CONTENTS

RME 602 : THEORY OF MACHINES

ANALYSIS OF AKTU PAPERS (2013-14 TO 2017-18) **(A–1 B to A–5 B)**

UNIT-1 : VELOCITY & ACCELERATION ANALYSIS **(1–1 B to 1–42 B)**

Introduction, mechanisms and machines, kinematics and kinetics, types of links, kinematic pairs and their classification, types of constraint, degrees of freedom of planar mechanism, Grubler's equation, mechanisms, inversion of four bar chain, slider crank chain and double slider crank chain.

Velocity analysis: Introduction, velocity of point in mechanism, relative velocity method, velocities in four barmechanism, instantaneous center.

Acceleration analysis: Introduction, acceleration of a point on a link, acceleration diagram, Coriolis component of acceleration, crank and slotted lever mechanism.

UNIT-2 : CAMS, GEARS & GEAR TRAINS **(2–1 B to 2–51 B)**

Cams: Introduction, classification of cams and followers, cam profiles for knife edge, roller and flat faced followers for uniform velocity, uniform acceleration.

Gears and gear trains: Introduction, classification of gears, law of gearing, tooth forms and their comparisons, systems of gear teeth, length of path of contact, contact ratio, minimum number of teeth on gear and pinion to avoid interference, simple, compound, reverted and planetary gear trains, sun and planet gear train.

UNIT-3 : FORCE ANALYSIS **(3–1 B to 3–37 B)**

Static force analysis of mechanisms, D'Alembert's Principle, dynamics of rigid link in plane motion, dynamic force analysis of planar mechanisms, piston force and crank effort. Turning moment on crankshaft due to force on piston, Turning moment diagrams for single cylinder double acting steam engine, four stroke IC engine and multi-cylinder engines, Fluctuation of speed, Flywheel.

UNIT-4 : BALANCING & GOVERNORS **(4–1 B to 4–51 B)**

Balancing: Introduction,static balance, dynamic balance, balancing of rotating masses,two plane balancing, graphical and analytical methods, balancing of reciprocating masses.

Governors: Introduction, types of governors, characteristics of centrifugal governors, gravitycontrolled and spring controlled centrifugal governors, hunting of centrifugal governors, inertia governors. Effort and Power of governor.

UNIT-5 : BRAKES AND DYNAMOMETERS **(5–1 B to 5–33 B)**

Introduction, Law of friction and types of lubrication, types of brakes, effect of braking on rear and front wheels of a four wheeler, dynamometers, belt transmission dynamometer, torsion dynamometer, hydraulic dynamometer.

SHORT QUESTIONS **(SQ-1B to SQ-18B)**

SOLVED PAPERS (2013-14 TO 2017-18) **(SP-1B to SP-16B)**

1
UNIT

Velocity and Acceleration Analysis

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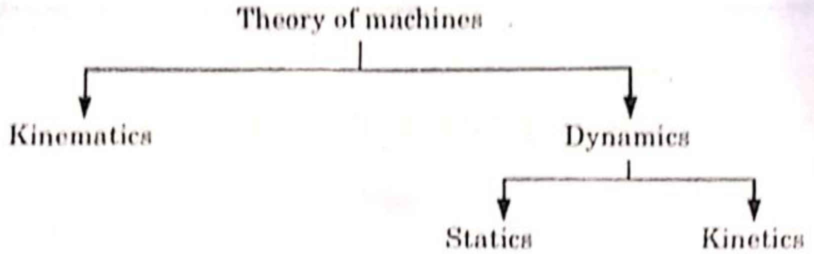
PART-1

Introduction, Mechanism and Machines.

CONCEPT OUTLINE

Theory of Machine : It is a science which deals with the study of structure, kinematics and dynamics of machines.



Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. **Write short note on mechanism and machines.**

Answer

A. Mechanism :

1. When one element or link of a kinematic chain is fixed, the arrangement may be used for transmitting or transforming motion, this arrangement is known as mechanism.
2. Examples of mechanism are typewriters, clocks, watches, spring toys, etc.

B. Machine :

1. It is an arrangement of various elements or links which are designed so as to carry with safety the forces, both static and kinetic, to which they are subjected, to transmit power or to do some particular kind of work.
2. A machine is a combination of resistant bodies, with successfully constrained relative motions, which is used for transmitting or transforming available energy so as to do some particular kind of work.
3. Examples of machines are reciprocating pump, steam engine etc.

PART-2

Kinematics and Kinetics, Types of Links.

CONCEPT OUTLINE

Kinematics of Machine : It is that branch of theory of machines which deals with the study of relative motion of parts of the machines, neglecting consideration of forces producing it.

Dynamics of Machine : It is the branch of theory of machines which deals with the study of motion of a machine under the forces acting on different parts of the machines.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.2. Explain the term kinematic link. Give the classification of kinematic link.

Answer

A. Kinematic Link or Element :

1. Each part of a machine which has motion relative to some other part is known as a kinematic link or element.
2. A link must be a resistant body *i.e.*, it must be capable of transmitting the required force with negligible deformation.
3. Each link or element may consist of several parts which are manufactured as separate units.

B. Classification of Link :

a. On the Basis of Transmission of the Motion :

i. **Rigid Link :** The link which does not undergo any deformation while transmitting motion e.g., connecting rod and crank of a reciprocating steam engine.

ii. **Flexible Link :** The link which is partly deformed in a manner not to affect the transmission of motion e.g., belts, ropes and chains are used to transmit tensile forces only.

iii. **Fluid Link :** The link which is formed by having a fluid in a vessel and the motion is transmitted through the compression of fluids e.g., hydraulic presses, jacks and brakes etc.

2. On the Basis of their Ends on which Turning Pairs can be Placed :

i. Binary link,

ii. Ternary link, and

iii. Quaternary link.

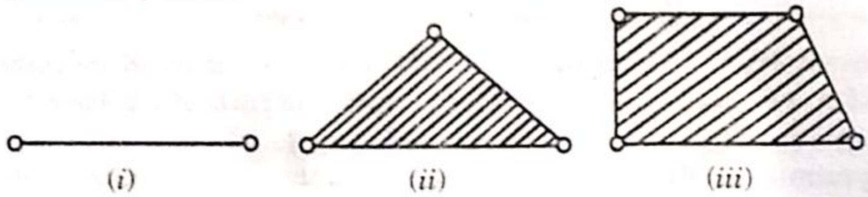


Fig. 1.2.1.

PART-3

Kinematic Pairs and their Classification.

CONCEPT OUTLINE

Kinematic Pair : A kinematic pair is a joint of two links having relative motion between them.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.3. **Explain the term kinematic pair. How are the kinematic pairs classified ? Explain with examples.**

Answer

A. Kinematic Pair :

1. The two links or elements of a machine which are connected together in such a way that their relative motion is completely constrained form a kinematic pair.

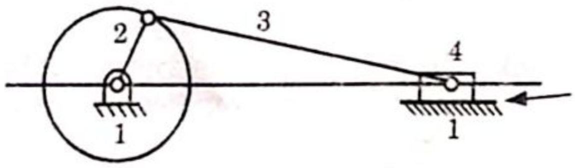


Fig. 1.3.1. Kinematic pair.

2. In Fig. 1.3.1, a slider crank mechanism is shown. The links 1 – 2 and 2 – 3 and 3 – 4 constitute turning pairs and link 4 reciprocates relative to the link 1 and is known as sliding pair.

B. Classification of Kinematic Pair :

I. On the Basis of Nature of Contact :

a. Lower Pair :

1. A pair of link having surface or area contact between the members is known as lower pair.
2. **Example :** All pairs of a slider crank mechanism, universal joint, shaft rotating in a bearing etc.

b. Higher Pair :

1. A pair of link having point or line contact between them is known as higher pair. They do not have similar contact surfaces.
2. **Example :** Cam and follower pair, gear teeth pair, ball and roller bearing etc.

II. On the Basis of Nature of Relative Motion :

a. Sliding Pair :

1. When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as sliding pair. A sliding pair has a completely constrained motion [Fig. 1.3.2(*a*)].

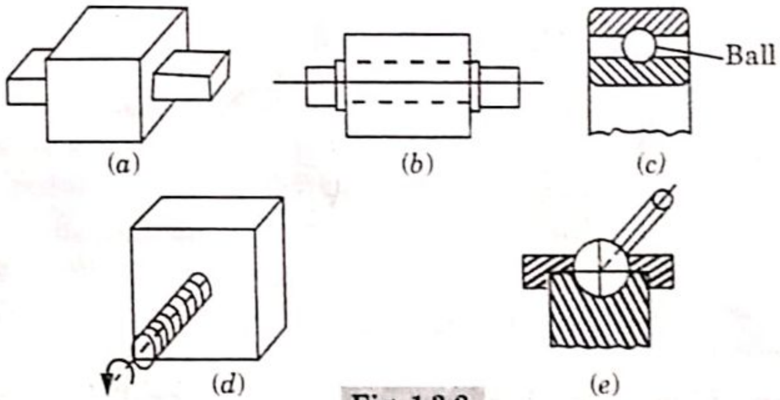


Fig. 1.3.2.

2. **Example :** A rectangular rod in a rectangular hole forms a sliding pair.

b. Turning Pair :

1. When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. It also has a completely constrained motion [Fig. 1.3.2(*b*)].
2. **Example :** A circular shaft revolving inside a bearing is a turning pair.

c. Rolling Pair :

1. When the links of a pair have a rolling motion relative to each other, they form a rolling pair.

2. In the Fig. 1.3.2(c) there are two rolling pairs present :
   i. The ball and the shaft constitute one rolling pair.
   ii. The ball and the bearing constitute other rolling pair.

d. Screw Pair (Helical Pair) :

1. If there is turning as well as sliding motion between two mating links, it is known as screw pair [Fig. 1.3.2(*d*)].
2. **Example :** The lead screw and the nut of a lathe is a screw pair.

e. Spherical Pair :

1. If one element (or link) turns about other fixed link, they constitute to form a spherical pair [Fig. 1.3.2(*e*)].
2. **Example :** The ball and socket joint is a spherical pair.

III. On the Basis of Nature of Mechanical Constraint :

a. Self Closed Pair (Closed Pair) : When the elements of a pair are held together mechanically, it is known as a closed pair [Fig. 1.3.3(*a*)]. The lower pairs are self closed pairs.

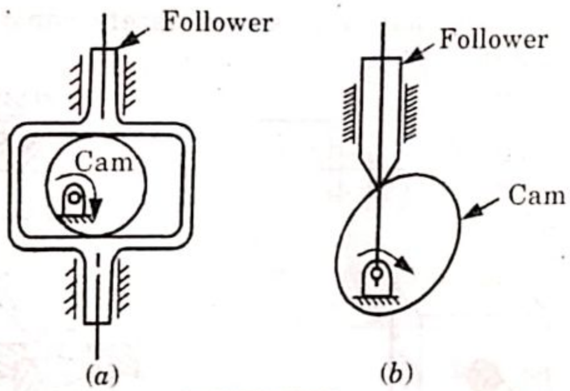


Fig. 1.3.3.

b. Force Closed Pair (Unclosed Pair) : When the two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair [Fig. 1.3.3(*b*)].

PART-4

Types of Constraint.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 1.4. **Define and explain with neat sketch types of constrained motions.**

Answer

A. Completely Constrained Motion :

1. When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, the motion is known as completely constrained motion.
2. **Example :** Motion of piston inside a cylinder in steam engine.
3. The motion of square bar in a square hole is completely constrained motion as it follows only one direction of motion irrespective of the force applied.

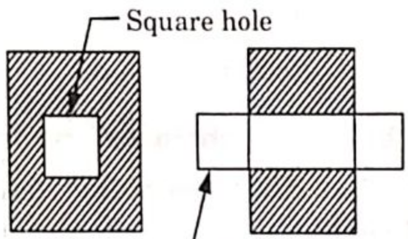


Fig. 1.4.1.

B. Incompletely Constrained Motion :

1. When the motion between the pair can take place in more than one direction, motion is called incompletely constrained motion.
2. The change in the direction of impressed force may change the direction of relative motion between the pair.
3. The motion of a circular shaft inside a circular hole can be sliding or rotational motion depending upon the direction of force applied. Hence it is an example of incomplete constrained motion.

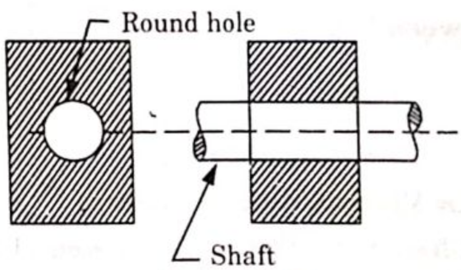
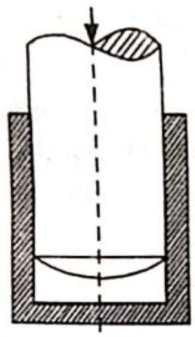


Fig. 1.4.2.

C. Successfully Constrained Motion :

1. When the motion between two elements of a pair is possible in more than one direction but is made to have motion only in one direction by using some external means, it is called successfully constrained motion.

2. Consider a shaft's motion in a footstep bearing, shaft may have rotary motion as well as vertical motion, by putting load on the shaft motion can be constrained only rotary thus it is called a successfully constrained motion.



Footstep bearing

Fig. 1.4.3.

Que 1.5. **Define kinematic chain and write the relation for a kinematic chain. Also describe the condition for locked, constrained and unconstrained chain.**

Answer

A. **Kinematic Chain :** A kinematic chain is an assembly of links in which the relative motions of the links is possible and the motion of each link relative to the other is definite.

B. **Various Relation for Kinematic Chain :**

a. **Relation between l and p :**

$$l = 2p - 4 \quad ...(1.5.1)$$

Where, l = Number of links, and

p = Number of pairs.

b. **Relation between l and j :**

$$j = \frac{3}{2}l - 2 \quad ...(1.5.2)$$

Where, j = Number of joint.

C. **Conditions for Various Kind of Chain :**

a. **For Locked Chain :** In both eq. (1.5.1) and eq. (1.5.2), if LHS > RHS, chain is called locked chain.

b. **For Constrained Chain :** In both eq. (1.5.1) and eq. (1.5.2), if LHS = RHS, chain is constrained kinematic chain.

c. **For Unconstrained Chain :** In both eq. (1.5.1) and eq. (1.5.2), if LHS < RHS, chain is called unconstrained chain.

PART-5

Degree of Freedom of Planer Mechanism, Gruebler's Equation.

CONCEPT OUTLINE

Degree of Freedom : Degree of freedom of a pair is defined as the number of independent relative motions, both translational and rotational a pair can have.

Degrees of freedom = 6 – Number of restraints

Kutzbach's Criterion for Degree of Freedom of Plane Mechanism :

$$\text{DOF} = 3(l-1) - 2j - h$$

Grubler's Criterion for Plane Mechanism :

$$3l - 2j - 4 = 0$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.6. **What is Kutzbach's criterion for degree of freedom of plane mechanism ? In what way Grubler's criterion is different from it ?**

Answer

A. Kutzbach's Criterion for Degree of Freedom of Plane Mechanism :

1. It is a method of determining the number of degree of freedom or movability (n) of a plane mechanism.
2. According to Kutzbach's criteria, degree of freedom is given as

$$n = 3(l-1) - 2j - h \quad \text{...(1.6.1)}$$

Where, n = Degree of freedom,

l = Number of link,

j = Number of binary joints, and

h = Number of higher pair.

B. Grubler's Criterion for Plane Mechanism :

1. This criteria is applicable for the mechanism having only one degree of freedom.
2. It can be obtained by substituting $n = 1$ and $h = 0$ in eq. (1.6.1),

$$3(l-1) - 2j = 1$$

or $$3l - 2j - 4 = 0$$

3. The above equation is known as Grubler's criterion for plane mechanism.

Que 1.7. **Determine the degree of freedom for the following mechanisms in Fig. 1.7.1 and Fig. 1.7.2.**

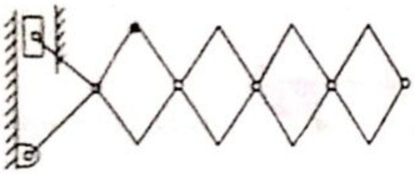
i. 

Fig. 1.7.1.

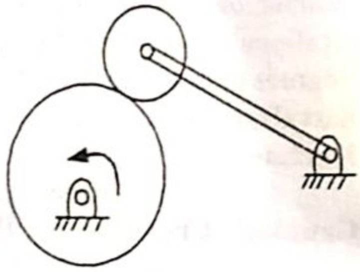
ii. 

Fig. 1.7.2.

Answer

Mechanism (i) :

1. Number of links, $l = 12$

   Number of binary joints, $j = 16$

   Number of higher pair, $h = 0$

2. Degree of freedom, $n = 3(l-1) - 2j - h$

$$n = 3(12-1) - 2 \times 16 - 0$$
$$= 3 \times 11 - 2 \times 16 - 0 = 33 - 32$$
$$n = 1$$

Mechanism (ii) :

1. Total number of links, $l = 4$

   Number of binary joints, $j = 3$

   Number of higher pair, $h = 1$

   Number of redundant degree of freedom, $F_r = 1$

2. Degree of freedom, $n = 3(l-1) - 2j - h - F_r$

$$= 3(4-1) - 2 \times 3 - 1 - 1$$
$$= 9 - 6 - 1 - 1$$
$$n = 1$$

PART-6

Mechanisms, Inversion of Four Bar Chain.

CONCEPT OUTLINE

Inversion of Mechanism : The method of obtaining different mechanism by fixing different links in a kinematic chain is known as inversion of the mechanism.

Inversion of Four Bar Chain Mechanism :

1. Crank lever mechanism.
2. Double crank mechanism.
3. Double lever mechanism.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.8. **Sketch and describe the four bar chain mechanism.**

Answer

1. Four bar chain mechanism is also known as quadric cycle chain.
2. It consists of four links, each of them forms a turning pair at A, B, C and D.
3. Each of four links may be of different length and it should follow Grashof's law. According to this law, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

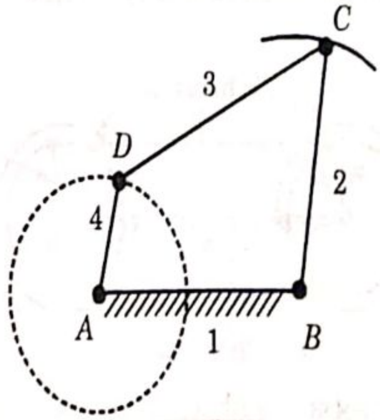


Fig. 1.8.1.

4. In a four bar chain, one of the shortest link will make a complete revolution relative to the other three links. Such a link is k

7. The fixed link AB (link 1) is known as frame of the mechanism.

Que 1.9. **Discuss the inversion of a four bar chain or mechanism.**

Answer

Following are the three inversion of a four bar chain :

a. **Crank Lever Mechanism (Beam Engine) :**

1. This mechanism is used to convert rotary motion into reciprocating motion.

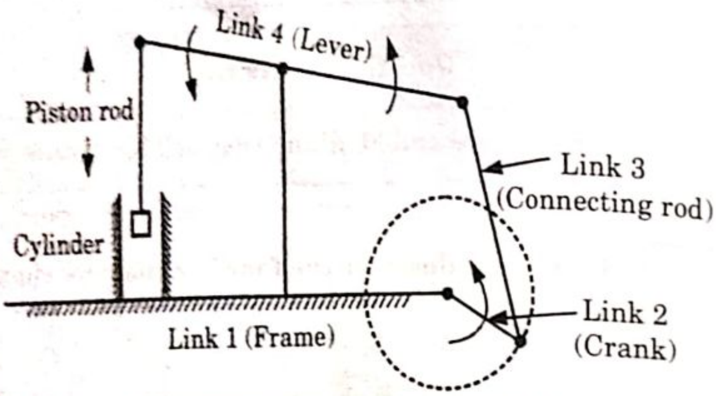


Fig. 1.9.1. Crank-lever mechanism.

2. In Fig. 1.9.1, link 1 is a fixed frame whose one end is connected with link 2 (crank) which rotates about a point A, link 3 transfers this motion to link 4 and link 4 converts it into a reciprocating motion of slider as shown in Fig. 1.9.1.

b. **Double Crank Mechanism (Coupling Rod of a Locomotive) :**

1. This mechanism transfers the rotary motion of one wheel to another wheel.

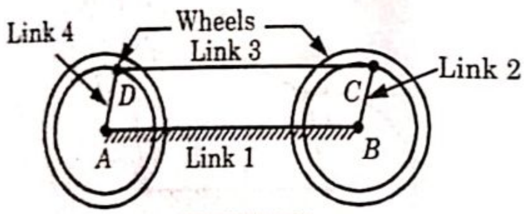


Fig. 1.9.2.

2. When the link BC rotates, link AD also rotates via link CD and link CD acts as a coupling rod.

3. This mechanism has two cranks as AD and BC, so it is called as double crank mechanism.

4. Double crank mechanism is generally seen in wheels of locomotive.

c. **Double Lever Mechanism (Watt's Indicator Mechanism) :**

1. A Watt's indicator mechanism which consists of four links is shown in Fig. 1.9.3.

2. The four links are : fixed link at A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers.
3. The displacement of the link BFD is directly proportional in the pressure of gas or steam which acts on the indicator plunger.
4. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.

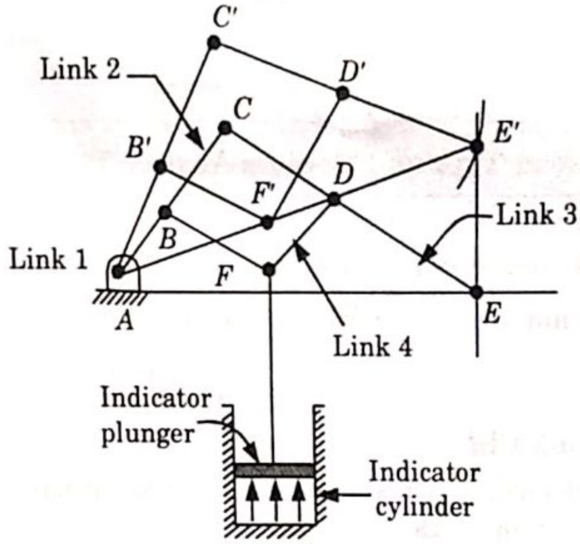


Fig. 1.9.3.

Que 1.10. **Fig. 1.10.1 shows four link mechanisms in which the figures indicate the dimensions in standard unit of length. Indicate the type of each mechanism whether crank rocker or double crank or double rocker.**

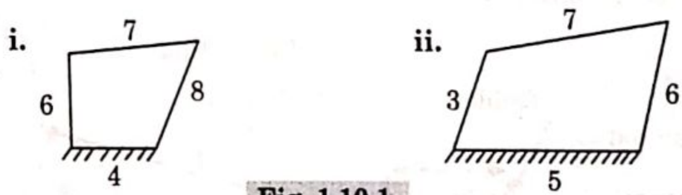


Fig. 1.10.1.

Answer

Mechanism (i) :

1. Length of the longest link = 8
Length of the shortest link = 4
Length of other links = 7 and 6
2. Since $8 + 4 < 7 + 6$, it belongs to the class-I mechanism. In this case as the shortest link is fixed so it is a double crank mechanism.

Mechanism (ii) :

1. Length of the longest link = 7
Length of the shortest link = 3
Length of other links = 5 and 6

2. Since 7 + 3 < 5 + 6, it belongs to class-I mechanism. In this case as the link adjacent to the shortest link is fixed so it is a crank rocker mechanism.

PART-7

Slider Crank Chain.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.11. **Sketch slider crank chain and its various inversions, stating actual machines in which these are used in practice.**

Answer

A. Slider Crank Chain :

1. Slider crank chain is a modified four bar chain. It has one sliding pair and three turning pairs.
2. It is used in the mechanisms where rotary motion converts into reciprocating motion or vice-versa.
3. In Fig. 1.11.1, links 1 and 2, links 2 and 3 and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.
4. With the rotation of crank, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.

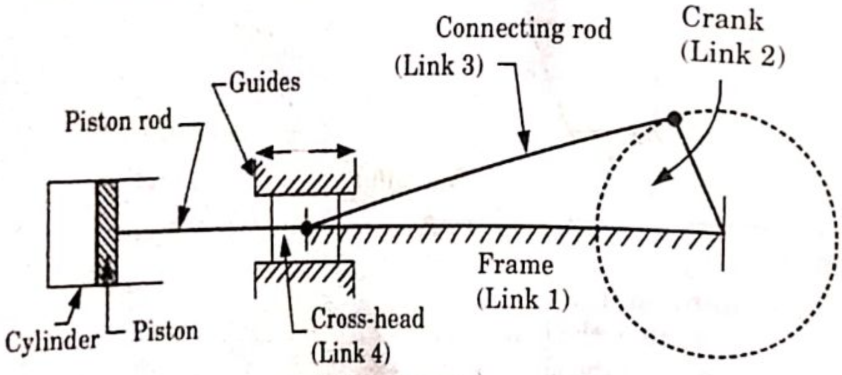


Fig. 1.11.1.

B. Inversions of Single Slider Crank Chain (or Slider Crank Chain) :

i. First Inversion :

1. It is obtained by fixing link 1 and links 2 and 4 are made crank and slider respectively.

2. **Application :** It is used in reciprocating engines and reciprocating compressor etc.

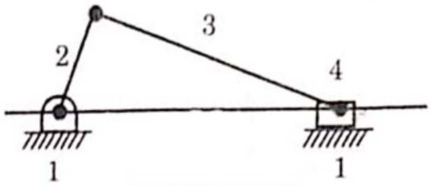


Fig. 1.11.2.

ii. **Second Inversion :**

1. By fixing the link 2 of a slider crank mechanism we get the second inversion of slider crank chain.

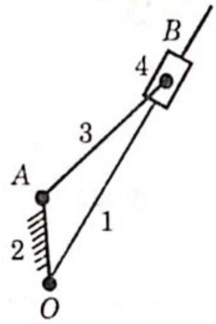


Fig. 1.11.3.

2. In this inversion, link 3 becomes crank and link 1 rotates about O.
3. **Application :** Whitworth quick-return motion mechanism and rotary engine.

iii. **Third Inversion :**

1. By fixing the link 3 of a slider crank mechanism, we get the third inversion of slider crank mechanism.
2. In this inversion link 2 acts as a crank and the link 4 oscillates.

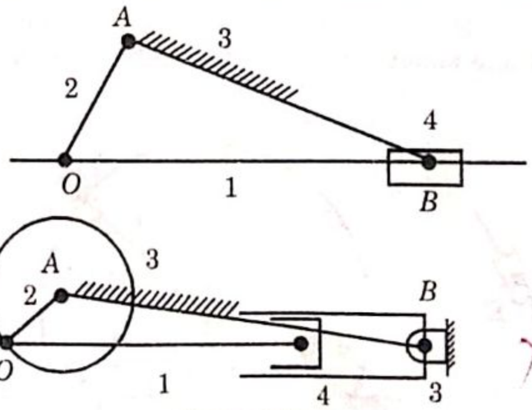


Fig. 1.11.4.

3. **Application :** Crank and slotted lever mechanism, oscillating cylinder engine.

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iv. Fourth Inversion :

1. By fixing the link 4 we get the fourth inversion of slider crank mechanism.

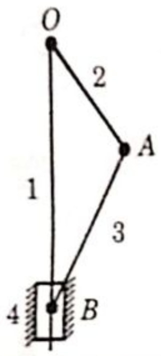


Fig. 1.11.5.

2. In this inversion, link 3 can oscillate about the fixed point B on the link 4 and link 2 also oscillates about the point B.
3. The end O reciprocates along the axis of the fixed link 4.
4. **Application :** Hand pump, duplex pump etc.

Que 1.12. **Describe the working of two different types of quick return mechanism with neat sketches. Write an expression for the ratio of time taken in forward and return stroke.**

Answer

Following are the two types of quick return mechanism :

A. Whitworth Quick Return Mechanism :

1. This mechanism is used in shaper or slotter machines in workshop.
2. The forward stroke cuts the metal whereas return stroke is idle.
3. Whitworth quick return mechanism is shown in the Fig. 1.12.1. Link 2, OA is fixed and slider 4 rotates in a circle about A and slides on the link 1.

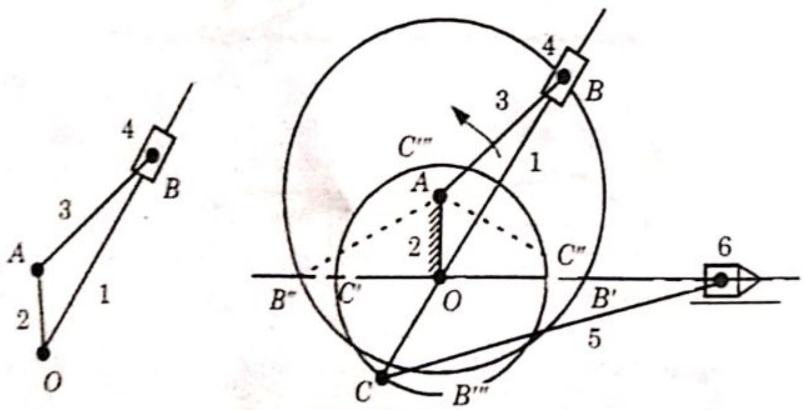


Fig. 1.12.1.

4. At point C link 5 is pivoted and its other end is connected to the tool (slider) 6, whose axis of motion is perpendicular to OA.
5. The crank 3 rotates in anticlockwise direction.
6. When slider B is at B', C at C', the cutting tool will be in the extreme left position.
7. As crank moves in anticlock wise direction point B' goes to point B'' via point B and C' goes to point C'' via point C and tool will be in the extreme right position and make forward stroke to cut the metal.
8. So the time taken for the forward stroke of the slider is proportional to the obtuse angle $B'AB''(\theta)$ at point A.
9. In backward stroke, the time taken for the backward stroke of the slider is proportional to acute angle $B''AB'(\beta)$ at point A.
10. So the time taken in forward stroke is longer than the time taken in backward stroke.

$$\frac{\text{Time of cutting}}{\text{Time of return stroke}} = \frac{\theta}{\beta} \quad \{\theta + \beta = 360°\}$$

B. Crank and Slotted Lever Quick Return Motion Mechanism :

1. In this mechanism link AC is fixed. Link AB has a slotted bar AP in which slider B slides.
2. Consider the crank BC rotates in clockwise direction. The cutting tool is connected with link AP through a link PR. Link PR transmits the motion from AP to the cutting tool and reciprocates along the line of stroke R_1R_2 which is perpendicular to AC.

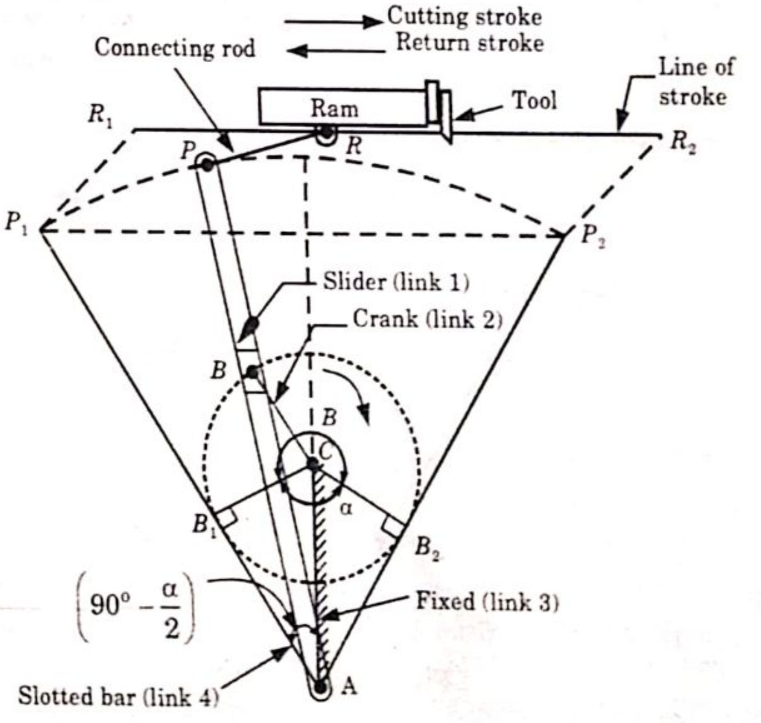


Fig. 1.12.2.

3. In extreme positions of cutting tool, AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of the stroke.

4. For cutting stroke, crank rotates from CB_1 to CB_2 (angle β at point C) and for return stroke, crank rotates from CB_2 to CB_1 (angle α at point C).

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{360° - \alpha}{\alpha} = \frac{\beta}{360° - \beta}$$

5. Length of stroke $= R_1R_2 = P_1P_2$

$$= 2P_1Q = 2AP_1 \sin \angle P_1AQ$$

Que 1.13. **In a crank and slotted lever quick return motion mechanism, the distance between the fixed centers is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke. If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.**

Answer

Given : $AC = 240$ mm, $CB_1 = 120$ mm, $AP_1 = 450$ mm

To Find :
i. Inclination of the slotted bar with the vertical.
ii. Time ratio of cutting stroke to the return stroke.
iii. Length of the stroke.

1. From Fig. 1.13.1

$$\sin \angle CAB_1 = \sin\left(90° - \frac{\alpha}{2}\right) = \frac{B_1C}{AC} = \frac{120}{240} = 0.5$$

$$\therefore \quad \angle CAB_1 = \sin^{-1}(0.5) = 30°$$

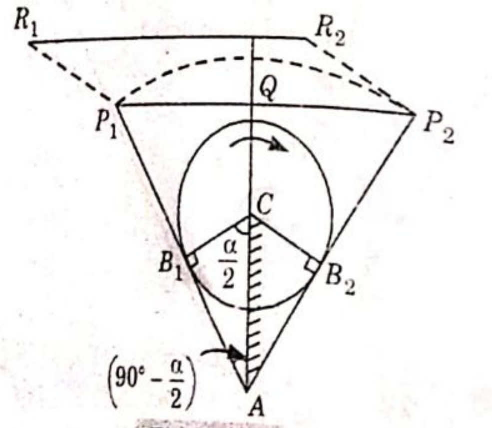


Fig. 1.13.1.

$$\therefore \quad 90° - \alpha/2 = 30°$$

$$\alpha/2 = 90° - 30° = 60°$$

Inclination of the slotter bar with vertical,

$$\alpha = 2 \times 60° = 120°$$

2. Time ratio $= \dfrac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \dfrac{360° - \alpha}{\alpha} = \dfrac{360° - 120°}{120°} = 2$

3. Length of the stroke,

$$R_1R_2 = P_1P_2 = 2\,P_1Q = 2\,AP_1 \sin (90° - \alpha/2)$$
$$= 2 \times 450 \times \sin (90° - 60°) = 900 \times 0.5 = 450 \text{ mm}$$

PART-8

Double Slider Crank Chain.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 1.14. **What is double slider crank chain ? Discuss its various inversions with neat sketches.**

Answer

A. **Double Slider Crank Chain :** A kinematic chain consists of two turning pairs and two sliding pairs is known as double slider crank chain. In this, two pairs of same kind are adjacent.

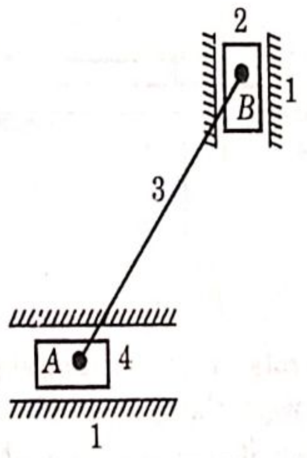


Fig. 1.14.1.

3. When sliders move, a point C (except midpoint of AB) on AB will trace an ellipse.

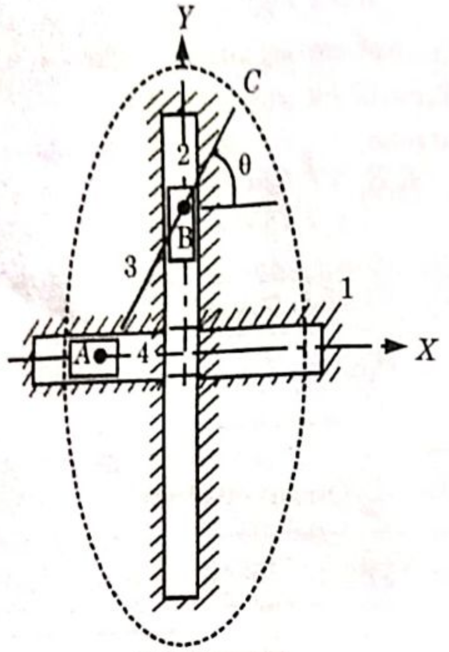


Fig. 1.14.2.

ii. Scotch Yoke Mechanism :

1. It is obtained by fixing any of the slide blocks of the chain.
2. When link 4 is fixed, the end B of the crank 3 rotates about A and link 1 reciprocates along horizontal direction.

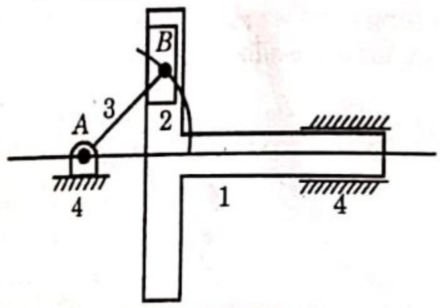


Fig. 1.14.3.

3. When link 3 (Crank) rotates, the horizontal portion of link 1 slides in the fixed link 4 and converts the rotary motion of crank into reciprocating motion of link 1. Hence, it is used to convert the rotary motion into a sliding motion.

iii. Oldham's Coupling :

1. An Oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart.
2. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed.

3. This inversion is obtained by fixing the link 2, as shown in Fig. 1.14.4(*a*).
4. The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.
5. The link 1 and link 3 form turning pairs with link 2. These flanges have diametrical slots cut in their inner faces, as shown in Fig. 1.14.4(*b*).
6. The intermediate piece (link 4) which is a circular disc, have two tongues (*i.e.*, diametrical projections) T_1 and T_2 on each face at right angles to each other, as shown in Fig. 1.14.4(*c*).
7. The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link 3). The link 4 can slide or reciprocate in the slots in the flanges.

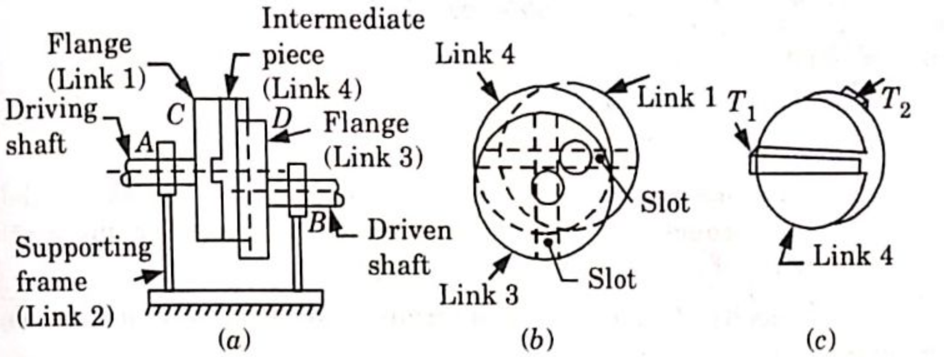


Fig. 1.14.4.

PART-9

Velocity Analysis Introduction, Velocity of Point in Mechanism, Relative Velocity Method, Velocities in Four Bar Mechanism.

Questions-Answers

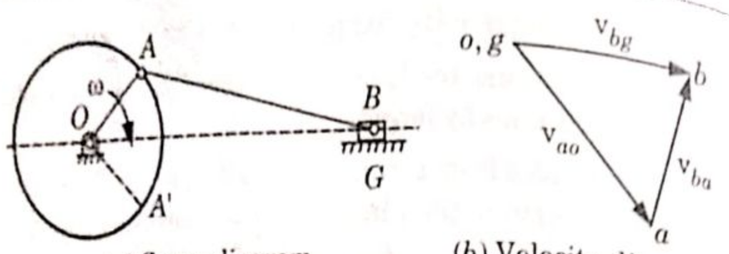
Long Answer Type and Medium Answer Type Questions

Que 1.15. **Find out the velocity of slider in a slider crank mechanism.**

Answer

1. Fig. 1.15.1(*a*) shows a slider crank mechanism. Consider crank *OA* is moving with uniform angular velocity ω rad/s in clockwise direction. This rotary motion transforms into linear velocity of slider *B*.
2. From vector equation,

$$v_{bo} = v_{ba} + v_{ao}$$



(a) Space diagram. (b) Velocity diagram.

Fig. 1.15.1.

3. As G is a fixed guide and O is also a fixed point, so these points can be considered as a single point in velocity vector diagram. Hence

$$v_{bo} = v_{bg} = v_{ba} + v_{ao}$$

$$\overline{gb} = \overline{ab} + \overline{oa}$$

4. Velocity diagram is drawn as discussed below :
   i. Draw $v_{ao} = \omega . OA$ and direction $\perp$ to OA.
   ii. v_{ba} is $\perp$ to AB, so draw a line $\perp$ to AB through a.
   iii. As slider moves in a linear direction, so draw a line ob parallel to OB through point o in velocity vector diagram. The intersection point of line ob and ab gives us a point b.
   iv. Velocity of slider can be determined by measuring the line $\overline{ob}$.

Que 1.16. **A single slider crank chain mechanism shown in Fig. 1.16.1 having crank OA = 20 cm, connecting rod AP = 70 cm and angular velocity of crank is 10 radian per second. Find the velocity of piston P, angular velocity of link PA and the velocity of point B at a distance of 20 cm from A on link AP when $\theta = 45°$.**

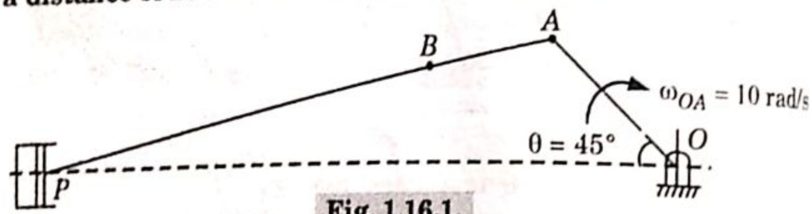


Fig. 1.16.1.

Answer

Given : OA = 20 cm, AP = 70 cm, ω_{OA} = 10 rad/s, $\theta = 45°$.

To Find :
i. Velocity of piston P.
ii. Angular velocity of link PA.
iii. Velocity of point B at a distance of 20 cm from A.

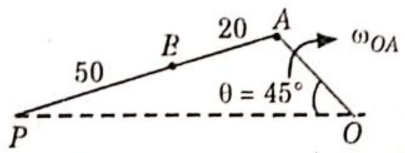
1. Velocity of crank OA, $v_{AO} = OA \times \omega_{OA}$
$= 0.2 \times 10 = 2$ m/s
2. Assuming suitable scale, the space and velocity diagram for the given problem are drawn as shown in Fig. 1.16.2(a) and (b) respectively.
3. By measurement from velocity diagram,

Velocity of piston P, v_p = Vector op = 3.5 cm = 3.5 × 0.5 = 1.75 m/s
Velocity of link PA, v_{PA} = Vector ap = 3 cm = 3 × 0.5 = 1.5 m/s

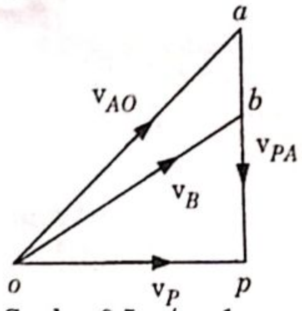
4. Angular velocity of link $PA = \dfrac{v_{PA}}{PA} = \dfrac{1.5}{0.7} = 2.14$ rad/s

5. Now,
$$\frac{AB}{AP} = \frac{ab}{ap}$$
$$\frac{20}{70} = \frac{ab}{3}$$
$$ab = 0.85 \text{ cm}$$

6. Velocity of B, v_B = Vector ob = 3.7 cm = 3.7 × 0.5 = 1.85 m/s



Scale : 1 mm = 1 cm
(a) Space diagram.



Scale : 0.5 m/s = 1 cm
(b) Velocity diagram.

Fig. 1.16.2.

Que 1.17. **In a slider crank mechanism, in Fig. 1.17.1, the length of crank *OB* and connecting rod *AB* are 125 mm and 500 mm respectively. The centre of gravity *G* of the connecting rod is 275 mm from the slider *A*. The crank speed is 600 rpm clockwise. When the crank has turned 45° from the inner dead center position, determine :**

i. Velocity of the slider *A*,
ii. Velocity of the point *G*, and
iii. Angular velocity of the connecting rod *AB*.

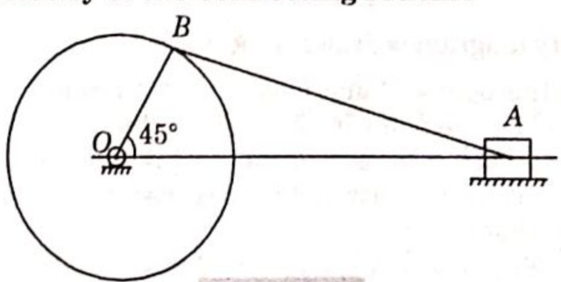


Fig. 1.17.1.

Answer

Same as Q. 1.16, Page 1–22B, Unit-1.

(**Answer :**
i. Velocity of slider A = 6.60 m/s
ii. Velocity of point u = 6.80 m/s
iii. Angular velocity of connecting rod AB = 11.6 rad/s.)

Que 1.18. In Fig. 1.18.1, the angular velocity of the crank OA is 600 rpm. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are : OA = 28 mm, AB = 44 mm, BC = 49 mm and BD = 46 mm. The centre distance between the centres of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.

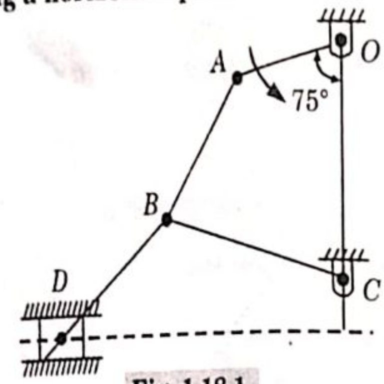


Fig. 1.18.1.

Answer

Given : N_{AO} = 600 rpm or $\omega_{AO} = 2\pi \times 600/60 = 62.84$ rad/s,
OA = 28 mm, AB = 44 mm, BC = 49 mm, BD = 46 mm.
To Find : i. The linear velocity of the slider D.
ii. Angular velocity of the link BD.

1. Since OA = 28 mm = 0.028 m, therefore velocity of A with respect to O or velocity of A,

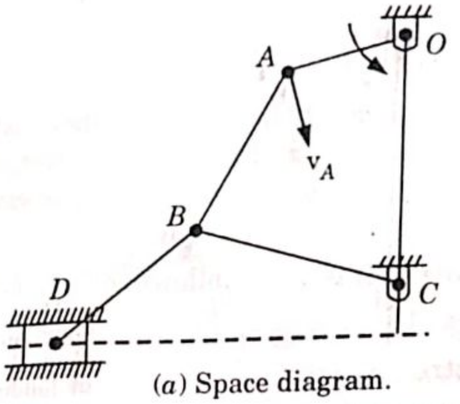
$$v_{AO} = v_A = \omega_{AO} \times OA = 62.84 \times 0.028$$
$$= 1.76 \text{ m/s} \quad \text{(Perpendicular to } OA)$$

2. The velocity diagram is drawn as discussed below :
   i. Since the points O and C are fixed, therefore these points are marked as one point in the velocity diagram. Now from point o, draw vector oa perpendicular to OA to some suitable scale to represent the velocity of A with respect to O or simply velocity of A such that

      Vector $oa = v_{AO} = v_A = 1.76$ m/s

   ii. From point a, draw vector ab perpendicular to AB to represent the velocity of B with respect to A and from point c, draw vector cb perpendicular to CB to represent the velocity of B. The vectors ab and cb intersect at b.
   iii. From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B and from point o, draw vector od parallel to the path of motion of the slider D which is horizontal.

to represent the velocity of D. The vectors bd and od intersect at d.



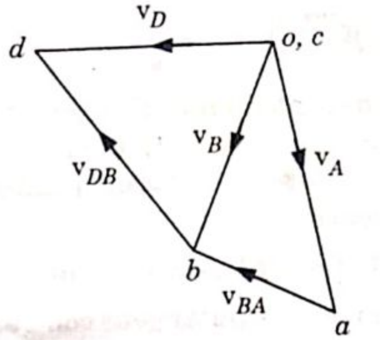


Fig. 1.18.2.

3. By measurement from velocity diagram,
   Velocity of the slider D, v_D = Vector od = 1.6 m/s
   Velocity of D with respect to B, v_{DB} = Vector bd = 1.7 m/s

4. Angular velocity of the link BD, $\omega_{BD} = \dfrac{v_{DB}}{BD} = \dfrac{1.7}{0.046}$
   = 36.96 rad/s (Clockwise about B)

PART-10

Instantaneous Centre.

CONCEPT OUTLINE

Formula for Finding the Number of Instantaneous Centre :

$$N = \frac{n(n-1)}{2}$$

Where, n = Number of links.

Arnold Kennedy Theorem : It states that "if three bodies move relative to each other, they have three instantaneous centres and lie on a straight line".

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.19. **Define instantaneous centre of rotation. Give the types of instantaneous centres. With the help of neat sketch show all the**

types of instantaneous centre of four bar mechanism and slider crank mechanism.

Answer

A. **Instantaneous Centre of Rotation :** It is the point about which motion of a body having both rotatory and translatory motion is assumed to be purely rotational. It is also known as centro or virtual centre.

B. **Types of Instantaneous Centre :** It is of the following three types :

1. Fixed instantaneous centre.
2. Permanent instantaneous centre.
3. Neither fixed nor permanent instantaneous centre.

C. **Instantaneous Centre of Rotation for a Four Bar Mechanism :**

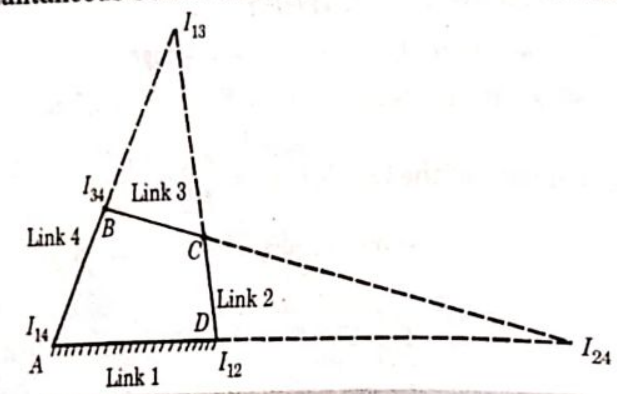


Fig. 1.19.1. Instantaneous centre of rotations in a four bar mechanism.

1. For a four bar mechanism, $n = 4$

$\therefore$ Number of instantaneous centres, $N = \dfrac{n(n-1)}{2} = \dfrac{4(4-1)}{2} = 6$

2. So, there are six instantaneous centres of rotation present in a four bar mechanism. In Fig. 1.19.1, these centres of rotation are shown.
3. I_{12} and I_{14} are called fixed instantaneous centres as they are at fixed joints.
4. The instantaneous centres I_{23} and I_{34} are permanent instantaneous centres as they are at joint and always remain at the joints.
5. Whereas I_{13} and I_{24} are neither fixed nor permanent instantaneous centres as they move with the motion of mechanism.

D. **Instantaneous Centre of Rotation for a Slider Crank Mechanism :**

1. For a slider crank mechanism, $n = 4$

$\therefore$ Number of instantaneous centres, $N = \dfrac{n(n-1)}{2} = \dfrac{4(4-1)}{2} = 6$

2. Locate the instantaneous centres of rotation. Here I_{12}, I_{34}, I_{23} are the primary (fixed and permanent type) centre of rotation. But slider moves on a straight line hence its instantaneous centre I_{14} will be at infinity.
3. According to Kennedy theorem, draw a circle and mark points equal to number of links in mechanism as 1, 2, 3, 4 on the circle to indicate I_{12}, I_{23}, I_{34} and I_{14}.
4. Joining 1 to 3 form two triangles 123 and 341 in the circle diagram. The side 13, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{13} will lie on the intersection of $I_{12}\,I_{23}$ and $I_{14}\,I_{34}$, produced if necessary. Thus centre I_{13} is located. Join 1 to 3 by a dotted line on the circle diagram and mark number I_{13} on it.
5. Join 2 to 4 by a dotted line to form two triangles 234 and 124. The side 24, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{24} lies on the intersection of $I_{23}\,I_{34}$ and $I_{12}\,I_{14}$. Join 2 to 4 by a dotted line and mark I_{24} on it.
6. For a particular link angular velocity will be same.

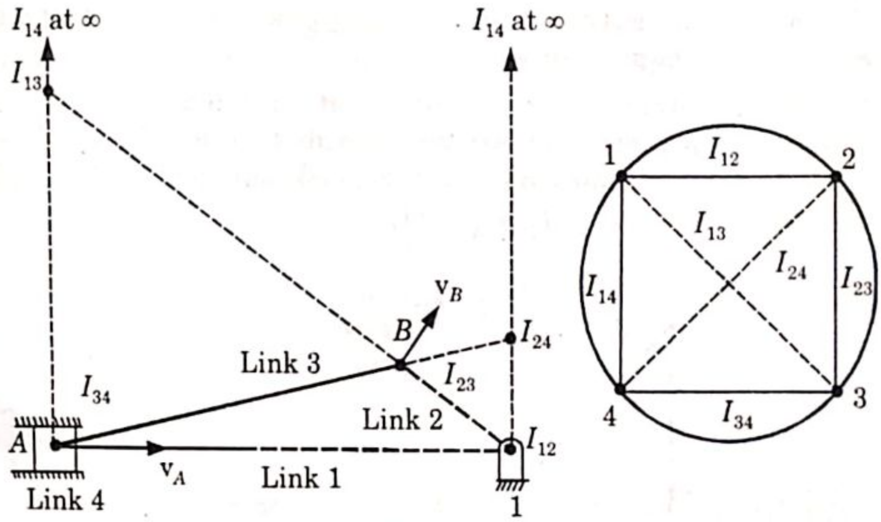


Fig. 1.19.2.

Que 1.20. **Locate all the instantaneous centres of the slider crank mechanism as shown in Fig. 1.20.1. The lengths of crank *OB* and connecting rod *AB* are 100 mm and 400 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s find : (i) Velocity of the slider '*A*' and (ii) Angular velocity of the connecting rod *AB*.**

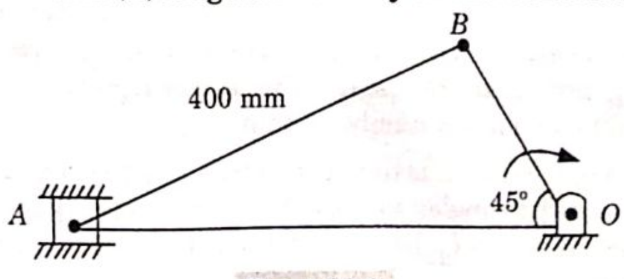


Fig. 1.20.1.

Answer

Given : $\omega_{OB} = 10$ rad/s, $OB = 100$ mm $= 0.1$ m,

$AB = 400$ mm $= 0.4$ m

To Find : i. Velocity of slider A.
ii. Angular velocity of connecting rod AB.

1. We know that linear velocity of the crank OB

$$v_{OB} = v_B = OB \times \omega_{OB} = 10 \times 0.1 = 1 \text{ m/s}$$

2. Since there are four links (*i.e.*, $n = 4$), therefore the number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12}, I_{23} and I_{34} as shown in Fig. 1.20.2. Since the slider (link 4) moves on a straight surface (link 1), therefore the instantaneous center I_{14} will be at infinity.

4. Locate the other two remaining neither fixed nor permanent instantaneous centres, by Arnold Kennedy theorem. This is done by circle diagram as shown in Fig. 1.20.2. Mark four points 1, 2, 3 and 4 on the circle to indicate I_{12}, I_{23}, I_{34} and I_{14}.

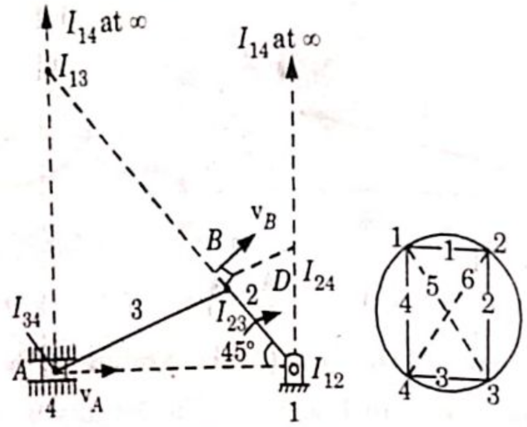


Fig. 1.20.2.

5. Join 1 to 3 to form two triangles 123 and 341 in the circle diagram. The side 13, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{13} will lie on the intersection of $I_{12}I_{23}$ and $I_{14}I_{34}$, produced if necessary. Thus centre I_{13} is located. Join 1 to 3 by a dotted line and mark number 5 on it.

6. Join 2 to 4 by a dotted line to form two triangles 234 and 124. The side 24, common to both triangles, is responsible for completing the two triangles. Therefore the centre I_{24} lies on it. Thus all the six instantaneous centres are located.

7. By measurement, we find that

$$I_{13}A = 460 \text{ mm} = 0.46 \text{ m and } I_{13}B = 560 \text{ mm} = 0.56 \text{ m}$$

8. We know that $\dfrac{v_A}{I_{13}A} = \dfrac{v_B}{I_{13}B}$

Velocity of slider A, $v_A = v_B \times \dfrac{I_{13}A}{I_{13}B} = 1 \times \dfrac{0.46}{0.56} = 0.82$ m/s

9. Also, $\dfrac{v_A}{IA} = \dfrac{v_B}{I_{13}B} = \omega_{AB}$

$\therefore$ $\omega_{AB} = \dfrac{v_B}{I_{13}B} = \dfrac{1}{0.56} = 1.78$ rad/s

Que 1.21. **Locate all the instantaneous centers for the crossed four bar mechanism as shown in Fig. 1.21.1. The dimensions of various links are : CD = 65 mm, CA = 60 mm, DB = 80 mm and AB = 55 mm. Find the angular velocities of the links AB and DB, if the crank CA rotates at 100 rpm in the anticlockwise direction.**

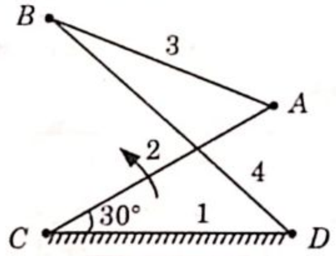


Fig. 1.21.1.

Answer

Given : CD = 65 mm, CA = 60 mm, DB = 80 mm, AB = 55 mm

N_{AC} = 100 rpm

To Find : Angular velocities of the link AB and DB.

1. Number of I-centre $= \dfrac{n(n-1)}{2} = \dfrac{4(4-1)}{2} = 2 \times 3 = 6$
2. All the instantaneous centres of given mechanism are shown in Fig. 1.21.2.

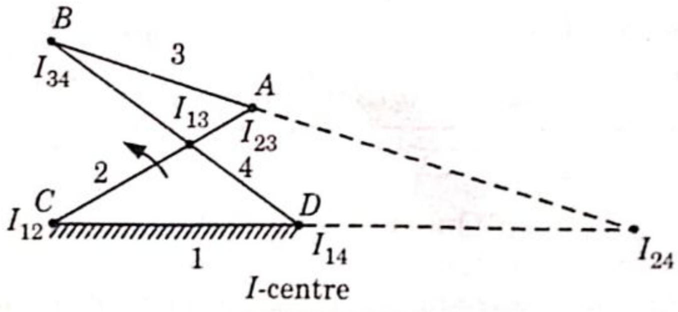


Fig. 1.21.2.

3. Angular velocity of link AC,

$$\omega_2 = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60} = 10.47 = 10.5 \text{ rad/s}$$

$\therefore$ $$v_{AC} = v_2 = \omega AC = 10.5 \times 60 = 630 \text{ mm/s}$$

4. We know that, $$\frac{v_2}{I_{23}C} = \frac{v_3}{I_{23}B}$$

$$\frac{630}{60} = \frac{v_3}{55}$$

$$v_3 = 577.5 \text{ mm/s}$$

$\therefore$ $$\omega_{AB} = \frac{v_3}{I_{24}A} = \frac{577.5}{10.5} = 55 \text{ rad/s}$$

5. Also, $$\frac{v_3}{I_{34}A} = \frac{v_4}{I_{34}D}$$

$$\frac{577.5}{55} = \frac{v_4}{80}$$

$$v_4 = 840 \text{ mm/s}$$

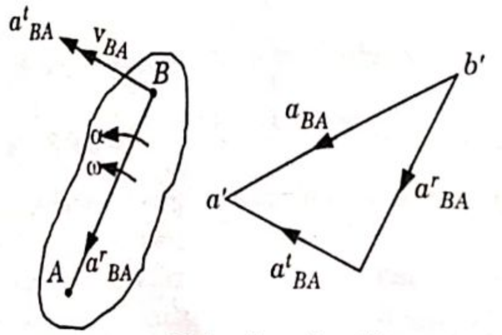
$\therefore$ $$\omega_{BD} = \frac{v_4}{I_{13}D} = \frac{840}{35} = 24 \text{ rad/s}$$

PART-11

Acceleration Analysis Introduction, Acceleration of a Point on a Link, Acceleration Diagram.

CONCEPT OUTLINE

Acceleration Diagram of a Link :



(*a*) Link (*b*) Acceleration diagram

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.22. **What do you understand by the term 'acceleration' ? What are the types of acceleration ?**

Answer

A. Acceleration :

1. The rate of change of velocity with respect to time is called acceleration.
2. It acts in the direction of the change in velocity. It is a vector quantity.
3. Acceleration comes into existence due to change in either magnitude or direction of the velocity or both magnitude and direction of the velocity.
4. To find acceleration of a point on a link (whether it is linear or angular), it is required to find velocity at that point (linear or angular) first.

B. Types of Acceleration : It is of three types :

i. **Tangential Acceleration :** Acceleration in tangential direction is called tangential acceleration. It is parallel to the velocity of the particle at the given instant.

ii. **Radial or Centripetal Acceleration :** Acceleration towards the centre of rotation is known as centripetal acceleration. It is perpendicular to the velocity of the particle at the given instant.

iii. **Coriolis Acceleration :** If the distance between two points varies that is the second point which was stationary now slides, the total acceleration will contain one additional components, known as coriolis component.

Que 1.23. **In a pin jointed four bar mechanism *ABCD*, the lengths of various links are as follows :**
AB = 25 mm; BC = 87.5 mm; CD = 50 mm and AD = 80 mm. The link AD is fixed and the angle BAD = 135°. If the velocity of B is 1.8 m/s in the clockwise direction, find velocity and acceleration of the mid point of BC.

Answer

Given : AB = 25 mm, BC = 87.5 mm, CD = 50 mm, AD = 80 mm, $\angle BAD$ = 135°, v_B = 1.8 m/s
To Find : Velocity and acceleration of the mid point of BC.

1. The space diagram, velocity diagram and acceleration diagram for the given problem are shown in Fig. 1.23.1(*a*), (*b*) and (*c*) respectively.
2. By measurement from velocity diagram, we have
   i. Velocity of mid point of BC *i.e.*, M,
   $$v_M = \text{Vector } am = 2.8 \text{ cm} = 2.8 \times 0.6 = 1.68 \text{ m/s}$$
   ii. Velocity of link BC,
   $$v_{CB} = \text{Vector } bc = 1.25 \text{ cm} = 1.25 \times 0.6 = 0.75 \text{ m/s}$$
   iii. Velocity of C, v_C = Vector dc = 2.8 cm = 2.8 × 0.6 = 1.68 m/s

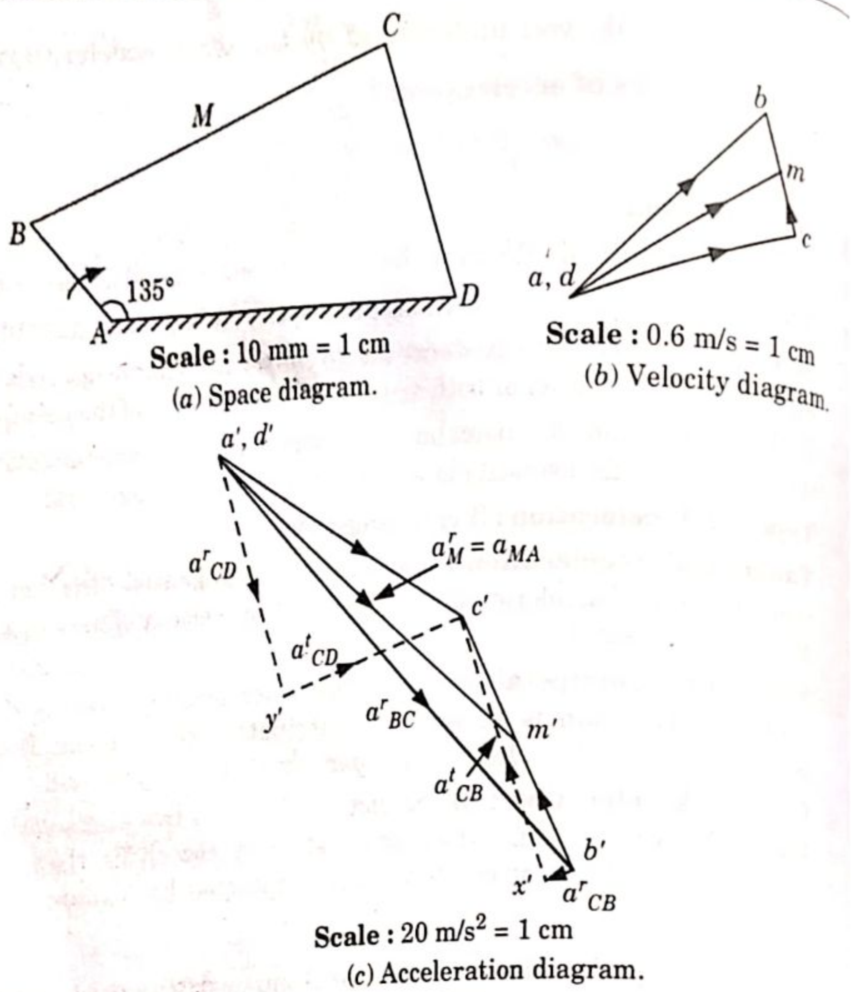


Fig. 1.23.1.

3. Now for drawing the acceleration diagram following component of acceleration are required :

S. No.	Acceleration Component	Magnitude	Direction
1.	Radial component of link AB *i.e.*, a^r_{BA}	$\dfrac{v_B^2}{AB} = \dfrac{(1.8)^2}{0.025} = 129.6\ \text{m/s}^2$	$\parallel$ to AB
2.	Radial component of link BC *i.e.*, a^r_{CB}	$\dfrac{v_{CB}^2}{BC} = \dfrac{(0.75)^2}{0.0875} = 6.43\ \text{m/s}^2$	$\parallel$ to BC
3.	Tangential component of link BC *i.e.*, a^t_{CB}	—	$\perp$ to BC
4.	Radial component of link CD *i.e.*, a^r_{CD}	$\dfrac{v_C^2}{CD} = \dfrac{(1.68)^2}{0.05} = 56.448\ \text{m/s}^2$	$\parallel$ to CD
5.	Tangential component of link CD *i.e.*, a^t_{CD}	—	$\perp$ to CD

4. By measurement from acceleration diagram,
Acceleration of mid point of BC,

$$a_M = \text{Vector } a'm' = 5.4 \text{ cm} = 5.4 \times 20 = 108 \text{ m/s}^2$$

Que 1.24. **Draw the acceleration diagram of a slider crank mechanism.**

Answer

1. A slider crank mechanism is shown in Fig. 1.24.1(a).
2. Let the crank OB makes an angle θ with the inner dead centre and rotates in clockwise direction about the fixed point O with uniform angular velocity ω_{BO} rad/s.

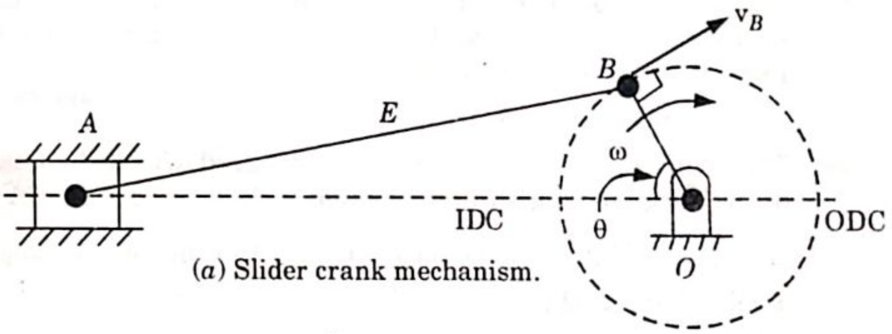
∴ Velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB, \text{ acting tangentially at } B.$$

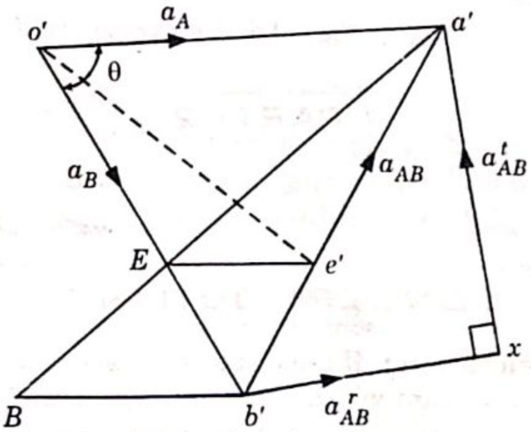
3. We know that centripetal or radial acceleration of B with respect to O or acceleration of B,

$$a_{BO}^r = a_B = \omega_{BO}^2 \times OB = \frac{v_{BO}^2}{OB}$$

4. The acceleration diagram, as shown in Fig. 1.24.1(b), may now be drawn as discussed below :



(a) Slider crank mechanism.



(b) Acceleration diagram.

Fig. 1.24.1. Acceleration in the slider crank mechanism.

i. Draw vector $o'b'$ parallel to BO and set off equal in magnitude to $a^r_{BO} = a_B$, to some suitable scale.

ii. From point b', draw vector $b'x$ parallel to BA. The vector $b'x$ represents the radial component of the acceleration of A with respect to B whose magnitude is given by,

$$a^r_{AB} = v^2_{AB} / AB$$

Since the point B moves with constant angular velocity, therefore there will be no tangential component of the acceleration.

iii. From point x, draw vector xa' perpendicular to $b'x$ (or AB). The vector xa' represents the tangential component of the acceleration of A with respect to B *i.e.*, a^t_{AB}.

iv. Since the point A reciprocates along AO, therefore the acceleration must be parallel to velocity. Therefore from o', draw $o'a'$ parallel to AO, intersecting the vector xa' at a'.

Now the acceleration of the piston or the slider A (a_A) and a^t_{AB} may be measured to the scale.

v. The vector $b'a'$, which is the sum of the vectors $b'x$ and xa', represents the total acceleration of A with respect to B *i.e.*, a_{AB}. The vector $b'a'$ represents the acceleration of the connecting rod AB.

vi. The acceleration of any other point on AB such as E may be obtained by dividing the vector $b'a'$ at e', in the same ratio as E divides AB in Fig. 1.24.1(a). In other words

$$a'e' / a'b' = AE/AB$$

vii. The angular acceleration of the connecting rod AB may be obtained by dividing the tangential component of the acceleration of A with respect to $B(a^t_{AB})$ to the length of AB. In other words, angular acceleration of AB,

$$\alpha_{AB} = a^t_{AB} / AB \text{ (Clockwise about } B)$$

PART-12

Coriolis Component of Acceleration, Crank and Slotted Lever Mechanism.

CONCEPT OUTLINE

Coriolis Acceleration : If the distance between two points varies that is the second point which was stationary now slides, the total acceleration will contain one additional components, known as coriolis component.

$$a^{cr} = 2\,v\omega$$

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Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.25. Explain Coriolis component of acceleration. Give the conditions for its existence and show that the magnitude of Coriolis component of acceleration is 2vω. Where v is the linear velocity of slider.

Answer

A. Coriolis Component of Acceleration :

1. When a slider is free to move on a link which is free to rotate, slider has a linear velocity as well as rotational velocity.
2. This type of motion provides an additional component of acceleration to the slider (or sleeve) which is known as Coriolis component of acceleration.

B. Condition for Coriolis Component : Coriolis component exists only if there are two coincident points which have two types of motion :

i. Linear relative velocity of sliding.

ii. Angular motion about fixed finite centres of rotation.

C. Derivation :

1. Let a link AR is fixed at A and P is a point on slider that is at point Q initially (Point Q is on link AR).
2. Let,

ω = Angular velocity of the link,

α = Angular acceleration of the link,

v = Linear velocity of the slider on the link,

a = Linear acceleration of the slider on the link, and

r = Radial distance of point P on the slider.

3. Let after δt time, $\delta\theta$ be the angular displacement of link AR and δr be the radial displacement of the slider P on link AR in outward direction.
4. After time δt,

Angular velocity of link AR, $\omega' = \omega + \alpha\delta t$

Linear velocity of slider P on link AR, $v' = v + a\delta t$

Radial distance moved by slider P on link AR, $r' = r + \delta r$

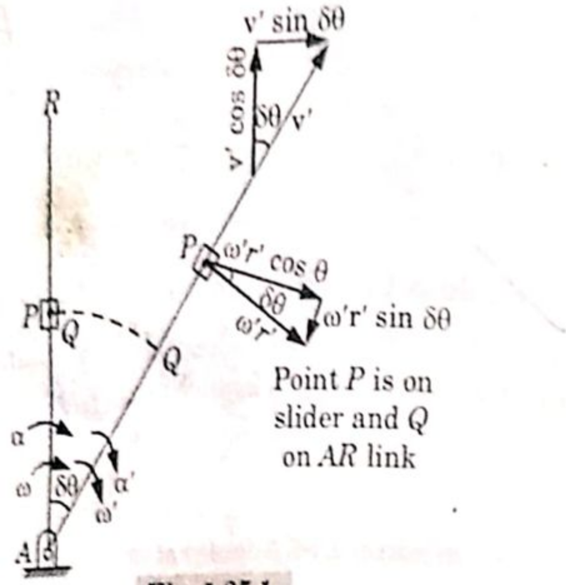


Fig. 1.25.1.

5. Now consider the acceleration of point P (or slider) in two directions.

i. Acceleration of P Parallel to AR :

1. Initial velocity of P along AR = v
   Final velocity of P along AR = $v' \cos \delta\theta - \omega' r' \sin \delta\theta$
2. Acceleration of P parallel to AR

$$= \frac{\text{Final velocity along } AR - \text{Initial velocity along } AR}{\delta t}$$

$$= \frac{(v' \cos \delta\theta - \omega' r' \sin \delta\theta) - v}{\delta t} \qquad \ldots(1.25.1)$$

3. Putting the values of v', ω' and r' in the eq. (1.25.1), we get
   Acceleration of P parallel to AR

$$= \frac{[(v + \alpha \delta t) \cos \delta\theta - (\omega + \alpha \delta t)(r + \delta r) \sin \delta\theta] - v}{\delta t}$$

4. For limiting conditions, as $\delta t \to 0$, $\cos \delta\theta \to 1$ and $\sin \delta\theta \to \delta\theta$ and neglecting the smaller terms, we have
   Acceleration of P parallel to AR

$$= \frac{[v + \alpha \delta t - \omega r \delta\theta] - v}{\delta t} = a - \omega r \frac{d\theta}{dt} = a - \omega^2 r$$

= Acceleration of slider – Centripetal acceleration.

ii. Acceleration of P Perpendicular to AR :

1. Initial velocity of $P \perp$ to $AR = \omega r$
   Final velocity of $P \perp$ to AR = $v' \sin \delta\theta + \omega' r' \cos \delta\theta$
   Change of velocity $\perp$ to AR = $(v' \sin \delta\theta + \omega' r' \cos \delta\theta) - \omega r$
2. Acceleration of $P \perp$ to AR

$$= \frac{(v + \alpha \delta t) \sin \delta\theta + (\omega + \alpha \delta t)(r + \delta r) \cos \delta\theta - \omega r}{\delta t}$$

3. For limiting conditions $\delta t \to 0$, $\cos \delta\theta \to 1$ and $\sin \delta\theta \to \delta\theta$

$$\text{Acceleration of } P \perp \text{ to } AR = \frac{v\,d\theta}{dt} + \frac{(\omega + \alpha\ \delta t)(r + \delta r) - \omega r}{\delta t}$$

$$= \frac{v\,d\theta}{dt} + \left[\frac{\omega r + \omega\delta r + r\alpha\delta t + \alpha\delta t\delta r - \omega r}{\delta t}\right]$$

(Neglecting the smaller terms)

$$= \frac{v\,d\theta}{dt} + \omega\frac{dr}{dt} + r\alpha$$

$$= v\omega + v\omega + r\alpha = 2v\omega + r\alpha$$

$$= 2v\omega + \text{Tangential acceleration.}$$

Here the component $2v\omega$ is known as Coriolis component of acceleration.

4. Coriolis component is positive if both v and ω are either positive or negative. Also, the Coriolis component is positive if :
   i. Link AR rotates clockwise and the slider moves radially outwards.
   ii. Link AR rotates anticlockwise and the slider moves radially inwards.

Otherwise, the Coriolis component will be negative.

Que 1.26. **In a quick return mechanism, as shown in Fig. 1.26.1, the driving crank OA is 60 mm long and rotates at a uniform speed of 200 rpm in a clockwise direction. For the position shown, find**

i. Velocity of the ram R,

ii. Acceleration of the ram R, and

iii. Acceleration of the sliding block A along the slotted bar CD.

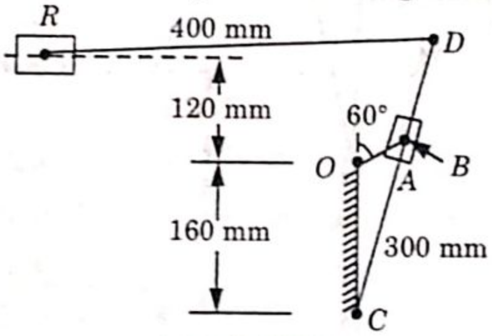


Fig. 1.26.1.

Answer

Given : N_{AO} = 200 rpm or $\omega_{AO} = 2\pi \times 200/60 = 20.95$ rad/s, OA = 60 mm = 0.06 m

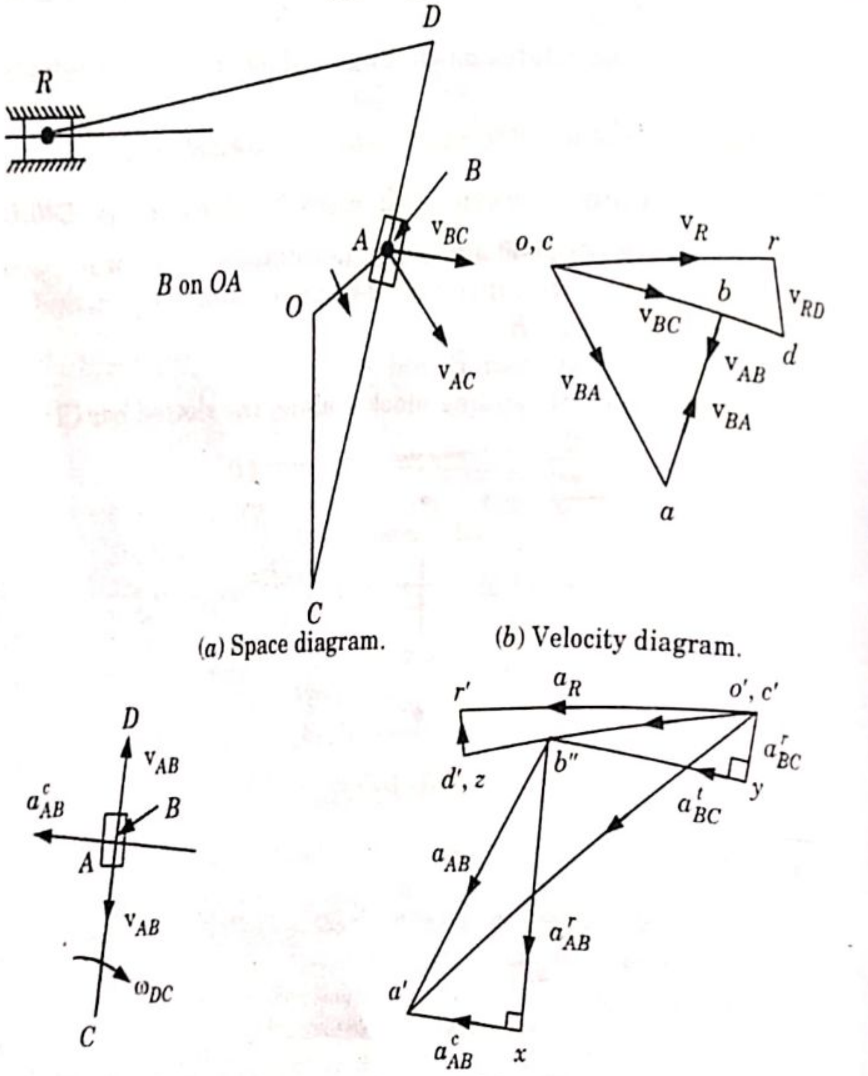
To Find :
i. Velocity of the ram R.
ii. Acceleration of the ram R.
iii. Acceleration of the sliding block A along the slotted bar CD.

1. We know that velocity of A with respect to O,

$$v_{AO} = \omega_{AO} \times OA = 20.95 \times 0.06 = 1.25 \text{ m/s}$$

(Perpendicular to OA)

2. The space diagram and velocity diagram drawn by following the usual procedure are shown in Fig. 1.26.2(*a*) and (*b*) respectively.
3. By measurement from velocity diagram,
   Velocity of the ram R, v_R = Vector or = 1.3 m/s
   Velocity of B with respect to A, v_{BA} = Vector ab = 0.875 m/s
   Velocity of B with respect to C, v_{BC} = Vector cb = 0.95 m/s
   Velocity of R with respect to D, v_{RD} = Vector dr = 0.325 m/s
   Velocity of D with respect to C,
   $$v_{DC} = \text{Vector } cd = 1.375 \text{ m/s}$$
4. Angular velocity of the link CD,
   $$\omega_{CD} = \frac{v_{DC}}{CD} = \frac{1.375}{0.3} = 4.58 \text{ rad/s} \qquad (\because \quad CD = 0.3 \text{ m})$$



(*a*) Space diagram. (*b*) Velocity diagram.

(*c*) Direction of coriolis component. (*d*) Acceleration diagram.

Fig. 1.26.2.

5. Now, for drawing the acceleration diagram as shown in Fig. 1.26.2(*d*), following components are needed :

Table 1.26.1.

S. No.	Acceleration Component	Magnitude	Direction
1.	Radial component of acceleration of A wrt O.	$a^r_{AO} = \omega^2_{AO} \times OA$ $= (20.95)^2 \times 0.06$ $= 26.33\text{ m/s}^2$	Parallel to OA.
2.	Coriolis component of acceleration of A wrt B.	$a^c_{AB} = 2\omega_{CD} \times v_{AB}$ $= 2 \times 4.58 \times 0.875$ $= 8.02\text{ m/s}^2$	Perpendicular to AC.
3.	Radial component of acceleration of A wrt B, *i.e.*, a^r_{BC}.	—	Parallel to BC.
4.	Radial component of the acceleration of B and C *i.e.*, a^r_{BC}.	$a^r_{BC} = \dfrac{v^2_{BC}}{CB} = \dfrac{(0.95)^2}{0.2}$ $= 4.5\text{ m/s}^2$	Parallel to BC
5.	Tangential component of acceleration of B wrt C *i.e.*, a^t_{BC}.	—	Perpendicular to BC.
6.	Radial component of acceleration of R wrt D *i.e.*, a^r_{RD}.	$a^r_{RD} = \dfrac{v^2_{RD}}{DR} = \dfrac{(0.325)^2}{0.4}$ $= 0.26\text{ m/s}^2$	Parallel to DR.
7.	Tangential component of acceleration of R wrt D *i.e.*, a^t_{BC}.	—	Perpendicular to DR.

6. By measurement from acceleration diagram,
   Acceleration of ram, a_R = Vector $o'r' = 9\text{ m/s}^2$
   Acceleration of sliding of the block A along the slotted lever CD,
   $$a_{AB} = \text{Vector } b''x' = 15\text{ m/s}^2$$

Que 1.27. **In a Whitworth quick return motion as shown in Fig. 1.27.1, *OA* is a crank rotating at 30 rpm in a clockwise direction. The dimensions of various links are *OA* = 150 mm, *OC* = 100 mm, *CD* = 125 mm and *DR* = 500 mm. Determine the acceleration of the sliding block *R* and angular acceleration of the slotted lever *CA*.**

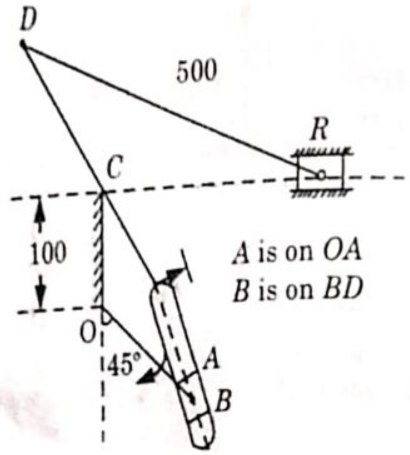


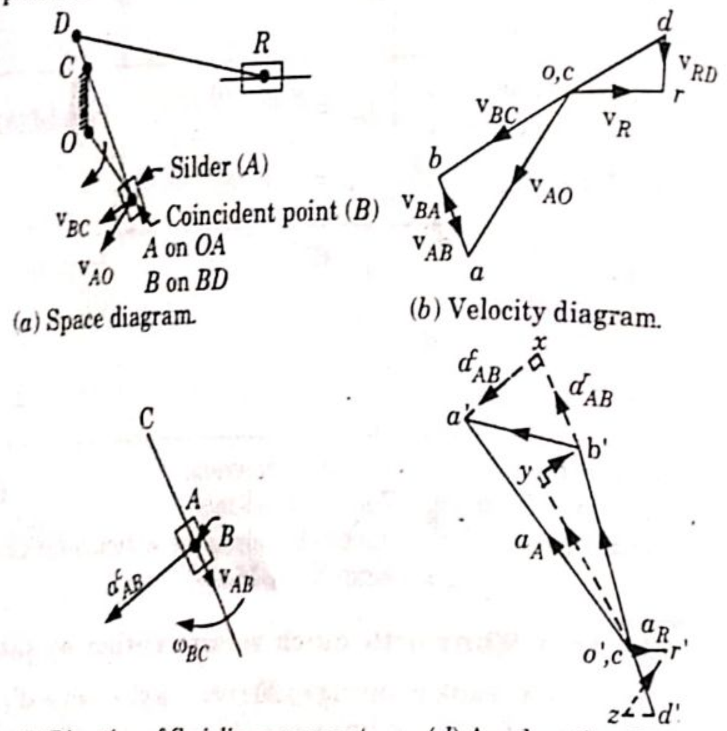
Fig. 1.27.1.

Answer

Given : $N_{AO} = 30$ rpm or $\omega_{AO} = 2\pi \times 30/60 = 3.142$ rad/s,
$OA = 150$ mm $= 0.15$ m, $OC = 100$ mm $= 0.1$ m, $CD = 0.125$ m,
$DR = 500$ mm $= 0.5$ m

To Find :
i. Acceleration of the sliding block R.
ii. Angular acceleration of the slotted lever CA.

1. We know that velocity of A with respect to O or velocity of A,
$v_{AO} = v_A = \omega_{AO} \times OA = 3.142 \times 0.15 = 0.47$ m/s (Perpendicular to O
2. The space diagram and velocity diagram drawn by following the us procedure are shown in Fig. 1.27.2(*a*) and (*b*) respectively.



(*a*) Space diagram. (*b*) Velocity diagram.

(*c*) Direction of Coriolis component. (*d*) Acceleration diagram.

Fig. 1.27.2.

3. By measurement from velocity diagram,
   Velocity of B with respect to C, v_{BC} = Vector cb = 0.46 m/s
   Velocity of A with respect B, v_{AB} = Vector ba = 0.15 m/s
   Velocity of R with respect to D, v_{RD} = Vector dr = 0.12 m/s
4. Angular velocity of the line BC,

$$\omega_{BC} = \frac{v_{BC}}{CB} = \frac{0.46}{0.24} = 1.92 \text{ rad/s (clockwise)}$$

5. Now, for drawing acceleration diagram as shown in Fig. 1.27.2(d), following components are required :

Table 1.27.1.

S. No.	Acceleration Component	Magnitude	Direction
1.	Radial component of acceleration of A wrt O *i.e.*, a^r_{AO} or a_A.	$a^r_{AO} = \frac{v^2_{AO}}{OA} = \frac{(0.47)^2}{0.15}$ $= 1.47 \text{ m/s}^2$	Parallel to OA.
2.	Coriolis component of acceleration of A wrt B *i.e.*, a^c_{AB}.	$a^c_{AB} = 2\omega_{BC}v_{AB}$ $= 2 \times 1.92 \times 0.15$ $= 0.576 \text{ m/s}^2$	Perpendicular to BC.
3.	Radial component of acceleration of A wrt B, *i.e.*, a^r_{AB}.	—	Parallel to BC.
4.	Radial component of acceleration of B wrt C *i.e.*, a^r_{BC}.	$a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(0.46)^2}{0.24}$ $= 0.88 \text{ m/s}^2$	Parallel to BC.
5.	Tangential component of acceleration of B wrt C *i.e.*, a^t_{BC}.	—	Perpendicular to BC.
6.	Radial component of acceleration of R wrt D *i.e.*, a^r_{RD}.	$a^r_{RD} = \frac{v^2_{RD}}{DR} = \frac{(0.12)^2}{0.5}$ $= 0.029 \text{ m/s}^2$	Parallel to RD.
7.	Tangential component of acceleration of R wrt D *i.e.*, a^t_{RD}.	—	Perpendicular to RD.

6. By measurement from acceleration diagram,

Acceleration of sliding block R, a_R = Vector $c'r' = 0.18 \text{ m/s}^2$

$$a^t_{BC} = \text{Vector } yb' = 0.14 \text{ m/s}^2$$

Angular acceleration of the slotted lever CA,

$$\alpha_{CA} = \alpha_{BC} = \frac{a^t_{BC}}{BC} = \frac{0.14}{0.24} = 0.583 \text{ rad/s}^2 \text{ (Anticlockwise)}$$

☺☺☺

2

UNIT

Cams, Gear and Gear Trains

CONTENTS

Part-1 : Cams : Introduction .... **2–2B to 2–6B**
Classification of Cams and Followers

Part-2 : Cam Profiles for Knife Edge .... **2–6B to 2–22B**
Roller and Flat Faced Followers for
Uniform Velocity, Uniform Acceleration

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Reverted and Planetary Gear
Trains, Sun and Planet Gear Train

PART-1

Cams : Introduction, Classification of Cams and Followers.

CONCEPT OUTLINE

Application of Cams : They are widely used in :

1. IC engines.
2. Paper cutting machines.
3. Spinning and weaving textile machineries.
4. Feed mechanism of automatic lathes etc.

Considerations affecting Choice of Cam Profile :

1. The lateral pressure is not excessive.
2. The force required to accelerate the reciprocating follower is as small as possible.
3. The base circle diameter is as large as possible.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **What is a cam ?**

Answer

1. Cam is a mechanical element which imparts desired motion to a follower by direct contact.
2. Cam may be rotating or reciprocating whereas the follower may be rotating, reciprocating or oscillating.
3. Cams are widely used in automatic machines, internal combustion engines, machine tools and printing control mechanisms.
4. Cams are manufactured usually by die casting, milling or by punching processes.

Que 2.2. **Explain with sketches the different types of cams and followers.**

Answer

A. **Classification of Followers :**

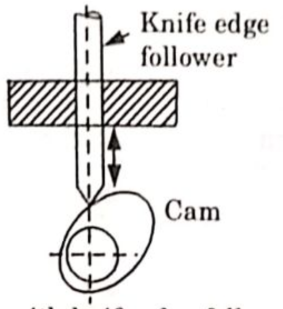
I. **According to the Surface in Contact :** The followers according to the surface in contact are as follows :

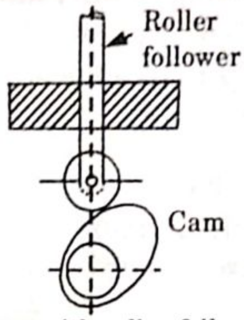
a. **Knife Edge Follower :** When the contacting end of the follower has a sharp knife edge, it is known as knife edge follower, as shown in Fig. 2.2.1(*a*).

b. **Roller Follower :** When the contacting end of the follower is a roller, it is called a roller follower, as shown in Fig. 2.2.1(*b*).

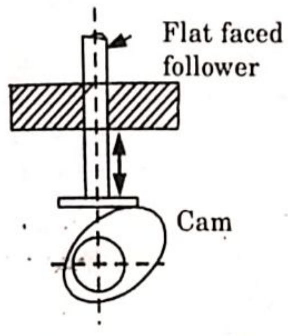
c. **Flat Faced or Mushroom Follower :** When the contacting end of the follower is a perfectly flat face, it is called a flat faced follower, as shown in Fig. 2.2.1(*c*).

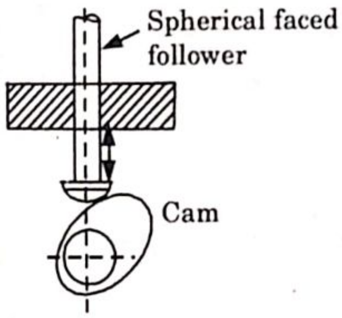
d. **Spherical Faced Follower :** When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, as shown in Fig. 2.2.1(*d*).





(*a*) Cam with knife edge follower. (*b*) Cam with roller follower.



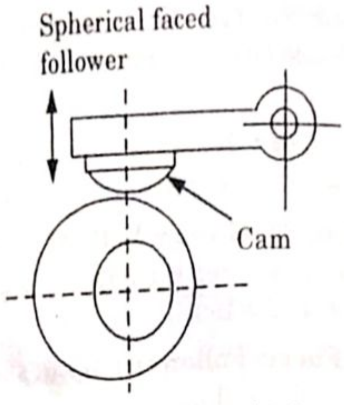


(*c*) Cam with flat faced follower. (*d*) Cam with spherical faced follower.

II. **According to the Motion of the Follower :** The followers according to its motion are of the following two types :

a. **Reciprocating or Translating Follower :** When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower. The followers as shown in Fig. 2.2.1(*a*) to (*d*) are all reciprocating or translating followers.

b. **Oscillating or Rotating Follower :** When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower as shown in Fig. 2.2.1(*e*).

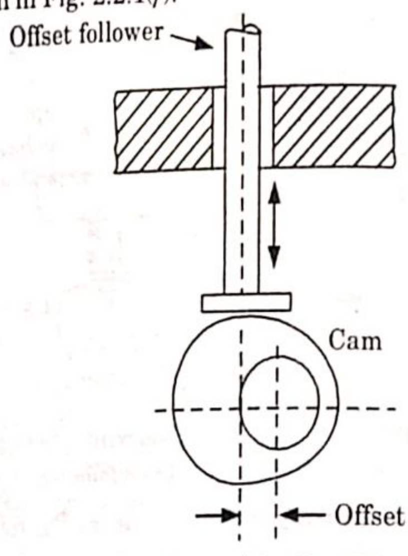


(e) Cam with spherical faced follower

III. According to the Path of Motion of the Follower : The followers, according to its path of motion are of the following two types :

a. **Radial Follower :** When the motion of the follower is along an axis passing through the center of the cam, it is known as radial follower. The followers shown in Fig. 2.2.1(*a*) to (*e*) are all radial follower.

b. **Off-set Follower :** When the motion of the follower is along an axis away from the axis of the cam center, it is called off-set follower as shown in Fig. 2.2.1(*f*).



(*f*) Cam with offset follower

Fig. 2.2.1. Classification of followers.

B. Classification of Cams : The cams may be classified as follows :

i. **Radial or Disc Cam :** In radial cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis. The cams as shown in Fig. 2.2.1(*a*), (*b*), (*c*), (*d*), (*e*) and (*f*) are all radial cams.

ii. **Cylindrical Cam :** In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis.

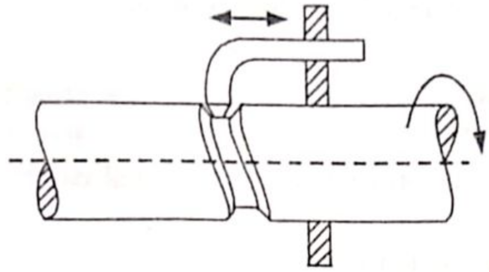


Fig. 2.2.2. Cylindrical cam with reciprocating follower.

Que 2.3. **Compare the performance of knife edge, roller and mushroom followers.**

Answer

S. No.	Knife Edge Follower	Roller Follower	Mushroom Follower
1.	It has limited use as it produces maximum wear.	It is widely used follower.	It also has limited use as it produces high surface stresses and wear.
2.	It has poor performance.	It has good performance.	Its performance lies in between the knife edge and roller follower.
3.	Problem of jamming.	Problem of jamming.	No problem of jamming.

Que 2.4. **Explain the terminology used in cam with a neat sketch.**

Answer

i. Trace Point :

1. Trace point is a reference point on the follower and is used to generate the pitch curve.
2. For knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the centre of the roller represents the trace point.

ii. Pitch Point : It is a point on the pitch curve having the maximum pressure angle.

iii. Pitch Curve and Pitch Circle :

1. Pitch curve is generated by the trace point as follower moves relative to the cam.

2. Pitch circle is drawn from the centre of the cam through the pitch points.

iv. **Lift or Stroke :** It is the maximum distance traversed by the follower from its lowest position to the topmost position.

v. **Base Circle :** It is the smallest circle that can be drawn to the cam profile.

vi. **Prime Circle :** It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve.

vii. **Pressure Angle :** It is the angle between the direction of the follower motion and a normal to the pitch curve. While designing a cam profile pressure angle is an important term. For a large value of pressure angle, a reciprocating follower will jam in its bearings.

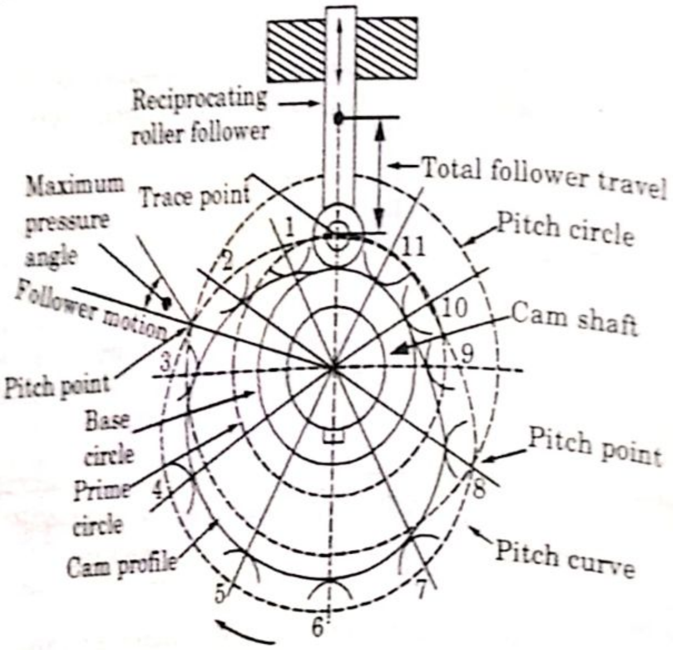


Fig. 2.4.1. Terms used in radial cams.

PART-2

Cam Profiles for Knife Edge, Roller and Flat Faced Followers for Uniform Velocity, Uniform Acceleration.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.5. **Draw the displacement, velocity and acceleration diagram for a follower when it moves with uniform velocity. Also draw the modified displacement, velocity and acceleration diagram. Why modifications in these diagrams are necessary ?**

Answer

A. **Displacement, Velocity and Acceleration Diagram when the Follower Moves with Uniform Velocity :**

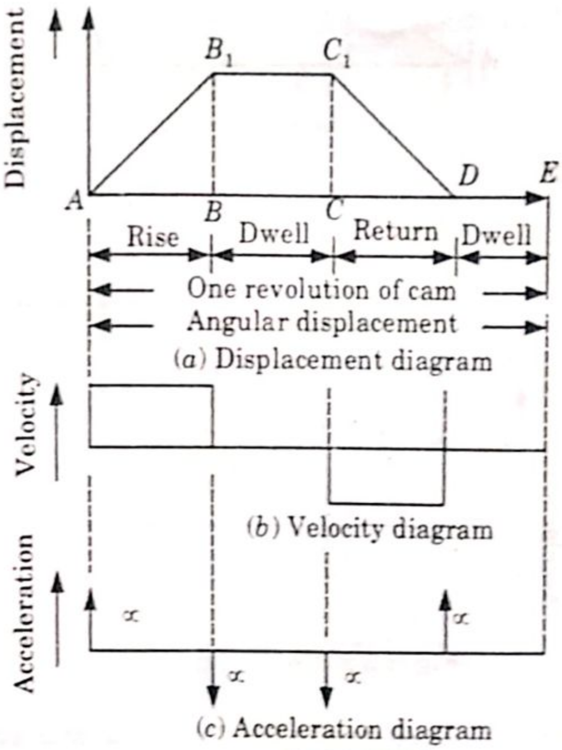


Fig. 2.5.1.

1. The abscissa represents the time or angular displacement of the cam in degrees. The ordinate represents the displacement, or velocity or acceleration of the follower.
2. Since the follower moves with uniform velocity during its rise and return stroke, therefore the slope of the displacement curves must be constant. In other words, AB_1 and C_1D must be straight line.
3. The periods during which the follower remains at rest are known as dwell periods, as shown by lines B_1C_1 and DE in Fig. 2.5.1(a).
4. From Fig. 2.5.1(c), we see that the acceleration or retardation of the follower at the beginning and at the end of each stroke is infinite. This is due to the fact that the follower is required to start from rest and has to gain a velocity within no time.
5. This is only possible if the acceleration or retardation at the beginning and at the end of each stroke is infinite.

B. Necessity of Modified Diagram :

1. In order to have the acceleration and retardation within the finite limits, it is necessary to modify the conditions which govern the motion of the follower.

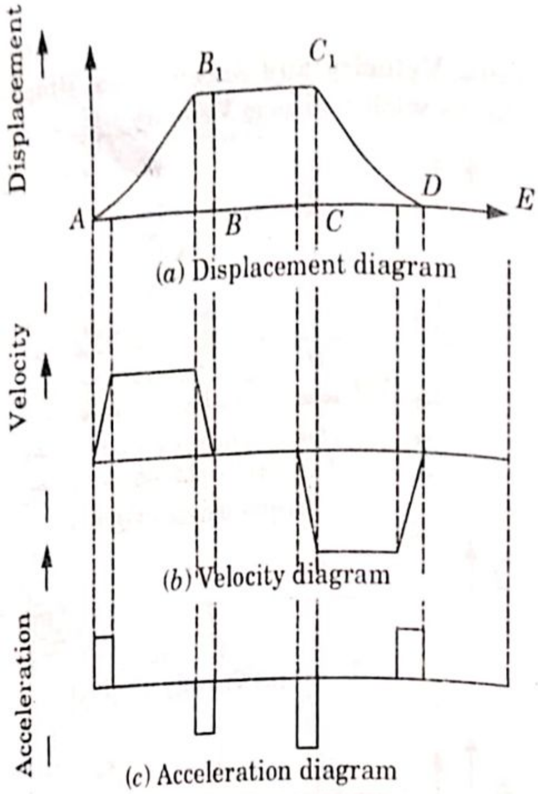


Fig. 2.5.2. Modified diagram.

2. Modification of the diagrams is done by making round corners in place of sharp corners of the displacement diagram at the beginning and at the end of each stroke.
3. This shows that the velocity of the follower increases gradually to its maximum value at the beginning of each stroke and decreases gradually to zero at the end of each stroke.

Que 2.6. **Discuss the displacement, velocity and acceleration diagram when the follower moves with uniform acceleration and retardation.**

Answer

1. Divide the angular displacement of the cam during out stroke (ϕ_o) into any number of equal parts (say eight) and draw the vertical lines through these points as shown in Fig. 2.6.1 and numbered these points as A, 1, 2, 3, 4, 5, 6, 7 and 8.

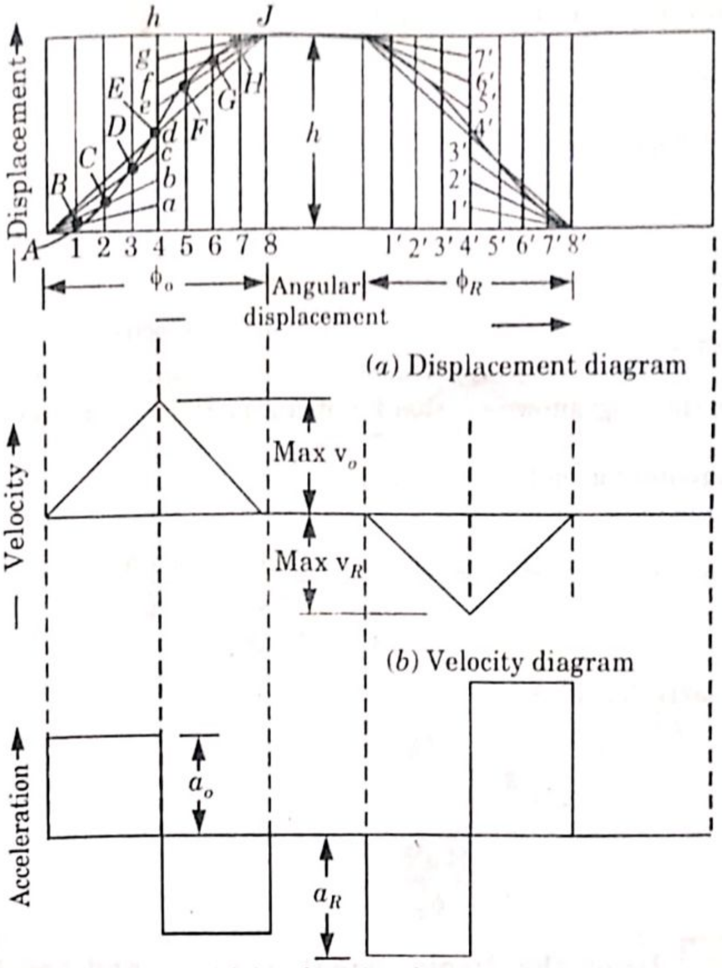


Fig. 2.6.1.

2. Divide the vertical line passes through point 4 into eight equal parts (Height = h = Stroke of cam) and numbered as a, b, c, d, e, f, g and h.
3. Joining point A to points a, b, c, d to obtain corresponding points B, C, D and E on the lines passing through 1, 2, 3 and 4 respectively.
4. Similarly joining point J to point's e, f and g to obtain corresponding points F, G and H on the lines passing through 5, 6 and 7 respectively.
5. Join these points with free hand to draw the required parabolic displacement curve.
6. Let, h = Stroke of the follower,

   ω = Angular velocity of cam, and

   ϕ_o and ϕ_R = Angular displacement during out stroke and return stroke of the follower.
7. Time required for the follower during outstroke,

$$t_o = \frac{\phi_o}{\omega}$$

8. Mean velocity of the follower during outstroke,

$$v_o = \frac{h}{t_o}$$

9. Since,the maximum velocity = 2 × Mean velocity

$$v_o = \frac{2h}{t_o} = \frac{2\omega h}{\phi_o}$$

10. As acceleration and deceleration are constant so,

$$\text{Maximum acceleration} = \frac{\text{Maximum velocity}}{\text{Time to attain maximum velocity}}$$

11. From the diagram we see that for attaining the maximum velocity t... time require is half of the total time $\left(i.e., \frac{t_o}{2}\right)$.

$$\therefore \quad \text{Maximum acceleration } a_o = \frac{v_o}{\left(\frac{t_o}{2}\right)} = \frac{2\omega h}{\phi_o\left(\frac{\phi_o}{2\omega}\right)} = \frac{4\omega^2 h}{\phi_o^2}$$

12. Similarly, for return stroke,

$$v_R = \frac{2\omega h}{\phi_R}$$

$$a_R = \frac{4\omega^2 h}{\phi_R^2}$$

Que 2.7. **Draw the displacement, velocity and acceleration diagram for a follower when it moves with simple harmonic motion.**

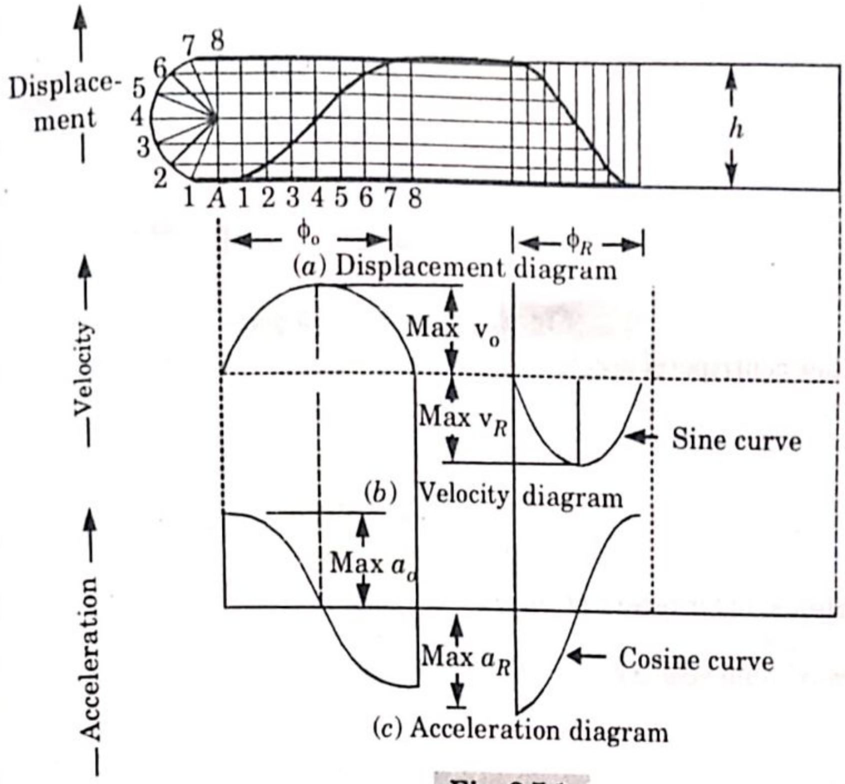
OR

Deduce expressions for the velocity and acceleration of the follower when it moves with simple harmonic motion.

Answer

1. Draw a semi circle with cam rise or fall (equals to stroke, h) as the diameter and divide this semicircle into n (say eight) even equal parts.
2. Divide the cam displacement (ϕ) interval into n (say eight) even equal parts.
3. Project the intercepts of the harmonic semi circle to the corresponding divisions of the cam displacement interval.
4. Join these points with free hand to draw the required harmonic displacement curve.
5. Let, s = Instantaneous follower displacement,
   v = Velocity of the follower,
   a = Acceleration of the follower,
   θ = Instantaneous cam rotation angle, and

ϕ = Cam rotation angle for the maximum follower displacement.



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

Fig. 2.7.1.

6. At any instant, time required for the out stroke of the follower in seconds,

$$t_o = \frac{\phi_o}{\omega}$$

7. Consider a point P moving at a uniform speed ω_P radians per sec around the circumference of a circle with stroke h as diameter.
8. Project a point P' on the diameter of the circle which executes a SHM as point P rotates.
9. Peripheral velocity of a point P,

$$v_P = \frac{\text{Distance}}{\text{Time}} = \left(\frac{\frac{\pi h}{2}}{t_o}\right) = \frac{\pi h \omega}{2\phi_o}$$

10. The maximum velocity of the follower on the out stroke,

$$v_o = \frac{\pi h \omega}{2\phi_o}$$

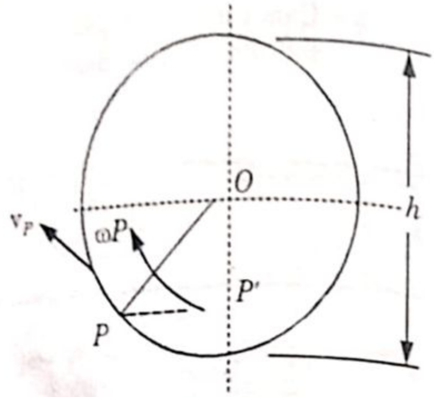


Fig. 2.7.2. Motion of a point.

11. Now centripetal acceleration of the point P,

$$a_P = \frac{v_P^2}{OP}$$

$$a_P = \left(\frac{\pi\omega h}{2\phi_o}\right)^2 \times \frac{2}{h} = \frac{\pi^2\omega^2 h}{2\phi_o^2}$$

12. Similarly for return stroke,

Maximum velocity, $v_R = \dfrac{\pi\omega h}{2\phi_R}$

Maximum acceleration, $a_R = \dfrac{\pi^2\omega^2 h}{2\phi_R^2}$

Que 2.8. **Discuss the displacement, velocity and acceleration diagram when the follower moves with cycloidal motion.**

Answer

1. The displacement, velocity and acceleration diagrams when the follower moves with cycloidal motion are shown in Fig. 2.8.1(a), (b) and (c) respectively.
2. The displacement diagram is drawn as discussed below :
   i. Draw a circle of radius $S/2\pi$ with A as centre.
   ii. Divide the circle into any number of equal even parts (say six). Project these points horizontally on the vertical centre line of the circle. These points are shown by a' and b' in Fig. 2.8.1(a).
   iii. Divide the angular displacement of the cam during out stroke into the same number of equal even parts as the circle is divided. Draw vertical lines through these points.
   iv. Join AB which intersects the vertical line through 3′ at c. From a' draw a line parallel to AB intersecting the vertical lines through 1′ and 2′ at a and b respectively.

v. Similarly, from b' draw a line parallel to AB intersecting the vertical lines through 4′ and 5′ at d and e respectively.

vi. Join the points $A\ a\ b\ c\ d\ e\ B$ by a smooth curve. This is the required cycloidal curve for the follower during outstroke.

3. Let ϕ = Angle through which the can rotates in time t seconds, and

ω = Angular velocity of the cam.

4. We know that displacement of the follower after time t seconds,

$$x = S\left[\frac{\phi}{\phi_o} - \frac{1}{2\pi}\sin\left(\frac{2\pi\phi}{\phi_o}\right)\right] \quad ...(2.8.1)$$

5. Velocity of the follower after time t seconds,

$$\frac{dx}{dt} = S\left[\frac{1}{\phi_o}\frac{d\phi}{dt} - \frac{2\pi}{2\pi\phi_o}\cos\left(\frac{2\pi\phi}{\phi_o}\right)\frac{d\phi}{dt}\right]$$

[Differentiating eq. (2.8.1)]

$$= \frac{S}{\phi_o}\frac{d\phi}{dt}\left[1-\cos\left(\frac{2\pi\phi}{\phi_o}\right)\right] = \frac{\omega S}{\phi_o}\left[1-\cos\left(\frac{2\pi\phi}{\phi_o}\right)\right]$$

...(2.8.2)

6. The velocity is maximum, when

$$\cos\left(\frac{2\pi\phi}{\phi_o}\right) = -1 \text{ or } \frac{2\pi\phi}{\phi_o} = \pi \quad \text{or} \quad \phi = \phi_o/2$$

Substituting $\phi = \phi_o/2$ in eq. (2.8.2), we have maximum velocity of the follower during outstroke,

$$v_o = \frac{\omega S}{\phi_o}(1+1) = \frac{2\,\omega S}{\phi_o}$$

7. Similarly, maximum velocity of the follower during return stroke,

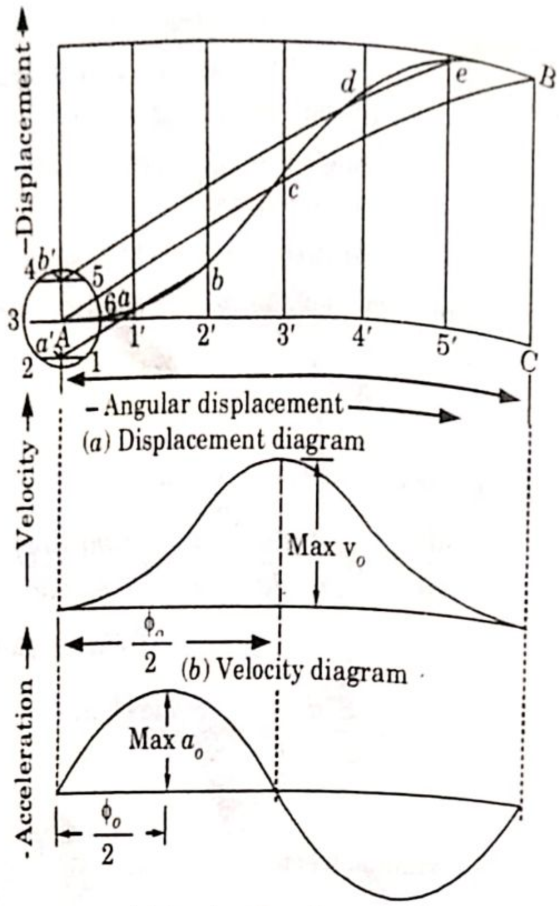
$$v_R = \frac{2\,\omega S}{\phi_R}$$

8. Now, acceleration of the follower after time t sec.

$$\frac{d^2x}{dt^2} = \frac{\omega S}{\phi_o}\left[\frac{2\pi}{\phi_o}\sin\left(\frac{2\pi\phi}{\phi_o}\right)\frac{d\phi}{dt}\right]$$

[Differentiating eq. (2.8.2)]

$$= \frac{2\pi\,\omega^2 S}{(\phi_o)^2}\sin\left(\frac{2\pi\phi}{\phi_o}\right) \qquad \left(\because \frac{d\phi}{dt} = \omega\right) \quad ...(2.8.3)$$



(c) Acceleration diagram

Fig. 2.8.1. Displacement, velocity and acceleration diagrams when the follower moves with cycloidal motion.

9. The acceleration is maximum, when

$$\sin\left(\frac{2\pi\phi}{\phi_o}\right) = 1 = \frac{2\pi\phi}{\phi_o} = \frac{\pi}{2} \quad \text{or} \quad \phi = \phi_o/4$$

Substituting $\phi = \phi_o/4$ in eq. (2.8.3), we have maximum acceleration of the follower during out stroke

$$a_o = \frac{2\pi\omega^2 S}{(\phi_o)^2}$$

10. Similarly, maximum acceleration of the follower during return stroke,

$$a_R = \frac{2\pi\omega^2 S}{(\phi_R)^2}$$

Que 2.9. **Explain the procedure to layout the cam profile for a reciprocating follower.**

Answer

1. A cam profile is constructed on considering the cam to be stationary and the follower to be rotating about it in the opposite direction of the cam rotation.
2. The following procedure is adopted to layout the cam profile for a reciprocating follower :
   i. First draw the displacement diagram of the follower.
   ii. Draw the prime circle of the cam with radius
      a. r_c if it is a knife edge or mushroom follower.
      b. $r_c + r_r$ if it is a roller follower.
   iii. Divide the prime circle into segments as follows :
      a. For a radial follower, divide the circle from the vertical position indicating the angles of ascent, dwell period and angle of descent in the opposite direction of the cam rotation.
      b. For an offset follower, draw another circle with radius equal to the offset of the follower and assume the initial position on the prime circle where the tangent to the horizontal radius of the circle meets the prime circle.
   iv. Now divide each segment of ascent and descent into the same number of angular parts as is done in the displacement diagram.
   v. On radial lines produced, mark distances equal to the lift of the follower beyond the circumference of the prime circle. For offset follower, the distances are marked on the tangents drawn to the circle with radius equal to the offset.
   vi. To obtain cam profile :
      a. For a knife edge follower, draw a smooth curve passing through the marked points to obtain required cam profile.
      b. For a roller follower, draw a series of arcs of radii equal to r_r on the inner side and draw a smooth curve tangential to all the arcs to get the required cam profile.
      c. For a mushroom follower, draw the follower in all the positions by drawing perpendiculars to the radial or tangent lines and draw a smooth curve tangential to the flat faces of the follower representing the cam profile.

Que 2.10. **Draw the profile of the cam when the roller follower moves with cycloidal motion during out stroke and return stroke as given below :**

i. Out stroke with maximum displacement of 31.4 mm during 180° of cam rotation.

ii. Return stroke for the next 150° of cam rotation.

Given : $h = 31.4$ mm, $\phi_o = 180°$, $\phi_R = 150°$, $\delta = 30°$, $r_c = 15$ mm, $d_r = 10$ mm, $x = 10$ mm

To Draw : Cam profile.

1. First of all, the displacement diagram, as shown in Fig. 2.10.1, is drawn as discussed in the following steps :

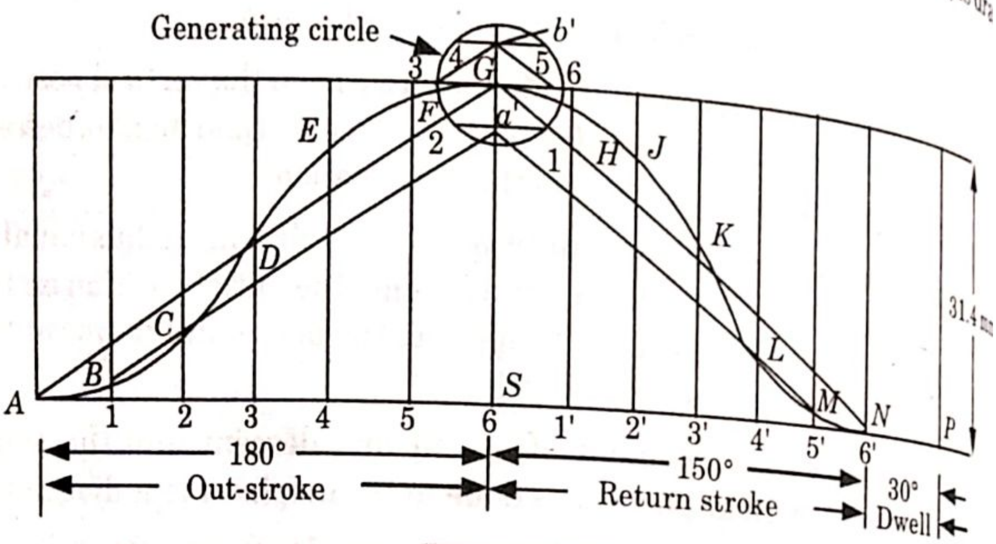


Fig. 2.10.1.

i. Draw horizontal line ASP, such that $AS = 180°$ to represent the out stroke, $SN = 150°$ to represent the return stroke and $NP = 30°$ to represent the dwell period.

ii. Divide AS and SN into any number of even equal parts (say six).

iii. From the points 1, 2, 3 etc., draw vertical lines and set off equal to the stroke of the follower.

iv. From a point G draw a generating circle of radius,

$$r = \frac{\text{Stroke}}{2\pi} = \frac{31.4}{2\pi} = 5 \text{ mm}$$

v. Divide the generating circle into six equal parts and from these points draw lines to meet the vertical diameter at a', G and b'.

vi. Join AG and GN. Form point a', draw lines parallel to AG and GN to intersect the vertical lines drawn through 1, 2, 4' and 5' at B, C, L and M respectively. Similarly draw parallel lines from b' intersecting the vertical lines through 4, 5, 1' and 2' at E, F, H and J respectively.

vii. Join the points $A, B, C \ldots L, M, N$ with a smooth curve.

viii. The curve $A\,B\,C \ldots L\,M\,N$ is the required displacement diagram.

2. Now the profile of the cam, when the line of stroke of the follower is off set 10 mm from the centre of the cam, as shown in Fig. 2.10.2, is drawn as discussed in the following steps :

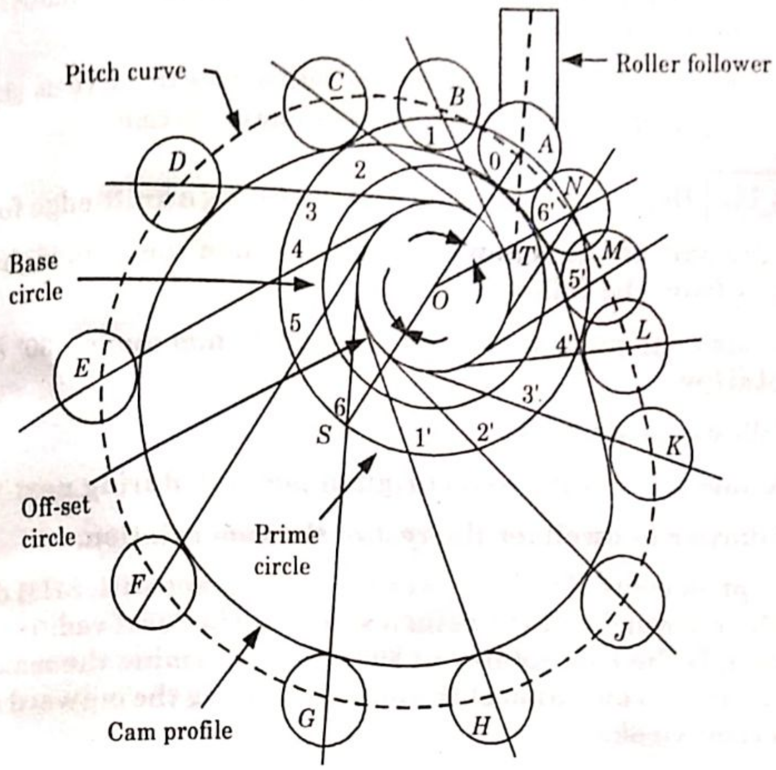


Fig. 2.10.2.

i. Draw a base circle with centre O and radius equal to the least radius of cam (*i.e.*, 15 mm).

ii. Draw a prime circle with centre O and radius,

$$OA = \text{Least radius of cam} + \text{Radius of roller}$$

$$= 15 + \frac{10}{2} = 20 \text{ mm}$$

iii. Draw an offset circle with centre O and radius equal to 10 mm

iv. Join OA. From OA draw angular displacement of the cam, *i.e.*, draw $\angle AOS = 180°$ to represent lift of the follower, $\angle TOS = 150°$ to represent fall of the follower and $\angle TOA = 30°$ to represent dwell.

v. Divide the angular displacement during lift and fall (*i.e.*, $\angle AOS$ and $\angle TOS$) into the same number of equal even parts (*i.e.* six parts) as in the displacement diagram.

vi. From points 1, 2, 3 ... etc., and 0′, 1′, 2′, 3′ .... etc., on the prime circle, draw tangents to the offset circle.

vii. Set off 1B, 2C, 3D ... etc., equal to the displacements as measured from the displacement diagram.

viii. By joining the points $A, B, C \ldots M, N, P$ with a smooth curve, we get a pitch curve.

ix. Now with $A, B, C \ldots$ etc., as centre, draw circles with radius equal to the radius of the roller.

x. Join the bottoms of the circles with a smooth curve as shown in Fig. 2.10.2. This is the required profile of the cam.

Que 2.11. **Draw the profile of a cam operating a knife edge follower when the axis of the follower is offset 20 mm towards right from cam axis from the following data :**

i. Follower to move outwards through 40 mm during 60° of cam rotation.

ii. Follower to dwell for the next 45°.

iii. Follower to return to its original position during next 90°.

iv. Follower to dwell for the rest of the cam rotation.

The displacement of the follower is to take place with SHM during both the outward and the return strokes. The least radius of cam is 50 mm. If the cam rotates at 300 rpm, determine the maximum velocity and acceleration of the follower during the outward stroke and return stroke.

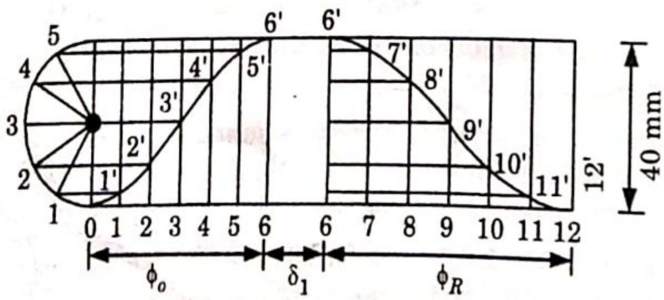
Answer

Given : $h = 40$ mm, $\phi_o = 60°$, $N = 300$ rpm, $\delta_1 = 45°$, $r_c = 50$ mm, $\phi_R = 90°$, $x = 20$ mm

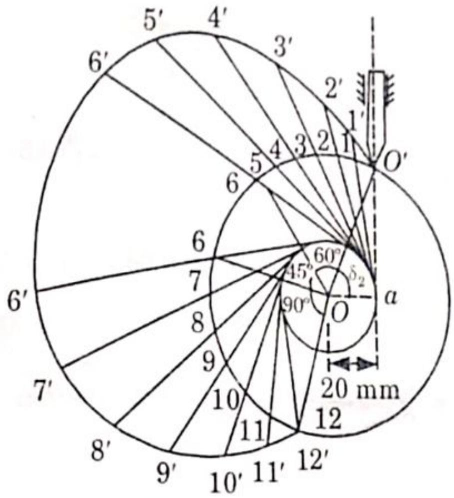
To Find : Maximum velocity and acceleration of the follower during the outward and return stroke.

To Draw : Cam profile.

1. The displacement diagram is drawn by following the usual procedure is shown in Fig. 2.11.1(*a*).



(*a*) Displacement diagram



(*b*) Cam profile.

Fig. 2.11.1.

2. Construct the cam profile shown in Fig. 2.11.1(*b*) as follows :
   i. Draw a circle with radius r_c (= 50 mm).
   ii. Draw another circle concentric with the previous circle with radius x (= 20 mm). If the cam is assumed stationary, the follower will be tangential to this circle in all the positions. Let the initial position be O- O'.
   iii. Join O-O'. Divide the circle of radius r_c into four parts as usual with angles ϕ_a, δ_1, ϕ_d and δ_2 starting from O-O'.
   iv. Divide the angles ϕ_a and ϕ_d into same number of parts as is done in the displacement diagram and obtain the points 1, 2, 3, etc., on the circumference of circle with radius r_c.
   v. Draw tangents to the circle with radius x from the points 1, 2, 3, etc.
   vi. On the extension of the tangent lines, mark the distances from the displacement diagram.
   vii. Draw a smooth curve through O', 1′, 2′, etc. This is the required cam profile.
3. During outward stroke,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 10\,\pi \text{ rad/s}$$

4. We know that velocity, $v_{max} = \dfrac{h}{2}\dfrac{\pi\omega}{\phi_o}$

$$v_{max} = \frac{40}{2} \times \left[\frac{\pi \times 10\pi}{60 \times \dfrac{\pi}{180}}\right] = 1884.96 \text{ mm/s} = 1.88 \text{ m/s}$$

$$a_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\phi_o}\right)^2$$

$$a_{max} = \frac{40}{2} \times \left[\frac{\pi \times 10\,\pi}{60 \times \frac{\pi}{180}}\right]^2$$

$$= 177652.88 \text{ mm/s}^2 = 177.652 \text{ m/s}^2$$

5. Similarly, during return stroke,

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\phi_R} = \frac{40}{2} \times \left[\frac{\pi \times 10\,\pi}{90 \times \frac{\pi}{180}}\right] = 1256.64 \text{ mm/s} = 1.25 \text{ m/s}$$

$$a_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\phi_R}\right)^2 = \frac{40}{2} \times \left[\frac{\pi \times 10\,\pi}{90 \times \frac{\pi}{180}}\right]^2 = 78956.84 \text{ mm/s}^2 = 78.956 \text{ m/s}^2$$

Que 2.12. **Draw the profile of a cam with oscillating roller follower for the following motion :**

a. Follower to move outwards through an angular displacement of 20° during 120° of cam rotation.

b. Follower to dwell for 50° of cam rotation.

c. Follower to return to its initial position in 90° of cam rotation with uniform acceleration and retardation.

d. Follower to dwell for the remaining period of cam rotation.

The distance between the pivot centre and the roller centre is 130 mm and the distance between the pivot centre and cam axis is 150 mm. The minimum radius of the cam is 80 mm and diameter of the roller is 50 mm.

Answer

Given : $r_c = 80$ mm, $r_r = \frac{50}{2} = 25$ mm, Distance between pivot centre and roller centre = 130 mm, Distance between pivot centre and cam axis = 150 mm, $\phi_o = 120°$, $\delta_1 = 50°$, $\phi_R = 90°$.

To Draw : Cam profile.

Note : In this question, motion of cam during out stroke is not mentioned, so we are assuming it simple harmonic motion during out stroke.

1. We know that the angular displacement of the roller follower,

$$= 20° \times \pi/180 = \pi/9 \text{ rad}$$

2. Since the distance between the pivot centre and the roller centre (*i.e.*, radius A_1A) is 130 mm, therefore length of arc AA_2, as shown in Fig. 2.12.1, along which the displacement of the roller actually take place

$$= 130 \times \pi/9 = 45.38 \text{ mm}$$

3. Since the angle is very small, therefore length of chord AA_2 is taken equal to the length of arc AA_2. Thus in order to draw the displacement

diagram, we shall take lift of the follower equal to the length of chord AA_2 *i.e.*, 45.38 mm.

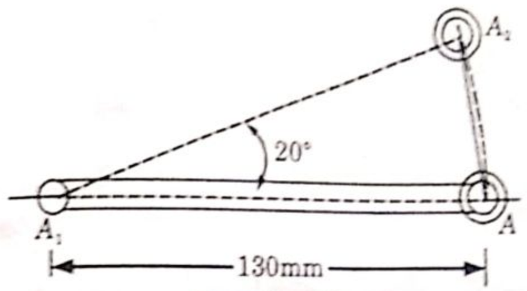
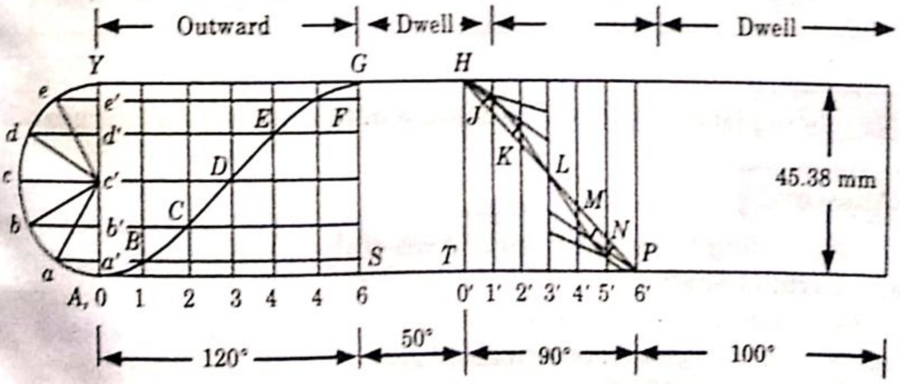


Fig. 2.12.1.

4. The displacement diagram drawn by following the usual procedure is shown in Fig. 2.12.2(*a*).
5. To draw the cam profile, proceed as follows :
   i. Draw a circle with radius ($r_c + r_r$) [Fig. 2.12.2(*b*)].
   ii. Assuming the initial position of the roller centre vertically above the cam centre *O*, locate the fulcrum centre as its distances from the cam centre and the roller centre (equal to length of follower arm) are known.
   iii. Draw a circle with radius *OA* and centre at *O*.
   iv. On the circle through *A*, starting from *OA*, take angles ϕ_o, δ_1 and ϕ_R as usual.
   v. Divide the angles ϕ_o and ϕ_R into same number of parts as is done in the displacement diagram and obtain the points *a*, *b*, *c*, *d*, etc., on this circle through *A*.
   vi. With centres *A*, *B*, *C*, etc., draw arcs with radii equal to length of the arm.
   vii. Mark distance 1-*B*, 2-*C*, 3-*D*, etc., on these arcs as shown in the diagram. It is on the assumption that for small angular displacements, the linear displacements on the arcs and on the straight lines are the same.
   viii. With 1′, 2′, 3′, etc., draw a series of arcs of radii equal of r_r.
   ix. Draw a smooth curve tangential to all the arcs and obtain the required cam profile.



(*a*)

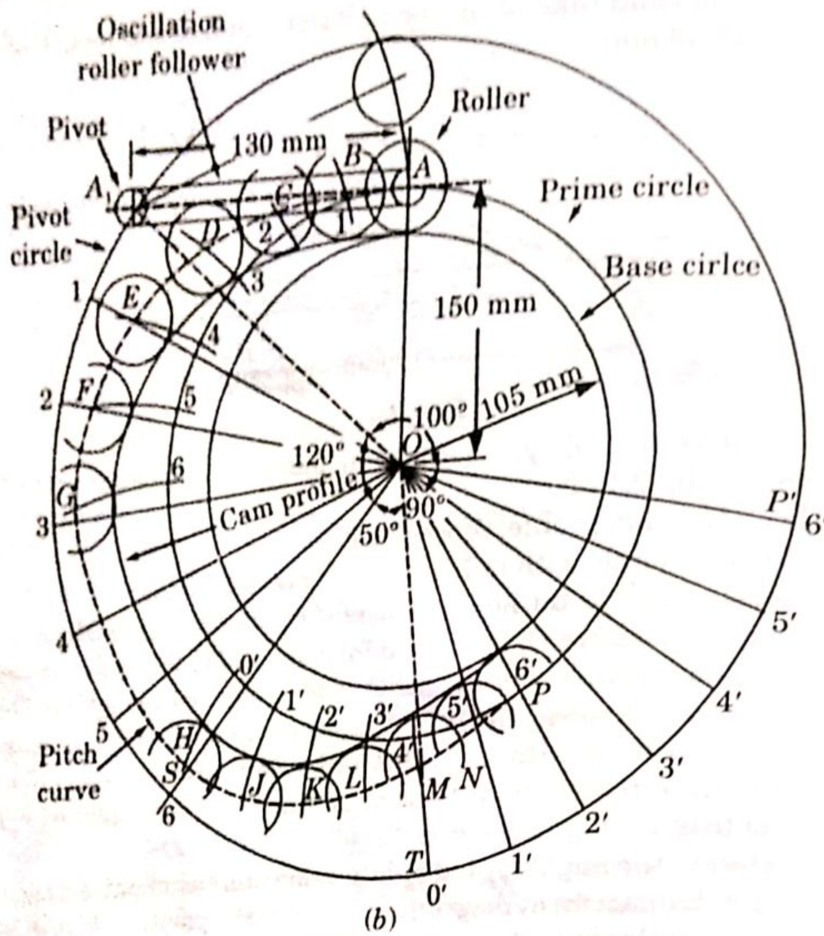


Fig. 2.12.2.

PART-3

Gears : Introduction, Classification of Gears.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.13. **Give a brief classification of toothed wheels (or gears).**

Answer

I. According to the Position of Axes of the Shafts :

a. Parallel Shaft :

1. Spur gear, and
2. Helical gear and herringbone gear.

b. Intersecting Shaft :

1. Bevel gear, and

2. Helical bevel gear.

c. **Non Intersecting Shaft :**

1. Spiral gears (or skew bevel gears).

II. **According to Peripheral Velocity of the Gears :**

a. **Low Velocity :** The gears having velocity less than 3 m/s.

b. **Medium Velocity :** The gears having velocity between 3 to 15 m/s.

c. **High Velocity :** The gears having velocity more than 15 m/s.

III. **According to the Type of Gearing :**

a. **External Gearing :** It provides unlike motion to the two wheels.

b. **Internal Gearing :** It provides like motion to the two wheels.

c. **Rack and Pinion :** It converts the rotary motion to linear motion.

IV. **According to Position of Teeth on the Gear Surface :**

a. Straight
b. Inclined
c. Curved

Que 2.14. **Discuss the terminology used in tooth gearing with suitable diagram.**

Answer

1. **Pitch Circle :** It is an imaginary circle which produces same motion as actual gear by pure rolling action.
2. **Pitch Circle Diameter :** It is the diameter of pitch circle. Gear sizes are usually specified by pitch circle diameter.
3. **Pressure Angle :** It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is also known as angle of obliquity. Standard pressure angles are $14\frac{1}{2}°$ and 20°.
4. **Addendum and Addendum Circle :** Addendum is the radial distance of a tooth from pitch circle to the top of tooth. Addendum circle is drawn through the top of the teeth and concentric with pitch circle.
5. **Dedendum and Dedendum Circle :** Dedendum is the radial distance between the pitch circle and bottom of the teeth. Dedendum circle is also known as root circle as it is drawn through the bottom of the teeth.
6. **Clearance :** It is the radial distance between the top of tooth and bottom of tooth of meshing gear. A circle drawn through top of meshing gear is known as clearance circle.
7. **Total Depth :** It is the radial distance between the addendum and the dedendum of a gear. In other words, it is the sum of addendum and dedendum.
8. **Working Depth :** It is the radial distance from the addendum circle to the clearance circle. It equals to the sum of addendum of the meshing gears.
9. **Backlash :** It is defined as the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, backlash should be zero, but it must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

10. **Circular Pitch :** It is the distance measured on the circumference the pitch circle from a point of one tooth to the corresponding point the next tooth. It is denoted by p_c.

Circular pitch, $p_c = \dfrac{\pi D}{T}$

Where, D = Diameter of pitch circle, and

T = Number of teeth on wheel.

11. **Diametral Pitch :** It is the ratio of number of teeth to the pitch circl diameter in millimeters. It is denoted by p_d.

Diametral pitch, $p_d = \dfrac{T}{D} = \dfrac{\pi}{p_c}$

12. **Module :** It is defined as the ratio of pitch circle diameter to the number of teeth. It is usually denoted by m. Mathematically,

Module, $m = \dfrac{D}{T}$

13. **Path of Contact :** It is the path traced by the point of contact of two teeth from the engagement of teeth to disengagement of teeth.
14. **Length of Path of Contact :** It is the length of the common normal cut off by the addendum circles of the wheel and the pinion. Pitch point is always one point on the path of contact.
15. **Arc of Contact :** It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of :
    a. **Arc of Approach :** It is the portion of the path of contact from the beginning of the engagement to the pitch point.
    b. **Arc of Recess :** It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.
16. **Contact Ratio :** Contact ratio is the ratio of the arc of contact to the circular pitch.

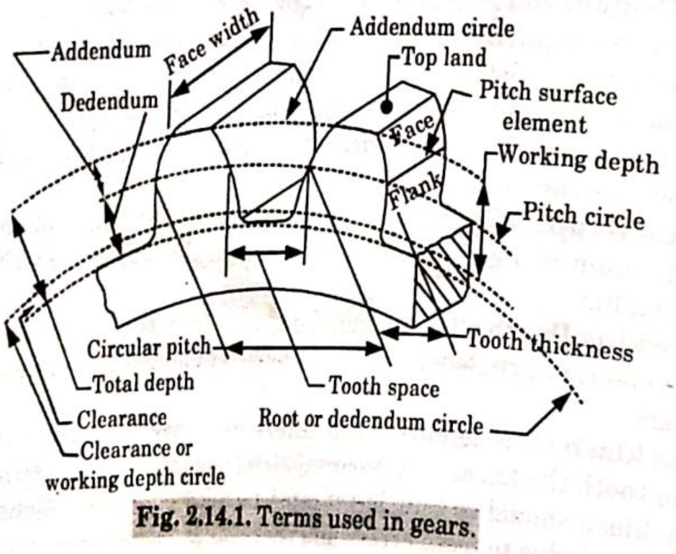


Fig. 2.14.1. Terms used in gears.

PART-4

Law of Gearing.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.15. **Define the fundamental law of gearing. Derive the condition that must be satisfied for two bodies having constant velocity ratio.**

Answer

A. **Law of Gearing :** It states that the common normal at the point of contact between a pair of teeth must always pass through the pitch point.

B. **Proof :**

1. Consider two bodies (1) and (2) representing a portion of the two gears in mesh.
2. At a point C on the tooth profile of gear (1) is in contact with a point D on the tooth profile of gear (2).
3. Let n-n be the common normal at point C and D on the respective curves.

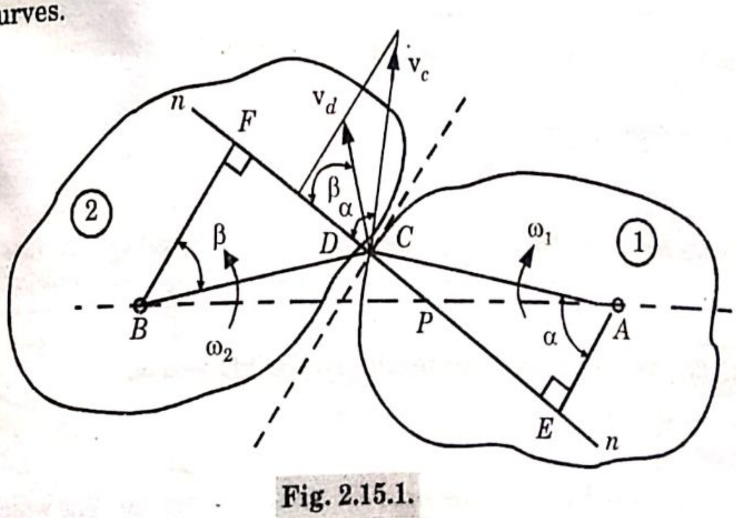


Fig. 2.15.1.

4. Let,

ω_1 = Instantaneous angular velocity of gear (1) (Clockwise),

ω_2 = Instantaneous angular velocity of gear (2) (Anticlockwise),

v_c = Linear velocity of C, and

v_d = Linear velocity of D.

5. Now, $v_c = \omega_1 AC$, in a direction perpendicular to AC or at an angle α to n-n, and
$v_d = \omega_2 BD$, in a direction perpendicular to BD or at an angle β to n-n.

6. As both gears should be in contact so one surface may slide relative to other along the common tangent. And there should be no motion along the common normal as it will lead to separation of gear teeth.

7. Component of v_c along n-n = $v_c \cos \alpha$
Component of v_d along n-n = $v_d \cos \beta$
Relative motion along n-n = $v_c \cos \alpha - v_d \cos \beta$

8. For proper contact :
$v_c \cos \alpha - v_d \cos \beta = 0$
$\omega_1 AC \cos \alpha - \omega_2 BD \cos \beta = 0$

$$\frac{\omega_1}{\omega_2} = \frac{BD \cos\beta}{AC \cos\alpha}$$

9. From ΔAEC and ΔBDF,

$$\cos \alpha = \frac{AE}{AC} \text{ and } \cos \beta = \frac{BF}{BD}$$

$$\therefore \quad \frac{\omega_1}{\omega_2} = \left(\frac{BD}{AC}\right)\left(\frac{BF}{BD}\right)\left(\frac{AC}{AE}\right) = \frac{BF}{AE} = \frac{BP}{AP}$$

This is the condition that must be satisfied for two bodies having constant velocity ratio.

PART-5

Tooth Forms and their Comparisons, System of Gear Teeth.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.16. **What is involute teeth profile ? Discuss.**

Answer

1. An involute is defined as the locus of a point on a straight line which rolls without slipping on the circumference of a circle.
2. Let A be the point on a base circle. First divide the base circle in equal number of parts e.g., AP_1, P_1P_2, P_2P_3, etc.
3. Draw the tangents at points P_1, P_2, P_3 etc.

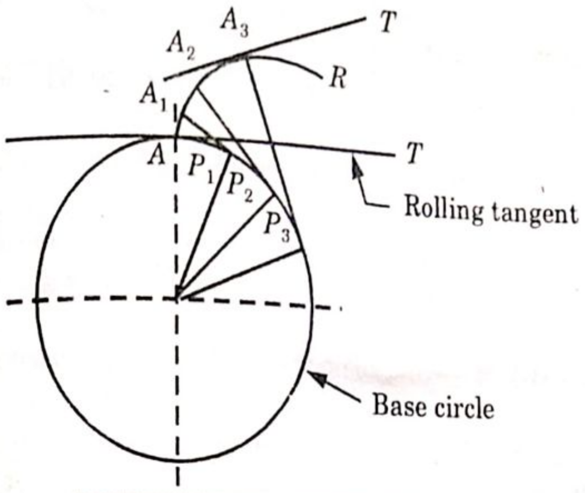


Fig. 2.16.1. Construction of involute.

4. Cut the length equals to AP_1, AP_2 and AP_3 on the tangents made from points P_1, P_2 and P_3 and mark the points as A_1, A_2, A_3.
5. Joining the points A, A_1, A_2 and A_3 will form an involute teeth profile.
6. For an involute teeth profile, normal at any point of an involute is a tangent to the base circle.

Que 2.17. **Explain the cycloid teeth profile.**

Answer

1. A cycloid is the locus of a point on the circumference of a circle that rolls without slipping on a fixed straight line.
2. When a circle rolls without slipping on the circumference of another circle, the locus of a point on the circle is known as epicycloid.
3. If a circle rolls inside the circumference of another circle without slipping, the locus of the point on the circle is known as hypocycloid.
4. A cycloidal tooth is formed as shown in Fig. 2.17.1.

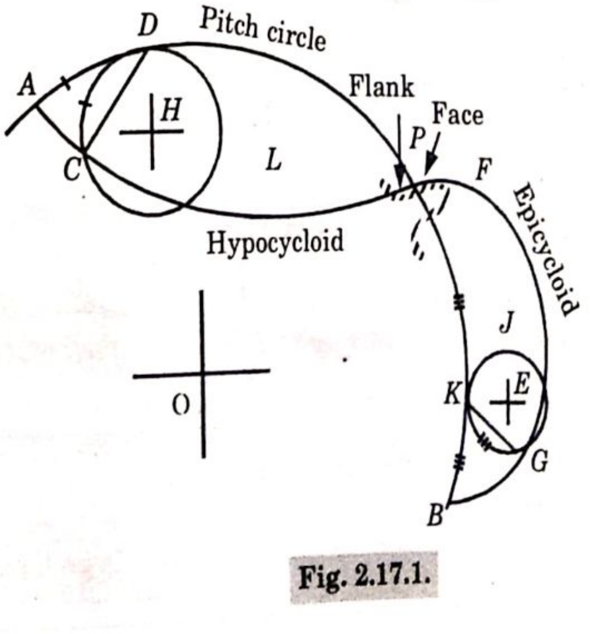


Fig. 2.17.1.

5. Let APB is a circle (or pitch circle) inside which a circle H rolls. When rolling starts, A is the point of contact.
6. As the circle rolls, trace the locus of point A, we get a path ALP which is a hypocycloid.
7. A small portion of the curve near the pitch circle is used for the flank of the tooth.
8. Let a circle E rotates or rolls outside the pitch circle, then the locus of its point of contact form an epicycloid PFB (Consider P be the starting point).
9. A small portion of the curve near the pitch circle is used for the face of the tooth.
10. Both epicycloid and hypocycloid has a property that at any instant, a line joining the generating point with the point of contact of two circles is normal to the epicycloid or hypocycloid.

Que 2.18. **Give the difference between involute tooth form and cycloidal tooth form.**

Answer

S. No.	Involute Tooth Form	Cycloidal Tooth Form
1.	Pressure angle remains constant.	Pressure angle varies.
2.	Smooth running of gears.	Less smooth running of gears.
3.	Easy to manufacture as single curve is required to generate tooth profile.	Difficult to manufacture as two curves are required to generate tooth profile.
4.	Less strong as compared to cycloidal tooth form.	Strong.
5.	More wear.	Less wear.
6.	Interference occurs.	Interference does not occur.

PART-6

Length of Path of Contact, Contact Ratio, Minimum Number of Teeth on Gear and Pinion to Avoid Interference.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.19. **Derive the expression for the path of contact and arc of contact for involute in contact.**

Answer

A. Path of Contact :

1. Let a pinion with centre O_1 driving a wheel with centre O_2.
2. When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at K (on the flank near the base circle of pinion) and ends at L (outer end of the tooth face on the pinion).
3. MN is the common normal at the point of contact as well as acts as a common tangent to the base circles.
4. Hence, KL is the length of path of contact where K is the intersection point of the addendum circle of wheel and common tangent and L is the intersection of the addendum circle of pinion and common tangent.
5. From Fig. 2.19.1,

$$KL = KP + PL$$

$$= \text{Path of approach} + \text{Path of recess}$$

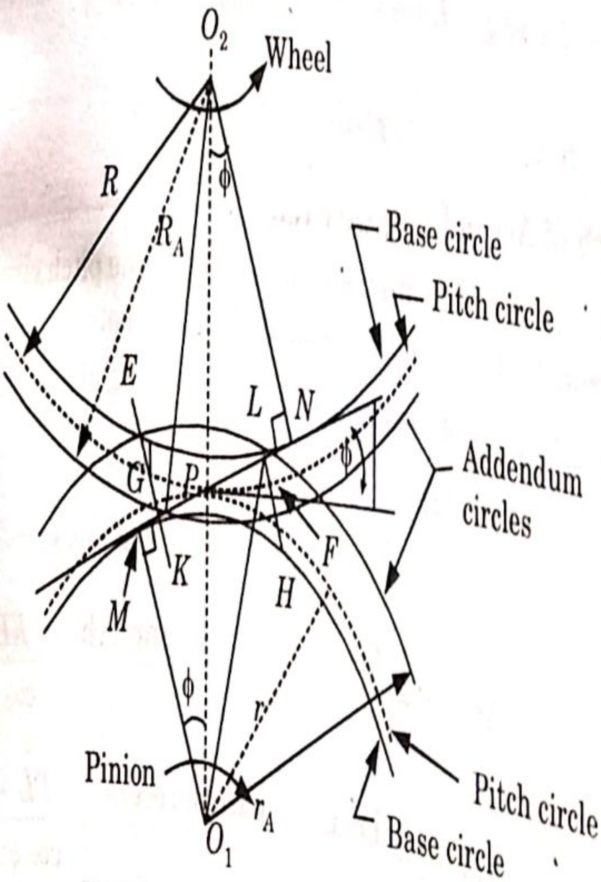


Fig. 2.19.1. Length of path of ...

7. From ΔO_1MP, $O_1M = O_1P \cos\phi = r\cos\phi$, $PM = r\sin\phi$

   From ΔO_2NP, $O_2N = O_2P\cos\phi = R\cos\phi$, $PN = R\sin\phi$

8. Now from right angled ΔO_2KN,

$$(O_2K)^2 = (O_2N)^2 + (KN)^2$$

$$KN = \sqrt{(O_2K)^2-(O_2N)^2} = \sqrt{R_A^2 - R^2\cos^2\phi}$$

9. Path of approach, $KP = KN - PN$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

10. Similarly from right angled ΔO_1ML,

$$ML = \sqrt{O_1L^2 - O_1M^2} = \sqrt{r_A^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

11. Path of recess, $PL = ML - MP$

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

12. Length of path of contact, $KL = KP + PL$

$$= \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi + \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - (R+r)\sin\phi$$

B. Length of Arc of Contact :

1. Arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth.
2. In Fig. 2.19.1, arc of contact is *EPF* or *GPH*.

$$\text{Arc } GPH = \text{Arc } GP + \text{Arc } PH$$

$$= \text{Arc of approach} + \text{Arc of recess}$$

$$\text{Arc } GP = \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

$$\text{Arc } PH = \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

3. So, length of arc of contact = Arc GP + Arc PH

$$= \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KP+PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

Que 2.20. **Prove that the maximum length of arc of contact between a pair of gear tooth to avoid interference is $(r + R) \tan \phi$.**

Answer

1. Fig. 2.20.1 shows a pinion with centre O_1, in mesh with wheel or gear with centre O_2. MN is the common tangent to the base circles and KL is the path of contact between the two mating teeth.
2. The maximum length of path of contact is MN, when the maximum addendum circles for pinion and wheel pass through the points of tangency N and M respectively as shown in Fig. 2.20.1.
3. In such a case :

   Maximum length of path of approach, $MP = r \sin \phi$

   and maximum length of path of recess, $PN = R \sin \phi$
4. Maximum length of path of contact,

$$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$$

and maximum length of arc of contact,

$$= \frac{(r + R) \sin \phi}{\cos \phi} = (r + R) \tan \phi$$

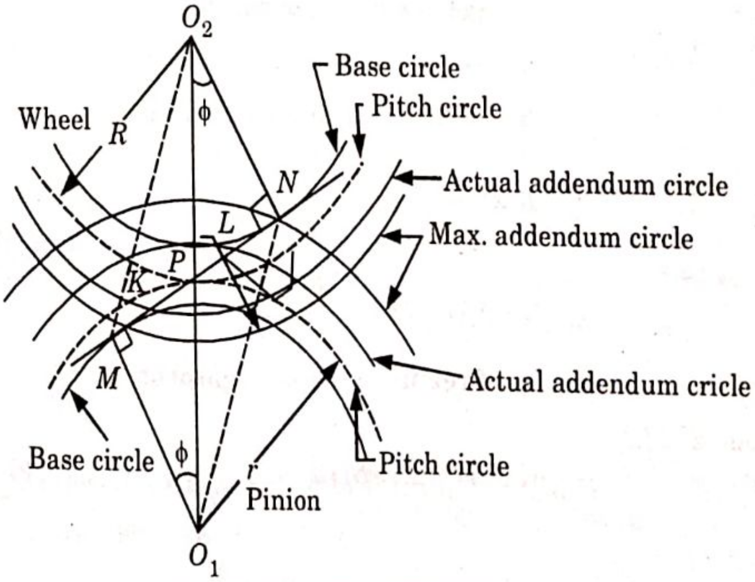


Fig. 2.20.1. Interference in involute gears.

Que 2.21. **Derive an expression for minimum number of teeth on the gear wheel to avoid interference.**

Answer

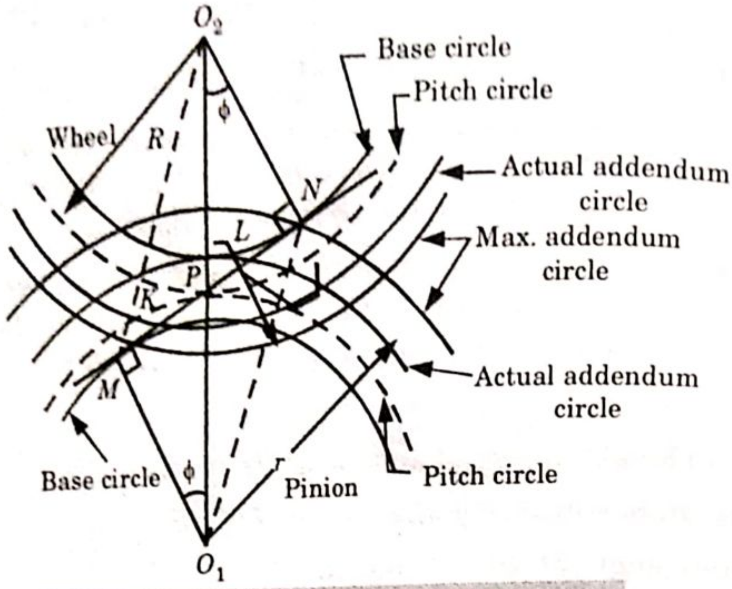


Fig. 2.21.1. Interference in involute gears.

1. Let T = Minimum number of teeth required on the wheel to avoid interference,

$A_w m$ = Addendum of the wheel,

Where, A_w = A constant, which is multiplied by module to get addendum of wheel,

m = Module,

R = Pitch circle radius of wheel,

$$R = \frac{mT}{2}$$

$$G = \text{Gear ratio} = \frac{T}{t} = \frac{R}{r}, \text{ and}$$

ϕ = Pressure angle or angle of obliquity.

2. From ΔO_2MP,

$$(O_2M)^2 = (O_2P)^2 + (PM)^2 - 2 \times O_2M \times PM \cos(\angle O_2PM)$$

$$= R^2 + r^2\sin^2\phi - 2Rr\sin\phi\cos(90° + \phi)$$

$$= R^2 + r^2\sin^2\phi + 2Rr\sin^2\phi$$

$$[\because PM = O_1P\sin\phi = r\sin\phi]$$

$$= R^2\left[1 + \frac{r^2\sin^2\phi}{R^2} + \frac{2r\sin^2\phi}{R}\right]$$

$$= R^2\left[1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi\right]$$

$$O_2M = \sqrt{R^2\left[1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi\right]}$$

$$= \frac{mT}{2}\sqrt{\left[1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi\right]}$$

$$\left[\because\ R=\frac{mT}{2}, \frac{r}{R}=\frac{t}{T}=\frac{1}{G}\right]$$

$$\therefore \quad O_2M = \frac{mT}{2}\sqrt{\left[1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi\right]}$$

3. Addendum of the gear wheel = $O_2M - O_2P$

$$A_wm = \frac{mT}{2}\sqrt{\left[1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi\right]} - \frac{mT}{2}$$

or

$$T = \frac{2A_w}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}\right]-1}$$

4. For $G = 1$, we have

$$T = \frac{2A_w}{\sqrt{1+3\sin^2\phi}-1}$$

Que 2.22. **Derive an expression for minimum number of teeth required on the pinion in order to avoid interference.**

Answer

1. Let t = Minimum number of teeth on the pinion required to avoid interference,

A_pm = Addendum of the pinion,

A_p = Fraction by which the standard addendum of one module for the pinion should be multiplied to avoid interference, and

$r = \frac{mt}{2}$ = Pitch circle radius of pinion.

2. From ΔO_1NP, (Refer Fig. 2.21.1),

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2\,O_1P \times PN \cos(\angle O_1PN)$$
$$= r^2 + R^2 \sin^2\phi - 2rR\sin\phi\cos(90° + \phi)$$
$$= r^2 + R^2 \sin^2\phi + 2rR\sin^2\phi$$
$$= r^2\left[1 + \frac{R^2}{r^2}\sin^2\phi + \frac{2R}{r}\sin^2\phi\right]$$
$$= r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$
$$= r^2\left[1 + \left(\frac{T}{t} + 2\right)\frac{T}{t}\sin^2\phi\right]$$
$$O_1N = r\sqrt{[1 + G(G+2)\sin^2\phi]}$$

3. Addendum of the pinion $= O_1N - O_1P$

$$A_p m = \frac{mt}{2}\sqrt{1 + G(G+2)\sin^2\phi} - \frac{mt}{2}$$
$$A_p m = \frac{mt}{2}\left[\sqrt{1 + G(G+2)\sin^2\phi} - 1\right]$$
$$t = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

4. For $G = 1$,

$$t = \frac{2A_p}{\sqrt{1 + 3\sin^2\phi} - 1}$$

Form above result, we concluded that for $G = 1$, both gear and pinion required same number of teeth to avoid interference.

Que 2.23. **Derive an expression for minimum number of teeth required on a pinion to avoid interference when it gears with a rack.**

Answer

1. Fig. 2.23.1 shows a rack and pinion in which pinion is rotating in the clockwise direction and driving the rack.
2. P is the pitch point and PE is the line of action. Engagement of the rack tooth with the pinion tooth occurs at C.

3. To avoid interference, the maximum addendum of the rack can be increased in such a way that C coincides with E. Thus, the addendum of the rack must be less than GE.

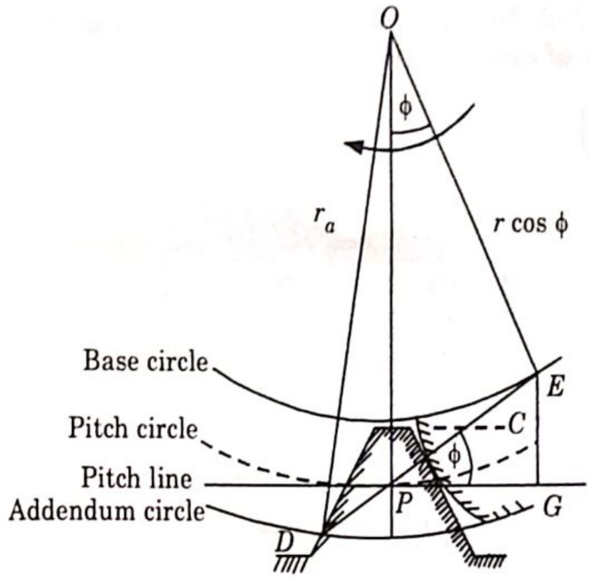


Fig. 2.23.1.

4. Let the adopted value of the addendum of the rack be $a_r m$ where a_r is the addendum coefficient by which the standard value of the addendum has been multiplied.

5. From Fig. 2.23.1, $GE = PE \sin \phi = (r \sin \phi) \sin \phi = r \sin^2 \phi$

$$GE = \frac{mt}{2} \sin^2 \phi$$

5. To avoid interference, $GE \geq a_r m$

$$\frac{mt}{2} \sin^2 \phi \geq a_r m$$

$$t \geq \frac{2a_r}{\sin^2 \phi}$$

When $a_r = 1$ *i.e.*, for standard addendum,

$$t_{\min} \geq \frac{2}{\sin^2 \phi}$$

6. Thus, the minimum number of teeth required on a pinion to avoid interference when it gears with a rack is given as,

$$t_{\min} \geq \frac{2}{\sin^2 \phi}$$

Que 2.24. **Two mating gears have 20 and 40 involute teeth of module 10 mm and 20° pressure angle. If the addendum on each wheel is such that the path of contact is maximum and interference is just avoided, find the addendum for each gear wheel, path of contact, arc of contact and contact ratio.**

Answer

Given : $t = 20$, $T = 40$, $m = 10$ mm, $\phi = 20°$

To Find :
i. Addendum for each gear wheel.
ii. Path of contact.
iii. Arc of contact.
iv. Contact ratio.

1. Pitch circle radius of the smaller gear,

$$r = mt/2 = 10 \times 20/2 = 100 \text{ mm}$$

Pitch circle radius of the larger gear,

$$R = mT/2 = 10 \times 40/2 = 200 \text{ mm}$$

2. For the condition of maximum path of contact and no interference :

i. Path of approach $= r \sin \phi$

$$\sqrt{R_A^2 - R^2 \cos^2 \phi} - R \sin \phi = r \sin \phi$$

$$\sqrt{R_A^2 - (200)^2 \cos^2 20°} - 200 \sin 20° = 100 \sin 20°$$

$$\sqrt{R_A^2 - 35320} = 300 \sin 20° = 300 \times 0.342$$

$$\sqrt{R_A^2 - 35320} = 102.6$$

$$R_A^2 - 35320 = 10526.76 \quad \text{(Squaring both sides)}$$

$$R_A^2 = 45846.76$$

or $$R_A = 214.12 \text{ mm}$$

$\therefore$ Addendum height for larger gear wheel

$$= R_A - R = 214.12 - 200 = 14.12 \text{ mm}$$

ii. Path of recess $= R \sin \phi$

$$\sqrt{r_A^2 - r^2 \cos^2 \phi} - r \sin \phi = R \sin \phi$$

$$\sqrt{r_A^2 - (100)^2 \cos^2 20} - 100 \sin 20° = 200 \sin 20°$$

$$\sqrt{r_A^2 - 8830.22} = 300 \sin 20°$$

$$\sqrt{r_A^2 - 8830.22} = 102.6$$

$$r_A^2 - 8830.22 = 10526.76$$

$$r_A^2 = 19356.98$$

or $$r_A = 139.12 \text{ mm}$$

$\therefore$ Addendum height for smaller gear wheel

$$= r_A - r = 139.12 - 100 = 39.12 \text{ mm}$$

3. Length of the path of contact $= (r + R) \sin \phi = (100 + 200) \sin 20°$

$$= 102.6 \text{ mm}$$

4. Length of the arc of contact $= (r + R) \tan \phi = (100 + 200) \tan 20°$

$$= 109.2 \text{ mm}$$

5. $$\text{Contact ratio} = \frac{\text{Length of arc of contact}}{p_c}$$

$$= \frac{109.2}{\pi m} = \frac{109.2}{\pi \times 10} = 3.46 \approx 4$$

Que 2.25. **A pair of involute spur gears with 16° pressure angle and pitch of module 6 mm in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 rpm. When the gear ratio is 1.75, find in order that interference is just avoided :**

i. The addenda on pinion and gear wheel,

ii. The length of path of contact, and

iii. The maximum velocity of sliding of teeth on either side of the pitch point.

Answer

Given : $\phi = 16°$, $N_1 = 240$ rpm, $m = 6$ mm, $t = 16$,

$G = T/t = 1.75$, $T = 1.75 \times 16 = 28$

To Find :
i. The addenda on pinion and gear wheel.
ii. The length of path of contact.
iii. The maximum velocity of sliding of teeth on either side of the pitch point.

1. Addendum on pinion $= \dfrac{mt}{2}\left[\sqrt{1 + \dfrac{T}{t}\left(\dfrac{T}{t} + 2\right)\sin^2 \phi} - 1\right]$

$$= \frac{6 \times 16}{2}\left[\sqrt{1+\frac{28}{16}\left(\frac{28}{16}+2\right)\sin^2 16^\circ} - 1\right]$$

$$= 48\,(1.224 - 1) = 10.76 \text{ mm}$$

2. Addendum on gear wheel $= \dfrac{mT}{2}\left[\sqrt{1+\dfrac{t}{T}\left(\dfrac{t}{T}+2\right)\sin^2\phi} - 1\right]$

$$= \frac{6 \times 28}{2}\left[\sqrt{1+\frac{16}{28}\left(\frac{16}{28}+2\right)\sin^2 16^\circ} - 1\right]$$

$$= 84\,(1.054 - 1) = 4.536 \text{ mm}$$

3. Pitch circle radius of wheel, $R = \dfrac{mT}{2} = \dfrac{6 \times 28}{2} = 84$ mm

Pitch circle radius of pinion, $r = \dfrac{mt}{2} = \dfrac{6 \times 16}{2} = 48$ mm

4. Addendum circle radius of wheel,

$$R_A = R + \text{Addendum of wheel}$$

$$= 84 + 10.76 = 94.76 \text{ mm}$$

Addendum circle radius of pinion,

$$r_A = r + \text{Addendum of pinion}$$

$$= 48 + 4.536 = 52.536 \text{ mm}$$

5. Length of path of approach,

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(94.76)^2 - (84)^2\cos^2 16^\circ} - 84\sin 16^\circ$$

$$= 49.6 - 23.16 = 26.45 \text{ mm}$$

Length of path of recess,

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(52.536)^2 - (48)^2\cos^2 16^\circ} - 48\sin 16^\circ$$

$$= 25.17 - 13.23 = 11.94 \text{ mm}$$

6. Length of path of contact

$$KL = KP + PL = 26.45 + 11.94 = 38.34 \text{ mm}$$

7. We know that $\dfrac{\omega_1}{\omega_2} = \dfrac{T}{t} = 1.75$

$$\omega_2 = \frac{\omega_1}{1.75} = \frac{25.132}{1.75} = 14.36 \text{ rad/s}$$

$$\left(\because \omega_1 = \frac{2\pi \times 240}{60} = 25.132 \text{ rad/s}\right)$$

∴ Maximum velocity of sliding of teeth on the left side of pitch point *i.e.*, at point $K = (\omega_1 + \omega_2)\, KP$

$$= (25.132 + 14.36) \times 26.45 = 1044.56 \text{ mm/s}$$

Maximum velocity of sliding of teeth on the right side of pitch point *i.e.*, at point $L = (\omega_1 + \omega_2)\, PL$

$$= (25.132 + 14.36) \times 11.89 = 469.6 \text{ mm/s}$$

PART-7

Gear Trains, Simple, Compound, Reverted and Planetary Gear Trains, Sun and Planet Gear Train.

CONCEPT OUTLINE

Gear train : It can be defined as a mechanism which transmits power or motion from one shaft to another with the help of gear wheels. Gear trains enable the speed of the follower or the driven wheel to be stepped up or down.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.26. **What are different types of gear train ? Write expression for velocity ratio.**

Answer

Gear trains may be classified into the following categories :

a. **Simple Gear Trains :**

1. When there is only one gear on each shaft, it is known as simple gear train. The gears are represented by their pitch circles.

2. Fig. 2.26.1(*a*) and (*b*) shows simple gear trains of two and three gears respectively.

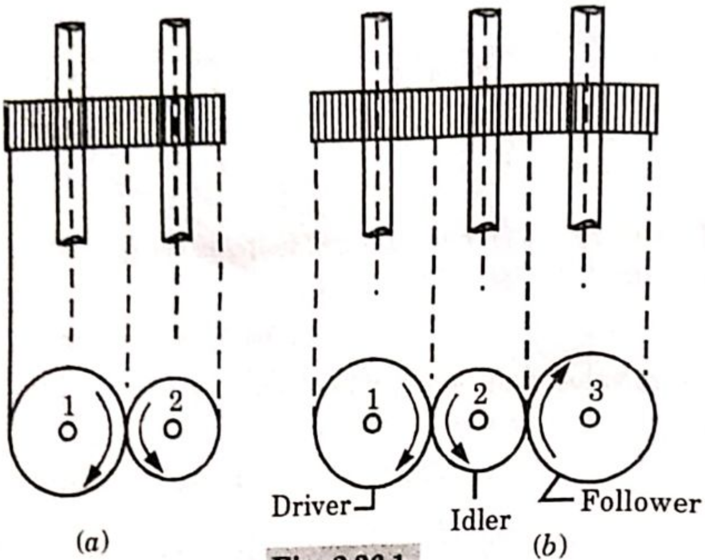


Fig. 2.26.1.

3. Let N_i = Rpm of gear numbered i.

T_i = Number of teeth on gear numbered i.

ω_i = Angular velocity of gear numbered i.

4. The speed ratio is given as,

$$\frac{\omega_2}{\omega_1} = \frac{T_1}{T_2} = \frac{N_2}{N_1}$$

$$\frac{N_3}{N_2} = \frac{\omega_3}{\omega_2} = \frac{T_2}{T_3}$$

5. The directions of rotation of 1 and 2 are opposite to each other and that of 2 and 3 are opposite so directions of 1 and 3 [Fig. 2.26.1(*b*)] are same.

b. Compound Gear Trains :

1. When there are more than one gear on a shaft, as shown in Fig. 2.26.2, it is called a compound gear train.

2. In a compound gear train, shown in Fig. 2.26.2, the gear 1 is the driving gear mounted on shaft *A*, gear 2 and 3 are compound gears which are mounted on shaft *B*. The gear 4 and 5 are also compound gears which are mounted on shaft *C* and the gear 6 is the driven gear mounted on shaft *D*.

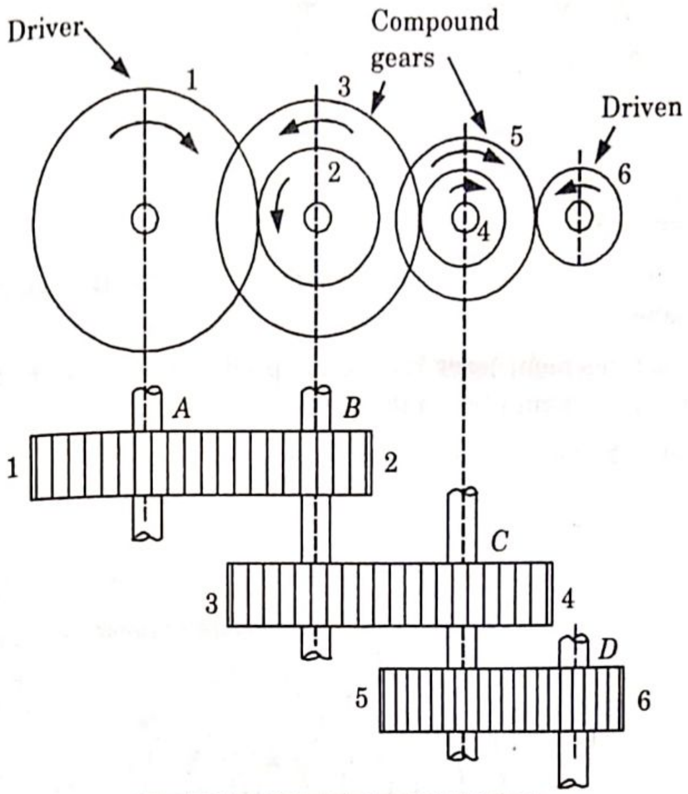


Fig. 2.26.2. Compound gear train.

3. Let N_1 = Speed of driving gear 1,

T_1 = Number of teeth on driving gear 1,

$N_2, N_3 ..., N_6$ = Speed of respective gears in rpm, and

$T_2, T_3 ..., T_6$ = Number of teeth on respective gear.

4. Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad ...(2.26.1)$$

5. Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad ...(2.26.2)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad ...(2.26.3)$$

6. The speed ratio of compound gear train is obtained by multiplying the eq. (2.26.1), eq. (2.26.2) and eq. (2.26.3),

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

or
$$\frac{N_1}{N_6} = \frac{T_2 T_4 T_6}{T_1 T_3 T_5}$$

c. **Reverted Gear Train :**

1. A reverted gear train is one in which the first and the last gear are on the same axis.
2. This arrangement finds its use in speed reducers, clocks to connect hours hand to minutes hand, machine tools etc.
3. The speed ratio of reverted gear train is given as

$$\frac{N_1}{N_4} = \frac{T_2}{T_1} \frac{T_4}{T_3}$$

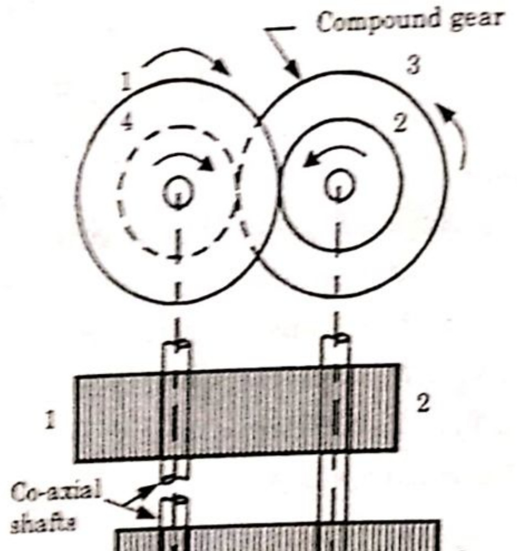


Fig. 2.26.3. Reverted gear train.

d. **Epicyclic Gear Trains :**

1. All the gear trains so far discussed have the common feature that the axes of revolution of the gears are stationary.
2. If one or more gears have axes which are not stationary the gear train is called an epicyclic gear train.

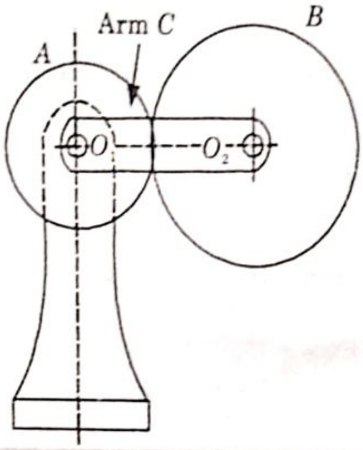


Fig. 2.26.4. Epicyclic gear train.

Que 2.27. **What do you understand by the term train value ? For what purpose idle gears are used ?**

Answer

A. Train Value :

1. It is the ratio of the speed of the driven or follower to the speed of the driver. It is the reciprocal of speed ratio.
2. Mathematically,

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}}$$

B. Purpose of Idle Gears : The idle gears have two purposes as follows :

1. These are used to connect the gears, which are at a large distance.
2. For rotating the follower into desired direction. As, if the number of intermediate gears are odd, the motion of both driver and follower is like and if the number of intermediate gears are even, the motion of both driver and follower is unlike (or in opposite direction).

Que 2.28. **Explain briefly the differences between simple, compound and epicyclic trains. What are the special advantages of epicyclic gear trains ?**

Answer

A. Difference between Simple, Compound and Epicyclic Gear Train :

S. No.	Simple Gear Train	Compound Gear Train	Epicyclic Gear Train
1.	Only one gear in each shaft and there is a relative motion between shaft axis.	In this there is more than one gear on the shaft which is rigidly fix and meshed with the gear on another shaft forming gear train.	The axis of shaft on which the gear is mounted may move relative to the fixed axis.
2.	For large speed reduction, large size of gear is required.	For large speed reduction, the small gear ratio is required.	For large speed reduction in high velocity, moderate size gear is required.

B. Advantages of Epicyclic Gear Train :

1. They have higher gear ratio.
2. They are popular for automatic transmission in automobiles.
3. They are also used in bicycles for controlling power of pedaling automatically or manually.
4. They are also used for power transmission between internal combustion engine and an electric motor.

Que 2.29. **The speed ratio of the reverted gear train, as shown in Fig. 2.29.1, is to be 12. The module of gears *A* and *B* is 3.125 mm and of gears *C* and *D* is 2.5 mm. Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth.**

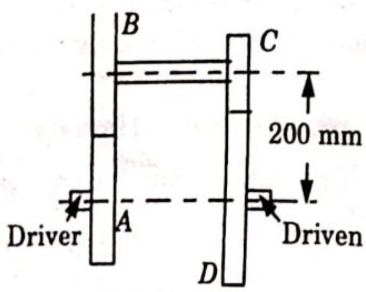


Fig. 2.29.1.

Answer

Given : Speed ratio, $N_A/N_D = 12$, $m_A = m_B = 3.125$ mm, $m_C = m_D = 2.5$ mm.
To Find : Number of teeth for the gears.

1. Since the speed ratio between the gears A and B and between the gears C and D are to be same, therefore

$$\frac{N_A}{N_D} = \frac{N_A}{N_B} \times \frac{N_C}{N_D} = \sqrt{12} \times \sqrt{12} = 12$$

$$\frac{N_A}{N_B} = \frac{N_C}{N_D} = \sqrt{12} = 3.464$$

2. Also the speed ratio of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\frac{T_B}{T_A} = \frac{T_D}{T_C} = 3.464 \qquad \text{...(2.29.1)}$$

3. We know that the distance between the shafts

$$x = r_A + r_B = r_C + r_D = 200 \text{ mm}$$

or $$\frac{m_A T_A}{2} + \frac{m_B T_B}{2} = \frac{m_C T_C}{2} + \frac{m_D T_D}{2} = 200 \qquad \left(\because\ r = \frac{mT}{2}\right)$$

$3.125\,(T_A + T_B) = 2.5\,(T_C + T_D) = 400 \qquad (\because\ m_A = m_B$ and $m_C = m_D)$

$\therefore \qquad T_A + T_B = 400/3.125 = 128 \qquad$ ...(2.29.2)

and $\qquad T_C + T_D = 400/2.5 = 160 \qquad$ ...(2.29.3)

5. From eq. (2.29.1), $T_B = 3.464\ T_A$. Substituting this value of T_B in eq. (2.29.2), we get

$$T_A + 3.464\ T_A = 128$$

$$T_A = 128/4.464 = 28.67 \text{ say } 28$$

and $$T_B = 128 - 28 = 100$$

6. Again from eq. (2.29.1), $T_D = 3.464\ T_C$. Substituting this value of T_D in eq. (2.29.3), we get

$$T_C + 3.464\ T_C = 160$$

$$T_C = 160/4.464 = 35.84 \text{ say } 36$$

and $$T_D = 160 - 36 = 124$$

Que 2.30. **Explain with a neat sketch the 'sun and planet wheel'.**

Answer

1. Fig. 2.30.1 shows a sun and planet wheel arrangement.
2. Sun and planet wheel arrangement consists of.
   i. Two coaxial shafts S_1 and S_2,
   ii. An annulus gear *A* with internal teeth,
   iii. A compound gear or planet gear *B-C*,
   iv. The sun gear *D*, and
   v. The arm *H*.
3. The annulus gear is connected to the compound gear *B-C* which are carried by the arm and revolves freely on a pin of the arm *H*.

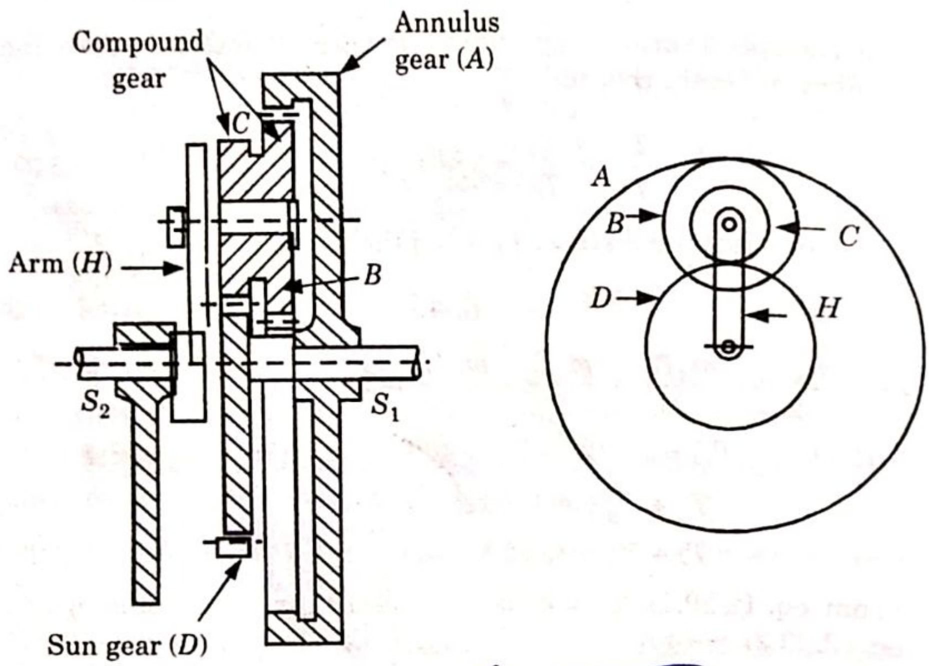


Fig. 2.30.1. Compound epicyclic gear train.

4. The sun gear is coaxial with the arm and the annulus gear.
5. The annulus gear *A* meshes with gear *B* and the sun sear *D* meshes with the gear *C*.
6. When the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive. In both cases, the arm acts as a follower.

Que 2.31. **In a reverted epicyclic gear train, the arm 'A' carries two gears *B* and *C* and a compound gear *D-E*. The gear *B* meshes with gear *E* and gear *C* meshes with gear *D*. The number of teeth on gears *B*, *C* and *D* are 75, 30 and 90 respectively. Find the speed and**

direction of gear C when gear B is fixed and the arm 'A' makes 100 rpm clockwise.

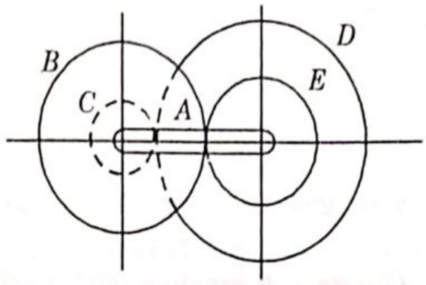


Fig. 2.31.1.

Answer

Given : $T_B = 75$, $T_C = 30$, $T_D = 90$, $N_A = 100$ rpm (Clockwise)

To Find : Speed and direction of gear C when gear B is fixed.

1. From the geometry of Fig. 2.31.1

$$d_B + d_E = d_C + d_D$$

2. Since, pitch circle diameter is proportional to number of teeth

So,
$$T_B + T_E = T_C + T_D$$
$$T_E = T_C + T_D - T_B$$
$$= 30 + 90 - 75 = 45$$

3. The table of motion is shown below :

S. No.	Condition of Motion	Revolution of Elements			
		Arm A	Compound Gear D-E	Gear B	Gear C
1.	Arm fixed, D-E rotates through +1 revolution.	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed, D-E rotates through + x revolution.	0	$+x$	$-x\frac{T_E}{T_B}$	$-x\frac{T_D}{T_C}$
3.	Add + y revolution to all elements.	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\frac{T_E}{T_B}$	$y-x\frac{T_D}{T_C}$

4 As gear B is fixed,

$$\therefore \quad y - x\frac{T_E}{T_B} = 0$$

$$y - x\frac{45}{75} = 0$$

$$y - 0.6\,x = 0$$

So, $x = \frac{y}{0.6} = \frac{-100}{0.6} = -166.67$ rpm ($\because N_A = y = -100$)

5. Now speed of gear C,

$$N_C = y - x\frac{T_D}{T_C} = -100 - (-166.67) \times \frac{90}{30} = 400.01$$

≈ 400 rpm (anti clockwise)

Que 2.32. **An epicyclic gear consists of three gears A, B and C as shown in Fig. 2.36.1. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 rpm. If the gear A is fixed, determine the speed of gears B and C.**

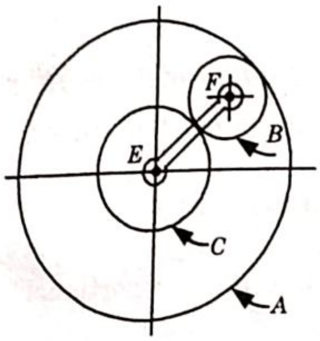


Fig. 2.32.1.

Answer

Given : $T_A = 72$, $T_C = 32$, $N_{EF} = 18$ rpm

To Find : Speed of gear B and C when gear A is fixed.

1. The table of motion is shown below :

S.No.	Condition of Motion	Revolution of Elements			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed, wheel rotates through +1 revolution anticlockwise	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2.	Arm fixed, wheel rotates through + x revolution	0	$+x$	$-x\frac{T_C}{T_B}$	$-x\frac{T_C}{T_A}$
3.	Add + y revolution to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\frac{T_C}{T_B}$	$y-x\frac{T_C}{T_A}$

2. We know that the speed of the arm is 18 rpm, therefore

$$y = 18 \text{ rpm}$$

and the gear A is fixed, therefore

$$y - x\,\frac{T_C}{T_A} = 0 \quad \text{or} \quad 18 - x \times \frac{32}{72} = 0$$

$$x = 18 \times 72/32 = 40.5$$

$\therefore$ Speed of gear $C = x + y = 40.5 + 18 = 58.5$ rpm

$= 58.5$ rpm in the direction of arm.

3. From the geometry of Fig. 2.36.1

$$d_B + \frac{d_C}{2} = \frac{d_A}{2} \quad \text{or} \quad 2d_B + d_C = d_A.$$

Since the number of teeth are proportional to their pitch circle diameters, therefore

$$2T_B + T_C = T_A \quad \text{or} \quad 2T_B + 32 = 72 \quad \text{or} \quad T_B = 20$$

4. Speed of gear $B = y - x\,\dfrac{T_C}{T_B} = 18 - 40.5 \times \dfrac{32}{20} = -46.8$ rpm

$= 46.8$ rpm in the opposite direction of arm.

Que 2.33. **A compound epicyclic gear is shown diagrammatically in Fig. 2.33.1. The gears *A*, *D* and *E* are free to rotate on the axis *P*. The compound gear *B* and *C* rotate together on the axis *Q* at the end of arm *F*. All the gears have equal pitch. The number of external teeth on the gears *A*, *B* and *C* are 18, 45 and 21 respectively. The gears *D* and *E* are annular gears. The gear *A* rotates at 100 rpm in the anticlockwise direction and the gear *D* rotates at 450 rpm clockwise. Find the speed and direction of the arm and the gear *E*.**

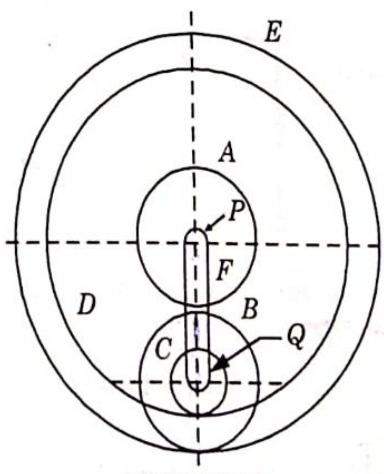


Fig. 2.33.1.

Answer

Given : $T_A = 18$, $T_B = 45$, $T_C = 21$, $N_A = 100$ rpm (anticlockwise), $N_D = 450$ rpm (clockwise)
To Find : Speed and direction of the arm and the gear E.

1. As all the gears have equal pitch so their diameters are directly proportional to their teeth.

∴ From geometry of Fig. 2.33.1,

$$\frac{d_A}{2} + d_B = \frac{d_E}{2} \text{ or } d_E = d_A + 2d_B$$

$$T_E = T_A + 2T_B = 18 + 2 \times 45 = 108$$

Also, $\frac{d_A}{2} + \frac{d_B}{2} + \frac{d_C}{2} = \frac{d_D}{2}$

$$T_A + T_B + T_C = T_D$$

$$T_D = 18 + 45 + 21 = 84$$

2. The table of motion is shown below,

S. No.	Conditions of Motion	Revolution of Elements				
		Arm F	Gear A	Compound Gear B–C	Gear D	Gear E
1.	Arm is fixed and gear A rotates through + 1 revolution anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$	$-\frac{T_C T_A}{T_D T_B}$	$-\frac{T_A}{T_E}$
2.	Arm is fixed and gear A rotates through + x revolution	0	x	$-\frac{xT_A}{T_B}$	$-\frac{xT_C T_A}{T_D T_B}$	$-\frac{xT_A}{T_E}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y	+ y
4.	Total motion	+ y	$x + y$	$y - \frac{xT_A}{T_B}$	$y - \frac{xT_C T_A}{T_D T_B}$	$y - \frac{xT_A}{T_E}$

3. Taking anticlockwise revolution positive and clockwise revolution negative. It is given that,

$$x + y = 100 \text{ (anticlockwise)} \quad ...(2.33.1)$$

$$y - \frac{xT_C T_A}{T_D T_B} = -450 \text{ (clockwise)}$$

$$y - \frac{x \times 21 \times 18}{84 \times 45} = -450$$

$$y - 0.1x = -450 \qquad \text{...(2.33.2)}$$

4. On solving eq. (2.33.1) and (2.33.2), we get

$$y = -400 \text{ rpm}$$

and $$x = 500 \text{ rpm}$$

5. Speed of arm $F = y = 400$ rpm (clockwise)

6. Speed of gear $E = y - \dfrac{xT_A}{T_E} = -400 - \dfrac{500 \times 18}{108}$

$$= -483.33 \text{ rpm}$$

$$= 483.33 \text{ rpm (clockwise)}$$

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3

UNIT

Force Analysis

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PART-1

Static Force Analysis of Mechanisms.

CONCEPT OUTLINE

Equilibrium : A rigid body is said to be in equilibrium if it continues its state of motion.

The conditions of equilibrium can be written mathematically as :

$$\Sigma F = 0$$
$$\Sigma M = 0$$

Static Force Analysis : This analysis is carried out by determining all the forces and couples acting on all the pairs. In this analysis, we ignore the forces due to acceleration. Only the effects of external forces applied to the mechanism are considered.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. Explain the necessary conditions for static equilibrium of a body for a planar system.

Answer

1. A body is said to be in static equilibrium if it remains in its state of rest or motion.
2. The necessary conditions of static equilibrium are as follows :
   i. The vector sum of all the forces acting on the body must be zero *i.e.*,
   $$\Sigma F = 0$$
   ii. The vector sum of all the moment about any arbitrary point must be zero *i.e.*,
   $$\Sigma M = 0$$
3. If body is in planar (or 2-Dimensional system) motion then the conditions of static equilibrium are as follows :
   i. $\Sigma F_x = 0$ *i.e.*, sum of all the horizontal components must be zero.
   ii. $\Sigma F_y = 0$ *i.e.*, sum of all the vertical components must be zero.
   iii. $\Sigma M = 0$ *i.e.*, sum of all the moment about any arbitrary point must be zero.

Que 3.2. **What are free body diagrams of a mechanism ? How are they helpful in finding the various forces acting on the various members of the mechanism ?** **AKTU 2013-14, Marks 3.5**

Answer

1. A free body diagram is a sketch or diagram of a part isolated from the mechanism in order to determine the nature of forces acting on it.
2. Fig. 3.2.1(*a*) shows a four link mechanism. The free body diagrams of its members 2, 3 and 4 are shown in Fig. 3.2.1(*b*), (*c*) and (*d*) respectively.
3. Various forces acting on each member are also shown. As the mechanism is in static equilibrium, each of its members must be in equilibrium individually.
4. Member 4 is acted upon by three forces F, F_{34} and F_{14}.
5. Member 3 is acted upon by two forces F_{23} and F_{43}.
6. Member 2 is acted upon by two forces F_{32} and F_{12} and a torque T.
7. Initially, the direction and the sense of some of the forces may not be known.
8. Assume that the forces F on the member 4 are known completely.
9. To know the other two forces acting on this member completely, the direction of one more force must be known.

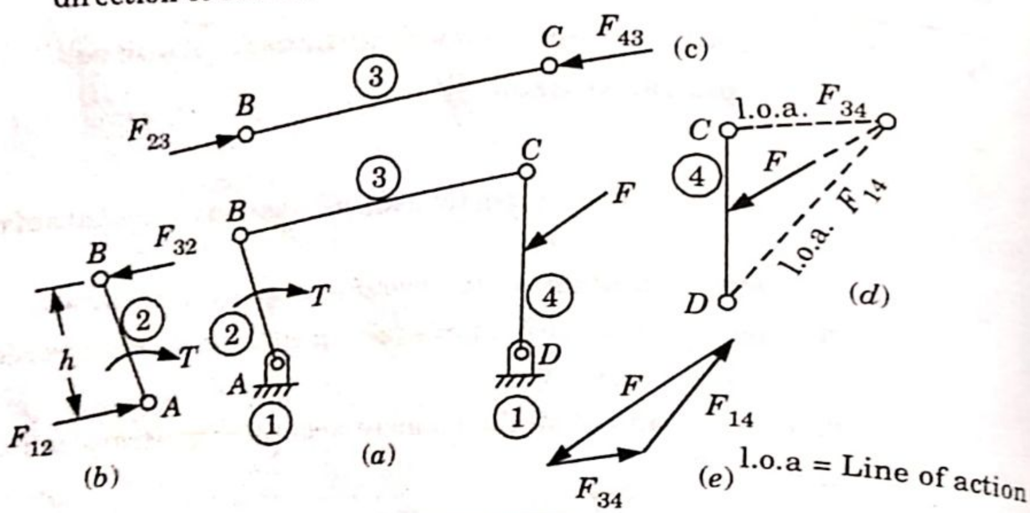


Fig. 3.2.1.

10. Link 3 is a two force member and for its equilibrium, F_{23} and F_{43} must act along BC. Thus F_{34}, being equal and opposite to F_{43}, also acts along BC. For the member 4 to be in equilibrium, F_{14} passes through the intersection of F and F_{34}. By drawing a force triangle (F is completely known), magnitude F_{14} and F_{34} can be known [Fig. 3.2.1(*e*)].

Now,
$$F_{34} = F_{43} = F_{23} = F_{32}$$

11. Member 2 will be in equilibrium if F_{12} is equal, parallel and opposite to F_{32} and

$$T = F_{12}h = F_{32}h$$

Que 3.3. What are the conditions for a body to be in equilibrium under the action of two forces, three forces and two forces and a torque ?

AKTU 2013-14, Marks 3.5

Answer

A. **Two Force System :** Fig. 3.3.1 shows a member under the action of two forces. The body will be in equilibrium if

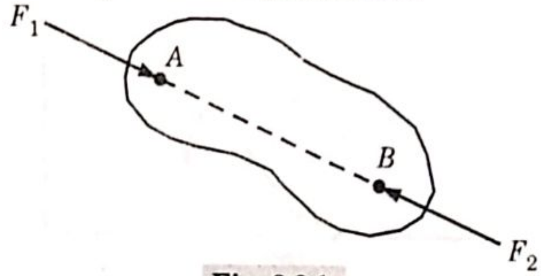


Fig. 3.3.1.

1. The forces are of the same magnitude, *i.e.*, $F_1 = F_2$
2. The forces have same line of action.
3. The forces have opposite directions to each other.

B. **Three Force System :** Fig. 3.3.2(*a*) shows a three force system. The body will be in equilibrium if
1. The resultant of the forces F_1, F_2 and F_3 are zero or the vector sum of all three forces should be zero.

   i.e., $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$
2. The three forces make a closed vector diagram as shown in Fig. 3.3.2(*b*).

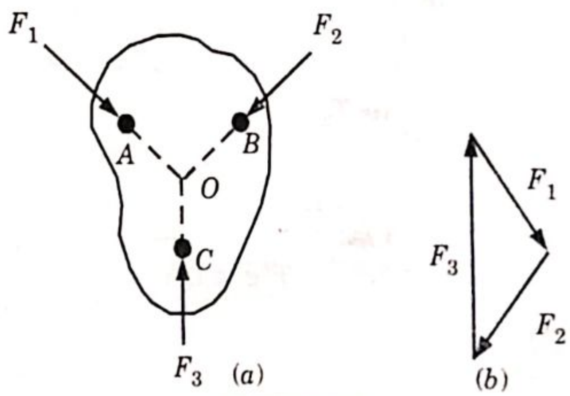


Fig. 3.3.2.

3. These forces intersect at a single point, that point is known as point of concurrency.

C. Two Forces and a Torque System :

1. Let us consider a link that is hinged at its one end and free to rot about the hinged point.
2. A force F_1 is acting at free end, a force F_2 is acting at the hinge point and a torque T is acting on the link in clockwise sense.
3. This system will be in equilibrium if
   i. The forces are equal in magnitude and parallel to each ot with opposite direction. *i.e.*, $F_1 = F_2$.
   ii. These forces F_1 and F_2 form a couple which is equal and oppo to the applied torque *i.e.*,

$$F_1 h = F_2 h = T$$

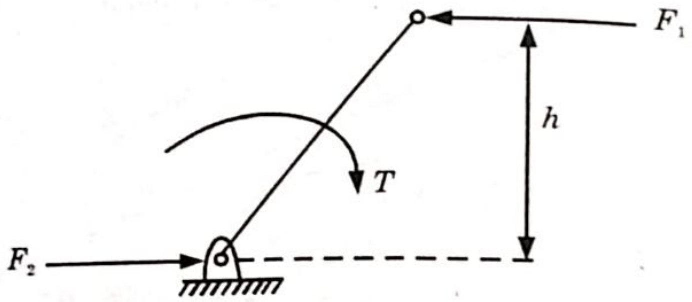


Fig. 3.3.3.

Que 3.4. **The four bar chain mechanism in which crank is driv by an input torque T_2 in clockwise direction and rocker link subjected to external force F = 500 N at mid point. Find all t constraint forces for static equilibrium of the mechanism. Li length are AB = 30 cm, BC = 70 cm, CD = 60 cm, AD = 50 c OD = 30 cm.**

AKTU 2014-15, Marks 0

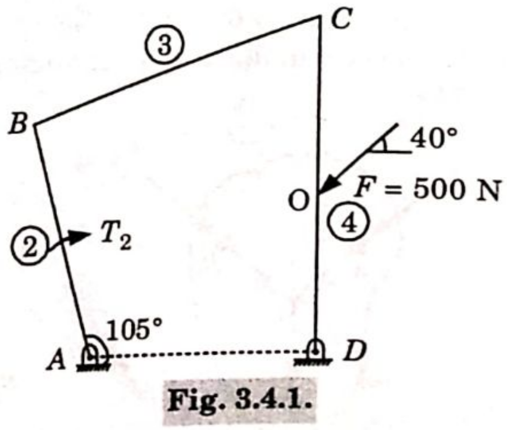


Fig. 3.4.1.

Answer

Given : F = 500 N, AB = 30 cm, BC = 70 cm, CD = 60 cm, AD = 50 cm
OD = 30 cm

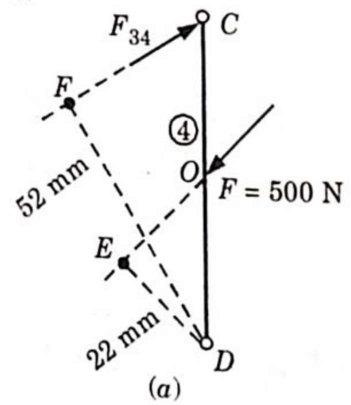
To Find : All the constraint forces for static equilibrium.

1. Draw the given mechanism, with given force F = 500 N with some suitable scale.
2. Draw FBD of all links with all applied forces on each links as shown in Fig. 3.4.2(*a*), (*b*) and (*c*).
3. Refer Fig. 3.4.2(*a*), link 4 = F_{34} acts along link 3 at point C and external force F = 500 N at point O. Taking moment about D,

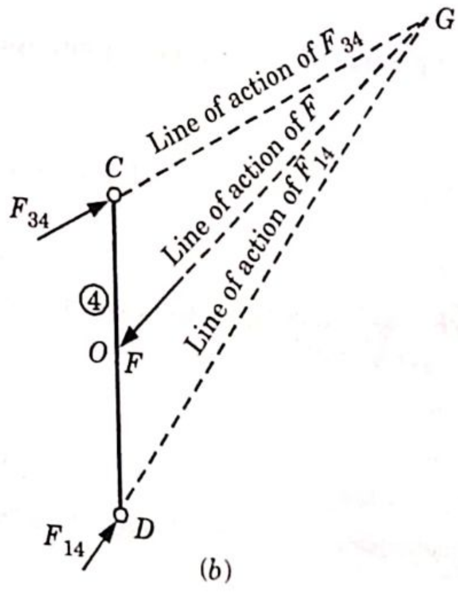
$$F \times DE - F_{34} \times DF = 0$$
$$500 \times 22 - F_{34} \times 52 = 0$$
$$F_{34} = 211.53 \text{ N}$$

4. We can also find magnitude of F_{34} by considering link 4 is a three force member as shown in Fig. 3.4.2 (*b*). Therefore extend line of action of force F_{34} and F intersect at point G. Join GD which gives line of action of force F_{14}. Now draw closed force polygon of F, F_{34} and F_{14} in which magnitude and directions of F is known and only directions of F_{34}, and F_{14} is known.
5. From force polygon, we get

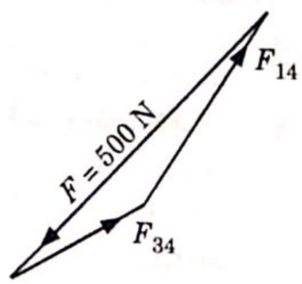
$$F_{34} = 210 \text{ N}$$
$$F_{14} = 290 \text{ N}$$



(*a*)



(*b*)



Force polygon of link 4

(*c*)

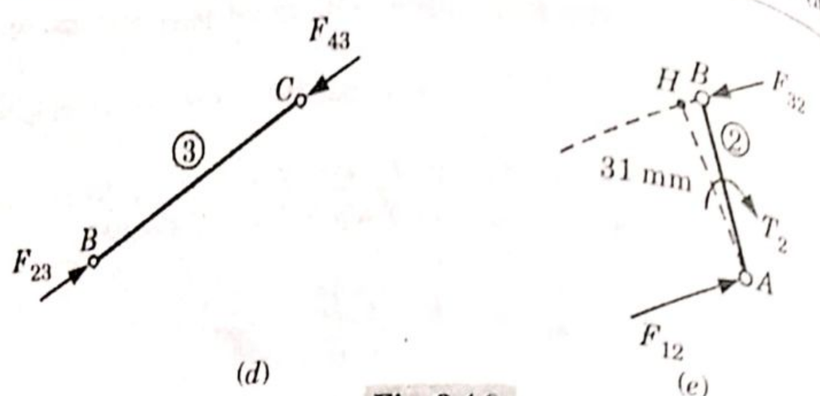


Fig. 3.4.2.

6. From FBD of link 4, 3 and 2 we can write

$$F_{34} = -F_{43} = F_{23} = -F_{32} = 210 \text{ N}$$

7. From FBD of link 2, Taking moment about A,

$$F_{32} \times AH - T_2 = 0$$

$$210 \times 31 - T_2 = 0$$

$$\therefore \quad T_2 = 6510 \text{ N-mm}$$

Positive sign gives clockwise sense of torque T.

PART-2

D' Alembert's Principle.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.5. **What is inertia ? Explain D' Alembert's principle.**

Answer

A. Inertia : It is a property of matter by virtue of which a body resists any change in its states (either it is in rest or motion).

B. D' Alembert's Principle :

1. It states that "the inertia forces and couples, the external forces and torques on a body together give statical equilibrium."

Inertia force, $F_i = -ma$

Inertia couple, $C_i = -I_g \alpha$

Where, m = Mass of body,

a = Acceleration of centre of mass of the body.

I_g = Moment of inertia about an axis passing through the centre of mass and perpendicular to plane of rotation of the body, and

α = Angular acceleration of the body.

2. Let F_1, F_2, F_3 = External forces acting on the body, and

T_1, T_2, T_3 = External torques on the body about its centre of mass.

3. Now, according to D' Alembert's principle,

$$\Sigma F + F_i = 0 \quad ...(3.5.1)$$

and,

$$\Sigma T + C_i = 0 \quad ...(3.5.2)$$

4. The above equations are similar to the equation of a body in static equilibrium.

PART-3

Dynamic of Rigid Link in Plane Motion, Dynamic Force Analysis of Planer Mechanisms.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.6. **Define equivalent offset inertia force.**

Answer

1. Equivalent offset inertia force is the inertia force which produces joint effect of inertia force and inertia couple, *i.e.*, an equivalent offset inertia force is used to replace inertia force and inertia couple.

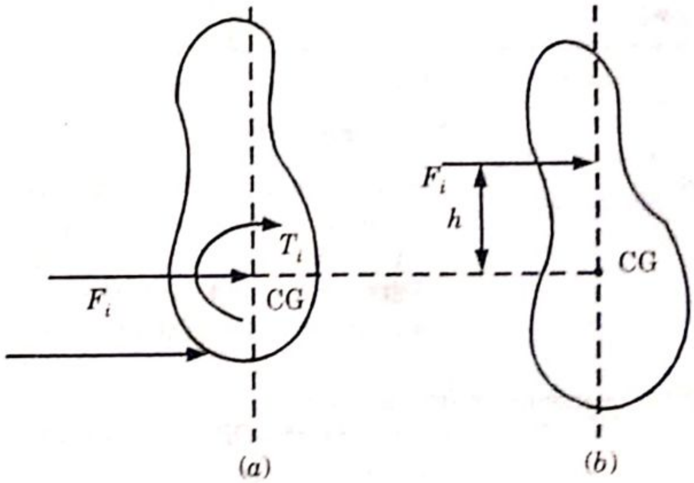


Fig. 3.6.1. Equivalent offset inertia force.

2. Equivalent offset inertia force can be found by displacing the line of action of inertia force from the centre of mass by a perpendicular displacement h such that the torque so produced is equal to the inertia couple acting on the body.
3. From the Fig. 3.6.1(b), it is clear that F_i acting at a distance h from CG of the body produces same effect as the effect is produced by F_i and T_i in Fig. 3.6.1(a).

i.e., T_i = Couple produced by force F_i

$$T_i = C_i$$
$$F_i \times h = C_i$$
$$h = \frac{C_i}{F_i} = \frac{-I_g\alpha}{-ma} = \frac{mk^2\alpha}{ma}$$
$$F_i = \frac{k^2\alpha}{a}$$

Que 3.7. **Deduce the expression for the inertia force in the reciprocating force neglecting the weight of the connecting rod.**

AKTU 2015-16, Marks 10

Answer

1. Consider the motion of a crank and connecting rod of a reciprocating steam engine as shown in Fig. 3.7.1.

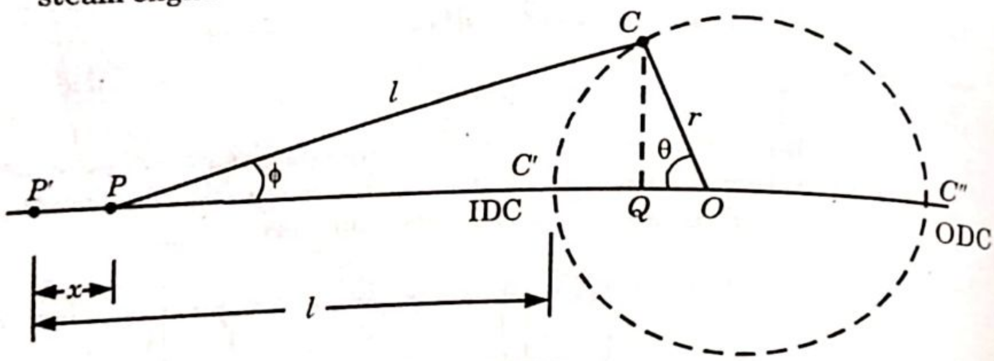


Fig. 3.7.1.

2. Let
   l = Length of connecting rod between the centres,
   r = Radius of crank or crank-pin circle,
   ϕ = Inclination of connecting rod to the line of stroke PO, and
   n = Ratio of length of connecting rod to the radius of crank = l/r.
3. From the geometry of Fig. 3.7.1.

$$x = P'P = OP' - OP = (P'C' + C'O) - (PQ + QO)$$
$$= (l + r) - (l\cos\phi + r\cos\theta)$$
$$= r(1 - \cos\theta) + l(1 - \cos\phi)$$

$$= r\left[(1-\cos\theta) + \frac{l}{r(1-\cos\phi)}\right]$$

$$= r[(1-\cos\theta) + n(1-\cos\phi)] \quad ...(3.7.1)$$

4. From triangles CPQ and CQO,

$$CQ = l\sin\phi = r\sin\theta \quad \text{or} \quad l/r = \sin\theta/\sin\phi$$

$$\therefore \quad n = \sin\theta/\sin\phi \quad \text{or} \quad \sin\phi = \sin\theta/n \quad ...(3.7.2)$$

5. We know that, $\cos\phi = (1-\sin^2\phi)^{\frac{1}{2}} = \left(1 - \frac{\sin^2\theta}{n^2}\right)^{\frac{1}{2}}$

6. Expanding the above expression by binomial theorem, we get

$$\cos\phi = 1 - \frac{1}{2}\times\frac{\sin^2\theta}{n^2} + ... \quad \text{(Neglecting higher terms)}$$

or

$$1-\cos\phi = \frac{\sin^2\theta}{2n^2} \quad ...(3.7.3)$$

7. Substituting the value of $(1-\cos\phi)$ from eq. (3.7.3) in eq. (3.7.1) we have

$$x = r\left[(1-\cos\theta) + n\times\frac{\sin^2\theta}{2n^2}\right]$$

$$x = r\left[(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right] \quad ...(3.7.4)$$

8. Differentiating eq. (3.7.4) with respect to θ,

$$\frac{dx}{d\theta} = r\left[\sin\theta + \frac{1}{2n}\times 2\sin\theta\cos\theta\right]$$

$$= r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) \quad ...(3.7.5)$$

$(\because \quad 2\sin\theta\cos\theta = \sin 2\theta)$

9. Velocity of the piston P,

$$v_{PO} = v_p = \frac{dx}{dt} = \frac{dx}{d\theta}\times\frac{d\theta}{dt} = \frac{dx}{d\theta}\times\omega$$

$(\because$ Ratio of change of angular velocity $= d\theta/dt = \omega)$

10. Substituting the value of $dx/d\theta$ from eq. (3.7.5) we have,

$$v_{PO} = v_p = \omega r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) \quad ...(3.7.6)$$

11. Since the acceleration is the rate of change of velocity, therefore acceleration of the piston P,

$$a_p = \frac{dv_p}{dt} = \frac{dv_p}{d\theta}\times\frac{d\theta}{dt} = \frac{dv_p}{d\theta}\times\omega \quad ...(3.7.7)$$

12. Differentiating eq. (3.7.6) with respect to θ,

$$\frac{dv_p}{d\theta} = \omega r\left[\cos\theta + \frac{\cos 2\theta\times 2}{2n}\right] = \omega r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

13. Substituting the value of $\frac{dv_p}{d\theta}$ in the eq. (3.7.7), we have

$$a_p = \omega r\left[\cos\theta + \frac{\cos 2\theta}{n}\right] \times \omega = \omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

14. Acceleration force or inertia force of reciprocating parts,

$$F_i = m_R\, a_p$$

$$= mr\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

Where, m = Mass of reciprocating parts.

Que 3.8. **The crank of a slider crank mechanism rotates at constant speed of 250 rpm. The crank is 150 mm and the connecting rod is 500 mm long. Determine the angular velocity and angular acceleration of the connecting rod at a crank angle of 45° from inner dead centre position.**

Answer

Given : $N = 250$ rpm, $r = 150$ mm $= 0.15$ m, $l = 500$ mm $= 0.5$ m, $\theta = 45°$
To Find : i. Angular velocity.
ii. Angular acceleration.

1. Speed of crank,

$$\omega = 2\pi N/60$$

$$= 2\pi \times \frac{250}{60} = 26.17 \text{ rad/s}$$

2. We know that, $n = \dfrac{l}{r} = \dfrac{0.50}{0.15} = \dfrac{10}{3}$

3. Now, angular velocity of connecting rod is,

$$\omega_{PC} = \frac{\omega\cos\theta}{(n^2 - \sin^2\theta)^{1/2}}$$

$$= \frac{26.17\cos 45^o}{\left[\left(\frac{10}{3}\right)^2 - \sin^2 45^o\right]^{1/2}} = \frac{18.50}{\left[\frac{100}{9} - \frac{1}{2}\right]^{1/2}}$$

$$\omega_{PC} = 5.68 \text{ rad/s}$$

4. Angular acceleration of connecting rod,

$$\alpha_{PC} = \frac{-\omega^2\sin\theta\,(n^2 - 1)}{(n^2 - \sin^2\theta)^{3/2}}$$

$$= \frac{-(26.17)^2\sin 45^o\left(\frac{100}{9} - 1\right)}{\left[\left(\frac{10}{3}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right]^{3/2}}$$

$$= -141.66 \text{ rad/s}^2$$

PART-4

Piston Force and Crank Effort, Turning Moment of Crank Shaft due to Force on Piston.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.9. **Consider a single cylinder horizontal engine. Derive the expression for net force acting on the piston, resultant load on the gudgeon pin and thrust on the cylinder walls and crank effort.**

AKTU 2014-15, 2016-17; Marks 10

OR

Describe forces on different parts of a slider crank mechanism.

AKTU 2017-18, Marks 10

Answer

The forces that act on an engine are as follows :

i. Piston Effort (Effective Driving Force) :

1. Piston effort is defined as the net or effective force applied on the piston.
2. Let, A_1 = Area of the cover end,
   A_2 = Area of the piston rod end,
   p_1 = Pressure on the cover end,
   p_2 = Pressure on the rod end, and
   m = Mass of reciprocating parts.
3. Force on piston due to gas pressure,
$$F_g = p_1A_1 - p_2A_2$$
4. Inertia force, $$F_i = mr\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$
5. Inertia force is in opposite direction of the acceleration of the piston.
   Hence, Net effective force on the piston,
$$F = F_g - F_i$$
6. If a friction force F_f is present, then
   Net effective force on the piston, $F = F_g - F_i - F_f$

7. In a vertical engine the weight of piston (or reciprocating parts) also acts on the piston, hence net effective force on the piston,

$$F = F_g + mg - F_i - F_f$$

ii. Force (or Thrust) along the Connecting Rod :

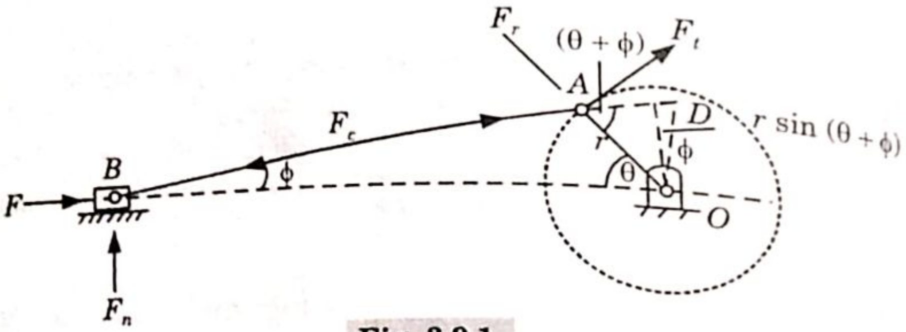


Fig. 3.9.1.

1. Let, F_c = Force along the connecting rod.
2. Equating the horizontal components of the forces,

$$F_c \cos\phi = F$$

$$F_c = \frac{F}{\cos\phi}$$

iii. Thrust on the Sides of Cylinder :

1. Thrust on the sides of the cylinder is the normal reaction of force F_c.
2. Thrust on the sides of cylinder, $F_n = F_c \sin\phi$

$$= \frac{F}{\cos\phi}\sin\phi$$

$$= F\tan\phi$$

iv. Crank Pin Effort and Crank Effort :

1. It is the net force applied at the crankpin perpendicular to the crank which gives the required turning moment on the crankshaft.
2. Let, F_t = Crank effort.

$$F_t \times r = F_c\, r \sin(\theta + \phi)$$

$$F_t = F_c \sin(\theta + \phi) = \frac{F\sin(\theta + \phi)}{\cos\phi}$$

3. Crank Effort, $T = F_t r$ driving Torque

$$= \frac{F\sin(\theta+\phi)\, r}{\cos\phi}$$

v. Thrust on the Bearings :

1. It is the radial components of force F_c along the crank that produces a thrust on the crankshaft bearings.

$$F_r = F_c \cos(\theta + \phi)$$

$$= \frac{F}{\cos\phi}\cos(\theta+\phi)$$

Que 3.10. **In Fig. 3.10.1 a slider crank mechanism is shown. $AB = 10$ cm, $BC = 30$ cm. The value of force applied on slider is 3000 N. Determine the forces on various links. Also calculate the driving torque applied on link AB.**

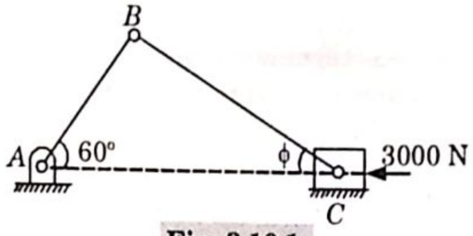


Fig. 3.10.1.

Answer

Given : $AB\ (r) = 10$ cm, $BC\ (l) = 30$ cm, $F = 3000$ N, $\theta = 60°$

To Find : i. Forces on various links.

ii. Driving torque applied on link AB.

1. Applying sine rule

$$\frac{\sin 60°}{BC} = \frac{\sin\phi}{AB} \Rightarrow \frac{\sqrt{3}}{2\times 30} = \frac{\sin\phi}{10}$$

$$\phi = 16.8°$$

$$n = \frac{l}{r} = \frac{30}{10} = 3$$

2. Force acting along the connecting rod,

$$F_c = \frac{F}{\cos\phi} = \frac{3000}{\cos 16.8°}$$

$$F_c = 3133.8 \text{ N}$$

3. Crank pin effort,

$$F_t = F_c \sin(\theta+\phi) = 3133.8 \sin(60° + 16.8°) = 3051 \text{ N}$$

4. Bearing thrust,

$$F_r = F_c \cos(\theta+\phi) = 3133.8 \cos(60° + 16.8°)$$

$$= 715.6 \text{ N}$$

5. Driving torque applied on link AB,

$$T = F_t\, r = 3051 \times \left(\frac{10}{100}\right) = 305.1 \text{ N-m}$$

Que 3.11. **Define the term turning moment. Show that the turning moment of a crank shaft in a reciprocating engine is given by**

$$T = Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

Answer

A. **Turning Moment :** The product of the crank pin effort and the crank pin radius is known as turning moment.

B. **Expression for Turning Moment of a Crank Shaft :**

1. Let, T = Turning moment of a crank shaft, and
   F = Net force acting on the piston.

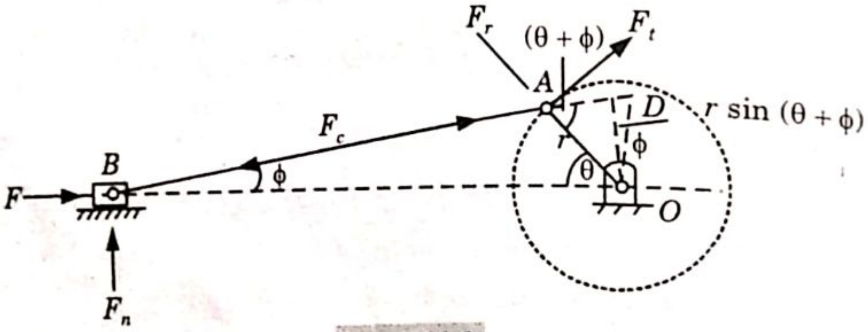


Fig. 3.11.1.

2. We know that, $\sin\phi = \dfrac{\sin\theta}{n}$ ...(3.11.1)

$$\therefore \quad \cos\phi = \sqrt{1 - \sin^2\phi} = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$

$$= \frac{1}{n}\sqrt{n^2 - \sin^2\theta} \quad ...(3.11.2)$$

3. On dividing eq. (3.11.1) and (3.11.2), we get

$$\frac{\sin\phi}{\cos\phi} = \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}} \quad ...(3.11.3)$$

4. Turning moment on crank shaft,

$$T = F_t r$$

$$T = \frac{F}{\cos\phi}\sin(\theta + \phi) r$$

$$= \frac{F[\sin\theta\cos\phi + \cos\theta\sin\phi]r}{\cos\phi}$$

$$T = Fr\left[\sin\theta + \cos\theta\frac{\sin\phi}{\cos\phi}\right] \quad ...(3.11.4)$$

5. Putting the value of $\frac{\sin\phi}{\cos\phi}$ from eq. (3.11.3) in eq. (3.11.4), we get

$$T = Fr\left[\sin\theta + \frac{2\cos\theta\sin\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

$$T = Fr\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

Que 3.12. **A vertical single cylinder gas engine has a bore of 8 cm and a stroke of 10 cm. The length of connecting rod is 20 cm. Reciprocating parts weigh 1.5 kg. Gas pressure on the piston is 6 kg/cm² when piston has moved downward by 1.5 cm from its IDC position in the power stroke. Determine the net vertical load on the gudgeon pin, when the engine runs at 2000 rpm.**
At what speed of engine, this load will become zero ?

Answer

Given : D = 8 cm, L = 10 cm, l = 20 cm, m = 1.5 kg, p = 6 kg/cm^2, x = 1.5 cm, N = 2000 rpm
To Find : i. Net vertical load on gudgeon pin
ii. Speed at which load will become zero.

1. Crank radius, $r = \frac{\text{Stroke}}{2}$ $r = \frac{10}{2} = 5$ cm

2. From configuration diagram, we have

$$\cos\theta = \frac{(OB')^2 + (OA)^2 - (AB')^2}{2(OB')(OA)}$$

$$= \frac{(23.5)^2 + (5)^2 - (20)^2}{2 \times 23.5 \times 5}$$

or $\theta = 41.04° \approx 41°$

and, $$\cos\phi = \frac{(OB')^2 + (AB')^2 - (OA)^2}{2(OB')(AB')}$$

$$= \frac{(23.5)^2 + (20)^2 - (5)^2}{2 \times 23.5 \times 20}$$

or $\phi = 9.447° \approx 9.5°$

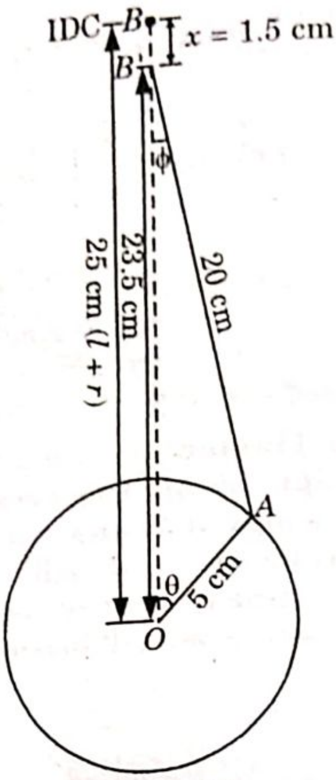


Fig. 3.12.1.

3. Force due to gas pressure,

$$F_g = \frac{\pi}{4} D^2 \, p = \frac{\pi}{4} \times (8)^2 \times 6 = 301.59 \text{ kg}$$

4. Inertia force, $F_i = mr\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$

$$F_i = 1.5 \times 5 \times (209.44)^2 \times \left[\cos 41° + \frac{\cos (2 \times 41°)}{4} \right]$$

$$\left(\begin{array}{l} \because \ \omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 2000}{60} = 209.44 \text{ rad/s} \\ n = \dfrac{l}{r} = \dfrac{20}{5} = 4 \end{array} \right)$$

$$F_i = 259737.24 \text{ kg-cm/s}^2 \approx 2597.37 \text{ N}$$

5. Net vertical load on the gudgeon pin

$$= F_g g - F_i + mg$$
$$= 301.59 \times 9.81 - 2597.37 + 1.5 \times 9.81$$
$$= 375.9429 \text{ N} \approx 376 \text{ N}$$

6. When load becomes zero, the equation is given as

$$F_g g - F_i + mg = 0$$

$$301.59\, g - mr\,\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n} \right) + 1.5\, g = 0$$

$$2973.3129 - 1.5 \times \left(\frac{5}{100}\right)\omega^2\left[\cos 41° + \frac{\cos 2(41°)}{4}\right] = 0$$

$$0.0592\omega^2 = 2973.3129$$

$$\omega = 224 \text{ rad/s}$$

7. Engine speed in rpm,

$$N = \frac{60\omega}{2\pi} = \frac{224 \times 60}{2\pi} = 2139 \text{ rpm}$$

Que 3.13. **A horizontal gas engine running at 200 rpm has a bore of 220 mm and a stroke of 480 mm. The connecting rod is 930 mm long and the reciprocating parts weigh 20 kg. When the crank has turned through an angle of 30° from the inner dead center, the gas pressure on the cover and the crank sides are 500 kN/m² and 50 kN/m² respectively. Diameter of the piston rod is 40 mm. Determine (*i*) turning moment on the crank shaft, (*ii*) thrust on the bearings.**

AKTU 2013-14, Marks 10

Answer

Given : $N = 200$ rpm, $D = 220$ mm $= 0.22$ m,
$L = 480$ mm $= 0.48$ m, $l = 0.93$ m, 930 mm $= m = 20$ kg, $\theta = 30°$,
$d = 40$ mm $= 0.04$ m, $p_1 = 500$ kN/m² $= 500 \times 10^3$ N/m²,
$p_2 = 50$ kN/m² $= 50 \times 10^3$ N/m².

To Find : i. Turning moment on the crank shaft.
ii. Thrust on the bearings.

1. We know that, area of the piston on the cover end side,

$$A_1 = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times (0.22)^2 = 0.038 \text{ m}^2$$

and, area of piston rod,

$$a = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times (0.04)^2 = 0.00126 \text{ m}^2$$

2. Net load on the piston due to gas pressure,

$$F_g = p_1A_1 - p_2A_2$$
$$= p_1A_1 - p_2(A_1 - a)$$
$$= 500 \times 10^3 \times 0.038 - 50 \times 10^3 (0.038 - 0.00126)$$
$$= 17163 \text{ N}$$

3. Ratio of length of the connecting rod and crank radius,

$$n = l/r = 0.93/0.24 = 3.875 \approx 4$$

4. Inertia force in the reciprocating parts,

$$F_i = m\omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= 20 \times (20.94)^2 \times 0.24 \left[\cos 30^\circ + \frac{\cos 60^\circ}{4}\right]$$

$$\left[\because \quad \omega = \frac{2\pi N}{60} = \frac{2\pi \times 260}{60} = 20.94 \text{ rad/s}\right]$$

$$= 2085.83 \text{ N}$$

5. Net force on the piston or piston effort,

$$F = F_g - F_i$$
$$= 17163 - 2085.83$$
$$= 15077.17 \text{ N} = 15.077 \text{ kN}$$

6. We know that, $\sin \phi = \dfrac{\sin \theta}{n} = \dfrac{\sin 30^\circ}{4} = 0.125$

$$\phi = 7.18^\circ$$

7. Turning moment on the crank shaft,

$$T = \frac{F \sin(\theta + \phi)}{\cos \phi} r$$
$$= \frac{15.077 \times \sin(30^\circ + 7.18^\circ)}{\cos 7.18^\circ} \times 0.24 \text{ kN-m}$$
$$= 2.204 \text{ kN-m} = 2204 \text{ N-m}$$

8. Thrust on the bearings,

$$F_r = \frac{F \cos(\theta + \phi)}{\cos \phi}$$
$$= \frac{15.077 \cos(30^\circ + 7.18^\circ)}{\cos 7.18^\circ} = 12.10 \text{ kN}$$

Que 3.14. **A vertical double acting steam engine has a cylinder 300 mm diameter and 450 mm stroke and runs at 200 rpm. The reciprocating parts have a mass of 225 kg and the piston rod is 50 mm diameter. The connecting rod is 1.2 m long. When the crank has turned through 125° from the top dead centre, the steam pressure above the piston is 30 kN/m² and below the piston is 1.5 kN/m². Calculate the effective turning moment on the crank shaft.**

AKTU 2017-18, Marks 10

Answer

Given : $D = 300$ mm $= 0.3$ m, $L = 450$ mm $= 0.45$ m, $N = 200$ rpm, $m = 225$ kg, $d = 50$ mm $= 0.05$ m, $l = 1.2$ m, $\theta = 125^\circ$, $p_1 = 30$ kN/m² $= 30 \times 10^3$ N/m², $p_2 = 1.5$ kN/m² $= 1.5 \times 10^3$ N/m²

To Find : The effective turning moment on the crank shaft.

1. We know that area of the piston,

$$A_1 = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times (0.3)^2 = 0.0707 \text{ m}^2$$

and, area of the piston rod,

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.05)^2 = 0.00196 \text{ m}^2$$

2. Force on the piston due to steam pressure,

$$F_g = p_1 A_1 - p_2(A_1 - a)$$
$$= 30 \times 10^3 \times 0.0707 - 1.5 \times 10^3 \times (0.0707 - 0.00196)$$
$$= 2121 - 103 = 2018 \text{ N}$$

3. Ratio of lengths of connecting rod and crank,

$$n = l/r = 1.2/0.225 = 5.33$$

$$\left(\because\ r = L/2 = \frac{0.45}{2} = 0.225 \text{ m}\right)$$

4. Inertia force in the reciprocating parts,

$$F_i = m\omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$= 225(20.95)^2 \times 0.225\left(\cos 125° + \frac{\cos 250°}{5.33}\right)$$

$$= -14172 \text{ N}$$

5. We know that for a vertical engine, net force on the piston or piston effort,

$$F = F_g - F_i + mg$$
$$= 2018 - (-14172) + 225 \times 9.81 = 18397 \text{ N}$$

6. We know that, $\sin\phi = \dfrac{\sin\theta}{n} = \dfrac{\sin 125°}{5.33} = \dfrac{0.8191}{5.33} = 0.1537$

$\therefore \quad \phi = 8.84°$

7. We know that effective turning moment on the crank shaft,

$$T = \frac{F \sin(\theta + \phi)}{\cos\phi} r$$

$$= \frac{18397 \sin(125° + 8.84°)}{\cos 8.84°} \times 0.225$$

$$= 3021.6 \text{ N-m}$$

PART-5

Turning Moment Diagrams for Single Cylinder Double Acting Steam Engine, Four Stroke IC Engine and Multi Cylinder Engines.

CONCEPT OUTLINE

Crank Effort Diagrams : Diagrams obtained on plotting crank effort or turning moment for various positions of crank are known as crank effort diagrams. These are sometimes referred to as turning moment diagrams.

Use of Crank Effort Diagrams : There are three uses of these diagrams :

1. Area of T-θ diagram per cycle represents work done per cycle.
2. By dividing the area of T-θ diagram with the length of base, mean turning moment can be found out.
3. The maximum ordinate of T-θ diagram gives the maximum torque to which the crank shaft is subjected. This can be used to determine diameter of crank shaft.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.15. **Define crank effort diagram also draw and explain the turning moment diagram for a single cylinder double acting steam engine.**

Answer

A. Crank Effort Diagram (Turning Moment Diagram) : It is the graphical representation of the turning moment or crank effort for various positions of the crank.

B. Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine :

1. Turning moment on the crankshaft,

$$T = Fr\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right] \quad ...(3.15.1)$$

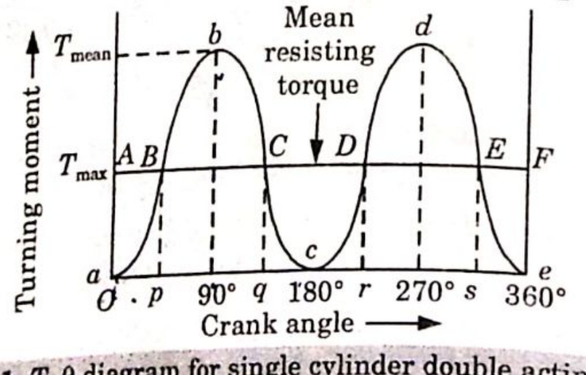


Fig. 3.15.1. T–θ diagram for single cylinder double acting engine.

2. From eq. (3.15.1),
   i. When $\theta = 0°, T = 0$
   ii. When $\theta = \frac{\pi}{2}, T = T_{max} = F_p r$
   iii. When $\theta = \pi, T = 0$
3. The curve *abc* represents the turning moment diagram for outstroke and a similar curve *cde* represent the turning moment diagram for instroke.
4. The area of turning moment diagram represents the work done per revolution.
   Work done by the turning moment per revolution
   = Work done against the mean resisting torque
5. Some noticeable points regarding the turning moment diagram are as follows :
   i. When the turning moment is positive, the crank shaft accelerates and the work is done by the steam.
   ii.. When the turning moment is negative, the crank shaft retards and the work is done on the steam.
   iii. The accelerating torque on the rotating parts of the engine is,
   $$= T - T_{mean}$$
   Where, T = Torque on the crankshaft at any instant, and
   T_{mean} = Mean resisting torque.
   iv. If $T - T_{mean} > 0 \Rightarrow$ The flywheel accelerates.
   If $T - T_{mean} < 0 \Rightarrow$ The flywheel retards.

Que 3.16. **Draw and explain the turning moment diagram for a four stroke IC engine.**

Answer

1. A four stroke IC engine has a working stroke after the crank has turned through two revolutions, *i.e.*, 720° (or 4π radians).

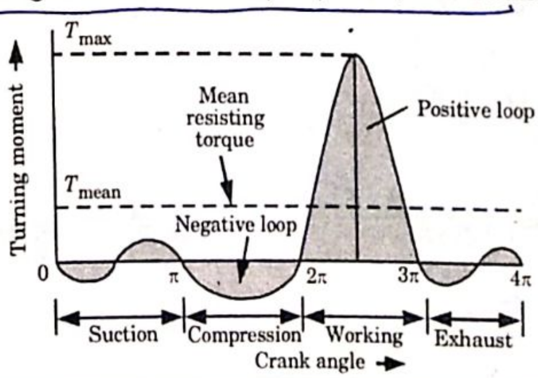


Fig. 3.16.1. Turning moment diagram for a four stroke cycle internal combustion engine.

2. During suction stoke, a negative loop is formed as the pressure in the engine cylinder is less than the atmospheric pressure.
3. During compression stroke a high negative loop is formed because work is done on the gases.
4. A large positive loop is formed during the working stroke and work done by the gases.
5. During exhaust stroke, the work is done on the gases therefore a negative loop is formed.

Que 3.17. **Draw and explain the crank effort diagram for multi cylinder steam engine.**

Answer

1. To draw the crank effort diagram for multi cylinder steam engine firstly we draw the turning moment diagrams for each cylinder separate on same graph.
2. The resultant turning moment diagram is the sum of the turning moment diagrams for each cylinder.
3. Fig. 3.17.1 shows a three cylinder steam engine crank effort diagram. In this, the first cylinder is the high pressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressure cylinder.
4. Usually the cranks, in case of three cylinders are placed at 120° to each other.

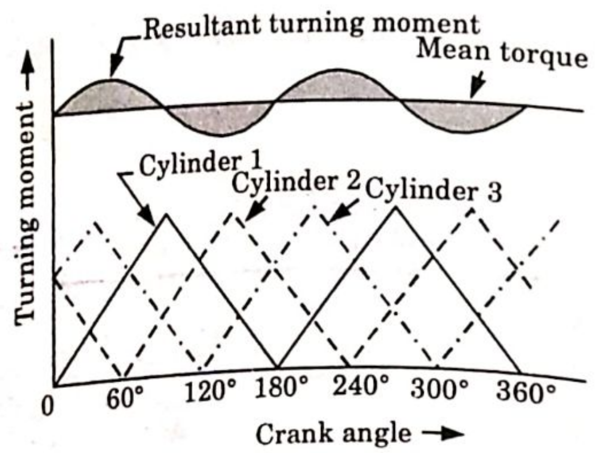


Fig. 3.17.1. Turning moment diagram for a multi cylinder engine.

PART-6

Fluctuation of Speed.

CONCEPT OUTLINE

Fluctuation of Energy : The variation of energy above and below the mean resisting torque line is called fluctuation of energy.

Fluctuation of Speed : The ratio of maximum fluctuation of speed to the mean speed is called the co-efficient of fluctuation of speed.

Co-efficient of Fluctuation of Speed :

$$C_s = \frac{N_1 - N_2}{N}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.18. **Give a short note on fluctuation of energy. Discuss and determine maximum fluctuation of energy.**

Answer

A. Fluctuation of Energy :

1. The variation of energy above and below the mean resisting torque line is known as fluctuation of energy.
2. From the Fig. 3.18.1, we see when the crank moves from *a* to *p*, the work done by the engine is equal to area *aBp*, but the energy required is represented by the area *aABp*. It means energy requirement is more than the work done by the engine.
3. This extra work or energy is taken from the flywheel, hence the speed of the flywheel decreases.
4. Now the crank moves from *p* to *q*, the work done by the engine is equal to the area *pBbCq*, whereas the requirement of energy is represent by the area *pBCq*.

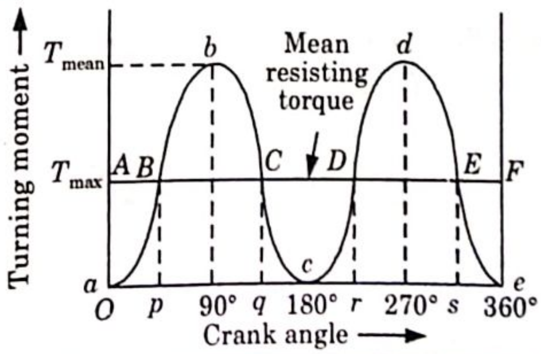


Fig. 3.18.1. Turning moment diagram for a single cylinder, double acting steam engine.

5. Therefore, the engine has done more work than the requirement. The excess work (equal to the area BbC) is stored in the flywheel and hence the speed of the flywheel increases.

B. Maximum Fluctuation of Energy :

1. The difference between the maximum and minimum energies is known as maximum fluctuation of energy.

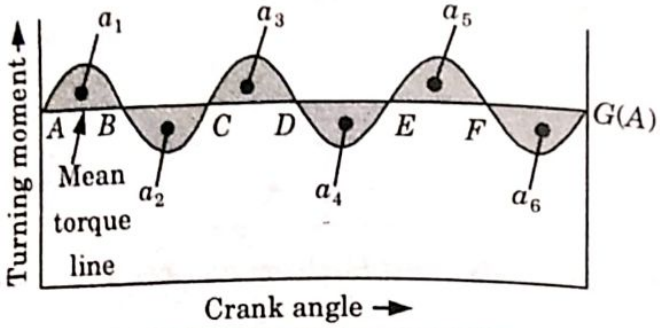


Fig. 3.18.2. Determination of maximum fluctuation of energy.

2. Let the line AG represents the mean torque line. Let a_1, a_3, a_5 are the areas above the mean torque line and a_2, a_4, a_6 are the areas below the mean torque line.

3. Let the energy in the flywheel at $A = E$

   Energy at $B = E + a_1$

   Energy at $C = E + a_1 - a_2$

   Energy at $D = E + a_1 - a_2 + a_3$

   Energy at $E = E + a_1 - a_2 + a_3 - a_4$

   Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

   Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

4. If the greatest energy is at B and least at E then,

   The maximum energy in flywheel, $E_{max} = E + a_1$

   The minimum energy in flywheel,

$$E_{min} = E + a_1 - a_2 + a_3 - a_4$$

5. The maximum fluctuation of energy,

$$\begin{aligned} E &= E_{max.} - E_{min} \\ &= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) \\ &= a_2 - a_3 + a_4 \end{aligned}$$

Que 3.19. **Define the following terms :**

i. Co-efficient of fluctuation of speed.

ii. Co-efficient of steadiness.

iii. Co-efficient of fluctuation of energy.

Answer

i. **Co-efficient of Fluctuation of Speed :**

1. The ratio of maximum fluctuation of speed to the mean speed is called the co-efficient of fluctuation of speed.

$$C_s = \frac{N_1 - N_2}{N}$$

Where, N_1 = Maximum speed during the cycle (in rpm),
N_2 = Minimum speed during the cycle (in rpm), and

$$N = \frac{N_1 + N_2}{2} = \text{Mean speed (in rpm).}$$

ii. **Co-efficient of Steadiness :**

1. The reciprocal of the coefficient of fluctuation of speed is known as co-efficient of steadiness.

$$m = \frac{1}{C_s} = \frac{N}{N_1 - N_2}$$

iii. **Co-efficient of Fluctuation of Energy :**

1. The ratio of the maximum fluctuation of energy to the work done per cycle is called the co-efficient of fluctuation of energy.

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

PART-7

Flywheel.

CONCEPT OUTLINE

Flywheel : It is used in machines to serve as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

Types of Flywheel :

1. Disc type.
2. Rim type.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.20. **Show that the maximum fluctuation of energy in a flywheel is given by**

$$\Delta E = \frac{\pi^2}{900} m k^2 N^2 C_s$$

Where, m = **Mass of the flywheel,**
k = **Radius of gyration of flywheel,**
N = **Mean speed during the cycle (in rpm), and**
C_s = **Co-efficient of fluctuation of speed.**

Answer

1. Let, N_1 = Maximum speed during the cycle.
N_2 = Minimum speed during the cycle.
N = Mean speed of flywheel, $= \frac{N_1 + N_2}{2}$

And, C_s = Co-efficient of fluctuation of speed $= \frac{N_1 - N_2}{N}$

2. The mean kinetic energy of the flywheel,

$$E = \frac{1}{2} I\omega^2 = \frac{1}{2} mk^2\omega^2 \qquad (\because \; I = mk^2)$$

3. The maximum fluctuation of energy in a flywheel,

$$\Delta E = \text{Maximum KE} - \text{Minimum KE}$$

$$= \frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2 = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2} mk^2(\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= mk^2\left(\frac{\omega_1 + \omega_2}{2}\right)\frac{\omega_1 - \omega_2}{\omega}(\omega)$$

$$= mk^2 \omega^2 C_s \qquad \left(\because \; C_s = \frac{\omega_1 - \omega_2}{\omega}\right)$$

$$= mk^2\left(\frac{2\pi N}{60}\right)^2 C_s \qquad \left(\because \; \omega = \frac{2\pi N}{60}\right)$$

$$\Delta E = \frac{\pi^2}{900} mk^2 N^2 C_s$$

Que 3.21. **Derive expression for dimensions of flywheel.**

AKTU 2017-18, Marks 10

Answer

1. Let,

D = Mean diameter of rim,
R = Mean radius of rim,
A = Cross-sectional area of rim,
ρ = Density of rim material,
N = Speed of flywheel,
ω = Angular velocity of flywheel,
σ = Tensile (or hoop) stress due to centrifugal force, and
v = Linear velocity at the mean radius.

$$= R\omega = \frac{\pi DN}{60}$$

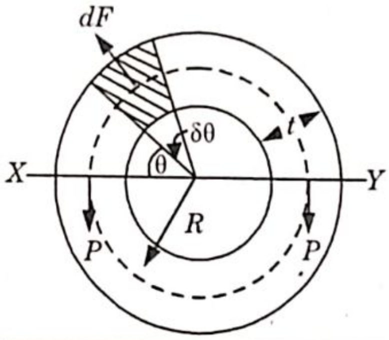


Fig. 3.21.1. Rim of a flywheel.

2. Consider a small element of rim which subtends an angle $\delta\theta$ at the centre of the flywheel.

3. Volume of the small element = $AR\,\delta\theta$
and, mass of the small element,

$$dm = \text{Density} \times \text{Volume}$$
$$= \rho AR\,\delta\theta$$

4. Centrifugal force that acts radially outwards,

$$dF = dm\,\omega^2 R$$
$$= \rho AR^2\omega^2\,\delta\theta$$

5. Vertical component of $dF = dF \sin\theta$

$$= \rho AR^2\omega^2\,\delta\theta \sin\theta$$

6. Total vertical upward force tending to burst the rim across the diameter XY,

$$F = \rho AR^2\omega^2 \int_0^{\pi} \sin\theta\, d\theta = 2\rho AR^2\omega^2 \quad ...(3.21.1)$$

7. This vertical upward force will produce hoop stress (or centrifugal stress or circumferential stress) and it is resisted by $2P$, such that

$$2P = 2\sigma A \quad ...(3.21.2)$$

8. On equating eq. (3.21.1) and eq. (3.21.2)

$$2\sigma A = 2\rho A R^2\omega^2$$

$$\sigma = \rho R^2\omega^2 \qquad [\because \; v = R\omega]$$

$$\sigma = \rho v^2$$

$$v = \sqrt{\frac{\sigma}{\rho}} \qquad ...(3.21.3)$$

9. The mass of rim, m = Volume × Density

$$= \pi D A \rho$$

$$A = \frac{m}{\pi D \rho} \qquad ...(3.21.4)$$

10. Using eq. (3.21.3) and eq. (3.21.4), we can find the value of the mean radius and cross-sectional area of the rim.
11. If the cross-sectional area of the rim is rectangular then,

$$A = bt$$

Where, b = Width of the rim, and

t = Thickness of the rim.

Que 3.22. **The radius of gyration of a flywheel is 1 metre and fluctuation of speed is not to exceed 1 % of the mean speed of the flywheel. If the mass of the flywheel is 3340 kg and the steam develops 150 kW at 135 rpm, then find**
i. Maximum fluctuation of energy.
ii. Co-efficient of fluctuation of energy.

AKTU 2015-16, Marks 10

Answer

Given : $k = 1$ m, $C_s = 0.01$, $m = 3340$ kg, $P = 150$ kW, $N = 135$ rpm
To Find : i. Maximum fluctuation of energy.
ii. Co-efficient of fluctuation of energy.

1. Maximum fluctuation of energy,

$$\Delta E = \frac{\pi^2}{900} mk^2N^2C_s$$

$$\Delta E = \frac{\pi^2}{900} \times 3340 \times (1)^2 \times (135)^2 \times 0.01$$

$$= 6675.307 \text{ Nm}$$

2. We know that, work done per cycle $= P \times \dfrac{60}{N}$

$$= 150 \times 10^3 \times \frac{60}{135}$$

$$= 66666.667 \text{ N-m}$$

3. Co-efficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

$$= \frac{6675.306}{66666.667} = 0.1$$

Que 3.23. **Torque exerted by a multi cylinder engine running at a mean speed of 240 rpm against a constant resistance is T(kg-m) = 350 + 560 sin θ + 84 sin 2θ + 8.4 sin 3θ. Find the HP of the engine and the minimum weight of the flywheel if the radius of gyration is 90 cm and the maximum fluctuation of speed is to be ± 1 % of the mean.**

Answer

Given : N = 240 rpm, k = 90 cm = 0.9 m, C_s = 0.02,
T = 350 + 560 sin θ + 84 sin 2θ + 8.4 sin 3θ
To Find : i. HP of engine,
ii. Minimum weight of flywheel.

1. Work done per revolution, $W = \int_0^{2\pi} T\, d\theta$

$$W = \int_0^{2\pi} (350 + 560 \sin\theta + 84 \sin 2\theta + 8.4 \sin 3\theta)\, d\theta$$

$$= \left[350\,\theta + 560(-\cos\theta) + \frac{84}{2}(-\cos 2\theta) + \frac{8.4}{3}(-\cos 3\theta)\right]_0^{2\pi}$$

$$= [350\,(2\pi) - 560\,(\cos 2\pi - \cos 0°) - 42\,(\cos 4\pi - \cos 0°) - 2.8\,(\cos 6\pi - \cos 0°)]$$

$$= 700\,\pi - 560\,(1 - 1) - 42\,(1 - 1) - 2.8\,(1 - 1)$$

$$= 700\,\pi$$

2. Also work done, $W = 2\pi\, T_{mean}$

$\therefore$ $700\pi = 2\pi\, T_{mean}$

or, $T_{mean} = 350 \text{ kg-m} = 350 \times 9.81 \text{ N-m}$

$= 3433.5 \text{ N-m}$

3. Power of engine, $P = T_{mean}\, \omega$

$$= \frac{3433.5 \times 2 \times \pi \times 240}{60}$$

$$= 86293.27 \text{ W} \qquad \left(\because \omega = \frac{2\pi N}{60}\right)$$

$$= \frac{86293.27}{746} = 115.67 \text{ HP} \qquad (\because 1 \text{ HP} = 746 \text{ W})$$

4. Maximum fluctuation of energy $\Delta E = \int_0^{\pi} (T - T_{mean})\, d\theta$

$$= \int_0^{\pi} (560 \sin\theta + 84 \sin 2\theta + 8.4 \sin 3\theta)\, d\theta$$

$$\Delta E = \left[560\{-\cos\theta\} + \frac{84}{2}\{-\cos 2\theta\} + \frac{8.4}{3}\{-\cos 3\theta\}\right]_0^{\pi}$$

$$= -560(\cos\pi - \cos 0°) - 42(\cos 2\pi - \cos 0°) - 2.8(\cos 3\pi - \cos 0°)$$

$$= -560(-1-1) - 42(1-1) - 2.8(-1-1)$$

$$= 1120 + 5.6 = 1125.6 \text{ kg-m}$$

$$= 1125.6 \times 9.81 \text{ N-m} = 11042.14 \text{ N-m}$$

5. Maximum fluctuation of energy is also given as

$$\Delta E = mk^2\omega^2 C_s$$

$$11042.14 = m\left(\frac{90}{100}\right)^2 \left(\frac{2\pi \times 240}{60}\right)^2 \times 0.02$$

$$m = 1079.092 \text{ kg} \approx 1079 \text{ kg}$$

6. Minimum weight of the flywheel,

$$w = mg = 1079 \times g = 1079 \times 9.81$$

$$= 10584.99 \text{ N} \approx 10585 \text{ N} = 10.59 \text{ kN}$$

Que 3.24. **The torque delivered by a two stroke engine is given by: $T = (1000 + 300 \sin 2\theta - 500 \cos 2\theta)$ N-m where θ is the angle turned by the crank from the inner dead center. The engine speed is 250 rpm. The mass of the flywheel is 400 kg and radius of gyration is 400 mm. Determine (*i*) the power developed (*ii*) total percentage fluctuation of speed (*iii*) angular acceleration of flywheel when the crank has rotated through an angle of 60° from the inner dead center.**

AKTU 2013-14, Marks 10

Answer

Given : $T = (1000 + 300 \sin 2\theta - 500 \cos 2\theta)$ N-m,
$N = 250$ rpm, $m = 400$ kg, $k = 400$ mm $= 0.4$ m

To Find :
i. Power developed.
ii. Total percentage fluctuation of speed.
iii. Angular acceleration of flywheel.

1. Work done per revolution $= \int_0^{2\pi} T d\theta$

$$= \int_0^{2\pi} (1000 + 300 \sin 2\theta - 500 \cos 2\theta) d\theta$$

$$= \left[1000\,\theta - 300 \times \frac{\cos 2\theta}{2} - 500 \times \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \left[1000 \times 2\pi - \frac{300}{2}\cos 4\pi - \frac{500}{2}\sin 4\pi + \frac{300}{2}\cos 0°\right]$$

$$= [1000 \times 2\pi - 150 + 150]$$

$$= 2000\pi \text{ N-m}$$

2. Mean resisting torque of the engine,

$$T_{mean} = \frac{\text{Work done per revolution}}{2\pi}$$

$$= \frac{2000\pi}{2\pi} = 1000 \text{ N-m}$$

3. Power developed by the engine

$$= T_{mean}\omega = 1000 \times 26.18$$

$$\left[\because \omega = \frac{2\pi N}{60} = \frac{2\pi \times 250}{60} = 26.18\right]$$

$$\tan 2\theta = 5/3$$

$$2\theta = 59.04° \text{ or } 239.04°$$

$\therefore$ $$\theta_B = 29.52°$$

and, $$\theta_D = 119.52°$$

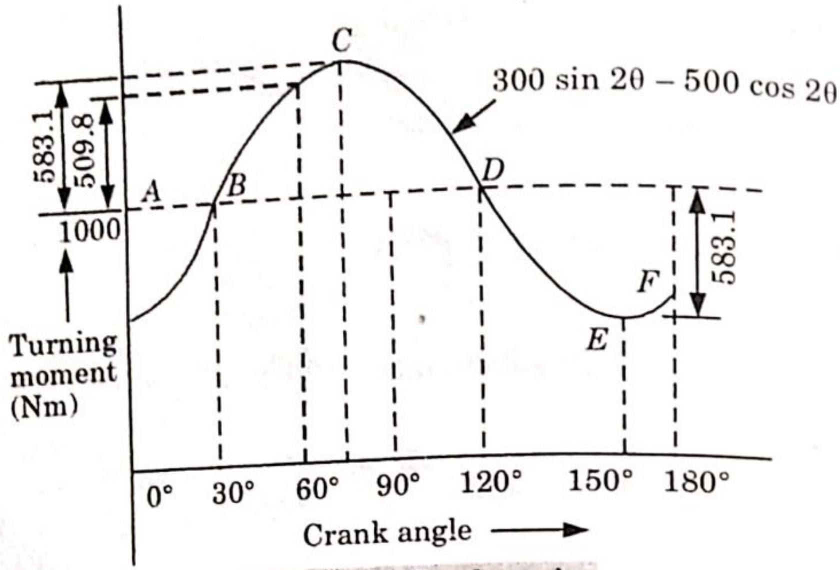


Fig. 3.24.1. Crank angle.

5. Maximum fluctuation of energy,

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{\text{mean}})\, d\theta$$

$$= \int_{29.52°}^{119.52°} (300 \sin 2\theta - 500 \cos 2\theta)\, d\theta$$

$$= \left[-\frac{300}{2} \cos 2\theta - \frac{500}{2} \sin 2\theta \right]_{29.52°}^{119.52°}$$

$$= -[150 \cos 239.04° + 250 \sin 239.04° - 150 \cos 59.04° - 250 \sin 59.04°]$$

$$= 583.09 \text{ N-m}$$

6. We know that, $\Delta E = I\omega^2 C_s$

$$583.09 = 64 \times 26.18^2 \times C_s$$

$$[\because \quad I = mk^2 = 400 \times (0.4)^2 = 64 \text{ kg-m}^2]$$

$$C_s = 0.013 \quad \text{or} \quad C_s = 1.3\ \%$$

7. Angular acceleration in the flywheel is produced by the excess torque over the mean torque,

$$T_{\text{excess}} = T - T_{\text{mean}}$$

$$= 300 \sin 2\theta - 500 \cos 2\theta$$

$$(T_{excess})_{\theta = 60°} = 300 \sin 120° - 500 \cos 120°$$

$$I\alpha = 509.81 \text{ N-m}$$

$$\alpha = \frac{509.81}{64} = 7.97 \text{ rad/s}^2$$

Que 3.25. **The turning moment diagram of a quadruple expansion marine engine (multi cylinder engine) drawn to the following scale : 1 cm = 15 ton-m and 1 cm = 15°. The areas of the loops above and below the mean turning moment line taken in order are 0.12, 0.34, 0.91, 0.81, 0.15, 0.18, 1.86, 1.71 cm². If the moment of inertia of the propeller and entrained water is 100 ton-m² and the mean speed of rotation is 100 rpm, determine the value of co-efficient of fluctuation of speed.**

AKTU 2014-15, Marks 10

Answer

Given : I = 100 ton-m², N = 100 rpm or $\omega = \dfrac{2\pi \times 100}{60}$ = 10.472 rad/sec

Vertical Scale : 1 cm = 15 ton-m, Horizontal scale : 1 cm = 15°

To Find : Co-efficient of fluctuation of speed.

1. Draw the turning moment diagram as shown in Fig. 3.25.1 on the basis of given data.
2. On turning moment diagram,

$$1 \text{ cm}^2 = 15 \times \frac{15\pi}{180} = \frac{5\pi}{4} \text{ ton-m}$$

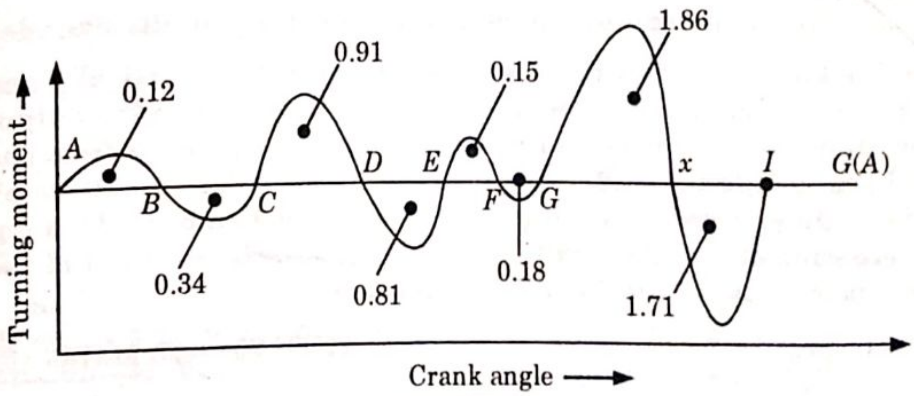


Fig. 3.25.1.

3. Let total energy at $A = E$, then rerfering to Fig. 3.25.1,

Energy at $B = E + 0.12$

Energy at $C = E + 0.12 - 0.34 = E - 0.22$ (Min. energy)

Energy at $D = E - 0.22 + 0.91 = E + 0.69$

Energy at $E = E + 0.69 - 0.81 = E - 0.12$

Energy at $F = E - 0.12 + 0.15 = E + 0.03$

Energy at $G = E + 0.03 - 0.18 = E - 0.15$

Energy at $H = E - 0.15 + 1.86 = E + 1.71$ (Max. energy)

Energy at $I = E + 1.71 - 1.71 = E$

4. Maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + 1.71) - (E - 0.22)$$
$$= 1.93 \text{ cm}^2$$

$$\Delta E = 1.93 \times \frac{5\pi}{4} = 7.58 \text{ ton-m}$$

5. We know that, $\Delta E = I\omega^2 C_s$

$$7.58 = 100 \times (10.472)^2 \times C_s$$

$$C_s = \frac{7.58}{100 \times (10.472)^2}$$

$$C_s = 0.000691 \text{ or } 0.069 \%$$

Que 3.26. **The turning moment diagram for a multi cylinder engine has been drawn to a scale 1 mm = 600 N-m vertically and 1 mm = 3° horizontally. The intercepted areas between the output torque curve and the mean resistance line, taken in order from one end are as follows : + 52, – 124, + 92, – 140, + 85, – 72 and + 107 mm², when the engine is running at a speed of 600 rpm. If the total fluctuation of speed is not to exceed ± 1.5 % of the mean, find the necessary mass of flywheel of radius 0.5 m.**

AKTU 2016-17, Marks 10

Answer

Given : $N = 600$ rpm, $R = 0.5$ m, $C_s = \pm 1.5\ \% = 3\ \% = 0.03$
Vertical scale : 1 mm = 600 N-m, Horizontal scale : 1 mm = 3°
To Find : Mass of flywheel.

1. Draw the turning moment diagram as shown in Fig. 3.26.1 on the basis of given data.

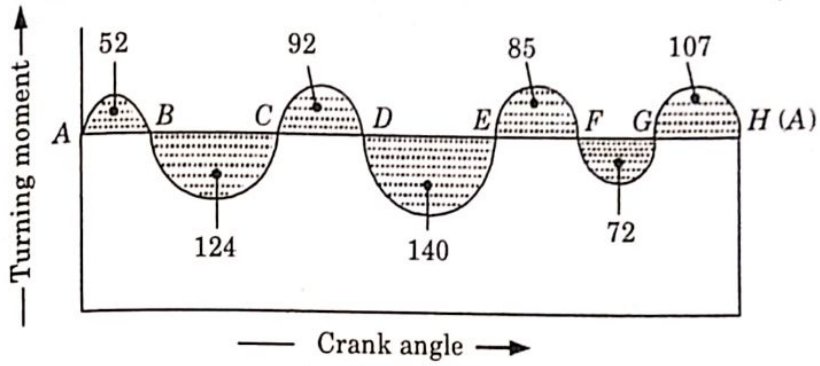


Fig. 3.26.1.

2. On turning moment diagram,

$$1\ \text{mm}^2 = 600 \times 3 \times \frac{\pi}{180} = 31.42\ \text{N-m}$$

3. Let the total energy at $A = E$, then referring to Fig. 3.26.1,

Energy at $B = E + 52$ (Max. energy)

Energy at $C = E + 52 - 124 = E - 72$

Energy at $D = E - 72 + 92 = E + 20$

Energy at $E = E + 20 - 140 = E - 120$ (Min. energy)

Energy at $F = E - 120 + 85 = E - 35$

Energy at $G = E - 35 - 72 = E - 107$

Energy at $H = E - 107 + 107 = E$ = Energy at A

4. We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + 52) - (E - 120) = 172 = 172 \times 31.42$$

$$= 5404 \text{ N-m}$$

5. We know that maximum fluctuation of energy (ΔE),

$$5404 = mR^2\omega^2C_s = m \times (0.5)^2 \times (62.84)^2 \times 0.03 = 29.6\ m$$

$$m = 5404/\ 29.6 = 183 \text{ kg}$$

☺☺☺

4

UNIT

Balancing and Governers

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PART-1

Introduction, Static Balance, Dynamic Balance

CONCEPT OUTLINE

Static Balancing : A system of rotating masses is said to be in static balance if the combined mass centre of the system lies on the axis of rotation.

Dynamic Balancing : A system of rotating masses is in dynamic balance when there does not exist any resultant centrifugal force as well as resultant couple.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. **Explain the term balancing. State the difference between static and dynamic balancing.**

Answer

A. **Balancing :** It is a process of designing or modifying machinery, so that the unbalance is reduced to an acceptable level and if possible is eliminated entirely.

B. **Difference between Static and Dynamic Balancing :**

S. No.	Static Balancing	Dynamic Balancing
1.	Net dynamic force acting on the system must be zero.	Net moments about any point in the plane must be zero.
2.	If a system is dynamically balanced, then it will also be statically balanced.	But if a system is statically balanced, it may be or may not dynamically balanced.

PART-2

...Rotating Masses, Two Plane Balancing, Graphical

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.2. **Explain the method of balancing of different masses revolving in the same plane.**

AKTU 2016-17, Marks 10

Answer

1. Consider four masses of magnitude m_1, m_2, m_3 and m_4 at distances of r_1, r_2, r_3 and r_4 from the axis of rotating shaft. Let θ_1, θ_2, θ_3 and θ_4 be the angles of these masses with the horizontal line OX as shown in Fig. 4.2.1.
2. These masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity ω.
3. The magnitude and position of the balancing mass can be obtained by following two methods :

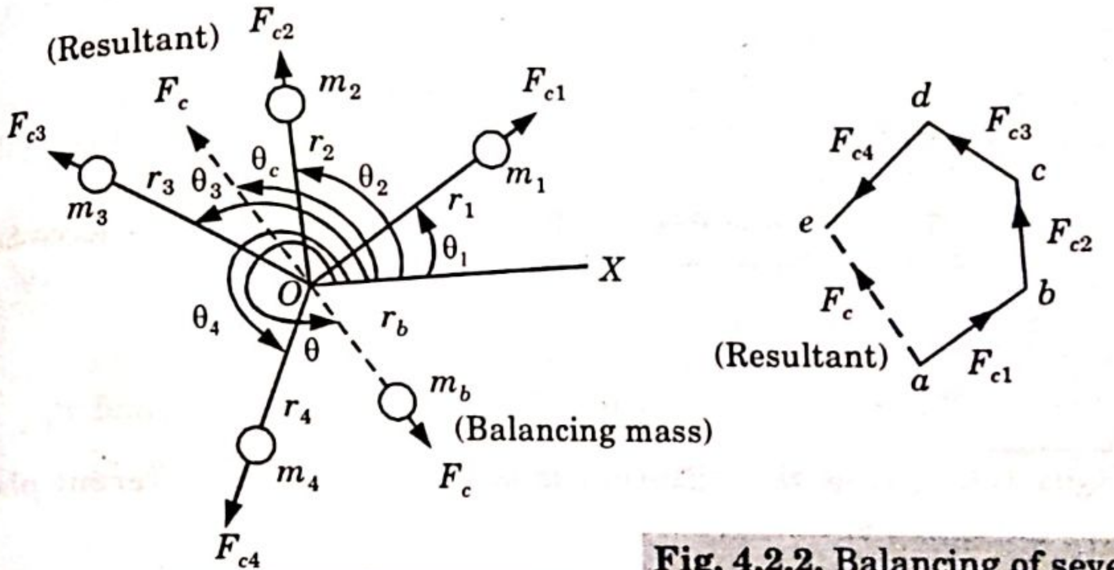


Fig. 4.2.1. Space diagram.

Fig. 4.2.2. Balancing of several masses rotating in same plane.

a. Analytical Method :

1. First of all, find out the centrifugal force exerted by each mass on the rotating shaft.
2. Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e.*, ΣH and ΣV.

   Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 r_1 \cos \theta_1 + m_2 r_2 \cos \theta_2 + ...$$

3. Magnitude of the resultant centrifugal force,

$$F_c = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan \theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in opposite direction.
6. Now the magnitude of the balancing mass is given as

$$F_c = mr$$

Where, m = Balancing mass, and

r = Radius of rotation.

b. Graphical Method :

1. Initially draw the space diagram of masses to show their positions.
2. Find the centrifugal forces exerted by each mass on the rotating shaft.
3. Draw the vector diagram using $mr\omega^2$ (or using only mr as ω is constant) of all the forces as shown in Fig. 4.2.2. Here ab represents m_1r_1 or $m_1r_1\omega^2$ in magnitude and direction to some suitable scale. Similarly draw bc, cd and de to represent centrifugal forces of other masses m_2, m_3 and m_4.
4. From polygon law of forces the closing side of polygon (*i.e.*, ae) represents the resultant force in magnitude and direction as shown in Fig. 4.2.2.
5. The balancing force is then equal to the resultant force but in opposite direction.
6. Now the value of balancing force is given as

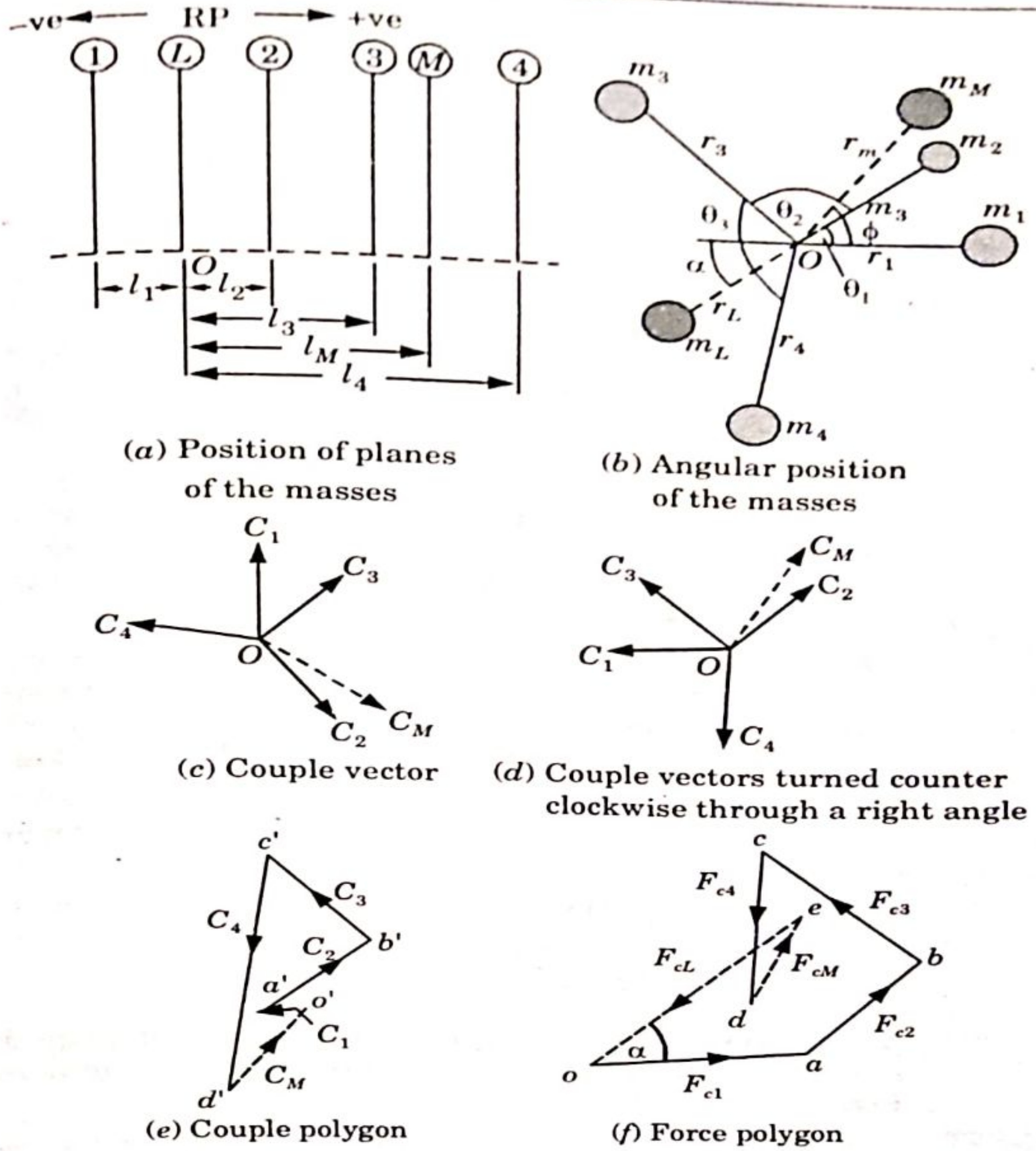
$$m_br_b\omega^2 = \text{Resultant centrifugal force.}$$

Where m_br_b = Resultant of m_1r_1, m_2r_2, m_3r_3 and m_4r_4.

Que 4.3. How the different masses rotating in different planes are balanced ?

Answer

1. For complete balancing of masses in different planes, following two conditions must be satisfied :
   i. The forces in the reference plane must be balanced *i.e.*, the resultant force must be zero.
   ii. The couples about the reference plane must be balanced *i.e.*, the resultant couple must be zero.
2. Consider the four masses m_1, m_2, m_3 and m_4 revolving in planes 1, 2, 3 and 4 respectively as shown in Fig. 4.3.1(a). The relative angular positions of these masses are shown in Fig. 4.3.1(b).



(a) Position of planes of the masses

(b) Angular position of the masses

(c) Couple vector

(d) Couple vectors turned counter clockwise through a right angle

(e) Couple polygon

(f) Force polygon

Fig. 4.3.1.

3. The magnitude of balancing masses m_L and m_M in planes L and M may be obtained as follows :
   i. Take a plane L as the reference plane. The distances of all the planes to the left of the reference plane may be taken as negative and those to the right of the reference plane as positive.
   ii. Tabulate the data in the same order in which they occur, heading from left to right.

Table 4.3.1.

Plane	Mass (m)	Radius (r)	mr	Distance from reference plane (*i.e.*, RP)(l)	mrl
1	m_1	r_1	m_1r_1	$-l_1$	$-m_1r_1l_1$
L (RP)	m_L	r_L	m_Lr_L	0	0
2	m_2	r_2	m_2r_2	l_2	$m_2r_2l_2$
3	m_3	r_3	m_3r_3	l_3	$m_3r_3l_3$
M	m_M	r_M	m_Mr_M	l_M	$m_Mr_Ml_M$
4	m_4	r_4	m_4r_4	l_4	$m_4r_4l_4$

iii. Draw the couple vector diagram. Draw OC_1 acting in the plane through Om_1 and perpendicular to the paper. This is represented by OC_1 in the plane of the paper and perpendicular to Om_1. Similarly OC_2, OC_3 and OC_4 are drawn perpendicular to Om_2, Om_3 and Om_4 respectively and in the plane of the paper.

iv. Now rotate the couple vector diagram in the counter clockwise direction such that the positions (or directions) of the couple vectors become in the parallel direction of masses (as OC_2, OC_3 and OC_4 are parallel and in the same direction as Om_2, Om_3 and Om_4).

v. Draw a couple polygon to a suitable scale. The vector $d'o'$ represents the balanced couple. C_M (balanced couple) is proportional to $m_M\, r_M\, l_M$. Therefore

$$C_M = m_M r_M l_M = \text{Vector } d'o'$$

$$m_M = \frac{\text{Vector } d'o'}{r_M l_M}$$

This expression gives value of balancing mass in plane M and the angle of inclination of this mass may be measured from Fig. 4.3.1 (*b*).

vi. Now draw a force polygon. The vector eo (direction from e to o) represents the balanced force. Since the balanced force is proportional to m_Lr_L, therefore

$$m_Lr_L = \text{Vector } eo$$

$$m_L = \frac{\text{Vector } eo}{r_L}$$

This expression provides the value of balancing mass in plane L and the angle of inclination of this mass can be measured from Fig. 4.3.1(*b*).

Que 4.4. Three masses of 8 kg, 12 kg and 15 kg attached at radial distances of 80 mm, 100 mm and 60 mm respectively to a disc on a shaft are in complete balance. Determine the angular positions of the masses of 12 kg and 15 kg relative to the 8 kg mass.

AKTU 2013-14, Marks 06

Answer

Given : $m_1 = 8$ kg, $m_2 = 12$ kg, $m_3 = 15$ kg, $r_1 = 80$ mm, $r_2 = 100$ mm, $r_3 = 60$ mm

To Find : Angular positions of mass 12 kg and 15 kg.

1. Three masses are in complete balance *i.e.*, they will form a close triangle with the sides of triangle as mr.

Table 4.4.1.

S. No.	Mass (m) kg	Radial distance (r) mm	(mr) kg-mm
1.	8	80	640
2.	12	100	1200
3.	15	60	900

2. According to question take 8 kg mass as reference and draw a horizontal (or in X-axis) line equal to 640 (on some suitable scale) and mark an arc of lengths 1200 and 900 to draw a triangle (keep the direction in same sense).

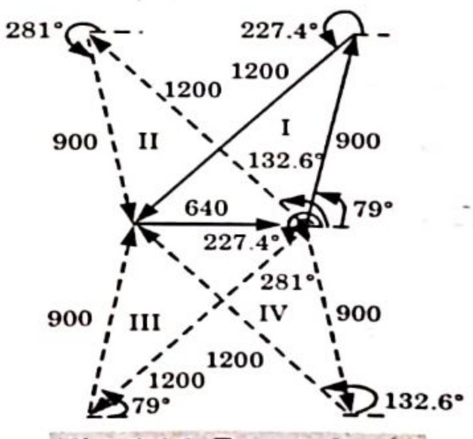


Fig. 4.4.1. Force polygon.

3. We have four triangles but two sets of solution are same. So we have only two solutions as (by measurement)

$\theta_2 = 227.4°$ and $\theta_3 = 79°$

$\theta_2 = 132.6°$ and $\theta_3 = 281°$

Here, θ_2 = Angle between 8 kg and 12 kg mass, and
θ_3 = Angle between 8 kg and 15 kg mass.

Que 4.5. A shaft carries four masses in parallel planes *A*, *B*, *C* and *D* in this order along its length. The masses at *B* and *C* are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at *A* and *D* have an eccentricity of 80 mm. The angle between the masses of *B* and *C* is 100° and that between the masses at *B* and *A* is 190°, both being measured in the same direction. The axial distance between the planes *A* and *B* is 100 mm and that between *B* and *C* is 200 mm. If the shaft is in complete dynamic balance, determine : 1. The magnitude of the masses at *A* and *D*; 2. The distance between planes *A* and *D*; and 3, the angular position of the mass at *D*.

AKTU 2017-18, Marks 10

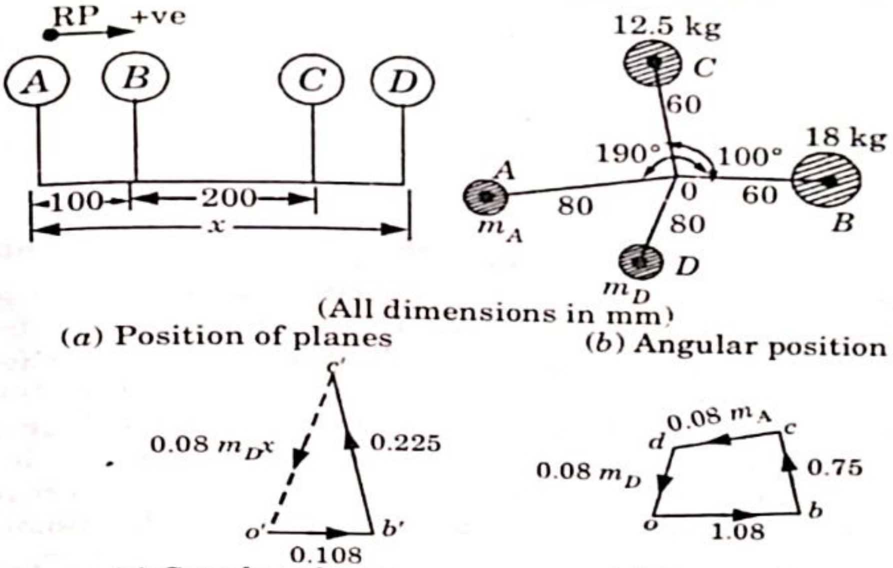
Answer

Given : $m_B = 18$ kg, $m_C = 12.5$ kg, $r_B = r_C = 60$ mm $= 0.06$ m, $r_A = r_D = 80$ mm $= 0.08$ m, $\angle BOC = 100°$, $\angle BOA = 190°$

To Find :
i. The magnitude of the masses at *A* and *D*.
ii. The distance between planes *A* and *D*.
iii. The angular position of the mass at *D*.

1. The position of the planes and angular position of the masses is shown in Fig. 4.5.1(*a*) and (*b*) respectively.
2. The position of mass *B* is assumed in the horizontal direction, *i.e.*, along *OB*. Taking the plane of mass *A* as the reference plane, the data may be tabulated as below :

Plane (1)	Mass (*m*) kg (2)	Eccentricity (*r*) m (3)	Cent. Force ÷ ω^2 (*mr*) kg-m (4)	Distance from Plane A (*l*) m (5)	Couple ÷ ω^2 (*mrl*) kg-m² (6)
A(RP)	m_A	0.08	$0.08\ m_A$	0	0
B	18	0.06	1.08	0.1	0.108
C	12.5	0.06	0.75	0.3	0.225
D	m_D	0.08	$0.08\ m_D$	x	$0.08\ m_D x$



(All dimensions in mm)

(*a*) Position of planes

(*b*) Angular position of masses

(*c*) Couple polygon

(*d*) Force polygon

Fig. 4.5.1.

3. First of all, the direction of mass D is fixed by drawing the couple polygon to some suitable scale, as shown in Fig. 4.5.1(*c*), from the data given in column 6 of table.
4. The closing side of the couple polygon (vector $c'\,o'$) is proportional to $0.08\, m_D x$. By measurement, we find that,

$$0.08\, m_D x = \text{Vector } c'\,o' = 0.235 \text{ kg-m}^2 \qquad ...(4.5.1)$$

5. In Fig. 4.5.1(*a*), draw OD parallel to vector $c'\,o'$ to fix the direction of mass D.
6. Now draw the force polygon to some suitable scale, as shown in Fig. 4.5.1(*d*), from the data given in column 4 of table, as discussed below :
   i. Draw vector ob parallel to OB and equal to 1.08 kg-m.
   ii. From point b, draw vector bc parallel to OC and equal to 0.75 kg-m.
   iii. For the shaft to be in complete dynamic balance, the force polygon must be a closed figure. Therefore from point c, draw vector cd parallel to OA and from point o draw vector od parallel to OD. The vector cd and od intersect at d. Since the vector cd is proportional to $0.08\, m_A$, therefore by measurement,

$$0.08\, m_A = \text{Vector } cd = 0.77 \text{ kg-m} \quad \text{or} \quad m_A = 9.625 \text{ kg}$$

Vector do is proportional to $0.08\, m_D$, therefore by measurement,

$$0.08\, m_D = \text{Vector } do = 0.65 \text{ kg-m} \quad \text{or} \quad m_D = 8.125 \text{ kg}$$

7. From eq. (4.5.1),

$$0.08\, m_D x = 0.235 \text{ kg-m}^2$$

$$0.08 \times 8.125 \times x = 0.235 \quad \text{or} \quad 0.65\, x = 0.235$$

$$\therefore \quad x = \frac{0.235}{0.65} = 0.3615 \text{ m} = 361.5 \text{ mm}$$

8. By measurement from Fig. 4.5.1(*b*), we find that the angular position of mass at D from mass B in the anticlockwise direction, *i.e.*, $\angle BOD = 251°$

Que 4.6. **A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.**

AKTU 2014-15, Marks 05

AKTU 2016-17, Marks 10

Answer

Given : $m_A = 200$ kg, $m_B = 300$ kg, $m_C = 400$ kg, $m_D = 200$ kg, $r_A = 80$ mm $= 0.08$ m, $r_B = 70$ mm $= 0.07$ m, $r_C = 60$ mm $= 0.06$ m, $r_D = 80$ mm $= 0.08$ m, $r_X = r_Y = 100$ mm $= 0.1$m, $\angle AOB = 45°$, $\angle BOC = 70°$, $\angle COD = 120°$

To Find : Magnitudes and angular positions of balancing masses.

1. The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. 4.6.1(*a*) and (*b*) respectively.
2. Assume the plane X as the reference plane (RP). The distances of the planes to the right of plane X are taken as + ve while the distances of the planes to the left of plane X are taken as – ve. The data may be tabulated as shown in table.

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. Force $\div \omega^2$ (mr) kg-m (4)	Distance from Plane $X(l)$ m (5)	Couple $\div \omega^2$ (mrl) kg-m^2 (6)
A	200	0.08	16	–0.1	–1.6
X(RP)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

3. First of all, draw the couple polygon from the data given in table (column 6) shown in Fig. 4.6.1(*c*) to some suitable scale.

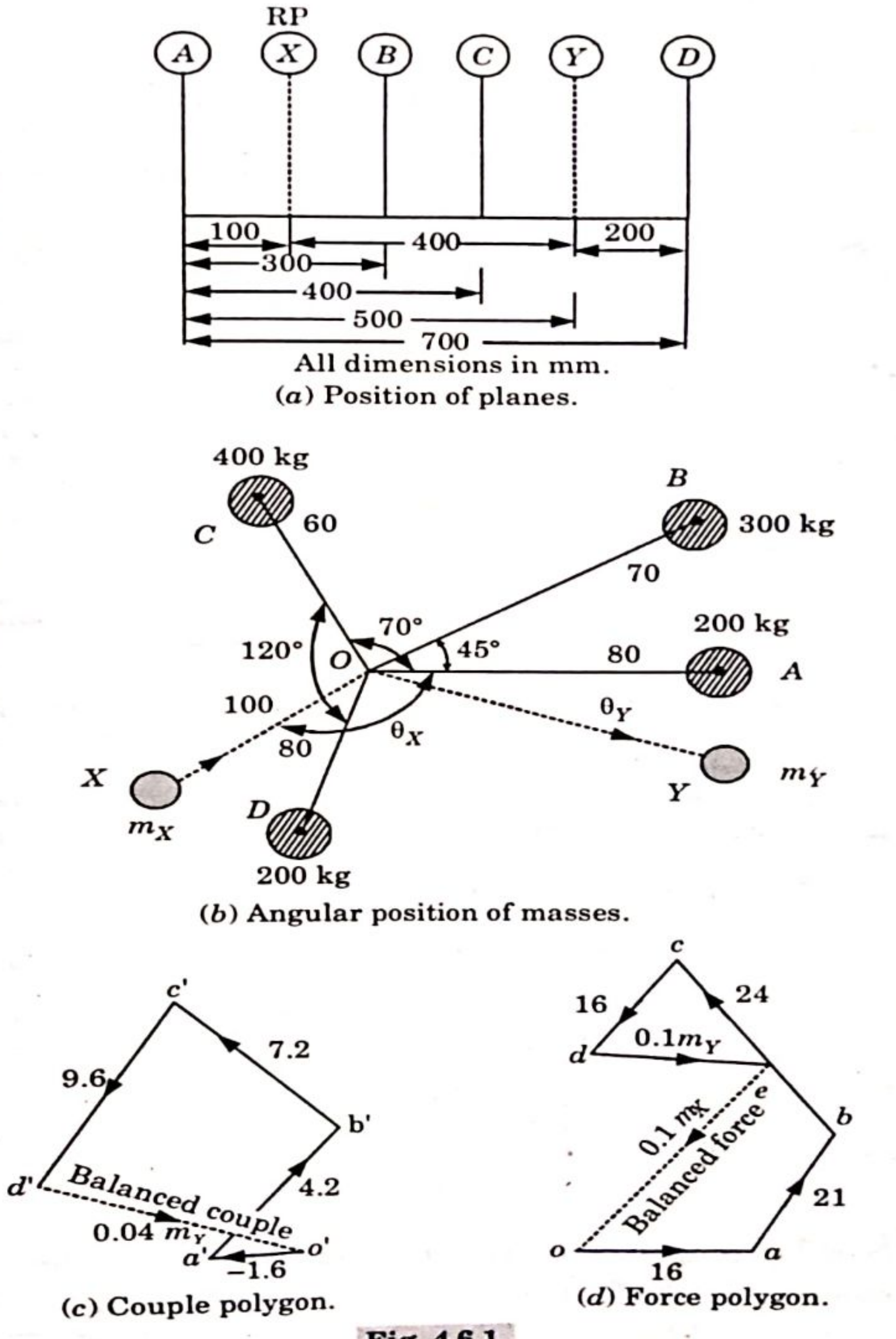


Fig. 4.6.1.

4. The vector $d'o'$ represents the balanced couple. Since the balanced couple is proportional to $0.04\ m_Y$, therefore by measurement

$$0.04\ m_Y = \text{Vector } d'o' = 7.3 \text{ kg-m}^2 \text{ or } m_Y = 182.5 \text{ kg}.$$

5. The angular position of the mass m_Y is obtained by drawing Om_Y in Fig. 4.8.1(*b*), parallel to vector $d'o'$. By measurement, the angular position of m_Y is $\theta_Y = 12°$ in the clockwise direction from mass m_A (*i.e.*, 200 kg).
6. Now draw the force polygon from the data given in table (column 4) as shown in Fig. 4.6.1(*d*).
7. The vector *eo* represents the balanced force. Since the balanced force is proportional to $0.1\ m_X$, therefore by measurement,

$$0.1\ m_X = \text{Vector } eo = 35.5 \text{ kg-m}$$

or $$m_X = 355 \text{ kg}$$

8. The angular position of the mass m_X is obtained by drawing Om_X in Fig. 4.6.1(*b*), parallel to vector *eo*. By measurement, the angular position of m_X is $\theta_X = 145°$ in the clockwise direction from mass m_A (*i.e.*, 200 kg).

PART-3

Balancing of Reciprocating Masses.

CONCEPT OUTLINE

Shaking Force : The resultant of all the forces acting on the body of the engine due to inertia force only is known as shaking force or unbalanced force.

Need of Balancing the Reciprocating Masses : The purpose of balancing the reciprocating masses is to eliminate the shaking force and a shaking couple.

Effect of Partial Balancing in Locomotive :

1. Hammer blow,
2. Variation of tractive force, and
3. Swaying couple.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.7. **Explain why only a part of the unbalanced force due to reciprocating masses is balanced by revolving mass.**

AKTU 2013-14, Marks 06

Answer

1. Consider a horizontal reciprocating engine mechanism as shown in Fig. 4.7.1.

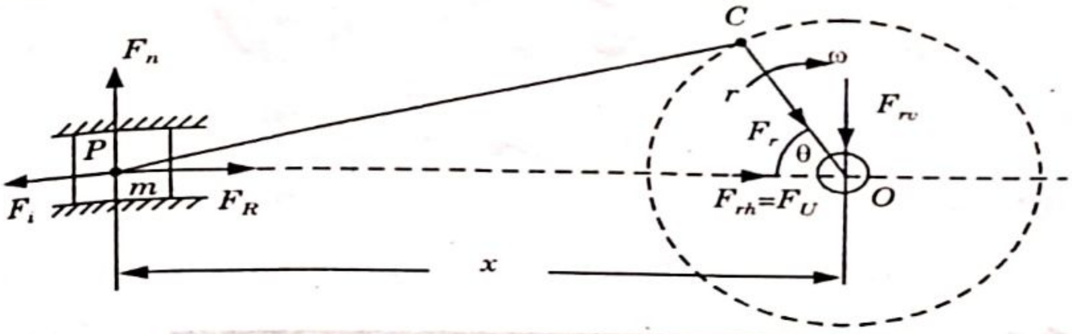


Fig. 4.7.1. Reciprocating engine mechanism.

2. Let,
   F_R = Force required to accelerate the reciprocating parts,
   F_i = Inertia force due to reciprocating parts,
   F_n = Normal force acting on the cross head guides, and
   F_r = Force acting on the main bearing (on crankshaft bearing).
3. F_R and F_i balance each other as they are equal in magnitude and opposite in direction.
4. The horizontal component of F_r acts along the line of reciprocation and also equal and opposite to F_i. Thus, F_{rh} is unbalanced or shaking force. This force is required to be balanced properly.
5. The force on the sides of the cylinder walls (F_n) and the vertical component of F_r are equal and opposite and they form a shaking couple.
6. These shaking force and shaking couple varies in magnitude and direction during the engine cycle and cause an objectional vibration.
7. In most of the mechanisms, we can reduce the shaking force and a shaking couple by adding appropriate balancing mass, but it is usually not practical to eliminate them completely. In other words, the reciprocating masses are only partially balanced.

Que 4.8. **What do you understand by primary and secondary unbalance in reciprocating engines ?**

Answer

1. Acceleration of the reciprocating mass of a slider crank mechanism is given by

$$a_R = r\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

2. Therefore, inertia force due to the reciprocating parts or the force required to accelerate mass m is

$$F_I = F_R = ma_R = mr\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$= mr\omega^2 \cos\theta + mr\omega^2 \frac{\cos 2\theta}{n}$$

Where, $mr\omega^2 \cos\theta$ = Primary accelerating force, and

$mr\omega^2 \dfrac{\cos 2\theta}{n}$ = Secondary accelerating force.

Que 4.9. **Derive the following expressions for an uncoupled two cylinder locomotive engine :**

i. Variation of tractive effort

ii. Swaying couple

iii. Hammer blow.

AKTU 2014-15, Marks 05

AKTU 2015-16, Marks 10

OR

Explain the terms in detail

i. Variation in tractive effort

ii. Swaying couple and

iii. Hammer blow and safe speed of locomotive.

AKTU 2017-18, Marks 10

OR

Derive an expression for variation in tractive force and hammer blow for an uncoupled two cylinder locomotive engine.

AKTU 2016-17, Marks 10

Answer

i. Variations in Tractive Force :

1. The resultant unbalanced force due to the two cylinders along the line of stroke is known as tractive force.
2. Consider the crank of first cylinder be inclined at an angle θ with the line of stroke as shown in Fig. 4.9.1. The second cylinder has the crank at right angle to the first crank so the angle of inclination for the second crank becomes $(90° + \theta)$.

T1

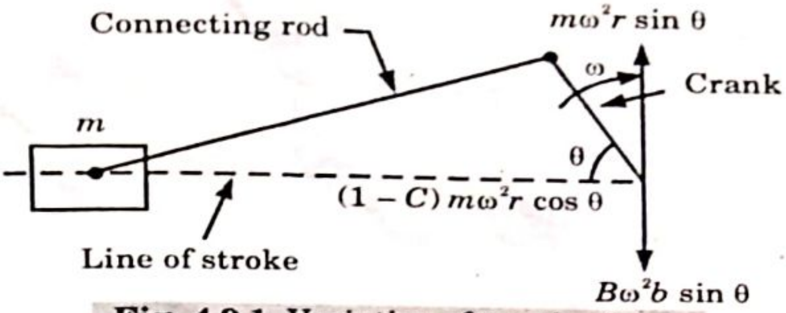


Fig. 4.9.1. Variation of tractive force.

3. Let, m = Mass of reciprocating parts per cylinder, and
C = Fraction of the reciprocating parts to be balanced.

4. The unbalanced force along the line of stroke of cylinder 1
$$= (1 - C)\, mr\omega^2 \cos\theta$$
Similarly, for cylinder 2, the unbalanced force along the line of stroke
$$= (1 - C)\, mr\omega^2 \cos(90° + \theta)$$

5. The tractive force,
$$F_T = (1 - C)\, mr\omega^2 \cos\theta + (1 - C)\, mr\omega^2 \cos(90° + \theta)$$
$$= (1 - C)\, mr\omega^2 [\cos\theta - \sin\theta]$$

6. For maximum or minimum tractive force, the term $(\cos\theta - \sin\theta)$ should be maximum or minimum. So,
$$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0$$
$$-\sin\theta - \cos\theta = 0$$
$$\tan\theta = -1$$
$$\theta = 135° \text{ or } 315°$$

7. The tractive force is maximum or minimum when
$$\theta = 135° \text{ or } 315°$$
Maximum and minimum value of the tractive force or variation in tractive force
$$= \pm (1 - C)\, mr\omega^2 (\cos 135° - \sin 135°)$$
$$= \pm \sqrt{2}\,(1 - C)\, mr\omega^2$$

ii. Swaying Couple :

1. The unbalanced forces along the line of stroke for the two cylinders constitute a couple about the centre line YY between the cylinders as shown in Fig. 4.9.2.
2. This couple tends to sway the engine alternately in clockwise and anticlockwise directions. Hence, the couple is known as swaying couple.

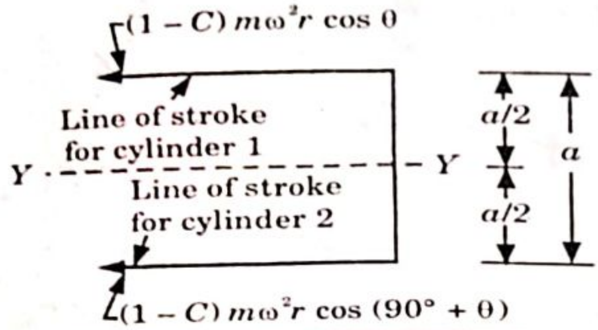


Fig. 4.9.2. Swaying couple.

3. Let, a = Distance between the centre lines of two cylinders.

4. Swaying couple $= (1 - C)\, mr\omega^2 \cos\theta \times \dfrac{a}{2} -$

$$(1 - C)\, mr\omega^2 \cos(90° + \theta) \times \frac{a}{2}$$

$$= (1 - C)\, mr\omega^2\, \frac{a}{2}(\cos\theta + \sin\theta)$$

5. For maximum or minimum value of swaying couple, the value of $(\cos\theta + \sin\theta)$ should be maximum or minimum. So,

$$\frac{d}{d\theta}(\cos\theta + \sin\theta) = 0$$

$$\tan\theta = 1 \Rightarrow \theta = 45°, 225°$$

6. Maximum or minimum value of the swaying couple

$$= \pm (1 - C)\, mr\omega^2 \times a/2(\cos 45° + \sin 45°)$$

$$= \pm \frac{a}{\sqrt{2}}(1 - C)\, mr\omega^2$$

iii. Hammer Blow :

1. The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as hammer blow.
2. We know that the unbalanced force along the perpendicular to the line of stroke is due to the balancing mass B, at a radius b, in order to balance reciprocating parts only a force $B\omega^2 b \sin\theta$ is required.
3. The quantity $B\omega^2 b \sin\theta$ will be maximum for $\sin\theta = 1$

i.e., $\theta = 90°$ or $270°$

$\therefore$ Hammer blow $= B\omega^2 b$

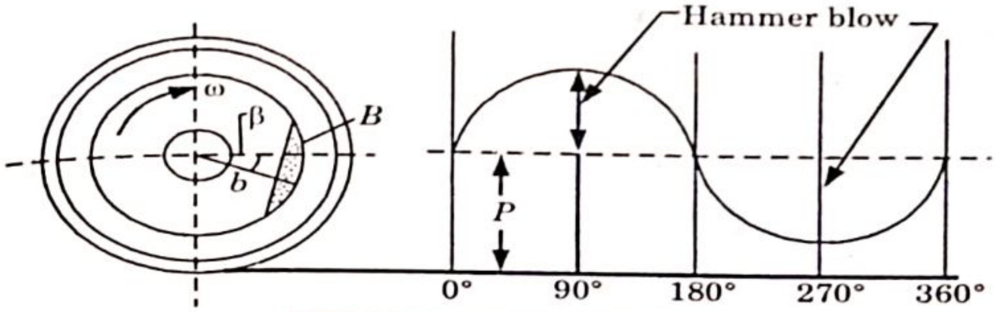


Fig. 4.9.3. Hammer blow.

4. The effect of hammer blow is to cause the variation in pressure between the wheel and the rail. The variation is shown in Fig. 4.9.3 for one revolution of the wheel.
5. Let P be the downward pressure on the rails.

$\therefore$ Net pressure between the wheel and the rails $= P \pm B\omega^2 b$

6. If $(P - B\omega^2 b) < 0$, the wheel will be lifted from the rails.

So, in limiting condition,

$$P - B\omega^2 b = 0 \quad \text{or} \quad P = B\omega^2 b$$

$$\omega = \sqrt{\frac{P}{Bb}}$$

Where, ω = Permissible value of the angular speed.

Que 4.10. **The following data refer to two cylinder locomotive with cranks at 90°. Reciprocating mass per cylinder = 300 kg; Crank radius = 0.3 m; Driving wheel diameter = 1.8 m; Distance between cylinder centre lines = 0.65 m; Distance between the driving wheel central planes = 1.55 m. Determine : 1. The fraction of the reciprocating masses to be balanced, if the hammer below is not to exceed 46 kN at 96.5 km/h; 2. The variation in tractive effort; and 3. The maximum swaying couple.**

AKTU 2017-18, Marks 10

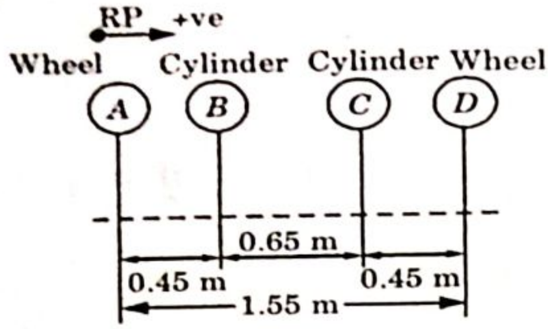
Answer

Given : m = 300 kg, r = 0.3 m, D = 1.8 m or R = 0.9 m, a = 0.65 m, Hammer blow = 46 kN = 46 × 10^3 N, v = 96.5 km/h = 26.8 m/s.

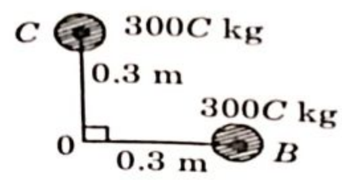
To Find :
- i. The fraction of the reciprocating masses to be balanced.
- ii. The variation in tractive effort.
- iii. The maximum swaying couple.

1. The mass of the reciprocating parts to be balanced = Cm

$$= 300C \text{ kg}$$



(*a*) Position of planes.



(*b*) Position of cranks.

Fig. 4.10.1.

2. The position of planes of the wheels and cylinders is shown in Fig. 4.10.1(*a*) and the position of cranks is shown in Fig. 4.10.1(*b*).
3. Assuming the plane of wheel *A* as the reference plane.

Plane (1)	Mass (*m*) kg (2)	Radius (*r*) m (3)	Cent. Force ÷ w^2 (*mr*) kg-m (4)	Distance from Plane A (*l*) m (5)	Couple ÷ $\omega^2(mrl)$ kg-m² (6)
A(RP)	*B*	*b*	*Bb*	0	0
B	300*C*	0.3	90*C*	0.45	40.5*C*
C	300*C*	0.3	90*C*	1.1	99*C*
D	*B*	*b*	*Bb*	1.55	1.55 *Bb*

4. Now the couple polygon, to some suitable scale, may be drawn with the data given in table (column 6), [Fig. 4.10.2].
5. The closing side of the polygon (vector *c′o′*) represents the balancing couple and is proportional to 1.55 *Bb*.
6. From the couple polygon,

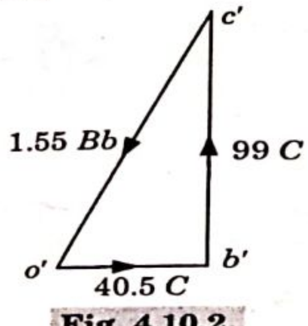


Fig. 4.10.2.

$$1.55\,Bb = \sqrt{(40.5\,C)^2 + (99\,C)^2} = 107\,C$$

$$\therefore \quad Bb = 107\,C/1.55 = 69\,C$$

7. Angular speed, $\omega = v/R = 26.8/0.9 = 29.8$ rad/s

8. Hammer blow,

$$46 \times 10^3 = B\omega^2 b = 69\ C(29.8)^2 = 61275\ C$$

$$\therefore \quad C = 46 \times 10^3/61275 = 0.751$$

9. We know that variation in tractive effort

$$= \pm \sqrt{2}(1 - C)m\omega^2 r$$

$$= \pm \sqrt{2}(1 - 0.751)\ 300(29.8)^2\ 0.3$$

$$= 28140\ \text{N} = 28.14\ \text{kN}$$

10. We know that maximum swaying couple

$$= \frac{a(1 - C)}{\sqrt{2}}\ m\omega^2 r$$

$$= \frac{0.65(1 - 0.751)}{\sqrt{2}} \times 300(29.8)^2 \times 0.3$$

$$= 9148\ \text{N-m} = 9.148\ \text{kN-m}$$

Que 4.11. **An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles. The whole of the rotating and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses and also find swaying couple at a crank speed of 300 rpm.**

AKTU 2016-17, Marks 15

Answer

Given : $a = 0.7$ m, $l_B = l_C = 0.6$ m, $m_1 = 150$ kg, $m = 180$ kg, $C = 2/3$, $r_A = r_D = 0.6$ m, $N = 300$ rpm

To Find :
- i. Magnitude of balancing masses.
- ii. Direction of balancing masses.
- iii. Swaying couple.

1. We know that the equivalent mass of the rotating parts to be balanced per cylinder at the crank pin,

$$m_2 = m_B = m_C = m_1 + Cm = 150 + \frac{2}{3} \times 180 = 270\ \text{kg}$$

2. First of all, draw the space diagram to show the positions of the planes of the wheels and the cylinders, as shown in Fig. 4.11.1(a). Since the cranks of the cylinders are at right angles, therefore assuming the position of crank of the cylinder B in the horizontal direction, draw OC and OB at right angles to each other as shown in Fig. 4.11.1(b).

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3. Tabulate the data as given in the table. Assume the plane of wheel *A* as the reference plane.

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. Force ÷ ω^2 (mr) kgm (4)	Distance from Plane $A(l)$ m (5)	Couple ÷ ω^2 (mrl) kg-m² (6)
A (RP)	m_A	0.6	0.6 m_A	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	0.6 m_D	1.5	0.9 m_D

4. Now, draw the couple polygon from the data given in table (column 6), to some suitable scale, as shown in Fig. 4.11.1(*c*). The closing side *c'o'* represents the balancing couple and it is proportional to 0.9 m_D. Therefore, by measurement,

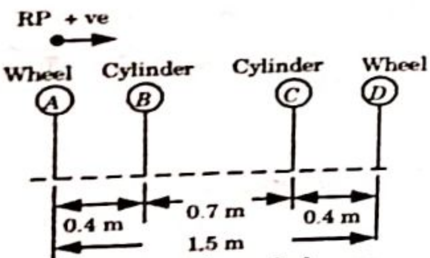
$$0.9\, m_D = \text{Vector } c'o' = 94.5 \text{ kg-m}^2 \quad \text{or} \quad m_D = 105 \text{ kg}$$

5. To determine the angular position of the balancing mass *D*, draw *OD* in Fig. 4.11.1(*b*) parallel to vector *c'o'*. By measurement,

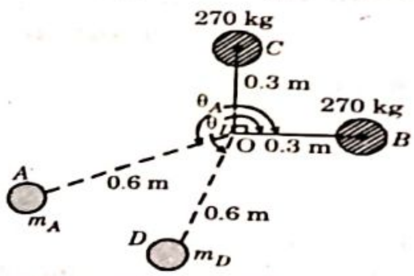
$$\theta_D = 250°.$$

6. In order to find the balancing mass *A*, draw the force polygon from the data given in table (column 4), to some suitable scale, as shown in Fig. 4.11.1(*d*). The vector *do* represents the balancing force and it is proportional to 0.6 m_A. Therefore by measurement,

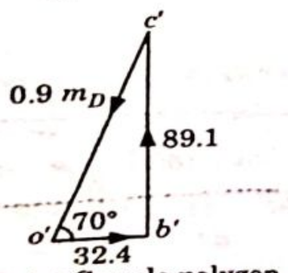
$$0.6\, m_A = \text{Vector } do = 63 \text{ kg-m or } m_A = 105 \text{ kg}$$



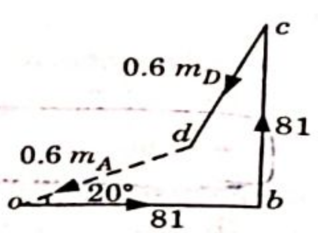
(*a*) Position of planes



(*b*) Angular position of masses



(*c*) Couple polygon



(*d*) Force polygon

Fig. 4.11.1.

7. To determine the angular position of the balancing mass A, draw OA in Fig. 4.11.1(b) parallel to vector do. By measurement,

$$\theta_A = 200°$$

8. We know that, maximum swaying couple,

$$= \frac{a(1-c)}{\sqrt{2}} m\omega^2 r = \frac{0.7 \times \left(1 - \frac{2}{3}\right)}{\sqrt{2}} \times 180 \times (31.42)^2 \times 0.3$$

$$= 8795.65 \text{ Nm.}$$

Que 4.12. **A single cylinder reciprocating engine has a reciprocating mass of 60 kg. The crank rotates at 60 rpm and the stroke is 320 mm. The mass of the revolving parts at 160 rpm is 50 kg. If two-thirds of the reciprocating parts and whole of the revolving parts are to be balanced, determine the balance mass required at a radius of 340 mm.** **AKTU 2013-14, Marks 06**

Answer

Given : $L = 320$ mm $= 0.32$ m, $m = 60$ kg, $m_1 = 50$ kg, $C = 2/3$, $N = 160$ rpm, $b = 340$ mm $= 0.34$ m
To Find : Balance mass required.

1. We know that,

$$Bb = (m_1 + Cm)r$$

$$B \times 0.34 = \left(50 + \frac{2}{3} \times 60\right) \times 0.16 \qquad \left[\because r = \frac{L}{2} = \frac{0.32}{2} = 0.16 \text{ m}\right]$$

$$B \times 0.34 = 14.4$$

$$B = 42.35 \text{ kg}$$

Que 4.13. **The three cranks of a three cylinder locomotive are all on the same axle and are set at 120°. The pitch of the cylinders is 1 metre and the stroke of each piston is 0.6 m. The reciprocating masses are 300 kg for inside cylinder and 260 kg for each outside cylinder and the planes of rotation of the balance masses are 0.8 m from the inside crank. If 40 % of the reciprocating parts are to be balanced, find : (i) the magnitude and the position of the balancing masses required at a radius of 0.6 m; and 2. The hammer blow per wheel when the axle makes 6 rps.** **AKTU 2014-15, Marks 05**

Answer

Given : $\angle AOB = \angle BOC = \angle COA = 120°$, $l_A = l_B = l_C = 0.6$ m or $r_A = r_B = r_C = 0.3$ m, $m_i = 300$ kg, $m_o = 260$ kg, $C = 40\ \% = 0.4$, $b_1 = b_2 = 0.6$ m, $N = 6$ rps $= 6 \times 2\pi = 37.7$ rad/s.

To Find :
i. Magnitude and position of balancing masses.
ii. Hammer blow per wheel.

1. Mass of the reciprocating parts to be balanced for each outside cylinder,

$$m_A = m_C = Cm_o = 0.4 \times 260 = 104 \text{ kg}$$

and mass of the reciprocating parts to be balanced for inside cylinder,

$$m_B = Cm_i = 0.4 \times 300 = 120 \text{ kg}$$

2. First of all, draw the position of planes and cranks as shown in Fig. 4.13.1(*a*) and (*b*) respectively. The position of crank A is assumed in the horizontal direction.

3. Tabulate the data as given in the Table. Assume the plane of balancing mass B_1 (*i.e.*, plane 1) as the reference plane.

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Centi. Force $\div \omega^2$ (mr) kg-m (4)	Distance from Plane 1 (l) m (5)	Couple $\div \omega^2$ (mrl) kg-m^2 (6)
A	104	0.3	31.2	– 0.2	– 6.24
1 (RP)	B_1	0.6	0.6 B_1	0	0
B	120	0.3	36	0.8	28.8
2	B_2	0.6	0.6 B_2	1.6	0.96 B_2
C	104	0.3	31.2	1.8	56.16

4. Now draw the couple polygon with the data given in Table (column 6), to some suitable scale, as shown in Fig. 4.13.1(*c*). The closing side $c'o'$ represents the balancing couple and it is proportional to $0.96 B_2$. Therefore, by measurement

$$0.96 B_2 = \text{Vector } c'o' = 55.2 \text{ kg-m}^2 \text{ or } B_2 = 57.5 \text{ kg}$$

5. To determine the angular position of the balancing mass B_2, draw OB_2 parallel to vector $c'o'$ as shown in Fig. 4.13.1(*b*). By measurement,

$$\theta_2 = 24°$$

6. In order to find the balance mass B_1, draw the force polygon with the data given in Table (column 4), to some suitable scale, as shown in Fig. 4.13.1(*d*). The closing side represents the balancing force and it is proportional to $0.6 B_1$. Therefore, by measurement,

$$0.6\,B_1 = \text{Vector } co = 34.5 \text{ kg-m} \quad \text{or} \quad B_1 = 57.5 \text{ kg}$$

To determine the angular position of the balancing mass B_1, draw OB_1 parallel to vector co, as shown in Fig. 4.13.1(b). By measurement,

$$\theta_1 = 215°$$

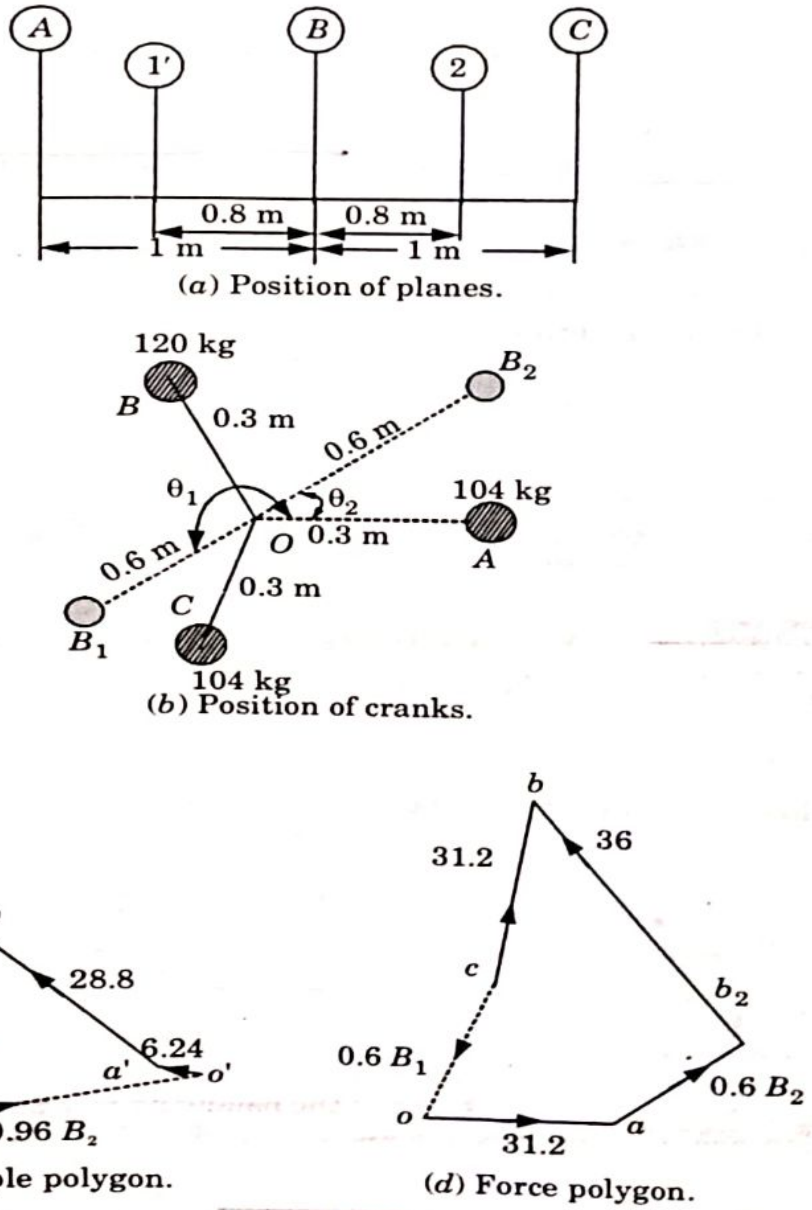


Fig. 4.13.1.

9. We know that, hammer blow per wheel

$$= B_1\omega^2 b_1 = 57.7 \times (37.7)^2 \times 0.6 = 49035 \text{ N.}$$

PART-4

Introduction, Types of Governors, Characteristics of Centrifugal Governors.

CONCEPT OUTLINE

Governor : It is a device which automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

Types of Governors :

1. Centrifugal governors, and
2. Inertia governors.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.14. **Give the classification of governors and discuss the centrifugal governor in brief.**

Answer

A. Classification of Governors :

- Governors
  - Centrifugal governor
    - Pendulum type
      - Watt governor
    - Loaded type
      - Dead weight governor
        - Porter governor
        - Proell governor
      - Spring controlled governor
        - Hartnell governor
        - Hartung governor
        - Wilson-Hartnell governor
        - Pickering governor
  - Inertia governor

B. Centrifugal Governor :

1. It is known as centrifugal governor because it is based on the balancing of centrifugal force on the rotating balls by equal and opposite force (*i.e.*, controlling force).
2. It works on centrifugal effects produced by the rotating balls. As speed of balls increases, the balls tend to rotate at a greater radius from the spindle axis.
3. This results into movement of sleeve to upward direction and this movement is attached to the throttle through a bell crank lever and it closes the throttle valve to required extent.
4. When speed of ball decreases, the balls tend to rotate at a smaller radius and valve is opened according to requirement.

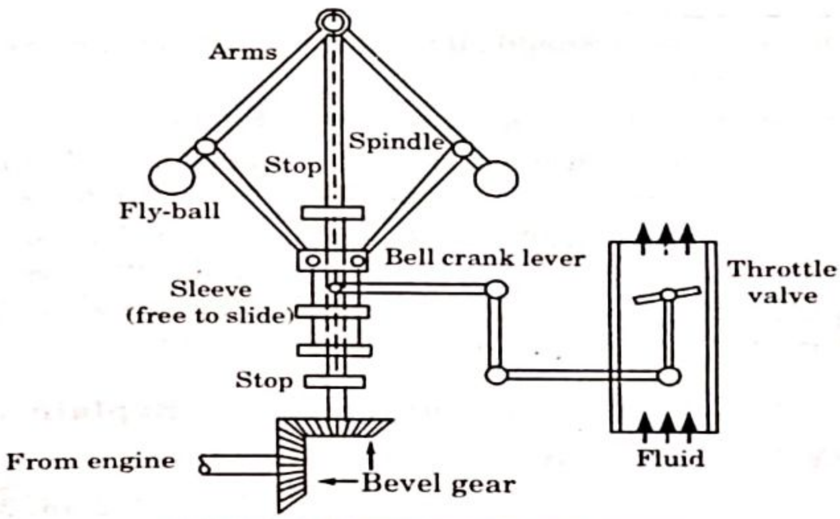


Fig. 4.14.1. Centrifugal governor.

Que 4.15. **Define the following terms related to governor :**

i. Height of a governor,
ii. Equilibrium speed,
iii. Mean equilibrium speed,
iv. Maximum and minimum equilibrium speed, and
v. Sleeve lift.

Answer

i. Height of Governor :

1. It is the perpendicular distance between the centre of the ball and the point where the axes of the arms intersect the spindle axis either in real or virtual (or producing the arms).
2. It is denoted by 'h'.

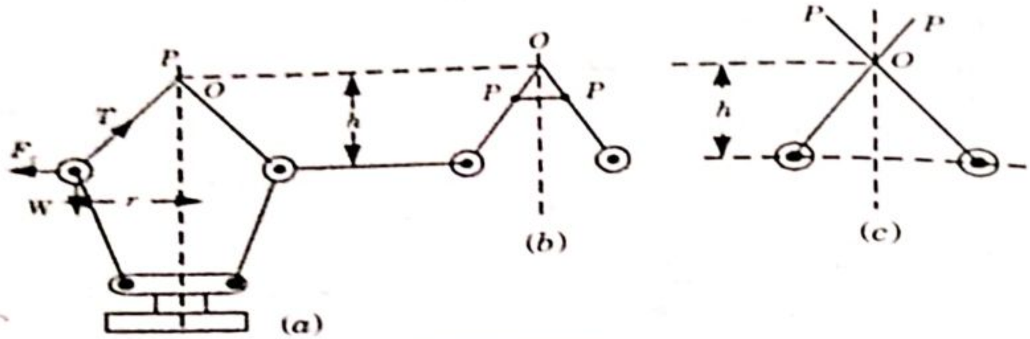


Fig. 4.15.1.

ii. **Equilibrium Speed :** When the governor balls, arms are in equilibrium such that sleeve does not tend to move upwards or downwards, then speed of governor is known as equilibrium speed.

iii. **Mean Equilibrium Speed :** It is the speed at the mean position of the balls or the sleeve.

iv. **Maximum and Minimum Equilibrium Speeds :**

1. When radius of rotation of the balls is maximum and balls do not tend to move either way, speed is called maximum equilibrium speed.
2. When radius of rotation of the balls is minimum and balls do not tend to move either way, speed is called minimum equilibrium speed.

v. **Sleeve Lift :** The vertical distance which is travelled by the sleeve due to change in equilibrium speed is known as sleeve lift.

Que 4.16. **What is function of governor ? Explain following terms : stability, isochronism and sensitiveness.**

AKTU 2013-14, Marks 06

OR

Explain the function of governor with neat sketch. State the different types of governors.

AKTU 2016-17, Marks 10

Answer

A. **Function of Governor :**

1. The function of a governor is to regulate the mean speed of an engine, when there are variations in the load e.g., when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid.
2. On the other hand when the load on the engine decreases, its speed increases and thus less working fluid is required.
3. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

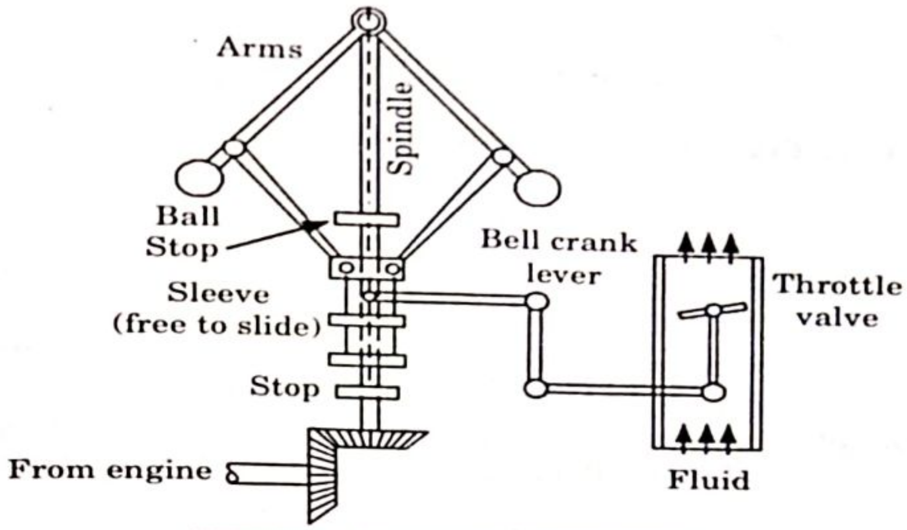


Fig. 4.16.1 Centrifugal governor.

B. Sensitiveness :

1. A governor is called sensitive when it readily responds to a small change of speed.
2. It is defined as the ratio of the difference between maximum and minimum speeds (range of speed) to mean equilibrium speed.
3. If, $N = \dfrac{N_1 + N_2}{2}$ = Mean speed,

   N_1 = Minimum speed corresponding to full load conditions, and

   N_2 = Maximum speed corresponding to no load conditions.

$$\text{Sensitiveness} = \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{(N_2 + N_1)}$$

C. Stability :

1. Stability and sensitivity are two opposite characteristics.
2. A governor is stable when for every speed within the working range, the configuration is constant *i.e.*, only one radius of rotation of the governor balls at which the governor is in equilibrium.
3. For a stable governor, the equilibrium speed increases with the radius of governor balls.
4. A governor is said to be unstable, if the radius of rotation decreases as the speed increases.

D. Isochronism :

1. Isochronous governor is a governor which has a zero range speed. It means the governor has constant equilibrium speed.

2. The isochronism is the stage of infinite sensitivity. Any change of speed results in moving the balls and the sleeve to their extreme positions.
3. However, an isochronous governor is not practical due to presence of friction.

E. Different Types of Governors : Refer Q. 4.14, Page 4–24B, Unit-4.

PART-5

Gravity Controlled and Spring Controlled Centrifugal Governors, Hunting of Centrifugal Governors.

CONCEPT OUTLINE

Hunting of Centrifugal Governors : A governor is said to be hunt if the speed of the engine fluctuates continuously above and below the mean speed.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.17. **What are the types of Watt governor ? Give an expression for height of Watt governor.**

OR

Explain term height of the governor. Derive an expression for the height in the case of a Watt governor. What are the limitations of a Watt governor ?

AKTU 2016-17, Marks 10

Answer

A. Types of Watt Governor :

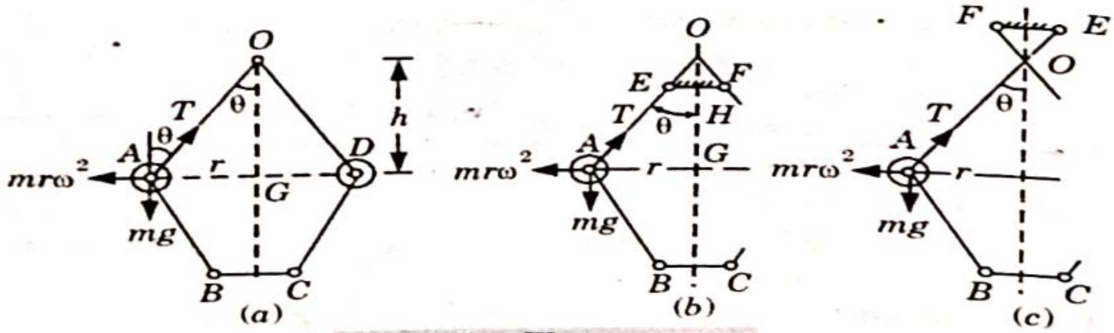


Fig. 4.17.1. Watt governor.

a. **Simple Watt Governor :** In Fig. 4.17.1(a), a simple Watt governor is shown. It has a pair of balls (masses) which is attached to a spindle by links. The upper links are pinned at point O.

b. **Open Arm Type Watt Governor :** The configuration of Watt governor shown in Fig. 4.17.1(b) is of open arm type watt governor. Here upper links are connected by a horizontal link EF.

c. **Crossed Arm Type Watt Governor :** The configuration of Watt governor shown in Fig. 4.17.1(c) is of crossed arm type watt governor. In this, upper links cross the spindle and connected by horizontal link forming a cross arm type configuration.

B. Expression for Height of Governor :

1. Let,
 m = Mass of each ball,
 h = Height of governor,
 w = Weight of each ball (= mg),
 ω = Angular velocity of the balls, arms and the sleeve,
 T = Tension in the arm, and
 r = Radial distance of ball centre from spindle axis.
2. Let us consider the links are massless and no friction is present on the sleeve.
3. Considering the equilibrium of the mass,

$$T \cos \theta = mg \qquad \text{...(4.17.1)}$$

and,

$$T \sin \theta = mr\omega^2 \qquad \text{...(4.17.2)}$$

4. Dividing eq. (4.17.2) by eq. (4.17.1),

$$\tan \theta = \frac{mr\omega^2}{mg}$$

$$\frac{r}{h} = \frac{r\omega^2}{g} \qquad \{\text{From } \Delta OAG, \tan \theta = r/h\}$$

$$h = \frac{g}{\omega^2} = \left(\frac{60}{2\pi}\right)^2 \times \frac{9.81}{N^2} \qquad \left\{\because \omega = \frac{2\pi N}{60}\right\}$$

$$h = \frac{895}{N^2}$$

5. Thus, the height of a Watt governor is inversely proportional to the square of the speed.

C. Limitations of Watt Governor :

1. It is limited to vertical position applications.
2. It is used in very slow speed engine. At higher speed, the sensitivity will decrease.

D. Height of Governor : Refer Q. 4.15, Page 4–25B, Unit-4.

Que 4.18. **Derive the relation between the height of the governor (h) and the angular speed of the balls for a Porter governor.**

Answer

1. Let M = Mass of sleeve,
m = Mass of each ball.
f = Force of friction at the sleeve.
h = Height of the governor.
r = Distance of the centre of each ball from axis of rotation.

2. When motion of sleeve is in upward direction, friction force will act in downward direction and vice-versa.

3. For upward motion of sleeve, the downward force acting on the sleeve is $(Mg + f)$ and for downward motion of sleeve, the downward force acting on the sleeve is $(Mg - f)$.

4. In general, we can take a force as $(Mg \pm f)$ whether the sleeve moves upwards or downwards. All the forces acting on the sleeve and ball are shown in Fig. 4.18.1(*b*).

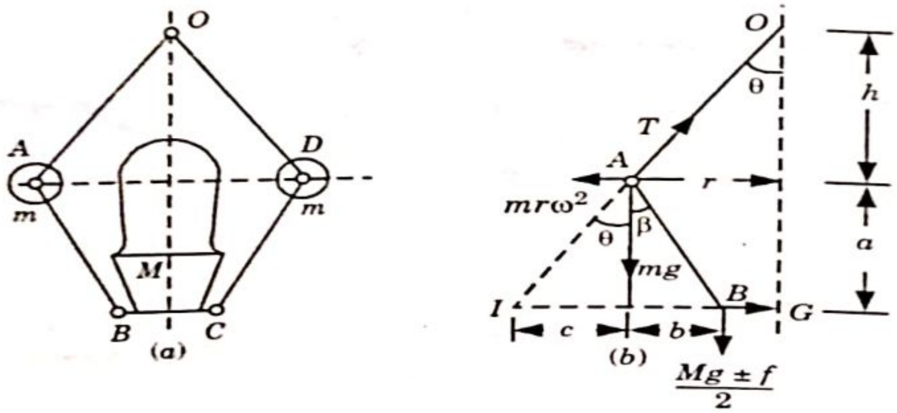


Fig. 4.18.1. Porter governor.

5. We find an instantaneous centre I lies at the point of intersection of OA produced and a line through G perpendicular to spindle axis as shown in Fig. 4.18.1(*b*).

6. Considering the equilibrium of the left half of the governor and taking moments about I, we have

$$mr\omega^2 a = mgc + \left(\frac{Mg \pm f}{2}\right)(c+b)$$

or

$$mr\omega^2 = mg\frac{c}{a} + \left(\frac{Mg \pm f}{2}\right)\left(\frac{c}{a} + \frac{b}{a}\right)$$

$$= mg\tan\theta + \left(\frac{Mg \pm f}{2}\right)(\tan\theta + \tan\beta)$$

($\because$ From Fig. 4.18.1 (*b*), $\tan\theta = c/a$ and $\tan\beta = b/a$)

$$= \tan\theta\left[mg + \frac{Mg \pm f}{2}(1+k)\right] \left\{\text{Let, } k = \frac{\tan\beta}{\tan\theta}\right\}$$

$$= \frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1+k)\right]$$

$$\omega^2 = \frac{1}{mh}\left[\frac{2mg + (Mg \pm f)(1+k)}{2}\right]$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h}\left[\frac{2mg + (Mg \pm f)(1+k)}{2mg}\right]$$

$$N^2 = \frac{895}{h}\left[\frac{2mg + (Mg \pm f)(1+k)}{2mg}\right]$$

8. If $k = 1$, $$N^2 = \frac{895}{h}\left[1 + \frac{Mg \pm f}{mg}\right]$$

9. If $f = 0$, $$N^2 = \frac{895}{h}\left[1 + \frac{M}{2m}(1+k)\right]$$

10. If $f = 0$ and $k = 1$, $$N^2 = \frac{895}{h}\left[\frac{m+M}{m}\right]$$

Que 4.19. **The length of the upper and lower arms of Porter governor are 200 mm and 250 mm respectively. Both the arms are pivoted on the axis of rotation. The central load is 150 N, the weight of the each ball is 20 N and the friction of the sleeve together with the resistance of the operating gear is equivalent to a force of 30 N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30° and 40° taking friction into account. Find the range of speed of tLe governor.**

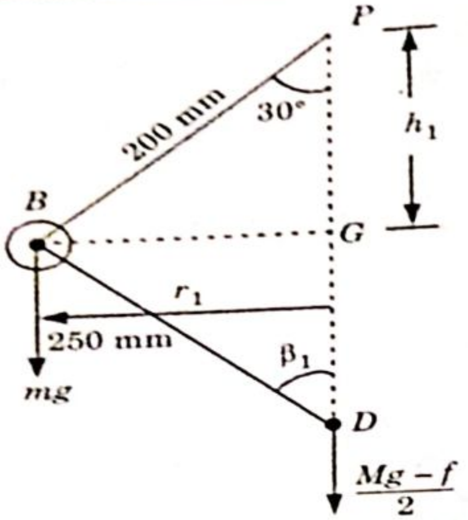
AKTU 2015-16, Marks 15

Answer

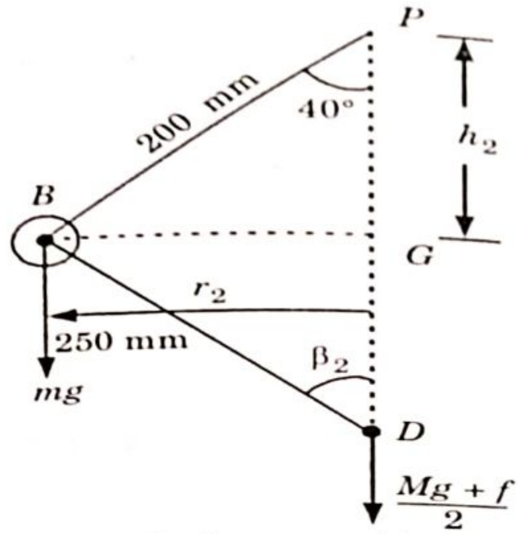
Given : Length of upper arm = 200 mm = 0.2 m,
Length of lower arm = 250 mm = 0.25 m, Mg = 150 N, mg = 20 N,
f = 30 N, $\theta_1 = 30°$, $\theta_2 = 40°$
To Find : Range of speed of governor.

1. From Fig. 4.19.1(a), the minimum radius of rotation of ball,

$$r_1 = BG = BP \sin 30° = 0.2 \sin 30° = 0.1 \text{ m}$$



(a) Minimum position.



(b) Maximum position.

Fig. 4.19.1.

2. Height of governor,

$$h_1 = PG = BP \cos 30° = 0.2 \cos 30° = 0.1732 \text{ m}$$

3. $$DG = \sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1)^2} = 0.23 \text{ m}$$

4. $$\tan \beta_1 = \frac{BG}{DG} = \frac{0.1}{0.23} = 0.4348$$

$$\tan \theta_1 = \tan 30° = 0.5774$$

$$k_1 = \frac{\tan \beta_1}{\tan \theta_1} = \frac{0.4348}{0.5774} = 0.753$$

5. For downward motion of sleeve, the frictional force will act in upward direction. Hence the minimum speed of governor will be,

$$N_1^2 = \frac{mg + \left(\frac{Mg - f}{2}\right)(1 + k_1)}{mg} \times \frac{895}{h_1}$$

$$N_1^2 = \left[\frac{20 + \left(\frac{150 - 30}{2}\right)(1 + 0.753)}{20}\right] \times \frac{895}{0.1732}$$

$$N_1^2 = 6.259 \times \frac{895}{0.1732} = 32342.985$$

$$N_1 = 179.84 \text{ rpm}$$

6. From Fig. 4.19.1(*b*), the maximum radius of rotation,

$$r_2 = BG = BP \sin 40° = 0.2 \times 0.643 = 0.1286 \text{ m}$$

7. Height of governor, $h_2 = PG = BP \cos 40° = 0.2 \times 0.766 = 0.1532$ m

8. $DG = \sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1286)^2} = 0.2144$ m

$$\tan \beta_2 = \frac{BG}{DG} = \frac{0.1286}{0.2144} = 0.599$$

$$\tan \theta_2 = \tan 40° = 0.839$$

$$k_2 = \frac{\tan \beta_2}{\tan \theta_2} = \frac{0.599}{0.839} = 0.714$$

10. For upward motion of sleeve, the frictional force will act in downward direction, hence the maximum speed of governor will be,

$$N_2^2 = \left[\frac{mg + \left(\frac{Mg + f}{2}\right)(1 + k_2)}{mg}\right] \times \frac{895}{h_2}$$

$$N_2^2 = \left[\frac{20 + \left(\frac{150 + 30}{2}\right) \times (1 + 0.714)}{20}\right] \times \frac{895}{0.1532}$$

$$N_2^2 = 8.713 \times \frac{895}{0.1532} = 50901.664$$

$$N_2 = 225.614 \text{ rpm}$$

11. Range of speed $= N_2 - N_1 = 225.614 - 179.84 = 45.774$ rpm

Que 4.20. **In a Porter governor, the upper and lower arms are each 250 mm long and are pivoted on the axis of rotation. The mass of each rotating ball is 3 kg and the mass of the sleeve is 20 kg. The sleeve is in its lowest position when the arms are inclined at 30° to the governor axis. The lift of the sleeve is 36 mm. Find the force of friction at the sleeve, if the speed at the moment it rises from the lowest position is equal to the speed at the moment it falls from the highest position. Also, find the range of speed of the governor.**

AKTU 2017-18, Marks 10

Answer

Given : $m = 3$ kg, $M = 20$ kg, $\theta_1 = 30°$, $x = 36$ mm,
Length of upper and lower arm, $AB = BC = 250$ mm
To Find : i. Force of friction at the sleeve.
ii. Range of speed of the governor.

1. Lift of sleeve is given as 36 mm, therefore ball is lifted by $\frac{x}{2}$. It follows that : $h_2 + \frac{x}{2} = h_1$

Hence, $x = 2(h_1 - h_2) = 36$ mm ...(4.20.1)

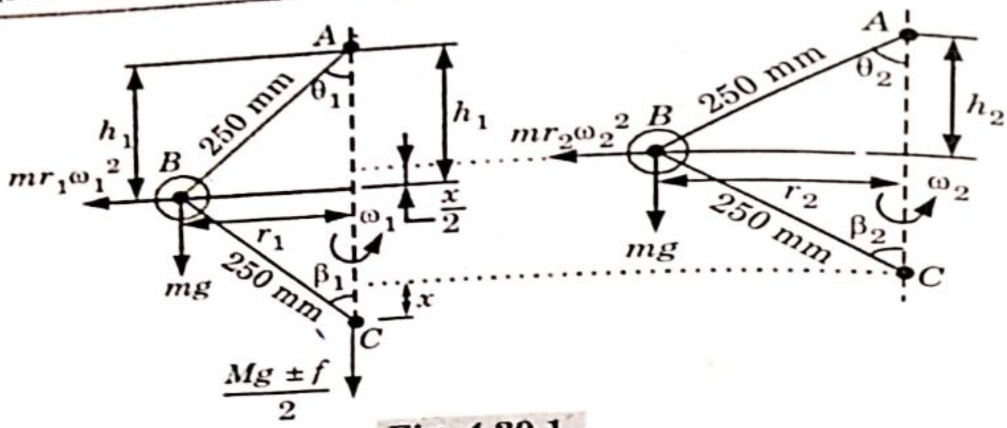


Fig. 4.20.1.

2. We know that,

$$\omega^2 = \frac{g}{h}\left[1 + \left(\frac{Mg \pm f}{2mg}\right)(1 + k)\right]$$

But, as arms are of same length [*i.e.*, $k = 1$]

$$\therefore \quad \omega^2 = \frac{g}{h}\left[1 + \left(\frac{Mg \pm f}{mg}\right)\right] \quad ...(4.20.2)$$

3. Lowermost position with sleeve having tendency to move upwar (*i.e.*, f is positive),

$$\therefore \quad r_1 = 250 \sin 30° = 125 \text{ mm} = 125 \times 10^{-3} \text{ m}$$

$$h_1 = \sqrt{(250)^2 - (125)^2} = 216.51 \text{ mm}$$

$$= 216.51 \times 10^{-3} \text{ m}$$

$$\therefore \quad \omega_1^2 = \frac{g}{h_1}\left[1 + \frac{(Mg + f)}{mg}\right]$$

$$\therefore \quad \omega_1^2 = \frac{9.81}{216.51 \times 10^{-3}}\left[1 + \frac{(20 \times 9.81 + f)}{3 \times 9.81}\right]$$

$$\omega_1^2 = 347.3742 + 1.5395 f \quad ...(4.20.3)$$

4. Uppermost position with sleeve having tendency to move dow (*i.e.*, f is negative),

$$36 = 2(h_1 - h_2) \text{ [From eq. (4.20.1)]}$$

$$36 = 2(216.51 - h_2)$$

$$h_2 = 198.51 \text{ mm} = 198.51 \times 10^{-3} \text{ m}$$

As arms are of same length [*i.e.*, $k = 1$]

$$\omega_2^2 = \frac{g}{h_2}\left[1 + \frac{(Mg - f)}{mg}\right]$$

$$\therefore \quad \omega_2^2 = \frac{9.81}{198.51 \times 10^{-3}}\left[1 + \frac{(20 \times 9.81 - f)}{3 \times 9.81}\right]$$

$$\omega_2^2 = 378.8726 - 1.67917 f \quad ...(4.20.4)$$

5. But, from given data these two speeds are same, *i.e.*,

$$\omega_1^2 = \omega_2^2$$

$\therefore$

$$347.3742 + 1.5395\, f = 378.8726 - 1.67917\, f$$

$$3.21867\, f = 31.4986$$

Friction force, $\quad f = 9.786$ N

6. We know that when the sleeve moves downwards, the frictional force 'f' acts upwards and the minimum equilibrium speed is given by,

$$\omega_1^2 = \frac{g}{h_1}\left[1 + \frac{(Mg - f)}{mg}\right]$$

$$\therefore \quad \omega_1^2 = \frac{9.81}{216.51 \times 10^{-3}}\left[1 + \left(\frac{20 \times 9.81 - 9.786}{3 \times 9.81}\right)\right]$$

$$= 332.3079$$

$$\therefore \quad \omega_1 = 18.2293 \text{ rad/sec} = \frac{2\pi N_1}{60}$$

$$\therefore \quad N_1 = 174.076 \text{ rpm}$$

7. We also know that when the sleeve moves upwards, the frictional force 'f' acts downwards and the maximum equilibrium speed is given by,

$$\omega_2^2 = \frac{g}{h_2}\left[1 + \frac{(Mg + f)}{mg}\right]$$

$$\omega_2^2 = \frac{9.81}{198.51 \times 10^{-3}}\left[1 + \frac{(20 \times 9.81 + 9.786)}{3 \times 9.81}\right]$$

$$= 395.305$$

$$\therefore \quad \omega_2 = 18.8822 \text{ rad/sec} = \frac{2\pi N_2}{60}$$

$$\therefore \quad N_2 = 189.861 \text{ rpm}$$

8. Range of governor $= N_2 - N_1 = 189.861 - 174.076 = 15.785$ rpm

Que 4.21. **A Porter governor has equal arms 100 mm and pivoted on the axis of rotation. The mass of each ball is 4 kg and the mass on the sleeve is 20 kg. The radius of rotation of the balls is 60 mm when the sleeve begins to rise and 80 mm at maximum speed. Determine range of speed.**

AKTU 2013-14, Marks 06

Answer

Given : $BP = BD = 100$ mm $= 0.1$ m, $m = 4$ kg, $M = 20$ kg,
$r_1 = 60$ mm $= 0.06$ m, $r_2 = 80$ mm $= 0.08$ m,
To Find : Range of speed.

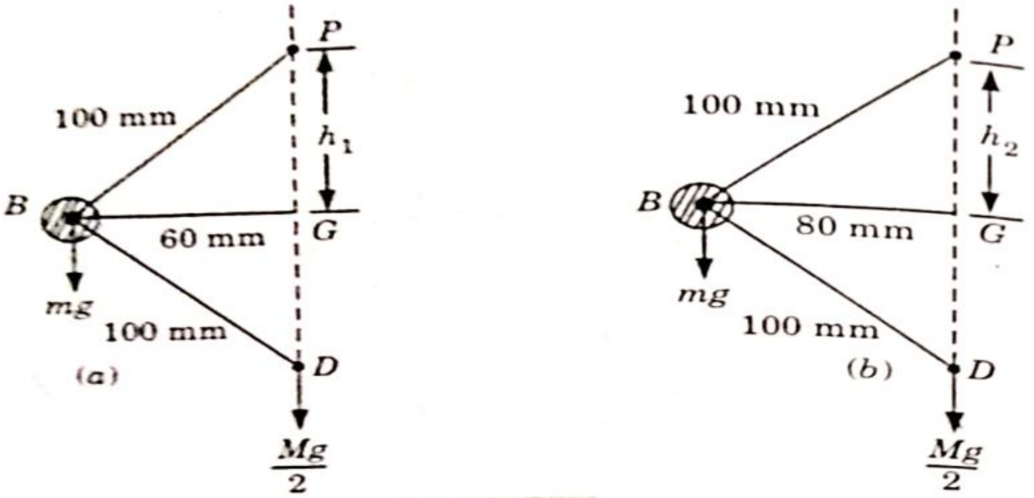


Fig. 4.21.1.

1. From Fig. 4.21.1(*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2}$$
$$= \sqrt{(0.1)^2 - (0.06)^2} = 0.08 \text{ m}$$

2. We know that,

$$(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1}$$
$$= \frac{4+20}{4} \times \frac{895}{0.08} = 67125 \text{ rpm}$$
$$N_1 = 259.08 \text{ rpm}$$

3. From Fig. 4.21.1(*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2}$$
$$= \sqrt{(0.1)^2 - (0.08)^2} = 0.06 \text{ m}$$

4. We know that,

$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{4+20}{4} \times \frac{895}{0.06} = 89500$$
$$N_2 = 299.16 \text{ rpm}$$

5. We know that, range of speed $= N_2 - N_1$

$$= 299.16 - 259.08 = 40.08 \text{ rpm}$$

Que 4.22. **Calculate the range of speed of Porter governor whicl has equal arms of each 200 mm long and pivoted on the axis o rotation. The mass of each ball is 4 kg and the central load of th sleeve is 20 kg. The radius of rotation of the ball is 100 mm when th governor being to lift and 130 mm when the governor is at maximur speed.**

AKTU 2015-16, Marks 10

Answer

Same as Q. 4.21, Page 4–35B, Unit-4.
[**Answer :** Range of speed = 11.78 rpm]

Que 4.23. **A Porter governor has equal arms each 250 mm long and pivoted on the axis rotation. Each ball has a mass of 5 kg and mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when governor is at maximum speed. Find the maximum and minimum speed and range of speed of the governor.**

AKTU 2015-16, Marks 10

Answer

Same as Q. 4.21, Page 4–35B, Unit-4.
[**Note :** Since radius of rotation at minimum speed is not given, so we are assuming, r_2 = 100 mm = 0.10 m]
[**Answer :** N_1 = 153.133 rpm, N_2 = 163.859 rpm,
Range of speed = 10.726 rpm]

Que 4.24. **Derive an expression for the height in the case of a Proell governor.**

AKTU 2016-17, Marks 15

Answer

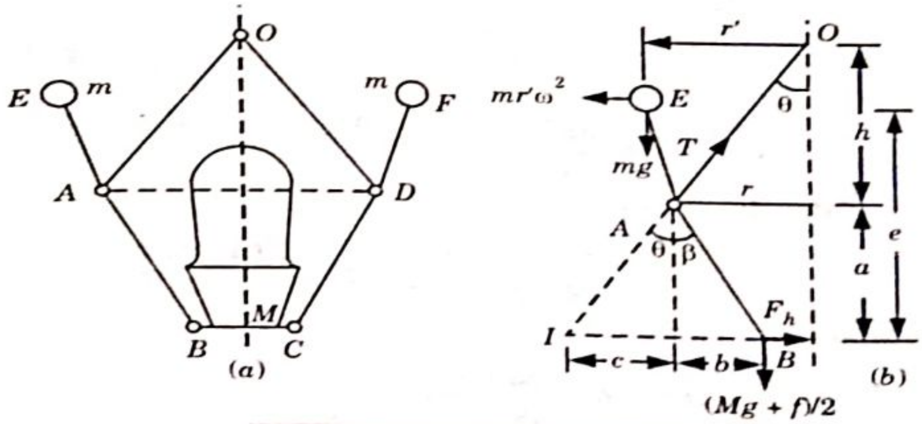


Fig. 4.24.1. Proell governor.

1. Consider the link *BAE* is in equilibrium which is under the action of following forces :
   i. The weight of the ball, mg,
   ii. The centrifugal force, $mr'\omega^2$,
   iii. Tension in the link *AO*,
   iv. Horizontal reaction of sleeve, and

v. Weight of sleeve and friction $\left(i.e., \frac{1}{2}(Mg \pm f)\right)$.

2. Find out I as instantaneous centre and take moment about I,

$$mr'\omega^2 e = mg\,(c + r - r') + \frac{Mg \pm f}{2}\,(c + b)$$

Where b, c, d and r are the dimensions as indicated in the diagram.

$$mr'\omega^2 = \frac{1}{e}\left[mg(c + r - r') + \frac{Mg \pm f}{2}(c + b)\right]$$

3. Considering AE as a vertical link ($i.e.$, $r = r'$) then,

$$mr'\omega^2 = \frac{1}{e}\left[mg\,c + \frac{Mg \pm f}{2}(c + b)\right]$$

$$= \frac{a}{e}\left[mg\,\frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{c}{a} + \frac{b}{a}\right)\right]$$

$$= \frac{a}{e}\left[mg\tan\theta + \frac{Mg \pm f}{2}(\tan\theta + \tan\beta)\right]$$

$$mr'\omega^2 = \frac{a}{e}\tan\theta\left[mg + \frac{Mg \pm f}{2}(1 + k)\right] \qquad \left\{\because k = \frac{\tan\beta}{\tan\theta}\right\}$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{a}{e}\,\frac{g}{h}\left[\frac{2mg + (Mg \pm f)(1 + k)}{2mg}\right]$$

$$\left\{\because \tan\theta = \frac{r}{h} = \frac{r'}{h}\right\}$$

$$N^2 = \frac{895}{h}\,\frac{a}{e}\left[\frac{2mg + (Mg \pm f)(1 + k)}{2mg}\right]$$

4. Height of Proell governor,

$$h = \frac{895}{N^2}\,\frac{a}{e}\left[\frac{2mg + (Mg \pm f)(1 + k)}{2mg}\right]$$

5. Special cases :

i. If $k = 1$, then, $h = \frac{895}{N^2}\,\frac{a}{e}\left[1 + \frac{Mg \pm f}{mg}\right]$

ii. If $f = 0$, then, $h = \frac{895}{N^2}\,\frac{a}{e}\left[1 + \frac{M}{2m}(1 + k)\right]$

iii. If $k = 1, f = 0$, $h = \frac{895}{N^2}\,\frac{a}{e}\left(\frac{m + M}{m}\right)$

Que 4.25. **In a Proell governor the mass of each ball is 8 kg and mass of sleeve is 120 kg. Each arm is 180 mm long. The length of extension of lower arms to which the balls are attached is 80 mm. The distance of pivots of arms from axis of rotation is 30 mm and the radius of rotation of the balls is 160 mm when the arms are inclined at 40° to the axis of rotation. Determine :**

i. **The equilibrium speed.**
ii. **The coefficient of insensitiveness if the friction of the mechanism is equivalent to 30 N.**
iii. **The range of speed when governor is inoperative.**

Answer

Given : m = 8 kg, M = 120 kg, $PF = DF$ = 180 mm, BF = 80 mm, $PQ = DK = HG$ = 30 mm, r = 160 mm, $\alpha = \beta = 40°$, f = 30 N

To Find : i. Equilibrium speed,
ii. Coefficient of insensitiveness,
iii. Range of speed.

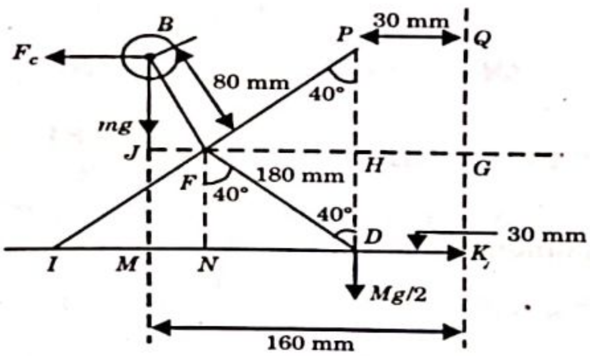


Fig. 4.25.1.

1. From the equilibrium position of the governor

$$PH = PF \cos 40°$$
$$= 180 \cos 40° = 137.89 \text{ mm} \cong 0.138 \text{ m}$$

and,
$$FH = PF \sin 40° = 180 \sin 40° = 115.70 \text{ mm}$$
$$= 0.1157 \text{ m} \simeq 0.116 \text{ m}$$
$$JF = JG - HG - FH = 160 - 30 - 115.7 = 14.3 \text{ mm}$$
$$BJ = \sqrt{(BF)^2 - (JF)^2} = \sqrt{(80)^2 - (14.3)^2}$$
$$= 78.71 \text{ mm}$$

2. We know that,

$$BM = BJ + JM = 78.71 + PH$$
$$= 78.71 + 137.89 \qquad \{\because JM = PH\}$$
$$= 216.6 \text{ mm}$$
$$IM = IN - NM = FH - JF = 115.7 - 14.3$$
$$= 101.4 \text{ mm}$$

and, $ID = IN + ND = 2 \times IN = 2 \times FH$

$= 2 \times 115.70 = 231.4$ mm

3. Now taking moments about the instantaneous centre I

$$F_c \times BM = mg \times IM + \frac{Mg}{2} \times ID$$

$$F_c \times 216.6 = 8 \times 9.81 \times 101.4 + \frac{120 \times 9.81}{2} \times 231.4$$

$$F_c = 665.56 \text{ N}$$

$$mr\omega^2 = 665.56$$

$$8 \times \left(\frac{2\pi N}{60}\right)^2 \times 0.16 = 665.56$$

$$N = 217.75 \text{ rpm}$$

4. The centrifugal force at the minimum speed,

$$F_{c1} = m(\omega')^2 r = 8\left(\frac{2\pi N'}{60}\right)^2 \times 0.16 = 0.0140\, N'^2$$

5. The centrifugal force at the maximum speed,

$$F_c'' = m(\omega'')^2 r = 8\left(\frac{2\pi N''}{60}\right)^2 \times 0.16 = 0.0140\, N''^2$$

6. Taking moments about I when sleeve moves downwards,

$$F_c' \times BM = mg \times IM + \left(\frac{Mg - f}{2}\right) ID$$

$$0.0140 N'^2 \times 216.6 = 8 \times 9.81 \times 101.4 + \left(\frac{120 \times 9.81 - 30}{2}\right) \times 231.4$$

$$N' = 215.39 \text{ rpm}$$

7. Again taking moments about I, when the sleeve moves upwards,

$$F_c'' \times BM = mg \times IM + \left(\frac{Mg + f}{2}\right) ID$$

$$0.0140\, N''^2 \times 216.6 = 8 \times 9.81 \times 101.4 + \left(\frac{120 \times 9.81 + 30}{2}\right) \times 231.4$$

$$N'' = 220.65 \text{ rpm}$$

8. Coefficient of insensitiveness,

$$= \frac{N'' - N'}{N} = \frac{220.65 - 215.39}{217.75} = 0.02415$$

$$= 2.415\ \%$$

9. Range of speed between when the governor is inoperative,

$$N'' - N' = 220.65 - 215.39 = 5.26 \text{ rpm}$$

Que 4.26. **Describe Hartnell type governor with the help of neat sketch. Drive the expression for stiffness of spring in Hartnell governor.**

AKTU 2014-15, Marks 05

Answer

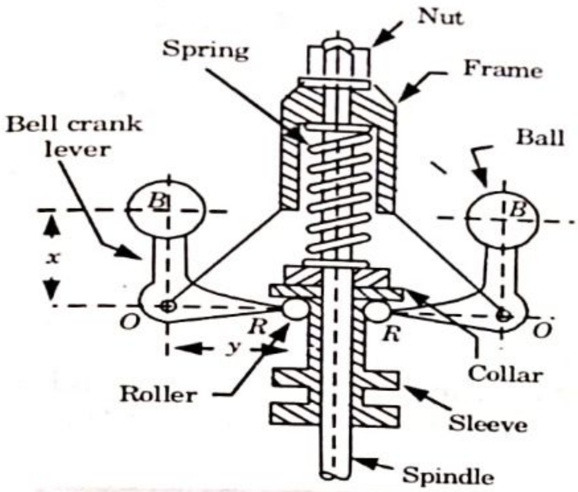


Fig. 4.26.1. Hartnell governor.

Hartnell Governor :

In this type of governor, the balls are controlled by a spring. Initially, the spring is fitted in compression so that a force is applied to the sleeve.

Two bell crank levers, each carrying a mass at one end and a roller at the other are pivoted to the pair of arms which rotate with the spindle. The rollers fit into a groove in the sleeve.

Expression for Stiffness of Spring :

Let,

m = Mass of each ball,

M = Mass of sleeve,

r_1, r_2 = Minimum and maximum radius of rotation,

ω_1, ω_2 = Angular speed of governor at minimum and maximum radius,

S_1, S_2 = Spring force exerted on the sleeve at ω_1 and ω_2,

F_{c1}, F_{c2} = Centrifugal force at ω_1 and ω_2,

s = Stiffness of the spring,

x = Length of vertical or ball arm of the lever,

y = Length of horizontal or sleeve arm of the lever,

r = Distance of fulcrum O from governor axis or radius of rotation when governor is in mid position, and

h = Compression of spring when radius of rotation changes from r_1 to r_2.

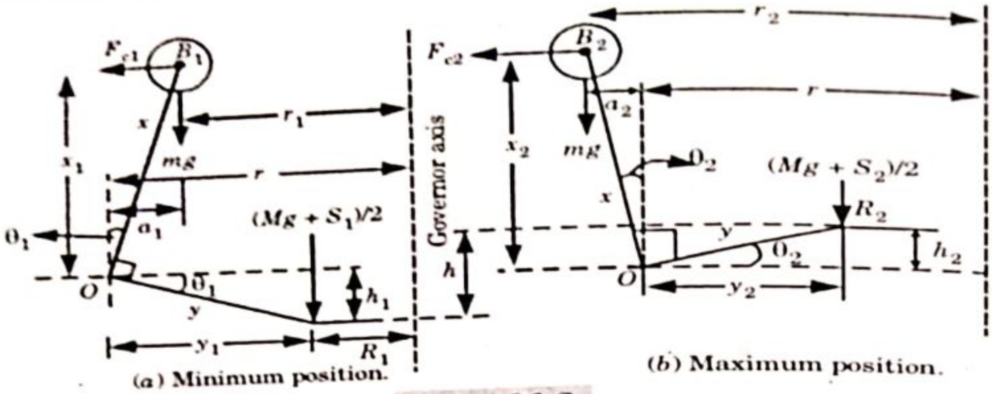


Fig. 4.26.2.

2. When sleeve is in minimum position *i.e.*, the radius of rotation changes from r to r_1, the compression of the spring or the lift of sleeve h_1 is given by

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \quad \text{...(4.26.1)}$$

3. Similarly, when sleeve is in maximum position *i.e.*, the radius of rotation changes from r to r_2, the compression of the spring or lift of sleeve h_2 is given by,

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \quad \text{...(4.26.2)}$$

4. Adding eq. (4.26.1) and eq. (4.26.2), we get

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x}$$

$$h = \left(\frac{r_2 - r_1}{x}\right) y \qquad \{\because \ h = h_1 + h_2\} \quad \text{...(4.26.3)}$$

5. Consider minimum position of sleeve and take moment about point O,

$$\left(\frac{Mg + S_1}{2}\right) y_1 = F_{c1}\, x_1 - mg\, a_1$$

$$Mg + S_1 = \frac{2}{y_1}(F_{c1} x_1 - mga_1) \quad \text{...(4.26.4)}$$

6. Similarly for maximum position of sleeve, taking moment about point O, we have

$$\left(\frac{Mg + S_2}{2}\right) y_2 = F_{c2}\, x_2 + mg\, a_2$$

$$Mg + S_2 = \frac{2}{y_2}(F_{c2}\, x_2 + mg\, a_2) \quad \text{...(4.26.5)}$$

7. Subtracting eq. (4.26.4) from eq. (4.26.5),

$$S_2 - S_1 = \frac{2}{y_2}(F_{c2}\,x_2 + mg\,a_2) - \frac{2}{y_1}(F_{c1}\,x_1 - mg\,a_1)$$

8. We know that, $S_2 - S_1 = hs$

$$\therefore \quad s = \frac{S_2 - S_1}{h} = \left(\frac{S_2 - S_1}{r_2 - r_1}\right)\frac{x}{y} \qquad \left\{\because h = \frac{(r_2 - r_1)y}{x}\right\}$$

9. Neglecting the obliquity effect of the arms and moment due to weight of the balls,

$$x_1 = x_2 = x, \quad y_1 = y_2 = y$$

10. Now for minimum position, we have,

$$\left(\frac{Mg + S_1}{2}\right) y = F_{c1}\,x$$

$$Mg + S_1 = 2F_{c1}\,\frac{x}{y} \qquad \text{...(4.26.6)}$$

11. Similarly, for maximum position,

$$Mg + S_2 = 2F_{c2}\,\frac{x}{y} \qquad \text{...(4.26.7)}$$

12. Subtracting eq. (4.26.6) from eq. (4.26.7),

$$S_2 - S_1 = 2\,(F_{c2} - F_{c1})\,\frac{x}{y}$$

We know that, $S_2 - S_1 = hs$

$$s = \frac{S_2 - S_1}{h} = 2\left(\frac{F_{c2} - F_{c1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2 \qquad \text{...(4.26.8)}$$

13. From the above formula we can find value F_{c2} and F_{c1} and corresponding to which we can find the equilibrium speed using formula

$$F_c = mr\omega^2$$

Que 4.27. **In a spring loaded governor of the Hartnell type, the mass of each ball is 1 kg, length of vertical arm of the bell crank lever is 100 mm and that of the horizontal arm is 50 mm. The distance of fulcrum of each bell crank lever is 80 mm from the axis of rotation of the governor. The extreme radii of rotation of the balls are 75 mm and 112.5 mm. The maximum equilibrium speed is percent greater than the minimum equilibrium speed which is 60 rpm. Find, neglecting obliquity of arms, initial compression of the spring and equilibrium speed corresponding to the radius of rotation of 100 mm.**

AKTU 2017-18, Marks 10

Answer

Given : $m = 1$ kg, $x = 100$ mm $= 0.1$ m, $y = 50$ mm $= 0.05$ m,
$r = 80$ mm $= 0.080$ m, $r_1 = 75$ mm $= 0.075$ m,
$r_2 = 112.5$ mm $= 0.1125$ m, $N_1 = 360$ rpm, $N_2 = 1.05\ N_1 = 1.05 \times 360$ $= 378$ rpm

To Find :
i. Initial compression of the spring.
ii. Equilibrium speed corresponding to the radius of rotation of 100 mm.

1. Centrifugal force at minimum speed,

$$F_{c1} = mr_1\omega_1^2$$

$$\left[\because \omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 360}{60} = 37.7 \text{ rad/s}\right]$$

$$= 1 \times 0.075\ (37.7)^2 = 106.6 \text{ N}$$

2. Centrifugal force at maximum speed,

$$F_{c2} = mr_2\omega_2^2$$

$$= 1 \times 0.1125 \times (39.58)^2 = 176.24 \text{ N}$$

$$\left[\because \omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 378}{60} = 39.58 \text{ rad/s}\right]$$

3. For minimum position,

$$Mg + S_1 = 2\,F_{c1}\ \frac{x}{y}$$

$$S_1 = 2 \times 106.6 \times \frac{0.1}{0.05} \qquad (\because M = 0)$$

$$S_1 = 426.4 \text{ N}$$

4. Similarly for maximum position,

$$Mg + S_2 = 2F_{c2}\ \frac{x}{y} \qquad (\because M = 0)$$

$$S_2 = 2 \times 176.24 \times \frac{0.1}{0.05}$$

$$S_2 = 704.96 \text{ N}$$

5. Lift of the sleeve,

$$h = (r_2 - r_1)\ \frac{y}{x}$$

$$= (0.1125 - 0.075) \times \frac{0.05}{0.1} = 0.01875 \text{ m}$$

6. Stiffness of the spring,

$$s = \frac{S_2 - S_1}{h}$$

$$= \frac{704.96 - 426.4}{0.01875} = 14856.533 \text{ N/m}$$

7. Initial compression of spring $= \dfrac{S_1}{s}$

$$= \frac{426.4}{14856.533} = 0.0287 \text{ m} = 28.7 \text{ mm}$$

8. As obliquity of the arms is neglected, therefore the centrifugal force at any instant,

$$F_c = F_{c1} + (F_{c2} - F_{c1}) \left(\frac{r - r_1}{r_2 - r_1}\right)$$

$$= 106.6 + (176.24 - 106.6) \left(\frac{0.1 - 0.075}{0.1125 - 0.075}\right)$$

$$= 153 \text{ N}$$

9. Centrifugal force, $F_c = mr\omega^2$

$$153 = 1 \times 0.1 \left(\frac{2\pi N}{60}\right)^2$$

$$153 = 1.0966 \times 10^{-3} N^2$$

$$N = 373.53 \text{ rpm}$$

Que 4.28. **In a spring controlled governor, the controlling force curve is a straight line. The balls are 400 mm apart when the controlling force is 1500 N and 240 mm when it is 800 N. The mass of each ball is 10 kg. Determine the speed at which the governor runs when the balls are 300 mm apart. By how much should the initial tension be increased to make the governor isochronous ? Also, find the isochronous speed.**

AKTU 2013-14, Marks 06

Answer

Given : $2y_1 = 400$ mm, $r_2 = 200$ mm, $F_{c2} = 1500$ N, $2y_2 = 240$ mm, $r_1 = 120$ mm, $F_{c1} = 800$ N, $m = 10$ kg, $r = 150$ mm $= 0.15$ m

To Find :
i. Speed of governor when balls are 300 mm apart.
ii. Initial tension in the spring for isochronism.
iii. Isochronous speed.

1. We know that for the stability of the spring controlled governors, the controlling force (F_c) is expressed in the form,

$$F_c = ar - b \qquad ...(4.28.1)$$

When $r = r_1 = 120$ mm $= 0.12$ m,

$$F_{c1} = ar_1 - b$$

$$800 = 0.12\,a - b \qquad ...(4.28.2)$$

When $r = r_2 = 200$ mm = 0.2 m,

$$1500 = 0.2\,a - b \quad \text{...(4.28.3)}$$

2. From eq. (4.28.2) and eq. (4.28.3), we get

$$a = 8750 \quad \text{and} \quad b = 250$$

3. Now eq. (4.28.1) may be written as,

$$F_c = 8750\,r - 250 \quad \text{...(4.28.4)}$$

$$F_c = 8750 \times 0.15 - 250 \quad (\because \quad R = 0.15)$$

$$F_c = 1062.5 \text{ N}$$

4. We know that,

$$F_c = m\,\omega^2 r = m\left(\frac{2\pi N}{60}\right)^2 r$$

$$1062.5 = 10 \times \left(\frac{2\pi}{60}\right)^2 N^2 \times 0.15$$

$$N^2 = 64592.25$$

$$N = 254.15 \text{ rpm}$$

5. We know that for an isochronous governor, the controlling force line passes through the origin (*i.e.*, $b = 0$). This is possible only by increasing the initial tension of the spring to 250 N.

$\therefore$ Initial tension in the spring for isochronism = 250 N

6. We know that for isochronism,

$$F_{c'} = ar \text{ or } m\omega'^2 r = ar$$

$$m\omega'^2 = a$$

$$m\left(\frac{2\pi N'}{60}\right)^2 = a$$

$$10 \times 0.0109\,(N')^2 = 8750$$

$$(N')^2 = 80275.23$$

$$N' = 283.33 \text{ rpm}$$

PART-6

Inertia Governors.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.29. **Discuss about inertia governor and also derive expression for moment of inertia for it.**

Answer

A. Inertia Governor :

1. It is known as inertia governor because balls change their position due to centrifugal forces and forces are set up by an angular acceleration or deceleration of the given spindle (*i.e.*, inertia effect).

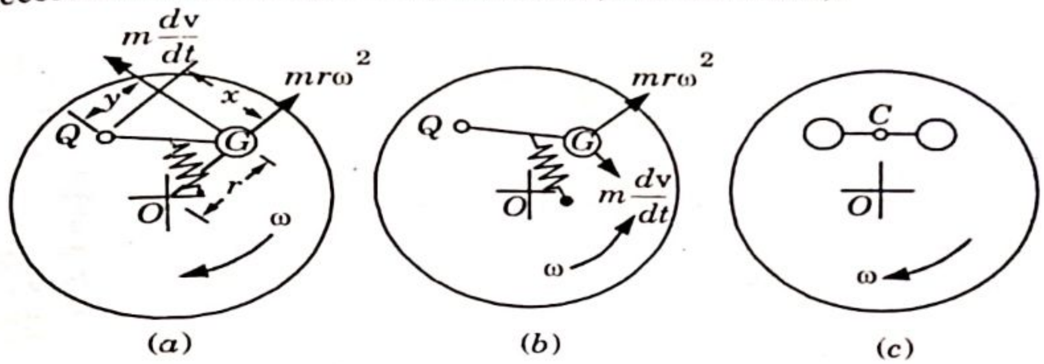


Fig. 4.29.1. Inertia governor.

2. In the Fig. 4.29.1, there is a rotating disc on which a mass m, with its centre at G, is fixed to an arm QG.
3. Point Q is pivoted on rotating disc on the engine shaft. Point O is the centre of rotation and point Q, G and O are not to be collinear.
4. Arm QG is connected to an eccentric which operates the fuel supply valve. When arm moves relative to disc, it results in shifting of eccentric's position which changes the fuel supply accordingly.

B. Expression for Moment of Inertia :

1. Let,
   r = Radial distance, OG
   ω = Angular velocity of disc, and
   v = Tangential velocity at $G = r\omega$
2. Due to rotation, centrifugal force acts radially outward on the mass G.
   Centrifugal force on rotating mass, $F_c = mr\omega^2$ (radially outward)
3. Acceleration of engine shaft, due to increase in speed and acceleration of rotating mass does not equal due to its inertia. So, the inertia force

$$F_i = mf = m\frac{dv}{dt}$$

4. Now, moment of centrifugal force F_c about Q,

$$= mr\omega^2 x \qquad \text{(counter clockwise)}$$

5. Moment of inertia force F_i about Q,

$$= m\frac{dv}{dt}y \qquad \text{(counter clockwise)}$$

6. The above discussion clears that moments due to F_c and F_i are acting in same direction which makes the response of inertia governors faster than that of centrifugal types.
7. Inertia governor acts as centrifugal governor when the acceleration or deceleration is very small because it makes inertia force zero (practically).

PART-7

Effort and Power of Governor.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 4.30. **Discuss and derive the effort and power for Porter governor.**

AKTU 2014-15, Marks 05

OR

Explain the terms and derive expressions for 'effort' and 'power' of a Porter governor.

AKTU 2017-18, Marks 10

Answer

A. Effort of a Governor :

1. The effort of the governor is the mean force acting on the sleeve to raise or lower it for a given change of speed. At constant speed, the governor is in equilibrium and the resultant force acting on the sleeve is zero.
2. However, when the speed of the governor increases or decreases, a force is exerted on the sleeve which tends to move it. When the sleeve occupies a new steady position, the resultant force acting on it again becomes zero.
3. If the force acting at the sleeve changes gradually from zero (when the governor is in the equilibrium position) to a value E for an increased speed of the governor, the mean force or the effort is $E/2$.
4. For a Porter governor, the height is given by

$$h = \frac{g}{\omega^2} + \frac{Mg(1+k)}{2m\omega^2} = \frac{2mg + Mg(1+k)}{2m\omega^2} \quad \text{...(4.30.1)}$$

5. Let ω be increased by c times ω where c is a factor and E be the force applied on the sleeve to prevent it from moving. Thus, the force on the sleeve is increased to $(Mg + E)$. Then

$$h = \frac{2mg + (Mg + E)(1 + k)}{2m(1 + c)^2\omega^2} \qquad ...(4.30.2)$$

6. Dividing eq. (4.30.2) by eq. (4.30.1), we have

$$\frac{2mg + (Mg + E)(1 + k)}{2mg + Mg(1 + k)} = \frac{(1 + c)^2}{1}$$

Subtracting 1 from both sides and solving,

$$\frac{[2mg + (Mg + E)(1 + k)] - [2mg + Mg(1 + k)]}{2mg + Mg(1 + k)} = \frac{1 + c^2 + 2c - 1}{1}$$

$$\frac{E(1 + k)}{2mg + Mg(1 + k)} = 2c$$

[$\because$ c^2 being a small quantity is usually neglected]

$$E = \frac{2c}{(1 + k)}[2mg + Mg(1 + k)]$$

7. Effort, $$\frac{E}{2} = \frac{cg}{1 + k}[2m + M(1 + k)]$$

B. Power of a Governor :

1. The power of a governor is the work done at the sleeve for a given percentage change of speed, *i.e.*, it is the product of the effort and the displacement of the sleeve.
2. For a Porter governor, having all equal arms which intersect on the axis or pivoted at points equidistant from the spindle axis,

$$\text{Power} = \frac{E}{2}(2 \times \text{Height of governor})$$

3. The height of the governor changes from h to h_1 when the speed changes from ω to $(1 + c)\omega$,

$$\therefore \qquad h = \frac{2m + Mg(1 + k)}{2m\omega^2} \text{ and } h_1 = \frac{2m + Mg(1 + k)}{2m(1 + c)^2\omega^2}$$

or $$\frac{h_1}{h} = \frac{1}{(1 + c)^2}$$

4. Displacement of sleeve = $2(h - h_1)$

$$= 2h\left(1 - \frac{h_1}{h}\right)$$

$$= 2h\left(1 - \frac{1}{(1 + c)^2}\right)$$

$$= 2h\left(1 - \frac{1}{1 + 2c}\right) \qquad [\because \text{ Neglecting } c^2]$$

$$= 2h\left(\frac{2c}{1+2c}\right)$$

5. $$\text{Power} = (m + M)cg \times 2h\left(\frac{2c}{1+2c}\right)$$

$$= (m + M)\,gh\left(\frac{4c^2}{1+2c}\right)$$

6. In case $k \neq 1$, displacement of sleeve,

$$= (1 + k)\,(h - h_1) = (1 + k)h\,\left(\frac{2c}{1+2c}\right)$$

$\therefore$ $$\text{Power} = \frac{cg}{1+k}[2m + M(1+k)]\,(1 + k)h\left(\frac{2c}{1+2c}\right)$$

$$= \left[m + \frac{M}{2}(1+k)\right]gh\left(\frac{4c^2}{1+2c}\right)$$

Que 4.31. **The upper arms of a Porter governor have lengths 350 mm and are pivoted on the axis of rotation. The lower arms have lengths 300 mm and are attached to the sleeve at a distance of 40 mm from the axis. Each ball has a mass of 4 kg and mass on the sleeve is 45 kg. Determine the equilibrium speed for a radius of rotation of 200 mm and find also the effort and power of the governor for 1 percent speed change.** **AKTU 2014-15, Marks 05**

Answer

Given : OA = 350 mm, AB = 300 mm, BG = 40 mm, m = 4 kg, M = 45 kg, r = 200 mm, c = 0.01.

To Find :
i. Equilibrium speed.
ii. Effort of governor.
iii. Power of governor.

1. From Fig. 4.31.1 $\tan\theta = \dfrac{r}{h} = \dfrac{200}{\sqrt{350^2 - 200^2}} = 0.6963$

and, $$\tan\beta = \frac{(200 - 40)}{\sqrt{(300)^2 - (200 - 40)^2}} = 0.6305$$

$\therefore$ $$k = \frac{\tan\beta}{\tan\theta} = \frac{0.6305}{0.6963} = 0.9055$$

2. We know that,

$$mr\omega^2 = \tan\theta\left[mg + \frac{Mg \pm f}{2}(1 + k)\right]$$

$$4 \times 0.2 \times \omega^2 = 0.6963 \left[4 \times 9.81 + \frac{45 \times 9.81}{2}(1 + 0.9055)\right]$$

[$\because$ $f = 0$]

$$0.8\,\omega^2 = 0.6963[39.24 + 420.59]$$
$$0.8\,\omega^2 = 320.179$$

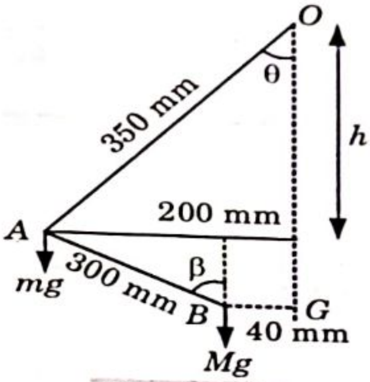


Fig. 4.31.1.

$$\omega = 20 = \frac{2\pi N}{60}$$

$$N = \frac{20 \times 60}{2\pi} = 190.98 \approx 191 \text{ rpm}$$

3. **Effort of governor,**

$$= \frac{cg}{1+k}\,[2m + M(1 + k)]$$
$$= \frac{0.01 \times 9.81}{1 + 0.9055}\,[2 \times 4 + 45\,(1 + 0.9055)]$$
$$= 0.0515\,(8 + 85.7475)$$
$$= 4.828 \text{ N}$$

4. **Power of governor,**

$$= \left[m + \frac{M}{2}(1 + k)\right] gh \left(\frac{4c^2}{1 + 2c}\right)$$
$$= \left[4 + \frac{45}{2}(1 + 0.9055)\right]\, 9.81 \times 0.287 \left(\frac{4 \times 0.01^2}{1 + 2 \times 0.01}\right)$$

$$[\because\ h = \sqrt{(350)^2 - (200)^2} = 287.23 \text{ mm} = 0.287 \text{ m}]$$

$$= 46.871 \times 9.81 \times 0.253 \times 0.000392$$
$$= 0.0517 \text{ N-m} = 51.7 \text{ N-mm}$$

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of the lubricant. This is known as film friction or viscous friction.

Que 5.2. **Explain :**

a. **Laws of static friction.**
b. **Laws of kinetic friction.**
c. **Laws of solid friction.**
d. **Laws of fluid friction.**

Answer

a. **Laws of Static Friction :**

1. The magnitude of the friction force is equal to the force and direction opposite to that force, which tends the body to move.
2. The magnitude of limiting friction (F) has a constant ratio with the normal reaction (N) between the two surfaces.

$$F/N = \text{Constant}$$

3. The friction force depends upon the roughness of the surfaces.
4. The friction force is independent of the area of contact between the two surfaces.

b. **Laws of Kinetic Friction :**

1. The friction force always acts in the direction opposite to that in which body is moving.
2. The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant but it decreases slightly with increase of speed.

c. **Laws of Solid Friction :**

1. The force of friction is directly proportional to the normal load between the surfaces.
2. The friction-force does not depend upon the area of the contact surface.
3. Friction depends upon the material of which the contact surfaces are made.

4. Friction does not depend upon the sliding velocity of one body relative to the other.

d. **Laws of Fluid Friction :**

1. The friction force is almost independent of the load.
2. With increase of the temperature of the lubricant, the friction force reduces.
3. Friction does not depend upon the substance of the materials.
4. Friction differs from one lubricant to other.

Que 5.3. **Explain the following :**

i. **Limiting friction,**
ii. **Co-efficient of friction,**
iii. **Angle of friction,**
iv. **Angle of repose, and**
v. **Cone of friction.**

Answer

i. **Limiting Friction :** It is the maximum value of friction between two surface after which motion of body occur.

ii. **Co-efficient of Friction :**

1. It is defined as the ratio of limiting friction force to the normal reaction.
2. Mathematically, coefficient of friction,

$$\mu = \frac{F}{N}$$

iii. **Angle of Friction :** This is the angle between the resultant reaction and normal to the plane on which motion of the body is impending.

iv. **Angle of Repose :**

1. Limiting angle upto which the body remains at rest or just about to move in absence of external forces is called angle of repose.
2. The body will begin to move down the plane when the angle of inclination of the plane is equal to the angle of friction.

v. **Cone of Friction :**

1. In Fig. 5.3.1, let ON represent normal reaction offered by a surface on the body and R is the resultant reaction of friction force and normal reaction. Let ϕ be the angle of friction.
2. Suppose the body tends to move in the direction of force, resultant reaction will make the same angle ϕ with the normal reaction.
3. Therefore, when the limiting friction is offered, the line of action of resultant reaction should always lie on the surface of an inverted right circular cone whose semi vertex angle is ϕ. This cone generated is called "cone of friction".

4. In other words, if we describe a right circular cone of apex angle about the line of action N as its axis, the cone produced is known "cone of friction."

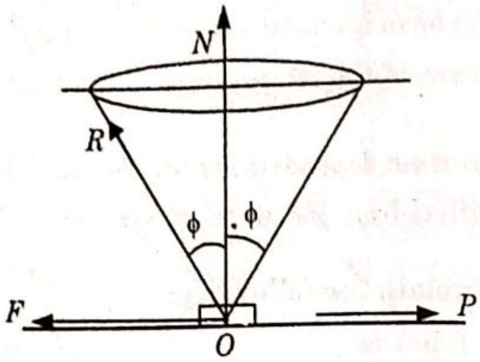


Fig. 5.3.1.

Que 5.4. **Show that the minimum force required to slide a bo on a rough horizontal plane will be $W \sin \theta$.**

Where, **W = Weight of the body.**

θ = Direction of the force from horizontal.

Answer

1. Consider a body A of weight W rests on a horizontal plane. Let a force acts at angle θ to the body and body A just moves.

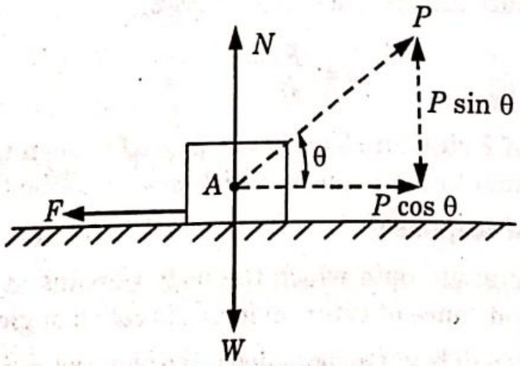


Fig. 5.4.1.

2. Resolving the force P and using condition of equilibrium, we have

i. $$\Sigma F_x = 0$$

$$F = P \cos \theta$$

$$P \cos \theta = \mu N \quad \text{...(5.4.1)}$$

ii. $$\Sigma F_y = 0$$

$$W = N + P \sin \theta$$

$$N = W - P \sin \theta \quad \text{...(5.4.2)}$$

3. From eq. (5.4.1) and eq. (5.4.2), we get

$$P\cos\theta = \mu(W - P\sin\theta)$$

$$P = \frac{\mu W}{\cos\theta + \mu\sin\theta}$$

$$P = \frac{W}{\frac{1}{\mu}\cos\theta + \sin\theta} \qquad \{\because \mu = \tan\phi\}$$

$$P = \frac{W\sin\phi}{\cos\theta\cos\phi + \sin\theta\sin\phi}$$

$$P = \frac{W\sin\phi}{\cos(\theta - \phi)} \qquad ...(5.4.3)$$

4. For minimum value of P, $\cos(\theta - \phi)$ of the eq. (5.4.3) should be maximum.

$$\cos(\theta - \phi) = 1$$

or

$$\theta = \phi$$

So, minimum force (or effort) required to slide a body on a rough horizontal plane is,

$$P_{min} = W\sin\phi$$

PART-2

Types of Lubrication.

CONCEPT OUTLINE

Objectives of Lubrication :

1. To reduce friction between the parts having relative motion.
2. To reduce wear of the moving part.
3. To cool the surface by carrying away heat generated due to friction.
4. To seal a space adjoining the surfaces.
5. To absorb shocks between bearings and other parts and consequently reduce noise.
6. To remove dirt and grit that might have crept between the rubbing parts.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.5. What do you mean by lubrication and explain its

Answer

A. **Lubrication :** It is the admittance of lubricating oil between two surfac having relative motion in order to reduce friction between them and t ensure smooth running.

B. **Types of Lubrication :**

a. **Hydrodynamic Lubrication :**

1. When the block is moved over the surface, a wedge-shaped oil film [Fig. 5.5.1(*b*)] is built-up between the moving block and the surface
2. This wedge shaped film is thicker at the leading edge than at the rear.
3. In other words, the moving block acts as a pump to force oil into clearance that narrows down progressively as the block moves.
4. This generates appreciable oil film pressure which carries the load
5. This type of lubrication where a wedge shaped oil film is formed between two moving surfaces is called hydrodynamic lubrication.

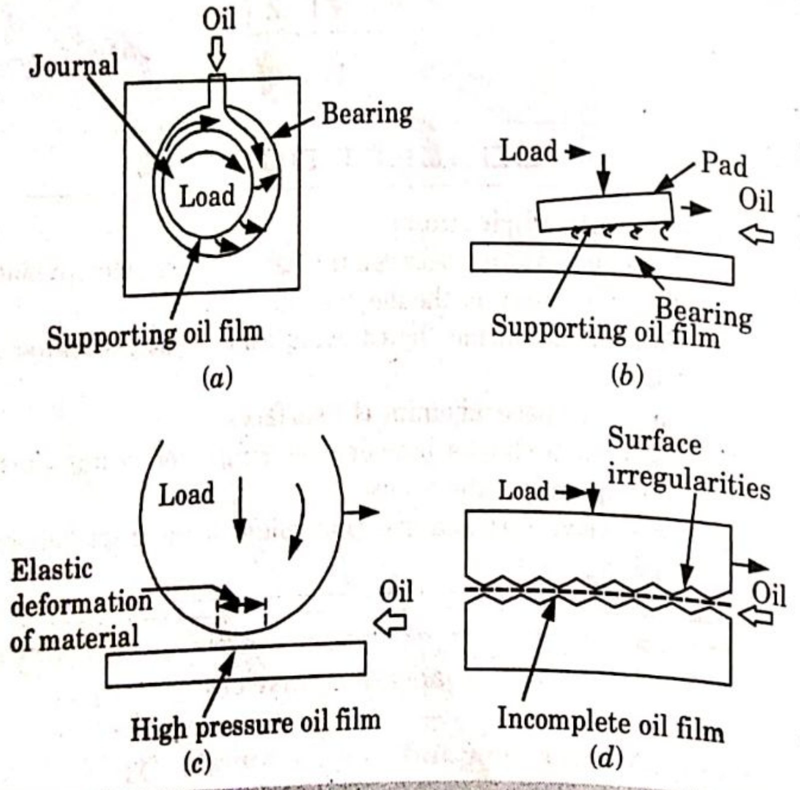


Fig. 5.5.1. (*a*) Action of lubrication in the bearing, (*b*) Hydrodynamic lubrication, (*c*) Elastohydrodynamic lubrication, (*d*) Boundary lubrication.

b. **Elastohydrodynamic Lubrication :**

1. When the load acting on the bearings is very high, the materia itself deforms elastically [Fig. 5.5.1(*c*)] against the pressure built up

of the oil film. This type of lubrication called elastohydrodynamic lubrication.

2. It occurs between cams and followers, gear teeth and rolling bearings where the contact pressures are extremely high.

c. **Boundary Lubrication :**

1. If the film thickness between the two surfaces in relative motion becomes so thin that formation of hydrodynamic oil film is not possible and the surface high spots or asperities penetrate this thin film to make metal-to-metal contact then such lubrication is called boundary lubrication [Fig. 5.5.1(*d*)].
2. Such a situation may arise due to too high load, too thin an oil or insufficient supply of oil due to low speed of movement.
3. Most of the wear associated with friction occurs during boundary lubrication due to metal-to-metal contact.
4. A condition of boundary lubrication always exists when the engine is first started.

d. **Hydrostatic Lubrication:** In this lubrication, a thin oil film resists its instantaneous squeezing out under reversal of loads with relative slow motions.

PART-3

Types of Brakes.

CONCEPT OUTLINE

Brake : It is an appliance used to apply frictional resistance to a moving body to stop or retard it by absorbing its kinetic energy.

Types of Brake :

1. Block or shoe brake,
2. Band brake,
3. Band and block brake, and
4. Internal expanding shoe brake.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.6. **What do you understand by brake ? Discuss its types.**

Answer

A. Brake :

1. A brake is an appliance used to apply frictional resistance to a movir body to stop or retard it by absorbing its kinetic energy.
2. The absorbed kinetic energy is dissipated in form of heat to tl surrounding atmosphere.
3. Brake provides an artificial frictional resistance to the moving body.

B. Types of Brake : The following are the different types of brakes :

1. Hydraulic brake
2. Electric brake
3. Mechanical brake : These can be further classified as follows :
   a. Block or shoe brake,
   b. Band brake,
   c. Band and block brake, and
   d. Internal expanding shoe brake.

Que 5.7. **Explain single block or shoe brake with neat sketcl Also derive the expression for braking torque for it.**

Answer

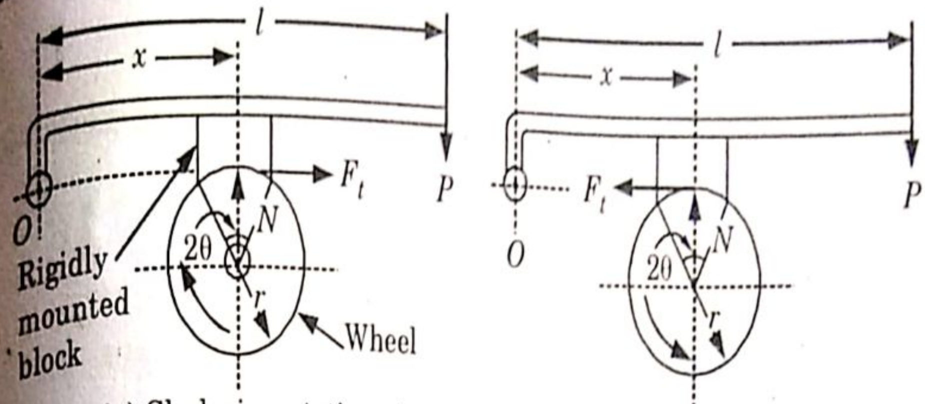
A. Single Block or Shoe Brake :

1. It consists of a block or shoe which is pressed against the rim of revolving brake wheel drum (Fig. 5.7.1).
2. The friction between the block and the wheel causes a tangential brakir force to act on the wheel, which retard the rotation of the wheel.

B. Expression for Braking Torque :

1. Let, P = Force applied at the end of the lever,
   N = Normal force pressing the brake block on tl wheel,
   r = Radius of the wheel,
   2θ = Angle of contact surface of the block,
   μ = Co-efficient of friction, and
   F_t = Tangential braking force.
2. The force P is applied at one end of lever to press the block against tl wheel. The other end of the lever is pivoted on a fixed fulcrum O.
3. For the contact angle less than 60°, it may be assumed that the norm pressure between the block and the wheel is uniform.
4. Tangential braking force on the wheel,

Case I : ...ngential Braking Force (F_t) Passes through the Fulcrum O of the Lever :



(a) Clockwise rotation of brake wheel

(b) Anticlockwise rotation of brake wheel

Fig. 5.7.1. Single block brake.

1. For equilibrium, taking moment about O when the brake wheel rotates clockwise,

$$Nx = Pl$$

$$N = \frac{Pl}{x}$$

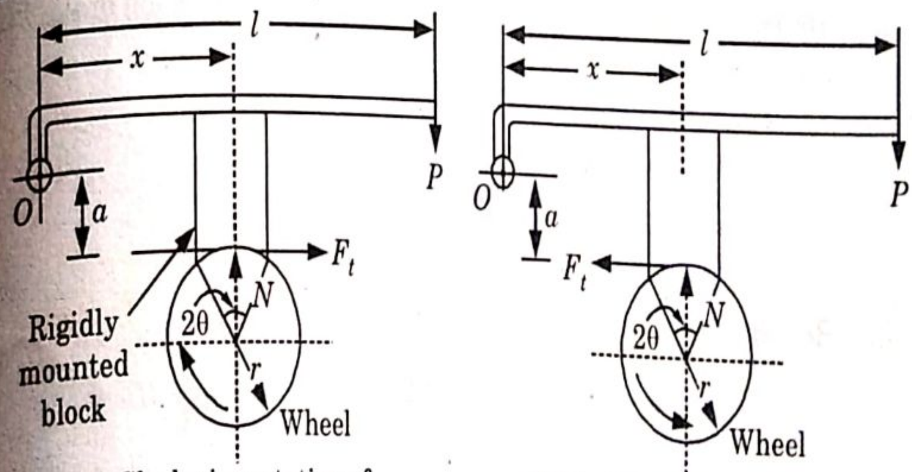
2. Braking torque,

$$T_B = \mu Nr = \mu \frac{Pl}{x} r$$

$$T_B = \frac{\mu Plr}{x}$$

3. The braking torque will be same when wheel rotates anticlockwise.

Case II : Line of Action of F_t Passes through a Distance 'a' below the Fulcrum O :



(a) Clockwise rotation of brake wheel

(b) Anticlockwise rotation of brake wheel

Fig. 5.7.2. Single block brake.

1. Taking moments about the fulcrum O, when the brake wheel rotates in clockwise direction,

$$Nx + F_t a = Pl$$

$$N = \frac{Pl}{x + \mu a} \qquad \{\because F_t = \mu N\}$$

2. Braking torque,

$$T_B = \mu N r = \frac{\mu P l r}{x + \mu a}$$

3. For equilibrium, taking moments about fulcrum O, when if wheel rotates in anticlockwise direction then,

$$Nx = Pl + F_t a$$

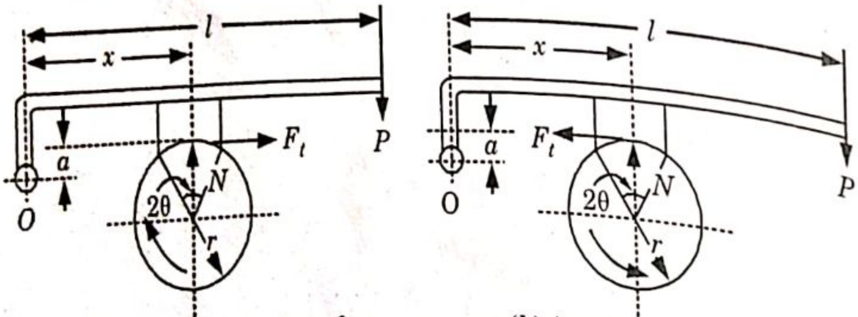
$$Nx = Pl + \mu N a$$

$$N = \frac{Pl}{x - \mu a}$$

4. Braking torque,

$$T_B = \mu N r = \frac{\mu P l r}{x - \mu a}$$

Case III : Line of Action of F_t Passes through a Distance 'a' above the Fulcrum O :



(*a*) Clockwise rotation of brake wheel

(*b*) Anticlockwise rotation of brake wheel

Fig. 5.7.3. Single block brake.

1. For equilibrium, taking moment about the fulcrum O when brake wheel rotates in clockwise direction,

$$Nx = Pl + F_t a$$

$$Nx = Pl + \mu N a$$

$$N = \frac{Pl}{(x - \mu a)}$$

2. Braking torque,

$$T_B = \mu N = \frac{\mu P l r}{(x - \mu a)}$$

3. Similarly, for anticlockwise rotation,

$$N = \frac{Pl}{x + \mu a}$$

4. Braking torque,

$$T_B = \frac{\mu P l r}{x + \mu a}$$

Que 5.8. **What are the drawbacks of single block or shoe brake and how it can be resolved ?**

OR

Discuss about double block or shoe brake.

Answer

1. When a single block or shoe brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to heavy normal force and produces bending of the shaft.
2. This drawback can be overcome by use of double block or shoe brake.
3. Fig. 5.8.1 shows a double block or shoe brake.
4. A double block brake consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate the unbalanced force on the shaft.
5. The brake is connected to a spring and spring pulls the upper ends of the brake arms together.
6. When force P is applied, spring gets compressed and brake is released and vice-versa.

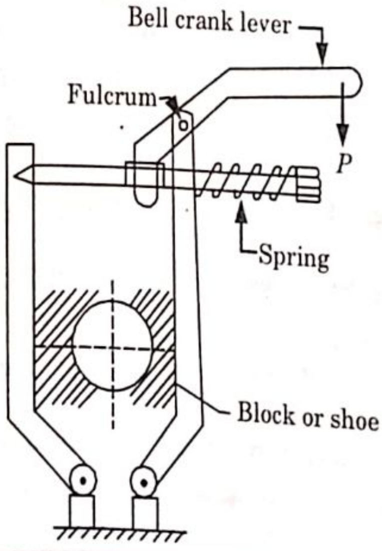


Fig. 5.8.1. Double block or shoe brake.

7. In case of double block or shoe brake, braking torque

$$T_B = (F_{t1} + F_{t2})\, r$$

Where, F_{t1} and F_{t2} = Braking forces on the two blocks.

Que 5.9. **Explain what happen when the angle of contact surface of block will become greater than 60° ?**

OR

Discuss the pivoted block or shoe brake.

Answer

1. If the angle of contact surface of block is less than 60° it is assumed that normal pressure between the block and wheel is uniform.
2. But when the angle of contact surface of block is greater than 60°, then the normal pressure at the ends of the contact surface will become lesser than that at the centre.
3. In this case, the block is pivoted to the lever and this gives uniform wear of the brake lining in the direction of the applied force.
4. The braking torque, $T_B = F_t r = \mu' N r$

Where,
$$\mu' = \frac{4\mu\sin\theta}{2\theta + \sin 2\theta}$$
= Equivalent co-efficient of friction, and

μ = Actual co-efficient of friction.

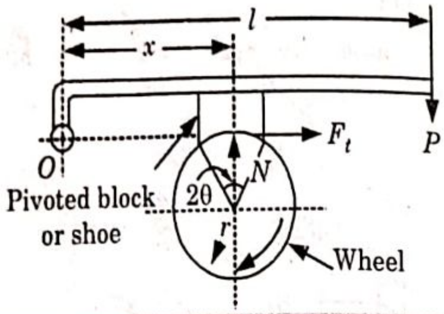


Fig. 5.9.1. Pivoted block or shoe brake.

Que 5.10. **A single block shoe is shown in Fig. 5.10.1. The brake drum is of radius 150 mm and angle of contact is 75°. If the operating force of 1000 N is applied at the end of lever. Determine the torque transmitted by the block brake.**
Take co-efficient of friction between the drum and lining 0.35.

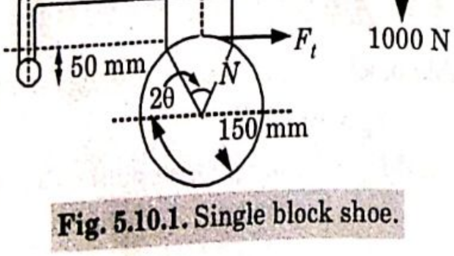


Fig. 5.10.1. Single block shoe.

Answer

Given : $r = 150$ mm, $2\theta = 75° = 75 \times (\pi/180)$ rad, $P = 1000$ N, $\mu = 0.35$
To Find : Torque transmitted by the block brake.

1. As $2\theta > 60°$,

$$\therefore \quad \mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.35 \sin\left(\frac{75°}{2}\right)}{\left(75 \times \frac{\pi}{180}\right) + \sin 75°} = 0.375$$

2. Taking moment about the fulcrum O,
$1000 \times (250 + 500) + F_t \times 50 = N \times 250$

$$750000 + F_t \times 50 = \frac{F_t}{\mu'} \times 250$$

$$F_t \left(\frac{250}{0.375} - 50\right) = 750000$$

$$F_t = \frac{750000}{616.67}$$

$$F_t = 1216.21 \text{ N}$$

3. Torque transmitted by the block brake,

$$T_B = F_t r = 1216.21 \times 150$$
$$= 182431.5 \text{ N-mm} = 182.431 \text{ N-m}$$

Que 5.11. **A braking system has its braking lever inclined at an angle 30° to the horizontal plane as shown in Fig. 5.11.1. The mass and diameter of the brake drum are 230 kg and 0.80 m respectively.**

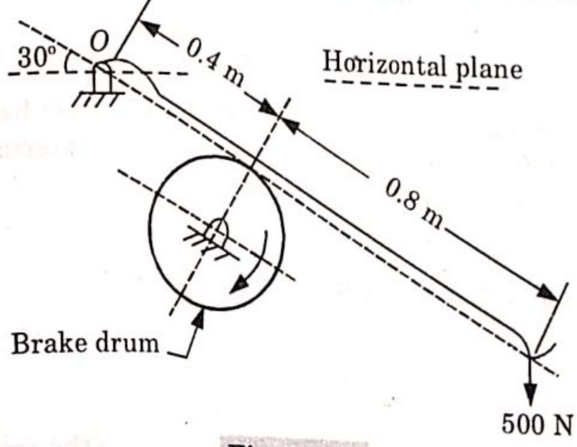


Fig. 5.11.1.

At the instant the lever is pressed on brake drum with a vertical force of 500 N, the drum rotates at 2500 rpm clockwise. Assume that the lever and brake shoe are perfectly rigid and possess negligible weight. Find :

1. **Braking torque, and**
2. **Number of revolutions that the drum will make before coming to rest from the instant of pressing the lever (Assume $\mu = 0.35$ between the brake shoe and brake drum).**

Answer

Given : $m = 230$ kg, $d = 0.80$ m or $r = 0.40$ m, $P = 500$ N, $N = 2500$ rpm, $\mu = 0.35$

To Find : i. Braking torque,

ii. Number of revolutions.

1. Taking moment about the fulcrum O,

$$500 \cos 30° \times 1.2 = N \times 0.4$$
$$N = 1299 \approx 1300 \text{ N}$$

2. Tangential braking force, $F_t = \mu N = 0.35 \times 1300 = 455$ N
3. Braking torque, $T_B = F_t r = 455 \times 0.40 = 182$ N-m
4. For number of revolution made by drum before coming to the rest,

Kinetic energy = Work done by braking torque

$$\frac{1}{2} m v^2 = T_B \times 2\pi n$$

$$\frac{1}{2} \times 230 \left(\frac{\pi d N}{60}\right)^2 = 182 \times 2\pi n$$

$$n = \frac{115(\pi \times 0.80 \times 2500)^2}{3600 \times 182 \times 2\pi} \approx 1103 \text{ revolutions.}$$

Que 5.12. **Explain the mechanism of band brake with neat sketch and expression.**

Answer

1. Band brake consists of a rope, belt or flexible steel band (lined with friction material) which is pressed against the external surface of a cylindrical drum when the brake is applied.
2. The force is applied at the free end of a lever.
3. Brake torque acting on the drum = $(T_1 - T_2)\, r$

Where, $\dfrac{T_1}{T_2} = e^{\mu\theta}$

r = Effective radius of the drum.

4. To apply the brake to the rotating drum, the band has to be tightened on the drum. This is possible :

Case i : When $a > b$, P acting Downwards :

a. **Anticlockwise Rotation :**

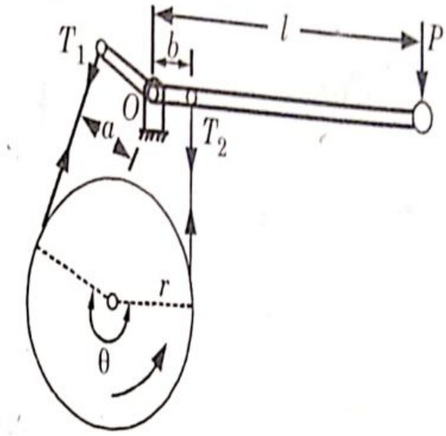


Fig. 5.12.1.

1. Let $T_1 > T_2$
2. Taking moment about pivot O,

$$Pl - T_1a + T_2b = 0$$

$$P = \frac{T_1a - T_2b}{l}$$

b. **Clockwise Rotation :**

1. In clockwise rotation, T_1 and T_2 changes their positions accordingly.

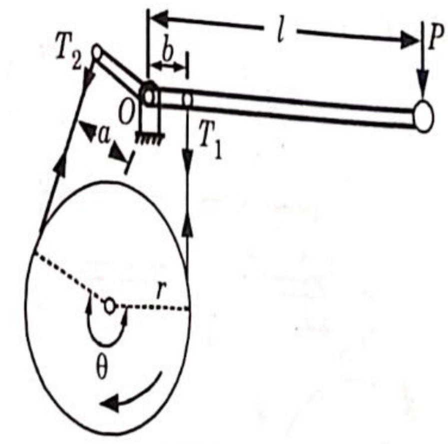


Fig. 5.12.2.

2. Now, $Pl - T_2a + T_1b = 0$

$$P = \frac{T_2a - T_1b}{l}$$

Case ii : When $a < b$, P acting Upwards :

a. Anticlockwise Rotation :

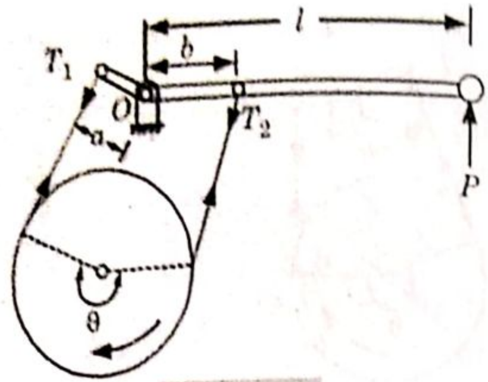


Fig. 5.12.3.

1. As $T_1 > T_2$, taking moment about pivot O,

$$Pl + T_1a - T_2b = 0$$

or $$P = \frac{T_2b - T_1a}{l}$$

$$T_1 > T_2 \text{ and } a < b$$

2. For $\frac{T_2}{T_1} = \frac{a}{b} \Rightarrow P = 0 \Rightarrow$ Brake becomes self locking.

b. Clockwise Rotation :

1. Taking moment about pivot O,

$$Pl - T_1b + T_2a = O$$

$$P = \frac{T_1b - T_2a}{l}$$

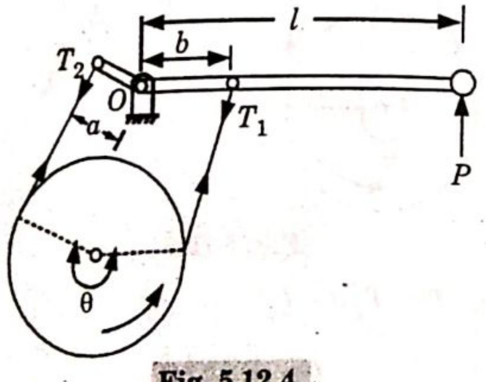


Fig. 5.12.4.

Que 5.13. **In a winch, the rope supports a load W and is wound round a barrel 450 mm diameter. A differential band brake acts on a drum 800 mm diameter which is keyed to the same shaft as the barrel. The two ends of the bands are attached to pins on opposite sides of the fulcrum of the brake lever and at distance of 25 mm and**

100 mm from the fulcrum. The angle of lap of the brake band is 250° and the co-efficient of friction is 0.25. What is the maximum load W which can be supported by the brake when a force of 750 N is applied to the lever at a distance of 3000 mm from the fulcrum ?

AKTU 2017-18, Marks 10

Answer

Given : $D = 450$ mm or $R = 225$ mm, $d = 800$ mm or $r = 400$ mm, $OB = 25$ mm, $OA = 100$ mm, $\theta = 250° = 250 \times \pi/180 = 4.364$ rad, $\mu = 0.25$, $P = 750$ N, $l = OC = 3000$ mm

To Find : The maximum load W which can be supported by the brake.

1. Since OA is greater than OB, therefore the operating force ($P = 750$ N) will act downwards.
2. We know that, when the drum rotates in clockwise direction, the end of band attached to A will be slack with tension T_2 and the end of the band attached to B will be tight with tension T_1, as shown Fig. 5.13.1.

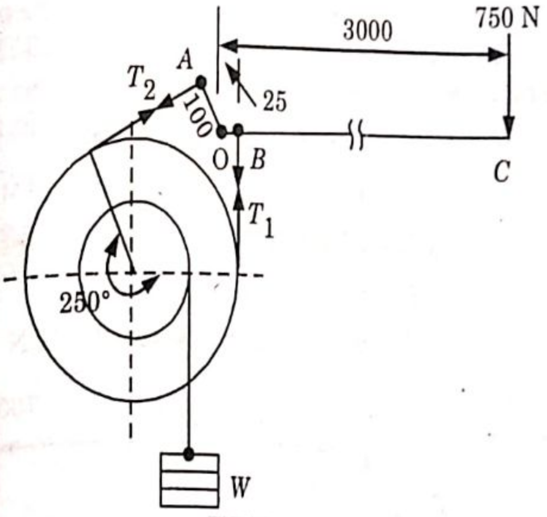


Fig. 5.13.1.

3. We know that,

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{(0.25 \times 4.364)}$$

$$\frac{T_1}{T_2} = 2.98 \quad \text{or} \quad T_1 = 2.98\, T_2 \qquad ...(5.13.1)$$

4. Now taking moments about the fulcrum O,

$$750 \times 3000 + T_1 \times 25 = T_2 \times 100$$

$$T_2 \times 100 - 2.98\, T_2 \times 25 = 2250 \times 10^3$$

$$25.5\, T_2 = 2250 \times 10^3 \qquad (\because T_1 = 2.98\, T_2)$$

$$T_2 = 88 \times 10^3 \text{ N}$$

and, $$T_1 = 2.98\, T_2 = 2.98 \times 88 \times 10^3 = 262 \times 10^3 \text{ N}$$

5. We know that braking torque,

$$T_B = (T_1 - T_2)r$$
$$= (262 \times 10^3 - 88 \times 10^3)\, 400$$
$$= 69.6 \times 10^6 \text{ N-mm} \quad ...(5.13.2)$$

And the torque due to the load W newtons,

$$T_W = WR = W \times 225 = 225\, W \text{ N-mm} \quad ...(5.13.3)$$

6. Since the braking torque must be equal to the torque due to load W therefore from eq. (5.13.2) and eq. (5.13.3),

$$W = 69.6 \times 10^6/225 = 309 \times 10^3 \text{ N} = 309 \text{ kN}$$

7. We know that, when the drum rotates in anticlockwise direction, the end of the band attached to A will be tight with tension T_1 and end of the band attached to B will be slack with tension T_2, as shown in Fig. 5.13.2. The ratio of tensions T_1 and T_2 will be same as calculated above, *i.e.*,

$$\frac{T_1}{T_2} = 2.98 \text{ or } T_1 = 2.98\, T_2$$

8. Now taking moments about the fulcrum O,

$$750 \times 3000 + T_2 \times 25 = T_1 \times 100$$
$$2.98\, T_2 \times 100 - T_2 \times 25 = 2250 \times 10^3 \quad (\because T_1 = 2.98\, T_2)$$
$$273\, T_2 = 2250 \times 10^3$$
$$T_2 = 8242 \text{ N}$$

and, $$T_1 = 2.98\, T_2 = 2.98 \times 8242 = 24561 \text{ N}$$

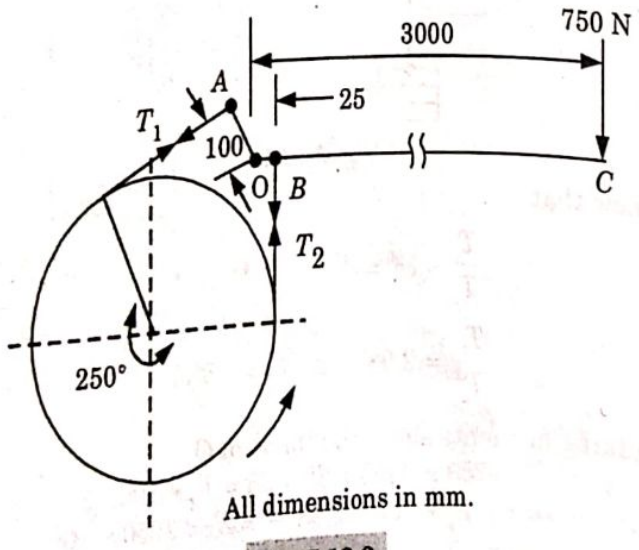


Fig. 5.13.2.

9. **Braking torque,** $T_B = (T_1 - T_2)r$

$$= (24561 - 8242)\,400 = 6.53 \times 10^6 \text{ N-mm} \quad ...(5.13.4)$$

10. **From eq.** (5.13.3) and eq. (5.13.4)

$$W = 6.53 \times 10^6/225 = 29 \times 10^3 \text{ N} = 29 \text{ kN}$$

11. **From above,** we see that the maximum load (W) that can be supported by the brake is 309 kN, when the drum rotates in clockwise direction.

Que 5.14. **What is band and block brake ? Obtain the expression for braking torque for the same.**

OR

Derive expression for the ratio of the maximum and minimum tensions of band and block brake. **AKTU 2017-18, Marks 10**

Answer

A. **Band and Block Brake :**

1. When the band brakes are lined with wooden blocks or other material, the brake is known as band and block brake.
2. Wooden blocks have a higher coefficient of friction, thus, increasing the effectiveness of the brake.

B. **Expression for the Maximum and Minimum Tensions of Band and Block Brake :**

1. Let there are 'n' number of blocks, each subtending an angle 2θ at the centre and the drum rotates in anticlockwise direction.
2. Let,

T_1 = Tension in the tight side,

T_2 = Tension in the slack side,

μ = Co-efficient of friction between the blocks and drum,

$T_{1'}$ = Tension in the band between the first and second block, and

$T_{2'}$, $T_{3'}$ = The tensions in the band between the second and third block and between the third and fourth block respectively.

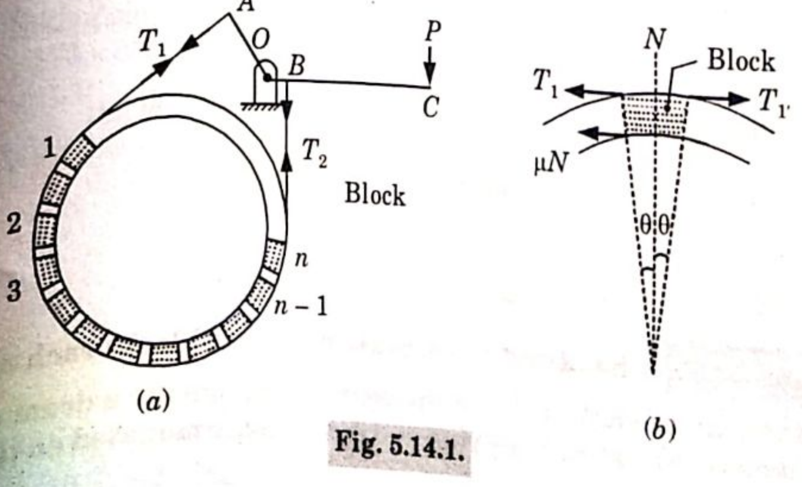


Fig. 5.14.1.

3. Consider block 1 as shown in Fig. 5.14.1(*b*), this is in equilibrium und the action of the following forces :
   i. Tension in the tight side (T_1),
   ii. Tension in the slack side ($T_{1'}$) or tension in the band between t first and second block,
   iii. Normal reaction of the drum on the block (N), and
   iv. The force of friction (μN).
4. For equilibrium, resolve the forces in vertical direction,

$$(T_1 + T_{1'}) \sin \theta = N \quad ...(5.14.$$

Similarly, resolving the forces in horizontal direction,

$$(T_1 - T_{1'}) \cos \theta = \mu N \quad ...(5.14.$$

5. Dividing eq. (5.14.2) by eq. (5.14.1),

$$\frac{(T_1 - T_{1'}) \cos \theta}{(T_1 + T_{1'}) \sin \theta} = \frac{\mu N}{N}$$

$$T_1 - T_{1'} = (T_1 + T_{1'})\, \mu \tan \theta$$

$$\frac{T_1}{T_{1'}} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

6. Similarly, for each block, we have

$$\frac{T_{1'}}{T_{2'}} = \frac{T_{2'}}{T_{3'}} = \frac{T_{3'}}{T_{4'}} = ... = \frac{T_{n-1}}{T_2} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

Now
$$\frac{T_1}{T_2} = \frac{T_1}{T_{1'}} \times \frac{T_{1'}}{T_{2'}} \times \frac{T_{2'}}{T_{3'}} \times \frac{T_{3'}}{T_{4'}} ... \times \frac{T_{n-1}}{T_2}$$

$$= \left(\frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}\right)^n$$

7. Now, braking torque on the drum of effective radius r_e,

$$T_B = (T_1 - T_2)\, r_e$$

Where, $r_e = r + \frac{t}{2}$ (t = Thickness of band)

8. If t is very small comparative to r then

$$r_e \approx r$$

$\therefore$ $T_B = (T_1 - T_2)\, r$

Que 5.15. **A band and block brake having 12 blocks each of whic subtends an angle of 15° at the centre, is applied to a drum of 1.5 effective diameter. The drum and flywheel are mounted on the sar**

shaft has a mass of 2000 kg and a combined radius of gyration of 450 mm. The two ends of the band are attached to pins on opposite sides of the brake lever at distances of 30 mm and 100 mm from the fulcrum. If a force of 200 N is applied at a distance of 650 mm from the fulcrum, find 1. Maximum braking torque, 2. Angular retardation of the drum and 3. Time taken by the system to come to rest from the rated speed of 360 rpm. The co-efficient of friction between blocks and drum may be taken as 0.30.

Answer

Given : $n = 12$, $2\theta = 15° \Rightarrow \theta = 7.5°$, $d = 1.5$ m $\Rightarrow r = 0.75$ m, $m = 2000$ kg, $k = 450$ mm $= 0.45$ m, $P = 200$ N, $N = 360$ rpm, $l = 650$ mm, $\mu = 0.30$.

To Find :
- i. Maximum braking torque.
- ii. Angular retardation of drum.
- iii. Time taken by drum to come to rest.

1. For maximum braking torque, $OB > OA$ and drum rotates anticlockwise and P acts upwards and $T_1 > T_2$.

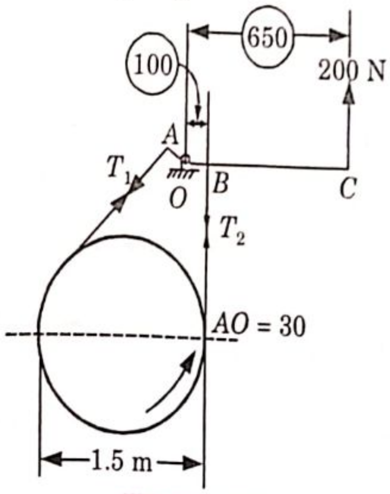


Fig. 5.15.1.

2. Taking moments about O,

$$200 \times 650 + T_1 \times 30 = T_2 \times 100$$

$$10\,T_2 - 3\,T_1 = 13000 \qquad ...(5.15.1)$$

3. We know that,

$$\frac{T_1}{T_2} = \left(\frac{1+\mu \tan\theta}{1-\mu\tan\theta}\right)^n = \left(\frac{1+0.30\tan 7.5°}{1-0.30\tan 7.5°}\right)^{12}$$

$$\frac{T_1}{T_2} = 2.58$$

$$T_1 = 2.58\,T_2 \qquad ...(5.15.2)$$

4. Putting $T_1 = 2.58T_2$ in eq. (5.15.1), we get

$$10\,T_2 - 3\,(2.58\,T_2) = 13000$$

$$T_2 = 5752 \text{ N}, T_1 = 14840 \text{ N}$$

5. For maximum braking torque,

$$T_B = (T_1 - T_2)r = (14840 - 5752) \times 0.75$$

$$T_B = 6816 \text{ N-m}$$

6. Also we know that,

$$T_B = I\alpha$$

$$= mk^2\alpha \qquad [\because I = mk^2]$$

$$6816 = 2000\,(0.45)^2\alpha$$

$$\alpha = 16.83 \text{ rad/s}^2$$

7. Initial angular speed, $\omega_1 = \dfrac{2\pi N_1}{60} = \dfrac{2\pi \times 360}{60} = 37.7$ rad/s

and $\omega_2 = 0$

8. $\omega_2 = \omega_1 - \alpha t$

$$0 = \omega_1 - \alpha t = 37.7 - (16.83)\,t$$

Time taken by drum to come to rest

$$t = 2.24 \text{ s}$$

PART-4

Effect of Braking on Rear and Front Wheels of a Four Wheeler.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.16. Explain effect of braking on a four wheeler moving up an inclined plane :

a. When brakes applied to rear wheels only.

b. When brakes applied to front wheels only.

c. When brakes applied to all four wheels.

Answer

1. Let us consider a vehicle moving up an inclined plane which is inclined at an angle α.
2. Let, m = Mass of vehicle,

   α = Inclination angle of the plane,

   R_A, R_B = Reactions of the ground on the front and rear wheels,

   a = Retardation of the vehicle,

   l = Wheel base of the car,

h = Perpendicular height between centre of mass of vehicle and inclined surface,

x = Distance between rear axle and centre of mass of vehicle, and

μ = Co-efficient of friction between the ground and tyres.

When Brakes Applied to Rear Wheels Only :

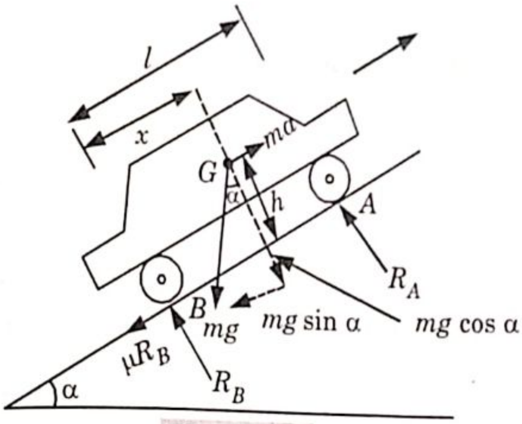


Fig. 5.16.1.

Let F_B = Total braking force (in newtons) acting at the rear wheels due to the application of the brakes. Its maximum value is μR_B.

The various forces acting on the vehicle are shown in Fig. 5.16.1. For the equilibrium of the vehicle, the forces acting on the vehicle must be in equilibrium.

i. Resolving the forces parallel to the plane,

$$F_B + mg \sin \alpha = ma \quad ...(5.16.1)$$

ii. Resolving the forces perpendicular to the plane,

$$R_A + R_B = mg \cos \alpha \quad ...(5.16.2)$$

Taking moments about G, the centre of gravity of the vehicle,

$$F_B h + R_B x = R_A(L - x) \quad ...(5.16.3)$$

Substituting the value of $F_B = \mu R_B$, and R_A from eq. (5.16.2) in eq. (5.16.3), we have

$$\mu R_B h + R_B x = (mg \cos \alpha - R_B)(L - x)$$

$$(\because \text{ From eq. (5.16.2), } R_A = mg \cos \alpha - R_B)$$

$$R_B(L + \mu h) = mg \cos \alpha (L - x)$$

$$\therefore \quad R_B = \frac{mg \cos \alpha (L - x)}{L + \mu h}$$

$$R_A = mg \cos \alpha - R_B = mg \cos \alpha - \frac{mg \cos \alpha (L - x)}{L + \mu h}$$

$$= \frac{mg \cos \alpha (x + \mu h)}{L + \mu h}$$

6. From eq. (5.16.1),

$$a = \frac{F_B + mg \sin \alpha}{m} = \frac{F_B}{m} + g \sin \alpha = \frac{\mu R_B}{m} + g \sin \alpha$$

$$= \frac{\mu g \cos \alpha (L - x)}{L + \mu h} + g \sin \alpha$$

b. When the Brakes are Applied to Front Wheels Only :

1. Let F_A = Total braking force (in newtons) acting at the front wheels due to the application of brakes. Its maximum value is μR_A.

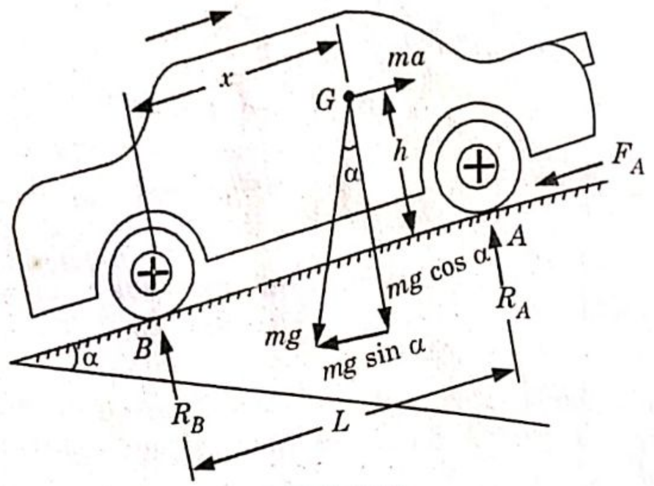


Fig. 5.16.2.

2. Resolving the forces parallel to the plane,

$$F_A + mg \sin \alpha = ma \quad \text{...(5.16.4)}$$

3. Resolving the forces perpendicular to the plane,

$$R_A + R_B = mg \cos \alpha \quad \text{...(5.16.5)}$$

4. Taking moments about G, the centre of gravity of the vehicle,

$$F_A h + R_B x = R_A (L - x) \quad \text{...(5.16.6)}$$

5. Substituting the value of $F_A = \mu R_A$ and R_B from eq. (5.16.5) in eq. (5.16.6), we have

$$\mu R_A h + (mg \cos \alpha - R_A)\, x = R_A (L - x)$$

($\because$ From eq. (5.16.6), $R_B = mg \cos \alpha - R_A$)

$$\mu R_A h + mg \cos \alpha \times x = R_A L$$

$$\therefore \quad R_A = \frac{mg \cos \alpha \times x}{L - \mu h}$$

$$R_B = mg\cos\alpha - R_A = mg\cos\alpha - \frac{mg\cos\alpha \times x}{L-\mu h}$$

$$= mg\cos\alpha\left(1-\frac{x}{L-\mu h}\right) = mg\cos\alpha\left(\frac{L-\mu h - x}{L-\mu h}\right)$$

From eq. (5.16.4),

$$a = \frac{F_A + mg\sin\alpha}{m} = \frac{\mu R_A + mg\sin\alpha}{m}$$

$$= \frac{\mu mg\cos\alpha \times x}{(L-\mu h)m} + \frac{mg\sin\alpha}{m}$$

$$= \frac{\mu g\cos\alpha \times x}{L-\mu h} + g\sin\alpha$$

When the Brakes are Applied to all the Four Wheels :

Let F_A = Braking force provided by the front wheels
$= \mu R_A$, and

F_B = Braking force provided by the rear wheels
$= \mu R_B$.

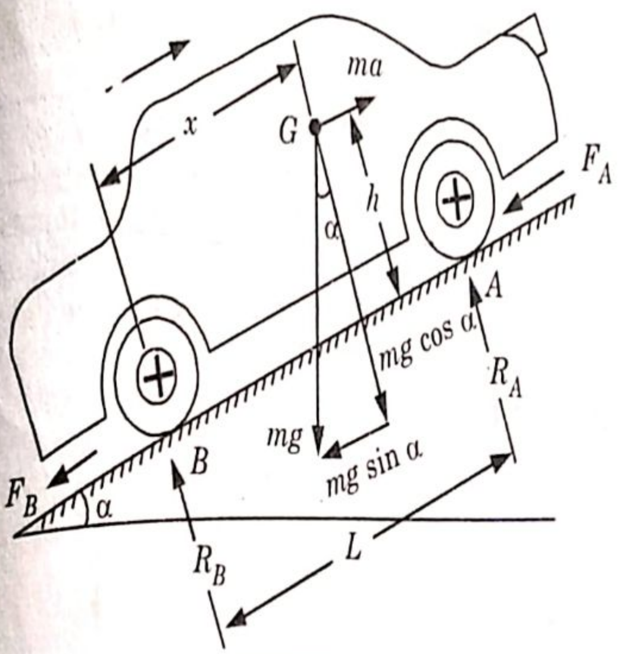


Fig. 5.16.3.

Resolving the forces parallel to the plane,

$$\mu(R_A + R_B)h + (mg\cos\alpha - R_A)\,x = R_A(L - x)$$

$$\mu(R_A + mg\cos\alpha - R_A x - R_A)h + (mg\cos\alpha - R_A)x = R_A(L - x)$$

(∵ From eq. (5.16.8), $R_B = mg\cos\alpha - R_A$)

$$\mu mg\cos\alpha \times h + mg\cos\alpha \times x = R_A L$$

∴ $$R_A = \frac{mg\cos\alpha(\mu h + x)}{L}$$

and $$R_B = mg\cos\alpha - R_A = mg\cos\alpha - \frac{mg\cos\alpha(\mu h + x)}{L}$$

$$= mg\cos\alpha\left[1 - \frac{\mu h + x}{L}\right] = mg\cos\alpha\left(\frac{L - \mu h - x}{L}\right)$$

6. Now from eq. (5.16.7),

$$\mu R_A + \mu R_B + mg\sin\alpha = ma$$

$$\mu(R_A + R_B) + mg\sin\alpha = ma$$

$$\mu mg\cos\alpha + mg\sin\alpha = ma$$

∴ $$a = g(\mu\cos\alpha + \sin\alpha)$$

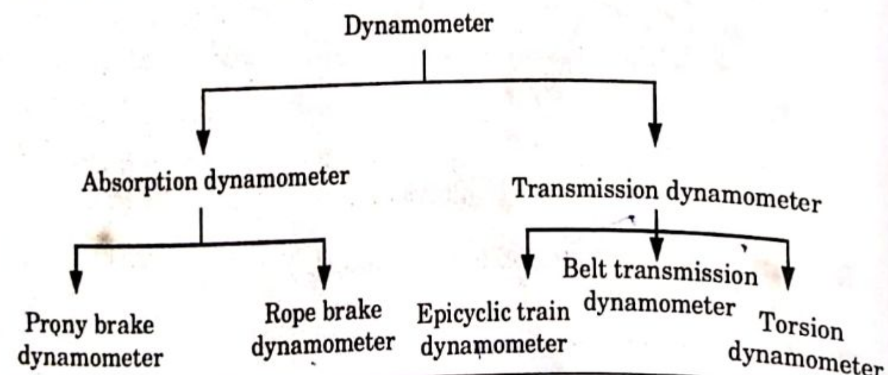
PART-5

Dynamometers, Belt Transmission Dynamometer, Torsion Dynamometer, Hydraulic Dynamometer.

CONCEPT OUTLINE

Dynamometer : It is a brake incorporating device to measure the frictional resistance applied. This is used to determine the power developed by the machine, while maintaining its speed at the rated value.

Classification of Dynamometer :



Questions-Answers

Long Answer Type and Medium Answer Type Questions

e 5.17. **Explain and discuss following in detail with examples :**

Absorption type brake dynamometer;
Transmission type brake dynamometer.

AKTU 2017-18, Marks 10

swer

Absorption Dynamometer :

In this type, the work done is converted into heat by friction while being measured. They can be used for the measurement of moderate powers only.

Following are the types of absorption dynamometer :

Prony Brake Dynamometer :

1. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two bolts and nuts, as shown in Fig. 5.17.1.
2. A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it and carries a weight W at its outer end.
3. A counter weight is placed at the other end of the lever which balances the brake when unloaded.
4. Two stops S, S are provided to limit the motion of the lever.
5. When the brake is to be put in operation, the long end of the lever is loaded with suitable weights W and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position.

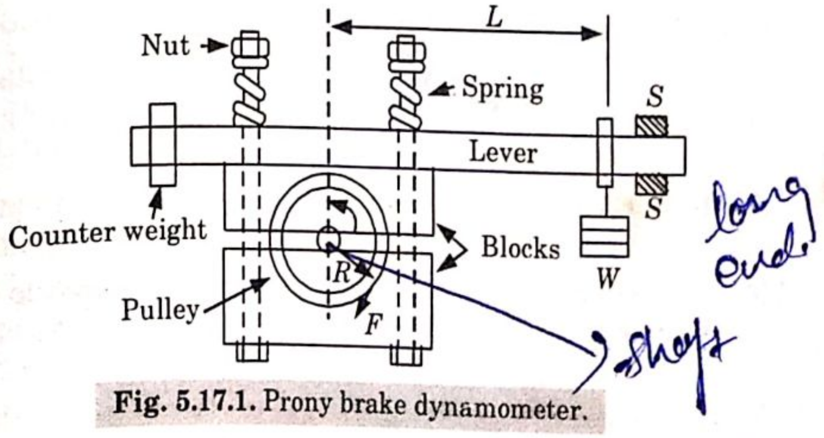


Fig. 5.17.1. Prony brake dynamometer.

6. Under these conditions, the moment due to the weight W must balance the moment of the frictional resistance between the blocks and the pulley.

$$P = \frac{2\pi N}{60}(WL)$$

b. Rope Brake Dynamometer :

1. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine.
2. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig. 5.17.2.

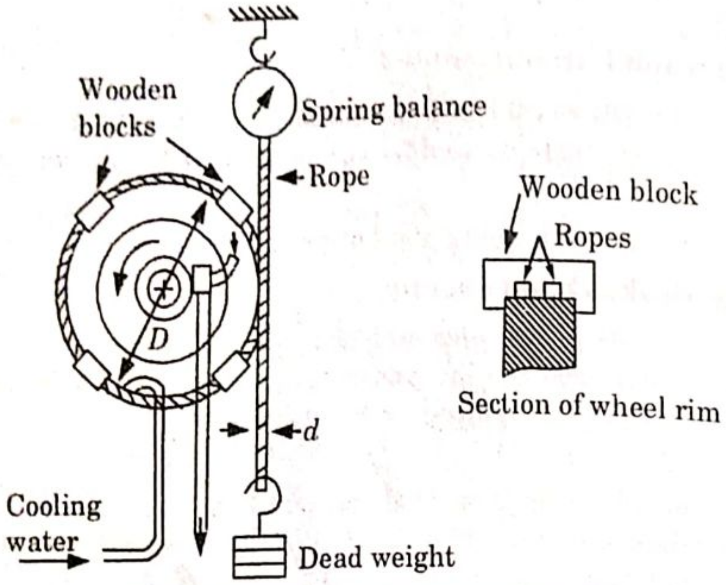


Fig. 5.17.2. Rope brake dynamometer.

3. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.
4. In the operations of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope must be equal to the torque being transmitted by the engine.

ii. Transmission Dynamometer :

1. In this type, the work is not absorbed in the process, but is utilized after the measurements.
2. Following are the types of transmission dynamometer :

a. Epicyclic Train Dynamometer :

1. It consists of a simple epicyclic train of gear, *i.e.*, a spur gear, an annular gear (a gear having internal teeth) and a pinion.
2. The spur gear is keyed to the engine shaft (*i.e.*, driving shaft) and rotates in anticlockwise direction. The annular gear is also keyed to the driving shaft and rotates in clockwise direction.
3. The pinion or the intermediate gear meshes with both the spur and annular gears. The pinion revolves freely on a lever which is pivoted to the common axis of the driving and driven shafts.

4. A weight W is placed at the smaller end of the lever in order to keep it in position.
5. A little consideration will show that if the friction of the pin on which the pinion rotates is neglected, then the tangential effort P exerted by the spur gear on the pinion and the tangential reaction of the annular gear on the pinion are equal.
6. Since these efforts act in the upward direction as shown, therefore total upward force on the lever acting through the axis of the pinion is $2P$.
7. This force tends to rotate the lever about its fulcrum and it is balanced by a dead weight W at the end of the lever.

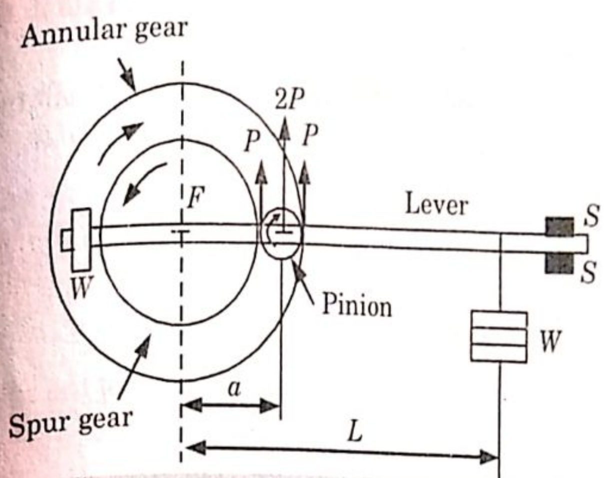


Fig. 5.17.3. Epicyclic train dynamometer.

b. **Belt Transmission Dynamometer-Froude or Thorneycroft Transmission Dynamometer :**

1. It consists of a pulley A (called driving pulley) which is rigidly fixed to the shaft of an engine whose power is required to be measured. There is another pulley B (called driven pulley) mou: ted on another shaft to which the power from pulley A is transm tted.
2. The pulleys A and B are connected by means of a continuous belt passing round the two loose pulleys C and D which are mounted on a T-shaped frame.
3. The frame is pivoted at E and its movement is controlled by two stops S, S.
4. Since the tension in the tight side of the belt (T_1) is greater than the total force acting on the pulley D (*i.e.*, $2T_2$). It is thus obvious that the frame causes movement about E in the anticlockwise direction.
5. In order to balance it, a weight W is applied at a distance L from E on the frame as shown in Fig. 5.17.4.

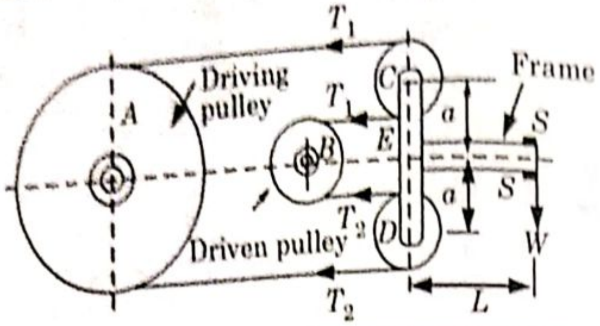


Fig. 5.17.4. Froude or Throneycroft transmission dynamometer.

c. **Torsion Dynamometer :**

1. It is used to measure the power transmitted along the propelle shaft of a turbine or motor vessel.
2. Due to power transmission, the driving end of a shaft twists throug a small angle relative to the driven end of the shaft.
3. From torsion equation,

$$\frac{T}{J} = \frac{C\theta}{l}$$

Where T = Torque acting on shaft,

J = Polar moment of inertia of the shaft,

C = Modulus of rigidity,

θ = Angle of twist, and

l = Length of the shaft.

$$\therefore \quad T = \frac{CJ}{l}\theta \text{ or } T = k\theta$$

Where $k = \frac{CJ}{l}$ = Constant for a particular shaft.

$$T \propto \theta$$

4. It means by measuring the angle of twist one can measure th torque acting on the shaft.
5. Power transmitted = $T\omega = \frac{2\pi NT}{60}$

Que 5.18. **A torsion dynamometer is fitted to a propeller shaft of marine engine. It is found that the shaft twists 2° in a length o 20 metres at 120 rpm. If the shaft is hollow with 400 mm externa diameter and 300 mm internal diameter, find the power of the engine Take modulus of rigidly for the shaft material as 80 GPa.**

Answer

Given : $\theta = 2^\circ = 2 \times \pi/180 = 0.035$ rad, $l = 20$ m, $N = 120$ rpm, $D = 400$ mm $= 0.4$ m, $d = 300$ mm $= 0.3$ m, $C = 80$ GPa $= 80 \times 10^9$ N/m^2

To Find : Power of the engine.

1. We know that polar moment of inertia of the shaft,

$$J = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}[(0.4)^4 - (0.3)^4]$$

$$= 0.0017 \text{ m}^4$$

2. Torque applied to the shaft,

$$T = \frac{CJ}{l}\theta = \frac{80 \times 10^9 \times 0.0017}{60} \times 0.035$$

$$= 238 \times 10^3 \text{ N-m}$$

3. We know that power of the engine,

$$P = \frac{2\pi NT}{60} = \frac{80 \times 10^9 \times 2\pi \times 120}{60}$$

$$= 2990 \times 10^3 \text{ W} = 2990 \text{ kW}$$

Que 5.19. **Explain the hydraulic dynamometer with suitable sketch.**

Answer

1. Hydraulic accumulator works on the principle of dissipating the power in fluid friction rather than in dry friction.
2. In principle, its construction is similar to that of a fluid flywheel.
3. It consists of an inner rotating member or impeller coupled to the output shaft of the engine. The impeller rotates in a casing filled with some hydraulic fluid.
4. The outer casing due to the centrifugal force developed tends to revolve with the impeller but is resisted by a torque arm supporting the balanced weight.
5. The frictional forces between the impeller and the fluid is measured by ... fitted on the casing.

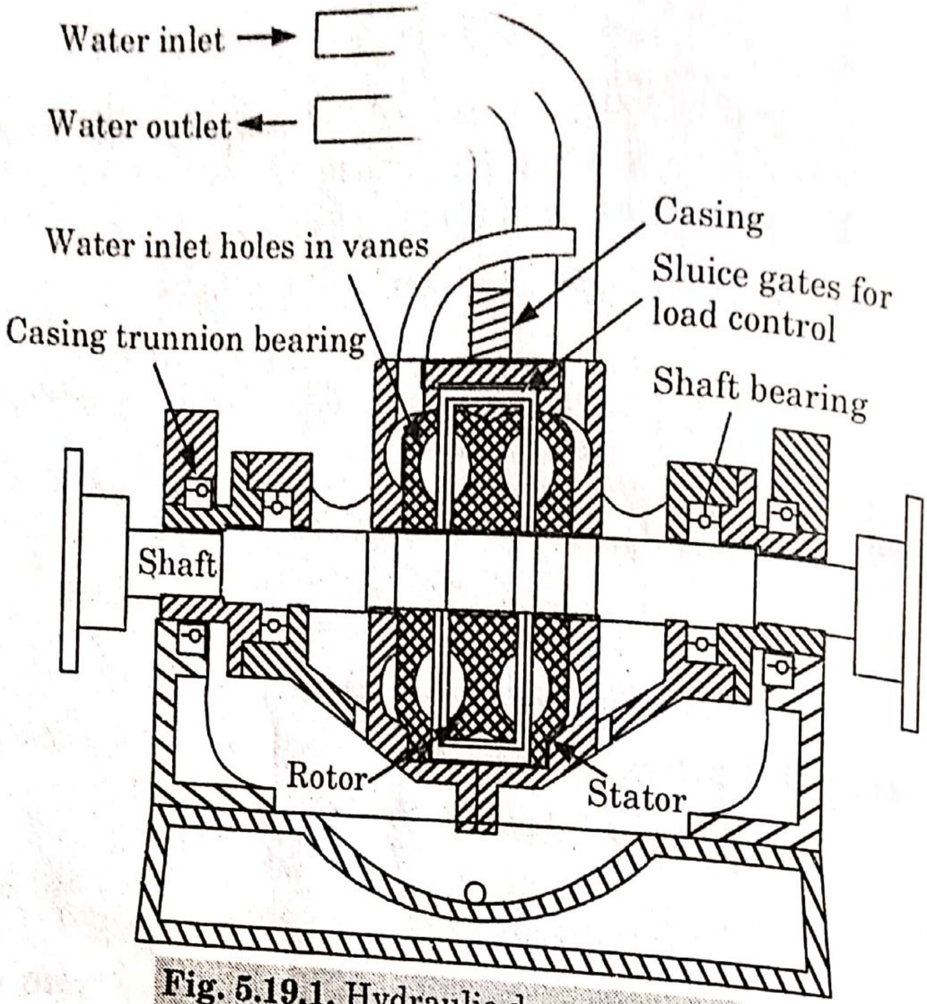


Fig. 5.19.1. Hydraulic dynamometer.

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5 UNIT

Brakes and Dynamometer

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PART-1

Introduction, Law of Friction.

CONCEPT OUTLINE

Friction : When a body slides over another, the motion is resisted by a force and that force is known as the force of friction or friction.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. Define friction. Give its classification in brief.

Answer

A. **Friction :**

1. When two bodies have relative motion between each other, a force acts in the opposite direction of their relative motion, which opposes their motion. This force is called friction force.
2. Friction force arises due to the relative motion between two bodies and hence some energy is wasted in overcoming the friction.

B. **Classification of Friction :**

a. **According to Motion of Body :**

i. **Static Friction :** When body is at rest, the friction acts on the body is called static friction.

ii. **Dynamic Friction :**

1. When body is in motion, the friction acts on the body is called dynamic friction.
2. It is also known as kinetic friction.
3. Dynamic friction is always less than static friction.
4. It is of the following three types :
   i. Sliding friction,
   ii. Rolling friction, and
   iii. Pivot friction.

b. **According to the Condition of Surface :**

i. **Dry Friction :**

1. When two dry and unlubricated surfaces are in contact, the friction acts between them is called dry friction.