TUTORIAL : 4

1. $T(n) = 3T(n/2) + n^2$

$a = 3, \quad b = 2, \quad f(n) = n^2$

$\Rightarrow n^{\log_b a} = n^{\log_2 3} = n^{1.5}$

$n^{1.5} < n^2 \Rightarrow n^2 < f(n)$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$a = 4, \quad b = 2, \quad f(n) = n^2$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$\therefore T(n) = \Theta(n^2 \log^k n)$

$T(n) = \Theta(n^2 \log n)$.

3. $T(n) = T(n/2) + 2^n$

$a = 1, \quad b = 2, \quad f(n) = 2^n$

$n^{\log_2 1} = n^0 = 1$

$f(n) > 1$

$T(n) = \Theta(f(n)) = \Theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

Invalid.

5. $T(n) = 16T(n/4) + n$

$a = 8, \quad b = 4, \quad f(n) = n$

$n^{\log_4 8} = \log_{2^2} 2^3 = \frac{\log 8}{\log 4} = \frac{\log 2^3}{\log_2 2}$

$3/2 = 1.5$

$n^{1.5} \quad f(n) < n^{1.5} = n < n^{1.5}$

$T(n) = \Theta(n^c) = \Theta(n^{1.5})$

6. $T(n) = 2T(n/2) + n \log n$

$a = 2, \quad b = 2, \quad F(n) = n \log n$

$n^{\log_2 2} = n$

$F(n) > n$

$T(n) = \Theta(F(n)) = \Theta(n \log n)$

7. $T(n) = 2T(n/2) + n/\log n$

Invalid.

8. $T(n) = 0.5\,T(n/2) + \frac{1}{n}$

Invalid.

9. $T(n) = 16T(n/4) + n$

$a = 16, \quad b = 4, \quad f(n) = n$

$n^{\log_4 16} = n^{\log_4 4^2} = n^2$

$f(n) > n^2 \quad T(n) = \Theta(n)$

13. $T(n) = 3T(n/2) + n$

$a = 3, \quad b = 2, \quad f(n) = n.$

$n^{\log_2 3} = n^{1.5} > f(n)$

$T(n) = \Theta(n^{1.5})$.

14. $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, \quad b = 3, \quad f(n) = \sqrt{n}$

$n^{\log_3 3} = n$

$n > f(n)$

$T(n) = \Theta(n)$.

15. $T(n) = 4T(n/2) + cn$

$a = 4, \quad b = 2, \quad f(n) = cn$

$n^{\log_2 4} = n^2$

$n^2 > f(n)$

$T(n) = \Theta(n^2)$.

16. $T(n) = 3T(n/4) + n \log n$

$a = 3, \quad b = 4, \quad f(n) = n \log n$.

$n^{\log_4 3} = n^{0.2}$

$f(n) > n^{\log_4 3}$

$T(n) = \Theta(n \log n)$.

17. $T(n) = 3T(n/3) + n/2$

$a = 3, \quad b = 3, \quad f(n) = n$

$n^{\log_3 3} = n$

$f(n) = n$

$T(n) = \Theta(n \log_k n) = \Theta(n \log n)$

18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \quad b = 3, \quad f(n) = n^2 \log n$

$n^{\log_3 6} =$

$f(n) > n^{\log_3 6}$

$T(n) = \Theta(n^2 \log n)$

19. $T(n) = 64T(n/8) - n^2 \log n$

invalid

20. $T(n) = 7T(n/3) + n^2$

$a = 7, \quad b = 3, \quad f(n) = n^2$

$n^{\log_3 7} \Rightarrow n^2 > n^{\log_3 7}$

$T(n) = \Theta(n^2)$

22. $T(n) = T(n/2) + n(2 - \log n)$

invalid