with Fault Impedance.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.23. **Explain double line-to-ground fault.**

Answer

A. Boundary conditions :

The boundary conditions at the fault point F are as follows :

$$I_a = 0 \quad \text{...(1.23.1)}$$

$$V_b = Z_f I_b + Z_g (I_b + I_c) = (Z_f + Z_g)\, I_b + Z_g I_c \quad \text{...(1.23.2)}$$

$$V_c = Z_f I_c + Z_g (I_b + I_c) = (Z_f + Z_g)\, I_c + Z_g I_b \quad \text{...(1.23.3)}$$

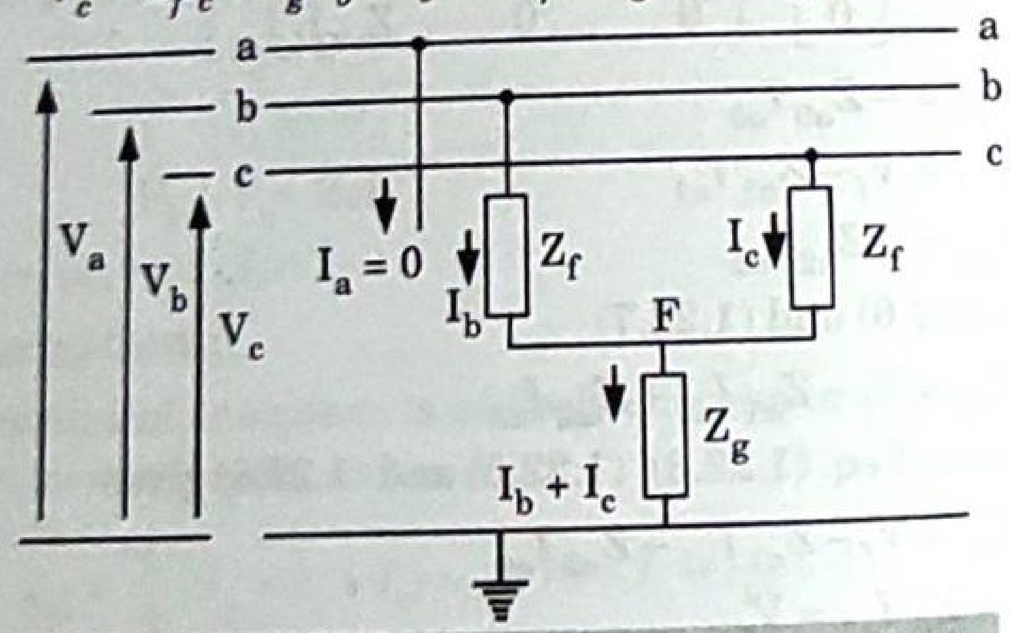

Fig. 1.23.1. Double line-to-ground (LLG) fault.

B. Transformation :

1. From eq. (1.23.2) and (1.23.3),

$$V_b - V_c = Z_f (I_b - I_c) \quad \text{...(1.23.4)}$$

2. Expressing V_b, V_c, I_b and I_c in terms of their sequence components we obtain

$$(V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}) - (V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2})$$

$$= Z_f[(I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}) - (I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2})]$$

$$(\alpha^2 - \alpha)(V_{a1} - V_{a2}) = Z_f(\alpha^2 - \alpha)(I_{a1} - I_{a2})$$

$$V_{a1} - V_{a2} = Z_f(I_{a1} - I_{a2})$$

$$V_{a1} - Z_f I_{a1} = V_{a2} - Z_f I_{a2} \quad \text{...(1.23.5)}$$

3. From eq. (1.23.2) and (1.23.3)

$$V_b + V_c = (Z_f + 2Z_g)(I_b + I_c)$$

Expressing V_b, V_c, I_b and I_c in terms of their sequence components we get

$$(V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2}) + (V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2})$$

$$= (Z_f + 2Z_g)\,[(I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}) + (I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2})]$$

$$2V_{a0} + (\alpha + \alpha^2)\, V_{a1} + (\alpha + \alpha^2)\, V_{a2}$$

$$= (Z_f + 2Z_g)\,[2I_{a0} + (\alpha + \alpha^2)\, I_{a1} + (\alpha + \alpha^2)\, I_{a2}]$$

$$2V_{a0} - V_{a1} - V_{a2} = (Z_f + 2Z_g)(2I_{a0} - I_{a1} - I_{a2})$$

Since $\quad I_a = 0$

$$I_{a0} + I_{a1} + I_{a2} = I_a = 0$$

$$2V_{a0} - V_{a1} - V_{a2} = (Z_f + 2Z_g)(3I_{a0}) \quad \text{...(1.23.6)}$$

4. Substituting the value of V_{a2} from eq. (1.23.5) in eq. (1.23.6), we get

$$2V_{a0} - 2V_{a1} + Z_f I_{a1} - Z_f I_{a2} = (Z_f + 2Z_g)(3I_{a0})$$

$$2V_{a0} - 2V_{a1} + Z_f I_{a1} + Z_f (I_{a0} + I_{a1}) = 3(Z_f + 2Z_g)\, I_{a0}$$

$$V_{a0} - (Z_f + 3Z_g)\, I_{a0} = V_{a1} - Z_f I_{a1} \quad \text{...(1.23.7)}$$

5. We know that

$$V_{a1} = E_{a1} - Z_{a1} I_{a1}$$

or

$$V_{a1} = V_f - Z_{a1} I_{a1}$$

$$V_{a2} = -Z_{a2} I_{a2}$$

$$V_{a0} = -Z_{a0} I_{a0}$$

6. Substitution of the values of V_{a0} and V_{a1} in eq. (1.23.7) gives

$$V_f - Z_{a1} I_{a1} - Z_f I_{a1} = -Z_{a0} I_{a0} - (Z_f + 3Z_g) I_{a0}$$

$$V_f - (Z_{a1} + Z_f) I_{a1} = -(Z_{a0} + Z_f + 3Z_g) I_{a0} \quad \text{...(1.23.8)}$$

7. Substituting the value of V_{a1} in (1.23.5) we get

$$V_{a1} - Z_f I_{a1} = -Z_{a2} I_{a2} - Z_f I_{a2}$$

$$V_{a1} - Z_f I_{a1} = -(Z_{a2} + Z_f) I_{a2} \quad \text{...(1.23.9)}$$

8. Combining eq. (1.23.8) and (1.23.9) we get

$$V_f - (Z_{a1} + Z_f) I_{a1} = -(Z_{a2} + Z_f) I_{a2}$$

$$= -(Z_{a0} + Z_f + 3Z_g)I_{a0} \quad ...(1.23.11)$$

Que 1.24. **Explain three-phase fault in detail.**

Answer

1. From Fig. 1.24.1, the boundary conditions are :

$$I_a + I_b + I_c = 0$$

$$V_a = V_b = V_c$$

$\because$ $|I_a| = |I_b| = |I_c|$ and if I_a is taken as reference

$$I_b = \alpha^2 I_a \text{ and } I_c = \alpha I_a$$

2. Using the relation

$$I_{a1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c) \quad ...(1.24.1)$$

and substituting the values of I_b and I_c in eq. (1.24.1),

$$I_{a1} = \frac{1}{3}(I_a + \alpha^3 I_a + \alpha^3 I_a) = I_a$$

3.
$$I_{a2} = \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c) \quad ...(1.24.2)$$

Substituting for I_b and I_c in terms of I_a in eq. (1.24.2)

$$I_{a2} = \frac{1}{3}(I_a + \alpha^4 I_a + \alpha^2 I_a)$$

$$= \frac{1}{3}(I_a + \alpha I_a + \alpha^2 I_a) = \frac{I_a}{3}(1 + \alpha + \alpha^2) = 0$$

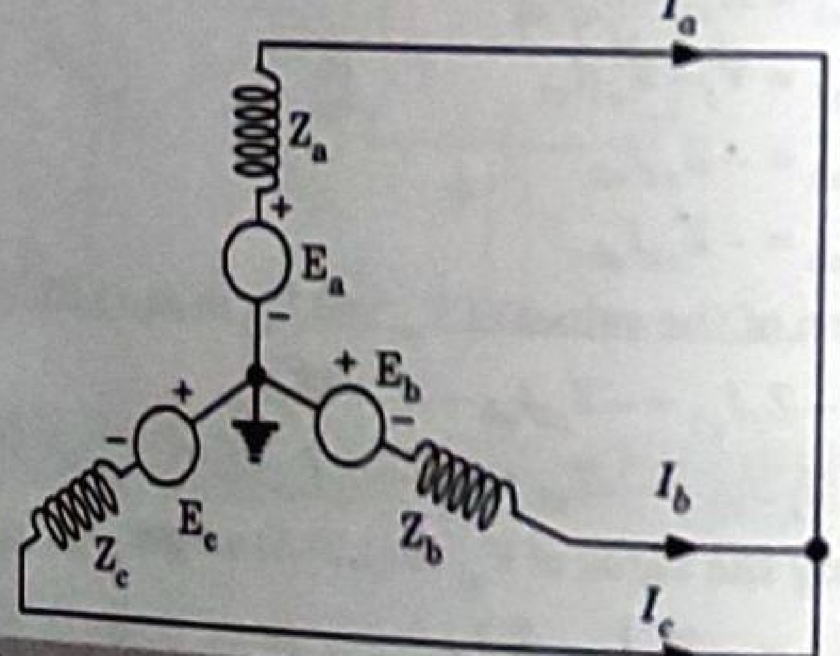

Fig. 1.24.1. 3ϕ neutral ground and unloaded alternator 3ϕ shorted.

4. Similarly, $I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = 0$
5. Now using the voltage boundary relation

$$V_{a1} = \frac{1}{3}(V_a + \alpha V_b + \alpha^2 V_c) = \frac{1}{3}(V_a + \alpha V_a + \alpha^2 V_a)$$

$$= \frac{V_a}{3}(1 + \alpha + \alpha^2) = 0$$

And, $V_{a2} = \frac{1}{3}(V_a + \alpha^2 V_b + \alpha V_c) = 0$

And, $V_{a0} = 0$

6. Since $V_{a1} = 0 = E_a - I_{a1}Z_a$

$\therefore$ $I_{a1} = \dfrac{E_a}{Z_a}$

Que 1.25. **Derive the expression for the symmetrical components of fault current of a power system for L-L fault through impedance. A 30 MVA, 11 kV generator has $Z_1 = Z_2 = j0.2$ pu, $Z_0 = j0.05$ pu. A line-to-ground fault occurs on the generator terminals. Find the fault current and line-to-line voltages during fault conditions.**

AKTU 2016-17, Marks 15

Answer

A. **Symmetrical components of fault current of a power system for L-L fault through impedance :** Refer Q. 1.22, Page 1–30B, Unit-1.

B. **Numerical :**

Given : Power = 30 MVA, V = 11 kV, $Z_1 = j0.2$ pu, $Z_2 = j0.2$ pu, $Z_0 = j0.05$ pu.

To Find : I_{a1}, I_{a2} and I_{a0}; V_{ab}, V_{ac} and V_{bc}.

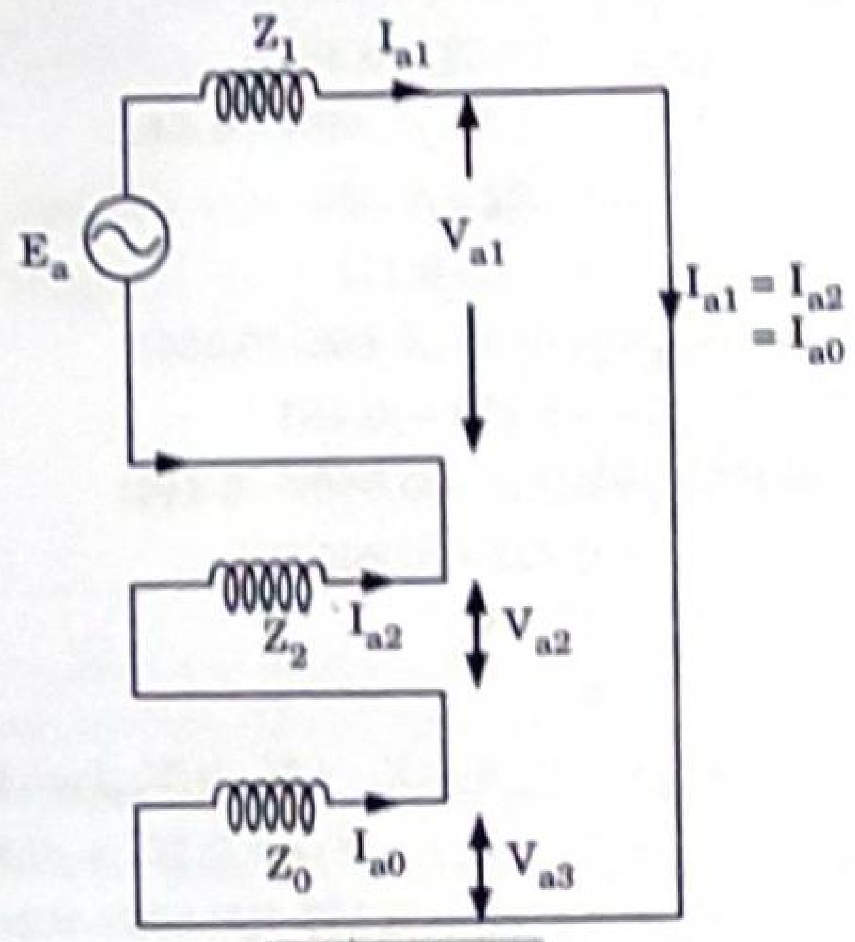

Fig. 1.25.1.

1. Fault impedance $= Z_1 + Z_2 + Z_3$

$$= j(0.2 + 0.2) + j0.05$$
$$= j0.45 \text{ pu}$$

$\therefore$
$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0} = \frac{1}{j0.45} = -j2.22$$

2. L-G conditions are

$$I_{a1} = I_{a2} = I_{a0} = -j2.22$$

Per unit fault current : $I_a = I_{a1} + I_{a2} + I_{a0} = 3I_{a1} = -j6.66$

3. Quantity of 30 MVA, 11 kV

$$\text{Base current} = \frac{30 \times 1000}{\sqrt{3} \times 13.2} = 1312.19 \text{ A}$$

Fault current in amp $= 1312.19 \times 6.66 = 8739.19$ A

4. For voltage (L-L),

$$V_{a1} = E_a - I_{a1}Z_1$$
$$= 1 + j0.0 - (-j2.22)(j0.2)$$
$$= 1 - 0.444 = 0.556$$
$$V_{a2} = -I_{a2}Z_2 = -(-j2.22)\,(j0.2) = -0.444$$
$$V_{a0} = -I_{a0}Z_0 = -(-j2.22)\,(j0.05) = -0.111$$

5. In L-G fault, $V_a = 0, V_b = V_{b1} + V_{b2} + V_{b0}$

$$V_c = V_{c1} + V_{c2} + V_{c0}$$

$\alpha^2 = -0.5 + j0.866$ and $\alpha = -0.5 + j0.866$

6. So, $V_{b1} = \alpha^2 V_{a1} = (-0.5 - j0.866)(0.556)$

$$= -0.278 - j0.481$$
$$V_{b2} = \alpha V_{a2} = (-0.5 + j0.866)(-0.444)$$
$$= 0.222 - j0.385$$
$$V_{b0} = V_{a0} = V_{c0} = -0.111$$
$$V_{c1} = \alpha V_{a1} = (-0.5 + j0.866)\,(0.556)$$
$$= -0.278 + j0.481$$
$$V_{c2} = \alpha^2 V_{a2} = (-0.5 - j0.866)(-0.444)$$
$$= 0.222 + j0.384$$

7.
$$V_{ab} = V_a - V_b$$

$\because$
$$V_a = 0$$
$$V_{ab} = -V_b = -(V_{b1} + V_{b2} + V_{b0}) = -0.167 - j0.866 \text{ kV}$$
$$V_{ac} = V_a - V_c = -V_c = -0.167 + j0.865 \text{ kV}$$
$$V_{bc} = V_b - V_c = 0.167 + j0.866 - 0.167 + j0.865$$

$$= j1.73 \text{ kV}$$

Que 1.26. **For a single line to ground fault at the terminals of an unloaded generator, positive sequence current was found to be 50 A. Determine sequence currents in phase b and c.**

AKTU 2018-19, Marks 07

Answer

Given : $I_{a1} = 50$ Amp
To Find : I_b, I_c.

1. As we know that in LG fault

$$I_{a0} = I_{a1} = I_{a2} = \frac{I_a}{3}$$
$$I_a = 3 \times 50 = 150 \text{ Amp}$$

2. And since only phase a is connected to ground at the fault, phase b and c are open circuited and carry no current that is $I_b = 0$ and $I_c = 0$ and the fault current is I_a.

PART-6

Reactors and Their Location, Short Circuit Capacity of a Bus.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.27. **Discuss the various strategic locational aspects of reactors for limiting the fault current and their advantages.**

AKTU 2016-17, Marks 10

AKTU 2018-19, Marks 07

Answer

Reactors used to limit the fault current in power system are known as current limiting reactors. Their types are :

A. Generator reactor (in series with generator) :

1. When the reactors are inserted between the generator and the generator bus, as shown in Fig. 1.27.1, the reactors are known as generator reactors.

2. Modern generators are designed to have sufficiently large reactance to protect them even in dead short-circuits at their terminals.
3. **Advantage :** It reduces the fault current to protect the generator.

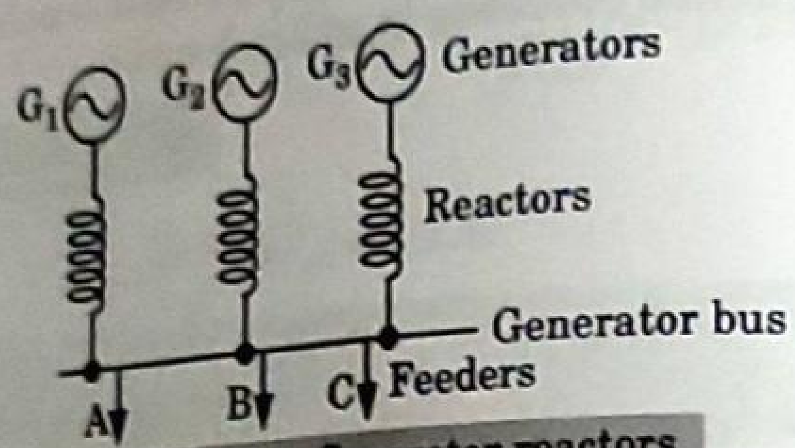

Fig. 1.27.1. Generator reactors.

B. Feeder reactors (in series with feeder) :

1. When the reactors are connected in series with the feeders, as shown in Fig. 1.27.1, the reactors are known as feeder reactors.
2. **Advantages :** In the event of fault on any one feeder, the main voltage drop is in its reactor only and the bus-bar voltage is not affected much, hence other machines continue supplying load.

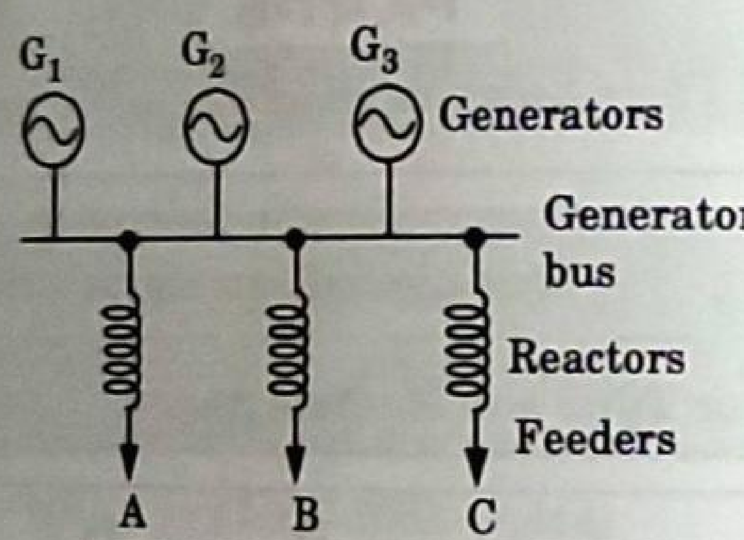

Fig. 1.27.2. Feeder reactors.

C. Bus-bar reactors (in series with bus bar) :

a. Bus-bar reactors (Ring system) :

1. Bus-bar reactors are used to tie together separate bus sections.
2. In this system sections are made of generators and feeders and these sections are connected to each other at the common bus-bar.
3. In this system normally one feeder is fed from one generator.
4. **Advantages :** Heavy currents and voltage disturbances caused by a short circuit on a bus section are reduced and confined primarily to that section due to feeder.

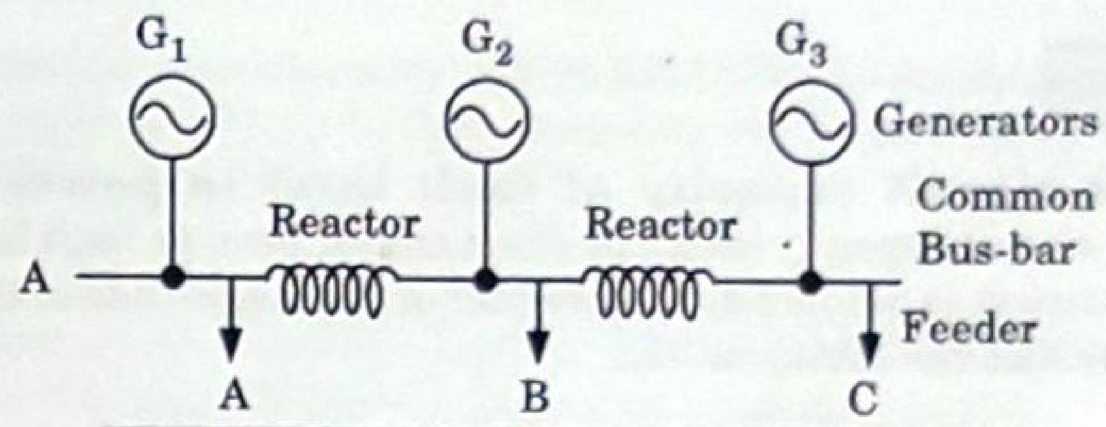

Fig. 1.27.3. Bus-bar reactors (ring system).

b. Bus-bar reactors (tie-bar system) :

1. This system is ideally suited to the generating systems where frequently new generators are being added.
2. In this system the generators are connected to the common bus-bar through the reactors but the feeders are fed from the generator side of the reactors.

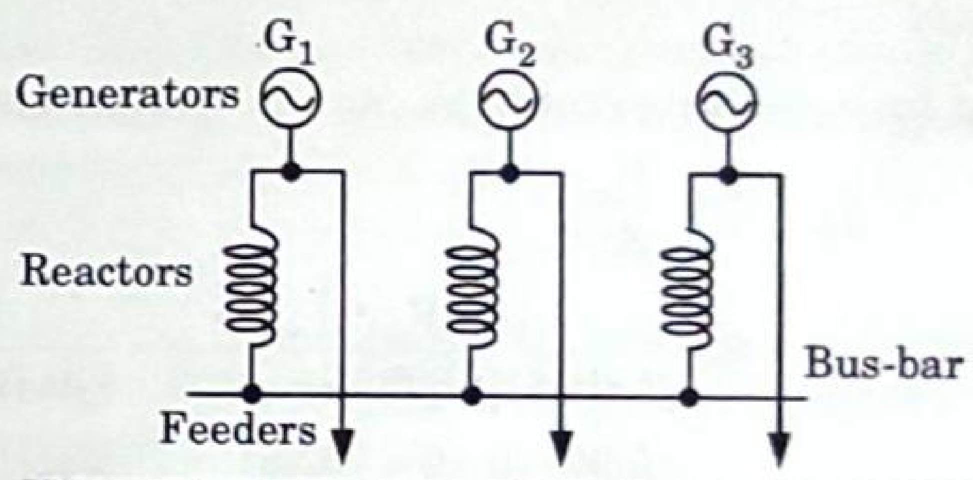

Fig. 1.27.4. Bus-bar reactors (tie-bar system).

3. The operation of this system is similar to ring system.
4. **Advantage :** If the number of sections is increased, the fault current will not exceed a certain value, which is fixed by the size of individual reactors.

Que 1.28. **Define the short-circuit capacity of fault level in power system. Find the fault current in the Fig. 1.28.1, if the pre-fault voltage at fault point is 0.97.**

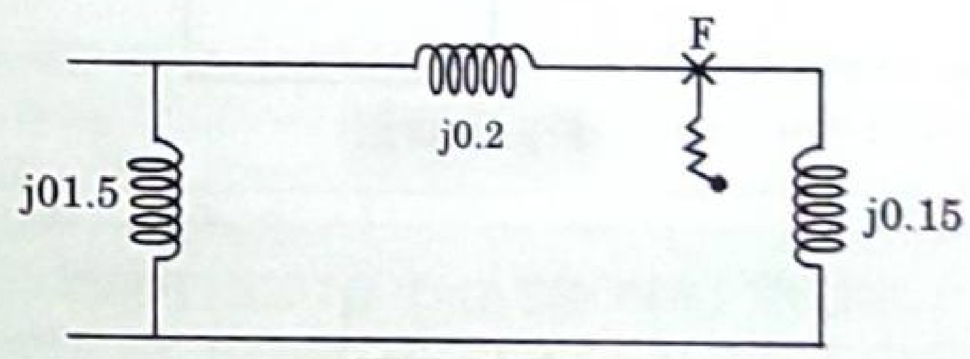

Fig. 1.28.1.

AKTU 2015-16, Marks 08

Answer

A. Short circuit capacity of fault level in power system : Short-circuit capacity (SCC) or short circuit level or fault level at a bus of a network is defined as the product of the magnitude of the pre-fault voltage and the fault current.

$$SCC = V_{th} I_f = \frac{V_{th}^2}{Z_{th}}$$

where, V_{th} = Pre-fault voltage

I_f = Fault current

B. Numerical :

Given : Pre-fault voltage, V_{th} = 0.97 V
To Find : I_f

1. For a solid fault, fault current in pu can be calculated by

$$I_f = \frac{V_{th}}{Z_{th}}$$

2.
$$I_f = \frac{V_{th}}{(j0.15) \,||\, (j0.15 + j0.2)} = \frac{0.97}{(j0.15 \,||\, j0.35)}$$

$$= \frac{0.97\,(j0.15 + j0.35)}{j0.15 \times j0.35} = -\,j9.23 \text{ pu}$$

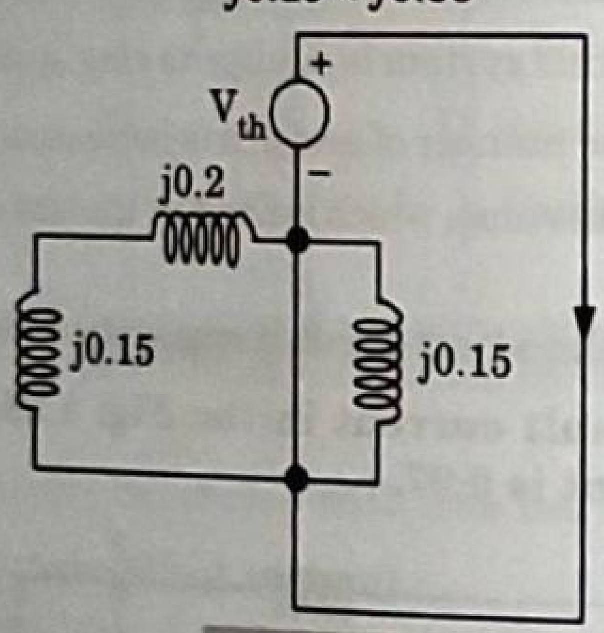

Fig. 1.28.2.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. What do you mean by "SINGLE LINE DIAGRAM" in power system analysis ? Also mention its importance in Power System Analysis.

Ans. Refer Q. 1.1.

Q. 2. What do you understand by "PER UNIT SYSTEM" ? What are the significances in power system analysis ?

Ans. Refer Q. 1.3.

Q. 3. Prove the per unit impedance of the transformer is independent of side it is referred to.
A generator is rated at 30 MVA, 11 kV and has a reactance of 20 %. Calculate its per unit reactance for 50 MVA, 10 kV base.

Ans. Refer Q. 1.5.

Q. 4. Discuss the principle of the symmetrical components. Derive the necessary equation to convert phase quantities into symmetrical components.

Ans. Refer Q. 1.11.

Q. 5. What do you understand by symmetrical components of unbalanced phasors ? Deduce the expression for symmetrical components.

Ans. Refer Q. 1.12.

Q. 6. Deduce the expression of power in terms of symmetrical components.

Ans. Refer Q. 1.13.

Q. 7. Explain the sequence impedances. Define balanced star connected load and sequence impedances of transmission lines.

Ans. Refer Q. 1.18.

Q. 8. Derive the relationship to determine the fault current for a single line to ground fault. Draw and equivalent network showing the interconnection of sequence network to stimulate LG fault.

Ans. Refer Q. 1.21.

Q. 9. Explain three-phase fault in detail.

Ans. Refer Q. 1.24.

Q. 10. Discuss the various strategic locational aspects of reactors for limiting the fault current and their advantages.

Ans. Refer Q. 1.27.

2

UNIT

Load Flow Analysis

CONTENTS

PART-1

Introduction, Formation of Z_{BUS} and Y_{BUS} Development of Load Flow Equations.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **What do you mean by "LOAD FLOW ANALYSIS" in power system networks ? What is the necessity of load flow study in power system ?**

OR

Classify and explain the various types of buses of a power system used in power flow analysis ?

AKTU 2018-19, Marks 07

Answer

A. Load flow analysis :

1. Load flow (or power flow) analysis is the determination of current, voltage, active power, and reactive voltamperes at various points in a power system operating under normal steady-state or static conditions.
2. Load flow studies are made to plan the best operation and control of the existing system as well as to plan the future expansion to keep pace with the load growth.
3. Such studies help in ascertaining the effects of new loads, new generating stations, new lines and new interconnections before they are installed.
4. The prior information serves to minimize the system losses and to provide a check on the system stability.

B. Necessity :

1. To get steady state solution of power system network.
2. To get information about V, P, Q, δ.
3. To monitor current state and analysing the effectiveness of plans for future.

C. Bus classification :

i. **Load Bus or *P-Q* Bus :** Load bus is nothing but the terminals of transformer at which the total active power and reactive power required is specified and voltage magnitude $|V|$, load angle δ are not specified.

ii. **Generator Bus or *P-V* Bus :**

1. Generator bus is nothing but the terminals of generator where P and $|V|$ are specified and Q and δ are not specified.
2. Even though the synchronous machine can generate reactive power, how much amount to be generated depends upon the loading condition.

iii. **Slack Bus :**

1. Until the completion of load flow, the power flowing through the various interconnected channels are not known.
2. But to keep the network alive, losses should be continuously supplied.
3. A special bus is engaged for this purpose called slack bus and in this bus, voltage and load angle are specified.

Que 2.2. **Derive and explain the formation of Z_{bus} by singular transformation method.**

Answer

Formation of Z_{bus} by using singular transformation method :

1. Consider that Z_{bus} has been formulated up to a certain stage and another branch is now added.

$$Z_{bus(old)} \xrightarrow{Z_b = \text{branch impedence}} Z_{bus(new)}$$

2. Upon adding a new branch, one of the following situations is presented :

i. Z_b is added from a new bus to the reference bus. This is type-1 modification.

ii. Z_b is added from a new bus to an old bus. This is type-2 modification.

iii. Z_b connected on old bus to the reference branch. This is type-3 modification.

iv. Z_b connects two old buses. This is type-4 modification.

v. Z_b connects two new buses.

3. **Notation :** $i, j \rightarrow$ old buses, $r \rightarrow$ reference bus, $k \rightarrow$ new bus.

Modification :

1. **Type-1 modification :**

i. Fig. 2.2.1, shows a passive n-bus network in which branch with impedance Z_b is added between new bus k and the reference bus, Now

$$V_k = Z_b I_k$$

$$Z_{ki} = Z_{ik} = 0;\ i = 1, 2, \ldots, n$$

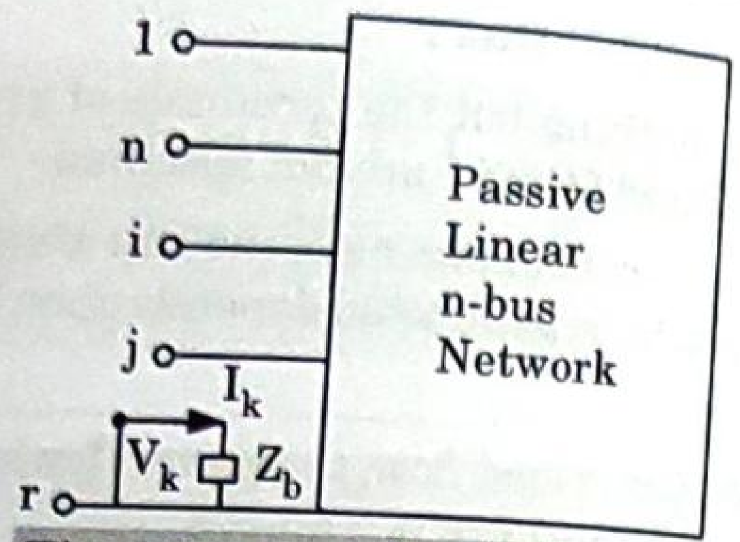

Fig. 2.2.1. Type-1 modification.

$$Z_{kk} = Z_b$$

ii. Hence, $Z_{\text{bus (new)}} = \begin{bmatrix} Z_{\text{bus(old)}} & 0 \\ & \vdots \\ 0 \cdots 0 & Z_b \end{bmatrix}$

2. **Type-2 modification :**

i. Z_b is added from new bus k to the old bus j as in Fig. 2.2.2. It follows t

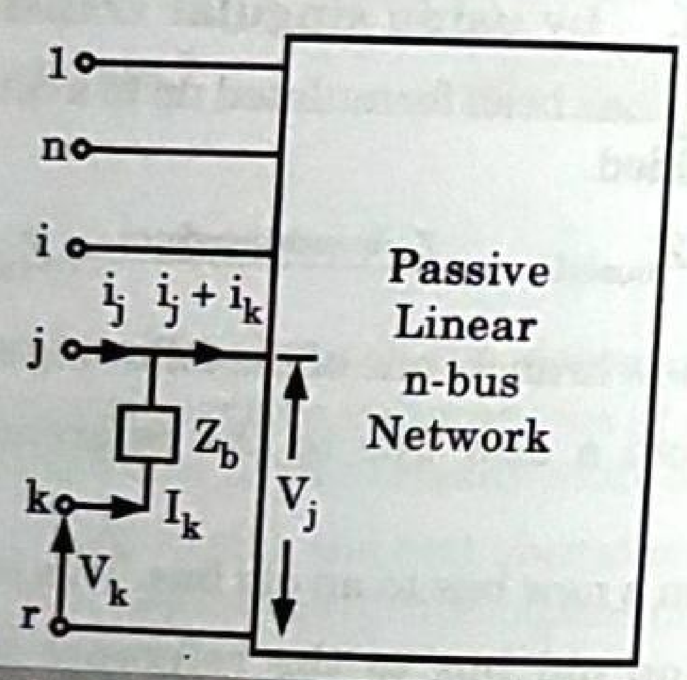

Fig. 2.2.2. Type-2 modification.

ii.

$$V_k = Z_b I_k + V_j$$
$$= Z_b I_k + Z_{j1} I_1 + Z_{j2} I_2 + \ldots\ldots$$
$$+ Z_{jj}(I_j + I_k) \ldots\ldots + Z_j$$

iii. Rearranging

$$V_k = Z_{j1} I_1 + Z_{j2} I_2 + \ldots\ldots Z_{jj} I_j + \ldots\ldots$$
$$+ Z_{jn} I_n + (Z_{jj} + Z_b$$

iv. Consequently $Z_{\text{bus (new)}} = \begin{bmatrix} Z_{\text{bus(old)}} & Z_{1j} \\ & Z_{2j} \\ & \vdots \\ & Z_{nj} \\ Z_{j1} Z_{j2} \cdots Z_{jn} & Z_{jj} + Z_b \end{bmatrix}$

3. **Type-3 modification :**

i. Z_b connects an old bus j to the reference bus as in Fig. 2.2.3.

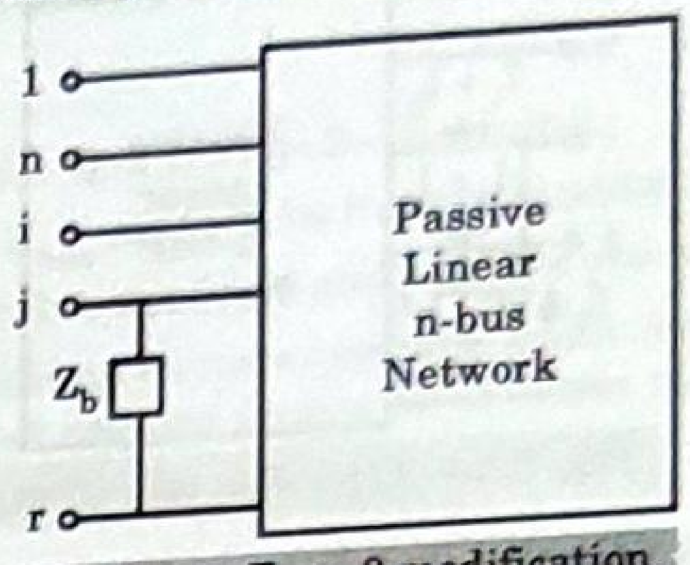

Fig. 2.2.3. Type-3 modification.

ii. This case follows by connecting bus k to the reference bus r, *i.e.*, by setting $V_k = 0$.

iii. Thus,

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{\text{Bus(old)}} & Z_{1j} \\ & Z_{2j} \\ & \vdots \\ & Z_{nj} \\ Z_{j1} Z_{j2} \cdots Z_{jn} & Z_{jj} + Z_b \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_k \end{bmatrix}$$

iv. Eliminate I_k in the set of equation contained in the matrix operation.

$$0 = Z_{j1} I_1 + Z_{j2} I_2 + \ldots\ldots + Z_{jn} I_n + (Z_{jj} + Z_b) I_k$$

or

$$I_k = -\frac{1}{Z_{jj} + Z_b}(Z_{j1} I_1 + Z_{j2} I_2 + \ldots\ldots + Z_{jn} I_n) \quad \ldots(2.2.1)$$

Now,

$$V_i = Z_{i1} I_1 + Z_{i2} I_2 + \ldots\ldots + Z_{in} I_n + Z_{ij} I_k \quad \ldots(2.2.2)$$

v. Substitute eq. (2.2.2) in eq. (2.2.1)

$$V_i = \left[Z_{i1} - \frac{1}{Z_{jj} + Z_b}(Z_{ij}\, Z_{j1})\right] I_1 + \left[Z_{i2} - \frac{1}{Z_{jj} + Z_b}(Z_{ij}\, Z_{j2})\right] I_2$$
$$+ \ldots\ldots + \left[Z_{in} - \frac{1}{Z_{jj} + Z_b}(Z_{ij}\, Z_{jn})\right] I_n$$

vi. The matrix form is $Z_{\text{bus (new)}} = Z_{\text{bus (old)}} - \dfrac{1}{Z_{jj} + Z_b}\begin{bmatrix} Z_{ij} \\ \vdots \\ Z_{nj} \end{bmatrix} [Z_{j1} \cdots Z_{jn}]$

4. **Type-4 modification :**

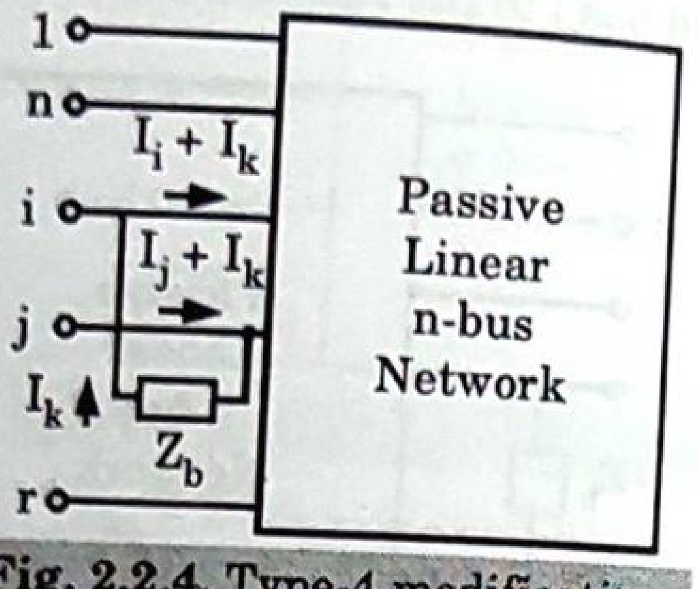

Fig. 2.2.4. Type-4 modification.

i. Z_b connects two old buses. Equation can be written as follows for all network buses.

$$V_i = Z_{i1}I_1 + Z_{i2}I_2 + \ldots\ldots Z_{1i}(I_i + I_k) + Z_{ij}(I_j - I_k) + \ldots\ldots + Z$$

ii. Similar equation follows for the other buses.

The voltages of the buses i and j are, however constrained by the equation.

$$V_j = Z_b I_k + V_i$$

or
$$Z_{j1}I_1 + Z_{j2}I_2 + \ldots\ldots + Z_{ji}(I_i + I_k) + Z_{jj}(I_j - I_k) + \ldots\ldots + Z_{jn}I_n$$
$$= Z_bI_k + Z_{i1}I_1 + Z_{i2}I_2 + \ldots\ldots + Z_{ii}(I_i + I_k) + Z_{ij}(I_j - I_k) + \ldots\ldots + Z$$

iii. Rearranging

$$0 = (Z_{i1} - Z_{j1})I_1 + \ldots\ldots + (Z_{ii} - Z_{ji})I_i + (Z_{ij} - Z_{jj})I_j + \ldots\ldots + (Z_{in} - Z_{jn}) + (Z_b + Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji})$$

iv. We can write

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{\text{bus (old)}} & \ldots & \ldots & (Z_{1i} - Z_{1j}) \\ \vdots & & & (Z_{2i} - Z_{2j}) \\ \vdots & & & \vdots \\ \vdots & & & (Z_{ni} - Z_{nj}) \\ (Z_{i1} - Z_{j1}) & \ldots & (Z_{in} - Z_{jn}) & Z_b + Z_{ii} + Z_{jj} - 2Z_{ij} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_j \end{bmatrix}$$

$$Z_{\text{bus (new)}} = Z_{\text{bus (old)}} - \frac{1}{Z_b + Z_{ii} + Z_{jj} - 2Z_{ij}} \begin{bmatrix} Z_{1i} - Z_{1j} \\ \vdots \\ Z_{ni} - Z_{nj} \end{bmatrix} [(Z_{i1} - Z_{j1}) \cdots (Z_{in} - Z_{jn})]$$

Que 2.3. **Explain the procedure of formation of Z_{bus} by using singular transformation and algorithms. What is the importance's of Z_{bus} matrix in Power System Analysis ?**

Answer

A. **Procedure :** Refer Q. 2.2, Page 2–3B, Unit-2.

B. **Importance of Z_{bus} matrix in Power System :**

1. Z_{bus} helps in calculating of real and reactive power.
2. In case of modification in power system, small changes need to be done in Z_{bus}. It does not require formation of whole new Z_{bus}. Hence, saves time.

Que 2.4. **Derive the expression of admittance bus matrix with resistive and reactive elements only.**

Answer

1. Writing of Y_{BUS} with R and X elements only.
2. By KCL at bus node (1)

$$I_1 = i_{12} + i_{13}$$
$$I_1 = Y_{12}(V_1 - V_2) + Y_{13}(V_1 - V_3)$$
$$= (Y_{12} + Y_{13})V_1 - Y_{12}V_2 - Y_{13}V_3 \quad \ldots(2.4.1)$$
$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 \quad \ldots(2.4.2)$$

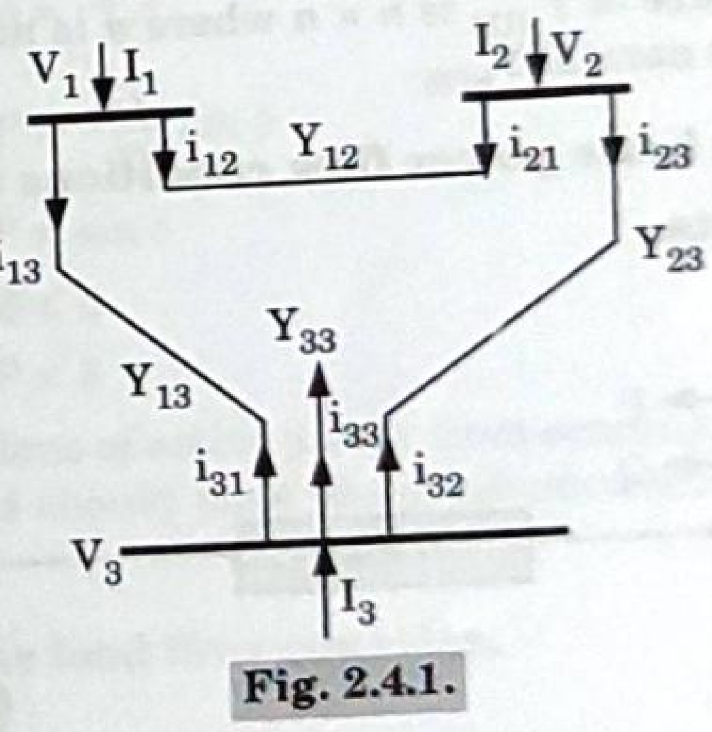

Fig. 2.4.1.

2. Comparing eq. (2.4.1) and (2.4.2), we get

$$Y_{11} = Y_{12} + Y_{13}$$
$$Y_{12} = -Y_{12}$$
$$Y_{13} = -Y_{13}$$

4. By KCL at bus node (2)

$$I_2 = i_{21} + i_{23}$$

$$I_2 = Y_{21}(V_2 - V_1) + Y_{23}(V_2 - V_3)$$

$$= -Y_{21}V_1 + (Y_{21} + Y_{23})V_2 - Y_{23}V_3 \quad ...(2.4.3)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 \quad ...(2.4.4)$$

5. Comparing eq. (2.4.3) and (2.4.4), we get

$$Y_{21} = -Y_{21}$$
$$Y_{22} = Y_{21} + Y_{23}$$
$$Y_{23} = -Y_{23}$$

6. Similarly,

$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3$$
$$Y_{31} = -Y_{31}$$
$$Y_{32} = -Y_{32}$$
$$Y_{33} = Y_{31} + Y_{32}$$

7. In matrix form

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$I_p = \sum_{q=1}^{n} Y_{pq} V_q$$

where, $p = 1, 2, ..., n$

8. It is clear that size of Y_{BUS} is $n \times n$ where n is number of buses. This Y_{BUS} is called as normal Y_{BUS}.

Que 2.5. **Define basic power flow conditions in power analysis. Also state its results.**

Answer

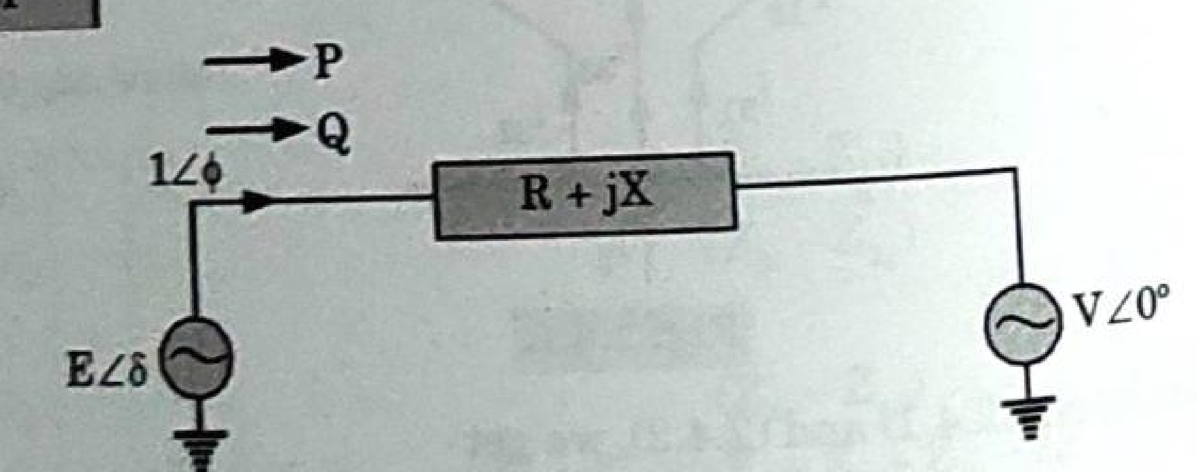

Fig. 2.5.1.

1. $E\angle\delta = V\angle 0° + I\angle -\phi\,(R + jX)$
2. $E\cos\delta + jE\sin\delta = V + I(\cos\phi - j\sin\phi)(R + jX)$

$$= V + IR\cos\phi + IX\sin\phi + j(IX\cos\phi - IR\sin\phi)$$

By separating the real and imaginary term, we get

$$E\cos\delta = V + IR\cos\phi + IX\sin\phi$$
$$E\sin\delta = IX\cos\phi - IR\sin\phi$$

3. If δ is so small, $E\cos\delta \cong E$

$$E - V = IR\cos\phi + IX\sin\phi$$
$$\Delta V = \frac{RP + XQ}{V}$$
$$E\sin\delta = IX\cos\phi - IR\sin\phi$$
$$E\sin\delta = \frac{XP - RQ}{V}$$

4. If the system is so large, $R \approx 0$

$$\Delta V = \frac{XQ}{V}$$
$$Q = \frac{(\Delta V)V}{X}$$
$$Q \propto (\Delta V) \quad ...(2.5.1)$$

5. Eq. (2.5.1) shows that for transmission of reactive power Q from sending end to receiving end, sending end potential should be more than receiving end potential.

6.
$$E\sin\delta = \frac{XP}{V}$$
$$P = \frac{EV}{X}\sin\delta$$
$$P \propto \sin\delta$$
$$\sin\delta \propto \delta$$
$$P \propto \delta$$

7. Hence for transmissions of active power from sending end to receiving end, the sending end should have phase advancement over receiving end.

Que 2.6. **Develop the load flow equation.**

Answer

1. The nodal current equation are written below for a n-bus system as :

$$I_i = \sum_{k=1}^{n} Y_{ik}V_k \; ; i = 1, 2, ..., n \quad ...(2.6.1)$$

$$I_i = Y_{ii}V_i + \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik}V_k \quad ...(2.6.2)$$

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$$V_i = \frac{I_i}{Y_{ii}} - \frac{1}{Y_{ii}} \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \quad ...(2.6$$

2. Now,
$$V_i^* I_i = P_i - jQ_i \quad ...(2.6$$
$$I_i = \frac{P_i - jQ_i}{V_i^*}$$

3. Substituting for I_i in eq. (2.6.3)

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad ...(2.6$$

where, $i = 1, 2, \ldots\ldots n$

I_i has been substituted by the real and reactive powers in eq. (2.6) because normally in a power system these quantities are specified.

Que 2.7. **Develop static load flow equations.**

Answer

1. From the nodal current equations, the total current entering the i^{th} bus of an n-bus system is given by

$$I_i = Y_{i1} V_1 + Y_{i2} V_2 + \ldots + Y_{ij} V_j + \ldots + Y_{in} V_n = \sum_{k=1}^{n} Y_{ik} V_k$$

2. Let
$$V_k = V_k \angle \delta_k$$
$$Y_{ik} = Y_{ik} \angle \theta_{ik}$$
$$\therefore \quad I_i = \sum_{k=1}^{n} Y_{ik} V_k \angle (\delta_k + \theta_{ik})$$

3. The complex power injected into the i^{th} bus is
$$S_i = P_i + j\,Q_i = V_i\, I_i^*$$

4. Since, $V_i = V_i \angle \delta_i$
$$S_i = P_i + j\,Q_i = \sum_{k=1}^{n} Y_{ik} V_k \angle (\delta_i - \delta_k - \theta_{ik})$$

5. Separation of real and imaginary parts gives

$$P_i = V_i \sum_{k=1}^{n} Y_{ik} V_k \cos(\delta_i - \delta_k - \theta_{ik}) \quad i = 1, 2, \ldots, n \quad ...(2.7$$

$$Q_i = V_i \sum_{k=1}^{n} Y_{ik} V_k \sin(\delta_i - \delta_k - \theta_{ik}) \quad i = 1, 2, \ldots, n \quad ...(2.7$$

6. Eq. (2.7.1) and (2.7.2) are called static load flow equations (SLFE).

Que 2.8. ***Develop Z_{Bus} matrix for the network shown in Fig. 2.8.1.***

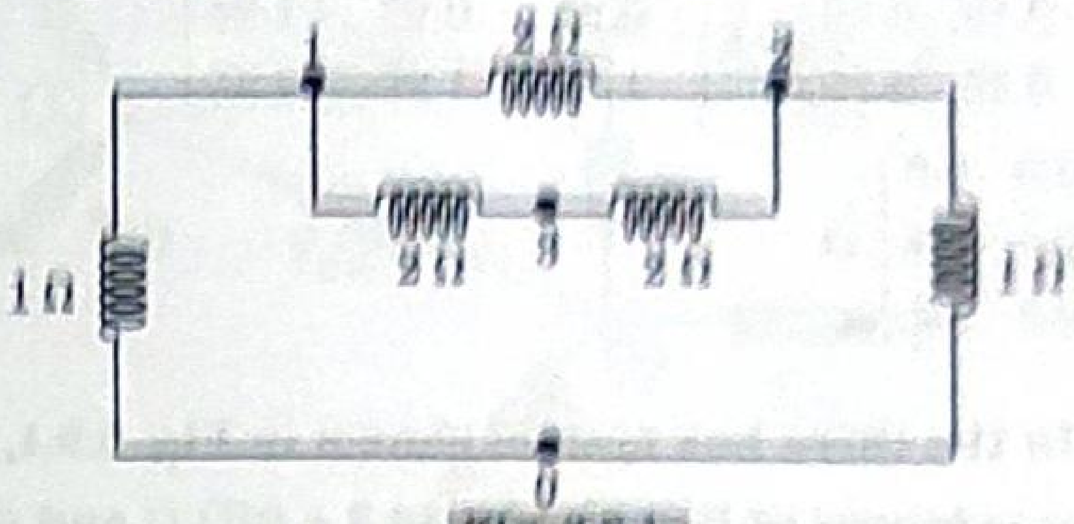

Fig. 2.8.1.

AKTU 2017-18, Marks 10

Answer

1. Add branch $Z_{1r} = 1$ [from bus 1 (new) to bus r]
$$Z_{Bus} = [1] \quad ...(2.8.1)$$

2. Add branch $Z_{21} = 2$ [from bus 2 (new) to bus 1 (old)] type-1 modification
$$Z_{Bus} = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \quad ...(2.8.2)$$

3. Add branch $Z_{13} = 2$ [from bus 3 (new) to bus 1 (old)] type-2 modification
$$Z_{Bus} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \quad ...(2.8.3)$$

4. Add branch Z_{2r} [from bus 2 (old) to bus r] type-3 modification
$$Z_{Bus} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} - \frac{1}{3+1} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} [1 \;\; 3 \;\; 1]$$
$$= \begin{bmatrix} 0.75 & 0.25 & 0.75 \\ 0.25 & 0.75 & 0.25 \\ 0.75 & 0.25 & 2.75 \end{bmatrix} \Omega$$

5. Add branch $Z_{23} = 2$ [from bus 2 (old) to bus 3 (old)] type-4 modification
$$Z_{Bus} = \begin{bmatrix} 0.75 & 0.25 & 0.75 \\ 0.25 & 0.75 & 0.25 \\ 0.75 & 0.25 & 2.75 \end{bmatrix}$$
$$- \frac{1}{2 + 0.75 + 2.75 - 2 \times 0.25} \begin{bmatrix} 0.25 - 0.75 \\ 0.75 - 0.25 \\ 0.25 - 2.75 \end{bmatrix} [-0.5 \quad 0.5 \quad -2.5]$$

$$= \begin{bmatrix} 0.75 & 0.25 & 0.75 \\ 0.25 & 0.75 & 0.25 \\ 0.75 & 0.25 & 2.75 \end{bmatrix} - \frac{1}{5}\begin{bmatrix} 0.25 & -0.25 & 1.25 \\ -0.25 & 0.25 & -1.25 \\ 1.25 & -1.25 & 6.25 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7 & 0.3 & 0.5 \\ 0.3 & 0.5 & 0.5 \\ 0.5 & 0.5 & 3 \end{bmatrix} \Omega$$

Que 2.9. **In the three bus system shown in Fig. 2.9.1, the series and shunt impedances of line (L_1) is (14.3 + *j*97) Ω and (– *j*3274) Ω. Line (L_2) is (7.13 + *j*48.60) Ω and (–*j*6547) Ω and line (L_3) is (9.38 + *j*64) Ω and (–*j*4976) Ω respectively, find [Y_{BUS}].**

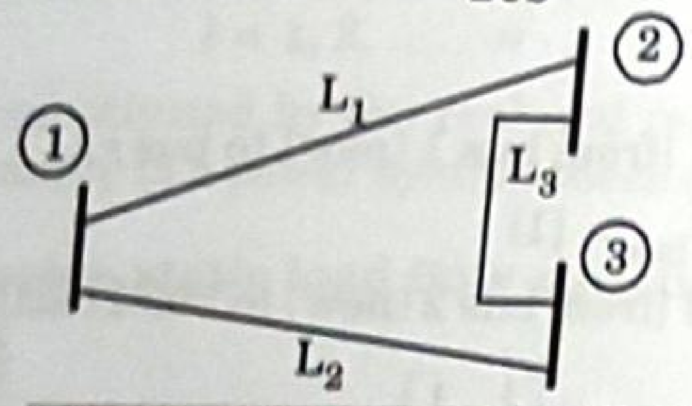

Fig. 2.9.1. Three bus system.

AKTU 2017-18, Marks 10

Answer

1. Each line can be represented as an equivalent circuit, having two shunt arms and one series arm. This has been shown in Fig. 2.9.2.
2. Equivalent representation of admittance elements,

Y_P = Shunt element

Y_S = Series element

$$Y_{S12} = \frac{1}{Z_{L1}} = \frac{1}{14.3 + j97} \text{ mho}$$

$$= (1.47 - j10) \times 10^{-3} \text{ mho}$$

$$Y_{S23} = \frac{1}{Z_{L3}} = \frac{1}{9.38 + j64} = (2.25 - j15.4)\ 10^{-3} \text{ mho}$$

$$Y_{P12} = \frac{1}{Z_{PL1}} = \frac{1}{-j3274} = j0.305 \times 10^{-3} \text{ mho}$$

$$Y_{P23} = \frac{1}{Z_{PL3}} = \frac{1}{-j4976} = j0.20 \times 10^{-3} \text{ mho}$$

$$Y_{S13} = \frac{1}{Z_{L2}} = \frac{1}{7.13 + j48.6} = (2.95 - j20.2)\ 10^{-3} \text{ mho}$$

$$Y_{P13} = \frac{1}{Z_{PL2}} = \frac{1}{-j6547} = j0.153 \times 10^{-3} \text{ mho}$$

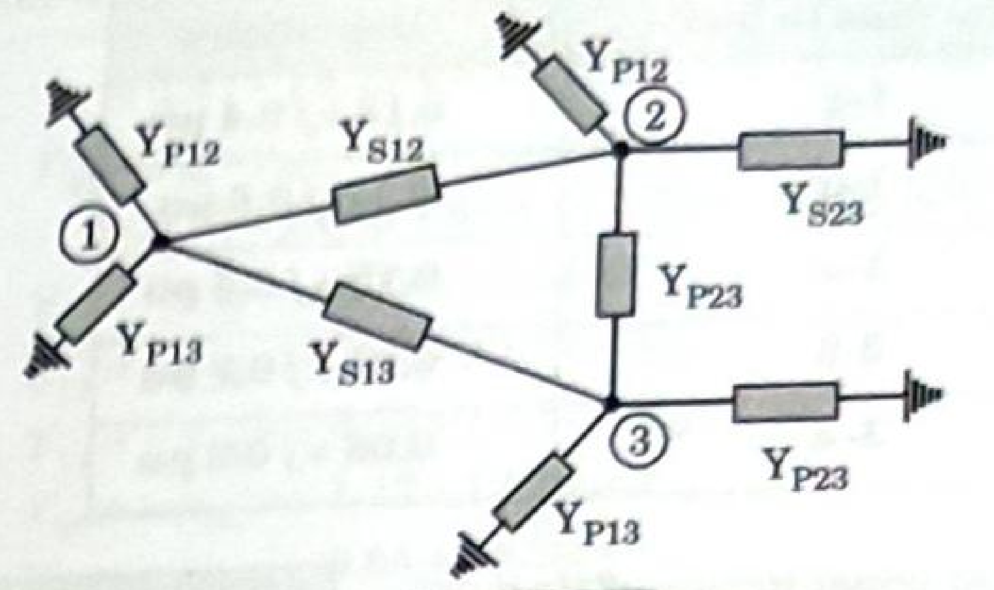

Fig. 2.9.2.

3. Also,

$$Y_{11} = Y_{P12} + Y_{P13} + Y_{S12} + Y_{S13}$$
$$= [j0.305 + j0.153 + (1.47 - j10) + (2.95 - j20.2)]\ 10^{-3} \text{ mho}$$
$$= (4.42 - j29.742) \times 10^{-3} \text{ mho}$$
$$Y_{12} = Y_{21} = -Y_{S12}$$
$$= -(1.47 - j10) \times 10^{-3} \text{ mho}$$
$$= (-1.47 + j10)\ 10^{-3} \text{ mho}$$
$$Y_{13} = Y_{31} = -Y_{S13}$$
$$= -(2.95 - j20.2)\ 10^{-3} \text{ mho}$$
$$= (-2.95 + j20.2) \times 10^{-3} \text{ mho}$$
$$Y_{22} = Y_{P12} + Y_{P23} + Y_{S12} + Y_{S23}$$
$$= [j0.305 + j0.2 + (1.47 - j10) + (2.25 - j15.4)] \times 10^{-3}$$
$$= (3.72 - j24.895)\ 10^{-3} \text{ mho}$$
$$Y_{23} = -Y_{S23} = (-2.25 + j15.4)\ 10^{-3} \text{ mho} = Y_{32}$$
$$Y_{33} = Y_{P13} + Y_{P23} + Y_{S23} + Y_{S13}$$
$$= [j0.153 + j0.2 + (2.25 - j15.4) + (2.95 - j20.2)]\ 10^{-3} \text{ mho}$$
$$= (5.20 - j35.24) \times 10^{-3} \text{ mho}$$

4. Now

$$[Y_{BUS}] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$= 10^{-3}\begin{bmatrix} (4.42 - j29.742) & (-1.47 + j10) & (-2.95 + j20.2) \\ (-1.47 + j10) & (3.72 - j24.895) & (-2.25 + j15.4) \\ (-2.95 + j20.2) & (-2.25 + j15.4) & (5.2 - j35.24) \end{bmatrix} \text{ mho}$$

Que 2.10. **Form Y_{bus} for 4-bus system. If the line series impedances are as follows :**

Line (bus to bus)	Impedance
1–2	$0.13 + j\,0.4$ pu
1–3	$0.1 + j\,0.6$ pu
1–4	$0.15 + j\,0.5$ pu
2–3	$0.05 + j\,0.2$ pu
3–4	$0.06 + j\,0.3$ pu

Neglect shunt capacitance of line. **AKTU 2018-19, Marks 07**

Answer

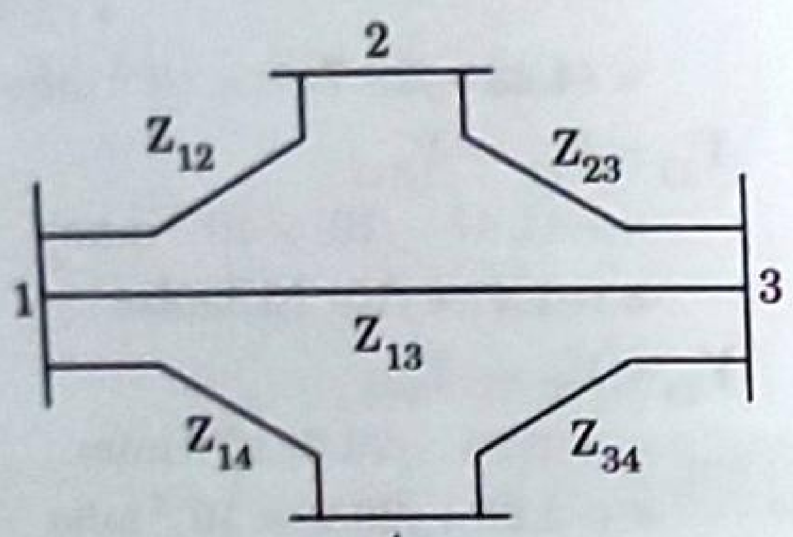

Fig. 2.10.1.

1.
$$y_{12} = \frac{1}{z_{12}} = \frac{1}{0.13 + j\,0.4} = 0.73 - j\,2.26$$
$$y_{13} = \frac{1}{z_{13}} = \frac{1}{0.1 + j\,0.6} = 0.27 - j\,1.62$$
$$y_{14} = \frac{1}{z_{14}} = \frac{1}{0.15 + j\,0.5} = 0.55 - j\,1.83$$
$$y_{23} = \frac{1}{z_{23}} = \frac{1}{0.05 + j\,0.2} = 1.17 - j\,4.70$$
$$y_{34} = \frac{1}{z_{34}} = \frac{1}{0.06 + j\,0.3} = 0.64 - j\,3.20$$

2.
$$Y_{11} = y_{12} + y_{13} + y_{14}$$
$$= 0.73 - j\,2.26 + 0.27 - j\,1.62 + 0.55 - j\,1.83 = 1.55 - j\,5.71$$
$$Y_{22} = y_{12} + y_{23}$$
$$= 0.73 - j\,2.26 + 1.17 - j\,4.70 = 1.9 - j\,6.96$$
$$Y_{33} = y_{23} + y_{13} + y_{34}$$

$$= 1.17 - j\,4.70 + 0.27 - j\,1.62 + 0.64 - j\,3.20 = 2.08 - j\,9.52$$
$$Y_{44} = y_{14} + y_{34}$$
$$= 0.55 - j\,1.83 + 0.64 - j\,3.20 = 1.19 - j\,5.03$$

3.
$$Y_{12} = -y_{12} = -0.73 + j\,2.26 = Y_{21}$$
$$Y_{13} = -y_{13} = -0.27 + j\,1.62 = Y_{31}$$
$$Y_{14} = -y_{14} = -0.55 + j\,1.83 = Y_{41}$$
$$Y_{23} = -y_{23} = -1.17 + j\,4.70 = Y_{32}$$
$$Y_{34} = -y_{34} = -0.64 + j\,3.20 = Y_{43}$$

$$Y_{bus} = \begin{bmatrix} 1.55 - j\,5.71 & 0.73 - j\,2.26 & 0.27 - j\,1.62 & 0.55 - j\,1.83 \\ -0.73 + j\,2.26 & 1.9 - j\,6.94 & 1.17 - j\,4.70 & 0 \\ -0.27 + j\,1.62 & -1.17 + j\,4.70 & 2.08 - j\,9.52 & 0.64 - j\,3.20 \\ -0.55 + j\,1.83 & 0 & -0.64 + j\,3.20 & 1.19 - j\,5.03 \end{bmatrix}$$

PART-2

Load Flow Solution using Gauss Siedel and Newton-Raphson Method, Comparison of Gauss Siedel and Newton Raphson Method.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.11. **Develop mathematical model for load flow analysis by using Gauss-Seidal method.**

OR

Develop and explain the load flow equation by Gauss method.

AKTU 2018-19, Marks 07

Answer

1. From the nodal current equations, the total current entering the k^{th} bus of an n-bus system is given by
$$I_k = Y_{k1} V_1 + Y_{k2} V_2 + \ldots + Y_{kk} + Y_{kn} V_n = \sum_{i=1}^{n} Y_{ki} V_i \quad \ldots(2.11.1)$$
2. The complex power injected into the k^{th} bus is
$$S_k = P_k + j\,Q_k = V_k\, I_k^* \quad \ldots(2.11.2)$$
3. The complex conjugate of eq. (2.11.2) gives

$$S_k^* = P_k - jQ_k = V_k^* I_k \quad \text{...(2.11.3)}$$

$$I_k = \frac{1}{V_k^*}(P_k - jQ_k) \quad \text{...(2.11.4)}$$

4. From eq. (2.11.1) and eq. (2.11.4),

$$Y_{k1}V_1 + Y_{k2}V_2 + ... + Y_{kk}V_k + ... + Y_{kn}V_n = \frac{1}{V_k^*}(P_k - jQ_k) \quad \text{...(2.11.5)}$$

5. Therefore, the voltage at any bus k where P_k and Q_k are specified is given as

$$V_k = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{V_k^*} - \sum_{\substack{i=1 \\ i \neq k}}^{n} Y_{ki}V_i\right] \quad \text{...(2.11.6)}$$

6. At bus 2, $V_2 = \dfrac{1}{Y_{22}}\left[\dfrac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3 - ... - Y_{2n}V_n\right]$ ...(2.11.7)

7. At bus 3, $V_3 = \dfrac{1}{Y_{33}}\left[\dfrac{P_3 - jQ_3}{V_3^*} - Y_{31}V_1 - Y_{32}V_2 - Y_{34}V_4 - ... - Y_{3n}V_n\right]$ ...(2.11.8)

8. For k^{th} bus the voltage at the $(r + 1)^{th}$ iteration is given by

$$V_k^{(r+1)} = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{(V_k^{(r)})^*} - \sum_{i=1}^{k-1} Y_{ki}V_i^{(r+1)} - \sum_{i=k+1}^{n} Y_{ki}V_i^{(r)}\right] \quad \text{...(2.11.9)}$$

9. In eq. (2.11.9), the quantities P_k, Q_k, Y_{kk} and Y_{ki} are known and do not vary during the iteration cycle. Let us define

$$C_k = \frac{P_k - jQ_k}{Y_{kk}} \text{ for } k = 2, 3, ..., n \quad \text{...(2.11.10)}$$

$$D_{ki} = \frac{Y_{ki}}{Y_{kk}} \text{ for } k = 2, 3, ..., n \quad \text{...(2.11.11)}$$

$$i = 1, 2, ..., n \text{ (except } i \neq k)$$

10. The values of C_k and D_{ki} are computed in the beginning and then used in every iteration.
11. For k^{th} bus the voltage at the $(r + 1)^{th}$ iteration can be written as

$$V_k^{(r+1)} = \frac{C_k}{(V_k^{(r)})^*} - \sum_{i=1}^{k-1} D_{ki}V_i^{(r+1)} - \sum_{i=k+1}^{n} D_{ki}V_i^{(r)}$$

$$V_k^{(r+1)} = \frac{C_k}{(V_k^{(r)})^*} - \sum_{\substack{i=1 \\ i \neq k}}^{n} D_{ki}V_i^{(r)} \quad k = 2, 3, ..., n$$

Que 2.12. **Develop the mathematical model for the load flow analysis of a power system using Gauss-Seidel method and discuss its solution algorithm.**

AKTU 2015-16, Marks 10

Answer

A. **Mathematical model :** Refer Q. 2.11, Page 2–15B, Unit-2.

B. **Load flow algorithm :**

1. Assume initial values of load bus voltages and angles for generator bus voltage (except for the slack bus). Let the assumed values be

$$V_2^{(0)}, V_3^{(0)}, ... V_n^{(0)}$$

The superscript (0) indicates an initial approximation.

2. Calculate $V_2^{(1)}$ in terms of the initial assumed voltages as follows :

$$V_2^{(1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(0)}\right)^*} - Y_{21}V_1 - Y_{23}V_3^{(0)} - ... - Y_{2n}V_n^{(0)}\right] \quad \text{...(2.12.1)}$$

3. When the corrected value of $V_2^{(1)}$ is determined, we determine $V_2^{(1)*}$. The corrected value $V_2^{(1)*}$ is then substituted back into the eq. (2.12.1) in place of $V_2^{(0)*}$. Thus a new corrected value of V_2 is obtained. This process is continued for a specified number of iterations.
4. Using the corrected value of the voltage from step 3 and other assumed values of voltages we perform calculations for bus 3. Here also we perform several iterations as we have done for bus 2.
5. We then go to buses 4, 5, 6, ..., etc., and continue the same procedure of iteration until all buses have been considered and we have obtained a new set of the values of all bus voltage in the network.
6. Repeat the iteration process from step 1 to step 5 until the difference ΔV in old and new values of bus voltages for all the buses of the network is within a specified limit of tolerance.
7. Let the iteration count be denoted by r. Then the magnitude of change in voltage at bus k between two consecutive iterations is given by

$$|\Delta V_k^{(r+1)}| = |V_k^{(r+1)} - V_k^{(r)}|$$

For all PQ buses the criterion for convergence is $|\Delta V_k| < \varepsilon$ where ε is the tolerance level.

Que 2.13. **What is the necessity of load flow study in a power system ? What is Newton-Raphson method ? How it is applied for the solution of power flow equation ? Explain with the help of an example.**

AKTU 2016-17, Marks 15

Answer

A. Necessity : Refer Q. 2.1, Page 2–2B, Unit-2.

B. Newton-Raphson (N-R) method :

1. It is a powerful method of solving non-linear algebraic equations. It works faster and it is the practical method of load flow solution of large power networks.

2. $$S_i = P_i + j\,Qi = V_i \sum_{k=1}^{n} Y_{ik} V_k$$
$$= \sum_{k=1}^{n} (V_i V_k Y_{ik}) / (\delta_i - \delta_k - \theta_{ik})$$

3. $$P_i = \sum_{k=1}^{n} (V_i V_k Y_{ik}) \cos(\delta_i - \delta_k - \theta_{ik}) \quad \text{...(2.13.1)}$$

4. $$Q_i = \sum_{k=1}^{n} (V_i V_k Y_{ik}) \sin(\delta_i - \delta_k - \theta_{ik}) \quad \text{...(2.13.2)}$$

5. Eq. (2.13.1) and (2.13.2) can also be written as
$$P_i = V_i V_i Y_{ii} \cos\theta_{ii} + \sum_{\substack{k=1\\k\neq i}}^{n} (V_i V_k Y_{ik}) \cos(\delta_i - \delta_k - \theta_{ik})$$
$$Q_i = -V_i V_i Y_{ii} \sin\theta_{ii} + \sum_{\substack{k=1\\k\neq i}}^{n} (V_i V_k Y_{ik}) \sin(\delta_i - \delta_k - \theta_{ik}) \quad \text{...(2.13.3)}$$

6. If $\Delta P_i = P_{i(sp)} - P_{i(cal)}$
then $i = 1, 2, ..., n, i \neq \text{slack}$

7. If $\Delta Q_i = Q_i(sp) - Q_{i(cal)}$
then, $i = 1, 2, ..., n, i \neq \text{slack}, i \neq PV$ bus
where the subscripts *sp* and *cal* denote the specified and calculated values respectively.

8. ΔP and ΔQ can be written as
$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix}$$
The off-diagonal and diagonal elements of the sub-matrices H, N, M and L are determined by differentiating eq. (2.13.1) and (2.13.2) with respect to δ and $|V|$.

9. Off-diagonal elements of H,
$$H_{ik} = \frac{\partial P_i}{\partial \delta_k} = V_i V_k Y_{ik} \sin(\delta_i - \delta_k - \theta_{ik}), \quad i \neq k$$

10. Diagonal elements,
$$H_{ii} = \frac{\partial P_i}{\partial \delta_i} = V_i \sum_{\substack{k=1\\k\neq 1}}^{n} Y_{ik} V_k \sin(\delta_i - \delta_k - \theta_i)$$

11. Using eq. (2.13.3), we have
$$\frac{\partial P_i}{\partial \delta_i} = -[Q_i + V_i^2 Y_{ii} \sin(-\theta_{ii})]$$
$$H_{ii} = -Q_i - V_i^2 Y_{ii} \sin\theta_{ii} = -Q_i - B_{ii} V_i^2$$

12. The off-diagonal and diagonal elements on N are given by
$$\frac{\partial P_i}{\partial |V_k|} = V_i V_k \cos(\delta_i - \delta_k - \theta_{ik})$$
$$\frac{\partial P_i}{\partial |V_i|} = 2V_i Y_{ii} \cos\theta_{ii} + \sum_{\substack{k=1\\k\neq i}}^{n} V_k Y_{ik} \cos(\delta_i - \delta_k - \theta_{ik})$$

13. The off-diagonal and diagonal elements of M matrix are
$$\frac{\partial Q_i}{\partial \delta_k} = -V_i V_k Y_{ik} \cos(\delta_i - \delta_k - \theta_{ik}) \text{ for } i \neq k$$
$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1\\k\neq i}}^{n} V_i V_k Y_{ik} \cos(\delta_i - \delta_k - \theta_{ik})$$

14. The off-diagonal and diagonal elements of L matrix are
$$\frac{\partial Q_i}{\partial V_k} = V_i Y_{ik} \sin(\delta_i - \delta_k - \theta_{ik}) \text{ for } i \neq k$$
$$\frac{\partial Q_i}{\partial V_i} = -2V_i Y_{ii} \sin\theta_{ii} + \sum_{\substack{k=1\\k\neq i}}^{n} V_k Y_{ik} \sin(\delta_i - \delta_k - \theta_{ik})$$

Que 2.14. **Draw and explain the algorithms and flow chart of Newton Raphson method used for load flow analysis in power system networks.**

OR

Explain clearly the computational procedure for load flow solution using Newton-Raphson method when the system contains only PQ buses.

AKTU 2017-18, Marks 10

Answer

A. Algorithm for N-R method :

1. Form Y_{bus}.
2. Assume initial values of bus voltages $|V_i|^0$ and phase angles δ_i^0 for $i = 2, 3, ..., n$ for load buses and phase angles for *PV* buses. Normally we set the assumed bus voltage magnitude and its phase angle equal to slack bus quantities $|V_1| = 1.0, \delta_1 = 0°$.

3. Compute P_i and Q_i for each load bus from the following equations :

$$P_i = \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\delta_i - \delta_k - \theta_{ik}) \quad ...(2.14.$$

$$Q_i = \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\delta_i - \delta_k - \theta_{ik}) \quad ...(2.14.$$

4. Compute the scheduled errors ΔP_i and ΔQ_i for each load bus from the following relations

$$\Delta P_i^{(r)} = P_{i\,sp} - P_{i\,(cal)}^{(r)} \quad i = 2, 3, ..., n \quad ...(2.14.$$

$$\Delta Q_i^{(r)} = Q_{i\,sp} - Q_{i\,(cal)}^{(r)} \quad i = 2, 3, ..., n \quad ...(2.14.4)$$

For *PV* buses, the exact value of Q_i is not specified, but its limits are known. If the calculated value of Q_i is within limits, only ΔP_i is calculated. If the calculated value of Q_i is beyond the limits, then an appropriate limit is imposed and ΔQ_i is also calculated by subtracting the calculated value of Q_i from the appropriate limit. The bus under consideration is now treated as a load (*PQ*) bus.

5. Compute the elements of the Jacobian matrix

$$\begin{bmatrix} H & N' \\ M & L' \end{bmatrix}$$

using the estimated $|V_i|$ and δ_i from step 2.

6. Obtain $\Delta\delta$ and $\Delta|V_i|$ from equation

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N' \\ M & L' \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V / V \end{bmatrix} \quad ...(2.14.5)$$

7. Using the values of $\Delta\delta_i$ and $\Delta|V_i|$ calculated in step 6, modify the voltage magnitude and phase angle at all load buses by the equations

$$|V_i^{(r+1)}| = |V_i^{(r)}| + \Delta|V_i^{(r)}| \quad ...(2.14.6)$$

$$\delta_i^{(r+1)} = \delta_i^{(r)} + \Delta\delta_i^{(r)} \quad ...(2.14.7)$$

Start the next iteration cycle at step 2 with these modified $|V_i|$ and δ_i.

8. Continue until scheduled errors $\Delta P_i^{(r)}$ and $\Delta Q_i^{(r)}$ for all load buses are within a specified tolerance, that is,

$$\Delta P_i^{(r)} < \varepsilon, \Delta Q_i^{(r)} < \varepsilon$$

where ε denotes the tolerance level for load buses.

9. Calculate the line flows and power at slack bus using Gauss-Seidal method.

B. Flow chart :

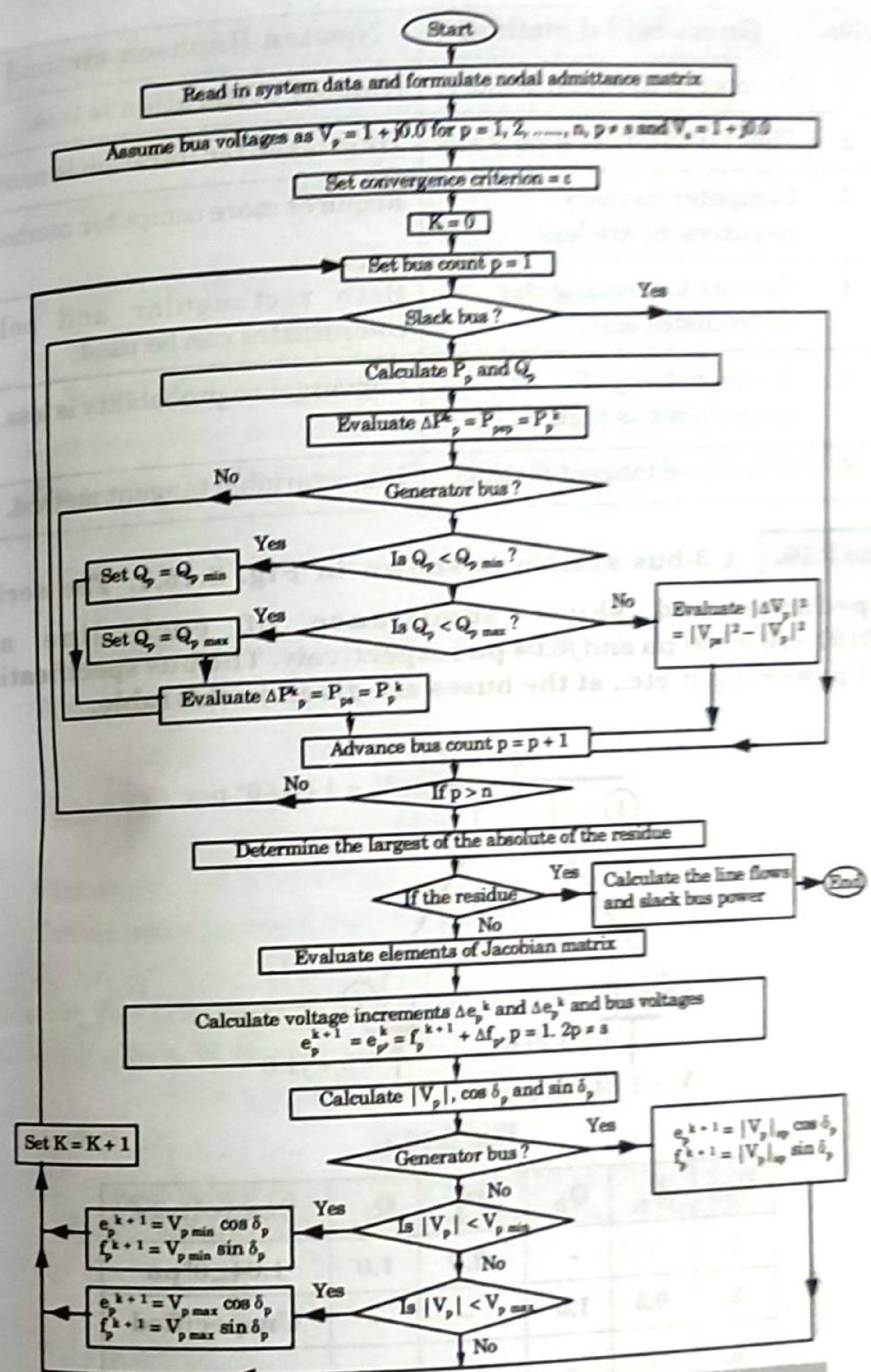

Fig. 2.14.1. Flow chart for load flow solution using Newton Raphson method.

Que 2.15. **Compare Gauss-Seidal and Newton Raphson method.**

Answer

S. No.	Gauss-Seidal method	Newton Raphson method
1.	Number of iteration is more.	Number of iteration is less.
2.	Time taken for iteration is less.	Time taken for iteration is more.
3.	Computer memory requirement are less.	Requires more computer memory.
4.	Favours for rectangular co-ordinates only.	Both rectangular and polar co-ordinates can be used.
5.	The probability of convergence is high.	Convergence probability is less.
6.	It is a fixed tangent method.	It is a variable tangent method.

Que 2.16. **A 3-bus system is shown in Fig. 2.16.1. The series impedance and shunt admittance of each line is (0.0197 + *j*0.0788) pu and *j*0.04 pu respectively. The bus specification and power input etc., at the buses are given in the table.**

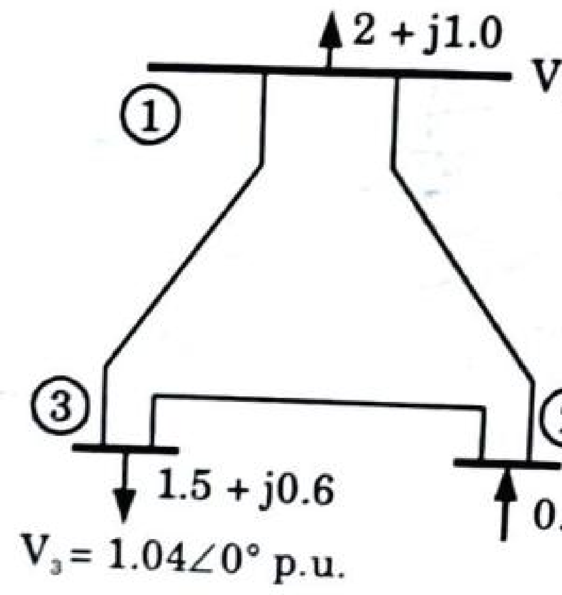

Fig. 2.16.1.

Bus	P_u	Q_0	P_L	Q_L	Bus voltage
1.	–	–	2.0	1.0	1.04∠0°pu
2.	0.5	1.0	–	–	Unspecified
3.	–	–	1.5	0.6	1.04∠0°pu

Form Y_{BUS} and calculate P_2°, Q_2° and P_3° by N-R Method.

AKTU 2015-16, Marks 10

Answer

Given : $Z_{sc} = (0.0197 + j0.0788)$ pu, $Z_{sh} = j0.04$ pu

To Find : Y_{BUS}, P_2°, P_2°, P_3°.

1. The series impedance of each line is $(0.0197 + j0.0788)$ pu.
   The series admittance of each line

$$\frac{1}{(0.0197 + j0.0788)} = 12.31 \angle -75.96^\circ = (2.98 - j11.94) \text{ pu}$$

So, $Y_{12} = Y_{21} = Y_{13} = Y_{31} = Y_{23} = Y_{32} = -(2.98 - j11.94)$
$= -2.98 + j11.94 = 12.31 \angle 104.04^\circ$ pu

2. Each line can be represented by a nominal π circuit.
3. The shunt admittance at each end of the line is $0.5 \times j0.04 = j0.02$ pu.
4. Since two lines are connected to each bus and shunt admittance between each bus and ground is $j0.04$ pu, so

$$Y_{11} = Y_{22} = Y_{33} = 2(0.298 - j11.94) + j0.04$$
$$= (0.596 - j23.84) = 24.23 \angle -75.95 \text{ pu}$$

5. Thus, the bus admittance matrix

$$Y_{BUS} = \begin{bmatrix} 24.23 \angle -75.95^\circ & 12.31 \angle 104.04^\circ & 12.31 \angle 104.04^\circ \\ 12.31 \angle 104.04^\circ & 24.23 \angle -75.95^\circ & 12.31 \angle 104.04^\circ \\ 12.31 \angle 104.04^\circ & 12.31 \angle 104.04^\circ & 24.23 \angle -75.95^\circ \end{bmatrix}$$

6. **Flat start :** $|V_2|^\circ = 1.0$ pu.
8. Taking bus 1 as slack bus, $k = 3$ and $i = 2$ and using

$$P_2^0 = |V_2|^0 |V_1| \, |Y_{21}| \cos(\delta_2^0 - \delta_1 - \theta_{21})$$
$$+ |V_2|^0 |V_2|^0 \, |Y_{22}| \cos(-\theta_{22}) \,| + |V_3|^0 \, |V_2|^0 \, |Y_{23}| \cos(\delta_3^0 - \delta_2^0 - \theta_{23})$$
$$= 1 \times 1.04 \times 12.31 \cos(-104.04^\circ)$$
$$+ 1 \times 1 \times 24.23 \cos(75.95^\circ + 1.04 \times 1 \times 12.31 \cos(-104.04^\circ)$$
$$= -0.33 \text{ pu}$$

9. $P_3^0 = |V_3|^0 \, |V_1| \, |Y_{31}| \cos(\delta_3^0 - \delta_1 - \theta_{31}) +$
$|V_3|^0 |V_3|^0 \, |Y_{33}| \cos(-\theta_{33}) + | \, |V_3|^0 \, |V_2|^0 \, |Y_{32}| \cos(\delta_2^0 - \delta_3^0 - \theta_{32})$

$$P_3^0 = 1.04 \times 1.04 \times 12.31 \cos(-104.04^\circ) + 1.04 \times$$
$$1.04 \times 24.23 \cos(75.95^\circ) + 1.04 \times 1 \times 12.31 \cos(-104.04^\circ)$$

$P_3^0 = 0.026$ pu.

10. $Q_2^0 = |V_2|^0 |V_2| \, |Y_{22}| \sin(-\theta_{22}) + |V_2|^0 |V_1|^0 |Y_{21}| \sin(\delta_2^0 - \delta_1 - \theta_{21})$
$+ |V_2|^0 |V_3|^0 |Y_{23}| \sin(\delta_2^0 - \delta_3^0 - \theta_{23})$

$$Q_2^0 = +1 \times 1 \times 24.23 \sin(+75.95^\circ) + 1 \times 1.04 \times 12.31$$
$$\sin(-104.04^\circ) + 1 \times 1.04 \times 12.31 \sin(-104.04^\circ)$$

$Q_2^0 = -1.33$ pu.

PART-3

Approximation to N-R method, Fast Decoupled Method.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.17. **Explain decoupled load flow method.**

Answer

1. $$\Delta P = H\Delta\delta + N\Delta V \quad ...(2.17.1)$$
$$\Delta Q = M\Delta\delta + L\Delta V \quad ...(2.17.2)$$

2. Multiplying and dividing by V the voltage magnitude increment ΔV to bring symmetry in the eq. (2.17.1) and (2.17.2) we have
$$\Delta P = H\,\Delta\,\delta + (VN)\frac{\Delta V}{V} \quad ...(2.17.3)$$
$$\Delta Q = M\,\Delta\,\delta + (VL)\frac{\Delta V}{V} \quad ...(2.17.4)$$

3. Let $VN = N'$ and $VL = L'$. We can write eq. (2.17.3) and (2.17.4),
$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}H & N'\\ M & L'\end{bmatrix}\begin{bmatrix}\Delta\,\delta\\ \dfrac{\Delta V}{V}\end{bmatrix} \quad ...(2.17.5)$$

4. In this case it is seen that
$$H_{ik} = L_{ik}' \text{ and } N_{ik}' = -M_{ik}$$

5. For a generator bus, the reactive power Q_i is not specified but the voltage magnitude $|\Delta V| = \Delta V$ is specified.
If
$$V_i = v_i' + j\,v_i''\,i$$
$$|V_i|^2 = v_i'^2 + v_i''^2$$
there at all generator buses, the variable $\Delta\,Q_i$ is to be replaced by $\Delta\,|V_i|^2$.

6. The elements of M are given by
$$M_{ik} = \frac{\partial(|V_i|)^2}{\partial\,\delta_k} = 0, \quad i \neq k$$
and
$$M_{ii} = \frac{\partial(|V_i|)^2}{\partial\,\delta_i} = 0$$

7. The element of L are given by
$$L_{ik} = |V_k|\,\frac{\partial(|V_i|)^2}{\partial|V_k|} = 0, \quad i \neq k$$
$$L_{ii} = |V_i|\,\frac{\partial(|V_i|)^2}{\partial|V_i|} = 2|V_i|^2$$

8. The coupling between active power P and the bus voltage magnitude $|V|$ is relatively weak. Similarly, the coupling between reactive power Q and bus voltage phase angle is also weak.

9. This weak coupling is utilized in the development of decoupled load flow (DLF) method in which P is decoupled from ΔV and Q is decoupled from $\Delta\delta$.

10. With these assumptions eq. (2.17.5) is reduced to
$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}H & 0\\ 0 & L\end{bmatrix}\begin{bmatrix}\Delta\,\delta\\ \dfrac{\Delta V}{V}\end{bmatrix} \quad ...(2.17.6)$$
where the N' and M are neglected.

11. Eq. (2.17.6) is the decoupled equation which can be expanded as
$$[\Delta\,P] = [H]\,[\Delta\,\delta] \quad ...(2.17.7)$$
$$[\Delta\,Q] = [L]\left[\frac{\Delta|V|}{|V|}\right] \quad ...(2.17.8)$$

12. We have relations :
$$L_{ik}' = H_{ik} = V_i\,V_k\,Y_{ik}\sin(\delta_i - \delta_k - \theta_{ik}) \quad ...(2.17.9)$$
$$H_{ii} = -Q_i - B_{ii}\,V_i^2 \quad ...(2.17.10)$$
$$L_i' = Q_i - B_{ii}\,V_i^2 \quad ...(2.17.11)$$
We can solve eq. (2.17.7) and (2.17.8) simultaneously at each iteration.

UNIT 3

Travelling Waves in Power System

CONTENTS

PART-1

Production of Travelling Waves, Open Circuited Line, Short Circuited Line, Line Terminated Through A Resistance.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. **Deduce the general wave equations for a lossless transmission line for propagation of voltage and current wave.**

AKTU 2017-18, Marks 10

Answer

1. The line parameters are uniformly distributed along the line. It may be assumed that the line is made up of short section of length dx.

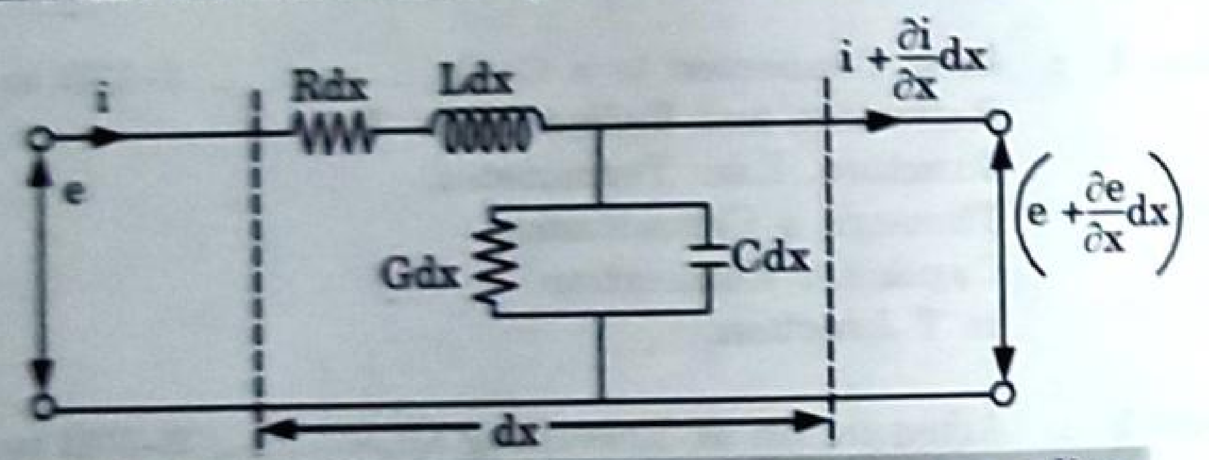

Fig. 3.1.1. Short section of a two-wire transmission line.

2. Suppose that
   R = Resistance of line per unit length.
   L = Inductance of line per unit length.
   C = Capacitance of line per unit length.
   G = Shunt conductance of line per unit length.
3. Consider a short section of the line of length dx. The instantaneous values of voltage and current are function of both distance x and time t.
4. At a distance x from the sending end, they can be represented as $e(x, t)$ and $i(x, t)$ respectively. The voltage at a neighbouring point distance $(x + dx)$ from the sending end is $e(x + dx)$.
5. By Taylor's theorem

$$e(x+dx) = e(x) + \frac{\partial e}{\partial x}dx \qquad ...(3.1.1)$$

6. The current i flowing through the resistance Rdx causes a voltage drop $i(Rdx)$ in it.
7. It changes at the rate of $\frac{\partial i}{\partial t}$ in the inductance Ldx to produce a voltage drop equal to $\left(L\frac{\partial i}{\partial t}dx\right)$ in it.
8. The difference in the voltage between the ends of the section is due to the voltage drops in the resistance Rdx and inductance Ldx.
9. Mathematically,

$$e - \left(e + \frac{\partial e}{\partial x}dx\right) = (Rdx)\, i + (Ldx)\frac{\partial i}{\partial t}$$

$$-\frac{\partial e}{\partial x}dx = (Rdx)\, i + (Ldx)\frac{\partial i}{\partial t}$$

$$-\frac{\partial e}{\partial x} = Ri + L\frac{\partial i}{\partial t} \qquad ...(3.1.2)$$

10. Also,

$$i - \left(i + \frac{\partial i}{\partial x}dx\right) = (Gdx)\, e + (Cdx)\frac{\partial e}{\partial t}$$

$$-\frac{\partial i}{\partial x}dx = (Gdx)\, e + (Cdx)\frac{\partial e}{\partial t}$$

$$-\frac{\partial i}{\partial x} = Ge + C\frac{\partial e}{\partial t} \qquad ...(3.1.3)$$

11. Since the losses in the line are much smaller than the energy travelling along the line, they can be neglected.
12. For a lossless line $R = 0$, $G = 0$, then eq. (3.1.2) and (3.1.3) becomes

$$\frac{\partial e}{\partial x} = -L\frac{\partial i}{\partial t} \qquad ...(3.1.4)$$

and

$$\frac{\partial i}{\partial x} = -C\frac{\partial e}{\partial t} \qquad ...(3.1.5)$$

13. Differentiating eq. (3.1.4) partially with respect to distance x and eq. (3.1.5) with respect to time t gives

$$\frac{\partial^2 e}{\partial x^2} = -L\frac{\partial^2 i}{\partial x \partial t} \qquad ...(3.1.6)$$

and

$$\frac{\partial^2 i}{\partial t\, \partial x} = -C\frac{\partial^2 e}{\partial t^2} \qquad ...(3.1.7)$$

Also,

$$\frac{\partial^2 i}{\partial t\, \partial x} = \frac{\partial^2 i}{\partial x\, \partial t}$$

14. Substituting the value of $\frac{\partial^2 i}{\partial x\,\partial t}$ in eq. (3.1.6) from eq. (3.1.7), we get

$$\frac{\partial^2 e}{\partial x^2} = LC\frac{\partial^2 e}{\partial t^2}$$

This is the voltage wave equation. ...(3.1.8)

15. Similarly differentiating eq. (3.1.4) with respect to t and eq. (3.1.5) w... respect to x,

$$\frac{\partial^2 e}{\partial x\,\partial t} = -L\frac{\partial^2 i}{\partial t^2} \quad ...(3.1.9)$$

and

$$\frac{\partial^2 i}{\partial x^2} = -C\frac{\partial^2 e}{\partial x\,\partial t} \quad ...(3.1.10)$$

16. Substituting the value of $\frac{\partial^2 i}{\partial x\,\partial t}$ from eq. (3.1.9) to eq. (3.1.10), we get

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} \quad ...(3.1.11)$$

This is the current wave equation.

17. Eq. (3.1.8) and eq. (3.1.11) are identical in form and give similar solution. They are called "the wave equation". They represent the distribution of voltage and current along the line in terms of time and distance.

Que 3.2. **Write a short note on surge phenomenon.**

Answer

Surge phenomenon :

1. A surge is a movement of charge along the conductor.
2. Such surges are characterized by a sudden very steep rise in voltage (the surge front) followed by a gradual decay in voltage (the surge tail).
3. The surges produced on the line due to lightning have great magnitude and different wave-shapes.

Que 3.3. **Show that the velocity of a travelling wave can be given by**

$$v = \frac{1}{\sqrt{LC}}$$

Answer

A. Velocity of propagation : Velocity of propagation is a measure of how fast a waves travels over a time.

B. Derivation of velocity of propagation a velocity of travelling waves :

1. For a short section dx of the line, total inductance = Ldx
   Change in flux linkage in time dt is $iLdx$.
2. Rate of change of flux linkage
$$= iL\frac{dx}{dt} = \text{Induced voltage}$$
3. But this induced voltage is equal to the voltage e
$$e = iL\frac{dx}{dt} = iLv \quad ...(3.3.1)$$
4. Total capacitance of the section
$$= Cdx$$
5. The charge dq delivered to the section
$$dq = eCdx$$
$$\therefore \quad i = \frac{dq}{dt} = eC\frac{dx}{dt} = eCv \quad ...(3.3.2)$$
6. Multiplication of eq. (3.3.1) and (3.3.2) gives
$$ei = eiLCv^2$$
$$v^2 = \frac{1}{LC}$$
$$v = \pm\frac{1}{\sqrt{LC}}$$
7. The double sign indicates that the surge is split into two components which travel along the line in opposite direction.

Que 3.4. **Give the evaluation of the velocity of the wave propagation for overhead transmission line and cable.**

Answer

A. Velocity of the wave propagation for over head transmission line :

1. $L = 2 \times 10^{-7} \ln (D/R)$ H/phase/m
2. $C = \frac{2\pi\varepsilon}{\ln(D/R)} = \frac{2\pi \times (10^{-9}/36\pi)}{\ln(D/R)} = \frac{10^{-9}}{18\ln(D/R)}$ F/phase/m
   where, D is the distance between the centres of the conductors and R is the radius of the conductor and $D > R$.
3. $v = \frac{1}{\sqrt{(LC)}} = \frac{1}{\sqrt{2 \times 10^{-7}\ln(D/R) \times 10^{-9}/18\ln(D/R)}} = 3 \times 10^8$ m/s

4. Thus, it is found that velocity of travelling waves on overhead lines with air as dielectric is the same as the speed of light in free space.

B. Velocity of the wave propagation for cable :

1. $$L = 2 \times 10^{-7} \ln (R/r) \text{ H/phase/m}$$

2. $$C = \frac{2\pi\varepsilon}{\ln (R/r)}$$

$$= \frac{10^{-9}\ \varepsilon_r}{18 \ln (R/r)} \text{ F/phase/m}$$

where, R is the radius of the cable and r is the radius of the conductor. Assuming a dielectric having a relative dielectric constant of ε_r.

3. $$v = \frac{1}{\sqrt{(LC)}} = \frac{1}{\sqrt{2 \times 10^{-7} \ln (R/r) \times [10^{-9}\ \varepsilon_r / 18 \ln (R/r)]}}$$

$$= \frac{3 \times 10^8}{\sqrt{\varepsilon_r}} \text{ m/s}$$

Que 3.5. **Give the evaluation of surge impedance.**

OR

Explain surge impedance and velocity of propagation of travelling waves.

AKTU 2018-19, Marks 3.5

Answer

A. **Surge impedance :** It is the ratio of magnitude of voltages and associated current and it is equivalent to $\sqrt{L/C}$ denoted by Z_0.

B. **Derivation of surge impedance :**

1. Consider the forward wave represented by

$$e = f_1\left(x - \frac{1}{\sqrt{LC}}t\right)$$

$$\frac{\partial i}{\partial x} = -C\frac{\partial e}{\partial t} \quad ...(3.5.1)$$

2. After integration, eq. (3.5.1) becomes

$$\int \frac{\partial i}{\partial x} dx = -\int C \frac{\partial e}{\partial t} dx \quad ...(3.5.2)$$

3. Consider the forward wave represented by

$$e = f_1\left(x - \frac{1}{\sqrt{LC}}t\right) = f_1 (x - ut) \quad ...(3.5.3)$$

where, $$u = \frac{1}{LC}$$

4. Differentiating eq. (3.5.3) with respect to x and t gives

$$\frac{\partial e}{\partial x} = \frac{\partial f_1}{\partial (x-ut)} \frac{\partial (x-ut)}{\partial x} = \frac{\partial f_1}{\partial (x-ut)} \quad ...(3.5.4)$$

and $$\frac{\partial e}{\partial t} = \frac{\partial f_1}{\partial (x-ut)} \frac{\partial (x-ut)}{\partial t} = -u\frac{\partial f_1}{\partial (x-ut)} \quad ...(3.5.5)$$

5. From eq. (3.5.4) and (3.5.5),

$$\frac{\partial e}{\partial t} = -u \frac{\partial e}{\partial x} \quad ...(3.5.6)$$

6. Using eq. (3.5.2) eq. (3.5.6),

$$\int \frac{\partial i}{\partial x} dx = -C(-u) \int \frac{\partial e}{\partial x} dx$$

$$i = Cue$$

7. Thus the ratio of the magnitude of voltages and associated current or surge impedance is given by

$$Z_0 = \frac{e}{i} = \frac{1}{Cu} = \frac{1}{C}\sqrt{LC} = \sqrt{\frac{L}{C}} \quad ...(3.5.7)$$

8. The quantity $\sqrt{\frac{L}{C}}$ is called as surge impedance Z_0 or wave impedance or natural impedance. It is also called the characteristic impedance for a lossless line. It is denoted by Z_0.

C. **Velocity of propagation :** Refer Q. 3.3, Page 3–4B, Unit-3.

Que 3.6. **Deduce the expression for reflection factor or reflection coefficients.**

Answer

1. Consider a general case of a line which is terminated through a Resistance (R)

2. By Ohm's law,

Resistance at the receiving end = $\dfrac{\text{The resultant voltage at the receiving end}}{\text{The resultant current at the receiving end}}$

i.e., $$\frac{e_R}{i_R} = R \quad ...(3.6.1)$$

3. And, $$e_R = e_f + e_r \quad ...(3.6.2)$$

$$i_R = i_f + i_r \quad ...(3.6.3)$$

4. Put the values of eq. (3.6.2) and (3.6.3) in eq. (3.6.1),

$$\frac{e_f + e_r}{i_f + i_r} = R \quad ...(3.6.4)$$

5. For incident wave

$$e_f = i_f Z_0$$

and for reflected wave ...(3.6.5)

$$e_r = -i_r Z_0 \quad ...(3.6.6)$$

6. The values of the currents obtained from eq. (3.6.5) and (3.6.6) when substituted in eq. (3.6.4) for R give

$$\frac{e_f + e_r}{e_f / Z_0 - e_r / Z_0} = R$$

7. Normally the value of the incident voltage being known and therefore the reflected voltage e_r is given by

$$e_r = e_f \frac{R - Z_0}{R + Z_0}$$

8. Similarly if the line terminated through an impedance (Z_t) then the reflected voltage e_r is given by

$$e_r = e_f \frac{Z_t - Z_0}{Z_t + Z_0}$$

9. The voltage at the end of the line is

$$e_R = e_f + e_r = 2e_f \frac{R}{R + Z_0}$$

10. The current through the resistance, R at the end of the line is

$$i_R \cong \frac{e_R}{R} = \frac{2e_f}{R + Z_0}$$

11. The ratio of the reflected voltage to the incident voltage is called the reflection coefficient or the reflection factor for the voltage. It is denoted by ρ_v.

$$\rho_v \cong \frac{e_r}{e_f} = \frac{R - Z_0}{R + Z_0}$$

Similarly for Z_t

$$\rho_v \cong \frac{e_r}{e_f} = \frac{Z_t - Z_0}{Z_t + Z_0}$$

12. This implies that a voltage wave is reflected at the termination and its amplitude becomes ρ_v times the incident wave. The ratio of the reflected current to the incident current is

$$\rho_i \cong \frac{i_r}{i_f} = \frac{-e_r / Z_0}{e_f / Z_0} = -\frac{e_r}{e_f} = -\rho_v$$

$$\rho_v = -\rho_i = \rho \text{ (say)}$$

Que 3.7. **Determine reflection coefficient and transmission coefficient for receiving end of transmission line terminated by resistance.**

AKTU 2017-18, Marks 10

OR

Develop wave equation for a uniform transmission line and find the velocity of its propagation. Derive the expression for reflection and refraction coefficients of voltages and current waves when a line terminated through a resistance or a cable.

AKTU 2016-17, Marks 15

OR

Derive the expression for reflection and refraction coefficients of voltage and current waves when a line terminated through a resistance or a cable.

AKTU 2018-19, Marks 07

Answer

A. **Derivation of wave equation :** Refer Q. 3.1, Page 3–2B, Unit-3.

B. **Derivation of reflection coefficient :** Refer Q. 3.6, Page 3–7B, Unit-3.

C. **Derivation of refraction coefficient or transmission coefficient :**

1. At the junction of two lines having different surge impedances, an incident wave is partially reflected and partially refracted on to the other line.
2. Let R_1 be the surge impedance of the line on which the incident wave e', i' approaches the junction and on which the reflected wave e'', i'' goes back.
3. Let R_2 be the surge impedance of the other line which carries the refracted wave e, i.
4. Consider conditions before any reflection comes back to the junction from the distant end of the later line.
5. During this time e and i are the only voltage and current on this line and they obey Ohm's law,

$$e = iR_2$$

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6. The voltage and current of the refracted wave are equal to resultant voltage and current of the incident and reflected waves.
7. Thus $e = e' + e'' = e' + ke'$
8. Since $e'' = ke'$ (where k = Reflection coefficient)

$$e = e'(1 + k) = e'\left(1 + \frac{R_2 - R_1}{R_2 + R_1}\right) = e'\frac{2R_2}{R_2 + R_1}$$

and $i = i' + i'' = i' + (-k)i'$

9. Since $i'' = -ki'$

$$i = i'(1 - k) = i'\frac{2R_1}{R_2 + R_1}$$

10. Therefore, the refraction coefficient for voltage is $(1 + k)$ and refraction coefficient for current is $(1 - k)$.

Que 3.8. **Discuss the behaviour of a travelling wave when it reaches the end of a**

i. Short-circuited

ii. Open-circuited transmission line

iii. A line terminated by an impedance equal to surge impedance (Z_0).

AKTU 2015-16, Marks 10

Answer

1. The ratio of the voltage at the termination to the incident voltage is called the transmission coefficient or refraction coefficient.
2. It is denoted by τ.

$$\frac{e_R}{e_f} = \tau$$

$$\frac{e_R}{e_f} - 1 = \tau - 1$$

$$\frac{e_R - e_f}{e_f} = \tau - 1$$

i.e., $\rho = \tau - 1$

3. The values of reflected and resultant current and voltage for their incident values of i_f and e_f can be conveniently written as,

Reflected voltage, $e_r = \rho e_f$

Reflected current, $i_r = -\rho i_f$

4. Resultant voltage at termination

$$e_R = e_f \frac{2Z_t}{Z_t + Z_0} = \tau e_f$$

5. Resultant current at termination

$$i_R = i_f \frac{2Z_0}{Z_t + Z_0}$$

6. **According to result some cases are as follows :**

i. **When $Z_t = 0$, *i.e.*, the line is short circuited :**

a. For a line which is short circuited at the receiving end, the voltage will be zero, *i.e.*,

$$e_f + e_r = e_R = 0$$

b. Thus $e_f = -e_r$ ...(3.8.1)

and $\rho = -1, \tau = 0$

c. Also $i_f = i_r$

and $i_R = i_f + i_r = 2i_f$

d. Eq. (3.8.1) shows that at the receiving point the reflected voltage will be equal to incident voltage but of opposite sign.

e. The reflected current is also equal to incident current but of the same sign.

f. The resultant voltage is zero at the termination while the resultant current is doubled there.

g. The line voltage becomes zero periodically, but the current is increased by $\frac{e}{Z_0}$ at each subsequent reflection.

ii. **When $Z_t = \infty$, *i.e.*, the line is open circuited :**

a. When the line is open at the receiving end, Z_t becomes infinite and

$$\rho = \frac{Z_t - Z_0}{Z_t + Z_0} = \frac{1 - Z_0 / Z_t}{1 + Z_0 / Z_t} = +1$$

Also, $\tau = 2$

b. The amplitude of the reflected wave is equal to that of the incident wave, the two waves being of the same sign.

c. The resultant voltage at the receiving end

$$e_R = e_f + e_r = e_f + e_f = 2e_f$$

d. Also for an open circuit

$$i_R = 0$$

And, $$i_f + i_r = 0$$

$$i_r = -i_f$$

e. The direction of reflected current is opposite to that of incident current.

f. Thus, the voltage at the receiving end of an open circuited line becomes equal to two times the incident voltage and the current there is zero.

iii. When $Z_t = Z_0$, *i.e.*, the line is terminated in its characteristic impedance :

a. If the line be terminated in its characteristic impedance Z_0, then $Z_t = Z_0$

$$\rho = \frac{Z_t - Z_0}{Z_t + Z_0} = 0$$

$$\tau = 1, e_r = 0, i_r = 0$$

which shows that there will be no reflected waves, *i.e.*, no reflection occurs.

b. The incident waves of voltage and current will be entirely absorbed by the load such that a line with the receiving end impedance equal to its characteristic impedance is said to be correctly terminated or matched.

c. It is also called a flat line or an infinite line.

Que 3.9. **A 300 kV, 5 μ sec rectangular surge travels along the line terminated by a capacitor of 1500 pF. Determine the voltage across the capacitance and reflected voltage wave if the surge impedance loading of line is 300 ohm.**

AKTU 2018-19, Marks 07

Answer

i. Voltage across the capacitance :

$$V(t) = 2V'(1 - e^{-t/CZ_0})$$

$$= 2 \times 300\,(1 - e^{-5 \times 10^{-6}/1500 \times 10^{-12} \times 300})$$

$$= 599.99 \text{ kV}$$

ii. Reflected voltage :

$$e_r = V'[1 - 2e^{-t/CZ_0}]$$

$$= 300\,[1 - 2e^{-5 \times 10^{-6}/1500 \times 10^{-12} \times 300}]$$

$$= 300[1 - 0.0000298]$$

$$= 299.99 \text{ kV}$$

PART-2

Line Connected to a Cable, Reflection and Refraction at T Junction, Line Terminated Through a Capacitance, Capacitor Connection at T Junction.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.10. **Derive the expression for reflected and transmitted voltage when a line connected to a cable.**

Answer

1. When a wave travels towards the cable from the line (Fig. 3.10.1) because of the difference in impedances at the junction, some part of the wave is reflected and the rest is transmitted.

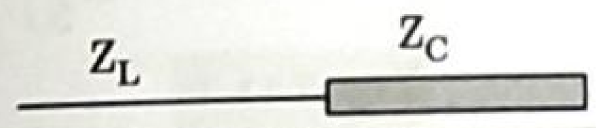


Fig. 3.10.1. Line connected to a cable.

2. The transmitted voltage is given by,

$$e = e' \times \frac{2Z_C}{Z_L + Z_C}$$

and, the transmitted current is given by,

$$i = i' \times \frac{2Z_C}{Z_L + Z_C}$$

3. Reflected voltage is given by,

$$e'' = e' \times \frac{Z_C - Z_L}{Z_C + Z_L}$$

and the reflected current is given by,

$$i'' = -i' \times \frac{Z_C - Z_L}{Z_C + Z_L}$$

Que 3.11. **Deduce the expression for reflected and transmitted voltage at T junction.**

Answer

1. Let us consider the case of a surge travelling along a line of surge impedance Z_1 reaching a junction of surge impedance Z_2, Z_3.
2. The wave will be partially reflected on the line of surge impedance Z_1 and partially transmitted in branches Z_2, Z_3.
3. For simplicity, let a line OA be divided into two branches AB and AC of surge impedances Z_2 and Z_3 respectively. It is shown in Fig. 3.11.1.
4. Since AB and AC are in parallel, the voltage transmitted in them will be the same.

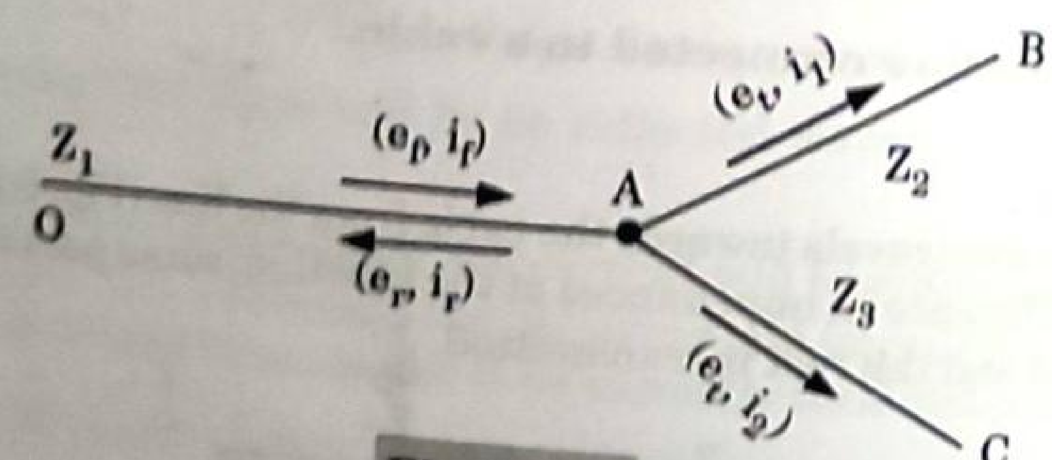


Fig. 3.11.1.

5. Let i_1 and i_2 be the currents transmitted in branches AB and AC. The following relations may be written as,

$$i_f = \frac{e_f}{Z_1} \quad ...(3.11.1)$$

$$i_r = \frac{-e_r}{Z_1} \quad ...(3.11.2)$$

$$i_1 = \frac{e_t}{Z_2} \quad ...(3.11.3)$$

$$i_2 = \frac{e_t}{Z_3} \quad ...(3.11.4)$$

6. The voltage at the junction is

$$e_f + e_r = e_t \quad ...(3.11.5)$$

Also,

$$i_f + i_r = i_1 + i_2 \quad ...(3.11.6)$$

7. Putting the values of eq. (3.11.1), (3.11.2), (3.11.3) and (3.11.4), in eq. (3.11.6), we get

$$\frac{e_f}{Z_1} - \frac{e_r}{Z_1} = \frac{e_t}{Z_2} + \frac{e_t}{Z_3} \quad ...(3.11.7)$$

8. Putting the values of e_r from eq. (3.11.5) in eq. (3.11.7)

$$\frac{e_f}{Z_1} - \frac{e_t - e_f}{Z_1} = \frac{e_t}{Z_2} + \frac{e_t}{Z_3}$$

9. The transmitted voltage is

$$e_t = \frac{\frac{2e_f}{Z_1}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = 2e_f \frac{\frac{1}{Z_1}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} \quad ...(3.11.8)$$

10. The reflected voltage, $e_r = e_t - e_f = e_f \dfrac{\frac{1}{Z_1} - \frac{1}{Z_2} - \frac{1}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$ ...(3.11.9)

Que 3.12. **Derive the expression for transmitted voltage when a line terminated through a capacitance.**

Answer

1. Suppose that a wave travels along the line of surge impedance Z_0 having at termination of a capacitor C.

Here,

$$e_f + e_r = e \quad ...(3.12.1)$$

$$i_f + i_r = i \quad ...(3.12.2)$$

$$\frac{e_f}{Z_0} + \frac{e_r}{Z_0} = \frac{e}{Z_0}$$

$$\frac{e_f}{Z_0} - \frac{e_r}{Z_0} = C\frac{de}{dt}$$

$$\frac{2e_r}{Z_0} = \frac{e}{Z_0} + C\frac{de}{dt}$$

$$\frac{de}{dt} + \frac{1}{CZ_0}e = \frac{2}{CZ_0}e_f \quad ...(3.12.3)$$

2. The solution is $e = \dfrac{b}{a} + A\,\varepsilon^{-at}$

3. Assuming that the capacitor cannot charge instantaneously, so that $e = 0$ at $t = 0$,

$$\therefore \quad A = -\frac{b}{a} \quad \text{...(3.12.4)}$$

4. Putting the value of eq. (3.12.4) in eq. (3.12.3), we get

$$e = \frac{b}{a}(1 - \varepsilon^{-at})$$

or

$$e = 2e_f\left(1 - \varepsilon^{-\frac{t}{CZ_0}}\right)$$

$$e_r = e - e_f$$

$$e_r = e_f\left(1 - 2\varepsilon^{-\frac{t}{CZ_0}}\right)$$

$$i = C\frac{de}{dt} = \frac{2e_f}{Z_0}\varepsilon^{-\frac{t}{CZ_0}} \quad \text{...(3.12.5)}$$

At $t = 0, e = 0$, $\quad e_r = -e_f, \quad i = \frac{2e_f}{Z_0} = 2i_f$ ...(3.12.6)

Que 3.13. **Give the expression for transmitted voltage across the capacitor at *T*-junction.**

Answer

1. Fig. 3.13.1 shows the connection diagram of capacitor at *T*-junction.

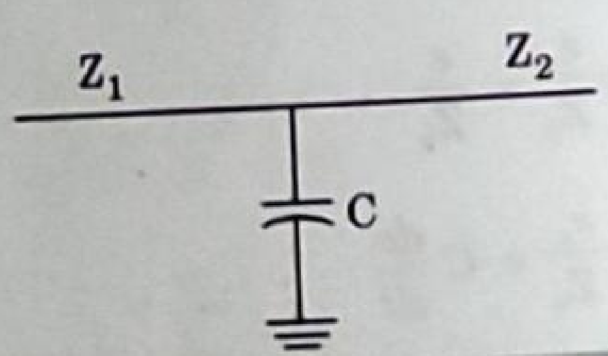


Fig. 3.13.1. Capacitor connected at *T*.

2. The transmitted voltage across the capacitor of the circuit is given by

$$e = \frac{2Z_1}{Z_0 + Z_t}e' \quad \text{...(3.13.1)}$$

$$e(s) = \frac{\frac{2e'}{Z_1 s}}{\frac{1}{Z_1} + \frac{1}{Z_2} + Cs} = \frac{2e'\,Z_2}{s}\,\frac{\frac{1}{Z_1Z_2C}}{\frac{Z_1 + Z_2}{Z_1Z_2C} + s}$$

$$= \frac{2e'}{sZ_1C} \times \frac{1}{s + \frac{Z_1 + Z_2}{Z_1Z_2\,C}} \quad \text{...(3.13.2)}$$

Let, $\quad \frac{Z_1 + Z_2}{Z_1Z_2C} = \alpha$ ...(3.13.3)

3. Putting the value of eq. (3.13.3) in eq. (3.13.2), we get

$$e(s) = \frac{2e'}{s} \times \frac{\frac{1}{Z_1C}}{s + \alpha}$$

$$e(s) = \frac{2e'}{s} \times \frac{Z_2}{Z_1 + Z_2} \times \frac{Z_1 + \frac{Z_2}{Z_1Z_2C}}{s + \alpha}$$

$$= \frac{2e'}{s} \times \frac{Z_2}{Z_1 + Z_2} \times \frac{\frac{\alpha}{Z_1Z_2C}}{s + \alpha}$$

$$= \frac{2e'Z_2}{Z_1 + Z_2}\left[\frac{1}{s} - \frac{1}{s + \alpha}\right]$$

4. Taking the inverse Laplace transform on both sides of the above equation,

$$e(t) = \frac{2e'Z_2}{Z_1 + Z_2}\left(1 - e^{-\left(\frac{Z_1 + Z_2}{Z_1 \times Z_2}\right)}\right)$$

Que 3.14. **A surge of 100 kV travelling in a line of natural impedance 600 Ω arrives at a junction with two lines of impedances 800 Ω and 200 Ω respectively. Find the surge voltages and currents transmitted into each branch line.**

Answer

Given : $Z_1 = 800\ \Omega$, $Z_2 = 200\ \Omega$, $Z_c = 600\ \Omega$ and
Surge magnitude, $V = 100$ kV
To Find : i. Surge voltages, V''
ii. Surge currents, I_1'' and I_2''.

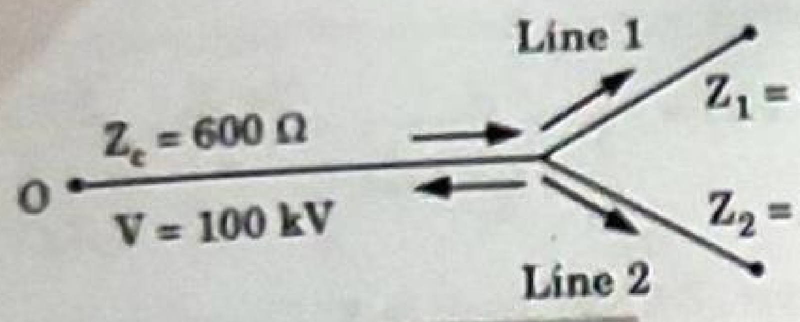


Fig. 3.14.1.

1. The surge as it reaches the joint suffers reflection and here the two lines are in parallel.
2. Therefore the transmitted voltage will have the same magnitude and is given by

$$\tau_V = \frac{V''}{V} = \frac{\left(\dfrac{2}{Z_c}\right)}{\left(\dfrac{1}{Z_1}+\dfrac{1}{Z_2}+\dfrac{1}{Z_c}\right)}$$

$$V'' = \frac{2V/Z_c}{\left(\dfrac{1}{Z_1}+\dfrac{1}{Z_2}+\dfrac{1}{Z_c}\right)} = \frac{2\times 100/600}{\left(\dfrac{1}{800}+\dfrac{1}{200}+\dfrac{1}{600}\right)}$$

$$= 42.04 \text{ kV}$$

3. The transmitted current in line 1,

$$I_1'' = \frac{42.04\times 1000}{800} = 52.55 \text{ A}$$

4. The transmitted current is line 2,

$$I_2'' = \frac{42.04\times 1000}{200} = 210.2 \text{ A.}$$

Que 3.15. **A battery with an emf E and series resistance R are connected at $t = 0$ to the sending end of lossless transmission line which is short circuited at far end. Plot the sending end current and voltage as the function of time for $R = \frac{R_0}{3}$. The time required for a wave to travel to full length for line is T.**

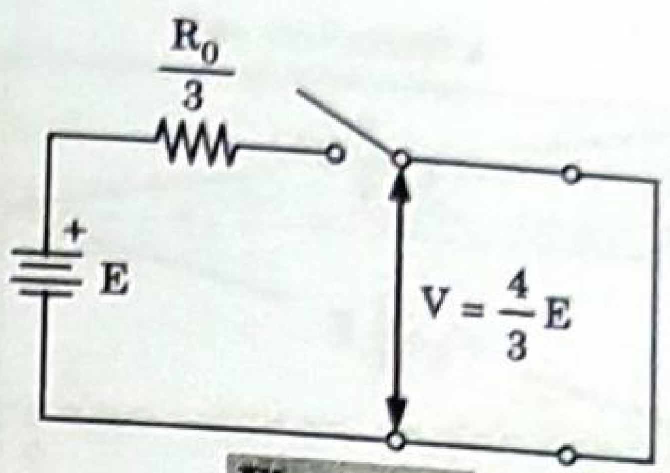


Fig. 3.15.1.

AKTU 2015-16, Marks 10

Answer

1. The first step is to find the reflection coefficient for the two ends. Reflection coefficient for the load end is :

$$K_L = \frac{R_L - R_0}{R_L + R_0} = -1$$

2. Reflection coefficient for the generator end is :

$$K_g = \frac{(R_0/3)-R_0}{(R/3)+R_0} = -\frac{1}{2}$$

3. The second step is to find the sending end voltage and current :

$$\text{Sending end voltage} = E - \frac{E}{R_0 + (R_0/3)}\times\frac{R_0}{3} = \frac{3}{4}E$$

$$\text{Sending end current} = \frac{3E}{4R_0}$$

4. The third step is to draw the zigzag diagrams, one of the voltage and the other for the current.
5. In a zigzag diagram, distance is plotted horizontally and time is downwards as shown in Fig. 5.15.1.
6. The zigzag lines are traces of the wave fronts of the various reflections.
7. The numbers attached to the line indicate the magnitude of individual waves.
8. The magnitude of each reflection is obtained by multiplying the magnitude of the preceding wave by the reflection coefficient at the point where reflection takes place.

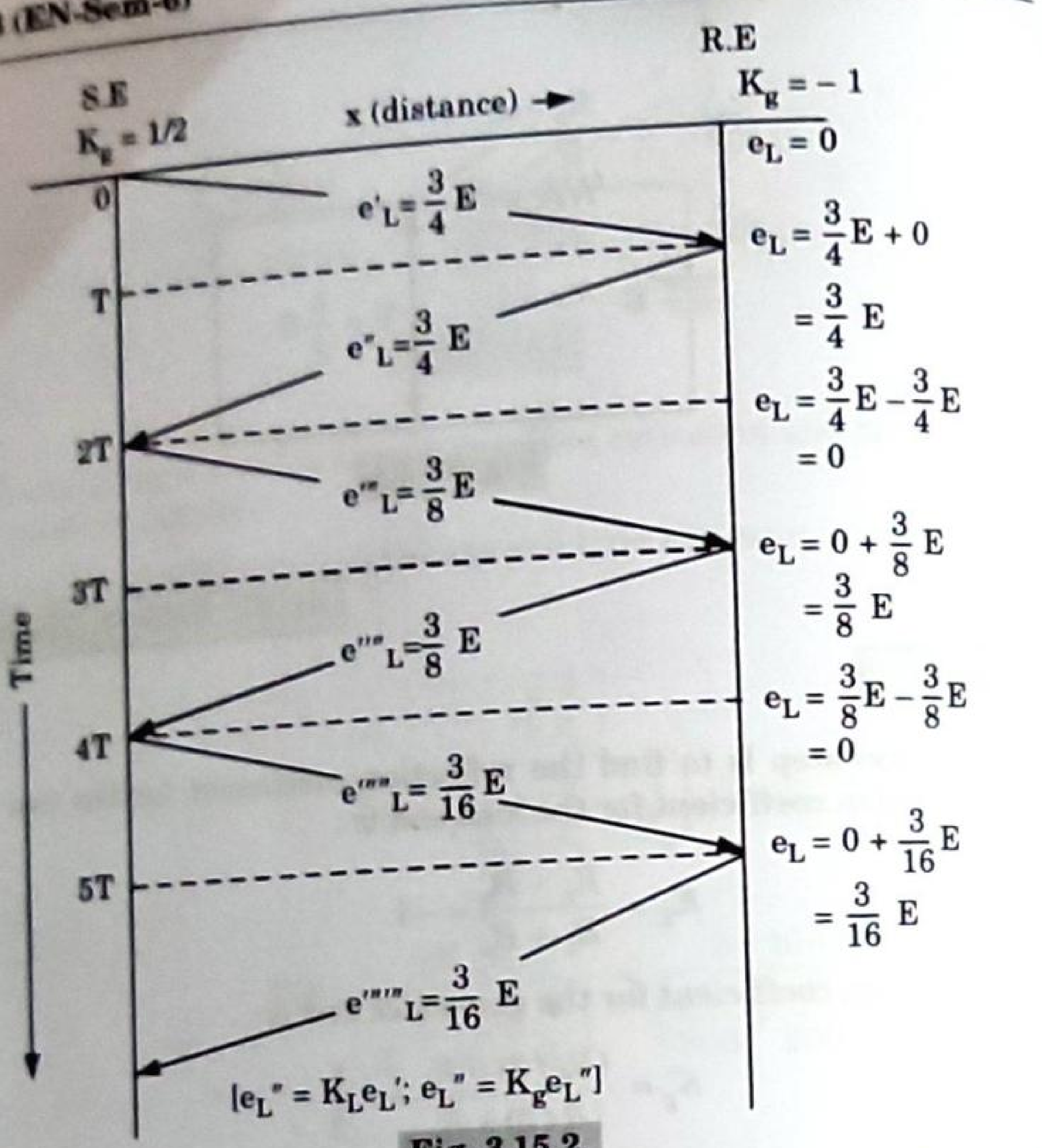


Fig. 3.15.2.

9. The number in each intervening space is the sum of the individual waves above the point and represents the net voltage or current in that region of the diagram.

PART-3

Attenuation of Travelling Wave, Bewley's Lattice Diagram.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.16. Discuss the attenuation of travelling wave.

Answer

Attenuation of travelling waves :

1. Consider r, L, C and g as the parameters per unit length of an overhead transmission line and V_0 and I_0 as the voltage and current waves at $x = 0$ as shown in Fig. 3.16.1.

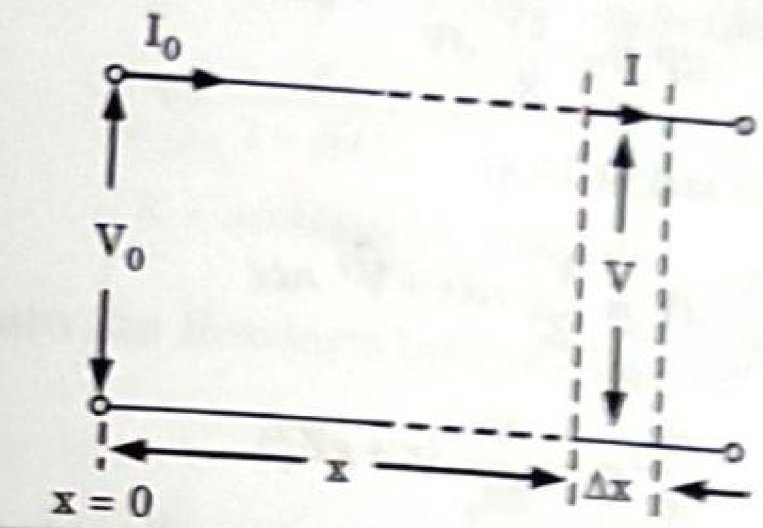


Fig. 3.16.1. Wave travelling on a lossy line.

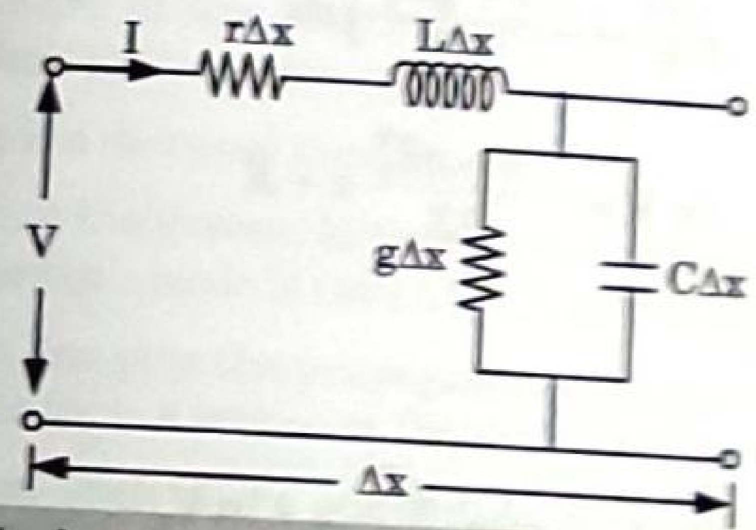


Fig. 3.16.2. Equivalent circuit of differential element of overhead line.

2. The aim is to determine the voltage (V) and current (I) waves after traveling a distance x with time t.
3. Let us consider a small distance dx traveled by the wave in time dt. The differential length, Δx, of the overhead line is shown in Fig. 3.16.2.

Power loss, $P = I^2 r + V^2 g = \frac{V^2}{Z_0^2} r + V^2 g$ ...(3.16.1)

On differentiation of eq. (3.16.1) with respect to x, we get

$$dP = \frac{V^2}{Z_0^2} rdx + V^2 rdx \quad ...(3.16.2)$$

4. Power at a distance x, $P = VI = -\frac{V^2}{Z_0}$ ...(3.16.3)

Negative sign indicates there is reduction in power as the wave travels with time.

5. Differentiation of eq. (3.16.3) with respect to V is

$$dP = -\frac{2V}{Z_0}dV \quad ...(3.16.4)$$

6. From eq. (3.16.2) and (3.16.4)

$$-\frac{2V}{Z_0}dV = \frac{V^2}{Z_0^2}rdx + V^2 rdx$$

$$\frac{dv}{dx} = -\frac{V}{2Z_0}(r + gZ_0^2)$$

$$\frac{dv}{V} = -\frac{r + gZ_0^2}{2Z_0}dx$$

$$\int\frac{dv}{V} = -\frac{r + gZ_0^2}{2Z_0}\int dx$$

$$\ln V = -\frac{r + gZ_0^2}{2Z_0}x + K$$

7. At $x = 0$, $V = V_0$

$\therefore K = \ln V_0$

$$\ln V = -\frac{r + gZ_0^2}{2Z_0}x + \ln V_0$$

$$\ln\frac{V}{V_0} = -\frac{r + gZ_0^2}{2Z_0}x$$

$$\frac{V}{V_0} = e^{-\left(\frac{r + gZ_0^2}{2Z_0}\right)x} = e^{-\alpha x}$$

where

$$\alpha = -\frac{r + gZ_0^2}{2Z_0} \quad ...(3.16.5)$$

$$\therefore \quad V = V_0e^{-\alpha x}$$

Similarly we can derive the expression for current, $I = I_0e^{-\alpha x}$ ...(3.16.6)

8. From eq. (3.16.5) and (3.16.6) the voltage and current waves are attenuated exponentially as they travel over the transmission line and the magnitude of attenuation depends upon the overhead line parameters.
9. From the empirical formula voltage and current at any point of the overhead line after traveling x distance can be calculated as

$$V = \frac{V_0}{1 + KxV_0}\ kV$$

where K = attenuation constant

Que 3.17. **Explain the Bewley's lattice diagram.**

Answer

Bewley's Lattice diagram :

1. It provides a simple and convenient method to study the effects of multiple or repeated reflections.
2. It gives the picture of the positions and direction of every incident, reflected and transmitted wave on the system at every instant.
3. Lattice diagram is a distance time graph.
4. The distance along the system in the direction of original surge is taken horizontally. The time scale is taken vertically downwards.
5. The slopes of the line give the propagation velocities in the system. The sloping lines thus form a lattice in the distance time plane.
6. The amplitude of each wave is marked on its line in terms of the amplitude of original wave and the coefficients of reflection and transmission.
7. Fig. 3.17.1 shows a lattice or zig-zag diagram.
8. In the diagram, AB represents the original wave of amplitude e_1. The slope of AB gives the velocity of propagation along the overhead line.
9. When the surge reaches the junction B, it splits up into waves BC and BD.
10. The wave $\rho_1 e_1$ represented by BC is reflected back in the overhead line and the wave $\tau_1 e_1$ represented by BD in the diagram is transmitted into the cable. The slope of BD gives velocity in the cable. The process is repeated.
11. The voltage or current at any distant can be found from the diagram. The incremental waves are projected on the time scale. The voltage at any point P is shown in the diagram to the right of the lattice diagram. The lattice diagram for current can also be drawn in a similar manner.

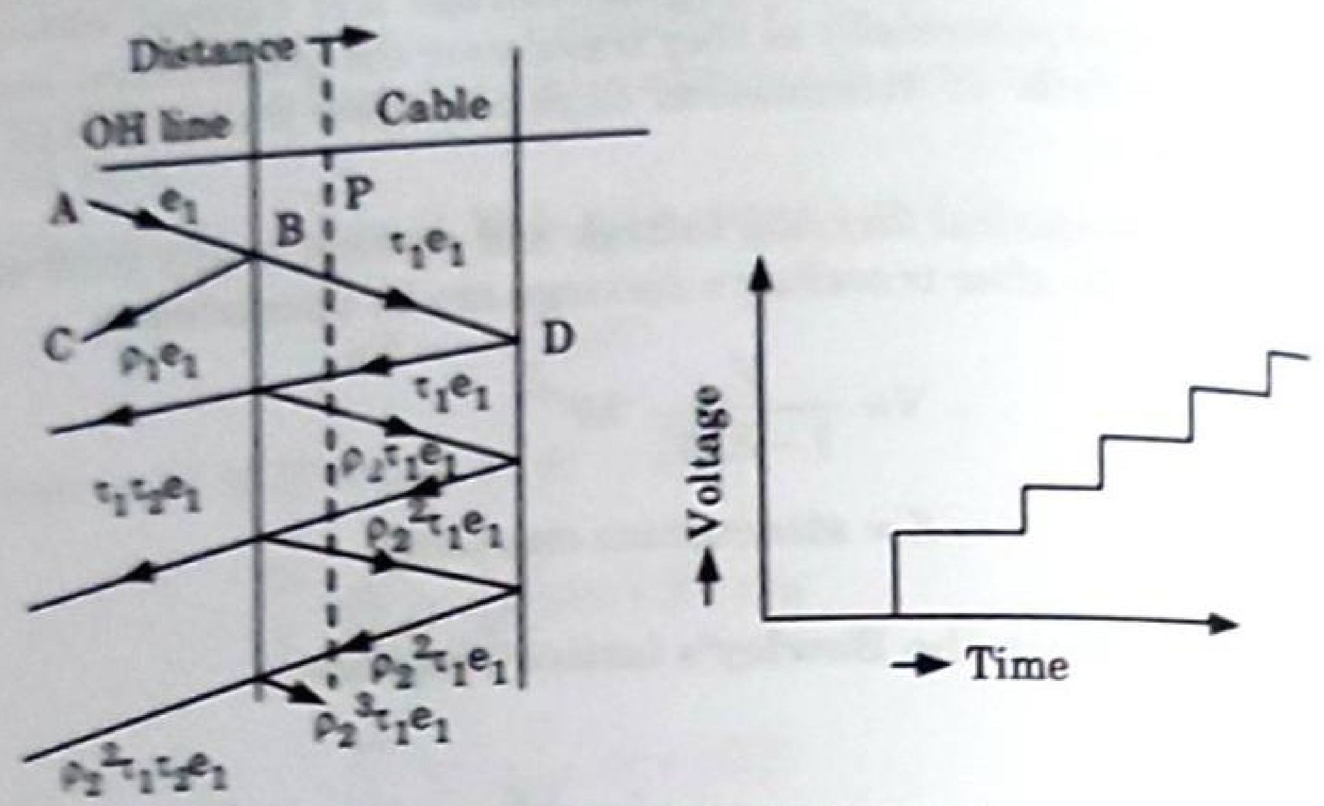


Fig. 3.17.1. Lattice diagram.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Deduce the general wave equations for a lossless transmission line for propagation of voltage and current wave.

Ans. Refer Q. 3.1.

Q. 2. Explain surge impedance and velocity of propagation of travelling waves.

Ans. Refer Q. 3.5.

Q. 3. Determine reflection coefficient and transmission coefficient for receiving end of transmission line terminated by resistance.

Ans. Refer Q. 3.7.

Q. 4. Discuss the behaviour of a travelling wave when it reaches the end of a

i. **Short-circuited**

ii. **Open-circuited transmission line**

iii. **A line terminated by an impedance equal to surge impedance (Z_0).**

Ans. Refer Q. 3.8.

Q. 5. Deduce the expression for reflected and transmitted voltage at *T* junction.

Ans. Refer Q. 3.11.

Q. 6. Discuss the attenuation of travelling wave.

Ans. Refer Q. 3.16.

Q. 7. Explain the Bewley's lattice diagram.

Ans. Refer Q. 3.17.

☺☺☺

4

UNIT

Stability in Power System

CONTENTS

PART-1

Power Flow Through a Transmission Line.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. **Derive the expression for active and reactive power when power flow through a transmission line.**

Answer

1. The transmission line equation are expressed in the form of voltage and current relationships between sending and receiving ends the loads are expressed in terms of active and reactive power.
2. The single transmission line of two bus system as shown in Fig. 4.1.1.

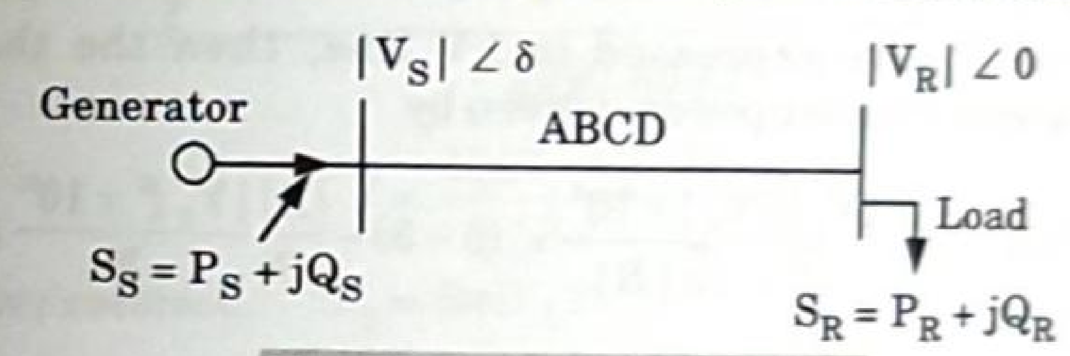


Fig. 4.1.1. A two-bus system.

3. Let us take the receiving-end voltage as a reference phasor ($V_R = |V_R| \angle 0°$) and let the sending-end voltage lead it by an angle δ ($V_S = |V_S| \angle \delta$). The angle δ is known as the torque angle.
4. The complex power leaving the receiving-end and entering the sending-end of the transmission line can be expressed as,

$$S_R = P_R + jQ_R = V_R I^*_R \quad ...(4.1.1)$$

$$S_S = P_S + jQ_S = V_S I^*_S \quad ...(4.1.2)$$

5. Receiving and sending-end currents can be expressed in terms of receiving and sending-end voltages,

$$I_R = \frac{1}{B} V_s - \frac{A}{B} V_R \quad ...(4.1.3)$$

$$I_S = \frac{D}{B} V_s - \frac{1}{B} V_R \quad ...(4.1.4)$$

6. Let A, B, D, the transmission line constants, be written as

$$A = |A| \angle \alpha, B = |B| \angle \beta, D = |D| \angle \alpha$$

(since $A = D$)

7. Therefore, we can write

$$I_R = \left|\frac{1}{B}\right| |V_S| \angle(\delta - \beta) - \left|\frac{A}{B}\right| |V_R| \angle(\alpha - \beta)$$

$$I_S = \left|\frac{D}{B}\right| |V_S| \angle(\alpha + \delta - \beta) - \left|\frac{1}{B}\right| |V_R| \angle -\beta$$

8. Putting the value of I_R in eq. (4.1.1), we get

$$S_R = |V_R| \angle 0 \left[\left|\frac{1}{B}\right| |V_S| \angle(\beta - \delta) - \left|\frac{A}{B}\right| |V_R| \angle(\beta - \alpha)\right]$$

$$= \frac{|V_S||V|}{|B|} \angle(\beta - \delta) - \left|\frac{A}{B}\right| |V_R|^2 \angle(\beta - \alpha) \quad ...(4.1.5)$$

Similarly,

$$S_S = \left|\frac{D}{B}\right| |V_S|^2 \angle(\beta - \alpha) - \frac{|V_S||V_R|}{|B|} \angle(\beta + \delta) \quad ...(4.1.6)$$

9. In the above equations S_R and S_S are per phase complex voltamperes, while V_R and V_S are expressed in per phase volts.

10. If V_R and V_S are expressed in kV line, then the three-phase receiving-end complex power is given by

$$S_R \text{ (3-phase } VA) = 3\left\{\frac{|V_S||V_R| \times 10^6}{\sqrt{3} \times \sqrt{3}|B|} \angle(\beta - \delta) - \left|\frac{A}{B}\right| \frac{|V_R|^2 \times 10^6}{3} \angle(\beta - \alpha)\right\}$$

$$S_R \text{ (3-phase } MVA) = \frac{|V_S||V_R|}{|B|} \angle(\beta - \delta) - \left|\frac{A}{B}\right| \angle(\beta - \alpha) \quad ...(4.1.7)$$

11. If eq. (4.1.5) is expressed in real and imaginary parts, we can write the real and reactive powers at the receiving-end as

$$P_R = \frac{|V_S||V_R|}{|B|} \cos(\beta - \delta) - \left|\frac{A}{B}\right| |V_R|^2 \cos(\beta - \alpha) \quad ...(4.1.8)$$

$$Q_R = \frac{|V_S||V_R|}{|B|} \sin(\beta - \delta) - \left|\frac{A}{B}\right| |V_R|^2 \sin(\beta - \alpha) \quad ...(4.1.9)$$

12. Similarly, the real and reactive powers at sending-end are

$$P_S = \left|\frac{D}{B}\right| |V_S|^2 \cos(\beta - \alpha) - \frac{|V_S||V_R|}{|B|} \cos(\beta + \delta) \quad ...(4.1.10)$$

$$Q_S = \left|\frac{D}{B}\right| |V_S|^2 \sin(\beta - \alpha) - \frac{|V_S||V_R|}{|B|} \sin(\beta + \delta) \quad ...(4.1.11)$$

13. It is easy to see from eq. (4.1.8) that the received power P_R will be maximum at

$$\delta = \beta$$

such that

$$P_R \text{ (max)} = \frac{|V_S||V_R|}{|B|} - \frac{|A||V_R|^2}{|B|} \cos(\beta - \alpha) \quad ...(4.1.12)$$

14. The corresponding Q_R (at max P_R) is

$$Q_R = -\frac{|A||V_R|^2}{|B|} \sin(\beta - \alpha)$$

Que 4.2. **Show that the steady state power which could be transmitted over a transmission line will be maximum when $X = \sqrt{3}\, R$, where X and R have their usual meaning.**

AKTU 2017-18, Marks 10

Answer

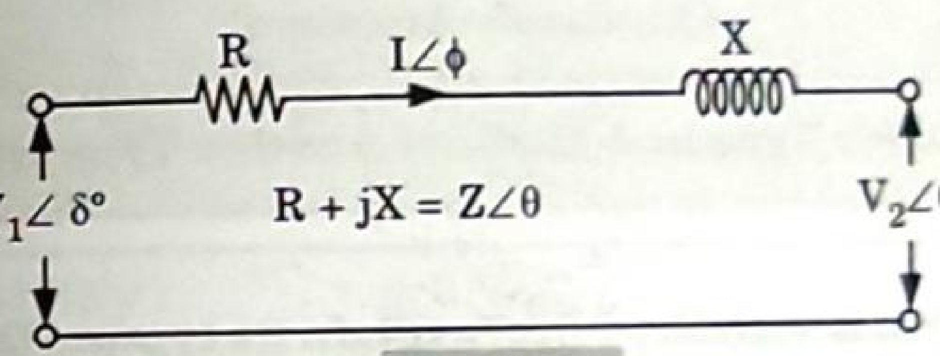


Fig. 4.2.1.

1. $$I = \frac{V_1\angle\delta - V_2\angle 0°}{Z\angle\theta} = \frac{V_1}{Z}\angle(\delta - \theta) - \frac{V_2}{Z}\angle -\theta$$

2. Power received, $P_2 = \text{Re}[V_2 I^*]$

$$= \text{Re}\left[V_2\left\{\frac{V_1}{Z}\angle(\theta - \delta) - \frac{V_2}{Z}\angle\theta\right\}\right] = \frac{V_1V_2}{Z}\cos(\theta - \delta) - \frac{V_2^2}{Z}\cos\theta$$

3. Let $\theta = 90° - \alpha$

$$\therefore \quad P_2 = \frac{V_1V_2}{Z}\cos(90° - \alpha - \delta) - \frac{V_2^2}{Z}\cos(90° - \alpha)$$

$$= \frac{V_1V_2}{Z}\sin(\alpha + \delta) - \frac{V_2^2}{Z}\sin\alpha$$

where, α = function of the impedance of the line

4. P_2 received is maximum when $\alpha + \delta = 90°$ or $\alpha = (90° - \delta)$

$$P_{2\,max} = \frac{V_1V_2}{Z} - \frac{V_2^2}{Z}\sin\alpha$$

$$\sin\alpha = \frac{R}{Z}$$

$$P_{2\,max} = \frac{V_1V_2}{\sqrt{R^2 + X^2}} - \frac{V_2^2}{\sqrt{R^2 + X^2}} \cdot \frac{R}{\sqrt{R^2 + X^2}}$$

5. When $V_1 = V_2$

$$P_{2\,max} = V_2^2\left[\frac{1}{\sqrt{R^2+X^2}} - \frac{R}{R^2+X^2}\right]$$

6. For $P_{2\,max}$ to be maximum

$$\frac{dP_{2\,max}}{dX} = V_2^2\left[\frac{X}{(R^2+X^2)^{3/2}} - \frac{2XR}{(R^2+X^2)^2}\right] = 0$$

$$\frac{V_2^2X}{(R^2+X^2)^2}[\sqrt{R^2+X^2} - 2R] = 0$$

$$R^2+X^2 = 4R^2 \quad \therefore X = \sqrt{3}\,R$$

PART-2

Stability and Stability Limit, Steady State Stability Study.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.3. What is stability ? And explain its types. What is stability limit ?

Answer

A. **Stability :** The ability of a system to reach a normal or stable condition after being disturbed is called stability.

B. **Type of stability :** Synchronous stability may be divided into two main categories depending upon the magnitude of the disturbance.

1. **Steady-state stability :**

i. Steady-state-stability refers to the ability of the power system to regain synchronism after small and slow disturbance, such as gradual power changes.

ii. Types of steady-state stability :

a. **Static stability :** Static stability refers to inherent stability that prevails without the aid of automatic control devices such as governors and voltage regulators.

b. **Dynamic stability :**

i. Dynamic stability refers to artificial stability given to an inherently unstable system by automatic control devices.

ii. Dynamic stability is concerned with small disturbances lasting for times of the order of 10 to 30 seconds with the inclusion of automatic control devices.

2. **Transient stability :**

i. The transient stability is the ability of the system to regain synchronism after a large disturbance.

ii. The large disturbance can occur due to sudden changes in application or removal of large loads line switching operations, faults on the system, sudden outage of a line, or loss of excitation.

C. **Stability limit :** The stability limit is the maximum power that can be transferred in a network between sources and loads without loss of synchronism and without the system becoming unstable when the load is increased gradually under steady-state-conditions.

Que 4.4. Define the following terms regarding power system stability :

i. Voltage stability

ii. Frequency stability

iii. Rotor angle stability

Also mention the significance of above terms related to power system.

Answer

i. **Voltage Stability :**

1. A power system at a given operating state is voltage stable if on being subjected to a certain disturbance, the voltages near load approach the post-disturbance equilibrium.
2. The concept of voltage stability is related to transient stability of a power system.
3. The analysis of voltage stability normally requires simulation of the system modelled by non-linear differential algebraic equations.
4. **Significance :** It helps in maintaining steady state voltage at all the buses.

ii. **Frequency stability :**

1. It refers to the ability of a power system to maintain steady frequency following a severe disturbance between generation and load.
2. It depends on the ability to restore equilibrium between system generation and load, with minimum loss of load.
3. Frequency instability may lead to sustained frequency swings leading to tripping of generating units or loads.
4. During frequency excursions, the characteristic times of the processes and devices that are activated will range from fraction of seconds like under frequency control to several minutes, corresponding to the response of devices such as prime mover and hence frequency stability may be a short-term phenomenon or a long-term phenomenon.
5. **Significance :** It helps in maintaining steady frequency and prevent imbalance between generator and load.

iii. Rotor angle stability :

1. Rotor angle stability is the ability of the interconnected synchronous machines running in the power system to remain in the state of synchronism.
2. **Significance :** It helps in maintaining synchronism between various synchronous machines.

PART-3

Derivation of Swing Equation, Transient Stability Studies By Equal Area Criterion.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.5. **Derive the swing equation for a machine connected to an infinite bus in a power system.** **AKTU 2017-18, Marks 10**

Answer

Swing equation :

1. The behaviour of a synchronous machine during transient is described by the swing equation.
2. $$\theta = \omega_s t + \delta \text{ (electrical radians)} \quad ...(4.5.1)$$
   where, θ = Angular position of rotor
   δ = Angular displacement of rotor
   ω_s = Synchronous speed
3. Differentiate eq. (4.5.1) with respect to t, we get
   $$\frac{d\theta}{dt} = \omega_s + \frac{d\delta}{dt} \quad ...(4.5.2)$$
4. Differentiation of eq. (4.5.2) gives
   $$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \quad ...(4.5.3)$$
5. Angular acceleration of rotor
   $$\alpha = \frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \text{ electrical radians/s}^2 \quad ...(4.5.4)$$
6. If damping is neglected

then acceleration torque, $T_a = T_s - T_e$ ...(4.5.5)

where, T_s = Shaft torque
T_e = Electromagnetic torque

7. Now, $M = J\omega$ ...(4.5.6)
   where, ω = Synchronous speed of the rotor
   J = Moment of inertia of the rotor
   M = Angular momentum of the rotor
8. Multiplying both the sides of eq. (4.5.5) by ω, we get
   $$\omega T_a = \omega T_s - \omega T_e$$
   $$P_a = P_s - P_e$$
   where, P_s = Mechanical power input
   P_e = Electrical power output
   P_a = Accelerating power
9. But,
   $$J\frac{d^2\theta}{dt^2} = T_a$$
   $$J\frac{d^2\delta}{dt^2} = T_a$$
   $$\omega J\frac{d^2\delta}{dt^2} = \omega T_a$$
   $$M\frac{d^2\delta}{dt^2} = P_a = P_s - P_e \quad ...(4.5.7)$$
10. Eq. (4.5.7) gives the relation between the accelerating power and angular acceleration. It is called "swing equation".

Que 4.6. **Explain the inertia constant and swing equations. Explain the terms swing curves in power system stability.** **AKTU 2016-17, Marks 10**

Answer

A. **Inertia constant :** Inertia constant is the ratio of kinetic energy of a rotor of a synchronous to the rating of a machine in (MVA).

B. **Swing equations :** Refer Q. 4.5, Page 4–7B, Unit-4.

C. **Swing curves :**

1. A graph of δ (usually in electrical radians) versus time in seconds is called the swing curve. Swing curves provide information regarding stability. It is shown in Fig. 4.6.1.
2. They show any tendency of δ to oscillate and/or increase beyond the point of return.
3. If δ increases continuously with time the system is unstable.

4. While if δ starts decreasing after reaching a maximum value it is inferred that the system will remain stable.

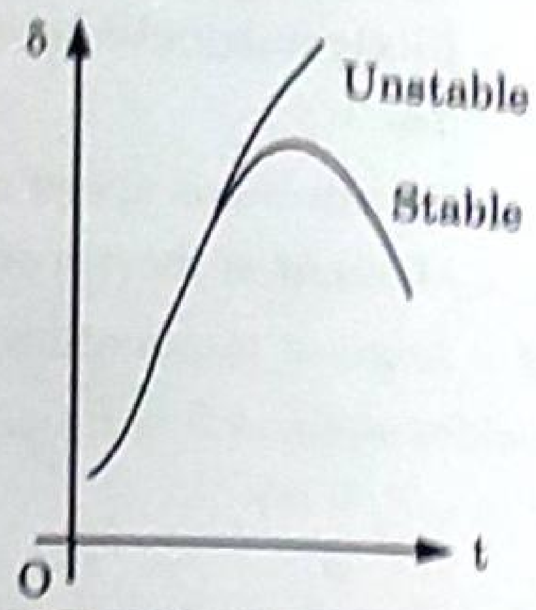


Fig. 4.6.1. Swing curves.

5. Swing curves are useful in determining the adequacy of relay protection on power systems with regard to the clearing of faults before one or more machines become unstable and fall out of synchronism.

Que 4.7. **Develop mathematical model for M constant and H constant.**

Answer

1. The transient conditions of synchronous machines depend on the mechanical constants of the rotor and load or prime mover.
2. Let
   - ω = Synchronous speed of the rotor in rad/s
   - m = Mass of the rotor in kg
   - r = Radius of gyration in m
   - J = Moment of inertia of the rotor in kgm²
   - M = Angular momentum of the rotor in Js/rad
   - W = Kinetic energy of the rotor in J
   - f = System frequency in Hz
   - T = Torque in N-m
   - P = Power in watts
   - α = Angular acceleration of the rotor

A. M constant :

1. Now,
$$J = mr^2$$
$$W = \frac{1}{2} J \omega^2$$
2.
$$M = J\omega = \frac{2W}{\omega}$$
$$\omega = 2\pi f \text{ rad/s} = 360 f \text{ elec. deg/s}$$

$$T = J\alpha$$
$$P = \omega T = \omega J \alpha = M\alpha$$
$$M = \frac{P}{\alpha}$$

3. Thus, M constant may be defined as the power in MW required to produce unit angular acceleration.

B. H constant or per unit inertia constant :

1. The per unit inertia constant, H is defined as the kinetic energy stored in the rotating parts of the machines at synchronous speed per unit megavoltamperes (MVA) of the machine. Thus
$$H = \frac{\text{Kinetic energy in MJ at rated speed}}{\text{Machine rating in MVA}}$$
It is expressed in MJ/MVA
2. If W is the stored energy in megajoules (MJ) and S is the rating of the machine in MVA, then
$$H = \frac{W}{S} = \frac{\omega M}{2S} = \frac{2\pi f M}{2S} = \frac{\pi f M}{S}$$
$$M = \frac{HS}{\pi f} \text{ MJs/ elec. radian}$$
or
$$M = \frac{HS}{180 f} \text{ MJs/ elec. degree}$$

Que 4.8. **What do you mean by "equal-area criterion" ? What are the basic roles of equal-area criterion in power system analysis ? Also mention its limitations.**

OR

Write a short note on equal area criterion.

AKTU 2015-16, Marks 05

OR

Explain equal-area criterion for the stability of an alternator supplying infinite busbar and inductive interconnector.

AKTU 2017-18, Marks 10

OR

Explain equal area criterion for stability by taking a suitable example of power system.

AKTU 2018-19, Marks 07

Answer

A. Equal-area criterion :

1. Consider a loss-free synchronous generator supplying an infinite bus through a purely reactive transmission line of reactance X_l as shown in Fig. 4.8.1(a).

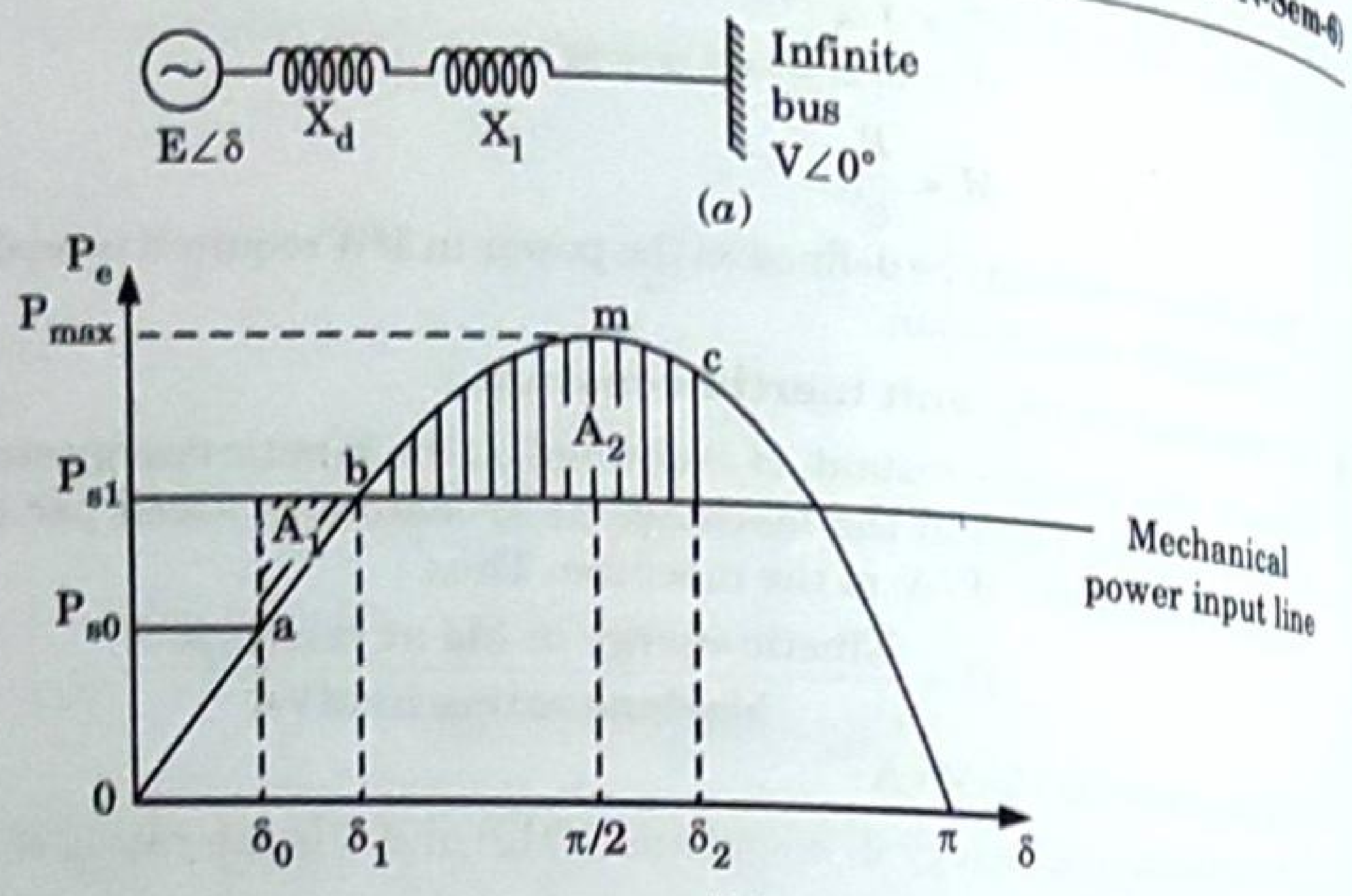


Fig. 4.8.1.

2. The electrical power transferred is given by equation

$$P_e = P_{e\,max} \sin \delta$$

3. The power angle curve is shown in Fig. 4.8.1(*b*).
4. Suppose that initially the mechanical input is P_{s0} at load angle δ_0. It is represented by point *a* on the power angle curve.
5. Let the mechanical input power suddenly increase to P_{s1}.
6. With the sudden increase of shaft power there is momentarily more shaft input than electrical output.
7. The increase in power $(P_{s1} - P_{s0})$ accelerates the rotor so that it is advanced with respect to the initial position with the result that load angle is increased. Let this new load angle be δ_1 corresponding to P_{s1}.
8. Since the rotor is in acceleration and running slightly above synchronous speed the load angle goes on increasing overshooting point *b*.
9. When the load angle is more than δ_1 the rotor retards since the power transferred to the busbars is greater than input power P_{s1}.
10. The rotor decelerates until it reaches same maximum point *c*, where it is again running at synchronous speed. The rotor swing starts in the reverse direction.
11. The load angle goes on decreasing until it is equal to δ_0 where again the rotor is running at synchronous speed.
12. Consider the swing equation

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} \qquad ...(4.8.1)$$

13. Multiplying both side by $2\frac{d\delta}{dt}$

$$2\frac{d\delta}{dt}\left(\frac{d^2\delta}{dt^2}\right) = \frac{2P_a}{M}\frac{d\delta}{dt}$$

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \frac{2P_a}{M}\frac{d\delta}{dt} \qquad ...(4.8.2)$$

14. The time rate of change of load angle $\frac{d\delta}{dt}$ is the speed of the machine with respect to the synchronously revolving reference frame.
15. For the stability, this speed must become zero at some time after disturbance.

That is, $\frac{d\delta}{dt} = 0$

16. It implies eq. (4.8.2) is integrated between the limits of swinging of δ.

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a\, d\delta$$

17. For stability $\frac{d\delta}{dt} = 0$

$$\int_{\delta_0}^{\delta_2} P_a\, d\delta = 0 \qquad ...(4.8.3)$$

$$\int_{\delta_0}^{\delta_1} P_a\, d\delta + \int_{\delta_1}^{\delta_2} P_a\, d\delta = 0$$

$$\int_{\delta_0}^{\delta_1} P_a\, d\delta = -\int_{\delta_1}^{\delta_2} P_a\, d\delta \qquad ...(4.8.4)$$

$$A_1 = -A_2 \qquad ...(4.8.5)$$

where, $A_1 = \int_{\delta_0}^{\delta_1} P_a\, d\delta$ = Positive or accelerating area

$A_2 = \int_{\delta_1}^{\delta_2} P_a\, d\delta$ = Negative or decelerating area

B. Basic role of equal area criteria in power system :

1. It is an easy means of finding the maximum angle of swing.
2. An estimate of whether synchronism will be maintained.
3. The maximum amount of disturbance that can be allowed without losing synchronism.

C. Limitations of equal-area criterion :

1. Equal-area criterion is only applicable to a single machine connected to an infinite bus system or a two-machine system.

2. In case of multi-machine problems, it would be necessary to have a detailed solution of swing curve for determining the system stability under transient conditions.

Que 4.9. **Find the power when there is sudden increase in mechanical power input ?**

Answer

1. The equal-area criterion is used to determine the maximum additional power P_s which can be applied for stability to be maintained.
2. With a sudden change in power input, the stability is maintained only if area A_2 is at least equal to A_1 can be located above P_s.
3. If area A_2 is less than area A_1, the accelerating momentum can never be overcome.
4. For the system to remain stable it is possible to find angle δ_2 such that $A_2 = A_1$.
5. As P_{s1} is increased a limiting condition is finally reached when A_1 equals the area above the P_{s1} line as shown in Fig. 4.9.1.
6. The limit of stability occurs when δ_{max} is at the intersection of line P_s and the power-angle curve for $90° < \delta < 180°$, as shown in Fig. 4.9.1.

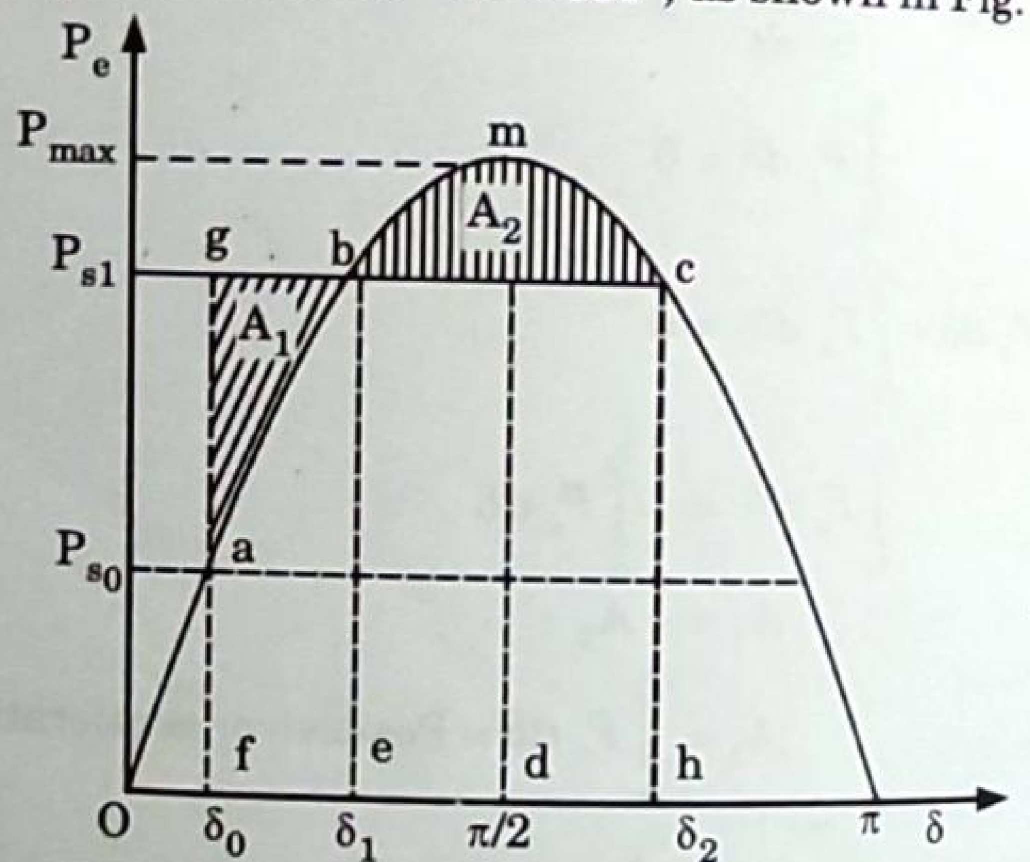


Fig. 4.9.1. Limiting case of transient stability with mechanical input suddenly increased.

7. Under this condition δ_2 acquires the maximum value δ_{max} such that

$$\delta_2 = \delta_{max} = 180° - \delta_1$$

8. Any further increase in P_{s1} means that the area available for A_2 is less than the area A_1, so that excess kinetic energy causes δ to increase beyond point c and the retarding power changes over to accelerating power with the system consequently become unstable.

9. It may also be noted from Fig. 4.9.1, that the system will remain stable even though the rotor may oscillate beyond $\delta = 90°$ as long as equal-area criterion is met.
10. The condition $\delta = 90°$ is meant for use in steady state stability only, and does not apply to the transient stability.
11. Applying the equal-area criterion to Fig. 4.9.1, we have

$$\text{Area } A_1 = \text{Area } A_2$$
$$\text{Area } agb = \text{Area } bmc$$
$$\text{Area } A_1 = \text{Area } agb$$
$$= \text{Area of the rectangle } gbef - \text{Area } abef \text{ under the sine curve}$$
$$= P_{s1}(\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_m \sin\delta\, d\delta$$
$$= P_{s1}(\delta_1 - \delta_0) + P_m(\cos\delta_1 - \cos\delta_0)$$
$$\text{Area } A_2 = \text{Area } bmc$$
$$= \text{Area } bmche \text{ under the sine curve} - \text{Area of rectangle } bche$$
$$= \int_{\delta_1}^{\delta_2} P_m \sin\delta\, d\delta - P_{s1}(\delta_2 - \delta_1)$$
$$= P_m(\cos\delta_1 - \cos\delta_2) - P_{s1}(\delta_2 - \delta_1)$$

12. By equal-area criterion

$$\text{Area } A_1 = \text{Area } A_2$$
$$P_{s1}(\delta_1 - \delta_0) + P_m(\cos\delta_1 - \cos\delta_0) = P_m(\cos\delta_1 - \cos\delta_2) - P_{s1}(\delta_2 - \delta_1)$$

or $$P_{s1}(\delta_2 - \delta_0) = P_m(\cos\delta_0 - \cos\delta_2) \quad ...(4.9.1)$$

13. Also, $$\delta_2 = \delta_{max}$$
and $$P_{s1} = P_m \sin\delta_{max}$$
at point c of the sine curve.
14. Substitution of value of δ_2 and P_{s1} in the eq. (4.9.1) gives

$$(\delta_{max} - \delta_0)\sin\delta_{max} + \cos\delta_{max} = \cos\delta_0 \quad ...(4.9.2)$$

eq. (4.9.2) can be solved by trial and error method for δ_{max}.
15. Once δ_{max} is obtained, the maximum permissible power or the transient stability limit is found from

$$P_{s1} = P_m \sin\delta_1$$
where $$\delta_1 = \pi - \delta_{max}$$

Que 4.10. **What is the effect when one of the parallel line is suddenly switched OFF ?**

Answer

1. Let us study the transient stability of the system when one of the lines is suddenly switched OFF with the system operating at a steady load.
2. Before switching OFF, power angle curve is given by

$$P_{eI} = \frac{|E'|\,|V|}{X_d + X_1 \,||\, X_2} \sin\delta = P_{\max I} \sin\delta \quad ...(4.10.1)$$

3. Immediately on switching OFF line 2, power angle curve is given by

$$P_{eII} = \frac{|E'|\,|V|}{X_d' + X_1} \sin\delta = P_{\max II} \sin\delta \quad ...(4.10.2)$$

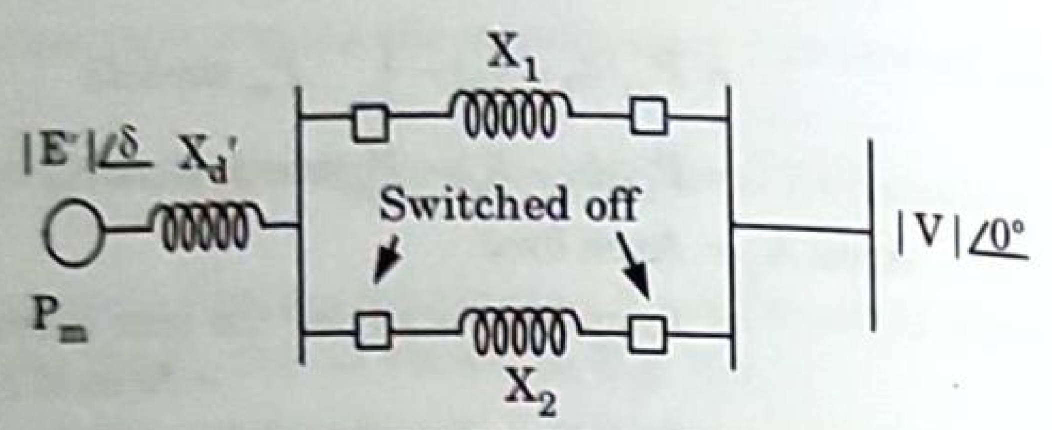


Fig. 4.10.1. Single machine tied to infinite bus through two parallel lines.

4. Both curves (eq. (4.10.1) and (4.10.2)) are plotted in Fig. 4.10.2, where in $P_{\max II} < P_{\max I}$ as $(X_d' + X_1) > (X_d' + X_1 \,||\, X_2)$.
5. The system is operating initially with a steady power transfer $P_e = P_m$ at a torque angle δ_0 on curve I.
6. Immediately on switching OFF line 2, the electrical operating point shifts to curve II (point *b*).
7. Accelerating energy corresponding to area A_1 is put into rotor followed by decelerating energy for $\delta > \delta_1$.
8. Assuming that an area A_2 corresponding to decelerating energy (energy out of rotor) can be found such that $A_1 = A_2$, the system will be stable and will finally operate at *c* corresponding to a new rotor angle $\delta_1 > \delta_0$.
9. This is so because a single line offers larger reactance and larger rotor angle is needed to transfer the same steady power.
10. If the steady load is increased (line P_m is shifted upwards in Fig. 4.10.2), a limit is finally reached beyond which decelerating area equal to A_1 cannot be found and therefore the system behaves as an unstable one.
11. For the limiting case of stability, δ_1 has maximum value given by

$$\delta_1 = \delta_{\max} = \pi - \delta_c$$

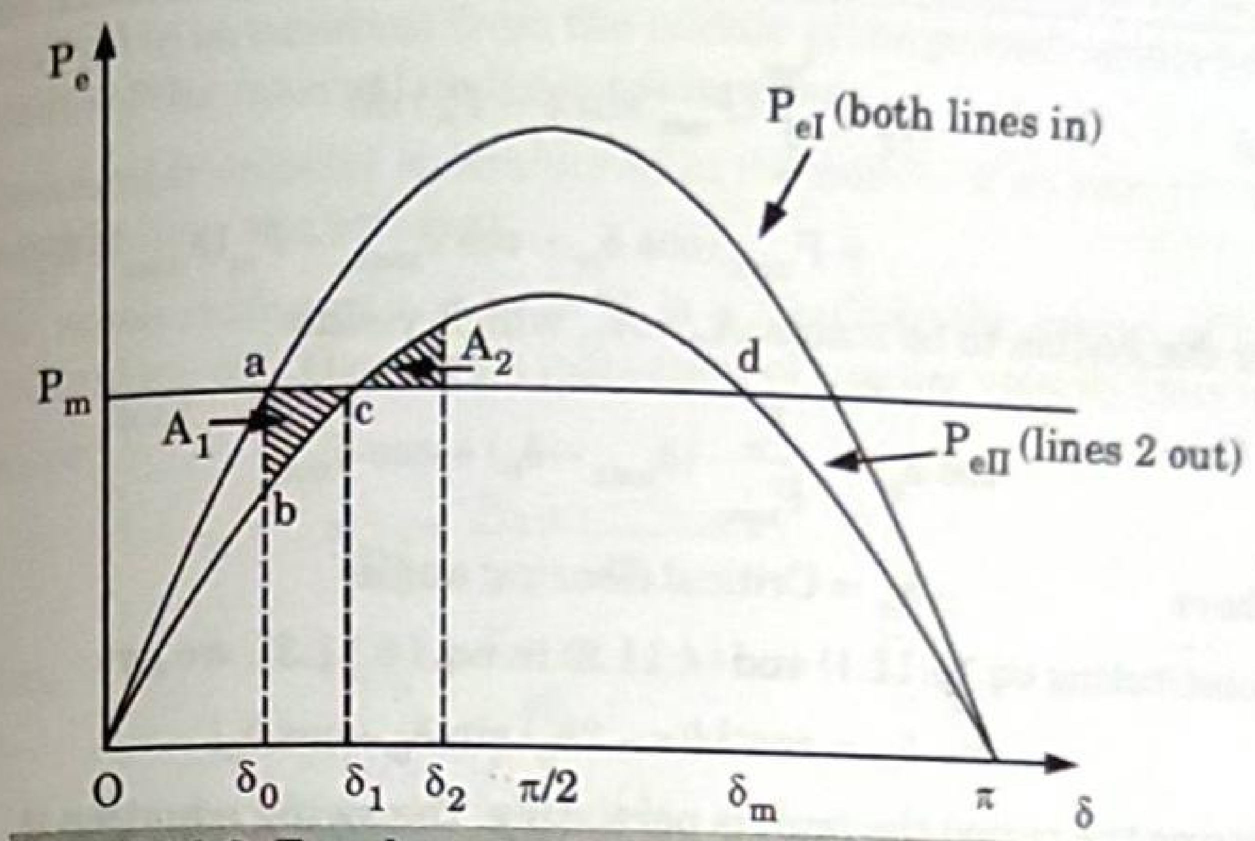


Fig. 4.10.2. Equal area criterion applied to the opening of one of the two lines in parallel.

Que 4.11. **Define and derive the expression for critical clearing time and critical clearing angle.**

Answer

1. The maximum allowable value of the clearing time and angle for the system to remain stable are known respectively as critical clearing time and angle.
2. From Fig. 4.11.1

$$\delta_{\max} = \pi - \delta_0 \quad ...(4.11.1)$$

and

$$P_m = P_{\max} \sin\delta_0 \quad ...(4.11.2)$$

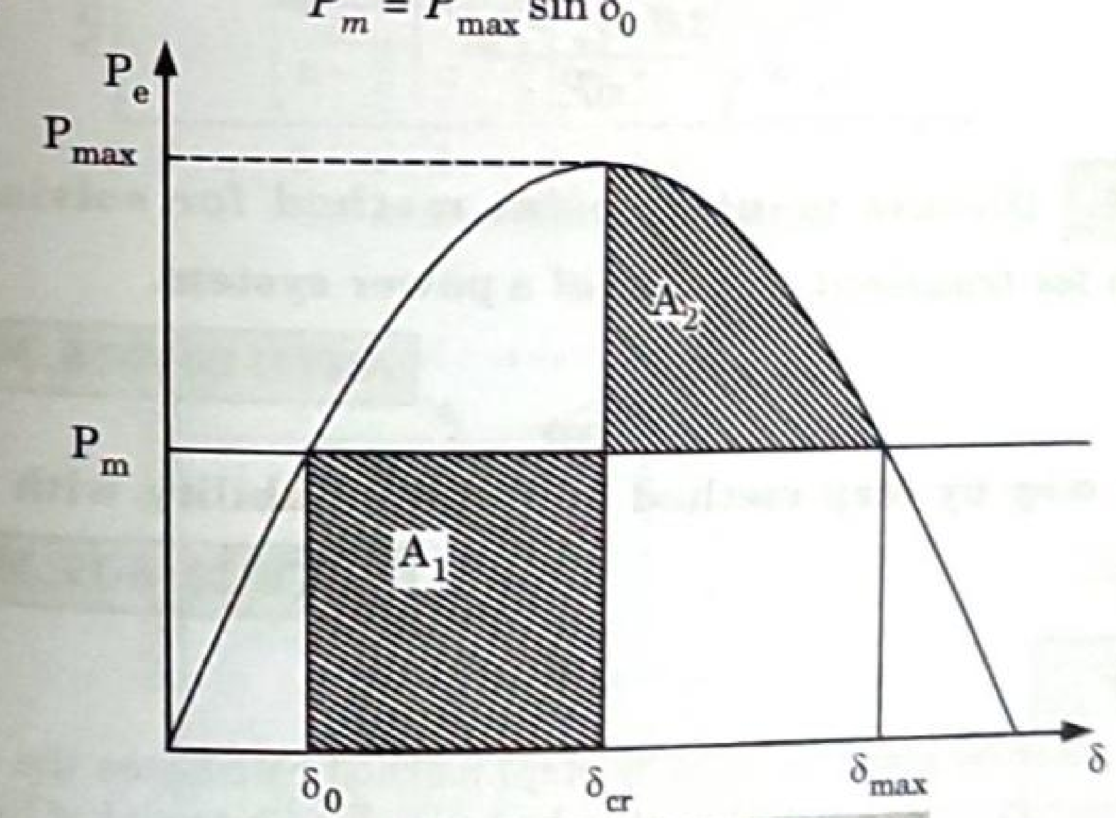


Fig. 4.11.1. Critical clearing angle.

3. Now

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - 0)\, d\delta = P_m (\delta_{cr} - \delta_0)$$

and $$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin \delta - P_m)\, d\delta$$

$$= P_{max} (\cos \delta_{cr} - \cos \delta_{max}) - P_m(\delta_{max} - \delta_{cr})$$

4. For the system to be stable, $A_2 = A_1$, which yields

$$\cos \delta_{cr} = \frac{P_m}{P_{max}} (\delta_{max} - \delta_0) + \cos \delta_{max} \quad ...(4.11.3)$$

where δ_{cr} = Critical clearing angle

5. Substituting eq. (4.11.1) and (4.11.2) in eq. (4.11.3), we get

$$\delta_{cr} = \cos^{-1} [(\pi - 2\delta_0) \sin \delta_0 - \cos \delta_0] \quad ...(4.11.4)$$

6. During the period the fault is persisting, the swing equation is

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_m; \quad P_e = 0 \quad ...(4.11.5)$$

7. Integrating eq. (4.11.5) twice

$$\delta = \frac{\pi f}{2H} P_m t^2 + \delta_0$$

or $$\delta_{cr} = \frac{\pi f}{2H} P_m t^2{}_{cr} + \delta_0 \quad ...(4.11.6)$$

where, t_{cr} = Critical clearing time

δ_{cr} = Critical clearing angle

8. From eq. (4.11.6)

$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m}}$$

Que 4.12. **Discuss point-by-point method for solving swing equation for transient stability of a power system.**

AKTU 2015-16, Marks 10

OR

Explain step by step method of system stability with suitable diagrams.

AKTU 2018-19, Marks 07

Answer

1. The point-by-point (or step-by-step) method calculates the change in the angular position of the rotor during a short interval of time.
2. The following assumptions are made during the computational procedure :

i. The accelerating power P_a computed at the beginning of an interval is assumed to be constant from the middle of the preceding interval to the middle of the interval under consideration.

ii. The angular velocity ω, computed at the middle of an interval, remains constant over the interval.

3. The accelerating at $t = (n - 1)$ is α_{n-1}. Over the region of constant acceleration α_{n-1} there is an increment of angular velocity from $\omega_{n-3/2}$ to $\omega_{n-1/2}$,

$$\therefore \quad \alpha_{n-1} = \frac{\omega_{n-1/2} - \omega_{n-3/2}}{\Delta t} \quad ...(4.12.1)$$

Also, $$\alpha_{n-1} = \frac{P_{a(n-1)}}{M} \quad ...(4.12.2)$$

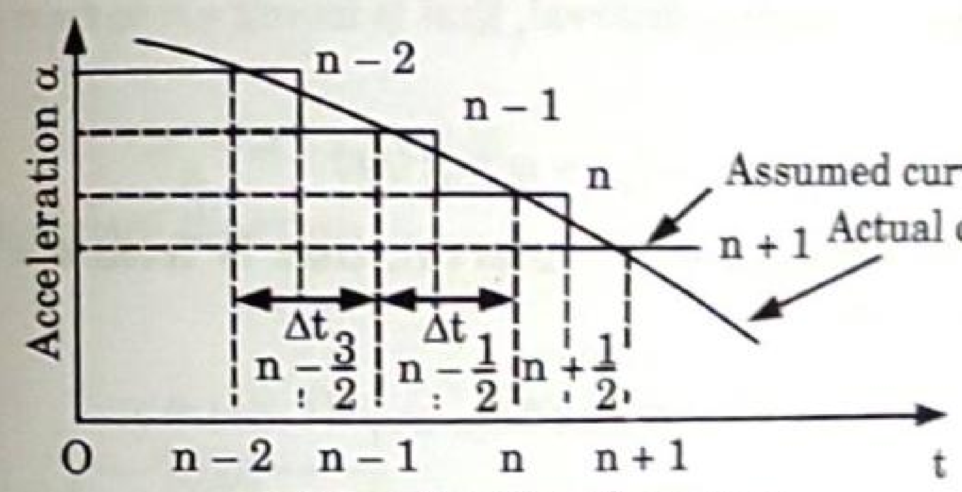
$$\therefore \quad \omega_{n-1/2} - \omega_{n-3/2} = \frac{P_{a(n-1)}}{M} (\Delta t) \quad ...(4.12.3)$$

4. Again, the angular velocity $\omega_{n-1/2}$ remains constant from $t = n - 1$ to $t = n$. The displacement angle increases from δ_{n-1} to δ_n during the time interval Δt.

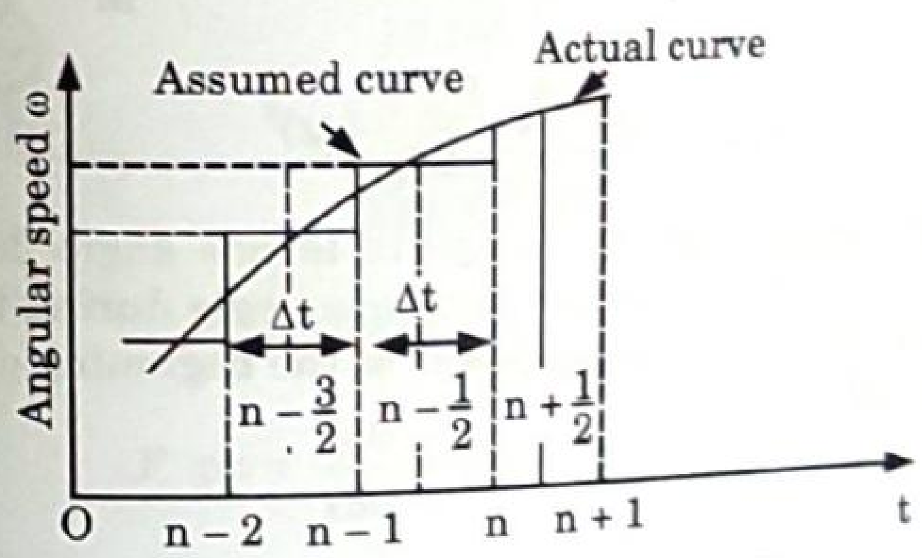
5. From the velocity / time curve Fig. 4.12.1(b)

$$\omega_{n-1/2} = \frac{\delta_n - \delta_{n-1}}{\Delta t}$$

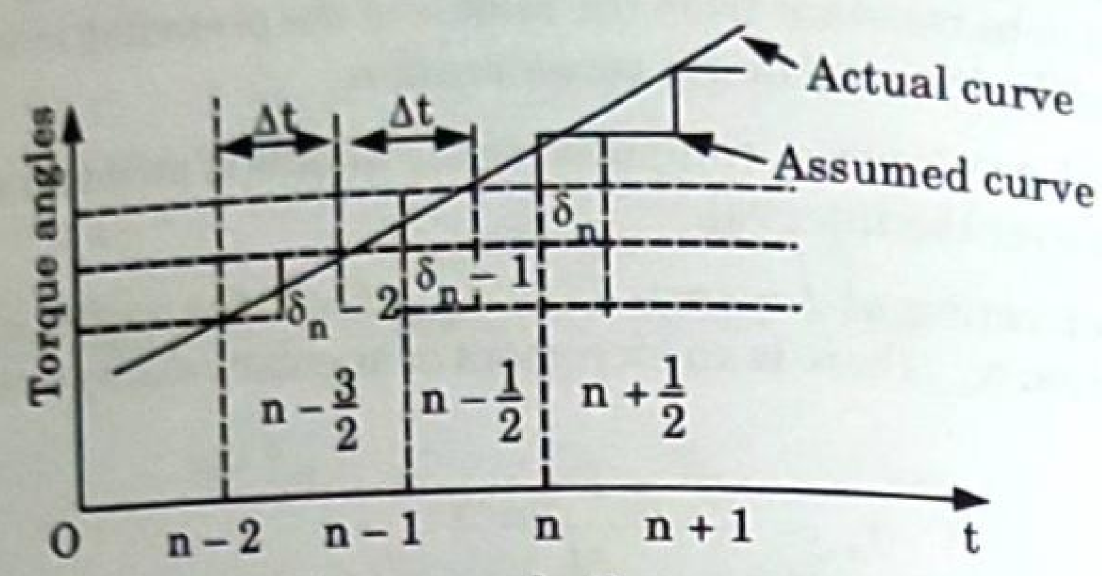
$$\therefore \quad \delta_n - \delta_{n-1} = \omega_{n-1/2}(\Delta t) \quad ...(4.12.4)$$



(a) Acceleration time curve



(b) Angular speed-time curve



(c) Torque angle-time curve

Fig. 4.12.1.

6. Similarly,

$$\omega_{n-3/2} = \frac{1}{\Delta t}(\delta_{n-1} - \delta_{n-2})$$

$$\therefore \quad \delta_{n-1} - \delta_{n-2} = \omega_{n-3/2}(\Delta t) \qquad ...(4.12.5)$$

The change in δ over any interval = Value of ω for that interval × Time of interval

7. But the change in δ over n_{th} interval, that is from $t = n$ to $t = n - 1$ is given by

$$(\Delta\delta)_n = \delta_n - \delta_{n-1} = \omega_{n-1/2}(\Delta t) \qquad ...(4.12.6)$$

8. The change in δ over $(n - 1)^{\text{th}}$ interval, that is from $t = (n - 1)$ to $t = (n-2)$ is given by

$$(\Delta\delta)_{n-1} = \delta_{n-1} - \delta_{n-2} = \omega_{n-3/2}(\Delta t) \qquad ...(4.12.7)$$

$$\therefore \quad (\Delta\delta)_n - (\Delta\delta)_{n-1} = \omega_{n-1/2}(\Delta t) - \omega_{n-3/2}(\Delta t) = \frac{P_{a(n-1)}}{M}(\Delta t)^2$$

$$\therefore \quad (\Delta\delta)_n = (\Delta\delta)_{n-1} + \frac{P_{a(n-1)}}{M}(\Delta t)^2 \qquad ...(4.12.8)$$

Eq. (4.12.8) shows that the change in torque angle during a given interval is equal to the change in torque angle during the preceding interval plus the accelerating power at the beginning of the interval times $(\Delta t)^2 / M$.

Que 4.13. **A 150 MVA generator-transformer unit having an overall reactance of 0.3 pu is delivering 150 MW to infinite bus bar over a double circuit 220 kV line having reactance per phase per circuit of 100 ohms. A 3 phase fault occurs midway along one of the transmission lines. Calculate the maximum angle of swing that the generator may achieve the fault is cleared without loss of stability.**

Answer

1. $$Z_{\text{Base}} = \frac{220^2 \times 10^6}{150 \times 10^6} = \frac{968}{3} = 322.67\ \Omega$$

2. $$X_{\text{line}}\ \text{pu} = \frac{100}{322.67} = 0.3099 = j0.31\ \text{pu}$$

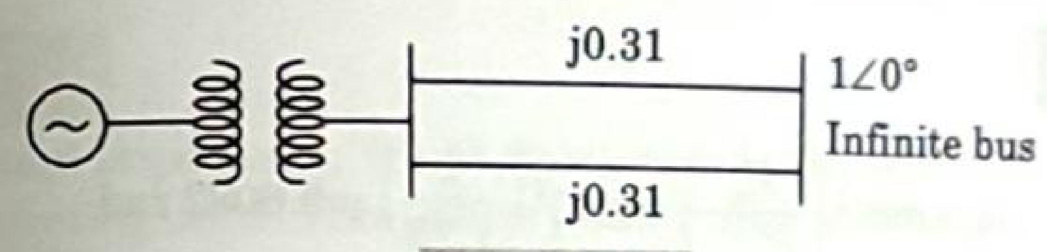


Fig. 4.13.1.

3. $$X'_{eq} = X_g + X_T + \frac{0.31}{2} = 0.3$$

$$X_g + X_T = j\,0.145$$

4. Per unit circuit diagram

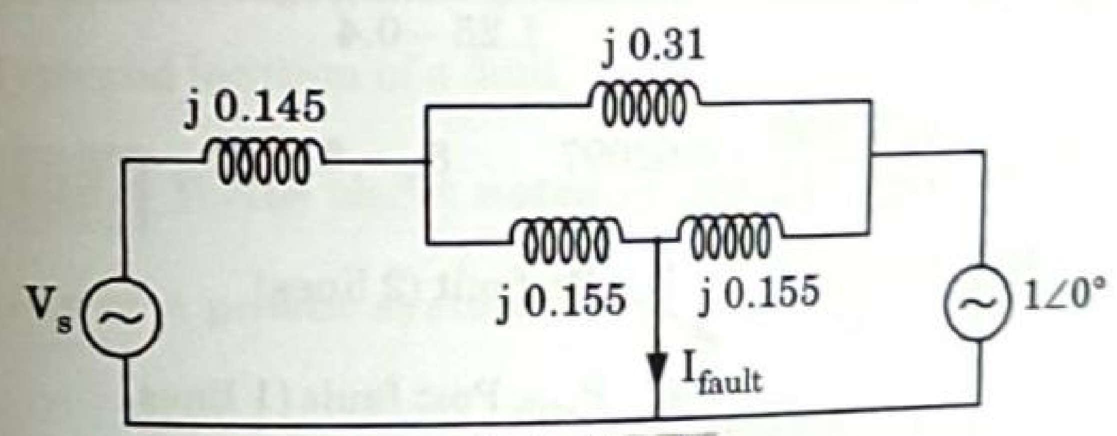


Fig. 4.13.2.

$$X_{eq} = j\,0.745$$

5. $$P_e\ \text{pu} = \frac{150}{150} = 1\ \text{pu}$$

6.
$$P_e = P_{em} \sin \delta = \frac{V_1 V_2}{X} \sin \delta$$

$$1 = \frac{1 \times 1}{0.745} \sin \delta$$

$$\sin \delta = 0.745 \quad \therefore \quad \delta = 48.15°$$

Que 4.14. **A generator operating at 50 Hz delivers 1 pu power to an infinite bus when a fault occurs and reduces the maximum power transferable to 0.4 pu. The maximum power transferable before the occurrence of fault was 1.75 pu. The maximum power transferable after clearance of fault is 1.25 pu. Compute critical clearing angles.** **AKTU 2015-16, Marks 10**

Answer

1. $$\delta_0 = \sin^{-1}\left(\frac{P_m}{P_{max\,I}}\right) = \sin^{-1}\left(\frac{1}{1.75}\right) = 0.6082 \text{ rad}$$

2. $$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{max\,III}}\right) = \pi - \sin^{-1}\left(\frac{1}{1.25}\right) = 2.2142 \text{ rad}$$

3. $$\cos \delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) - P_{max\,II} \cos \delta_0 + P_{max\,III} \cos \delta_{max}}{P_{max\,III} - P_{max\,II}}$$

$$\cos \delta_{cr} = \frac{1(2.2142 - 0.6082) - 0.4 \cos 0.6082 + 1.25 \cos 2.2142}{1.25 - 0.4}$$

$$\cos \delta_{cr} = \frac{0.527826}{1.25 - 0.4} = 0.62097 \quad \therefore \delta_{cr} = 51.61°$$

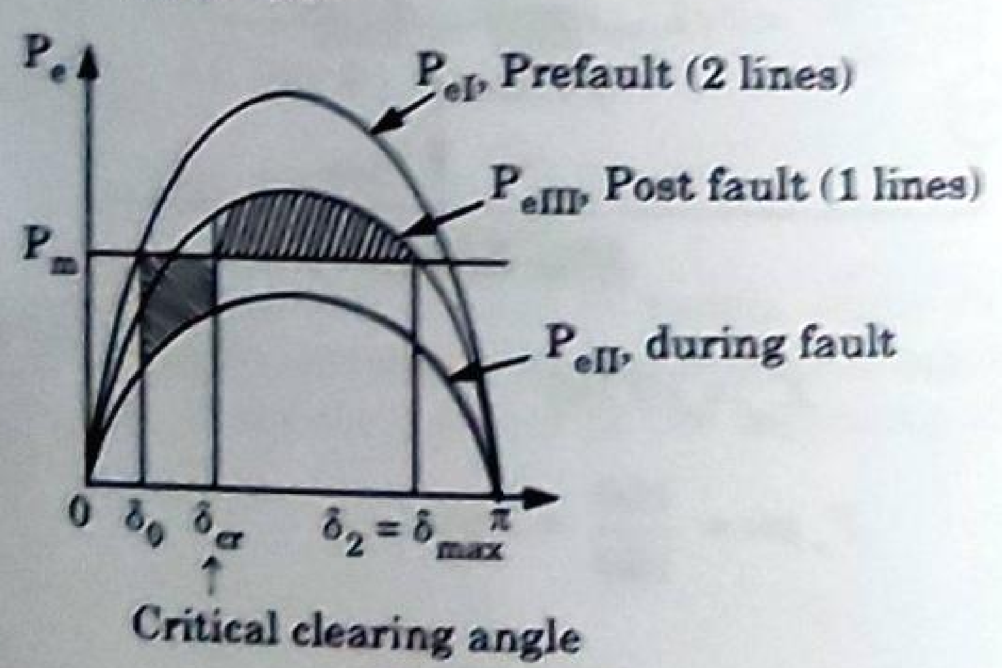


Fig. 4.14.1.

PART-4

Factors Affecting Steady State and Transient Stability and Methods of Improvement.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.15. **Discuss the factors which affect transient state stability of a power system.** **AKTU 2016-17, Marks 10**

Answer

1. System impedance, which must include the transient reactance of all generating units. This affects phase angles and the flow of synchronising power.
2. Duration of the fault, chosen as the criterion for stability. If fault persists for longer duration it makes system unstable.
3. Generator loadings prior to the fault which will determine the internal voltages and the change in output.
4. System loading which will determine the phase angles among the various internal voltages of the generators.
5. Type and location of a fault.

Que 4.16. **Write short notes on methods of improving transient stability of a power system.** **AKTU 2015-16, Marks 05**

OR

What is transient stability ? Describe different methods of improving transient stability of a power system. **AKTU 2018-19, Marks 07**

Answer

A. **Transient stability :** Refer Q. 4.3, Page 4–5B, Unit-4.

B. Methods of improving stability :

1. **Use of double-circuit lines :** The impedance of a double-circuit line is less than that of a single circuit line. A double-circuit line doubles the transmission capability.
2. **Use of bundled conductors :** Bundling of conductors reduces to a considerable extent the line reactance and so increases the power limit of the line.
3. **Series compensation of the lines :** The inductive reactance of a line can be reduced by connecting static capacitors in series with the line.
4. **High speed excitation systems :** High speed excitation helps to maintain synchronism during a fault by quickly increasing the excitation voltage.
5. **Fast switching :** Rapid isolation of faults is the principal way of improving transient stability. The fault should be cleared as fast as possible.
6. **Braking resistors :** In this method an artificial electric load in the form of shunt resistors is temporarily connected at or near the generator bus. Such resistors partially compensate the reduction of load on a generator following a fault. The acceleration of the generator rotor is therefore reduced.
7. **Turbine fast valving or bypass valving :** In the event of a fault, the generator output is reduced resulting in a high accelerating power. If the mechanical input power to the turbine could be momentarily reduced, the acceleration could be reduced.
8. **Single-pole switching :**

i. Majority of the line faults are single line to ground (LG) faults.

ii. In single-pole switching the three phases of the circuit breaker are closed or opened independently of each other.

iii. In the event of an LG fault, the circuit breaker pole corresponding to the faulty line is opened and the remaining two healthy phases continue to transfer power.

iv. Since most of the faults are transitory, this phase can be reclosed after it has been open for a predetermined time. The system should not be operator for long periods with one phase open.

v. Therefore, provision should be made to trip the whole line if one phase remains open for a predetermined time.

9. **HVDC Links :** High voltage direct current (HVDC) links are helpful in maintaining stability due to the DC tie link.

i. A DC tie link provides a loose coupling between two AC systems to be interconnected.

ii. A DC link may interconnect two AC systems at different frequencies.

iii. There is no transfer of fault energy from one AC system to another if they are interconnected by a DC tie link.

10. **Load shedding :** If there is insufficient generation to maintain system frequency, some of the generators are disconnected during or immediately after a fault. Thus the stability of the remaining generators is improved.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Derive the expression for active and reactive power when power flow through a transmission line.

Ans. Refer Q. 4.1.

Q. 2. What is stability ? And explain its types. What is stability limit ?

Ans. Refer Q. 4.3.

Q. 3. Derive the swing equation for a machine connected to an infinite bus in a power system.

Ans. Refer Q. 4.5.

Q. 4. Explain the inertia constant and swing equations. Explain the terms swing curves in power system stability.

Ans. Refer Q. 4.6.

Q. 5. Explain equal area criterion for stability by taking a suitable example of power system.

5
UNIT

Introduction to Power System Protection

CONTENTS

PART-1

Relays : Operating Principle of a General Relay.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. Discuss the operating principle of a relay.

Answer

Relay :

1. Relay is a device which detects the fault and is responsible for energizing the trip circuit of a circuit breaker. This isolates the faulty part from rest of the system. The relay operates when the resultant torque is positive.
2. Circuit for relay is shown in Fig. 5.1.1. Relay circuit is a 3-phase circuit and its contact circuit is complicated.

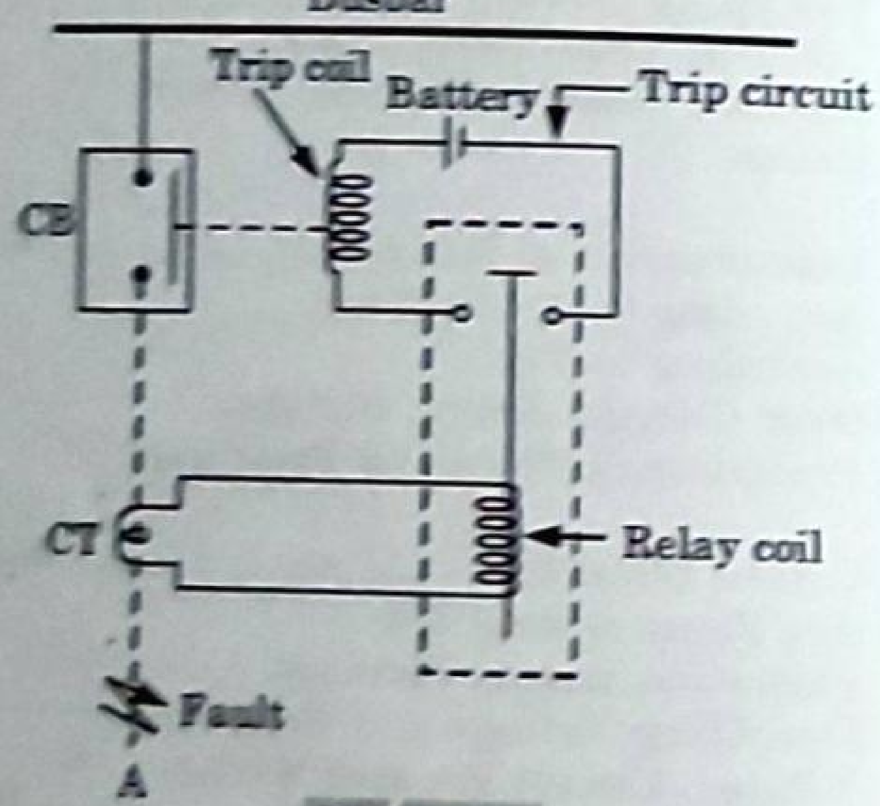


Fig. 5.1.1.

3. Let part A is to be protected. The current transformer is connected with its primary around the line to be protected. The secondary of CT is connected in series with the relay coil.
4. Relay contacts are part of the trip circuit of circuit breaker. In addition to trip contacts, trip circuit consists of trip coil and a battery.
5. If the fault is as shown in Fig. 5.1.1, then current through the line connected to A increases to a very high value. Accordingly secondary current of the CT increases which is nothing but the current through relay coil.

6. In the influence of such high current, relay contacts mechanically get closed.
7. So trip circuit of circuit breaker gets closed and current starts flowing from battery through trip coil, thus trip coil gets energized making the circuit breaker open. This isolates the faulty part from rest of the healthy part.

Que 5.2. Explain what do you understand by primary and backup protection. What are the various methods of providing backup protection ?

Answer

Primary and backup protection :

1. A power system is divided into various zones for its protection. There is a particular scheme for each zone.
2. If the fault occurs in a particular zone, it is the duty of primary relays of that zone to isolate the faulty portion.
3. If because of some reason the primary protection fails to clear the fault, then the backup protection has to clear it.
4. The backup relays are made independent of those factors because of which the primary relay failed. A backup relay operates after some time delay so as to give the primary relay sufficient time to operate.
5. When a backup relay operates, a large part of the power system is disconnected from the power source, which is unavoidable. A backup relay is usually placed at different stations.

Methods of providing backup protection :

1. **Relay backup :** This is a local backup in which an additional relay is provided for backup protection. If the primary relay fails, it trips the same circuit breaker and it does so without any additional delay. This backup is costly, so preferred only when remote backup is not possible.
2. **Breaker backup :**
   i. This is also a local backup. It is necessary for busbar system where a number of circuit breakers are connected to it. When a protective relay operates in response to a fault but circuit breaker fails to operate, the fault is treated as busbar faults.
   ii. It is necessary that all the circuit breakers on the busbar should trip. If the proper breaker does not trip within a specified time, the main relay closes the contact of the backup which trips all the circuit breakers.
3. **Remote backup :** When backup relays are located at neighbouring stations, they backup the entire primary scheme which includes relay, circuit breaker, PT and CT. It is the cheapest form of backup protection and used for transmission lines.

PART-2

Basic Terminology : Relay, Energizing Quantity, Setting, Pickup, Drop Out, Flag, Fault Clearing Time, Relay Time, Breaker Time, Overreach, Under Reach.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.3. Explain in detail the basic terminologies.

Answer

1. **Relay :** Relay is a device which detects the fault and is responsible for energizing the trip circuit of a circuit breaker. This isolates the faulty part from rest of the system.
2. **Energizing quantity :** Energizing quantity is defined as the current, voltage or frequency which is used to operate the relay under abnormal condition.
3. **Setting :** The value of the actuating quantity at which the relay is set to operate. This process is called setting.
4. **Pickup :** A relay is said to be picked up when it moves from the OFF position to ON position. Thus when relay operates it is said that relay has picked up.
5. **Drop out or reset value :** A relay is said to dropout or reset when it comes back to original position *i.e.*, when relay contacts get opened from its closed position. The value of an actuating quantity current or voltage below which the relay resets is called reset value of that relay.
6. **Flag or target :** Flag is a device which gives visual indication whether a relay has operated or not.
7. **Fault clearing time :** The total time required between the instant of fault and the instant of final arc interruption in the circuit breaker is fault clearing time. It is sum of the relay time and circuit breaker time.
8. **Relay time :** It is the time between the instant of fault occurrence and the instant of closure of relay contacts.
9. **Breaker time :** It is the time between the instant at circuit breaker operates and opens the contacts to the instant of extinguishing the arc completely.
10. **Overreach :** Sometimes a relay may operate even when a fault point is beyond its present reach. This phenomena is called overreach.

11. **Underreach :** Sometimes a relay may fail to operate even when the fault point is within its reach, but it is at the far end of the protected line. This phenomenon is called underreach.

Que 5.4. Differentiate the following :

i. **Primary and Backup Protection**
ii. **Pickup and Reset value**
iii. **Operating time and Reset time**
iv. **Normal and Abnormal conditions.**

Answer

i. **Primary and Backup protection :**

S. No.	Primary protection	Backup protection
1.	It is first stage of protection.	It is second stage of protection. It operates only when primary protection fails.

ii. **Pickup and Reset value :**

S. No.	Pickup value	Reset value
1.	It is the minimum value of an actuating quantity at which relay starts operating.	The value of an actuating quantity current or voltage below which the relay resets is called reset value of that relay.

iii. **Operating time and Reset time :**

S. No.	Operating time	Reset time
1.	This is the time between the instant at which the actuating quantity crosses its pick up value and final operation of protective component.	This is the time which elapses from the moment the actuating quantity falls below its reset value to the instant when the relay comes back to its normal value.

iv. **Normal and abnormal conditions :**

S. No.	Normal conditions	Abnormal conditions
1.	Normal condition is the condition in which any system operates under desired level of voltage, frequency, current, power factor etc.	Abnormal condition arises due to system fault.

PART-3

Classification of Relays According to Applications, According to Time, Over Current Relay, Distance Protection, Differential Protection.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.5. **Give the classification of relays according to applications.**

Answer

According to applications the relays may be classified as :

1. **Overvoltage, overcurrent and overpower relay :** The relay operates when the voltage, current or power rises above a specified value.
2. **Under-voltage, under-current and under-power relay :** The relay operates when the voltage, current or power falls below a specified value.
3. **Directional or reverse current relay :** The relay operates when the applied current assumes a specified phase displacement with respect to the applied voltage and the relay is compensated for fall in voltage.
4. **Directional or reverse power relay :** The relay operates when the applied current and voltage assume specified phase displacement and no compensation is allowed for fall in voltage.
5. **Differential relay :** The relay operates when some specified phase or magnitude difference between two or more electrical quantities occurs.
6. **Distance relay :** In this relay the operation depends upon the ratio of the voltage to the current.

Que 5.6. **What do you understand by overcurrent relay ? And explain its type.**

Answer

A. **Overcurrent relay :** The overcurrent relay is defined as the relay, which operates only when the value of the current is greater than the relay setting time. It protects the equipment of the power system from the fault current.

B. **Types of overcurrent relay :**

The overcurrent relays are classified depending upon the time of operation. These are classified as :

1. **Instantaneous overcurrent relays :**
   i. These relays operate very fast and there is no time delay. The operating time can be as low as 0.01 sec. This speed of operation can be achieved by hinged armature type electromagnetic relay.
   ii. These are effective only when the impedance between the relay and source Z_s is very small as compared to the impedance of protected section Z_L.

   Applications : For protection of outgoing feeders.
2. **Inverse definite time relays :**
   i. All overcurrent relay have inverse time characteristics means as the fault current level increases, the operating time of the relay decreases.
   ii. These characteristics are more near the pickup value of the actuating quantity and become less inverse as it is increased. The nature of these characteristics can be obtained by using the suitable core and by varying the point of saturation of this core.
   iii. If the saturation occurs at a very early stage, then the time of operation almost remains same over the active working range of the relay. This is called definite time characteristics.
3. **Inverse definite minimum time relay (IDMT) :**
   i. When the core saturates at later stage, then the operating time becomes inversely proportional to the fault current near the pickup value and then becomes constant.
   ii. The core saturation current is slightly higher than the pickup value of current. This is called inverse definite minimum time (IDMT) characteristics.

   Applications : Used in utility and industrial circuits.
4. **Very inverse relays :** As the core saturation occurs at a further later stage, then the inverse nature of characteristics continues for longer range and takes the shape. This is called very inverse time characteristics. After saturation, the curve tends to definite time.
5. **Extremely inverse relays :** In this, the saturation occurs at very later stage and the curve has an inverse nature for almost entire working range. The equation is $I^2t = K$ where I is the operating current and t is the operating time. This characteristic is called extremely inverse time characteristics.

   Applications : Suitable for protection of distribution feeders with peak currents during switching ON (refrigerators, pumps, water heaters etc.).

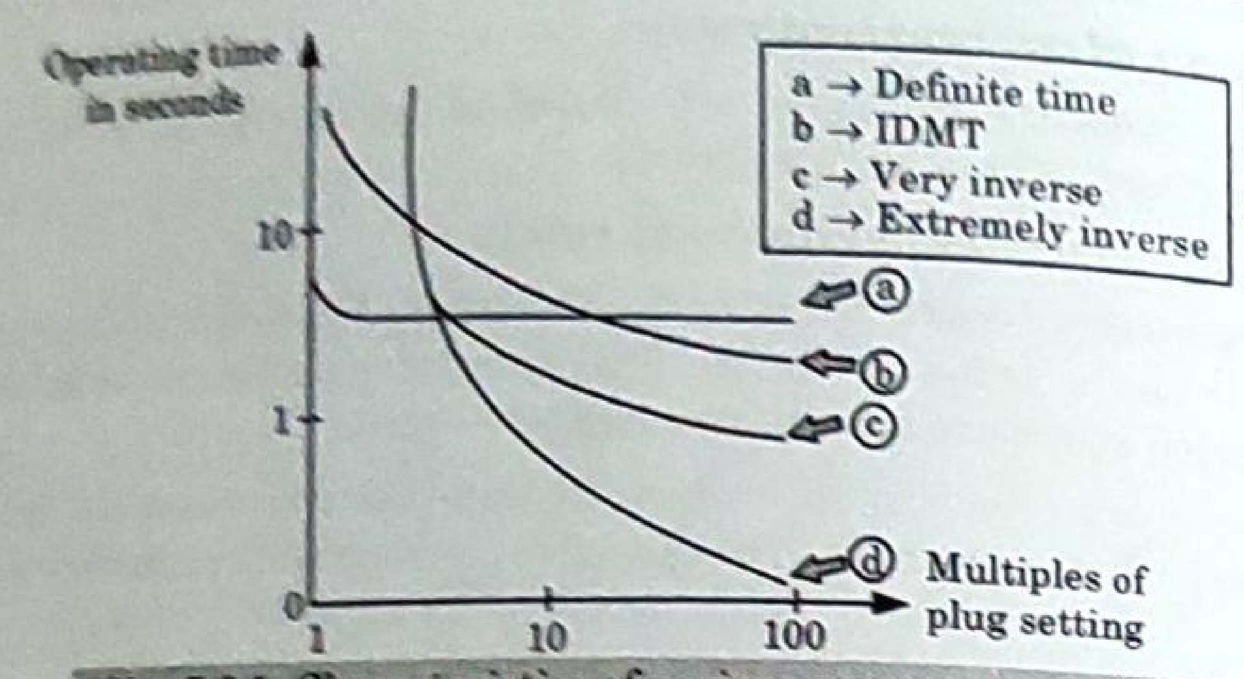


Fig. 5.6.1. Characteristics of various overcurrent relays.

Que 5.7. **With the help of block diagram discuss differential protection (Pilot wire protection).**

Answer

Differential protection (Pilot wire protection) :

1. In differential protection scheme, the current entering at one end of line and leaving for other end of line is compared. The pilot wires are used to connect the relays.
2. Under normal operation, the two currents at both ends are equal and pilot wires do not carry any current, keeping relays inoperative.
3. When fault occurs, both the currents are different, this causes circulating current to flow and relays trip which operate the circuit breakers to isolate the faulty section.
4. **The various schemes are :**

A. Merz-Price voltage balance system :

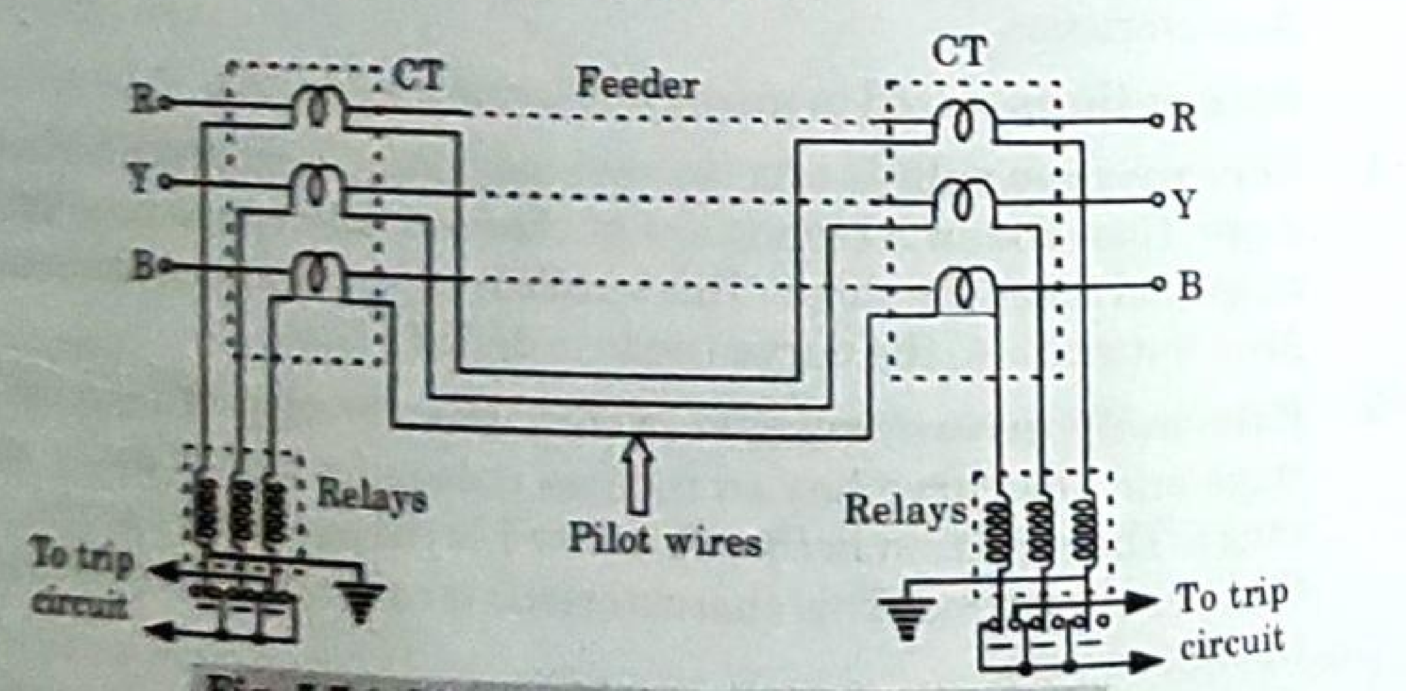


Fig. 5.7.1. Merz-Price Protection for feeder.

1. Under normal condition, both the currents are equal. Thus equal and opposite voltage are induced in the secondary CT at the two ends. Hence no current flows through relays.

2. Under fault condition, currents are different and secondary voltage of the two CTs also differs. Thus circulating current flows and trips the relay.

Advantages :

i. It can be used for parallel as well as ring main system.
ii. It provides instantaneous protection to the ground faults.

Limitations :

i. The CT must match accurately.
ii. The pilot wires must be healthy without discontinuity.
iii. Cost is high.
iv. Due to long pilot wires, capacitive effects may affect the operation of the relays.

B. Longitudinal differential protection system :

a. Translay Scheme :

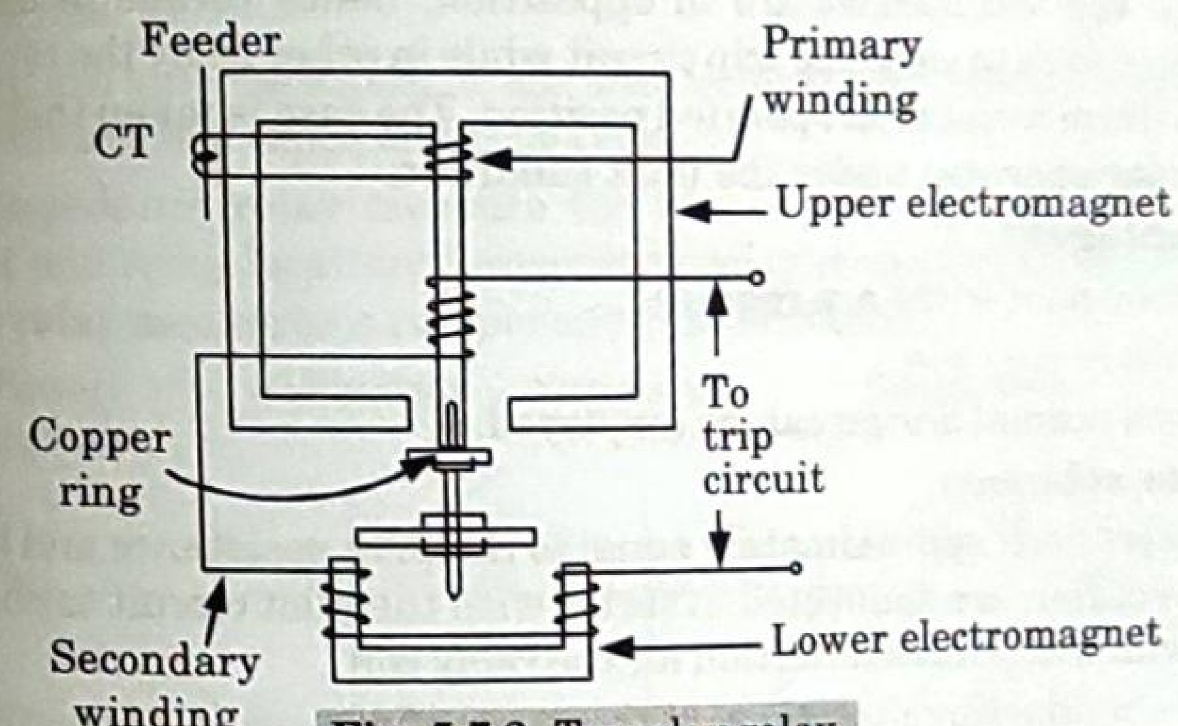


Fig. 5.7.2. Translay relay.

1. These relays are used in feeder protection and in this two such relays are employed at the two ends of feeder as shown in Fig. 5.7.3.

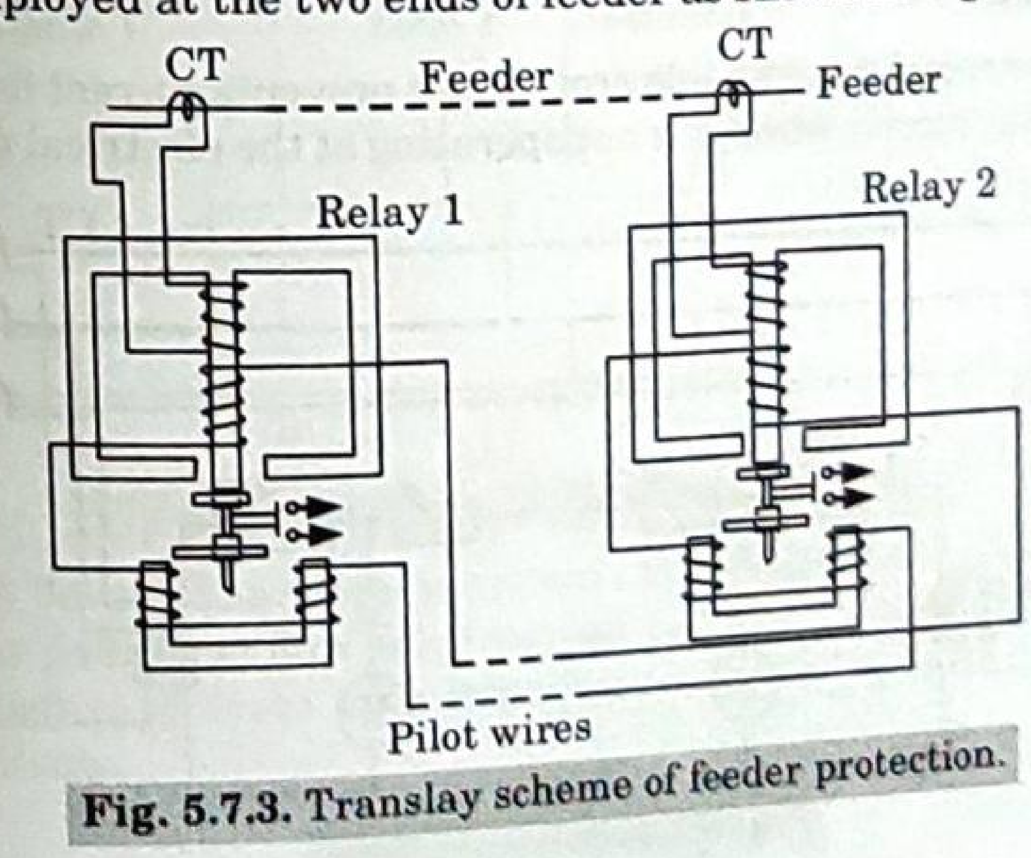


Fig. 5.7.3. Translay scheme of feeder protection.

2. The secondaries are connected to each other using pilot wires. The connection is such that the voltages induced in the two secondaries oppose each other.
3. The copper coils are used to compensate the effect of pilot wire capacitance currents and unbalance between two current transformers.
4. Under normal condition, the two end currents are same. The primaries of the two relays carry the same currents inducing the same voltage in the secondaries.
5. As these two voltages are in opposition, no current flows through the two secondary circuits and no torque is exerted on the discs of both the relays.
6. When fault occurs, the two currents are different. Hence unequal voltages are induced in the secondaries due to which circulating current flows causing torque to be exerted on the disc of each relay.
7. But as the secondaries are in opposition, hence torque in one relay operates so as to close the trip circuit while in other relay the torque just holds the movement in operated position. The case is taken that at least the relay operates under the fault condition.

Advantages :

i. Only two pilot wires are required.
ii. The cost is very low.
iii. CT with normal design can be employed.

b. Solkor scheme :

1. Resistors each approximately equal to the pilot resistance and bridged by a rectifier, are connected in series with the pilot circuit to obtain an artificial mid-point connection for the relay coil.
2. As the conducting directions of these rectifiers are in opposition, the equipotential point alternates between the two ends during through-fault conditions, each relay being at the mid-point on alternate half-cycles.
3. The series rectifier in the relay coil circuit prevents current from flowing in the relay circuit when it is not operating at the electrical mid-point.

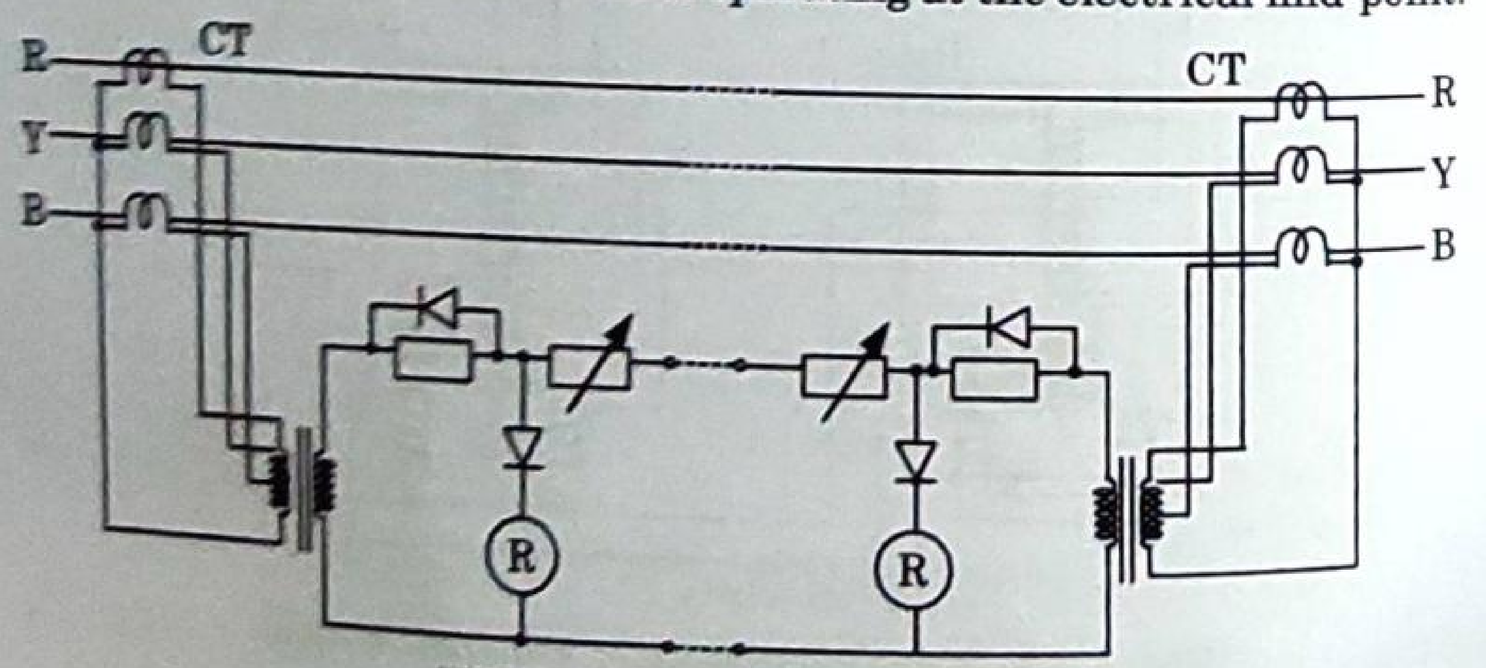


Fig. 5.7.4. Solkor scheme.

4. Under internal fault conditions current flow in the relay coils causing operation. Padding resistors are induced to enable the resistance of the complete pilot circuit between the relays to be adjusted to 1000 Ω.

Que 5.8. **Describe distance protection.**

Answer

Distance protection :

1. Distance protection is used for the protection of transmission or sub-transmission lines; usually 33 kV, 66 kV and 132 kV lines.
2. It includes a number of distance relays of the same or different types.
3. A distance relay measures the distance between the relay location and the point of fault in terms of impedance, reactance, etc.
4. The relay operates if the point of fault lies within the protected section of the line.
5. There are various kinds of distance relays. The important types are impedance, reactance and mho type.
6. An impedance relay measure the line impedance between the fault point and relay location; a reactance relay measures reactance, and a mho relay measures a component of admittance.

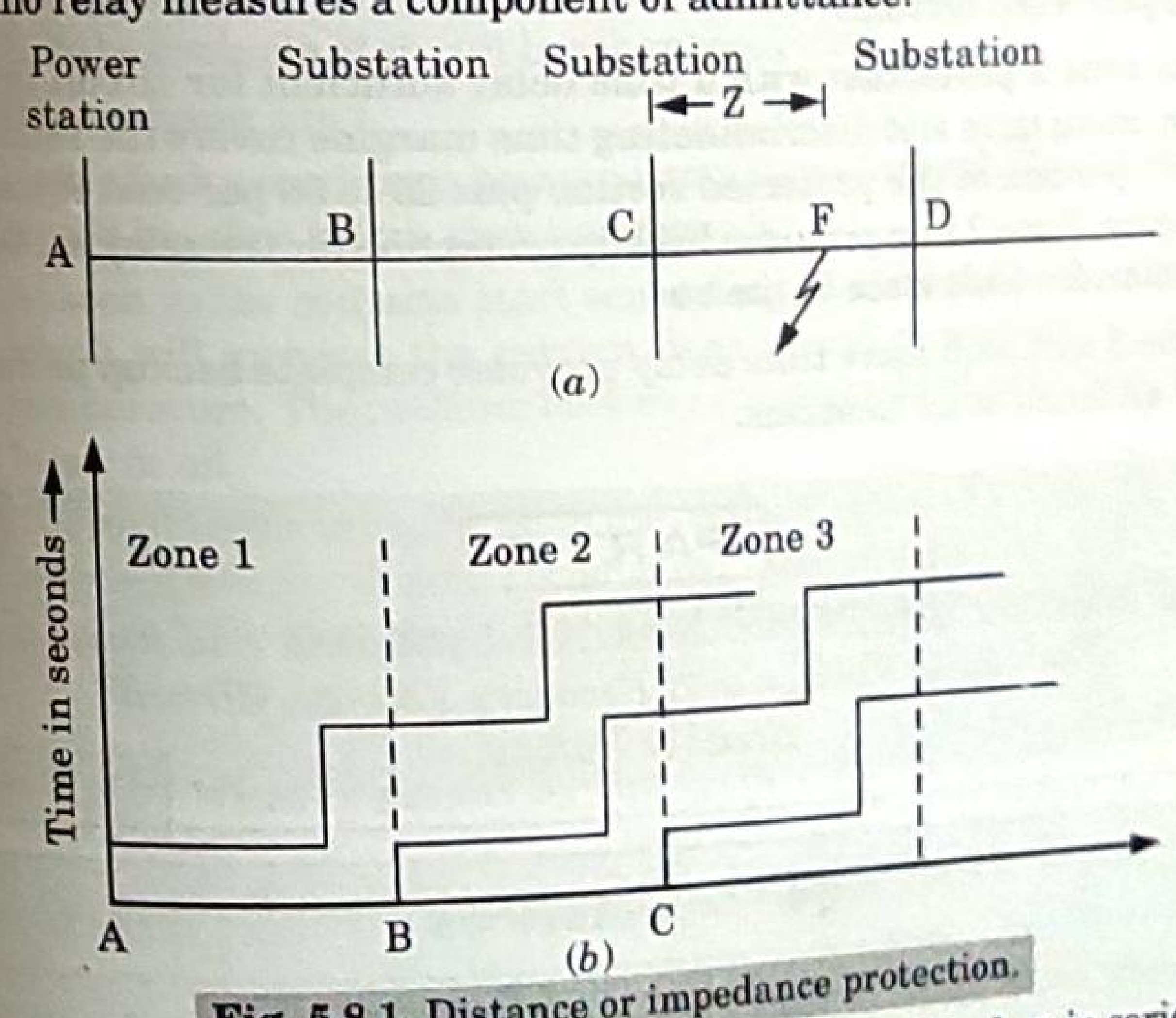


Fig. 5.8.1. Distance or impedance protection.

7. Fig. 5.8.1 shows the simplest system consisting of feeders in series such that the power can flow only from left to right. The relay at *A*, *B*, *C* and *D* are set to operate with impedances less than Z_A, Z_B, Z_C and Z_D respectively.

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8. For a short-circuit fault at point F between substations C and D, the fault loop impedances at power station A and substations B and C are $(Z_1 + Z_2 + Z)$, $(Z_2 + Z)$, and Z respectively. Now only relay at substation C will operate.
9. Similarly for short-circuit faults between substations B and C, and between power station and substation B only relays at substation B and power station A respectively will operate.
10. A system with instantaneous impedance relays, set to act on impedances less than or equal to the impedance of a section, as shown in Fig. 5.8.1(a), would be difficult to adjust; a fault near the junction of two sections is likely to cause the operation of two relays.
11. Furthermore, if a fault of finite resistance occurs near the end of a section, it is possible that total impedance is greater than that for relay operation. For these reasons it is advantageous to use impedance time relays.
12. A number of distance relays are used in association with timing relays so that the power system is divided into a number of zones with varying tripping times associated with each zone.
13. The first zone tripping which is instantaneous is normally set to 80 % of the protected section.
14. The zone 2 protection with a time delay sufficient for circuit breaker operating time and discriminating time margins covers the remaining 20 % portion of the protected section plus 25 to 50 per cent of the next section. Zone 2 also provides backup protection for the relay in the next section for fault close to the bus.
15. Zone 3 with still more time delay provides complete backup protection for all faults at all locations.

PART-4

Circuit Breakers : Arc Phenomenon, Arc Extinction and its Methods, Restriking Voltage and Recovery Voltage, Circuit Breaker Rating.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.9. **Describe the principle of operation of a circuit breaker. Also explain the phenomena of arc.**

Answer

A. Operation of circuit breaker :

1. The circuit breaker mainly consists of fixed contacts and moving contacts. In normal ON condition of the circuit breaker, these two contacts are physically connected to each other due to applied mechanical pressure on the moving contacts.
2. There is an arrangement stored potential energy in the operating mechanism of circuit breaker which is released if the switching signal is given to the breaker.
3. The potential energy can be stored in the circuit breaker by different ways like by deforming metal spring, by compressed air, or by hydraulic pressure.
4. All circuit breaker have operating coils (tripping coils and close coil), whenever these coils are energized by switching pulse, and the plunger inside them displaced.
5. This operating coil plunger is typically attached to the operating mechanism of circuit breaker; as a result the mechanically stored potential energy in the breaker mechanism is released in forms of kinetic energy, which makes the moving contact to move.
6. And the contacts of circuit breaker separated.

B. Phenomena arc :

1. Under faulty conditions heavy current flows through the contacts of the circuit breaker before they are opened.
2. As soon as the contacts start separating, the area of contact decreases which will increase the current density and consequently rise in the temperature. The medium between the contacts of circuit breaker may be air or oil.
3. The heat which is produced in the medium is sufficient enough to ionize air or oil which will act as conductor. Thus an arc is struck between the contacts. The potential difference between the contacts is sufficient to maintain the arc.

Que 5.10. **What is meant by the term arc extinction ?**

OR

How do you quench an arc in a circuit breaker ?

Answer

A. Arc extinction : Arc interruption (quenching) is a process in which path of arc is interrupted for the purpose to extinguish it. For arc interruption, different processes like air blast, high air pressure turbulence and arc splitting are used.

B. Arc quenching methods (interruption) : There are two methods of extinguishing the arc in circuit breakers which are :

A. **High resistance method :**

a. In this the arc resistance is increased with time. This will reduce the current to such a value which will be sufficient to maintain the arc.

b. Thus the arc is interrupted and extinguished. It is employed in only DC circuit breakers.

c. **The various methods of high resistance arc interruption are :**

i. **Lengthening the arc :**

1. In this method the arc length is increased by using arc runners which are horn like blades of conducting material. The arc runners are connected to arcing contacts and it is in shape of letter 'V'.
2. The arc is initiated at the bottom and blows upwards due to electromagnetic force. Due to this arc length increases and arc is extinguished.

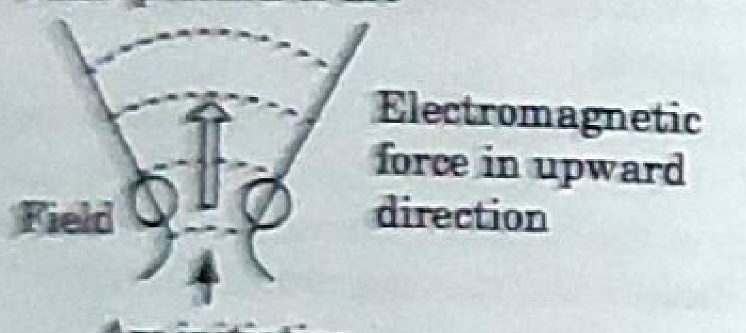

Fig. 5.10.1. Arc lengthening.

ii. **Splitting of arc :**

1. In this method elongation of arc is done and the arc is split using arc splitters which are specially made plates of resin bonded fibre glass.
2. These plates are placed in perpendicular path to arc so that it will be pulled towards it by electromagnetic force.
3. When the arc is pulled upwards it gets elongated, then split and cooled due to which it gets extinguished.

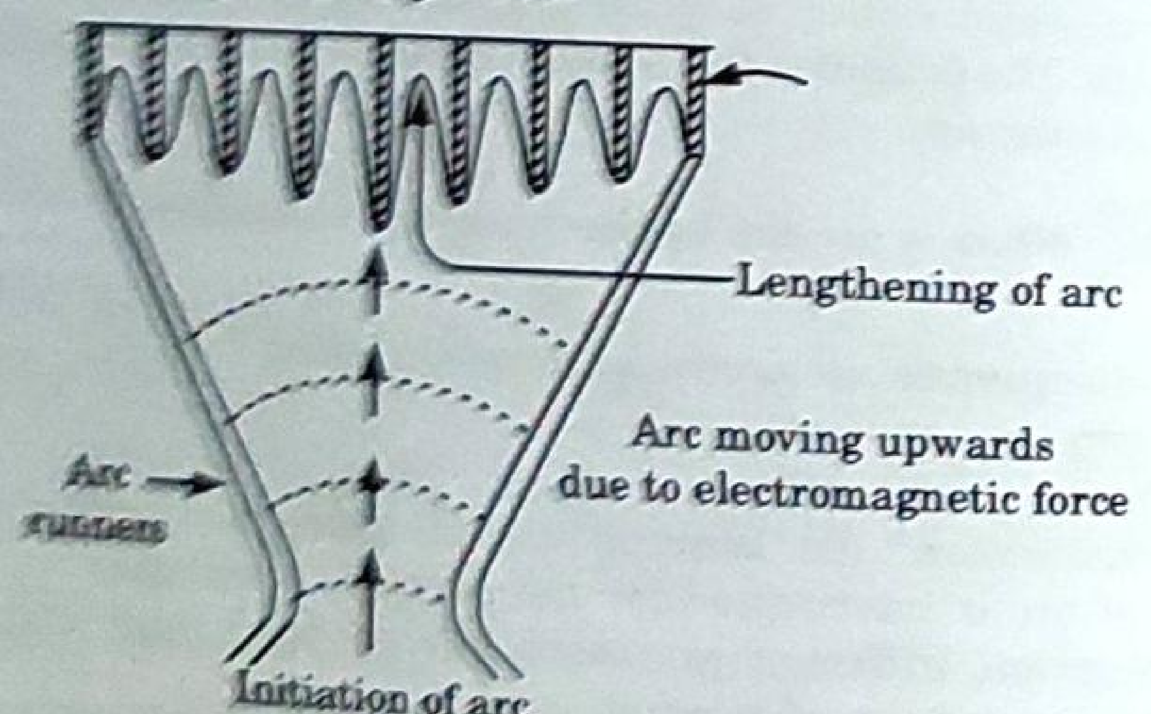

Fig. 5.10.2. Arc splitting.

iii. **Cooling of arc :**

1. The recombination of ionized particles can be done by cooling the arc which removes heat from the arc. This is done by bringing the arc in contact with cooled air.
2. Hence the arc diameter reduces which will increase its resistance. Hence arc extinguishes.

B. **Low resistance method :**

a. It is employed for arc extinction in AC circuits. In this method arc resistance is kept low until current zero where extinction of arc takes place naturally and is prevented from restriking.

b. **The process is divided in three parts :**

i. **Arcing phase :**

1. In this, the temperature of the contact space is increased due to the arc. The heat produced must be removed quickly by providing radial and axial flow to gases.
2. The arc cannot be broken abruptly but its diameter can be reduced by the passage of gas over the arc.

ii. **Current zero phase :** In this, when AC current wave is near its zero, the diameter of the arc is very less and consequently arc is extinguished.

iii. **Post arc phase :**

1. In this, to avoid the re-establishment of arc, the contact space must be filled with dielectric medium having high dielectric strength.
2. Hot gases are removed and fresh dielectric medium is introduced.

Que 5.11. Write a short note on :

i. **Restriking voltage.**
ii. **Recovery voltage.**
iii. **RRRV.**

Answer

i. **Restriking voltage :** The transient voltage that appears across the circuit breaker contacts at the instant of arc extinction is called restriking voltage.

ii. **Recovery voltage :** Power frequency rms voltage which appears across the circuit breaker contacts after the transient oscillations die out and final extinction of arc has resulted is known as recovery voltage.

iii. **RRRV :** It is the rate of rise of restriking voltage which is expressed in volts per microsecond. This will represent the rate at which transient recovery voltage is increasing. The rate of rise of TRV is dependent on system parameters.

Que 5.12. **Derive the expression for the restriking voltage across the contact of circuit breaker.**

Answer

i. **Restriking voltage :**

1. When the breaker contacts are opened and the arc finally extinguishes at some current zero, a voltage v is suddenly applied across capacitor and therefore, across the circuit breaker contacts.
2. The current i which would flow to the fault is not injected in the capacitor and inductor. Thus

$$i = i_L + i_c$$

or
$$i = \frac{1}{L}\int v\, dt + C\frac{dv}{dt}$$

$$\frac{di}{dt} = \frac{v}{L} + C\frac{d^2v}{dt^2} \quad ...(5.12.1)$$

3. Assuming zero currents when $t = 0$, and further

$$v = V_{max} \cos \omega t$$

$$i = \frac{V_{max}}{\omega L} \sin \omega t \text{ before opening of circuit breaker}$$

$$\frac{di}{dt} = \frac{V_{max}}{\omega L} \times \omega \cos \omega t$$

At
$$t = 0; \left|\frac{di}{dt}\right| = \frac{V_{max}}{L}$$

4. Substituting in eq. (5.12.1), we get

$$\frac{V_{max}}{L} = \frac{v}{L} + C\frac{d^2v}{dt^2} \quad ...(5.12.2)$$

5. The solution of this standard equation is

$$v = V_{max}\left[1 - \cos\frac{t}{\sqrt{LC}}\right]$$

ii. **RRRV :** $$\text{RRRV} = \frac{dv}{dt} = \frac{V_{max}}{\sqrt{LC}} \sin\frac{t}{\sqrt{LC}}$$

Que 5.13. **Give the important ratings of a circuit breaker.**

Answer

A. **Rated current, frequency and voltage :**

1. The rated current is the rms value of the current that circuit breaker can carry continuously without any temperature rise.
2. Rated frequency is the frequency at which the circuit has been designed to operate.
3. Rated voltage is the maximum voltage at which the operation of the circuit breaker is generated.

B. **Breaking capacity :**

1. Breaking capacity of circuit breaker is of two types :
   a. **Symmetrical breaking capacity :** Symmetrical breaking capacity is the rms value of the AC component of the fault current that the circuit breaker can break under specified conditions of recovery voltage.
   b. **Asymmetrical breaking capacity :** Asymmetrical breaking capacity is the rms value of the total current comprising of both AC and DC currents that the circuit breaker can break under specified conditions of recovery voltage.
2. The breaking capacity of circuit breaker is generally in MVA.
3. For a three phase circuit breaker :

$$\text{Breaking capacity} = \sqrt{3} \times \text{Rated voltage in kV} \times \text{Rated current in kA}$$

$$\text{Rated } I_{asym.} = 1.6 \times (\text{Rated } I_{sym.})$$

C. **Making capacity :** The rated making capacity is defined as the peak value of current at which the circuit breaker can be closed into a short circuit.

$$\text{Making current} = \sqrt{2} \times 1.8 \times \text{Symmetrical breaking current.}$$

$$\text{Making capacity} = 2.55 \times \text{Symmetrical breaking capacity.}$$

D. **Short time current rating :**

1. The short time current rating is based on thermal and mechanical limitations. The circuit breaker must be capable of carrying short circuit current for a short period while another circuit breaker is clearing the fault.
2. The rated short time current is the rms value of the total current that the circuit breaker can carry safely for specified short period.

Que 5.14. **A 50 Hz, 400 kV, three phase alternator with earthed neutral has a reactance of 10 ohm per phase and is connected to busbar through a circuit breaker. The capacitance to earth between the alternator and the circuit breaker is 0.05 μF per phase. Assuming the resistance of the generator to be negligible, calculate the following :**

i. **Maximum restriking voltage across the contact of circuit breaker.**
ii. **Frequency of oscillations.**
iii. **Maximum value of RRRV.**
iv. **The average value of RRRV up to the first peak.**

Answer

$$E_{rms} = 400 \text{ kV},$$

$$\therefore \quad E_{peak} = \sqrt{2}\, E_{rms} = \sqrt{2} \times \frac{400}{\sqrt{3}} = 326.59 \text{ kV}$$

R/phase = 10 Ω, C/phase = 0.05 μF

i. Maximum restriking voltage across the contact of circuit breaker :

$$= 2\, E_{peak} = 2 \times 326.59 \text{ kV} = 653.18 \text{ kV}$$

ii.

$$L = \frac{R}{2\pi \times 50} = \frac{10}{2\pi \times 50} = 0.032 \text{ H}$$

$\therefore$ Frequency of oscillation,

$$f_n = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.032 \times 0.05 \times 10^{-6}}} = 3.98 \text{ kHz}$$

iii. Maximum value of RRRV = $\omega_m\, E_{peak} = 2\pi \times 50 \times 326.59 = 102.6$ kV

iv. Average value of RRRV up to first peak

$$= \frac{\text{Maximum restriking voltage}}{\text{Time upto first peak}} = \frac{653.18 \text{ kV}}{t}$$

$$= 653.18 \times 2f_n$$

$$= 653.18 \times 2 \times 3.98 \times 10^3 \text{ kV/sec}$$

$$= 5.19 \times 10^6 \text{ kV/sec}$$

Que 5.15. **Calculate the RRRV of 132 kV circuit breaker with neutral earthed. Given data as follows : Broken current is symmetrical; restriking voltage has frequency 20 kHz, pf = 0.15. Assume fault is also earthed.**

Answer

Given : System voltage = 132 kV, $\cos\phi = 0.15$, $f_n = 20$ kHz
To Find : RRRV.

1.
$$K_1 = \sin\phi = \sin(\cos^{-1} 0.15) = 0.9886$$
$$K_2 = 1 \text{ and } K_3 = 1 \text{ both grounded}$$

2.
$$E_m = \frac{\sqrt{2} \times 132}{\sqrt{3}} = 107.77 \text{ kV}$$

3.
$$V_{ar} = K_1 K_2 K_3 E_m = 106.54 \text{ kV}$$

$\therefore$ Maximum voltage, $e_m = 2\, V_{ar} = 213.09$ kV

4.
$$t_m = \pi\sqrt{LC}$$

and
$$f_n = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \quad \pi\sqrt{LC} = t_m = \frac{1}{2f_n} \text{ sec}$$

5. Maximum time,
$$t_m = \frac{1}{2 \times 20 \times 10^3} \text{ sec}$$

$$\therefore \quad \text{RRRV} = \frac{e_m}{t_m} = \frac{213.09}{[1/(20 \times 10^3 \times 2)]} = 8.52 \text{ kV/μsec}$$

Que 5.16. **In 130 kV transmission system, the phase to ground capacitance is 0.02 μF. The inductance being 8 H. Calculate the voltage appearing across the pole of a circuit breaker if a magnetizing current of 12 A is interrupted. Find the value of resistance to be used across contact space to eliminate the striking voltage transient.**

Answer

Given : $L = 8$ H, $C = 0.02\ \mu F = 0.02 \times 10^{-6}$ F, $i = 12$ A

To Find : V and R.

1. As we know, $\frac{1}{2}Li^2 = \frac{1}{2}CV^2$

$$V = i\sqrt{\frac{L}{C}} = 12\sqrt{\frac{8}{0.02 \times 10^{-6}}} = 12\sqrt{400 \times 10^6}$$

$$= 240000 \text{ V}$$

2. $$R = \frac{1}{2}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{8}{0.02 \times 10^{-6}}} = 10000\ \Omega$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Discuss the operating principle of a relay.

Ans. Refer Q. 5.1.

Q. 2. Explain in detail the basic terminologies.

Ans. Refer Q. 5.3.

Q. 3. What do you understand by over current relay ? And explain its type.

Ans. Refer Q. 5.6.

Q. 4. With the help of block diagram discuss differential protection (Pilot wire protection).

Ans. Refer Q. 5.7.

Q. 5. Describe the principle of operation of a circuit breaker. Also explain the phenomena of arc.

Ans. Refer Q. 5.9.

1 UNIT

Fault Analysis in Power System (2 Marks Questions)

1.1. What is a one-line diagram ?

Ans.
1. One-line diagram of a power system shows the main connections and arrangements of components.
2. They represent using suitable symbols for generators, motors, transformers and loads.

1.2. What do you mean by per unit system ?

Ans. The per unit value of a quantity is defined as the ratio of the actual value in any unit to the base or reference value in same unit.

1.3. Give the formula for per unit impedance.

Ans.
1. $$Z(\text{pu}) = \frac{Z_\Omega \times (MVA)_B}{(kV)_B^2}$$
where, $(MVA)_B$ = Base megavoltamperes
$(kV)_B$ = Base kilovolts.
2. When $(MVA)_B$ and $(kV)_B$ is changed from old to new value, the new per unit impedance is given by
$$Z(\text{pu})_{\text{new}} = Z(\text{pu})_{\text{old}} \times \frac{(MVA)_{B\,\text{new}}}{(MVA)_{B\,\text{old}}} \times \frac{(kV)^2_{B\,\text{old}}}{(kV)^2_{B\,\text{new}}}$$

1.4. Define the symmetrical components.

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Ans. The three-phase voltages and currents which are unbalanced are transformed into three sets of balanced voltages and currents. These are called symmetrical components.

1.5. Give the expression for power in terms of symmetrical components.

Ans. Power is given by
$$S = 3V_{a_1}I_{a_1}^* + 3V_{a_2}I_{a_2}^* + 3V_{a_0}I_{a_0}^*$$
= Sum of symmetrical component powers

1.6. The neutral grounding impedance Z_n appears as $3Z_n$ in the zero-sequence equivalent circuit. Why ?

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Ans. A fault current of $3\,I_{R0}$ produces a drop of $3\,I_{R0} \times Z_n$ and to show in the equivalent zero sequence network the same drop where current of I_{R0} flows, the impedance should be $3Z_n$. Therefore, the neutral grounding impedance is $3Z_n$ in zero sequence equivalent circuit.

1.7. Write the relationship between base kVA, base kV and percentage reactance.

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Ans.
1. Let actual reactance = X_Ω
2. Base Impedance, $Z_b = \dfrac{(\text{Base } kVA)^2}{\text{Base } MVA}$
3. $$\% X_{\text{pu}} = X_{\text{pu}} \times 100 = \frac{X_\Omega}{Z_b} \times 100 = \frac{X_\Omega \times 100}{(kV)^2 / MVA}$$
$$= \frac{X_\Omega \times 100}{(kV)^2 / 1000 \times kVA} = \frac{X_\Omega \times kVA \times 10^5}{(kV)^2}$$

1.8. Discuss the representation of power system network by reactance diagram.

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Ans. Reactance diagram :

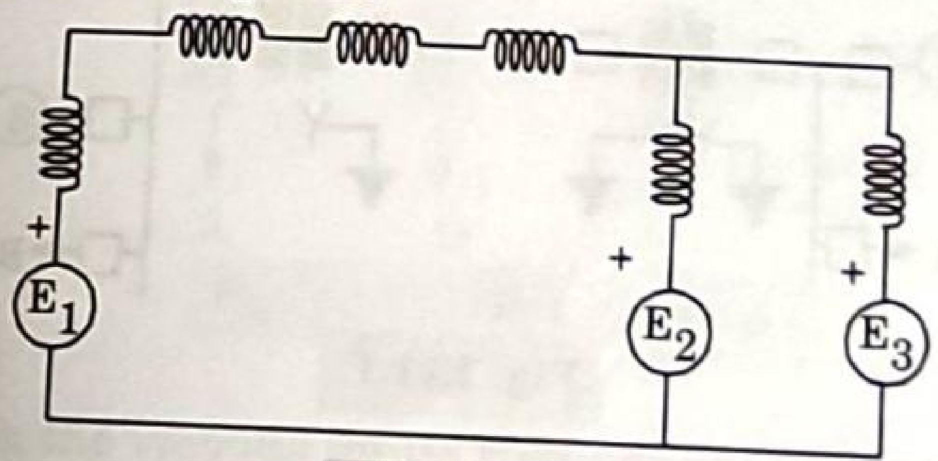

Fig. 1.8.1.

1.9. Discuss advantages and disadvantages of per unit system.

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OR

Give the advantages of per unit system.

AKTU 2018-19, Marks 02

Ans.

A. Advantages :

1. Per unit system leads to great simplification of 3ϕ networks involving transformers.
2. Unit values can be obtained directly by using 3ϕ base quantities.

B. Disadvantages : For transmission lines, its value of impedance and admittances in physical unit that are of same magnitude regardless of voltage level or *MVA* rating.

1.10. Prove that $1 + \alpha + \alpha^2 = 0$. The symbols having their usual meanings.

AKTU 2016-17, Marks 02

Ans.

1. $\alpha = 1\angle 120° = -0.5 + j0.866$
2. $\alpha^2 = 1\angle -120° = -0.5 - j0.866$
3. $\alpha^3 = 1$

$$\alpha^3 - 1 = 0$$

$$(\alpha - 1)(\alpha^2 + \alpha + 1) = 0$$

$\alpha \neq 1$, so complex quantity can be defined as

$$\alpha^2 + \alpha + 1 = 0$$

1.11. What is single line diagram of power system from generating station to utilization level ?

AKTU 2017-18, Marks 02

Ans.

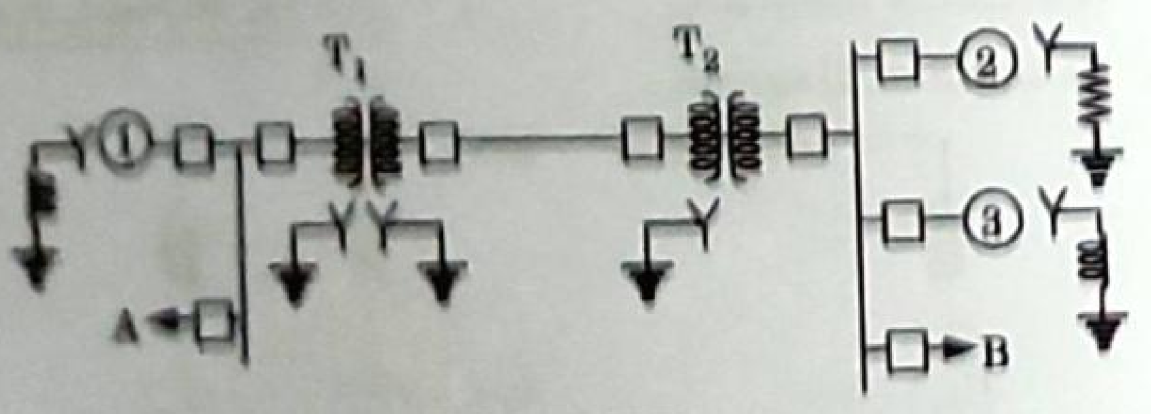

Fig. 1.11.1.

1.12. What is impedance and reactance diagram ?

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Ans.

i. **Impedance diagram :** A diagram in which different components of power system are replaced by their equivalent circuit is known as impedance diagram.

ii. **Reactance diagram :** If in impedance diagram line generator resistance, resistance of transformer windings, resistance of transmission line, line charged and magnetizing circuits of

transformers are neglected the impedance diagram then becomes reactance diagram.

1.13. Give the function of current limiting reactors.

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Ans. Current-limiting reactors are coils used to limit current during fault conditions. Such reactors have large values of inductive reactance and low ohmic resistance.

1.14. Name the fault in which all the three sequence currents are present and are equal.

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Ans. Single line to ground fault.

1.15. Write a short note on feeder reactors.

AKTU 2017-18, Marks 02

Ans.

1. When the reactors are connected in series with the feeders, as shown in Fig. 1.15.1, the reactors are known as feeder reactors.
2. They are used to limit fault current.

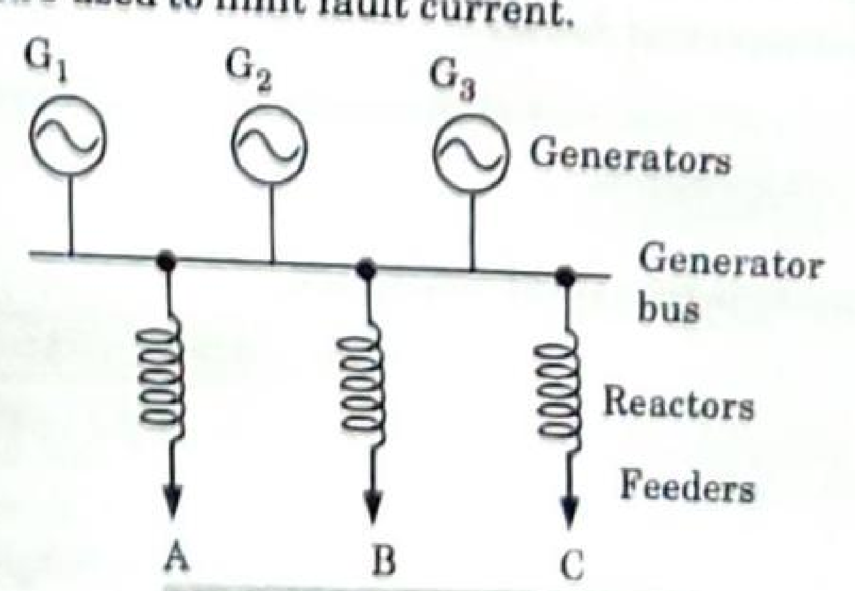

Fig. 1.15.1. Feeder reactors.

1.16. Rank the various faults that can occur in power system in the order of severity.

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Ans. The various types of faults in the order of decreasing severity are :

1. 3ϕ fault
2. LLG fault
3. LL fault
4. LG fault

1.17. Draw the zero sequence network of delta-delta connection.

AKTU 2018-19, Marks 02

Ans. **Δ-Δ transformer bank :** Since a delta circuit provides no return path, the zero sequence currents cannot flow in or out of Δ-Δ transformer, however, it can circulate in the delta windings.

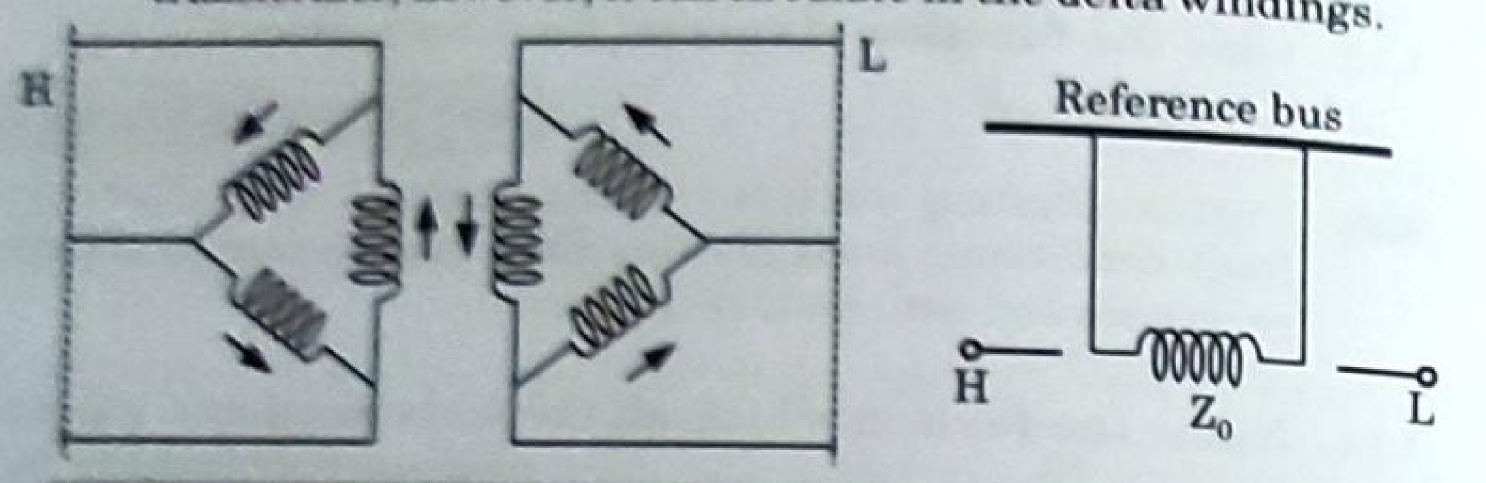

Fig. 1.17.1. D-D Transformer bank and its zero sequence network.

1.18. Name the symmetrical and unsymmetrical faults.

AKTU 2018-19, Marks 02

Ans.

1. **Symmetrical faults :**
   LLL
   LLLG
2. **Unsymmetrical faults :**
   LG
   LL
   LLG

1.19. Define sub-transient reactance.

AKTU 2017-18, Marks 02

Ans. The reactance presented by the machine in the initial period of the short circuit, *i.e.,*

$$X_l + \frac{1}{(1/X_a + 1/X_f + 1/X_{dw})} = X_d''$$

is called the sub-transient reactance of the machine.

☺☺☺

2 UNIT

Load Flow Analysis (2 Marks Questions)

2.1. What are the data obtained from the load flow study ?

Ans. The data comprises of magnitude and phase angles of load bus voltages, reactive powers and voltage phase angles at generator buses and real and reactive power flow on transmission lines together with power at the reference bus.

2.2. Name the types of buses.

Ans.

1. Slack bus or swing bus or reference bus
2. P-Q bus or load bus
3. P-V bus or generator bus.

2.3. Define : Load bus, Generator bus and Slack bus.

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Ans.

A. **Load bus :** Load bus is the terminals of transformer at which the total active power and reactive power required are specified and voltage magnitude and load angle are not specified.
B. **Slack bus :** Slack bus is to supply the losses in the power system network. In this bus, voltage and load angle are specified.
C. **Generator bus :** This bus is always connected to a generator. Here net power and voltage magnitude are known and reactive power and load angle are unknown.

2.4. State Gauss-Seidal load flow formula.

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Ans.

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}} Y_{ik} V_k\right]$$

where, $i = 2, 3, ..., n$

V_i = Voltage at i^{th} bus

P_i and Q_i = Real and reactive power respectively

Y_{ii} = Admittance matrix

3 UNIT Travelling Waves in Power System (2 Marks Questions)

3.1. Give the expression of wave equations.

Ans. These are $\frac{\partial^2 e}{\partial x^2} = LC\frac{\partial^2 e}{\partial t^2}$

and $\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2}$

3.2. What is the velocity of traveling wave ?

Ans. The velocity of traveling wave is

$$v = \frac{1}{\sqrt{LC}}$$

3.3. Write the expression for surge impedance.

Ans. The ratio of the magnitude of voltage and associated current surges is called surge impedance and is given by

$$\frac{e}{i} = \sqrt{\frac{L}{C}}$$

3.4. Give the general expression for reflection coefficient.

Ans. The ratio of reflected voltage to the incident voltage is called the reflection coefficient and is denoted by r_V or r.

$$\rho_V = \frac{e_r}{e_f} = \frac{Z_t - Z_0}{Z_t + Z_0}$$

where, Z_0 = Characteristic impedance
Z_t = Terminal impedance.

3.5. Give the general expression for refraction coefficient.

Ans. The ratio of voltage at the termination to the incident voltage is called refraction coefficient. It is denoted by τ.

$$\tau = \frac{e_r}{e_f}$$

3.6. What is the relation between reflection and refraction coefficient ?

Ans. Reflection coefficient = Refraction coefficient – 1

$$\rho = \tau - 1$$

3.7. Define Bewley's lattice diagram.

Ans.
1. It provides a simple and convenient method to study the effects of multiple or repeated reflections.
2. It gives the picture of the positions and direction of every incident, reflected and transmitted wave on the system at every instant.

3.8. Find the CIL of 200 kV transmission line.

AKTU 2017-18, Marks 02

Ans. Let $Z_0 = 400\ \Omega$ for transmission line

CIL (characteristic impedance loading)

$$= \frac{(kV_L)^2}{Z_0} = \frac{(200)^2}{400}$$

$$= 100 \text{ MW}$$

3.9. What is meant by voltage surge ?

AKTU 2016-17, Marks 02

AKTU 2018-19, Marks 02

Ans.
1. Voltage surge is defined as the sudden rise in excessive voltage which damages the electrical equipment of an installation.
2. The over voltage in line occurs because of a rise in voltage between both phases and between phase and ground.

3.10. Define characteristics impedance loading or surge impedance loading.

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Ans. Surge impedance loading (SIL) of a transmission line is the power (MW) loading of a transmission when the line is lossless.

3.11. What do the double sign indicate in the velocity of traveling wave ?

Ans. The double sign indicates that the surge is split into two components which travel along the line in opposite direction.

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4 UNIT

Stability in Power System (2 Marks Questions)

4.1. Define stability of a power system.

AKTU 2015-16, Marks 02

Ans. **Stability :** The ability of a system to reach a normal or stable condition after being disturbed is called stability.

4.2. What is stability limit ?

Ans. The stability limit is the maximum power that can be transferred in a network between sources and loads without loss of synchronism.

4.3. Define inertia constant of a synchronous machine and write the units for inertia constant. **AKTU 2016-17, Marks 02**

Ans. **Inertia constant (H) :** It is the ratio of kinetic energy of a rotor of synchronous machine to the rating of a machine (in MVA).

Units : The unit of inertia constant H is mega joule/MVA or MW-sec/MVA.

4.4. Define steady state stability.

Ans. The phenomenon of stability under slow and small changes like slight variation in excitation and small increase in load is called steady state stability.

4.5. Explain transient stability.

Ans. The phenomenon of stability under the large changes within a small time is called transient stability. Parameter variation in this phenomenon is from 5 % to maximum.

4.6. Define transient stability limit.

Ans. Transient stability limit is the maximum power that can be transferred without the system becoming unstable when a sudden or large disturbance occurs.

4.7. What is swing equation ?

Ans. The equation

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = (P_m - P_e) \text{ pu}$$

is called the swing equation and it describes the rotor dynamics for a synchronous machine.

4.8. Explain the swing curve. **AKTU 2017-18, Marks 02**

Ans. The swing curve is the plot between the power angle and time. It is usually plotted for a transient state study the nature of variation in angle for a sudden large disturbance.

4.9. What is the significance of equal-area criterion ?

Ans.

1. It is an easy means of finding the maximum angle of swing.
2. An estimate of whether synchronism will be maintained.
3. The maximum amount of disturbance that can be allowed without losing synchronism.

4.10. If the two machines with inertias M_1 and M_2 are swinging together, what will be the inertia of the equivalent machine ? **AKTU 2015-16, Marks 02**

Ans.

$$M' = \frac{M_1 M_2}{M_1 + M_2}$$

M' = The inertia of the equivalent machine.

4.11. Write the power angle equation of a synchronous machine connected to an infinite bus. **AKTU 2016-17, Marks 02**

Ans.

1. $$P_e = \frac{EV}{X}\sin\delta$$
2. $$Q_e = \frac{EV}{X}\cos\delta - \frac{V^2}{X}$$

4.12. Explain the methods of improving steady state stability. **AKTU 2017-18, Marks 02**

Ans.

1. Higher excitation voltage.
2. Reducing the impedance between the stations.
3. Quick response excitation system.

4.13. Differentiate between stability and instability. **AKTU 2018-19, Marks 02**

Ans.

Stability	Instability
The ability of a system to reach a normal or stable condition after being disturbed is called stability.	If the system not able to reach a normal or stable condition after being disturbed is called instability.

4.14. On what factor does maximum power transfer depend ?

AKTU 2018-19, Marks 02

Ans.

1. Maximum power transfer depends on power angle.
2. At $\delta = 90°$ maximum power is transferred.

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5 UNIT

Introduction to Power System Protection (2 Marks Questions)

5.1. What do you understand by relay ?

Ans. Relay is a device which detects the fault and is responsible for energizing the trip circuit of a circuit breaker. This isolates the faulty part from rest of the system.

5.2. What is dropout or reset in relay ?

Ans. A relay is said to dropout when it moves from ON position to OFF position. The value of the characteristics quantity below which this change occur is known as dropout or reset value.

5.3. Write the types of overcurrent relay.

Ans.

i. Instantaneous overcurrent relays.
ii. Inverse definite time relays.
iii. Inverse definite minimum time (IDMT) relays.
iv. Very inverse relays.
v. Extremely inverse relays.

5.4. What is high resistance interruption method ?

Ans. In this method, the arc is controlled by increasing the resistance such that the current decreases and its value reduce to insufficient value to maintain the arc.

5.5. Define low resistance interruption method.

Ans. In this method, arc resistance is kept low until current become zero where extinction of arc takes place naturally and is prevented from restriking.

5.6. What do you mean by breaker time ?

Ans. It is the time between the instant at circuit breaker operates and opens the contacts to the instant of extinguishing the arc completely.

5.7. Explain the term restriking voltage.

Ans. The transient voltage that appears across the circuit breaker contacts at the instant of arc extinction is called restriking voltage.

5.8. Define arc extinction.

OR

What is meant by the term arc quenching ?

Ans. Arc interruption (quenching) is a process in which path of arc is interrupted for the purpose to extinguish it. For arc interruption, different processes like air blast, high air pressure turbulence and arc splitting are used.

5.9. Write the methods of arc extinction.

Ans.

1. High resistance
2. Low resistance

5.10. Write the name of circuit breaker ratings.

Ans.

a. Rated current, frequency and voltage.
b. Breaking capacity.
c. Making capacity.
d. Short time current rating.

5.11. What do you understand by primary and backup protection ?

Ans.

1. **Primary protection :** It is used to protect any equipment by isolating it from the system or it is the protection provided by each zone to its elements. It is the first line of defense.
2. **Backup protection :** It is the one which came into play when primary protection fails. It operates after a time delay to give primary relay sufficient time to operate.

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