**PUBLISHED BY :** **Apram Singh**
**Quantum Publications®**
**(A Unit of Quantum Page Pvt. Ltd.)**
Plot No. 59/2/7, Site - 4, Industrial Area,
Sahibabad, Ghaziabad-201 010

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**Engineering Mechanics (CE : Sem-3)**

1st Edition : *2019-20*
2nd Edition : *2020-21*
3rd Edition : *2021-22*

*Price: Rs. 135/- only*

*Printed at :* Balajee Offset, Delhi.



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## KCE 301 : Engineering Mechanics

1

UNIT

# Introduction to Engineering Mechanics

## CONTENTS







## PART-1

*Introduction to Engineering Mechanics, Force Systems, Basic Concepts.*

### CONCEPT OUTLINE

**Engineering Mechanics :** It is that branch of science which deals with the behaviour of a body when the body is at rest or in motion.

**Branches of Mechanics :**

i. **Statics :** Branch of mechanics which deals with the study of body when the body is at rest is known as statics.

ii. **Dynamics :** Branch of mechanics which deals with the study of body when the body is in motion is known as dynamics. It is further divided into kinematics (force not considered) and kinetics (force considered).

**Scalar Quantity :** A quantity which is completely specified by magnitude only is known as scalar quantity.

**Example :** Mass, length, time, etc.

**Vector Quantity :** A quantity which is specified by both magnitude and direction is known as vector quantity.

**Example :** Velocity, force, displacement, etc.

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 1.1.** **Define free, fixed and forced vectors.**

**Answer**

i. **Free Vector :** A vector which can be moved parallel to its position anywhere in space provided its magnitude, direction and sense remain the same is known as free vector. Fig. 1.1.1($a$) shows free vector.

ii. **Fixed Vector :** A vector whose initial point is fixed, is known as fixed vector. Fig. 1.1.1($b$) shows fixed vector.

iii. **Forced Vector :** A vector which can be applied anywhere along its line of action is known as forced vector. Fig. 1.1.1($c$) shows a forced vector.

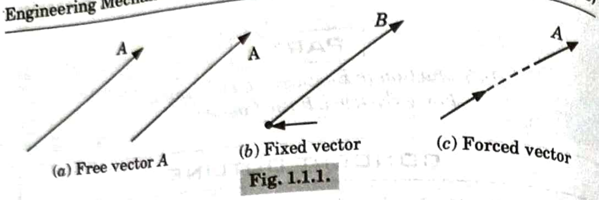



**Fig. 1.1.1.**

**Que 1.2.** **State and prove parallelogram law of forces.**

**Answer**

A. **Statement :** Parallelogram law states that if two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.

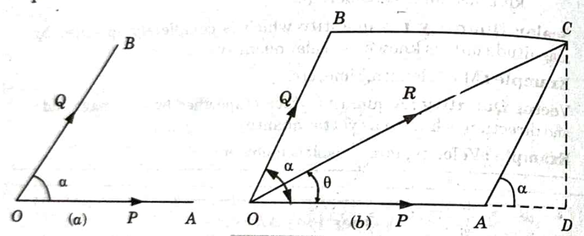



**Fig. 1.2.1.**

B. **Proof :**

1. Let two forces $P$ and $Q$ act at a point $O$ as shown in Fig. 1.2.1($a$). The force $P$ is represented in magnitude and direction by $OA$ whereas the force $Q$ is represented in magnitude and direction by $OB$.
2. Let the angle between the two forces be '$\alpha$'. The resultant of these two forces will be obtained in magnitude and direction by the diagonal (passing through $O$) of the parallelogram of which $OA$ and $OB$ are two adjacent sides. Hence draw the parallelogram with $OA$ and $OB$ as adjacent sides as shown in Fig. 1.2.1($b$).
3. The resultant $R$ is represented by $OC$ in magnitude and direction.
4. From $C$ draw $CD$ perpendicular to $OA$ produced.
5. Let,
   $\alpha$ = Angle between two forces $P$ and $Q = \angle AOB$
   $\theta$ = Angle made by resultant with $OA$.
6. In parallelogram $OACB$, $AC$ is parallel and equal to $OB$.



$\therefore \quad AC = Q$

7. In triangle $ACD$, $AD = AC \cos \alpha = Q \cos \alpha$
   and $CD = AC \sin \alpha = Q \sin \alpha$
8. In triangle $OCD$, $OC^2 = OD^2 + DC^2$
   But $OC = R,\ OD = OA + AD = P + Q \cos \alpha$
   $DC = Q \sin \alpha$

$$\therefore \quad R^2 = (P + Q\cos\alpha)^2 + (Q \sin\alpha)^2$$
$$= P^2 + Q^2\cos^2\alpha + 2PQ\cos\alpha + Q^2\sin^2\alpha$$
$$= P^2 + Q^2(\cos^2\alpha + \sin^2\alpha) + 2PQ\cos\alpha$$
$$= P^2 + Q^2 + 2PQ\cos\alpha \quad (\because \cos^2\alpha + \sin^2\alpha = 1)$$
$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \quad \text{...(1.2.1)}$$

Eq. (1.2.1) gives the magnitude of resultant force $R$.

9. Now from triangle $OCD$,

$$\tan\theta = \frac{CD}{OD} = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$
$$\therefore \quad \theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right) \quad \text{...(1.2.2)}$$

Eq. (1.2.2) gives the direction of resultant ($R$).

**Que 1.3.** **Discuss the law of parallelogram of forces. Two forces equal to $P$ and $2P$ act on a rigid body. When the first force is increased by 100 N and the second force is doubled, the direction of the resultant remains unchanged. Determine the value of $P$.**

**AKTU 2013-14, (I) Marks 05**

**Answer**

A. **Parallelogram Law of Forces :** Refer Q. 1.2, Page, Unit-1.

B. **Numerical :**

**Given :** $F_1 = P,\ F_2 = 2P,\ F_1' = P + 100,\ F_2' = 2P$
**To Find :** Value of $P$.

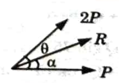



**Fig. 1.3.1.**

1. We know that,

$$\tan \alpha = \frac{2P \sin \theta}{P + 2P \cos \theta} \quad ...(1.3.1)$$

2. According to question if $P$ is now changed to $P$ + 100 and $2P$ is now changed to $4P$ then again direction of resultant remains same *i.e.*,

$$\tan \alpha = \frac{4P \sin \theta}{(P + 100) + 4P \cos \theta} \quad ...(1.3.2)$$

3. From eq. (1.3.1) and eq. (1.3.2), we have

$$\frac{2P \sin \theta}{P + 2P \cos \theta} = \frac{4P \sin \theta}{(P + 100) + 4P \cos \theta}$$

$$\sin \theta \, [P + 100 + 4P \cos \theta] = 2 \sin \theta \, [P + 2P \cos \theta]$$

$$\sin \theta \, [P + 100 + 4P \cos \theta - 2P - 4P \cos \theta] = 0$$

Either $\sin \theta = 0$ or $P + 100 - 2P = 0$

$$P = 100 \text{ N}$$

So, the value of $P$ = 100 N

**Que 1.4.** **Two forces $P$ and $Q$ are inclined at an angle of 75°, magnitude of their resultant is 100 N. The angle between the resultant and the force $P$ is 45°. Determine the magnitude of $P$ and $Q$.**

**AKTU 2016-17, (II) Marks 10**

**Answer**

**Given :** $\alpha = 75°$, $\theta = 45°$, $R = 100$ N
**To Find :** Magnitude of $P$ and $Q$.

1. The resultant $R$ of $P$ and $Q$ is given by,

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$$

$$100 = \sqrt{P^2 + Q^2 + 2PQ \cos 75°}$$

$$(100)^2 = P^2 + Q^2 + 0.517\,PQ \quad ...(1.4.1)$$

2. The inclination of $R$ to the direction of the force $P$ is given by,

$$\tan \theta = \frac{Q \sin \alpha}{P + Q \cos \alpha}$$

$$\tan 45° = \frac{Q \sin 75°}{P + Q \cos 75°}$$

$$P + 0.259\,Q = 0.966\,Q$$

$$P = 0.707\,Q \quad ...(1.4.2)$$



3. Putting value of $P$ from eq. (1.4.2) in eq. (1.4.1), we get

$$(100)^2 = (0.707\,Q)^2 + Q^2 + 0.517\,(0.707)Q^2$$

$$(100)^2 = 1.865Q^2$$

$$Q = 73.22 \text{ N}$$

4. From eq. (1.4.2), we have

$$P = 0.707 \times 73.22 = 51.76 \text{ N}$$

**Que 1.5.** **What are the basic laws of mechanics ?**

**Answer**

Following are the basic laws of mechanics :

i. **Newton's First Law of Motion :** It states that every body continues in a state of rest or uniform motion in a straight line unless it is compelled to change that state by some external force acting on it.

ii. **Newton's Second Law of Motion :** It states that, the net external force acting on a body in the direction of motion is directly proportional to the rate of change of momentum in that direction.

iii. **Newton's Third Law of Motion :** It states that to every action there is always equal and opposite reaction.

iv. **Gravitational Law of Attraction :** It states that two bodies will be attracted towards each other along their connecting line with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Mathematically, $F = G \dfrac{m_1 m_2}{r^2}$

where, $G$ = Universal gravitational constant of proportionality.

**Que 1.6.** **What do you understand by resolution of force ?**

**Answer**

1. Resolution of a force means finding the components of a given force in two given directions.
2. Let a given force be $R$ which makes an angle $\theta$ with $X$-axis as shown in Fig. 1.6.1. It is required to find the components of the force $R$ along $X$-axis and $Y$-axis.

   Components of $R$ along $X$-axis = $R \cos \theta$
   Components of $R$ along $Y$-axis = $R \sin \theta$
3. Hence, the resolution of force is the process of finding components of forces in specified directions.

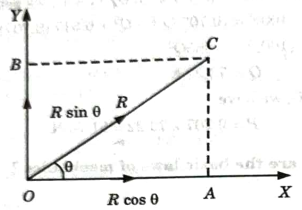



Fig. 1.6.1.

**PART-2**

*Rigid Body Equilibrium, System of Forces, Coplanar Concurrent Forces, Components in Space, Resultant.*

**CONCEPT OUTLINE**

**Rigid Body :** A body which does not deform under the action of external forces is known as rigid body.

**System of Forces :** When several forces act on a body then, they are said to form a system of forces.

**Coplanar Force System :** If in a system, all the forces lie in the same plane, then the force system is known as coplanar.

**Non-Coplanar Force System :** If in a system, all the forces lie in different planes, then the force system is known as non-coplanar.

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 1.7.** **Discuss in short about rigid body equilibrium.**

**Answer**

1. The external forces acting on a rigid body can be reduced to a force-couple system at some arbitrary point.
2. When the force and the couple are both equal to zero, the external forces form a system equivalent to zero, and the rigid body is said to be in equilibrium.



3. The necessary and sufficient conditions for the equilibrium of a rigid body are :

$$\Sigma F = 0$$
$$\Sigma M_O = \Sigma(r \times F) = 0$$

4. In general, the point $O$ should be fixed with respect to an inertial reference frame.
5. Resolving each force and each moment into its rectangular components, we can express the necessary and sufficient conditions for the equilibrium of a rigid body with following six scalar equations :

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$$
$$\Sigma M_x = 0, \Sigma M_y = 0, \Sigma M_z = 0$$

**Que 1.8.** **Give the classification of system of forces and also explain the systems involved.**

**Answer**

A. **Classification of System of Forces :**

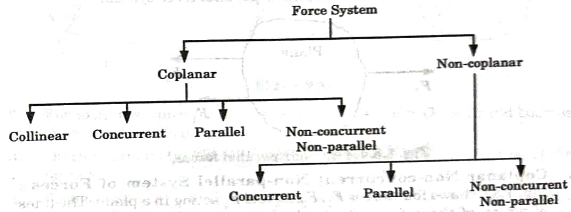



Fig. 1.8.1.

B. **Explanation :**

a. **Coplanar Collinear System of Forces :** Fig. 1.8.2 shows three forces $F_1$, $F_2$ and $F_3$ acting in the same plane. These three forces are in the same line, *i.e.*, these three forces are having a common line of action. This system of forces is known as coplanar collinear force system.

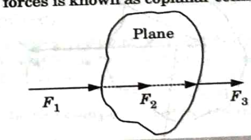



Fig. 1.8.2. Coplanar collinear forces.

b. **Coplanar Concurrent System of Forces :** Fig. 1.8.3 shows three forces $F_1$, $F_2$ and $F_3$ acting in the same plane and these forces intersect or meet at a common point $O$. This system of forces is known as coplanar concurrent force system.

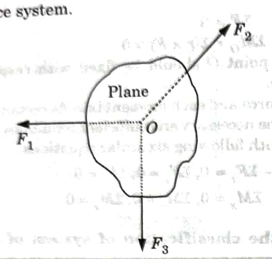



Fig. 1.8.3. Concurrent coplanar forces.

c. **Coplanar Parallel System of Forces :** Fig 1.8.4 shows three forces $F_1$, $F_2$ and $F_3$ acting in the same plane and these forces are parallel. This system of forces is known as coplanar parallel force system.

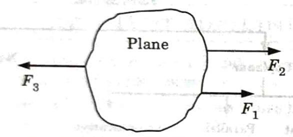



Fig. 1.8.4. Coplanar parallel forces.

d. **Coplanar Non-concurrent Non-parallel System of Forces :** Fig. 1.8.5 shows four forces $F_1$, $F_2$, $F_3$ and $F_4$ acting in a plane. The lines of action of these forces lie in the same plane but they are neither parallel nor meet or intersect at a common point. This system of forces is known as coplanar non-concurrent non-parallel force system.

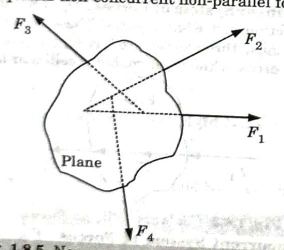



Fig. 1.8.5. Non-concurrent non-parallel forces.



**Que 1.9.** **Define the principle of transmissibility of forces.**

**AKTU 2011-12, Marks 02**

**Answer**

1. Principle of transmissibility of forces states that if force acting at a point on a rigid body is shifted to any other point which is on the line of action of the force, the external effect of the force on the body remains unchanged.

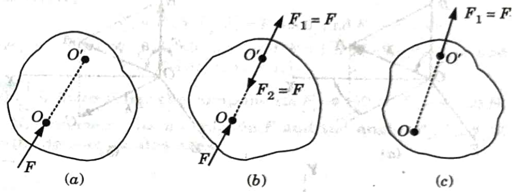



Fig. 1.9.1.

2. For example, consider a force $F$ acting at a point $O$ on a rigid body as shown in Fig. 1.9.1(*a*).
3. On this rigid body, "there is another point $O'$ in the line of action of the force $F$.
4. Suppose at this point $O'$, two equal and opposite forces $F_1$ and $F_2$ (each equal to $F$ and collinear with $F$) are applied as shown in Fig. 1.9.1(*b*).
5. The force $F$ and $F_2$ being equal and opposite will cancel each other leaving a force $F_1$ at point $O'$ as shown in Fig. 1.9.1(*c*). But force $F_1$ is equal to force $F$.
6. The original force $F$ acting at point $O$ has been transferred to point $O'$ which is along the line of action of $F$ without changing the effect of the force on the rigid body.
7. Hence any force acting at a point on a rigid body can be transmitted to act at any other point along its line of action without changing its effect on the rigid body. This proves the principle of transmissibility of a force.

**Que 1.10.** **Describe the component of forces in space and also give the formula for resultant.**

**Answer**

1. Consider a force $F$ acting at the origin $O$ of the system of rectangular coordinates $X$, $Y$ and $Z$.

2. To define the direction of $F$, we draw the vertical plane $OBAC$ containing $F$ [Fig. 1.10.1 (*a*)]. This plane passes through the vertical $Y$-axis; its orientation is defined by the angle $\phi$ it forms with the $XY$ plane.
3. The direction of $F$ within the plane is defined by the angle $\theta_y$ that $F$ forms with $Y$-axis. The force $F$ may be resolved into a vertical component $F_y$ and a horizontal component $F_h$, this operation is shown in Fig. 1.10.1(*b*), is carried out in plane $OBAC$.

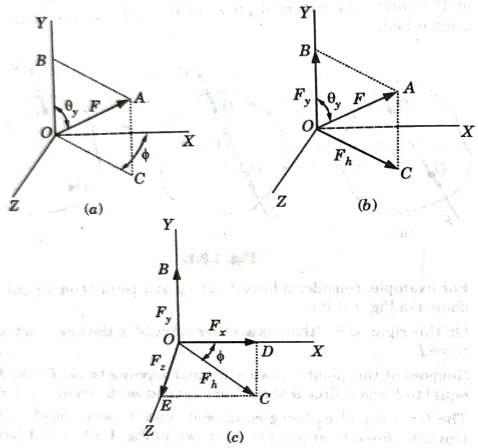



**Fig. 1.10.1.**

4. The corresponding scalar components are :

$$F_y = F\cos\theta_y \quad F_h = F\sin\theta_y \qquad \text{...(1.10.1)}$$

5. But $F_h$ may be resolved into two rectangular components $F_x$ and $F_z$ along the $X$ and $Z$ axes, respectively. This operation shown in Fig. 1.10.1(*c*) is carried out in the $XZ$ plane.
6. We obtain the following expression for the corresponding scalar components :

$$F_x = F_h\cos\phi = F\sin\theta_y\cos\phi$$
$$F_z = F_h\sin\phi = F\sin\theta_y\sin\phi \qquad \text{...(1.10.2)}$$

7. The given force $F$ has thus been resolved into three rectangular vector components $F_x$, $F_y$, $F_z$ which are directed along the three coordinate axes.



8. Applying the Pythagorean Theorem to the triangles $OAB$ and $OCD$ of Fig. 1.10.1, we write

$$F^2 = (OA)^2 = (OB)^2 + (BA)^2 = F_y^2 + F_h^2$$
$$F_h^2 = (OC)^2 = (OD)^2 + (DC)^2 = F_x^2 + F_z^2$$

9. Eliminating $F_h^2$ from these two equations and solving for $F$, we obtain the following relation between the magnitude of $F$ and its rectangular scalar components,

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

10. We also have,

$$F_x = F\cos\theta_x,\ F_y = F\cos\theta_y \text{ and } F_z = F\cos\theta_z$$

where, $\theta_x$, $\theta_y$, $\theta_z$, = Angle made of $F$ with $X$-axis, $Y$-axis and $Z$-axis, respectively.

**Que 1.11.** **A force $F$ has the components $F_x = 100$ N, $F_y = -150$ N, $F_z = 300$ N. Determine its magnitude $F$ and the angles $\theta_x$, $\theta_y$, $\theta_z$ it forms with the coordinates axes.**

**Answer**

**Given :** $F_x = 100$ N, $F_y = -150$ N, $F_z = 300$ N
**To Find :** $F$, $\theta_x$, $\theta_y$ and $\theta_z$

1. We know that,

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$
$$= \sqrt{(100)^2 + (-150)^2 + (300)^2}$$
$$= \sqrt{122500} = 350 \text{ N}$$

2. Also, we know that

$$\cos\theta_x = \frac{F_x}{F} = \frac{100}{350} \Rightarrow \theta_x = 73.4°$$
$$\cos\theta_y = \frac{F_y}{F} = \frac{-150}{350} \Rightarrow \theta_y = 115.4°$$
$$\cos\theta_z = \frac{F_z}{F} = \frac{300}{350} \Rightarrow \theta_z = 31.0°$$

## PART-3

*Moment of Forces and its Applications.*

## Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 1.12.** **Define moment of forces. Also give its applications.**

**Answer**

A. **Moment of Forces :** The product of a force and the perpendicular distance of the line of action of the force from a point is known as moment of the force about that point.

Moment ($M$) of the force $F$ about $O$ is given by,

$$M = Fr$$

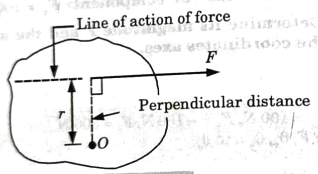



**Fig. 1.12.1.**

B. **Applications :** Following are the applications of moment of forces :

1. Used in levers.
2. Used in levers safety valve.
3. Used in balancing.

**Que 1.13.** **State and prove Varignon's theorem.**

**AKTU 2011-12, Marks 05**

**Answer**

A. **Statement :** Varignon's theorem states that the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.

B. **Proof :**

1. Let $R$ be the resultant of forces $F_1$ and $F_2$ and $B$ the moment centre.
2. Let $d, d_1$ and $d_2$ be the moment arms of the forces, $R, F_1$ and $F_2$, respectively from the moment centre $B$. Then in this case, we have to prove that :

$$Rd = F_1 d_1 + F_2 d_2$$



3. Join $AB$ and consider it as $Y$-axis and draw $X$-axis at right angle to it at $A$ [Fig. 1.13.1($b$)]. Denoting by $\theta$ the angle that $R$ makes with $X$-axis noting that the same angle is formed by perpendicular to $R$ at $B$ with $AB_1$, we can write :

$$Rd = R \times AB \cos\theta$$
$$= AB \times (R \cos\theta)$$
$$= AB \times R_x \quad ...(1.13.1)$$

where $R_x$ denotes the component of $R$ in $X$ direction.

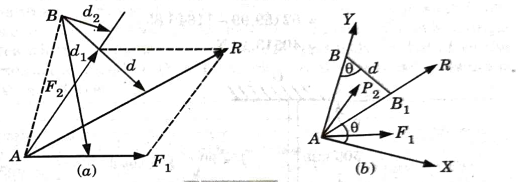



**Fig. 1.13.1.**

4. Similarly, if $F_{1x}$ and $F_{2x}$ are the components of $F_1$ and $F_2$, in $X$ direction, respectively, then

$$F_1 d_1 = AB \times F_{1x} \quad ...(1.13.2)$$

and
$$F_2 d_2 = AB \times F_{2x} \quad ...(1.13.3)$$

5. From eq. (1.13.2) and eq. (1.13.3), we have

$$F_1 d_1 + F_2 d_2 = AB\,(F_{1x} + F_{2x}) = AB \times R_x \quad ...(1.13.4)$$

6. Since, the sum of $x$ components of individual forces is equal to the $x$ component of the resultant $R$. From eq. (1.13.1) and eq. (1.13.4), we can conclude :

$$Rd = F_1 d_1 + F_2 d_2$$

**Que 1.14.** **Calculate the moment of 90 N force about point $O$ for the condition $\theta = 15°$. Also, determine the value of $\theta$ for which the moment about $O$ is zero.**

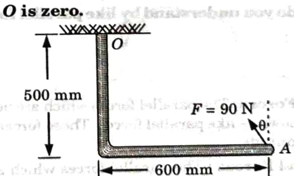



**Fig. 1.14.1.**

**AKTU 2013-14, (II) Marks 05**

**Answer**

**Given :** $\theta = 15°$, $F = 90$ N
**To Find :** i. Moment.
ii. Value of $\theta$.

1. Taking moment about $O$ by 90 N force,

$$M = 90 \cos 15° \times 600 - 90 \sin 15° \times 500$$
$$= 52159.99 - 11646.85$$
$$= 40513.14 \text{ N}$$

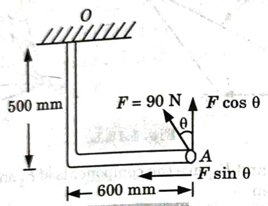



**Fig. 1.14.2.**

2. According to the question, moment about $O$ due to 90 N is zero.

$$\therefore \quad \Sigma M_O = 0$$
$$90 \cos \theta \times 600 - 90 \sin \theta \times 500 = 0$$
$$54 \cos \theta = 45 \sin \theta$$
$$\tan \theta = \frac{54}{45} = \frac{6}{5}$$
$$\tan \theta = 1.2$$
$$\theta = 50.19°$$

**Que 1.15.** **What do you understand by like parallel forces and unlike parallel forces ?**

**Answer**

i. **Like Parallel Forces :** The parallel forces which are acting in the same direction are known as like parallel forces. These forces may be equal or unequal in magnitude.

ii. **Unlike Parallel Forces :** The parallel forces which are acting in the opposite direction are known as unlike parallel forces.



## PART-4

*Couples and Resultant of Force System.*

### CONCEPT OUTLINE

**Couple :** Two parallel forces equal in magnitude and opposite in direction and separated by a definite distance are said to form a couple.

**Resultant of Several Forces :** When a number of coplanar forces are acting on a rigid body, then these forces can be replaced by a single force which has the same effect on the rigid body as that of all the forces acting together, then this single force is known as the resultant of several forces.

### Questions-Answers

**Long Answer Type and Medium Answer Type Questions**

**Que 1.16.** **Derive an expression for the resultant of collinear coplanar forces.**

**Answer**

1. The resultant is obtained by adding all the forces if they are acting in the same direction. If any one of the forces is acting in the opposite direction, then resultant is obtained by subtracting that force.
2. Fig. 1.16.1, shows three collinear coplanar forces $F_1$, $F_2$ and $F_3$ acting on a rigid body in the same direction, their resultant $R$ will be the sum of these forces.

$$\therefore \quad R = F_1 + F_2 + F_3$$

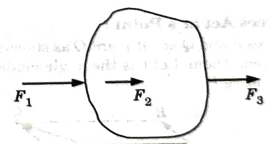



**Fig. 1.16.1.**

3. If any one of these forces (say force $F_2$) is acting in the opposite direction, as shown in Fig. 1.16.2, then their resultant will be given by,

$$R = F_1 - F_2 + F_3$$

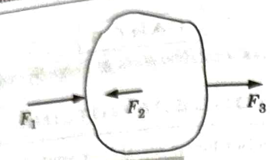



Fig. 1.16.2.

**Que 1.17.** **Three collinear horizontal forces of magnitude 200 N, 100 N and 300 N are acting on a rigid body. Determine the resultant of the forces analytically when**
**i. All the forces are acting in the same direction.**
**ii. The force 100 N acts in the opposite direction.**

**Answer**

**Given :** $F_1 = 200$ N, $F_2 = 100$ N and $F_3 = 300$ N
**To Find :** Resultant, when
i. All the forces are acting in the same direction.
ii. The force 100 N acts in the opposite direction.

1. When all the forces are acting in the same direction, then resultant is given as,

$$R = F_1 + F_2 + F_3 = 200 + 100 + 300 = 600 \text{ N}$$

2. When the force 100 N acts in the opposite direction, then resultant is given as,

$$R = F_1 + F_2 + F_3 = 200 - 100 + 300 = 400 \text{ N}$$

**Que 1.18.** **Derive an expression for the resultant of concurrent coplanar forces when two or more than two forces act on a point.**

**Answer**

**A. When Two Forces Act at a Point :**

1. Suppose two forces $P$ and $Q$ act at point $O$ as shown in Fig. 1.18.1 and $\alpha$ is the angle between them. Let $\theta$ is the angle made by the resultant $R$ with direction of force $P$.

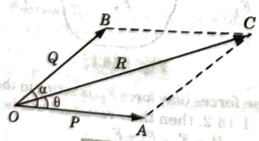



Fig. 1.18.1.



2. Forces $P$ and $Q$ form two sides of a parallelogram and according to the law, the diagonal through the point $O$ gives the resultant $R$ as shown. Thus, the magnitude of resultant is given by,

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$$

3. The direction of the resultant with the force $P$ is given by,

$$\theta = \tan^{-1}\left(\frac{Q \sin \alpha}{P + Q \cos \alpha}\right)$$

**B. When More than Two Forces Act at a Point :**

1. According to this method, all the forces acting at a point are resolved into horizontal and vertical components and then algebraic summation of horizontal and vertical components is done separately.
2. The summation of horizontal component is written as $\Sigma F_H$ and that of vertical $\Sigma F_V$. Then resultant $R$ is given by,

$$R = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2}$$

3. The angle made by the resultant with horizontal is given by,

$$\tan \theta = \frac{\Sigma F_V}{\Sigma F_H}$$

4. Let four forces $F_1$, $F_2$, $F_3$ and $F_4$ act at a point $O$ as shown in Fig. 1.18.2.

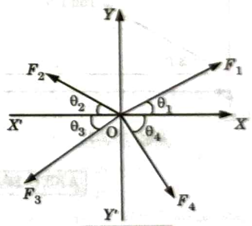



Fig. 1.18.2.

5. The inclination of the forces is indicated with respect to horizontal direction. Let,

$\theta_1$ = Inclination of force $F_1$ with $OX$.
$\theta_2$ = Inclination of force $F_2$ with $OX'$.
$\theta_3$ = Inclination of force $F_3$ with $OX'$.
$\theta_4$ = Inclination of force $F_4$ with $OX$.

6. Summation or algebraic sum of horizontal components,

$$\Sigma F_H = F_1 \cos\theta_1 - F_2 \cos\theta_2 - F_3 \cos\theta_3 + F_4 \cos\theta_4$$

7. Summation or algebraic sum of vertical components,

$$\Sigma F_V = F_1 \sin\theta_1 + F_2 \sin\theta_2 - F_3 \sin\theta_3 - F_4 \sin\theta_4$$

8. Then the resultant will be given by,

$$R = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2}$$

And the angle ($\theta$) made by resultant with $X$-axis is given by,

$$\tan\theta = \frac{(\Sigma F_V)}{(\Sigma F_H)}$$

**Que 1.19.** **The force system applied to an angle bracket is shown in Fig. 1.19.1. Determine the magnitude, direction and line of action of the resultant force.**

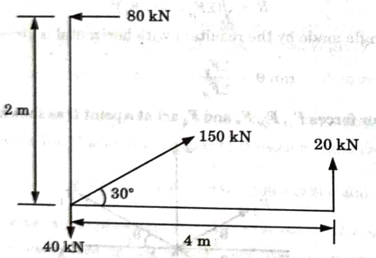



Fig. 1.19.1.

**AKTU 2013-14, (I) Marks 10**

**Answer**

**Given :** Fig. 1.19.1.

**To Find :** Magnitude, direction and line of action of the resultant force.

1. Considering the equilibrium of force system, we have

$$\Sigma F_H = 0 \Rightarrow -80 + 150 \cos 30° + R \cos\theta = 0$$

$$R \cos\theta = -49.9 \text{ kN (towards negative } X\text{-axis)}$$

$$\Sigma F_V = 0 \Rightarrow 150 \sin 30° + R \sin\theta + 20 - 40 = 0$$

$$R \sin\theta = -55 \text{ kN (towards negative } Y\text{-axis)}$$



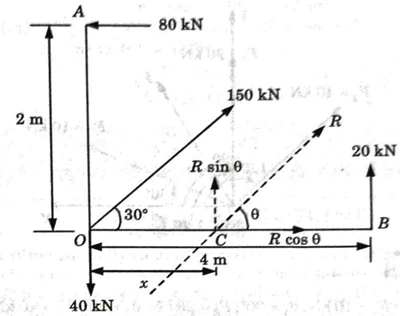



Fig. 1.19.2.

2. Resultant magnitude, $R = \sqrt{(R\cos\theta)^2 + (R\sin\theta)^2}$

$$R = \sqrt{(-49.9)^2 + (-55)^2} = 74.26 \text{ kN}$$

3. Direction of the resultant,

$$\tan\theta = \frac{R\sin\theta}{R\cos\theta} = \frac{-55}{-49.9}$$

$$\tan\theta = 1.1022$$

$$\theta = 47.78°$$

4. Now for line of action of the resultant taking moment about $O$, we have

$$\Sigma M_O = 0$$

$$80 \times 2 - R\sin\theta \times OC + 20 \times 4 = 0$$

$$160 - 74.26 \times \sin 47.78° \times x + 80 = 0$$

$$x = 4.36 \text{ m}$$

Resultant will act at a distance 4.36 m from point $O$ towards $B$ and it will lie outside the frame.

**Que 1.20.** **The resultant of four forces which are acting at a point $O$ as shown in Fig. 1.20.1 is along $Y$-axis. The magnitude of forces $F_1$, $F_3$ and $F_4$ are 10 kN, 20 kN and 40 kN respectively. The angles made by 10 kN, 20 kN and 40 kN with $X$-axis are 30°, 90° and 120° respectively. Find the magnitude and direction of force $F_2$ if resultant is 72 kN.**

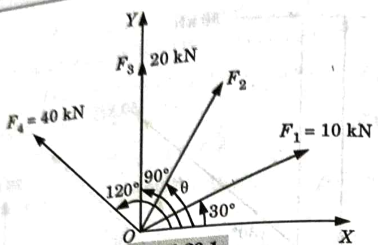



Fig. 1.20.1.

**Answer**

**Given :** $F_1 = 10$ kN, $\theta_1 = 30°$, $F_3 = 20$ kN, $\theta_3 = 90°$, $F_4 = 40$ kN, $\theta_4 = 120°$, $R = 72$ kN
**To Find :** Magnitude and direction of force $F_2$.

1. Resultant is along $Y$-axis hence the algebraic sum of horizontal component should be zero and algebraic sum of vertical components should be equal to the resultant.

$\therefore$ $\quad \Sigma F_H = 0$ and $\Sigma F_V = R = 72$ KN

2. But $\quad \Sigma F_H = F_1 \cos 30° + F_2 \cos \theta + F_3 \cos 90° + F_4 \cos 120°$

$$= 10 \times 0.866 + F_2 \cos \theta + 20 \times 0 + 40 \times \left(-\frac{1}{2}\right)$$

$$= 8.66 + F_2 \cos \theta + 0 - 20$$

$$= F_2 \cos \theta - 11.34$$

$\therefore$ $\quad \Sigma F_H = 0$

$$F_2 \cos \theta - 11.34 = 0$$

$$F_2 \cos \theta = 11.34 \quad ...(1.20.1)$$

3. Now, $\quad \Sigma F_V = F_1 \sin 30° + F_2 \sin \theta + F_3 \sin 90° + F_4 \sin 120°$

$$= 10 \times \frac{1}{2} + F_2 \sin \theta + 20 \times 1 + 40 \times 0.866$$

$$= 5 + F_2 \sin \theta + 20 + 34.64$$

$$= F_2 \sin \theta + 59.64$$

4. But $\quad \Sigma F_V = R$

$\therefore$ $\quad F_2 \sin \theta + 59.64 = 72$

$$F_2 \sin \theta = 72 - 59.64 = 12.36 \quad ...(1.20.2)$$

5. Dividing eq. (1.20.2) by the eq. (1.20.1), we get

$$\frac{F_2 \sin \theta}{F_2 \cos \theta} = \frac{12.36}{11.34} \text{ or } \tan \theta = 1.0899$$



$\therefore$ $\quad \theta = \tan^{-1} 1.0899 = 47.46°$

6. Substituting the value of θ in eq. (1.20.2), we get

$$F_2 \sin (47.46°) = 12.36$$

$$F_2 = \frac{12.36}{\sin(47.46°)} = \frac{12.36}{0.7368} = 16.77 \text{ kN}$$

## PART-5

*Equilibrium of System of Forces, Free Body Diagrams.*

### CONCEPT OUTLINE

**Equilibrium of System of Forces :** When some external forces act on a body but it does not start moving and also does not start rotating about any point, then the body is said to be in equilibrium.

**Free Body Diagram :** A diagram in which the body under consideration is freed from all the contact surfaces and all the forces acting on it are shown on it, is known as free body diagram (FBD).

### Questions-Answers

**Long Answer Type and Medium Answer Type Questions**

**Que 1.21.** **State and prove Lami's Theorem.**

**AKTU 2011-12, Marks 05**

**Answer**

A. **Statement :** Lami's theorem states that if three forces acting at a point are in equilibrium, then each force will be proportional to the sine of the angle between the other two forces.

B. **Proof of Lami's Theorem :**

1. The three forces acting on a point are in equilibrium and hence they can be represented by the three sides of the triangle taken in the same order.
2. Now draw the force triangle as shown in Fig. 1.21.1(*b*).
3. Now applying sine rule, we get

$$\frac{P}{\sin(180° - \beta)} = \frac{Q}{\sin(180° - \gamma)} = \frac{R}{\sin(180° - \alpha)}$$

4. This can also be written as,

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha}$$

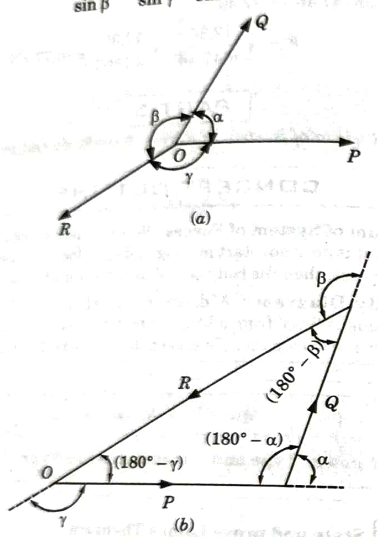



**Fig. 1.21.1.**

**Que 1.22.** **Write in short about principle of equilibrium.**

**Answer**

1. The principle of equilibrium states that, a stationary body which is subjected to coplanar forces (concurrent or parallel) will be in equilibrium if the algebraic sum of all the external forces is zero and also the algebraic sum of moments of all the external forces about any point in their plane is zero.
2. Mathematically, it is expressed by the following equations
$$\Sigma F = 0 \quad ...(1.22.1)$$
$$\Sigma M = 0 \quad ...(1.22.2)$$
3. The eq. (1.22.1) is also known as force law of equilibrium whereas the eq. (1.22.2) is known as moment law of equilibrium.
4. The forces are generally resolved into horizontal and vertical components. Hence eq. (1.22.1) is written as



$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

where, $\Sigma F_x$ = Algebraic sum of all horizontal components.

$\Sigma F_y$ = Algebraic sum of all vertical components.

**Que 1.23.** **Two slender rods of negligible weight are pin connected at *C* and attached to two blocks. *A* and *B* each of weight 100 N is shown in Fig. 1.23.1. If coefficient of friction is 0.3 at all surfaces of contact, find largest value of *P* for which equilibrium is maintained.**

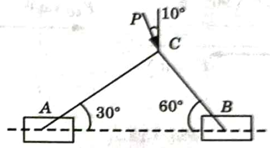



**Fig. 1.23.1.**

**AKTU 2013-14, (I) Marks 05**

**Answer**

**Given :** $W = 100$ N, $\mu = 0.3$
**To Find :** Value of $P$.

1. Considering FBD of pin $C$ (Fig. 1.23.2), we have
$$F_{CB} = P\cos 10°$$
$$F_{CA} = P\sin 10°$$

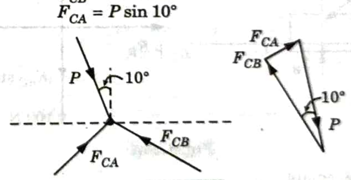



**Fig. 1.23.2.**

2. Now, considering the FBD of block $A$ (Fig. 1.23.3).
3. For vertical force equilibrium,
$$\Sigma F_V = 0$$
$$R_A - 100 - F_{AC}\cos\theta = 0$$
$$R_A = 100 + P\sin 10°\cos\theta \quad [\because F_{AC} = F_{CA}]$$
4. Now, $\Sigma F_H = 0,$
$$0.3\,R_A - F_{AC}\sin\theta = 0$$
$$0.3\,[100 + P\sin 10°\cos\theta] - P\sin 10°\sin\theta = 0$$

$$30 + 0.3\,P \sin 10^\circ \cos \theta - P \sin 10^\circ \sin \theta = 0$$

$$P \sin 10^\circ (\sin \theta - 0.3 \cos \theta) = 30$$

$$P = \frac{30}{\sin 10^\circ(\sin \theta - 0.3 \cos \theta)}$$

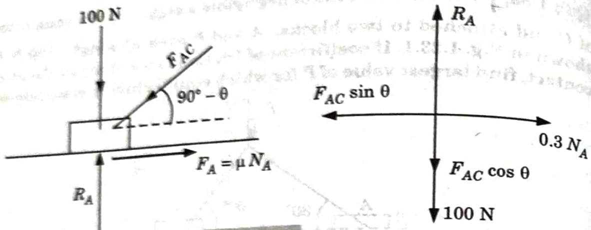



Fig. 1.23.3.

5. Let,

$\theta = 60°$, so

$$P = \frac{30}{\sin 10^\circ(\sin 60^\circ - 0.3 \cos 60^\circ)}$$

$$P = 241.28 \text{ N}$$

6. Considering FBD of block $B$ (Fig. 1.23.4).

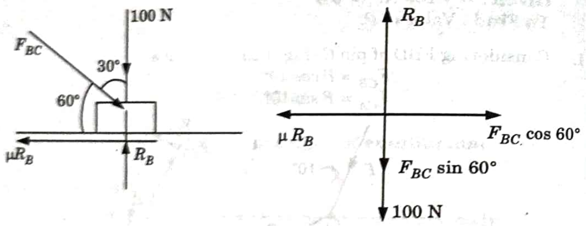



Fig. 1.23.4.

7. For vertical equilibrium,

$$100 + F_{BC} \sin 60^\circ = R_B$$

$$R_B = 100 + P \cos 10^\circ \sin 60^\circ \qquad \{\because F_{BC} = F_{CB}\}$$

8. For horizontal equilibrium,

$$0.3\,R_B - F_{BC} \cos 60^\circ = 0$$

$$0.3\,(100 + P \cos 10^\circ \sin 60^\circ) - P \cos 10^\circ \cos 60^\circ = 0$$

$$30 + P \cos 10^\circ\,(0.3 \sin 60^\circ - \cos 60^\circ) = 0$$

$$P = \frac{30}{\cos 10^\circ(\cos 60^\circ - 0.3 \sin 60^\circ)}$$



$$P = 126.83 \text{ N}$$

9. So the largest value of $P$ for which equilibrium is maintained will be,

$$P = 126.83 \text{ N}$$

**Que 1.24.** **Two smooth spheres each of radius 100 mm and weight 100 N, rest in a horizontal channel having vertical walls, the distance between which is 360 mm. Find the reactions at the points of contacts *A*, *B*, *C*, and *D* shown in Fig. 1.24.1 below.**

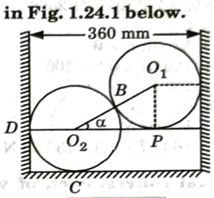



Fig. 1.24.1.

**AKTU 2016-17, (II) Marks 07**

**Answer**

**Given :** $r = 100$ mm $= 0.1$ m, $W = 100$ N, $l = 360$ mm $= 0.36$ m
**To Find :** Reaction at $A$, $B$, $C$ and $D$.

1. From Fig. 1.24.1, we have

$$\cos \alpha = \frac{O_2P}{O_1O_2} = \frac{360 - O_1A - O_2D}{O_1B + O_2B}$$

$$= \frac{360 - 100 - 100}{100 + 100} = \frac{160}{200}$$

$$\cos \alpha = 0.8$$

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - (0.8)^2}$$

$$= \sqrt{0.36} = 0.6$$

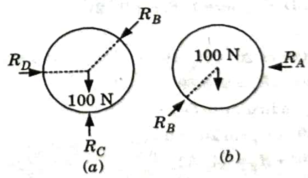



Fig. 1.24.2.

2. Considering FBD of sphere 1 [Fig. 1.24.2(*b*)].

$$\Sigma F_V = 0$$
$$R_B \times \sin\alpha = W$$
$$R_B \times 0.6 = 100$$
$$R_B = 166.67 \text{ N}$$
$$\Sigma F_H = 0$$
$$R_A = R_B \times \cos\alpha$$
$$R_A = 166.667 \times 0.8 = 133.33 \text{ N}$$

3. Considering FBD of sphere 2 [Fig. 1.24.2(*a*)].

$$\Sigma F_V = 0$$
$$R_C = R_B \sin\alpha + W$$
$$R_C = 166.67 \times 0.6 + 100$$
$$R_C = 200 \text{ N}$$
$$\Sigma F_H = 0$$
$$R_D = R_B \cos\alpha$$
$$R_D = 166.67 \times 0.8 = 133.33 \text{ N}$$

**Que 1.25.** **Two identical rollers, each of weights 1000 N are supported by an inclined plane as shown in Fig. 1.25.1. Assuming smooth surfaces, find the reactions induced at the points of supports.**

**AKTU 2015-16, (I) Marks 10**

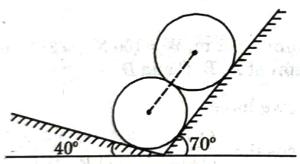



**Fig. 1.25.1.**

**Answer**

**Given :** Fig. 1.25.1, w = 1000 N
**To Find :** Reactions at the point of supports

1. Considering FBD of sphere 1 (Fig. 1.25.2).

Along axis *OO′* :

$$R_1 \cos 20° - 1000 \cos 20° - R_4 = 0$$
$$R_1 \cos 20° - R_4 = 939.69 \qquad ...(1.25.1)$$

Along axis perpendicular to *OO′* :

$$R_2 - 1000 \sin 20° - R_1 \sin 20° = 0$$
$$-R_1 \sin 20° + R_2 = 342.02 \qquad ...(1.25.2)$$



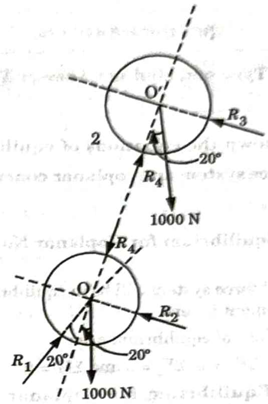



**Fig. 1.25.2.**

2. Considering FBD of sphere 2 (Fig. 1.25.2).

Along axis *OO′* :

$$R_4 - 1000 \cos 20° = 0$$
$$R_4 = 939.69 \text{ N} \qquad ...(1.25.3)$$

Along axis perpendicular to *OO′* :

$$R_3 - 1000 \sin 20° = 0$$
$$R_3 = 342.02 \text{ N} \qquad ...(1.25.5)$$

3. On putting the value of $R_4$ in eq. (1.25.1) from eq. (1.25.3), we get

$$R_1 \cos 20° = 939.69 + 939.69$$
$$R_1 = \frac{1879.38}{\cos 20°}$$
$$R_1 = 1999.99$$
$$R_1 \approx 2000 \text{ N}$$

4. Now putting the value of $R_1$ in eq. (1.25.2), we get

$$R_2 = 342.0201 + 2000 \sin 20°$$
$$R_2 = 1026.06 \text{ N}$$

## PART-6

***Equations of Equilibrium of Coplanar Systems.***

## Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 1.26.** **Write down the equations of equilibrium for coplanar non-concurrent force system and coplanar concurrent force system.**

**Answer**

i. **Equations of Equilibrium for Coplanar Non-concurrent Force System :**

1. A non-concurrent force system will be in equilibrium if the resultant of all forces and moment is zero.
2. Hence the equations of equilibrium are :

$$\Sigma F_x = 0, \Sigma F_y = 0 \text{ and } \Sigma M = 0$$

ii. **Equations of Equilibrium for Coplanar Concurrent Force System :**

1. For the concurrent forces, the lines of action of all forces meet at a point, and hence the moment of those forces about that point will be zero or $\Sigma M = 0$ automatically.
2. Thus for concurrent force system, the condition $\Sigma M = 0$ becomes redundant and only two conditions, *i.e.*, $\Sigma F_x = 0$ and $\Sigma F_y = 0$ are required.

**Que 1.27.** **Three parallel forces $F_1$, $F_2$ and $F_3$ are acting on a body as shown in Fig. 1.27.1 and the body is in equilibrium. If force $F_1$ = 250 N and $F_3$ = 1000 N and the distance between $F_1$ and $F_2$ = 1.0 m, then determine the magnitude of force $F_2$ and the distance of $F_2$ from force $F_3$.**

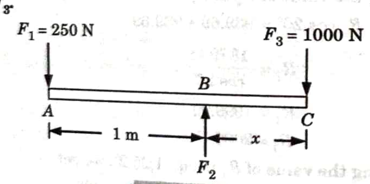



**Fig. 1.27.1.**

**Answer**

Given : $F_1$ = 250 N, $F_3$ = 1000 N, $AB$ = 1.0 m
To Find : $F_2$ and $BC$.



1. For the equilibrium of the body, the resultant force in the vertical direction should be zero.

$$\therefore \quad \Sigma F_V = 0$$
$$F_1 + F_3 - F_2 = 0$$
$$250 + 1000 - F_2 = 0$$
$$F_2 = 250 + 1000 = 1250 \text{ N}$$

2. For the equilibrium of the body, the moment of all forces about any point must be zero. Taking moments of all forces about $A$ and considering distance $BC = x$, we have

$$F_2 \times AB - AC \times F_3 = 0$$
$$1250 \times 1 - (1 + x) \times 1000 = 0 \qquad (\because AC = AB + BC = 1 + x)$$
$$250 = 1000\,x$$
$$x = \frac{250}{1000} = 0.25 \text{ m}$$

## PART-7

*Friction, Types of Friction, Limiting Friction, Laws of Friction Static and Dynamic Friction, Motion of Bodies.*

### CONCEPT OUTLINE

**Force of Friction :** When a solid body slides over a stationary solid body, a force is exerted at the surface of contact by the stationary body on the moving body, this force is called force of friction.

**Static Friction :** The force of friction up to which body does not move is called static friction.

**Limiting Friction :** The force of friction at which body just tends to start moving is called limiting friction.

**Kinetic Friction :** The force of friction acting on the body when the body is moving is called kinetic friction.

## Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 1.28.** **Define friction. Also explain its types.**

**Answer**

A. **Friction :** The property of the bodies by virtue of which a force is exerted by a stationary body on the moving body to resist the motion of the moving body is called friction. Friction acts parallel to the surface of contact and depends upon the nature of surface of contact.

B. **Types of Friction :**

a. **Static and Dynamic Friction :** If the two surfaces which are in contact, are at rest, the force experienced by one surface is called static friction. But if one suface starts moving and the other is at rest, the force experienced by the moving surface is called dynamic friction.

b. **Wet and Dry Friction :** If between two surfaces, which are in contact, lubrication is used, the fricion, that exists between two surfaces is known as wet friction. But if no lubricantion is used, then the friction between two surfaces is called dry friction or solid friction.

**Que 1.29.** **Write down the laws of friction.**

**Answer**

Following are the laws of friction :

1. The force of friction acts in the opposite direction in which surface is having tendency to move.
2. The force of friction is equal to the force applied to the surface, so long as the surface is at rest.
3. The limiting frictional force bears a constant ratio to the normal reaction between two surfaces.
4. The limiting frictional force does not depend upon the shape and areas of the surfaces in contact.
5. The ratio between limiting friction and normal reaction is slightly less when the two surfaces are in motion.
6. The force of friction is independent of the velocity of sliding.

**Que 1.30.** **Define the following terms :**

i. **Coefficient of friction,**
ii. **Angle of friction, and**
iii. **Angle of repose.**

**Answer**

i. **Coefficient of Friction :** It is defined as the ratio of the limiting force of friction ($F$) to the normal reaction ($R$) between two bodies. It is denoted by $\mu$.

Mathematically, $$\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}} = \frac{F}{R}$$



ii. **Angle of Friction :** It is defined as the angle made by the resultant of the normal reaction ($R$) and the limiting force of friction ($F$) with the normal reaction ($R$). It is denoted by $\phi$.

Mathematically, $\tan \phi = \dfrac{F}{R} = \dfrac{\mu R}{R} = \mu$

iii. **Angle of Repose :** It is defined as the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by the assistance of friction only.

Also, Angle of repose = Angle of friction

**Que 1.31.** **Two blocks, as shown in Fig. 1.31.1 slide down at 30° incline. If coefficient of friction at all contact surfaces is 0.2, determine the pressure between the blocks.**

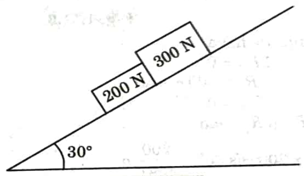



**Fig. 1.31.1.**

**AKTU 2013-14, (I) Marks 10**

**Answer**

**Given :** $\mu = 0.2$, $\theta = 30°$, Weight of blocks = 200 N and 300 N
**To Find :** Pressure between two blocks.

1. Considering FBD of block of 300 N (Fig. 1.33.2).

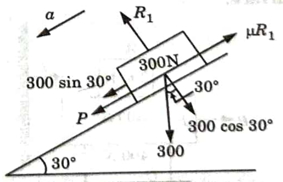



**Fig. 1.31.2.**

2. For equilibrium, we have

$$\Sigma F_V = 0$$
$$R_1 = 300 \cos 30°$$

Also
$$\Sigma F_H = 0$$
$$300 \sin 30° + P - \mu R_1 = ma$$

$$300 \sin 30° + P - 0.2 \times 300 \cos 30° = \frac{300}{9.81} a \qquad \left(\because m = \frac{W}{g}\right)$$

$$\frac{300}{9.81} a - P = 98.04 \qquad ...(1.31.1)$$

3. Now considering the FBD of block of 200 N (Fig. 1.31.3).

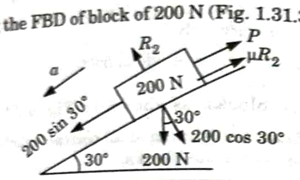



**Fig. 1.31.3.**

4. For equilibrium, we have
$$\Sigma F_V = 0$$
$$R_2 = 200 \cos 30°$$
$$\Sigma F_H = 0$$
$$200 \sin 30° - P - \mu R_2 = ma$$

$$100 - P - 0.2 \times 200 \cos 30° = \frac{200}{9.81} a$$

$$\frac{200}{9.81} a + P = 65.36 \qquad ...(1.31.2)$$

5. After solving eq. (1.31.1) and eq. (1.31.2), we have
$$a = 3.206 \text{ m/sec}^2 \text{ and } P = 0$$

So, no pressure will act between the blocks.

**Que 1.32.** **Determine the force $P$ required to impend the motion of the block $B$ shown in Fig. 1.32.1. Take coefficient of friction as 0.3 for all contact surface.**

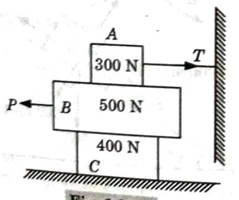



**Fig. 1.32.1.**

**AKTU 2014-15, (II) Marks 10**



**Answer**

**Given :** $W_A = 300$ N, $W_B = 500$ N, $W_C = 400$ N, $\mu = 0.3$

**To Find :** Value of $P$.

1. Considering the FBD of block $A$ (Fig. 1.32.2).
$$\Sigma F_V = 0$$
$$R_1 = 300 \text{ N}$$
Since $F_1$ is limiting friction,
$$\therefore \quad F_1 = \mu R_1 = 0.3 \times 300 = 90 \text{ N}$$
$$\Sigma F_H = 0, \text{ gives}$$
$$T = F_1 = 90 \text{ N}$$

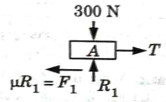



**Fig. 1.32.2.**

2. Considering the FBD of block $B$ (Fig. 1.32.3).

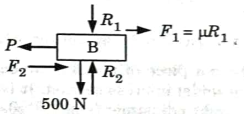



**Fig. 1.32.3.**

$$\Sigma F_V = 0$$
$$R_2 - 500 - R_1 = 0$$
$$R_2 - 500 - 300 = 0$$
$$R_2 = 800 \text{ N}$$
$$\therefore \quad F_2 = \mu R_2 = 0.3 \times 800 = 240 \text{ N}$$
$$\Sigma F_H = 0$$
$$P = F_1 + F_2$$
$$P = 240 + 90$$
$$P = 330 \text{ N}$$

**Que 1.33.** **What are the different types of motion of bodies ?**

**Answer**

Following are the different types of motion of bodies :

i. **Linear Motion :** When a body moves in a straight line only, the motion is called linear motion.

ii. **Curvilinear Motion :** When a body moves along a curved path, the motion is called curvilinear motion.

iii. **Rectilinear Motion :** When a body posses both linear and circular motion, it is said to be in rectilinear motion.

iv. **Periodic Motion :** When the motion of a body repeats over a period of time, it is called periodic motion.

v. **Oscillatory Motion :** To and fro motion of a body about a point is called oscillatory motion.

## PART-8

*Wedge Friction.*

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 1.34.** **Define wedge and discuss about the equilibrium of body placed on wedge.**

**Answer**

A. **Wedge :** A wedge is a piece of metal or wood which is usually of a triangular or trapezoidal in cross-section. It is used for either lifting loads or used for slight adjustments in the position of a body *i.e.*, for tightening fits or keys for shafts.

B. **Equilibrium of Body Placed on Wedge :**

1. Considering the equilibrium of the wedge. The forces acting on the wedge are shown in Fig. 1.34.1. They are :
   i. The force $P$ applied horizontally on face $BC$.
   ii. Reaction $R_1$ on the face $AC$ (The reaction $R_1$ is the resultant of normal reaction on the rubbing face $AC$ and force of friction on surface $AC$). The reaction $R_1$ will be inclined at an angle $\phi_1$ with the normal.
   iii. Reaction $R_2$ on the face $AB$ (The reaction $R_2$ is the resultant of normal reaction on the rubbing face $AB$ and force of friction on surface $AB$). The reaction $R_2$ will be inclined at an angle $\phi_2$ with the normal.
2. When the force $P$ is applied on the wedge, the surface $CA$ will be moving towards left and hence force of friction on this surface will be acting towards right.



3. Similarly, the force of friction on face $AB$ will be acting from $A$ to $B$. These forces are shown in Fig. 1.34.1.
4. Resolving the forces horizontally, we get

   $R_1 \sin \phi_1 + R_2 \sin (\phi_2 + \alpha) = P$

   Resolving the forces vertically, we get

   $R_1 \cos \phi_1 = R_2 \cos (\phi_2 + \alpha)$

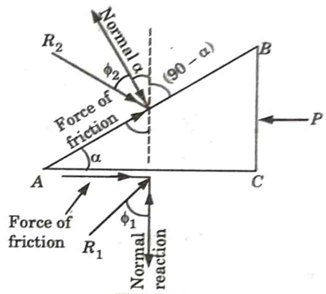



**Fig. 1.34.1.**

**Que 1.35.** **A uniform ladder 5 m long weighs 180 N. It is placed against a wall making an angle of 60° with floor. The coefficient of friction between the wall and ladder is 0.25 and between the floor and the ladder is 0.35. The ladder has to support a mass 900 N at its top. Calculate the horizontal force $P$ to be applied to the ladder at the floor level to prevent slipping.** **AKTU 2014-15, (II) Marks 10**

**Answer**

**Given :** $W_1 = 180$ N, $W_2 = 900$ N, $\mu_a = 0.35$, $\mu_b = 0.25$, $l = 5$ m, $\alpha = 60°$
**To Find :** Horizontal force $P$ to prevent slipping.

1. According to Fig. 1.35.1 for the ladder $AB$ placed against a wall and various force acting on it. $P$ is the horizontal force which has been applied on the ground level to prevent slipping.
2. Resolving all the forces along horizontal and vertical directions, we have

$$P + \mu_a R_a = R_b \quad ...(1.35.1)$$

$$R_a + \mu_b R_b = W_1 + W_2 = 180 + 900 = 1080 \text{ N} \quad ...(1.35.2)$$

3. Taking moments about the end $A$,

$$W_2 \times OA + W_1 \times DA = R_b \times OB + \mu_b R_b \times OA$$

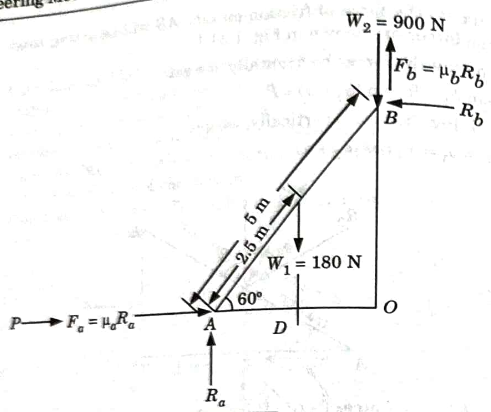



Fig. 1.35.1.

4. From the geometrical configuration,

$$OA = 5 \cos 60° = 2.5 \text{ m}, DA = 2.5 \cos 60° = 1.25 \text{ m}$$

$$OB = 5 \sin 60° = 4.33 \text{ m}$$

$$\therefore \; 900 \times 2.5 + 180 \times 1.25 = R_b \times 4.33 + 0.25 R_b \times 2.5$$

$$R_b (4.955) = 2475$$

$$R_b = \frac{2475}{4.955} = 499.495 \text{ N}$$

5. From eq. (1.35.2) and eq. (1.35.1), we have

$$R_a = 1080 - \mu_b R_b = 1080 - 0.25 \times 499.495$$

$$R_a = 955.13 \text{ N}$$

$$P = R_b - \mu_a R_a = 499.495 - 0.35 \times 955.13$$

$$P = 165.2 \text{ N}$$

## PART-9

*Screw Jack and Differential Screw Jack.*

### CONCEPT OUTLINE

**Screw Jack :** It is a device used for lifting heavy weights or loads with the help of a small effort applied at its handle.



## Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 1.36.** **Derive an expression for the effort applied to lift or lower the load.**

**Answer**

**I. Effort Applied at the End of Handle to Lift the Load :**

1. Let, $W$ = Weight placed on the screw head,
   $P$ = Effort applied at the end of the handle,
   $L$ = Length of handle,
   $p$ = Pitch of the screw,
   $d$ = Mean diameter of the screw,
   $\alpha$ = Angle of the screw or helix angle,
   $\phi$ = Angle of friction, and
   $\mu$ = Coefficient of friction between screw and nut = $\tan \phi$

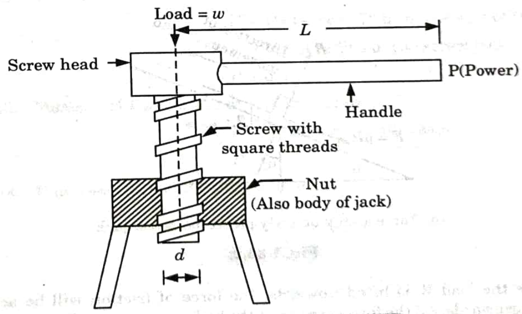



Fig. 1.36.1. Simple screw-jack.

2. When the handle is rotated through one complete turn, the screw is also rotated through one turn. Then the load is lifted by a height $p$ (pitch of screw).

3. The development of one complete turn of a screw thread is shown in Fig. 1.36.2(*a*). This is similar to the inclined plane. The distance *AB* will be equal to the circumference ($\pi d$) and distance *BC* will be equal to the pitch ($p$) of the screw.

4. From the Fig. 1.36.2(*a*), we have

$$\tan \alpha = \frac{BC}{AC} = \frac{p}{\pi d} \quad \text{...(1.36.1)}$$

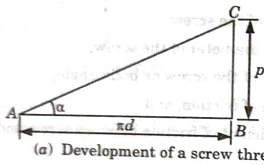
5. Let,

$P'$ = Effort applied horizontally at the mean radius of the screw jack to lift the load $W$,

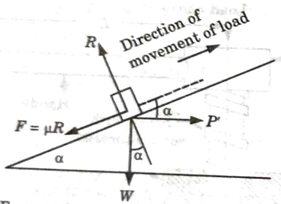
$r$ = Mean radius of the screw jack = $d/2$,

$R$ = Normal reaction, and

$F$ = Force of friction = $\mu R$.




(*a*) Development of a screw thread




(*b*) Force acting on body placed on screw jack

**Fig. 1.36.2.**

6. As the load $W$ is lifted upwards, the force of friction will be acting downwards. All the forces acting on the body are shown in Fig. 1.36.2(*b*).

7. Resolving forces along the inclined plane, we have

$$F + W \sin \alpha = P' \cos \alpha$$

$$\mu R + W \sin \alpha = P' \cos \alpha \quad (\because F = \mu R) \quad \text{... (1.36.2)}$$



8. Resolving forces normal to the inclined plane, we have

$$R = W \cos \alpha + P' \sin \alpha$$

9. Substituting the value of $R$ in eq. (1.36.2), we get

$$\mu(W \cos \alpha + P' \sin \alpha) + W \sin \alpha = P' \cos \alpha$$

$$\frac{\sin \phi}{\cos \phi}(W \cos \alpha + P' \sin \alpha) + W \sin \alpha = P' \cos \alpha \quad \left(\because \mu = \tan \phi = \frac{\sin \phi}{\cos \phi}\right)$$

$$W\frac{\sin \phi \cos \alpha}{\cos \phi} + P'\frac{\sin \phi \sin \alpha}{\cos \phi} + W \sin \alpha = P' \cos \alpha$$

10. Multiplying by $\cos \phi$, we get

$$W \sin \phi \cos \alpha + P' \sin \phi \sin \alpha + W \sin \alpha \cos \phi = P' \cos \alpha \cos \phi$$

$$W (\sin \phi \cos \alpha + \sin \alpha \cos \phi) = P' (\cos \alpha \cos \phi - \sin \alpha \sin \phi)$$

$$W \sin(\alpha + \phi) = P' \cos(\alpha + \phi)$$

$$P' = W \frac{\sin (\alpha + \phi)}{\cos(\alpha + \phi)} = W \tan (\alpha + \phi) \quad \text{...(1.36.3)}$$

11. Now $P'$ is the effort applied at the mean radius of the screw-jack. But in case of screw-jack, effort is actually applied at the end of the handle as shown in Fig. 1.36.1. The effort applied at the end of the handle is $P$.

12. Moment of $P'$ about the axis of the screw

= $P'$ × Distance of $P'$ from the axis of the screw

= $P'$ × Mean radius of the screw jack

= $P' \times d/2$

13. Moment of $P$ about the axis of the screw

= $P$ × Distance of $P$ from axis

= $P \times L$

14. Equating the two moments, we get

$$P' \times \frac{d}{2} = P \times L$$

$$\therefore \quad P = P' \times \frac{d}{2L} = \frac{P}{2L} \times P' \quad \text{...(1.36.4)}$$

15. Substituting the value of $P'$ from eq. (1.36.3) into eq. (1.36.4), we get

$$P = \frac{d}{2L} \times W \tan (\alpha + \phi) \quad \text{...(1.36.5)}$$

Eq. (1.36.5) gives the relation between the effort required at the end of the handle and the load lifted.

16. Torque required to work the jack, $T = PL = \frac{d}{2} W \tan(\alpha + \phi)$

17. Now,
$$P = \frac{d}{2L} W \tan(\alpha + \phi)$$
$$= \frac{Wd}{2L}\left(\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi}\right)$$
$$= \frac{Wd}{2L}\left(\frac{\frac{p}{\pi d} + \mu}{1 - \frac{p}{\pi d}\mu}\right) \qquad \left(\because \tan\alpha = \frac{p}{\pi d}, \tan\phi = \mu\right)$$
$$= \frac{Wd}{2L}\left(\frac{p + \mu\pi d}{\pi d - p\mu}\right) \qquad ...(1.36.6)$$

Eq. (1.36.6) gives the value of $P$ in terms of coefficient of friction and pitch of the screw.

**II. Effort Required at the End of Screw Jack to Lower the Load :**

1. The screw jack is also used for lowering the heavy load. When the load is lowered by the screw jack, the force of friction ($F = \mu R$) will act upwards. Fig. 1.36.3 shows all the forces acting on the body.

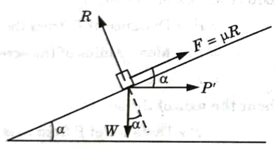



**Fig. 1.36.3.** Body moving down.

2. Resolving forces along the inclined plane,
$$F + P' \cos\alpha = W \sin\alpha$$
$$\mu R + P' \cos\alpha = W \sin\alpha \qquad ...(1.36.7)$$

3. Resolving forces normal to the plane
$$R = W\cos\alpha + P' \sin\alpha$$

4. Substituting the value of $R$ in eq. (1.36.7), we get
$$\mu(W\cos\alpha + P'\sin\alpha) + P'\cos\alpha = W\sin\alpha$$
$$\mu W\cos\alpha + \mu P'\sin\alpha + P'\cos\alpha = W\sin\alpha$$
$$\mu P'\sin\alpha + P'\cos\alpha = W\sin\alpha - \mu W\cos\alpha$$



$$P'(\mu\sin\alpha + \cos\alpha) = W(\sin\alpha - \mu\cos\alpha)$$
$$P'\left[\frac{\sin\phi}{\cos\phi}\sin\alpha + \cos\alpha\right] = W\left[\sin\alpha - \frac{\sin\phi}{\cos\phi}\cos\alpha\right] \qquad \left(\because \mu = \tan\phi = \frac{\sin\phi}{\cos\phi}\right)$$

5. Multiplying by $\cos\phi$, we get
$$P'(\sin\phi\sin\alpha + \cos\alpha\cos\phi) = W(\sin\alpha\cos\phi - \sin\phi\cos\alpha)$$
$$P'[\cos(\phi - \alpha)] = W[\sin(\phi - \alpha)]$$
$$\therefore \quad P' = W\frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)} = W\tan(\phi - \alpha)$$

If $\alpha > \phi$, then $\quad P' = W\tan(\alpha - \phi) \qquad ...(1.36.8)$

6. But $P'$ is the effort applied at the mean radius of the screw jack. But in actual case, effort is applied at the handle of the jack. Let the effort applied at the handle is $P$. Equating the moment of $P$ and $P'$ about the axis of the jack, we get
$$P \times L = P' \times \frac{d}{2}$$
$$\therefore \quad P = \frac{d}{2L} \times P' = \frac{d}{2L} \times W\tan(\phi - \alpha) \qquad ...(1.36.9)$$

Eq. (1.36.9) gives the relation between the efforts required at the end of the handle to lower the load ($W$).

7. Expression for $P$ in terms of coefficient of friction and pitch of the screw,
$$P = \frac{Wd}{2L}\tan(\phi - \alpha) = \frac{Wd}{2L}\left(\frac{\tan\phi - \tan\alpha}{1 + \tan\phi\tan\alpha}\right)$$
$$= \frac{Wd}{2L}\left(\frac{u - \frac{p}{\pi d}}{1 + \mu\frac{p}{\pi d}}\right) \qquad \left(\because \tan\phi = \mu, \tan\alpha = \frac{d}{\pi d}\right)$$
$$= \frac{Wd}{2L}\left(\frac{\mu\pi d - p}{\pi d + \mu p}\right)$$

**Que 1.40.**

a. **Find the effort required to apply at the end of a handle, fitted to the screw head of screw jack to lift a load of 1500 N. The length of the handle is 70 cm. The mean diameter and the pitch of the screw jack are 6 cm and 0.9 cm respectively. The coefficient of friction is given as 0.095.**

b. **If instead of raising the load of 1500 N, the same load is lowered, determine the effort required so apply at the end of the handle.**

**Answer**

**Given :** $W = 1500$ N, $L = 70$ cm $= 0.7$ m, $d = 6$ cm $= 0.06$ m
$p = 0.9$ cm $= 0.009$ m, $\mu = 0.095$
**To Find :** i. Effort required to raise the load.
ii. Effort required to lower the load.

1. Effort required to raise the load is given by,

$$P = \frac{Wd}{2L}\left(\frac{p + \mu\pi d}{\pi d - p\mu}\right)$$

$$= \frac{1500 \times 0.06}{2 \times 0.70}\left(\frac{0.009 + 0.095 \times \pi \times 0.06}{\pi \times 0.06 - 0.009 \times 0.095}\right) = 9.22 \text{ N}$$

2. Effort required for lowering the load is given by,

$$P = \frac{Wd}{2L}\left(\frac{\mu\pi d - p}{\pi d + \mu p}\right)$$

$$= \frac{1500 \times 0.06}{2 \times 0.70}\left(\frac{0.095 \times \pi \times 0.06 - 0.009}{\pi \times 0.06 + 0.009 \times 0.095}\right)$$

$$= 3.024 \text{ N}$$

**Que 1.38.** **Write a short note on differential screw jack with neat diagram.**

**Answer**

1. Differential screw jack consists of two spindles $A$ and $B$. $B$ externally threaded and $A$ both internally and externally threaded.
2. The internal threads of spindle $A$ meshes with internal threads of spindle $B$. Spindle $A$ is screwed to fixed base.
3. When the lever is rotated such that spindle $A$ rises, spindle $B$ also rotates and it will come down.



4. Velocity ratio, $VR = \dfrac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$

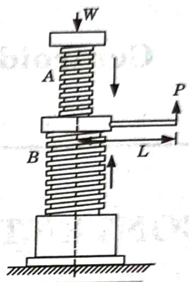



**Fig. 1.38.1.**

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2

UNIT

# Centroid and Centre of Gravity

## CONTENTS







## PART-1

*Centroid, Centre of Gravity, Centroid of Simple Figures from First Principle.*

### CONCEPT OUTLINE

**Centre of Gravity :** It is the point at which the whole weight of the body acts. A body is having only one centre of gravity for all positions of the body.

**Centroid :** The point at which the total area of a plane figure (like triangle, rectangle, circle, etc.) is assumed to be concentrated is known as the centroid of that area.

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 2.1.** **Derive the coordinates for the centroid of :**

**i. a line,**
**ii. a straight line, and**
**iii. a composite line.**

**Answer**

i. **Centroid of a Line :**

1. Consider a homogenous wire of uniform cross-sectional area $A$, total length $L$ and density $\rho$. If we divide it into infinitesimally small elements then the weight of an element of length $dL$ is given as,

$$dW = \rho A(dL)g$$

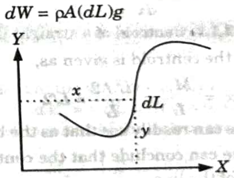

**Fig. 2.1.1. Centroid of a line.**

2. Hence, the weight of the entire wire is obtained by integrating the above expression over the length,

$\therefore$ $$W = \rho AgL$$

3. The first moment of weight of the infinitesimally small element about the $X$-axis is given as the weight multiplied by the perpendicular distance *i.e.*, $\rho Ag(dL)y$.
4. Using the principle of moments, the $y$-coordinate of location of centre of gravity of the entire wire is determined as

$$\bar{y}W = \int \rho Ag(dL)y$$

$$\bar{y}\rho AgL = \int \rho Ag(dL)y \qquad (\because W = \rho AgL)$$

5. Since the density $\rho$ and cross-sectional area $A$ are constant throughout the length of the wire, they can be taken outside the integral sign.

$$\bar{y} = \frac{\int y dL}{L}$$

6. Similarly, the $x$-coordinate of location of centre of gravity of the wire can be determined as,

$$\bar{x} = \frac{\int x dL}{L}$$

**ii. Centroid of a Straight Line :**

1. Consider a straight line of length $L$ along the $X$-axis. If we take an infinitesimally small length $dx$ at a distance $x$ from the origin then its first moment about the $Y$-axis is,

$$dM_y = x\, dx$$

2. Therefore, the first moment of the entire length about the $Y$-axis is,

$$M_y = \int_0^L x dx = \frac{L^2}{2}$$

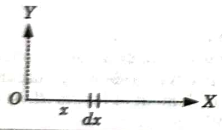


**Fig. 2.1.2. Centroid of a straight line.**

3. The $x$-coordinate of the centroid is given as,

$$\bar{x} = \frac{M_y}{L} = \frac{L^2/2}{L} = L/2$$

4. From figure 2.1.2, we can readily see that as the line is along the $X$-axis, $\bar{y} = 0$. Therefore, we can conclude that the centroid of a straight line lies at the midpoint of the line.

**iii. Centroid of a Composite Line :**

1. In general, a given curve may not be of regular shape then in that case, it is divided into finite segments of regular shapes for which positions of centroids are readily known.



2. Let $L_i$ be the length of a segment for which the centroid is known and $(\bar{x}_i, \bar{y}_i)$ be the location of its centroid.
3. Then the centroid of the composite line is given by,

$$\bar{x} = \frac{\sum L_i \bar{x}_i}{L}$$

and

$$\bar{y} = \frac{\sum L_i \bar{y}_i}{L}$$

**Que 2.2.** **Derive an expression for the centroid of an arc of a circle.**

**Answer**

1. Consider an arc of a circle symmetric about the $X$-axis as shown in Fig. 2.2.1. Let $R$ be the radius of the arc and $2\alpha$ be the subtended angle.
2. Consider an infinitesimally small length $dL$ such that the radius to the length makes an angle $\theta$ with the $X$-axis. Then its length $dL$ is given as,

$$dL = R\, d\theta$$

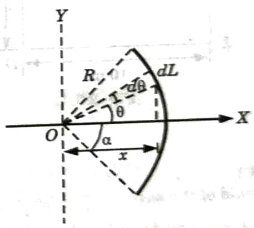


**Fig. 2.2.1. Centroid of an arc of a circle.**

3. Therefore, the total length of the arc is

$$L = \int_{-\alpha}^{\alpha} R\, d\theta = 2\alpha R$$

4. The first moment of the infinitesimally small length about the $Y$-axis is,

$$dM_y = x\, dL = (R\cos\theta)(R\, d\theta) = R^2 \cos\theta\, d\theta$$

5. Hence, the first moment of the entire arc about the $Y$-axis is given as,

$$M_y = \int_{-\alpha}^{\alpha} R^2 \cos\theta\, d\theta$$

$$= R^2 [\sin\theta]_{-\alpha}^{\alpha} = 2R^2 \sin\alpha$$

6. Therefore, the $x$-coordinate of centroid of the arc is given as,

$$\bar{x} = \frac{M_y}{L} = \frac{2R^2 \sin\alpha}{2\alpha R} = \frac{R\sin\alpha}{\alpha} \qquad ...(2.2.1)$$

7. From the Fig. 2.2.1, we can see that due to symmetry of the arc about $X$-axis, $\bar{y} = 0$.
8. For a semicircular arc, $\theta$ varies from $-\pi/2$ to $\pi/2$ hence the location of its centroid is obtained by substituting $\alpha = \pi/2$ in eq. (2.2.1), we get

$$\bar{x} = 2R/\pi \quad \text{and} \quad \bar{y} = 0$$

**Que 2.3.** **A wire is bent into a closed loop *A–B–C–D–E–A* as shown in Fig. 2.3.1 in which portion *AB* is circular arc. Determine the centroid of the wire.**

**AKTU 2011-12, Marks 05**

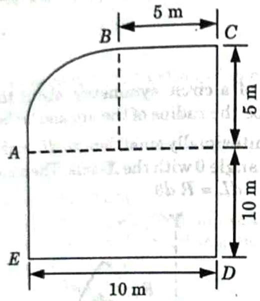


**Fig. 2.3.1.**

**Answer**

**Given :** Fig. 2.3.1.
**To Find :** Centroid of the wire.

1. Consider $ED$ as $X$-axis and $AE$ as $Y$-axis or $AE$ and $ED$ as reference axes to determine the centroid.
2. $$\text{Length of arc } AB = \frac{\pi r}{2} = \frac{5}{2}\pi = 7.85 \text{ m}$$

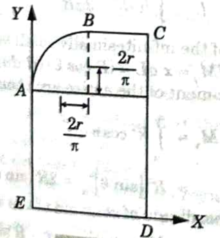


**Fig. 2.3.2.**



3. Position of centroid for arc,

$$x_i = 5 - \frac{2r}{\pi} = 5 - \frac{2\times 5}{\pi} = 1.82 \text{ m}$$

$$y_i = 10 + \frac{2r}{\pi} = 10 + \frac{2\times 5}{\pi} = 13.18 \text{ m}$$

4. The coordinates for the centroid of various lines and curves are shown in table given below :

| S. No. | Curve/Line | Length ($L_i$) (in mm) | Centroid Co-ordinate (in mm) | | | |
|---|---|---|---|---|---|---|
| | | | $x_i$ | $y_i$ | $L_i x_i$ | $L_i y_i$ |
| 1. | AB | 7.85 | 1.82 | 13.18 | 14.287 | 103.463 |
| 2. | BC | 5 | $5 + \frac{5}{2} = 7.5$ | $10 + 5 = 15$ | 37.5 | 75 |
| 3. | CD | $5 + 10 = 15$ | 10 | $\frac{15}{2} = 7.5$ | 150 | 112.5 |
| 4. | DE | 10 | $\frac{10}{2} = 5$ | 0 | 50 | 0 |
| 5. | EA | 10 | 0 | $\frac{10}{2} = 5$ | 0 | 50 |
| | | $\Sigma L_i = 47.85$ | | | 251.787 | 340.963 |

5. Centroid of the given figure is,

$$(\bar{x}, \bar{y}) = \left(\frac{\Sigma L_i x_i}{\Sigma L_i}, \frac{\Sigma L_i y_i}{\Sigma L_i}\right)$$

$$= \left(\frac{251.787}{47.85}, \frac{340.963}{47.85}\right) = (5.26, 7.13)$$

**Que 2.4.** **Prove that centroid of a rectangle lies at the intersection of its diagonals.**

**Answer**

1. Consider a rectangle of base length $b$ and height $h$. If we take a thin strip parallel to the $X$-axis at a distance $y$ from the $X$-axis and of infinitesimally small thickness $dy$ then its area is given as,

$$dA = b\,dy$$

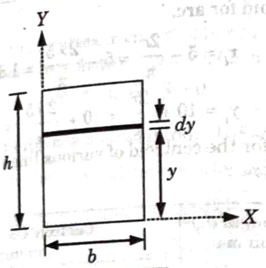


**Fig. 2.4.1.**

2. Hence, the area of the rectangle is,

$$A = \int_0^h dA = \int_0^h b\, dy = bh$$

3. As each point on this strip is at the same distance $y$ from the $X$-axis, we can take moment of area of the strip about the $X$-axis as,

$$dM_x = y dA = yb\, dy$$

4. Therefore, the first moment of the entire area about the $X$-axis is,

$$M_x = \int_0^h y(b\, dy) = \frac{bh^2}{2}$$

5. Hence, the $y$-coordinate of the centroid of the rectangle is given as,

$$\bar{y} = \frac{M_x}{A} = \frac{bh^2/2}{bh} = \frac{h}{2}$$

6. In a similar manner, we can consider a vertical strip at a distance $x$ from the $Y$-axis and of infinitesimally small thickness $dx$, and obtain the $x$-coordinate of the centroid as,

$$\bar{x} = \frac{b}{2}$$

7. Thus, we can see that the centroid of a rectangle lies at the midpoint or in other words, at the intersection of its two diagonals.

**Que 2.5.** **Show that centroid of a right angled triangle lies at (*b*/3, *h*/3) where *b* and *h* are the base and height of the triangle respectively.**

**Answer**

1. Consider a right angled triangle of base $b$ and height $h$. If we take a thin strip parallel to the base at a distance $y$ from the $X$-axis and of infinitesimally small thickness $dy$ then its area is $dA = b' \, dy$, where $b'$ is the width of the strip.



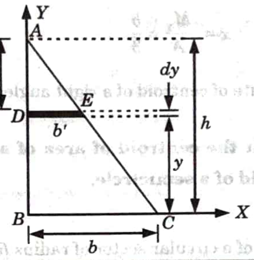


**Fig. 2.5.1. Centroid of a right angled triangle.**

2. From similar triangles $ABC$ and $ADE$, we have

$$\frac{b'}{h-y} = \frac{b}{h} \Rightarrow b' = \frac{b}{h}(h-y)$$

$$\therefore \quad dA = b'dy = \frac{b}{h}(h-y)dy$$

3. Then area of the entire triangle is obtained as,

$$A = \frac{b}{h}\int_0^h (h-y)dy$$

$$= \frac{b}{h}\left[hy - \frac{y^2}{2}\right]_0^h = \frac{bh}{2}$$

4. The first moment of the strip with respect to the $X$-axis is,

$$dM_x = y dA = y\left[\frac{b}{h}(h-y)\right]dy$$

5. Therefore, the first moment of the entire area about the $X$-axis is given as,

$$M_x = \int_0^h y\, dA = \int_0^h y\frac{b}{h}(h-y)dy$$

$$= \frac{b}{h}\int_0^h (hy - y^2)dy$$

$$= \frac{b}{h}\left[h\frac{y^2}{2} - \frac{y^3}{3}\right]_0^h = \frac{bh^2}{6}$$

6. Therefore, the $y$-coordinate of the centroid is given as,

$$\bar{y} = \frac{M_x}{A} = \frac{bh^2/6}{bh/2} = \frac{h}{3}$$

7. In a similar manner, we can consider a vertical strip of area $dA$ parallel to the $Y$-axis and obtain the $x$-coordinate of the centroid as,

$$\bar{x} = \frac{M_y}{A} = \frac{b}{3}$$

Thus the coordinate of centroid of a right angled triangle is $\left(\frac{b}{3}, \frac{h}{3}\right)$

**Que 2.6.** **Find out the centroid of area of a circular sector and also find the centroid of a semicircle.**

**Answer**

1. Consider an area of a circular sector of radius $R$ with subtended angle $2\alpha$, and symmetric about the $X$-axis. If we take an element of area $OCD$ at an angle $\theta$ from the $X$-axis then its area can be determined by considering $OCD$ as a triangle and is given as,

$$dA = (1/2)\, R \times R d\theta = \frac{R^2}{2} d\theta$$

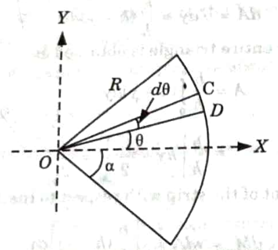


**Fig. 2.6.1.** A circular sector.

2. The centroid of this triangle lies at a distance of $(2/3)\, R$ from $O$. Hence, the $x$ and $y$-coordinates of the centroid are,

$$x = \frac{2}{3} R \cos\theta \text{ and } y = \frac{2}{3} R \sin\theta$$

3. Area of the entire circular sector is obtained by integrating the expression for $dA$ between limits, *i.e.*,

$$A = \int_{-\alpha}^{\alpha} \frac{R^2}{2} d\theta = R^2\alpha$$

4. Taking the first moment of the triangle $OCD$ about the $Y$-axis,

$$dM_y = x\, dA = \frac{2}{3} R \cos\theta \frac{R^2}{2} d\theta$$

5. Therefore, the first moment of the entire area about the $Y$-axis is,

$$M_y = \int x\, dA$$



$$= \int_{-\alpha}^{\alpha} \frac{2}{3} R \cos\theta \frac{R^2}{2} d\theta$$

$$= \frac{R^3}{3} [\sin\theta]_{-\alpha}^{\alpha} = \frac{2R^3 \sin\alpha}{3}$$

6. Therefore, the $x$-coordinate of the centroid is

$$\bar{x} = M_y / A = \frac{2}{3} \frac{R \sin\alpha}{\alpha} \quad ...(2.6.1)$$

7. As the sector is symmetric about $X$-axis,

$$\therefore \quad \bar{y} = 0$$

8. For a semicircular area, we know that $\theta$ varies from $-\pi/2$ to $\pi/2$. Hence, its centroid is obtained by substituting $\alpha = \pi/2$ in eq. (2.6.1) for $\bar{x}$. Therefore, we get

$$\bar{x} = \frac{4R}{3\pi} \text{ and } \bar{y} = 0$$

9. Similarly, if the area is symmetric about $Y$-axis then the centroidal coordinates are

$$\bar{x} = 0 \text{ and } \bar{y} = \frac{4R}{3\pi}$$

**Que 2.7.** **Derive the expression for the centroid of a parabola.**

**Answer**

1. Consider a shaded area bounded by a parabola of equation $y = kx^2$, $X$-axis and line $x = b$ as shown in Fig. 2.7.1. Then we see that at $x = 0$, $y = 0$ and at $x = b$, $y = h$. Therefore,

$$k = \frac{h}{b^2}$$

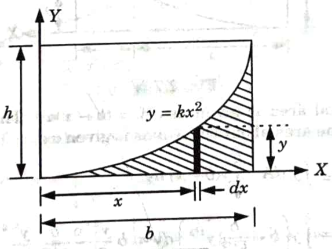


**Fig. 2.7.1.**

2. Hence, we can write the equation of the curve as,

$$y = \frac{h}{b^2} x^2$$

3. Consider a vertical strip parallel to the $Y$-axis at a distance $x$ from the origin and of infinitesimally small thickness $dx$ as shown in the Fig. 2.7.1. Then its elemental area is given as $dA = y\,dx = (h/b^2)x^2\,dx$. Therefore, the area under the entire curve is,

$$A = \int_0^b \left(\frac{h}{b^2}\right) x^2\,dx$$

$$= \frac{h}{b^2} \times \frac{b^3}{3} = \frac{bh}{3}$$

We see that the area of the curve is 1/3rd of the area of the enclosed rectangle.

4. The first moment of the area about the $Y$-axis is given as,

$$M_y = \int x\,dA$$

$$= \int_0^b x \frac{h}{b^2} x^2\,dx$$

$$= \frac{h}{b^2} \times \frac{b^4}{4} = \frac{b^2 h}{4}$$

5. Therefore, the $x$-coordinate of the centroid is given as,

$$\bar{x} = \frac{M_y}{A} = \frac{b^2 h/4}{bh/3} = \frac{3}{4}b$$

6. In a similar manner, we can consider a thin strip parallel to the $X$-axis and of infinitesimally small thickness $dy$ as shown in Fig. 2.8.2.

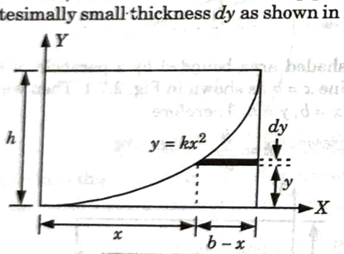


Fig. 2.7.2.

7. The elemental area is given as $dA = (b - x)dy$. Therefore, the first moment of the area about the $X$-axis is given as,

$$M_x = \int y\,dA = \int_0^h y(b - x)\,dy$$

$$= \int_0^h y\left(b - \frac{b}{h^{1/2}} y^{1/2}\right) dy = \left[b\frac{y^2}{2} - \frac{b}{h^{1/2}}\frac{y^{5/2}}{5/2}\right]_0^h = \frac{bh^2}{10}$$

8. Therefore, the $y$-coordinate of the centroid is given as,

$$\bar{y} = \frac{M_x}{A} = \frac{bh^2/10}{bh/3} = \frac{3}{10}h$$



**Que 2.8.** **Determine the centroid of a semi circular segment given that $a$ = 100 mm and $\alpha$ = 45°.**

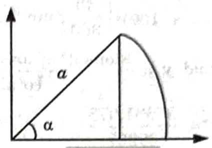


Fig. 2.8.1.

**AKTU 2013-14 (I), Marks 05**

**Answer**

**Given :** $a$ = 100 mm = 0.1 m, $\alpha$ = 45°
**To Find :** Centroid of semi circular segment.

1. Let us consider an element at a distance $r$ from the centre $O$ of the semi circle, radial width being $dr$ and bound by radii at $\theta$ and $\theta + d\theta$.
Area of element = $r d\theta dr$
2. Its moment about $X$-axis is given by,

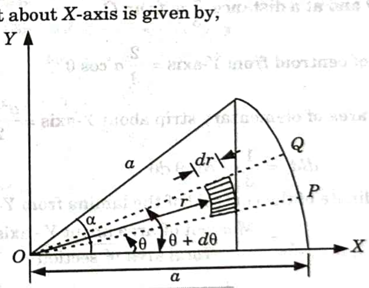


Fig. 2.8.2.

$$r d\theta\, dr \times r \sin\theta = r^2 \sin\theta\, dr\, d\theta$$

3. Total moment of area about $X$-axis is,

$$\int_0^\alpha \int_0^a r^2 \sin\theta\, dr\, d\theta = \int_0^\alpha \left[\frac{r^3}{3}\right]_0^a \sin\theta\, d\theta$$

$$= \frac{a^3}{3}[-\cos\theta]_0^\alpha = \frac{a^3}{3}[-\cos\alpha + \cos 0°]$$

$$= \frac{(100)^3}{3}[-\cos 45° + 1] = 97631.073 \text{ mm}^3$$

4. Area of the sector $= \pi a^2 \left(\frac{\alpha}{360}\right)$

$$= \pi (100)^2 \left(\frac{45}{360}\right) \text{mm}^2 = 3927 \text{ mm}^2$$

5. The position of centroid $\bar{y} = \frac{\text{Moment of area about } X\text{-axis}}{\text{Total area}}$

$$= \frac{97631.073}{39267}$$

$$\bar{y} = 24.86 \text{ mm}$$

6. Now consider an elementary strip $OPQ$ that subtends an angle $d\theta$ at $O$.

$$PQ = a\, d\theta$$

7. As angle $d\theta$ is very small, consider it as a triangle.

$\therefore$ Area of the elementary strip $= \frac{1}{2}(a d\theta)a$

$$dA = \frac{a^2}{2} d\theta$$

8. Centroid of this triangular strip lies on a line that joins $O$ to the mid point of $PQ$ and at a distance $\frac{2}{3}a$ from $O$.

9. Distance $x$ of centroid from $Y$-axis $= \frac{2}{3} a \cos\theta$

10. Moment of area of elementary strip about $Y$-axis $= \frac{a^2 d\theta}{2} \times \frac{2}{3} a \cos\theta$

$$dM_y = \frac{1}{3} a^3 \cos\theta\, d\theta$$

11. The $x$-coordinate of the centroid of the lamina from $Y$-axis will be,

$$\bar{x} = \frac{\text{Moment of area about } Y\text{-axis}}{\text{Total area of section}}$$

$$= \frac{\int_0^\alpha \frac{1}{3} a^3 \cos\theta\, d\theta}{\int_0^\alpha \frac{a^2}{2} d\theta} = \frac{2}{3} a \frac{[\sin\theta]_0^\alpha}{[\theta]_0^\alpha}$$

$$= \frac{2a}{3} \frac{\sin\alpha}{\alpha}$$

$$\bar{x} = \frac{2 \times 100}{3} \frac{\sin 45°}{\left(45 \times \frac{\pi}{180}\right)} = 60.02 \text{ mm}$$



## PART-2

### *Centroid of Composite Sections, Centre of Gravity and its Implications.*

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 2.9.** **Discuss in brief about centroid of composite figures.**

**Answer**

1. In engineering work, we frequently need to locate the centroid of a composite area. Such an area may be composed of regular geometric shapes such as rectangle, triangle, circle, semicircle, quarter circle, etc.
2. In such cases, we divide the given area into regular geometric shapes for which the positions of centroids are readily known.
3. Let $A_i$ be the area of an element and $(\bar{x}_i, \bar{y}_i)$ be the respective centroidal coordinates. Then for the composite area,

$$A\bar{x} = A_1\bar{x}_1 + A_2\bar{x}_2 + ... + A_n\bar{x}_n$$

$$\therefore \quad \bar{x} = \frac{\sum A_i \bar{x}_i}{A}$$

4. Similarly,

$$\bar{y} = \frac{\sum A_i \bar{y}_i}{A}$$

where the total area, $A = \Sigma A_i$, in which the areas are added up algebraically.

**Que 2.10.** **Find out the centroid of an $L$-section of 120 mm × 80 mm × 20 mm as shown in Fig. 2.10.1.**

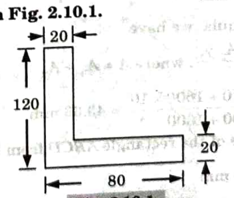


**Fig. 2.10.1.**

**AKTU 2014-15 (I), Marks 10**

**Answer**

**Given :** Fig. 2.10.1.
**To Find :** Centroid of $L$-section.

1. The given $L$-section is not symmetrical about any section. Hence, in this case, there will be two axes of references. The lowest line of the figure (*i.e.*, line $GF$) will be taken as axis of reference for calculating $\bar{y}$ and the left line of the $L$-section (*i.e.*, line $AG$) will be taken as axis of reference for calculating $\bar{x}$.

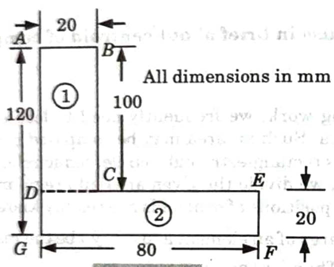


**Fig. 2.10.2.**

2. The given $L$-section is split up into two rectangles $ABCD$ and $DEFG$, as shown in Fig. 2.10.2.
3. $A_1$ = Area of rectangle $ABCD = 100 \times 20 = 2000\ \text{mm}^2$
   $y_1$ = Distance of centroid of rectangle $ABCD$ from bottom line $GF$.
   $$y_1 = 20 + \frac{100}{2} = 20 + 50 = 70\ \text{mm}$$
   $A_2$ = Area of rectangle $DEFG = 80 \times 20 = 1600\ \text{mm}^2$
   $y_2$ = Distance of centroid of rectangle $DEFG$ from bottom line $GF$.
   $$= \frac{20}{2} = 10\ \text{mm}$$
4. By using the formula, we have
   $$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A}, \text{ where } A = A_1 + A_2$$
   $$= \frac{2000 \times 70 + 1600 \times 10}{2000 + 1600} = 43.33\ \text{mm}$$
5. Let, $x_1$ = Distance of the rectangle $ABCD$ from left line $AG$.
   $$= \frac{20}{2} = 10\ \text{mm}$$
   $x_2$ = Distance of the rectangle $DEFG$ from left line $AG$.



$$= \frac{80}{2} = 40\ \text{mm}$$

6. Using formula, we have
   $$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A}$$
   $$= \frac{2000 \times 10 + 1600 \times 40}{2000 + 1600} = 23.33\ \text{mm}$$
   Hence, the centroid of the $L$-section is at a distance of 43.33 mm from the bottom line $GF$ and 23.33 mm from the left line $AG$.

**Que 2.11.** **Locate the centroid of the shaded area shown in Fig. 2.11.1. All dimensions are in meters.**

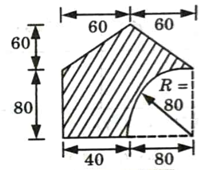


**Fig. 2.11.1.**

**AKTU 2014-15 (II), Marks 10**

**Answer**

**Given :** Fig. 2.11.2
**To Find :** Centroid of the shaded area.

1. Shaded area, $ABCED$ = Rectangle $AOCD$ + Triangle $DCE$ – Quarter circle $OBC$

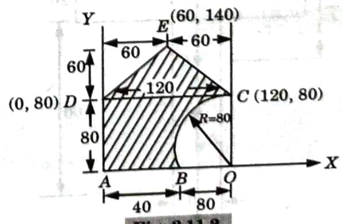


**Fig. 2.11.2.**

2. The coordinates of the centroid for various sections are shown in the table given below :

| Shape | Area, $A_i$ (mm²) | Centroid coordinate | | | |
|---|---|---|---|---|---|
| | | $x_i$ (mm) | $y_i$ (mm) | $A_ix_i$ (mm³) | $A_iy_i$ (mm³) |
| Rectangle *AOCD* | $120 \times 80 = 9600$ | $120/2 = 60$ | $80/2 = 40$ | $576 \times 10^3$ | $384 \times 10^3$ |
| Triangle *DEC* | $\frac{1}{2} \times 120 \times 60 = 3600$ | $\frac{0+120+60}{3} = 60$ | $\frac{80+140+80}{3} = 100$ | $216 \times 10^3$ | $360 \times 10^3$ |
| Quarter circle *BOC* | $\frac{-\pi(80)^2}{4} = -5026.55$ | $40 + \frac{4 \times 80}{3\pi} = 73.95$ | $\frac{4 \times 80}{3\pi} = 33.95$ | $-371.7 \times 10^3$ | $-170.65 \times 10^3$ |
| | $\Sigma A_i = 8173.4$ | | | $\Sigma A_ix_i = 420 \times 10^3$ | $\Sigma A_iy_i = 573.35 \times 10^3$ |

3. Centroid of shaded portion, $(\bar{x}, \bar{y})$

$$= \left(\frac{\Sigma A_ix_i}{\Sigma A_i}, \frac{\Sigma A_iy_i}{\Sigma A_i}\right)$$

$$= \left(\frac{420 \times 10^3}{8173.45}, \frac{573.35 \times 10^3}{8173.45}\right)$$

$$= (51.4, 70.15)$$

**Que 2.12.** Locate the centroid of the *T*-section shown in the Fig. 2.12.1.

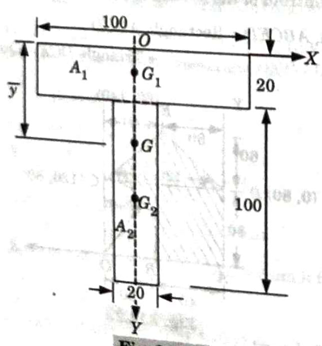


**Fig. 2.12.1.**



**Answer**

**Given :** Fig. 2.12.1.
**To Find :** Centroid of *T* section.

1. Selecting the axis as shown in Fig. 2.12.1, we can say due to symmetry centroid lies on *Y*-axis, *i.e.*, $\bar{x} = 0$.
2. Now the given *T*-section may be divided into two rectangles $A_1$ and $A_2$ each of size 100 × 20 mm and 20 × 100 mm. The centroid of $A_1$ and $A_2$ are $G_1(0, 10)$ and $G_2(0, 70)$ respectively.
3. The distance of centroid from top is given by,

$$\bar{y} = \frac{100 \times 20 \times 10 + 20 \times 100 \times 70}{100 \times 20 + 20 \times 100} = 40 \text{ mm}$$

Hence, centroid of *T*-section is on the symmetric axis at a distance 40 mm from the top.

**Que 2.13.** For the semi-annular area shown in Fig. 2.13.1, determine the ratio of *a* to *b* so that $\bar{y} = \frac{3}{4}b$.

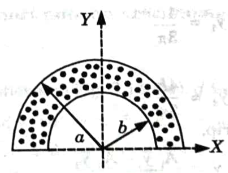


**Fig. 2.13.1.**

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**Answer**

**Given :** $\bar{y} = \frac{3}{4}b$
**To Find :** Ratio of *a* to *b*.

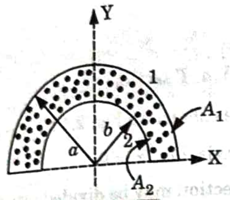


**Fig. 2.13.2.**

1. Area of semicircle (1),

$$A_1 = \frac{\pi a^2}{2}$$

2. Area of semicircle (2),

$$A_2 = \frac{\pi b^2}{2}$$

3. Net area of strip, $A = A_1 - A_2$

$$A = \frac{\pi}{2}(a^2 - b^2)$$

4. Due to symmetry, centroid will lie on Y-axis.
For semicircle (1),

$$\bar{y}_1 = \frac{4a}{3\pi}$$

For semicircle (2),

$$\bar{y}_2 = \frac{4b}{3\pi}$$

5. Then centroid of strip,

$$\bar{y} = \frac{A_1 \bar{y}_1 - A_2 \bar{y}_2}{A_1 - A_2}$$

6. On putting the values of $A_1, A_2, \bar{y}_1$ and $\bar{y}_2$, we have

$$\bar{y} = \frac{\frac{\pi a^2}{2} \times \frac{4a}{3\pi} - \frac{\pi b^2}{2} \times \frac{4b}{3\pi}}{\frac{\pi}{2}(a^2 - b^2)}$$

$$\bar{y} = \frac{\frac{\pi}{2}\left[\frac{4a^3}{3\pi} - \frac{4b^3}{3\pi}\right]}{\frac{\pi}{2}(a^2 - b^2)} = \frac{4}{3\pi}\frac{(a^3 - b^3)}{(a^2 - b^2)}$$

$$= \frac{4}{3\pi}\left[\frac{(a-b)(a^2 + b^2 + ab)}{(a-b)(a+b)}\right]$$



$$\frac{3}{4}b = \frac{4}{3\pi}\left[\frac{(a+b)^2 - ab}{a+b}\right] = \frac{4}{3\pi}\left[a + b - \frac{ab}{a+b}\right] \quad \left(\because \bar{y} = \frac{3}{4}b\right)$$

$$\frac{3^2\pi}{4^2} = \left[\frac{a}{b} + 1 - \frac{a}{a+b}\right] \quad ...(2.13.1)$$

7. After solving eq. (2.3.1), we get

$$\frac{a}{b} = 1.34$$

## PART-3

*Area Moment of Inertia-Definition, Moment of Inertia of Plane Sections from First Principle.*

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 2.14.** **Write a short note on area moment of inertia.**

**Answer**

1. Consider the area shown in Fig. 2.14.1(*a*). *dA* is an elemental area with coordinates as *x* and *y*. The term $\Sigma y_i^2 \, dA_i$ is called moment of inertia of the area about *X* axis and is denoted as $I_{XX}$. Similarly, the moment of inertia about *y* axis is

$$I_{YY} = \Sigma y_i^2 \, dA_i$$

2. In general, if *r* is the distance of elemental area *dA* from the axis *AB* [Fig. 2.14.1(*b*)], the sum of the terms $\Sigma r^2 \, dA$ to cover the entire area is called moment of inertia of the area about the axis *AB*.

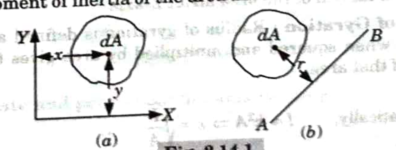


**Fig. 2.14.1.**

3. Though moment of inertia of plane area is a purely mathematical term, it is one of the important properties of areas. The strength of members subject to bending depends on the moment of inertia of its cross-sectional area.

4. The moment of inertia is a fourth dimensional term since it is a term obtained by multiplying area by the square of the distance. Hence, its SI unit is $m^4$.

**Que 2.15.** **Define the following terms :**

i. **Polar moment of inertia, and**

ii. **Radius of gyration.**

**Answer**

i. **Polar Moment of Inertia :**

1. If an elemental area $dA$ is at a distance $r$ from origin of the coordinate axes then its polar moment of inertia is given by,

$$J_O = \int r^2 dA$$

where, $J_O$ = Polar moment of inertia of the area $A$ with respect to the pole $O$.

2. As $r^2 = x^2 + y^2$

Hence, $J_O = \int (x^2 + y^2)\, dA = \int y^2\, dA + \int x^2\, dA$

$$J_O = I_X + I_Y$$

where $I_X$ = Moment of inertia of the area about $X$-axis.

$I_Y$ = Moment of inertia of the area about $Y$-axis.

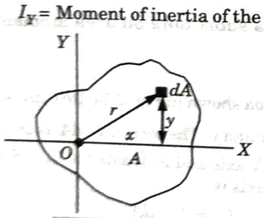


Fig. 2.15.1.

3. In other words we can say that polar moment of inertia of an area is the moment of inertia of the area about $Z$-axis.

ii. **Radius of Gyration :** Radius of gyration is defined as the distance which is when squared and multiplied by area gives the moment of inertia of that area.

Mathematically, $I = k^2 A \Rightarrow k = \sqrt{\dfrac{I}{A}}$

where, $k$ = Radius of gyration,

$I$ = Moment of inertia, and

$A$ = Cross-sectional area.



## PART-4

*Theorems of Moment of Inertia.*

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 2.16.** **State and prove perpendicular axis theorem.**

**Answer**

A. **Perpendicular Axis Theorem :**

1. The moment of inertia of an area about an axis perpendicular to its plane (*i.e.*, polar moment of inertia) at any point $O$ is equal to the sum of moment of inertia about any two mutually perpendicular axis through the same point $O$ and lying in the plane of the given area.

B. **Proof :**

1. Consider an elemental area $dA$ at distance $r$ from point $O$.

2. Let $dA$ have coordinates $x$ and $y$, then from the definition,

$$I_{ZZ} = \Sigma r^2 dA$$

$$= \Sigma(x^2 + y^2)dA = \Sigma x^2 dA + \Sigma y^2 dA \quad (\because r^2 + x^2 + y^2)$$

$$I_{ZZ} = I_{XX} + I_{YY}$$

$I_{ZZ}$ is also called polar moment of inertia.

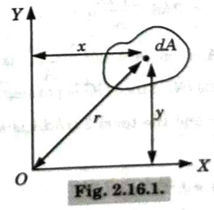


Fig. 2.16.1.

**Que 2.17.** **State and prove parallel axis theorem.**

**Answer**

A. **Parallel Axis Theorem :**

1. According to this theorem, moment of inertia about any axis in the plane of an area (or lamina) is equal to the sum of moment of inertia

about a parallel centroidal axis and the product of area and square of distance between the two parallel axes.

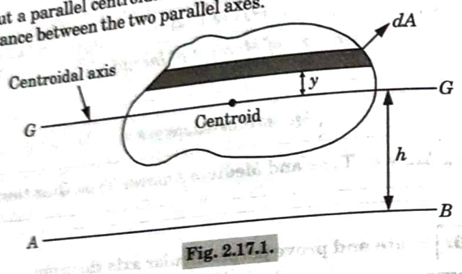


Fig. 2.17.1.

2. According to definition,

$$I_{AB} = I_G + Ah^2$$

where, $I_{AB}$ = Moment of inertia about the line $AB$,
$A$ = Area of the plane figure, and
$h$ = Distance between the axis $AB$ and parallel centroidal axis $GG$.

**B. Proof :**

1. Let us consider an elemental parallel strip $dA$ at '$y$' distance from axis $GG$.

$$I_{AB} = \Sigma(y + h)^2 dA$$
$$= \Sigma y^2 dA + \Sigma 2yh dA + \Sigma h^2 dA$$

2. Here 1[st] term $\Sigma y^2 dA$ is the moment of inertia about $GG$ axis.

$$I_{GG} = \Sigma y^2 dA$$

3. The 2[nd] term,

$$\Sigma 2yhdA = 2h\Sigma ydA$$
$$= 2hA\frac{\Sigma ydA}{A}$$

4. In the above term $2hA$ is constant and $\frac{\Sigma y\, dA}{A}$ is the distance of centroid from the reference axis $GG$. Since $GG$ is passing through centroid itself, hence $\frac{\Sigma y\, dA}{A}$ is zero and the term $\Sigma 2yhdA$ is zero.

5. The 3[rd] term,

$$\Sigma h^2 dA = h^2 \Sigma dA = Ah^2$$

6. Therefore, $I_{AB} = I_G + Ah^2$

**PART-5**

*Moment of Inertia of Standard Sections and Composite Sections.*



**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 2.18.** **Find the moment of inertia of following shapes about base and centroidal axis :**
**i. Rectangle,**
**ii. Triangle, and**
**iii. Circle.**

**Answer**

**i. Rectangle :**

1. Consider a rectangle of width $b$ and depth $d$ (Fig. 2.18.1).

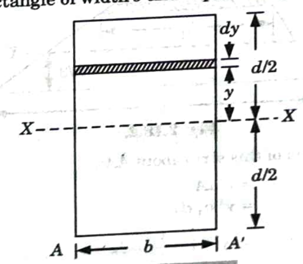


Fig. 2.18.1.

2. Consider an elemental strip of width $dy$ at a distance $y$ from the $X$-$X$ axis. Moment of inertia of the elemental strip about the centroidal axis $X$-$X$ is,

$$dI_{XX} = y^2 dA = y^2 b\, dy$$

$$\therefore \quad I_{XX} = \int_{-d/2}^{d/2} y^2 b\, dy = b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= b\left[\frac{d^3}{24} + \frac{d^3}{24}\right] = \frac{bd^3}{12}$$

Similarly, $I_{YY} = \dfrac{db^3}{12}$

3. Now moment of inertia about base,

$$I_{AA'} = I_{CG} + Ah^2$$

$$= I_{XX} + bd\left(\frac{d}{2}\right)^2 \qquad \left(\because h = \frac{d}{2}\right)$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{bd^3}{3}$$

**ii. Triangle :**

1. Consider an elemental strip at a distance $y$ from the base $AA'$. Let $dy$ be the thickness of the strip and $dA$ its area. Width of this strip is given by,

$$b_1 = \frac{(h-y)}{h} \times b = \left(1 - \frac{y}{h}\right)b$$

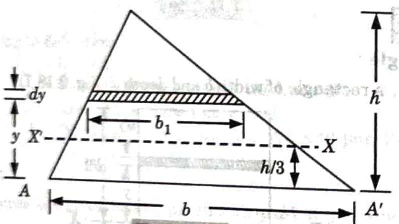


**Fig. 2.18.2.**

2. Moment of inertia of this strip about $AA'$,

$$= y^2 dA$$
$$= y^2 b_1 dy$$
$$= y^2\left(1 - \frac{y}{h}\right) bdy$$

3. Moment of inertia of the triangle about $AA'$,

$$I_{AA'} = \int_0^h by^2\left(1 - \frac{y}{h}\right)dy = \int_0^h b\left(y^2 - \frac{y^3}{h}\right)dy$$

$$= b\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_0^h$$

$$I_{AA'} = \frac{bh^3}{12}$$

4. By parallel axis theorem,

$$I_{AA'} = I_{XX'} + Ay^2$$
$$I_{XX'} = I_{AA'} - Ay^2$$

$$= \frac{bh^3}{12} - \frac{1}{2}bh\left(\frac{h}{3}\right)^2 \qquad (\because y = h/3)$$

$$= \frac{bh^3}{12} - \frac{bh^3}{18}$$



$$I_{XX'} = \frac{bh^3}{36}$$

**iii. Circle :**

1. Let $dA$ be an elemental ring of radius $r$ and thickness $dr$.
So, elemental area, $dA = 2\pi r dr$
2. Now, moment of inertia of thin ring about its central axis or polar moment of inertia,

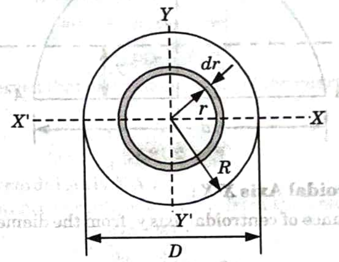


**Fig. 2.18.3.**

$$I_{ZZ'} = \int_0^R r^2\, dA = \int_0^R r^2\,(2\pi r)\,dr$$

$$I_{ZZ'} = \frac{\pi R^4}{2}$$

$$= \frac{\pi D^4}{32} \qquad \left\{\because R = \frac{D}{2}\right\}$$

3. By perpendicular axis theorem,

$$I_{ZZ'} = I_{XX'} + I_{YY'}$$

4. Due to symmetry along $X$-$X'$ and $Y$-$Y'$ axes we have

$$I_{XX'} = I_{YY'}$$

$$I_{XX'} = I_{YY'} = \frac{I_{ZZ'}}{2} = \frac{\pi D^4}{64}$$

**Que 2.19.** **Find the moment of inertia of a semicircle and quarter circle.**

**Answer**

**i. Moment of Inertia of a Semicircle :**

**a. About Diametral Axis :**

1. If the limit of integration is put as 0 to $\pi$ instead of 0 to $2\pi$ in the derivation for the moment of inertia of a circle about diametral axis the moment of inertia of a semicircle is obtained.

2. It can be observed that the moment of inertia of a semicircle (Fig. 2.19.1) about the diametral axis $AA'$ is,

$$I_{AA'} = \frac{1}{2} \times \frac{\pi d^4}{64} = \frac{\pi d^4}{128}$$

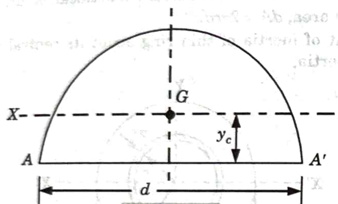


Fig. 2.19.1.

**b. About Centroidal Axis X-X :**

1. Now, the distance of centroidal axis $y_c$ from the diametral axis is given by,

$$y_c = \frac{4R}{3\pi} = \frac{2d}{3\pi}$$

$$\text{Area, } A = \frac{1}{2} \times \frac{\pi d^2}{4} = \frac{\pi d^2}{8}$$

2. From parallel axis theorem,

$$I_{AA'} = I_{XX} + Ay_c^2$$

$$\frac{\pi d^4}{128} = I_{XX} + \frac{\pi d^2}{8} \times \left(\frac{2d}{3\pi}\right)^2$$

$$I_{XX} = \frac{\pi d^4}{128} - \frac{d^4}{18\pi}$$

$$= 0.00686\, d^4$$

**ii. Moment of Inertia of a Quarter of a Circle :**

**a. About the Base :**

1. If the limit of integration is put as 0 to $\pi/2$ instead of 0 to $2\pi$ in the derivation for moment of inertia of a circle, the moment of inertia of a quarter of a circle is obtained.



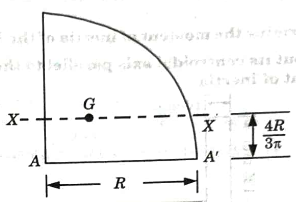


Fig. 2.19.2.

2. It can be observed that moment of inertia of the quarter of a circle about the base $AA'$ is,

$$I_{AA'} = \frac{1}{4} \times \frac{\pi d^4}{64} = \frac{\pi d^4}{256}$$

**b. About Centroidal Axis X-X :**

1. Now, the distance of centroidal axis $y_c$ from the base is given by,

$$y_c = \frac{4R}{3\pi} = \frac{2d}{3\pi}$$

$$\text{Area, } A = \frac{1}{4} \times \frac{\pi d^2}{4} = \frac{\pi d^2}{16}$$

2. From parallel axis theorem,

$$I_{AA'} = I_{XX} + Ay_c^2$$

$$\frac{\pi d^4}{256} = I_{XX} + \frac{\pi d^2}{16}\left(\frac{2d}{3\pi}\right)^2$$

$$I_{XX} = \frac{\pi d^4}{256} - \frac{d^4}{36\pi} = 0.00343\, d^4$$

**Que 2.20.** **Discuss the procedure of finding the moment of inertia of composite sections.**

**Answer**

Moment of inertia of composite sections about an axis can be found by the following steps :

1. Divide the given figure into a number of simple figures.
2. Locate the centroid of each simple figure by inspection or using standard expressions.
3. Find the moment of inertia of each simple figure about its centroidal axis. Add the term $Ay^2$, where $A$ is the area of the simple figure and $y$ is the distance of the centroid of the simple figure from the reference axis. This gives moment of inertia of the simple figure about the reference axis.
4. Sum up moments of inertia of all simple figures to get the moment of inertia of the composite section.

**Que 2.21.** **Determine the moment of inertia of the *L* section shown in Fig. 2.21.1 about its centroidal axis parallel to the legs. Also find the polar moment of inertia.**

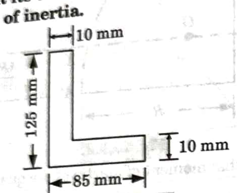


Fig. 2.21.1.

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**Answer**

**Given :** Fig. 2.21.1.
**To Find :** i. Moment of inertia about centroid axis.
ii. Polar moment of inertia.

1. The given section is divided into two rectangles $A_1$ and $A_2$.
   Area, $A_1 = 125 \times 10 = 1250\ mm^2$
   Area, $A_2 = 75 \times 10 = 750\ mm^2$
   Total Area $= 2000\ mm^2$
2. First, the centroid of the given section is to be located. Two reference axis (1)-(1) and (2)-(2) are chosen as shown in Fig. 2.21.2.

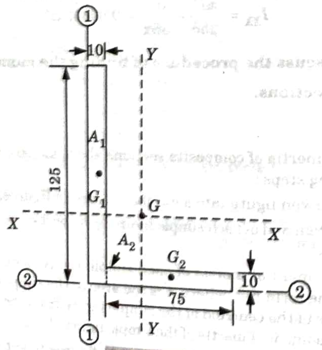


Fig. 2.21.2.

3. The distance of centroid from the axis (1)-(1),

$$\bar{x} = \frac{\text{Sum of moment of areas } A_1 \text{and } A_2 \text{ about (1)-(1)}}{\text{Total area}}$$

$$\bar{x} = \frac{1250 \times 5 + 750\left(10 + \frac{75}{2}\right)}{2000} = 20.94 \text{ mm}$$

4. Similarly, the distance of the centroid from the axis (2)-(2),

$$\bar{y} = \frac{1250 \times \frac{125}{2} + 750 \times 5}{2000} = 40.94 \text{ mm}$$

5. With respect to the centroidal axis *X-X* and *Y-Y*, the centroid of $A_1$ is $G_1$ (15.94, 21.56) and that of $A_2$ is $G_2$ (26.56, 35.94).

$\therefore$ $I_{XX}$ = Moment of inertia of $A_1$ about *X-X* axis + Moment of inertia of $A_2$ about *X-X* axis

$$\therefore \quad I_{XX} = \frac{10 \times 125^3}{12} + 1250 \times 21.56^2 + \frac{75 \times 10^3}{12} + 750 \times 35.94^2$$

*i.e.*, $I_{XX} = 3183658.9\ mm^4$

6. Similarly,

$$I_{YY} = \frac{125 \times 10^3}{12} + 1250 \times 15.94^2 + \frac{10 \times 75^3}{12} + 750 \times 26.56^2$$

$I_{YY} = 1208658.9\ mm^4$

7. Polar moment of inertia, $I_{zz} = I_{xx} + I_{yy}$
   $= 3183658.9 + 1208658.9$
   $I_{ZZ} = 4392317.8\ mm^4$

**Que 2.22.** **Determine the area moment of inertia of the composite area *ABOC* about given *X* and *Y* axes.**

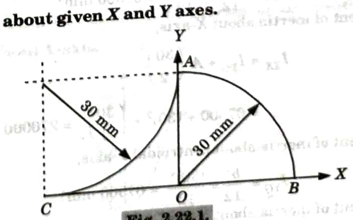


Fig. 2.22.1.

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**Answer**

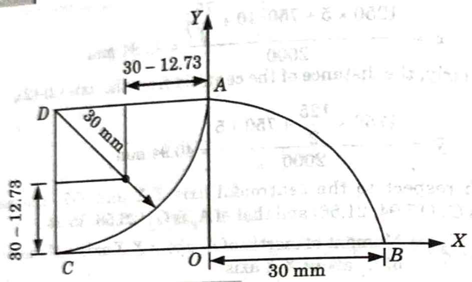



Fig. 2.22.2.

**Given :** Fig. 2.22.1.
**To Find :** Area moment of inertia.

1. For quarter circle $OAB$ :
   i. Moment of inertia about $X$-axis,
   $$I_{XX} = \frac{\pi R^4}{16} = \frac{\pi(30)^4}{16} = 159043.13 \text{ mm}^4$$
   ii. Moment of inertia about $Y$-axis,
   $$I_{YY} = \frac{\pi R^4}{16} = \frac{\pi(30)^4}{16} = 159043.13 \text{ mm}^4$$
2. For square $AOCD$ :
   i. Moment of inertia about, centroidal $X$-axis,
   $$I_{XG} = \frac{b^4}{12} = \frac{(30)^4}{12} = 67500 \text{ mm}^4$$
   ii. Moment of inertia about $X$-axis,
   $$I_{XX} = I_{XG} + A\left(\frac{30}{2}\right)^2$$
   $$= 67500 + (30)^2 \times \left(\frac{30}{2}\right)^2 = 270000 \text{ mm}^4$$
   iii. Moment of inertia about centroidal $Y$-axis,
   $$I_{YG} = \frac{b^4}{12} = \frac{(30)^4}{12} = 67500 \text{ mm}^4$$
   iv. Moment of inertia about $Y$-axis,
   $$I_{YY} = I_{YG} + A\left(\frac{30}{2}\right)^2 = 67500 + (30)^2 \times \left(\frac{30}{2}\right)^2$$
   $$= 270000 \text{ mm}^4$$



3. For quarter circle $DAC$ :
   i. Moment of inertia about centroidal $X$-axis,
   $$I_{XG} = 0.055R^4 = 0.055\,(30)^4 = 44550 \text{ mm}^4$$
   ii. Moment of inertia about $X$-axis,
   $$I_{XX} = I_{XG} + Ah^2$$
   Here $$h = 30 - \frac{4R}{3\pi} = 30 - \frac{4 \times 30}{3\pi} = 17.27 \text{ mm}$$
   $$I_{XX} = 44550 + \frac{\pi}{4}(30)^2 \times (17.27)^2$$
   $$= 255372.55 \text{ mm}^4$$
   iii. Moment of inertia about centroidal $Y$-axis,
   $$I_{YG} = 0.055R^4 = 0.055\,(30)^4 = 44550 \text{ mm}^4$$
   iv. Moment of inertia about $Y$-axis,
   $$I_{YY} = I_{YG} + Ah^2$$
   Here, $$h = 30 - \frac{4R}{3\pi} = 30 - \frac{4 \times 30}{3\pi} = 17.27 \text{ mm}$$
   $$I_Y = 44550 + \frac{\pi}{4}(30)^2 \times (17.27)^2$$
   $$= 255372.55 \text{ mm}^4$$
4. Moment of inertia of the composite area
   i. About $X$-axis, $I_{XX} = (I_{XX})_{OAB} + (I_{XX})_{AOCD} - (I_{XX})_{DAC}$
   $$= 159043.13 + 270000 - 255372.55$$
   $$I_{XX} = 173670.58 \text{ mm}^4$$
   ii. About $Y$-axis, $I_{YY} = (I_{YY})_{OAB} + (I_{YY})_{AOCD} - (I_{YY})_{DAC}$
   $$= 159043.13 + 270000 - 255372.55$$
   $$I_{YY} = 173670.58 \text{ mm}^4$$

**Que 2.23.** **Find the moment of inertia of the section shown in Fig. 2.23.1 about $X$-axis.**

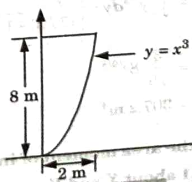



Fig. 2.23.1.

**AKTU 2013-14 (I), Marks 10**

**Answer**

**Given :** Fig. 2.23.1.
**To Find :** Moment of inertia.

1. Let's consider a horizontal strip of small thickness $dy$ at distance $y$ from $X$-axis, as shown in Fig. 2.23.2.

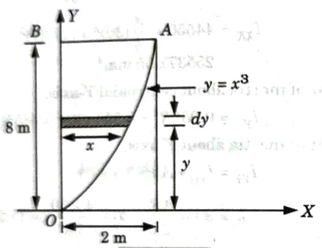



**Fig. 2.23.2.**

2. The area of the strip is,

$$dA = x\,dy$$

3. The moment of inertia about the $X$-axis,

$$dI_{XX} = y^2 dA$$

4. We know that, $\quad y = x^3 \Rightarrow x = y^{1/3}$

So, $$dI_{XX} = y^2\,x\,dy$$

$$dI_{XX} = y^2\,y^{1/3}\,dy \qquad \text{...(2.23.1)}$$

5. Integrating the eq. (2.23.1) within the limits 0 to 8, we get

$$I_{XX} = \int_0^8 y^{7/3} dy = \left[\frac{y^{(7/3)+1}}{(7/3)+1}\right]_0^8$$

$$I_{XX} = \frac{3}{10}(8)^{10/3}$$

$$I_{XX} = 307.2 \text{ m}^4$$

**Que 2.24.** **Determine the area moment of inertia of the composite area shown in Fig. 2.24.1 about $X$ and $Y$ axis.**



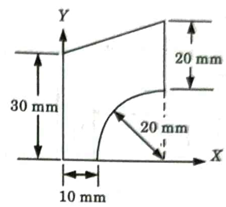



**Fig. 2.24.1.**

**AKTU 2013-14 (I), Marks 10**

**Answer**

**Given :** Fig. 2.24.1.
**To Find :** Area moment of inertia.

1. The given area can be obtained by subtracting a quarter circle and a triangle from a rectangle.
2. Area moment of inertia of rectangle $OEBF$ :
   i. About $X$-axis,

$$(I_{XX})_{OEBF} = \frac{1}{3}\,bh^3 = \frac{1}{3} \times 30 \times (40)^3 = 6.4 \times 10^5 \text{ mm}^4$$

   ii. About $Y$-axis,

$$(I_{YY})_{OEBF} = \frac{1}{3}\,b^3h = \frac{1}{3} \times (30)^3 \times 40 = 3.6 \times 10^5 \text{ mm}^4$$

3. Moment of inertia of triangle $ABF$ :
   i. Moment of inertia of right angled triangle $ABF$ about its centroidal axis along $X_1$,

$$I_{X_1X_1} = \frac{hb^3}{36} = \frac{1}{36} \times 30 \times (10)^3 = 833.33 \text{ mm}^4$$

   ii. About $X$-axis,

$$(I_{XX})_{\Delta ABF} = I_{X_1X_1} + A d_{X_1}^{\,2}$$

$$A = \text{Area of } \Delta = \frac{1}{2}\,bh = \frac{1}{2} \times 30 \times 10 = 150 \text{ mm}^2$$

$$d_{X_1} = 36.67 \text{ mm}$$

$$(I_{XX})_{\Delta ABF} = 833.33 + 150 \times (36.67)^2$$

$$(I_{XX})_{\Delta ABF} = 2.0254 \times 10^5 \text{ mm}^4$$

   iii. About $Y$-axis,

$$(I_{YY})_{ABF} = \frac{bh^3}{12} = \frac{10 \times (30)^3}{12} = 0.225 \times 10^5 \text{ mm}^4$$

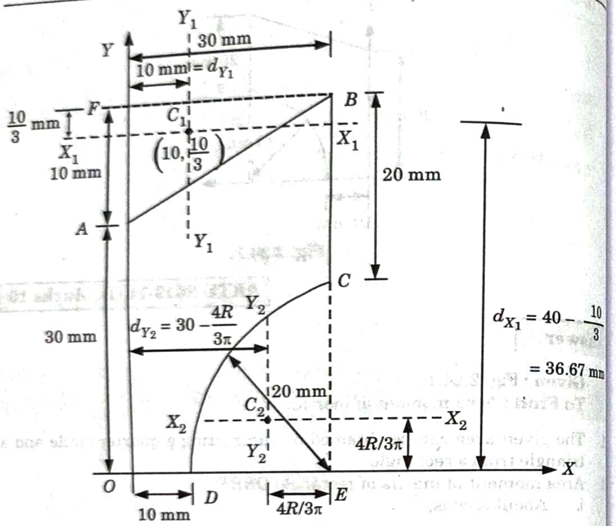



**Fig. 2.24.2.**

4. Moment of inertia of quarter circle *CDE* :
   i. Moment of inertia about *X*-axis,
   $$(I_{XX})_{CDE} = \frac{\pi R^4}{16} = \frac{\pi(20)^4}{16} = 0.3142 \times 10^5 \text{ mm}^4$$
   ii. Moment of inertia about centroidal axis, $Y_2Y_2$
   $$(I_{Y_2Y_2}) = 0.055\,R^4 = 0.055\,(20)^4 = 8800 \text{ mm}^4$$
   iii. Now moment of inertia about *Y*-axis using parallel axis theorem,
   $$(I_{YY})_{CDE} = I_{Y_2Y_2} + Ad_{Y_2} = 8800 + \frac{1}{4}\pi(20)^2\left(30 - \frac{4 \times 20}{3\pi}\right)^2$$
   $$(I_{YY})_{CDE} = 1.542 \times 10^5 \text{ mm}^4$$
5. Now moment of inertia for the given area,
   i. About *X*-axis, $I_X = (I_{XX})_{OEBF} - (I_{XX})_{ABF} - (I_{XX})_{CDE}$
   $$= 6.4 \times 10^5 - 2.0254 \times 10^5 - 0.3142 \times 10^5$$
   $$= 4.0604 \times 10^5 \text{ mm}^4$$
   ii. About *Y*-axis, $I_Y = (I_{YY})_{OEBF} - (I_{YY})_{ABF} - (I_{YY})_{CDE}$
   $$= 3.6 \times 10^5 - 0.225 \times 10^5 - 1.542 \times 10^5$$
   $$= 1.833 \times 10^5 \text{ mm}^4$$



**Que 2.25.** **Determine the moment of inertia about *X-X* and *Y-Y* axis passing through the centroid of the symmetrical *I*- section as shown in Fig. 2.25.1.**

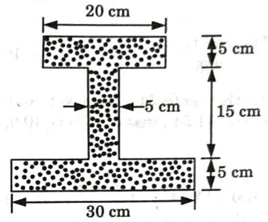



**Fig. 2.25.1.**

**AKTU 2015-16 (I), Marks 15**

**Answer**

**Given :** Fig. 2.25.1.
**To Find :** Moment of inertia about *X-X* and *Y-Y* axis.

1. Since the section is divided into three rectangles as shown in Fig. 2.27.2.
   $$A_1 = 20 \times 5 = 100 \text{ cm}^2$$
   $$A_2 = 15 \times 5 = 75 \text{ cm}^2$$
   $$A_3 = 30 \times 5 = 150 \text{ cm}^2$$
   $$\text{Total Area, } A = A_1 + A_2 + A_3 = 325 \text{ cm}^2$$
2. Due to symmetry, centroid lies on axis *Y-Y*. The bottom fiber 1-1 may be chosen as reference axis to locate the centroid.

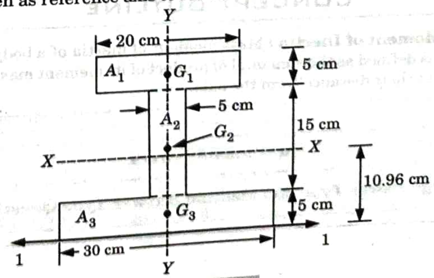



**Fig. 2.25.2.**

3. The distance of the centroid from 1-1,

$$\bar{y} = \frac{\text{Sum of moments of the areas of the rectangles about axis 1-1}}{\text{Total area of section}}$$

$$= \frac{100 \times (20 + 2.5) + 75 \times (7.5 + 5) + 150 \times 2.5}{325} = 10.96 \text{ cm}$$

4. With reference to the centroidal axis $X$-$X$ and $Y$-$Y$, the centroid of rectangle $A_1$ is $G_1$ (0.0, 11.54), that of $A_2$ is $G_2$ (0.0, 1.54) and that of $A_3$ is $G_3$ (0.0, – 8.46).

5. $$I_{XX} = \left(\frac{20 \times 5^3}{12} + 100 \times 11.54^2\right) + \left(\frac{5 \times 15^3}{12} + 75 \times 1.54^2\right) + \left(\frac{30 \times 5^3}{12} + 150 \times (-8.46)^2\right)$$

$$I_{XX} = 26157.8533 \text{ cm}^4$$

6. $$I_{YY} = \frac{5 \times 20^3}{12} + \frac{15 \times 5^3}{12} + \frac{5 \times 30^3}{12}$$

$$I_{YY} = 14739.5833 \text{ cm}^4$$

## PART-6

*Mass Moment Inertia of Circular Plate, Cylinder, Cone, Sphere, Hook.*

### CONCEPT OUTLINE

**Mass Moment of Inertia :** Mass moment of inertia of a body about an axis is defined as the sum total of product of its element masses and square of their distance from the axis.

### Questions-Answers

**Long Answer Type and Medium Answer Type Questions**



**Que 2.26. Derive the expression of mass moment of inertia of circular disc about its diametral axis.**

**AKTU 2014-15 (II), Marks 10**

**Answer**

1. Consider an elemental area $r\, d\theta\, dr$ and thickness $dr$ as shown in Fig. 2.26.1.

Mass of the element, $dm = \rho r\, d\theta\, dr\, t = \rho\, t\, r\, d\theta\, dr$

where, $\rho$ = Density of the circular plate.

$t$ = Thickness of the plate.

Its distance from $X$ axis $= r \sin \theta$

2. Now, $$I_{XX} = \oint (r \sin \theta)^2\, dm$$

$$= \int_0^R \int_0^{2\pi} r^2 \sin^2 \theta\, \rho\, t\, r\, d\theta\, dr$$

$$= \rho t \int_0^R \int_0^{2\pi} r^3 \left(\frac{1 - \cos 2\theta}{2}\right) dr d\theta$$

$$= \rho t \int_0^R \frac{r^3}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} dr = \rho t \int_0^R \frac{r^3}{2} \times 2\pi\, dr$$

$$= \rho t \pi \left[\frac{r^4}{4}\right]_0^R = \rho t\, \frac{\pi R^4}{4}$$

3. Mass of the plate, $M = \rho \times \pi R^2 t$

$$\therefore \quad I_{XX} = \frac{MR^2}{4}$$

Similarly, $$I_{YY} = \frac{MR^2}{4}$$

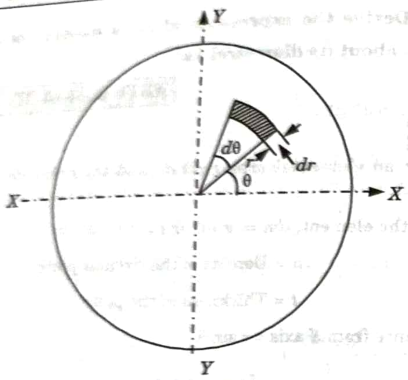



Fig. 2.26.1.

Actually $I = \frac{MR^2}{4}$ is the moment of inertia of circular plate about any diametral axis in the plate.

4. To find $I_{ZZ'}$, consider the same element,

$$I_{ZZ} = \oint r^2\, dm = \int_0^R \int_0^{2\pi} r^2 \rho\, t\, r\, dr\, d\theta$$

$$= \rho t \int_0^R r^3 [\theta]_0^{2\pi}\, dr = \rho t \int_0^R 2\pi r^3\, dr$$

$$= \rho t\, 2\pi \left[\frac{r^4}{4}\right]_0^R = \rho t 2\pi \frac{R^4}{4} = \rho t \frac{\pi R^4}{2}$$

5. But total mass, $M = \rho t \pi R^2$

$\therefore$ $$I_{ZZ} = \frac{MR^2}{2}$$

**Que 2.27.** **Derive an expression for mass moment of inertia of a solid cylinder about its longitudinal axis and its centroidal axes.**

**Answer**

Let us consider a solid cylinder of base radius $R$, length $L$ and uniform mass density $\rho$.



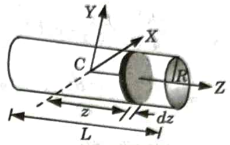



Fig. 2.27.1.

**i. Mass Moment of Inertia about Longitudinal Axis :**

1. In the Fig. 2.27.1, $Z$-axis which passes from the centroid of the cylinder and is along the length of cylinder is termed as longitudinal axis of cylinder.
2. Now consider a solid circular disc of infinitesimal thickness $dz$ perpendicular to $Z$-axis of a distance $z$ from the origin.
3. Mass of the infinitesimal disc, $dm = \rho\pi R^2 dz$
4. Mass moment of inertia about the $Z$-axis, $dI_{ZZ'} = dm \frac{R^2}{2}$
5. Now,

$$dI_{ZZ'} = dm \frac{R^2}{2} = \rho\, \pi R^2\, dz \frac{R^2}{2} = \rho\, \pi \frac{R^4}{2} dz$$

$$\int dI_{ZZ'} = \int_{-L/2}^{L/2} \frac{\rho \pi R^4}{2} dz = \frac{\rho \pi R^4}{2} [z]_{-L/2}^{L/2}$$

$$I_{ZZ'} = \frac{\rho \pi R^4}{2} L$$

$$I_{ZZ'} = \frac{MR^2}{2} \qquad \{\because M = \rho \pi R^2 L\}$$

Here, $M$ = Mass of the solid cylinder.

**ii. Mass Moment of Inertia about Centroidal Axes :**

1. Mass moment of inertia of the solid circular disc about an axis (*i.e.*, $X$-$X$ or $Y$-$Y$ axis) lying on its plane is,

$$dI_{X'X'} = dm \frac{R^2}{4}$$

2. Now using parallel axis theorem, we have

$$dI_{XX} = dI_{X'X'} + z^2\, dm = dm \frac{R^2}{4} + z^2\, dm$$

$$\int dI_{XX} = \int_{-L/2}^{L/2} \rho \pi R^2 dz \frac{R^2}{4} + \int_{-L/2}^{L/2} \rho \pi R^2 z^2 dz$$

$$I_{XX} = \frac{\rho\pi R^4}{4}[z]_{-L/2}^{L/2} + \rho\pi R^2\left[\frac{z^3}{3}\right]_{-L/2}^{L/2}$$

$$= \frac{\rho\pi R^4}{4}L + \frac{\rho\pi R^2}{12}L^3 = \frac{\rho\pi R^2 L}{12}[3R^2 + L^2]$$

$$I_{XX} = \frac{M}{12}[3R^2 + L^2] \qquad [\because M = \rho\pi R^2 L]$$

3. As the cylinder is symmetrical about X-Z and Y-Z plane,

$$I_{XX} = I_{YY} = \frac{M}{12}[3R^2 + L^2]$$

**Que 2.28.** **Find the mass moment of inertia of a hollow cylinder about its axis. The mass of cylinder is 5 kg, inner radius 10 cm, outer radius 15 cm and height 20 cm.**

**AKTU 2012-13, Marks 05**

**Answer**

Given : $M = 5$ kg, $R_2 = 10$ cm $= 0.1$ m, $R_1 = 15$ cm $= 0.15$ m, $L = 20$ cm $= 0.2$ cm
To Find : Mass moment of inertia of hollow cylinder.

1. Mass moment of inertia of hollow cylinder about longitudinal axis is given by,

$$I_{ZZ} = \frac{M}{2}[R_1^2 + R_2^2] = \frac{5}{2}[(0.15)^2 + (0.1)^2]$$

$$I_{ZZ} = 0.08125 \text{ kg-m}^2$$

2. Mass moment of inertia of hollow cylinder about its centroidal axis is given by,

$$I_{XX} = I_{YY} = \frac{M}{12}[3(R_1^2 + R_2^2) + L^2]$$

$$= \frac{5}{12}[3(0.15)^2 + 3(0.1)^2 + (0.2)^2]$$

$$= 0.05729 \text{ kg-m}^2 \approx 0.0573 \text{ kg-m}^2$$

**Que 2.29.** **Calculate the mass moment of inertia of the cylinder of radius 0.5 m, height 1 m and density 2400 kg/m³ about the centroidal axis Fig. 2.29.1.**

**AKTU 2013-14 (I), Marks 10**



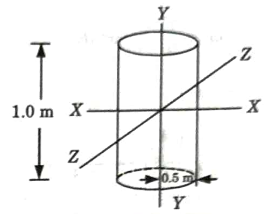



**Fig. 2.29.1.**

**Answer**

Given : $R = 0.5$ m, $L = 1$ m, $\rho = 2400$ kg/m³
To Find : Mass moment of inertia of the cylinder about centroidal axis.

1. We know that, $I_{zz} = \frac{1}{6}M(3R^2 + L^2)$

$$= \frac{1}{6}\rho\pi R^2 L(3R^2 + L^2) \qquad (\because M = \rho\pi R^2 L)$$

$$= \frac{1}{6} \times 2400 \times \pi \times 0.5^2 \times 1 \times (3 \times 0.5^2 + 1^2)$$

$$= 549.78 \text{ kg-m}^2$$

**Que 2.30.** **Determine the mass moment of inertia of a right circular solid cone of base radius R and height h about the axis of rotation.**

**AKTU 2013-14 (I), Marks 10**

**Answer**

1. Consider a solid cone of height $h$ and radius $R$. If $\rho$ is the density of the material of the cone, then
   Mass of the cone, $M$ = Density × Volume

$$M = \rho \times \frac{1}{3}\pi R^2 h$$

2. Consider an element of thickness $dy$ and radius $r$ at distance $y$ from the apex $A$.
3. Mass of the elemental strip, $dm = \rho\pi r^2 dy$
4. Mass moment of inertia of the elemental strip about axis $YY$
   $= (1/2) \times$ Mass moment of inertia about polar axis

$$= \frac{1}{2}(r^2 dm) = \frac{1}{2} r^2 (\rho \pi r^2\, dy)$$

$$= \frac{1}{2}(\rho\, \pi r^4\, dy)$$

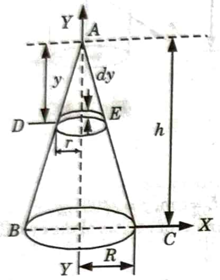



**Fig. 2.30.1.**

5. Since the integration is to be done with respect to $y$ within the limits 0 to $h$.
   In triangles $ADE$ and $ABC$

$$\frac{r}{R} = \frac{y}{h}, \quad r = R \times \frac{y}{h}$$

$$I_{YY} = \int_0^h \frac{1}{2} \rho\, \pi \left(\frac{Ry}{h}\right)^4 dy$$

$$= \frac{\rho \pi R^4}{2h^4}\left[\frac{y^5}{5}\right]_0^h = \frac{\rho \pi R^4 h}{10}$$

$$= \frac{\rho \pi R^2 h}{3} \times \frac{3}{10} R^2$$

$$= \frac{3}{10} MR^2 \qquad \left(\because M = \frac{1}{3}\pi \rho R^2 h\right)$$

**Que 2.31.** **Derive the expression for mass moment of inertia of a sphere about centroidal axis.**

**AKTU 2015-16 (I), Marks 10**

**Answer**

1. Consider a solid sphere of radius $R$ with $O$ as centre. If $\rho$ is the density of the material of the sphere, then



Mass of the sphere, $M$ = Density × Volume

$$M = \rho \times \frac{4}{3}\pi R^3$$

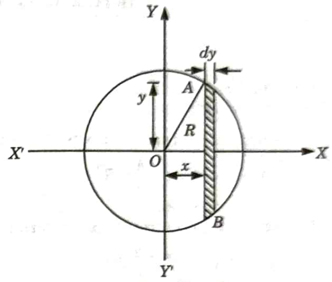



**Fig. 2.31.1.**

2. Let us focus on a thin disc $AB$ of thickness $dx$ at radius $x$ from the centre.

   Radius of the disc, $y = \sqrt{R^2 - x^2}$

   Mass of the disc, $dm = \rho \times \pi y^2\, dx = \rho\, \pi (R^2 - x^2)\, dx$

3. Mass moment of inertia of this elementary disc about the polar axis $ZZ'$

$$= y^2\, dm = \rho\, \pi (R^2 - x^2)\, dx \times (R^2 - x^2)$$

$$= \rho\, \pi (R^2 - x^2)^2\, dx = \rho\, \pi (R^4 + x^4 - 2R^2 x^2)\, dx$$

4. The mass moment of inertia of the whole sphere can be worked out by integrating the above expression between the limits $-R$ to $R$.

   ∴ Mass moment of inertia of the sphere about polar axis $Z$-$Z'$,

$$I_{ZZ'} = \rho\, \pi \int_{-R}^{R} (R^4 + x^4 - 2R^2 x^2)\, dx$$

$$I_{ZZ'} = \rho\, \pi \left[R^4 x + \frac{x^5}{5} - 2R^2 \frac{x^3}{3}\right]_{-R}^{R}$$

$$= \frac{16 \rho \pi R^5}{15} = \frac{4}{5} MR^2$$

5. According to perpendicular axis theorem, the mass moment of inertia of a solid sphere about $X$-$X'$ or $Y$-$Y'$ axis is,

$$I_{XX'} = I_{YY'} = \frac{I_{ZZ'}}{2} = \frac{2}{5} MR^2$$

**Que 2.32.** **Determine the mass moment of inertia of uniform density sphere of radius 5 cm about its centroidal axes.**

**AKTU 2013-14 (II), Marks 10**

**Answer**

**Given :** $R = 5$ cm = 0.05 m
**To Find :** Mass moment of inertia.

1. Assume uniform density of solid sphere is $\rho$. So, mass of sphere,

$$M = \rho \times V = \rho \times \frac{4}{3}\pi R^3 = \rho \times \frac{4}{3} \times \pi \times 5^3$$
$$= 523.6 \rho \text{ kg}$$

2. Mass moment of inertia about centroidal axis in terms of mass $M$,

$$I_{XX'} = I_{YY'} = \frac{I_{ZZ'}}{2} = \frac{2}{5} MR^2$$
$$= \frac{2}{5} \times 523.6 \rho \times (5)^2 = 5236 \rho \text{ cm}^4$$

☺☺☺

# 3 UNIT

# Basic Structural Analysis

## CONTENTS

PART-1

*Basic Structural Analysis.*

## CONCEPT OUTLINE

**Truss :** A structure made up of several members riveted or welded together is known as truss.
**Frame :** If the members of the structure are hinged or pin-joined, then the structure is known as frame.

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 3.1.** **What are the different types of frames ?**

**Answer**

Following are the different types of frames :

i. **Perfect Frame :**

1. The frame which is composed of such members, which are just sufficient to keep the frame in equilibrium, when the frame is supporting an external load is known as perfect frame.
2. For a perfect frame, the number of joints and number of members are given by,

$$n = 2j - 3$$

where, $n$ = Number of members.
$j$ = Number of joints.

ii. **Imperfect Frame :**

1. A frame in which number of members and number of joints are not given by $n = 2j - 3$ is known as imperfect frame. This means that number of members in an imperfect frame will be either more or less than $(2j - 3)$.
2. If the number of members in a frame are less than $(2j - 3)$, then the frame is known as deficient frame.
3. If the number of members in a frame are more than $(2j - 3)$, then the frame is known as redundant frame.

**Que 3.2.** **What do you understand by the analysis of frame ? Also write down the assumptions made in the analysis of frame.**



**Answer**

a. **Analysis of a Frame :**

1. Analysis of a frame consists of :
   i. Determinations of the reactions at the supports.
   ii. Determinations of the axial forces in the members of the frame.
2. The reactions are determined by the condition that the applied load system and the induced reactions at the supports form a system in equilibrium.
3. The forces in the members of the frame are determined by the condition that every joint should be in equilibrium and so, the forces acting at every joint should form a system in equilibrium.

b. **Assumptions made in the Analysis of Frame :**

1. The frame should be perfect.
2. The frame carries load at the joints.
3. All the members are pin-joined and joints are smooth.

PART-2

*Equilibrium in Three Dimensions.*

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 3.3.** **Write down the equations for the equilibrium of a body in three dimension.**

**Answer**

1. There are six equations expressing the equilibrium of a body in three dimensions. These are :
   i. Sum of forces : $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma F_z = 0$
   ii. Sum of moments : $\Sigma M_x = 0$, $\Sigma M_y = 0$ and $\Sigma M_z = 0$
2. The above six equations can be resolved into components to solve the given problems.

PART-3

*Analysis of Simple Trusses by Method of Sections.*

## Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 3.4.** **Write the procedure of method of section in truss analysis.**

**Answer**

Procedure of method of sections is as follows :

**Step 1 :** The truss is split into two parts by passing an imaginary section.

**Step 2 :** The imaginary section has to be such that it does not cut more than three members in which the forces are to be determined.

**Step 3 :** The conditions of equilibrium $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M = 0$ are applied for one part of the truss and the unknown forces in the member is determined.

**Step 4 :** While considering equilibrium, the nature of force in any member is chosen arbitrarily to be tensile or compressive.

i. If the magnitude of a particular force comes out positive, the assumption in respect of its direction is correct.

ii. However, if the magnitude of the force comes out to be negative, the actual direction of the force is opposite to that what has been assumed.

**Que 3.5.** **A truss of 12 m span is loaded as shown in Fig. 3.5.1. Determine the forces in the members *DG*, *DF* and *EF*, using method of sections.**

**Answer**

**Given :** Length of truss = 12 m, Fig. 3.5.1.
**To Find :** Forces in members *DG*, *DF* and *EF*.

1. In triangle *AEC*, $AC = AE \cos 30°$

$$= 4 \times 0.866 = 3.464 \text{ m}$$

2. Now length, $AD = 2 \times AC = 2 \times 3.464 = 6.928$ m

3. Now taking the moments about *A*, we get

$$R_B \times 12 = 2 \times AC + 1 \times AD + 1 \times AE$$
$$= 2 \times 3.464 + 1 \times 6.928 + 1 \times 4 = 17.856$$

$$\therefore \quad R_B = \frac{17.856}{12} = 1.49 \text{ kN}$$



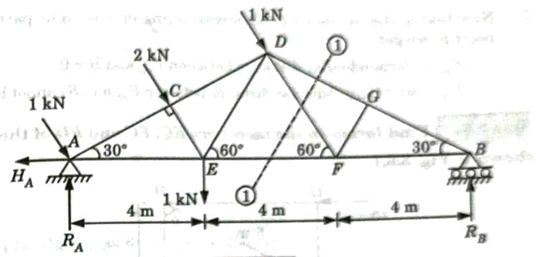



**Fig. 3.5.1.**

4. Now draw the section line (1-1), passing through members *DG*, *DF* and *EF* in which the forces are to be determined. Consider the equilibrium of the right part of the truss. This part is shown in Fig. 3.5.2.

5. Taking moments of all forces acting on right part about point *F*, we get

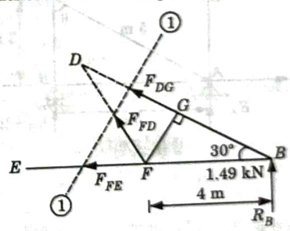



**Fig. 3.5.2.**

$$R_B \times 4 + F_{DG} \times FG = 0 \qquad (\because FG = 4 \times \sin 30°)$$
$$1.49 \times 4 + F_{DG} \times (4 \times \sin 30°) = 0$$
$$F_{DG} = \frac{-1.49 \times 4}{4 \times \sin 30°} = -2.98 \text{ kN}$$
$$\therefore \quad F_{DG} = 2.98 \text{ kN (Compressive)}$$

6. Now taking the moments about point *D*, we get

$$R_B \times BD \cos 30° = F_{FE} \times BD \sin 30°$$
$$R_B \times \cos 30° = F_{FE} \times \sin 30°$$
$$\therefore \quad F_{FE} = \frac{1.49 \times \cos 30°}{\sin 30°} = \frac{1.49 \times 0.866}{0.5}$$
$$= 2.58 \text{ kN (Tensile)}$$

7. Now taking the moments of all forces acting on the right part about point $B$, we get

$F_{FD} \times$ Perpendicular distance between $F_{FD}$ and $B = 0$

$\therefore \; F_{FD} = 0$ ($\because$ Perpendicular distance between $F_{FD}$ and $B$ cannot be zero)

**Que 3.6.** **Find forces in the members *EC*, *FC* and *FD* of the truss shown in Fig. 3.6.1.**

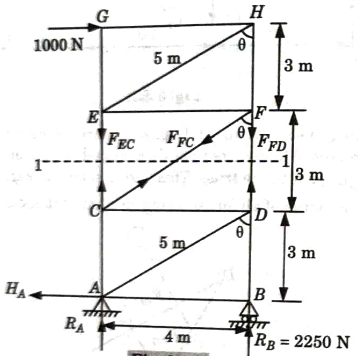



Fig. 3.6.1.

**Answer**

**Given :** Fig. 3.6.1.
**To Find :** Forces in the members *EC*, *FC* and *FD*.

1. From geometry of Fig. 3.6.1.

$$\cos\theta = \frac{3}{5} = 0.6$$

and
$$\sin\theta = \frac{4}{5} = 0.8$$

2. Draw the FBD of the portion above section 1 – 1 (Fig. 3.6.2).
3. Consider the equilibrium of the FBD of the drawn portion,

$$\Sigma M_F = 0$$
$$-F_{EC} \times 4 + 1000 \times 3 = 0$$
$$F_{EC} = 750 \text{ N}$$



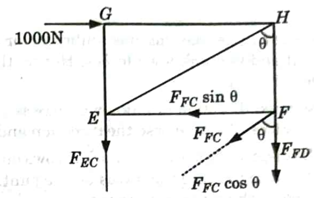



Fig. 3.6.2.

4. Consider the condition of equilibrium at point $F$,

$$\Sigma F_x = 0$$
$$F_{FC} \sin\theta = 1000$$
$$F_{FC} \times 0.8 = 1000$$
$$\therefore \quad F_{FC} = 1250 \text{ N}$$

and
$$\Sigma F_y = 0$$
$$F_{EC} + F_{FD} + F_{FC}\cos\theta = 0$$
$$750 + F_{FD} + 1250 \times 0.6 = 0$$
$$F_{FD} = -1500 \text{ N}$$

5. So, direction of $F_{FD}$ is opposite to our assumed direction hence it is compressive in nature.

## PART-4

*Analysis of Simple Trusses by Method of Joints.*

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 3.7.** **Write the procedure of method of joints in truss analysis.**

**Answer**

Procedure of method of joints is as follows :

**Step 1 :** Determine the inclinations of all inclined members.

**Step 2 :**

1. Look for a joint at which there are only two unknowns.
2. If such a joint is not available, determine the reactions at the supports, and then at the supports these unknowns may reduce to only two.

**Step 3 :**

1. Now there are two equations of equilibrium for the forces meeting at the joint and two unknown forces. Hence, the unknown forces can be determined.
2. If the assumed direction of unknown force is opposite, the value will be negative. Then reverse the direction and proceed.

**Step 4 :** On the diagram of the truss, mark arrows on the members near the joint analysed to indicate the forces on the joint. At the other end, mark the arrows in the reverse direction.

**Step 5 :** Look for the next joint where there are only two unknown forces and analyse that joint.

**Step 6 :** Repeat steps 4 and 5 till forces in all the members are found.

**Step 7 :**

1. Determine the nature of forces in each member and tabulate the results.
2. Note that if the arrow marks on a member are towards each other, then the member is in tension and if the arrow marks are away from each other, the member is in compression as shown in Fig. 3.7.1.

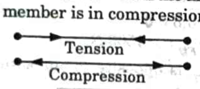



**Fig. 3.7.1.**

**Que 3.8.** **Using method of joint determine the forces in each member of the truss shown in Fig. 3.8.1.**

**AKTU 2013-14, (II) Marks 10**

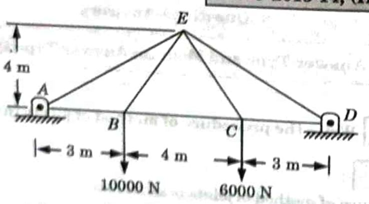



**Fig. 3.8.1.**

**Answer**

**Given :** Fig. 3.8.1.
**To Find :** Forces in each member of truss.



1. From $\Delta AFE$, $\tan\theta = \dfrac{EF}{AF} = \dfrac{EF}{AB + BF} = \dfrac{4}{3+2} = \dfrac{4}{5}$

$$\theta = 38.66°$$

2. In $\Delta BEF$, $\tan\phi = \dfrac{EF}{BF} = \dfrac{4}{2} = 2$

$$\phi = 63.43°$$

$$\Sigma F_y = 0$$

$$R_A + R_D = 10000 + 6000 = 16000 \text{ N} \quad ...(3.8.1)$$

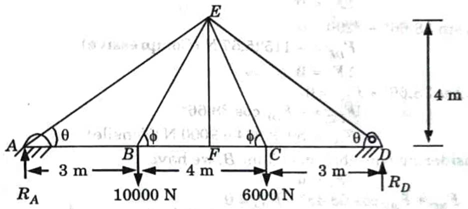



**Fig. 3.8.2.**

3. Taking moment about $A$, $\Sigma M_A = 0$

$$10000 \times 3 + 6000 \times 7 - R_D \times 10 = 0$$

$$R_D = \frac{72000}{10} = 7200 \text{ N}$$

4. From eq. (3.8.1), we have

$$R_A = 8800 \text{ N}$$

5. Considering equilibrium of joint $A$,

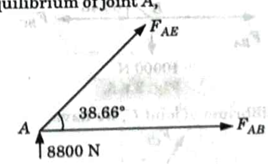



**Fig. 3.8.3.**

$$\Sigma F_x = 0$$

$$F_{AE} \cos 38.66° + F_{AB} = 0 \quad ...(3.8.2)$$

$$\Sigma F_y = 0$$

$$F_{AE} \sin 38.66° + 8800 = 0$$

$$F_{AE} = -14086.81 \text{ N(Compressive)}$$

From eq. (3.8.2), we get

$$F_{AB} = -F_{AE} \cos 38.66° = 10999.92 \approx 11000 \text{ N (Tensile)}$$

6. Considering equilibrium of joint $D$,

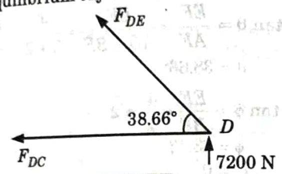



Fig. 3.8.4.

$$\Sigma F_y = 0$$
$$F_{DE} \sin 38.66° + 7200 = 0$$
$$F_{DE} = -11525.57 \text{ N (Compressive)}$$
$$\Sigma F_x = 0$$
$$F_{DE} \cos 38.66° + F_{DC} = 0$$
$$F_{DC} = -F_{DE} \cos 38.66°$$
$$F_{DC} = 8999.934 \approx 9000 \text{ N (Tensile)}$$

7. Considering equilibrium of joint $B$, we have

$$\Sigma F_x = 0$$
$$F_{BC} + F_{BE} \cos 63.43° - F_{BA} = 0$$
$$\Sigma F_y = 0$$
$$F_{BE} \sin 63.43° = 10000$$
$$F_{BE} = 11180.82 \text{ N (Tensile)}$$
$$F_{BC} = -F_{BE} \cos 63.43° + F_{BA}$$
$$= 5998.92 \text{ N (Tensile)}$$

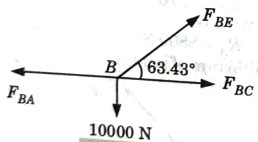



Fig. 3.8.5.

8. Considering equilibrium of joint $C$, we have

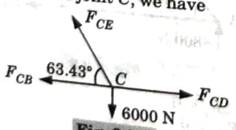



Fig. 3.8.6.

$$\Sigma F_y = 0$$
$$F_{CE} \sin 63.43° = 6000$$
$$F_{CE} = 6708.5 \text{ N (Tensile)}$$



**Que 3.9.** **Determine the force in each member of the simple equilateral truss Fig. 3.9.1.**

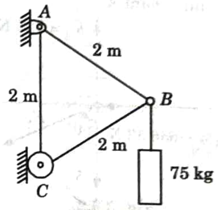



Fig. 3.9.1.

**AKTU 2014-15, (I) Marks 10**

**Answer**

**Given :** Fig. 3.9.1.
**To Find :** Force in each member of truss.

1. Consider the equilibrium of the entire frame,

$$\Sigma M_A = 0$$
$$R_C \times 2 - 735 \times 2 \cos 30° = 0$$
$$R_C = 636.53 \text{ N}$$
$$\Sigma F_x = 0$$
$$R_C + H_A = 0$$

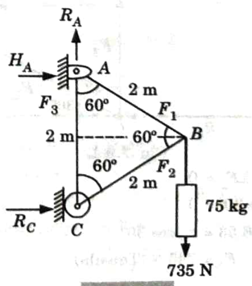



Fig. 3.9.2.

$$H_A = -R_C = -636.53 \text{ N}$$
$$H_A = -636.53 \text{ N}$$
$$\Sigma F_y = 0$$

$R_A - 735 = 0$

$R_A = 735$ N

2. Considering equilibrium of joint $A$,

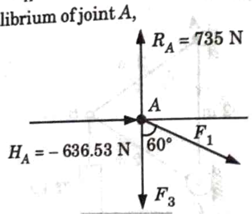



**Fig. 3.9.3.**

$$\Sigma F_y = 0$$

$$F_3 + F_1 \cos 60° = 735 \quad ...(3.9.1)$$

$$\Sigma F_x = 0$$

$$-636.53 + F_1 \sin 60° = 0$$

$$F_1 = \frac{636.53}{\sin 60°} = 735 \text{ N (Tensile)}$$

3. From eq. (3.9.1), we get

$$F_3 = 735 - F_1 \cos 60°$$

$$= 735 - 735 \cos 60°$$

$$F_3 = 367.5 \text{ N (Tensile)}$$

4. Considering equilibrium of joint $C$,

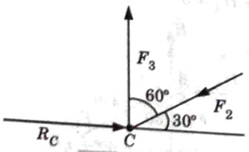



**Fig. 3.9.4.**

$$\Sigma F_x = 0$$

$$R_C - F_2 \cos 30° = 0$$

$$636.53 = F_2 \cos 30°$$

$$F_2 = 735 \text{ N (Tensile)}$$

**Que 3.10.** **Compute the forces in all the members for the given truss as shown in Fig. 3.10.1. Distance between $A$ and $C$ is 12 m.**



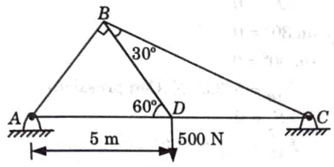



**Fig. 3.10.1.**

**AKTU 2015-16, Marks 10**

**Answer**

**Given :** Fig. 3.10.1, $AC = 12$ m

**To Find :** Forces in all members.

1. Consider the equilibrium of entire truss,

$$\Sigma F_y = 0$$

$$R_A + R_C = 500 \text{ N} \quad ...(3.10.1)$$

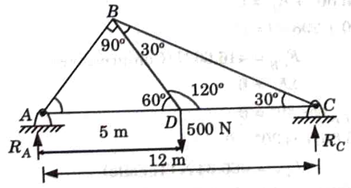



**Fig. 3.10.2.**

2. Taking moment about $A$, $\Sigma M_A = 0$

$$500 \times 5 = R_C \times 12$$

$$R_C = 208.33 \text{ N}$$

3. From eq. (3.10.1), we get

$$R_A = 500 - 208.33 = 291.67 \text{ N}$$

4. Considering equilibrium of joint $A$,

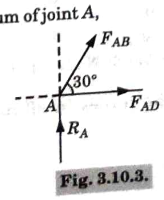



**Fig. 3.10.3.**

$$\Sigma F_y = 0$$
$$R_A + F_{AB} \sin 30° = 0$$
$$291.67 + F_{AB} \times \sin 30° = 0$$
$$F_{AB} = 583.34 \text{ N (Compressive)}$$
$$\Sigma F_x = 0$$
$$F_{AD} + F_{AB} \cos 60° = 0$$
$$F_{AD} + (-583.34) \cos 30° = 0$$
$$F_{AD} = 505.19 \text{ N (Tensile)}$$

5. Considering the equilibrium of joint $C$,

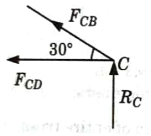



**Fig. 3.10.4.**

$$\Sigma F_y = 0$$
$$F_{CB} \sin 30° + R_C = 0$$
$$F_{CB} \times \sin 30 + 208.33 = 0$$
$$F_{CB} = 416.66 \text{ N (Compressive)}$$
$$\Sigma F_x = 0$$
$$F_{CD} + F_{CB} \cos 30° = 0$$
$$F_{CD} + (-416.66) \cos 30° = 0$$
$$F_{CD} = 360.84 \text{ N (Tensile)}$$

6. Considering the equilibrium of joint $D$,

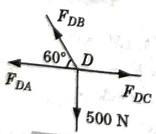



**Fig. 3.10.5.**

$$\Sigma F_y = 0$$
$$F_{DB} \sin 60° = 500$$
$$F_{DB} = 577.35 \text{ N (Tensile)}$$

**Que 3.11.** **Determine the forces in all members of the truss as shown in Fig. 3.11.1.**



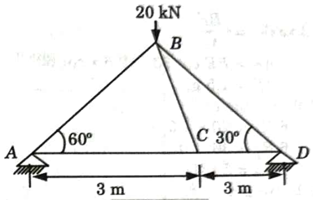



**Fig. 3.11.1.**

**AKTU 2014-15, (II) Marks 10**

**Answer**

**Given :** Fig. 3.11.1.
**To Find :** Forces in all the members of truss.

1. Consider the equilibrium of entire truss,

$$\Sigma F_y = 0$$
$$R_A + R_B = 20 \text{ kN} \quad \text{...(3.11.1)}$$

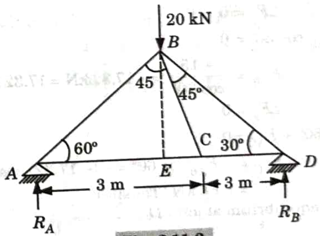



**Fig. 3.11.2.**

2. Taking moment about $A$,

$$\Sigma M_A = 0$$
$$6R_B = 20 \times AE \quad \text{...(3.11.2)}$$

3. In $\Delta ABD$, $\angle B = 90°$

$$\therefore \quad \sin 30° = \frac{AB}{AD} = \frac{AB}{6}$$
$$AB = 6 \sin 30° = 3 \text{ m}$$

4. In $\Delta ABE$, $\angle E = 90°$

$$BE = 3 \sin 60° = 2.6 \text{ m}$$

$$\tan 60° = \frac{BE}{AE}$$

$\therefore$ $AE = BE \cot 60° = 2.6 \times \cot 60°$

$AE = 1.5$ m

5. From eq. (3.11.2), we have

$$6\,R_B = 20 \times 1.5$$
$$6\,R_B = 30$$
$$R_B = 5 \text{ kN}$$

6. From eq. (3.11.1), we get

$$R_A = 20 - R_B$$
$$= 20 - 5$$
$$= 15 \text{ kN}$$

7. Consider the equilibrium at joint $A$,

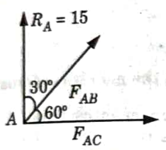



**Fig. 3.11.3.**

$$\Sigma F_y = 0$$
$$15 + F_{AB} \cos 30° = 0$$
$$F_{AB} = \frac{-15}{\cos 30°} = -17.32 \text{ kN} = 17.32 \text{ kN (Compressive)}$$
$$\Sigma F_x = 0$$
$$F_{AB} \cos 60° + F_{AC} = 0$$
$$F_{AC} = -F_{AB} \cos 60° = -(-17.32) \cos 60°$$
$$= 8.66 \text{ kN (Tensile)}$$

8. Consider the equilibrium at joint $D$,

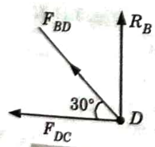



**Fig. 3.11.4.**

$$\Sigma F_y = 0$$
$$5 + F_{BD} \sin 30° = 0$$
$$F_{BD} = -5/\sin 30° = 10 \text{ kN (Compressive)}$$
$$\Sigma F_x = 0$$
$$F_{DC} + F_{BD} \cos 30° = 0$$



$$F_{DC} = 10 \cos 30° = 8.66 \text{ kN (Tensile)}$$

9. Consider the equilibrium at joint $B$,

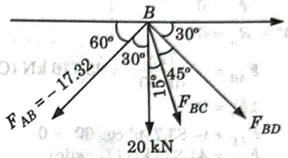



**Fig. 3.11.5.**

$$\Sigma F_y = 0$$
$$20 + F_{AB} \cos 30° + F_{BC} \cos 15° + F_{BD} \cos 60° = 0$$
$$20 - 17.32 \cos 30° + F_{BC} \cos 15° - 10 \cos 60° = 0$$
$$F_{BC} \approx 0$$

**Que 3.12.** **Determine the forces in all member of the truss shown in Fig. 3.12.1 and indicate the magnitude and nature of forces on the diagram of the truss. All inclined members are at 60° to horizontal and length of each member is 2 m.** **AKTU 2016-17, (I) Marks 10**

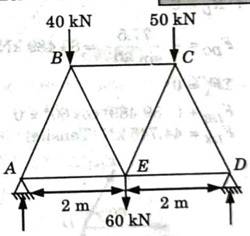



**Fig. 3.12.1.**

**Answer**

**Given :** Fig. 3.12.1, length of each member = 2 m
**To Find :** Magnitude and nature of all the forces in the members of truss.

1. Consider the equilibrium of the entire frame,

$$\Sigma M_A = 0$$
$$R_D \times 4 - 40 \times 1 - 60 \times 2 - 50 \times 3 = 0$$
$$\therefore \quad R_D = 77.5 \text{ kN}$$
$$\Sigma F_y = 0$$
$$R_A + 77.5 = 40 + 60 + 50$$

$R_A = 72.5$ kN

2. **Considering equilibrium at joint $A$,**

$$\Sigma F_y = 0$$

$$F_{AB} \sin 60° + R_A = 0$$

$$F_{AB} = -\frac{72.5}{\sin 60°} = 83.716 \text{ kN (Compressive)}$$

$$\Sigma F_x = 0$$

$$F_{AE} + (-83.716) \cos 60° = 0$$

$$F_{AE} = 41.858 \text{ kN (Tensile)}$$

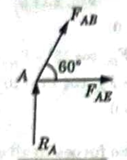



**Fig. 3.12.2.**

4. **Considering equilibrium at joint $D$,**

$$\Sigma F_y = 0$$

$$F_{DC} \sin 60° + R_D = 0$$

$$F_{DC} = -\frac{77.5}{\sin 60°} = 89.489 \text{ kN (Compressive)}$$

$$\Sigma F_x = 0$$

$$F_{DE} + (-89.489) \cos 60° = 0$$

$$F_{DE} = 44.745 \text{ kN (Tensile)}$$

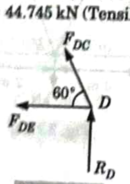



**Fig. 3.12.3.**

5. **Considering equilibrium at joint $B$,**

$$\Sigma F_y = 0$$

$$F_{BE} \sin 60° + F_{AB} \sin 60° + 40 = 0$$

$$\therefore \quad F_{BE} = \frac{-(-72.5) - 40}{\sin 60°} = 37.528 \text{ (Tensile)}$$

$$\Sigma F_x = 0$$

$$F_{BC} - F_{AB} \cos 60° + F_{BE} \cos 60° = 0$$

$$F_{BC} = (-83.716 - 37.528) \times 0.5$$



$$F_{BC} = 60.622 \text{ kN (Compressive)}$$

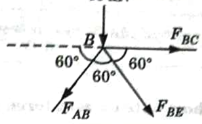



**Fig. 3.12.4.**

6. **Considering equilibrium at joint $C$,**

$$\Sigma F_y = 0$$

$$F_{CE} \sin 60° + 50 + F_{DC} \sin 60° = 0$$

$$F_{CE} = \frac{-(-77.5) - 50}{\sin 60°} = 31.754 \text{ kN (Tensile)}$$

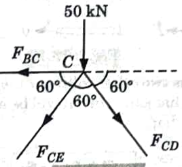



**Fig. 3.12.5.**

7. **Now the forces in all the members are known. The results are shown on the diagram of the truss in Fig. 3.12.6.**

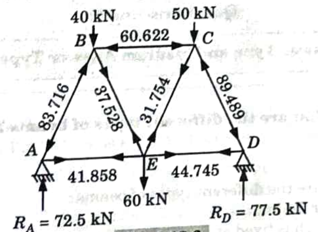



**Fig. 3.12.6.**

**PART-5**

*Zero Force Member.*

Questions-Answers

Long Answer Type and Medium Answer Type Questions

**Que 3.13.** **Write a short note on zero force members.**

**Answer**

1. Zero force members are the members in which there is no force.
2. After knowing the members of zero forces, they can be eliminated while calculating the forces in the members.

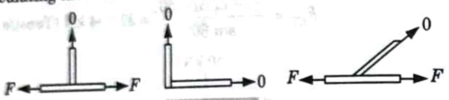



Fig. 3.13.1.

3. A member that joins two other collinear members, at right angles and if no load is acting at that joint, then it will be a zero force member (member with 'L', 'T' and 'Y' shapes).

PART-6

*Simple Beams and Support Reactions.*

Questions-Answers

Long Answer Type and Medium Answer Type Questions

**Que 3.14.** **What are the different types of beams ?**

**Answer**

Following are the different types of beams :

i. **Cantilever Beam :**
1. A beam which is fixed at one end and free at the other end is known as cantilever beam (Fig. 3.14.1).
2. At the fixed end, there will be fixing moment. Also at the fixed end, there can be horizontal and vertical reactions, depending upon the type of load acting on the beam.



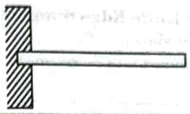


Fig. 3.14.1. Cantilever beam.

ii. **Simply Supported Beam :** A beam supported or resting freely on the supports at its both ends is known as simply supported beam (Fig. 3.14.2).

Fig. 3.14.2. Simply supported beam.

iii. **Overhanging Beam :** If the end portion of a beam is extended beyond the support, such beam is known as overhanging beam (Fig. 3.14.3).

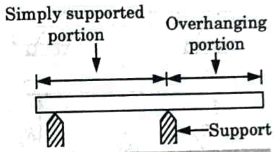



Fig. 3.14.3. Overhanging beam.

iv. **Fixed Beam :** A beam whose both ends are fixed or built-in-walls, is known as fixed beam (Fig. 3.14.4). A fixed beam is also known as a built-in or encastre beam. At the fixed ends, there will be fixing moments and reactions.

Fig. 3.14.4. Fixed beam.

v. **Continuous Beam :** A beam which is provided more than two supports as shown in Fig. 3.14.5, is known as continuous beam.

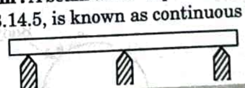


Fig. 3.14.5. Continuous beam.

**Que 3.15.** **Discuss in short about the various types of supports.**

**Answer**

Following are the various types of supports :

i. **Simple Support or Knife Edge Support :** A beam supported on the knife edges $A$ and $B$ is shown in Fig. 3.15.1. The reactions at $A$ and $B$ in case of knife edge support will be normal to the surface of the beam.

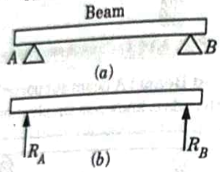



**Fig. 3.15.1.**

ii. **Roller Support :** A beam supported on the rollers at points $A$ and $B$ is shown in Fig. 3.15.2(*a*). The reaction in case of roller supports will be normal to the surface on which roller is placed as shown in Fig. 3.15.2(b).

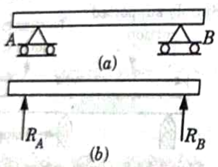



**Fig. 3.15.2.**

iii. **Pin Joint (or Hinged) Support :** A beam, which is hinged (or pin-joint) at point $A$, is shown in Fig. 3.15.3. The reaction at the hinged end may be either vertical or inclined depending upon the type of loading.


**Fig. 3.15.3.**

iv. **Smooth Surface Support :** Fig. 3.15.4 shows a body in contact with a smooth surface. The reaction will always act normal to the support as shown in Fig. 3.15.4.

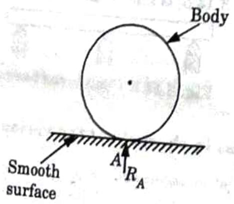



**Fig. 3.15.4.**



v. **Fixed or Built-in Support :**

1. Fig. 3.15.5, shows the end $A$ of a beam, which is fixed. Hence the support at $A$ is known as a fixed support.
2. The fixed support prevents the vertical movement and rotation of the beam. Hence at the fixed support there can be horizontal reaction and vertical reaction. Also there will be fixing moment at the fixed end.


**Fig. 3.15.5.**

**Que 3.16.** **What are the different types of loading ? Explain.**

**Answer**

Following are the different types of loading :

i. **Concentrated or Point Load :**

1. Fig. 3.16.1 shows a beam $AB$, which is simply supported at the ends $A$ and $B$. A load $W$ is acting at the point $C$. This load is known as point load (or concentrated load).
2. Hence any load acting at a point on a beam, is known as point load.

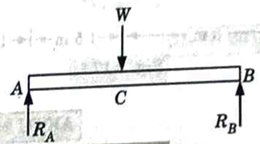



**Fig. 3.16.1.**

ii. **Uniformly Distributed Load :**

1. If a beam is loaded in such a way that each unit length of the beam carries same intensity of the load, then that type of load is known as uniformly distributed load which is written as UDL.
2. Fig. 3.16.2 shows a beam $AB$, which carries a uniformly distributed load.

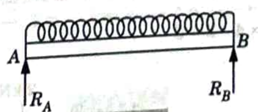



**Fig. 3.16.2.**

iii. **Uniformly Varying Load :**

1. Fig. 3.16.3 shows a beam $AB$, which carries load in such a way that the rate of loading on each unit length of the beam varies uniformly. This type of load is known as uniformly varying load.

2. The total load on the beam is equal to the area of the load diagram. The total load acts at the center of gravity of the load diagram.

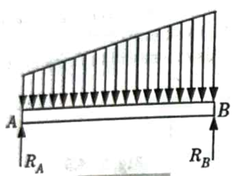



Fig. 3.16.3.

**Que 3.17.** **Determine the reactions at *A* for the cantilever beam subjected to the distributed and concentrated loads Fig. 3.17.1.**

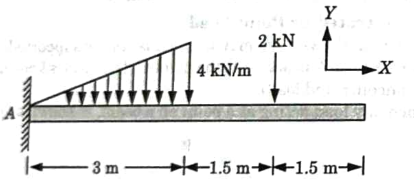



Fig. 3.17.1.

**AKTU 2014-15, (I) Marks 10**

**Answer**

**Given :** Fig. 3.17.1.
**To Find :** Reaction at *A*.

1. Consider the equilibrium of the beam,

$$\Sigma F_y = 0$$

$$R_A - \frac{1}{2} \times 3 \times 4 - 2 = 0$$

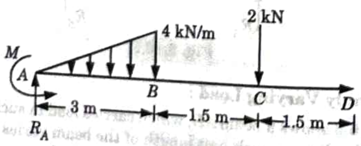



Fig. 3.17.2.



$$R_A = 2 + 6 = 8 \text{ kN}$$

2. Taking moments about *A*,

$$\Sigma M_A = 0$$

$$M - \left(\frac{1}{2} \times 3 \times 4 \times \frac{2}{3} \times 3\right) - 2 \times 4.5 = 0$$

$$M = 21 \text{ kN-m}$$

3. So, the reaction at *A*, $R_A = 8$ kN
Moment, $M = 21$ kN-m

**Que 3.18.** **Determine the reaction at support *A* and *D* in the structure shown in Fig. 3.18.1.**

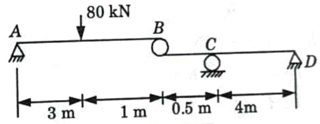



Fig. 3.18.1.

**AKTU 2014-15, (I) Marks 10**

**Answer**

**Given :** Fig. 3.18.1.
**To Find :** Reaction at support *A* and *D*.

1. Consider the FBD of the given beam for the section *AB* and consider its equilibrium,

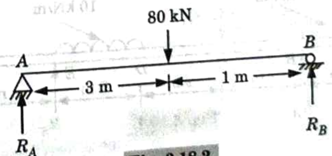



Fig. 3.18.2.

$$\Sigma F_y = 0$$

$$R_A + R_B = 80 \text{ kN}$$

2. Taking moment about *A*, $\Sigma M_A = 0$

$$80 \times 3 = 4 \times R_B$$

$$R_B = 60 \text{ kN}$$

$\therefore \quad R_A = 80 - R_B = 80 - 60 = 20 \text{ kN}$

3. Consider the FBD of the given beam for section $BD$ and consider its equilibrium,

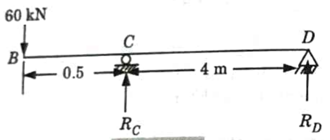



Fig. 3.18.3.

$$\Sigma F_y = 0$$

$$R_C + R_D = 60 \text{ kN}$$

4. Taking moment about $D$, we have

$$\Sigma M_D = 0$$

$$R_C \times 4 = 60 \times 4.5$$

$$R_C = 67.5 \text{ kN}$$

$$\therefore \quad R_D = 60 - R_C = 60 - 67.5 = -7.5 \text{ kN}$$

Here negative sign means that reaction will at $C$ in downward direction.

**Que 3.19.** **Determine the reactions at $B$ and $E$ of the beam, loaded as shown in Fig. 3.19.1 below.**

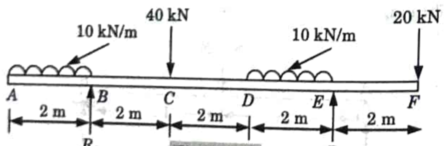



Fig. 3.19.1.

**AKTU 2016-17, (II) Marks 10**

**Answer**

**Given :** Fig. 3.19.1.
**To Find :** Reactions at $B$ and $E$.



1. Considering the equilibrium of the beam,

$$\Sigma F_y = 0$$

$$R_B + R_E = 10 \times 2 + 40 + 10 \times 2 + 20$$

$$R_B + R_E = 100 \text{ kN} \quad ...(3.19.1)$$

2. Now taking moment about $B$, we have

$$\Sigma M_B = 0$$

$$-10 \times 2 \times 1 + 40 \times 2 + 10 \times 2 \times 5 - R_E \times 6 + 20 \times 8 = 0$$

$$R_E = 53.33 \text{ kN}$$

3. From eq. (3.19.1), we get

$$R_B = 100 - R_E = 100 - 53.33$$

$$R_B = 46.67 \text{ kN}$$

**Que 3.20.** **Calculate the support reactions in the given cantilever beam as shown in Fig. 3.20.1.**

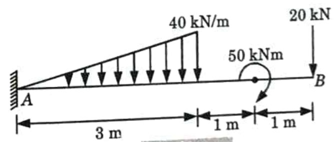



Fig. 3.20.1.

**AKTU 2015-16, (I) Marks 10**

**Answer**

**Given :** Fig. 3.20.1.
**To Find :** Support reactions.

1. Considering the equilibrium of the beam,

$$\Sigma F_y = 0$$

$$R_A - \frac{1}{2} \times 3 \times 40 - 20 = 0$$

$$R_A = 80 \text{ kN}$$

2. Taking moment about point $A$, $\Sigma M_A = 0$

$$M_A - \frac{1}{2} \times 3 \times 40 \times \frac{2}{3} \times 3 - 50 - 20 \times 5 = 0$$

$$M_A = 120 + 50 + 100$$

$$M_A = 270 \text{ kN-m}$$

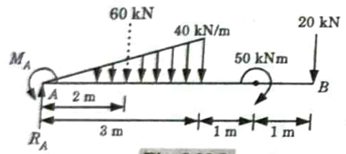



**Fig. 3.20.2.**

☺☺☺

4 UNIT

# Review of Particle Dynamics

## CONTENTS

## PART-1

*Review of Particle Dynamics – Rectilinear Motion.*

### CONCEPT OUTLINE

**Dynamics of Particle :** The study of motion of a particle is known as dynamics of particle. It is further divided into kinematics and kinetics.

**Rectilinear Motion :** The motion of the body along a straight line is called rectilinear motion. It is also known as one-dimensional motion.

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 4.1. What are the parameters used for defining the rectilinear motion of a particle ?**

**OR**

**Define the following terms :**
**i. Displacement.**
**ii. Velocity.**
**iii. Acceleration.**

**Answer**

Following are the parameters used for defining the rectilinear motion :

i. **Displacement :** It is defined as the change in position of the particle during given interval of time. The displacement is a vector quantity and it is dependent only on the initial and final position of the particle.

ii. **Velocity :** Velocity of a particle can be defined as the rate of change of displacement with time.

Mathematically, $v = \frac{ds}{dt}$

iii. **Acceleration :** Acceleration of a particle can be defined as the rate of change of the velocity with time.

Mathematically, $a = \frac{dv}{dt}$



**Que 4.2. Derive the equation of motion for a body moving in a straight line by the method of integration.**

**Answer**

The equation of motion of a body moving in a straight line may be derived by integration as given below :

i. **Derivation of $s = ut + \frac{1}{2}at^2$:**

1. Let a body is moving with a uniform acceleration $a$.
2. We know that,

$$\frac{d^2s}{dt^2} = a \text{ or } \frac{d}{dt}\left(\frac{ds}{dt}\right) = a$$

or $$d\left(\frac{ds}{dt}\right) = a\,dt$$

3. Integrating the above equation,

$$\int d\left(\frac{ds}{dt}\right) = \int a\,dt \text{ or } \frac{ds}{dt} = at + C_1 \quad ...(4.2.1)$$

where, $C_1$ = Constant of integration.

4. But $\frac{ds}{dt}$ = Velocity at any instant.

When $t = 0$, the velocity is known as initial velocity which is represented by $u$.

$\therefore$ At, $t = 0$, $\frac{ds}{dt}$ = Initial velocity = $u$

5. Substituting these values in eq. (4.2.1), we get

$$u = a \times 0 + C_1$$
$$C_1 = u$$

Substituting the value of $C_1$ in eq. (4.2.1), we get

$$\frac{ds}{dt} = at + u \quad ...(4.2.2)$$

6. Now, integrating eq. (4.2.2), we get

$$s = \frac{at^2}{2} + ut + C_2 \quad ...(4.2.3)$$

where, $C_2$ = Another constant of integration.

7. When $t = 0$, then $s = 0$. Substituting these values in eq. (4.2.3), we get

$$0 = \frac{a}{2} \times 0 + u \times 0 + C_2$$
$$\therefore \quad C_2 = 0$$

8. Substituting this value of $C_2$ in eq. (4.2.3), we get

$$s = ut + \frac{1}{2}at^2$$

**ii. Derivation of $v = u + at$ :**

1. From eq. (4.2.2), we have

$$\frac{ds}{dt} = at + u \quad ...(4.2.4)$$

2. But $\frac{ds}{dt}$ represents the velocity at any time. After the time '$t$' the velocity is known as final velocity, which is represented by v.

$\therefore$ $\frac{ds}{dt}$ after time '$t$' = Final velocity = v

3. Substituting the value of $\frac{ds}{dt}$ = v in eq. (4.2.4), we get

$$v = u + at$$

**iii. Derivation of $v^2 = u^2 + 2as$ :**

1. We know that, acceleration $a$ is given by

$$a = \frac{v\,dv}{ds}$$

$$v\,dv = a\,ds \quad ...(4.2.5)$$

2. Integrating eq. (4.2.5), we get

$$\frac{v^2}{2} = as + C_3 \quad ...(4.2.6)$$

where, $C_3$ = Constant of integration.

3. When $s = 0$, the velocity is known as initial velocity.

$\therefore$ At $s = 0$, $v = u$

4. Substituting these values in eq. (4.2.6), we get

$$\frac{u^2}{2} = a \times 0 + C_3$$

$$\therefore \quad C_3 = \frac{u^2}{2}$$

5. Substituting the value of $C_3$ in eq. (4.2.6), we get

$$\frac{v^2}{2} = as + \frac{u^2}{2} \text{ or } v^2 = u^2 + 2as$$

**Que 4.3.** **Acceleration of a ship moving along a straight curve varies directly as the square of its speed. If the speed drops from 3 m/sec to 1.5 m/sec in one minute, find the distance moved in this period.**

**AKTU 2013-14, (II) Marks 10**



**Answer**

**Given :** $a \propto v^2$, $v_1 = 3$ m/sec, $v_2 = 1.5$ m/sec, $t = 1$ min = 60 s

**To Find :** Distance moved.

1. Acceleration is given by,

$$a = Kv^2 \quad ...(4.3.1)$$

$$\frac{dv}{dt} = Kv^2$$

$$\frac{dv}{v^2} = Kdt$$

2. On integrating both sides,

$$\int_3^{1.5} \frac{dv}{v^2} = K\int_0^{60} dt$$

$$\left[\frac{v^{-2+1}}{-2+1}\right]_3^{1.5} = K[t]_0^{60}$$

$$\left[\frac{1}{-v}\right]_3^{1.5} = K \times 60$$

$$\left[-\frac{1}{1.5} + \frac{1}{3}\right] = 60\,K$$

$$-\frac{1}{3} = 60\,K$$

$$K = -5.55 \times 10^{-3}$$

3. From eq. (4.3.1.), we get

$$a = -5.55 \times 10^{-3}\,v^2$$

$$\frac{v\,dv}{ds} = -5.55 \times 10^{-3}\,v^2$$

$$\frac{dv}{v} = -5.55 \times 10^{-3}\,ds$$

4. On integrating both sides,

$$[\ln v]_3^{1.5} = [-5.55 \times 10^{-3}\,s]_0^s$$

$$\ln 1.5 - \ln 3 = -5.55 \times 10^{-3}\,s$$

$$s = 124.89 \text{ m}$$

**Que 4.4.** **Derive the formula for the distance travelled in $n^{th}$ second.**

**Answer**

1. Let, $u$ = Initial velocity of a body.

$a$ = Acceleration of the body.

$s_n$ = Distance covered in $n$ second.

$s_{n-1}$ = Distance covered in $(n-1)$ seconds.

2. Then distance travelled in the $n^{th}$ seconds

$$= \text{Distance travelled in } n \text{ seconds} - \text{Distance travelled in } (n-1) \text{ seconds}$$

$$= s_n - s_{n-1}$$

3. Distance travelled in $n$ seconds is obtained by substituting $t = n$ in the following equation,

$$s = ut + \frac{1}{2}at^2$$

$$\therefore \quad s_n = un + \frac{1}{2}an^2$$

Similarly, $$s_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

4. Distance travelled in the $n^{th}$ seconds

$$= s_n - s_{n-1}$$

$$= \left(un + \frac{1}{2}an^2\right) - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$= un + \frac{1}{2}an^2 - \left[un - u + \frac{1}{2}a(n^2 + 1 - 2n)\right]$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 - \frac{1}{2}a + \frac{1}{2}a \times 2n$$

$$= an + u - \frac{1}{2}a = u + \frac{a}{2}(2n-1)$$

**Que 4.5.** **Write down the equation of motion due to gravity.**

**Answer**

Following are the equation of motion due to gravity :

i. **For Downward Motion :**

$$a = +g$$
$$v = u + gt$$
$$s = h = ut + \frac{1}{2}gt^2$$
$$v^2 - u^2 = 2gh$$

ii. **For Upward Motion :**

$$a = -g$$
$$v = u - gt$$
$$s = h = ut - \frac{1}{2}gt^2$$
$$v^2 - u^2 = -2gh$$



**Que 4.6.** **A stone is dropped into a well and is heard to strike the water after 4 seconds. Find the depth of the well if the velocity of sound is 350 m/sec.** **AKTU 2014-15, (II) Marks 05**

**Answer**

**Given :** $t = 4$ sec, velocity of sound = 350 m/sec
**To Find :** Depth of the well.

1. Let, $h$ = Depth of well.
$t_1$ = Time taken by stone to strike water.
$t_2$ = Time taken by sound to reach from surface of water to top of well.

2. So, total time, $$t = t_1 + t_2 = 4 \quad \text{...(4.6.1)}$$

3. Considering downward motion of stone and using the relation $s = ut + \frac{1}{2}gt^2$, we have

$$h = 0 \times t_1 + \frac{1}{2} \times 9.81 \times t_1^2$$
$$h = 4.905\, t_1^2$$

4. Considering the motion of sound, the time taken by the sound to reach from surface of water to top of well is given by,

$$t_2 = \frac{\text{Depth of well}}{\text{Speed of sound}} = \frac{h}{350} = \frac{4.905\, t_1^2}{350}$$
$$(\because h = 4.905\, t_1^2)$$

5. From eq. (4.6.1), we have

$$t_1 + \frac{4.905\, t_1^2}{350} = 4$$
$$350\, t_1 + 4.905\, t_1^2 = 1400$$
$$\therefore \quad 4.905\, t_1^2 + 350\, t_1 - 1400 = 0$$

6. Solution of the quadratic equation given as,

$$t_1 = \frac{-350 \pm \sqrt{350^2 + 4 \times 4.905 \times 1400}}{2 \times 4.905} = \frac{-350 \pm 387.26}{9.81}$$

Taking the +ve root ; $t_1 = 3.798$ sec

7. Depth of well, $h = 4.905\, t_1^2 = 4.905 \times (3.798)^2 = 70.75$ m

**Que 4.7.** **Acceleration of particle is defined by $a = 21 - 21s^2$, where $a$ is acceleration in m/sec$^2$ and $s$ is in metres. The particle starts with rest at $s = 0$, Determine (a) velocity when $s$ = 1.5 m, (b) the position where velocity is again zero, (c) the position where the velocity is maximum.**

**AKTU 2013-14, (I) Marks 10**

**Answer**

Given : $a = 21 - 21s^2$, $u = 0$

To Find :
i. Velocity when $s$ = 1.5 m.
ii. The position where velocity is again zero.
iii. The position where the velocity is maximum.

1. Velocity when $s$ = 1.5 m,

$$a = v\frac{dv}{ds} = 21 - 21s^2$$

$$\int_0^v v\,dv = \int_0^{1.5} (21 - 21s^2)ds$$

$$\frac{v^2}{2} = \left[21s - \frac{21s^3}{3}\right]_0^{1.5}$$

$$v^2 = 2\left[21 \times 1.5 - 21 \times \frac{(1.5)^3}{3}\right]$$

$$v^2 = 15.75$$

$$v = 3.97 \text{ m/sec}$$

2. Position where velocity is again zero,

$$v\frac{dv}{ds} = 21 - 21s^2$$

$$\int v\,dv = \int (21 - 21s^2)\,ds$$

$$\frac{v^2}{2} = 21s - \frac{21s^3}{3} + C \quad ...(4.7.1)$$

Here $C$ is integration constant.

3. At $s = 0$, $v = 0$, from eq. (4.7.1), we have

$$C = 0$$

Put the value of $C$ in eq. (4.7.1), we get

$$v^2 = 2\left(21s - \frac{21s^3}{3}\right) \quad ...(4.7.2)$$

4. For $v = 0$, we have

$$21s - \frac{21s^3}{3} = 0$$

$$3s - s^3 = 0$$

$$s(3 - s^2) = 0$$



$$s = 0, \qquad s^2 = 3 \Rightarrow s = \pm\sqrt{3}$$

The velocity will be again zero at $s$ = 1.732 m

5. On differentiating the eq. (4.7.2) w.r.t $s$,

$$2v\frac{dv}{ds} = 2\left(21 - 21 \times 3\frac{s^2}{3}\right)$$

$$\frac{dv}{ds} = \frac{21 - 21s^2}{v} \quad ...(4.7.3)$$

6. For maximum or minimum velocity, $\frac{dv}{ds} = 0$

$$0 = (21 - 21s^2)$$

$$s^2 = 1 \Rightarrow s = \pm 1$$

7. At $s$ = 1 m, from eq. (4.7.2)

$$v = \sqrt{2\left[(21 \times 1) - \left(\frac{21}{3}\right)\right]} = 5.29 \text{ m/sec}$$

8. Now, again differentiating the eq. (4.7.3) w.r.t $s$, we get

$$\left[v\frac{d^2v}{ds^2} + \left(\frac{dv}{ds}\right)^2\right] = (-21 \times 2s) \quad ...(4.7.4)$$

9. At $s$ = 1 m, from eq. (4.7.3)

$$\frac{dv}{ds} = \frac{21 - 21s^2}{v} = \frac{21 - 21}{v} = 0$$

10. Now substituting $\frac{dv}{ds} = 0$ and $s = 1$ in eq. (4.7.4), we get

$$\frac{d^2v}{ds^2} = -\frac{42}{v} = -\frac{42}{5.29} = -7.94$$

As $\frac{d^2v}{ds^2}$ at $s$ = 1 m is negative, therefore velocity is maximum at 1 m.

**Que 4.8.** **A car starts from rest on a curved road of 200 m radius and accelerates at a constant tangential acceleration of 0.5 m/sec$^2$. Determine the distance and time which the car will travel before the total acceleration attained by it becomes 0.75 m/sec$^2$.**

**AKTU 2013-14, (II) Marks 05**

**Answer**

Given : $R$ = 200 m, $a_t$ = 0.5 m/sec$^2$, $a$ = 0.75 m/sec$^2$, $u = 0$
To Find : Distance and time for which the car travel.

1. We know that,

Final acceleration$^2$ = Normal acceleration$^2$ + Tangential acceleration$^2$

$$a^2 = a_n^2 + a_t^2$$
$$(0.75)^2 = a_n^2 + (0.5)^2$$
$$a_n^2 = 0.3125 \text{ m/sec}^2$$
$$a_n = 0.559 \text{ m/sec}^2$$

2. Normal acceleration is given by,

$$a_n = \frac{v^2}{R}$$
$$v^2 = a_n \times R = 0.559 \times 200$$
$$v^2 = 111.8$$
$$v = 10.57 \text{ m/sec}$$

3. We know that, $v = u + a_t t$ (∵ Initial velocity, $u = 0$)

$$10.57 = 0.5\,t$$
$$t = \frac{10.57}{0.5} = 21.14 \text{ sec}$$

4. Also we known that,

$$v^2 = u^2 + 2a_t s$$
$$s = \frac{v^2}{2a_t} = \frac{(10.57)^2}{2 \times 0.5} \quad (\because u = 0)$$
$$s = 111.725 \text{ m}$$

**Que 4.9.** **An automobile is accelerated at the rate of 0.8 m/sec² as it travels from station *A* to station *B*. If the speed of the automobile is 36 km/h as it passes station *A*, determine the time required for automobile to reach *B* and its speed as it passes station *B*. The distance between *A* and *B* is 250 m.** **AKTU 2013-14, (II) Marks 05**

**Answer**

**Given :** $a = 0.8 \text{ m/sec}^2$, $s = 250$ m, $u = 36 \text{ km/h} = 36 \times \frac{5}{18} = 10$ m/sec

**To Find :** Time taken to reach *B* and speed as it passes station *B*.

1. We know that,

$$s = ut + \frac{1}{2}at^2$$
$$250 = 10t + \frac{1}{2} \times 0.8 \times t^2$$
$$250 = 10t + 0.4t^2$$
$$0.4t^2 + 10t - 250 = 0$$
$$t = \frac{-10 \pm \sqrt{(10)^2 - 4 \times 0.4 \times (-250)}}{2 \times 0.4}$$



$$= \frac{-10 \pm \sqrt{100 + 400}}{0.8}$$
$$t = 15.45 \text{ sec}$$

2. Also, we know that

$$v^2 = u^2 + 2as$$
$$v^2 = (10)^2 + 2 \times 0.8 \times 250 = 500$$
$$v = 22.36 \text{ m/sec}$$

## PART-2

*Plane Curvilinear Motion (Rectangular, Path and Polar Coordinates).*

### CONCEPT OUTLINE

**Curvilinear Motion :** The motion of a body in a plane along a circular path is known as plane curvilinear motion.

**Equation of Motion for Curvilinear Motion :**

$$\omega = \omega_0 + \alpha t$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta$$
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

where, $\omega_0$ = Initial angular velocity (rad/sec)

$\omega$ = Final angular velocity (rad/sec)

$\alpha$ = Angular acceleration (rad/sec²)

$\theta$ = Angular displacement (rad)

$t$ = Time (sec)

**Projectile Motion :** Curvilinear motion with constant acceleration can be considered as the combination of two rectilinear motions occurring simultaneously along two mutually perpendicular $x$ and $y$ directions. This motion is known as projectile motion.

**Example :** Motion of a missile or a ball hit in air.

### Questions-Answers

**Long Answer Type and Medium Answer Type Questions**

**Que 4.10.** **What are the parameters required for defining the curvilinear motion of a body ?**

**OR**

**Define the following terms :**
**i. Angular displacement.**
**ii. Angular velocity.**
**iii. Angular acceleration.**

**Answer**

Following are the parameters required for defining the curvilinear motion of the body :

i. **Angular Displacement :** The displacement of a body in rotation is called angular displacement, and it is measured in terms of the angle through which the body moves from the initial state.

ii. **Angular Velocity :** The rate of change of angular displacement of a body with respect to time is called angular velocity. If the body traverses angular distance $d\theta$ over a time interval $dt$, then the average angular velocity $\omega$ is given by,

$$\omega = \frac{d\theta}{dt}$$

iii. **Angular Acceleration :** The rate of change of angular velocity of a body with respect to time is called angular acceleration.

Mathematically, $$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$$

**Que 4.11.** **Write down the relationship between angular motion and linear motion.**

**Answer**

1. If $r$ is the distance of the particle from the centre of rotation, then
$$s = r\theta$$
2. The tangential velocity of the particle is called as linear velocity and is denoted by v. Then
$$v = \frac{ds}{dt} = r\frac{d\theta}{dt}$$
3. The linear acceleration of the particle in tangential direction $a_t$ is given by
$$a_t = \frac{dv}{dt} = r\frac{d^2\theta}{dt^2}$$

**Que 4.12.** **If crank $OA$ rotates with an angular velocity of $\omega$ = 12 rad/sec, determine the velocity of piston $B$ and the angular velocity of rod $AB$ at the instant shown in the Fig. 4.12.1.**

**AKTU 2014-15, (I) Marks 10**



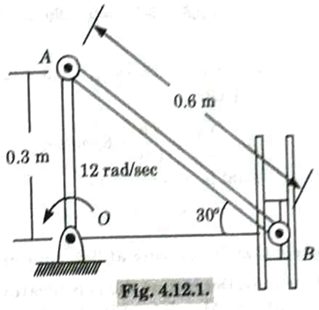



**Fig. 4.12.1.**

**Answer**

**Given :** $\omega_{OA}$ = 12 rad/sec, $OA$ = 0.3 m, $AB$ = 0.6 m, $\phi$ = 30°,
**To Find :** i. Velocity of piston $B$.
ii. Angular velocity of rod $AB$.

1. Applying the sine rule in the $\Delta OAB$ (Fig. 4.12.2),

$$\frac{OA}{\sin 30^\circ} = \frac{AB}{\sin\theta}$$

$$\frac{0.3}{(1/2)} = \frac{0.6}{\sin\theta}$$

$$\sin\theta = 1$$

$$\theta = 90^\circ$$

So this is the right angle triangle.

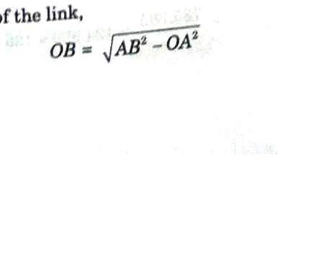



**Fig. 4.12.2.**

2. The length of the link,
$$OB = \sqrt{AB^2 - OA^2}$$

$$= \sqrt{0.6^2 - 0.3^2} = 0.5196 \text{ m} = 0.52 \text{ m}$$

3. Velocity of point $A$,

$$v_A = r\omega_{OA} = OA \times \omega_{OA} = 0.3 \times 12 = 3.6 \text{ m/sec}$$

4. Angular velocity of rod $AB$,

$$\omega_{AB} = \frac{v_A}{AB} = \frac{3.6}{0.6} = 6 \text{ rad/sec}$$

5. And the velocity at point $B$, $v_B = OB \times \omega_{OA}$

$$= 0.52 \times 12 = 6.24 \text{ m/sec}$$

**Que 4.13.** **A wheel that is rotating at 300 rpm attains a speed of 180 rpm after 20 seconds. Determine the acceleration of the flywheel assuming it to be uniform. Also determine the time taken to come to rest from a speed of 300 rpm if the acceleration remains the same and number of revolutions made during this time.**

**AKTU 2015-16, (I) Marks 10**

**Answer**

**Given :** $N_0 = 300$ rpm, $\omega_0 = \dfrac{2\pi \times (300)}{60} = 31.4159$ rad/sec

$N = 180$ rpm, $\omega = \dfrac{2\pi \times 180}{60} = 18.8495$ rad/sec, $t = 20$ sec

**To Find :** i. Acceleration of the flywheel.

ii. Time taken to come to rest from a speed of 300 rpm.

iii. Number of revolutions.

1. We know that, $\omega = \omega_0 + \alpha t$ ...(4.13.1)

$$18.8495 = 31.4159 + \alpha \times (20)$$

$$\alpha = -0.62832 \text{ rad/sec}^2$$

2. If $\omega = 0$, (for rest) then from eq. (4.13.1),

$$0 = 31.4159 + (-0.62832) \times t$$

$$t = 49.99984 = 50 \text{ sec}$$

Hence time taken to come to rest = 50 sec.

3. Also we know that, $\omega^2 = \omega_0^2 + 2\alpha\theta$

$$0 = (31.4159)^2 + 2 \times (-0.62832) \times \theta$$

$$\theta = 785.395 \text{ rad}$$

4. Total revolutions made by flywheel

$$= \frac{785.395}{2\pi} = 124.999 \approx 125$$



**Que 4.14.** **Discuss the curvilinear motion of a body in rectangular coordinates.**

**Answer**

1. Consider a particle moving in the $XY$-plane. Let its position at an instant of time be $A$, whose position vector is $\vec{r}$ as shown in Fig. 4.14.1.

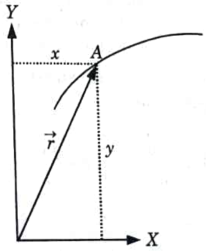



**Fig. 4.14.1.**

2. If $x$ and $y$ be the rectangular coordinates of the point $A$, then its position vector $\vec{r}$ can be expressed as

$$\vec{r} = x\hat{i} + y\hat{j} \quad \text{...(4.14.1)}$$

3. Then velocity vector can be obtained by differentiating eq. (4.14.1) with respect to time, *i.e.*,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$= v_x\hat{i} + v_y\hat{j} \quad \text{...(4.14.2)}$$

where $v_x$ and $v_y$ are $x$ and $y$ components of velocity $\vec{v}$ (Fig. 4.14.2).


**Fig. 4.14.2.**

4. The magnitude and direction of instantaneous velocity can be expressed in terms of its components as,

$$v = \sqrt{v_x^2 + v_y^2} \text{ and } \theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

The direction of this instantaneous velocity is tangential to the path of the particle at that instant.

5. If the equation of path of the particle is known in the form, $y = f(x)$, then it can be proved that the direction of velocity vector coincides with the slope of the curve or tangent to the curve at that point.

6. Similarly, the acceleration vector can be obtained by differentiating eq. (4.14.2) with respect to time, *i.e.*,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{v}_x}{dt}\hat{i} + \frac{d\vec{v}_y}{dt}\hat{j}$$

$$= \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

$$= a_x\hat{i} + a_y\hat{j}$$

where $a_x$ and $a_y$ are $x$ and $y$ components of acceleration.

7. The magnitude and direction of instantaneous acceleration in terms of its components are,

$$a = \sqrt{a_x^2 + a_y^2} \text{ and } \theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

**Que 4.15.** **The $x$ and $y$ coordinates of the position of a particle moving in curvilinear motion are defined by $x = 2 + 3t^2$ and $y = 3 + t^3$. Determine the particle's position, velocity and acceleration at $t = 3$ sec.**

**Answer**

**Given :** $x = 2 + 3t^2$, $y = 3 + t^3$
**To Find :** Particle's position, velocity and acceleration at $t = 3$ sec.

1. It is given that,

$$x = 2 + 3t^2 \text{ and } y = 3 + t^3$$

Therefore, the $x$ and $y$ components of velocity and acceleration can be obtained by differentiating successively the above expressions with respects to time.

$$\therefore \quad v_x = \frac{dx}{dt} = 6t,\ v_y = \frac{dy}{dt} = 3t^2$$



and $$a_x = \frac{d^2x}{dt^2} = 6,\ a_y = \frac{d^2y}{dt^2} = 6t$$

2. Particle's position at $t = 3$ sec,

$$x(3) = 2 + 3(3)^2 = 29 \text{ m}$$
$$y(3) = 3 + (3)^3 = 30 \text{ m}$$

3. Magnitude and direction of position vector at $t = 3$ sec are,

$$r = \sqrt{x^2 + y^2} = \sqrt{29^2 + 30^2} = 41.73 \text{ m}$$

and $$\theta_r = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{30}{29}\right) = 45.97°$$

4. Particle's velocity at $t = 3$ sec,

$$v_x(3) = 6(3) = 18 \text{ m/sec}$$
$$v_y(3) = 3(3)^2 = 27 \text{ m/sec}$$

5. Magnitude of velocity at time $t = 3$ sec is given by,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{18^2 + 27^2} = 32.45 \text{ m/sec}$$

6. Its inclination with respect to the $X$-axis is given by,

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{27}{18}\right) = 56.31°$$

7. Particle's acceleration at $t = 3$ sec,

$$a_x(3) = 6 \text{ m/sec}^2$$
$$a_y(3) = 6(3) = 18 \text{ m/sec}^2$$

8. Magnitude of acceleration at $t = 3$ sec is given by,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{6^2 + 18^2} = 18.97 \text{ m/sec}^2$$

9. Its inclination with respect to the $X$-axis is given by,

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{18}{6}\right) = 71.57°$$

**Que 4.16.** **Write down the equations of projectile motion and derive expression for the various terms associated with projectile motion.**

**Answer**

**A. Equation of Motion for Projectile Motion :**

**i. Motion along the $X$-direction (Uniform Motion) :**

$$a_x = 0 \quad ...(4.16.1)$$

$$v_x = v_0 \cos \alpha \quad ...(4.16.2)$$

$$x = (v_0 \cos \alpha)\, t \quad ...(4.16.3)$$

**ii. Motion along the Y-direction (Uniform Accelerated Motion) :**

$$a_y = -g \quad ...(4.16.4)$$

$$v_y = v_0 \sin \alpha - gt \quad ...(4.16.5)$$

$$v_y^2 = (v_0 \sin \alpha)^2 - 2gy \quad ...(4.16.6)$$

$$y = (v_0 \sin \alpha)\, t - \frac{1}{2} gt^2 \quad ...(4.16.7)$$

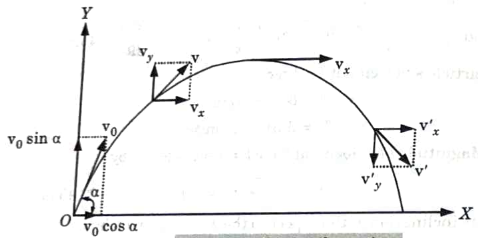



**Fig. 4.16.1.** Projectile motion.

**B. Derivation of Various Terms :**

**i. Time Taken to Reach Maximum Height and Time of Flight :**

1. When the particle reaches the maximum height, we know that the vertical component of velocity *i.e.*, $v_y$ is zero. Therefore, from the eq. (4.16.5), we have

$$0 = v_0 \sin \alpha - gt$$

2. Hence, the time taken to reach the maximum height is,

$$t = \frac{v_0 \sin \alpha}{g} \quad ...(4.16.8)$$

3. Since the time of ascent is equal to the time of descent, the total time taken for the projectile to return to the same level of projection is,

$$T = \frac{2v_0 \sin \alpha}{g}$$

**ii. Maximum Height Reached :**

1. Substituting the value of time of ascent in the eq. (4.16.7), we get

$$y = v_0 \sin \alpha \left(\frac{v_0 \sin \alpha}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin \alpha}{g}\right)^2$$



$$= \frac{v_0^2 \sin^2 \alpha}{g} - \frac{1}{2} g \left(\frac{v_0^2 \sin^2 \alpha}{g^2}\right) = \frac{v_0^2}{2g} \sin^2 \alpha$$

2. Hence, the maximum height reached is,

$$h_{max} = \frac{v_0^2 \sin^2 \alpha}{2g}$$

**iii. Range :**

1. The horizontal distance between the point of projection and point of return of projectile to the same level of projection is termed as range.
2. Hence, range is obtained by substituting the value of total time of flight in the eq. (4.16.3),

$$R = (v_0 \cos \alpha) T$$

$$= (v_0 \cos \alpha)\left[\frac{2v_0 \sin \alpha}{g}\right]$$

3. Since, $\sin 2\alpha = 2 \sin \alpha \cos \alpha$, we can write,

$$R = \frac{v_0^2 \sin 2\alpha}{g}$$

**Que 4.17.** **A ball is thrown from the ground with a velocity of 20 m/sec at an angle of 30° to the horizontal. Determine :**

**i The velocity of the ball at $t = 0.5$ sec and $t = 1.5$ sec.**
**ii. Total time of flight of the ball.**
**iii. Maximum height reached.**
**iv. Range of the ball.**
**v. Maximum range.**

**Answer**

**Given :** $v_0 = 20$ m/sec, $\alpha = 30°$

**To Find :** i. The velocity of the ball at $t = 0.5$ s and $t = 1.5$ sec.
ii. Total time of flight of the ball.
iii. Maximum height reached.
iv. Range of the ball.
v. Maximum range.

1. The initial velocity of the ball can be resolved into horizontal and vertical components as,

$$v_{0x} = v_0 \cos \alpha = 20 \cos 30° = 17.32 \text{ m/sec}$$

and

$$v_{0y} = v_0 \sin \alpha = 20 \sin 30° = 10 \text{ m/sec}$$

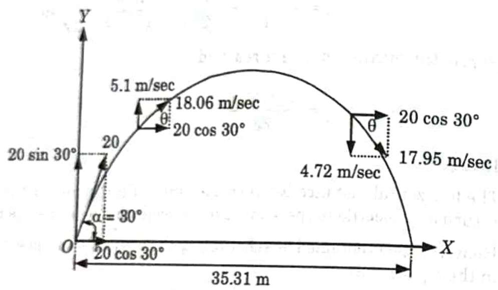



**Fig. 4.17.1.**

2. We know that the horizontal component of velocity always remains constant and only the vertical component of velocity varies with time. Thus,

$$v_{y(0.5)} = v_0 \sin \alpha - gt$$

$$= 10 - 9.81\,(0.5) = 5.1 \text{ m/sec}$$

3. The total velocity at that instant is obtained by,

$$v_{(0.5\ sec)} = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(17.32)^2 + (5.1^2)} = 18.06 \text{ m/sec}$$

And its inclination with respect to the $X$-axis is obtained by,

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{5.1}{17.32}\right) = 16.41°$$

4. Similarly, $v_{y(1.5\ sec)} = 10 - 9.81(1.5) = -4.72$ m/sec

$\therefore$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(17.32)^2 + (-4.72)^2} = 17.95 \text{ m/sec}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4.72}{17.32}\right) = 15.24°$$

5. We know that total time of flight of the ball is given by,

$$T = \frac{2v_0 \sin \alpha}{g} = \frac{2(10)}{9.81} = 2.04 \text{ sec} \quad (\because v_0 \sin \alpha = 10)$$



6. Maximum height reached by the ball is given by,

$$h = \frac{v_0^2 \sin^2 \alpha}{2g} = \frac{(10)^2}{2 \times 9.81} = 5.1 \text{ m}$$

7. Range of the projectile is given by,

$$R = \frac{v_0^2 \sin 2\alpha}{g} = \frac{(20)^2 \sin 60°}{9.81} = 35.31 \text{ m}$$

8. Maximum range, $R_{max} = \dfrac{v_0^2}{g} = \dfrac{(20)^2}{9.81} = 40.77$ m $\quad (\because 2\alpha = 90°)$

**Que 4.18.** **Discuss the curvilinear motion of a body in polar coordinates.**

**Answer**

1. Consider a particle moving in a curvilinear path as shown in Fig. 4.18.1(*a*).
2. Let it be at a point $A$ at a particular instant of time. Its position is then specified by the radial vector $\vec{r}$ and inclination or $\vec{r}$ with respect $X$-axis, *i.e.*, $\theta$. The instantaneous velocity $\vec{v}$ of the particle is tangential to the path at that instant.
3. This tangential velocity can be resolved into orthogonal components along the radial and transverse directions.
4. For this, let us consider unit vector $\hat{e}_r$ and $\hat{e}_\theta$ along the radial and transverse directions respectively as shown in Fig. 4.18.1 (*a*).

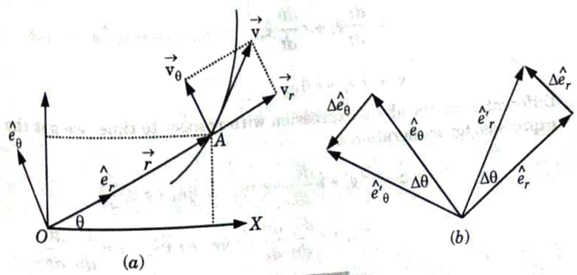



**Fig. 4.18.1.**

5. As the particle moves from point $A$ to another point in a small interval of time, we can see that the directions of unit vectors also change. To determine this change in unit vectors, we proceed as follows.

6. Draw the unit vectors with a common origin as shown in Fig. 4.18.1(*b*). Let the unit vector along the direction of radial vector at a later instant of time be $\hat{e}_r$ and along the transverse direction $\hat{e}'_\theta$. As we let the time interval $\Delta t \to 0$ then the angle $\Delta\theta \to 0$.
7. In the limiting case, we have

$$\lim_{\Delta\theta \to 0} \frac{\Delta \hat{e}_r}{\Delta\theta} = \frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta$$

and

$$\lim_{\Delta\theta \to 0} \frac{\Delta \hat{e}_r}{\Delta\theta} = \frac{d\hat{e}_\theta}{d\theta} = -\hat{e}_r$$

8. That is, in the limiting case, the change in radial unit vector points in the direction of angular unit vector and the change in angular unit vector points in the direction opposite to that of the radial unit vector.
9. The radius vector can be expressed as a product of the radial distance and the unit vector along that direction, *i.e.*,

$$\vec{r} = r\hat{e}_r$$

10. Differentiating it with respect to time, we can get the expression for velocity as,

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$= \frac{dr}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{dt}$$

$$= \frac{dr}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{d\theta}\frac{d\theta}{dt}$$

$$= \frac{dr}{dt}\hat{e}_r + r\frac{d\theta}{dt}\hat{e}_\theta$$

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

1. Differentiating the above expression with respect to time, we get the expression for acceleration as,

$$\vec{a} = \ddot{r}\hat{e}_r + \dot{r}\frac{d\hat{e}_r}{dt} + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d\hat{e}_\theta}{dt}$$

$$= \ddot{r}\hat{e}_r + \dot{r}\frac{d\hat{e}_r}{d\theta}\frac{d\theta}{dt} + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\frac{d\hat{e}_\theta}{d\theta}\frac{d\theta}{dt}$$

$$= \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta - r(\dot{\theta})^2\hat{e}_r$$

$$\vec{a} = [\ddot{r} - r(\dot{\theta})^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

Here single and double dots shows the single and double differentiation respectively.



**Que 4.19.** **The motion of a particle is defined as $r = 2t^2$ and $\theta = t$, where $r$ is in metres, $\theta$ is in radians and $t$ is in seconds. Determine the velocity and acceleration of the particle at $t = 2$ sec.**

**Answer**

**Given :** $r = 2t^2$, $\theta = t$

**To Find :** Velocity and acceleration of the particle at $t = 2$ sec.

1. Differentiating the radial and angular displacement functions, we have

$$\dot{r} = 4t,\ \dot{\theta} = 1$$

$$\ddot{r} = 4,\ \ddot{\theta} = 0$$

2. We know that velocity vector is given as,

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

3. Substituting the values, we have

$$\vec{v} = (4t)\hat{e}_r + (2t^2)(1)\hat{e}_\theta$$

4. Hence, the velocity at $t = 2$ sec is obtained by,

$$\vec{v} = (4 \times 2)\hat{e}_r + 2 \times (2)^2 \times (1)\hat{e}_\theta$$

$$= 8\hat{e}_r + 8\hat{e}_\theta$$

$$|\vec{v}| = \sqrt{8^2 + 8^2} = 11.31 \text{ m/sec}$$

5. The acceleration vector is given by,

$$\vec{a} = [\ddot{r} - r(\dot{\theta})^2]\hat{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\hat{e}_\theta$$

$$= [(4) - 2t^2(1)^2]\hat{e}_r + [2t^2(0) + 2(4t)(1)]\hat{e}_\theta$$

6. Hence, the acceleration at $t = 2$ s is obtained by,

$$\vec{a} = [(4) - 2(2)^2]\hat{e}_r + [(8)(2)]\hat{e}_\theta = -4\hat{e}_r + 16\hat{e}_\theta$$

$$|\vec{a}| = \sqrt{(-4)^2 + (16)^2} = 16.49 \text{ m/sec}^2$$

## PART-3

*Work, Kinetic Energy, Power, Potential Energy.*

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 4.20.** **Define work done and discuss its special cases.**

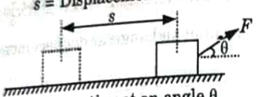
**Answer**

A. **Work Done :** Work done in general is defined as a product of the component of the force in the direction of motion and the displacement.

Mathematically, $W = (F \cos \theta)s$

where, $F$ = Force in the direction of motion.

$s$ = Displacement.




Force acting at an angle θ

**Fig. 4.20.1.**

B. **Special Cases :**

i. **When the Displacement (*s*) is Zero :**

1. Even though forces may act on a particle, if there is no displacement of the particle then no work is done on the particle.
2. Consider a block resting on a table. In its free-body diagram, we see that even though its weight $W$ and normal reaction $R$ are acting on it, they do no work on the block as there is no displacement of the block.

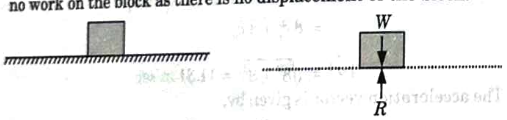



**Fig. 4.20.2.**

ii. **When the Motion is at Right Angle to the Direction of the Forces :**

1. When the motion is at right angle to the direction of the forces, we see that $\theta = 90°$ and hence, $\cos \theta = 0$. Thus, work done is zero.
2. Consider a block moving along a horizontal plane as shown in Fig. 4.20.3. Since the displacement is at right angles to the direction of the forces, namely, its weight and normal reaction, the two forces do not work on the block.

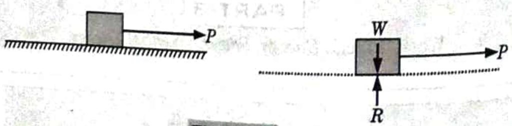



**Fig. 4.20.3.**

iii. **When the Motion is in the Direction of Force :**

1. In this case, $\theta = 0°$

$\therefore$ $W = Fs$ $(\because \cos 0° = 1)$



**Que 4.21.** **A block of 10 kg mass resting on a rough horizontal plane is pulled by an inclined force *P* as shown Fig. 4.21.1, at a constant velocity over a distance of 5 m. The coefficient of kinetic friction between the contact planes is 0.2. Sketch the free body diagram of the block showing all the forces acting on it. Also, determine (i) the work done by each force acting on the free body, and (ii) the total work done on the block.**

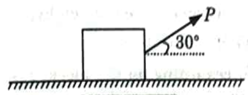



**Fig. 4.21.1.**

**Answer**

**Given :** $m = 10$ kg, $s = 5$ m, $\theta = 30°$, $\mu = 0.2$

**To Find :** i. Work done by each force acting on the free body.
ii. Total work done on the block.

1. The free-body diagram of the block is shown in Fig. 4.21.2. As there is no motion along the Y-direction,

$$\Sigma F_y = 0$$

$\therefore$ $$R + P \sin 30° - 10\,g = 0$$

$\therefore$ $$R = 10\,g - P \sin 30° \quad ...(4.21.1)$$

2. Since the block is moving with constant velocity along the horizontal direction, its acceleration in that direction is zero. Hence, we can write

$$\Sigma F_x = 0$$

$$P \cos 30° - F = 0$$

$$P \cos 30° - \mu R = 0 \quad (\because F = \mu R) \quad ...(4.21.2)$$

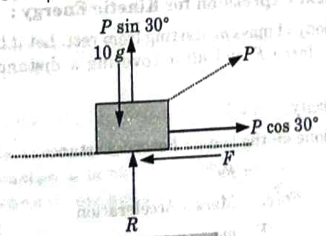



**Fig. 4.21.2.**

3. Substituting the value of $R$ from eq. (4.21.1) in eq. (4.21.2), we have

$$P \cos 30° - \mu\,(10g - P \sin 30°) = 0$$

$$P = \frac{10\,\mu g}{[\cos 30° + \mu \sin 30°]} = \frac{10(0.2)(9.81)}{[\cos 30° + (0.2)\sin 30°]}$$

$= 20.31$ N

4. Force of friction is given by,
$F = P \cos 30° = 20.31 \cos 30° = 17.59$ N
5. Work done by the horizontal component of $P$, *i.e.*, $P \cos \theta$ is,
$(W)_{P\cos\theta} = 20.31 \cos 30° \times 5 = 87.95$ J
6. Work done by the frictional force is given by,
$W_F = -17.59 \times 5 = -87.95$ J
7. Since the other forces acting on the block, $P \sin \theta$, $mg$ and $R$ are all perpendicular to the direction of displacement of the block, the work done by each of them is zero.
8. The total work done on the block is the algebraic sum of works done by each of the forces acting on the block.

$\therefore$ $W = 87.95 - 87.95 = 0$

Alternatively, we could say that as the block moving with constant velocity, the resultant force acting on it is zero, hence, the work done on the block is zero.

**Que 4.22.** **Define kinetic energy and also derive an expression for it.**

**Answer**

**A. Kinetic Energy :** The energy that a body possesses by the virtue of its motion is known as kinetic energy.

Mathematically, $\text{KE} = \frac{1}{2} m v^2$

**B. Mathematical Expression for Kinetic Energy :**

1. Consider a body of mass $m$ starting from rest. Let it be subjected to an accelerating force $F$ and after covering a distance $s$, its velocity becomes v.

$\therefore$ Initial velocity, $u = 0$

2. Now, work done on the body = Force × Distance
$= Fs$ ...(4.22.1)
3. But, Force = Mass × Acceleration

$\therefore$ $F = ma$

4. Substituting the value of $F$ in eq. (4.22.1), we get
Work done $= m \times (as)$ ...(4.22.2)
5. But from equation of motion, we have

$$v^2 - u^2 = 2as \quad \text{or} \quad v^2 - 0^2 = 2as \qquad (\because\; u = 0)$$

$$as = \frac{v^2}{2}$$

6. Substituting the value of $as$ in eq. (4.22.2), we have

Work done $= m\dfrac{v^2}{2}$

7. But work done on the body is equal to KE possessed by the body.

$\therefore$ $\text{KE} = \frac{1}{2} m v^2$



**Que 4.23.** **Write a short note on power.**

**Answer**

1. Power is defined as the rate at which work is done. The capacity of an engine or a machine used to do work is normally expressed as its rated power.
2. If $W$ is the total work done in a time interval $t$, then average power is given by,

$$P_{avg} = \frac{\text{Total work done}}{\text{Time taken}} = \frac{W}{t} \qquad ...(4.23.1)$$

3. The instantaneous power, *i.e.*, power at a particular instant of time is given by,

$$P = \frac{dW}{dt} = \frac{d(Fs)}{dt} \qquad ...(4.23.2)$$

4. The force can be assumed to be constant over this infinitesimally small time interval $dt$. Hence, we can write the above expression as :

$$P = \frac{Fds}{dt} = F\text{v} \qquad ...(4.23.3)$$

5. In SI system of units, the unit of power is Joule per second (J/sec), also called watt (W).

**Que 4.24.** **A car of 2 ton mass starts from rest and accelerates at a uniform rate to reach a speed of 60 kmph in 20 seconds. If the frictional resistance is 600 N/ton, determine the driving power of the engine when it reaches a speed of 60 kmph.**

**Answer**

**Given :** $u = 0$, $v = 60$ kmph $= 16.67$ m/sec, $m = 2$ ton $= 2000$ kg, $f = 600$ N/ton
**To Find :** Power.

1. We know that,

$$v = u + at$$

$$\therefore \quad a = \frac{v - u}{t} = \frac{16.67 - 0}{20} = 0.8335 \text{ m/sec}^2$$

2. The kinetic equation of motion of the car is given by,

$$F - f = ma$$

where, $F$ = Driving force.

$f$ = Force of friction.

$$\therefore \quad F = f + ma$$

$$= (600)(2) + (2 \times 10^3)(0.8335) = 2867 \text{ N}$$

3. Driving power of the engine when the car is moving at 60 kmph is given by,

$$P = Fv$$

$$= (2867)(16.67) = 47792.89 \text{ W} = 47.8 \text{ kW}$$

**Que 4.25.** **Define potential energy and also give principle of conservation of mechanical energy.**

**Answer**

A. **Potential Energy :** It is defined as the capacity to do work by virtue of its position. There are many types of potential energies such as gravitational, electrical, elastic, etc.

Mathematically, PE = $mgh$

B. **Principle of Conservation of Mechanical Energy :**

1. If a body is subjected to a conservative system of forces, (say gravitational force) then its mechanical energy remains constant for any position in the force field.
2. Consider a body either sliding down a smooth incline or freely falling. Since it is initially at rest, all of its energy is potential energy.
3. As it accelerates downwards, some of its potential energy is converted into kinetic energy.
4. At the bottom of the incline or at the ground level, the energy will be purely kinetic, assuming the bottom of the slope or the ground level as the datum for potential energy.
5. By the principle of conservation of energy, we see that the loss in potential energy is equal to the gain in kinetic energy.

Mathematically,

$$(PE)_i - (PE)_f = (KE)_f - (KE)_i$$

6. On rearranging, we have



$$(PE)_i + (KE)_i = (PE)_f + (KE)_f$$

$$(PE) + (KE) = \text{Constant}$$

7. Thus, we see that the total mechanical energy, *i.e.*, sum of potential and kinetic energies remain constant. This is known as principle of conservation of mechanical energy.

**Que 4.26.** **A ball is dropped from the top of a tower. If it reaches the ground with a velocity of 30 m/sec, determine the height of the tower by the conservation of energy method.**

**Answer**

**Given :** v = 30 m/sec
**To Find :** Height of the tower.

1. By the principle of conservation of energy, we know that the total mechanical energy remains constant. Hence, the total energy at the top of the tower must be equal to that at the base of the tower *i.e.*,

$$(KE + PE)_{top} = (KE + PE)_{base}$$

2. Since the ground surface is taken as the datum, the potential energy at the top is $mgh$ [where $h$ is height of the tower] and that at the bottom is zero. If v is the velocity of the ball at the base, we can write

$$0 + mgh = \frac{1}{2} m v^2 + 0$$

$$h = \frac{v^2}{2g} = \frac{(30)^2}{2(9.81)} = 45.87 \text{ m}$$

## PART-4

*Impulse, Momentum (Linear and Angular).*

### CONCEPT OUTLINE

**Momentum :** The product of mass and velocity of a body is known as momentum. Mathematically, $p = mv$

**Impulse :** The product of the force and time is known as impulse. Mathematically, $I = Ft$

**Conservation of Linear Momentum :** When no external forces act on bodies forming a system, the momentum of the system is conserved *i.e.*, the initial momentum of the system is equal to final momentum of the system.

## Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 4.27.** **Derive impulse-momentum equation.**

**Answer**

1. Let, $F$ = Net force acting on a rigid body in the direction of motion through CG of the body.
   $m$ = Mass of the rigid body.
   $a$ = Acceleration of the body.
2. We know that,
$$F = ma = m\frac{dv}{dt} \qquad \left(\because a = \frac{dv}{dt}\right)$$
$$Fdt = mdv$$
3. Integrating the above equation, we get
$$\int_{t_1}^{t_2} Fdt = \int_{v_1}^{v_2} mdv$$
$$= m(v_2 - v_1)$$
Impulse $= mv_2 - mv_1$
Impulse = Final momentum – Initial momentum

**Que 4.28.** **A football of mass 200 gm is at rest. A player kicks the balls which move with a velocity of 20 m/sec at an angle of 30° with respect to ground level. Find the force exerted by the player on the ball of duration of strikes is 0.02 seconds.**

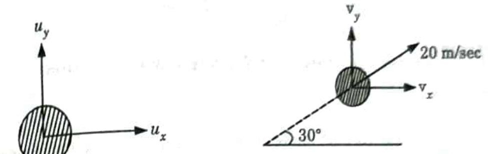
**Answer**

**Given :** $m$ = 200 gm = 0.2 kg , $t$ = 0.02 sec, $\theta$ = 30°
**To Find :** Force exerted on the ball.

1. Initially the ball is at rest. Hence, $u_x = 0$ and $u_y = 0$.
2. The ball leaves with a velocity of 20 m/sec at an angle of 30° (Fig. 4.28.1(b)).
3. Writing impulse-momentum equation along $X$- and $Y$-directions, we get
   i. For $X$-direction,
$$F_x t = m\,(v_x - u_x),$$
$$F_x \times 0.02 = 0.2\,(20\cos 30° - 0)$$



$$F_x = \frac{0.2 \times 20\cos 30°}{0.02} = 173.2 \text{ N}$$




(a) Initial position of ball. Ball at rest ($u_x = 0$, $u_y = 0$)

(b) Final position of ball. Ball moves with a velocity of 20 m/sec ($v_x$ = 20 cos 30°, $v_y$ = 20 sin 30°)

**Fig. 4.28.1.**

   ii. For $Y$-direction,
$$F_y t = m\,(v_y - u_y)$$
$$F_y \times 0.02 = 0.2\,(20\sin 30° - 0)$$
$$F_y = \frac{0.2 \times 20\sin 30°}{0.02} = 100 \text{ N}$$
4. Hence, the resultant impulse force exerted by the player on the ball,
$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(173.2)^2 + 100^2} = 199.99 \approx 200 \text{ N}$$

**Que 4.29.** **A bullet of mass 50 gm is fired into a freely suspended target to mass 5 kg. On impact, the target moves with a velocity of 7 m/sec along with the bullet in the direction of firing. Find the velocity of bullet.**

**Answer**

**Given :** $m_1$ = 50 gm = 0.05 kg, $m_2$ = 5 kg, $u_2$ = 0, $m$ = 5 + 0.05 = 5.05 kg, v = 7 m/sec
**To Find :** Velocity of bullet.

1. Total initial momentum (*i.e.*, momentum before impact),
$$= m_1u_1 + m_2u_2 = 0.05 \times u_1 + 5 \times 0$$
$$= 0.05\,u_1$$
2. Total final momentum (*i.e.*, momentum after impact),
$$= \text{Total mass} \times \text{Common velocity} = mv$$
$$= (5.05) \times 7$$
3. According to conservation of momentum,

Initial momentum = Final momentum

$$0.05\,u_1 = 0.05 \times 7$$

$$\therefore \quad u_1 = \frac{5.05 \times 7}{0.05} = 707 \text{ m/sec}$$

**Que 4.30.** **Derive an expression for angular momentum.**

**Answer**

1. The product of mass moment of inertia and angular velocity of a rotating body is known as moment of momentum or angular momentum.
2. If $\omega$ = Angular velocity of a body rotating about an axis.
   $I$ = Moment of inertia of the body about the axis.

   Then, angular momentum = $\omega I$
3. Consider a body of mass '$m$' rotating in a circle about its centre $O$.
4. Let, $dm$ = Mass of the elementary strip.
   $r$ = Radius of the mass $dm$.
   $\omega$ = Angular velocity of the body or angular velocity of the mass $dm$.
   v = Linear velocity of mass $dm$.

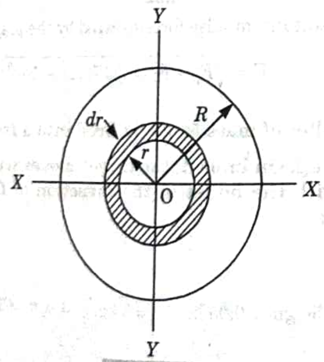



Fig. 4.30.1.

5. Now momentum of elementary mass
   = Elementary mass × Velocity = $dm$ × v
   = $dm \times \omega r$ $\quad (\because \text{v} = \omega r)$
6. Moment of momentum of elementary mass $dm$ about $O$
   = Elementary mass × Radius
   = $(dm \times \omega r) \times r$
   = $dm \times \omega r^2$ ...(4.30.1)



7. The moment of momentum of the entire mass about $O$ is obtained by integrating eq. (4.30.1).
   Moment of momentum of the entire mass

   $$= \int dm \times \omega r^2 = \omega \int r^2 dm \quad ...(4.30.2)$$

   But $\int r^2\, dm$ = Moment of inertia of the whole body about $O = I$.
8. Substituting the value in eq. (4.30.2), we get
   Moment of momentum of the entire mass = $\omega I$

**Que 4.31.** **At a given instant the 5 kg slender bar has the motion shown in Fig. 4.31.1. Determine the angular momentum about point $G$ ($v_A$ = 2 m/sec).**

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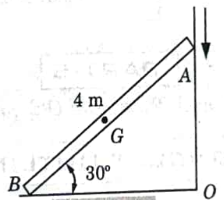



Fig. 4.31.1.

**Answer**

**Given :** $m$ = 5 kg, $v_A$ = 2 m/sec, $L$ = 4 m
**To Find :** Angular momentum about point $G$.

1. In $\Delta OBA$, $AB$ = 4 m
   $OA = 4 \sin 30° = 2$ m
   $OB = 4 \cos 30° = 3.464$ m

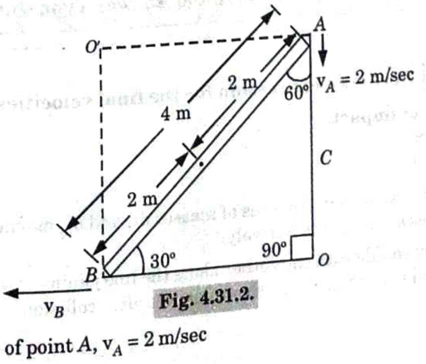



Fig. 4.31.2.

2. Velocity of point $A$, $v_A$ = 2 m/sec

$$\omega_{AB} = \frac{v_A}{OA} = \frac{2}{2} = 1 \text{ rad/sec}$$

3. Velocity at point $B$, $\omega_{AB} = \dfrac{v_B}{OB}$

$$1 = \frac{v_B}{3.464}$$

$$v_B = 3.464 \text{ rad/sec}$$

4. Angular momentum about $G$

$$= I\omega = \frac{ML^2}{12} \times 1 \qquad \left(\because I = \frac{ML^2}{12}\right)$$

$$= \frac{5 \times 4^2}{12} \times 1 = 6.67 \text{ rad/sec}^2$$

## PART-5

*Impact (Direct and Oblique).*

### CONCEPT OUTLINE

**Direct Impact :** During collision, when the direction of motion of each body is along the line joining their centres, the impact is called direct impact

**Oblique Impact :** During collision, when the direction of motion of either one or both bodies is inclined to the line joining their centres, the impact is called oblique impact.

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 4.32.** **Derive an expression for the final velocities of the body during direct impact.**

**Answer**

1. Consider two smooth spheres of masses $m_1$ and $m_2$ moving with initial velocities $u_1$ and $u_2$ respectively.
2. Let them collide with each other along the line joining their centres and let $v_1$ and $v_2$ be their respective velocities after collision.



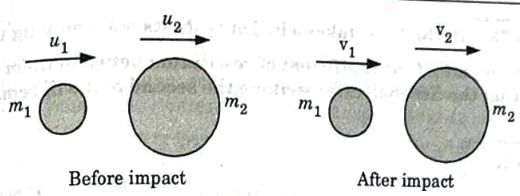



**Fig. 4.32.1.**

3. As the impulsive force exerted by each body on the other during the collision is equal and opposite, we know that the total momentum of the system is conserved. Thus, we can write

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{...(4.32.1)}$$

4. We know that,

$$-e = \frac{v_1 - v_2}{u_1 - u_2} \qquad \text{...(4.32.2)}$$

where, $e$ = Coefficient of restitution.

5. Solving for $v_1$ and $v_2$ from eq. (4.32.1) and eq. (4.32.2), we have

$$v_1 = \frac{m_1u_1 + m_2u_2 - m_2e\,(u_1 - u_2)}{m_1 + m_2} \qquad \text{...(4.32.3)}$$

and

$$v_2 = \frac{m_1u_1 + m_2u_2 + m_2e\,(u_1 - u_2)}{m_1 + m_2} \qquad \text{...(4.32.4)}$$

The above two expression shows the final velocities after collision.

6. If we assume that the collision is inelastic then substituting the value of the coefficient of restitution $e = 0$ in eq. (4.32.3) and eq. (4.32.4), we get

$$v_1 = v_2 = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

Thus, we see that if the collision is inelastic then after impact, the two bodies coalesce as one body and move with the same velocity.

7. If we assume that the collision is elastic then substituting the value of the coefficient of restitution $e = 1$ in the eq. (4.32.3) and eq. (4.32.4), we get

$$v_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2}$$

$$v_2 = \frac{2m_1u_1 + u_2(m_2 - m_1)}{m_1 + m_2}$$

8. Further, if the masses of the two colliding bodies are equal, *i.e.*, $m_1 = m_2$, then we get

$$v_1 = u_2 \text{ and } v_2 = u_1$$

9. Thus, when the collision is elastic between two equal masses, the two bodies exchange their velocities after impact.

**Que 4.33.** **If a ball overtakes a ball of twice its mass moving 1/7th of its velocity and if the coefficient of restitution between them is 3/4, show that the first ball after striking the second ball will remain at rest.**

**Answer**

**Given :** $m_1 = m$, $m_2 = 2\,m$, $u_1 = u$, $u_2 = u/7$, $e = 3/4$

**To Prove :** First ball after striking the second ball will remain at rest *i.e.,* $v_1 = 0$

1. It is given that the velocity of the second ball is 1/7th of the velocity of the first ball. Hence, applying the conservation of momentum equation,

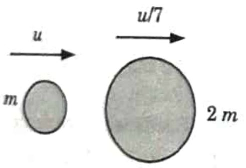



**Fig. 4.33.1.**

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$mu + 2m\,\frac{u}{7} = mv_1 + 2mv_2$$

$$v_1 + 2v_2 = \frac{9u}{7} \qquad ...(4.33.1)$$

2. Coefficient of restitution is given as,

$$-e = \frac{v_1 - v_2}{u_1 - u_2} = \frac{v_1 - v_2}{u - u/7}$$

$$v_1 - v_2 = -\frac{6}{7}\,eu$$

$$= -\frac{6}{7}\left[\frac{3}{4}\right]u = -\frac{9}{14}u \qquad ...(4.33.2)$$

3. From eq. (4.33.1) and eq. (4.33.2) solving for $v_1$, we get

$$v_1 = 0$$

**Que 4.34.** **Discuss in brief about oblique impact.**



**Answer**

1. Consider two smooth spheres of masses $m_1$ and $m_2$ approaching each other with velocities $u_1$ and $u_2$ such that their directions are inclined to the line joining their centres at the instant of impact at $\theta$ and $\phi$ respectively.
2. Let $v_1$ and $v_2$ be the respective velocities immediately after impact and their directions be inclined to the line joining centres at $\alpha$ and $\beta$ respectively as shown in Fig. 4.34.1.

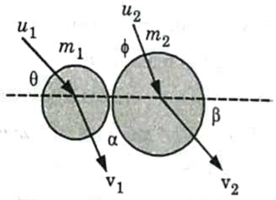



**Fig. 4.34.1.** Oblique impact.

3. As the spheres are smooth, there is no impulsive force acting on each body along their common tangential plane during their time of collision thus, there is no change in momentum of individual bodies in that direction.
4. Hence, we can write,

$$v_1 \sin \alpha = u_1 \sin \theta$$

and $$v_2 \sin \beta = u_2 \sin \phi$$

5. As the impulsive force exerted by each sphere on the other in the direction of line joining their centres is equal and opposite, the momentum of the system is conserved. Thus, we can write

$$m_1(u_1 \cos \theta) + m_2(u_2 \cos \phi) = m_1(v_1 \cos \alpha) + m_2(v_2 \cos \beta)$$

6. We know that,

$$-e = \frac{v_1 \cos \alpha - v_2 \cos \beta}{u_1 \cos \theta - u_2 \cos \phi}$$

**Que 4.35.** **A smooth sphere moving at 10 m/sec in the direction shown in Fig. 4.35.1 collides with another smooth sphere of double its mass and moving with 5 m/sec in the direction shown. If the coefficient of restitution is 2/3, determine their velocities after collision.**

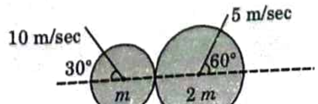



**Fig. 4.35.1.**

**Answer**

**Given :** $m_1 = m$, $m_2 = 2\,m$, $u_1 = 10$ m/sec, $u_2 = 5$ m/sec, $e = 2/3$, $\theta = 30°$, $\phi = 60°$

**To Find :** Velocities after collision.

1. We know that,

$$v_1 \sin \alpha = u_1 \sin \theta = 10 \sin 30° \quad ...(4.35.1)$$

$$v_2 \sin \beta = u_2 \sin \phi = 5 \sin 60° \quad ...(4.35.2)$$

2. According to conservation of momentum,

$$m_1 (u_1 \cos \theta) + m_2 (u_2 \cos \phi) = m_1 (v_1 \cos \alpha) + m_2 (v_2 \cos \beta)$$

$$m(10 \cos 30°) - 2m(5 \cos 60°) = m(v_1 \cos \alpha) + 2m(v_2 \cos \beta)$$

$$v_1 \cos \alpha + 2v_2 \cos \beta = 3.66 \quad ...(4.35.3)$$

3. Also, we know that,

$$-e = \frac{v_1 \cos \alpha - v_2 \cos \beta}{10 \cos 30° - (-5 \cos 60°)} \quad (\because e = 2/3)$$

$$v_1 \cos \alpha - v_2 \cos \beta = -7.44 \quad ...(4.35.4)$$

4. From eq. (4.35.3) and eq. (4.35.4) solving for $v_1 \cos \alpha$ and $v_2 \cos \beta$, we get

$$v_1 \cos \alpha = -3.74 \text{ m/sec} \quad ...(4.35.5)$$

and

$$v_2 \cos \beta = 3.7 \text{ m/sec} \quad ...(4.35.6)$$

5. From eq. (4.35.1) and eq. (4.35.5), we get $v_1 = 6.24$ m/sec in the direction opposite to that of the initial velocity at an angle of $\alpha = 53.2°$ to the line joining their centres.

6. Similarly, from eq. (4.35.2) and eq. (4.35.6), we get $v_2 = 5.7$ m/sec at an angle of $\beta = 49.49°$ of the line joining their centres.

☺☺☺

# 5 UNIT

# Introduction to Kinetics of Rigid Bodies

## CONTENTS

## PART-1

*Introduction to Kinetics of Rigid Bodies, Basic Terms, General Principles in Dynamics, Types of Motion.*

### CONCEPT OUTLINE

**Kinetics :** It is that branch of engineering mechanics which deals with the force system which produces acceleration and resulting motion of bodies.

**Newton's Laws of Motion :** When a body is at rest or moving in a straight line or rotating about an axis, the body obeys certain laws of motion. These laws are called Newton's law of motion.

### Questions-Answers

### Long Answer Type and Medium Answer Type Questions

**Que 5.1. Discuss the various terminologies related with kinetics of rigid body.**

**Answer**

Following are the some terminologies related with the kinetics of rigid body :

i. **Force :** It is defined as an agent which tends to change the state of rest or motion of a body to which it is applied. The SI unit of force is Newton (N).

ii. **Mass :** The quantity of matter combined in a body is known as the mass of the body. Mass is a scalar quantity. The SI unit of mass is kilogram (kg).

iii. **Acceleration :** It is defined as the rate of change of velocity of a body. Its SI unit is m/sec².

$$\therefore \quad \text{Acceleration} = \frac{\text{Change of velocity}}{\text{Time}} = \frac{dv}{dt}$$

iv. **Weight :** Weight of a body is defined as the force by which the body is attracted towards the centre of the earth. Mathematically weight of a body is given by,

$$\text{Weight} = \text{Mass} \times \text{Acceleration due to gravity} = mg$$

v. **Momentum :** The product of the mass of a body and its velocity is known as momentum of the body. Momentum is a vector quantity. Mathematically, momentum is given by,

$$\text{Momentum} = \text{Mass} \times \text{Velocity} = mv$$



**Que 5.2. State the various Newton's law of motion.**

**Answer**

Various laws of motion are as follows :

i. **Newton's First Law of Motion :** It states that a body continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external force to change that state.

ii. **Newton's Second Law of Motion :** It states that the rate of change of momentum of a body is proportional to the external force applied on the body and takes place in the direction of the force.

iii. **Newton's Third Law of Motion :** It states that to every action, there is always an equal and opposite reaction.

**Que 5.3. Discuss in detail about Newton's second law of motion.**

**Answer**

1. Newton's second law of motion enables us to measure a force.
2. Let a body of mass $m$ is moving with a velocity $u$ along a straight line. It is acted upon by a force $F$ and the velocity of the body becomes v in the time $t$.
3. Initial momentum of the body = Mass × Initial velocity = $mu$

   Final momentum of the body = $mv$

   $\therefore$ Change in momentum = Final momentum – Initial momentum
   $$= mv - mu = m(v - u)$$
4. $$\text{Rate of change of momentum} = \frac{\text{Change of momentum}}{\text{Time}} = \frac{m(v-u)}{t} \quad ...(5.3.1.)$$
5. But we know that,
   $$\frac{v-u}{t} = a \quad (i.e., \text{ linear acceleration})$$
6. Substituting the value of $\left(\frac{v-u}{t}\right)$ in eq. (5.3.1), we get

   Rate of change of momentum = $ma$
7. But according to Newton's second law of motion, the rate of change of momentum is directly proportional to the external force acting on the body.
   $$\therefore \quad F \propto ma \text{ or } F = kma \quad ...(5.3.2.)$$
   where, $k$ = Constant of proportionality.

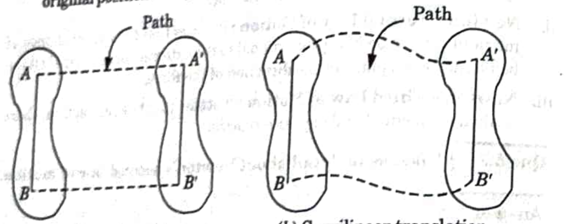
**Que 5.4. Discuss the various types of plane motion.**

**Answer**

Generally a body undergoes the following three types of plane motion :

i. **Translation :**

1. During translation, the particles have the same velocity and acceleration, and a straight line drawn on the moving body remains parallel to its original position at any time.




(a) Rectilinear translation (b) Curvilinear translation

**Fig. 5.4.1.**

2. If path traced by the particles during motion is a straight line, then the motion is said to be rectilinear translation (Fig. 5.4.1(*a*)).
3. If particle traces a curved path, the motion is called curvilinear translation (Fig. 5.4.1(*b*)).

ii. **Rotation :**

1. During rotation, the body rotates about a fixed point and all the particles constituting the body move in a circular path.
2. The fixed point about which the body rotates is called the point of rotation and the axis passing through the point of rotation is called the axis of rotation.

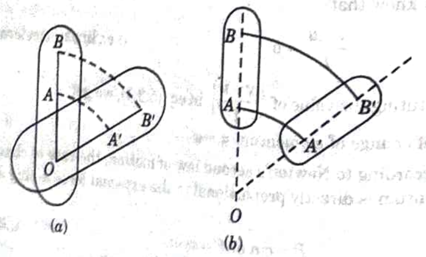



**Fig. 5.4.2.**



iii. **General Plane Motion (Combined Motion of Translation and Rotation) :**

1. When a body possess both translation and rotation motions simultaneously at a particular instant, the motion is called general plane motion.

**Example :** (i) Motion of roller without slipping, motion of wheel of a locomotive train, truck and car etc., (ii) A rod sliding against a wall at one end and floor at the other end.

**Que 5.5.** **A particle of mass 1 kg moves in a straight line under the influence of a force which increases linearly with time at the rate of 60 N per sec. At time $t = 0$, the initial force may be taken as 50 N. Determine the acceleration and velocity of the particle 4 sec after it started from the rest at the origin.**

**Answer**

**Given :** $m = 1$ kg, $\dfrac{dF}{dt} = 60$ N/sec, At $t = 0$, $F = 50$ N, $t = 4$ sec

**To Find :** i. Velocity

ii. Acceleration

1. Force is increasing linearly with time. Hence applied force on the particle is a function of time.

Let, $$F = At + B \qquad ...(5.5.1)$$

where, $A$ and $B$ are constant.

2. When $t = 0$, $F = 50$ N. Now eq. (5.5.1) becomes,

$$50 = A \times 0 + B = B$$

$$\therefore \quad B = 50 \text{ N}$$

3. Differentiating eq. (5.5.1), we get

$$\therefore \quad \frac{dF}{dt} = A + 0$$

But $$\frac{dF}{dt} = 60 \text{ N/sec}$$

$$A = 60 \text{ N/s}$$

4. Substituting the value $A$ and $B$ in eq. (5.5.1), we get

$$F = 60t + 50 \qquad ...(5.5.2)$$

5. We know that, $$F = ma = m\frac{dv}{dt} \qquad \left(\because a = \frac{dv}{dt}\right)$$

Substituting this value of $F$ in eq. (5.5.2), we get

$$m \times \frac{dv}{dt} = 60t + 50 \quad ...(5.5.3)$$

$$1 \times \frac{dv}{dt} = 60t + 50 \quad (\because m = 1 \text{ kg})$$

$$\frac{dv}{dt} = 60t + 50 \quad ...(5.5.4)$$

6. Integrating the eq. (5.5.4) w.r.t time, we get

$$\int dv = \int (60t + 50)\, dt$$

$$v = \int_0^4 (60t + 50)\, dt$$

$$v = \left[\frac{60t^2}{2} + 50t\right]_0^4 = 30 \times 4^2 + 50 \times 4 = 480 + 200$$

$$= 680 \text{ m/sec}$$

7. From eq. (5.5.3), we have

$$\frac{dv}{dt} = 60t + 50$$

$$a = 60t + 50 \quad \left(\because \frac{dv}{dt} = a\right)$$

8. Acceleration after 4 sec, $a = 60 \times 4 + 50 = 290$ m/sec$^2$

## PART-2

*Instantaneous Centre of Rotation in Plane Motion and Simple Problems.*

### Questions-Answers

**Long Answer Type and Medium Answer Type Questions**

**Que 5.6.** **Define instantaneous centre of rotation and also write the procedure for locating the position of instantaneous centre of rotation.**

**Answer**

**A. Instantaneous Centre of Rotation :**

1. Instantaneous centre is the point about which motion of a body having both rotatory and translatory motion is assumed to be purely rotational. It is also known as virtual centre.



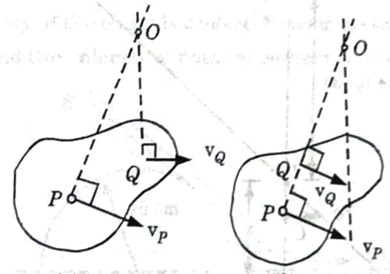
2. The angular velocity of any point about instantaneous centre is given by,

$$\omega = \frac{v}{I}$$

where, $\omega$ = Angular velocity.
v = Linear velocity.
$I$ = Instantaneous centre.

**B. Locating the Position of Instantaneous Centre of Rotation :**

1. If the directions of the velocities of two particles $P$ and $Q$ of the body are known and if they are different, the instantaneous centre is obtained by drawing the perpendicular to $v_P$ through $P$ and perpendicular to $v_Q$ through $Q$. The intersection point of these two perpendiculars is known as instantaneous centre of rotation.
2. If the velocities $v_P$ and $v_Q$ of two particles $P$ and $Q$ are perpendicular to the line $PQ$ and the magnitudes of $v_P$ and $v_Q$ are known, the instantaneous centre of rotation can be found by intersection point of line $PQ$ with the line joining the extremities of the vectors $v_P$ and $v_Q$.
3. If the velocities $v_P$ and $v_Q$ are parallel and have different magnitude or if the velocities $v_P$ and $v_Q$ are perpendicular to line $PQ$ and have equal magnitude, the instantaneous centre $O$ will be at an infinite distance and $\omega$ will be zero and all the points of the body will have the same velocity.




(a) (b)

**Fig. 5.6.1.**

**Que 5.7.** **A compound wheel rolls without slipping between two parallel plates $A$ and $B$ as shown in Fig. 5.7.1. At the instant $A$ moves to the right with a velocity of 1.2 m/sec and $B$ moves to the left with a velocity of 0.6 m/sec. Calculate the velocity of centre of wheel and the angular velocity of wheel. Take $r_1$ = 120 mm and $r_2$ = 360 mm.**

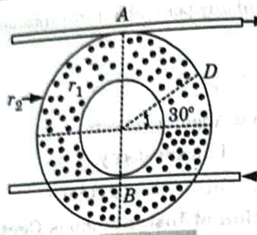



Fig. 5.7.1.

**AKTU 2015-16, (I) Marks 10**

**Answer**

**Given :** $v_A = 1.2$ m/sec, $v_B = 0.6$ m/sec, $r_1 = 120$ mm $= 0.12$ m, $r_2 = 360$ mm $= 0.36$ m

**To Find :** Velocity of centre of wheel and angular velocity of wheel.

1. The instantaneous centre $I$ is the point of intersection of the line joining $A$ and $B$ with line joining the extremities of the velocity vectors $v_A$ and $v_B$ as shown in Fig. 5.7.2.

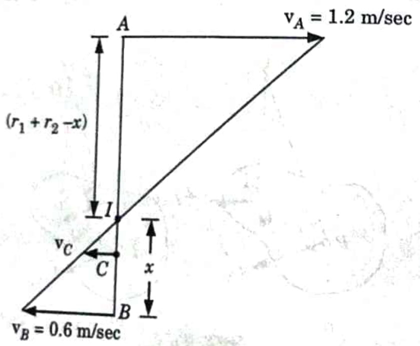



Fig. 5.7.2.

2. Every vector on the wheel will appear to rotate about the instantaneous centre $I$ with an angular velocity $\omega$.

$$\omega = \frac{v_A}{IA} = \frac{v_B}{IB}$$

$$\therefore \quad \frac{v_A}{(r_1 + r_2 - x)} = \frac{v_B}{x}$$



$$\frac{1.2}{(480 - x)} = \frac{0.6}{x} \quad ...(5.7.1)$$

3. Solving eq. (5.7.1), we get

$$x = 160 \text{ mm}$$

$$\therefore \quad IB = 160 \text{ mm}$$

Also $\quad IA + IB = 480$

$$\therefore \quad IA = 480 - IB = 480 - 160 = 320 \text{ mm}$$

4. Now angular velocity of the disc,

$$\omega = \frac{v_A}{IA} = \frac{1.2}{\left(\frac{320}{1000}\right)} = 3.75 \text{ rad/sec}$$

5. From Fig. 5.7.2, $\quad IC = x - CB$

$$= 160 - 120 = 40 \text{ mm}$$

6. Velocity of the centre $C$,

$$v_C = \omega\, IC = 3.75 \times \frac{40}{1000}$$

$$v_C = 0.15 \text{ m/sec}$$

**Que 5.8.** **A slender bar $AB$ slides down a circular surface and on a horizontal surface as shown in Fig. 5.8.1. At an instant, when $\theta = 45°$, velocity of the end $A$ is 2 m/sec. Determine the angular velocity of the bar and the velocity of point of contact on the circular surface.**

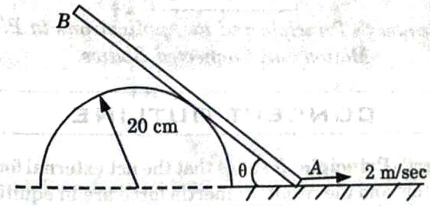



Fig. 5.8.1.

**AKTU 2012-13, Marks 10**

**Answer**

**Given :** $\theta = 45°$, $v_A = 2$ m/sec

**To Find :** Angular velocity of the bar and velocity of point of contact on the circular surface.

1. Instantaneous centre $I$ is obtained by drawing perpendicular on $v_A$ and $v_C$.

2. Now,
$$v_C = v_A \times \frac{IC}{IA} = v_A \times \cos 45°$$

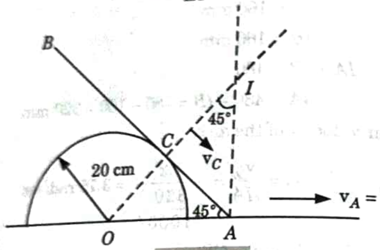



Fig. 5.8.2.

$$\therefore \quad v_C = 2 \times \frac{1}{\sqrt{2}} = 1.414 \text{ m/sec}$$

3. Also,
$$v_A = \omega_o \times IA$$
$$\omega_o = \frac{v_A}{IA}$$

From $\Delta ICA$,
$$IA = 20\sqrt{2} \text{ cm} = 28.28 \text{ cm} = 0.2828 \text{ m} \quad (\because CA = 20 \text{ cm})$$

Hence
$$\omega_o = \frac{2}{0.2828} = 7.072 \text{ rad/sec}$$

## PART-3

*D'Alembert's Principle and its Applications in Plane Motion and Connected Bodies.*

### CONCEPT OUTLINE

**D'Alembert's Principle :** It states that the net external forces acting on the system and the resultant inertia force are in equilibrium.

Mathematically, $F - ma = 0$

where, $F$ = External force.

$ma$ = Resulting inertia force.

### Questions-Answers

**Long Answer Type and Medium Answer Type Questions**



**Que 5.9.** Illustrate D'Alembert's principle with respect to connected bodies.

**Answer**

Following cases can be considered for illustrating D'Alembert's principle :

a. **Motion of a Lift :**

1. Let the tension in the string be $T$, acceleration of the lift be $a$ and $W$ be the weight of lift plus persons in the lift.

2. Considering upward motion of the lift (Fig. 5.9.1 (a)).
$$T - W - \frac{W}{g}a = 0$$
$$T = W\left[1 + \frac{a}{g}\right]$$
$$T = m(g + a) \quad (\because W = mg)$$
where, $m$ = Mass equivalent of weight $W$.

3. Considering downward motion of the lift (Fig. 5.9.1 (a))
$$W - T - \frac{W}{g}a = 0$$
$$T = W\left[1 - \frac{a}{g}\right]$$
$$T = m(g - a)$$

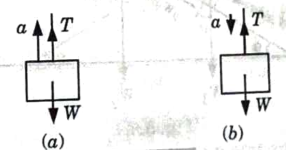



Fig. 5.9.1.

b. **Motion of Two Connecting Weights over a Smooth Pulley :**

1. Let $m_1 > m_2$ and the acceleration of the system be $a$, $m_1$ obviously moving downwards. According to D'Alembert's principle,

For block of mass $m_1$,
$$m_1 g - T = m_1 a \quad ...(5.9.1)$$
For block of mass $m_2$,
$$T - m_2 g = m_2 a \quad ...(5.9.2)$$

2. Adding eq. (5.9.1) and eq. (5.9.2), we get

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$$

3. Subtracting eq. (5.9.1) from eq. (5.9.2), we get

$$T = \frac{2gm_1m_2}{m_1 + m_2}$$

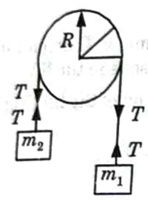



Fig. 5.9.2.

c. **Motion of Two Interconnected Bodies on an Inclined Plane :**

This motion can be divided in two cases.

**Case I :**

1. Let two bodies $A$ and $B$ be joined by any inextensible string and the composite system moved down the inclined with a common acceleration

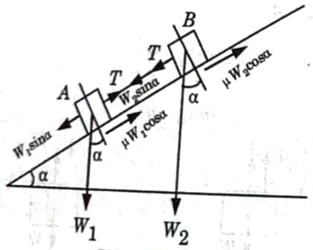



Fig. 5.9.3.

For block $B$,

$$T + W_2 \sin\alpha - \mu W_2 \cos\alpha - \frac{W_2}{g} a = 0 \quad ...(5.9.3)$$

For block $A$,

$$W_1 \sin\alpha - T - \mu W_1 \cos\alpha - \frac{W_1}{g} a = 0 \quad ...(5.9.4)$$

2. From eq. (5.9.3) and eq. (5.9.4), we get

$$(W_1 + W_2)\sin\alpha - \mu(W_1 + W_2)\cos\alpha - \frac{a}{g}(W_1 + W_2) = 0$$



$$\sin\alpha - \mu\cos\alpha - \frac{a}{g} = 0$$

$$\frac{a}{g} = \frac{\sin(\alpha - \phi)}{\cos\phi} \quad (\because \mu = \tan\phi)$$

where, $\phi$ = Angle of friction.

3. If the coefficients of friction are different for $A$ and $B$, *i.e.*, $\mu_1$ and $\mu_2$, then

$$T + W_2 \sin\alpha - \mu_2 W_2 \cos\alpha - \frac{W_2 a}{g} = 0$$

and $W_1 \sin\alpha - T - \mu_1 W_1 \cos\alpha - \dfrac{W_1 a}{g} = 0$

4. Which on simplification gives

$$\frac{a}{g} = \frac{(W_1 + W_2)\sin\alpha - (\mu_1 W_1 + \mu_2 W_2)\cos\alpha}{W_1 + W_2}$$

and

$$T = \frac{W_1 W_2(\mu_2 - \mu_1 \cos\alpha)}{W_1 + W_2}$$

**Case II :**

1. Motion of two connected masses, one of which moves on the inclined plane, while the other falls freely being connected to the former by a string running over a pulley.

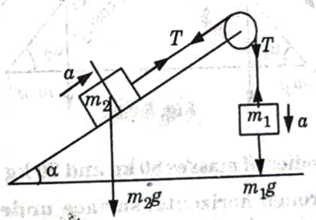



Fig. 5.9.4.

2. Let the two masses accelerate with acceleration $a$ in the direction of $m_1$ as shown in Fig. 5.9.4. Considering no friction,

For block of mass $m_1$,

$$m_1 g - T - m_1 a = 0 \quad ...(5.9.5)$$

For block of mass $m_2$,

$$m_2 g \sin\alpha - T - m_2 a = 0 \quad ...(5.9.6)$$

3. From eq. (5.9.5) and eq. (5.9.6), we have

$$a = \frac{g(m_1 - m_2 \sin\alpha)}{m_1 + m_2}$$

$$T = \frac{2gm_1m_2 \sin\alpha}{m_1 + m_2}$$

**d. Motion of Two Connected Bodies One on each of the Two Smooth Inclined Planes :**

1. Let the motion be on the $m_2$ side of the body as shown in Fig. 5.9.5.
2. Then by D'Alembert's principle,

For block of mass $m_2$,

$$m_2 g \sin \alpha_2 - T - m_2 a = 0 \quad ...(5.9.7)$$

For block of mass $m_1$,

$$T - m_1 g \sin \alpha_1 - m_1 a = 0 \quad ...(5.9.8)$$

3. From eq. (5.9.7.) and eq. (5.7.8), we get

$$a = \frac{g(m_2 \sin \alpha_2 - m_1 \sin \alpha_1)}{m_2 + m_1}$$

$$T = \frac{m_1 m_2 (\sin \alpha_1 + \sin \alpha_2) g}{m_2 + m_1}$$

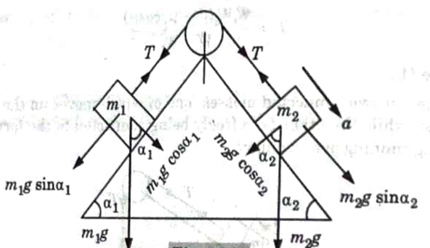



Fig. 5.9.5.

**Que 5.10.** **Two bodies of masses 80 kg and 20 kg are connected by a thread along a rough horizontal surface under the action of a force 400 N applied to the first body of mass 80 kg as shown in Fig 5.10.1. The coefficient of friction between the sliding surfaces of the bodies and plane is 0.3. Determine the acceleration of two bodies and tension in the thread using D'Alembert's principle.**

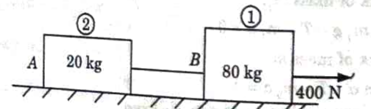



Fig. 5.10.1.

**AKTU 2014-15, (II) Marks 10**



**Answer**

**Given :** $m_1 = 80$ kg, $W_1 = 80 \times 9.81 = 784.8$ N, $m_2 = 20$ kg, $W_2 = 20 \times 9.81 = 196.2$ N, $F = 400$ N, $\mu = 0.3$

**To Find :** i. Acceleration of two bodies, ii. Tensions in the thread.

1. Let us consider, both the blocks are moving with acceleration $a$ and tension developed in thread is $T$.
2. Considering FBD of Block 1 (Fig. 5.10.2 ($a$))

Using D'Alembert's principle,

$$400 - T - \mu R = 80\,a$$

$$400 - T - 0.3 \times 784.8 = 80\,a$$

$$164.56 - T = 80\,a \quad ...(5.10.1)$$

3. Considering FBD of Block 2 (Fig. 5.10.2($b$))

Using D'Alembert's principle,

$$T - \mu R = 20\,a$$

$$T - 0.3 \times 196.2 = 20\,a$$

$$T - 58.86 = 20\,a \quad ...(5.10.2)$$

4. On solving the eq. (5.10.1) and eq. (5.10.2), we get

$$a = 1.057 \text{ m/sec}^2$$

and $$T = 80 \text{ N}$$

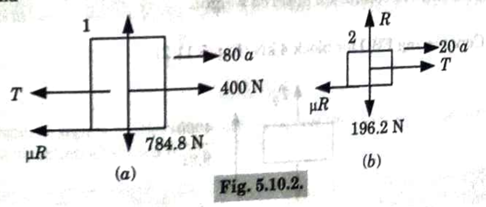



Fig. 5.10.2.

**Que 5.11.** **A system of weight connected by string passing over pulleys $A$ and $B$ shown in Fig. 5.11.1. Find the acceleration of three weights. Assuming string is weightless and ideal condition for pulleys.**

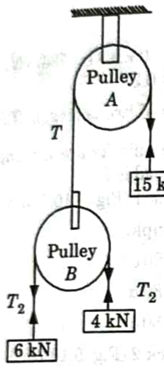



Fig. 5.11.1.

**AKTU 2014-15, (II) Marks 10**

**Answer**

**Given :** Fig. 5.11.1.

**To Find :** Acceleration of three weight.

1. Considering FBD for block 4 kN (Fig. 5.11.2)

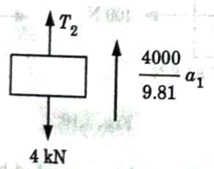



Fig. 5.11.2.

$$T_2 - 4000 = \frac{4000}{9.81} a_1 \quad ...(5.11.1)$$

2. Considering FBD for block 6 kN (Fig. 5.11.3)

$$6000 - T_2 = \frac{6000}{9.81} a_1 \quad ...(5.11.2)$$



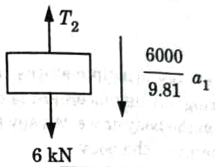



Fig. 5.11.3.

3. From eq. (5.11.1) and eq. (5.11.2), we get

$$T_2 = 4800 \text{ N}, \quad a_1 = 1.962 \text{ m/sec}^2$$

4. Considering FBD for pulley A,

$$T = 2\,T_2$$

$$T = 2T_2 = \frac{15000}{9.81} a_2 + 15000$$

$$2 \times 4800 = \frac{15000}{9.81} a_2 + 15000$$

$$-5400 = \frac{15000}{9.81} a_2$$

$$a_2 = -3.5316 \text{ m/sec}^2$$

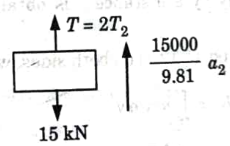



Fig. 5.11.4.

Negative sign of accelerations indicates that the direction is opposite to the direction as shown in Fig. 5.11.4.

## PART-4

*Work-Energy Principle and its Application in Plane Motion of Connected Bodies.*

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 5.12.** **State and prove work-energy principle.**

**Answer**

A. **Statement :** Work-energy principle states that the change in kinetic energy of a body during any displacement is equal to the work done by the net force acting on the body or we can say that work done is equal to change in kinetic energy of the body.

B. **Proof :**

1. We know that, $F = ma$ ...(5.12.1)

where, $F$ = Resultant of all forces acting on a body.

$m$ = Mass of the body.

$a$ = Acceleration in the direction of resultant force.

$$a = v\frac{dv}{ds}$$

2. Substituting the value of $a$ in eq. (5.12.1), we get

$$F = m \times \left(v\frac{dv}{ds}\right) \text{ or } F\,ds = mv\,dv \quad ...(5.12.2)$$

3. But $F\,ds$ is the work done by the resultant force $F$ in displacing the body by a small distance $ds$. The total work done by the resultant force $F$ in displacing the body by a distance $s$ is obtained by integrating the eq. (5.12.2).

4. Hence, integrating eq. (5.12.2) on both sides, we get

$$\int_0^s F\,ds = \int_u^v mv\,dv$$

$$\therefore \quad Fs = m\left[\frac{v^2}{2}\right]_u^v = \frac{m}{2}[v^2 - u^2] = \frac{mv^2}{2} - \frac{mu^2}{2}$$

Work done by resultant force = Change in kinetic energy

**Que 5.13.** **A body of mass 30 kg is projected up an incline of 30° with an initial velocity of 10 m/sec. The friction coefficient between the contacting surfaces is 0.2. Determine distance travelled by the body before coming to rest.**

**AKTU 2013-14, (II) Marks 05**

**Answer**

**Given :** $m$ = 30 kg, $u$ = 10 m/sec, v = 0 (rest), $\mu$ = 0.2

**To Find :** Distance travelled by the body before coming to rest.



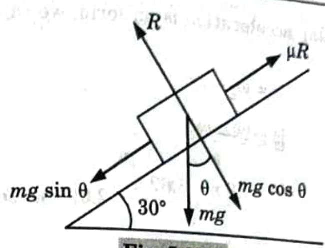



**Fig. 5.13.1.**

1. Resultant force acting on the block,

$$F = mg \sin\theta - \mu R$$
$$= mg \sin\theta - \mu\, mg \cos\theta$$
$$= 30 \times 10 \times \sin 30° - 0.2 \times 30 \times 10 \cos 30°$$
$$F = 98.04 \text{ N}$$

2. Using the work-energy balance equation,

Work done by the block = Kinetic energy of the block

$$Fx = \frac{1}{2}m(u^2 - v^2)$$

$$98.04 \times x = \frac{1}{2} \times 30\,[10^2 - 0^2]$$

$$x = 15.30 \text{ m}$$

**Que 5.14.** **The speed of a flywheel rotating at 200 rpm is uniformly increased to 300 rpm in 5 seconds. Determine the work done by the driving torque and the increase in kinetic energy during this time. Take mass of the flywheel as 25 kg and its radius of gyration as 20 cm.**

**Answer**

**Given :** $N_0$ = 200 rpm, $\omega_0 = \frac{2 \times \pi \times 200}{60} = 6.67\,\pi$ rad/sec,

$t$ = 5 sec, $m$ = 25 kg, $k$ = 20 cm = 0.2 m, $N$ = 300 rpm,

$\omega = \frac{2 \times \pi \times 300}{60} = 10\,\pi$ rad/sec

**To Find :** i. Work done by the driving torque.

ii. Increase in kinetic energy.

1. Mass moment of inertia of the flywheel about its centroidal axis is,

$$I = mk^2 = (25)(0.2)^2 = 1 \text{ kg m}^2$$

2. Since the angular acceleration is uniform, we can use the kinematic equation,

$$\omega = \omega_0 + \alpha t$$

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$= \frac{10\pi - 6.67\pi}{5} = 2.09 \text{ rad/sec}^2$$

3. Also we know that,

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha}$$

$$= \frac{(10\pi)^2 - (6.67\pi)^2}{2(2.09)} = 131.07 \text{ rad}$$

4. Since the angular acceleration is constant, the driving torque is constant and hence applying the kinetic equation of motion about fixed axis, we have

$$M = I\alpha = (1)(2.09) = 2.09 \text{ N-m}$$

5. Work done by the driving torque is given by,

$$W = M(\theta_2 - \theta_1)$$

$$= (2.09)(131.07) = 273.94 \text{ J}$$

6. The increase in kinetic energy is given by,

$$\Delta(KE) = (KE)_f - (KE)_i$$

$$= \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2$$

$$= \frac{1}{2}I(\omega^2 - \omega_0^2)$$

$$= \frac{1}{2}(1)[(10\pi)^2 - (6.67\pi)^2] = 273.94 \text{ J}$$

**Que 5.15.** **A constant force of 100 N is applied as shown tangentially on a cylinder at rest, whose mass is 50 kg and radius is 10 cm, for a distance of 5 m. Determine the angular velocity of the cylinder and the velocity of its centre of mass. Assume that there is no slip.**

**Answer**

**Given :** $F = 100$ N, $m = 50$ kg, $r = 10$ cm $= 0.1$ m, $s = 5$ m

**To Find :** i. Angular velocity of the cylinder.

ii. Velocity of centre of mass.



1. Since the applied force is horizontal and the displacement is in the direction of the force, the work done by the force in causing a displacement $s$ is given by,

$$W = Fs$$

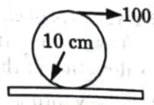



**Fig. 5.15.1.**

2. Applying the work-energy principle, we have

Work done = Change in kinetic energy

$$Fs = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$Fs = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}\frac{mr^2}{2}\omega^2 \qquad \left(\because v = r\omega, I = \frac{mr^2}{2}\right)$$

$$Fs = \frac{3}{4}mr^2\omega^2$$

$$\omega^2 = \frac{4Fs}{3mr^2} = \frac{4(100)(5)}{3(50)(0.1)^2} = 1333.33$$

$$\therefore \quad \omega = 36.51 \text{ rad/sec}$$

3. Velocity of the centre of mass is given as,

$$v_{cm} = r\omega$$

$$= (0.1)(36.51) = 3.651 \text{ m/sec}$$

## PART-5

*Kinetics of Rigid Body Rotation.*

### Questions-Answers

**Long Answer Type and Medium Answer Type Questions**

**Que 5.16.** **Discuss and describe the laws of motion applied to planar translation and rotation.** **AKTU 2014-15, (II) Marks 05**

**Answer**

A. **Laws of Translation :** Refer Q. 5.2, Page 5–3C, Unit-5.

**B. Laws of Rotation :** Following are the laws as applied to rotary motion :

i. **First Law :** It states that a body continues in its state of rest or of rotation about an axis with constant angular velocity unless it is compelled by an external torque to change the state.

ii. **Second Law :** It states that the rate of change of angular momentum of a rotating body is proportional to the external torque applied on the body and takes place in the direction of the torque.

iii. **Third Law :** It states that to every torque there is always an equal and opposite torque.

**Que 5.17.** **Derive an expression for kinetic energy due to rotation.**

**Answer**

1. Consider a rigid body rotating about $O$ as shown in Fig. 5.17.1.
2. Let, $\omega$ = Angular velocity of the body.
   $dm$ = Elementary mass of the body.
   $r$ = Radius of elementary mass from $O$.
   v = Tangential velocity of elementary mass.
3. KE of the elementary mass is,
$$= \frac{1}{2} \times \text{Mass} \times \text{Velocity}^2 = \frac{1}{2}\, dm\text{v}^2 \qquad ...(5.17.1)$$
4. KE of the whole body is obtained by integrating the eq. (5.17.1). Hence KE of the body,
$$= \int \frac{1}{2} dm\text{v}^2 = \frac{1}{2}\int dm\,(\omega r)^2 \qquad (\because \text{v} = \omega r)$$
$$= \frac{1}{2}\int \omega^2 r^2\, dm = \frac{1}{2}\omega^2 \int r^2\, dm \quad (\because \omega \text{ is a constant})$$

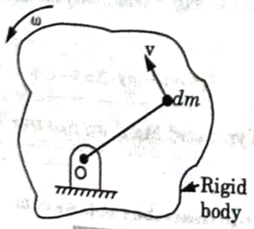



Fig. 5.17.1.

5. But $\int r^2\, dm = I$ = Moment of inertia of the body about $O$.

$\therefore$ KE of the body $= \frac{1}{2}\omega^2 I$



**Que 5.18.** **A uniform homogeneous cylinder rolls without slip along a horizontal level surface with a translational velocity of 20 cm/sec. If its weight is 0.1 N and its radius is 10 cm, what is its total kinetic energy ?**

**Answer**

**Given :** v = 20 cm/sec = 0.20 m/sec, W = 0.1 N, $m = \frac{W}{g} = \frac{0.1}{9.81}$ kg
$r$ = 10 cm = 0.1 m
**To Find :** Total kinetic energy.

1. We know that, $I = \frac{mr^2}{2}$
$$= \frac{0.1}{9.81} \times \frac{0.1^2}{2} = 0.000051$$
$$\omega = \frac{\text{v}}{r} = \frac{0.20}{0.10} = 2 \text{ rad/sec}$$
2. Total kinetic energy $= \frac{1}{2}I\omega^2 + \frac{1}{2}m\text{v}^2$
$$= \frac{1}{2} \times 0.000051 \times 2^2 + \frac{1}{2} \times \frac{0.1}{9.81} \times 2^2$$
$$= 0.000102 + 0.0204 = 0.020502 \text{ N-m}$$

**Que 5.19.** **Derive an expression for the acceleration of system in which weights are attached to the two ends of a string which passes over a rough pulley.**

**Answer**

1. Fig. 5.19.1 shows the two weights $W_1$ and $W_2$ attached to the two ends of a string, which passes over a rough pulley of radius $R$.
2. As pulley is rough and having certain weight, the tensions on both sides of the string will not be same. If $W_1 > W_2$, the weight $W_1$ will move downwards whereas the weight $W_2$ will move upwards with the same acceleration.
3. Let, $a$ = Acceleration of the system.
   $T_1$ = Tension in the string to which weight $W_1$ is attached.
   $T_2$ = Tension in the string to which weight $W_2$ is attached.
   $R$ = Radius of the pulley.

$I$ = Moment of inertia of the pulley about the axis of rotation.

$\alpha$ = Angular acceleration.

$W_0$ = Weight of the pulley.

4. Considering the motion of weight $W_1$, let it is moving downwards with an acceleration $a$.
The net downwards force on weight $W_1 = (W_1 - T_1)$

Mass of weight, $m_1 = \frac{W_1}{g}$

5. We know that,

Net force = Mass × Acceleration

$$(W_1 - T_1) = \frac{W_1}{g} a \quad \text{...(5.19.1)}$$

6. Considering the motion of weight $W_2$, let it is moving upwards with an acceleration $a$.
Net upward force = $(T_2 - W_2)$

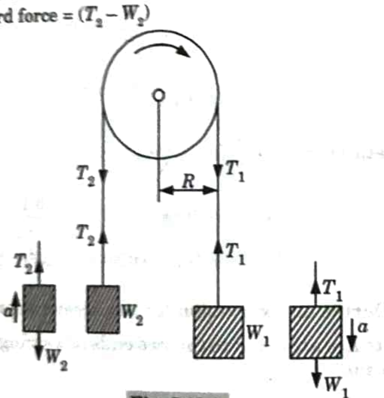



Fig. 5.19.1.

7. Using, net force = Mass × Acceleration

$$(T_2 - W_2) = \frac{W_2}{g} a \quad \text{...(5.19.2)}$$

8. Now considering the rotation of the pulley, let it is rotating with an angular acceleration $\alpha$.

9. If the pulley is considered as a solid disc, then moment of inertia of the pulley is given by,

$$I = \frac{mR^2}{2} \quad (\because \text{Solid disc is like a cylinder})$$

$$I = \frac{W_0}{g}\frac{R^2}{2} \quad \left(\because m = \frac{W_0}{g}\right)$$



10. The torque on the pulley is given by,

$$T = I\alpha = \frac{W_0}{g} \times \frac{R^2}{2} \times \frac{a}{R} \quad \left(\because \alpha = \frac{a}{R}\right) \quad \text{...(5.19.3)}$$

11. But torque on the pulley = Torque due to $T_1$ – Torque due to $T_2$

$$= T_1 \times R - T_2 \times R = R(T_1 - T_2)$$

12. Substituting the value of torque in eq. (5.19.3), we get

$$R(T_1 - T_2) = \frac{W_0}{g} \times \frac{R^2}{2} \times \frac{a}{R}$$

$$T_1 - T_2 = \frac{W_0}{2g} a \quad \text{...(5.19.4)}$$

13. Adding eq. (5.19.1), eq. (5.19.2) and eq. (5.19.4), we get

$$W_1 - W_2 = \frac{W_1}{g} a + \frac{W_2}{g} a + \frac{W_0}{2g} a = \frac{a}{g}\left(W_1 + W_2 + \frac{W_0}{2}\right)$$

$$a = \frac{g(W_1 - W_2)}{\left(W_1 + W_2 + \frac{W_0}{2}\right)}$$

**Que 5.20.** **Two weights of 8 kN and 5 kN are attached at the ends of a flexible cable. The cable passes over a pulley of diameter 1 m. The weight of the pulley is 500 N and radius of gyration is 0.5 m about its axis of rotation. Find the torque which must be applied to the pulley to raise the 8 kN weight with an acceleration of 1.2 m/sec². Neglect the friction in the pulley.**

**AKTU 2013-14, (I) Marks 10**

**Answer**

Given : $W_1$ = 8 kN, $W_2$ = 5 kN, $D$ = 1 m, $W_0$ = 500 N, $k$ = 0.5 m, $a$ = 1.2 m/sec²

To Find : Torque applied to pulley.

1. As we need to raise 8 kN weight with an acceleration of 1.2 m/sec², then we must apply a torque on the pulley which will be given as

$$\text{Torque} = (T_1 - T_2)\,r + I\alpha$$

2. Applying equilibrium equation on block of 8 kN, we get

$$T_1 - 8000 = \frac{8000}{9.81} a$$

$$T_1 = \frac{8000}{9.81} \times 1.2 + 8000$$

$$T_1 = 8978.59 \text{ N}$$

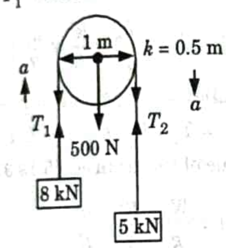



**Fig. 5.20.1.**

3. Applying equilibrium equation on block of 5 kN, we get

$$5000 - T_2 = \frac{5000}{9.81} a$$

$$T_2 = 5000 - \frac{5000}{9.81} \times 1.2$$

$$T_2 = 4388.38 \text{ N}$$

4. Torque applied on the pulley $= I\alpha$

$$I\alpha = mk^2 \frac{a}{r} \qquad \left(\because\ I = mk^2, \alpha = \frac{a}{r}\right)$$

$$I\alpha = \left(\frac{500}{9.81}\right)(0.5)^2 \times \frac{1.2}{(1/2)}$$

$$I\alpha = 30.58 \text{ N-m}$$

5. Now total applied torque $= (T_1 - T_2)\, r + I\alpha$

$$= (8978.59 - 4388.38) \times \left(\frac{1}{2}\right) + 30.58$$

$$= 2325.685 \text{ N-m}$$

**PART-6**

*Virtual Work and Energy Method, Virtual Displacements, Principle of Virtual Work for Particle and Ideal System of Rigid Bodies.*

**CONCEPT OUTLINE**

**Virtual Displacement :** The displacement of a particle or a rigid body in equilibrium is not at all possible. However we can assume an imaginary displacement to occur, particularly if the system is partially constrained, this displacement is known as virtual displacement.

**Virtual Work :** The total work done by the system of forces causing the virtual displacement is termed as virtual work.



**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 5.21.** **Discuss in short about work done on a particle and work done on a rigid body.**

**Answer**

i. **Work Done on a Particle :**

1. When a force acts on a particle, which is not constrained to move, it causes a displacement of the particle. The force is then said to have done work on the particle.
2. We then define work done on the particle as a product of magnitude of the force and the displacement. Mathematically, we can write this as

$$W = Fs$$

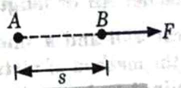



**Fig. 5.21.1.**

ii. **Work Done on a Rigid Body :**

1. We know that a rigid body is subjected to moments in addition to the forces. Just as the forces cause linear displacements, moments cause angular displacements.
2. If a moment $M$ acting on a rigid body causes an angular displacement $\theta$ then work done by the moment on the rigid body is defined as the product of moment and angular displacement, *i.e.*,

$$W = M\theta$$

**Que 5.22.** **Give the principle of virtual work for a particle and a rigid body.**

**Answer**

1. For the particle or rigid body to remain in equilibrium in the displaced position also, we know that the resultant force acting on it must be zero. Thus, we say that work done in causing this virtual displacement is also zero. This is known as principle of virtual work.
2. For a system of concurrent forces $F_1, F_1, \ldots F_1$, the virtual work done is given by,

$$\delta U = F_1\, \delta r + F_2\, r + \ldots + F_n\, \delta r$$

$$= (F_1 + F_2 + ..... + F_n)\, \delta r$$

$$= \sum \vec{F} \delta \vec{r}$$

3. As a system of concurrent force can be replaced by a single resultant force, the virtual work done is equal to the work done by the resultant.
4. For the body to remain in equilibrium in the displaced position, we know that the resultant must be zero. Hence, virtual work done in causing this virtual displacement is also zero, *i.e.*,

$$\delta U = \left(\sum F\right) \delta r = 0$$

5. The necessary and sufficient condition for the equilibrium of a particle is zero virtual work done by all external forces acting on the particle during any virtual displacement consistent with the constraints imposed on the particle.
6. Similarly, for a rigid body, we can write the principle of virtual work as

$$\delta U = \sum F\, \delta r + \sum M\, \delta\theta = 0$$

**Que 5.23.** **A uniform ladder *AB* of length *l* and weight *W* leans against a smooth vertical wall and a smooth horizontal floor as shown in Fig. 5.23.1. By the method of virtual work, determine the horizontal force *P* required to keep the ladder in equilibrium position.**

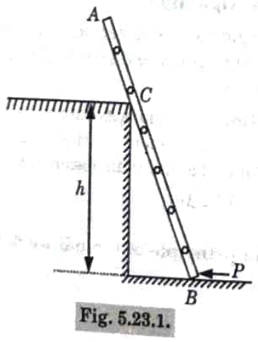



Fig. 5.23.1.

**Answer**

**Given :** Fig. 5.23.1.
**To Find :** Horizontal force, *P*.

1. Under its own weight, the ladder tries to slide down, but the horizontal force *P* holds it in equilibrium. The free body diagram of the ladder is shown in Fig. 5.23.2.



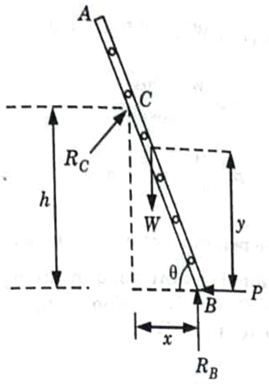



Fig. 5.23.2.

2. Let $\theta$ be inclination of the ladder with respect to the horizontal. From the geometry of the triangle, we see that the location $x$ of the end $B$ and the location $y$ of the centre of gravity of ladder with respect to the origin are :

$$x = \frac{h}{\tan\theta} \quad ...(5.23.1)$$

$$y = \frac{l}{2}\sin\theta \quad ...(5.23.2)$$

3. The virtual displacement are obtained by differentiating eq. (5.23.1) and eq. (5.23.2) as,

$$\delta x = -\, h\, \text{cosec}^2\, \theta\, \delta\theta \text{ and } \delta y = \frac{l}{2} \cos\theta\, \delta\theta$$

4. From Fig. 5.23.2 we see that as $\theta$ decreases, $y$ also decreases but $x$ increases. Hence, considering only positive virtual displacements, the above expressions reduce to

$$\delta x = h\, \text{cosec}^2\, \theta\, \delta\theta \text{ and } \delta y = \frac{l}{2} \cos\theta\, \delta\theta$$

5. Now applying the principle of virtual work, we have

$$\delta U = 0$$

$$-\, P\, \delta x + W\, \delta y = 0$$

6. It should be noted that reaction $R_B$ and $R_C$ do no work, as the virtual displacement of the contact points $B$ and $C$ are perpendicular to the direction of the forces. Therefore

$$-P[h \operatorname{cosec}^2 \theta\, \delta\theta] + W\left[\frac{l}{2} \cos\theta\, \delta\theta\right] = 0$$

$$P = \frac{Wl}{2h} \frac{\cos\theta}{\operatorname{cosec}^2 \theta}$$

$$P = \frac{Wl}{2h} \sin^2\theta \cos\theta$$

**Que 5.24.** **Using the principle of virtual work, determine the angle $\theta$ for which equilibrium is maintained in the mechanism shown for given values of forces $P_1$ and $P_2$ applied. Length of the longer links is $l$ and that of the shorter links is $l/2$.**

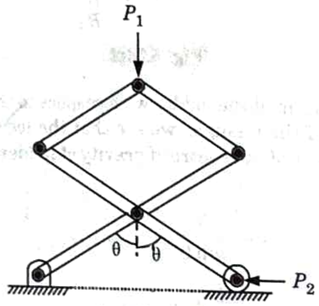



**Fig. 5.24.1.**

**Answer**

**Given :** Fig. 5.24.1, Length of longer link = $l$, Length of shorter link = $l/2$
**To Find :** Angle $\theta$.

1. Choosing the hinge point as the origin, the point of application of the forces $P_1$ and $P_2$ are $y$ and $x$ respectively. Expressing these positions $x$ and $y$ in terms of $\theta$, we have

$$y = \frac{l}{2}\cos\theta + \frac{l}{2}\cos\theta + \frac{l}{2}\cos\theta = \frac{3}{2} l \cos\theta \quad \text{...(5.24.1)}$$

and $$x = 2 \times \frac{l}{2}\sin\theta = l \sin\theta \quad \text{...(5.24.2)}$$

2. The virtual displacements are obtained by differentiating eq. (5.24.1) and eq. (5.24.2) as,

$$\delta y = -\frac{3l}{2}\sin\theta\, \delta\theta$$

and $$\delta y = l \cos\theta\, \delta\theta$$



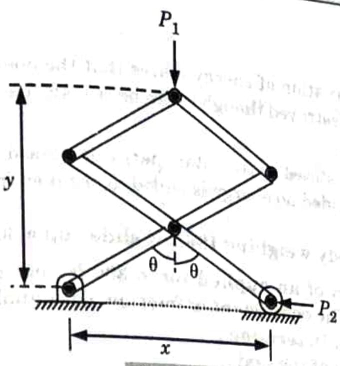



**Fig. 5.24.2.**

3. From Fig. 5.24.2, we see that as $\theta$ increases, $x$ increases while $y$ decreases. Hence, considering only positive values of virtual displacements, the above expressions reduce to

$$\delta y = \frac{3l}{2}\sin\theta\, \delta\theta \text{ and } \delta x = l\cos\theta\, \delta\theta$$

4. Applying the principle of virtual work, we get

$$P_1 \delta y - P_2 \delta x = 0$$

$$P_1\left(\frac{3l}{2}\sin\theta\, \delta\theta\right) - P_2(l\cos\theta\, \delta\theta) = 0$$

$$\frac{3P_1}{2}\sin\theta = P_2\cos\theta$$

$$\theta = \tan^{-1}\left[\frac{2P_2}{3P_1}\right]$$

## PART-7

*Applications of Energy Method for Equilibrium.*

### Questions-Answers

**Long Answer Type and Medium Answer Type Questions**

**Que 5.25.** **State law of conservation of energy.**

**Answer**

1. Law of conservation of energy states that the energy can neither be created nor destroyed though it can be transformed from one form to another form.
2. It can also be stated as the total energy possessed by a body remains constant provided no energy is added to or taken from it.

**Que 5.26.** **A body weighing 196.2 N slides up a 30° inclined plane under the action of an applied force 300 N acting parallel to the inclined plane. The coefficient of friction, μ is equal to 0.2. The body moves from rest. Determine :**

i. **Acceleration of the body.**
ii. **Distance travelled by body in four seconds.**
iii. **Velocity of body after four seconds.**
iv. **Kinetic energy of the body after four seconds.**
v. **Work done on the body in four seconds.**
vi. **Momentum of the body after four seconds.**
vii. **Impulse applied in four seconds.**

**Answer**

**Given :** $W = 196.2$ N, $m = \frac{W}{g} = \frac{196.2}{9.81} = 20$ kg, Applied force = 300 N, $\theta = 30°$, $\mu = 0.2$.

**To Find :**
i. Acceleration of the body.
ii. Distance travelled by body in 4 sec.
iii. Velocity of body after 4 sec.
iv. Kinetic energy of the body after 4 sec.
v. Work done on the body in 4 sec.
vi. Momentum of the body after 4 sec.
vii. Impulse applied in 4 sec.

1. As body moves from rest, hence initial velocity ($u$) will be zero.

$$u = 0$$

2. Fig. 5.26.1 shows the free body diagram. The net force in the direction of motion is given by,

$$F = \text{Applied force} - W \sin\theta - \mu R$$
$$= 300 - 196.2 \times \sin 30° - 0.2 \times W \cos\theta \quad (\because R = W\cos\theta)$$
$$= 300 - 98.1 - 0.2 \times 196.2 \times \cos 30°$$
$$= 300 - 98.1 - 33.98 = 167.92 \text{ N}$$



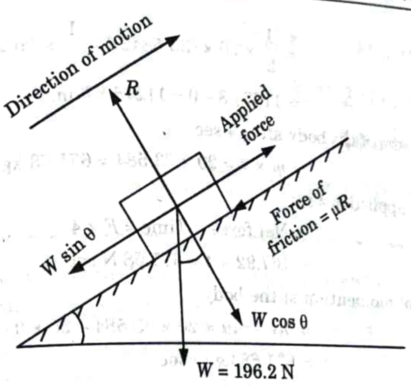



**Fig. 5.26.1.**

3. We know that, $F = m \times a$

$$167.92 = 20 \times a$$
$$a = \frac{167.92}{20} = 8.396 \text{ m/sec}^2.$$

4. Distance travelled in 4 sec,

$$s = ut + \frac{1}{2}at^2$$
$$= 0 \times 4 + \frac{1}{2} \times 8.396 \times 4^2 = 67.168 \text{ m}$$

5. Velocity after 4 sec, $v = u + at$

$$= 0 + 8.396 \times 4 = 33.584 \text{ m/sec}$$

6. The kinetic energy after 4 sec is given by,

$$\text{KE} = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 20 \times (33.584)^2 = 11278.8 \text{ N-m}$$

7. Work done on the body in 4 sec

$$= \text{Net force} \times \text{Distance moved in 4 sec}$$
$$= 167.92 \times 67.168 = 11278.8 \text{ Nm}$$

8. The work done on the body is equal to the change of kinetic energy of the body.

$$\text{Change of KE} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$= \frac{1}{2} \times 20 \times (33.5842)^2 - \frac{1}{2} \times 20 \times 0^2 \quad (\because u = 0)$$

$$= 11278.8 - 0 = 11278.8 \text{ Nm.}$$

9. Momentum of the body after 4 sec

$$= m \times v = 20 \times 33.584 = 671.68 \text{ kg m/sec.}$$

10. Impulse applied in 4 sec

$$= \text{Net force} \times \text{Time} = F \times 4$$

$$= 167.92 \times 4 = 671.68 \text{ N sec}$$

11. Change of momentum of the body

$$= mv - mu = 20 \times 33.584 - 20 \times 0$$

$$= 671.68 \text{ kg m/sec}$$

$$= 671.68 \text{ Nsec}$$

**PART-8**

*Stability of Equilibrium.*

**Questions-Answers**

**Long Answer Type and Medium Answer Type Questions**

**Que 5.27. Write a short note on stability of equilibrium.**

**Answer**

1. Equilibrium is a state of a system which does not change.
2. An equilibrium is considered stable, if the system always returns to its initial stage after small disturbances. If the system moves away from the equilibrium after small disturbances, then the equilibrium is unstable.
3. For example, the equilibrium of a pencil standing on its tip is unstable while the equilibrium of a picture on the wall is (usually) stable.

☺☺☺



**1 UNIT**

# Introduction to Engineering Mechanics (2 Marks Questions)

**1.1. What do you understand by a particle and a rigid body ?**

**Ans. Particle :** A particle is a body of infinitely small volume and the mass of the particle is considered to be concentrated at a point.

**Rigid Body :** A body which does not deform under the action of external forces is known as rigid body.

**1.2. Give the effect of force and moment on a body.**

**Ans.** The force acting on a body causes linear displacement while moment causes an angular displacement.

**1.3. What are the steps in making of a free body diagram ?**

**AKTU 2013-14, (I) Marks 02**

**Ans.** The steps in making a free body diagram are as follows :

i. A sketch of the body is drawn by removing the supporting surfaces.
ii. Indicate on this sketch all the applied or active forces, which tend to set the body in motion, such as those caused by weight of the body or applied forces, etc.
iii. Also indicate on this sketch all the reactive forces, such as those caused by the constraints or supports that tend to prevent motion.
iv. All relevant dimensions and angles, reference axes are shown on the sketch.

**1.4. Define resultant of forces.**

**Ans.** A single force which can replace a number of forces acting on a body and gives same effect is called resultant of forces.

**1.5. The resultant of two forces 3$P$ and 2$P$ is $R$. If the first force is doubled the resultant is also doubled, determine the angle between the two forces.**

**AKTU 2013-14, (II) Marks 02**

**Ans.**

**Given :** $P = 3P$, $Q = 2P$, $P' = 6P$, $R' = 2R$

**To Find :** Angle between the two forces, $\theta$.

1. From parallelogram law of forces,

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

So, $$R = \sqrt{(3P)^2 + (2P)^2 + 2 \times 3P \times 2P \times \cos \theta}$$

$$R = \sqrt{9P^2 + 4P^2 + 12P^2 \cos \theta} \quad ...(1.5.1)$$

2. Now according to changed values,

$$R' = \sqrt{P'^2 + Q^2 + 2P'Q \cos \theta}$$

$$2R = \sqrt{(6P)^2 + (2P)^2 + 2 \times 6P \times 2P \cos \theta}$$

$$2R = \sqrt{36P^2 + 4P^2 + 24P^2 \cos \theta} \quad ...(1.5.2)$$

3. From eq. (1.5.1) and eq. (1.5.2), we have

$$2\sqrt{9P^2 + 4P^2 + 12P^2 \cos \theta} = \sqrt{36P^2 + 4P^2 + 24P^2 \cos \theta}$$

$$4(9P^2 + 4P^2 + 12P^2 \cos \theta) = 36P^2 + 4P^2 + 24P^2 \cos \theta$$

$$12P^2 + 24P^2 \cos \theta = 0$$

$$12P^2(1 + 2 \cos \theta) = 0$$

4. Since, $12P^2 \neq 0$, $1 + 2 \cos \theta = 0$

$$\cos \theta = -1/2$$

$$\theta = 120°$$

**1.6. What is static equilibrium ? Write down sufficient condition of static equilibrium for a coplanar concurrent and non-concurrent force system.**

**AKTU 2015-16, (I) Marks 02**

**OR**

**Explain condition of equilibrium of coplanar-non concurrent forces.** **AKTU 2016-17, (II) Marks 02**

**Ans.** **Static Equilibrium :** A body is said to be in static equilibrium if all the forces acting on the body are balanced whether the body is at rest or in motion.

**Conditions of Static Equilibrium for a Coplanar Concurrent Force System :**

$$\Sigma F_x = 0, \text{ and}$$
$$\Sigma F_y = 0$$

**Conditions of Static Equilibrium for a Coplanar Non-Concurrent Force System :**

$$\Sigma F_x = 0,$$
$$\Sigma F_y = 0, \text{ and}$$
$$\Sigma M = 0$$

**1.7. How do you find the resultant of non-coplanar concurrent force system ?** **AKTU 2014-15, (II) Marks 02**



**Ans.** The resultant of several forces in non coplanar concurrent force system can be found analytically by summing the components of forces along X, Y and Z directions, *i.e.*, resultant R can be obtained by,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

**1.8. "Friction is both desirable and undesirable". Explain**

**AKTU 2014-15, (II) Marks 02**

**Ans.** Friction helps in working of friction brakes and clutches, belt and rope drives, holding and fastening devices while it may also deteriorates the working of power screws, bearing and gears, flow of fluids in pipes. So we can say that friction is both desirable and undesirable.

**1.9. Explain the relationship between angle of friction and angle of repose.** **AKTU 2013-14, (I) Marks 02**

**Ans.** Angle of friction = Angle of repose

**1.10. A block of mass *m* on an inclined plane is kept in equilibrium and prevented from sliding down by applying a force of 500 N. If the angle of the inclination is 30° and coefficient of friction for the contact surface is 0.35, determine the weight of the block.** **AKTU 2013-14, (II) Marks 02**

**Ans.**

**Given :** $F = 500$ N, $\theta = 30°$, $\mu = 0.35$
**To Find :** Weight of block.

1. Fig. 1.10.1 shows the block resting on inclined plane.

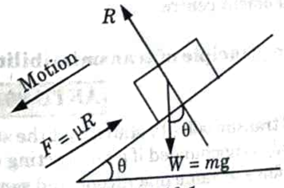



**Fig. 1.10.1.**

2. FBD of block is as shown in Fig. 1.10.2.

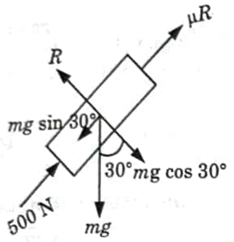



Fig. 1.10.2.

3. Equation of equilibrium along plane,

$$500 + \mu R = mg \sin 30°$$

$$500 + 0.35 \times mg \cos 30° = mg \sin 30° \qquad (\because R = mg \cos 30°)$$

$$500 = mg (\sin 30° - 0.35 \cos 30°)$$

$$500 = mg (0.5 - 0.30) \Rightarrow 500 = 0.2\, mg$$

$$mg = 2500 \text{ N}$$

**1.11. Write any four engineering applications of friction.**

**AKTU 2015-16, (I) Marks 02**

**Ans.** Following are the engineering applications of friction :

i. In producing relative motion between bodies.
ii. In transmitting power.
iii. In braking system to stop the vehicle.
iv. In lifting the heavy blocks, machinery etc., over wedges.

**1.12. State Varignon's theorem of moments.**

**AKTU 2016-17, (I) Marks 02**

**Ans.** Varignon's theorem of moments states that the algebraic sum of the moments of a system of coplanar forces about a moment centre in their plane is equal to the moment of their resultant force about the same moment centre.

**1.13. Define the principle of transmissibility.**

**AKTU 2016-17, (II) Marks 02**

**Ans.** Principle of transmissibility states that the state of rest or of motion of a rigid body is unchanged if a force acting on the body is replaced by another force of same magnitude and same direction but acting anywhere on the body along the line of action of the replaced force.

**1.14. Explain free body diagram with example.**

**AKTU 2016-17, (II) Marks 02**

**Ans.** **Free Body Diagram :** A body may consist of more than one element and supports. Each element or support can be isolated from the



rest of system by properly incorporating the effect of forces. The diagram of the isolated element or a portion of the body along with the net effect of forces is known as free body diagram (FBD).

**Example :**

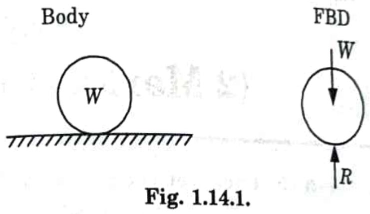



Fig. 1.14.1.

**1.15. Define parallelogram law of forces.**

**AKTU 2016-17, (II) Marks 02**

**Ans.** Parallelogram law of forces states that if two forces acting simultaneously on a body at a point are represented in magnitude and direction by two adjacent sides of a parallelogram, their resultant will be represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of two sides representing the forces.

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# 2 UNIT

# Centroid and Centre of Gravity (2 Marks Questions)

**2.1. What is the difference between centroid and centre of gravity ?** **AKTU 2014-15, (II) Marks 02**

**Ans.** The term centre of gravity applies to bodies with weight while centroid applies to lines, planes, areas and volumes.

**2.2. Define axis of symmetry.**

**Ans.** The line about which the figure can be cut into equal halves is known as axis of symmetry.

**2.3. Determine the centroid of a circular arc having radius 20 mm and central angle 180°.** **AKTU 2013-14, (I) Marks 02**

**Ans.**

**Given :** $2\alpha = 180°$, $\alpha = 90° = \pi/2$ rad, $R = 20$ mm
**To Find :** Centroid of circular arc.

1. Position of the centroid for circular arc is given as,

$$\bar{x} = \frac{R \sin \alpha}{\alpha} = \frac{20 \sin(\pi/2)}{\frac{\pi}{2}}$$

$$= \frac{40}{\pi} = 12.73 \text{ mm}$$

$$\bar{y} = 0 \text{ (due to symmetry)}$$

**2.4. What is the centroid of segment of a circular disc of radius 5 cm and subtended angle of 120° ?** **AKTU 2013-14, (II) Marks 02**

**Ans.**

**Given :** $R = 5$ cm, $2\alpha = 120°$, $\alpha = 60° = \pi/3$ rad
**To Find :** Centroid of segment of a circular disc.



1. Centroid of circular lamina is given as,

$$\bar{x} = \frac{2R}{3\alpha} \sin \alpha$$

$$\bar{x} = \frac{2 \times 5}{3 \times \frac{\pi}{3}} \sin 60° = \frac{10}{\pi} \sin 60° \qquad (\because \alpha = \pi/3)$$

$$\bar{x} = 2.75 \text{ cm}$$

$$\bar{y} = 0 \text{ (due to symmetry)}$$

**2.5. Explain polar moment of inertia.** **AKTU 2013-14, (II) Marks 02**

**Ans.** Moment of inertia about an axis perpendicular to the plane of an area is known as polar moment of inertia.

**2.6. Find the polar moment of inertia of a circular area of diameter 5 mm.** **AKTU 2013-14, (I) Marks 02**

**Ans.**

**Given :** $D = 5$ mm
**To Find :** Polar moment of inertia.

1. Polar moment of inertia of a circular disc is given as,

$$J = \frac{\pi D^4}{32} = \frac{\pi \times 5^4}{32} = 61.36 \text{ mm}^4$$

**2.7. What do you understand by radius of gyration ?** **AKTU 2015-16, (I) Marks 02**

**Ans.** Radius of gyration is the distance which is when squared and multiplied by area gives the moment of inertia of that area.

**2.8. State perpendicular axis theorem.** **AKTU 2015-16, (I) Marks 02**

**OR**

**State and explain perpendicular axis theorem.** **AKTU 2014-15, 2016-17, (II) Marks 02**

**Ans.** Perpendicular axis theorem states that the moment of inertia of an area about an axis perpendicular to its plane (polar moment of inertia) at any point $O$ is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point $O$ and lying in the plane of the area.

Mathematically, $I_{ZZ} = I_{XX} + I_{YY}$

**2.9. State parallel axis theorem.** **AKTU 2016-17, (I) Marks 02**

**Ans.** Parallel axis theorem states that the moment of inertia about any axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance between the two parallel axis.

**2.10. Define mass moment of inertia.**

**Ans.** Mass moment of inertia of a body about an axis is defined as the sum total of product of its elemental masses and square of their distance from the axis.

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# 3 UNIT

# Basic Structural Analysis (2 Marks Questions)

**3.1. Write the different types of support.**

**Ans.** Following are the different types of support :

i. Simple support or knife edge support,
ii. Roller support,
iii. Pin joint or hinged support,
iv. Smooth surface support, and
v. Fixed or built-in support.

**3.2. List the various types of loads to which the beam can be subjected.** **AKTU 2016-17, (I) Marks 02**

**Ans.** Following are the different types of loads to which the beam can be subjected :

i. Concentrated or point load,
ii. Uniformly distributed load (UDL), and
iii. Uniformly varying load (UVL).

**3.3. Differentiate between perfect and imperfect truss.** **AKTU 2015-16, (I) Marks 02**

**Ans.**

| S. No. | Perfect Truss | Imperfect Truss |
|---|---|---|
| 1. | Prefect trusses always retain their shape. | Imperfect trusses cannot retain their shape when loaded and get distorted. |
| 2. | Number of members in perfect truss are equals to $(2j - 3)$, where $j$ is number of joints. | Numbers of members are either more or less than $2j - 3$. |

**3.4. What do you understand by point of contraflexure ?** **AKTU 2015-16, (I) Marks 02**

**Ans.** The point of contraflexure is a point which represents the section on the beam where bending moment is zero or bending moment changes its sign.

**3.5. A truss structure is made up of five members. If the number of joints in the truss is four then state the nature of truss.**

**AKTU 2013-14, (II) Marks 02**

**Ans.**

**Given :** $m = 5, j = 4$
**To Find :** Nature of truss.

1. Nature of truss can be determine by the following formula,

$$m = 2j - 3$$
$$m = 2j - 3$$
$$= 2 \times 4 - 3 = 5$$
$$\text{LHS} = \text{RHS}$$

So, the given truss is a perfect truss.

**3.6. What are the different methods of analysing a frame ?**

**Ans.** A frame is analysed by the following methods :

i. Method of joints,
ii. Method of section, and
iii. Graphical method.

**3.7. What assumptions are made while determining stresses in a truss ?**

**AKTU 2014-15, (II) Marks 02**

**Ans.** Following are the assumptions made while determining stresses in a truss :

i. The frame should be a perfect frame.
ii. The frame carries load at the joints.
iii. All the members are pin-joined.

**3.8. Discuss the conditions under which the method of section is preferred over method of joints in analysis of truss.**

**AKTU 2013-14, (I) Marks 02**

**Ans.** Under the following two conditions the method of section is preferred over the method of joints :

i. In a large truss in which forces in only few members are required.
ii. In the situation where the method of joints fails to start/proceed with analysis.

**3.9. Define zero force members.**

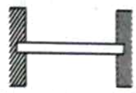
**Ans.** The members of a truss in which net force is zero are known as zero force members.



**3.10. With neat sketches describe in brief different types of beams.**

**AKTU 2014-15, (II) Marks 02**

**Ans.** Following are the different types of beams :

| S.No. | Type of beam | Diagram |
|---|---|---|
| i. | Cantilever beam |  |
| ii. | Simply supported beam |  |
| iii. | Overhanging beam |  |
| iv. | Fixed beam |  |
| v. | Continuous beam |  |

**3.11. Determine the maximum bending moment in a simply supported beam having span of 5 m and carrying a uniformly distributed load of 10 kN/m throughout its span.**

**AKTU 2013-14, (I) Marks 02**

**Ans.**

**Given :** $l = 5$ m, $w = 10 \times 10^3$ kN/m
**To Find :** Maximum bending moment.

1. We know maximum bending moment for simply supported beam carrying uniformly distributed load is given as,

$$(\text{BM})_{max} = \frac{wl^2}{8} = \frac{10 \times 10^3 \times 5^2}{8} = 31250 \text{ N-m}$$

**3.12. Determine the maximum bending moment in a simply supported beam of span 5 m, carrying uniformly distributed load of 2 kN/m over its entire span.**

**AKTU 2013-14, (II) Marks 02**

**Ans.**

**Given :** $w = 2$ kN/m, $l = 5$ m
**To Find :** Maximum bending moment.

1. Maximum bending moment for a simply supported beam carrying uniformly distributed load is given by as,

$$(BM)_{max} = \frac{wl^2}{8} = \frac{2 \times 5^2}{8} = \frac{25}{4} = 6.25 \text{ kN-m}$$

☺☺☺



4 UNIT

# Review of Particle Dynamics (2 Marks Questions)

**4.1. Define rectilinear motion.**

**Ans.** The motion of a body along a straight line is known as rectilinear motion.

**4.2. A mass of 3 kg is dropped from a height from rest. Find the distance travelled in 5 seconds.**

**AKTU 2013-14, (I) Marks 02**

**Ans.**

**Given :** Mass = 3 kg, $t$ = 5 sec,
**To Find :** Distance travelled in 5 sec.

1. We know that, $s = ut + \frac{1}{2}gt^2$

But, $u = 0$ (body is initially at rest)

$\therefore$ $s = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 5^2$ (Using, $g = 10$ m/sec$^2$)

$= 125$ m

**4.3. The equation of motion for motion of a particle is given by $s = 18t + 3t^2 - 2t^3$. Find acceleration and velocity at $t = 2$ sec.**

**AKTU 2014-15, (II) Marks 02**

**Ans.**

**Given :** $s = 18t + 3t^2 - 2t^3$
**To Find :** Acceleration and velocity at $t = 2$ sec.

1. We know that, $v = \frac{ds}{dt} = 18 + 6t - 6t^2$

Velocity at, $t = 2$ sec

$v = 18 + 6 \times 2 - 6 \times 4$

$= 6$ m/sec

2. Also, acceleration, $a = \frac{d^2s}{dt^2} = 6 - 12t$

Acceleration at, $t = 2$ sec

$$a = 6 - 12 \times 2 = -18 \text{ m/sec}^2$$

**4.4. What do you understand by plane curvilinear motion ?**

**Ans.** The motion of a body in a plane along a circular path is known as plane curvilinear motion.

**4.5. Define relative motion.**

**Ans.** The motion of a moving body with respect to another moving body is known as the relative motion of the first body with respect to second body.

**4.6. Define work.**

**Ans.** Work is defined as the product of force and displacement. Its unit is joule (J).

**4.7. What do you mean by energy ?**

**Ans.** The capacity of doing work is known as energy. It is the product of power and time.

**4.8. Define kinetic energy and potential energy.**

**Ans.** **Kinetic Energy :** The energy possessed by a body by virtue of its motion is known as kinetic energy. It is given by,

$$KE = \frac{1}{2} mv^2$$

**Potential Energy :** The energy by virtue of position of a body with respect to any given reference or datum is known as potential energy. It is given by,

$$PE = mgh$$

**4.9. Define impulse and momentum.**

**Ans.** **Impulse :** The product of force and time is known as impulse.
**Momentum :** The product of mass and velocity of a body is known as momentum.

**4.10. What do you understand by angular momentum ?**

**Ans.** The product of mass moment of inertia and angular velocity of rotating body is known as angular momentum.

**4.11. State the law of conservation of energy.**

**Ans.** Law of conservation of energy states that the energy can neither be created nor destroyed, though it can be converted from one form into another form.

☺☺☺



# 5 UNIT

# Introduction to Kinetics of Rigid Bodies (2 Marks Questions)

**5.1. State Newton's second law of motion.**

**Ans.** Newton's second law of motion states that the rate of change of momentum of a body is proportional to the external force applied on the body and takes place in the direction of the force.

**5.2. Define instantaneous centre of rotation.**

**Ans.** The point about which motion of a body having both translational and rotational motion is assumed to be pure rotational is known as instantaneous centre of rotation.

**5.3. State and explain D'Alembert's principle.**

**AKTU 2014-15, (II) Marks 02**

OR

**State D-Alembert's principle.** **AKTU 2015-16, (I) Marks 02**

**Ans.** D'Alembert's principle states that the net external force acting on the system and the resultant inertia force are in equilibrium.

**5.4. What do you understand by work-energy principle ?**

**AKTU 2015-16, (I) Marks 02**

**Ans.** Work-energy principle states that the change in kinetic energy of a body during any displacement is equal to the work done by the body.

**5.5. Write D'Alembert's principle for rotary motion.**

**Ans.** According to D'Alembert's principle, when external torques acts on a system having rotating motion, then the algebraic sum of all the torques acting on the system due to external forces and reversed active forces including the inertia torque is zero.

**5.6. Give the expression for the kinetic energy of rotating bodies.**

**Ans.** $KE = \frac{1}{2} \omega^2 I$

Where, $\omega$ = Angular velocity, and
$I$ = Moment of inertia.

**5.7. Define virtual displacement.**

**Ans.** The displacement of a partially constrained body which is occurring only in imagination but not in reality is known as virtual displacement.

**5.8. Write principle of virtual work for a particle and for a rigid body.**

**Ans.** **For Particle :**

$$\delta U = \Sigma F \delta r = 0$$

**For Rigid Body :**

$$\delta U = \Sigma F \delta r + \Sigma M \delta \theta = 0$$

**5.9. Write the different types of motion.**

**AKTU 2015-16, (I) Marks 02**

**Ans.** Following are the different types of motion :
i. Translation,
ii. Rotation, and
iii. General plane motion (combined motion of translation and rotation).

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# B.Tech.
# (SEM. III) ODD SEMESTER THEORY EXAMINATION, 2019-20
# ENGINEERING MECHANICS

**Time : 3 Hours** **Max. Marks : 100**

**Note :** 1. Attempt **all** section. If require any missing data; then choose suitably.

## Section-A

1. Attempt **all** questions in brief. **(2 × 10 = 20)**

**a. Define shear force and bending moment.**

**Ans.** **Shear Force :** It is the force that tries to shear off the section of a beam. It is obtained as algebraic sum of all forces acting normal to axis of beam, either to the left or to the right of section.

**Bending Moment :** It is the moment that tries to bend the beam and it is obtained as algebraic sum of moment of all forces about the section, acting either to left or to the right of section.

**b. How does a rigid body differ from an elastic body ?**

**Ans.**

| S.No. | Elastic Body | Rigid Body |
|---|---|---|
| 1. | On applying load it undergoes deformation. | On applying load it does not deform. |
| 2. | On removal of load, comes back to its original size and shape. | On removal of load shape and size remains unchanged. |
| 3. | Deformation is temporary. | No deformation. |

**c. Define center of mass and write down the co-ordinates of center of gravity of trapezoid.**

**Ans.** **Center of Mass :** Refer Concept Outline-1, Page 2-2C, Unit-2.

**Co-ordinates of Centers of Gravity :** From Fig. 1.

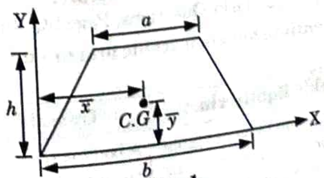



**Fig. 1.**

$$\bar{x} = b/2, \bar{y} = \frac{2a+b}{a+b} \times \frac{h}{3}$$

**d. Define work and power. Write the mathematical relation and SI unit.**

**Ans.** **Work :** Refer Q. 4.6, 2 Marks Questions, Page SQ–14C, Unit-4.
**Power and Relation :** Refer Q. 4.23, Page 4–27C, Unit-4.

**e. State and prove law of conservation of momentum.**

**Ans.** **State :** Refer Concept Outline Part-4, Page 4–29C, Unit-4.
**Proof :** Refer Q. 4.27, Page 4–30C, Unit-4.

**f. Enlist different types of supports and loading system.**

**Ans.** **Types of Support :** Refer Q. 3.1, 2 Marks Questions, Page SQ–9C, Unit-3.
**Types of Load :** Refer Q. 3.2, 2 Marks Questions, Page SQ–9C, Unit-3.

**g. Explain with the help of neat diagram, the concept of limiting friction.**

**Ans.** The force of friction has a remarkable property of adjusting its magnitude, so as to become exactly equal and opposite to the applied force, which tends to produce motion. There is, however, a limit beyond which the force of friction cannot increase. If the applied force exceeds this limit, the force of friction cannot balance it and the body begins to move in the direction of the applied force. This maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as limiting friction.

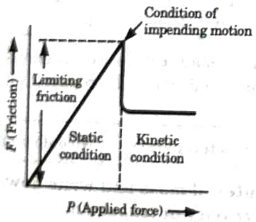



**Fig. 2.**

**h. Write down D'Alembert's Principle.**

**Ans.** Refer Q. 5.3, 2 Marks Questions, Page SQ–15C, Unit-5.

**i. Differentiate between stable and unstable equilibrium.**

**Ans.**

| S. No. | Stable Equilibrium | Unstable Equilibrium |
|---|---|---|
| 1. | A body is said to be in stable equilibrium, if it returns back to its original position, after it is slightly displaced from its position of rest. | A body is said to be in an unstable equilibrium, if it does not return back to its original position and heels farther away, after slightly displaced from its position of rest. |
| 2. | This happens when some additional force sets up due to displacement and brings the body back to its original position. | This happens when the additional force moves the body away from its position of rest. |
| 3. | A smooth cylinder, lying in a concave surface, is in stable equilibrium. | A smooth cylinder lying on a convex surface is in unstable equilibrium. |



**j. State parallel axis theorem. Define radius of gyration.**

**Ans.** **Parallel Axis Theorem :** Refer Q. 2.9, 2 Marks Questions, Page SQ–8C, Unit-2.
**Radius of Gyration :** Refer Q. 2.7, 2 Marks Questions, Page SQ–7C, Unit-2.

## Section-B

2. Attempt any **three** of the following : **(10 × 3 = 30)**

**a. State and prove Lami's theorem.**
**The greatest and least resultant of two forces acting on body are 35 kN and 5 kN respectively. Determine the magnitude of the forces. What would be the angle between these forces if the magnitude of the resultant is stated to be 25 kN ?**

**Ans.** **State and Prove Lami's Theorem :** Refer Q. 1.21, Page 1–22C, Unit-1.

**Numerical :**

**Given :** Maximum resultant = 35 kN, Minimum resultant = 5 kN
**To Find :** Angle between two forces, magnitude of forces.

1. Let $P$ and Q be the two forces and $\theta$ be the angle of indication between them. According to the parallelogram law of forces, the resultant $R$ is
$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$
2. The resultant will be maximum when the forces are collinear and in the same direction, *i.e.*, $\theta = 0°$. The gives
$$R^2 = \sqrt{P^2 + Q^2 + 2PQ\cos 0°} = \sqrt{P^2 + Q^2 + 2PQ} = P + Q \quad ...(1)$$
$$\therefore \quad 35 = P + Q$$
3. The resultant will be minimum when the forces are collinear and act in the opposite direction, *i.e.*, $\theta = 180°$. That gives
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180°} = \sqrt{P^2 + Q^2 - 2PQ} = P - Q \quad ...(2)$$
$$\therefore \quad 5 = P - Q$$
4. From the eq. (1) and (2), we get
$$P = 20 \text{ kN and } Q = 15 \text{ kN}$$
5. Let $\theta$ be the angle between the force $P = 20$ kN and $Q = 15$ kN when their resultant is 25 kN, then
$$25^2 = 20^2 + 15^2 + 2 \times 20 \times 15\cos\theta$$

$$625 = 400 + 225 + 600 \cos \theta$$
$$\cos \theta = 0;\ \theta = 90°$$

Thus the given system of forces is at right angles to each other when the resultant is 25 kN.

**b. Calculate the centroid of a semi-circular ring of radius 'r', using method of moments.**

**Ans.**

1. Consider a semicircular arc of radius $r$ as shown in Fig. 3.

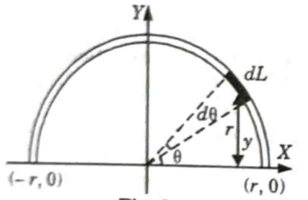



**Fig. 3.**

2. Let us take an elemental strip of thickness $dL$ at a distance '$y$' from the $X$-axis.
3. Solving the problem using polar co-ordinates.

   Integral to be evaluated is $y_c = \dfrac{\int y dL}{\int dL}$
4. From Fig. 3. $y = r \sin \theta$ and $dL = r\, d\theta$

$$y_c = \frac{\int_0^{\pi} (r \sin \theta) r d\theta}{\int_0^{\pi} r d\theta} = \frac{r^2[-\cos \theta]_0^{\pi}}{r[\theta]_0^{\pi}}$$

$$= \frac{-r^2[\cos \pi - \cos 0]}{r\pi} = \frac{-r^2[-1-1]}{r\pi} = \frac{2r}{\pi}$$

5. Thus the centroid of a semicircle of radius $R$ is at a distance $2r/\pi$ from the base.

**c. Find moment of inertia of the Fig. 4 about X-X axis, thickness of member is 20 mm.**

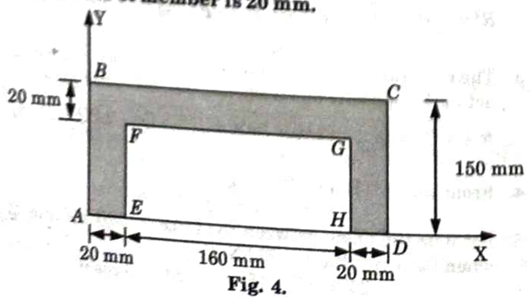



**Fig. 4.**



**Ans.**

**Given :** Thickness of member = 20 mm
**To Find :** Moment of inertia about XX axis.

1. We know that, moment of inertia of rectangular section about its base = $bd^3/3$
2. Moment of inertia of hatch portion = Moment of inertia of rectangle $ABCD$ – Moment of inertia of rectangle $EFGH$ about its base.

$$= \frac{200 \times 150^3}{3} - \frac{160 \times 130^3}{3} = 107.83 \times 10^6 \text{ mm}^4$$

**d. Differentiate between rectilinear and curvilinear motion. Also derive the expression for the horizontal range, Time of flight and maximum height of a projectile with initial velocity $u$ and inclined at an angle "α" with the horizontal.**

**Ans. Difference :**

| S.No. | Rectilinear Motion | Curvilinear Motion |
|---|---|---|
| 1. | The motion of the body along a straight line is called rectilinear motion. | The motion of the body along a curved path is called curvilinear motion. |
| 2. | It is also known as one dimensional motion. | It is also known as multi dimensional motion. |
| 3. | Equations of motion for rectilinear motion are given by,<br>$v = u + at$<br>$s = ut + 1/2\ at^2$<br>$v^2 = \mu^2 + 2as$ | Equations of motion for curvilinear motion are given by,<br>$w = w_0 + \alpha t$<br>$\theta = w_0 t + 1/2\ \alpha t^2$<br>$w^2 = w_0^{\ 2} + 2\alpha\theta$ |
| 4. | **Example :** A ball thrown vertically upward, a car travelling on a straight road. | **Example :** A golf ball hit from the ground, a motion travelling on a curved road. |

**Expression :** Refer Q. 4.16, Page 4–17C, Unit-4.

**e. State Work Energy principle.**
**A uniform cylinder of 125 mm radius has a mass of 0.15 kg. This cylinder rolls without slipping along a horizontal surface with a translation velocity of 20 cm/sec. Determine its total kinetic energy.**

**Ans. Principle :** Refer Q. 5.12, Page 5–17C, Unit-5.

**Numerical :**

**Given :** Translation velocity, v = 20 cm/sec = 0.20 m/sec, Mass of cylinder, $m$ = 0.15 kg, Radius of cylinder, $R$ = 0.125 m.
**To Find :** Kinetic energy.

1. Total KE of rotating body is given by, $= \frac{1}{2} I\omega^2 + \frac{1}{2} m v^2$ ...(1)
2. Moment of inertia of solid cylinder,

$$I = \frac{MR^2}{2} = 0.15 \times \frac{0.125^2}{2} = 1.172 \times 10^{-3}$$

3. Angular velocity, $\omega = \frac{v}{R} = \frac{0.20}{0.125} = 1.6$ rad/ sec
4. Substituting these values in eq. (1), we get

Total KE $= \frac{1}{2} \times 1.172 \times 10^{-3} \times 1.6^2 + \frac{1}{2} \times 0.15 \times 0.2^2 = 4.5 \times 10^{-3}$ Joule

## Section-C

**3.** Attempt any **one** part of the following : **(10 × 1 = 10)**

**a. Explain how a wedge is used for raising heavy loads. Also mention the principle.**
**A body resting on a rough horizontal plane required a pull of 24 N inclined at 30° to the plane just to move it. It was also found that a push of 30 N at 30° to the plane was just enough to cause motion to impend. Make calculations for the weight of body and the coefficient of friction.**

**Ans. Wedge :**

1. A wedge is, usually, of a triangular or trapezoidal in cross-section. It is, generally, used for slight adjustments in the position of a body *i.e.*, for tightening fits or keys for shafts. Sometimes, a wedge is also used for lifting heavy weights as shown in Fig. 5.

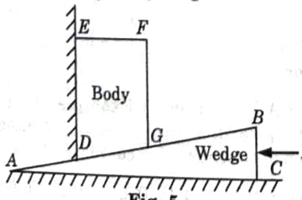


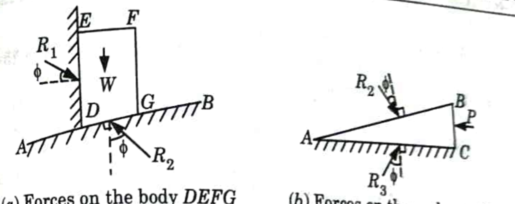

**Fig. 5.**

2. It will be interesting to know that the problems on wedges are basically the problems of equilibrium on inclined planes.
3. Thus these problems may be solved either by the equilibrium method or by applying Lami's theorem.
4. Now consider a wedge *ABC*, which is used to lift the body *DEFG*

Let, $W$ = Weight fo the body *DEFG*.
$P$ = Force required to lift the body.
$\mu$ = Coefficient of friction on the planes *AB*, *AC* and *DE* such that, $\tan \phi = \mu$.

5. A little consideration will show that when the force is sufficient to lift the body, the sliding will take place along three planes *AB*, *AC* and *DE* will also occur as shown in Fig. 6(*a*) and (*b*).






(*a*) Forces on the body *DEFG* (*b*) Forces on the wedge *ABC*

**Fig. 6.**

6. The three reactions and the horizontal force ($P$) may now be found out either by graphical method or analytical method as discussed below :

**Analytical Method :**

i. First of all, consider the equilibrium of the body *DEFG* and resolve the forces $W$, $R_1$ and $R_2$ horizontally as well as vertically.
ii. Now consider the equilibrium of the wedge *ABC*, and resolve the forces $P$, $R_2$ and $R_3$ horizontally as well as vertically.

**B. Numerical :**

**Given :** Pull = 24 N, Push = 30 N, Angle inclination with horizontal plane ($\alpha$) = 30°
**To Find :** Weight of body and the coefficient of friction.

1. Let, $W$ = Weight of the body.
$R$ = Normal reaction.
$\mu$ = Coefficient of friction.
2. Firstly we consider a pull of 24 N acting on the body. We know that in this case, the force of friction ($F_1$) will act towards left as shown in Fig. 7(*a*).
3. Resolving the forces horizontally,
$$F_1 = 24 \cos 30° = 24 \times 0.866 = 20.785 \text{ N}$$
4. Resolving the forces vertically,
$$R_1 = W - 24 \sin 30° = W - 24 \times 0.5 = W - 12$$
5. We know that the force of friction ($F_1$), ...(1)
$$20.785 = \mu R_1 = \mu(W - 24)$$

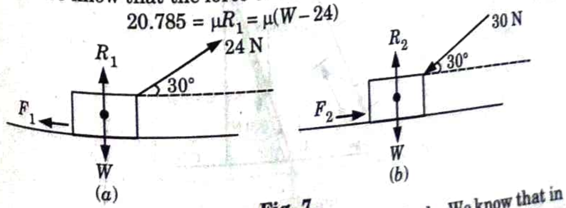



**Fig. 7.**

6. Now consider a push of 30 N acting on the body. We know that in this case, the force of friction ($F_2$) will act towards right as shown in Fig. 7(*b*).
7. Resolving the forces horizontally,
$$F_2 = 30 \cos 30° = 30 \times 0.866 = 25.98 \text{ N}$$

8. Resolving the forces horizontally,
$$R_2 = W + 30 \sin 30° = W + 30 \times 0.5 = W + 15$$
9. We know that the force of friction ($F_2$),
$$25.98 = \mu R_2 = \mu(W + 15) \quad ...(2)$$
10. Dividing eq. (1) and eq. (2), we get
$$\frac{20.785}{25.98} = \frac{\mu(W - 24)}{\mu(W + 15)} = \frac{W - 24}{W + 15}$$
$$20.785\ W + 311.775 = 25.98\ W - 623.52$$
$$5.195\ W = 935.295$$
Weight of body, $W = 935.295/5.195 = 180.04$ N
11. Now substituting the value of $W$ in eq. (1), we get
$$20.785 = \mu(180.04 - 24) = 156.04\ \mu$$
Coefficient of friction, $\mu = 20.785/156.04 = 0.133$ N

**b. A ladder 5m long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750N stands 1.5m from the bottom of the ladder. Calculate coefficient of friction between the ladder and the floor.**

**Ans.**

**Given :** Length of the ladder ($l$) = 5 m; Angle which the ladder makes with the horizontal ($\alpha$) = 70°; Weight of the ladder ($w_1$) = 900 N; Weight of man ($w_2$) = 750 N and distance between the man and bottom of ladder = 1.5 m.
**To Find :** Calculate coefficient of friction between ladder and the floor.

1. Forces acting on the ladder are shown in Fig. 8,

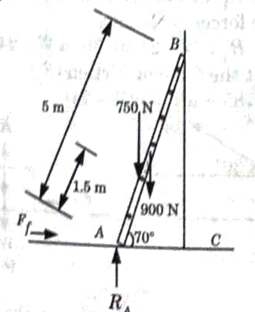



**Fig. 8.**

2. Let,
$\mu_f$ = Coefficient of friction between ladder and floor.
$R_A$ = Normal reaction at point $A$.
3. Resolving the forces vertically, $R_A = 900 + 750 = 1650$ N ...(1)
4. Force of friction at $A$, $F_f = \mu_f \times R_A = \mu_f \times 1650$ ...(2)



5. Now taking moments about $B$, and equating the same,
$$R_A \times 5 \sin 20° = (F_f \times 5 \cos 20°) + (900 \times 2.5 \sin 20°) + (750 \times 3.5 \sin 20°)$$
$$= (F_f \times 5 \cos 20°) + (4875 \sin 20°)$$
6. Now substituting the values of $R_A$ and $F_f$ from eq. (1) and (2), we get
$$1650 \times 5 \sin 20° = (\mu_f \times 1650 \times 5 \cos 20°) + (4875 \sin 20°)$$
7. Dividing both sides by 5 sin 20°,
$$1650 = (\mu_f \times 1650 \times \cot 20°) + 975$$
$$= (\mu_f \times 1650 \times 2.7475) + 975 = 4533.375\ \mu_f + 975$$
$$\therefore \quad \mu_f = 0.15$$

4. Attempt any **one** part of the following : **(10 × 1 = 10)**

**a. Draw the SF and BM diagram for the simply supported beam loaded as shown in Fig. 9.**

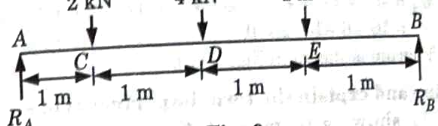



**Fig. 9.**

**Ans.**

**Given :** Load on beam shown in Fig. 10.
**To Find :** Draw SFD and BMD.

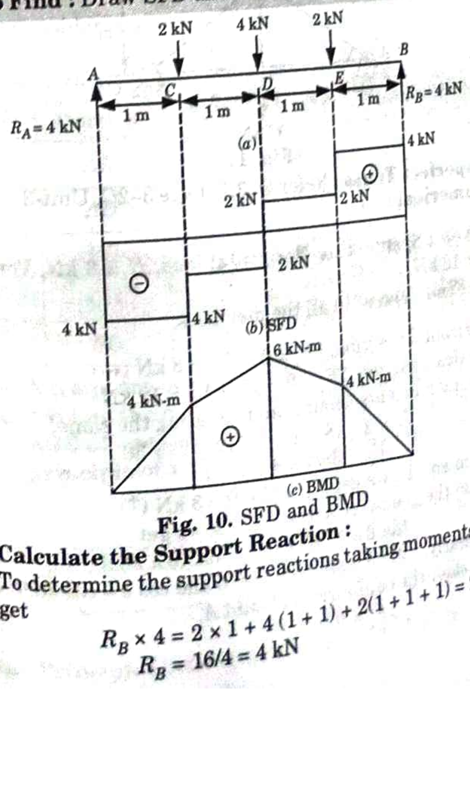



**Fig. 10.** SFD and BMD

**1. Calculate the Support Reaction :**
i. To determine the support reactions taking moments about $A$, we get
$$R_B \times 4 = 2 \times 1 + 4\,(1 + 1) + 2(1 + 1 + 1) = 2 + 8 + 6 = 16$$
$$R_B = 16/4 = 4 \text{ kN}$$

ii. $\Sigma F_y = 0 \Rightarrow R_A + R_B = 2 + 4 + 2 = 8$ kN
$\therefore$ $R_A = 8 - R_B = 8 - 4 = 4$ kN

2. **Shear Force Calculations :**

$S_{B-E} = +4$ kN
$S_{E-D} = 4 - 2 = 2$ kN
$S_{D-C} = 2 - 4 = -2$ kN
$S_{C-A} = -2 - 2 = -4$ kN

SF at point $A$, $S_A = -4 + 4 = 0$ kN
SF diagram is shown in Fig. 8(*b*).

3. **Calculate the Bending Moment :**

$M_B = 0$
$M_E = 4 \times 1 = 4$ kN-m
$M_D = 4(1+1) - 2 \times 1 = 8 - 2 = 6$ kN-m
$M_C = 4(1+1+1) - 2(1+1) - 4 \times 1 = 12 - 4 - 4 = 4$ kN-m
$M_A = 4(1+1+1+1) - 2(1+1+1) - 4(1+1) - 2 \times 1$
$= 16 - 6 - 8 - 2 = 0$

BM diagram is shown in Fig. 10(*c*).

**b. Define and explain the term imperfect truss. Fig. 11 shows a framed of 4 m span and 1.5 m height subjected to two point loads at *B* and *D*. Find the forces in all the members of the structure.**

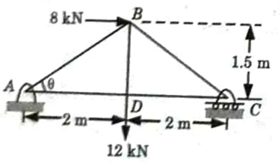


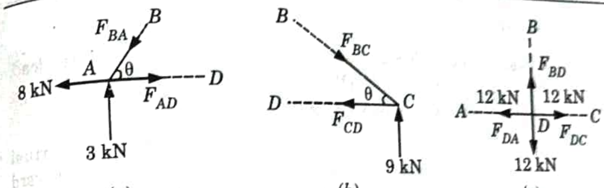

**Fig. 11.**

**Ans. Imperfect Truss :** Refer Q. 3.1, Page 3–2C, Unit-3.
**Numerical :**

**Given :** Span = 4 m, Horizontal load, $H$ = 8 kN, Vertical load, $V$ = 12 kN.
**To Find :** Forces in all the members.

1. Horizontal reaction, $\Sigma F_x = 0$, $H_A = 8$ kN ($\leftarrow$)
2. Vertical reaction, $\Sigma F_y = 0$, $V_A + V_C = 12$ kN ...(1)
3. Taking moments about $A$ and equating the same,
$V_C \times 4 = (8 \times 1.5) + (12 \times 2) = 36$
$V_C = 36/4 = 9$ kN ($\uparrow$)
4. From eq. (1), we get $V_A = 12 - 9 = 3$ kN ($\uparrow$)
5. From the geometry of the Fig. 12, we get
$\tan\theta = 1.5/2 = 0.75$ or $\theta = 36.9°$
Similarly, $\sin\theta = \sin 36.9° = 0.6$ and $\cos\theta = \cos 36.9° = 0.8$
6. Consider the equilibrium at joint $A$,
i. $\Sigma F_x = 0$






(*a*) (*b*) (*c*)

**Fig. 12.**

$F_{AD} = 8 \text{ kN} + F_{BA}\cos\theta = 8 + \cos 36.9°\, F_{BA}$ ...(2)

ii. $\Sigma F_y = 0$
$F_{BA}\sin\theta = 3$ kN
$F_{BA} = 3/\sin 36.9° = 5$ kN

iii. From eq. (2) we get
$F_{AD} = 8 + 5 \times 0.8 = 12$ kN

7. Consider the equilibrium at joint $C$,
i. $\Sigma F_x = 0$
$F_{CD} = F_{BC}\cos\theta = F_{BC}\cos 36.9°$ ...(3)
ii. $\Sigma F_y = 0$
$F_{BC}\sin\theta = 9$ kN
$F_{BC} = 9/\sin 36.9° = 15$ kN
iii. From eq. (3) we get
$F_{CD} = 15 \times 0.8 = 12$ kN
8. Considering the equilibrium of joint $D$, $\Sigma F_y = 0$
$F_{DB} = 12$ kN
9. Now tabulate the results as given below :

| S. No. | Member | Magnitude of force in kN | Nature of Force |
|---|---|---|---|
| 1. | *AB* | 5.0 | Compression |
| 2. | *AD* | 12.0 | Tension |
| 3. | *BC* | 15.0 | Compression |
| 4. | *CD* | 12.0 | Tension |
| 5. | *BD* | 12.0 | Tension |

5. Attempt any **one** part of the following : **(10 × 1 = 10)**

**a. Explain the principle of virtual work. An overhanging beam ABC of span 3 m is loaded as shown in Fig. 13. Using the principle of virtual work, find the reactions at *A* and *B*.**

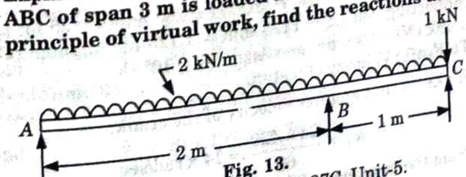



**Fig. 13.**

**Ans. Principle :** Refer Q. 5.22, Page 5–27C, Unit-5.

**Numerical :**

**Given :** Span, $AB = 2$ m and span, $BC = 1$ m, Concentrated load, $W = 1$ kN, Intensity of UDL, $w = 2$ kN/m
**To Find :** Reaction at $A$ and $B$.

1. From the geometry of the Fig. 14, we find that when the virtual upward displacement of the beam at $B$ is $y$, then the virtual upward displacement of the beam at $C$ is $1.5\,y$ as shown in Fig. 14.

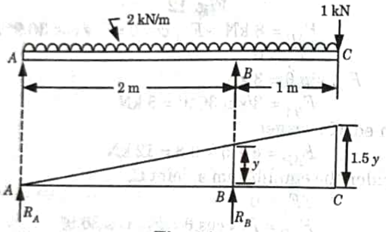



**Fig. 14.**

2. Total virtual work done by the two reactions $R_A$ and $R_B$

$$= +[(R_A \times 0) + (R_B \times y)] = +R_B \times y$$

(Plus sign due to reactions acting upwards)

3. Total virtual work done by the point load at $C$ and uniformly distributed load between $A$ and $C$

$$= -\left[(1 \times 1.5\,y) + 2\left(\frac{0 + 1.5y}{2} \times 3\right)\right] = -(1.5y + 4.5y) = -6y$$

(Minus sign due to loads acting downwards)

4. We know that from the principle of virtual work, the algebraic sum of the total virtual works done is zero, therefore

$$R_B \times y - 6y = 0,\; R_B = 6 \text{ kN}$$

5. $\Sigma F_y = 0,\quad R_A + R_B = 2 \times 3 + 1 = 7$ kN, $\quad R_A = 7 - 6 = 1$ kN

**b. In a reciprocating pump, the lengths of connecting rod and crank is 1125 mm and 250 mm respectively. The crank is rotating at 420 rpm. Find the velocity with which the piston will move, when the crank has turned through an angle of 40° from the inner dead centre.**

**Ans.**

**Given :** Radius of the crank ($r$) = 250 mm = 0.25 m; Length of connecting rod ($l$) = 1125 mm = 1.125 m; Angular rotation of crank ($N$) = 420 rpm and Angle ($\theta$) = 40°
**To Find :** Velocity of piston.

1. We know that angular velocity of the crank,

$$\omega_1 = \frac{2\pi \times 420}{60} = 14\,\pi \text{ rad/sec}$$

2. From the geometry of the Fig. 15, we get



$$\sin\phi = \frac{BM}{BC} = \frac{AB \sin 40°}{BC} = \frac{0.25 \times 0.643}{1.125} = 0.143 \text{ or } \phi = 8.125°$$

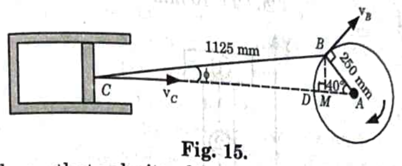



**Fig. 15.**

3. We know that velocity of the piston, $v_c = \omega_1(l \sin\phi + r\cos\theta \tan\phi)$

$$= 14\pi[1.125 \sin(8.22°) + 0.25 \cos 40° \tan(8.22°)] = 8.286 \text{ m/sec}$$

**6.** Attempt any **one** part of the following : **(10 × 1 = 10)**

**a. Derive an equation for moment of inertia of triangle centroidal axis and about its base.**

**Ans.** Refer Q. 2.18, Page 2–24C, Unit-2.

**b. An I-section is made up of three rectangles as shown in Fig. 16. Find the moment of inertia of the section about the horizontal axis passing through the centre of gravity of the section.**

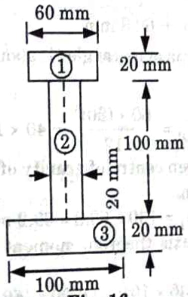



**Fig. 16.**

**Ans.**

**Given :** *I*-section is shown in Fig. 16.
**To Find :** Moment of inertia about its centroidal axis.

1. As the section is symmetrical about $Y$-$Y$ axis, therefore its centre of gravity will lie on this axis. Let bottom face of the bottom flange be the axis of reference.
2. **Rectangle 1 :**
   Area, $a_1 = 60 \times 20 = 1200$ mm
   and $\bar{y}_1 = 20 + 100 + 20/2 = 130$ mm
3. **Rectangle 2 :**
   Area, $a_2 = 100 \times 20 = 2000$ mm²
   and $\bar{y}_2 = 20 + 100/2 = 70$ mm

4. **Rectangle 3 :**

Area, $a_3 = 100 \times 20 = 2000 \text{ mm}^2$

and $\bar{y}_3 = 20 / 2 = 10 \text{ mm}$

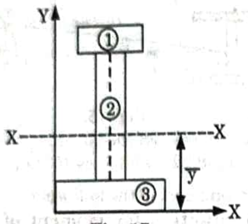



**Fig. 17.**

5. Centre of gravity of the section from bottom face,

$$\bar{y} = \frac{a_1 \bar{y}_1 + a_2 \bar{y}_2 + a_3 \bar{y}_3}{a_1 + a_2 + a_3}$$

$$= \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000} \text{ mm}$$

$$\bar{y} = 60.8 \text{ mm}$$

5. Moment of inertia of rectangle (1) about an axis through its centre of gravity,

$$I_{G1} = \frac{60 \times (20)^3}{12} = 40 \times 10^3 \text{ mm}^4$$

Distance between centre of gravity of rectangle (1) and *X-X* axis of whole section,

$$h_1 = 130 - 60.8 = 69.2 \text{ mm}$$

6. From parallel axis theorem moment of inertia of rectangle (1) about *X-X* axis,

$= I_{G1} + a_1 h_1^2 = (40 \times 10^3) + [1200 \times (69.2)^2] = 5786.37 \times 10^3 \text{ mm}^4$

7. Similarly, moment of inertia of rectangle (2) about an axis through its centre of gravity,

$$I_{G2} = \frac{20 \times (100)^3}{12} = 1666.67 \times 10^3 \text{ mm}^4$$

Distance between centre of gravity of rectangle (2) and *X-X* axis

$$h_2 = 70 - 60.8 = 9.2 \text{ mm}$$

8. Moment of inertia of rectangle (2) about *X-X* axis,

$= I_{G2} + a_2 h_2^2 = (1666.67 \times 10^3) + [2000 \times (9.2)^2] = 1836.95 \times 10^3 \text{ mm}^4$

9. Moment of inertia of rectangle (3) about an axis through its centre of gravity,

$$I_{G3} = \frac{100 \times (20)^3}{12} = 66.67 \times 10^3 \text{ mm}^4$$

Distance between centre of gravity of rectangle (3) and *X-X* axis,



$$h_3 = 60.8 - 10 = 50.8 \text{ mm}$$

Moment of inertia of rectangle (3) about *X-X* axis,

$= I_{G3} + a_3 h_3^2 = (66.67 \times 10^3) + [2000 \times (50.8)^2] = 5227.95 \times 10^3 \text{ mm}^4$

Now moment of inertia of the whole section about *X-X* axis,

$$I_{XX} = 5786.37 \times 10^3 + 1836.95 \times 10^3 + 5227.95 \times 10^3$$

$$= 12851.27 \times 10^3 \text{ mm}^3$$

7. Attempt any **one** part of the following : **(10 × 1 = 10)**

**a. A body of mass 20 kg moving towards with a velocity of 16 m/sec strikes with another body of 40 kg mass moving towards left with 50 m/sec. Determine**

**i. Final velocity of the two bodies.**

**ii. Loss in kinetic energy due to impact.**

**iii. Impulse acting on either body during impact.**

**Take coefficient of restitution as 0.65**

**Ans.**

**Given :** $m_1 = 20$ kg, $u_1 = 16$ m/sec, $m_2 = 40$ kg, $u_2 = 50$ m/sec, $e = 0.65$

**To Find :** i. Find velocities of both bodies.
ii. Loss in kinetic energy.
iii. Impulse acting on either body during impact.

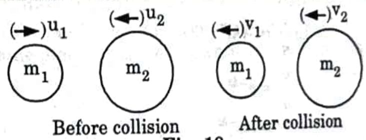



**Fig. 18.**

1. Applying movement conservation of system

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$20 \times 16 + 40 \times (-50) = 20(-v_1) + 40(-v_2)$$

$$v_1 + 2v_2 = 84 \quad ...(1)$$

2. We know that, $-e = \dfrac{v_1 - v_2}{u_1 - u_2}$

$$-0.65 = \frac{-v_1 - (-v_2)}{16 - (-50)}$$

$$-v_1 + v_2 = -42.9 \quad ...(2)$$

$$v_1 - v_2 = 42.9$$

3. Solving the eq. (1) and eq. (2), we get

$$v_1 = 56.6 \text{ m/sec, and } v_2 = 13.7 \text{ m/sec}$$

4. Loss in kinetic energy = Kinetic energy before collision – Kinetic energy after collision

$$= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2\right)$$

$$= \frac{1}{2} \times 20 \times (16)^2 + \frac{1}{2} \times 40(-50)^2 - \frac{1}{2} \times 20 \times (-56.6)^2 - \frac{1}{2} \times 40 \times (-13.7)^2$$

$= 2560 + 50000 - 32035.6 - 3753.8 = 16770.6$ J

5. Impulse of first body,
$I_1 = m_1 \Delta v_1 = m_1(v_1 - u_1) = 20(-56.6 - 16) = -1452$ kg-m/sec
6. Impulse of second body,
$I_2 = m_2 \Delta v_2 = m_2(v_2 - u_2) = 40[-13.7 - (-50)] = -1452$ kg-m/sec

**b. A particle start with velocity $u$ and the acceleration-velocity relationship is prescribed as $a = -kv$ where $k$ is a constant. Set up an expression that prescribes the displacement time relation for the particle.**

**Ans.**

**Given :** Acceleration, $a = -kv$, Initial velocity $= u$.
**To Find :** Expression for displacement and time.

1. Acceleration of particle is given by, $a = \dfrac{dv}{dt} = -kv$ ...(1)
2. Upon rearranging, we get
$$\frac{dv}{v} = -kdt$$
3. Integrating both side, $\ln v = -kt + C_1$ ...(2)
Since $v = u$ at $t = 0$, we have $C_1 = \ln u$. Therefore, the eq. (2) can be written as
$$\ln\frac{v}{u} = -kt$$
or
$$\frac{v}{u} = e^{-kt}$$
$$\therefore \quad v = ue^{-kt} \quad ...(3)$$
4. Further, we can write velocity as :
$$v = \frac{dx}{dt} = ue^{-kt}$$
On rearranging, $dx = ue^{-kt}dt$
5. Integrating both side, we get
$$x = -\frac{u}{k}e^{-kt} + C_2 \quad ...(4)$$
6. At $t = 0$, $x = 0$, we get
$$C_2 = \frac{u}{k}$$
7. Put the value of $C_2$ in eq. (4), we get
$$x = \frac{u}{k}[1 - e^{-kt}]$$

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# B.Tech.
# (SEM. III) ODD SEMESTER THEORY EXAMINATION, 2020-21
# ENGINEERING MECHANICS

**Time : 3 Hours** **Max. Marks : 100**

**Note :** 1. Attempt **all** section. If require any missing data; then choose suitably.

## SECTION-A

1. Attempt **all** questions in brief. **(2 × 10 = 20)**

**a. What do you understand by line of action of force ?**

**Ans.** The line of action of a force $F$ is a geometric representation of how the force is applied. It is the line through the point at which the force is applied in the same direction as the vector $F$.

**b. Define non-concurrent non-parallel coplanar force system.**

**Ans. Non-Coplanar Non-Concurrent Forces :** All forces do not lie in the same plane, and their lines of action do not meet at a single point.

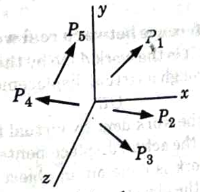



**Fig. 1.**

**Example :** Forces acting on a moving bus.

**c. What is the difference between center of gravity and centroid ?**

**Ans.** Refer Q. 2.1, 2 Marks Questions, Page SQ–6C, Unit-2.

**d. How will you find co-ordinates of centroid of an area ?**

**Ans.** Co-ordinate of centroid of an area can be calculated as :
$$\bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i}, \bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}$$
where, $x_i$ = Distance of CG of area $a_i$ from vertical axis.
$y_i$ = Distance of CG of area $a_i$ from horizontal axis.

**e. What is perfect truss ?**

**Ans. Perfect Truss :**

1. The truss which is composed of such members, which are just sufficient to keep the truss in equilibrium, when the truss is supporting an external load, is known as perfect truss.
2. For a perfect truss, the number of joints and number of members are given by,

$$m = 2j - 3$$

where, $m$ = Number of members.
$j$ = Number of joints.

**f. Define redundant frame.**

**Ans.** If the number of members are more than that required by equation $m = 2j - 3$, then such frames will be called as redundant frame.

**g. Define uniform motion.**

**Ans.** Uniform motion is defined as the motion of an object in which the object travels in a straight line and its velocity remains constant along that line as it covers equal distances in equal interval of time irrespective of the length of the time.

**h. What do you understand by impulse momentum for rigid bodies ?**

**Ans.** Refer Q. 4.9, 2 Marks Questions, Page SQ–14C, Unit-4.

**i. What is the difference between real work and virtual work ?**

**Ans. Virtual Work :** It is the work done by the actual forces acting on body moving through a virtual displacement.

OR

Virtual work is the work done by virtual force acting on the body moving through the actual displacements.

**Real Work :** Work is done on an object when a force causes a displacement of the object.

**j. What is the application of energy method for equilibrium ?**

**Ans.** Energy methods are used to obtain solution to elasticity problems and determine deflection of statically determinate structure and redundant reaction and deflection of statically indeterminate structures.

## SECTION-B

2. Attempt any **three** of the following : **(10 × 3 = 30)**

**a. Write the steps for finding the resultant of concurrent coplanar force system.**



**Ans.** The resultant of a number of a coplanar-concurrent forces acting on a body is worked out analytically by adopting the step-by-step procedure as given below :

1. Find the components of each force in the system in two mutually perpendicular $X$ and $Y$ directions.
2. Make algebraic addition of components in each direction to get two components $\Sigma F_x$ and $\Sigma F_y$.
3. Obtain the resultant both in magnitude and direction by combining the two component forces $\Sigma F_x$ and $\Sigma F_y$ which are mutually perpendicular.
4. Resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

and its inclination $\theta$ to $X$-axis is given by,

$$\tan \theta = \frac{\Sigma F_y}{\Sigma F_x}$$

This analytical method is based on theorem of resolved parts which states that "the algebraic sum of the resolved parts of two forces in a given direction is equal to the resolved part of their resultant in the same direction".

**b. Block $A$ of weight 520 N rest on the horizontal top of block $B$ having weight 700 N as shown in Fig. 2. Block $A$ is tied to a support $C$ by a cable at 30° horizontally. Coefficient of friction is 0.4 for all contact surface. Determine the minimum value of the horizontal force $P$ just to move the block $B$. How much the tension in the cable then ?**

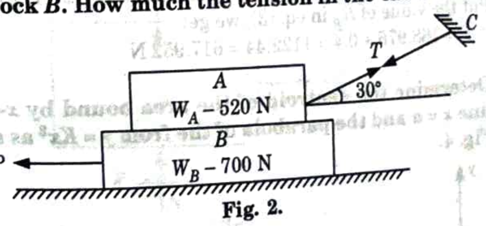



Fig. 2.

**Ans.**

**Given :** Weight of block $A$ = 520 N, Weight of block $B$ = 700 N, Angle of cable, $\theta$ = 30°, Coefficient of friction = 0.4

**To Find :** Value of Forces $P$, Tension in the cable.

**A. Considering Equilibrium of Block $A$ [Fig. 3(a)] :**

1. Resolving the forces along the horizontal and vertical directions,

$T \cos 30° - \mu R_A = 0;\quad T \cos 30° = \mu R_A$ ...(1)

$R_A + T \sin 30° - W_A = 0;\quad T \sin 30° = W_A - R_A$ ...(2)

2. Divide the eq. (2) by eq. (1), we get

$$\tan 30° = \frac{W_A - R_A}{\mu R_A} = \frac{520 - R_A}{0.4 R_A}$$

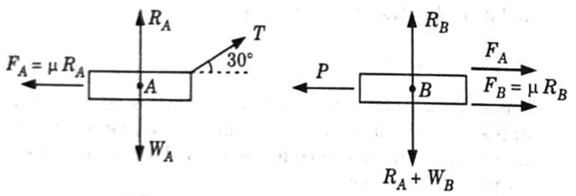
$$0.5773 = \frac{520 - R_A}{0.4\, R_A}\ ;\ 0.23094\, R_A = 520 - R_A$$

$$R_A = \frac{520}{1.23094} = 422.44 \text{ N}$$

3. $F_A = \mu R_A = 0.4 \times 422.44 = 168.976$ N
4. Tension in the cable, $T \sin 30 = W_A - R_A$

$$T = \frac{520 - 422.44}{\sin 30°}$$

$$T = 195.12 \text{ N}$$




(a) (b)

Fig. 3.

**B. Considering Equilibrium of Block B [Fig. 3(b)] :**

1. Resolving the forces along the horizontal and vertical directions,
   $F_A + \mu R_B - P = 0\ ;\ P = F_A + \mu R_B$ ...(3)
   $R_B - R_A - W_B = 0;\ R_B = R_A + W_B = 422.44 + 700 = 1122.44$ N
2. Put the value of $R_B$ in eq. (3), we get
   $\therefore P = 168.976 + 0.4 \times 1122.44 = 617.952$ N

**c. Determine the centroid of the area bound by x-axis, the line $x = a$ and the parabola of the from $y = Kx^2$ as shown in Fig. 4.**

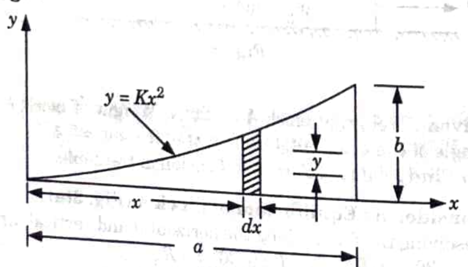



Fig. 4.

**Ans.** Refer Q. 2.7, Page 2–10C, Unit-2.



**d. A motorist travelling at a speed of 70 km/h, suddenly applies brakes and halts after skidding 50 m. Determine :**

**i. The time required to stop the car.**

**ii. The coefficient of friction between tyres and the road.**

**Ans.**

**Given :** Speed, $u = 70$ kmph, Skid, $s = 50$ m
**To Find :** Time required to stop a car, coefficient of friction.

1. Initial velocity, $u = 70 \text{ kmph} = \frac{70 \times 1000}{60 \times 60} = 19.44$ m/sec
2. Final velocity, v = 0
3. Displacement, s = 50 m
4. Using the equation of linear motion,
   $$v^2 = u^2 + 2as$$
   $$0 = 19.44^2 + 2a \times 50$$
   $$a = -3.78 \text{ m/sec}^2$$
   *i.e.*, the retardation is 3.78 m/sec².
5. Using the first equation of motion, $v = u + at$
   $$0 = 19.44 - 3.78\, t$$
   $$t = 5.14 \text{ sec.}$$
6. Inertia force must be applied in the opposite direction of acceleration, which means, it should be applied in the direction of motion while retarding. Fig. 5 shows the free body of the motor along with inertia force.

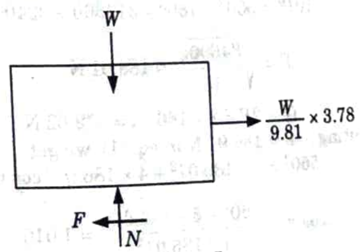



Fig. 5.

7. $\Sigma F_y = 0$, we get
   $$N = W$$
8. From the law of friction, $F = \mu N = \mu W$
9. $\Sigma F_x = 0$, we get
   $$F = \frac{W}{9.81} \times 3.78$$
   $$\mu W = \frac{W \times 3.78}{9.81}$$
   $$\therefore \mu = 0.385$$

**e. State and explain work energy equation.**

**Ans.** Refer Q. 5.12, Page 5–17C, Unit-5.

## SECTION-C

3. Attempt any **one** part of the following : **(10 × 1 = 10)**

**a. The resultant of two forces, one of which is double the other is 560 N. If the direction of larger force is reversed and the other remain unaltered, the resultant reduces to 180 N. Determine the magnitude of the forces and the angle between the forces.**

**Ans.**

**Given :** Resultant of two forces = 560 N, Resultant reduced = 180 N, Q = 2P
**To Find :** Magnitude of forces and angle between them.

1. Let $P$ and $Q$ $(= 2P)$ be the two forces and $\theta$ be the angle of inclination between them. From parallelogram law of forces
$$R^2 = P^2 + Q^2 + 2PQ \cos \theta$$
2. Then according to the given conditions,
$$560^2 = P^2 + (2P)^2 + 2P\,(2P) \cos \theta$$
$$= P^2 + 4P^2 + 4P^2 \cos \theta = 5P^2 + 4P^2 \cos \theta \quad ...(1)$$
3. And
$$180^2 = P^2 + (-2P)^2 + 2P\,(-2P) \cos \theta$$
$$= P^2 + 4P^2 - 4P^2 \cos \theta = 5P^2 - 4P^2 \cos \theta \quad ...(2)$$
4. Adding eq. (1) and eq. (2), we get
$$10P^2 = 560^2 + 180^2 = 313600 + 32400 = 346000$$
$$\therefore \quad P = \sqrt{\frac{346000}{10}} = 186.01 \text{ N}$$
5. $$Q = 2P = 2 \times 186.01 = 372.02 \text{ N}$$
6. Substituting $P = 186.01$ N in eq. (1), we get
$$560^2 = 5 \times 186.01^2 + 4 \times 186.01^2 \cos \theta$$
$$\cos \theta = \frac{560^2 - 5 \times 186.01^2}{4 \times 186.01^2} = 1.016$$

**Note :** There is some problem in given data because the value of $\cos \theta$ is greater than 1. That's why $\theta$ cannot be found accurately.

**b. Derive the expression to find the effort required to raise load using a screw jack.**

**Ans.** Refer Q. 1.36, Page 1–38C, Unit-1.

4. Attempt any **one** part of the following : **(10 × 1 = 10)**

**a. Determine the moment of inertia of a solid sphere radius *R* about diametral axis.**

**Ans.** Refer Q. 2.31, Page 2–43C, Unit-2.



**b. Locate the centroid of the area shown in Fig. 6 with respect to the axis shown in Fig. 6.**

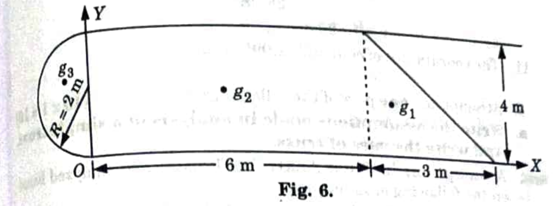



**Fig. 6.**

**Ans.**

**Given :** Fig. 6.
**To Find :** Centroid of the Fig. 6.

1. The composite section is divided into three simple figures, a triangle, a rectangle and a semicircle.
2. Now, area of triangle, $A_1 = \frac{1}{2} \times 3 \times 4 = 6 \text{ m}^2$
3. Coordinates of centroid of triangle, $g_1$ :
$$x_1 = 6 + \frac{1}{3} \times 3 = 7 \text{ m}$$
$$y_1 = \frac{4}{3} \text{ m}$$
4. Area of rectangle, $A_2 = 6 \times 4 = 24 \text{ m}^2$
5. Coordinates of centroid of rectangle, $g_2$ :
$$x_2 = 3 \text{ m}$$
$$y_2 = 2 \text{ m}$$
6. Area of semicircle, $A_3 = \frac{1}{2} \times \pi \times 2^2 = 6.283 \text{ m}^2$
7. Coordinates of centroid of semicircle, $g_3$ :
$$x_3 = \frac{-4R}{3\pi} = -\frac{4 \times 2}{3\pi} = -0.849 \text{ m}$$
$$y_3 = 2 \text{ m}$$
8. $\therefore$ Total area, $A = A_1 + A_2 + A_3 = 6 + 24 + 6.283 = 36.283 \text{ m}^2$
9. $x$ coordinate, $$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A}$$
$$= \frac{6 \times 7 + 24 \times 3 + 6.283 \times (-0.849)}{36.283}$$
*i.e.,* $$\bar{x} = 2.995 \text{ m}$$
10. $y$ coordinate, $$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A}$$

$$= \frac{\frac{6\times4}{3}+24\times2+6.283\times2}{36.283}$$

*i.e.,* $\bar{y} = 1.890$ m

11. The coordinates of centroid (2.995, 1.890).

5. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. **Write the assumptions made in analysis of a simple truss. And write the uses of truss.**

**Ans.** **Assumptions in Truss Analysis :** Trusses are analyzed based on the following assumptions :

i. Each member of the truss is connected at its end by frictionless pins.
ii. The truss is loaded as well as supported only at its joints.
iii. The forces in the members of the truss are axial.
iv. The self-weight of the members is neglected.

**Uses of Truss :**

i. Trusses are used as roof trusses to support sloping roof and as bridge trusses to support deck.
ii. In many machines steel trusses are used.
iii. Transmission towers are also example of trusses.

b. **Determine the forces in all the members of the truss loaded and supported as shown in Fig. 7.**

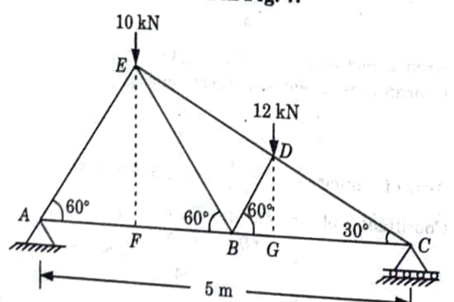


Fig. 7.

**Ans.**

**Given :** Fig. 7.
**To Find :** Forces in members of truss.



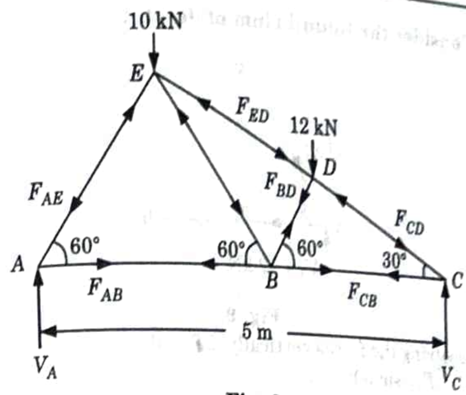


Fig. 8.

1. Triangle $ACE$ is a right-angled triangle having angle $AEC = 90°$.
   $\therefore$ $AE = AC \cos 60° = 5 \times 0.5 = 2.5$ m
2. The distance of the line of action of the vertical load 10 kN from point $A$ will be $AE \cos 60°$
   or $2.5 \times 0.5 = 1.25$ m.
3. From triangle $ABE$, we have $AB = AE = 2.5$ m
   $\therefore$ $BC = 5 - 2.5 = 2.5$ m
4. In right-angled triangle $BCD$, we have
$$CD = BC \cos 30° = 2.5 \times \frac{\sqrt{3}}{2}$$
5. The distance of the line of action of the vertical load of 12 kN from point $C$ will be $CD \times \cos 30°$
$$\text{or} \quad CD \times \frac{\sqrt{3}}{2} = \left(2.5\times\frac{\sqrt{3}}{2}\right)\times\frac{\sqrt{3}}{2} = 1.875 \text{ m.}$$
6. $\therefore$ The distance of the line of action of the load of 12 kN from point $A$ will be $(5 - 1.875) = 3.125$ m.
7. $\Sigma F_y = 0$, $V_A + V_C = 10 + 12 = 22$ kN ...(1)
8. Now taking the moments about $A$, we get
$$V_C \times 5 = 10 \times 1.25 + 12 \times 3.125 = 50$$
$$\therefore \quad V_C = \frac{50}{5} = 10 \text{ kN}$$
$$\therefore \quad V_A = \text{Total load} - V_C = (10 + 12) - 10 = 12 \text{ kN}$$

**9. Consider the Equilibrium of Joint $A$ :**

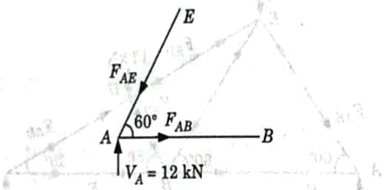



**Fig. 9.**

i. Resolving the forces vertically, $\Sigma F_y = 0$

$F_{AE} \sin 60° = 12$

$$\therefore \quad F_{AE} = \frac{12}{\sin 60°} = 13.856 \text{ kN (Compressive)}$$

ii. Resolving the forces horizontally, $\Sigma F_x = 0$

$F_{AB} = F_{AE} \cos 60° = 13.856 \times 0.5 = 6.928$ kN (Tensile)

**10. Consider the Equilibrium of Joint $C$ :**

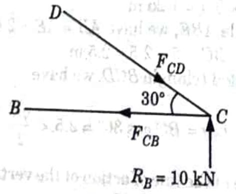



**Fig. 10.**

i. Resolving the forces vertically, $\Sigma F_y = 0$

$F_{CD} \sin 30° = 10$

$$\therefore \quad F_{CD} = \frac{10}{\sin 30°} = 20 \text{ kN} \quad \text{(Compressive)}$$

ii. Now resolving the forces horizontally, $\Sigma F_x = 0$

$F_{CB} = F_{CD} \cos 30° = 20 \times 0.866 = 17.32$ kN (Tensile)

**11. Consider the Equilibrium of Joint $B$ :**

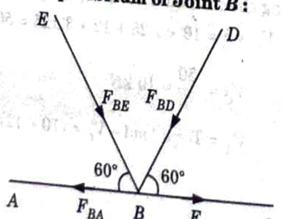



**Fig. 11.**



i. Resolving the forces vertically, $\Sigma F_y = 0$

$F_{BE} \sin 60° + F_{BD} \sin 60° = 0$

$$F_{BE} = -F_{BD} \quad \text{...(2)}$$

ii. Resolving the forces horizontally, $\Sigma F_x = 0$

$F_{BA} - F_{BE} \cos 60° = F_{BC} - F_{BD} \cos 60°$

$$6.928 - \frac{F_{BE}}{2} = 17.32 - \frac{F_{BD}}{2}$$

$$\frac{-F_{BE} + F_{BD}}{2} = 17.32 - 6.928 = 10.392$$

$$-F_{BE} + F_{BD} = 10.392 \times 2 = 20.784$$

$$F_{BD} + F_{BD} = 20.784 \quad \text{...(3)}$$

iii. From eq. (2) and eq. (3), we get

$$\therefore \quad F_{BD} = \frac{20.784}{2} = 10.392 \text{ kN}$$

and $\quad F_{BE} = -F_{BD} = -10.392$ kN

The magnitude of $F_{BE}$ is –ve, hence the assumed direction of $F_{BF}$ is wrong.

$\therefore \quad F_{BD} = 10.392$ (Compressive)

and $\quad F_{BE} = 10.392$ (Tensile)

**12. Consider the Equilibrium of Joint $D$ :**

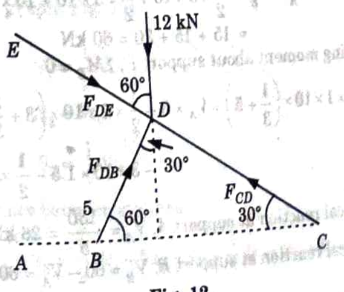



**Fig. 12.**

Resolving the forces along $CDE$, we get $F_{DE} + 12 \cos 60° = F_{CD}$

$F_{ED} = F_{CD} - 12 \times 0.5$

$= 20 - 6 = 14$ kN (Compressive)

**6. Attempt any one part of the following :** **(10 × 1 = 10)**

**a. Determine the reaction at supports $A$ and $B$ of the loaded beam as shown in Fig. 13.**

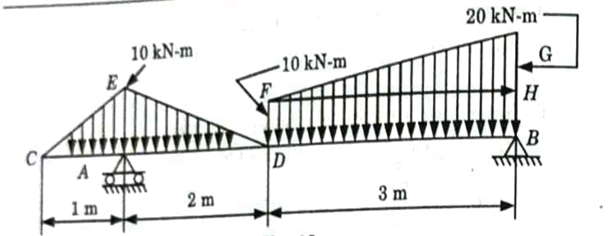



Fig. 13.

**Ans.**

**Given :** Fig. 13.
**To Find :** Reaction at support *A* and *B*.

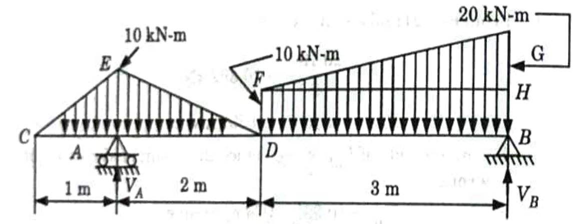



Fig. 14.

1. From Fig. 14, $\Sigma F_y = 0$

$$V_A + V_B = \frac{1}{2}\times 3\times 10 + \frac{1}{2}\times 3\times 10 + 10\times 3$$
$$= 15 + 15 + 30 = 60 \text{ kN}$$

2. Taking moment about support *B*, $\Sigma M_B = 0$

$$-\frac{1}{2}\times 1\times 10\times\left(\frac{1}{3}+5\right) + V_A\times 5 - \frac{1}{2}\times 2\times 10\times\left(3+\frac{2}{3}\times 2\right)$$
$$- 3\times 10\times 1.5 - \frac{1}{2}\times 3\times 10\times\frac{1}{3}\times 3 = 0$$

Vertical reaction at support *A*, $V_A = \frac{130}{5} = 26$ kN

3. Vertical reaction at support *B*, $V_B = 60 - V_A = 60 - 26 = 34$ kN

**b. The greatest possible acceleration and deceleration that a train may have is α and its maximum speed is *V*. Find the maximum time in which the train can get one station to the next, if they are '*S*' distance apart.**



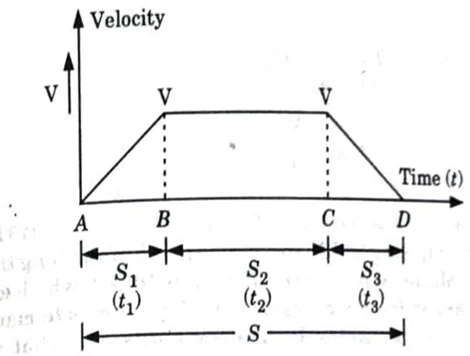



Fig. 15.

**Ans.**

**Given :** Acceleration and deceleration = α, Maximum speed = V, Distance = S.
**To Find :** Maximum time.

1. Using the velocity-time graph with slope of velocity-time graph represents the acceleration or deceleration and area under diagram is known as distance.

Thus, from graph (Fig. 15),

Distance, $S_1 = \frac{1}{2}Vt_1$

$S_2 = Vt_2$

$S_3 = \frac{1}{2}Vt_3$

2. Total distance between stations,

$$S = S_1 + S_2 + S_3 = \frac{1}{2}Vt_1 + Vt_2 + Vt_3 = \frac{V}{2}[t_1 + 2t_2 + t_3] \quad ...(1)$$

3. Also slope give acceleration, $\alpha = \frac{V}{t_1} = \frac{V}{t_3}$

$$t_1 = t_3 = \frac{V}{\alpha} \quad ...(2)$$

4. From eq. (1) and eq. (2), we get

$$S = \frac{V}{2}[2t_1 + 2t_2]$$

$$t_1 + t_2 = \frac{S}{V} \quad ...(3)$$

5. Total time, $T = t_1 + t_2 + t_3$
6. From eq. (2) and (3), we get

$$T = \frac{S}{V} + \frac{V}{\alpha}$$

7. Attempt any **one** part of the following : **(10 × 1 = 10)**

**a. A car weighing 50 kN and moving 54 kmph along the main road collides with a lorry of weight 100 kN which emerges at 18 kmph from a crossroad at right angles to main road. If the two vehicles lock after collision, what will be magnitude and direction of the resulting velocity ?**

**Ans.**

**Given :** Weight of car, $M_C$ = 50 kN, Speed of car, $V_C$ = 54 kmph, Weight of lorry, $M_L$ = 100 kN, Speed of lorry, $V_L$ = 18 kmph
**To Find :** Magnitude and direction of resulting velocity.

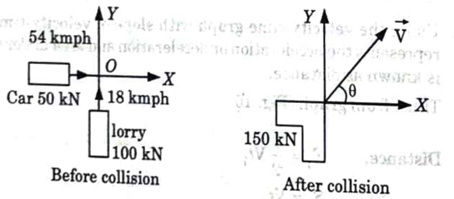



**Fig. 16.**

1. Conservation of momentum implies

$$M_C V_C \hat{i} + M_L V_L \hat{j} = (V\cos\theta\hat{i} + V\sin\theta\hat{j})$$

2. and thus we have two equations,

$$M_c V_c = (M_c + M_L)V\cos\theta$$
$$M_L V_L = (M_c + M_L)V\sin\theta$$

3. with two unknown V and θ

$$\tan\theta = \frac{M_L V_L}{M_c V_c} = \frac{100 \times 18}{50 \times 54} = 0.667$$
$$\theta = 33.7°$$



4. Velocity,
$$V = \frac{M_c V_L}{\sin\theta(M_c + M_L)} = \frac{100 \times 18}{\sin 33.7°(100 + 50)}$$
$$= 21.628 \text{ kmph}$$

**b. Establish the relationship between linear velocity of point on a body rotating about fixed axis and its angular velocity. Also describe the two velocities.**

**Ans.**

**A. Velocity/Linear Velocity :**

1. Velocity is defined as the rate of change of displacement of a body moving in a straight line. It is measured in metre per second. Velocity is a vector quantity. It is denoted by v.
2. Let, $s$ = Distance travelled by a body in a straight line.
   $t$ = Time taken to travel the distance.

   Then velocity of the body = $\frac{s}{t} = \frac{ds}{dt}$.

**B. Angular Velocity :**

1. It is defined as the rate of change of angular displacement of a body. Angular displacement is always measured in terms of angle covered by the body from the initial position.
2. Let a body is moving along a circular path as shown in Fig. 16. Let initially the body is at $A$ and after time '$t$', the body is at $B$. Let $\angle AOB = \theta$.

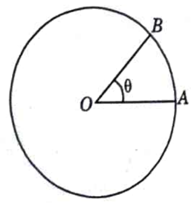



**Fig. 17.** Body moving in a circle.

3. Then angular displacement = $\angle AOB = \theta$.
   Time taken = $t$.
4. ∴ Angular velocity, ω

$$= \frac{\text{Angular displacement}}{\text{Time}} = \frac{\theta}{t} \text{ rad/sec}$$

Mathematically, angular velocity = $\frac{d\theta}{dt}$.

C. **Relation between Linear Velocity and Angular Velocity :**

1. Consider the body moving in a circle as shown in Fig. 17. The initial position of the body is at $A$ and after time '$t$' the body is at $B$. The angle $AOB$ is equal to $\theta$.

   Angular velocity = $\theta / t$.

2. Let, $\quad v = \text{linear velocity} = \dfrac{\text{Linear displacement}}{\text{time}}$

3. But linear displacement = Arc $AB = OA \times \theta = r \times \theta$

   ($\because OA$ = radius of circle = $r$)

$$\therefore \quad v = \frac{r \times \theta}{t} = r \times \text{Angular velocity}$$

$$v = r \times \omega$$

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